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Reliability of Navigation Service Provided by the Global Positioning System

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16. Abstract

The planned NAVSTAR/GPS satellite constellation of 18 satellites plus 3 active will provide excellent coverage over the continental United States (CONUS) if all are operating properly. This report examines the coverage under conditions of one satellite failure. It turns out that the failure of any satellite results in service outages of up to half an hour somewhere in the CONUS. While altimeter aiding eliminates most of these outages, there still remain 14 outage events each day. Furthermore, "coasting" the navigation solution with a stable clock is not effective in handling these outages. One technique which does appear to be effective is using a mask angle of 5 degrees or less. At these low angles the signal is down in amplitude due to antenna rolloff, and it is more affected by tropospheric refraction and multipath, but it is better to use even a corrupted signal than to suffer a service outage. Also examined in this report is the impact of adding satellites to the constellation. It is shown that a 24-satellite constellation still provides 100% availability in the CONUS when one satellite fails.

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PREFACE

This document contains an analysis of the susceptibility of navigation service for aviation users to a satellite failure in the planned NAVSTAR/Global Positioning Service (GPS) system. A number of different operational scenarios are explored: different numbers of satellites in the constellation, different receiver designs, and aiding techniques involving altimeter or high-quality clock inputs.

The work is motivated by the desire to clarify the conditions that must be met by the NAVSTAR/GPS system if it is to serve as a primary navigation source. While the service availability is 100% in the continental United States if all 18 satellites and 3 active spares are working, the loss of any satellite will result in service outages somewhere in the country. These outages can last up to half an hour. Unless some means is found to avoid this situation, the system could not be used as a sole means of navigation. Some alternatives are explored in this report.

The analysis was prepared under the sponsorship of the Federal Aviation Administration, Systems Engineering Service, Advanced Concepts Divisfon. The author wishes to thank Anthony Buige and Jerry Bradley of that division, who provided valuable suggestions and support. The author also wishes to thank Shirish Dandekar, who performed many of the computations and plotted the results.

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1. INTRODUCTION

1.1 BACKGROUND

The Global Positioning System (GPS) is a satellite system which will provide global, continuous navigation service when it becomes operational in about 1988. The GPS program has been in existence for a decade now; several experimental satellites are operating and available for intermittent use. The system development is managed by the GPS Joint Program Office (JPO) of the Air Force Systems Command Space Division in Los Angeles, California. The management team consists primarily of Department of Defense staff, but Department of Transportation and NATO liaisons are stationed there as well. Policy is set by the Office of the Secretary of Defense (OSD) in Washington, D.C.

Figure 1-1 shows the planned GPS constellation, consisting of 18 satellites in 6 planes, plus 3 spares. The three in-orbit spares will be positioned in such a way as to provide optimum coverage for the continental United States (CONUS). In the event of a satellite failure in one of the three planes containing spares, the spare will be moved to a position close to the defunct satellite. If a failure occurs in one of the other planes, the adjacent plane satellites will be moved within their planes to provide the best possible coverage, until a new satellite has been launched and placed in orbit.

The accuracy of the GPS appears to be adequate for en-route, terminal, and non-precision approach phases of flight. From the standpoint of reliability and continuous, uninterrupted service, however, the case is not so clear. While the system will provide continuous service over the CONUS if all 18 satellites and 3 spares are satisfactorily working, the loss of any of the 21 satellites will cause periods where service is not fully available at some locations.

The Federal Aviation Administration has taken the position that the navigation service must be reliable enough that the probability of a satellite failure is small, and two satellite failures negligible. Furthermore, the satellite constellation should be robust enough that the loss of a satellite does not cause an interruption of service. This kind of redundancy is characteristic of the current VOR/DME National Air Space navigation system, which has proven itself to be a quite satisfactory navigation system. As a consequence of this, the FAA has asked that five satellites be in view at all times in the CONUS, so that a satellite failure leaves four available, the minimum required for unaided operation. Furthermore, the geometry of the remaining four satellites should support the stated 100-meter accuracy of the GPS.

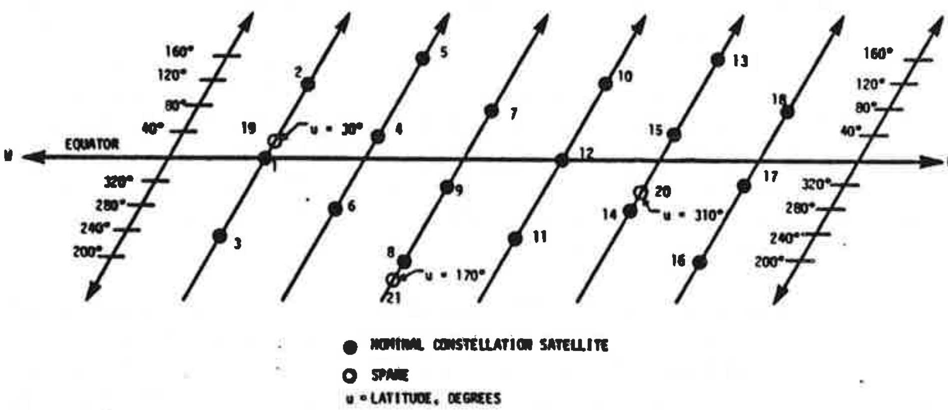


FIGURE 1-1. PLANNED GPS SATELLITE CONSTELLATION

The failure of a satellite will result in a loss of accuracy at certain locations and times. The resulting accuracy is still adequate for en-route and terminal guidance, but not for non-precision approach guidance. In order to provide adequate non-precision approach accuracy, there are three major options available:

1. Increase the number of satellites in the constellation to the point where there are no such service outage periods.

2. Require airborne GPS equipment to be aided by the input of barometric altimeter measurements.

3. Require airborne GPS sets to incorporate a clock of sufficient stability to maintain the required accuracy for the duration of the service outage.

This report examines the implications of each of these alternatives.

1.2 SCOPE .

This report addresses the capability of GPS to provide adequately reliable service for airborne navigation. Section 2 describes the approach used, the parameters examined, and the type of results obtained. Sections 3-5 discuss the major alternatives and their implications. Section 6 summarizes the conclusions of the study. A number of detailed appendices are provided that enable the reader to review the data on which the conclusions are based.

2. APPROACH

2.1 CHOICE OF PARAMETERS

In any study of this type, which involves tradeoffs among several different parameters, it is necessary to limit the number of combinations by judicious choice of the parameters. In this report the following parameter values are used:

1. Number of satellites in the constellation: 18, 21, and 24.

2. Mask angle (the minimum elevation angle that a satellite must have to be considered to be "visible"): 5, 7.5, and 10 degrees.

3. Area of coverage: CONUS.

4. Number of satellite failures: 0 and 1.

The choice of the number of satellites was premised on the stated policy of the DOD with respect to the satellite constellation. Eighteen satellites represent the baseline, while three spares will usually be operational and available. Since there are six orbital planes, the addition of one satellite per plane to the baseline constellation gives 24 satellites. Investigation of the latter option provided adequate information, so that consideration of more satellites proved unnecessary for the study.

The choices of mask angle were based on different considerations. The DOD plans to use 5-degree mask angles in their equipment. At low elevation angles, it is known that the signal is reduced due to antenna pattern falloff near the horizon. The signal from a satellite near the horizon is more subject to multipath and to temporary signal reductions caused by aircraft roll. As a result the FAA has indicated an initial preference for a 10-degree mask angle to be used for evaluation purposes. Earlier studies at TSC indicated that a mask angle of 7.5 degrees has advantages that make it attractive [Kalafus, et al., 1983, p. 5-7].

While a number of computations have been made for world-wide coverage, they are not included in this report. However, the results are quite similar, and the conclusions would not be different.

Results were obtained for full 18, 21, and 24-satellite constellations, and for the case of one satellite failure. The implications of multiple satellite failures were not considered.

2.2 METHOD OF TREATING SATELLITE FAILURES

In treating the failure of a single satellite in the 18-satellite constellation, each of the satellites was faulted for a a period of 24 hours, and the dilution-of-precision parameters computed over the CONUS grid. Actually, the symmetry of the constellation made it possible to perform the computations using 12-hour periods, which reduced the computer run costs.

The 21-satellite constellation had been treated on a previous project, using a coarser grid, namely ten degrees in longitude and latitude. The results from that effort were used in this study, initially. However, the coarseness of the grid proved inadequate to detect some troublesome situations. For this reason the critical situations were examined more closely using a five-degree grid. Each of the satellites was faulted to determine if there were outages that could not be handled by altimeter aiding or clock coasting.

With 24 satellites there would be 6 or more satellites in view about 99% of the time. That is, the locations where only 4 or 5 satellites would be visible would be few. Rather than faulting the satellites one at a time, a detailed study was made of these infrequent periods to determine the effect of a satellite failure.

2.3 PERFORMANCE MEASURES UTILIZED

The computer program developed at TSC outputs the number of visible satellites and the dilution-of-precision (DOP) values at each location on a latitude/longitude grid at time intervals specified by the operator. In this study, a 5-degree lat/lon grid was used to cover most of the CONUS: latitudes varied from 30 to 50 degrees north, and longitudes varied from 70 to 125 degrees west. The values were calculated at 6 minute intervals.

Since four satellites must be in view to obtain an unaided solution, the fraction of time a given number of satellites are visible from the CONUS is an important variable. For example, if only four satellites are visible, a failure of any satellite would make it impossible to obtain a solution without additional inputs. Also, if five satellites are visible, the loss of a satellite could be problemmatical if the remaining four are in an unfavorable geometry. The unfavorable geometry most frequently experienced in the GPS system is the case where all four satellites are almost coplanar. This results in large errors, which are reflected in the GDOP values.

The primary anticipated use of GPS for air navigation is that of obtaining horizontal position information. As a consequence, the horizontal DOP measure (HDOP) is the one most relevant to this study. HDOP distributions are computed along with the number of visible satellites for each of the configurations considered. The HDOP distribution gives the percentage of time that the HDOP is less than a given value. The 100-meter position accuracy measure attributed to the GPS is premised on an HDOP of about 1.72 [Kalafus, et al, 1983, p. 7-3]. The HDOP distribution curve can be used to determine the percentage of time that CONUS users would experience 100 meter accuracy (2drms) or better, by determining the point on the curve that corresponds to an HDOP of 1.72.

It should be pointed out that the 100 meter accuracy value attributed to GPS is a "2drms" value, which means that about 97% of the time the accuracy should be 100 meters or better. However, an elaborate computation would be required to obtain an accurate measure of the probability of exceeding 100 meters in the CONUS over a 24-hour period . It is considered more relevant to deal with short-term values of HDOP, since they are usually well-behaved, except under well-defined conditions that occur in predictable locations at predictable times.

In addition to HDOP values and the number of visible satellites, the fraction of time the HDOP is within acceptable limits is called the "system availability". For this study, an HDOP of 6 is chosen as the value which defines a service "outage". The results are not highly sensitive to this value.

It is shown in the next section that short-term outages can be handled by "coasting" with the user clock, if the outages are not too long. Thus it is important to determine the length of outages that could be encountered under the various conditions of the study.

2.4 TREATMENT OF ALTIMETER INPUTS AND CLOCK COASTING

The technique of augmenting the navigation solution by inputting altitude from a barometric altimeter is treated here by postulating an extra "satellite" at the center of the earth. This provides a line-of-position which defines a constant altitude. Of course, considering the solution in terms of a resulting HDOP presupposes that the error due to the altimeter is similar to the error in the pseudoranges to the various satellites. The standard deviation in pseudorange is about 27 meters. Altimeters vary in accuracy from about 20 meters to 1000 meters (1-sigma), depending on whether a local correction has been made, and on the distance from the local reference. Since the altitude error can differ significantly from the pseudorange error, the effect of an altitude error on the estimate of horizontal position is addressed by considering its effect separately from that of pseudorange errors.

If the receiver has been obtaining good navigation information with 4 or more satellites visible, and loses a satellite from view or enters a region of high HDOP, it can go into a "coast" mode, whereby the clock is used to estimate system time, and position is computed by assuming that range, rather than pseudorange, is known to each satellite in view. Under these conditions only 3 satellites are required to obtain a solution. However, as the clock frequency drifts from its original value, an error in position is accrued which grows in time. A sufficiently stable clock could enable coasting through a given period of time before unacceptably large errors have occurred. Thus it is important to know the length of any service outages that can occur. Also treated in this report are the length of outages, the number of visible satellites and HDOP distributions.

3. GPS SERVICE RELIABILITY UNDER CONDITIONS OF UNAIDED RECEIVER OPERATION

3.1 18-SATELLITE CONSTELLATION

Under unaided conditions, where the receiver relies solely on the GPS for navigation positioning, the 18-satellite constellation can not provide adequate service if one satellite fails. This is apparent from the visibility histogram in Figure 3-1. Depending on the mask angle, there are only 4 satellites visible from 2% to 17% of the time. Thus the loss of one satellite would result in a total loss of navigation guidance to the unaided receiver somewhere in the CONUS at some point in time during the day.

Examination of the data revealed that the loss of one satellite could produce periods which last well over an hour during which less than four satellites are visible.

Since there could be appreciable periods of time when navigation service is not available under the condition of a satellite failure, 18 satellites are not considered adequate for unaided receiver operation.

3.2 21-SATELLITE CONSTELLATION

The current plan of the GPS JPO is to locate the three spares where they provide optimum service in the CONUS. With 18 satellites all operating, there are always at least 4 satellites in view at all times within the CONUS. However, there are periods which last up to 30 minutes during which the HDOP becomes excessive due to the almost coplanar alignment of the satellites. By placing the spares in the locations indicated in Figure 1-1 and activating them, the outages will be eliminated for user receivers having mask angles of 8 degrees or less [Kalafus, et al., 1983, p.5-3].

It can be seen from the visibility histogram in Figure 3-2 that the spares greatly reduce the fraction of time that only 4 or 5 satellites are in view. For example, for a 10-degree mask angle, the percentage of time that 4 satellites are in view is reduced from 17% to less than 3%. For a 5-degree mask angle, the same fraction is less than 0.5%.

The effect of satellite faults on the navigation service provided by GPS was examined for the 21-satellite constellation in a previous project at TSC [Kalafus, et al., 1983, Ch. 6]. The following results are taken from that effort. The satellites were faulted one at a time, and the resulting HDOP's were computed over the CONUS for a 24-hour period. Here a 10-degree latitude/longitude grid was used, and HDOP's were computed every 10 minutes. Examination of the data showed that a failure of satellite #2 (see Figure 1-1) produced the worst outages. Figure 3-3 shows the resulting outage times for a mask angle of 8 degrees.

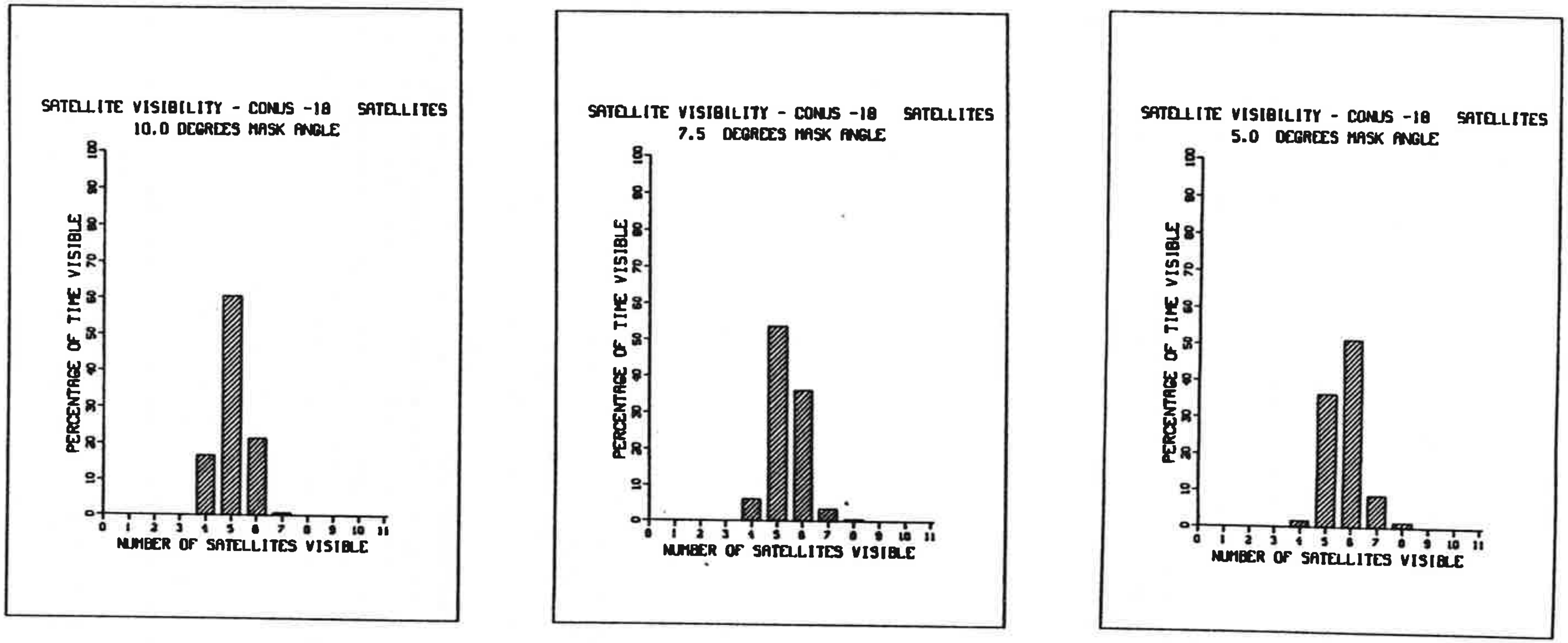


FIGURE 3-1. SATELLITE VISIBILITY FROM CONUS - 18 SATELLITE CONSTELLATION

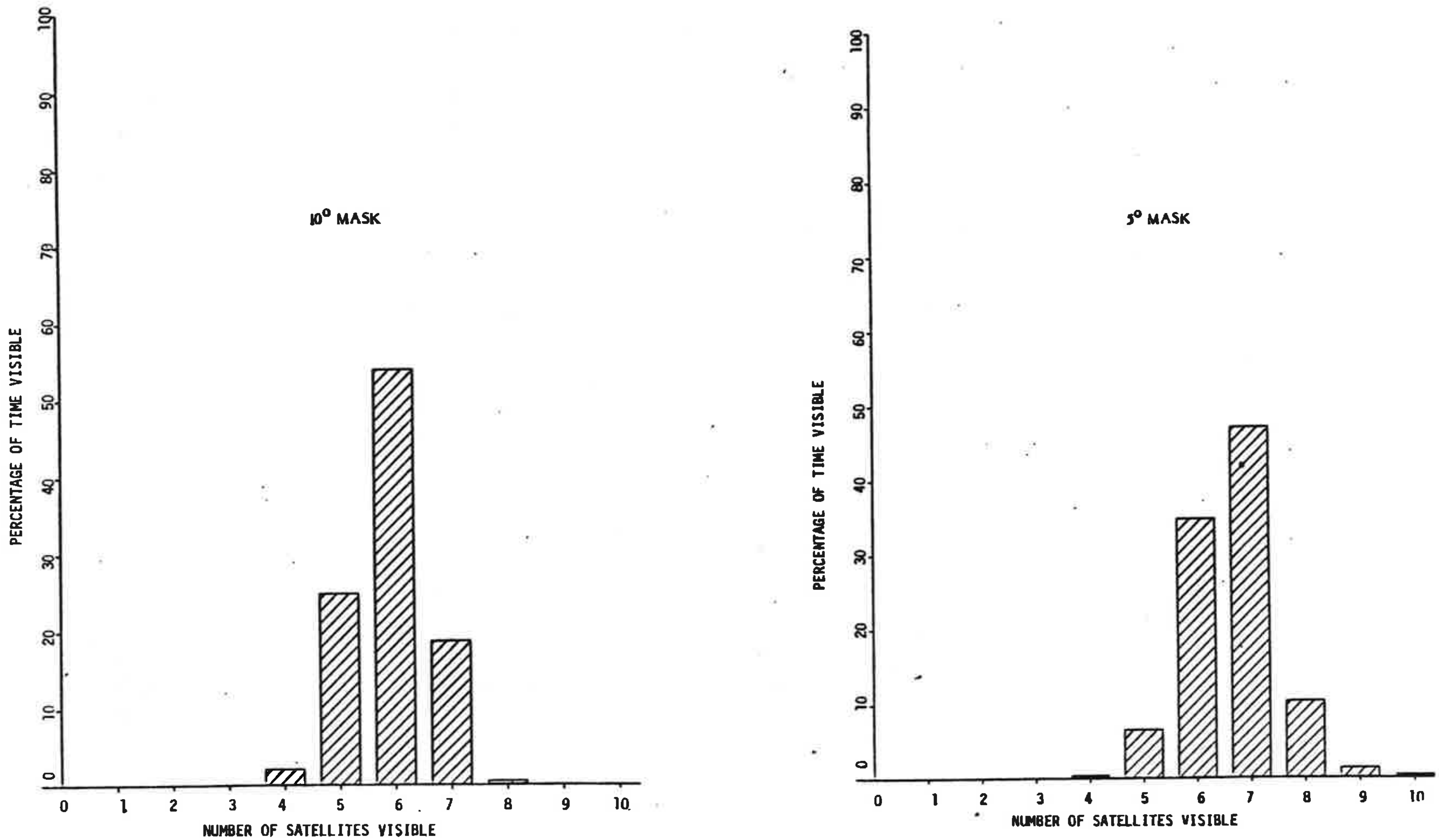


FIGURE 3-2. SATELLITE VISIBILITY FROM CONUS - 21 SATELLITE CONSTELLATION

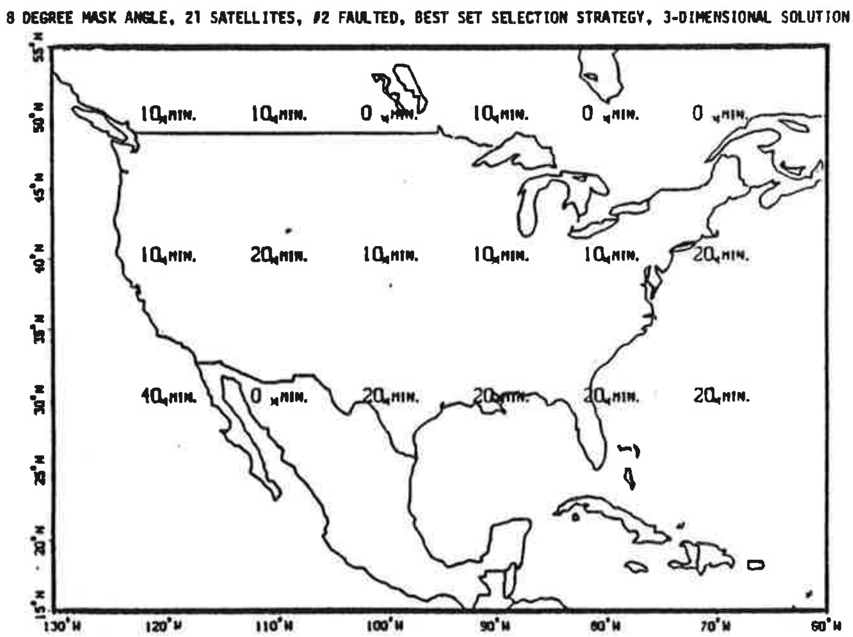


FIGURE 3-3. MAXIMUM OUTAGE TIMES - CONUS
8 DEGREE MASK ANGLE - 21 SATELLITE CONSTELLATION

It can be seen that the values range from 0 to 20 minutes, although there is a longer outage off the coast of California. For a 10-degree mask angle, the outages can last as long as 40 minutes. On the average, however, the fraction of outage time is less than 0.3% over the CONUS over a 24-hour period.

What this means is that service outages are quite rare, but when they do occur, they can last for periods of up to 30 minutes or so. In section 5 it will be shown that in order to "coast" with a clock, the clock stability must be considerably better than that provided by the $50 oscillator usually envisioned for airborne equipment.

Unaided operation with the 21-satellite constellation would have to be judged marginal. It is true that the information available to the receiver is more than adequate to alert the user that there is a service outage; and in fact these periods can be predicted, once it is known that a satellite has failed. However, to suffer 20 or 30-minute periods during which non-precision approach guidance is questionable is not advisable.

The problem could be further exacerbated when the process of reconfiguring the constellation is considered. Suppose a satellite fails in an orbital plane containing a spare satellite. The current plan calls for moving the spare to the location of the failed satellite. If during this period the spare was not active, it would mean that for a period of a few weeks, two satellites would be out. A similar problem would exist for failures in non-spare orbital planes. However, conversations with GPS JPO staff indicate that the energy required for repositioning is small, and is quite modellable. As a result, it is claimed that the spares will remain active during the repositioning period, and that the ephemeris data broadcast during the repositioning period will be more than adequate for aircraft navigation.

3.3 24-SATELLITE CONSTELLATION

One of the major alternatives available is to add satellites until the point is reached that the system could sustain the loss of one satellite without causing outages to unaided receivers. Assuming that the 6-plane configuration is maintained, having 4 satellites per plane requires 24 satellites in all. This configuration was chosen for further evaluation. The location of the satellites in this 6 x 4 constellation is shown in Figure 3-4. The phasing between adjacent planes is 15 degrees, a value that was determined to give the best overall performance in terms of satellite visibility and GDOP.

It is interesting to note that the average improvement in HDOP by the addition of 6 satellites to the constellation is not significant for receivers employing the best-set-of-four satellite selection strategy. Figure 3-5 shows the HDOPs for a mask angle of 10 degrees for both 18 and 24 satellites. However, while the overall gain in accuracy is not great, there is a significant improvement in system robustness.

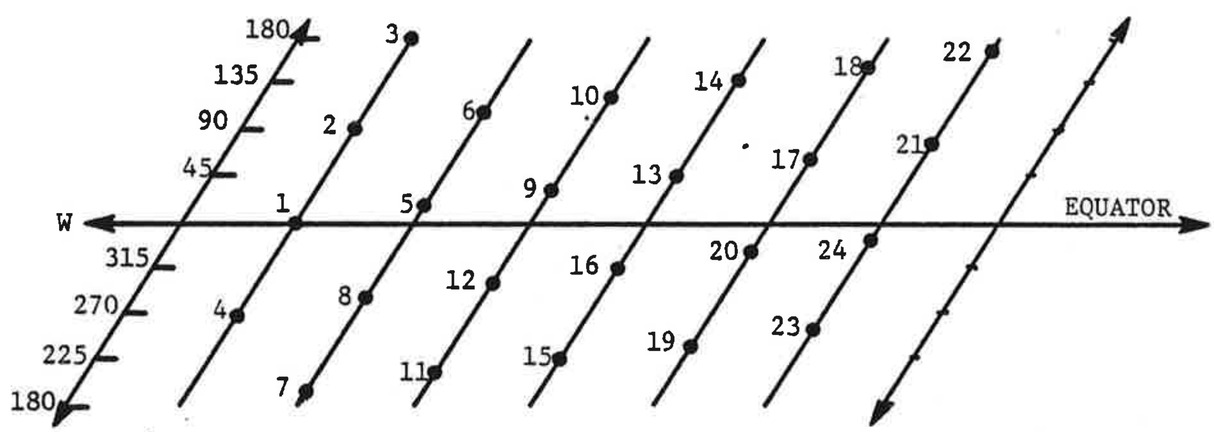


FIGURE 3-4. SATELLITE CONSTELLATION - 24 SATELLITES

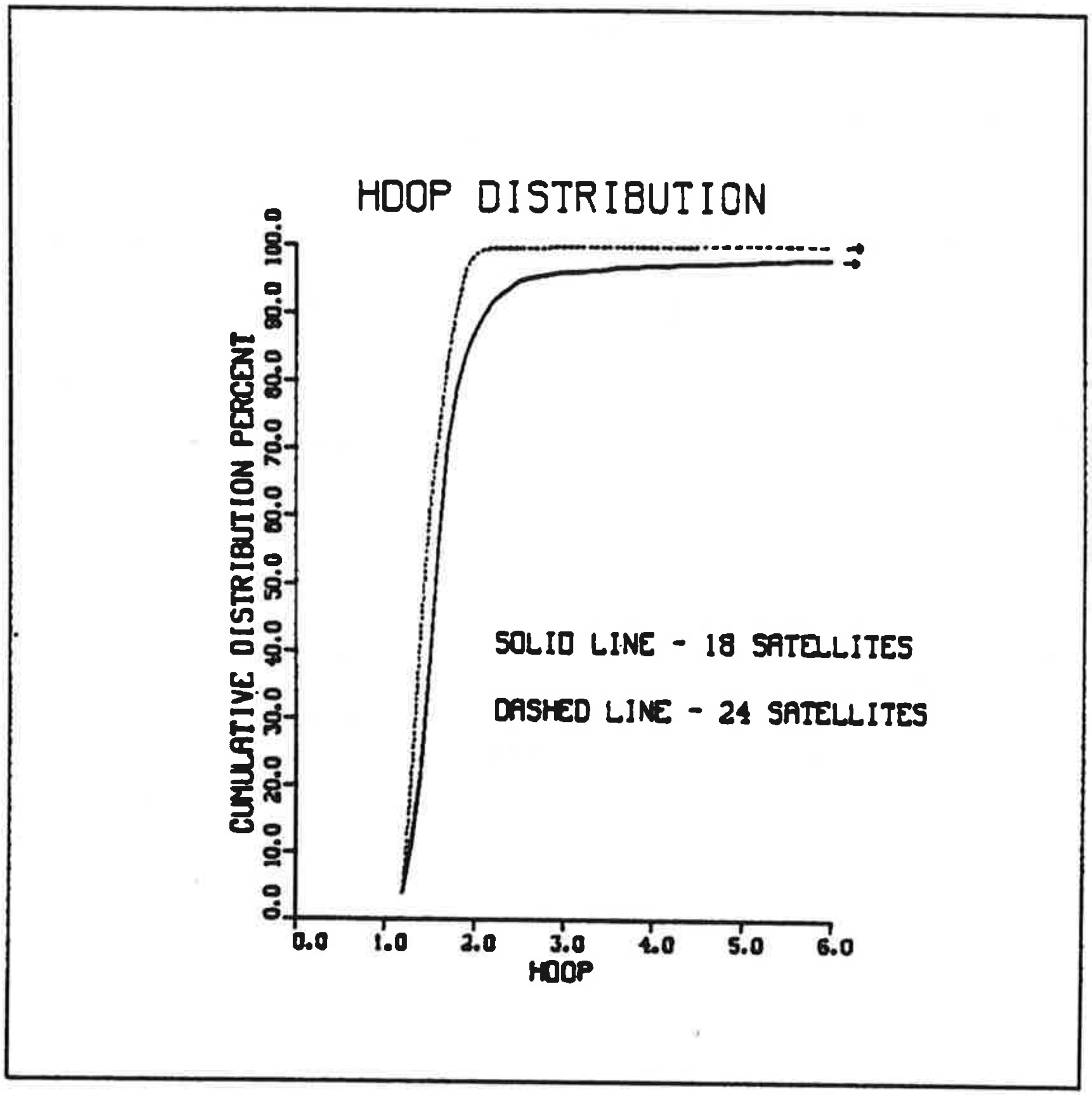


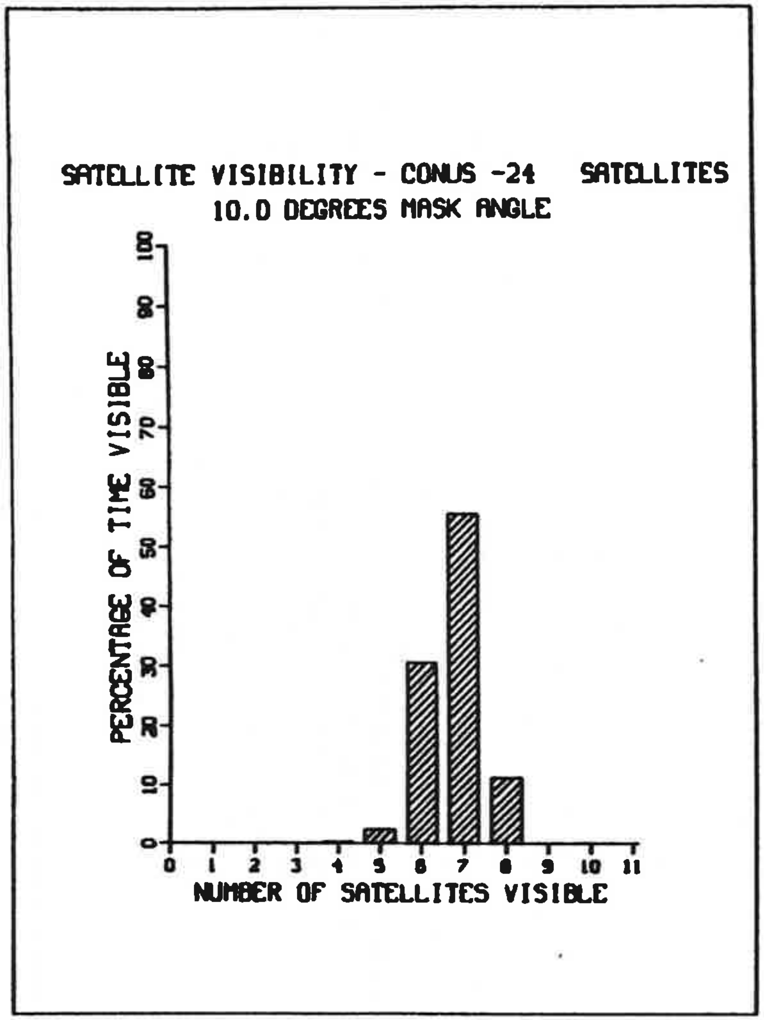
FIGURE 3-5. HDOP PROBABILITY DISTRIBUTIONS
10 DEGREE MASK ANGLE - 18 AND 24 SATELLITES

Figure 3-6 shows the visibility histogram for the 24-satellite constellation. It can be seen that even for a 10-degree mask angle, there are 6 or more satellites available 98% of the time. Even with 24 satellites, however, there are a few periods, highly localized, where only 4 satellites are in view, and where large HDOP's are experienced. They last for only about 3 minutes, which means they can be "coasted" through by even an inexpensive user receiver clock. These periods disappear for mask angles of 5 degrees.

Satellite failures were treated by examining the data at the few times where less than 6 satellites would be in view. (Faulting the satellites one at a time was considered to be unduly expensive from the point of view of computer time.)

For a mask angle of 7.5 degrees, the longest period of time when 4 or 5 satellites are in view is about 18 minutes. Thus, a satellite failure would result in at most an 18-minute outage. This is a highly localized event, occurring only at 50 degrees latitude, and affecting a region of about 60 by 120 miles. Three such regions would be affected during a 24-hour period. With a mask angle of 5 degrees, no such outages occur.

Since these regions are so small, and since they are located well away from major population centers, the 24-satellite constellation is judged to be robust enough to tolerate a satellite failure. Therefore, the 24-satellite constellation is considered to adequately support air navigation requirements with unaided receivers.



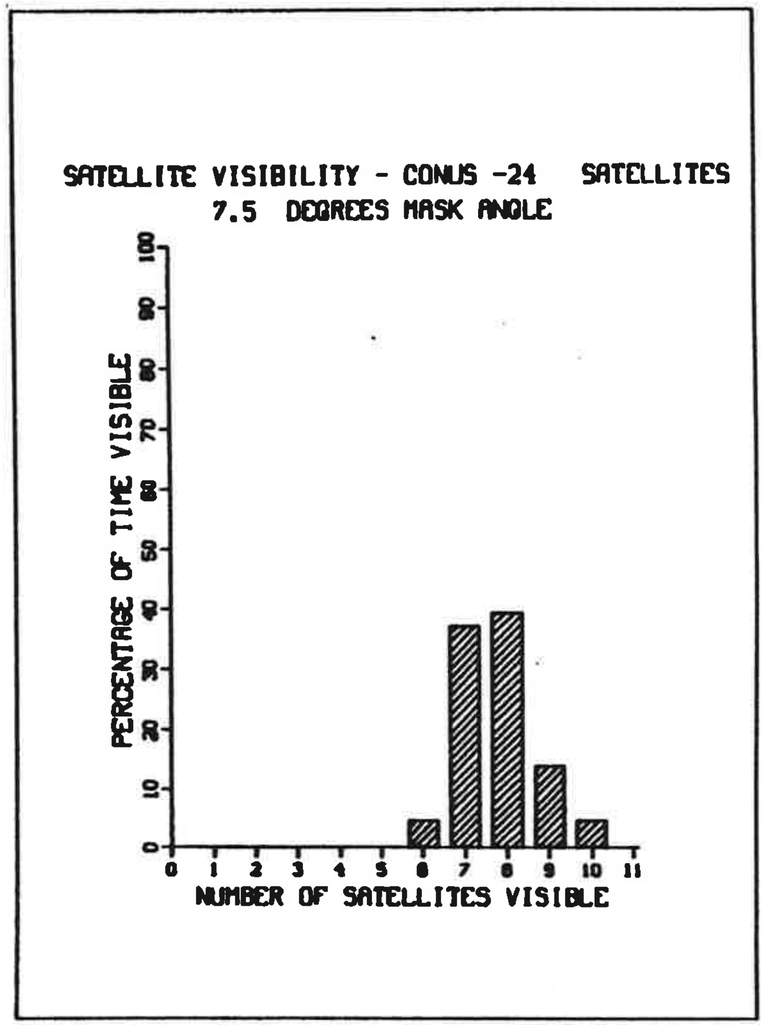


FIGURE 3-6. SATELLITE VISIBILITY FROM CONUS - 24 SATELLITE CONSTELLATION

4. GPS SERVICE RELIABILITY UNDER CONDITIONS OF ALTIMETER AIDING OF USER RECEIVER

4.1 TREATMENT OF ALTIMETER INPUT

Under unaided operation of GPS the navigation solution provides estimates of vertical position. Along with the 100 meter horizontal positioning accuracy, the vertical accuracy will be about 165 meters (2-sigma), which translates to a 3-sigma value of 800 feet. Barometric altimeters provide 3-sigma accuracies of 200-2000 feet, depending on the calibration and the altitude of the aircraft. Above 18000 feet altitude no calibration is used, and all aircraft fly baro altitude. That is, the reported altitude is the reading of the barometric altimeter, not the height above mean sea level.

Barometric altitude can differ from height above mean sea level by as much as 2000 feet, due to deviations of actual pressures from the standard atmospheric conditions on which the altitude-pressure transformation is premised. GPS altitude, on the other hand, is really a measurement of distance from the center of the earth, corrected for the WGS-72 model of the earth for mean sea level. As a result, there can be differences of up to 2000 feet or more between GPS altitude and baro altitude, at high altitudes. These differences must be taken into consideration when evaluating the effectiveness of altimeter aiding of the GPS solution.

If GPS altitude is known independently of the GPS receiver, the information can be used to augment, or aid, the solution. In effect, the altitude measurement is similar to a satellite measurement. An altimeter input is treated by adding a "satellite" at the center of the earth. This defines a sphere of constant distance from the center of the earth, or a constant altitude contour. The resulting HDOP's are always better than the HDOP's without the altimeter input [Milliken & Kizner, 1978].

There is an inherent assumption involved in treating the altimeter in this way. HDOP is a measure of the ratio of the positional error to the pseudorange error, which is assumed to be the same for all the "satellites" used in the solution. Thus this treatment assumes that altimeter errors are comparable to pseudorange errors, which are about 25-30 meters (1-sigma). If the aircraft is below about 5000 feet and the altimeter is calibrated to a local reference, this assumption is reasonable. At higher altitudes the assumption breaks down.

In order to better understand the effects of altimeter error on the horizontal position accuracy, a study was performed of the ratio of horizontal position error to altimeter error for satellite configurations typically encountered in the CONUS. Figure 4-1 shows this as a probability distribution. Depending on the geometry of the satellites, the horizontal position error can vary

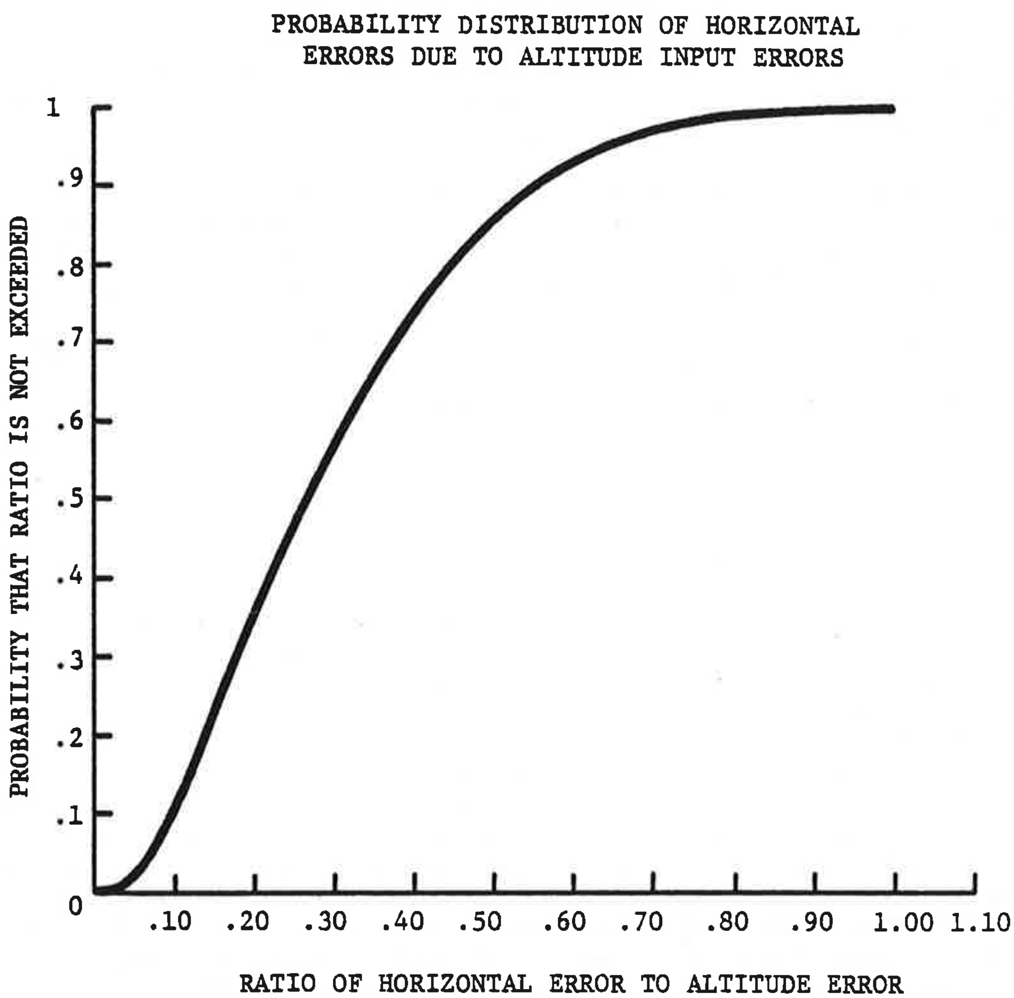


FIGURE 4-1. EFFECT OF ALTIMETER ERRORS ON HORIZONTAL POSITION ERRORS

from zero (where an overhead satellite already provides the same information) to 1.1 times the altitude error. For 50% of the points computed the horizontal position error resulting from a 100 meter altimeter input error is 27 meters or better, and 95% of the time it is 65 meters or better. This means that for altimeter input errors less than about 50 meters (1-sigma, or 500 feet 3-sigma) HDOP is an adequate measure of comparative accuracy under conditions of altimeter aiding.

At high altitudes aircraft are in the en-route or terminal phases of flight, where the accuracy requirements are not stringent: 500 meters or more is adequate. A 2000-foot altimeter error would result 95% of the time in an error of 400 meters or less.

On a non-precision approach, the altimeter is assumed to be calibrated to a local reference, thus providing altitude accuracies of 30 meters or better (1-sigma). The resulting horizontal errors are small compared to the 100 meter requirement for horizontal navigation guidance.

4.2 18-SATELLITE CONSTELLATION

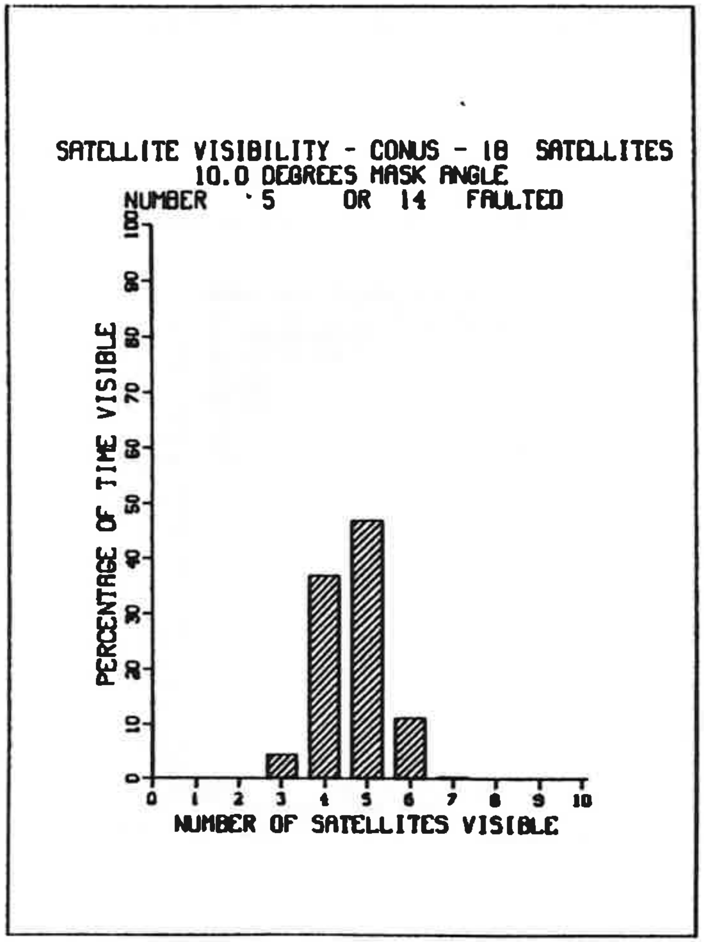
If all 18 satellites are operational a receiver aided by an altimeter input will experience no outages. The outages encountered by an unaided receiver (see section 3.1) are eliminated by the altimeter input.

To check the ability of an altimeter-aided receiver to function in the presence of a satellite failure, the HDOP's were computed over the CONUS, assuming the failure of one satellite. Figure 4-2 shows some representative visibility histograms. It turns out that the failure of any satellite causes outages somewhere in the CONUS sometime each day. Figure 4-3 shows the length of the outages and the number of grid locations experiencing each length of outage for two representative satellite failure conditions. It should be remembered that the grid is 5- by 5-degrees in latitude and longitude, using 6-minute intervals. It can be seen that the longest outage encountered is about 36 minutes.

There are outages that could be experienced by an altimeter-aided receiver operating in the CONUS, and which last significantly more than 5-10 minutes; the outages affect large areas over the country, and are not highly localized in low-traffic density areas. Consequently, the 18-satellite constellation cannot adequately support operation even with altimeter aiding.

4.3 21-SATELLITE CONSTELLATION

The addition of 3 active spares significantly improves the situation for altimeter-aided receivers. The outages described in the section above are largely, but not entirely, eliminated. Table 4-1 shows 14 outage events associated with the loss of a particular satellite. The outage durations associated with a 7.5



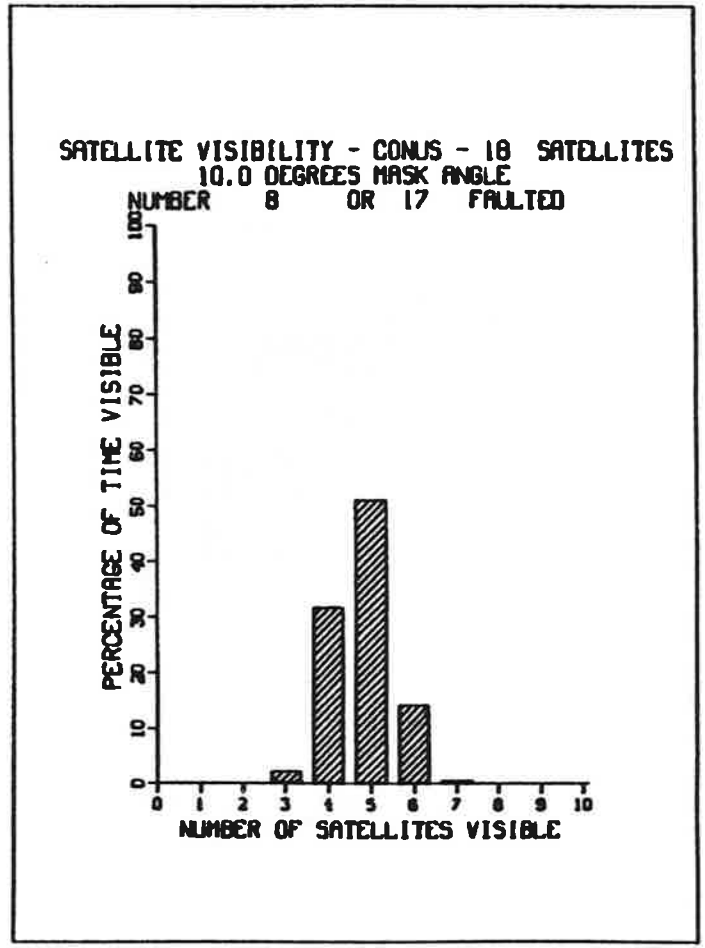
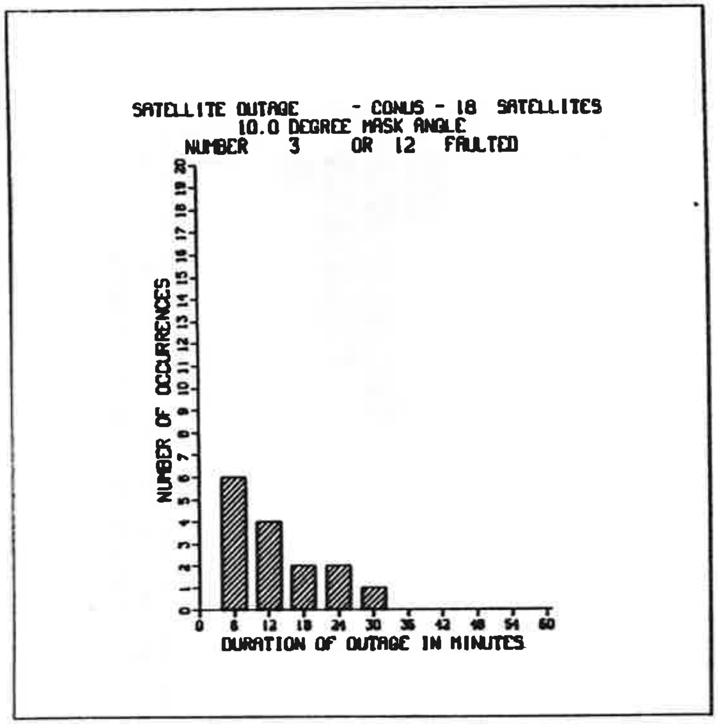


FIGURE 4-2. TYPICAL VISIBILITY HISTOGRAMS WITH ONE SATELLITE FAILURE
18 SATELLITE CONSTELLATION



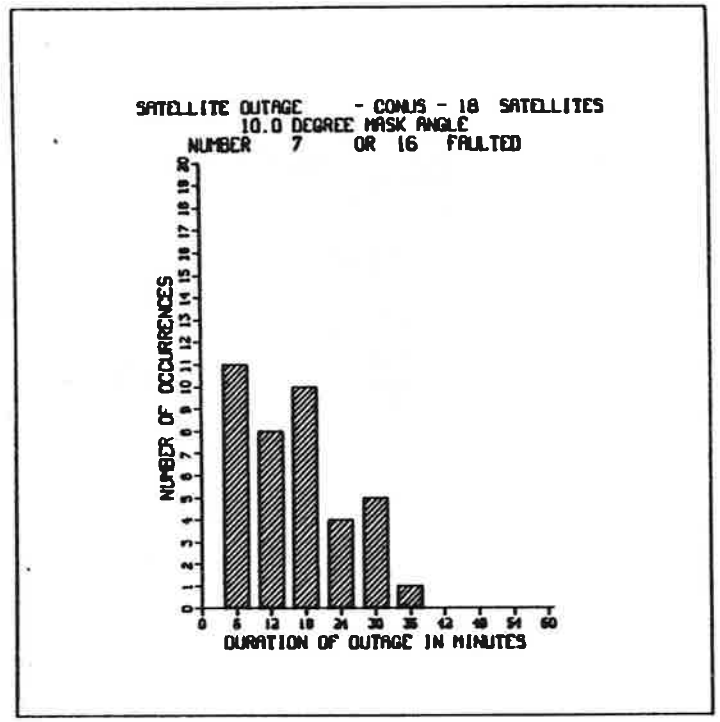


FIGURE 4-3. SERVICE OUTAGE OCCURRENCES CAUSED BY SATELLITE FAILURE
18 SATELLITE CONSTELLATION

TABLE 4-1. OUTAGE LOCATIONS FOR ALTIMETER-AIDED OPERATION

FAILED SATELLITE NUMBER	GPS TIME MINUTES	LOCATION LATITUDE N. DEGREES	LOCATION LONGITUDE W. DEGREES	WORST * OUTAGE MINUTES
1	1092	45-50	70-85	18
2	468	45-50	125	12
3	228	45-50	70-80	12
	426	40-45	70	6
4	630	45-50	105-115	12
5	60	45-50	115-125	12
6	NONE			
7	222	45-50	90-105	12
8	NONE			
9	786	45-50	85-100	12
	984	40-45	80-90	12
10	NONE			
11	1158	35-45	70	18
12	NONE			
13	576	35-50	115-125	24
	774	45-50	110-125	12
14	NONE			
15	1338	40-50	100-120	18
16	NONE			
17	66	45-50	85-95	12
18	NONE			

* Using a 7.5 degree mask angle

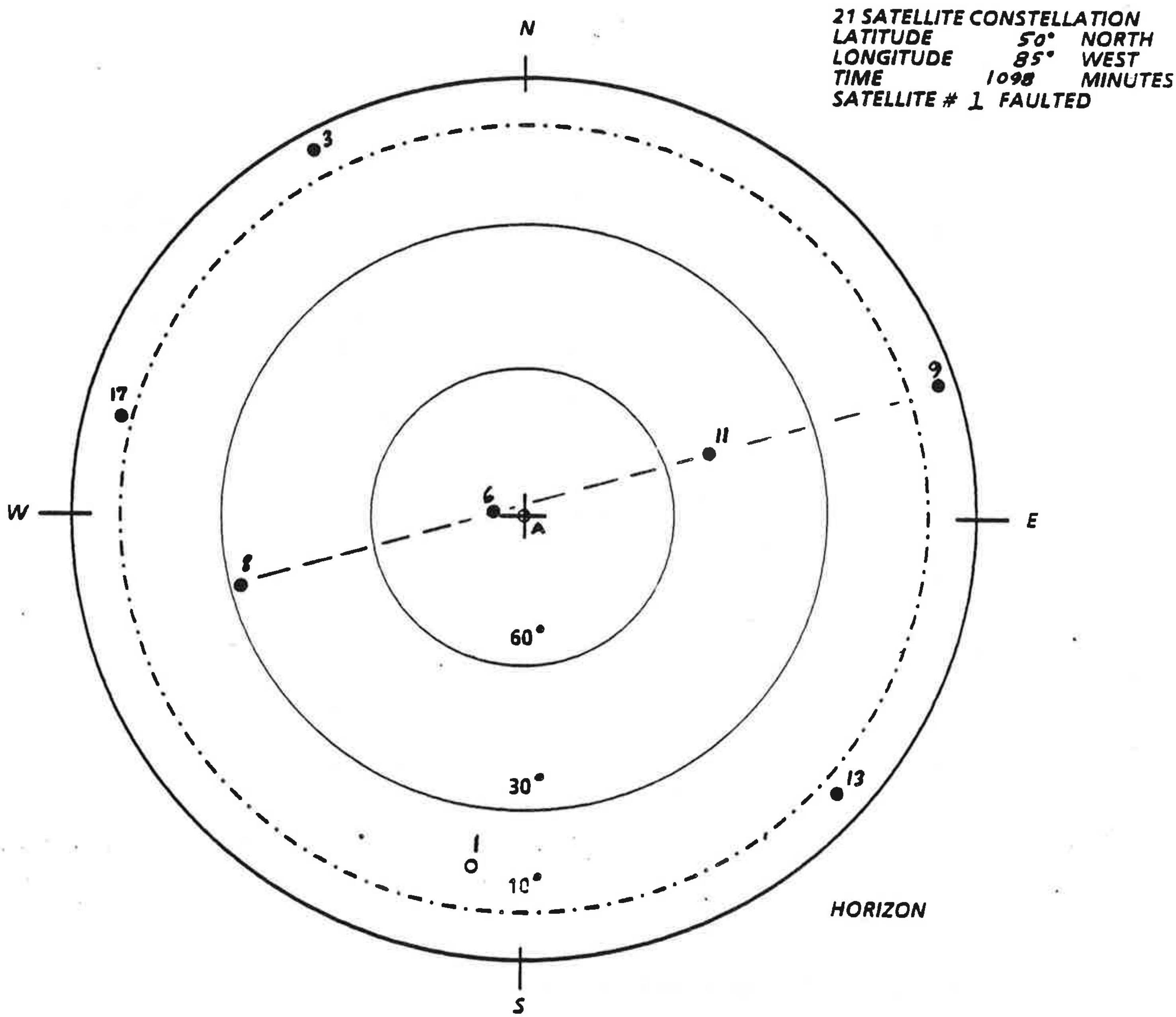


FIGURE 4-4. CRITICAL SATELLITE GEOMETRY UNDER FAILURE CONDITIONS
ALTITUDE-AIDED RECEIVER - 21 SATELLITE CONSTELLATION

degree mask angle range from 6 to 24 minutes. Whereas a failure of certain satellites (specifically, 6,8,10,12,14, and 16) causes no outages, a failure of any of the others will cause at least one outage somewhere in the CONUS. It can be seen from the table that most of the outages take place between 45 and 50 degrees latitude.

In each outage event the satellites are aligned very close to a plane that includes the user. This causes a large uncertainty in the direction perpendicular to plane, and thus a large HDOP. This alignment is evident in Figure 4-4, which shows an example of the satellite geometry during such an outage. Here satellite #1, indicated by the open circle in the south, is assumed to fail. With a 10 degree mask angle (indicated by the dot-dash circle) only satellites 6,8 and 11 are visible. It can be seen that they lie almost exactly in a plane with the user.

The figure also shows that the effect of a altitude aiding does not cure the situation. To a first approximation, the altimeter acts much like a satellite at the zenith (at the center of the chart). It is evident that the plane is very close to the zenith. The observed HDOP for this geometry, including the altitude, has the value 32, which is very large.

Appendix C provides similar depictions of all 14 outage situations. It can be noted that in several outage situations 4 and even 5 satellites are visible, but still lie in a plane. It is also apparent that in every case there are satellites above about 5 degrees that would ameliorate the bad HDOP situation. Thus the use of a 5 degree mask angle (or no mask angle at all) would eliminate the outage problem and enable operation with the 21-satellite constellation during a single satellite fault.

4.4 24-SATELLITE CONSTELLATION

Examination of the satellite visibility data for 24-satellite operation shows that 24 satellites would provide quite adequate service for altimeter-aided receivers. Even in the highly localized regions described in 3.3 the failure of a satellite would be accommodated by the altimeter input. No outages would occur.

5. CLOCK QUALITY REQUIRED FOR "COASTING" DURING OUTAGES

5.1 METHOD OF USING THE CLOCK DURING OUTAGE PERIODS

During normal operation the receiver computes latitude, longitude, and altitude (if unaided), using the pseudorange measurements from 4 satellites or more. Additionally, the receiver navigation processor computes time corrections by which the receiver clock tracks GPS time. Indeed, some currently available receivers use the GPS to compute time, but do not compute position at all. The resulting time accuracy is typically 100 nanoseconds or better. One could estimate satellite range by adding the time correction to the pseudorange measurement, and perform a rho-rho-rho type of position estimate.

If the number of available satellites drops to three, there are no longer 4 independent measurements available to the receiver to compute position and time. In this case the receiver, which tracks time and estimates frequency, could take the last time and frequency measurements from the point in time that 4 satellites were available, and perform the rho-rho-rho position estimate. If the user clock frequency did not drift, the solution would continue to be as good as it had been, and the receiver would "coast" through the outage period until another satellite appeared, or until the 4 satellites became sufficiently non-coplanar. At such time the position algorithm would revert to the usual 4-satellite pseudorange solution.

5.2 EFFECT OF CLOCK DRIFT ON POSITION ERROR

Since users do not usually carry rubidium or cesium atomic standard clocks on board, but rather employ various grades of crystal-controlled oscillators, there can be significant frequency drifts over a period of 5-60 minutes or more. These frequency drifts are caused by temperature changes, pressure changes, aging of the crystal, accelerations, and vibrations. A constant frequency offset error causes a positional error which grows linearly in time. Thus for any level of frequency drift, there is a limited amount of time that the clock can "coast" the solution through an outage.

The positional error depends not only on the frequency drift and the coast time, but also on the geometry of the satellites available during the coasting period. This can be illustrated by the limiting case where there are three satellites which lie in a plane containing the user position. In such a case only two satellites are independently affecting the position solution; there are 3 unknowns and 2 equations, and therefore no solution is possible. The ability of a clock to coast the solution therefore depends on the satellite locations. The formulation is given in Appendix C, and is similar to that of Mark Sturza of Litton Corporation [Sturza, 1983].

It turns out that when the receiver processor switches to the clock-coasting algorithm, the horizontal positioning error consists of two terms. The first term is the new HDOP term, which relates to the geometry but not the clock. It is numerically different (usually much better) than the HDOP in the normal navigation mode, which is in an outage condition (see equation C-13). The second term contains the effect of clock delay error. It is equal to the product of the accumulated time delay error and a geometric factor which depends on the satellite locations. This geometric factor is called the "time-position error multiplier", referred to in this report as the TPEM. The time delay error itself is equal to the product of the time elapsed, the frequency stability of the oscillator, and a factor which accounts for the frequency drift behavior during a particular period of time (from C-21). In good geometries the TPEM typically takes on values between 0.5 and 1.2.

The values of TPEM during the outage periods defined in section 4.3 for failures in the 21-satellite constellation using altitude aiding turn out to be unfavorable. Since in those situations three or more satellites are almost in a plane with the user, and since the altitude measurement is effectively in or near that plane, there are really only two independent measurements. In the limit of actual coplanarity, the TPEM would blow up. The values of TPEM behave roughly as the square root of the HDOP in the near-coplanar geometries. As a result, the clock coasting mode is not effective in providing accurate navigation through the outages.

Figure 5-1 shows the 2drms horizontal positioning error due to clock drift for typical values of the TPEM and frequency drift factors. Figure 5-2 gives the same information in a different way. It plots the clock frequency stability required to keep the error under a specified value for the required period.

Figure 5-3 shows the sensitivity of the results to the frequency drift factor. Similarly, Figure 5-4 shows the sensitivity to the TPEM value. It can be seen that large changes in these factors change the results by less than 30%.

Figure 5-5 shows the combined positional error due to geometry and clock effects for a typical outage situation (see equation C-22). It can be seen that there is an interval of time after which the clock error effects predominate.

This analysis did not fully consider the effects of vibrations and other environmental factors on clock stability. Also, the TPEM values were based on a small sampling of potential outage geometries. To the extent to which the values of the factors used here are valid, it appears that to keep clock-induced errors to less than 100 meters (and thus the total errors to less than 141 meters), the clock frequency stability (in an Allan Variance sense) should be 1.6×10^{-10} or better, using an averaging time of 36 minutes.

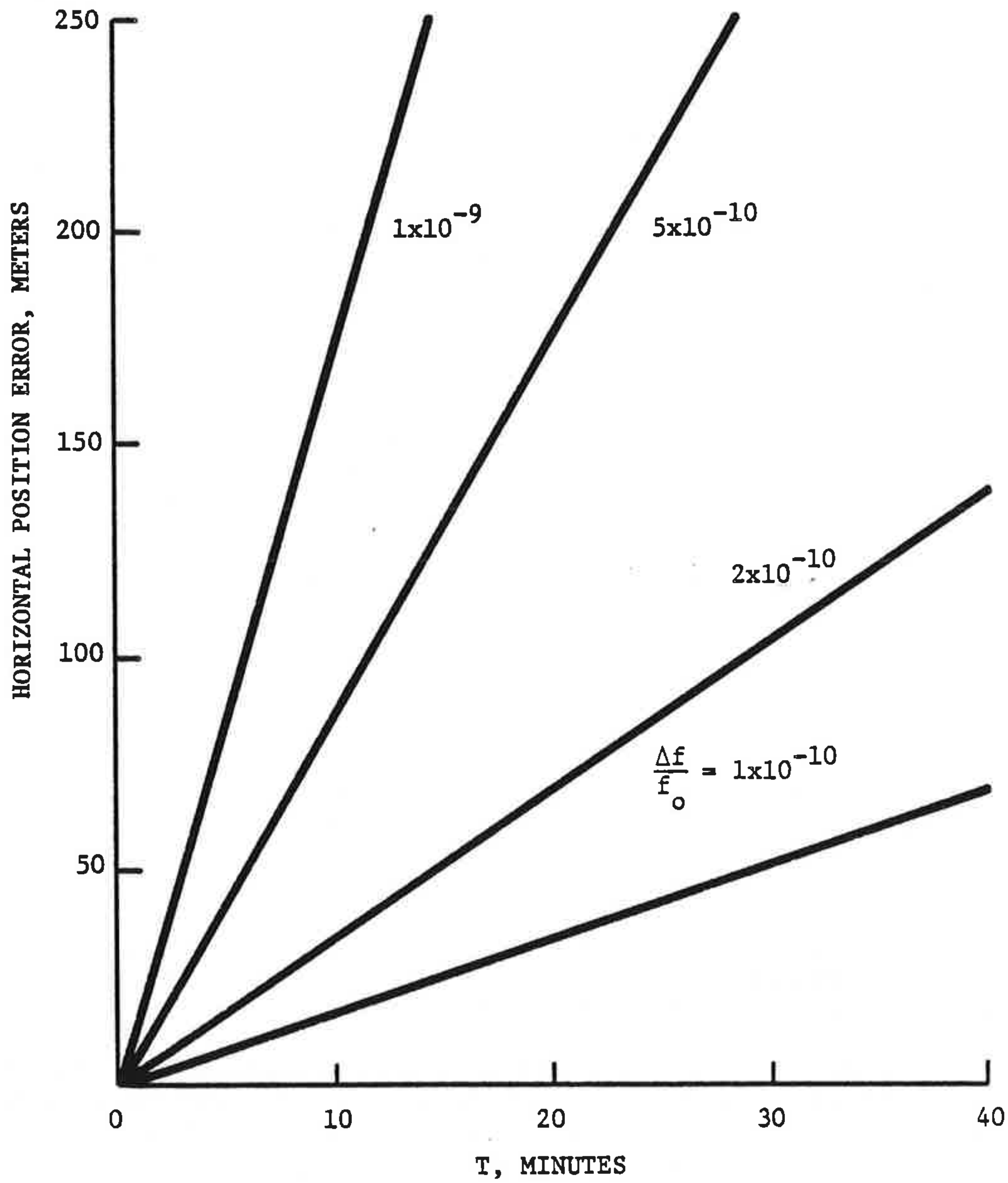


FIGURE 5-1. POSITION ERROR GROWTH DUE TO CLOCK DRIFT

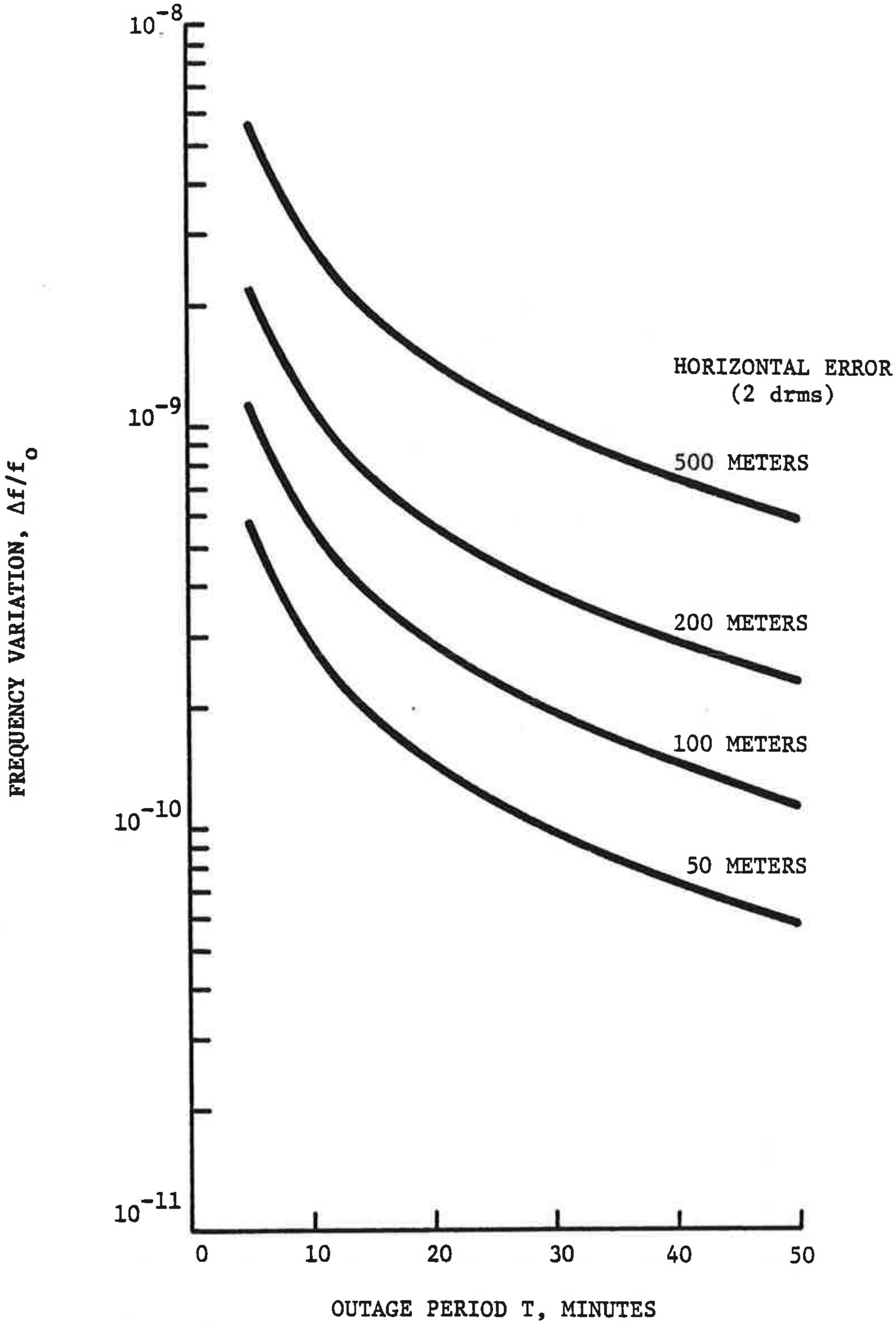


FIGURE 5-2. CLOCK STABILITY REQUIRED TO LIMIT POSITION ERRORS

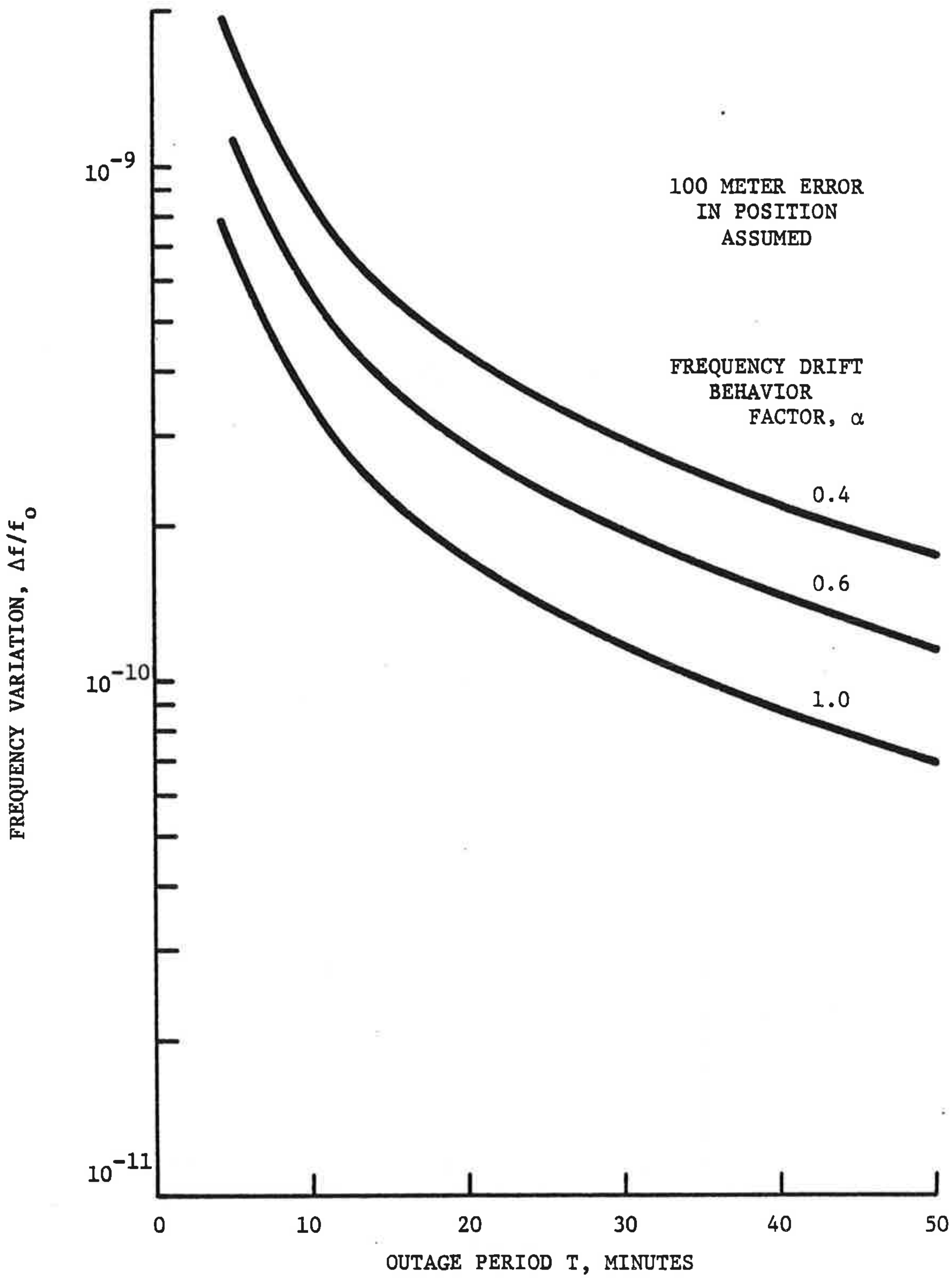


FIGURE 5-3. VARIATION OF CLOCK STABILITY REQUIREMENT WITH FREQUENCY DRIFT BEHAVIOR

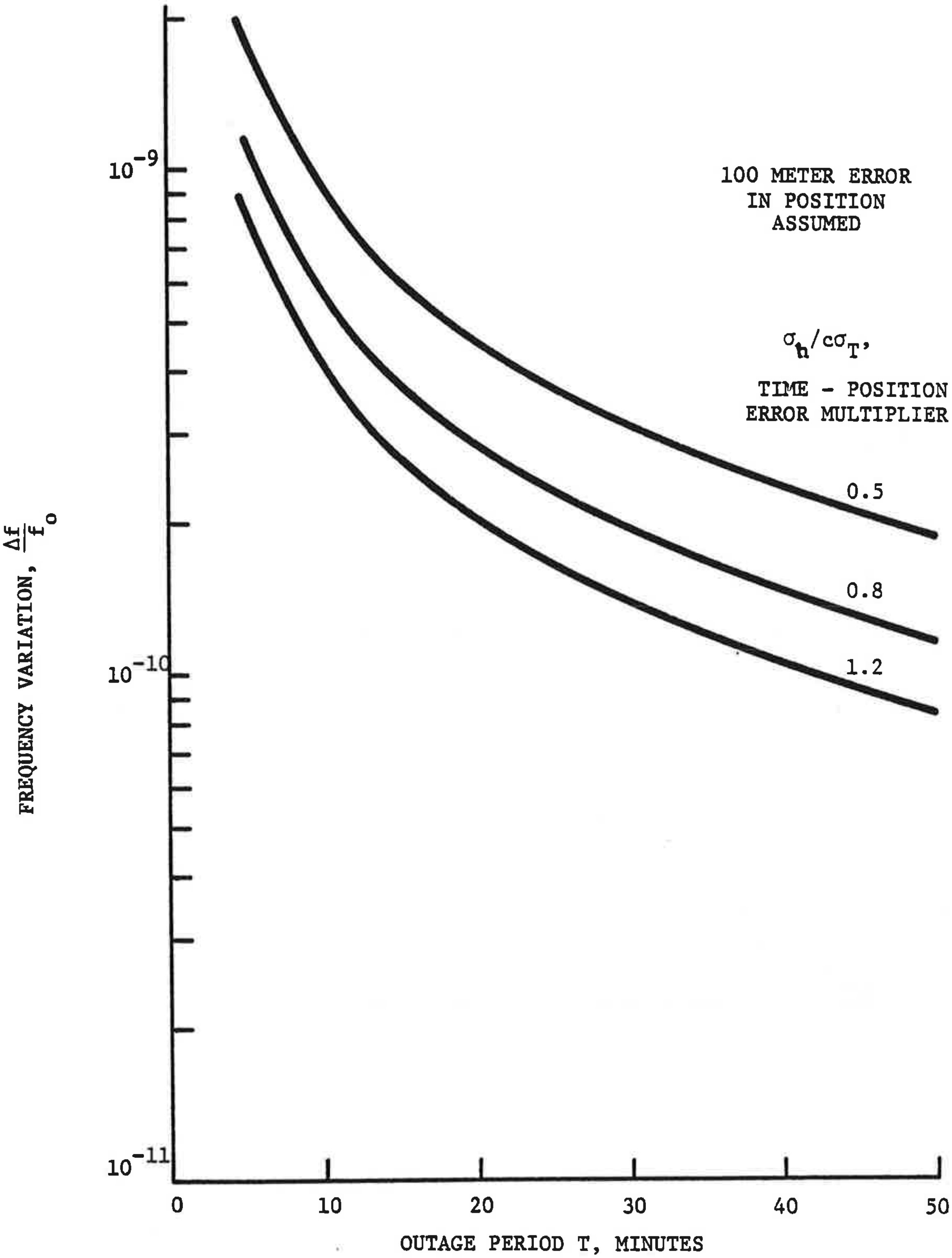


FIGURE 5-4. VARIATION OF CLOCK STABILITY REQUIREMENT WITH GEOMETRIC FACTOR

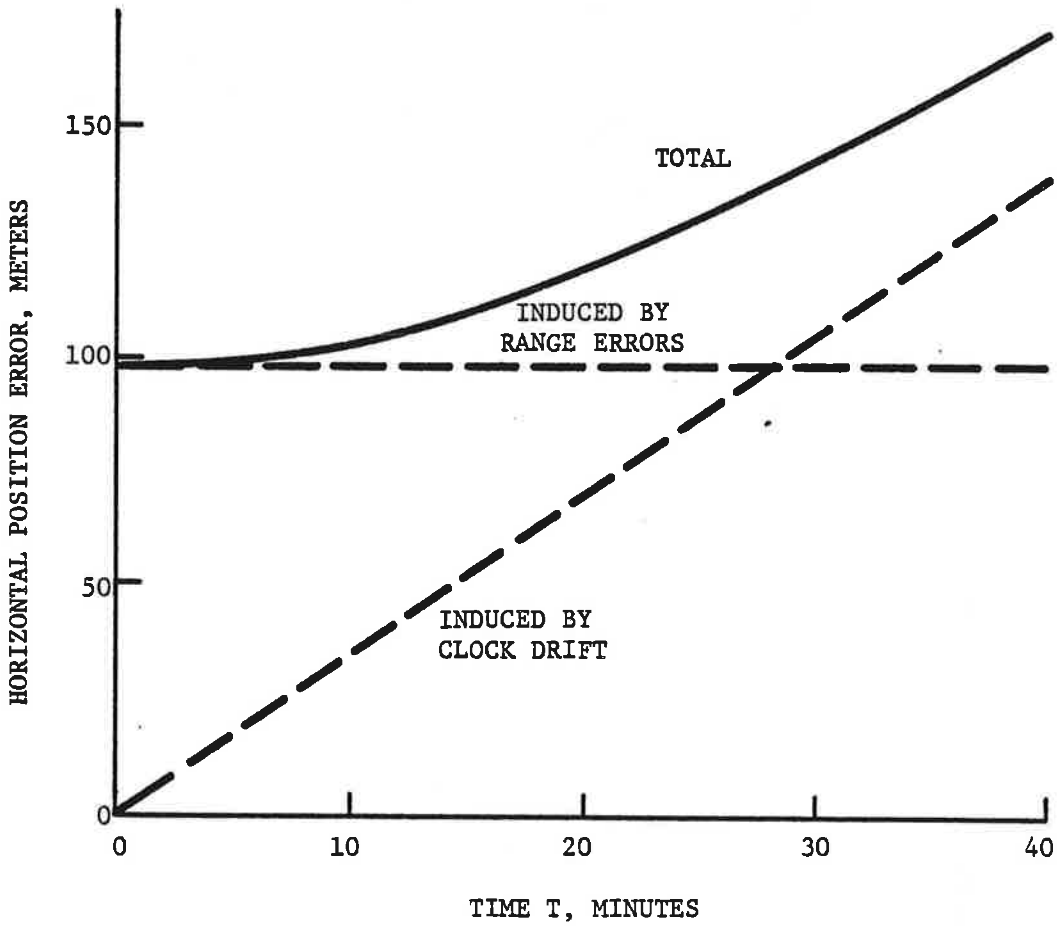


FIGURE 5-5. TYPICAL POSITION ERROR GROWTH DURING A COASTING PERIOD

Nothing has been said so far about the effect of a Kalman filter or other estimating algorithm on the clock coasting mode. Implicit in the discussion is the assumption that at the moment the receiver processor switches over to the coasting algorithm, the receiver's estimates of time delay and frequency are assumed to be perfect. Frequency has to be estimated in order to coast at all.

It has been suggested that by performing carrier Doppler processing in the receiver, the frequency drift rate could be continuously estimated. When the receiver switched to a coast mode, this frequency drift rate could be used to reduce the clock-induced position error. The efficacy of this technique depends on the behavior of the clock as it responds to environmental changes. Whether this can significantly reduce the clock oscillator stability requirements has not been established.

6. CONCLUSIONS AND RECOMMENDATIONS

From the analysis performed here, the following conclusions can be drawn:

1. For unaided user receiver operation, the planned GPS constellation cannot provide adequate service in the event of a satellite failure. An augmented constellation consisting of an additional satellite in each plane (24 satellites) appears to provide adequate service with one failed satellite.

2. If an altimeter input is used to augment, or aid, the user receiver during a non-precision approach, then the 21-satellite constellation (18 normal, plus 3 active spares) comes close, but does not quite provide adequate service everywhere in the CONUS, under conditions of a single satellite failure.

3. Under conditions of one failed satellite, there are periods of up to 24 minutes in the proposed 21-satellite constellation where there are outages even with altimeter aiding. Furthermore, during these periods, the use of a high-quality clock does not fully solve the problem.

4. If the user mask angle can be lowered to 5 degrees, the outages do not occur. For 7.5 and 10 degree mask angles, however, they do occur.

5. In order to operate with the 18-satellite constellation with or without a satellite failure, the user receiver must be able to "coast" through a period of 15-36 minutes. By having the user receiver processor switch into a coast mode algorithm when a service outage is sensed, a stable clock would enable the receiver to provide an accurate navigation solution throughout the period for most outage situations. The clock stability required to achieve this appears to be about 1.6 parts in ten billion (1.6×10^{-10}). However, for the coplanar alignment described in 3, even a clock an order of magnitude more stable would still not be adequate for the higher mask angles.

APPENDIX A. EXAMPLES OF SATELLITE VISIBILITY / HDOP DATA

The basic data consists of a grid covering most of the CONUS:

LATITUDE: 30 to 50 degrees North in 5 degree increments;
LONGITUDE: 70 to 125 degrees West in 5 degree increments.

A grid of values of the number of satellites visible and the HDOP at that time and location is printed for each point in time. Time is incremented for every 6 minutes.

Figure A-1 shows an example of the output data for an outage condition encountered for a satellite failure in the 18-satellite constellation. The areas where fewer than 6 satellites are visible are highlighted.

Figure A-2 shows an example of the output data for the 24-satellite constellation. Comparison with Figure A-1 shows that the number of satellites visible is significantly higher.

Figure A-3 shows a closeup of the highly localized area where brief outages occur in the 24-satellite constellation for user mask angles of 7.5 degrees. All of these small outages areas are at about 50 degrees north latitude.

GPS TIME: 156.000 MINUTES

	-125.0	-120.0	-115.0	-110.0	-105.0	-100.0	-95.0	-90.0	-85.0	-80.0	-75.0	-70.0
50.0	6* 1.57	6* 1.57	6* 1.64	6* 1.64	* 1.63	5* 1.64	6* 1.66	5* 1.88	5* 1.85	5* 1.83	5* 1.84	5* 1.86
45.0	6* 1.65	6* 1.63	6* 1.62	6* 1.62	6* 1.63	5* 1.83	4* 2.03	4* 2.01	5* 2.08	5* 2.07	5* 2.08	5* 2.11
40.0	6* 1.65	6* 1.64	6* 1.63	6* 1.63	6* 1.64	5* 1.66	4* 2.45	4* 2.43	4* 2.48	4* 2.59	5* 2.34	6* 1.54
35.0	5* 1.43	6* 1.66	6* 1.65	5* 1.65	5* 1.66	5* 1.69	4* 2.91	4* 2.90	4* 2.94	4* 3.04	6* 1.52	7* 1.36
30.0	5* 1.45	5* 1.53	5* 1.69	5* 1.69	5* 1.71	5* 1.73	5* 1.45	5* 1.44	5* 1.44	6* 1.46	6* 1.49	6* 1.53

THERE ARE 0 OUTAGES ON THIS MAP

GPS TIME: 162.000 MINUTES

	-125.0	-120.0	-115.0	-110.0	-105.0	-100.0	-95.0	-90.0	-85.0	-80.0	-75.0	-70.0
50.0	6* 1.60	6* 1.60	6* 1.60	6* 1.62	6* 1.62	5* 1.52	6* 1.53	5* 1.75	5* 1.73	5* 1.73	5* 1.73	5* 1.76
45.0	6* 1.60	6* 1.63	6* 1.61	6* 1.60	6* 1.61	6* 1.62	4* 1.78	4* 1.75	5* 1.89	5* 1.89	5* 1.90	5* 1.93
40.0	5* 1.50	6* 1.63	6* 1.61	6* 1.61	6* 1.61	5* 1.62	4* 2.01	4* 1.98	4* 2.00	4* 2.07	5* 2.09	5* 2.12
35.0	5* 1.48	5* 1.56	6* 1.64	5* 1.63	5* 1.64	5* 1.65	4* 2.30	4* 2.27	5* 1.48	5* 1.49	5* 1.51	6* 1.54
30.0	5* 1.51	5* 1.59	4* 1.74	5* 1.68	5* 1.68	5* 1.70	6* 1.28	5* 1.44	5* 1.44	5* 1.45	6* 1.47	6* 1.50

THERE ARE 0 OUTAGES ON THIS MAP

FIGURE A-1. CONUS GRID OF SATELLITE VISIBILITY & HDOP VALUES
18 SATELLITE CONSTELLATION

GPS TIME: 156.000 MINUTES

	-125.0	-120.0	-115.0	-110.0	-105.0	-100.0	-95.0	-90.0	-85.0	-80.0	-75.0	-70.0
50.0	7* 1.31	7* 1.33	7* 1.36	7* 1.41	5* 2.01	6* 1.40	7* 1.40	7* 1.38	8* 1.36	8* 1.35	8* 1.34	8* 1.34
45.0	9* 1.35	8* 1.30	8* 1.32	7* 1.88	7* 1.87	6* 1.88	6* 1.34	7* 1.30	8* 1.37	8* 1.36	8* 1.35	8* 1.34
40.0	8* 1.25	8* 1.26	8* 1.29	7* 1.72	7* 1.71	7* 1.72	7* 1.70	8* 1.32	8* 1.32	7* 1.24	8* 1.37	8* 1.36
35.0	8* 1.23	8* 1.25	7* 1.62	7* 1.57	7* 1.56	8* 1.54	7* 1.56	8* 1.30	8* 1.29	8* 1.30	8* 1.31	8* 1.39
30.0	8* 1.23	8* 1.24	7* 1.50	7* 1.45	7* 1.43	8* 1.42	8* 1.43	7* 1.48	8* 1.27	8* 1.28	8* 1.29	8* 1.32

THERE ARE 0 OUTAGES ON THIS MAP

GPS TIME: 162.000 MINUTES

	-125.0	-120.0	-115.0	-110.0	-105.0	-100.0	-95.0	-90.0	-85.0	-80.0	-75.0	-70.0
50.0	7* 1.32	7* 1.34	7* 1.55	7* 1.61	4*24.63	5* 2.11	7* 1.40	7* 1.38	8* 1.36	8* 1.35	8* 1.34	7* 1.31
45.0	8* 1.35	8* 1.36	8* 1.37	8* 1.38	7* 1.86	6* 1.95	8* 1.38	8* 1.37	8* 1.37	8* 1.36	8* 1.35	8* 1.34
40.0	8* 1.34	8* 1.33	8* 1.34	7* 1.72	7* 1.70	7* 1.69	7* 1.70	8* 1.33	8* 1.33	8* 1.33	8* 1.36	8* 1.36
35.0	8* 1.32	8* 1.31	8* 1.31	7* 1.57	7* 1.55	8* 1.55	7* 1.56	7* 1.59	8* 1.30	8* 1.30	8* 1.32	7* 1.23
30.0	8* 1.23	8* 1.30	7* 1.50	7* 1.45	8* 1.44	8* 1.42	7* 1.43	7* 1.47	8* 1.29	8* 1.29	8* 1.30	8* 1.32

THERE ARE 1 OUTAGES ON THIS MAP

FIGURE A-2. CONUS GRID OF SATELLITE VISIBILITY & HDOP VALUES
24 SATELLITE CONSTELLATION

GPS TIME: 161.000 MINUTES

	-105.0	-104.0	-103.0	-102.0	-101.0	-100.0
51.0	5* 6.58	5* 6.59	5* 6.61	4*16.82	5* 6.39	5* 6.40
50.0	5* 2.12	5* 2.11	5* 2.10	5* 2.10	5* 2.09	5* 2.09
49.0	5* 2.08	5* 2.07	5* 2.06	5* 2.05	5* 2.04	5* 2.04

THERE ARE 6 OUTAGES ON THIS MAP

GPS TIME: 162.000 MINUTES

	-105.0	-104.0	-103.0	-102.0	-101.0	-100.0
51.0	5* 6.97	5* 6.98	5* 7.00	4*23.29	5* 6.74	5* 6.75
50.0	4*24.63	4*24.53	5* 2.13	5* 2.12	5* 2.11	5* 2.11
49.0	5* 2.10	5* 2.09	5* 2.08	5* 2.07	5* 2.07	5* 2.06

THERE ARE 8 OUTAGES ON THIS MAP

GPS TIME: 163.000 MINUTES

	-105.0	-104.0	-103.0	-102.0	-101.0	-100.0
51.0	5* 7.41	5* 7.42	5* 7.44	5* 7.46	5* 7.13	5* 7.14
50.0	5* 7.63	4*38.36	4*38.18	4*38.02	4*37.88	5* 2.13
49.0	6* 2.17	6* 2.16	6* 2.16	6* 2.15	5* 2.09	5* 2.08

THERE ARE 11 OUTAGES ON THIS MAP

FIGURE A-3. CLOSEUP OF HIGHLY LOCALIZED OUTAGES
24 SATELLITE CONSTELLATION

APPENDIX B. OUTAGE OCCURRENCES & DURATIONS FOR SINGLE SATELLITE FAILURES 18-SATELLITE CONSTELLATION

When altimeter aiding is employed, the failure of a single satellite causes service outages in the 18 satellite constellation. Figures B-1 through B-9 show the number of grid points on the maps of Appendix A that experience service outages of different periods. The number of grid points gives a measure of the area affected.

Note that while there is some variation in the impact of specific satellite failures, the longest outages are 36 minutes. These values were derived for mask angles of 10 degrees, the worst case. The periods are shorter for lower mask angles. The symmetry of the 18 satellite constellation is such that there are nine satellite pairs, such that the track of the first member of each pair during the first 12 hours and the track of the second member during the next 12 hours is the same.

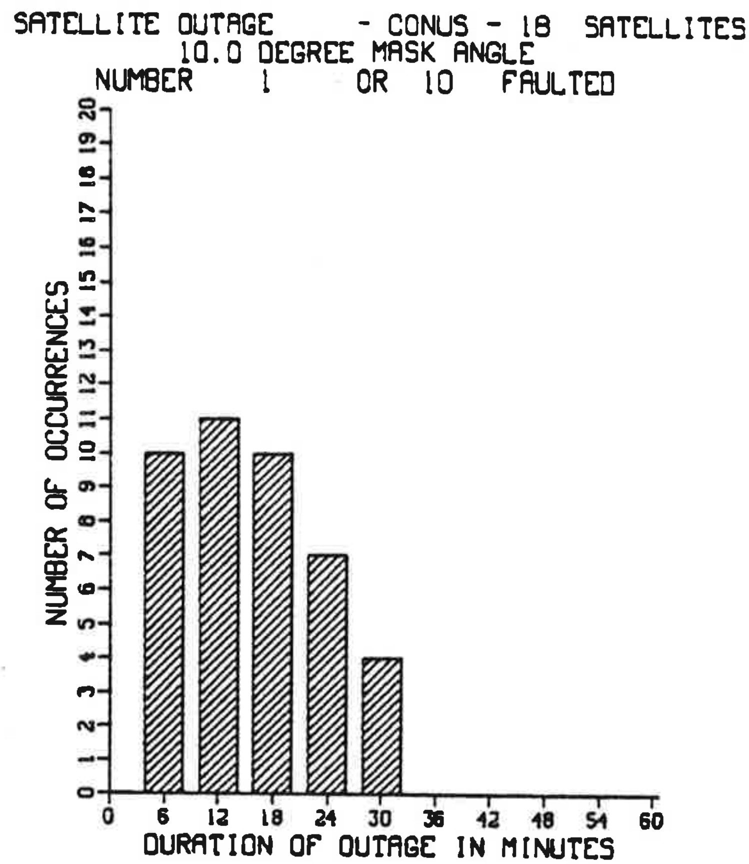


FIGURE B-1. OUTAGE DURATIONS AND OCCURRENCES
18 SATELLITE CONSTELLATION - SATELLITE 1 OR 10 FAULTED

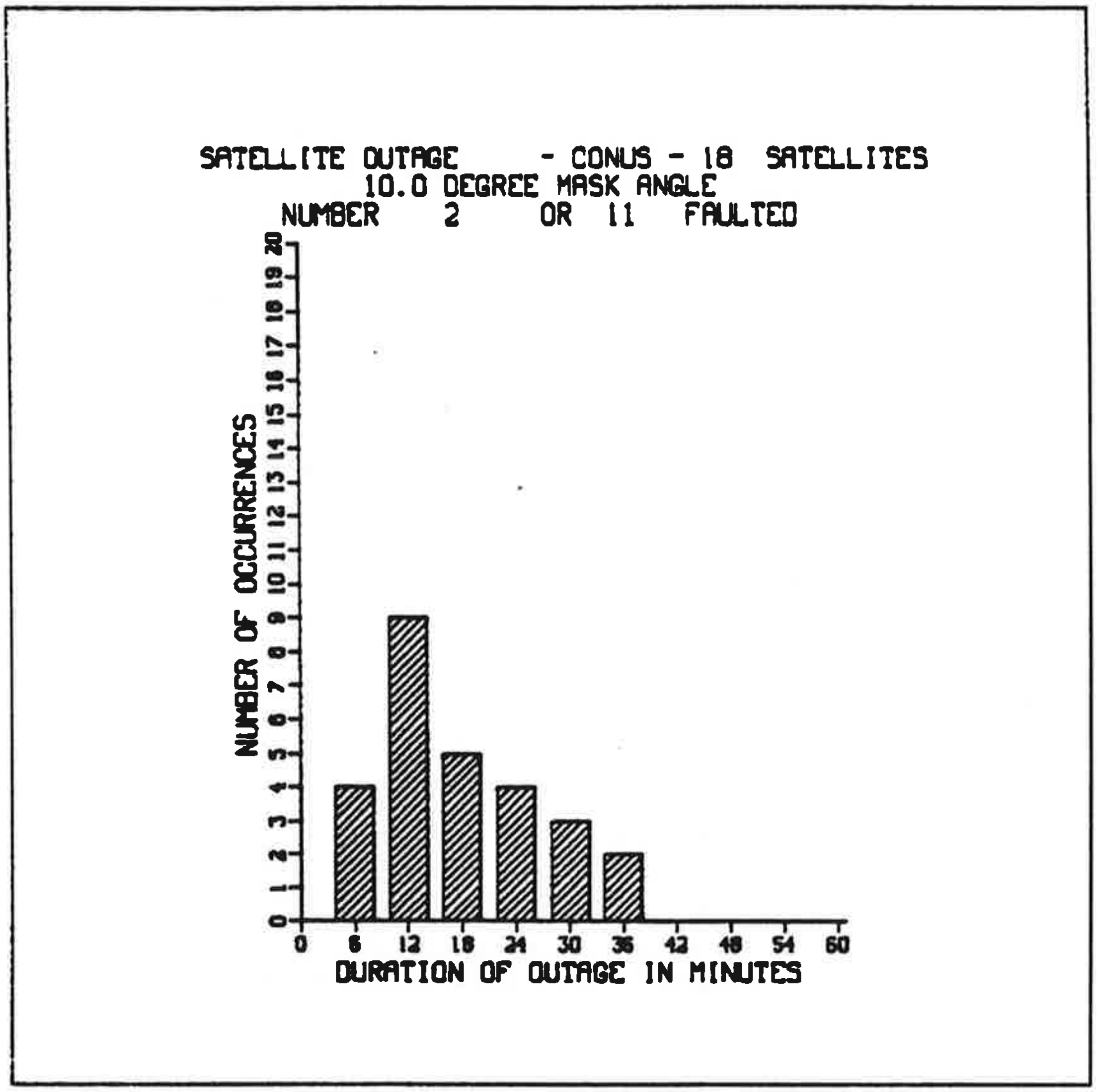


FIGURE B-2. OUTAGE DURATIONS AND OCCURRENCES
18 SATELLITE CONSTELLATION - SATELLITE 2 OR 11 FAULTED

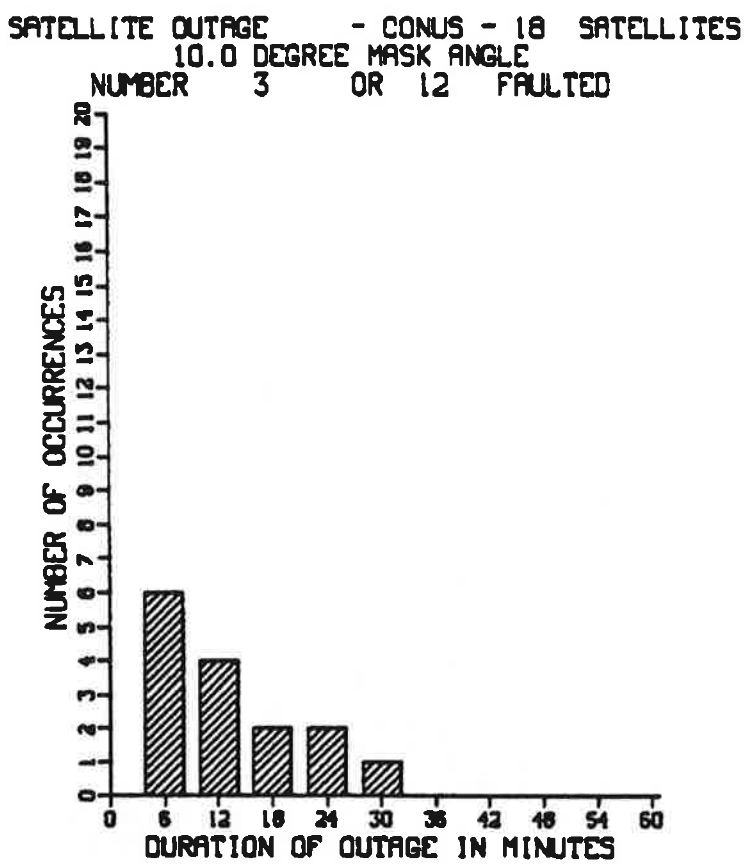


FIGURE B-3. OUTAGE DURATIONS AND OCCURRENCES
18 SATELLITE CONSTELLATION - SATELLITE 3 OR 12 FAULTED

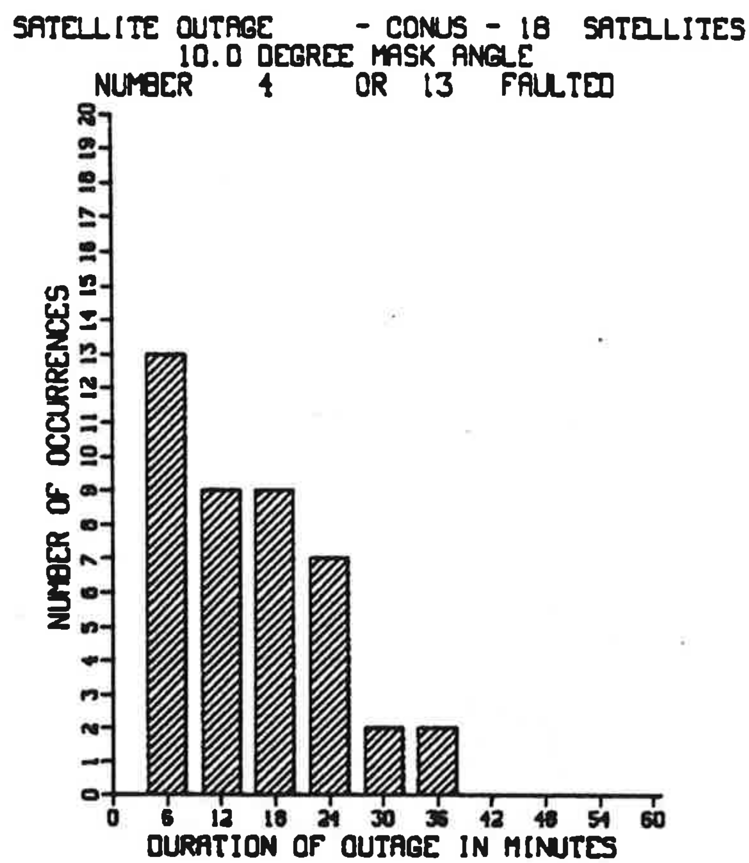


FIGURE B-4. OUTAGE DURATIONS AND OCCURRENCES
18 SATELLITE CONSTELLATION - SATELLITE 4 OR 13 FAULTED

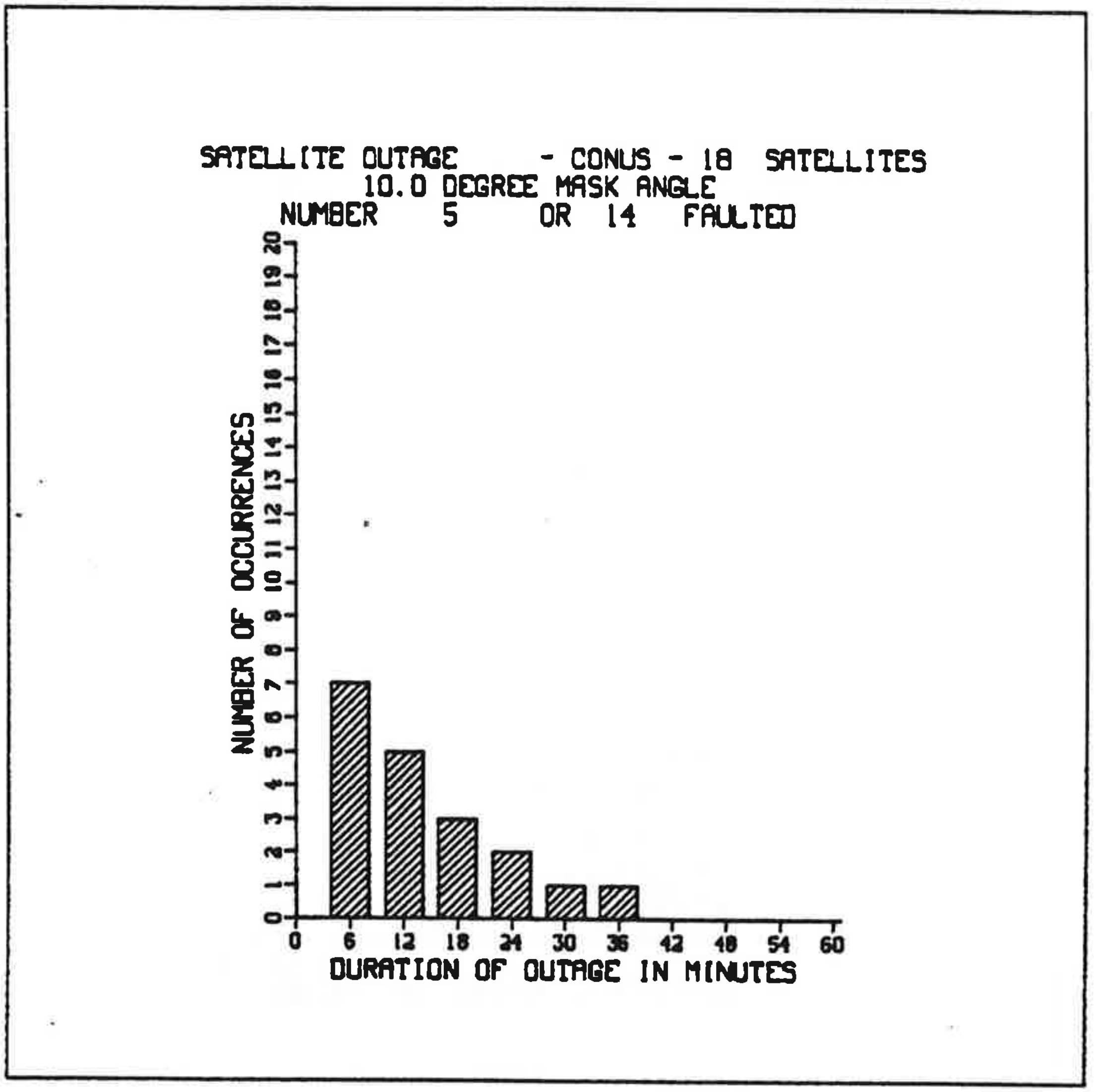


FIGURE B-5. OUTAGE DURATIONS AND OCCURRENCES
18 SATELLITE CONSTELLATION - SATELLITE 5 OR 14 FAULTED

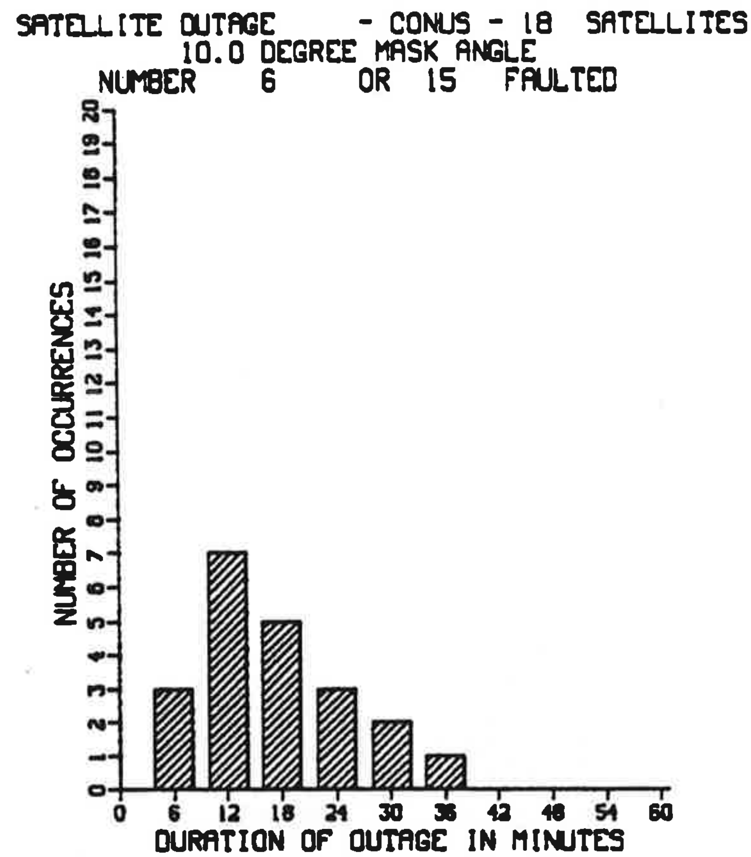


FIGURE B-6. OUTAGE DURATIONS AND OCCURRENCES
18 SATELLITE CONSTELLATION - SATELLITE 6 OR 15 FAULTED

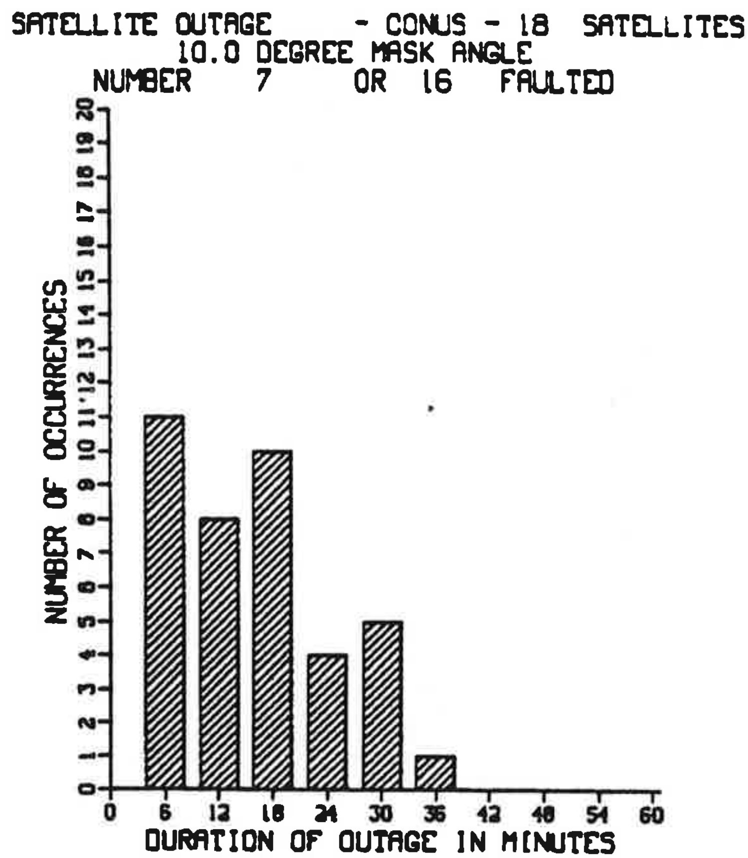


FIGURE B-7. OUTAGE DURATIONS AND OCCURRENCES
18 SATELLITE CONSTELLATION - SATELLITE 7 OR 16 FAULTED

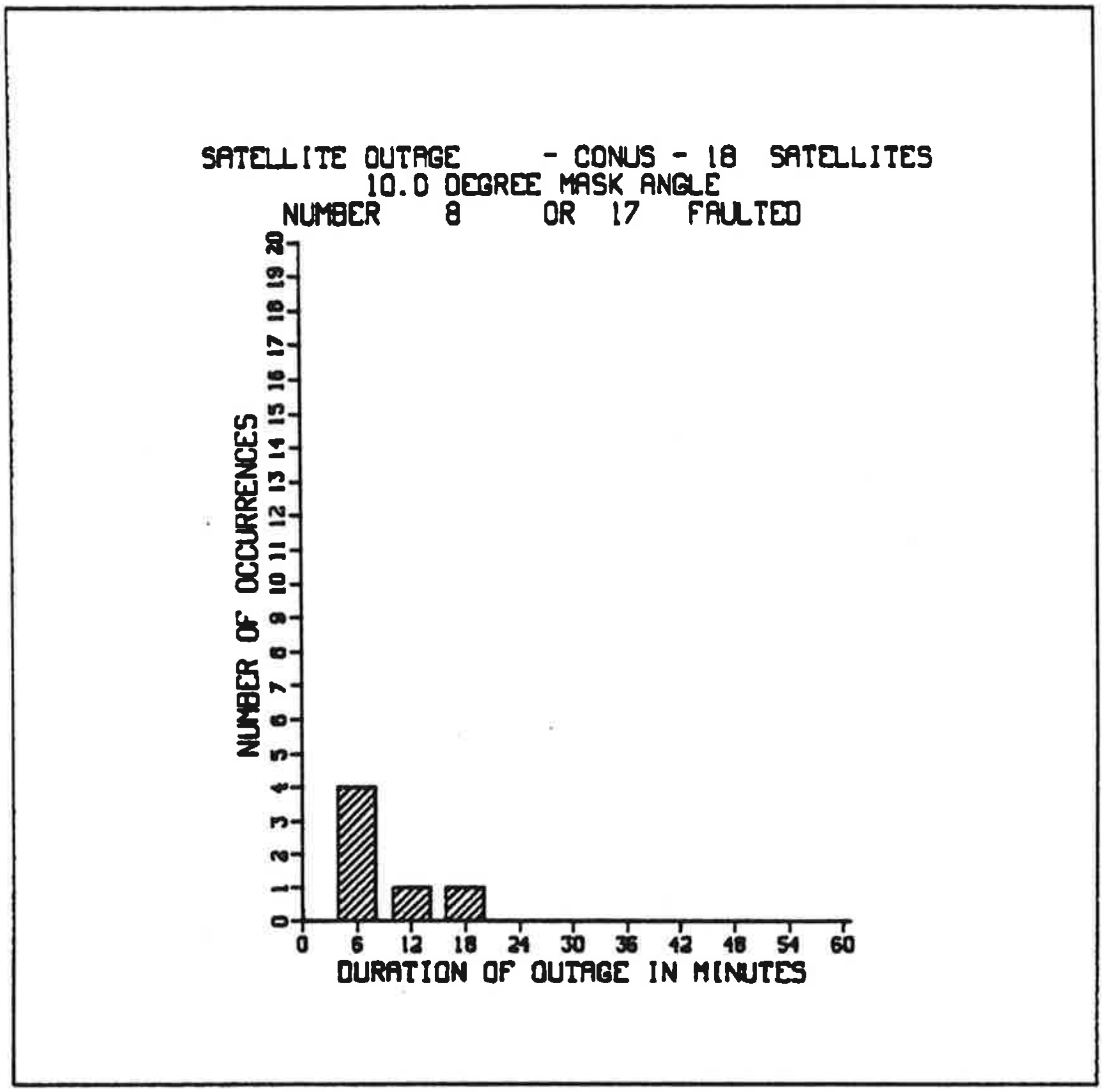


FIGURE B-8. OUTAGE DURATIONS AND OCCURRENCES
18 SATELLITE CONSTELLATION - SATELLITE 8 OR 17 FAULTED

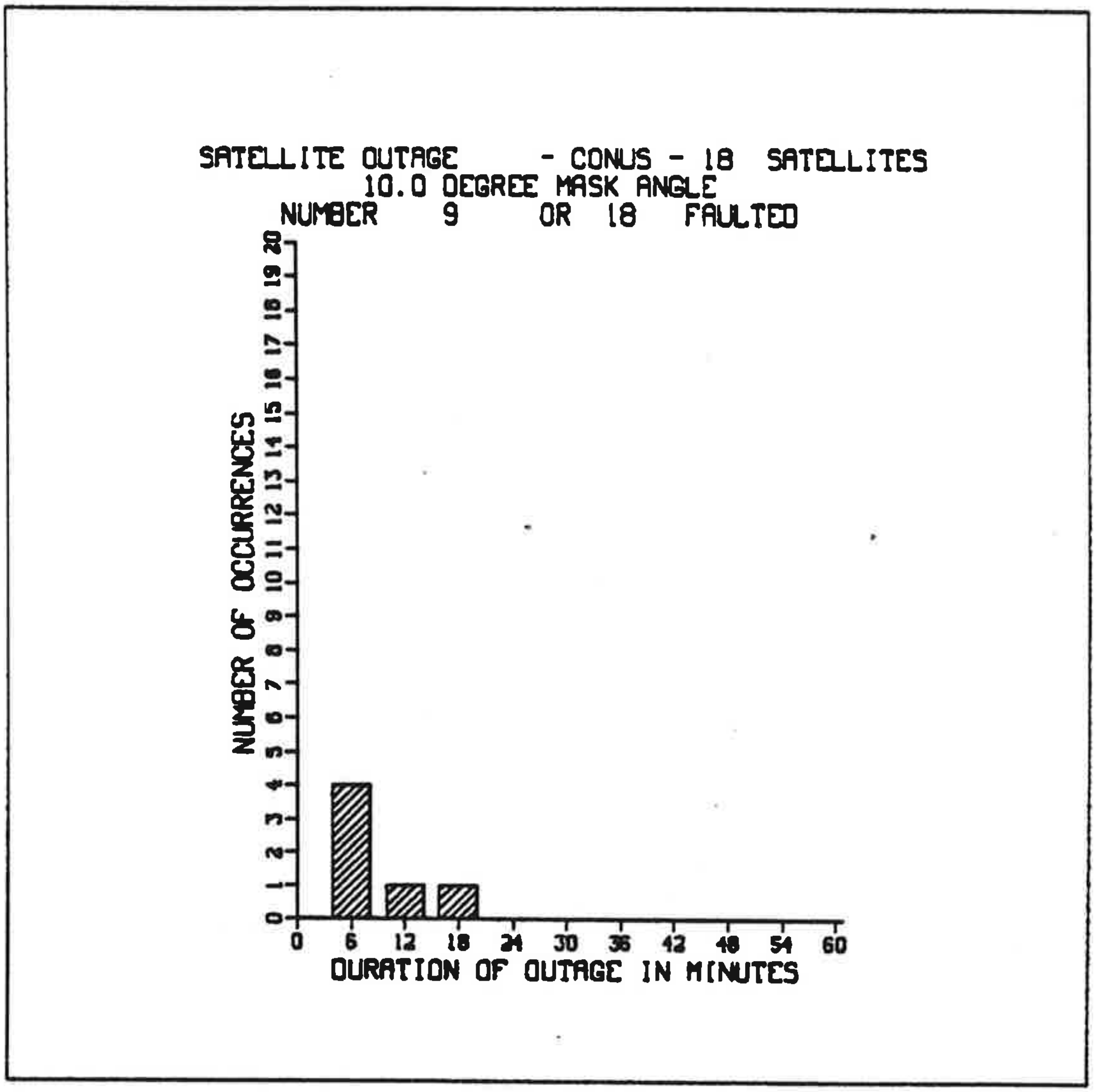


FIGURE B-9. OUTAGE DURATIONS AND OCCURRENCES
18 SATELLITE CONSTELLATION - SATELLITE 9 OR 18 FAULTED

APPENDIX C. SATELLITE GEOMETRIES DURING CRITICAL OUTAGES INVOLVING A FAILURE OF ONE SATELLITE - 21 SATELLITE CONSTELLATION

The outage events tabulated in Table 4-1, repeated here as Table C-1, have the peculiar property that even with altimeter aiding the navigation error can become unacceptably large, if one satellite fails. This appendix shows the satellite geometry associated with each of these outage events.

In the figures, the center of the concentric circles is directly over the user, at the zenith. The horizon is represented by the largest of the circles, and the elevation angle of each satellite as seen by the user is measured linearly from the horizon (zero degrees) to the zenith (ninety degrees). The azimuth angle of each satellite is its angle in the plan view, with north shown toward the top of the page. The 10-degree elevation is also shown, with a dash-dot circle. With a mask angle of 10 degrees, only satellites within that circle would be included in the receiver's solution algorithm. The active satellites are represented by solid dots, the faulted satellites by open dots, and the altimeter by an equivalent open dot at the zenith.

Actually the outage events affect a region, not a single location. However, the figures provide information for nearby users within a few degrees latitude and longitude. For example, a user 5 degrees to the east will see approximately the same satellite geometry moved 5 degrees to the west. In Figure C-1 such a user would see satellite #13 as above a 10 degree mask angle, which would eliminate the outage for him. Similar reasoning holds for users in nearby locations in other directions.

Furthermore, the satellites move in time. Some satellites are rising, while some are setting. This motion is not shown in the figures.

An inspection of the figures reveals that in all cases there are satellites above 5 degrees elevation. This means that a user receiver having a mask angle of 5 degrees will experience no outage, unless the critical satellites are blocked from view by aircraft wings, tail, or some other obstacle.

From Table C-1 it can be seen that certain satellites can fail without causing an outage in the CONUS, specifically numbers 6,8,10,12,14 and 16. Double outages are associated with satellites 3, 9 and 13. The longest observed outage for a mask angle of 7.5 degrees was about 24 minutes, at 125 W, 40 N, at GPS time of 576 minutes. It should also be noted that there are frequently 4 and even 5 satellites in view, all nearly coplanar (see Figures C-3, C-4, C-5, C-7, C-8, C-9, C-10, C-12,and C-14).

In all the outage events depicted, the active satellites lie nearly in a plane with the user. In this way of showing the satellite geometry, such a plane going through the zenith could be represented by a straight line. A plane well away from the zenith would involve satellites lying on an arc, as in Figures C-4, C-9, C-10 and C-11. When the plane passes near the zenith, it means that the addition of an altimeter input, which to a first approximation can be represented by a satellite at the zenith, does not greatly improve the solution. This reasoning is borne out by the observed high values of HDOP that occur in the outage regions.

The TPEMs associated with the outage events are also high, indicating that a high-quality clock does not completely solve the problem of coasting through the outages. At first this is surprising, but it can be explained by examining the geometry involved. If only three satellites are visible (one of which could be the "altimeter" at the zenith), a clock can coast the solution, which would be computed not on the basis of pseudoranges, but range estimates. If the geometry is good, this is satisfactory. However, if the satellites approach coplanarity with the user, one of the satellites becomes redundant, i.e., effectively only two satellites are available, and no solution is possible. The effectiveness of the third satellites is measured by the TPEM, where a large TPEM indicates near-coplanarity.

TABLE C-1. OUTAGE LOCATIONS FOR ALTIMETER-AIDED OPERATION

FAILED SATELLITE NUMBER	GPS TIME MINUTES	LOCATION LATITUDE N. DEGREES	LOCATION LONGITUDE W. DEGREES	WORST * OUTAGE MINUTES
1	1092	45-50	70-85	18
2	468	45-50	125	12
3	228	45-50	70-80	12
	426	40-45	70	6
4	630	45-50	105-115	12
5	60	45-50	115-125	12
6	NONE			
7	222	45-50	90-105	12
8	NONE			
9	786	45-50	85-100	12
	984	40-45	80-90	12
10	NONE			
11	1158	35-45	70	18
12	NONE			
13	576	35-50	115-125	24
	774	45-50	110-125	12
14	NONE			
15	1338	40-50	100-120	18
16	NONE			
17	66	45-50	85-95	12
18	NONE			

* Using a 7.5 degree mask angle

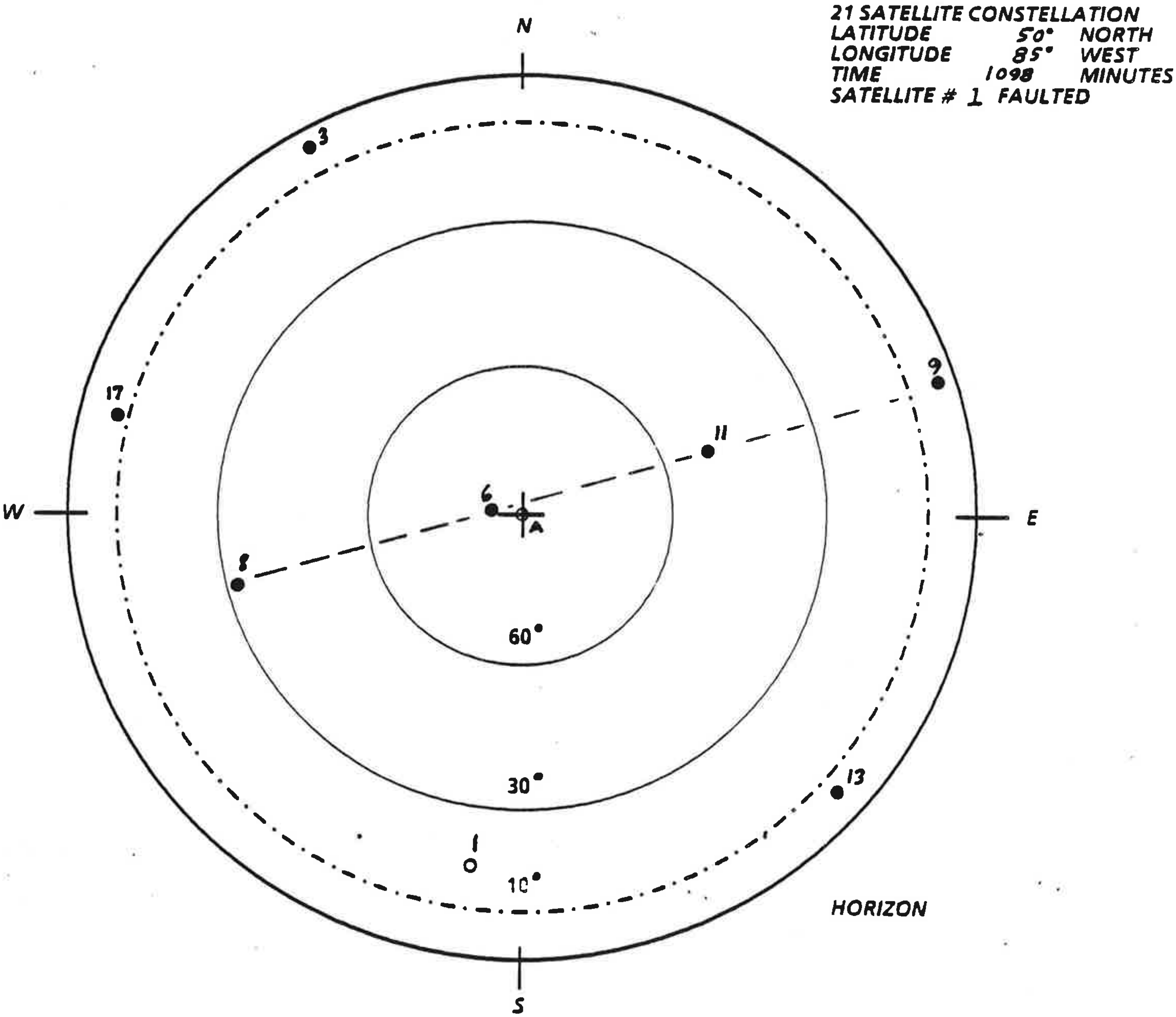


FIGURE C-1. SATELLITE GEOMETRY DURING OUTAGE ASSOCIATED WITH A FAILURE OF SATELLITE #1

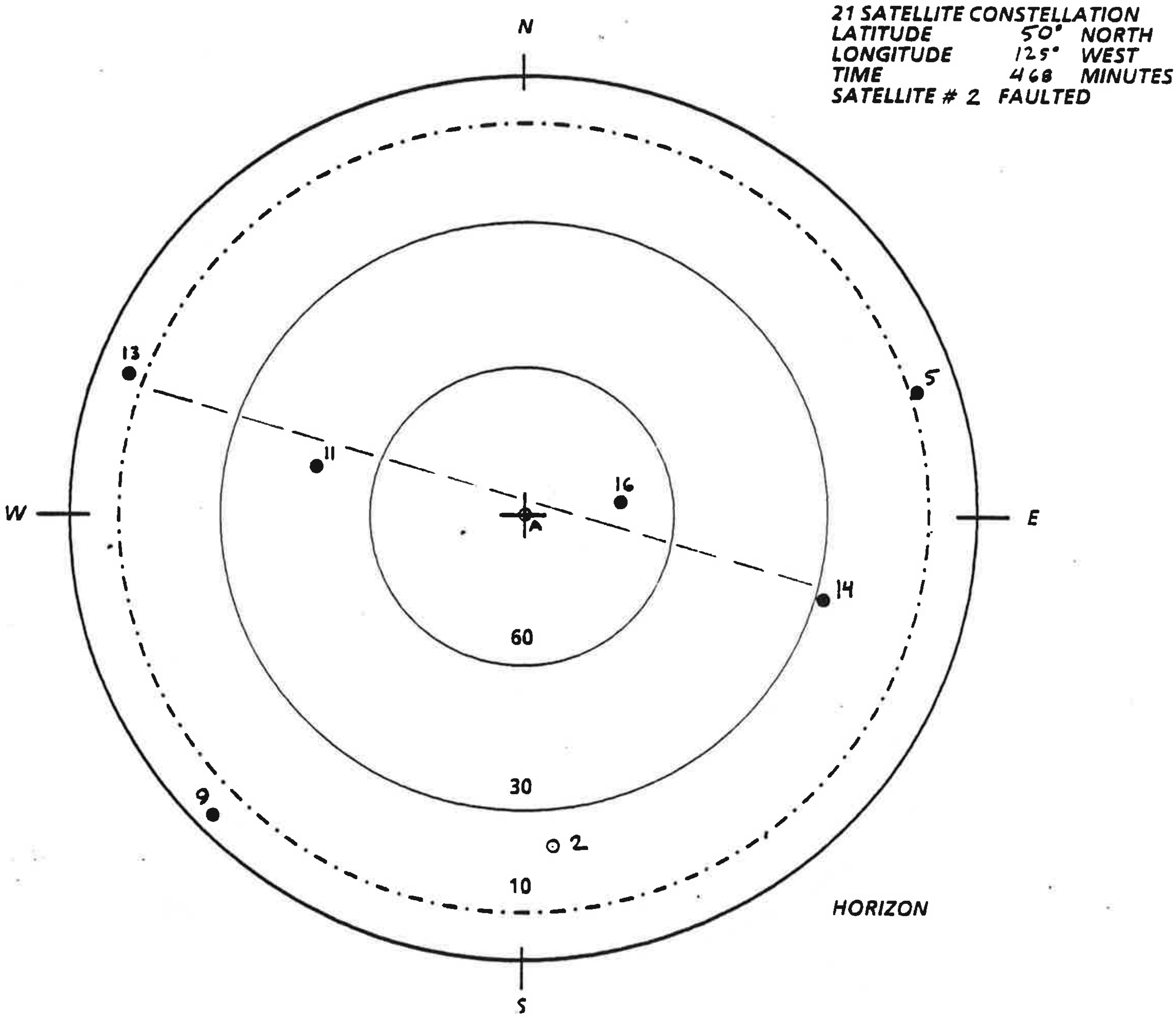


FIGURE C-2. SATELLITE GEOMETRY DURING OUTAGE ASSOCIATED WITH A FAILURE OF SATELLITE #2

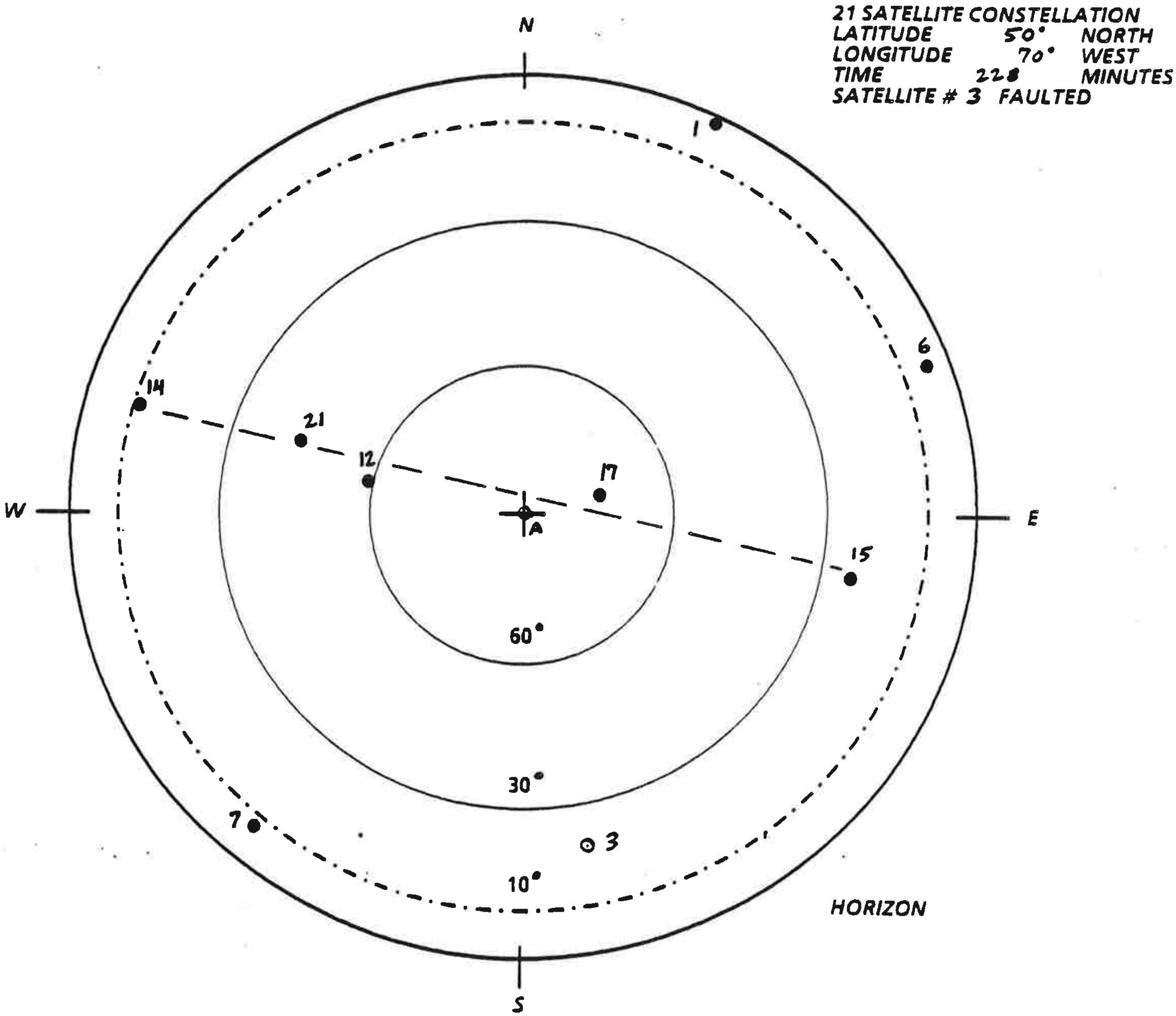


FIGURE C-3. SATELLITE GEOMETRY DURING OUTAGE ASSOCIATED WITH A FAILURE OF SATELLITE #3

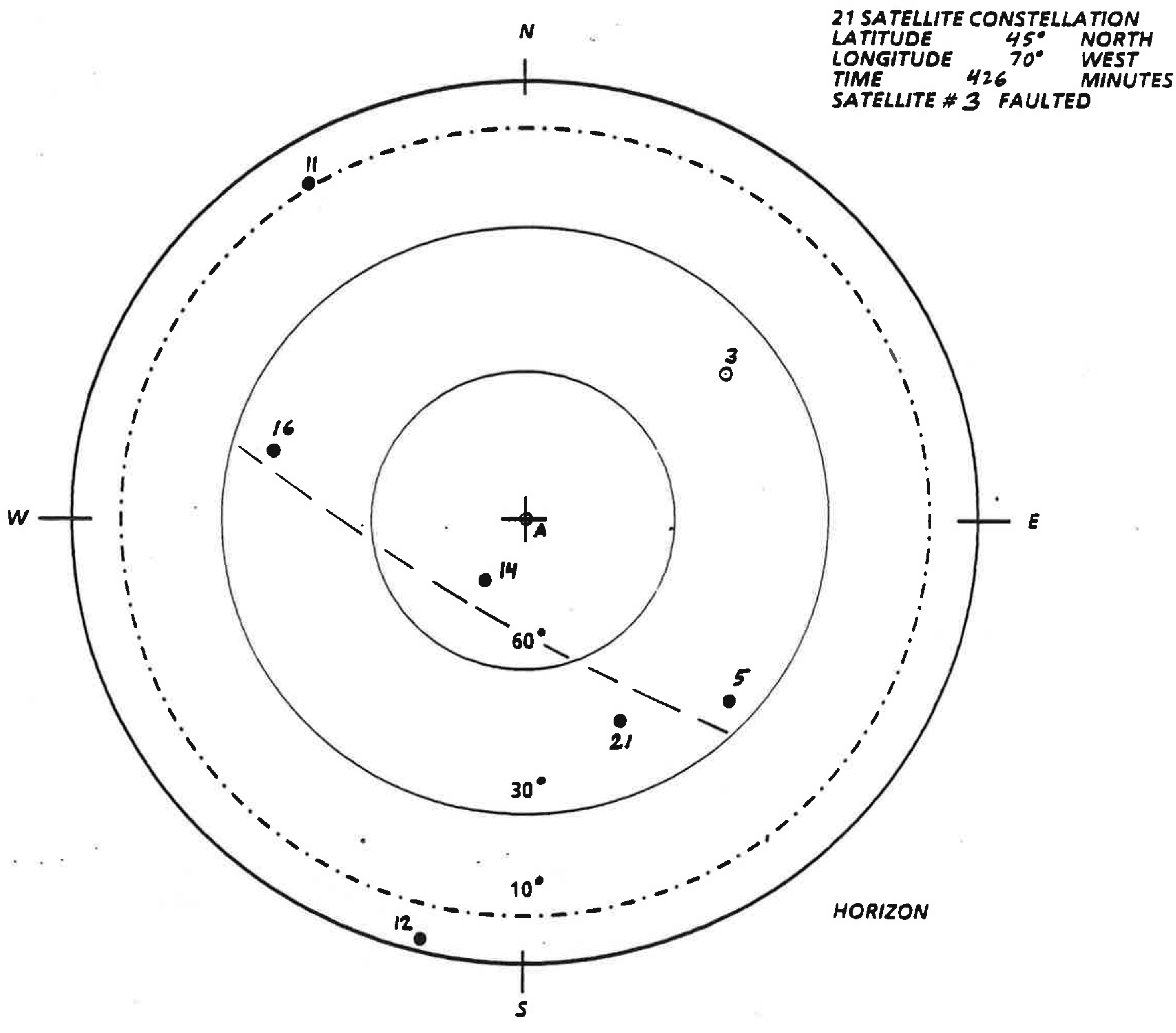


FIGURE C-4. SATELLITE GEOMETRY DURING OUTAGE ASSOCIATED WITH A FAILURE OF SATELLITE #3

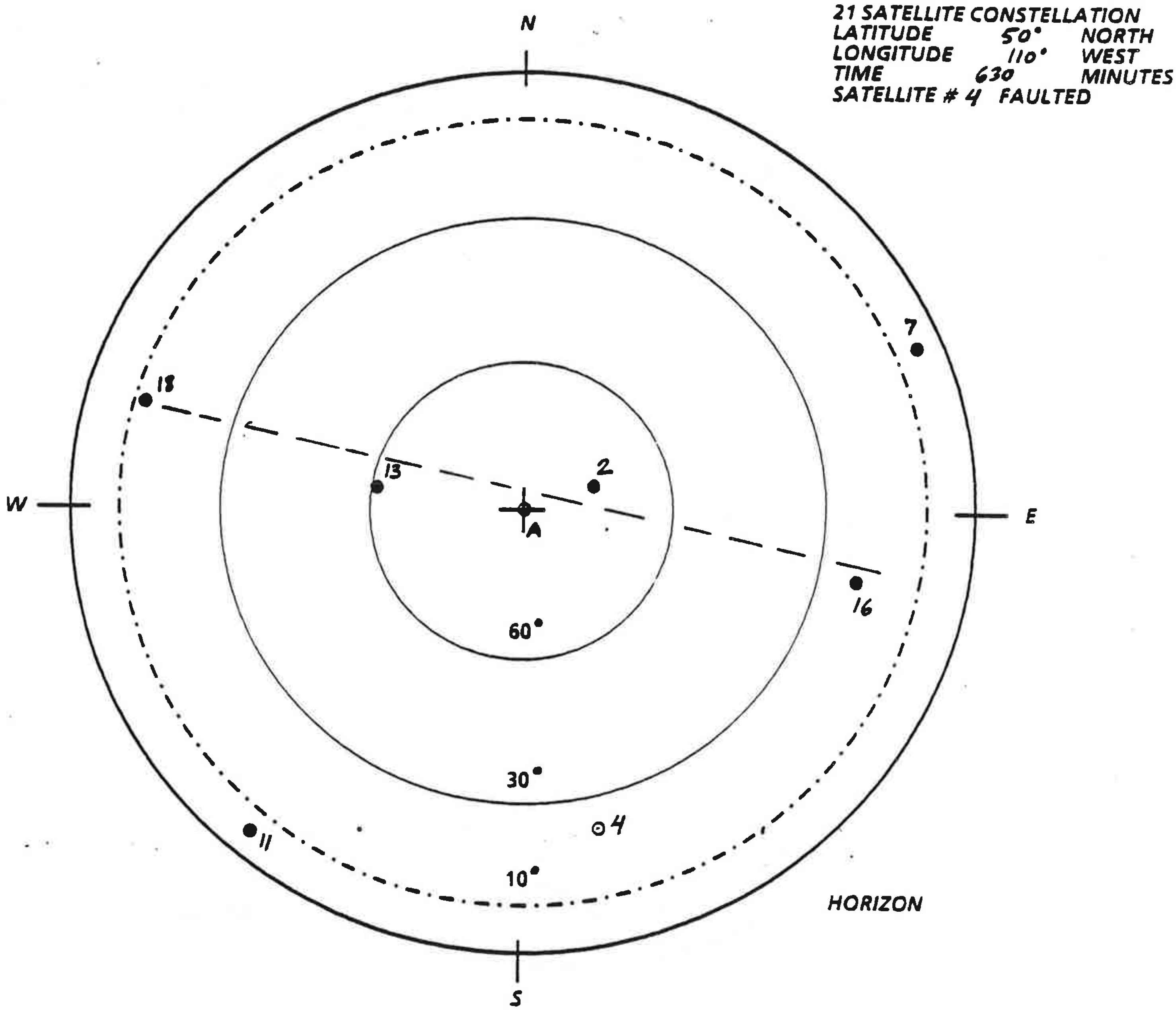


FIGURE C-5. SATELLITE GEOMETRY DURING OUTAGE ASSOCIATED WITH A FAILURE OF SATELLITE #4

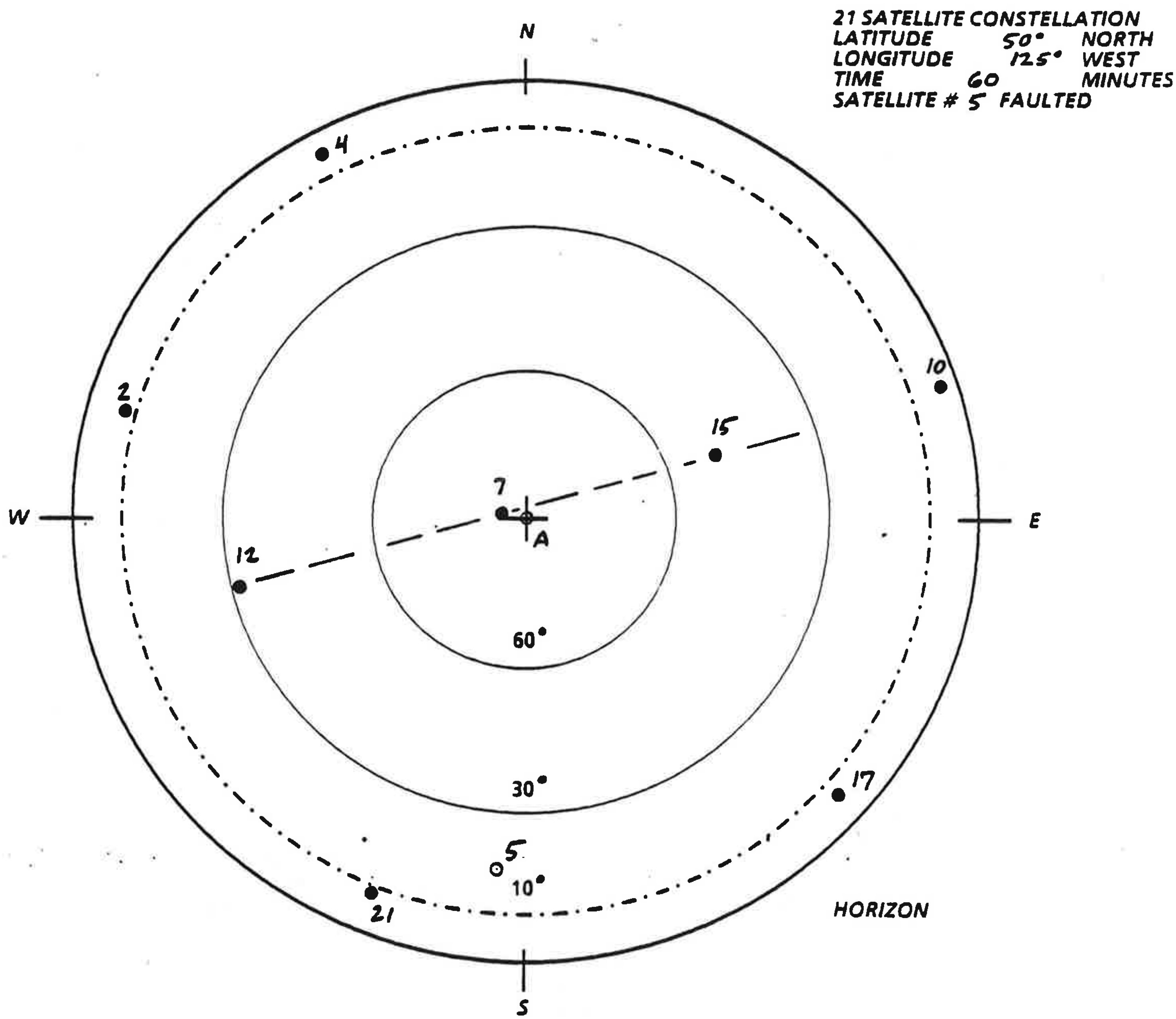


FIGURE C-6. SATELLITE GEOMETRY DURING OUTAGE ASSOCIATED WITH A FAILURE OF SATELLITE #5

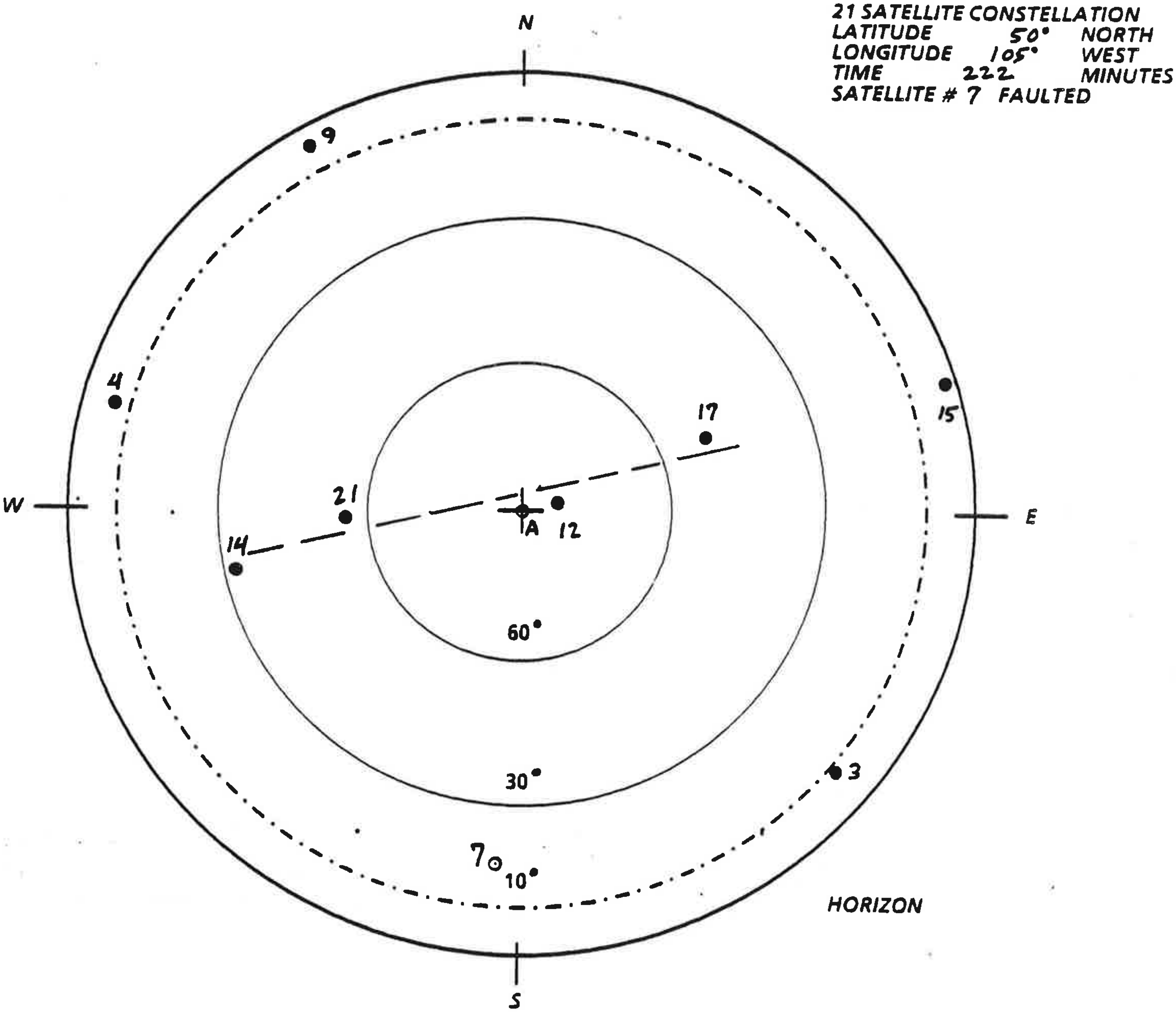


FIGURE C-7. SATELLITE GEOMETRY DURING OUTAGE ASSOCIATED WITH A FAILURE OF SATELLITE #7

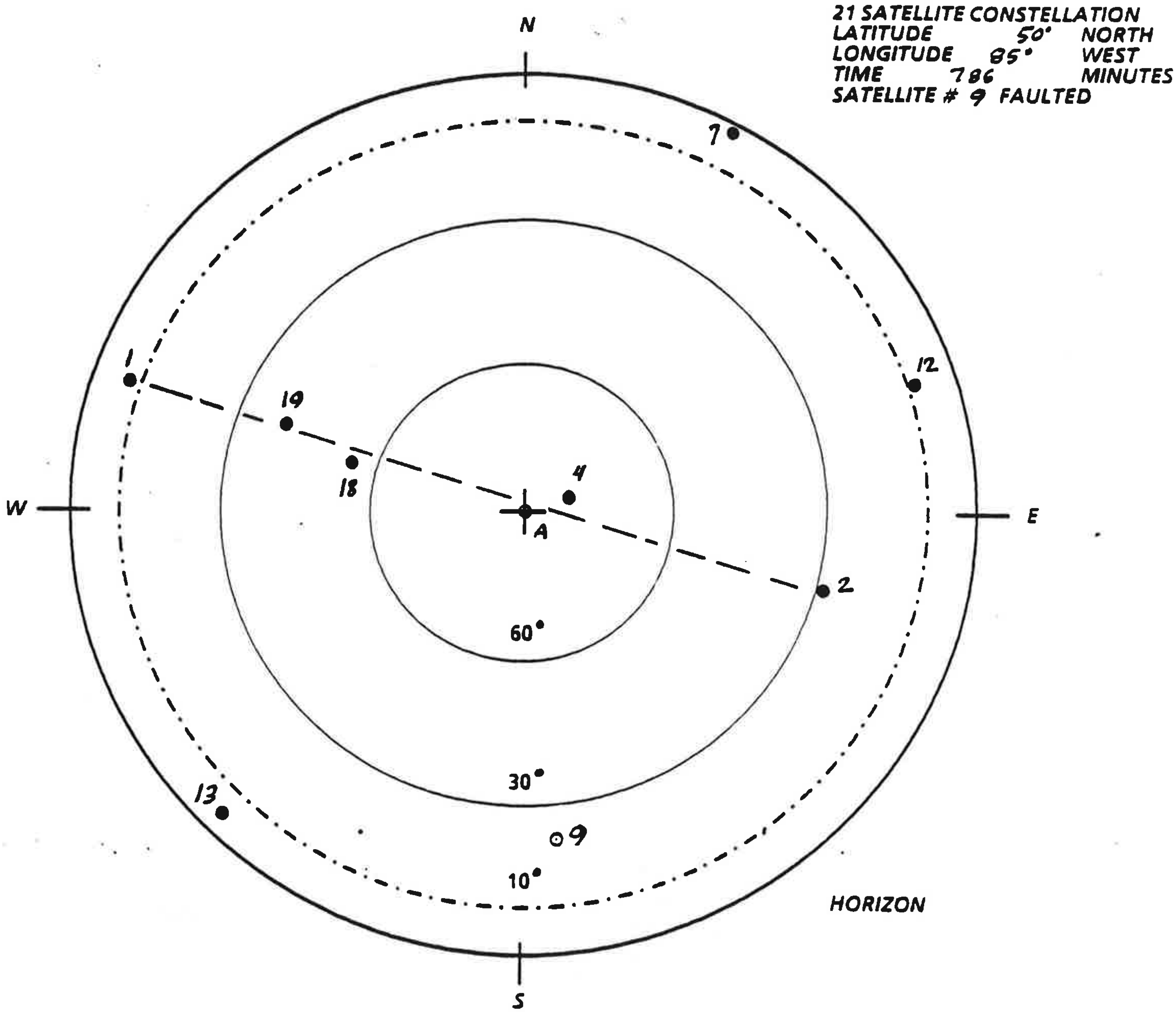


FIGURE C-8. SATELLITE GEOMETRY DURING OUTAGE ASSOCIATED WITH A FAILURE OF SATELLITE #9

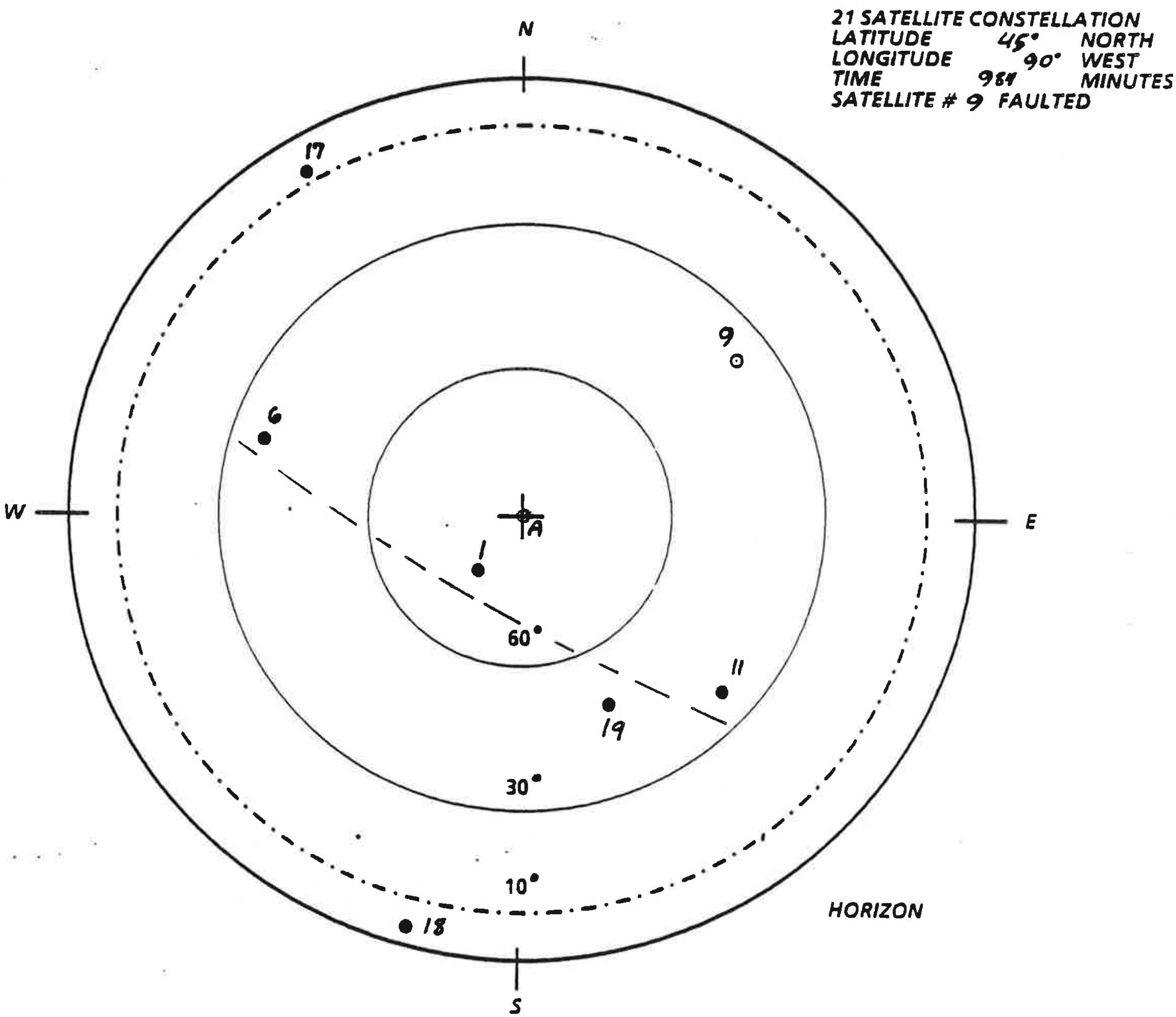


FIGURE C-9. SATELLITE GEOMETRY DURING OUTAGE ASSOCIATED WITH A FAILURE OF SATELLITE #9

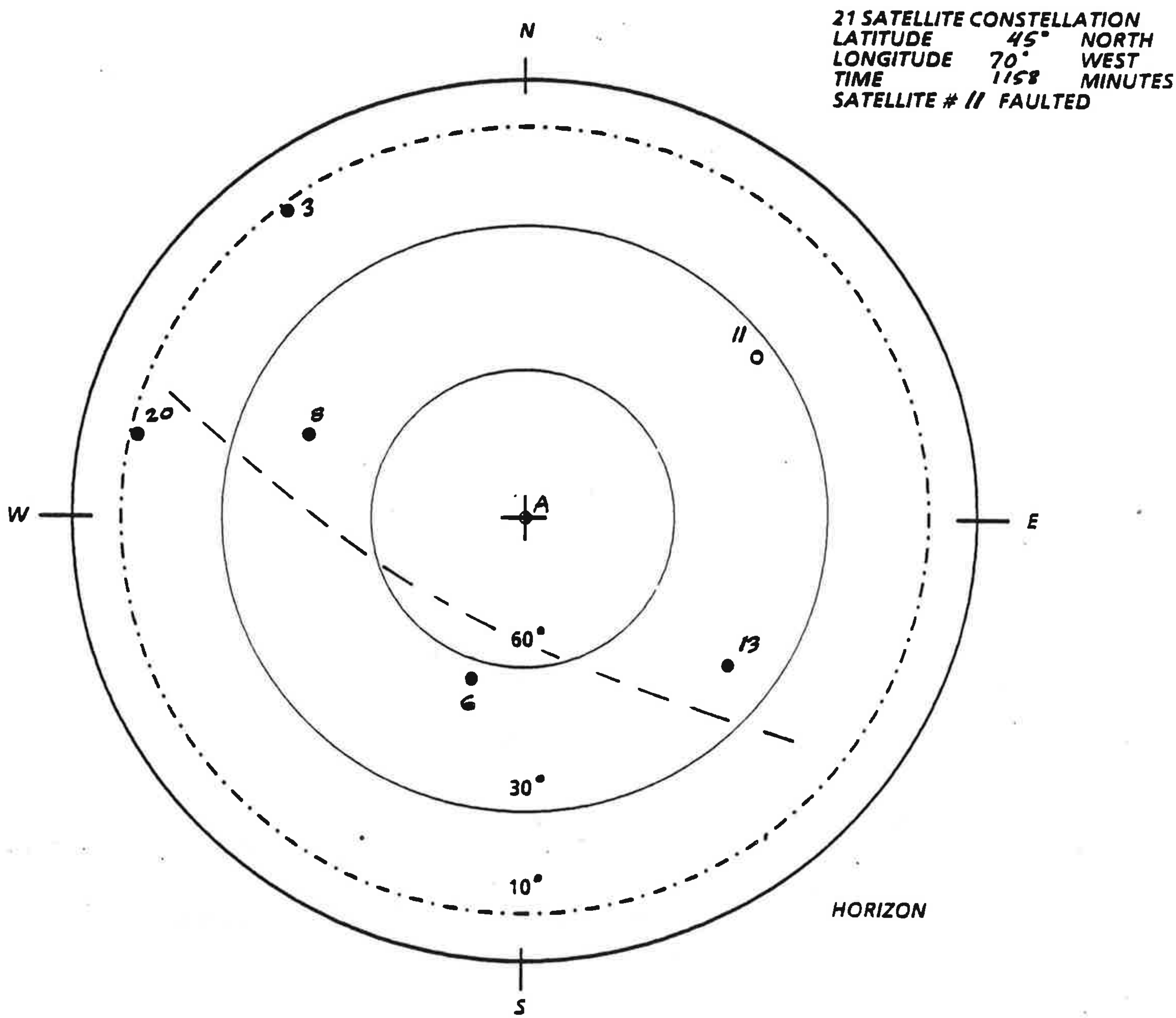


FIGURE C-10. SATELLITE GEOMETRY DURING OUTAGE ASSOCIATED WITH A FAILURE OF SATELLITE #11

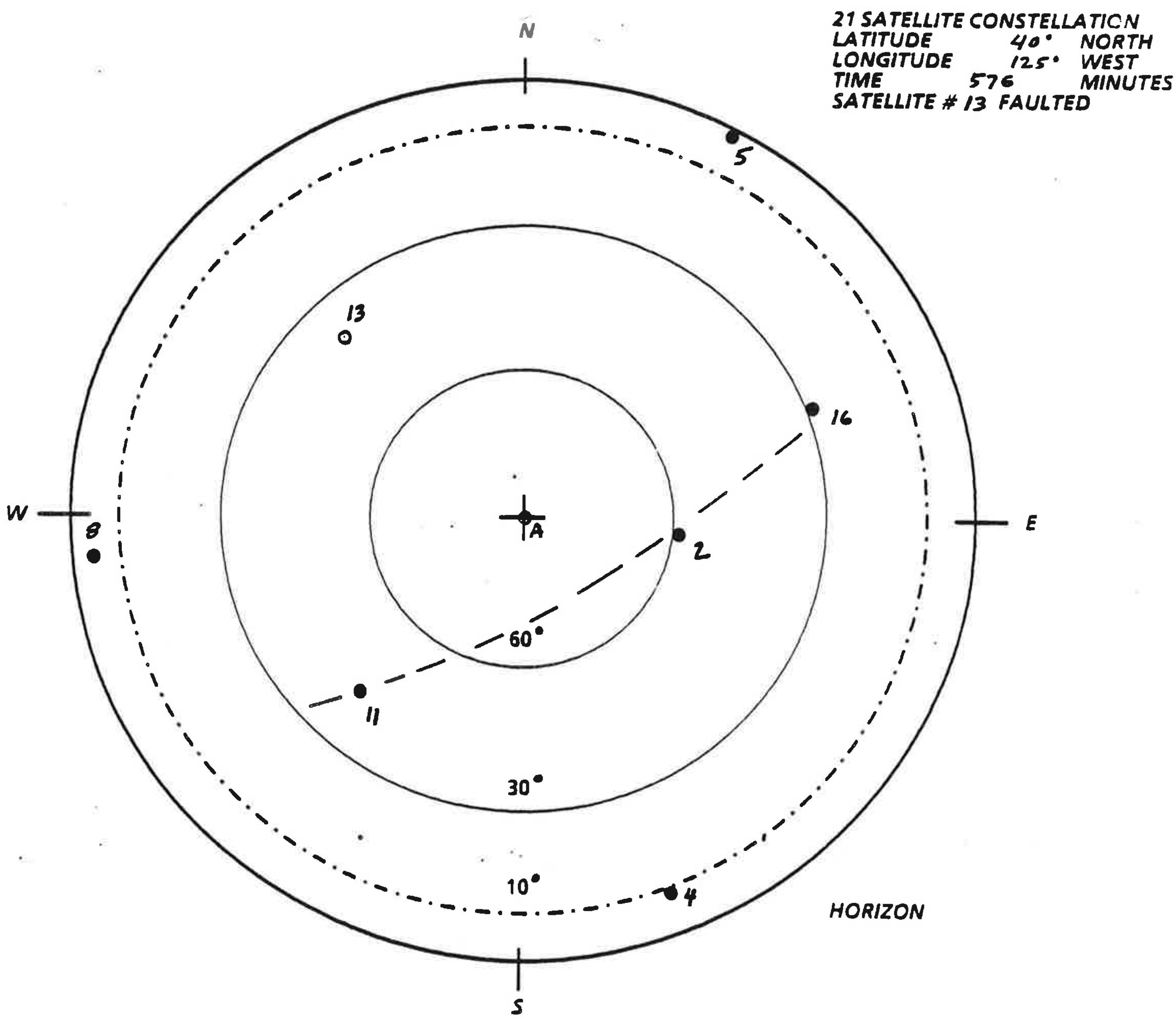


FIGURE C-11. SATELLITE GEOMETRY DURING OUTAGE ASSOCIATED WITH A FAILURE OF SATELLITE #13

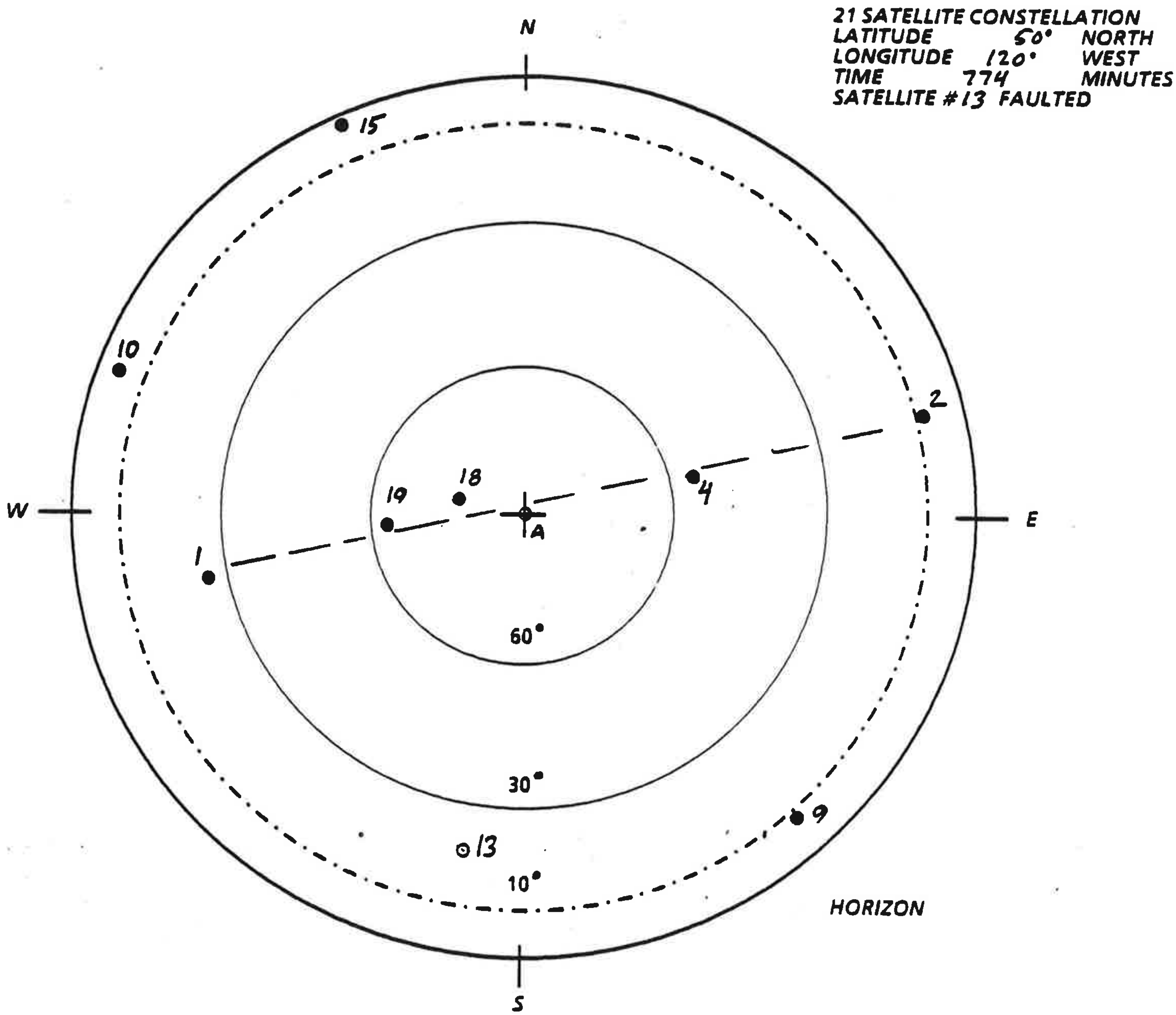


FIGURE C-12. SATELLITE GEOMETRY DURING OUTAGE ASSOCIATED WITH A FAILURE OF SATELLITE #13

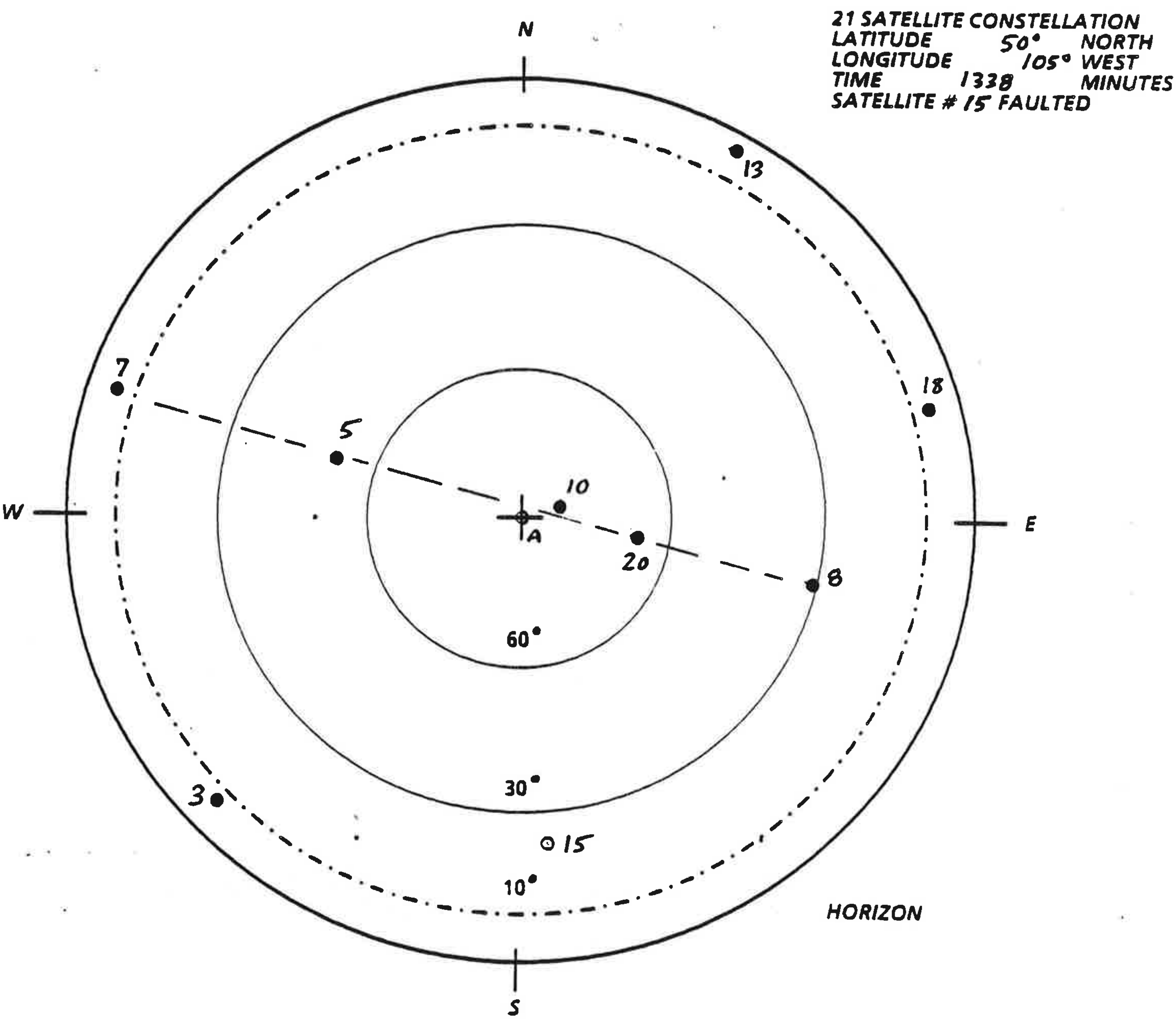


FIGURE C-13. SATELLITE GEOMETRY DURING OUTAGE ASSOCIATED WITH A FAILURE OF SATELLITE #15

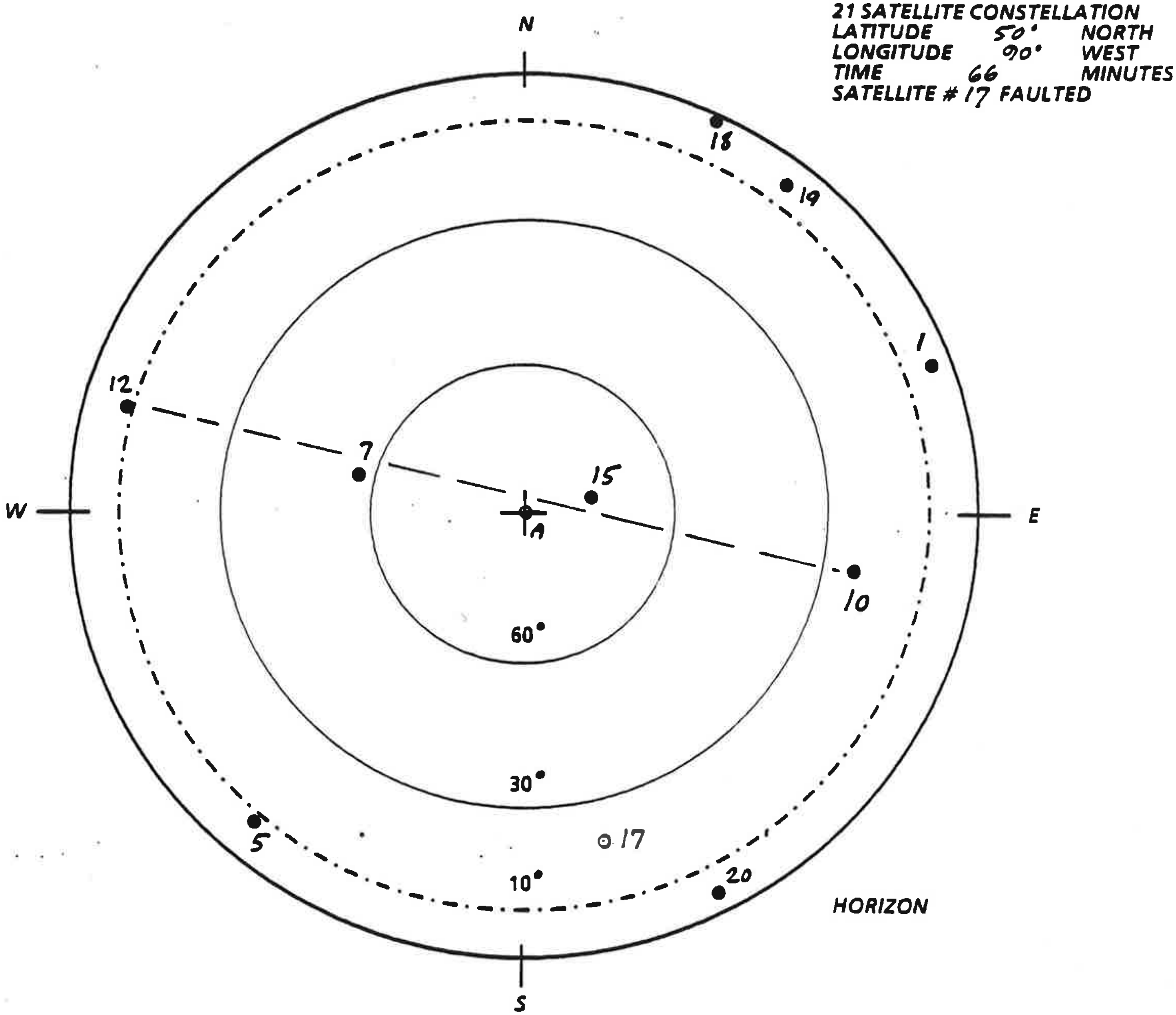


FIGURE C-14. SATELLITE GEOMETRY DURING OUTAGE ASSOCIATED WITH A FAILURE OF SATELLITE #17

APPENDIX D. FORMULATION OF THE EFFECTS OF CLOCK ERRORS

During normal GPS operation, the user navigation processor computes position and time based on 4 or more satellite pseudo-ranges ρ_i, where

$$\rho_i = D_i + \Delta T \tag{D-1}$$

Here

D_i = true range to the i-th satellite
ΔT = user clock offset from GPS time.

This notation is the same as Milliken & Zollner [1978]. Normally pseudorange is a measured quantity, while D_i and ΔT are estimated from the solution.

Position Solution - 4 or More Satellites

The solution for user position and time is indicated by the 4-dimensional vector $\underline{X}$ (i.e., x, y, z, and time), which must be found from the N matrix equations (one for each satellite, so N>3 and $\underline{\rho}$ is N-dimensional):

$$G \cdot \underline{X} = A \cdot \underline{s} - \underline{\rho} \tag{D-2}$$

Here G is an Nx4 matrix made up of the user-to-satellite line-of-sight directional cosines, augmented by the number 1 for time:

$$G = \begin{bmatrix} \hat{e}_1 & 1 \\ \hat{e}_2 & 1 \\ \vdots & \\ \hat{e}_n & 1 \end{bmatrix} \tag{D-3}$$

The vector $A \cdot \underline{s}$ describes the geometry of the user and satellites. However, this term is of no concern here, because it does not figure in the effect of pseudorange errors on the position estimates $\underline{X}$. In the general formulation this term describes the effect of satellite ephemeris errors.

A small change in the pseudorange vector, $d\underline{\rho}$, causes a change $d\underline{X}$ in the position estimates such that:

$$G \cdot d\underline{X} = -d\underline{\rho} \tag{D-4}$$

The matrix G is an Nx4 matrix and therefore not generally square, so there is no inverse matrix. To get a square matrix, both sides are multiplied by the transpose of G , namely G^T :

$$G^T G \cdot d\underline{X} = -G^T \cdot d\underline{\rho} \tag{D-5}$$

Regardless of the number of satellites N, G^TG is a 4x4 square matrix, and usually has an inverse. The solution is then given by:

$$\underline{dX} = - [G^TG]^{-1} G^T \cdot \underline{d\rho} \tag{D-6}$$

That is, this equation relates position and time estimate errors ($\underline{dX}$) to pseudorange errors ($\underline{d\rho}$). Since we are dealing with small changes, the elements of G are essentially constant, i.e., the equations are linear about the current operating point.

If all satellite pseudorange errors have zero means and the same distribution, then:

$$\sigma(\underline{d\rho}) = \sigma(d\rho)\, \underline{1} = \sigma_\rho \underline{1} \tag{D-7}$$

where $d\rho$ has a zero mean and a measurable variance σ_ρ^2. The resulting estimate of position has components with variances proportional to σ_ρ^2.

Covariance of Position/Time - 4 or More Satellites

The variances associated with a vector are elements of a matrix called the "covariance" matrix. For example, the covariance of the pseudorange error vector $\underline{d\rho}$ is given by

$$\mathrm{cov}(\underline{d\rho}) = E\{\underline{d\rho}\ \underline{d\rho}^T\} = \sigma_\rho^2 I \quad , \text{where } I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

where $d\rho$ has a zero mean and a measurable variance σ_ρ^2. The simplification takes advantage of the fact that the pseudorange errors are uncorrelated, and thus the cross-terms are zero.

Up to now the coordinate system of the position-time vector $\underline{X}$ has not been specified. However, if x is taken to be the east direction, y the north direction, z the up direction, and t to be time, the covariance matrix is composed of the following expected values:

$$\mathrm{cov}(\underline{dX}) = E\{\underline{dX}\ \underline{dX}^T\} = \begin{bmatrix} E\{dx^2\} & E\{dx\,dy\} & E\{dx\,dz\} & E\{dx\,dt\} \\ E\{dy\,dx\} & E\{dy^2\} & E\{dy\,dz\} & E\{dy\,dt\} \\ E\{dz\,dx\} & E\{dz\,dy\} & E\{dz^2\} & E\{dz\,dt\} \\ E\{dt\,dx\} & E\{dt\,dy\} & E\{dt\,dz\} & E\{dt^2\} \end{bmatrix} \tag{D-8}$$

This matrix is obviously symmetric. The diagonal terms are the variances of the position/time estimates σ_x^2, σ_y^2, σ_z^2 and σ_t^2. The off-diagonal terms are cross-correlation terms.

The conventional use of this matrix is to define HDOP, VDOP, PDOP, etc. HDOP, for example, is defined as:

$$HDOP \triangleq \sqrt{\frac{\sigma_x^2 + \sigma_y^2}{\sigma_\rho^2}} = \frac{1}{\sigma_\rho}\sqrt{E\{dx^2\} + E\{dy^2\}} \tag{D-9}$$

HDOP is used as an estimate of the ratio of horizontal position error to pseudorange measurement error.

To get the covariance matrix, equation (D-6) must be solved:

$$\begin{aligned} cov(\underline{dX}) &= E\{\underline{dx}\ \underline{dx}^T\} \\ &= [G^TG]^{-1} G^T\ cov(\underline{d\rho})\ G\,[G^TG]^{-T} \\ &= \sigma_\rho^2\ [G^TG]^{-1}\ [G^T \cdot I \cdot G]\ [G^TG]^{-T} \\ &= \sigma_\rho^2\ [G^TG]^{-1} \end{aligned} \tag{D-10}$$

HDOP is then the square root of the sum of the x-x and y-y diagonal elements of $[G^TG]^{-1}$.

Algorithm for "Coasting" with the Clock

If there is a service outage caused by either a coplanar geometry of 4 or more satellites or by satellite failures that result in only 3 visible satellites, the navigation processor should be designed to switch into a "coast" mode. In the coast mode time is not computed, but rather the user clock reading is used along with each satellite pseudorange measurement to estimate satellite range. Then a rho-rho-rho solution is employed.

Formally it looks very similar. Equation (D-6) applies, except that the solution vector $\underline{X}$ is 3-dimensional rather than 4-dimensional. It has no time component, and G is an Nx3 matrix:

$$G = \begin{bmatrix} \hat{e}_1 \\ \hat{e}_2 \\ \vdots \\ \hat{e}_n \end{bmatrix} \tag{D-11}$$

Equation (D-6) becomes a 3-dimensional equation:

$$\begin{aligned} \underline{dX} &= [G^TG]^{-1} G^T \cdot (\underline{dD} + c\Delta T\ \underline{1}) \\ &\triangleq \underline{dX}_{geom} + \underline{dX}_{clock} \end{aligned} \tag{D-12}$$

where $\underline{1}$ is an N-dimensional vector:

$$\underline{1} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$$

The errors in position, given by $\underline{dX}$, thus depend on the geometry of the satellites, the range errors $\underline{dD}$ (the same as the pseudorange errors in D-4), and the time error ΔT.

Satellite Geometry Errors

The errors in satellite range measurements are given by $\underline{dD}$, and are the same as the pseudorange errors for the 4-dimensional solution. The covariance of the errors due to satellite geometry and range errors are given by the first term in (D-12):

$$cov(\underline{dX}_{geom}) = \sigma_{\rho}^2 \left[G^T G\right]^{-1} \tag{D-13}$$

which is similar in form to (D-10).

Clock-Induced Errors

From (D-12) the time delay error ΔT, caused by frequency drift of the user clock, causes positional errors given by:

$$\underline{dX}_{Clock} = -c\,\Delta T \left[G^T G\right]^{-1} G^T \cdot \underline{1} \quad , \; c = \text{speed of light} \tag{D-14}$$

In a specific outage, one would expect the clock error to grow in time. A constant frequency offset, for example, would cause a linearly growing clock error ΔT, while a constant frequency drift would cause a clock error which grows as $(\Delta T)^2$. Either behavior is quite different from the pseudorange errors caused by rapidly varying thermal noise errors and uncorrelated atmospheric delays.

Equation (D-14) gives the positional error which would result from a given time error ΔT. That is, one could postulate a time behavior of ΔT as a function of time and compute the resulting error.

If a statistical answer is desired, the covariance must be computed. The covariance formulation answers the question, "Over a period of time T, what is the relation between the variance of the time errors and the resulting positional error variances?" The formulation is different from that given in (D-10), but follows the same steps:

$$\begin{aligned} cov(\underline{dX}_{clock}) &= E\{\underline{dX}_{clock}\,\underline{dX}^T_{clock}\} \\ &= c^2 \left[G^T G\right]^{-1} G^T cov(\Delta T\, \underline{1})\, G \left[G^T G\right]^{-T} \\ &= c^2 \sigma_T^2 \left[G^T G\right]^{-1} G^T \left[\underline{1}\,\underline{1}^T\right] G \left[G^T G\right]^{-T} \end{aligned} \tag{D-15}$$

where σ_T^2 is the variance in time error for a period T. The matrix $[\underline{1}\,\underline{1}^T]$ is not the identity matrix; it has all 1's for components.

This complex expression is not as formidable as it first appears. To see this, let F_{ij} be the elements of the covariance matrix $[G^T G]$, and let G_{ij} indicate the elements of the matrix $[G]$. Also let the elements of the covariance of clock-induced errors be indicated by C_{in}. Then

$$C_{in} \triangleq \left[\mathrm{cov}\left(\underline{dX}_{clock}\right)\right]_{in} = c^2\sigma_T^2 \sum_{j}^{3} \sum_{k}^{N} \sum_{l}^{N} \sum_{m}^{3} F_{ij}\, G^T_{jk}\, 1_k\, 1_l\, G_{lm}\, F^T_{mn}$$

or

$$C_{in} = c^2\sigma_T^2 \left(\sum_{j}^{3} F_{ij} \sum_{k}^{N} G_{kj} \right) \left(\sum_{m}^{3} F_{nm} \sum_{l}^{N} G_{lm} \right)$$

The diagonal elements are just:

$$C_{ii} = c^2\sigma_T^2 \left(\sum_{j}^{3} F_{ij} \sum_{k}^{N} G_{kj} \right)^2 \qquad \text{(D-16)}$$

Since G is made up of the direction cosines, $\sum_{k}^{N} G_{k1}$ is just the sum of the x-components of the user-to-satellite line-of-sight vector directional cosines. Similarly $\sum G_{k2}$ is the sum of the corresponding y-components. $\sigma_\rho^2 F_{ij}$ is the covariance matrix of the pseudorange errors (see D-13).

The ratio of the standard deviation of the horizontal position error (σ_h) to the standard deviation of the clock error ($c\sigma_T$) is given by:

$$\frac{\sigma_h}{c\sigma_T} = \sqrt{\frac{C_{11} + C_{22}}{c^2\sigma_T^2}} = \sqrt{\left(\sum_{j}^{3} F_{1j} \sum_{k}^{N} G_{kj} \right)^2 + \left(\sum_{j}^{3} F_{2j} \sum_{k}^{N} G_{kj} \right)^2} \qquad \text{(D-17)}$$

This term depends only on satellite geometry. We define it to be the "time-position error multiplier", or TPEM. It indicates how a clock error translates into a position error. The TPEM was evaluated for the outage conditions for the 18-satellite constellation. Typical values of HDOP obtained from equation (D-13) and TPEM obtained from (D-17) are tabulated in Table D-1. Figure D-1 shows the distribution of TPEM during the times only three satellites are visible. Note that there are a few situations where the TPEM is greater than 3; these correspond to geometries described in Appendix C.

TABLE D-1. TYPICAL VALUES OF HDOP AND TIME-POSITION ERROR MULTIPLIER

GPS TIME	LATITUDE	LONGITUDE	HDOP	TPEM
0 min.	50N	100W	5.26	1.23
138 min.	35N	100W	3.32	.71
"	"	"	1.80	.67
"	"	"	1.77	.63
"	"	"	3.04	.61
258 min.	45N	125W	2.79	.85
"	"	"	1.64	.83
"	"	"	1.66	.84
300 min.	40N	75W	1.82	.52
"	"	"	3.10	.49

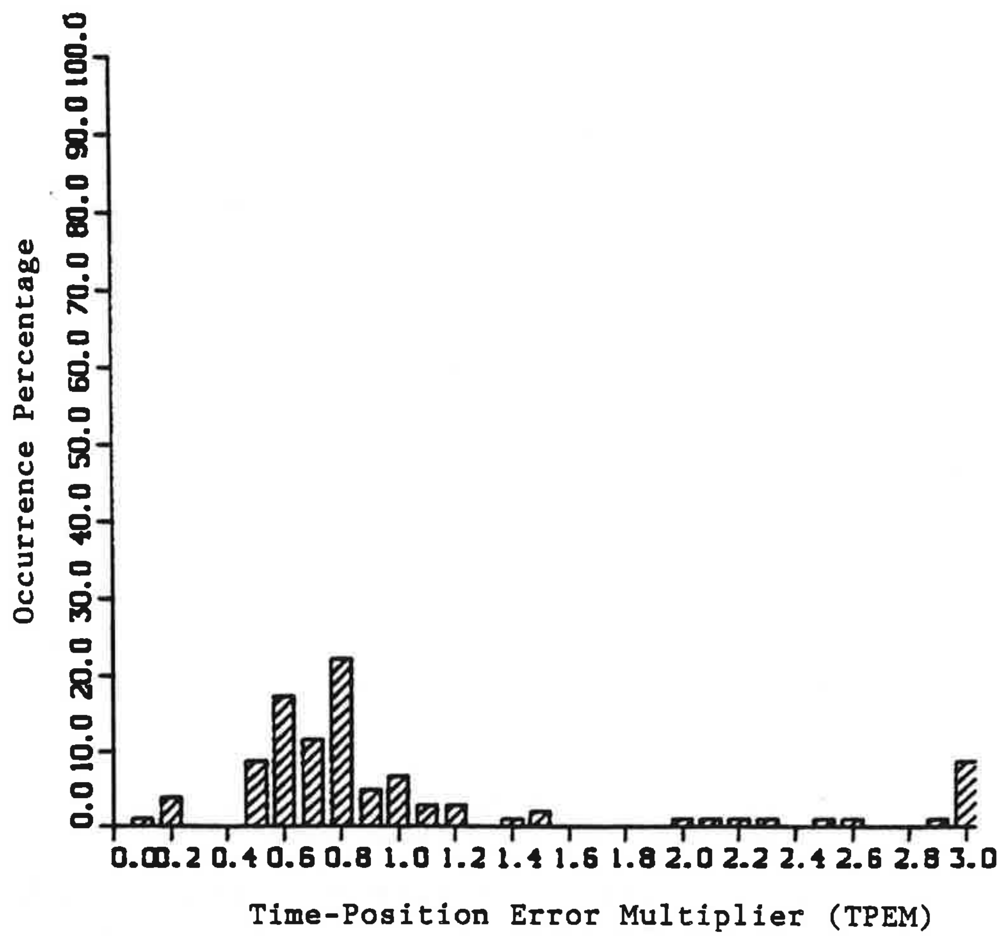


FIGURE D-1. TPEM VALUES OBSERVED IN ADVERSE SATELLITE GEOMETRIES

Clock Errors - Sources

The time errors themselves are caused by the response of the clock crystal to temperature, pressure, and vibration variations, as well as acceleration and crystal aging. For an unprotected crystal, thermal effects predominate over periods of less than an hour. To reduce the thermal effects, temperature compensation techniques and temperature control techniques are often used to improve stability. These add cost to the unit; temperature control adds weight to the unit as well. Acceleration effects can be reduced by proper orientation of the crystal in the cockpit.

The effect of a constant clock oscillator frequency offset Δf on the time error ΔT is given by:

$$\Delta T = \frac{\Delta f}{f_0}(t - t_0) \qquad \text{(D-18)}$$

where f_0 is the base frequency of the oscillator, and t_0 is the last time at which the clock was updated.

If Δf varies with time, then a more accurate expression is the following:

$$\Delta T = \frac{1}{f_0}\int_{t_0}^{t} \Delta f(t)\, dt \qquad \text{(D-19)}$$

It is apparent from this that knowledge of the behavior of the frequency of the clock during the period under consideration is crucial to assessing the effects of clock-induced errors and establishing clock stability requirements.

Modelling the Clock Drift

Figure D-2 shows the Allan Variance plot of a representative high quality crystal clock and a rubidium standard [Sturza, 1983]. The Allan Variance is a statistical measure of clock stability, derived by taking sequential measurements of a clock at different time intervals T, where T is the abscissa of the graph. It provides information on the percentage drift of clock frequency during a given period T. It does not provide information on drift rate, nor is the behavior of the clock frequency during the interval directly provided. These plots do not usually provide information on the impact of other than thermal variations.

Figure D-3 shows some theoretically possible frequency error profiles, given that after some period T that the frequency has drifted to some value Δf . It is not likely that the frequency error would rapidly oscillate during this period, nor is it likely that it would swing to values much larger than the final value. One possible behavior is shown in curve A. A constant frequency drift rate, which is possible, is given as curve B. Following the shape of the Allan Variance gives curve C. Intuitively, this would appear to be the one most likely to represent a large number of cases, since more data available suggests it. Curve D shows what is considered to be a kind of "worst case" among those likely to be observed; in this behavior, the frequency quickly changes to the full value Δf and remains at that value for the rest of the period.

If an acceleration took place during the period the frequency would shift somewhat until the acceleration ended, after which it would return to some other value. This is shown in Figure D-4. Such a variation might occur during a turn. The dotted line shows another possible frequency behavior which combines drift and acceleration.

From this discussion it appears that the most likely behavior would be best represented by the Allan Variance curve; other behaviors would probably be bracketed by the constant drift curve B and the "worst case" curve D. From (D-19) the resulting time delay error ΔT can be related to the total frequency change during the period by using a shape factor α, given by:

$$\Delta T = \alpha T \frac{\Delta f(T)}{f_0}, \qquad T \triangleq t - t_0 \tag{D-20}$$

where the coefficient α takes on the following values for the different curves of Figure D-3:

A: $\alpha \approx 0.4$
B: $\alpha = 0.5$
C: $\alpha \approx 0.6$
D: $\alpha = 1.0$

The 2drms error is given approximately by:

$$\varepsilon_h = 2 \cdot \frac{\sigma_h}{c\sigma_T} \cdot c\alpha T \frac{\Delta f(T)}{f_0} \tag{D-21}$$

The combined effects of geometry and clock-induced errors are derived by combining equations D-13 and D-18. For a frequency behavior like that of curve C in Figure D-3, this gives:

$$\varepsilon_h = 2\sqrt{(F_{11}+F_{22})\sigma_\xi^2 + \left(TPEM \cdot \alpha \cdot \frac{\Delta f(T)}{f_0}\right)^2 c^2 T^2} \tag{D-22}$$

These expressions are used to develop the plots shown in Figures 5-1 to 5-5.

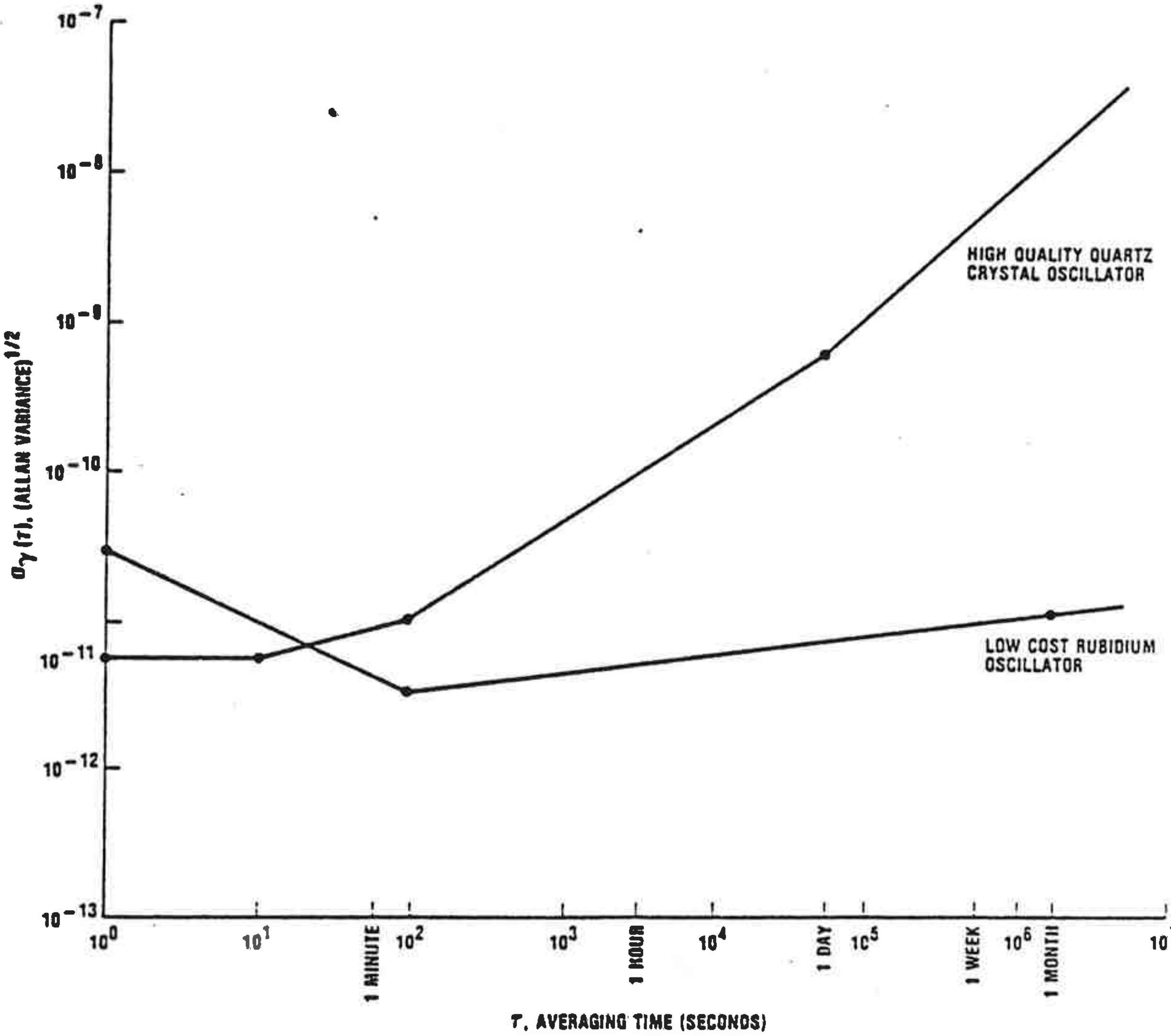


FIGURE D-2. ALLAN VARIANCE OF HIGH QUALITY CLOCKS

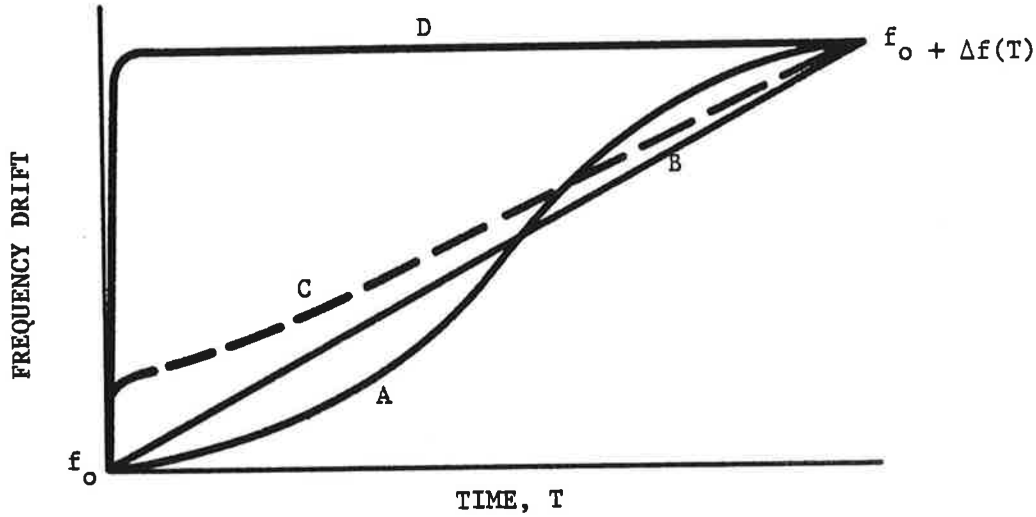


FIGURE D-3. POSSIBLE FREQUENCY DRIFT PROFILES

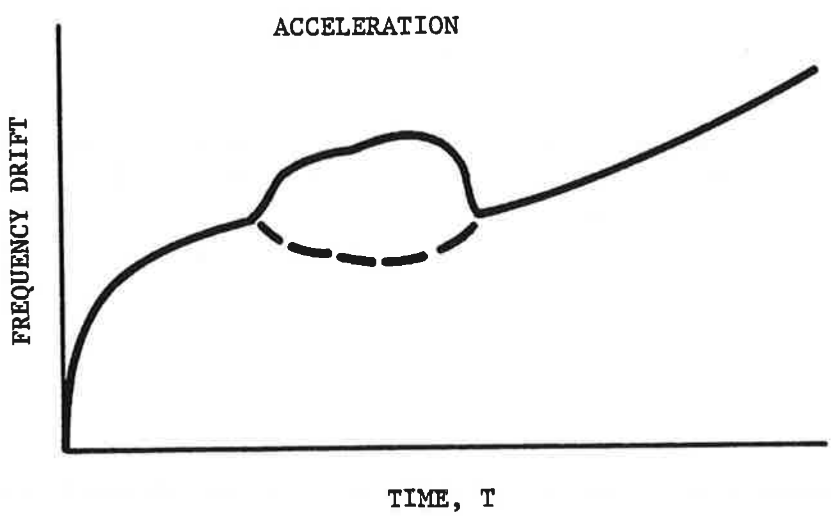


FIGURE D-4. FREQUENCY DRIFT PROFILE DURING ACCELERATION

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