justification for substituting c_{eff} for c in the linear wave equation can only come from a rigorous treatment of the problem using the nonlinear wave equation.

At first glance, it would appear that inclusion of the factor $1+B/2A$ would result in a decrease in the threshold of parametric oscillation for fluids with larger values of B/A. This effect, however, may be vitiated by the increased effective attenuation for the more nonlinear fluids. Nevertheless, the parameter of nonlinearity of the fluid must be a determining factor in the threshold.

The generation of fractional harmonics in a resonant cavity is described by Adler and Breazeale as a parametric phenomenon due to the periodic variation of the resonant frequencies

$$\omega_n = n\pi c/l. \tag{13}$$

It is proposed here, that the variation of ω_n arises from two sources: first, directly from the variation of l, as per Adler and Breazeale, and second, from the periodic variation of the sound velocity caused by the variation in the mean density of the fluid. Since $B/2A$ for liquids is greater than unity, the second mechanism would, in fact, be the dominant one.

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[3] L. Adler and M. A. Breazeale, Naturwissenschaften 8, 385 (1968).

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Observations of Acoustic Ray Deflection by Aircraft Wake Vortices

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Acoustic ray deflection by aircraft wake vortex flows has been observed during landing operations of large aircraft. The phenomenon has been used to detect and locate vortex traces in a plane perpendicular to the runway centerline. The maximum deflection angles observed for a variety of aircraft show qualitative agreement with values predicted for a viscous core.

We wish to report that we have observed experimentally the acoustic ray deflection described theoretically by Georges.[1] The experiments are being sponsored by the Federal Aviation Administration as part of their Aircraft Wake Vortex Detection Program. The first observation of this effect was made at Logan International Airport on 19 January 1971 on acoustic rays deflected by one vortex of the pair generated by a landing aircraft. The observation was reported at the time.[2] Subsequent data have been obtained on many types of commercial and military heavy jet aircraft, as well as on some lighter piston aircraft, at Logan and Kennedy Airports and at the National Aviation Facilities Experimental Center (NAFEC) in Atlantic City, N. J.

Briefly, the apparatus consists of a pulsed acoustic transmitter and a receiver located on opposite sides of the runway glide path (Fig. 1). Transmit and receive transducers are mounted at the focus of identical reflectors made of plastic sheets bent to the shape of parabolic cylinders. The antennas are set with their focal axes approximately perpendicular to the ground

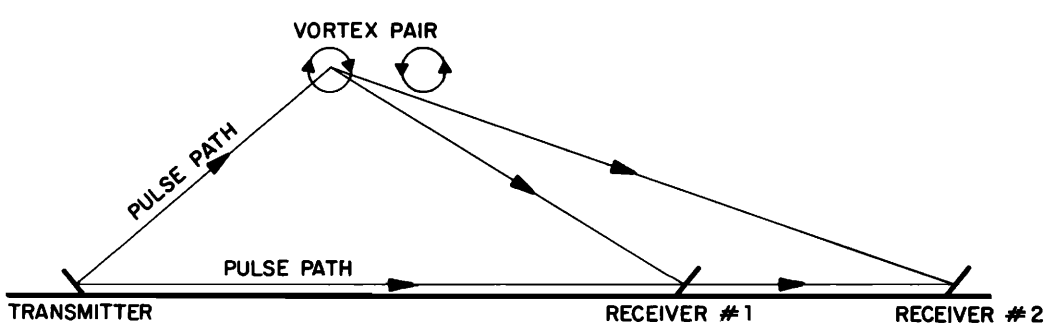

FIG. 1. Pulsed acoustic vortex sensor.

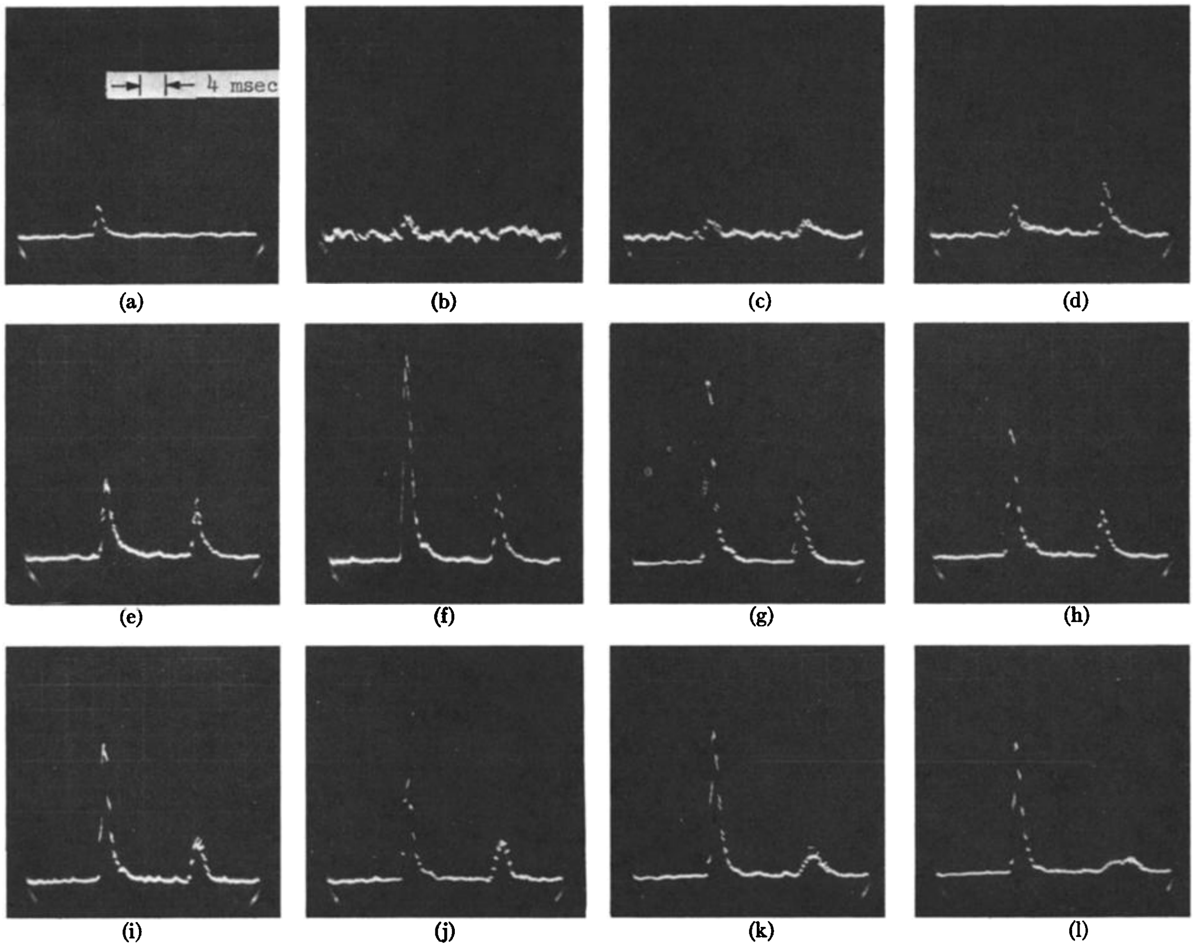

FIG. 2. Acoustic pulses recorded during landing pass of a Boeing 727 jet aircraft. Integration time, 1 sec (20 pulses); pulse width, 2.5 msec; acoustic frequency, 3 kHz; baseline, 350 ft; peak power input, 30 W. (a) −10 sec; (b) 4 sec; (c) 5 sec; (d) 6 sec; (e) 7 sec; (f) 8 sec; (g) 9 sec; (h) 10 sec; (i) 11 sec; (j) 12 sec; (k) 13 sec; (l) 14 sec.

in a plane which is perpendicular to the centerline of the runway. Thus, the two antennas have vertical fan-shaped beams whose ray paths intersect in a sensitive common volume containing a section of the glide-slope window.

In order to detect the presence of a vortex, short pulses of acoustic energy are transmitted over the runway (typical pulse width is 2.5 msec at a frequency of 3 kHz with 30 W of peak power). When there are no disturbances within the sensitive volume, the receiver detects only that portion of the emitted pulses which propagates parallel to the ground. The detection of these pulses provides a time reference. If a vortex, with circulation vector pointing to the left as viewed from the transmitter, is in the sensitive volume, a portion of the pulse energy is deflected downward, in accordance with the model described by Georges, and is detected in the receiver. This second received pulse is delayed relative to the ground pulse by a time corresponding to the difference in path length. In this way the presence of a vortex in the sensitive volume can be detected and the

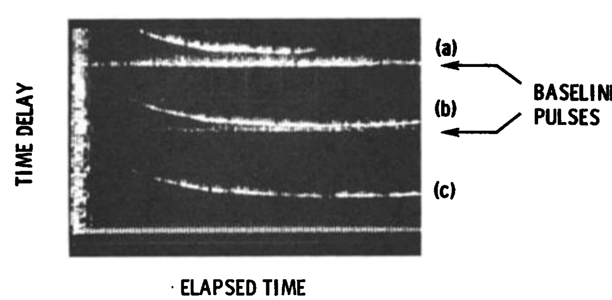

FIG. 3. Acoustic pulse deflection by the starboard vortex of a B-727 aircraft over the middle marker at JFK Runway 31R. Receiver located 28 ft upwind of the runway centerline. Upper trace (a) shows pulses from transmitter 358 ft downwind of the centerline; middle trace (b) from transmitter 688 ft downwind; lower trace (c) from transmitter 1018 ft downwind. Corresponding bistatic baselines are: (a) 385 ft; (b) 716 ft; (c) 1046 ft. Horizontal scale: elapsed time, 5 sec/cm; vertical scale: time delay of pulses deflected by vortex relative to pulses propagating parallel to the ground (horizontal bands), 75 msec/cm. Bright vertical band at the left is noise due to aircraft passage. Note that pulse strength diminishes rapidly with increasing baseline for the ground path, but *not* for the deflected path. Signal is acquired in each channel as the vortex loses altitude and the deflection angles decrease. Signal is lost in channel (a) when the maximum deflection angle is again exceeded (i.e., the vortex has been blown too close to the near transmitter).

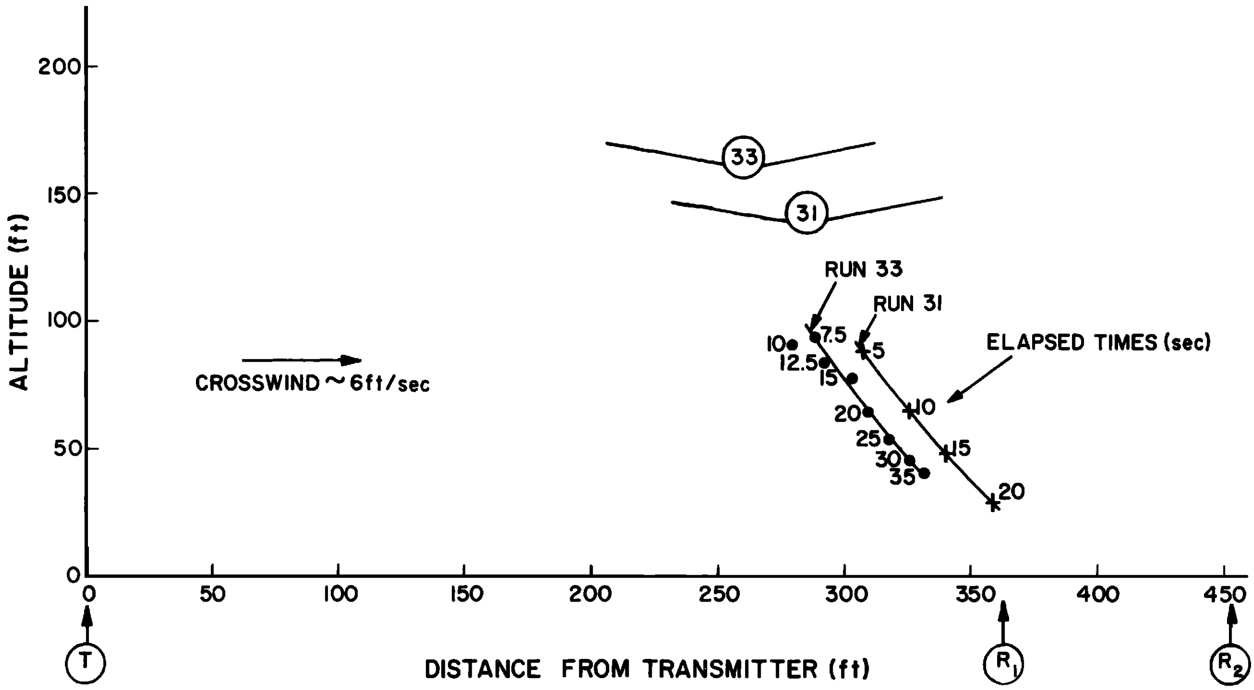

FIG. 4. Left wing vortex tracks. Two B-727 aircraft, landing configuration.

TABLE I. Observed angles for landing aircraft.

Aircraft type	Max. scattering angle
DC-9, B-727, BAC-111	1.2 rad
B-747	1.0 rad
DC-8, B-707	0.5 rad

volume monitored until the vortex has either decayed or drifted off.

A refinement of this system consists of adding a second transmitter or receiver aligned with the first but displaced from it. The data then comprise two different time delays which are sufficient to locate the vortex and follow its motion. A typical "A-scope" presentation of the echo data from one receiver is shown in Fig. 2. An alternate presentation in the form of an "acoustogram" is shown in Fig. 3. Typical measured tracks for two different B-727 passes are shown in Fig. 4. These correlate well with the theoretical motions of a pair of line vortices over the ground.

In his paper Georges shows that for a particular model of a vortex with a viscous core, the maximum angular deflection for acoustic rays passing through the core depends only on the maximum velocity in the vortex. The observed maximum scattering angles are in qualitative agreement with this model, although further refinements can be made. Some observed angles for various aircraft in a landing configuration appear in Table I.

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