PB-201 878

PROPOSED METHOD FOR AERODYNAMIC MATHEMATICAL ANALYSES

Institute for Rapid Transit
Washington, D. C.

October 1970

Distributed ... 'to foster, serve and promote the nation's economic development and technological advancement.'

NATIONAL TECHNICAL INFORMATION SERVICE

U.S. DEPARTMENT OF COMMERCE

Technical Report No. UMTA–DC–MTD–7–71–4

Institute for Rapid Transit
Washington, D.C.

PROPOSED METHOD FOR AERODYNAMIC MATHEMATICAL ANALYSES

Interim Report

ASSOCIATED ENGINEERS/A JOINT VENTURE
Parsons, Brinckerhoff, Quade & Douglas, Inc.
De Leuw, Cather & Company
Kaiser Engineers

OCTOBER 1970

[redacted]

Prepared by
PARSONS, BRINCKERHOFF, QUADE & DOUGLAS, ~~INC.~~
Consulting Engineers • New York

for
UNITED STATES DEPARTMENT OF TRANSPORTATION
Urban Mass Transportation Administration
Washington, D.C. 20590

Technical Report No. UMTA-DC-MTD-7-71-4

Institute for Rapid Transit
Washington, D.C.

PROPOSED METHOD FOR AERODYNAMIC MATHEMATICAL ANALYSES

Interim Report

ASSOCIATED ENGINEERS/A JOINT VENTURE
Parsons, Brinckerhoff, Quade & Douglas, Inc.
De Leuw, Cather & Company
Kaiser Engineers

OCTOBER 1970

[redacted]

Prepared by
PARSONS, BRINCKERHOFF, QUADE & DOUGLAS, INC.
Consulting Engineers • New York

for

UNITED STATES DEPARTMENT OF TRANSPORTATION
Urban Mass Transportation Administration
Washington, D.C. 20590

PREFACE

At the outset of the Institute for Rapid Transit's research and development program on ventilation and environmental control in subway rapid transit systems, a meeting of the program participants engaged in the theoretical and experimental investigations of subway environment has been scheduled. The purpose of this meeting is to present and discuss the methodologies to be pursued by these participants in meeting their objectives and to further define the informational interfaces among these participants. Pursuant to the goals of this meeting, Parsons, Brinckerhoff, Quade and Douglas, Inc., which is responsible for the development of mathematical models for the computerized simulation of subway environment, has prepared this document for discussion purposes. This report outlines the methodology proposed for the development of the mathematical model which describes unsteady air flow throughout a subway system and is to be regarded as a working document, subject to revision and updating as the program advances.

Table of Contents

A. SUMMARY

The primary objective of this report is to establish and assure an understanding of the methodology to be employed in the aerodynamic mathematical model development and of the dependence of these mathematical models on the physical models. This dependence is characterized by the requirement of empirical input for the mathematical model.

For any underground subway system, or portion thereof, for which air flow rates resulting from train piston action are to be analyzed, all physical and geometrical parameters and all train performance characteristics must be taken into account, as exemplified by the numerical analysis contained in the Appendix of this report.

The analytical development outlined herein relates at present to the evaluation of air flow rates resulting from piston action of trains moving in single track tunnel. Future efforts will be geared towards expanding the analysis to take into account the interaction of piston effect and forced mechanical ventilation, as well as multiple track subway systems, with and without barrier walls between tracks.

The analysis assumes that air behaves as a perfect gas and that the air flow is isothermal, unsteady, viscous and compressible. The method of solution is based on the application of the principle of conservation of mechanical energy to finite system segments, which results in a set of simultaneous, non-linear, ordinary differential equations which in turn are solved by an iterative procedure.

Physical model investigations, which are scheduled concurrently with the mathematical analysis, will develop data not included in this analysis and will supply empirical input for it.

B. OBJECTIVE

The purpose of this report is to outline a proposed mathematical analysis for evaluating air flow rates in a single track underground subway system. An experimental evaluation on the basis of model tests will be undertaken concurrently with this effort. These model tests, in turn, will serve a dual purpose: (1) to develop data, such as air distribution patterns through a subway station, not included in the scope of the mathematical analyses, (2) to determine experimentally the magnitude of certain coefficients used in the mathematical analysis.

Since the mathematical model is dependent upon empirical input from physical model data it is important to assure an understanding of the methodology to be employed in the aerodynamic mathematical model development among all parties involved. The material contained herein is presented with that primary purpose in mind.

Alternately, the results of the aerodynamic analysis or parts thereof will be coupled with a thermodynamic analysis in order to evaluate the overall system environment and/or requirements for control of such environment.

C. DEFINITIONS AND SYMBOLS

1. Definitions:

System	- The physical underground structure of a subway system or part thereof which includes stations and tunnels.
Station	- A complete underground structure which serves as a means of egress to and from trains and as storage for passengers using the system. It includes platform,partial or full mezzanine, entrances, equipment rooms, concession areas and all ventilation shafts attached to or protruding from the station.
Tunnel	- The underground trainway between two stations, between two portals, or between station and portal.
Ventilation Shaft	- Any structure provided for the purpose of conveying air exchange between the system or any part thereof, and the outside atmosphere. The total length of a shaft is measured between the point of intersect with the system and the shaft terminal at free atmosphere.
Portal	- That point on a track profile which coincides with free atmosphere.
Train	- Any number of cars which operate as a unit. (For the purpose of this analysis, as well as other methods of evaluation of train

performance which take into account the instantaneous location of a train, the train location is considered to be a point corresponding to the train head on its longitudinal axis).

Section - A grouping of all those segments which are contained between the centerlines of two adjacent ventilation shafts.

Segment - Any length of station, tunnel, shaft or station entrance with statistically uniform physical and geometric properties. Accordingly, segments may be further identified as station segment, tunnel segment, shaft segment, or entrance segment, whichever is appropriate.

Finite Segment - A subdivision of a segment which in addition to statistically uniform physical and geometrical properties has statistically uniform fluid properties.

Node - The interface between two finite segments.

Frontal Area - The area contained within the projected perimeter of car and truck body.

Blockage Ratio - The ratio of the car portal area to the cross-sectional area of the tunnel or station segment.

Junction - Any interface between three or more segments; for example the interface between two tunnel segments and a ventilation shaft.

2. Symbols

English:

A_i - Tunnel cross section area at node i.

a_j - Frontal area of train j.

C_d - Total Aerodynamic Drag Coefficient.

C_{DF} - Coefficient of skin friction acting on the surface of the train.

C_{DP} - Aerodynamic Drag Coefficient of a train running in an open space.

C_F - Coefficient of skin friction acting on the tunnel wall.

D - Aerodynamic Drag.

d_i - Tunnel equivalent diameter defined as four times the tunnel cross section area divided by the tunnel perimeter at a node i.

D_r - Aerodynamic drag of a train in open space.

f_i - Force due to flow contraction at front of train.

f_f^k - Energy loss or gain at junction between tunnel sections K and K+1.

f_f^{sk} - Energy loss or gain at junction between tunnel K and shaft K.

f_k - Darcy-Weisbach friction factor at node k.

F_m - Skin friction force acting on the tunnel wall.

f_m - Term including skin friction force and forces acting on top and under body of train.

English cont'd

f_m^k - Energy input from mechanical devices in tunnel section k.

f_m^{sk} - Energy input from mechanical devices in shaft k.

f_x - Force due to flow expansion at rear of train.

i_{km} - m th node in segment k.

k - Segment number.

K_c - Contraction coefficient over front of train.

K_e - Entrance loss coefficient at portal

K_i - Kinetic Energy Flux correction factor at node i.

$K_{l_{sk}}$ - Loss Coefficient for Shaft k.

K_{sk} - Kinetic Energy Correction Factor for Shaft k.

K_x - Expansion coefficient over rear of train.

L_k - Length of tunnel segment k.

l_k - The length of shaft segment k.

L_{km} - Length of finite segment m in segment k.

l_{km} - The length of the mth finite segment in shaft segment k.

M - Denotes junction at section ends. M=0 is the left or entrance portal; M=N is the right or exist portal.

N - Number of sections in system.

N_k - Number of finite segments in segment k.

N_{sk} - Number of finite segment in shaft k.

o - Subscript denoting ambient conditions.

P_i - Mean Static pressure at node i.

P^a_b - The value of parameter P @ a divided by the value of parameter P @ b.

$\dot{Q}_m$ - The air mass flow rate at the m^{th} node.

R - Gas constant for air.

s_{km} - m^{th} node on shaft segment k.

t - time

T - Temperature

U_j - Velocity of j^{th} train.

V_i - Mean Air velocity at node i.

v_i - Radially dependent axial velocity at a node i.

V_{ka} - The mean air velocity at node a in segment k.

V_o - Velocity at time (t) = 0

X - Coordinate designating tunnel and station segment.

Y - Coordinate designating shafts.

<u>Greek</u>

δ_k - Perimeter of segment k.

ϵ_{jk} - Operator with value of unity when train j is in segment k - otherwise 0.

ζ_H - Length of head of train.

ζ_j - Length of train j.

ζ_{jk} - Length of train j in tunnel k.

ζ_m - Length of main portion of train.

ρ_i - Mean air mass density at node i.

ϕ_{jk} - Blockage ratio of the frontal area of the j^{th} train to the cross sectional area of segment k.

D. INTRODUCTION

The method outlined herein relates at present to the evaluation of air flow rates resulting from piston action of trains moving in a single-track tunnel. In future efforts the analysis will be expanded to take into account the interaction of piston effect with forced mechanical ventilation as well as multiple track systems, with and without structural separation of trainways. The analysis is based on the assumption that air behaves as a perfect gas and that air flow is one-dimensional, isothermal, unsteady, viscous and compressible.

The analysis, as outlined herein, requires that the system be divided into a series of finite segments. Application of the principle of conservation of mechanical energy to these finite segments results in a set of simultaneous, non-linear, ordinary differential equations which reflect the interdependence of pressure, density and velocity.

The proposed method of solution involves an iterative process: (1) an initial distribution of density is assumed; (2) using a numerical technique such as the Runge-Kutta method, the simultaneous equations are solved for the velocity distribution; (3) these velocities in turn are utilized to calculate the pressure distribution; (4) the density distribution is then computed utilizing the thermodynamic equation of state; (5) this density distribution is compared with that originally assumed; (6) if the computed densities do not match the assumed densities within a certain tolerance, the new density distribution is used and the process repeated until the tolerance between the new and previous densities is acceptable. The time is then stepped by some time increment and the process is repeated.

E. ANALYTICAL DEVELOPMENT

The descriptive geometry of an arbitrary subway system is illustrated by Figure 1. From this diagram it is seen that the origin, 0, of the coordinate frame is the left portal. The abscissa, X, designates the tunnel and station segments and the ordinate, Y, designates the shaft segments. A shaft segment may be connected to tunnel segments and/or station segments.

Figure 2 illustrates the finite segments into which the subway system is divided to facilitate treatment of spatial variations in physical parameters. It should be noted that, for ease of presentation, only one segment per section has been depicted. A finite segment must lie wholly within a segment; thus they will have in common statistically uniform physical properties such as wall roughness and cross section shape and area.

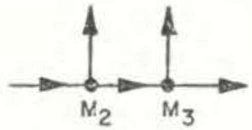
Application of the mechanical energy equation between the two adjacent nodes that bound a finite segment yields the following relationship between the mean air velocity and the mean static pressure:

$$\frac{K_B V_B^2}{2} + RT \ln p_B = \frac{K_A V_A^2}{2} + RT \ln p_A - \int_A^B \frac{\partial V}{\partial t} dx - \frac{1}{\rho_B} \int_A^B \frac{\rho f |V| V dx}{2d} \quad \text{(E-1)}$$

The subscripts A and B denote the adjacent nodes of the finite segment. The subscript B is used to "tag" the finite segment and always represents the node with the more positive X value.

The kinetic energy flux correction factor, K_i, is defined as:

$$K_i = \frac{\iint_{A_i} \frac{\rho_i v_i^2 (v_i)\, dA_i}{2}}{\frac{\rho V_i^3 A_i}{2}} \qquad \text{(E-2)}$$

Notations: 

Denotes Junctions of Tunnel Segments and/or Station Segments with Shaft Segments

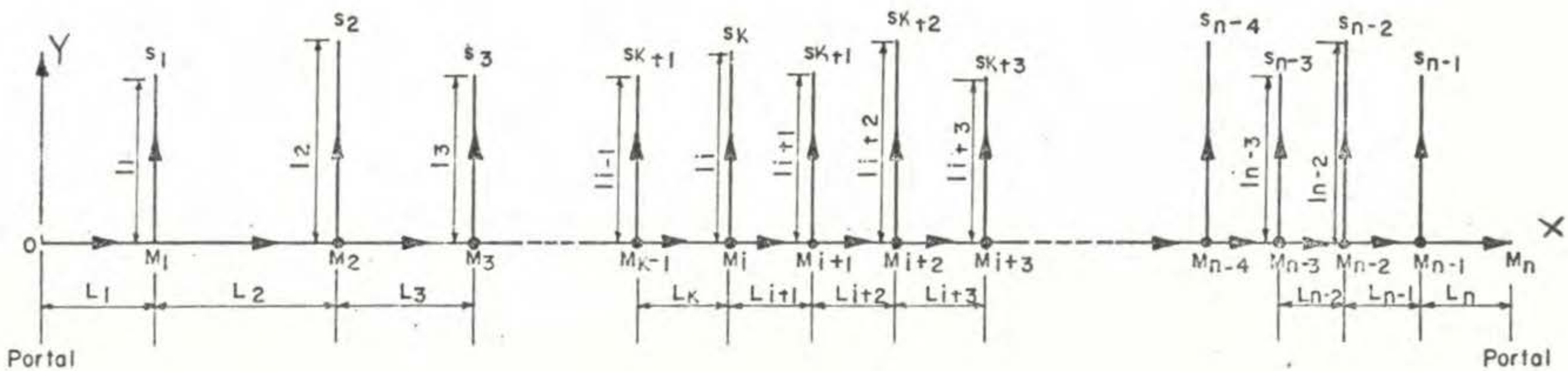


Fig. 1 SCHEMATIC REPRESENTATION OF A SUBWAY SYSTEM
(SINGLE TRACK)

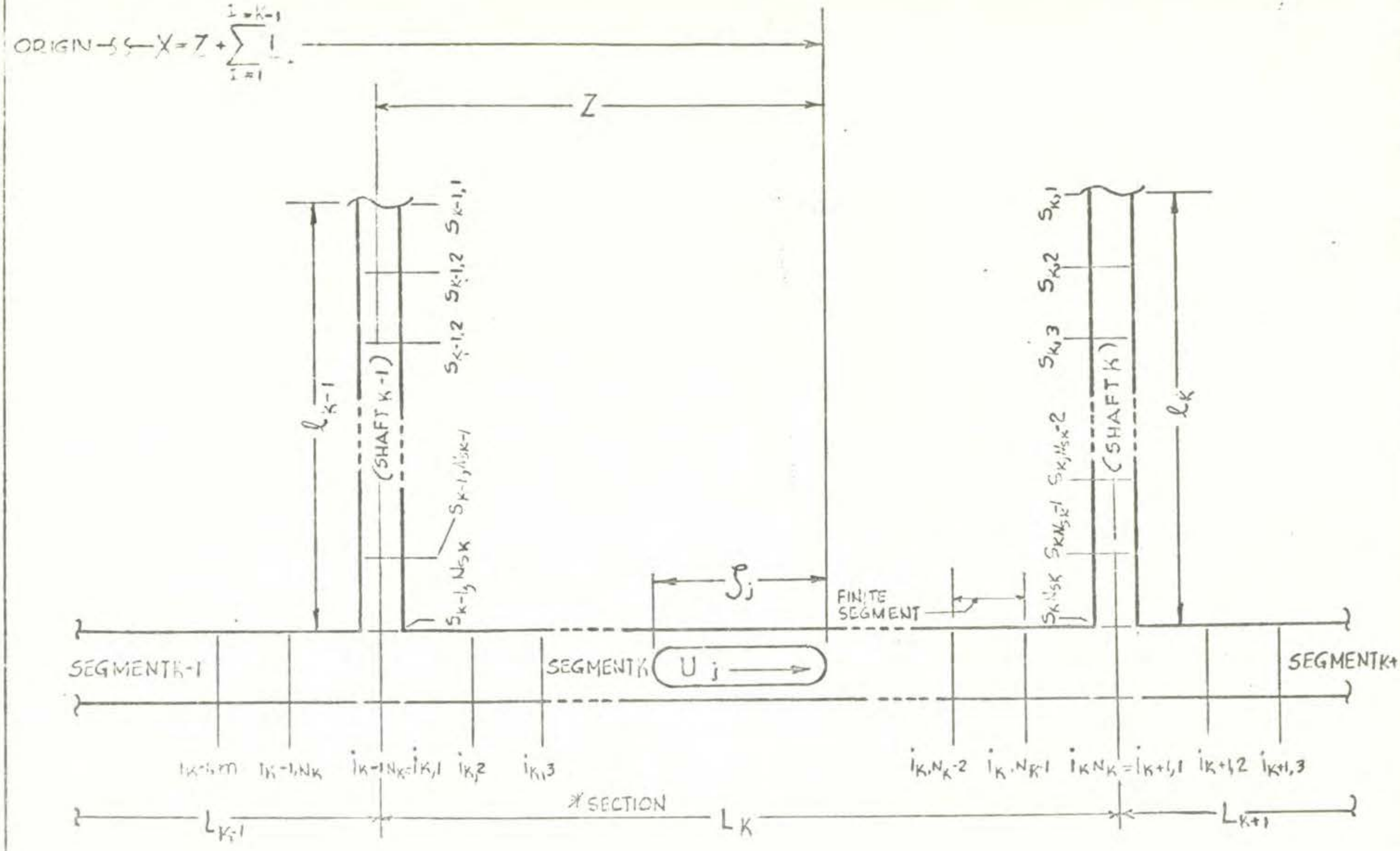


FIG. 2 SCHEMATIC PRESENTATION OF A TRAIN MOVING IN A SECTION WITH FINITE SEGMENT ARRANGEMENT SHOWN

* IN THIS ILLUSTRATION ONLY, THE SEGMENT LENGTH IS EQUAL TO THE SECTION LENGTH

Implicit in the formulation of equation (E-1) are the following assumptions:

1. The air behaves as a ideal gas
2. The flow is one-dimensional, unsteady, turbulent, and isothermal with negligible energy changes due to body forces.
3. The rate of change of the air mass in a finite segment is negligibly small in comparison with the net air mass flux through the finite segment.

By virtue of the defined statistical uniformity for a finite segment and assumption 3 above, the integrals in equation (E-1) may be evaluated in the following manner:

$$-\int_A^B \frac{\partial V}{\partial t} dx \cong (A-B)\frac{\partial V_B}{\partial t} \tag{E-3}$$

$$-\frac{1}{\rho_B}\int_A^B \frac{\rho f|V|V}{2d} dx \cong \frac{f_B(A-B)|V_B|V_B}{2d_B} \tag{E-4}$$

Substituting equations (E-3) and (E-4) into Equation (E-1) yields:

$$\frac{K_B V_B|V_B|}{2} + RT\ln P_B = \frac{K_A V_A|V_A|}{2} + RT\ln P_A + (A-B)\frac{\partial V_B}{\partial t} + \frac{f_B(A-B)|V_B|V_B}{2d_B} \tag{E-5}$$

The principle of conservation of mass may be applied at nodes and junctions to provide the following expression:

$$\sum_{m=1}^{m=N} Q_m = 0 \tag{E-6}$$

where N is the total number of finite segments meeting at a node or junction.

Equations (E-5) and (E-6) together with thermal equation of state, can be utilized to describe the behavior of air flowing through the subway system, assuming the existence of some forcing function to create the air flow. In this case, the forcing function is train piston effect, which is attributable to aerodynamic drag.

A pre-requisite to the evaluation of piston effect is a knowledge of the position, velocity, and acceleration of all trains in the subway system. Such information is illustrated by the Train Situation Diagram shown in Figure 3. At any instant the head of a train, j, is moving as depicted in Figure 3 in the positive X direction in the i-th tunnel section. By denoting this history curve of train j in functional form, F_j (t), the train velocity can be expressed as:

$$U_j(t) = \frac{\partial F_j(t)}{\partial t} = f_j(t) \qquad \text{(E-7)}$$

where f_j (t) is a known function of time.

The approach utilized in this analysis for the evaluation of aerodynamic drag and the pressure change along the length of a train traveling in a subway system involves an examination of the unsteady flow in the gap between the train and the tunnel wall. This method closely parallels Hara's work (7) which treats the steady drag case in a single tube. For the conditions described in Figure 2, which provides a schematic representation of a train moving in a tunnel section, the unsteady pressure change along the train length may be approximated by:

$$RT \ln \frac{P_A}{P_B} = \frac{C_{DF}\,\phi_{Kj}}{(1-\phi_{Kj})^3}\,\frac{(U_j - V_{KA})\left|(U_j - V_{KA})\right|}{2} + \frac{\overset{\varphi}{\ell}_j}{(1-\phi_{Kj})}\left(\frac{\partial U_j}{\partial t} - \frac{\partial V_{KA}}{\partial t}\right)$$

$$+ \frac{C_F\,\delta_K\,\overset{\varphi}{\ell}_j\,(U_j - V_{KA})\left|(U_j - V_{KA})\right|}{2A_K(1-\phi_{Kj})^3} + \frac{(K_c + K_x)(U_j - V_{KA})\left|(U_j - V_{KA})\right|}{2(1-\phi_{Kj})} \qquad \text{(E-8)}$$

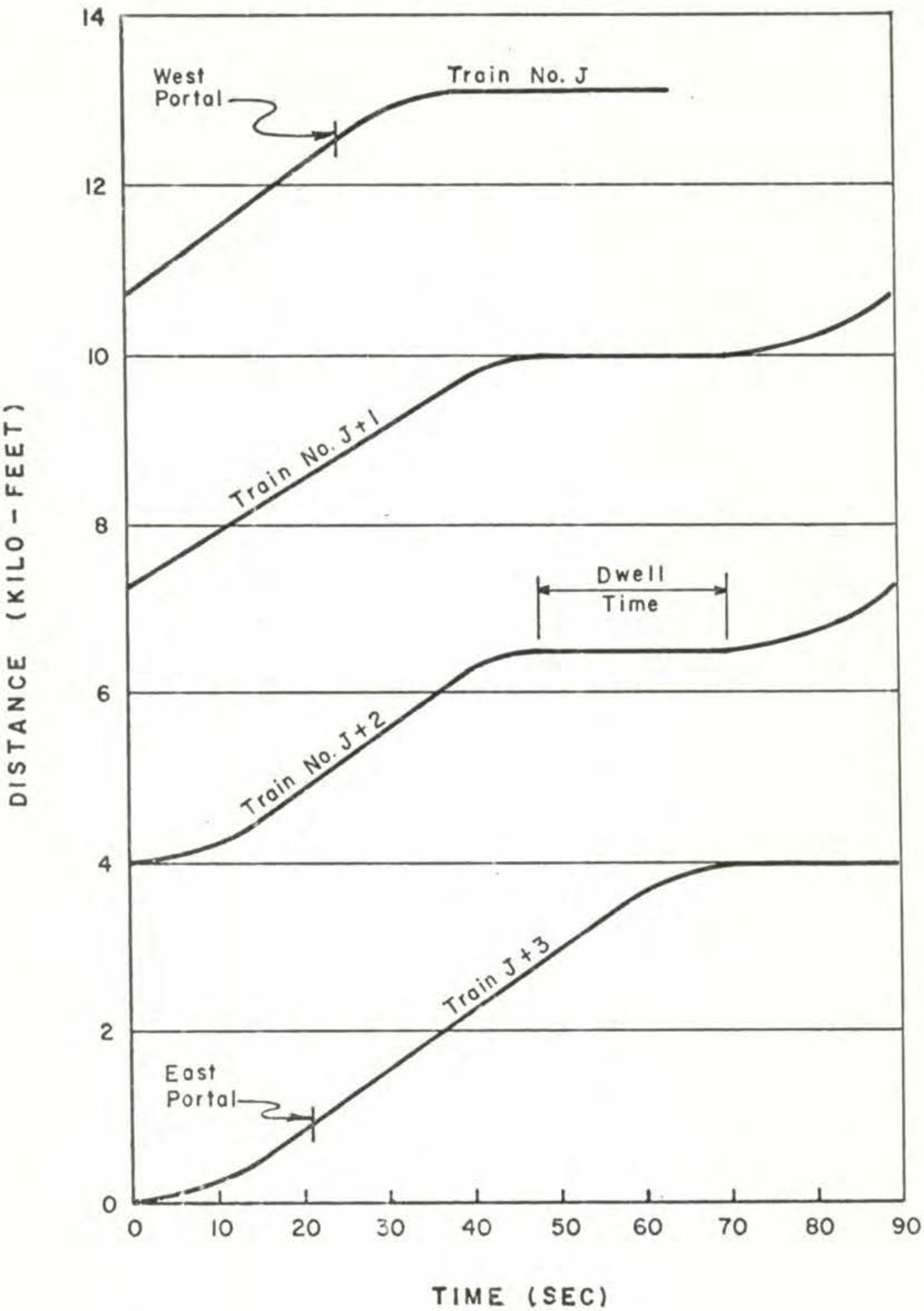


FIG. 3 TYPICAL TRAIN SITUATION DIAGRAM FOR A 90 SECOND HEADWAY

Where the subscripts A and B are the nodes ahead of and behind the train respectively.

The unsteady aerodynamic drag on the train is given by:

$$D = \frac{C_D \rho_A U_j^2 a_j}{2} \tag{E-9}$$

Where:

$$C_D = \frac{1}{(1-\phi_{Kj})^2}\left[1-\frac{V_{KA}}{U_j}\right]^2 + \frac{C_{DF}}{(1-\phi_{Kj})^3}\left[1-\frac{V_{KA}}{U_j}\right]^2 + \frac{C_{DP}}{(1-\phi_{Kj})^2}\left[1-\frac{V_{KA}}{U_j}\right]^2$$
$$+ \frac{C_F \delta_K \mathcal{S}_j \phi_{Kj}}{a_j(1-\phi_{Kj})}\left(\phi_{Kj}-\frac{V_{KA}}{U_j}\right)\left|\left(\phi_{Kj}-\frac{V_{KA}}{U_j}\right)\right| \tag{E-10}$$
$$+ \left(\frac{\partial U_j}{\partial t}-\frac{\partial V_{KA}}{\partial t}\right)\left(\frac{2}{\phi_{Kj}U_j^2}\right)\left(\frac{\mathcal{S}_H \phi_{Kj}-\mathcal{S}_m}{1-\phi_{Kj}}\right)$$

The coefficients in equation E-10 are defined as follows:

$$C_{DF} = \frac{f_m}{\frac{\rho_A}{2}\left(\frac{U_{Kj}-V_{KA}}{1-\phi_{Kj}}\right)^2 a_j} \tag{E-11a}$$

$$C_F = \frac{F_m}{\delta_K \mathcal{S}_j \rho_A\left(\frac{U_j-V_{KA}}{1-\phi_{Kj}} - U_{Kj}\right)^2} \tag{E-11b}$$

$$C_{DP} = \frac{D_r}{a_j \rho_A\left(\frac{U_j-V_{KA}}{1-\phi_{Kj}}\right)^2} \tag{E-11c}$$

$$K_C = \frac{F_C}{\frac{\rho_A}{2}\left(\frac{U_j - V_{KA}}{1-\phi_{Kj}}\right)^2} \tag{E-11d}$$

$$K_x = \frac{F_x}{\frac{\rho_A}{2}\left(\frac{U_j - V_{KA}}{1-\phi_{Kj}}\right)^2} \tag{E-11e}$$

The coefficients in equations (E-10) and (E-11) are functions of Reynolds number, and local Mach number for given train and system geometries. The coefficient C_{Dp} depends also on the Strouhal number which is a unique function of the Reynolds number (8). Therefore, the drag coefficient, C_d, depends upon Reynolds number, Mach number, blockage ratio, general porosity of the subway system, train history and ventilation (initial Kinematic condition):

The development to this point has been concerned with flow in statistically uniform finite segments, while no mention has been made of viscous losses due to abrupt changes in cross sectional area, and changes in flow direction. These losses are incorporated in the analysis as localized losses at nodes and junctions, since such area and flow direction changes are, by definition, nodes or junctions. Mathematically, this means that the fluid properties are piecewise continuous at a node or junction where such a change occurs.

The equation of energy can be combined with the equation of continuity, with reference to the specific geometry of the subway system. As Figure 1 illustrates, the tunnel sections are numbered sequentially from left to right, as are the nodes in each tunnel section. Note that the node numbers begin anew in each segment. The node number in each shaft progresses from the top to the bottom of the shaft. By writing the equation of continuity for each node in a section, the result can be manipulated such that all of the velocities for this group of nodes are expressed in terms of the velocity at the reference node, t_{ir}. These velocities are then substituted into the energy equation (E-5). The unsteady pressure change (equation E-8) is also substituted into the energy equation for the two nodes immediately preceeding and following the train. The pressures are eliminated by this process and the equations are expressed only in terms of the velocities and densities. In matrix form the equations are as follows:

$$\bar{B}\,\frac{\partial \bar{V}}{\partial t} = \bar{\Omega} \qquad \text{(E-12)}$$

Which is subject to the initial conditions:

$$\bar{V}(t_0) = \bar{V}_0 \qquad \text{(E-12a)}$$

$\bar{B}$ is a square matrix (B_{Mk}) with its dimensions being equal to the number of segments and thus the number of unknown velocities at the reference nodes. $\bar{V}$ and Ω are column matrices (V_k and Ω_m) whose lengths are equal to the dimension of $\bar{B}$ - N. Thus:

$$\begin{bmatrix} B_{11} & B_{12} & 0 & \cdot\cdot & \cdot\cdot & 0 \\ B_{21} & B_{22} & B_{23} & 0 & \cdot\cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & B_{N-1\,N} \\ B_{N1} & \cdot & \cdot & \cdot & \cdot & B_{NN} \end{bmatrix} \times \begin{bmatrix} \frac{\partial V_1}{\partial t} \\ \cdot \\ \cdot \\ \cdot \\ \frac{\partial V_N}{\partial t} \end{bmatrix} = \begin{bmatrix} \Omega_1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \Omega_N \end{bmatrix} \qquad \text{(E-13)}$$

1) IF $0 < M < N-1$ AND $K > M+1$

$$\text{THEN} \quad B_{MK} = 0 \tag{E-13a}$$

2) IF $K = M = N$ OR $0 < M < N$ WITH $K < M$

$$\text{THEN} \quad B_{MK} = \frac{\mathcal{g}_{jK}\phi_{Kj}}{1-\phi_{Kj}} + \sum_{m=2}^{m=N_K} L_{Km} A_{i_{Km}}^{i_{KN_K}} \rho_{i_{Km}}^{i_{KN_K}} \tag{E-13b}$$

3) IF $0 < M < N$ AND $K = M+1$

$$\text{THEN} \quad B_{MK} = A_{S_{KN_{SK}}}^{i_{K+1},N_K} \rho_{S_{KN_{SK}}}^{i_{K+1},N_K} \sum_{m=2}^{m=N_{SK}} \ell_{Km} \rho_{S_{Km}}^{S_{KN_{SK}}} \tag{E-13c}$$

4) IF $0 < M < N$ AND $K = M$

$$\text{THEN} \quad B_{MK} = \frac{\mathcal{g}_{jK}\phi_{Kj}}{(1-\phi_{Kj})} + \sum_{m=2}^{m=N_K} L_{Km} A_{i_{Km}}^{i_{KN_K}} \rho_{i_{Km}}^{i_{KN_K}} \tag{E-13d}$$

$$+ A_{S_{KN_{SK}}}^{i_{KN_K}} \rho_{S_{KN_{SK}}} \sum_{m=2}^{m=N_{SK}} \ell_{Km} \rho_{S_{Km}}^{S_{KN_{SK}}} A_{S_{Km}}$$

IF $0 < M < N$

THEN $$\Omega_M = \sum_{K=1}^{K=M} \epsilon_{jK}\Bigg[\left(\frac{C_{DF}\,\phi_{Kj}}{(1-\phi_{Kj})^3} + \frac{K_x + K_c}{(1-\phi_{Kj})^2}\right)\times$$

$$\left(\frac{\left(U_j - \rho^{i_{KN_K}}_{i_{MN_K}} A^{i_{KN_K}}_{i_{MN_K}} V_{i_{KN_K}}\right)\left|\left(U_j - \rho^{i_{KN_K}}_{i_{MN_K}} A^{i_{KN_K}}_{i_{MN_K}} V_{i_{KN_K}}\right)\right|}{2}\right)$$

$$+ \frac{C_F\,\delta_K\,\varrho_{jK}\left(U_j - \rho^{i_{KN_K}}_{i_{MN_K}} A^{i_{KN_K}}_{i_{MN_K}} V_{i_{KN_K}}\right)\left|\left(U_j - \rho^{i_{KN_K}}_{i_{MN_K}} A^{i_{KN_K}}_{i_{MN_K}} V_{i_{KN_K}}\right)\right|}{2a_j(1-\phi_{Kj})^3}$$

$$+ \frac{\phi_{Kj}\,\varrho_{jK}}{1-\phi_{Kj}}\frac{\partial U_j}{\partial t}\Bigg] + \sum_{K=1}^{K=M} f_m^K + \sum_{K=1}^{K=M-1}\left(f_f^K + \left\{f_f^{SK}\right\}\right) + \left\{f_m^{SK}\right\}$$

$$- \sum_{K=1}^{K=M}\sum_{m=2}^{m=N_K} \frac{f_{Km} L_{Km}\left(A^{i_{KN_K}}_{i_{Km}}\rho^{i_{KN_K}}_{i_{Km}}\right)^2 \left|V_{i_{KN_K}}\right|\left(V_{i_{KN_K}}\right)}{2d_{Km}}$$

$$- \left\{\sum_{m=1}^{m=N_{SK}}\left[\frac{f_{S_{Km}}\,\ell_{Km}\left(A^{S_{KN_{SK}}}_{S_{Km}}\rho^{S_{KN_{SK}}}_{S_{Km}}\right)^2}{2d_{S_{Km}}} + K_{SK}K_{LSK}\,\rho^{S_{KN_K}}_{S_{KN_K}} A^{S_{KN_K}}_{S_{KN_K}}\right]\times\right.$$

$$\left[\frac{\left(A^{i_{KN_K}}_{S_{KN_K}}\rho^{i_{KN_K}}_{S_{KN_{SK}}} V_{S_{KN_K}} - A^{i_{K+1,N_K}}_{S_{KN_{SK}}}\rho^{i_{K+1,N_K}}_{S_{KN_{SK}}} V_{S_{K+1,N_K}}\right)}{2}\right]\times$$

$$\left.\left[\left|A^{i_{KN_K}}_{S_{KN_K}}\rho^{i_{KN_K}}_{S_{KN_{SK}}} V_{S_{KN_K}} - A^{i_{K+1,N_K}}_{S_{KN_{SK}}}\rho^{i_{K+1,N_K}}_{S_{KN_{SK}}} V_{S_{K+1,N_K}}\right|\right]\right\}$$

$$- \frac{K_E K_1 A^{i_{KN_K}}_{i_{11}}\rho^{i_{KN_K}}_{i_{11}}\left|V_{KN_K}\right| V_{KN_K}}{2}$$

(E-14)

If M = N then equation (E-14) is modified by removing the terms enclosed by brackets of type $\{\ \}$ and adding the term $\frac{K_N V_{NK_N}^2}{2}$ to the right hand side of the equation.

Equation (E-12) can be solved for the velocities at each reference node at any instant in time. Utilizing the equation of continuity, the velocities at each node are then found from the velocities at the reference nodes. The nodal velocities are substituted into the energy equation yielding the pressures at each node. For an isothermal process, assuming an ideal gas, the pressure - density relationship takes the form:

$$\rho_{km} = \frac{P_{km}}{RT_{km}} \tag{E-15}$$

where T_{KM} has a given value at any instant.

Using this state relationship, the density at each node is evaluated and compared with the assumed densities. If the difference between succeeding densities is greater than a specified tolerance, the new densities are used and the entire process is repeated. If the densities are within the tolerance, the time is stepped and the process is repeated.

F. APPENDIX

1. Numerical Example Assuming Incompressible Flow

A previous modeling of the aerodynamic behavior of a subway system was based on the assumption of incompressible flow (1). The assumption of incompressibility may be restrictive in the sense of limiting the range of applicability of the results (low blockage ratios, low velocities and short section lengths); nevertheless, the results obtained through such an approach can serve to provide the analyst with qualitative information as to aerodynamic behavior. The following numerical example is presented with this thought in mind.

There are six coefficients, C_{Df}, C_F, C_{DP}, K_X, K_C and f_F in the governing equations which must be determined through experiment. Literature shows that available date are limited. The following data are used in this numerical example.

The coefficient of the drag force acting on the side surface of a train, C_{Df}, was measured by Hara in tests (7) with actual trains. The result is:

$$C_{Df} = 0.0045\varsigma \text{ to } 0.0079\varsigma \qquad (1)$$

where ς is the length of the train in meters.

The coefficient of skin friction on the tunnel wall, C_F, measured by Maekawa (9) and rearranged by Miyata (10) falls in the range

$$C_F = 0.0035 \text{ to } 0.0087 \qquad (2)$$

The coefficient of air drag acting on both the front and the rear surfaces of an operated train without attachments on both and surfaces of a train is approximately (7).

$$C_{DP} = 0.12 \qquad (3)$$

The suggested values of the friction factor, f_f, (11) are 0.02 for concrete subway segments and concrete shafts, and 0.08 for ribbed steel lined segments. A value of 0.012 for a tunnel friction factor is reported in the Libby Dam test (12).

The expansion pressure loss coefficient, K_x, over the rear of the train may be evaluated based on the Carnot-Borda equation:

$$h_{loss_{1,2}} = \frac{(V_1 - V_2)^2}{2g} \qquad (4)$$

where V_1 and V_2 are the air velocities at both sides of an abrupt increase of the gap flow. Using Eq. (1) and the equation of continuity, K_x is given by:

$$K_x = \phi_{kj}^2$$

where $\phi_{ij} = \dfrac{a_j}{A_k}$ (5)

The contraction pressure loss coefficient, K_c, over the front of the train is given a value of 0.5 to describe the maximum head loss involved when a flowing constant density fluid encounters an abrupt decrease in the area of its system boundaries (13, 14). In the present numerical example, the following is used:

$$K_c = 0.5\ \phi_{kj} \qquad (6)$$

The coefficients of energy loss and gain of flow in the entrance region of tunnels and shafts are dependent largely on the configuration of the entrance region (15, 16).

In utilizing the mathematical model, the user specifies the subway layout, subway geometry (length, cross-sectional area, and perimeter of trainways, stations and shafts), train operation schedule, train geometry (length, cross-sectional area, and perimeter), physical coefficients (energy losses and gains) and initial ventilation conditions. The model output consists of a report of the direction and magnitude of the air velocities at each section of the subway system, and the locations of trains in the system with respect to time.

Samples of input data for a subway system are given in Table. 1. Fig. 4 is the alignment of the subway system described in Table 1. The system has three stations, eleven station and tunnel sections and ten shafts.

Figs. 5 and 6 are histories of air flows in the tunnels and the shafts for the train traveling with a maximum speed, $U_{max.}$ from the left portal toward the right portal. Initial air flow in the system is assumed to be zero. As the train enters the left portal, substantial air flow is generated in tunnel section No. 1 and shaft No. 1. This is due to the fact that positive pressure created in front of the train pushes the air through the shafts and tunnels. Once the train is fully in section No. 1 air flow in shafts No. 1 and No. 2 is fully built up.

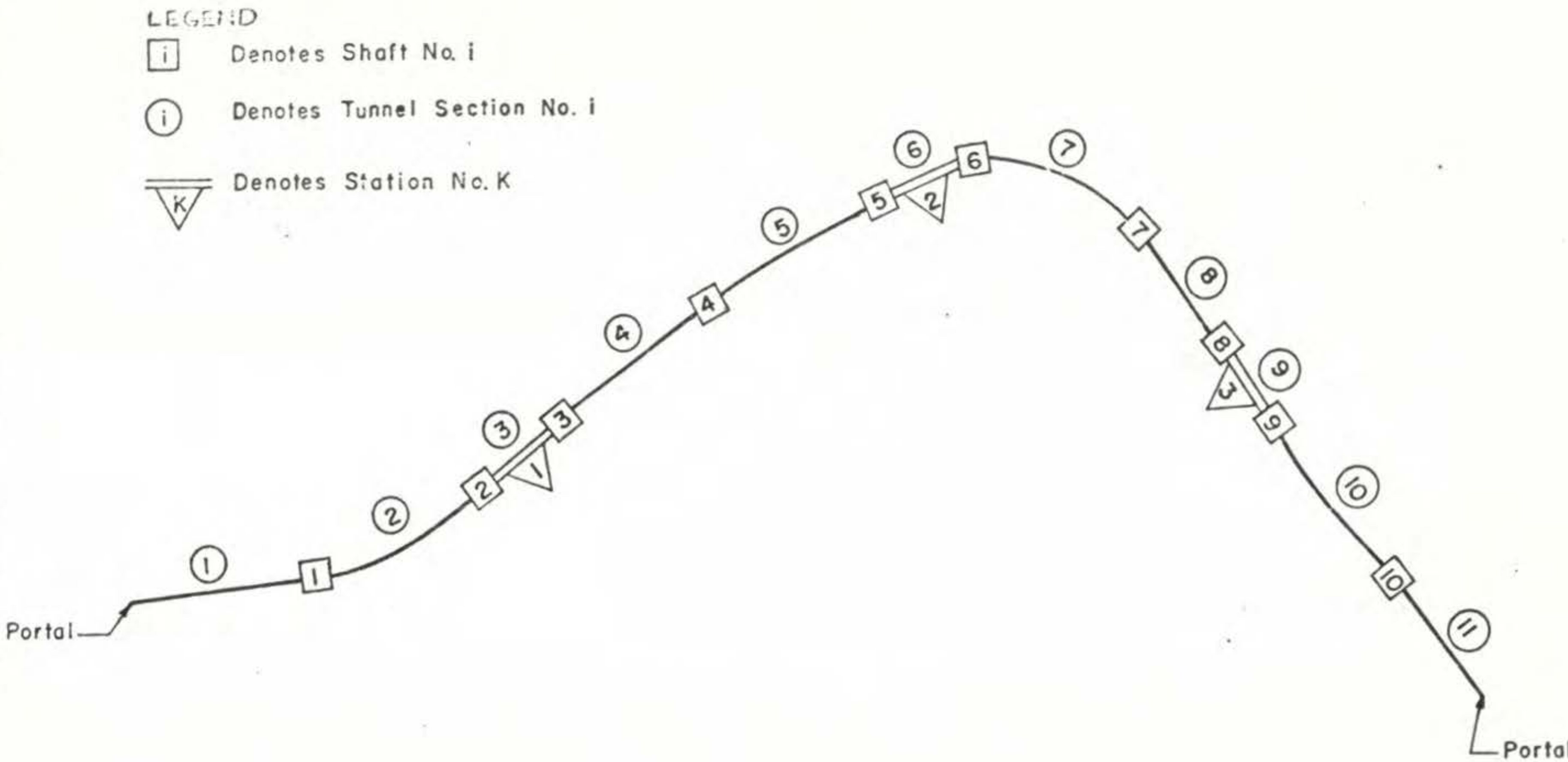


Fig. 4 ROUTE OF A SUBWAY SYSTEM FOR NUMERICAL EXAMPLE

TABLE 1.

Input Data for a Train-Tunnel System

System Sections

Section Number	Length (ft)	Cross Section Area (sq. ft.)	Perimeter (ft)
1	1501.5	209.5	52.5
2	1501.5	209.5	52.5
3	500.0	795.9	133.0
4	1006.5	209.5	52.5
5	1006.5	209.5	52.5
6	500.0	795.9	133.0
7	1521.0	209.5	52.5
8	1521.0	109.5	52.5
9	500.0	795.9	133.0
10	1113.0	193.7	56.1
11	1113.0	193.7	56.1

Shafts

Number	Length (ft)	Cross Section Area (sq. ft.)	Perimeter (ft)
1	32.0	72.5	34.3
2	91.0	152.5	58.4
3	91.0	152.5	58.4
4	32.0	71.5	34.3
5	91.0	152.5	58.4
6	91.0	152.5	58.4
7	32.0	72.5	34.3
8	91.0	152.5	58.4
9	91.0	152.5	58.4
10	23.1	57.0	30.4

Train

Number	Length (ft)	Frontal Area (sq. ft.)	Perimeter (ft)	Speed (MPH)
1 (only)	400.0	93.7	36.4	44

As the train enters section No. 2, the air flow decreases in section No. 1, and increases in section No. 2. The air flow in shaft No. 1 changes its direction, reacting to the negative pressure generated at the rear of the train.

The maximum negative air velocity in shaft 1 appears to be greater than the maximum positive air velocity for this case. However, the relative magnitude of the air flow in the shaft depends on the subway geometry as a whole (i.e., relative location, length and size of shafts and sections). Usually the positive pressure generated in front of train appears to be smaller in absolute magnitude than the negative pressure at the rear of the train.

The effect of changing the system size on the air flows in tunnel sections and shafts is seen to be small in these sections which are in the immediate vicinity of a moving train. In the computation, alternative systems with three stations (Fig. 4) two stations (station No. 3 omitted), and one station (stations No. 3 and 2 omitted) were used. The air flow in sections No. 1, 2 and 3 as well as in shafts No. 1, 2 and 3 in three subways are nearly the same. There is a minor change of air flow in the other sections. However, the magnitude of air flow in these sections is negligibly small. This implies that the air motion generated by a train is confined to the three sections associated with a moving train. This apparent characteristic of the air motion will perhaps allow the flow analysis of a large subway system to be performed on a smaller system.

The sensitivity of air flow to changes in train speed is illustrated in Figs. 5 and 6. For a given system, the pattern of the dimensionless air velocity with respect to the dimensionless time is nearly identical. The maximum air flow generated by a train is proportional to the maximum train speed. Figs. 5 and 6 also show that the total volume of air motion in a tunnel or a shaft generated by a moving train with a headway is largely independent of the maximum train speed. This implies that shorter headways will result in the generation of greater air circulation in the system.

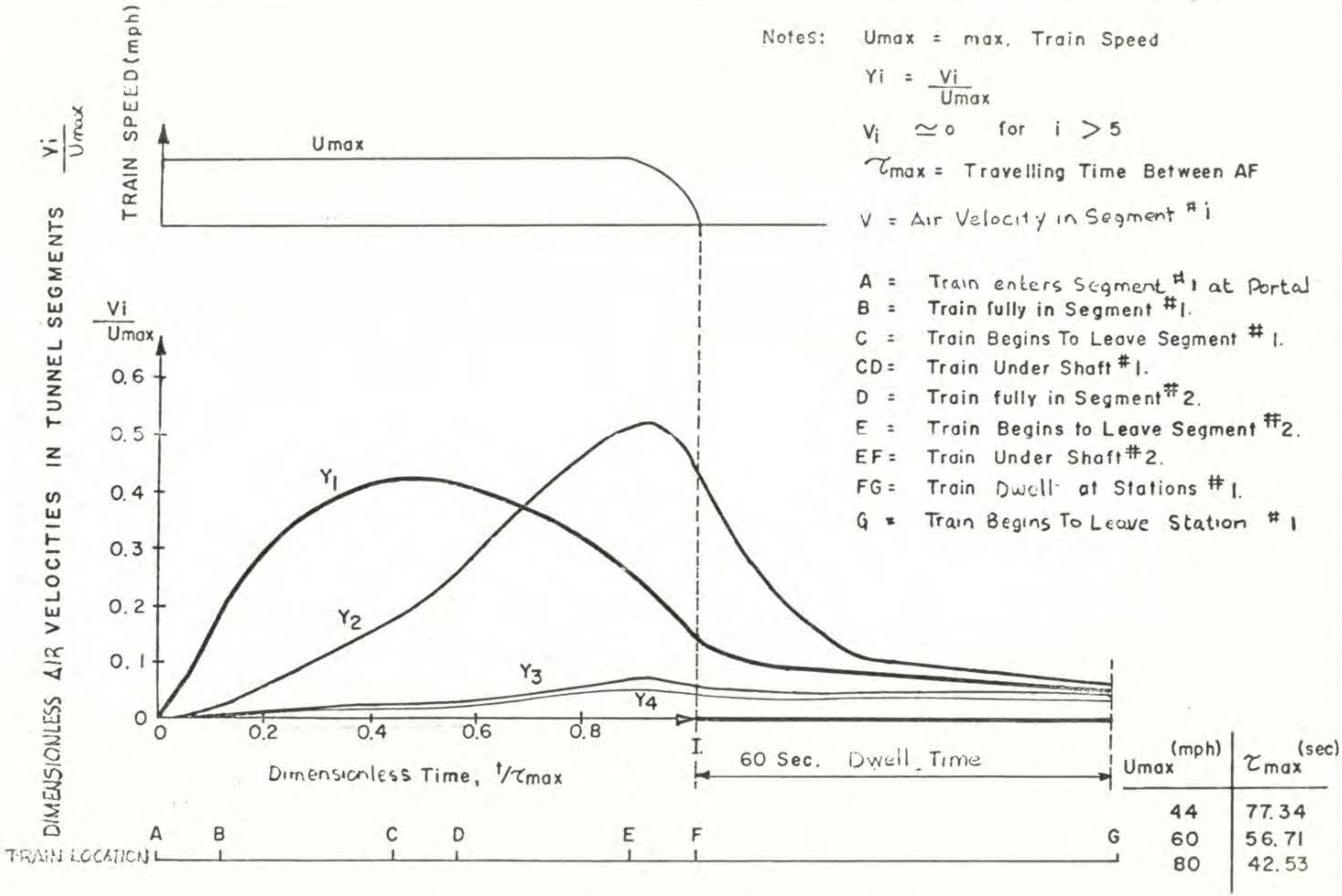
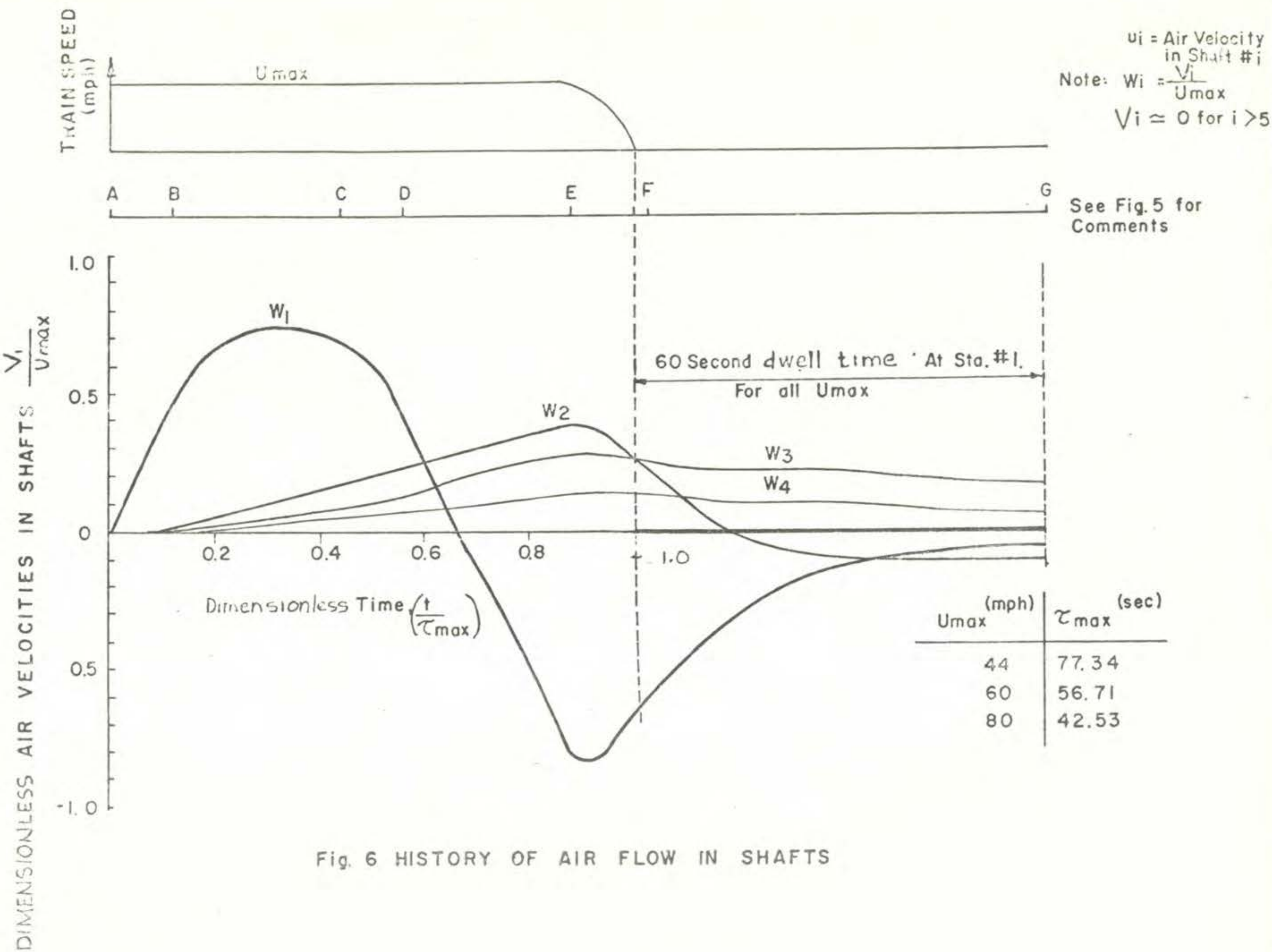


Fig. 5 HISTORY OF AIR FLOW IN TUNNEL SEGMENTS



Umax (mph)	τ_{max} (sec)
44	77.34
60	56.71
80	42.53

Fig. 6 HISTORY OF AIR FLOW IN SHAFTS

2. References:

1. T. C. Chen, "Subway Environmental System Analysis, Part I, Fluid Mechanics of Train-Tunnel Systems", the Eighth Conference of Engineering Science, Washington, D.C. to be published in November, 1970.

2. H. W. Emmons (Editor) Foundations of Gas Dynamics: Vo.. III of High Speed Aerodynamics and Jet Propulsion Princeton, 1956.

3. V. L. Streeter, Handbook of Fluid Dynamics, McGraw-Hill, New York, 1961.

4. R. P. Benedict, and N. A. Carlucci, "Flow with Losses" Transactions of the ASME, Journal of Engineering for Power, Jan. 1965, pp. 37-49.

5. S. W. Gouse, Jr., and associates, "Aerodynamic Drag on Vehicles in Tunnels" Transactions of the ASME, Dec. 1969, pp. 694-706.

6. T. C. Chen, "Draft on Subway Ventilation Analysis", Parsons, Brinckerhoff, Quade & Douglas, Inc., 111 John St., New York, March 1969.

7. T. Hara, "Aerodynamic Drag on Trains in Tunnels" Quarterly Report RTRI.

8. H. Schlichting, Boundary Layer Theory, 4th Edition, McGraw-Hill, 1955.

9. J. Maekawa, Railway Engineering Research, Vol. 8, No. 3, (1951) P. 15 & Vol. 8, No. 10 (1951), P. 11.

10. Miyata, Pre. Report of RTRI No. 58-62 (1958).

11. PBQ&D, Inc., "Subway Ventilation by Piston Effect - Designer's Manual", Report prepared by PBQ&D, Inc. for the SFBA Rapid Transit District, Sept. 1965.

12. E. G. Aisiks and N. H. Danziger, "Ventilation Research Program at Cascade Tunnel, Great Northern Railway" American Railway Engineering Association, September-October 1969.

13. R. C. Binder, Fluid Mechanics, Prentice-Hall Book Co., N.Y., N.Y. 3rd Edition, 1955, p. 128.

14. R. P. Benedict, N. A. Carlucci, and S. D. Swetz, "Flow Losses in Abrupt Enlargements and Contractions", Journal of Eng. for Power, ASME Trans., Vol. 88, January 1966, pp. 73-81.

15. W. D. Campbell, and J. C. Slattery, "Flow in the Entrance of a Tube", Journal of Basic Eng., ASME Trans., Vol. 85. pp. 41-46 1963.

16. E. H. Sparrow, S. H. Lin, and T. S. Lundgren, "Flow Development in the Hydrodynamic Entrance Region of Tubes and Ducts", The Physics of Fluids, Vol. 7, No. 3, 1964, pp. 338-347.

STANDARD TITLE PAGE FOR TECHNICAL REPORTS

1. Report No. UMTA-DC-MTD-7-71-4
2. Govt. Accession No.
3. Recipient's Catalog No.
4. Title and Subtitle: PROPOSED METHOD FOR AERODYNAMIC MATHEMATICAL ANALYSES
5. Report Date: October 1970
6. Performing Organization Code
7. Author(s): Parsons, Brinckerhoff, Quade & Douglas, Inc.
8. Performing Organization Rept. No. (From IRT)
9. Performing Organization Name and Address: Institute for Rapid Transit, 1612 K Street, N. W. , Suite 508, Washington, D. C. 20006
10. Project/Task/Work Unit No. DC-MTD 7
11. Contract/Grant No. UT-290
12. Sponsoring Agency Name and Address: Urban Mass Transportation Administration, 400 Seventh Street, S. W., Washington, D. C. 20590
13. Type of Report & Period Covered: INTERIM
14. Sponsoring Agency Code
15. Supplementary Notes

16. Abstracts

The primary objective of this report is to establish and assure an understanding of the methodology to be employed in the aerodynamic mathematical model development and of the dependence of these mathematical models on the physical models. This dependence is characterize by the requirement of empirical input for the mathematical model. The analytical development outlined herein relates at present to the evaluation of air flow rates resulting from piston action of trains moving in single track tunnel. Future efforts will be geared towards expanding the analysis to take into account the interaction of piston effect and forced mechanical ventilation, as well as multiple track subway systems, with and without ventilation, as well as multiple tracks.

17. Key Words and Document Analysis. 17a. Descriptors

1306 Subways
1505 Urban Transportation
0505 Environmental Engineering
1301 Engineering
1313 Tunnels
1201 Mathematical Models

17b. Identifiers/Open-Ended Terms

Single track
Vehicle in Confined Spaces
Vehicles in Tubes

17c. COSATI Field/Group 13F, 05E

18. Distribution Statement: **Releasable to the public. Available from NTIS, Springfield, Va. 22151**
19. Security Class (This Report) UNCLASSIFIED
20. Security Class (This Page) UNCLASSIFIED
21. No. of Pages 38
22. Price $3.00

FORM CFSTI-35 (4-70) USCOMM-DC 65002