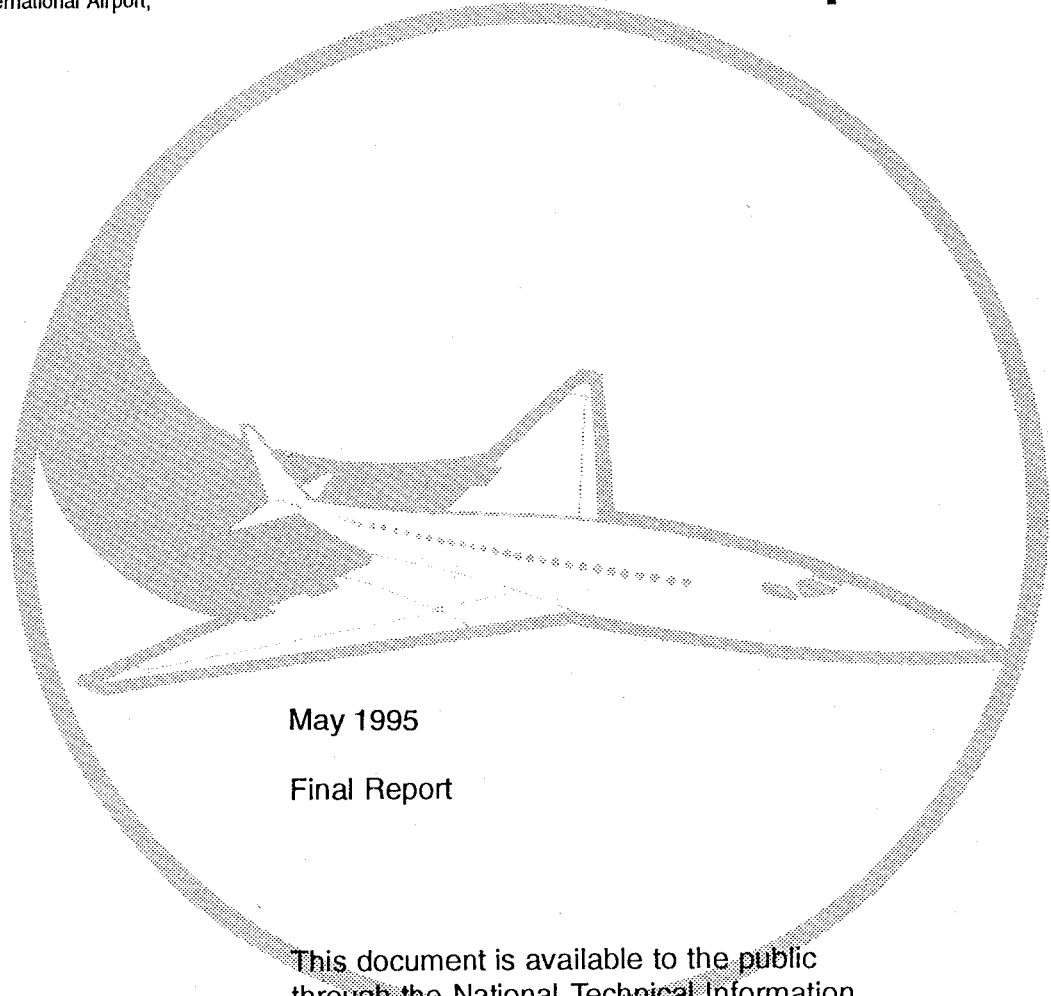


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Fatigue Reliability Method With In-Service Inspections



May 1995

Final Report

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EXECUTIVE SUMMARY

The first order reliability method (FORM) has traditionally been applied to probabilistic fatigue analyses for single inspection intervals. Accounting for inspections then requires several sequential FORM analyses and that the random distribution of crack lengths be recharacterized following each inspection. The augmented FORM presented here allows the reliability analysis to span several inspection periods without explicit characterization of the crack length distribution subsequent to each inspection. The method thereby preserves the attractive feature of FORM in that relatively few realizations in the random variable space need to be considered. Examples are given which show that the present methodology gives estimates which are in good agreement with Monte Carlo simulations and is efficient even for complex components.

1. INTRODUCTION.

Probabilistic fatigue methods are often applied in a setting where critical structural components are subjected to inspections by non-destructive evaluation (NDE) techniques so that crack can be identified and the component can be repaired or replaced. Inspections can significantly reduce the probability of fatigue failure of structures, as well as increase the useful service lives. Quantified measures of reliability (provided by probabilistic methods) can optimize the inspection schedule and allow comparison of effectiveness for various inspection methods.

A risk analysis methodology for the assessment of structural integrity of aircraft structures has been outlined by Berens, et al. (1991). This methodology, which is based on the direct integration of the probability of failure, works well when the number of random variables is relatively small and a single parameter characterization of crack size is adequate. However, when it is useful to characterize crack growth in detail, other modeling techniques such as Monte Carlo simulations (MCS) and the first order reliability method (FORM) are needed for calculating probabilities of failure (see Harkness, et al. (1992) and references therein). Traditionally, these latter techniques have been applied to one inspection interval at a time. However, that approach requires characterization of the crack size distribution (i.e., crack size probability density function) following each inspection, which is difficult. Techniques for recharacterizing crack size distributions after each inspection with FORM are discussed in Rahman and Rice (1992).

An alternative approach is given here which does not require recharacterization of the crack size distribution. The first order reliability method is augmented to account for the effects of the inspections so that the crack size distribution need only be characterized at an initial state. This is of considerable advantage since recharacterizations of the crack length distribution are often tedious or impractical. In the present work, it is assumed that components with detected cracks are repaired such that their subsequent likelihood of failure is negligible.

In Section 2, the standard FORM and its application to fatigue reliability are reviewed. The introduction of non-destructive evaluation (NDE) into the fatigue reliability problem is discussed in Section 3 along with a description of the augmentation of

FORM to efficiently treat multiple inspections. The first part of section 3 is devoted to identifying the quantities of interest in a fatigue reliability analysis with in-service inspections; techniques to evaluate these quantities are given in the remainder of the section. An important aspect of FORM is finding the so-called design points. An algorithm for this task which is applicable to both the standard FORM and the augmented FORM is provided in the Appendix A.

Two numerical examples are presented in Section 4. The first example investigates the accuracy of augmented FORM by comparison to MCS results. A more complex fatigue problem is studied in the second example where the MCS approach is not computationally feasible; here the augmented FORM requires only minutes of CPU time on a workstation. In these examples, the inspection schedule is adjusted so that inspections occur when the probability of failure reaches a specified value. Further discussion and concluding remarks are given in Section 5.

2. FIRST ORDER RELIABILITY METHOD.

2.1 STANDARD RELIABILITY STATEMENT.

We begin by defining a performance function, $G(x)$, which distinguishes between safe and unsafe realizations of the random variables x (see figure 1a). Performance functions are typically defined so that positive outcomes indicate safe realizations and negative outcomes indicate unsafe realizations (the limit case of $G(x)=0$ is often included in the failure domain). The objective of a reliability analysis is to determine the probability of failure

$$P_f = P[G(x) \leq 0], \quad (2.1)$$

which is given by

$$P_f = \int_{\Omega_f^*} f_x(x) dx, \quad (2.2)$$

where Ω_f^x is the failure space ($G(x) \leq 0$), and $f_X(x)$ is the joint probability density function for realizations in the space of random variables x .

The random variables x are often non-normally distributed, rendering the integral in equation (2.2) difficult to evaluate. To circumvent this difficulty, random variables x are mapped to standardized equivalent normal random variables r , where each component r_i is an independent Gaussian variable with zero mean and unit variance. This mapping can be achieved via the Rosenblatt transformation (see Rosenblatt 1952; Ang and Tang 1984)

$$r_i = \Phi^{-1} \left[F_i(x_i | x_1, x_2, \dots, x_{i-1}) \right], \quad (2.3)$$

where F_i is the conditional cumulative probability at x_i given $x_1, x_2, \dots, x_{i-1}$. In standardized space, the performance function is transformed to $g(r) = G(x(r))$ and equation (2.2) becomes

$$P_f = \int_{\Omega_f^r} f_R(r) dr, \quad (2.4)$$

where Ω_f^r is the failure space ($g(r) \leq 0$) as in figure 1b, and $f_R(r)$ is the joint probability density function for realizations of random variables r . The joint probability density function, $f_R(r)$, is simply the product of the probability density functions for all random variables r_i and is given by

$$f_R(r) = \prod_{i=1}^n \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}r_i^2\right), \quad (2.5)$$

The essence of FORM is to approximate the limit surface $g(r)=0$ by a tangent hyperplane at the most likely point of failure, r_d . In standard FORM the most likely failure point is the point which minimizes the distance $|r|$ to the failure surface. The resulting "first order" estimate of the probability of failure is then given by

$$P_f^1 = \Phi(-\beta), \quad (2.6)$$

where $\beta = |r_d|$ and Φ is the standard normal cumulative distribution function. Because the random variables r are Gaussian and normal, the decay of $f_R(r)$ is exponential and identical for all random variables. Therefore, the closest point is the point at which failure is most probable and it provides a good point for approximating the failure surface to calculate the probability of failure using standard FORM. It will be shown in Section 3.3 that the closest point may not be the most likely failure point when in-service inspections are accounted for.

2.2 FATIGUE RELIABILITY.

In a fatigue setting, the failure set contains all realizations that result in fatigue lives less than a desired service life, so an appropriate performance function is

$$G(x) = N_f(x) - N, \quad (2.7)$$

or in standard Gaussian space

$$g(r) = N_f(x(r)) - N, \quad (2.8)$$

where N_f is the fatigue life (which is influenced by several random variables) and N is the desired service life. Note that the fatigue life may be defined as the number of cycles for a crack to reach a specified critical size, which may not necessarily correspond to catastrophic failure of the component. Failure is deemed to occur when the fatigue life is shorter than the desired service life ($g < 0$). Standard FORM described above is often effective for estimating failure probability versus service life in the absence of inspections or for estimating the probability of failure with inspections when the crack length distribution following an inspection is known. In the latter, a standard FORM analysis is performed for each inspection interval, and the probability of failure is determined in each analysis. This approach requires knowledge of the crack size distribution following each inspection. A more efficient method for treating inspections is introduced in the following Section.

3. FATIGUE RELIABILITY AND IN-SERVICE NDE INSPECTIONS.

Consider a fatigue reliability setting where the component is subjected to in-service NDE inspections according to some schedule. The inspection schedule may be prescribed and the failure probabilities sought or conversely, the aim of the analysis may be to determine an inspection schedule which will keep failure probabilities below a specified level. Let N denote the service life (in cycles) and N_I denote the number of cycles to the inspection prior to N . We seek a method to determine the fatigue reliability (or alternatively failure probability) over the service life, N . As mentioned previously, standard FORM techniques require a complete characterization of the probability density function for the crack size following each inspection. However, an exact determination of the crack size probability density function may be extremely tedious or impractical to obtain especially for cracks in complicated geometries. Rahman and Rice (1992) discuss a method based on the standard FORM to recharacterize the crack size distribution at each inspection, but this method may require extensive computations. In the following subsection, we introduce a straightforward and efficient method based on the initial crack size distribution for determining fatigue reliability with NDE inspections; explicit knowledge of the crack size distribution at each inspection is not required.

3.1 STATEMENT OF RELIABILITY PROBLEM WITH NDE INSPECTIONS.

The probability of failure is defined here as the probability that the service life of the component exceeds the fatigue life, where the fatigue life is determined strictly by undetected cracks. Thus the probability of failure, P_f , after N fatigue cycles, can be written as

$$P_f(N) = P\left[\left(N_f \leq N\right) \cap \left(O_i = 0, i = 1, 2, \dots, I\right)\right], \quad (3.1)$$

where I is the number of previous inspections at N cycles ($N > N_I$), O_i indicates the outcome of the i -th inspection which is given by:

$$O_i = \begin{cases} 0 & \text{no crack detected in } i^{\text{th}} \text{ inspection} \\ 1 & \text{crack detected in } i^{\text{th}} \text{ inspection} \end{cases} \quad (3.2)$$

and N_f is the fatigue life. The outcome of each inspection is random due to uncertainties in the inspection technique and the crack length at the time of the inspection. Other functions which may be of particular interest can be derived from P_f . The hazard function (or failure rate) $h(N)$ is given by

$$h(N) = \frac{1}{[1-P_f(N)]} \frac{\partial P_f(N)}{\partial N}, \quad (3.3)$$

The factor $[1-P_f]$ in equation 3.3 is typically very close to unity in a reliability analysis, so the hazard function is effectively equal to the derivative of the probability of failure. Another useful failure probability is the probability of failure since the last inspection (due to an undetected crack), i.e.,

$$P_{\#}(N) = P[(N_I \leq N_f \leq N) \cap (O_i = 0, i = 1, 2, \dots, I)], \quad (3.4)$$

where I is the number of previous inspections at N cycles, and N_I and O_i are as previously defined.

As in the standard FORM the random variable space can be transformed to standardized Gaussian variable space. The probability density function for realizations of r in this space is $f_R(r)$ (see equation 2.5). To derive an expression for the probability of failure, P_f , at cycle N which accounts for inspections, we require the probability density function associated with realizations, r , for which the associated cracks are undetected. The probability density function for realizations with undetected cracks after the first I inspections, is given by

$$f_U(r) = f_R(r)P_{nd}(r, N). \quad (3.5)$$

Here, $P_{nd}(r, N)$ is the probability that cracks associated with the realization r are not detected in all of the inspections prior to the current cycle, N , i.e., $P_{nd}(r, N) = P[O_i = 0 \text{ for } i = 1, 2, \dots, I \mid r]$, and is given by

$$P_{nd}(r, N) = \prod_{i=1}^I \{1 - POD[a(r, N_i)]\}, \quad (3.6)$$

where $POD[a(r, N_i)]$ is the probability of detection for the inspection method and $a(r, N_i)$ is the crack length upon the i^{th} inspection for the realization r .

The probability of failure is therefore given by

$$P_f(N) = \int_{\Omega_f} f_R(r) P_{nd}(r, N) dr, \quad (3.7)$$

where Ω_f includes all r such that $g \leq 0$ (i.e. $N_f \leq N$). The probability of failure since the last inspection is given by

$$P_{fl}(N) = \int_{\Omega_{fl}} f_R(r) P_{nd}(r, N) dr, \quad (3.8)$$

where Ω_{fl} includes all r such that $N_I < N_f \leq N$. Note that the computation of N_f is not influenced by the inspections, i.e., the failure surface $g(r)=0$ is not influenced by in-service inspections. Rather, it is the integrands in equations (3.7) and (3.8) which incorporate the effect of inspections on failure probability.

3.2 EVALUATION OF THE INTEGRAL FOR FAILURE PROBABILITY.

The integrals in Eqs. (3.7) and (3.8) differ from that which arises in standard FORM in that the integrands have been multiplied by the non-Gaussian function $P_{nd}(r, N)$ and, therefore, the integration technique must be modified to accurately integrate the functions. A simple modification of the standard FORM integration procedure is introduced here for the evaluation of the integral in equation (3.7). As in standard FORM, the failure surface is approximated by a tangent hyperplane at the most likely failure point, r_d (the so-called design point). The sharp variation of $P_{nd}(r, N)$ in the direction of the gradient of g prevents the direct evaluation of the integral via the standard normal cumulative distribution function, as in equation (2.6). Nevertheless, we wish to maintain the essential structure of the FORM integration procedure and, thus, it is convenient to discretize the domain Ω_f into subdomains as shown in figure 2. By approximating $P_{nd}(r, N)$ as constant over each subdomain we obtain

$$P_f(N) \approx \sum_{j=1}^{n_s} \left[\bar{P}_{nd}^j \int_{\Omega_j} f_R(r) d\Omega \right], \quad (3.9)$$

where n_s is the number of subdomains and $\bar{P}_{nd}^j$ is the approximation of $P_{nd}(r, N)$ in the subdomain Ω_j . The remaining integrands are the Gaussian distributed $f_R(r)$, so the standard cumulative normal function Φ is useful for "first order" approximations of these integrals. Thus, equation (3.9) leads to approximations of the form

$$P_f^1(N) = \sum_{j=1}^{n_s} \left\{ \bar{P}_{nd}^j \left[\Phi(-|r^j|) - \Phi(-|r^{j+1}|) \right] \right\} + \bar{P}_{nd}^{n_s} \Phi(-|r^{n_s}|), \quad (3.10)$$

where r^j is the integration point on the surface of the subdomain Ω_j as shown in figure 2. In this numerical integration scheme, the design point is taken as the first integration point, i.e., $r^1 = r^d$. Subsequent integration points are found by

$$r^j = [1 + (j-1)\delta] r^d, \quad (3.11)$$

where r^j is the position of the j^{th} integration point and δ is the desired step size for the integration. The number of subdomains, the magnitude of δ , and the location, r^{n_s} of the last integration point depend on the accuracy desired and can be deduced through numerical experimentation and comparison with known solutions. A non-uniform subdomain discretization can be used in place of equation (3.11) if desired.

3.3 LOCATION OF DESIGN POINTS.

In standard applications of FORM, the most likely failure point can be shown to be the point closest to the origin on the surface $g=0$ (Ang and Tang, 1984). The approximation of the failure surface as a tangent hyperplane at the design point leads to the first order approximation in equation (2.6). In the present application of FORM with in-service inspections, the integrand in equation (3.7) contains the non-Gaussian variable $P_{nd}(r, N)$ and the most likely failure point is not, in general, the closest point to the origin on $g=0$. The question then arises as to which point provides the most appropriate first order approximation to the failure surface, i.e., where to locate the design point. One possibility is to take the closest point to the origin on $g=0$ as in standard FORM. However, due to variations in $P_{nd}(r, N)$ along the surface $g=0$, an alternative point is suggested which will maximize the integrand in equation (3.7).

A procedure for locating the design point at the most likely failure point on $g=0$ is introduced here. By suitable choice of parameters, the algorithm can also be used to locate the design point at the closest point to the origin. Consider β such that

$$\Phi(-\beta) = P_{nd}(r, N)\Phi(-|r|). \quad (3.12)$$

The design point is then taken to be the point on the surface $g=0$ which minimizes β (or maximizes the product $P_{nd}(r, N)\Phi(-|r|)$). Typically, the variation in $P_{nd}(r, N)$ is small in the direction of constant g and the design point and the closest point are nearly coincidental. However, in some problems, a significant difference in the location of these points is found. The Rackwitz algorithm (Rackwitz and Fiessler, 1978) can be used to find the design point for standard FORM, but this algorithm may fail to converge for high values of β if the failure surface is not relatively flat. A variation of the Rackwitz algorithm which corrects for this problem and has been generalized for use with both standard and augmented FORM is presented in the Appendix A.

4. NUMERICAL EXAMPLES.

4.1 EDGE CRACK.

We first consider the fatigue of an edge crack in a semi-infinite plate to investigate the accuracy of the augmented FORM based on comparisons with Monte Carlo simulations (MCS). It is assumed that the cracks propagate according to the Paris model (Paris and Erdogan, 1963)

$$\frac{da}{dN} = D(\Delta K)^m, \quad (4.1)$$

where da/dN is the rate of crack growth, D and m are material parameters, and ΔK is the range of the stress-intensity factor. The stress-intensity factor range for an edge crack of length a is given by $\Delta K = 1.12\sigma(\pi a)^{1/2}$ where σ is the amplitude of the applied stress. Using this relation, equation 4.1 can be integrated to obtain the number of cycles for a crack of initial length a_i to grow to a crack of length a_f

$$N_f = \frac{a_f^{\kappa} - a_i^{\kappa}}{D\kappa(1.12\sigma\sqrt{\pi})^m} \quad \text{for } m \neq 2 \quad (4.2)$$

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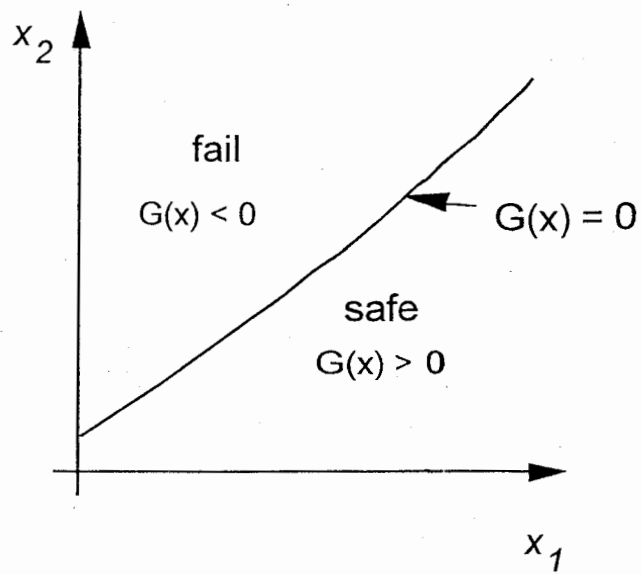


Figure 1a. Failure surface in original space

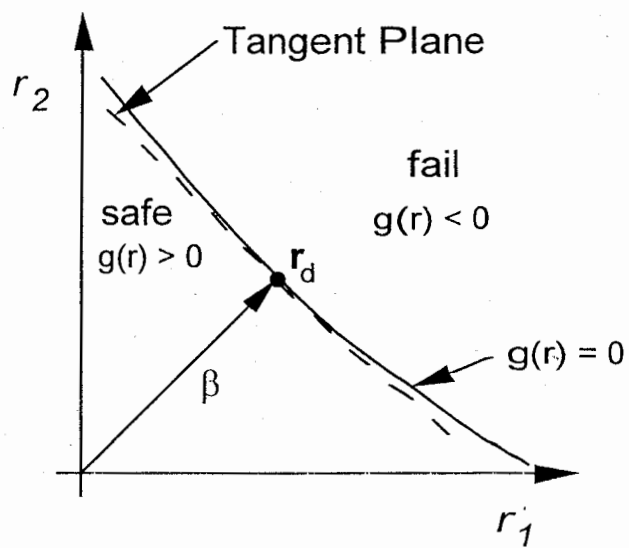


Figure 1b. Failure surface in standardized space

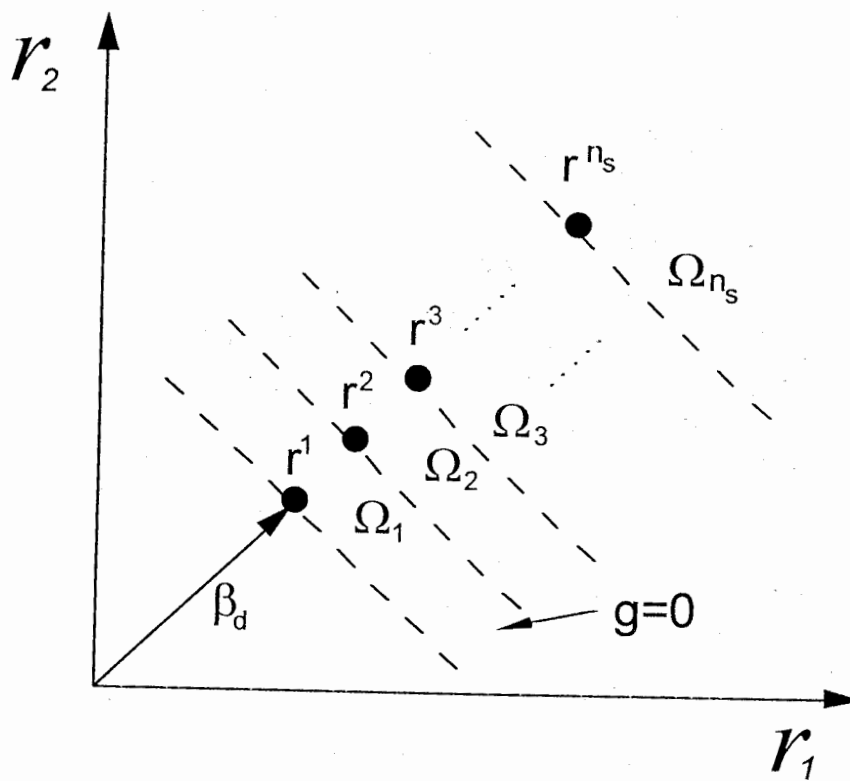


Figure 2. Modified FORM integration scheme

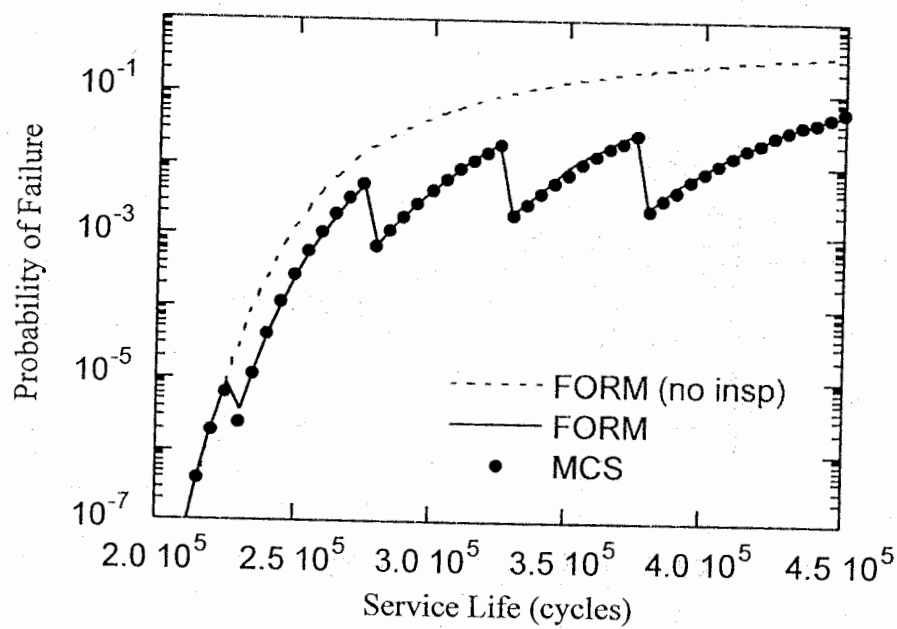


Figure 3. Probability of Failure for test case with and without inspections.

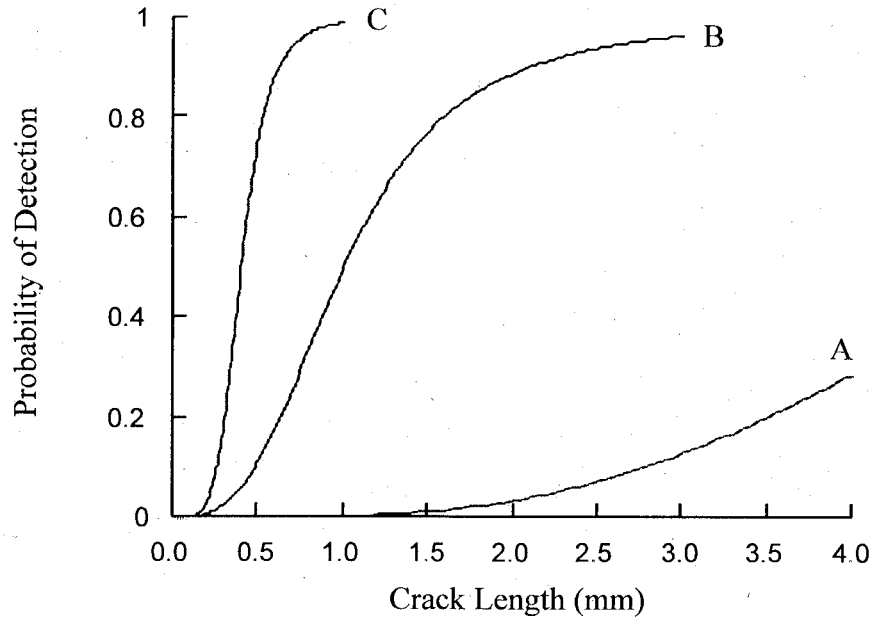


Figure 4. Probability of detection curves (A: $\alpha=0.0032 \text{ mm}^{-\beta}$, $\beta=3.5$; B: $\alpha=1 \text{ mm}^{-\beta}$, $\beta=3$; C: $\alpha=100 \text{ mm}^{-\beta}$, $\beta=5$)

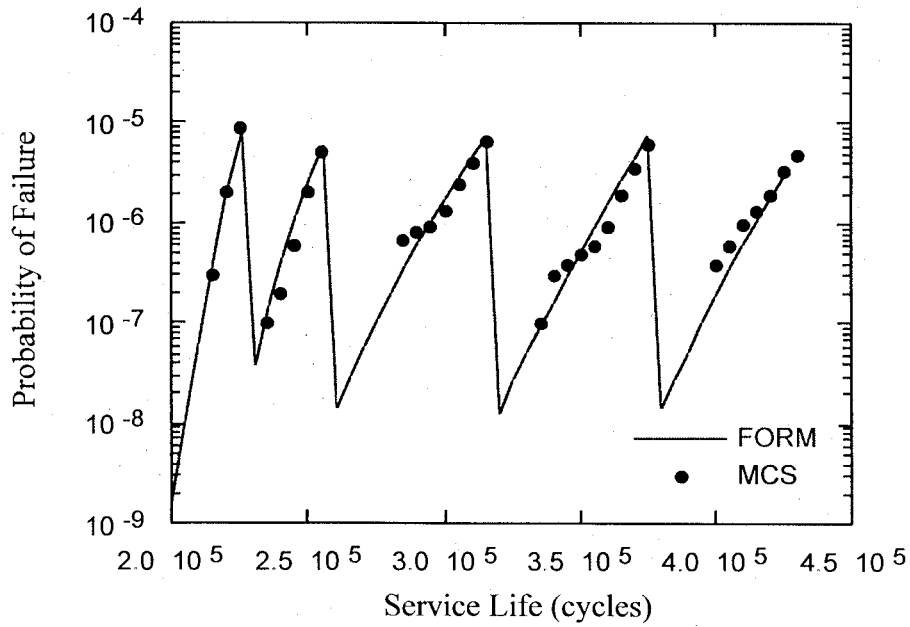


Figure 5. Results for edge crack using augmented FORM and uneven inspection intervals.

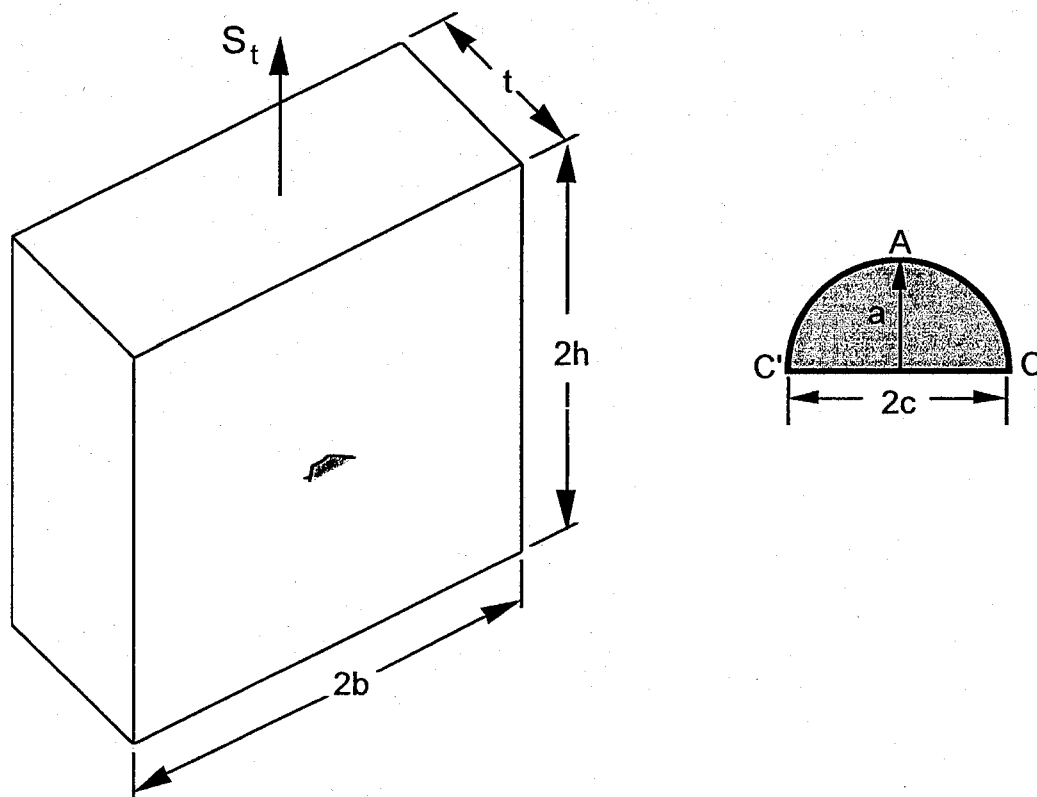


Figure 6. Rectangular plate with surface-breaking semi-elliptical crack.

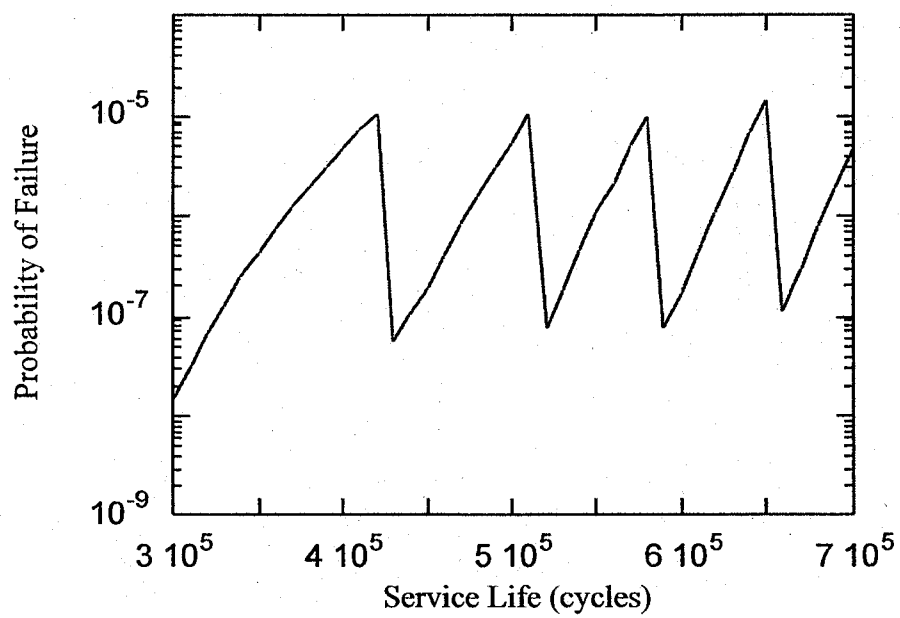


Figure 7. Probability of failure for surface-breaking crack in a plate with inspections.

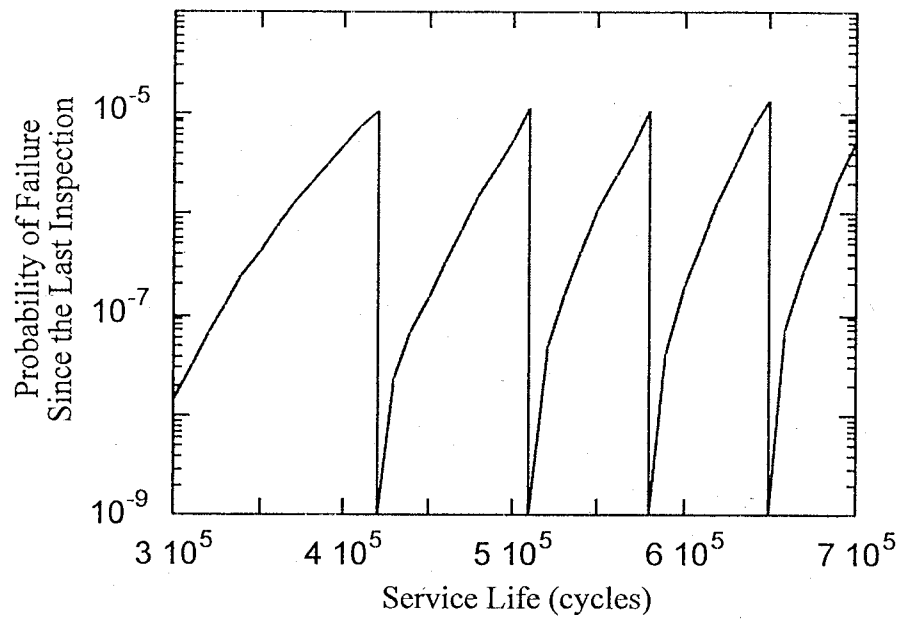


Figure 8. Probability of failure since the last inspection for surface-breaking crack in a plate

APPENDIX A: ALGORITHM TO LOCATE DESIGN POINTS.

A variation on the Rackwitz algorithm (Rackwitz and Fiessler, 1978) is presented in the following four steps (Harkness, 1993):

1) define failure function $G(x)$; initialize iteration count: $v=0$; estimate design point coordinates r_i^0 ; transform r_i^0 to x_i^0 .

2) evaluate g , $\frac{\partial \beta}{\partial r_i}$, and $\frac{\partial G}{\partial r_i}$ at x_i^v , r_i^v and compute:

$$I_i = -\lambda \frac{\partial G}{\partial r_i} \bigg/ \frac{\partial \beta}{\partial r_i}, \text{ where } \lambda = |\nabla_r \beta| / |\nabla_r g|$$

3) increment iteration count: $v=v+1$; update r_i values in two steps:

a) $r_i^{temp} = r_i^{v-1} - (G - G^*) \frac{\partial G}{\partial r_i} \bigg/ (\nabla_r G)^2$; adjusts magnitude of r

b) $r_i^v = r_i^{temp} (\xi + (1 - \xi) I_i)$, $0 < \xi < 1$; adjusts direction of r

4) transform r_i^v to x_i^v ; check for convergence:

If not converged, go to step 2.

If converged, design point found; reliability index $\beta_d = |r^v|$.

This algorithm minimizes β on any surface of constant g (equal to g^*). Other distinctions between this algorithm and the Rackwitz algorithm are:

1) $G=G^*$ is not enforced on each iteration. Instead, step 3a just brings r^v toward the limit surface. In practice, $G \approx G^*$ after several iterations (i.e., after $v > 4$) with the algorithm presented above.

2) The iteration parameter ξ is introduced to avoid the large angular corrections in r^v which lead to non-convergence.

An intermediate value of the iteration parameter, such as $\xi=0.7$, is recommended. For practical purposes, convergence can be assumed to have occurred when each of the standardized variables changes by less than 0.01 in an iteration. Convergence is typically achieved within ten to twenty iterations with this algorithm, and lack of convergence is uncommon.

The algorithm can be used to find design points for standard or augmented FORM. The algorithm calls for partial derivatives of $\beta = |r|$ and G with respect to r_i . For standard FORM, these derivatives are given by

$$\frac{\partial \beta}{\partial r_i} = \frac{r_i}{\beta} \quad (\text{A.1})$$

$$\frac{\partial G}{\partial r_i} = \frac{\partial G}{\partial x_i} \frac{\partial x_i}{\partial r_i} \quad (\text{A.2})$$

If $G = N_f - N$ and the desired service life is deterministic, then

$$\frac{\partial G}{\partial x_i} = \frac{\partial N_f}{\partial x_i} \quad (\text{A.3})$$

and

$$\frac{\partial x_i}{\partial r_i} = \sigma_i^N, \quad (\text{A.4})$$

where σ_i^N are the equivalent normal standard deviations in the Rosenblatt transformation (see Ang and Tang, 1984). With these substitutions,

$$\frac{\partial G}{\partial r_i} = \frac{\partial N_f}{\partial x_i} \sigma_i^N. \quad (\text{A.5})$$

For augmented FORM, the derivatives of β with respect to r_i are modified to account for the affect of the $P_{nd}(r, N)$ function on the design points (see Section 3.3). The derivatives can now be calculated as

$$\frac{\partial \beta}{\partial r_i} = \frac{r_i}{\beta} + \frac{R}{\beta} \frac{\partial R}{\partial r_i} \quad (\text{A.6})$$

where $R = \sqrt{\beta^2 - |r|^2}$. The derivatives of the “augmented component” R can be calculated using finite differences.

For both standard and augmented FORM, $|\nabla_r \beta| = 1$.

