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AIRPORT FACILITY QUEUING MODEL VALIDATION

Li Shin Yuan
Lawrence J. McCabe

U.S. Department of Transportation
Transportation Systems Center
Kendall Square
Cambridge MA 02142

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INTERIM REPORT

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16. Abstract

Criteria are presented for selection of analytic models to represent waiting times due to queing processes. An existing computer model by M.F. Neuts which assumes general nonparametric distributions of arrivals per unit time and service times for a single service was envisioned as best fulfilling requirements.

Data obtained from Denver Stapleton Airport were applied to this model. Service times and arrival rates at an express baggage check facility, a security station, and a gate were used as inputs. Delay times corresponding to the observed arrival rates were recorded and compared to model outputs. Using the T-test, agreement was obtained at the 5 percent level of significance for the mean values of the first two facilities. Predictions of waiting time distribution, however, did not pass the Kolmogroff test at the same level of significance. Discrepancies are due to a lack of time resolution in arrival times and the application of this model to multiserver situations.

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PREFACE

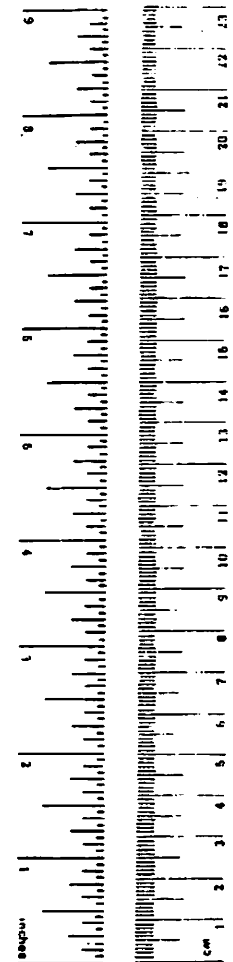
This study was undertaken as part of Project Plan Agreement FA-632. One objective of this program is the development and testing of techniques to evaluate airport landside congestion parameters such as delay. This analysis provides a method of determining delays arising from queuing. Results contained in this report should be considered as preliminary and indicative of further development, especially where multi-server landside facilities require analysis.

The authors would like to express their appreciation to the Program Manager, Mr. Mark Gorstein, for his support and constructive criticsm.

METRIC CONVERSION FACTORS

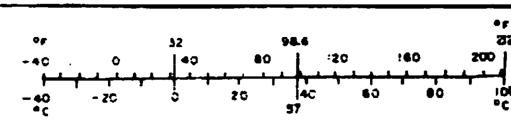
Approximate Conversions to Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
	LENGTH			
in	inches	2.5	centimeters	cm
ft	feet	30	centimeters	cm
yd	yards	0.9	meters	m
mi	miles	1.6	kilometers	km
	AREA			
in^2	square inches	6.5	square centimeters	cm^2
ft^2	square feet	0.09	square meters	m^2
yd^2	square yards	0.8	square meters	m^2
mi^2	square miles	2.5	square kilometers	km^2
	acres	0.4	hectares	ha
	MASS (weight)			
oz	ounces	28	grams	g
lb	pounds	0.45	kilograms	kg
	short tons (2000 lb)	0.9	tonnes	t
	VOLUME			
tsp	teaspoons	5	milliliters	ml
Tbsp	tablespoons	15	milliliters	ml
fl oz	fluid ounces	30	milliliters	ml
c	cups	0.24	liters	l
pt	pints	0.47	liters	l
qt	quarts	0.95	liters	l
gal	gallons	3.8	liters	l
ft^3	cubic feet	0.03	cubic meters	m^3
yd^3	cubic yards	0.76	cubic meters	m^3
	TEMPERATURE (exact)			
°F	Fahrenheit temperature	5/9 (after subtracting 32)	Celsius temperature	°C



Approximate Conversions from Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
	LENGTH			
mm	millimeters	0.04	inches	in
cm	centimeters	0.4	inches	in
m	meters	3.3	feet	ft
m	meters	1.1	yards	yd
km	kilometers	0.6	miles	mi
	AREA			
cm^2	square centimeters	0.16	square inches	in^2
m^2	square meters	1.2	square yards	yd^2
km^2	square kilometers	0.4	square miles	mi^2
ha	hectares (10 000 m^2)	2.5	acres	
	MASS (weight)			
g	grams	0.035	ounces	oz
kg	kilograms	2.2	pounds	lb
t	tonnes (1000 kg)	1.1	short tons	
	VOLUME			
ml	milliliters	0.03	fluid ounces	fl oz
l	liters	2.1	pints	pt
l	liters	1.06	quarts	qt
l	liters	0.26	gallons	gal
m^3	cubic meters	35	cubic feet	ft^3
m^3	cubic meters	1.3	cubic yards	yd^3
	TEMPERATURE (exact)			
°C	Celsius temperature	9/5 (then add 32)	Fahrenheit temperature	°F



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EXECUTIVE SUMMARY

An analytical queuing model was examined for accuracy in predicting one of the major measures of airport congestion, namely, the waiting time at a landside facility. The particular model chosen is applicable to a single channel server whose arrival and service time processes have general distributions. This model was selected because

a. Arrival time distributions obtained from measurements were tested and found to be non-Poisson.

b. A potentially wide range of applicability was desired because of the many service facilities present at the landside.

c. This model was available as a computer program which furnished waiting time and queue length distributions directly.

In order to investigate the feasibility of using this queuing model as a tool for evaluating landside congestion, statistical comparisons were performed to determine whether there is any correspondence between observed field data and model predictions. Using observed passenger arrival rates and service time distributions obtained at Denver-Stapleton Airport as the inputs, the model validation was done by comparing predicted waiting times at the security station, express check-in, and one boarding gate. Using the t-test, agreement was obtained at the 5 percent level of significance for the mean values of the first two facilities. More comprehensive data collection is required to validate the probability distributions of the waiting time and queue length.

This single channel queuing model may be viewed as the first step in evaluating landside congestion using an analytical approach. Further developments in this area would include multi-channel facility modeling and the linking of various facilities to formulate a congestion model for the complete landside.

1. INTRODUCTION

Some of the Denver-Stapleton International Airport passenger data which were collected for simulation model validation were used to determine the validity of an analytic model. This was done by comparing the analytically computed queue length and waiting time distributions with the actual observed queue length and waiting time distributions in the field. It was hoped that this validation work would lead to the determination of the feasibility of using analytic models for estimating the level-of-service of the landside facility.

The input data for the analytic model are the actual arrival rates and the actual service time as observed in Stapleton International Airport in Denver. The outputs of the model are the queue length and waiting time distributions computed for each facility under study. The waiting time output was compared with its field observed counterpart for model validation. It is believed that this quantity is one of the most meaningful indicators of the level of service.

2. THE QUEUING MODEL

2.1 MODEL SELECTION

There are six basic characteristics which can be used to specify a queuing model:

1. Arrival Pattern: The arrival pattern of the "customers" could be deterministic (D) or described parametrically as a Poisson Distribution (or exponential arrival time) (M) or general (G). For many airport landside facilities the arrival pattern is most likely general (G).

2. Service Pattern: The service time pattern of each server channel could also be deterministic (D), exponential (M), or general (G). In the airport landside situation it is most likely general (G).

3. Number of Service Channels: The queuing process may take place at a facility with a single channel or a number of parallel channels. In the airport landside both the single and parallel channel cases are applicable.

4. System Capacity: Facilities may accommodate a limited queue size or the maximum queue size may be unbounded. Since it is most likely that no airport landside customers will be turned back, the maximum possible queue length may be considered infinite for most queuing situations of interest.

5. Number of Service Stages (in series per each channel): Each channel may have a single stage or multiple stages connected in series. Both single stage and multi-stage processes are manifested by the airport landside.

6. Queue Discipline: Queue discipline can be classified into (a) first-come-first-served (FCFS), alias first-in-first-out (FIFO); (b) service in random (SIRO); (c) last-come-first-served (LCFS); (d) priority (PRI); and (e) general discipline (GD). The airport landside system operates mostly on a first-come-first-served (FCFS or FIFO) basis, with some priority (PRI) treatments.

Based on the six basic characteristics, the <u>most desirable</u> model for the airport landside system would be a model with (1) general arrival pattern, (2) general service time pattern, (3) multiple channels, (4) infinite or finite maximum queue length, (5) single or multiple stages, per channel, and (6) first-come-first-served and/or priority service discipline. In queuing theory notation this <u>most desirable</u> model is represented as:

$$G/G/N_1/\binom{\infty}{N_2}/\binom{FCFS}{PRI} \text{ with multiple-stage per channel}$$

It should be realized that the so called most desirable model is probably the most complicated model one would ever attempt to utilize.

The development of analytic queuing models appear to have been influenced strongly by two factors: the origins of models in the study of congestion in telephone systems and the question of what is easy and possible in mathematical analysis. Therefore, many assumptions commonly made in queuing analyses are precisely those which seem reasonable in constructing queuing models associated with telephone systems and those which make the models mathematically solvable. As a result, analytic models which deal with multiple channels or servers are generally restricted to Poisson arrivals and exponential service times (i.e., M/M/N). Models which deal with general arrivals and general service times are presently limited to single channel (G/G/1) cases.

The simplifying assumptions used in M/M/N model development are not always applicable to queues other than those encountered in telephone systems; they are particularly inappropriate when the customers and servers are really people as in the airport landside. In a telephone system the process of switching channels or jockeying is carried out automatically and rapidly, whereas in many multi-counter operations there are no easily defined rules, and people move with inertia. This was observed visually at Denver-Stapleton International Airport where queues were formed at ticket counters at the same time that servers were observed idle. This point was further verified numerically by the field data. Out of five sets of arrival data tested, only one set was Poisson, and none of the service times observed were exponential. This was the main reason why G/G/1 model was selected over M/M/N for validation. Although this single channel model can represent only a limited number of facilities in the airport landside, it was chosen to determine whether there is any resemblance between observed field data and the model predictions. In addition, a multiple-channel G/G/N model is currently under investigation.

2.2 NEUTS G/G/1 MODEL

A readily available G/G/1 queuing model was chosen for model validation. This model, developed by Professor Marcel F. Neuts of Purdue University, accepts general arrivals and general service times and computes the time dependency features of a single server discrete time queue with a finite queue length. The model was coded in Fortran IV and it resides on the DEC-10 computer system at Transportation Systems Center/U.S. Department of Transportation (TSC), Cambridge, Mass. The program listing is included in the appendix.

In the model, the numbers of arrivals during successive unit time intervals are independent, identically distributed random variables. These are expressed as the probability of a number of customers joining the queue during a specified unit time. The service times of successive customers are also independent and identically distributed random variables with a given probability density. The queue discipline is FCFS.

For any given time, n, the queue length is denoted as X_n and the residual service time (i.e., the number of additional units of service time required by the customer in service) is denoted by Y_n. The joint probability density function of queue length X_n and residual service time Y_n is denoted as $P_n(i,j)$, where

$$P_n(i,j) = P\left\{X_n = i,\ Y_n = j\right\}.$$

Recurrence relations were derived which compute $P_{n+1}(i,j)$ from $P_n(i,j)$ under all possible conditions of i and j. This joint probability density function is used to compute the distributions of queue length and waiting time. These are the basic outputs of the model.* As stated before, the model has been translated into a Fortran IV program. The inputs to the program are:

a. The size of the waiting room L1. In the program, L1 is limited to 100 customers in the system.

b. The maximum number, K, of arrivals per unit time. In the program, K is limited to be less than L1 arrivals.

c. The probability, P(J), that J customers join the queue per unit time, where J = 1,2, ... K.

d. The maximum service time, L2, in terms of time units. In the program, L2 is limited to be less than 31 time units.

e. The probability, R(J), that a service time lasts for J units of time, where J = 1, 2, ... L2.

f. The probability, PO, that no customers arrive during a unit of time.

g. The initial queue length, IO, and the initial residual service time, JO.

h. The program control parameters, including the number of time units to be run and the output option selection parameters.

The full output of this program includes the following:

a. The mean queue length as a function of time.

b. The distribution of queue length as a function of time.

c. The mean waiting time as a function of time.

d. The distribution of waiting time as a function of time.

e. The joint density of the queue length and the residual service time as a function of time.

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*For details, see M.F. Nuets, "The Single Server Queue in Discrete Time-Numerical Analysis, I," Naval Research Logistics Quarterly, 20 (1973), 297-304.

Any one or the combination of the above output items can be printed out according to the user-selected options.

2.3 SUPPORTING PROGRAMS

In addition to the queuing model itself, two supporting programs were written. One of them is used to process the raw data for queuing model input. The other program is used to compare the model-predicted waiting time distribution with the field-observed waiting time distribution.

2.3.1 Preprocessing Program CHISQ.F4

This program was written for preprocessing the raw arrival data which were gathered in Stapleton Airport. The functions of this program are:

a. Convert observed arrivals into an arrival distribution (histogram).

b. Determine the sample mean and variance of the data.

c. Perform X^2-test (chi square test) on raw data to determine whether the arrivals are from a Poisson process.

d. Convert the arrival distribution into compatible units for the G/G/1 model.

This program was necessary because the raw data were not directly applicable to the G/G/1 model. The test of Poisson was incorporated in this program, because if the majority of the arrivals were Poisson and the service times were exponential the applicability of the analytic approach would be greatly broadened by using M/M/N models. This program was written in Fortran IV language and resides on the DEC-10 computer system of TSC under the file name CHISQ.F4. The program listing is included in the appendix.

2.3.2 Postprocessing Program KOLMOG.F4

This program was written to compare the model-predicted waiting time distribution with the field-observed data. The computed and the observed waiting time distributions are the inputs to this program. The Kolmogroff statistic* is computed from the input data. Based on this statistic a decision is made whether to accept or reject the hypothesis that the predicted waiting time distribution can be used to represent the field observed waiting time distributions. The outcome of this test determines whether or not the queuing model can be used to represent airport landside facilities.

As before, the program was written in Fortran IV and resides on the DEC-10 computer system at TSC under the name of KOLMOG.F4. The program listing is also included in the appendix.

*K.V. Bury, Statistical Models in Applied Science, John Wiley and Sons, New York, 1975.

3. MODEL VALIDATION

3.1 DEFICIENCIES IN THE FIELD-OBSERVED DATA

Data deficiencies are discussed here for two reasons. First, because of these imperfect data a judgement of the validation results should be qualified accordingly. Second, if more data are to be obtained for further analytic model validation, those deficiencies are the pitfalls to be avoided. It may be noted that the field data were not taken specifically for the purpose of analytic model validation but rather for a simulation validation. This would probably account for some of the reasons for a deficient data set. The major problems of the data, in addition to the expected nonstationarity, are listed as follows:

a. The majority of arrival data was recorded as number of arrivals per 5 minutes. That means that inter-arrival times can be erroneous by as much as 5 minutes. However, because the service times are on the order of seconds, there is an incompatibility in the accuracy of the two sets of data. A 5-minute inaccuracy in data is too coarse to draw definitive conclusions.

Waiting times were grouped in 5-second intervals to form distributions. Again, compared to the expected service time this is also too coarse for detailed validation. It is suggested that the arrival times and the waiting times for each customer should be recorded and aggregated in 1-second intervals.

b. A variable number of service channels were used during the data-taking period. The number of servers changes as the demand slackened or increased.

c. The data are incomplete. For all the facilities, the queue length data are missing.

d. Observed data were arbitrarily sampled. This may or may not give a reasonable representation of the population on which the data were taken.

In spite of these deficiencies, three facilities were selected for model validation. They are the UAL express check-in, the security check, and the Branniff gate.

3.2 TEST PROCEDURE

The field observed arrival data and the service time data were preprocessed to conform with the input requirements of the G/G/1 model. Based on the preprocessed data the waiting time distribution and the queue length distribution are computed by the model. The predicted waiting time distributions and their expected values are compared with the field-observed counterparts. The comparison of the mean is done by the t-test and the Kolmogroff test is used for the distribution.

3.2.1 T-Test for Means

The mean value of the predicted waiting time is compared with the observed mean waiting time using the t-test. The null hypothesis for this test is that the population mean of the observed waiting times is equal to the predicted value on the basis of a random sample. The t-statistic is defined as:

$$t = \frac{\bar{x} - a}{s/\sqrt{n}}$$

where

$\bar{x}$ = sample mean

a = predicted value of mean

s = standard deviation of sample

n = number of sample points

The absolute value of the t-statistic is compared with a critical value $t_{\alpha/2,\ n-1}$ for a given significance level α. The null hypothesis will not be rejected (i.e., it will be accepted that the mean waiting time equals a) if the absolute value of t-statistic is less than $t_{\alpha/2,\ n-1}$. The numerical values of this critical statistic may be found from standard statistical tables.* The significance level α is usually set at 0.05.

3.2.2 Kolmogroff Test for Distributions

For a given sample x of size n, such that

$$x_1 \leq x_2 \leq \cdots \leq x_n$$

a cumulative distribution function, $G_n(x)$, is defined by the following:

$$G_n(x) = 0, \; x < x_1$$

$$= r/n, \; x_r \leq x \leq x_{v+1}$$

$$= 1, \; x \geq x_n .$$

If the sample x comes from the completely specified postulate, F(x), then

$$\lim_{n \to \infty} \Pr \left\{ G_n(x) - F(x) = 0 \right\} = 1, \text{ for all } x.$$

Based on this fact, a test of fit for F(x) is constructed by defining the Kolmogroff statistic,

$$D_n = \sup_x \left| G_n(x) - F(x) \right|$$

If the value D_n is larger than a critical value (as a function of n) $(d_n)_c$, the null hypothesis that the sample x comes from the postulate F(x) will be rejected. For significance level $\alpha = 0.05$ the critical value is†

*See, for example, E.L. Crow, F.A. Davis, and M.W. Maxfield, Statistics Manual, Dover Publications, New York, 1955.

†Bury, Statistics Models, op. cit.

$$(d_n)_c = \frac{1.36}{\sqrt{n}}, \text{ for } \alpha = 0.05 .$$

The observed waiting time distributions are tested by the Kolmogroff test.

3.3 FACILITIES TESTED

3.3.1 Security Check

3.3.1.1 Observed Data - Data were taken on January 23, 1976, between 1500 and 2000 hours. The arrivals at this station were aggregated over 5-minute intervals, which resulted in 59 data points. From these, the histogram shown as Figure 1 was produced. A chi-square test of the data showed no agreement with a Poisson process, and thus the G/G/1 model was applied. Two channels were generally in service; however, at infrequent intervals only one channel was operating. The service time per channel was operating. The service time per channel was assumed to be a constant 8 seconds per customer. Waiting times were recorded for 726 arrivals sampled from a total of 3,511 actual patrons. The frequency distribution of waiting times is shown in Table 1.

3.3.1.2 Test Results - In order to get better resolutions, the arrival rate was converted from the number of arrivals per 5 minutes to number of arrivals per 4 seconds. This is done by dividing the observed number of arrivals (per 5 minutes) by 75. It is true that the resulting arrivals may not represent the actual arrivals if better resolution were used when the data were taken. However, this is probably the best that can be done with the existing data.

In the majority of times, two channels were in operation, each with a constant 8 seconds service time (estimated). The service time used in the analytic model was assumed to be 4 seconds for 97 percent of time, 12 seconds for 2 percent, and 20 seconds for 1 percent of time. It is believed that this service time distribution used in the analytic model validation is more realistic than the estimated constant service time.

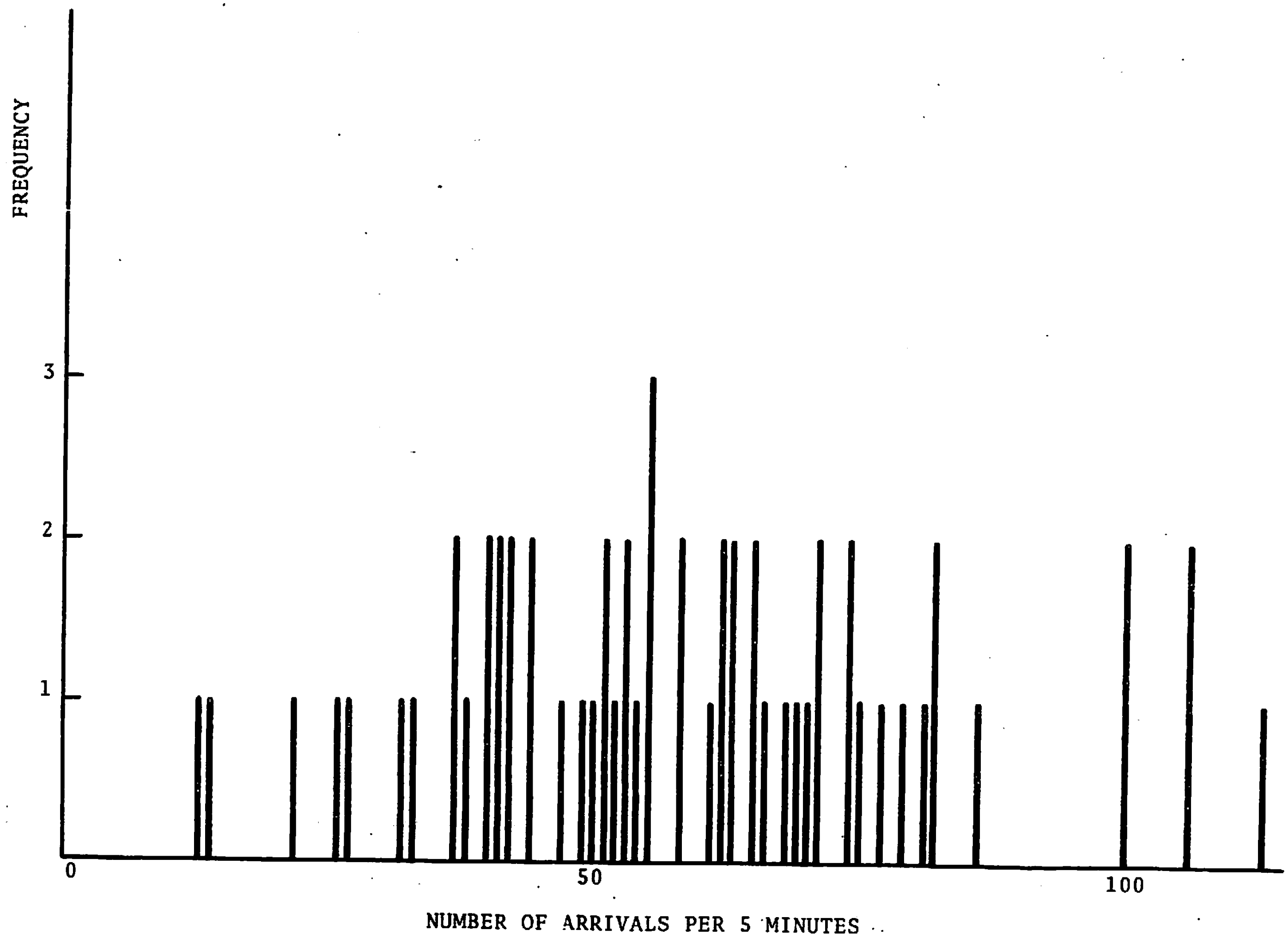

FIGURE 1. OBSERVED ARRIVALS AT SECURITY CHECK

TABLE 1. OBSERVED WAITING TIME (IN SECONDS) AT SECURITY CHECK, FRIDAY, JANUARY 23, 1976, 1500-2000 HOURS

FREQUENCY DISTRIBUTION

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1- 20	0	0	0	0	0	0	0	0	0	3	2	5	5	2	3	7	4	10	5	13
21- 40	1	10	6	5	8	4	6	4	2	6	3	7	3	3	7	3	5	2	2	4
41- 60	3	4	3	2	3	1	5	2	1	3	2	3	0	1	2	1	3	1	2	1
61- 80	1	1	0	0	1	1	0	0	2	1	1	0	1	0	1	0	0	0	1	0
81-100	0	0	0	0	0	0	0	0	0	0	0	0	0	2	1	1	0	0	0	1
101-120	0	1	1	2	1	0	0	1	0	0	1	0	1	0	0	0	0	0	0	0
121-140	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
141-160	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0
161-180	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

No. of values = 726 (sampled from 3511 passengers)

No. of zeros = 505

Mean = 11.63 seconds

STD. Dev. = 23.05 seconds

Minimum = 0, Maximum = 163 seconds

69.5% waited less than 10 seconds

99% waited less than 105 seconds

The comparison of mean and distribution are done separately as follows:

a. Test for Mean Wating Time: The observed 11.63 seconds mean waiting time at security check is compared with the model-predicted mean waiting time of 10.41 seconds by the t-test. The t-statistic is computed as

$$t = \frac{\bar{x} - a}{s/\sqrt{n}}$$

$$= \frac{11.63 - 10.41}{23.05/\sqrt{726}}$$

$$= 1.425.$$

The critical value is

$$t_{\alpha/2,n-1} = t_{0.025,725} = 1.960 .$$

Since t is less than $t_{\alpha/2,n-1}$, there is no reason to reject the hypothesis that the G/G/1 model can be used to predict the mean waiting time for the security check tested.

b. Test for Distribution: The observed and the computed waiting time distributions are compared by Kolmogroff test. The Kolmogroff statistic is computed by the program KOLMOG.F4 as follows:

$$D_n = 0.08099, \text{ with } n = 726$$

The critical value is

$$(d_n)_c = \frac{1.36}{\sqrt{n}} = \frac{1.36}{\sqrt{726}}$$

$$= 0.055, \text{ for } \alpha = 0.05.$$

Since D_n is greater than $(d_n)_c$, it is concluded that the G/G/1 model cannot be used to predict the waiting time distribution at the security check tested.

The field observed and the computed waiting time distributions are plotted in Figure 2. It is noticed that for waiting time less than 10 seconds the field observed curve is flat. This flat portion is the cause of the high value of Kolmogroff statistic D_n. Checking with the observed raw data it is found that there are many (505 out of 726) who went through security check without any wait. All of the rest waited 10 seconds or more. However, there were no persons recorded as waiting less than 10 seconds if they were waiting in line. This most likely indicates that the person who took the data arbitrarily recorded anything less than 10 seconds as 0 seconds. This casts some doubt on the validity of this portion of the field data, which therefore was omitted in comparison. The observed and the computed results are summarized in Table 2.

3.3.2 <u>UAL Express Check-In</u>

3.3.2.1 Observed Data - Data were taken on January 23, 1976, between 1500 and 2000 hours. The arrivals were taken in 5-minute intervals. The histogram of the arrivals is shown in Figure 3. The arrivals do not agree with a Poisson process.

A total of 58 arrival intervals were observed. The number of channels in service varied between one and two. The service time per channel was in cumulative distribution form. (See Figure 4.) There were 192 arrivals sampled from a total of 337 actual passengers. (See Table 3 for details.)

3.3.2.2 Test Results - As before, the arrival rate was converted from number of arrivals per 5 minutes to number of arrivals per 10 seconds for better resolution. The service time distribution was deduced from the observed cumulative distribution plot (Figure 4).

The effect of the variable number of channels in operation was compensated by multiplying the arrival rate by a factor corresponding to the average number of servers in operation. The comparison of mean and distribution are done separately as follows:

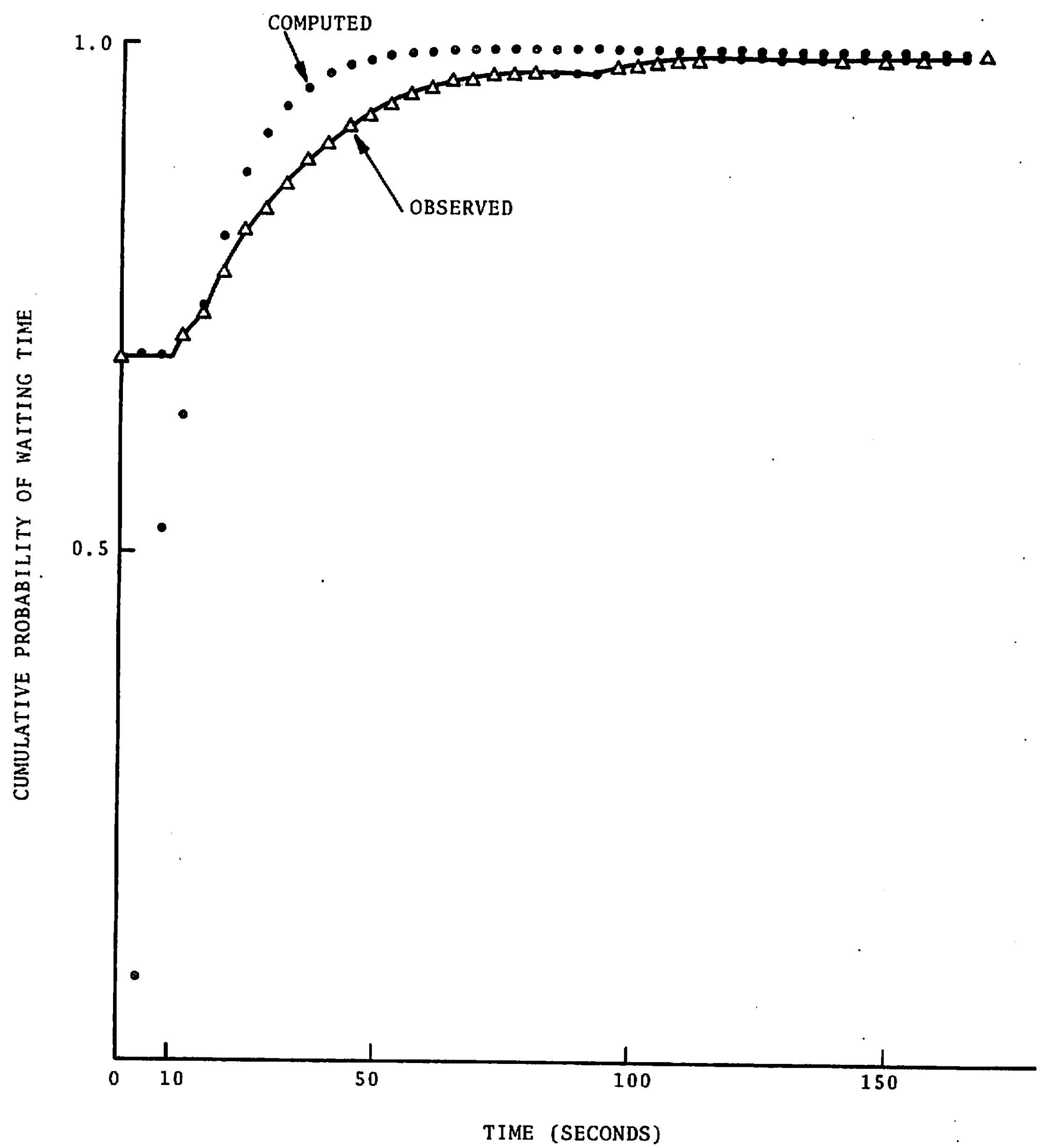

FIGURE 2. SECURITY CHECK WAITING TIME DISTRIBUTION

TABLE 2. SECURITY CHECK COMPARISON

OBSERVED FIELD DATA	COMPUTED RESULTS	CONSISTENCY
Mean Waiting Time 11.63 seconds	10.41 seconds	yes (α = 0.05)
Waiting Time Distribution Std. Dev. = 23.05 sec. 69.5% with zero wait 69.5% waited less than 10 sec. 99% waited less than 105 sec.	 8.4% waited less than 4 seconds 57.9% waited less than 10 seconds 99% waited less than 48 seconds	no (α = 0.05)
Queue Length = No Data	mean = 2.35 99% of time queue length is less than 11.	-

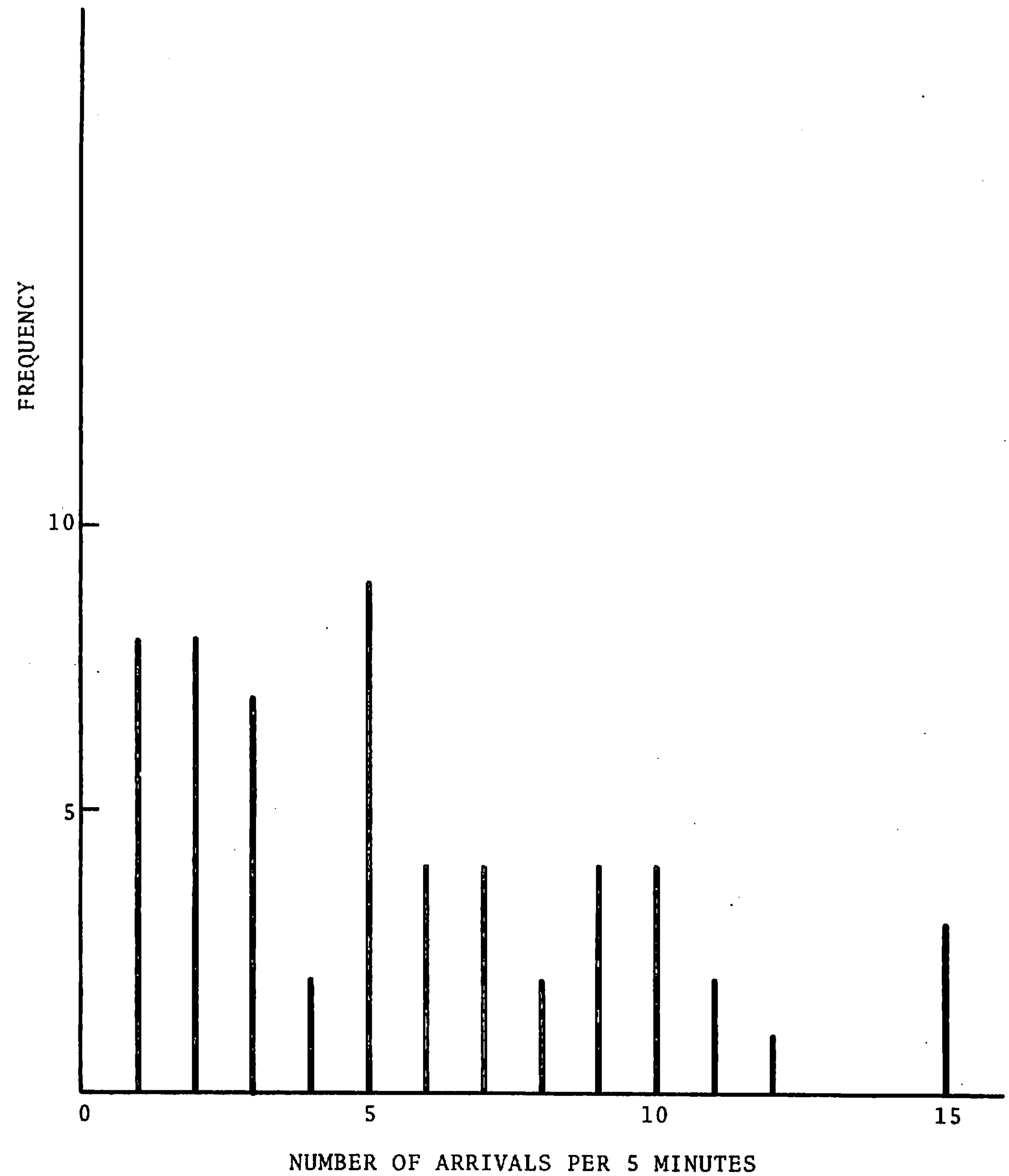

FIGURE 3. OBSERVED ARRIVALS AT UAL EXPRESS CHECK-IN

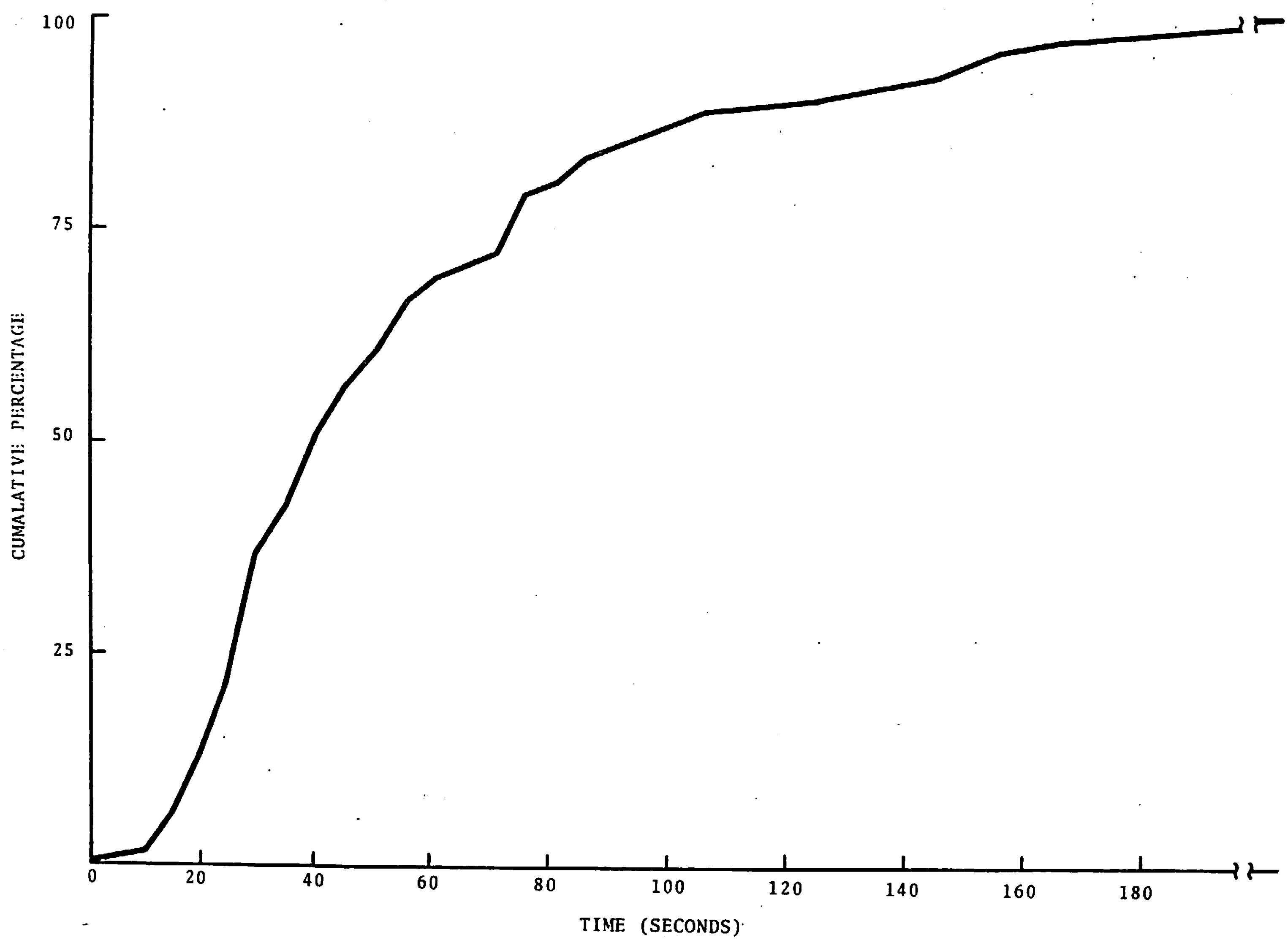

FIGURE 4. UAL EXPRESS CHECK-IN SERVICE TIME DISTRIBUTION

TABLE 3. OBSERVED WAITING TIME (IN SECONDS) AT UAL EXPRESS CHECK-IN, FRIDAY, JANUARY 23, 1976, 1500-2000 HOURS

FREQUENCY DISTRIBUTION

	1-5	6-10	11-15	16-20	21-25	26-30	31-35	36-40	41-45	46-50	51-55	56-60
1- 60	10	6	11	6	7	8	5	8	7	7	8	7
61-120	3	3	6	6	0	4	5	2	3	2	1	2
121-180	5	2	0	1	3	1	0	3	3	0	1	0
181-240	2	1	1	0	0	1	0	1	0	0	0	0
241-300	0	0	0	0	1	0	0	0	0	1	0	0
301-360	0	0	0	1	0	1	0	0	0	0	0	0
361-420	0	0	0	0	0	0	0	0	0	0	0	0
421-480	0	1	0	0	0	0	0	0	0	0	0	0

No. of values = 192 (sampled from 337 passengers)

No. of zeros = 35

Mean = 59.75 seconds

Std. Dev. = 68.15 seconds

Minimum = 0, Maximum = 427 seconds

18.23% with zero wait

26.6.% waited less than 10 seconds

99% waited less than 330 seconds

a. Test for Mean Waiting Time: The observed 59.75 seconds mean waiting time at UAL Express check-in is compared with the model-predicted mean waiting time of 52.68 seconds by the t-test. The t-statistic is computed as:

$$t = \frac{\bar{x} - a}{s/\sqrt{n}}$$

$$= \frac{59.75 - 52.68}{68.15/\sqrt{192}}$$

$$= 1.437.$$

The critical value is

$$t_{\alpha/2,n-1} = t_{0.025,191} = 1.960$$

Since t is less than $t_{\alpha/2,n-1}$, there is no reason to reject that the G/G/1 model can be used to predict the mean waiting time for the UAL express check-in tested.

b. Test for Distribution: The observed and the computed waiting time distributions are compared by Kolmogroff test. The Kolmogroff statistic is computed by the program KOLMOG.F4 as follows:

$$D_n = 0.11894, \text{ with } n = 192$$

The critical value is

$$(d_n)_c = \frac{1.36}{\sqrt{n}} = \frac{1.36}{\sqrt{192}}$$

$$= 0.098, \text{ for } \alpha = 0.05$$

Since D_n is greater than $(d_n)_c$, it is concluded that the G/G/1 model cannot be used to predict the waiting time distribution at the UAL express check-in tested.

The field observed and the computed waiting time distributions are plotted in Figure 5. Note that if the waiting times which are less than 30 seconds are treated the same, the predicted

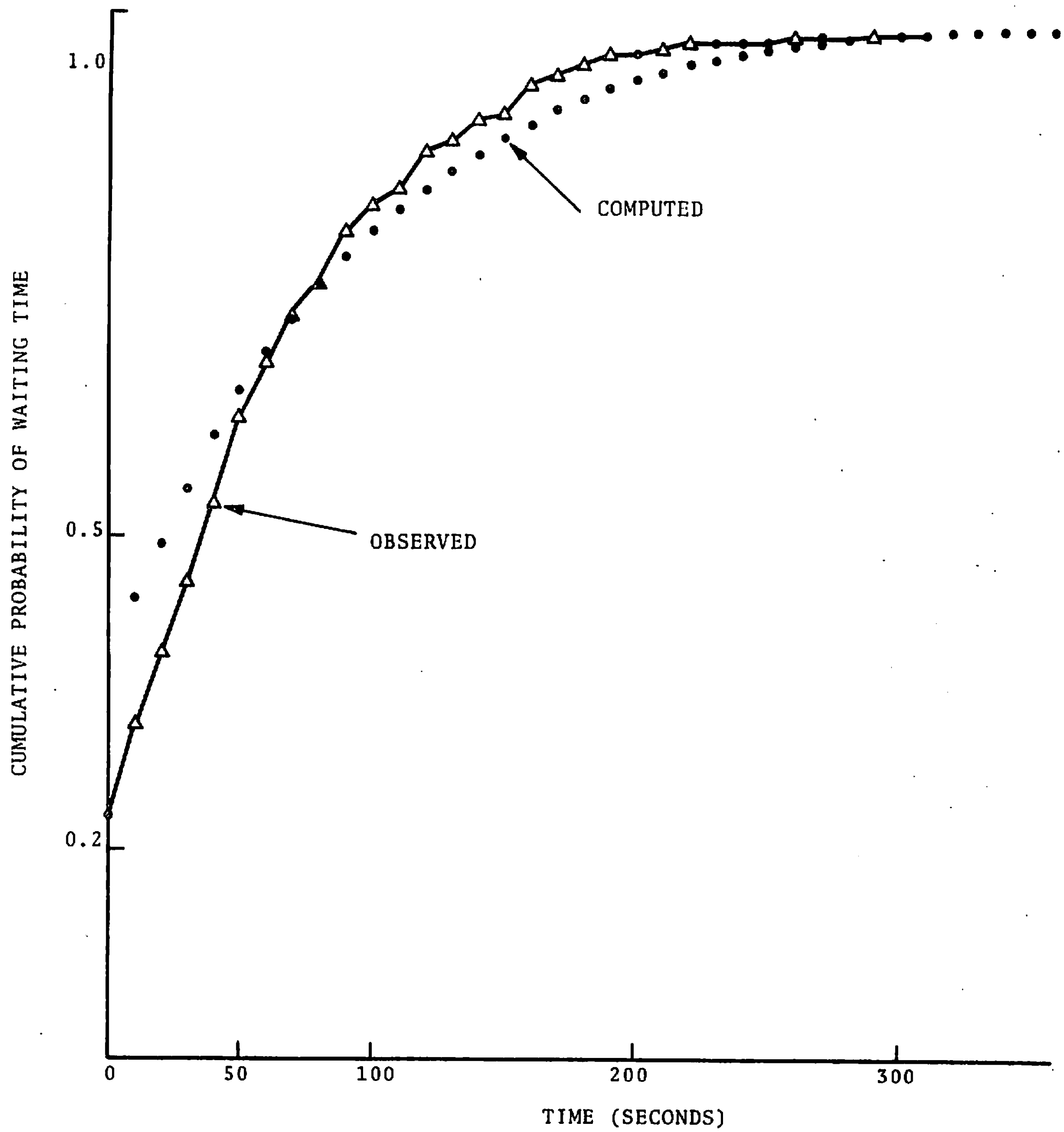

FIGURE 5. UAL EXPRESS CHECK-IN WAITING TIME DISTRIBUTION

waiting time distribution is a rather close fit to the observed waiting times.

The observed and the computed results are summarized in Table 4 for better comprehension.

3.3.3 Braniff Gate

3.3.3.1 Observed Data - Data were taken on January 25, 1976, between 1000 and 1500 hours. Forty-four arrivals were observed and recorded to the nearest minute. The histogram of these is shown in Figure 6. A chi-square test showed a lack of agreement with a Poisson process. The service time per channel is shown as Figure 7 in cumulative distribution form. In this case, only one channel was in service. Braniff gate waiting time data were combined with all other gate waiting times to form a single distribution shown in Table 5. There were 368 customers sampled from a total of 1,498. This combined dsitribution was used for comparison with the G/G/1 model.

3.3.3.2 Test Results - For better resolution, as before, the arrival rate was converted from numbers of arrivals per minute into number of arrivals per 10 seconds. The service time distribution was deduced from the observed cumulative distribution plot (Figure 7). The mean waiting time predicted by the G/G/1 model at Braniff gate was 33.85 seconds, which is significantly lower than the observed mean waiting time at the gate (68.96 seconds). However, it is emphasized that the field observed data were not observed at Braniff gate alone. They were accumulated for all gates in service during the observation period. Since the Braniff gate was relatively lightly used and the service time was generally lower than the rest of the gates. The predicted lower waiting time at Braniff gate was consistent. Because of the lack of field data, the statistical tests were not done. However, the observed (for all gates) and the computed (for Braniff gate) results are summarized in Table 6.

TABLE 4. UAL EXPRESS CHECK-IN COMPARISON

OBSERVED FIELD DATA	COMPUTED RESULTS	CONSISTENCY
Mean Waiting Time 59.75 seconds	50.93 seconds	yes ($\alpha = 0.05$)
Waiting Time Distribution Std. Dev. = 68.15 sec. 18.23% with zero wait 26.6% waited less than 10 sec. 99% waited less than 330 sec.	45.1% waited less than 10 sec. 99% waited less than 340 sec.	no ($\alpha = 0.05$)
Queue Length: No Data	mean = 1.0 99% of time the queue length is less than 5.	-

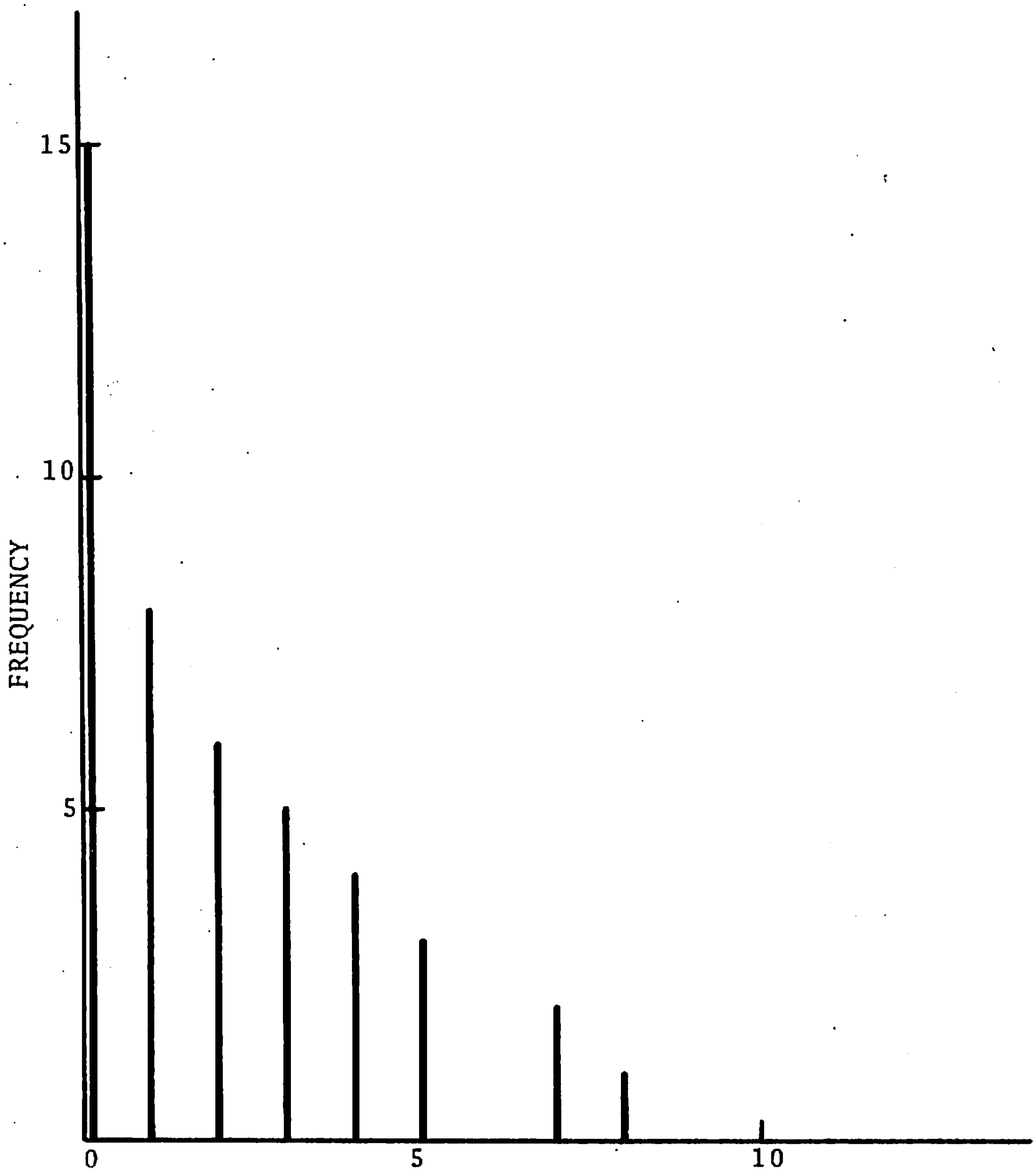

FIGURE 6. OBSERVED ARRIVALS AT BRANIFF GATE

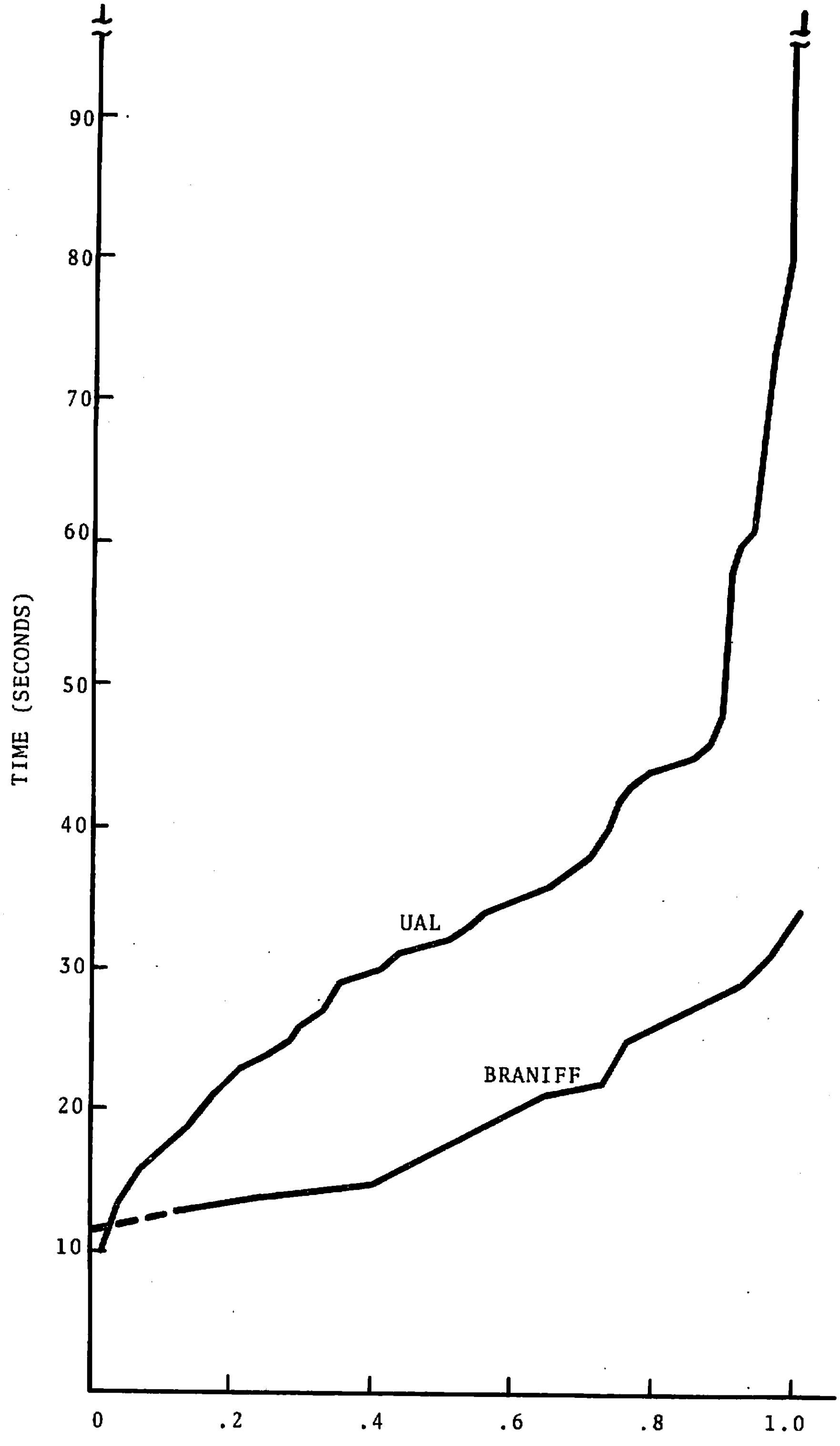

FIGURE 7. BRANIFF GATE SERVICE TIME DISTRIBUTIONS

TABLE 5. OBSERVED WAITING TIME (IN SECONDS) FOR ALL GATES, SUNDAY, JANUARY 25, 1976, 1000-1500 HOURS

FREQUENCY DISTRIBUTION

	1-5	6-10	11-15	16-20	21-25	26-30	31-35	36-40	41-45	46-50	51-55	56-60
1-60	10	12	22	17	9	17	7	15	7	4	10	8
61-120	6	2	8	3	2	7	5	5	4	0	2	0
121-180	2	1	3	2	1	2	0	2	3	1	0	1
181-240	1	2	1	0	0	2	2	2	0	1	1	0
241-300	4	3	2	1	0	1	1	0	0	0	1	1
301-360	1	1	0	0	1	0	0	1	2	0	0	0
361-420	2	0	1	0	1	0	2	0	1	0	1	1
421-480	1	0	1	0	0	1	0	0	0	0	1	0
481-540	0	0	0	1	0	0	0	0	0	0	0	0
541-600	0	0	0	0	0	0	1	0	1	0	0	0

No. of values = 368 (sampled from 1498 passengers)

No. of zeros = 120

Mean = 68.96 seconds

Std. Dev. = 108.79 seconds

Minimum = 0, Maximum = 585 seconds

32.6% with zero wait

38.5% waited less than 10 seconds

99% waited less than 455 seconds

TABLE 6. BRANIFF GATE COMPARISON

OBSERVED RESULTS (ALL GATES)	COMPUTED RESULT (BRANIFF GATE)
Waiting Time:	Mean = 33.85 seconds
Mean = 68.96 seconds	
Std. Dev. = 108.79 seconds	
32.6% with zero wait	25.1% waited less than 10 seconds
38.58% waited less than 10 seconds	99% waited less than 160 seconds
99% waited less than 455 seconds	Mean = 1.95
Queue Length:	
(No Information)	99% of time the queue length was less than 9.

4. CONCLUSION

As mentioned before, the field data were not collected specifically for the analytic model validation and thus were not ideal for the purpose of this study. Instead of taking the number of arrivals per 5 minutes and taking the number of delays in a time bracket, the data for actual delay in seconds for each passenger would be more suitable for analytic model validation.

Because of the single channel limitations, there are many facilities in the airport landside which may not be represented by the G/G/1 model (e.g., ticket counter, baggage claims, curb sides). However, despite the less than desirable field data, for all three facilities tested the G/G/1 model adequately predicted mean values of the waiting time. Moreover, the tested facilities include those whose numbers of servers were changed to accommodate demand changes. It seems to suggest that as far as the mean values are concerned, the G/G/1 model can be used for waiting time predictions. The predictions of waiting time distributions, however, were not good. The model did not do well enough to pass the Kolmogroff test, possibly because of the reasons described above.

APPENDIX
PROGRAM LISTINGS

A.1 YNEVTS.F4 - G/G/1 MODEL

```
C THIS PROGRAM WAS DEVELOPED BY MARCEL F. NEUTS - DEPART-
C MENT OF STATISTICS - PURDUE UNIVERSITY - WEST LAFAYETTE-
C INDIANA. OCTOBER 1971.
C
C THIS PROGRAM COMPUTES THE TIME DEPENDENT FEATURES OF A
C SINGLE SERVER DISCRETE TIME QUEUE WITH A FINITE WAITING-
C ROOM.
C WITH THE PRESENT DIMENSION STATEMENTS A WAITINGROOM OF
C SIZE UP TO ONE HUNDRED MAY BE STUDIED. THE DENSITY OF THE
C SERVICE TIME CAN BE CONCENTRATED ON UP TO THIRTY POINTS.
C
C THE FULL OUTPUT OF THIS PROGRAM INCLUDES THE FOLLOWINGV
C
C 1. THE MEAN QUEUE LENGTH AT TIME N.
C 2. THE DISTRIBUTION OF THE QUEUE LENGTH AT TIME N.
C 3. THE MEAN WAITINGTIME AT TIME N.
C 4. THE DISTRIBUTION OF THE WAITINGTIME AT TIME N.
C 5. THE JOINT DENSITY OF THE QUEUE LENGTH AND THE RESIDUAL
C    SERVICE TIME AT TIME N.
C
C ALL THESE ARE COMPUTED FOR N UP TO A SPECIFIED VALUE NNN.
C
C BY USE OF THE VARIOUS OPTIONS, LISTED BELOW, SOME OF THESE
C FEATURES MAY BE DELETED FROM THE WRITTEN OUTPUT.
C
C THE THEORETICAL DEVELOPMENT OF THE DISCRETE TIME QUEUE
C WITH AN UNBOUNDED QUEUE LENGTH MAY BE FOUND INV
C
C * STELLA C. DAFERMOS AND MARCEL F. NEUTS
C * A SINGLE SERVER QUEUE IN DISCRETE TIME *
C * CAHIERS DU CENTRE DE RECHERCHE OPERATIONNELLE - 1971.
C
C THE FOLLOWING IS A COMPANION PAPER TO THE PRESENT PROGRAMV
C
C * MARCEL F. NEUTS
C * THE SINGLE SERVER QUEUE IN DISCRETE TIME - NUMERICAL
C * ANALYSIS *
C * PURDUE MIMEOGRAPH SERIES - DEPT. OF STATISTICS.
C * PURDUE UNIVERSITY - WEST LAFAYETTE - IN - 47907
C
      DIMENSION P(100),H(30),PP(100,30),X(100),Y(100,30)
      DIMENSION Z(100),W(100),WT(3000)
      INTEGER OPT(10)
```

```
C
C     ***** T H E   O P T I O N S *****
C
C IN ORDER TO WRITE OUT THE JOINT DENSITY OF THE QUEUE LENGTH
C AND THE RESIDUAL SERVICE TIME, SET OPT(1)=1 - OTHERWISE
C SET OPT(1)=0
C
      READ(5,999) OPT(1)
C
C IN ORDER TO WRITE OUT THE DISTRIBUTION OF THE WAITING-
C TIME, SET OPT(2)=1 - OTHERWISE SET OPT(2)=0
C
      READ(5,999) OPT(2)
C
C THE USER MAY WISH TO COMPUTE AND WRITE THE DISTRIBUTION OF
C THE WAITINGTIME ONLY AT TIME POINTS WHICH ARE A MULTIPLE OF
C A CONSTANT NR. THIS TO SAVE ON PROCESSING TIME AND ON THE
C NUMBER OF LINES OF OUTPUT. IN THIS CASE THE IDENTIFIER
C OPT(3) SHOULD BE SET EQUAL TO ONE AND THE NUMBER NR SHOULD BE
C GIVEN. OPT(3) AND NR ARE TO BE GIVEN IN AN I1,I2 FORMAT.
C
      READ(5,997) OPT(3),NR
      XNR=NR
      KTEST1=OPT(1)
      KTEST2=OPT(2)
      KTEST3=OPT(3)
      IF(KTEST3.EQ.1) KTEST2=0
      IF(KTEST3.EQ.0) KTEST4=0
C
C     ***** T H E   D A T A *****
C
C L1 IS THE SIZE OF THE WAITINGROOM. L1 IS LT. 101
C
      READ(5,1001) L1
C
C L2 IS THE NUMBER OF POINTS ON WHICH THE DENSITY OF THE
C SERVICE TIME IS CONCENTRATED. L2 IS LT. 31
C
      READ(5,1001) L2
C
C K IS THE MAXIMUM NUMBER OF ARRIVALS PER UNIT OF TIME.
C K SHOULD BE AT LEAST ONE AND STRICTLY LESS THAN L1.
C
      READ(5,1001) K
C
C R(J) IS THE PROBABILITY THAT A SERVICE TIME LASTS FOR J
C UNITS OF TIME.
C ONE SHOULD VERIFY BEFOREHAND THAT THE SUM R(1)+...R(L2)
C IS EQUAL TO ONE.
C
      READ(5,1002) (R(J),J=1,L2)
C
C P(J) IS THE PROBABILITY THAT J CUSTOMERS JOIN THE QUEUE
C DURING A UNIT OF TIME.
C THE INDEX J RUNS FROM ONE TO K.
```

```
      READ(5,1002) (P(J),J=1,K)
C
C PO IS THE PROBABILITY THAT NO CUSTOMERS ARRIVE DURING A
C UNIT OF TIME.
C ONE SHOULD VERIFY BEFOREHAND THAT P0 + P(1) +...+P(K)
C IS EQUAL TO ONE.
C
      READ(5,1003) P0
C
C I0 IS THE INITIAL QUEUE LENGTH.
C J0 IS THE INITIAL RESIDUAL SERVICE TIME.
C IF I0=0, THEN J0=0 AND CONVERSELY.
C I0 SHOULD NOT EXCEED L1.
C J0 SHOULD NOT EXCEED L2.
C
      READ(5,1001) I0
      READ(5,1001) J0
C
C NNN IS THE MAXIMUM TIME POINT FOR WHICH THE QUEUE
C FEATURES ARE COMPUTED. NNN SHOULD BE AT LEAST ONE AND
C AT MOST 9999. NOTE HOWEVER THAT THE PROCESSING TIME
C AND THE NUMBER OF LINES OF OUTPUT GROW PROPORTIONATELY TO
C THE VALUE OF NNN.
C
      READ(5,1004) NNN
      IF(I0.EQ.0.AND.J0.EQ.0)GOTO 2001
      PP(I0,J0)=1.
      GOTO 2002
 2001 P00=1.
 2002 N=0
      X11=P0
      DO 21 I=1,K
      XI=I
      X11=X11+P(I)
      X1=X1+XI*P(I)
   21 CONTINUE
      DO 22 J=1,L2
      XJ=J
      X11=X11+R(J)
      X2=X2+XJ*R(J)
   22 CONTINUE
      X11=X11-2.
      X11=ABS(X11)
      WRITE(3,1014)
      WRITE(3,1022)K,L2,L1
      N1=0
      WRITE(3,1007)N1,P0,(J,P(J),J=1,K)
      WRITE(3,1012)
      WRITE(3,1010)(J,R(J),J=1,L2)
      WRITE(3,1012)
      WRITE(3,1008)I0,J0
      WRITE(3,1012)
      WRITE(3,1009)NNN
      WRITE(3,1012)
      WRITE(3,1013)X1,X2
      WRITE(3,1006)
```

```
C      ***** D I A G N O S T I C S *****
C
       IF(L1.LE.K) GOTO 2005
       IF(I0.EQ.0.AND.J0.NE.0.OR.I0.NE.0.AND.J0.EQ.0)GOTO 2005
       IF(X11.GT..000001) GOTO 2005
       IF(KTEST3.EQ.1.AND.NR.LT.2) GOTO 2005
       WRITE(4,3200)
 3200  FORMAT(/,1X,≠THE PROGRAM HAS RUN≠)
C
C
C
       L11=L1-1
       L21=L2-1
       L22=L2+1
       M1=L1*L2
       K1=K+1
       DO 13 I=1,L1
       Y(I,L2)=0.0
   13  W(I)=1.-P0
       IF(K.EQ.1)GOTO 2003
       DO 20 I=2,K
       W(I)=W(I-1)-P(I-1)
   20  CONTINUE
C
C AT THIS STAGE THE INPUT DATA HAVE BEEN READ IN, THE
C PP-ARRAY HAS BEEN INITIALIZED AND THE INPUT DATA HAVE
C BEEN WRITTEN OUT AND SUBJECTED TO SOME RUDIMENTARY
C DIAGNOSTIC TESTS. THE NEXT LINE STARTS THE MAIN LOOP
C WHICH IS REPEATED NNN TIMES.
C
C      ***** T H E   M A I N   L O O P *****
C
 2003  N=N+1
       IF(N.GT.NNN) STOP
C
C THIS PORTION OF THE PROGRAM COMPUTES THE NEW PP-ARRAY.
C PP(I,J) IS THE PROBABILITY THAT AT THE TIME CONSIDERED
C THERE ARE I CUSTOMERS IN THE SYSTEM AND THE RESIDUAL
C SERVICE TIME OF THE CUSTOMER BEING SERVED IS J.
C THIS IS FOR I BETWEEN ONE AND L1, FOR J BETWEEN ONE AND
C L2. THE IDENTIFIER P00 CONTAINS THE PROBABILITY THAT THE
C QUEUE IS EMPTY.
C
       WRITE(3,1000)
       WRITE(3,1017)N
       WRITE(3,1012)
       X00=P00+PP(1,1)
       DO 1 I=1,K
       X(I)=P(I)*P00+P0*PP(I+1,1)
       II=I-1
       DO 3 NU=1,I
       X(I)=X(I)+P(I+1-NU)*PP(NU,1)
    3  CONTINUE
```

```
    4  DO 2 J=1,L21
       Y(I,J)=P0*PP(I,J+1)
       IF(I.EQ.1)GOTO 51
       DO 5 NV=1,II
       Y(I,J)=Y(I,J)+P(I-NV)*PP(NV,J+1)
    5  CONTINUE
   51  CONTINUE
    2  CONTINUE
    1  CONTINUE
       DO 6 I=K1,L11
       NU1=I-K
       NU2=NU1+1
       II=I-1
       X(I)=P0*PP(I+1,1)
       DO 7 NU=NU2,I
       X(I)=X(I)+P(I+1-NU)*PP(NU,1)
    7  CONTINUE
       DO 9 J=1,L21
       Y(I,J)=P0*PP(I,J+1)
       DO 8 NV=NU1,II
       Y(I,J)=Y(I,J)+P(I-NV)*PP(NV,J+1)
    8  CONTINUE
    9  CONTINUE
    6  CONTINUE
       X(L1)=0.0
       DO 10 NU=1,K
       X(L1)=X(L1)+W(NU)*PP(L1+1-NU,1)
   10  CONTINUE
       DO 11 J=1,L21
       Y(L1,J)=PP(L1,J+1)
       DO 12 NU=1,K
       Y(L1,J)=Y(L1,J)+W(NU)*PP(L1-NU,J+1)
   12  CONTINUE
   11  CONTINUE
       P00=P0*X00
C
C Z(I) CONTAINS FIRST THE DENSITY AND NEXT THE DISTRIBUTION
C OF THE QUEUE LENGTH AT THE TIME POINT CONSIDERED.
C XMN CONTAINS THE MEAN QUEUE LENGTH AT THE TIME POINT
C CONSIDERED.
C
       XMN=1.-P00
       DO 14 I=1,L1
       Z(I)=0.0
       DO 15 J=1,L2
       PP(I,J)=Y(I,J)+R(J)*X(I)
       Z(I)=Z(I)+PP(I,J)
   15  CONTINUE
   14  CONTINUE
       Z(1)=Z(1)+P00
       XMN=XMN+1.-Z(1)
       DO 18 I=2,L1
       Z(I)=Z(I)+Z(I-1)
       XMN=XMN+1.-Z(I)
   18  CONTINUE
```

```
      IF(KTEST3.EQ.0) GOTO 2008
      XN=N
      U1=AMOD(XN,XNE)
      IF(U1.LT.0.5) KTEST4=1
      IF(U1.GT.0.9) KTEST4=0
      IF(KTEST4.EQ.1) GOTO 2010
      IF(KTEST4.EQ.0) GOTO 2009
 2008 CONTINUE
C
C THIS PORTION OF THE PROGRAM COMPUTES THE DISTRIBUTION OF
C THE VIRTUAL WAITINGTIME AT TIME N. THE ALGORITHM IS AN
C ANALOGUE OF HORNER*S METHOD FOR THE EVALUATION OF ORDI-
C NARY POLYNOMIALS, BUT ADAPTED HERE TO CONVOLUTION PRODUCTS.
C
      IF(KTEST2.EQ.0) GOTO 2006
 2010 CONTINUE
      DO 32 J=1,L2
      WT(J)=PP(L1,J)
   32 CONTINUE
      DO 33 J=L22,M1
      WT (J)=0.0
   33 CONTINUE
      MN1=1
      MN3=2
      MN4=L2
      DO 34 JX=1,L11
      JX1=L1-JX
      MN2=MN4
      MN4=MN2+L2
      MN5=MN4+2
      DO 35 J=2,MN4
      JR=MN5-J
      WT(JR)=0.0
      MN6=MAX0(1,JR-MN2)
      MN7=MIN0(L2,JR-1)
      DO 35 NU=MN6,MN7
      WT(JR)=WT(JR)+R(NU)*WT(JR-NU)
   35 CONTINUE
      WT(1)=0.0
      DO 37 JJ=1,L2
      WT(JJ)=WT(JJ)+PP(JX1,JJ)
   37 CONTINUE
   34 CONTINUE
      WT(1)=P00+WT(1)
      ZMN=2.-P00-WT(1)
      DO 36 J=2,M1
      WT(J)=WT(J)+WT(J-1)
      ZMN=ZMN+1.-WT(J)
   36 CONTINUE
 2006 CONTINUE
C
C THE WRITE STATEMENTS FOR THE REQUIRED OUTPUT.
C
 2009 WRITE(3,1016)N,XMN
      WRITE(3,1012)
```

```
      WRITE(3,1018)N
      WRITE(3,1012)
      DO 4000 I=1,L1
      IF(Z(I).LE.0.99999) GO TO 4000
      L1P=I+1
      GO TO  4010
 4000 CONTINUE
 4010 CONTINUE
      WRITE(3,1011)N1,P00,(I,Z(I),I=1,L1P)
      WRITE(3,1012)
      IF(KTEST2.EQ.0.AND.KTEST4.EQ.0) GOTO 2007
      WRITE(3,1021)N,ZMN
      WRITE(3,1012)
      DO 4100 I=1,M1
      IF(WT(I).LE.0.99999) GOTO 4100
      M1P=I+1
      GO TO 4110
 4100 CONTINUE
 4110 CONTINUE
      WRITE(3,1020)N,N1,P00,(J,WT(J),J=1,M1P)
 2007 CONTINUE
      IF(KTEST1.EQ.0) GOTO 2004
      WRITE(3,1012)
      WRITE(3,1019)N
      WRITE(3,1015)P00
      WRITE(3,1012)
      DO 17 I=1,L1
      WRITE(3,1005)I,(PH(I,J),J=1,L2)
   17 CONTINUE
 2004 GO TO 2003
 2005 WRITE(3,998)
      WRITE(3,3000)X11
 3000 FORMAT(/,1X,≠X11 = ≠,E20.10)
      WRITE(4,3100)X11
 3100 FORMAT(/,1X,≠ERROR, X11 = ≠,E20.10)
C
C THE FORMAT STATEMENTS.
C
  997 FORMAT(I1,I2)
  998 FORMAT(≠  ATTENTION THERE ARE ERRORS IN THE INPUT≠,
     *≠ DATA. PLEASE CHECK. ≠)
  999 FORMAT(I1)
 1000 FORMAT(≠1≠)
 1001 FORMAT(I3)
 1002 FORMAT(3F7.5)
 1003 FORMAT(F7.5)
 1004 FORMAT(I4)
 1005 FORMAT(3X,I3,10F7.4,(6X,10F7.4))
 1006 FORMAT(//)
```

```
1007 FORMAT(≠  THE DENSITY OF THE NUMBER OF ARRIVALS PER≠,
    *≠ UNIT OF TIME≠,//(2X,10(I4,F8.5)))
1008 FORMAT(≠  THE INITIAL QUEUE LENGTH IS ≠,I3,/≠  THE≠,
    *≠ INITIAL RESIDUAL SERVICE TIME IS ≠,I3)
1009 FORMAT(≠ THE NUMBER OF TIME POINTS COMPUTED IS≠,
    *I4)
1010 FORMAT(≠ THE DENSITY OF THE SERVICE TIMES≠,//2X,
    *(10(I4,F8.5)))
1011 FORMAT(3X,10(I4,F8.5))
1012 FORMAT(/)
1013 FORMAT(2X,≠THE MEAN NR. OF ARRIVALS PER UNIT-TIMEY≠,
    *F10.4,/≠ THE MEAN SERVICE TIMEY≠,F10.4)
1014 FORMAT(≠1≠,////≠ THE TRANSIENT BEHAVIOR OF A ≠,
    *≠DISCRETE TIME QUEUE WITH A FINITE WAITINGROOM≠,
    *//)
1015 FORMAT(≠ THE QUEUE IS EMPTY WITH PROBABILITY≠,
    *F9.5)
1016 FORMAT(≠ AT TIME N =≠,I4,≠ THE MEAN QUEUE LENGTH≠,
    *≠ EQUALS≠,F10.4)
1017 FORMAT(≠ THE QUEUE CHARACTERISTICS AT TIME N = ≠,
    *I4)
1018 FORMAT(≠ THE DISTRIBUTION OF THE QUEUE LENGTH ≠,
    *≠AT TIME N = ≠,I4/)
1019 FORMAT(≠ THE JOINT DENSITY OF THE QUEUE LENGTH ≠,
    *≠AND THE RESIDUAL SERVICE TIME AT TIME N =≠,I4/)
1020 FORMAT(≠ THE DISTRIBUTION OF THE WAITINGTIME AT ≠,
    *≠TIME N = ≠,I4,//(3X,10(I4,F8.5)))
1021 FORMAT(≠ THE MEAN WAITINGTIME AT TIME N =≠,I4,
    *≠ IS≠,F12.4)
1022 FORMAT(≠ THE UPPER LIMIT OF THE NUMBER OF ARRIVALS≠,
    *≠ PER UNIT OF TIME IS≠,I3,/≠ THE UPPER LIMIT OF THE≠,
    *≠ NUMBER OF UNITS OF SERVICE-TIME PER CUSTOMER IS≠,
    *I3,/≠ THE UPPER LIMIT TO THE NUMBER OF CUSTOMERS≠,
    *≠ IN THE SYSTEM IS≠,I4/)
     END
```

A.2 CHISQ.F4 — PREPROCESSING PROGRAM

```
C1. READ IN THE SAMPLE OF ARRIVALS PER UNIT TIME.
C2. SORT THE SAMPLE BY NUMBER OF ARRIVALS.
C3. CHECK IF THE SAMPLE IS A POISSON PROCESS.
      DIMENSION A1(101),P1(101),TP(10)
C. A1(I)=NUMBER OF OCCURANCES OF I-1 ARRIVALS PER UNIT TIME.
C. N=TOTAL SAMPLE POINTS.
      READ(20,100)N
      SUM=0.
      SUM2=0.
      JMAX=0
      SMALL=0.02*N
      DO 900 I=1,101
900   A1(I)=0.
      DO 1000 I=1,N
      READ(20,100)IA
      IF(IA.NE.0)GO TO 4000
      A1(1)=A1(1)+1
      GO TO 1000
4000  CONTINUE
      DO 5000 J=1,100
      IF(IA.GT.J)GO TO 5000
      A1(J+1)=A1(J+1)+1
      SUM=SUM+IA
      SUM2=SUM2+IA*IA
      IF(J.GT.JMAX)JMAX=J
      GO TO 1000
5000  CONTINUE
1000  CONTINUE
      FMEAN=SUM/N
      VAR=SUM2/N-FMEAN*FMEAN
      RATIO=VAR/FMEAN
      TYPE 300,FMEAN,VAR,RATIO
      IF(RATIO.LT.0.8.OR.RATIO.GT.1.2)GO TO 9999
C. VERIFY IF THE SAMPLE IS POISSON
      JMIN=1
      FACT=1.
      EXPA=EXP(-FMEAN)
      FMEANI=1.
      P1(1)=EXPA*N
6100  CONTINUE
      DO 6000 I=JMIN,JMAX
      FACT=FACT*I
      FMEANI=FMEANI*FMEAN
      P=EXPA*FMEANI/FACT
6000  P1(I+1)=P*N
      IF(P1(JMAX).LE.SMALL)GO TO 6200
      JMIN=JMAX+1
      JMAX=JMAX+1
      GO TO 6100
6200  CONTINUE
      DOF=JMAX
      P1L=P1(1)
      A1L=A1(1)
```

```
      PIH=PI(JMAX)
      AIH=AI(JMAX)
      DO 6300 I=2,JMAX
      IF(PIL.GE.SMALL)GO TO 6310
      PIL=PIL+PI(I)
      AIL=AIL+AI(I)
      IL=I
      DOF=DOF-1
6310  CONTINUE
      IF(PIH.GE.SMALL)GO TO 6320
      PIH=PIH+PI(JMAX-I)
      AIH=AIH+AI(JMAX-I)
      IH=JMAX-I
      DOF=DOF-1
6320  IF(PIL.GE.SMALL.AND.PIH.GE.SMALL)GO TO 6400
6300  CONTINUE
6400  CONTINUE
      DOF=DOF-1
      IL1=IL+1
      IH1=IH-1
      CHI2=(AIL-PIL)**2/PIL+(AIH-PIH)**2/PIH
      DO 6500 I=IL1,IH1
      CHI2=CHI2+(AI(I)-PI(I))**2/PI(I)
6500  CONTINUE
      TYPE 400,CHI2,DOF
      GO TO 9998
9999  TYPE 700
9998  CONTINUE
      TYPE 450
      DO 8000 I=1,JMAX,10
      DO 8010 J=1,10
8010  IP(J)=(I-1)+J-1
      TYPE 500,(IP(J),J=1,10)
      TYPE 600,(PI(IP(J)+1),J=1,10)
8000  TYPE 600,(AI(IP(J)+1),J=1,10)
100   FORMAT(I5)
300   FORMAT(/,1X,≠MEAN,VAR,V/M = ≠,3F10.3)
400   FORMAT(/,1X,≠CHI SQ,DOF = ≠,2F10.4)
450   FORMAT(//,1X,≠= ARVLS/EXPTD FREQ/OBSVD FREQ TABLE V≠,/)
500   FORMAT(/,1X,10(1X,I5))
600   FORMAT(1X,10(1X,F5.1))
700   FORMAT(//,1X,≠THE SAMPLE CAN NOT BE POISSON !≠)
      END
```

A.3 KOLMOG.F4 POSTPROCESSING PROGRAM

```
C
C      THIS PROGRAM WAS DEVELOPED BY LI SHIN YUAN, TRANSPORTATION
C      SYSTEMS CENTER. DOT, CAMBRIDGE, MASS., MAY 1976.
C1.    THIS PROGRAM COMPARES THE OBSERVED WAITING TIME DISTRIBUTION
C1.    WITH THE QUEUEING MODEL GENERATED WAITING TIME DISTRIBUTION USING
C1.    KOLMOGOROFF TEST.
        DIMENSION OBFRQ(200),QFRQ(200)
        DO 1000 I=1,200
        OBFRQ(I)=0.
        QFRQ(I)=0.
1000     CONTINUE
4000     FORMAT(F5.0)
C.    INPUTS FROM QUEUEING MODEL :
         TYPE 6000
6000      FORMAT(/,1X,≠NUMBER OF CLASSES BY QUEUEING MODEL = ≠,/,4X)
        ACCEPT 4100,NQ
        WRITE(11,4100)NQ
        TYPE 6100
6100     FORMAT(/,1X,≠TYPE IN THEM ONE AT A TIME WITH -1. AS THE 1ST≠,/)
        ACCEPT 4200,(QFRQ(I),I=1,NQ)
        WRITE(11,4210)(QFRQ(I),I=1,NQ)
4210     FORMAT(F7.5)
4100     FORMAT(I3)
4200     FORMAT(F7.0)
C.   FACTOR=QUEUEING UNIT TIME/OBSERVED UNIT TIME.
         TYPE 6400
6400     FORMAT(/,1X,≠FACTOR = ≠,/,4X)
        ACCEPT 4000,FACTOR
        WRITE(11,4000)FACTOR
        FACTOR=1./FACTOR
C.    INPUTS FROM OBSERVED DATA :
        J=1
        SFREQ=0.
        TYPE 6300
6300     FORMAT(/,1X,≠TYPE IN OBS VALUE + FREQ 1 PAIR AT A TIME ≠,/,4X)
2000     CONTINUE
        ACCEPT 4220,OBSV,FREQ
        WRITE(12,4220)OBSV,FREQ
4220     FORMAT(2F5.0)
        IF(OBSV.LT.-0.01)OBFRQ(J)=SFREQ
        IF(OBSV.LT.-0.01)GO TO 2110
        IOBSV=OBSV*FACTOR+1.5
        IF(IOBSV.GT.J)GO TO 2100
        SFREQ=SFREQ+FREQ
        GO TO 2000
2100     OBFRQ(J)=SFREQ
        J=IOBSV
        SFREQ=SFREQ+FREQ
        GO TO 2000
2110     CONTINUE
        N=SFREQ
        SFREQ=1./SFREQ
        DO 2200 I=1,IOBSV
        OBFRQ(I)=OBFRQ(I)*SFREQ
```

```
2200      CONTINUF
C.     GET MAX ABS DEVIATE FOR TEST :
         DMAX=0.
         DO 2300 I=2,IOBSV
         IF(OBFRQ(I).LE.0.000001)GO TO 2300
         IF(QFRQ(I).LE.0.000001)QFRQ(I)=1.
         DEV=ABS(OBFRQ(I)-QFRQ(I))
         IF(DEV.GT.DMAX)DMAX=DEV
2300      CONTINUF
         FN=N
         SQRTN=SQRT(FN)
         TYPE 5000,N,SQRTN,DMAX
5000      FORMAT(/,1X,≠N,SQRTN,DMAX = ≠,I4,F10.5,F9.5)
         TYPE 5100
5100     FORMAT(/,1X,≠OBSERVED : ≠)
         TYPE 5200,(OBFRQ(I),I=1,IOBSV)
5200     FORMAT(5F9.5,/)
         TYPE 5300
5300      FORMAT(/,1X,≠COMPUTED : ≠)
         IF(IOBSV.GT.NQ)NQ=IOBSV
         TYPE 5200,(QFRQ(I),I=1,NQ)
         END
```

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