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USERS' MANUAL FOR KALKER'S SIMPLIFIED NONLINEAR CREEP THEORY

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16. Abstract The conversion of the computer program, "Simplified Theory of Rolling Contact," (used for calculation of a nonlinear creep force-creepage relationship) from the original Algol language to Fortran is considered. The Algol program was written by Professor J. J. Kalker and was derived from the paper, "Simplified Theory of Rolling Contact," Delft Progr. Rep., Series C: Mechanical and Aeronautical Engineering and Shipbuilding, 1 (1973), pp. 1-10. A significant number of changes was made in the program for more convenient use; however, the fundamental equations remain unchanged. The results were checked in detail to insure agreement with the original solution. The program gives an appropriate solution for the resultant tangential creep forces and spin moment acting between two bodies of equal linearly elastic material properties. The creep forces and spin moment are due to lateral, longitudinal, and spin creepages. Assumptions corresponding to the Hertz contact theory are implied and two additional simplifying assumptions are made, resulting in a significant reduction in computation time as contrasted with previous solutions. Two separate computer codes were developed, the first being the general solution with extended input and output, and the second a shortened version primarily intended for use as a subroutine. Surprisingly good agreement is found to exist between the "Simplified Theory" and published experimental results for a wide range of contact ellipse eccentricity.			
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Last, but certainly not least, we would like to thank Professor J. J. Kalker of Delft who very graciously sent us copies of his papers and computer programs. We have tried, in this Users' Manual, to make the results of his work more widely available to the rail vehicle dynamics community. Although we have checked carefully the Fortran version of Professor Kalker's program, any errors in the conversion are ours and not Professor Kalker's.

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I. INTRODUCTION

Background

The forces and moments due to shear stresses in the contact area between wheel and rail play a major role in rail vehicle dynamics. These shear stresses arise, in part, due to relative linear and angular motions (lateral, longitudinal, and spin creepage) between the wheel and rail. Hobbs [1] presents a review of the analytical and experimental work concerned with the creep force/creepage phenomenon.

For many problems in rail vehicle dynamics a linear creep force/creepage relationship has been used. Typical of these are eigenvalue/eigenvector analyses of lateral stability, lateral forced response studies, and estimation of slip and flange contact boundaries for steady state curving. It is widely recognized that the best available linear creep law is that due to Kalker [2] and called the "linearized theory" (see equations (12) and (13) of [2]). Recently, however, more and more questions are being asked of rail vehicle dynamicists that require more sophisticated models of the wheel/rail interaction process. Factors that should be considered in these models are: (1) the nonlinear wheel/rail geometric constraint functions arising from curved or worn wheel and rail profiles; and, (2) the effects of adhesion limits on the creep force/creepage relationship (i.e. a nonlinear creep law).

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- [1] A.E.W. Hobbs, "A Survey of Creep", DYN/52, April 1967, British Railways Research Dept., Derby, England.
 - [2] J.J. Kalker, "Simplified Theory of Rolling Contact," Delft. Progr. Rep., Series C: Mechanical and Aeronautical Engineering and Shipbuilding, 1 (1973), pp. 1-10.

Attempts have been made to formulate a nonlinear creep law. Johnson's theory [3,4] has been confirmed by laboratory experiments but does not account for spin creepage*. Unfortunately, the effects of spin creepage are expected to predominate for contact areas in the wheel flange region - precisely the situation where a nonlinear creep law is needed. The Levi-Chartet creep law [5,6] used by some researchers is empirically based and does not account for spin creepage.

Professor Kalker of Delft University has formulated two nonlinear creep laws that incorporate the effects of spin creepage and that have been found to compare well with results of laboratory experiments. These two creep laws are generally referred to as the "Simplified theory of rolling contact" [2] and the "exact solution for rolling contact" [7]. The differences in the solutions presented in [2] and [7] lie in two simplifying assumptions made in [2] concerning the tangential displacement-stress relations and the normal stress distribution on the contact surface. These assumptions shorten the computation time required by a factor of approximately 100.

* Spin creepage is the nondimensional relative angular velocity between wheel and rail in the contact zone.

- [3] K.L. Johnson, "Adhesion", Proc. Inst. Mech. Engrs., Vol. 178, part 3E (1964), pp. 208, 209.
- [4] P.J. Vermeulen and K.L. Johnson, "Contact of Nonspherical Elastic Bodies Transmitting Tangential Forces," J. Appl. Mechanics, Vol. 31 (1964), pp. 338-340.
- [5] R. Levi, "Le roulement avec glissement", Compt. rend. Acad. Science 199, 1934, pp. 119-120.
- [6] A. Chartet, "Proprietes generales des contacts de roulement. Theorie des similitudes." Compt. rend. Acad. Science 225, 1947, pp. 986-988.
- [7] J.J. Kalker, "On the Rolling Contact Between Two Elastic Bodies in the Presence of Dry Friction," Ph.D. Thesis, Delft University of Technology (1967).

A portion of the work being conducted under contract DOT-OS-40018, Freight Car Dynamics, deals with developing models for the lateral dynamic response of North American freight cars during curve entry and negotiation. These models will be used to predict vehicle response and wheel/rail forces during hard curving where severe flange contact is anticipated. Consequently, it is expected that creep forces may approach the limits of adhesion and a nonlinear creep law will be required for accurate modeling.

The object of the work reported in this Users' Manual was to convert the Algol program developed by Professor Kalker for the "Simplified theory of rolling contact" to Fortran and to check the resulting program by direct comparison with the results calculated by the original Algol program and with available experimental results. It is anticipated that a Fortran version of this computer program will prove quite valuable to rail vehicle dynamics researchers in the United States where most scientific programs are written in Fortran.

To further aid in the use of Kalker's theory by rail vehicle dynamics researchers, a subroutine called FØRCES was developed based on the "Simplified theory of rolling contact". This subroutine can be included within Fortran programs that are used to obtain the lateral dynamic response of rail vehicles. This subroutine together with the program for the complete solution of the "Simplified theory of rolling contact" are discussed in this manual.

Summary of Users' Manual

It is intended that Kalker's original paper [2] be read concurrently with this manual. References to equations in [2] are made directly by equation number both in the present text and in the computer code.

The problem analysed in [2] and considered in the computer code is for steady rolling contact of two elastic bodies of equal linearly elastic material properties and having both longitudinal and lateral creepage and spin about an axis normal to the contact surface. The appropriate geometry is given in Figure 1 of [2].

Approximate solutions to three special problems of steady state rolling contact are presented in [2]. The first case is that of infinitesimal slip in which the area of slip is vanishingly small and the resultant tangential creep forces and torsional moment are linearly related to the creepage and spin parameters. This is Kalker's "Linearized theory" widely used by rail vehicle dynamics researchers. It is presented on page 4, equations (11) to (13) of [2]. The second solution, "Steady-state rolling with pure creepage", is presented on page 5, equations (15) to (21) and considers finite slip with a resulting nonlinear relationship between the resultant creep force and the creepage. The present computer code is for the third solution, "Combined creepage and spin: a numerical method", in which finite slip is assumed and the resultant creep forces and moment are nonlinearly related to the creepage and spin parameters.

The problem may be stated as follows. Given two bodies of equal elastic properties and known dimensions, normal force, rolling velocity, creepage and spin, determine the resultant creep forces tangent to the contact surface and the resultant moment about a normal to the contact surface. The region of slip within the contact surface is also determined. In the actual solution, the static Hertzian contact problem is

first solved (see [7] page 55, or [8] page (414) to determine the dimensions of the contact ellipse, a and b . The resultant creep forces and moment, F_x , F_y , and M_z are then determined knowing the parameters a , b , N , G , ν , μ , ν_x , ν_y , and ϕ where:

F_x = longitudinal creep force (in the direction of rolling)

F_y = lateral creep force

M_z = spin creep moment about normal to contact surface

a = semi-axis of contact ellipse in longitudinal direction

b = semi-axis of contact ellipse in lateral direction

N = resultant normal load on the contact region

G = shear modulus

ν = Poisson's ratio

ν_x , ν_y = longitudinal and lateral creepage

ϕ = spin creepage

This is the same problem considered in [7] and referred to as the "exact" solution. The only differences in the solutions presented in [2] and [7] lie in two simplifying assumptions concerning the tangential displacement - stress relations and the normal stress distribution on the contact surface. These two assumptions considerably reduce the complexities in obtaining a numerical solution and shorten the computation time by approximately a factor of 100.

The first assumption regarding the tangential displacement-stress relation is common to all three solutions developed in [2]. This is:

$$\begin{aligned} u(x,y) &= S_x X = -S_x \tau_{xz} \\ v(x,y) &= S_y Y = -S_y \tau_{yz} \end{aligned} \quad \text{equation (9), [2]}$$

[8] S.P. Timoshenko and J.N. Goodier, Theory of Elasticity, 3rd Ed., McGraw-Hill Book Company (1970).

where $u(x,y)$ and $v(x,y)$ are the tangential displacements in the longitudinal and lateral directions and τ_{xz} and τ_{yz} are the shear stresses. The "exact" relationships for the tangential displacements as given in [7] are

$$u(x,y) = \sum_{m=0}^M \sum_{n=0}^{M-m} a_{mn} x^m y^n$$

equation (2.2), [7]

$$v(x,y) = \sum_{m=0}^M \sum_{n=0}^{M-m} b_{mn} x^m y^n$$

The two elastic constants S_x and S_y of [2] are determined explicitly in terms of the elastic properties G and ν , the contact ellipse dimensions a and b and the creepage and spin coefficients C_{ij} (see equations (13) and (41) - (47) of [2]). It is important to note that S_x and S_y have different values if forces are to be computed than they have when the moment is to be determined.

The method of determination of the constants a_{mn} and b_{mn} in [7] is much more complicated than that used to determine S_x and S_y in [2] and is the significant difference in the solutions.

The simplified theory also may be used to investigate the effects of a very thin elastic layer covering the bodies and having a tangential displacement-stress relation as given by equation (45) of [2].

$$u_\ell = L_x X = -L_x \tau_{xz}, \text{ and}$$

$$v_\ell = L_y Y = -L_y \tau_{yz},$$

where L_x and L_y are the inverse stiffnesses of the layer. The combined effective stiffnesses of the wheel-rail with an elastic layer are given by equations (46) and (47) of [2]. These are, for moments

$$S_x = 8b/(15C_{33}G) + L_x \quad \text{and,}$$

$$S_y = \pi a^{3/2}/(4b^{1/2}C_{23}G) + L_y$$

and in the calculations of forces

$$S_x = 8a/(3C_{11}G) + L_x \quad \text{and}$$

$$S_y = 8a/(3C_{22}G) + L_y.$$

If no layer is present one then takes $L_x = L_y = 0$.

The effect of changes in L_x and L_y on the resulting solution has not been investigated; however, some observations should be noted. First, the layer is assumed to be so thin that its presence does not influence the determination of the contact ellipse dimensions or the pressure distribution. That is, a and b are still computed from the static Hertz solution in terms of G , ν and N . The effect of a finite thickness work-hardened layer could not then be accounted for by including L_x and L_y . Further, it seems to the writers that if the effect of a contaminated rail is desired, it is more directly accounted for by an appropriate change in the coefficient of friction than in a layer as defined by equation (45). The utility of modifying the elastic properties by adding L_x and L_y is not clear to the writers at this time.

The additional simplification made in the combined creepage and spin solution of [2] is that the normal stress distribution over the contact region is assumed to be of the form given by (14.III) rather than the Hertz stress distribution of (14.I). It should be noted that the

Hertzian distribution is used to determine the contact region dimensions a and b . Equation (14.III) is chosen so as to have bounded derivatives at the edge of the contact region and to still be similar to the Hertzian distribution over most of the contact area. The functions $A(y)$ and $B(y)$ in (14.III) are

$$A(y) = 0.5 (1-(y/b)^2)^{-\frac{1}{2}}(1-(0.9)^2)^{-\frac{1}{2}} \text{ and}$$

$$B(y) = -0.5 (1-(y/b)^2)^{\frac{1}{2}}(1-(0.9)^2)^{\frac{1}{2}}.$$

Numerous changes were made in the computer code in order to make the program more convenient to use. The Algol version was, however, fundamentally correct and numerous checks were made to insure that the Fortran and Algol codes gave the same results. The use of the Fortran code is considered in the next sections. The complete solution is discussed first, followed by the subroutine, "SUBROUTINE FØRCES".

II. DESCRIPTION OF COMPUTER CODE FOR COMPLETE SOLUTION

A. PURPOSE

This program and associated subroutines compute the lateral and longitudinal creep forces and the spin creep moment acting between two elastic bodies in steady state rolling contact. The bodies are of equal linearly elastic material properties and have longitudinal and lateral creepage and spin creepage about an axis normal to the contact region. Kalker's theory of simplified rolling contact [2] is the basis of the program.

B. PROGRAM DESCRIPTION

- 1) Usage: The program consists of a main program and three subroutines.

The main program, MAIN, coordinates the input and outputs the results. Subroutine MAAKZ computes the normal stress as given by equation (14.III). Subroutine RØL is the solution portion of the program and determines the region of slip or adhesion within the contact zone. Subroutine CØNST determines the linear creepage and spin coefficients, C_{ij} , and the normalized modulus GS by linear and quadratic interpolation from Kalker's table [7].

- 2) Subroutines Required:

SUBROUTINE MAAKZ (P, Q, WZ, DZ, D2Z, A, B, MUZ) determines the assumed normal stress as given by equation (14.III).

SUBROUTINE RØL (CS, GEL, MUZ, NX, NY, X, Y, VX, VY, G, FX, FY, MZ) determines the region of slip or adhesion and computes the

tangential stresses and relative velocity at points within the contact zone. The resultant creep forces and moment are also computed. Uses subroutine MAAKZ.

SUBROUTINE CØNST (A, B, NU, C11, C22, C23, C33, GS) determines the linear creepage and spin coefficients, C11, C22, C23, and C33 and the normalized modulus, GS, by linear and quadratic interpolation from Kalker's table, [7]. These values are used in MAIN to determine the normalized stiffness SXN and SYN and the spin constant HC.

3) Description of Input Parameters:

- NV1 NV1 is an integer denoting the number of complete problems to be solved. Input.
- A,B $A = a/c$, $B = b/c$, where a and b are the actual contact dimensions determined from the static Hertz solution and $c = \sqrt{ab}$ is the normalized unit of length. a is the longitudinal and b is the lateral semi-axis of the contact ellipse. Input.
- NU $NU = \nu$ = Poisson's ratio. Input.
- LXN, LYN $LXN = L_x \rho N / c^4$, $LYN = L_y \rho N / c^4$. Inverse stiffnesses of an elastic layer covering the bodies. N = resultant normal force and $1/\rho = 1/4 (1/R_1^+ + 1/R_1^- + 1/R_2^+ + 1/R_2^-)$ with R_1^+ , R_1^- , R_2^+ , R_2^- being the principal radii of curvature of the two elastic bodies. See equation (45). For no layer, take $LXN = LYN = 0$. Input.
- NX, NY Lattice points in the normalized contact region with $X = (I)(A)/NX$, $Y = (J)(B)/NY$ and $-NX \leq I \leq NX$, $-NY \leq J \leq NY$. Accuracy increases with increasing values of NX , NY . Maximum values NX , $NY = 40$. Typical values:

$A/B = 10.0$, $NX = 30$, $NY = 10$,
 $A/B = 0.1$, $NX = 10$, $NY = 30$,
 $A/B = 1.0$, $NX = NY = 20$. Input.

- DM DM is an incremental step in the computation. Accuracy increases with decreasing values of DM. Typical value, $DM = 0.02A$. Input.
- NF If the resultant forces are desired, take $NF = 1$. For the resultant moment take $NF = 2$. The appropriate values of SXN , SYN and HC will then be computed. Input.
- NS To print all output including stresses and displacements on the contact region take $NS = 1$. To suppress all output except the resultant forces or moment take $NS = 2$. Input.
- NV2 NV2 is the integral number of sets of UXN , UYN , PHN to be considered. Input.
- UXN, UYN $UXN = v_x \rho / \mu c$, $UYN = v_y \rho / \mu c$ where v_x , v_y are the longitudinal and lateral creepages, μ = coefficient of friction. See equation (6). Input.
- PHN $PHN = \phi \rho / \mu$ where ϕ is the spin creepage. See equation (6). Input.

4) Input Format:

A sample deck set up is listed in Appendix A of this manual. The program requires contact region dimensions, elastic properties, wheel/rail creepages and program control information. The following format is for $NV1 = 1$. If $NV1 > 1$, there would be $NV1$ sets of the group of cards after the first card.

Card Number	Input Data
1	NV1 = Integer. Program solves NV1 complete problems, Typical card: 1
2	A, B, NU, LXN, LYN Typical card: 2.5980 0.3849 0.28 0.250 0.125
3	NX, NY, DM, NF, NS Typical card: 30 10 0.04 1 2
4	NV2 = Integer. Program solves NV2 problems for different values of creepage and spin given on NV2 cards starting with 5. Typical card: 1
5 to NV2	UXN, UYN, PHN Typical card: 0.0 2.0 0.4

Note: The input is free format with a space needed between each input parameter.

5) Description of Other Parameters in Program:

C11, C22, Longitudinal, lateral, lateral/spin, and spin
C23, C33
creepage coefficients, respectively; tabulated
in [7].

GS $GS = Gc^3/\rho N$ where G = shear modulus. GS may also
be computed from $GS = 3(1-\nu) \tilde{E}/(4\pi\sqrt{g})$ where $\tilde{E}$ =
complete elliptic integral of the second kind,
see [7] page 58, and g = axial ratio of the contact
ellipse = $\min(a/b, b/a)$. GS is determined within
the computer program in terms of A, B and NU.

MU $\mu = \mu$ = coefficient of friction. All variables are normalized so that μ does not explicitly appear.

SXN, SYN $SXN = S_x \rho N / c^4$, $SYN = S_y \rho N / c^4$. Inverse stiffnesses of the elastic bodies. See equations (9), (43), (44), and (47) for the form of S_x , S_y to be used to determine the resultant forces and equations (42) and (46) for the appropriate form to determine the resultant moment.

For forces let

$$SXN1 = 8A / (3C_{11}GS), \quad SYN1 = 8A / (3C_{22}GS)$$

and

$$HC1 = 32 \sqrt{B/A} C_{23} / (3\pi C_{22}), \text{ then}$$

$$SXN = SXN1 + LXN$$

$$SYN = SYN1 + LYN$$

$$HC = (SYN1 + LYN) / (SYN1/HC1 + LYN).$$

For moments,

$$SXN = 8B / (15C_{33}GS) + LYN,$$

$$SYN = \pi A / (4\sqrt{B/A} C_{23}GS) + LYN, \text{ and}$$

$$HC = 1.0$$

The C_{ij} are the linear creepage and spin coefficients, [7]. SXN , SYN , HC and the C_{ij} are determined within the program in terms of A , B , NU , LXN and LYN .

- 6) Output: NV2 sub-cases of NV1 cases are calculated. For each of the NV1 cases, the input parameters A , B , NU , LXN , LYN , NX , NY , DM , NF , and NS are printed. The linear creepage

coefficients, calculated within the program, are also printed out as are the normalized shear modulus, GS, and normalized inverse stiffness, SXN and SYN. For each of the NV1 cases, there will be NV2 sets of output corresponding to the NV2 sets of normalized creepages and spin, UXN, UYN, and PHN. For each of the NV2 cases, the inputs UXN, UYN, and PHN are printed out together with the computed values of the normalized longitudinal and lateral creep forces, FXN and FYN, or the computed value of the spin creep moment, MZN (depending on whether $NF = 1$ or $NF = 2$). If $NS = 2$, the output is as described above. If $NS = 1$, the normalized coordinate points X, Y over the contact region and the values of the stresses (TX, TY, TZH, TZK) and slip components (VX, VY) are given at each point.

The Fortran names used in the program output are the following:

UXN, UYN,	Repeated program input variables.
PHN	
FXN, FYN	$FXN = F_x/\mu N$, $FYN = F_y/\mu N$. Normalized resultant longitudinal and lateral forces. Computed.
MZN	$MZN = M_z c/\mu N$. Normalized resultant moment. Computed.
X, Y	$X = x/c$, $Y = y/c$. $-A \leq X \leq A$, $-B \leq Y \leq B$. Normalized coordinates where x and y are longitudinal and lateral distances from the center of the contact ellipse.

TX, TY	Normalized shear stresses. $TX, TY = -\tau_{xz}c^3/\rho N, -\tau_{yz}c^3/\rho N,$ $\sqrt{TX^2 + TY^2} < TZK \text{ for no slip,}$ $\sqrt{TX^2 + TY^2} = TZK \text{ for slip.}$
TZH	$TZH = 3/(2\pi) \sqrt{1 - (X/A)^2 - (Y/B)^2} = \text{Normalized Hertzian stress on the contact region. Given for reference only. See equation (14.I).}$
TZK	TZK is the assumed normal stress distribution over the contact region, see equation (14.III). $TZK = (F)(A) A1(Y)(1 - (X/A)^2 - (Y/B)^2), X \geq 0.9L(Y)$ $TZK = (F)(A)(\sqrt{1 - (X/A)^2 - (Y/B)^2} + B1(Y)), X < 0.9L(Y)$ where $L(Y) = A \sqrt{1 - (Y/B)^2}$ $A1(Y) = 0.5 (1 - (Y/B)^2)^{-1/2} (1 - (0.9)^2)^{-1/2}$ $B1(Y) = -0.5 (1 - (Y/B)^2)^{1/2} (1 - (0.9)^2)^{1/2}$ and $(F)(A) = 0.656773$, such that the resultant normal force = 1.0.
VX, VY	Normalized relative slip components. $VX, VY = v_{x\rho}/(V_{\mu c}), v_{y\rho}/(V_{\mu c})$ where V is the rolling velocity and v_x and v_y are the longitudinal and lateral components of the relative slip velocity.

7) Summary of User Requirements and Recommendations

All input data is on cards in free format as shown. As A and B are normalized, the product of A and B must be unity. LXN and LYN are taken as zero if no elastic layer is to be considered. Maximum values for NX and NY are 40. Accuracy increases with increasing values of NX and NY . Typical values are:

$A/B = 10.0$ $NX = 30, NY = 10$

$A/B = 1.0$ $NX = NY = 20$

$A/B = 0.1$ $NX = 10, NY = 30$

DM is an incremental step size in the computation. Accuracy is improved with smaller sizes of DM. A typical value is $DM = 0.2 \cdot A$.

C. TEST PROBLEM

The following test problem is given to demonstrate the program. The calculation were performed on an IBM-370/3165-II computer.

$A = 2.598, B = 0.3849, NU = 0.28, LXN = 0, LYN = 0$

$NX = 10, NY = 10, DM = 0.04, NF = 1, NS = 1$

$UXN = 0, UYN = -1.4, PHN = 0.8$

D. PROGRAM LISTINGS WITH EXAMPLE INPUT AND OUTPUT

A listing of the program for the sample problem with input and output is given in Appendix B.

III. DESCRIPTION OF COMPUTER CODE FOR SUBROUTINE FORCES

The subroutine FØRCES is a version of the complete code discussed in Chapter II that has been converted to subroutine form. All the WRITE statements have been deleted as has the calculation of the pure spin creep moment. In almost all cases of interest to rail vehicle dynamicists, the pure spin creep moment contributions from the two wheels comprising the wheelset are much smaller than the yaw moment about the wheelset center of gravity due to the longitudinal creep forces. Thus, this calculation was deleted in the interests of computational time savings.

The subroutine and its argument list are:

```
SUBRØUTINE FØRCES (A, B, NU, UXN, UYN, PHN, NX, NY, DM, FXN, FYN)
```

The input parameters are: A, B, NU, UXN, UYN, PHN, NX, NY and DM and are as defined in Chapter II. The outputs are FXN and FYN and are as defined in Chapter II. Stresses and slip values over the contact region are not returned. All discussion of users' requirements and other program descriptive material is as outlined for the complete code in Chapter II.

The purpose of FØRCES is to compute lateral and longitudinal creep forces acting between two elastic bodies in steady rolling contact. The bodies have relative longitudinal and lateral creepage as well as spin creepage about an axis normal to the contact region.

FØRCES may be used as a subroutine within other Fortran programs developed for calculating the lateral dynamic response of rail vehicles. It addresses only one wheel and must be called for each wheel separately.

The input parameters must be evaluated for each wheel/rail contact condition considered and the outputs FXN and FYN are appropriate obviously for only those input parameters.

A listing and a sample test problem using SUBROUTINE FØRCES is given in Appendix B.

```

C      THEN SXN1=8*B/(15*C33*GS), SYN1=PI*A/(4*SQR(B/A)) 000000590
C      *C23*GS), AND SXN=SXN1+LXN, SYN=SYN1+LYN, HC=1.0 00000600
C      SEE EQUATIONS (42),(43),(46), AND (47). 00000610
C      NS (TO PRINT OUTPUT ON THE CONTACT REGION,NS=1, 00000620
C      TO SUPPRESS ALL OUTPUT EXCEPT THE RESULTANT 00000630
C      FORCES OR MOMENT, TAKE NS=2), INTEGER 00000640
C      NOTE: FXN=FX/(MU*N), FYN=FY/(MU*N), 00000650
C      MZN=MZ*C/(MU*N) 00000660
C      DATA CARD #4 NV2 00000670
C      TYPICAL CARD: 1 00000680
C      SOLVES NV2 PROBLEMS FOR DISTINCT VALUES OF 00000690
C      CREEPAGE AND SPIN GIVEN ON NV2 CARDS 5), INTEGER 00000700
C      DATA CARD #5 LXN,UYN,PHN 00000710
C      TYPICAL CARD: 0.0 2.0 0.4 00000720
C      LXN AND UYN ARE NORMALIZED CREEPAGES, PHN 00000730
C      IS THE NORMALIZED SPIN), REAL 00000740
C      LXN=UX*RHO/(MU*C), UYN=UY*RHO/(MU*C), 00000750
C      PHN=PH*RHO/MU 00000760
C      ***** NOTE: ALL VARIABLES HAVE BEEN NORMALIZED SUCH 00000770
C      ***** THAT THE COEFFICIENT OF FRICTION, MU, DOES NOT 00000780
C      ***** EXPLICITLY APPEAR. 00000790
C      COMMON A,B 00000800
C      DATA X,Y,Z,ZH,VX,VY,C/6561*C.0,6561*C.0,6561*0.0,6561*0.0,6561*0.0 00000810
C      $,6561*0.0,610*C.C/ 00000820
C      PI=3.14159 00000830
C      GR=PI/180.0 00000840
C      READ(1,*)NV1 00000850
C      DO 999 I=1,NV1 00000860
C      WRITE(3,968) 00000870
C      1040 READ(1,*,END=9999)A,B,NU,LXN,LYN 00000880
C      IF(A/B.LT.0.1) GO TO 998 00000890
C      SUBROUTINE CONST COMPUTES THE LINEAR CREEPAGE AND 00000900
C      SPIN COEFFICIENTS, AND THE NORMALIZED MODULUS 00000910
C      FROM KALKER'S TABLES AND ASYMPTOTIC EXPANSIONS. 00000920
C      VALID FOR A/B EQUAL TO OR GREATER THAN 0.1 . 00000930
C      THESE VALUES ARE USED BELOW TO COMPUTE THE 00000940
C      INVERSE STIFFNESSES SXN AND SYN. 00000950
C      CALL CONST(A,B,NU,C11,C22,C23,C33,GS) 00000960
C      READ(1,*,END=9999) NX,NY,DM,NF,NS 00000970
C      IF(NS.EQ.1) GO TO 1021 00000980
C      MX=3*NX 00000990
C      MY=3*NY 00001000
C      LY=3*NY 00001010
C      GO TO 1022 00001020
C      1021 MX=1 00001030
C      MY=1 00001040

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	LY=1	00001170
1022	MUZ=3.C/(2.C*PI)	00001180
	IF(NF.EQ.2) GC TC 1025	00001190
	SXN=8.0*A/(3.0*C11*GS)+LXN	00001200
	SYN=8.0*A/(3.C*C22*GS)	00001210
	SYN=SYN+LYN	00001220
	HC1=32.0*SQRT(B/A)*C23/(3.0*PI*C22)	00001230
	FC=(SYN+LYN)/(SYN/HC1+LYN)	00001240
	GO TO 1027	00001250
1025	SXN=8.0*B/(15.0*C33*GS)+LXN	00001260
	SYN=PI*A/(4.C*SQRT(B/A)*C23*GS)+LYN	00001270
	FC=1.0	00001280
1027	GEL(5)=DM	00001290
	WRITE(3,969)	00001300
	WRITE(3,970)A,B,NU,LXN,LYN	00001310
	WRITE(3,972)NX,NY,DM,NF,NS	00001320
	WRITE(3,973)C11,C22,C23,C33,GS,SXN,SYN	00001330
	GEL(1)=SXN	00001340
	GEL(2)=SYN	00001350
	GEL(3)=A	00001360
	GEL(4)=B	00001370
	I=-NX-1	00001380
1074	I=I+1	00001390
	IF(I.GT.NX)GC TC 1078	00001400
	J=-NY-1	00001410
1075	J=J+1	00001420
	IF(J.GT.NY)GC TC 1074	00001430
	P=1.-FLOAT(I*I)/FLOAT(NX)/FLCAT(NX)-FLOAT(J*J)/FLCAT(NY)/FLOAT(NY)	00001440
	IF(P.GT.C.C) GC TC 1076	00001450
	GO TO 1075	00001460
1076	ZH(I+NX+1,J+NY+1)=MUZ*SQRT(P)	00001470
	P=FLOAT(I)*A/FLCAT(NX)	00001480
	C=FLOAT(J)*B/FLCAT(NY)	00001490
	CALL PAAKZ(P,Q,WZ,DZ,D2Z,A,E,MUZ)	00001500
	Z(I+NX+1,J+NY+1)=WZ	00001510
	GO TO 1075	00001520
1078	CONTINUE	00001530
	READ(1,*)NV2	00001540
	WRITE(3,974) NV2	00001550
	DO 997 I12=1,NV2	00001560
1090	READ(1,*,END=9999)UXN,UYN,PHN	00001570
	WRITE(3,975)UXN,UYN,PHN	00001580
	PHNN=FC*PHN	00001590
	CS(1)=UXN	00001600
	CS(2)=UYN	00001610
	CS(3)=PHNN	00001620
	CALL ROL(CS,GEL,MUZ,NX,NY,X,Y,VX,VY,G,FX,FY,MZ)	00001630
	IF(NF.EQ.2) GC TC 1091	00001640
	RES=SQRT(FX*FX+FY*FY)	00001650
	WRITE(3,977)FX,FY,RES	00001660
	GO TO 1092	00001670
1091	WRITE(3,978)MZ	00001680
1092	IF(NS.EQ.2) GC TC 1530	00001690
	WRITE(3,9004)	00001700
	J=NY	00001710
1260	J=J-LY	00001720
	IF(J.LT.-NY+1)GC TC 1345	00001730
	Q=FLOAT(J)*B/FLCAT(NY)	00001740

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      IF(G(J+NY+1,1).LT.0.C) WRITE(3,9007)C
      IF(G(J+NY+1,1).GE.0.C) WRITE(3,9008)C
C DOOR:
1340 GO TO 1260
1345 CONTINUE
      IF(MY.GT.2*NY)GO TC 1530
      J=NY
1350 J=J-MY
      IF(J.LT.1-NY)GO TC 1520
      C=FLCAT(J)*B/FLCAT(NY)
      WRITE(3,9005)
      WRITE(3,9006)C
      WRITE(3,9009)
      I=-NX-MX
1400 I=I+MX
      IF(I.GT.NX)GO TC 1515
      IF(Z(I+NX+1,J+NY+1).LT.1.E-8*MUZ)GO TO 1510
      P=FLOAT(I)*A/FLCAT(NX)
      FIX1=Z(I+NX+1,J+NY+1)
      FIX6=ZH(I+NX+1,J+NY+1)
      TX=X(I+NX+1,J+NY+1)
      TY=Y(I+NX+1,J+NY+1)
      UX=VX(I+NX+1,J+NY+1)
      UY=VY(I+NX+1,J+NY+1)
      FIX2=SQRT(TX*TX+TY*TY)
      ARG=1.0
      IF(TX.LT.0.C)ARG=-1.C
      ARG=ARG*ATAN(TY/(ABS(TX)+1.E-8))/GR+90.C*(1.0-ARG)
      FIX3=ARG
      ARG=1.0
      IF(UX.LT.0.0) ARG=-1.0
      ARG=ARG*ATAN(UY/(ABS(UX)+1.E-8))/GR+90.C*(1.0-ARG)
      FIX4=SQRT(UX*UX+UY*UY)
      FIX5=ARG
      WRITE(3,9010)P,FIX6,FIX1,FIX2,FIX3,FIX4,FIX5
C VERDER:
1510 GO TO 1400
1515 GO TC 1350
1520 WRITE(3,9001)
C NEXT:
1530 CONTINUE
997 CONTINUE
      GO TO 999
998 WRITE(3,979)
999 CONTINUE
9999 WRITE(3,9998)
      STOP
968 FORMAT('1',///,T63,'PROGRAM WITA-SRT',/,T53,'SIMPLIFIED THEORY OF
$ROLLING CONTACT',/,T64,'BY J.J. KALKER',/,T56,'MODIFIED AT CLEMSON
$ UNIVERSITY',/,T61,'DEPT. OF MECH. ENGR.',/,T66,'CLEMSON, SC',/)
969 FORMAT(///,58X,'***** INPUT PARAMETERS *****',/)
970 FORMAT(16X,'NORMALIZED CONTACT DIMENSIONS      A=',1PE11.4,10X,'(00002260
$ A=A1/C1, B=B1/C1, WHERE C1=SQRT(A1*B1),',/,32X,'(CARD #2)'
$,11X,'B=',1PE11.4,10X,'( A1,B1 ARE ACTUAL CONTACT DIMENSIONS',/,
$ 30X,'POISSON S RATIO      NU=',1PE11.4,/,33X,'(CARD #2)'
$',/,28X,'LAYER STIFFNESSES      LXN=',1PE11.4,/,33X,'(CARD #2)',
$ 8X,'LYN=',1PE11.4,/)
972 FORMAT(26X,'NUMERICAL CONSTANTS      NX=',13,/,31X,'(CARD #3)',

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$11X,'NY=',I3,/,51X,'DM=',1PE11.4,/,51X, 00CC2330
$'NF=',I3,10X,'(NF=1, FORCES COMPUTED; NF=2, MOMENTS COMPUTED)', 00CC2340
$/,51X, 00CC2350
$'NS=',I3,10X,'(NS=1, FULL OUTPUT; NS=2 ONLY FORCES OR MOMENTS)',/ 00002360
$///) 00CC2370
973 FORMAT(47X,'***** PARAMETERS COMPUTED AND USED IN PROGRAM *****' 00002380
$,//, 15X,'CREEPAGE AND SPIN COEFFICIENTS C11=',1PE11.4,/, 00CC2390
$50X,'C22=',1PE11.4,/,50X,'C23=',1PE11.4,/,50X,'C33=',1PE11.4,/, 00CC2400
$21X,'NORMALIZED SHEAR MODULUS GS=',1PE11.4,/,15X, 00CC2410
$NORMALIZED INVERSE STIFFNESSES SXN=',1PE11.4,/,50X,'SYN=', 00CC2420
$1PE11.4,////////) 00CC2430
974 FORMAT(42X,'***** NV2=',I2,' DISTINCT PROBLEMS FOLLOW FOR DIFFEREN 00CC2440
$T *****',/45X,'***** VALUES OF NORMALIZED CREEPAGE AND SPIN ***** 00002450
$',//) 00CC2460
975 FORMAT(//,17X,'NORMALIZED CREEPAGE AND SPIN UXN=',1PE11.4,/, 00002470
$23X,'(INPUT ON CARD #5)', 00CC2480
$ 9X,'UYN=',1PE11.4,/,50X,'PTN=',1PE11.4,//) 00CC2490
977 FORMAT( 24X,'NORMALIZED FORCES ARE FXN=',1PE11.4,/,25X, 00CC2500
$'(COMPUTED)',11X,'FYN=',1PE11.4,/,24X,'RESULTANT FORCE 00CC2510
$RES=',1PE11.4,/,24X,'(RES=SQRT(FXN**2+FYN**2))',//) 00CC2520
978 FORMAT( 25X,'NORMALIZED MOMENT IS MZN=',1PE11.4,/, 00CC2530
$30X,'(COMPUTED)',//) 00CC2540
979 FORMAT(//,58X,'***** A/B LESS THAN 0.1 *****',/, 00CC2550
$58X,'***** WORK NEXT PROBLEM *****',//) 00002560
9001 FORMAT(1H1) 00CC2570
9004 FORMAT(//,53X,'***** CONTACT REGION FOLLOWS *****',/, 00002580
$10X,'X AND Y ARE NORMALIZED COORDINATES, X IN THE ROLLING',/, 00CC2590
$10X,'DIRECTION, X,Y=X1/C1,Y1/C1 WHERE X1,Y1 ARE DIM. COORD.',/, 00CC2600
$10X,'TZK=HERTZ STRESS =3/(2*PI)*SQRT(P), FOR REFERENCE ONLY',/, 00CC2610
$10X,'TZK=KALKER NORMAL STRESS AS ASSUMED IN THE PROGRAM',/, 00002620
$10X,'TZK=F*A*A1(Y)*P, FOR X.GE.0.9*L(Y)',/, 00CC2630
$10X,'TZK=F*A*(SQRT(P)+B1(Y)), FOR X.LE.0.9*L(Y)',/, 00CC2640
$10X,'WHERE P=1.0-X*X/(A*A)-Y*Y/(B*B), L(Y)=A*SQRT(1.0-Y*Y/(B*B))',00002650
$/,10X,'A1(Y)=0.5/SQRT((1.0-Y*Y/(B*B))*(1.0-(C.9)**2))',/, 00CC2660
$10X,'B1(Y)=-0.25/A1(Y), F*A=0.656773,SUCH THAT RES NORMAL FORCE=1'00002670
$/,10X,'TX AND TY ARE NORMALIZED SHEAR STRESSES',/,10X,'TX=-TAUXZ*00CC2680
$C**3/(RHO*N), TY=-TALYZ*C**3/(RHO*N)',/, 00002690
$10X,'ABS(TX, TY) LESS THAN TZK FOR NO SLIP, EQUAL TO TZK FOR SLIP',00CC2700
$/,10X,'VX, VY ARE NORMALIZED SLIP COMPONENTS, VX=VX1/V*RHO/(MU*C)',00002710
$/,10X,'VY=VY1/V*RHO/(MU*C), WHERE VX1,VY1=REL. VEL. BETWEEN',/,10X00CC2720
$,'ADJACENT POINTS AND V=ROLLING VEL.',////////) 00CC2730
9005 FORMAT(1H1) 00002740
9006 FORMAT( 1X,'***** Y=',1F11.4) 00CC2750
9007 FORMAT(10X,'AT Y=',1F11.4,5X,'THE LEADING EDGE SLIPS') 00002760
9008 FORMAT(10X,'AT Y=',1F11.4,5X,'THE LEADING EDGE STICKS') 00002770
9009 FORMAT( 7X,'X',9X,'TZH',9X,'TZK',5X,'ABS(TX, TY)',1X,'ARG(TX, TY)', 00002780
$ 1X,'ABS(VX, VY)',1X,'ARG(VX, VY)') 00CC2790
9010 FORMAT(1CF11.4) 00002800
9998 FORMAT(////16F INPUT EXHAUSTED///) 00CC2810
END 00002820

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MAAKZ

	SUBROUTINE MAAKZ(P,C,WZ,DZ,C2Z,A,B,MUZ)	00002830
C	MAAKZ COMPUTES THE NORMAL STRESS AS GIVEN BY EQUATION 14.III	00002840
C	AW=L(Y) OF EQ. 14.III	00002850
C	WZ=TZK=ASSUMED NORMAL STRESS OF EQ. 14.III	00002860
C	DZ= FIRST DER. OF TZK WITH RESPECT TO X	00002870
	REAL MUZ	00002880
	AL=0.9	00002890
	S=SQRT(1.0-AL*AL)	00002900
	AW=A*SQRT(1.0-C*C/B/B)	00002910
	PI=3.1415926536	00002920
	F=MUZ*PI/2.0/A/(ATAN(AL/S)+(2.0/3.0-AL+AL*AL*AL/3.0)/S)	00002930
	IF(P.LE.AL*AW) GO TO 10	00002940
	WZ=F*(AW*AW-P*P)/2.0/AW/S	00002950
	DZ=F*P/AW/S	00002960
	D2Z=F/AW/S	00002970
	GO TO 11	00002980
10	WZ=F*SQRT(AW*AW-P*P)-F/2.0*AW*S	00002990
	IF(ABS(AW).EQ.ABS(P)) GO TO 12	00003000
	DZ=F*P/SQRT(AW*AW-P*P)	00003010
	GO TO 11	00003020
12	DZ=1.0	00003030
11	RETURN	00003040
	END	00003050

	SUBROUTINE ROL(CS,GEL,MUZ,NX,NY,X,Y,VX,VY,G,FX,FY,MZ)	03003060
	REAL MUZ,MZ,K,NU	00003070
	INTEGER PIJL	00003080
	REAL LE(81)	00003090
	INTEGER E(81)	00003100
	COMMON A,E	00003110
	DIMENSION X(81,81),Y(81,81),VX(81,81),VY(81,81),	00003120
	\$ G(81,10),CS(3),GEL(5)	00003130
	UX=CS(1)	00003140
	UY=CS(2)	00003150
	PH=CS(3)	00003160
	DM=GEL(5)	00003170
	SX=GEL(1)	00003180
	SY=GEL(2)	00003190
	A=GEL(3)	00003200
	B=GEL(4)	00003210
	H=A/FLOAT(NX)	00003220
	K=B/FLOAT(NY)	00003230
	PI=3.1415926536	00003240
	CZP=MUZ*2.0/A/A	00003250
	J=-NY	00003260
380	J=J+1	00003270
	IF(J.GT.NY-1)GO TO 100	00003280
	LE(J+NY+1)=A*SQRT(1.0-FLOAT(J*J)/FLOAT(NY)/FLOAT(NY))	00003290
	P=LE(J+NY+1)	00003300
	E(J+NY+1)=0.99*P/H	00003310
	I=-NX-1	00003320
401	I=I+1	00003330
	IF(I.GT.NX)GO TO 200	00003340
	IF(I.LT.-E(J+NY+1).CR.1.GT.E(J+NY+1))GO TO 15	00003350
	GO TO 10	00003360
15	VY(I+NX+1,J+NY+1)=0.0	00003370
	VX(I+NX+1,J+NY+1)=0.0	00003380
	Y(I+NX+1,J+NY+1)=0.0	00003390
	X(I+NX+1,J+NY+1)=0.0	00003400
10	GO TO 401	00003410
200	CONTINUE	00003420
	DO 20 I=1,10	00003430
20	G(J+NY+1,I)=-3.0*A	00003440
	GO TO 380	00003450
100	CONTINUE	00003460
	THV=-1.0	00003470
	IF(UX-PH*B.GE.C.C) THV=1.0	00003480
	THV=THV*ATAN(UY/(ABS(UX-PH*E)+1.E-08))+PI/2.0*(1.0-THV)	00003490
C		00003500
	J=-NY	00003510
470	J=J+1	00003520
	IF(J.GT.NY-1)GO TO 300	00003530
	Q=J*K	00003540
	P=LE(J+NY+1)	00003550
	PG=P	00003560
	CUX=UX-PH*Q	00003570
	PIJL=2	00003580
	CUY=UY+PH*P	00003590
	CALL MAAKZ(P,Q,ZN,CZ,CZP,A,E,MUZ)	00003600
	XG=0.0	00003610
	YG=0.0	00003620
	XV=0.0	00003630

	YV=0.0	00003640
	VV=0.0	00003650
	I=E(J+NY+1)	00003660
	G(J+NY+1,1)=1.0	00003670
	IF(CZ*CZ-CUX*CUX/SX/SX-CUY*CUY/SY/SY.GT.0.0) GO TO 1001	00003680
C	SLIP AT THE LEADING EDGE. DETERMINE THETA.	00003690
	G(J+NY+1,1)=-1.0	00003700
	T=-1.0	00003710
	IF(CUX.GE.0.0) T=1.0	00003720
	T=T*ATAN(CUY/(ABS(CUX)+1.E-08))+PI/2.0*(1.0-T)	00003730
C	BEPT	00003740
1002	S=SIN(T)	00003750
	C=CCS(T)	00003760
	NU=(CUX*S-CUY*C-CZ*(SX-SY)*C*S)/(CUX*C+CUY*S-CZ*(SX-SY)*(C*C-S*S))	00003770
	T=T-NU	00003780
	IF(ABS(NU).GT.1.E-03) GO TO 1002	00003790
	THV=T	00003800
	C=CCS(T)	00003810
	S=SIN(T)	00003820
C	THE STARTING VALUE OF T HAS BEEN FOUND. THE	00003830
C	DERIVATIVE IS DETERMINED IN A SPECIAL WAY.	00003840
	TP=(PH*C+CZP*(SX-SY)*C*S)/(CUX*C+CUY*S-CZ*((3.0*C*C-1.0)*(SX-SY)	00003850
	\$ -SX))	00003860
C	NEXTGL	00003870
1003	D=-DM	00003880
	IF(P-DM.LE.I*H+1.E-06) D=I*H-P	00003890
	PN=P+D	00003900
	CALL NAAKZ(PN,C,ZN,CZ,CZP,A,B,MUZ)	00003910
	TN=T+D*TP	00003920
	S=SIN(TN)	00003930
	C=CCS(TN)	00003940
	CUY=UY+PH*PN	00003950
	TPN=(CUX*S-CUY*C-CZ*(C*S*(SX-SY)))/ZN/(SY*C*C+SX*S*S)	00003960
	T=T+0.5*D*(TP+TPN)	00003970
	S=SIN(T)	00003980
	C=CCS(T)	00003990
	XN=ZN*C	00004000
	YN=ZN*S	00004010
	V=CUX*SY*C+CUY*SX*S-CZ*SX*SY	00004020
	IF(V.GE.-4.E-05) GO TO 1004	00004030
	AN=VV/(VV-V)	00004040
	IF(ABS(VV).LT.1.E-10) AN=0.9	00004050
	VV=0.0	00004060
	AV=1.0-AN	00004070
	G(J+NY+1,PIJL)=AV*P+AN*PN	00004080
	PG=G(J+NY+1,PIJL)	00004090
	PIJL=PIJL+1	00004100
	P=PG	00004110
	IF(PIJL.GT.10)PIJL=10	00004120
	XV=AV*XV+AN*XN	00004130
	XG=XV	00004140
	YV=AV*YV+AN*YN	00004150
	YG=YV	00004160
	GO TO 1006	00004170
1004	CONTINUE	00004180
C	SLIP IN THE NEW POINT	00004190
	XV=XN	00004200
	YV=YN	00004210

	P=PN	00004220
C SLIP		00004230
1005	TP=(CUX*S-CUY*C-CZ*C*S*(SX-SY))/ZN/(SY*C*C+SX*S*S)	00004240
	IF(P.GT.I*H+1.E-C) GC TO 1003	00004250
	X(I+NX+1,J+NY+1)=XN	00004260
	Y(I+NX+1,J+NY+1)=YN	00004270
	XP=-ZN*S*TP-CZ*C	00004280
	YP=ZN*C*TP-CZ*S	00004290
	VX(I+NX+1,J+NY+1)=CLX+SX*XP	00004300
	VY(I+NX+1,J+NY+1)=CUY+SY*YP	00004310
	VV=V	00004320
	I=I-1	00004330
	IF(I.LT.-E(J+NY+1)) GC TO 1007	00004340
	GO TO 1003	00004350
C ADH		00004360
1001	T=-1.0	00004370
	IF(CUX.GT.C.0) T=1.0	00004380
	THV=T*ATAN((UY+PH*P)/(ABS(CUX)+1.E-08))+PI*(1.0-T)/2.0	00004390
C ADHERE		00004400
1006	PN=I*H	00004410
	XN=CUX*(PG-PN)/SX+XC	00004420
	YN=(UY+0.5*PH*(PG+PN))*(PG-PN)/SY+YC	00004430
	CALL MAAKZ(PN,G,ZN,CZ,CZP,A,E,MUZ)	00004440
	V=ZN-SQRT(XN*XN+YN*YN)	00004450
	IF(V.GE.(-4.E-05)*MUZ)GO TO 1008	00004460
	AN=VV/(VV-V)	00004470
	IF(ABS(VV).LT.1.E-10) AN=0.5	00004480
	AV=1.C-AN	00004490
	P=AV*P+AN*PN	00004500
	G(J+NY+1,PIJL)=F	00004510
	VV=0.0	00004520
	PIJL=PIJL+1	00004530
	XV=AV*XV+AN*XN	00004540
	YV=AV*YV+AN*YN	00004550
	IF(PIJL.GT.10) PIJL=10	00004560
	T=-1.0	00004570
	IF(XV.GE.0.0) T=1.0	00004580
	CUY=UY+PH*P	00004590
	T=T*ATAN(YV/(ABS(XV)+1.E-08))+PI/2.0*(1.C-T)	00004600
	C=CCS(T)	00004610
	S=SIN(T)	00004620
	CALL MAAKZ(P,G,ZN,CZ,CZP,A,E,MUZ)	00004630
	GO TO 1005	00004640
1008	VV=V	00004650
	X(I+NX+1,J+NY+1)=XN	00004660
	XV=XN	00004670
	Y(I+NX+1,J+NY+1)=YN	00004680
	YV=YN	00004690
	VY(I+NX+1,J+NY+1)=C.C	00004700
	VX(I+NX+1,J+NY+1)=C.C	00004710
	I=I-1	00004720
	P=PN	00004730
	IF(I.CE.-E(J+NY+1)) GO TO 1006	00004740
1007	GO TO 470	00004750
300	CONTINUE	00004760
C	THE ARRAYS ARE FILLED. THE INTEGRALS ARE	00004770
C	DETERMINED	00004780
	T=4.0*K/3.C	00004790

	TN=2.C*K	00004800
	FX=0.0	00004810
	FY=0.C	00004820
	MZ=0.0	00004830
	YN=2.C*H	00004840
	J=-NY	00004850
940	J=J+1	00004860
	IF (J.GT.NY-1)GC TC 400	00004870
	I=E(J+NY+1)	00004880
	PIJL=I	00004890
	P=(LE(J+NY+1)-I*H)/2.0-H/3.C	00004900
	YP=J*K	00004910
	C=P*(X(I+NX+1,J+NY+1)+X(NX+1-I,J+NY+1))	00004920
	S=P*(Y(I+NX+1,J+NY+1)+Y(NX+1-I,J+NY+1))	00004930
	D=P*I*H*(Y(I+NX+1,J+NY+1)-Y(NX+1-I,J+NY+1))	00004940
	P=2.0*H/3.C	00004950
	I=-PIJL-1	00004960
500	I=I+1	00004970
	IF (I.GT.PIJL)GC TC 550	00004980
	C=C+P*X(I+NX+1,J+NY+1)	00004990
	D=D+P*I*H*Y(I+NX+1,J+NY+1)	00005000
	S=S+P*Y(I+NX+1,J+NY+1)	00005010
	P=YN-F	00005020
	GO TO 500	00005030
550	FX=FX+T*C	00005040
	FY=FY+T*S	00005050
	MZ=MZ+T*(D-YP*C)	00005060
	T=TN-T	00005070
	GO TO 940	00005080
400	CONTINUE	00005090
	RETURN	00005100
	END	00005110

```

SUBROUTINE CONST(A,B,NU,C11,C22,C23,C33,GS)
DIMENSION AL(5),BE(5),D(15),E(15,20),AR(20),CNT(5),D1(15,9),
$D2(15,9),C3(15),C4(15)
C ***** DATA E(I,J) GIVES LINEAR CREEPAGE AND SPIN COEFFICIENTS*****
C *****AND GS FROM KALKER REPORT TABLE 1 *****
C ***** VALID FOR A/E GREATER THAN CR EQUAL TO 0.1
REAL NU
DATA C1/
$ 2.51, 3.31, 4.85, 2.51, 2.52, 2.53, 0.334, 0.473, 0.731, 6.42,
$ 8.28, 11.7, 0.7670, 0.5752, 0.3835,
$ 2.59, 3.37, 4.81, 2.59, 2.63, 2.66, 0.483, 0.603, 0.809, 3.46,
$ 4.27, 5.66, 0.5608, 0.4206, 0.2804,
$ 2.68, 3.44, 4.80, 2.68, 2.75, 2.81, 0.607, 0.715, 0.889, 2.49,
$ 2.96, 3.72, 0.4775, 0.3584, 0.2390,
$ 2.78, 3.53, 4.82, 2.78, 2.88, 2.98, 0.720, 0.823, 0.977, 2.02,
$ 2.32, 2.77, 0.4343, 0.3257, 0.2172,
$ 2.88, 3.62, 4.83, 2.88, 3.01, 3.14, 0.827, 0.925, 1.07, 1.74,
$ 1.93, 2.22, 0.4089, 0.3066, 0.2044,
$ 2.98, 3.72, 4.91, 2.98, 3.14, 3.31, 0.930, 1.03, 1.18, 1.56,
$ 1.68, 1.66, 0.3934, 0.2950, 0.1967,
$ 3.09, 3.81, 4.97, 3.09, 3.28, 3.48, 1.03, 1.14, 1.29, 1.43,
$ 1.50, 1.60, 0.3840, 0.2880, 0.1920,
$ 3.19, 3.91, 5.05, 3.19, 3.41, 3.65, 1.13, 1.25, 1.40, 1.34,
$ 1.37, 1.42, 0.3765, 0.2839, 0.1892,
$ 3.29, 4.01, 5.12, 3.29, 3.54, 3.82, 1.23, 1.36, 1.51, 1.27,
$ 1.27, 1.27, 0.3758, 0.2818, 0.1879/
DATA C2/
$ 3.40, 4.12, 5.20, 3.40, 3.67, 3.98, 1.33, 1.47, 1.63, 1.21,
$ 1.19, 1.16, 0.3750, 0.2812, 0.1875,
$ 3.51, 4.22, 5.30, 3.51, 3.81, 4.16, 1.44, 1.59, 1.77, 1.16,
$ 1.11, 1.06, 0.3758, 0.2818, 0.1879,
$ 3.65, 4.36, 5.42, 3.65, 3.99, 4.39, 1.58, 1.75, 1.94, 1.10,
$ 1.04, 0.954, 0.3785, 0.2839, 0.1892,
$ 3.82, 4.54, 5.58, 3.82, 4.21, 4.67, 1.76, 1.95, 2.18, 1.05,
$ 0.965, 0.852, 0.3840, 0.2880, 0.1920,
$ 4.06, 4.78, 5.80, 4.06, 4.50, 5.04, 2.01, 2.23, 2.50, 1.01,
$ 0.892, 0.751, 0.3934, 0.2950, 0.1967,
$ 4.37, 5.10, 6.11, 4.37, 4.90, 5.56, 2.35, 2.62, 2.96, 0.958,
$ 0.819, 0.650, 0.4089, 0.3066, 0.2044,
$ 4.84, 5.57, 6.57, 4.84, 5.48, 6.31, 2.88, 3.24, 3.70, 0.912,
$ 0.747, 0.549, 0.4343, 0.3257, 0.2172,
$ 5.57, 6.34, 7.34, 5.57, 6.40, 7.51, 3.79, 4.32, 5.01, 0.868,
$ 0.674, 0.446, 0.4775, 0.3584, 0.2390,
$ 6.96, 7.78, 8.82, 6.96, 8.14, 9.79, 5.72, 6.63, 7.89, 0.828,
$ 0.601, 0.341, 0.5608, 0.4206, 0.2804/
DATA C3/
$ 10.7, 11.7, 12.9, 10.7, 12.8, 16.0, 12.2, 14.6, 18.0, 0.795,
$ 0.562, 0.228, 0.7670, 0.5752, 0.3835 /
DATA C4/
$ 11.08, 12.01, 13.10, 11.08, 13.38, 16.90, 13.72, 16.34, 20.20,
$ 0.785, 0.552, 0.208, 0.7918, 0.5938, 0.3959/
DATA AR / 0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9,1.0,1.111111,
$1.25,1.428571,1.666667,2.0,2.5,3.333333,5.0,10.0,11.0/
DO 6 I=1,15
DO 5 J=1,9
E(I,J)=D1(I,J)
E(I,J+9)=D2(I,J)
E(I,15)=C3(I)

```

6	E(I,20)=D4(I)	00005700
	PI=3.14159	00005710
	RG=A/B	00005720
	IF(RG.GT.AR(20)) GC TO 14	00005730
	GO TO 15	00005740
14	SG=B/A	00005750
	GAM=ALOG(16.0/(SG*SG))	00005760
	C11=2.0*PI/(SG*(GAM-2.0*NU))*(1.0+(1.613706)/(GAM-2.0*NU))	00005770
	C22=((1.613706)*(1.0-NU))/(2.0*NU+GAM*(1.0-NU))	00005780
	C22=2.0*PI*(1.0+C22)/(SG*(2.0*NU+GAM*(1.0-NU)))	00005790
	C23=2.0*PI/((SQRT(SG)*SG*3.0)*((1.0-NU)*GAM-2.0+4.0*NU))	00005800
	C33=PI/4.0*(GAM*(1.0-2.0*NU)-2.0+6.0*NU)/(GAM*(1.0-NU)-2.0+4.0*NU)	00005810
	GS=3.0*(1.0-NU)/(4.0*PI*SQRT(SG))	00005820
	GO TO 80	00005830
15	DO 20 I=2,20	00005840
	IF(RG.LE.AR(I)) GC TO 25	00005850
20	CONTINUE	00005860
25	J=I	00005870
	DO 30 I=1,15	00005880
30	D(I)=E(I,J-1)+(E(I,J)-E(I,J-1))*(RG-AR(J-1))/(AR(J)-AR(J-1))	00005890
	DO 40 I=1,5	00005900
	AL(I)=8.0*(C(3*I)-2.0*C(3*I-1)+D(3*I-2))	00005910
	BE(I)=2.0*(-D(3*I)+4.0*D(3*I-1)-3.0*D(3*I-2))	00005920
40	CNT(I)=AL(I)*NU**2+BE(I)*NU+C(3*I-2)	00005930
	C11=CNT(1)	00005940
	C22=CNT(2)	00005950
	C23=CNT(3)	00005960
	C33=CNT(4)	00005970
	GS=CNT(5)	00005980
80	CONTINUE	00005990
	RETURN	00006000
	END	00006010

PROGRAM WITA-SRT
SIMPLIFIED THEORY OF ROLLING CONTACT
BY J.J. KALKER
MODIFIED AT CLEMSON UNIVERSITY
DEPT. OF MECH. ENGR.
CLEMSON, SC

***** INPUT PARAMETERS *****

NORMALIZED CONTACT DIMENSIONS (CARD #2)	A= 2.5980E+00 B= 3.8490E-01	(A=A1/C1, B=B1/C1, WHERE C1=SQRT(A1*B1), (A1,B1 ARE ACTUAL CONTACT DIMENSIONS
POISSON'S RATIO (CARD #2)	NU= 2.8000E-01	
LAYER STIFFNESSES (CARD #2)	LXN= 0.0 LYN= 0.0	
NUMERICAL CONSTANTS (CARD #3)	NX= 10 NY= 10 DM= 4.0000E-02 NF= 1 NS= 1	(NF=1, FORCES COMPUTED; NF=2, MOMENTS COMPUTED) (NS=1, FULL OUTPUT; NS=2 ONLY FORCES OR MOMENTS)

***** PARAMETERS COMPUTED AND USED IN PROGRAM *****

CREEPAGE AND SPIN COEFFICIENTS	C11= 9.2721E+00 C22= 9.9975E+00 C23= 9.6298E+00 C33= 5.5604E-01
NORMALIZED SHEAR MODULUS	GS= 4.5572E-01
NORMALIZED INVERSE STIFFNESSES	SXN= 1.6396E+00 SYN= 1.5206E+00

***** NV2= 1 DISTINCT PROBLEMS FOLLOW FOR DIFFERENT *****
 ***** VALUES OF NORMALIZED CREEPAGE AND SPIN *****

NORMALIZED CREEPAGE AND SPIN
 (INPUT ON CARD #5)

UXN= 0.0
 UYN=-1.4000E+00
 PHN= 8.0000E-01

NORMALIZED FORCES ARE
 (COMPUTED)

FXN= 8.9034E-07
 FYN=-3.4703E-01

RESULTANT FORCE
 (RES=SQRT(FXN**2+FYN**2))

RES= 3.4703E-01

***** CONTACT REGION FOLLOWS *****

X AND Y ARE NORMALIZED COORDINATES, X IN THE ROLLING
 DIRECTION, $X, Y = X_1/C_1, Y_1/C_1$ WHERE X_1, Y_1 ARE DIM. COORD.
 TZN=HERTZ STRESS $= 3/(2*\pi)*\sqrt{P}$, FOR REFERENCE ONLY
 TZK=KALKER NORMAL STRESS AS ASSUMED IN THE PROGRAM
 $TZK = F*A*A1(Y)*P$, FOR $X \geq 0.9*L(Y)$
 $TZK = F*A*(\sqrt{P}+B1(Y))$, FOR $X \leq 0.9*L(Y)$
 WHERE $P = 1.0 - X*X/(A*A) - Y*Y/(B*B)$, $L(Y) = A*\sqrt{(1.0 - Y*Y/(B*B))}$
 $A1(Y) = 0.5/\sqrt{((1.0 - Y*Y/(B*B))*(1.0 - (0.9)**2))}$
 $B1(Y) = -0.25/A1(Y)$, $F*A = 0.656773$, SUCH THAT RES NORMAL FORCE=1
 TX AND TY ARE NORMALIZED SHEAR STRESSES
 $TX = -TAUXZ*C**3/(RHO*N)$, $TY = -TAUYZ*C**3/(RHO*N)$
 ABS(TX, TY) LESS THAN TZK FOR NO SLIP, EQUAL TO TZK FOR SLIP
 VX, VY ARE NORMALIZED SLIP COMPONENTS, $VX = VX1/V*RHO/(MU*C)$
 $VY = VY1/V*RHO/(MU*C)$, WHERE $VX1, VY1 =$ REL. VEL. BETWEEN
 ADJACENT POINTS AND $V =$ ROLLING VEL.

AT Y=	0.3464	THE LEADING EDGE STICKS
AT Y=	0.3079	THE LEADING EDGE STICKS
AT Y=	0.2694	THE LEADING EDGE STICKS
AT Y=	0.2309	THE LEADING EDGE STICKS
AT Y=	0.1924	THE LEADING EDGE SLIPS
AT Y=	0.1540	THE LEADING EDGE SLIPS
AT Y=	0.1155	THE LEADING EDGE SLIPS
AT Y=	0.0770	THE LEADING EDGE SLIPS
AT Y=	0.0385	THE LEADING EDGE SLIPS
AT Y=	0.0	THE LEADING EDGE SLIPS
AT Y=	-0.0385	THE LEADING EDGE SLIPS
AT Y=	-0.0770	THE LEADING EDGE SLIPS
AT Y=	-0.1155	THE LEADING EDGE SLIPS
AT Y=	-0.1540	THE LEADING EDGE SLIPS
AT Y=	-0.1924	THE LEADING EDGE SLIPS
AT Y=	-0.2309	THE LEADING EDGE STICKS
AT Y=	-0.2694	THE LEADING EDGE STICKS
AT Y=	-0.3079	THE LEADING EDGE STICKS
AT Y=	-0.3464	THE LEADING EDGE STICKS

```

***** Y=      0.3464
      X      TZH      TZK      ABS(TX,TY)  ARG(TX,TY)  ABS(VX,VY)  ARG(VX,VY)
-1.0392    0.0827    0.0514    0.0514    261.5010    3.3606    261.5010
-C.7794    0.1510    0.1453    0.1453    260.1367    2.5782    260.1367
-C.5196    0.1849    0.1920    0.1920    258.2253    2.1538    258.2253
-C.2598    0.2026    0.2163    0.2163    255.5179    1.7885    255.5178
  0.0      0.2081    0.2239    0.2239    251.7460    1.4411    251.7460
  C.2598    0.2026    0.2163    0.2163    246.7664    1.0954    246.7663
  0.5196    0.1849    0.1920    0.1920    240.7860    0.7374    240.7860
  C.7794    0.1510    0.1453    0.1263    233.5065    0.0       0.0
  1.0392    0.0827    0.0519    0.0273    223.4534    0.0       0.0

```

```

***** Y=      0.3079
      X      TZH      TZK      ABS(TX,TY)  ARG(TX,TY)  ABS(VX,VY)  ARG(VX,VY)
-1.2990    0.1584    0.1319    0.1319    263.0781    3.3061    263.0781
-1.0392    0.2135    0.2078    0.2078    262.0066    2.8105    262.0066
-C.7794    0.2481    0.2554    0.2554    260.4182    2.4291    260.4182
-C.5196    0.2701    0.2856    0.2856    257.9551    2.0834    257.9551
-C.2598    0.2825    0.3027    0.3027    254.1517    1.7515    254.1517
  0.0      0.2865    0.3082    0.3082    248.5337    1.4219    248.5337
  C.2598    0.2825    0.3027    0.3027    240.8285    1.0879    240.8285
  C.5196    0.2701    0.2856    0.2856    231.2778    0.7493    231.2778
  C.7794    0.2481    0.2554    0.1864    217.7482    0.0       0.0
  1.0392    0.2135    0.2078    0.1032    197.7118    0.0       0.0
  1.2990    0.1584    0.1319    0.0496    172.2837    0.0       0.0
  1.5588    0.0002    0.0000    0.0       0.0       0.0       0.0

```

```

***** Y=      0.2694
      X      TZH      TZK      ABS(TX,TY)  ARG(TX,TY)  ABS(VX,VY)  ARG(VX,VY)
-1.5588    0.1849    0.1521    0.1521    264.4822    3.5782    264.4822
-1.2990    0.2435    0.2327    0.2327    263.6963    3.0991    263.6963
-1.0392    0.2825    0.2863    0.2863    262.4902    2.7217    262.4902
-C.7794    0.3094    0.3234    0.3234    260.5039    2.3791    260.5039
-C.5196    0.3273    0.3480    0.3480    257.2070    2.0507    257.2070
-C.2598    0.3376    0.3622    0.3622    251.9587    1.7242    251.9587
  0.0      0.3410    0.3668    0.3668    244.1799    1.3889    244.1798
  C.2598    0.3376    0.3622    0.3622    233.6683    1.0382    233.6683
  C.5196    0.3273    0.3480    0.2846    219.0531    0.0       0.0
  C.7794    0.3094    0.3234    0.1855    196.2500    0.0       0.0
  1.0392    0.2825    0.2863    0.1385    167.1360    0.0       0.0
  1.2990    0.2435    0.2327    0.1150    143.1956    0.0       0.0
  1.5588    0.1849    0.1521    0.0792    128.2595    0.0       0.0
  1.8186    0.0675    0.0211    0.0125    119.2189    0.0       0.0

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```

***** Y=      0.2309
      X      TZH      TZK      ABS(TX,TY)  ARG(TX,TY)  ABS(VX,VY)  ARG(VX,VY)
-1.8186    0.1849    0.1399    0.1399    265.6860    3.9349    265.6860
-1.5588    0.2527    0.2330    0.2330    265.1428    3.4150    265.1428
-1.2990    0.2982    0.2956    0.2956    264.3091    3.0261    264.3091
-1.0392    0.3308    0.3405    0.3405    262.8872    2.6793    262.8872
-C.7794    0.3541    0.3726    0.3726    260.3867    2.3505    260.3867
-C.5196    0.3698    0.3942    0.3942    256.1316    2.0266    256.1316
-C.2598    0.3790    0.4068    0.4068    249.3674    1.6948    249.3674
  0.0      0.3820    0.4109    0.4109    239.5138    1.3409    239.5138
  C.2598    0.3790    0.4068    0.3707    225.9109    0.0       0.0
  C.5196    0.3698    0.3942    0.2403    203.0654    0.0       0.0
  C.7794    0.3541    0.3726    0.1872    169.7594    0.0       0.0
  1.0392    0.3308    0.3405    0.1876    141.7920    0.0       0.0
  1.2990    0.2982    0.2956    0.1896    125.6612    0.0       0.0
  1.5588    0.2527    0.2330    0.1648    116.5637    0.0       0.0
  1.8186    0.1849    0.1399    0.1029    110.9889    0.0       0.0

```

***** Y= 0.1924						
X	TZH	TZK	ABS(TX,TY)	ARG(TX,TY)	ABS(VX,VY)	ARG(VX,VY)
-2.0784	0.1584	0.0939	0.0939	266.7043	4.4259	266.7043
-1.8186	0.2435	0.2109	0.2109	266.3472	3.7651	266.3472
-1.5588	0.2982	0.2862	0.2862	265.8198	3.3456	265.8198
-1.2990	0.3376	0.3404	0.3404	264.9233	2.9870	264.9233
-1.0392	0.3667	0.3805	0.3805	263.2856	2.6536	263.2856
-0.7794	0.3879	0.4096	0.4096	260.3276	2.3308	260.3276
-0.5196	0.4023	0.4294	0.4294	255.2953	2.0065	255.2953
-0.2598	0.4107	0.4410	0.4410	247.4055	1.6657	247.4055
0.0	0.4135	0.4448	0.4448	236.1813	1.2922	236.1813
0.2598	0.4107	0.4410	0.2952	217.1355	0.0	0.0
0.5196	0.4023	0.4294	0.2047	181.6965	0.0	0.0
0.7794	0.3879	0.4096	0.2120	145.0758	0.0	0.0
1.0392	0.3667	0.3805	0.2493	125.0430	0.0	0.0
1.2990	0.3376	0.3404	0.2670	114.9075	0.0	0.0
1.5588	0.2982	0.2862	0.2493	109.1405	0.0	0.0
1.8186	0.2435	0.2109	0.1911	105.4864	0.0	0.0
2.0784	0.1584	0.0957	0.0904	102.9887	0.0	0.0

***** Y= 0.1540						
X	TZH	TZK	ABS(TX,TY)	ARG(TX,TY)	ABS(VX,VY)	ARG(VX,VY)
-2.0784	0.2135	0.1625	0.1625	267.3408	4.1843	267.3408
-1.8186	0.2825	0.2574	0.2574	267.0288	3.6901	267.0288
-1.5588	0.3308	0.3238	0.3238	266.5320	3.3068	266.5320
-1.2990	0.3667	0.3733	0.3733	265.6252	2.9627	265.6252
-1.0392	0.3937	0.4104	0.4104	263.9319	2.6366	263.9319
-0.7794	0.4135	0.4376	0.4376	260.8638	2.3174	260.8638
-0.5196	0.4271	0.4562	0.4562	255.7088	1.9940	255.7088
-0.2598	0.4350	0.4672	0.4672	247.7752	1.6519	247.7752
0.0	0.4376	0.4708	0.3962	235.6525	0.0	0.0
0.2598	0.4350	0.4672	0.2275	208.9920	0.0	0.0
0.5196	0.4271	0.4562	0.1850	160.4578	0.0	0.0
0.7794	0.4135	0.4376	0.2414	128.3532	0.0	0.0
1.0392	0.3937	0.4104	0.2995	114.7184	0.0	0.0
1.2990	0.3667	0.3733	0.3261	107.9860	0.0	0.0
1.5588	0.3308	0.3238	0.3129	104.0802	0.0	0.0
1.8186	0.2825	0.2574	0.2573	101.5555	0.0	0.0
2.0784	0.2135	0.1625	0.1623	99.7412	0.0	0.0
2.3382	0.0827	0.0247	0.0247	98.4557	0.0996	98.4598

***** Y= 0.1155						
X	TZH	TZK	ABS(TX,TY)	ARG(TX,TY)	ABS(VX,VY)	ARG(VX,VY)
-2.3382	0.1510	0.0711	0.0711	268.1663	4.8506	268.1663
-2.0784	0.2481	0.2047	0.2047	267.9902	4.0869	267.9902
-1.8186	0.3094	0.2891	0.2891	267.7378	3.6488	267.7378
-1.5588	0.3541	0.3505	0.3505	267.3147	3.2831	267.3147
-1.2990	0.3879	0.3970	0.3970	266.5255	2.9471	266.5255
-1.0392	0.4135	0.4322	0.4322	265.0417	2.6257	265.0417
-0.7794	0.4324	0.4582	0.4582	262.3895	2.3102	262.3895
-0.5196	0.4453	0.4761	0.4761	258.0063	1.9924	258.0063
-0.2598	0.4530	0.4865	0.4865	251.3702	1.6618	251.3702
0.0	0.4555	0.4900	0.3397	239.8070	0.0	0.0
0.2598	0.4530	0.4865	0.1707	206.7441	0.0	0.0
0.5196	0.4453	0.4761	0.1645	144.5747	0.0	0.0
0.7794	0.4324	0.4582	0.2510	117.4232	0.0	0.0
1.0392	0.4135	0.4322	0.3206	107.6421	0.0	0.0
1.2990	0.3879	0.3970	0.3525	102.9081	0.0	0.0
1.5588	0.3541	0.3505	0.3422	100.1453	0.0	0.0
1.8186	0.3094	0.2891	0.2891	98.3403	0.0273	98.3404
2.0784	0.2481	0.2047	0.2047	97.0495	0.1093	97.0495
2.3382	0.1510	0.0790	0.0790	96.1155	0.1288	96.1155

**** Y= J.J77J 40

X	TZH	TZK	ABS(TX, TY)	ARG(TX, TY)	ABS(VX, VY)	ARG(VX, VY)
-2.3382	0.1849	0.1141	0.1141	268.7739	4.6488	268.7739
-2.0784	0.2701	0.2313	0.2313	268.6521	4.0376	268.6521
-1.8186	0.3273	0.3100	0.3100	268.4727	3.6249	268.4727
-1.5588	0.3698	0.3685	0.3685	268.1626	3.2686	268.1626
-1.2990	0.4023	0.4132	0.4132	267.5769	2.9373	267.5769
-1.0392	0.4271	0.4472	0.4472	266.4773	2.6189	266.4773
-0.7794	0.4453	0.4723	0.4723	264.5308	2.3070	264.5308
-0.5196	0.4580	0.4897	0.4897	261.3491	1.9956	261.3491
-0.2598	0.4654	0.4999	0.4999	256.5828	1.6790	256.5828
0.0	0.4678	0.5033	0.2951	247.0065	0.0	0.0
0.2598	0.4654	0.4999	0.1167	208.0296	0.0	0.0
0.5196	0.4580	0.4897	0.1483	127.7151	0.0	0.0
0.7794	0.4453	0.4723	0.2570	107.7684	0.0	0.0
1.0392	0.4271	0.4472	0.3341	101.4152	0.0	0.0
1.2990	0.4023	0.4132	0.3695	98.3826	0.0	0.0
1.5588	0.3698	0.3685	0.3613	96.6089	0.0	0.0
1.8186	0.3273	0.3100	0.3100	95.4402	0.0438	95.4402
2.0784	0.2701	0.2313	0.2313	94.5983	0.1530	94.5983
2.3382	0.1849	0.1153	0.1153	93.9837	0.1473	93.9837

**** Y= 0.0385

X	TZH	TZK	ABS(TX, TY)	ARG(TX, TY)	ABS(VX, VY)	ARG(VX, VY)
-2.3382	0.2026	0.1362	0.1362	269.3855	4.5703	269.3857
-2.0784	0.2825	0.2461	0.2461	269.3232	4.0131	269.3232
-1.8186	0.3376	0.3220	0.3220	269.2300	3.6122	269.2300
-1.5588	0.3790	0.3789	0.3789	269.0657	3.2606	269.0657
-1.2990	0.4107	0.4226	0.4226	268.7532	2.9318	268.7532
-1.0392	0.4350	0.4559	0.4559	268.1680	2.6154	268.1680
-0.7794	0.4530	0.4806	0.4806	267.1382	2.3059	267.1382
-0.5196	0.4654	0.4977	0.4977	265.4653	1.9999	265.4653
-0.2598	0.4727	0.5077	0.5077	262.9739	1.6942	262.9739
0.0	0.4751	0.5111	0.2654	257.3679	0.0	0.0
0.2598	0.4727	0.5077	0.0668	219.0571	0.0	0.0
0.5196	0.4654	0.4977	0.1378	109.3834	0.0	0.0
0.7794	0.4530	0.4806	0.2605	98.7453	0.0	0.0
1.0392	0.4350	0.4559	0.3419	95.6179	0.0	0.0
1.2990	0.4107	0.4226	0.3792	94.1317	0.0	0.0
1.5588	0.3790	0.3789	0.3722	93.2624	0.0	0.0
1.8186	0.3376	0.3220	0.3220	92.6873	0.0521	92.6873
2.0784	0.2825	0.2461	0.2461	92.2718	0.1741	92.2718
2.3382	0.2026	0.1363	0.1363	91.9662	0.1576	91.9662

**** Y= 0.0

X	TZH	TZK	ABS(TX, TY)	ARG(TX, TY)	ABS(VX, VY)	ARG(VX, VY)
-2.3382	0.2081	0.1431	0.1431	-90.0000	4.5484	-90.0000
-2.0784	0.2865	0.2509	0.2509	-90.0000	4.0056	-90.0000
-1.8186	0.3410	0.3259	0.3259	-90.0000	3.6082	-90.0000
-1.5588	0.3820	0.3823	0.3823	-90.0000	3.2581	-90.0000
-1.2990	0.4135	0.4256	0.4256	-90.0000	2.9301	-90.0000
-1.0392	0.4376	0.4588	0.4588	-90.0000	2.6143	-90.0000
-0.7794	0.4555	0.4834	0.4834	-90.0000	2.3058	-90.0000
-0.5196	0.4678	0.5004	0.5004	-90.0000	2.0017	-90.0000
-0.2598	0.4751	0.5103	0.5103	-90.0000	1.7003	-90.0000
0.0	0.4775	0.5136	0.2548	-90.0000	0.0	0.0
0.2598	0.4751	0.5103	0.0380	-89.9998	0.0	0.0
0.5196	0.4678	0.5004	0.1342	90.0000	0.0	0.0
0.7794	0.4555	0.4834	0.2616	90.0000	0.0	0.0
1.0392	0.4376	0.4588	0.3443	90.0000	0.0	0.0
1.2990	0.4135	0.4256	0.3824	90.0000	0.0	0.0
1.5588	0.3820	0.3823	0.3757	90.0000	0.0	0.0
1.8186	0.3410	0.3259	0.3259	90.0000	0.0546	90.0000
2.0784	0.2865	0.2509	0.2509	90.0000	0.1805	90.0000
2.3382	0.2081	0.1431	0.1431	90.0000	0.1610	90.0000

***** Y= -0.0385

X	TZH	TZK	ABS(TX,TY)	ARG(TX,TY)	ABS(VX,VY)	ARG(VX,VY)
-2.3382	0.2026	0.1362	0.1362	-89.3857	4.5703	-89.3857
-2.0784	0.2825	0.2461	0.2461	-89.3234	4.0131	-89.3234
-1.8186	0.3376	0.3220	0.3220	-89.2301	3.6122	-89.2301
-1.5588	0.3790	0.3789	0.3789	-89.0656	3.2606	-89.0656
-1.2990	0.4107	0.4226	0.4226	-88.7532	2.9318	-88.7532
-1.0392	0.4350	0.4559	0.4559	-88.1679	2.6154	-88.1679
-0.7794	0.4530	0.4806	0.4806	-87.1380	2.3059	-87.1380
-0.5196	0.4654	0.4977	0.4977	-85.4652	1.9999	-85.4652
-0.2598	0.4727	0.5077	0.5077	-82.9735	1.6942	-82.9735
0.0	0.4751	0.5111	0.2654	-77.3669	0.0	0.0
0.2598	0.4727	0.5077	0.0668	-39.0543	0.0	0.0
0.5196	0.4654	0.4977	0.1378	70.6146	0.0	0.0
0.7794	0.4530	0.4806	0.2605	81.2535	0.0	0.0
1.0392	0.4350	0.4559	0.3419	84.3813	0.0	0.0
1.2990	0.4107	0.4226	0.3792	85.8676	0.0	0.0
1.5588	0.3790	0.3789	0.3722	86.7368	0.0	0.0
1.8186	0.3376	0.3220	0.3220	87.3120	0.0521	87.3120
2.0784	0.2825	0.2461	0.2461	87.7278	0.1741	87.7278
2.3382	0.2026	0.1363	0.1363	88.0336	0.1576	88.0336

***** Y= -0.0770

X	TZH	TZK	ABS(TX,TY)	ARG(TX,TY)	ABS(VX,VY)	ARG(VX,VY)
-2.3382	0.1849	0.1141	0.1141	-88.7741	4.6488	-88.7741
-2.0784	0.2701	0.2313	0.2313	-88.6521	4.0376	-88.6521
-1.8186	0.3273	0.3100	0.3100	-88.4728	3.6249	-88.4728
-1.5588	0.3698	0.3685	0.3685	-88.1628	3.2686	-88.1628
-1.2990	0.4023	0.4132	0.4132	-87.5770	2.9373	-87.5770
-1.0392	0.4271	0.4472	0.4472	-86.4774	2.6189	-86.4774
-0.7794	0.4453	0.4723	0.4723	-84.5309	2.3070	-84.5309
-0.5196	0.4580	0.4897	0.4897	-81.3491	1.9956	-81.3491
-0.2598	0.4654	0.4999	0.4999	-76.5824	1.6790	-76.5824
0.0	0.4678	0.5033	0.2951	-67.0059	0.0	0.0
0.2598	0.4654	0.4999	0.1167	-28.0250	0.0	0.0
0.5196	0.4580	0.4897	0.1483	52.2839	0.0	0.0
0.7794	0.4453	0.4723	0.2570	72.2309	0.0	0.0
1.0392	0.4271	0.4472	0.3341	78.5804	0.0	0.0
1.2990	0.4023	0.4132	0.3695	81.6169	0.0	0.0
1.5588	0.3698	0.3685	0.3613	83.3906	0.0	0.0
1.8186	0.3273	0.3100	0.3100	84.5593	0.0438	84.5592
2.0784	0.2701	0.2313	0.2313	85.4013	0.1530	85.4013
2.3382	0.1849	0.1153	0.1153	86.0161	0.1473	86.0161

***** Y= -0.1155

X	TZH	TZK	ABS(TX,TY)	ARG(TX,TY)	ABS(VX,VY)	ARG(VX,VY)
-2.3382	0.1510	0.0711	0.0711	-88.1665	4.8506	-88.1665
-2.0784	0.2481	0.2047	0.2047	-87.9903	4.0869	-87.9903
-1.8186	0.3094	0.2891	0.2891	-87.7378	3.6488	-87.7378
-1.5588	0.3541	0.3505	0.3505	-87.3148	3.2831	-87.3148
-1.2990	0.3879	0.3970	0.3970	-86.5260	2.9471	-86.5260
-1.0392	0.4135	0.4322	0.4322	-85.0418	2.6257	-85.0418
-0.7794	0.4324	0.4582	0.4582	-82.3900	2.3102	-82.3900
-0.5196	0.4453	0.4761	0.4761	-78.0063	1.9924	-78.0063
-0.2598	0.4530	0.4865	0.4865	-71.3700	1.6618	-71.3700
0.0	0.4555	0.4900	0.3397	-59.8068	0.0	0.0
0.2598	0.4530	0.4865	0.1707	-26.7440	0.0	0.0
0.5196	0.4453	0.4761	0.1645	35.4246	0.0	0.0
0.7794	0.4324	0.4582	0.2510	62.5763	0.0	0.0
1.0392	0.4135	0.4322	0.3206	72.3575	0.0	0.0
1.2990	0.3879	0.3970	0.3525	77.0915	0.0	0.0
1.5588	0.3541	0.3505	0.3422	79.8503	0.0	0.0

1.8186	0.3094	0.2891	0.2891	81.6592	0.0273	81.6590
2.0784	0.2481	0.2047	0.2047	82.9503	0.1093	82.9503
2.3382	0.1510	0.0790	0.0790	83.8840	0.1288	83.8840

***** Y= -0.1540

X	TZH	TZK	ABS(TX,TY)	ARG(TX,TY)	ABS(VX,VY)	ARG(VX,VY)
-2.0784	0.2135	0.1625	0.1625	-87.3408	4.1843	-87.3408
-1.8186	0.2825	0.2574	0.2574	-87.0290	3.6901	-87.0290
-1.5588	0.3308	0.3238	0.3238	-86.5320	3.3068	-86.5320
-1.2990	0.3667	0.3733	0.3733	-85.6292	2.9627	-85.6292
-1.0392	0.3937	0.4104	0.4104	-83.9320	2.6366	-83.9320
-0.7794	0.4135	0.4376	0.4376	-80.8637	2.3174	-80.8637
-0.5196	0.4271	0.4562	0.4562	-75.7088	1.9940	-75.7088
-0.2598	0.4350	0.4672	0.4672	-67.7751	1.6519	-67.7751
0.0	0.4376	0.4708	0.3962	-55.6524	0.0	0.0
0.2598	0.4350	0.4672	0.2275	-28.9918	0.0	0.0
0.5196	0.4271	0.4562	0.1850	19.5421	0.0	0.0
0.7794	0.4135	0.4376	0.2414	51.6465	0.0	0.0
1.0392	0.3937	0.4104	0.2995	65.2814	0.0	0.0
1.2990	0.3667	0.3733	0.3261	72.0138	0.0	0.0
1.5588	0.3308	0.3238	0.3129	75.9196	0.0	0.0
1.8186	0.2825	0.2574	0.2573	78.4443	0.0	0.0
2.0784	0.2135	0.1625	0.1623	80.2587	0.0	0.0
2.3382	0.0827	0.0247	0.0247	81.5402	0.0996	81.5401

***** Y= -0.1924

X	TZH	TZK	ABS(TX,TY)	ARG(TX,TY)	ABS(VX,VY)	ARG(VX,VY)
-2.0784	0.1584	0.0939	0.0939	-86.7044	4.4255	-86.7044
-1.8186	0.2435	0.2109	0.2109	-86.3472	3.7651	-86.3472
-1.5588	0.2982	0.2862	0.2862	-85.8198	3.3456	-85.8198
-1.2990	0.3376	0.3404	0.3404	-84.9235	2.9870	-84.9235
-1.0392	0.3667	0.3805	0.3805	-83.2858	2.6536	-83.2858
-0.7794	0.3879	0.4096	0.4096	-80.3276	2.3308	-80.3276
-0.5196	0.4023	0.4294	0.4294	-75.2952	2.0065	-75.2952
-0.2598	0.4107	0.4410	0.4410	-67.4096	1.6657	-67.4096
0.0	0.4135	0.4448	0.4448	-56.1814	1.2922	-56.1814
0.2598	0.4107	0.4410	0.2952	-37.1395	0.0	0.0
0.5196	0.4023	0.4294	0.2047	-1.6965	0.0	0.0
0.7794	0.3879	0.4096	0.2120	34.9202	0.0	0.0
1.0392	0.3667	0.3805	0.2493	54.9570	0.0	0.0
1.2990	0.3376	0.3404	0.2670	65.0925	0.0	0.0
1.5588	0.2982	0.2862	0.2493	70.8595	0.0	0.0
1.8186	0.2435	0.2109	0.1911	74.5116	0.0	0.0
2.0784	0.1584	0.0957	0.0904	77.0113	0.0	0.0

***** Y= -0.2309

X	TZH	TZK	ABS(TX,TY)	ARG(TX,TY)	ABS(VX,VY)	ARG(VX,VY)
-1.8186	0.1849	0.1399	0.1399	-85.6862	3.9349	-85.6862
-1.5588	0.2527	0.2330	0.2330	-85.1429	3.4150	-85.1429
-1.2990	0.2982	0.2956	0.2956	-84.3052	3.0261	-84.3052
-1.0392	0.3308	0.3405	0.3405	-82.8873	2.6793	-82.8873
-0.7794	0.3541	0.3726	0.3726	-80.3867	2.3505	-80.3867
-0.5196	0.3698	0.3942	0.3942	-76.1317	2.0266	-76.1317
-0.2598	0.3790	0.4068	0.4068	-69.3675	1.6948	-69.3675
0.0	0.3820	0.4109	0.4109	-59.5139	1.3409	-59.5139
0.2598	0.3790	0.4068	0.3707	-45.9109	0.0	0.0
0.5196	0.3698	0.3942	0.2403	-23.0654	0.0	0.0
0.7794	0.3541	0.3726	0.1872	10.2406	0.0	0.0
1.0392	0.3308	0.3405	0.1876	38.2080	0.0	0.0
1.2990	0.2982	0.2956	0.1896	54.3388	0.0	0.0
1.5588	0.2527	0.2330	0.1648	63.4363	0.0	0.0
1.8186	0.1849	0.1399	0.1029	69.0111	0.0	0.0

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***** Y= -0.2694
      X      TZH      TZK      ABS(TX,TY) ARG(TX,TY) ABS(VX,VY) ARG(VX,VY)
-1.5588    0.1849    0.1521    0.1521   -84.4823    3.5782   -84.4823
-1.2990    0.2435    0.2327    0.2327   -83.6963    3.0991   -83.6963
-1.0392    0.2825    0.2863    0.2863   -82.4903    2.7217   -82.4903
-C.7794    0.3094    0.3234    0.3234   -80.5040    2.3791   -80.5040
-C.5196    0.3273    0.3480    0.3480   -77.2072    2.0507   -77.2072
-C.2598    0.3376    0.3622    0.3622   -71.9587    1.7242   -71.9587
  C.0      0.3410    0.3668    0.3668   -64.1800    1.3890   -64.1800
  C.2598    0.3376    0.3622    0.3622   -53.6684    1.0382   -53.6684
  C.5196    0.3273    0.3480    0.2846   -39.0531    0.0       0.0
  C.7794    0.3094    0.3234    0.1855   -16.2500    0.0       0.0
  1.0392    0.2825    0.2863    0.1385    12.8640    0.0       0.0
  1.2990    0.2435    0.2327    0.1150    36.8044    0.0       0.0
  1.5588    0.1849    0.1521    0.0792    51.7405    0.0       0.0
  1.8186    0.0675    0.0211    0.0125    60.7811    0.0       0.0

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***** Y= -0.3079
      X      TZH      TZK      ABS(TX,TY) ARG(TX,TY) ABS(VX,VY) ARG(VX,VY)
-1.2990    0.1584    0.1319    0.1319   -83.0782    3.3061   -83.0782
-1.0392    0.2135    0.2078    0.2078   -82.0067    2.8105   -82.0067
-C.7794    0.2481    0.2554    0.2554   -80.4182    2.4291   -80.4182
-C.5196    0.2701    0.2856    0.2856   -77.9553    2.0834   -77.9553
-C.2598    0.2825    0.3027    0.3027   -74.1517    1.7515   -74.1517
  C.0      0.2865    0.3082    0.3082   -68.5338    1.4219   -68.5338
  C.2598    0.2825    0.3027    0.3027   -60.8286    1.0879   -60.8286
  C.5196    0.2701    0.2856    0.2856   -51.2779    0.7493   -51.2779
  C.7794    0.2481    0.2554    0.1864   -37.7482    0.0       0.0
  1.0392    0.2135    0.2078    0.1032   -17.7118    0.0       0.0
  1.2990    0.1584    0.1319    0.0496    7.7163    0.0       0.0
  1.5588    0.0002    0.0000    0.0       0.0       0.0       0.0

```

```

***** Y= -0.3464
      X      TZH      TZK      ABS(TX,TY) ARG(TX,TY) ABS(VX,VY) ARG(VX,VY)
-1.0392    0.0827    0.0514    0.0514   -81.5012    3.3606   -81.5012
-C.7794    0.1510    0.1453    0.1453   -80.1367    2.5782   -80.1367
-C.5196    0.1849    0.1920    0.1920   -78.2255    2.1538   -78.2255
-C.2598    0.2026    0.2163    0.2163   -75.5179    1.7885   -75.5179
  C.0      0.2081    0.2239    0.2239   -71.7461    1.4411   -71.7461
  C.2598    0.2026    0.2163    0.2163   -66.7664    1.0954   -66.7664
  C.5196    0.1849    0.1920    0.1920   -60.7860    0.7374   -60.7860
  C.7794    0.1510    0.1453    0.1263   -53.5065    0.0       0.0
  1.0392    0.0827    0.0519    0.0273   -43.4534    0.0       0.0

```

APPENDIX B

LISTING AND TEST PROBLEM FOR
SUBROUTINE FØRCES

(FØRTRAN IV G1 RELEASE 2.0)

REAL NU

TO USE SUBROUTINE FORCES, THE PARAMETERS TRANSFERRED FROM THE MAIN PROGRAM ARE (A,B,NU,UXN,UYN,PHN,NX,NY,DM) THE SUBROUTINE THEN DETERMINES THE NORMALIZED RESULTANT FORCES FXN AND FYN. THIS PROGRAM CONTAINS SUBROUTINES FORCES,MAAKZ,RCL,AND CONST

(A AND B ARE THE NORMALIZED CONTACT ELLIPSE DIMENSIONS, WHERE IF A1 AND B1 ARE THE ACTUAL DIMENSIONS THEN $A=A1/\text{SQRT}(A1*B1)$ AND $B=B1/\text{SQRT}(A1*B1)$. NOTE $A/B \geq 0.1$ NU IS POISSON'S RATIO. THIS IS THE ONLY INFORMATION NEEDED TO COMPUTE (INTERNALLY) FROM KALKER'S TABLES AND ASYMPTOTIC EXPANSIONS, (SEE SUBROUTINE CONST), THE LINEAR CREEPAGE AND SPIN COEFFICIENTS, CIJ, AND THE NORMALIZED MODULUS, GS. THE CONSTANT $GS=G*(C**3)/(RHC*N)$, WHERE $C=\text{SQRT}(A1*B1)$, $1/RHC=1/4*(1/R1+ + 1/R1- + 1/R2+ + 1/R2-)$, AND N=RESULTANT NORMAL FORCE. THESE ARE USED IN THE PROGRAM TO COMPUTE THE INVERSE STIFFNESSES SX AND SY. THEN $SX=8*A/(3*C11*GS)$, $SY=8*A/(3*C22*GS)$, $HC=32*\text{SQRT}(B/A)*C23/(3*PI*C22)$ AND $PHNN=HC*PHN$ NOTE, THE OPERATION $PHNN=HC*PHN$ IS DONE IN THE PROGRAM. THE NORMALIZED SPIN $PHN=PH*RHO/MU$ IS THE VALUE TRANSFERRED TO THE SUBROUTINE UXN AND UYN ARE NORMALIZED CREEPAGES, PHN IS THE NORMALIZED SPIN), $UXN=UX*RHO/(MU*C)$, $UYN=UY*RHC/(MU*C)$, $PHN=PH*RHO/MU$ NX,NY (LATTICE POINTS IN CONTACT REGION, $(I*A/NX, J*B/NY)$, $-NX < I < NX$, $-NY < J < NY$, ACCURACY INCREASES WITH INCREASING NX,NY TYPICAL VALUES NX=30,NY=10. MAXIMUM VALUES; NX,NY=40. FOR A/B=10 NX=40,NY=10, FOR A/B=0.1 NX=10,NY=40, FOR A/B=1 NX=NY=20, ARE TYPICAL VALUES. DM (AN INCREMENTAL STEP IN THE COMPUTATION, ACCURACY INCREASES WITH DECREASING DM, TYPICAL VALUE=0.02*A) THE NORMALIZED FORCES RETURNED ARE $FXN=FX/(MU*N)$, $FYN=FY/(MU*N)$

***** NOTE: ALL VARIABLES HAVE BEEN NORMALIZED SUCH
***** THAT THE COEFFICIENT OF FRICTION, MU, DOES NOT
***** EXPLICITLY APPEAR.

```

      READ(1,*) NV1
      DO 999 I=1,NV1
      READ(1,*) A,B,NU,UXN,UYN,PHN,NX,NY,DM
      CALL FORCES(A,B,NU,UXN,UYN,PHN,NX,NY,DM,FXN,FYN)
      WRITE(3,850)A,B,NU,UXN,UYN,PHN,NX,NY,DM,FXN,FYN
850  FORMAT(/,12X,'A=',1PE11.4,/,12X,'B=',1PE11.4,/,11X,'NU=',1PE11.4,
/,10X,'UXN=',1PE11.4,/,10X,'UYN=',1PE11.4,/,10X,'PHN=',1PE11.4,/,
$,11X,'NX=', I3,/,11X,'NY=', I3,/,11X,'DM=',1PE11.4,/,10X,'F
$XN=',1PE11.4,/,10X,'FYN=',1PE11.4,/)
999  CONTINUE
      STOP
      END

```

46.

```

C SEE "SIMPLIFIED THEORY OF ROLLING CONTACT", BY J.J. KALKER 00000001
C DELFT PRGR. REP., SERIES C: MECHANICAL AND AERONAUTICAL 00000020
C ENGINEERING AND SHIPBUILDING, 1 (1973) PP.1-10 00000030
C SUBROUTINE FORCES(A,B,NU,UXN,UYN,PHN,NX,NY,DM,FXN,FYN) 00000040
C ***** A AND B ARE THE NORMALIZED CONTACT ELLIPSE DIMENSIONS, NU 00000050
C ***** IS POISSON'S RATIO. NOTE, A/B MUST BE .GE. 0.1 00000060
C ***** UXN AND UYN ARE THE NORMALIZED CREEPAGES AND PHN IS THE 00000070
C ***** NORMALIZED SPIN. UXN=UX*RHC/(MU*C), UYN=UY*RHC/(MU*C), 00000080
C ***** PHN=PH*RHC/MU 00000090
C ***** NX AND NY ARE LATTICE POINTS IN THE CONTACT REGION, 00000100
C ***** (I*A/NX,J*B/NY),-NX .GE. I .LE. NX, -NY .GE. J .LE. NY 00000110
C ***** ACCURACY INCREASES WITH INCREASING NX,NY. TYPICAL VALUES 00000120
C ***** NX=30,NY=10, MAXIMUM VALUES NX,NY=40. DM IS A STEP 00000130
C ***** ACCURACY INCREASES WITH DECREASING DM, TYPICAL VALUE=0.02*A 00000140
C ***** NORMALIZED FORCES ARE RETURNED WHERE FXN=FX/(MU*N) 00000150
C ***** FYN=FY/(MU*N) WITH N=RESULTANT NORMAL FORCE 00000160
C REAL X(81,81),Y(81,81),VX(81,81),VY(81,81),G(81,10) 00000170
C REAL CS(3),GEL(5) 00000180
C REAL CNT(5) 00000190
C REAL UX,UY,PH,SX,SY,A,B,MUZ,FX,FY,MZ,U,UE,UE,UD,DM,AB,AD,AE,TX,TY, 00000200
C $P,Q,ARG,GR,AL,NU 00000210
C INTEGER NX,NY,I,J,MX,MY,LY,NF,NS 00000220
C DATA X,Y,VX,VY,G/6561*0.0,6561*0.0,6561*0.0,6561*0.0, 00000230
C $ 810*0.0/ 00000240
C CALL CONST(A,B,NU,C11,C22,C23,C33,GS) 00000250
C PI=3.14159 00000260
C GR=PI/180.0 00000270
C MX=3*NX 00000280
C MY=3*NY 00000290
C LY=3*NY 00000300
1022 MUZ=3.0/(2.0*PI) 00000310
C SX=8.0*A/(3.0*C11*GS) 00000320
C SY=8.0*A/(3.0*C22*GS) 00000330
C HC=32.0*SQRT(E/A)*C23/(3.0*PI*C22) 00000340
1027 GEL(5)=DM 00000350
C GEL(1)=SX 00000360
C GEL(2)=SY 00000370
C GEL(3)=A 00000380
C GEL(4)=B 00000390
C PHNN=HC*PHN 00000400
C CS(1)=UXN 00000410
C CS(2)=UYN 00000420
C CS(3)=PHNN 00000430
C CALL ROL(CS,GEL,MUZ,NX,NY,X,Y,VX,VY,G,FX,FY,MZ) 00000440
C FXN=FX 00000450
C FYN=FY 00000460
C RETURN 00000470
C END 00000480

```

	SUBROUTINE MAAKZ(P,C,WZ,DZ,C2Z,A,E,MUZ)	00000490
	REAL MUZ	00000500
	AL=C.9	00000510
	S=SQRT(1.0-AL*AL)	00000520
	AW=A*SQRT(1.0-Q*Q/B/B)	00000530
	PI=3.1415926536	00000540
	F=MUZ*PI/2.0/A/(ATAN(AL/S)+(2.0/3.0-AL+AL*AL*AL/3.0)/S)	00000550
	IF(P.LE.AL*AW) GO TO 10	00000560
	WZ=F*(AW*AW-P*P)/2.0/AW/S	00000570
	DZ=F*P/AW/S	00000580
	C2Z=F/AW/S	00000590
	GO TO 11	00000600
10	WZ=F*SQRT(AW*AW-P*P)-F/2.0*AW*S	00000610
	DZ=F*P/SQRT(AW*AW-P*P)	00000620
11	RETURN	00000630
	END	00000640

	SUBROUTINE ROL(CS,GEL,MUZ,NX,NY,X,Y,VX,VY,G,FX,FY,MZ)	00000650
	REAL MUZ,MZ,K,NU	00000660
	INTEGER PIJL	00000670
	REAL LE(81)	00000680
	INTEGER E(81)	00000690
	COMMON A,E	00000700
	DIMENSION X(81,81),Y(81,81),VX(81,81),VY(81,81),	00000710
	\$ G(81,10),CS(3),GEL(5)	00000720
	UX=CS(1)	00000730
	UY=CS(2)	00000740
	PH=CS(3)	00000750
	DM=GEL(5)	00000760
	SX=GEL(1)	00000770
	SY=GEL(2)	00000780
	A=GEL(3)	00000790
	B=GEL(4)	00000800
	H=A/LOAT(NX)	00000810
	K=B/LOAT(NY)	00000820
	PI=3.1415926536	00000830
	CZP=MUZ*2.C/A/A	00000840
	J=-NY	00000850
380	J=J+1	00000860
	IF(J.GT.NY-1)GO TC 100	00000870
	LE(J+NY+1)=A*SQRT(1.C-LOAT(J*J)/LOAT(NY)/LOAT(NY))	00000880
	P=LE(J+NY+1)	00000890
	E(J+NY+1)=C.SS*P/H	00000900
	I=-NX-1	00000910
401	I=I+1	00000920
	IF(I.GT.NX)GO TC 200	00000930
	IF(I.LT.-E(J+NY+1).CR.I.GT.E(J+NY+1))GO TC 15	00000940
	GO TC 10	00000950
15	VY(I+NX+1,J+NY+1)=C.C	00000960
	VX(I+NX+1,J+NY+1)=C.C	00000970
	Y(I+NX+1,J+NY+1)=C.C	00000980
	X(I+NX+1,J+NY+1)=C.C	00000990
10	GO TC 401	00001000
200	CONTINUE	00001010
	DO 20 I=1,10	00001020
20	G(J+NY+1,I)=-3.C*A	00001030
	GO TC 380	00001040
100	CONTINUE	00001050
	THV=-1.0	00001060
	IF(UX-PH*B.GE.C.C) THV=1.0	00001070
	THV=THV*ATAN(UY/(ABS(UX-PH*E)+1.E-08))+PI/2.0*(1.0-THV)	00001080
C		00001090
	J=-NY	00001100
470	J=J+1	00001110
	IF(J.GT.NY-1)GO TC 300	00001120
	C=J*K	00001130
	P=LE(J+NY+1)	00001140
	PG=P	00001150
	CUX=UX-PH*G	00001160
	PIJL=2	00001170
	CUY=UY+PH*P	00001180
	CALL MAAKZ(P,G,ZN,CZ,CZP,A,E,MUZ)	00001190
	XG=0.0	00001200
	YG=0.0	00001210
	XV=0.0	00001220

```

C
  YV=0.0
  WV=0.0
  I=E(J+NY+1)
  G(J+NY+1,1)=1.0
  IF(CZ+CZ-CUX+CUX/SX/SX-CUY*CUY/SY/SY.GT.0.0) GO TO 1001
  SLIP AT THE LEADING EDGE. DETERMINE THETA).
  G(J+NY+1,1)=-1.0
  T=-1.0
  IF(CUX.GE.0.0) T=1.0
  T=ATAN(CUY/(ABS(CUX)+1.E-01))+PI/2.0*(1.0-T)

C BEPT
1002 S=SIN(T)
      C=CCS(T)
      NU=(CUX*S-CUY*C-CZ*(SX-SY)*C*S)/(CUX*C+CUY*S-CZ*(SX-SY)*(C*C-S*S))
      T=T-NU
      IF(ABS(NU).GT.1.E-03) GO TO 1002
      THV=T
      C=CCS(T)
      S=SIN(T)

C THE STARTING VALUE CF T HAS BEEN FOUND. THE
C DERIVATIVE IS DETERMINED IN A SPECIAL WAY.
      TP=(PH*C+CZP*(SX-SY)*C*S)/(CUX*C+CUY*S-CZ*(3.0*C*C-1.0)*(SX-SY)
      *-SX))

C NEXT IGL
1003 D=-DM
      IF(P-DM.LE.1*H+1.E-06) D=1*H-P
      PN=P+D
      CALL MAKZ(PN,C,ZN,CZ,CZP,A,E,MUZ)
      TN=T+D*TP
      S=SIN(TN)
      C=CCS(TN)
      CUY=UY+PH*PN
      TPN=(CUX*S-CUY*C-CZ*(C*S*(SX-SY)))/ZN/(SY*C*C+SX*S*S)
      T=T+0.5*D*(TP+TPN)
      S=SIN(T)
      C=CCS(T)
      XN=ZN*C
      YN=ZN*S
      V=CUX*SY*C+CUY*SX*S-CZ*SX*SY
      IF(V.GE.-4.E-05) GC TO 1004
      AN=V/(V-V)
      IF(ABS(VV).LT.1.E-10) AN=0.5
      VV=0.0
      AV=1.0-AN
      G(J+NY+1,PIJL)=AV*P+AN*PN
      PG=G(J+NY+1,PIJL)
      PIJL=PIJL+1
      P=PG
      IF(PIJL.GT.10) PIJL=10
      XV=AV*XV+AN*XN
      XG=XV
      YV=AV*YV+AN*YN
      YG=YV
      GO TO 1006

1004 CONTINUE
C SLIP IN THE NEW PCINT
      XV=XN
      YV=YN

```

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00001800

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	P=PN	00001810
C SLIP		00001820
1005	TP=(CLX*S-CUY*C-CZ*(S*(SX-SY))/ZN/(SY*C*C+SX*S*S)	00001830
	IF(P.GT.I*H+1.E-C6) GO TO 1003	00001840
	X(I+NX+1,J+NY+1)=XN	00001850
	Y(I+NX+1,J+NY+1)=YN	00001860
	XP=-ZN*S*TP-CZ*C	00001870
	YP=ZN*C*TP-CZ*S	00001880
	VX(I+NX+1,J+NY+1)=CLX+SX*XP	00001890
	VY(I+NX+1,J+NY+1)=CUY+SY*YP	00001900
	VV=V	00001910
	I=I-1	00001920
	IF(I.LT.-E(J+NY+1)) GO TO 1007	00001930
	GO TO 1003	00001940
C ADH		00001950
1001	T=-1.0	00001960
	IF(CUX.GT.C.C) T=1.0	00001970
	THV=T*ATAN((UY+PF*P)/(ABS(CUX)+1.E-08))+PI*(1.0-T)/2.0	00001980
C ADHERE		00001990
1006	PN=I*H	00002000
	XN=CUX*(PG-PN)/SX+XC	00002010
	YN=(UY+0.5*PH*(PG+PN))*(PG-PN)/SY+YG	00002020
	CALL MAAKZ(PN,C,ZN,CZ,CZP,A,E,MUZ)	00002030
	V=ZN-SQRT(XN*XN+YN*YN)	00002040
	IF(V.GE.(-4.E-05)*MUZ)GO TO 1008	00002050
	AN=VV/(VV-V)	00002060
	IF(ABS(VV).LT.1.E-10) AN=0.5	00002070
	AV=1.0-AN	00002080
	P=AV*P+AN*PN	00002090
	G(J+NY+1,PIJL)=P	00002100
	VV=0.0	00002110
	PIJL=PIJL+1	00002120
	XV=AV*XV+AN*XN	00002130
	YV=AV*YV+AN*YN	00002140
	IF(PIJL.GT.10) PIJL=10	00002150
	T=-1.0	00002160
	IF(XV.GE.C.C) T=1.0	00002170
	CUY=UY+PF*P	00002180
	T=T*ATAN(YV/(ABS(XV)+1.E-08))+PI/2.0*(1.0-T)	00002190
	C=COS(T)	00002200
	S=SIN(T)	00002210
	CALL MAAKZ(P,Q,ZN,CZ,CZP,A,E,MUZ)	00002220
	GO TO 1005	00002230
1008	VV=V	00002240
	X(I+NX+1,J+NY+1)=XN	00002250
	XV=XN	00002260
	Y(I+NX+1,J+NY+1)=YN	00002270
	YV=YN	00002280
	VY(I+NX+1,J+NY+1)=C.C	00002290
	VX(I+NX+1,J+NY+1)=C.C	00002300
	I=I-1	00002310
	P=PN	00002320
	IF(I.GE.-E(J+NY+1)) GO TO 1006	00002330
1007	GO TO 470	00002340
300	CONTINUE	00002350
C	THE ARRAYS ARE FILLED. THE INTEGRALS ARE	00002360
C	DETERMINED	00002370
	T=4.0*K/3.0	00002380

	TN=2.C*K	00002390
	FX=0.C	00002400
	FY=0.C	00002410
	MZ=0.C	00002420
	YN=2.C*H	00002430
	J=-NY	00002440
940	J=J+1	00002450
	IF (J.GT.NY-1)GC TC 400	00002460
	I=E (J+NY+1)	00002470
	PIJL=I	00002480
	P=(LE (J+NY+1)-I*H)/2.0-H/3.C	00002490
	YP=J*K	00002500
	C=P*(X (I+NX+1,J+NY+1)+X (NX+1-I,J+NY+1))	00002510
	S=P*(Y (I+NX+1,J+NY+1)+Y (NX+1-I,J+NY+1))	00002520
	D=P*I*H*(Y (I+NX+1,J+NY+1)-Y (NX+1-I,J+NY+1))	00002530
	P=2.0*H/3.C	00002540
	I=-PIJL-1	00002550
500	I=I+1	00002560
	IF (I.GT.PIJL)GC TC 550	00002570
	C=C+P*X (I+NX+1,J+NY+1)	00002580
	D=D+P*I*H*Y (I+NX+1,J+NY+1)	00002590
	S=S+P*Y (I+NX+1,J+NY+1)	00002600
	P=YN-P	00002610
	GO TO 500	00002620
550	FX=FX+I*C	00002630
	FY=FY+I*S	00002640
	MZ=MZ+I*(C-YP*C)	00002650
	T=TN-I	00002660
	GO TO 940	00002670
400	CCNTINUE	00002680
	RETURN	00002690
	END	00002700

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C
L

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6      E(I,20)=D4(I)
      PI=3.14159
      RG=A/B
      IF(RG.GT.AR(20)) GO TO 14
      GC TC 15
14     SG=B/A
      GAM=ALOG(16.0/(SG*SG))
      C11=2.0*PI/(SG*(GAM-2.0*NU))*(1.0+(1.613706)/(GAM-2.0*NU))
      C22=((1.613706)*(1.0-NU))/(2.0*NU+GAM*(1.0-NU))
      C22=2.0*PI*(1.0+C22)/(SG*(2.0*NU+GAM*(1.0-NU)))
      C23=2.0*PI/(SG*SG*(1.0-NU))*GAM*(1.0-NU)
      C33=PI/4.0*(GAM*(1.0-2.0*NU)-2.0+6.0*NU)/(GAM*(1.0-NU)-2.0+4.0*NU)
      GS=3.0*(1.0-NU)/(4.0*PI*SQR1(SG))
      GC TC 80
15     DO 20 I=2,20
      IF(RG.LE.AR(I)) GC TC 25
20     CONTINUE
25     J=I
      DO 30 I=1,15
30     D(I)=E(I,J-1)+(E(I,J)-E(I,J-1))*(RG-AR(J-1))/(AR(J)-AR(J-1))
      AL 40 I=1,5
      AL(I)=8.0*(D(3*I)-2.0*(D(3*I-1)+D(3*I-2)))
      BE(I)=2.0*(D(3*I)+4.0*(D(3*I-1)-3.0*(D(3*I-2)))
40     CNT(I)=AL(I)*NU*2+BE(I)*NU+C(3*I-2)
      C11=CNT(I)
      C22=CNT(2)
      C23=CNT(3)
      C33=CNT(4)
      GS=CNT(5)
80     CONTINUE
      RETURN
END

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00003300
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00003370
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00003590
00003600

```

A= 2.5980E+00
B= 3.8490E-01
NU= 2.8000E-01
UXN= 0.0
UYN=-1.4000E+00
PHN= 8.0000E-01
NX= 10
NY= 10
DM= 4.0000E-02
FXN= 8.9034E-07
FYN=-3.4703E-01

A= 2.5980E+00
B= 3.8490E-01
NU= 2.8000E-01
UXN= 0.0
UYN=-1.4000E+00
PHN= 8.0000E-01
NX= 40
NY= 20
DM= 4.0000E-02
FXN= 8.6427E-07
FYN=-3.4689E-01

Users' Manual for Kalkers
NonLinear Creep Theory

Users' Manual for Kalkers Simplified
NonLinear Creep Theory (Interim Report),
1977
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