Optimal Alpha-Beta Parameter Selection For Altitude Rate Tracking Of Digital Air Data Computer Derived Altitude

Barry R. Billmann

March 1983

DOT/FAA/CT-82/163

Document is on file at the Technical Center
Library, Atlantic City Airport, N.J. 08405

U.S. Department of Transportation
Federal Aviation Administration

Technical Center
Atlantic City Airport, N.J. 08405

NOTICE

Technical Report Documentation Page

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
DOT/FAA/CT-82/163		
4. Title and Subtitle OPTIMAL ALPHA-BETA PARAMETER SELECTION FOR ALTITUDE RATE TRACKING OF DIGITAL AIR DATA COMPUTER DERIVED ALTITUDE		5. Report Date April 1983 6. Performing Organization Code
7. Author(s) Barry R. Billmann		8. Performing Organization Report No. DOT/FAA/CT-82/163
9. Performing Organization Name and Address Federal Aviation Administration Technical Center Atlantic City Airport, New Jersey 08405		10. Work Unit No. (TRAIS) 11. Contract or Grant No. 052-244-340 13. Type of Report and Period Covered
12. Sponsoring Agency Name and Address U.S. Department of Transportation Federal Aviation Administration Technical Center Atlantic City Airport, New Jersey 08405		Technical Note 14. Sponsoring Agency Code

15. Supplementary Notes

16. Abstract

The Digital Air Data Computer (DADC) can provide more precise altitude information on the "own" aircraft's altitude than is available by monitoring Mode C altitude output. Review of DADC specifications indicates DADC vertical velocity information may be too noisy for direct use by the collision avoidance logic in the Traffic Alert and Collision Avoidance System (TCAS). This report identifies the optimal alpha-beta parameters which should provide the smoothed vertical velocity estimates required by the collision avoidance logic. The only remaining task is to obtain sufficient DADC altitude data to insure the resulting variance reduction is sufficient. If not adequate or if the error in the DADC altitude is autocorrelated, slight adjustment to the selected parameters may be necessary.

17. Key Words	18. Distribution Statement
Digital Air Data Computer Tracking Digital Filter	

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price
Unclassified	Unclassified	19	

Form DOT F 1700.7 (8-72) Reproduction of completed page authorized

PREFACE

The author wishes to express appreciation to Dr. R. E. Lefferts of the FAA Technical Center for his assistance in the literature research.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

LIST OF TABLES

INTRODUCTION

The Digital Air Data Computer (DADC) can provide more precise altitude information on the "own" aircraft's altitude than is available by monitoring Mode C altitude output. Review of DADC specifications indicates DADC vertical velocity information may be too noisy for direct use by the collision avoidance logic in the Traffic Alert and Collision Avoidance System (TCAS). This report identifies the optimal alpha-beta parameters which should provide the smoothed vertical velocity estimates required by the collision avoidance logic.

The DADC output, as specified in reference 1, was used to determine the quality of altitude information that is available from the DADC. Table 1 identifies the characteristics of the DADC output that was used in the analysis presented in this report.

TABLE 1. CHARACTERISTICS OF DADC DATA

Parameter	Range	Resolution	Update Rate
Altitude	-1,000 to 50,000 feet	1 foot	16 hertz
Altitude Rate	-20,000 to 20,000 ft/min	10 ft/min	16 hertz

Optimality is defined as the quality which provides the best balance between transient response delay, peak error in the velocity estimate, and noise rejection capability of the alpha-beta tracking system. Previously, work by Andrews (reference 2) and Billmann (reference 3) had identified problems, with the use of alpha-beta tracking, of highly quantized altitude inputs (Mode C altitude). However, the DADC can provide altitude inputs with 1-foot resolution. This granularity is assumed to have little impact on alpha-beta tracking performance. Following analytical selection of the theoretically optimal parameters, extensive numerical testing was performed to verify the validity of the granularity assumption. Although the DADC can provide altitude data at a 16-hertz rate, a 1-hertz data rate was analyzed since this matched the cycle time of the collision avoidance logic and was considered adequate.

The noise rejection performance of the alpha-beta tracker can be evaluated with the use of the measure called the variance reduction ratio. This measure is defined as the ratio of the variation in the tracker output to the variation in the tracker input. The lower this ratio is, the better the noise rejection performance of the tracker. Settling time is the time required for the tracker to respond to a transient input and the velocity estimated to settle within a given tolerance of the true velocity. Current nonlinear tracking of Mode C altitude results in settling times in the range of 9 to 15 seconds for 10 percent tolerance. The goal was to obtain parameters such that alpha-beta tracking of DADC altitude input would yield settling times of 6 seconds and variance reduction ratios ≤ 0.3 for the range of rates of interest (0 to 6,000 ft/min) and 0.1 G accelerations.

Following analytical derivation of optimal alpha and beta parameters, numerical methods were used to determine the effects of input granularity and finite precision arithmetic on tracker performance.

The analytically derived values of alpha and beta were $\alpha = 0.50$ and $\beta = 0.13$. Numerical analysis resulted in a slight shift in the value of β. The values selected for alpha-beta tracking are $\alpha = 0.50$ and $\beta = 0.15$. With these values of alpha and beta, several examples of tracking performance using input data provided by National Transportation Safety Board (NTSB) were reviewed. One example is discussed in this report.

METHODOLOGY

Since only a limited amount of DADC altitude data was available, selection of alpha and beta was based on theoretical and numerical methods rather than empirical methods. Once sufficient data become available, the only remaining task is to determine if sufficient noise rejection exists with the selected parameters. Minor adjustments, possibly at the cost of increased settling times, may be necessary to provide the noise rejection needed.

ANALYTICAL RESULTS.

A literature review was made to identify the initial ranges of alpha and beta for the numerical study which followed. In reference 4, Lefferts provides the following equation for determining the normalized error in the velocity response of an alpha-beta tracker to a ramp input.

$$h_v(t)=(1-\beta)\cdot \exp\{-a\cdot(t-1)\}\{\cos\{\omega(t-1)\}-\kappa\cdot\sin\{\omega(t-1)\}\} \qquad t\geq 1$$

where: $\quad = 0 \qquad t\leq 0 \qquad (1)$

$$a=-\ln(1-\alpha)/2$$

$$\omega=\arccos\{\{1-(\alpha+\beta)/2\}/\sqrt{1-\alpha}\}$$

$$\kappa=\{(\beta-\alpha)/(1-\beta)+(\alpha+\beta)/2\}/\sqrt{\beta-(\alpha+\beta)^2/4}$$

From this equation Lefferts was able to obtain the settling time to a given tolerance as a function of alpha. Beta is constrained so that

$$\beta = \alpha^2/(2-\alpha) \qquad (2)$$

This constraint is called the Benedict and Bordner relationship (reference 5). The constraint yields the selection of beta which minimizes, during transient periods, the mean square error in the velocity response to a unit ramp input. Figure 1, provided by Lefferts from reference 4, identifies the minimum size alpha required to obtain a given settling time for a fixed tolerance level. The normalized time scale represents the number of equally spaced sampling periods following the initiation of the transient input. For 1-second sampling periods, as in the case of TCAS, the abscissa value of 6 represents a 7-second settling time. As a result, alpha must be greater than or equal to 0.4 to obtain a 7-second settling time to 10 percent tolerance. With $\alpha = 0.4$, equation (2) yields $\beta = 0.10$. With this information, numerical testing began with $\alpha \geq 0.35$ and $\beta \geq 0.08$.

Equation (1) holds for systems which are at least critically damped (reference 4). That is

$$\beta - (\alpha+\beta)^2/4 > 0 \qquad (3)$$

This condition imposes a second constraint on beta

$$\beta > 2-\alpha-2(1-\alpha)^{1/2} \qquad (3a)$$

Due to the collision avoidance logic's sensitivity to peak error in the velocity estimate, a final consideration is the selection of alpha and beta to control maximum transient error in the velocity response. This consideration prevents the automatic selection of alpha considerably larger than 0.40. Navarro in (reference 6) identifies the time, t_{max}, at which the peak error in response to a ramp input occurs.

$$t_{max} = \omega^{-1}\left\{\arctan\left\{(c\omega-a)/(\omega+ca)\right\} + \pi\right\} \qquad (4)$$

where a and ω are defined as in equation (1) and

$$c = \alpha/\sqrt{4\beta-(\alpha+\beta)^2}$$

The normalized peak error in the velocity response was obtained by evaluating h (t_{max}). The peak error is obtained by multiplying h (t_{max}) by the true rate. Table 2 presents the analytically derived peak errors as a function of alpha and beta. The peak errors were obtained from a 6,000-ft/min rate, the maximum rate investigated.

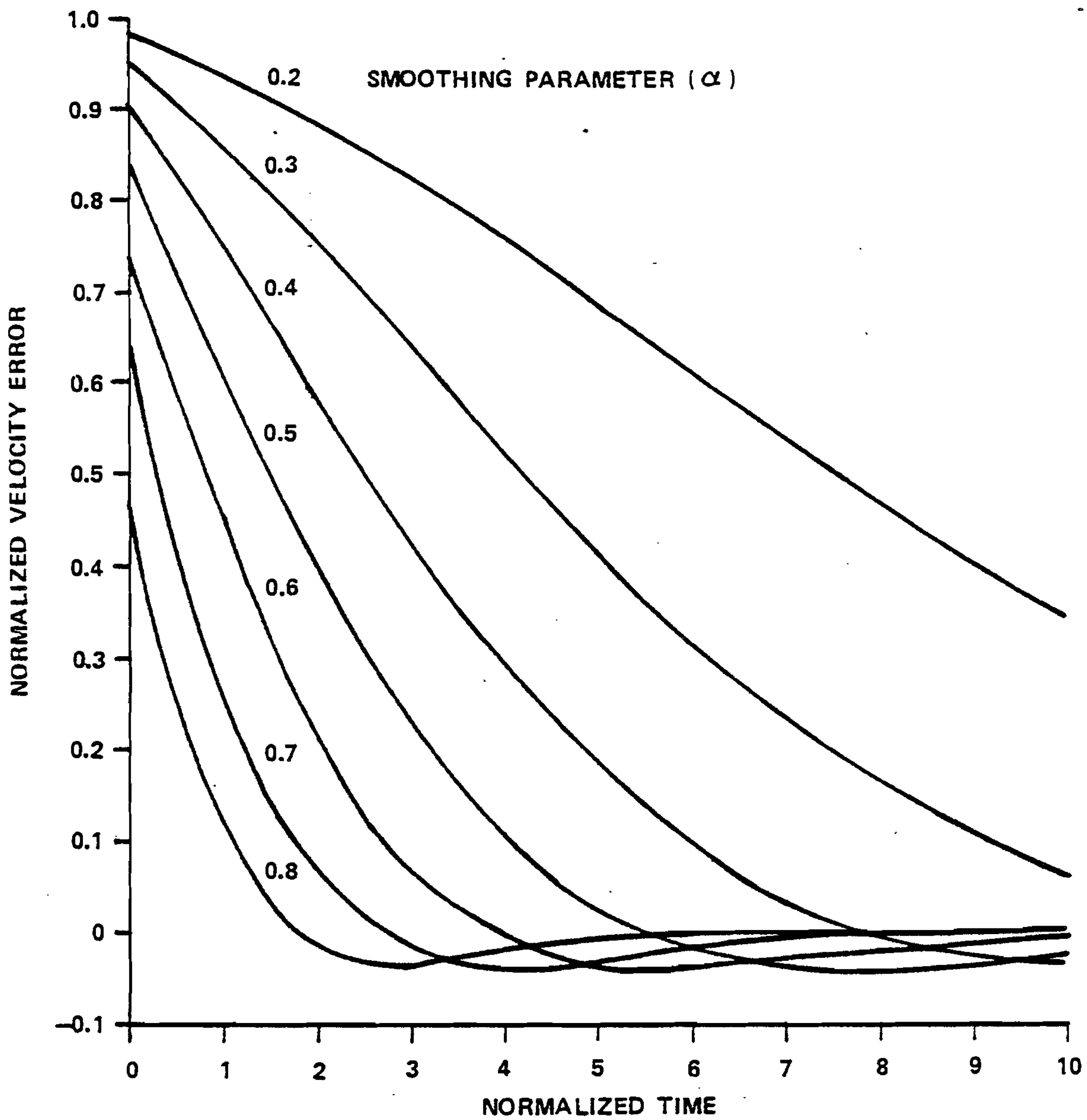

FIGURE 1. SETTLING TIME TO A GIVEN TOLERANCE IN ALPHA-BETA TRACKING

Beta	Alpha 0.35	0.40	0.45	0.50	0.55	0.60
0.10	7.08	2.84	0.50	*	*	*
0.11	8.84	4.17	1.19	0.01	*	*
0.13	12.13	6.87	3.03	0.71	0.01	*
0.15	15.09	9.47	5.08	1.97	0.32	0.01
0.17	17.75	11.89	7.14	3.50	1.10	0.07
0.19	20.13	14.14	9.13	5.11	2.17	0.45
0.21	22.33	16.22	11.02	6.73	3.39	1.13

* Beta does not satisfy constraint (3a).

Theoretically, if $0.40 \leq \alpha \leq 0.55$ and $0.15 \leq \beta \leq 0.20$, the peak error can be contained below 10 ft/sec.

The measure of noise rejection, the variance reduction ratio, V_s, is the noise reduction in the position estimate. From reference 7,

$$V_s = (2\alpha^2 - 3\alpha\beta + 2\beta)/(4\alpha - 2\alpha^2 - \alpha\beta) \quad (5)$$

The variance reduction ratio for velocity, V_ν, is also taken from reference 7 and can be expressed as

$$V_\nu = (2\beta^2)/(4\alpha - 2\alpha^2 - \alpha\beta) \quad (6)$$

Both equations (5) and (6) hold only if the input noise is not serially correlated. Table 3 presents V_s as a function of alpha and beta.

A review of table 3 indicates theoretically the settling time of 7 seconds or less ($\alpha \geq 0.4$), and $V_s \leq 0.3$ cannot be simultaneously obtained. However, the selection of $\alpha = 0.50$ and $\beta = 0.13$ satisfies the settling time requirement and causes only a slight increase in the variance reduction ratio to 0.394.

Beta	Alpha 0.35	0.40	0.45	0.50	0.55	0.60
0.09	0.294	0.315	0.342	0.343	*	*
0.11	0.313	0.330	0.354	0.384	*	*
0.13	0.332	0.345	0.366	0.394	0.427	*
0.15	0.351	0.361	0.379	0.404	0.435	0.472
0.17	0.371	0.376	0.391	0.413	0.443	0.478
0.19	0.391	0.392	0.404	0.423	0.451	0.484
0.21	0.411	0.408	0.416	0.434	0.459	0.490

* Beta does not satisfy constraint (3a).

NUMERICAL RESULTS.

Numerical testing was performed to identify the impact of 1-foot resolution and finite precision arithmetic on the the theoretical results. Twelve ramps were used as sample input to the tracker. The ramps represented the following rates: 500, 1,000, 1,500, 2,000, 2,500, 3,000, 3,500, 4,000, 4,500, 5,000, 5,500, and 6,000 ft/min. The acceleration was fixed at 0.1 G. The velocity response is obtained from equation (1) and can be expressed as

$$R_{\nu}(T) = P(T) \cdot \{1-h_{\nu}(T)\} \quad \text{for } T = 1, 2, 3, \ldots \tag{7}$$

where

$P(T)$ = magnitude of the input at time T.

Using equation (7), measures of maximum settling time and peak error in the velocity estimate were obtained for the ensemble of quantized sample inputs. Figure 2 presents the results obtained through numerical testing. The curves on the lower portion of figure 2 present the maximum settling times which were observed for the ensemble of quantized inputs. Generally, better performance in terms of shorter settling times was observed as beta was increased. As alpha was increased, better performance was also observed. Table 4 presents a comparison between theoretical and numerically observed settling times for serveral combinations of alpha and beta. Beta has been selected to conform to the Benedict and Bordner relationship, equation (2), so that the theoretical settling times could be obtained directly from figure 1.

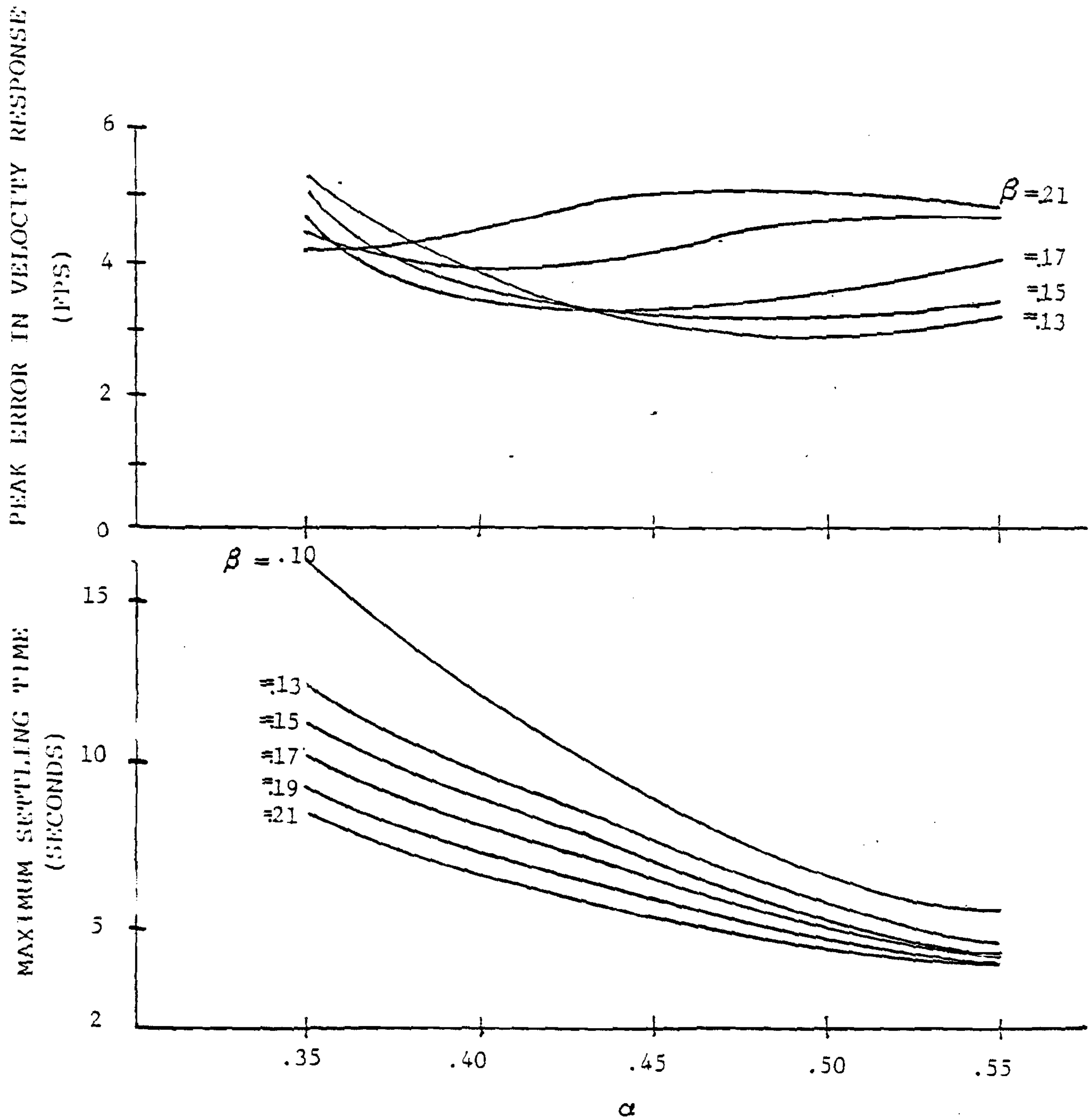

FIGURE 2. NUMERICAL RESULTS

Alpha	0.35	0.40	0.45	0.50	0.55
Beta	0.0792	0.1000	0.1306	0.1667	0.2086
Theoretical	8.6	7.0	6.2	5.0	4.4
Observed	16	12	7	6	5

The additional noise added by the 1-foot quantitization in the altitude input and the impact of finite precision arithmetic results in a slight increase in the observed settling time above for $\alpha < 0.50$. For $\alpha \geq 0.50$ the observed settling times converge to the theoretical values.

The group of curves at the top of figure 2 represent the observed peak errors in the velocity estimate as a function of alpha and beta. For values of $\alpha \leq 0.45$, the observed peak errors are considerably less than the predicted values shown in table 2. This difference can be explained since the values in table 2 were derived based on a continuous variable t_{max}, while results in figure 2 were based on a discrete time sample (1 second). For values of $\alpha \geq 0.45$, and $\beta \leq 0.15$ the observed peak errors approach the theoretical values. For large alpha, the observed peak errors (3 to 5 ft/sec) exceed the theoretical values (1 to 2 ft/sec). This difference is again caused by the additional noise induced by the quantized input and finite precision arithmetic.

EMPIRICAL RESULTS.

Approximately 30 minutes of DADC altitude data were obtained from NTSB. The data were used to verify the performance of an alpha-beta tracker with alpha = 0.50 and beta = 0.15. In general, excellent performance resulted. An example will be presented. Figure 3 depicts a 120-second segment of the data obtained from the NTSB. The aircraft initially was descending through 23,000 feet at an average rate of 2,000 ft/min.

In order to evaluate the performance of the tracker, a method of determining a "true" rate was needed. Using all 120 position inputs, a 7-point Graham first-order polynomial fit (reference 8) provided an estimate of the true vertical rate. The Graham polynomial fit, along with the resulting alpha-beta tracked vertical rate, is shown on figure 4. The tracked vertical rate exhibits a nearly constant 3-second lag when compared to the polynomial fit. The tracked vertical rate ranged from -47 ft/sec to -24 ft/sec. The peak error was 8.32 ft/sec and occurred at time = 56151. The average error in the rate estimate was 0.94 ft/sec. The standard deviation was 2.39 ft/sec.

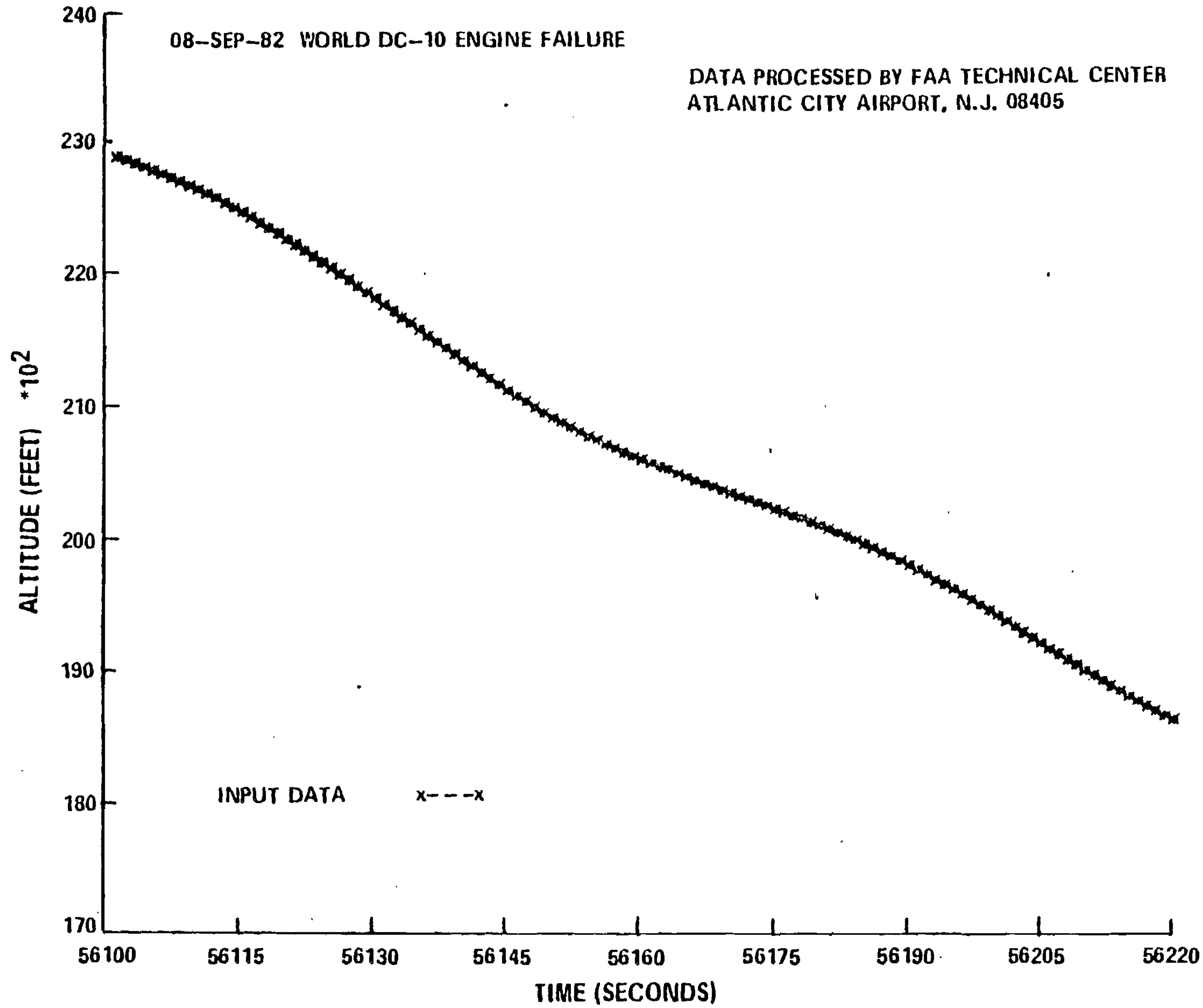

FIGURE 3. DIGITAL AIR DATA COMPUTER ALTITUDE INPUT TO ALPHA-BETA TRACKER

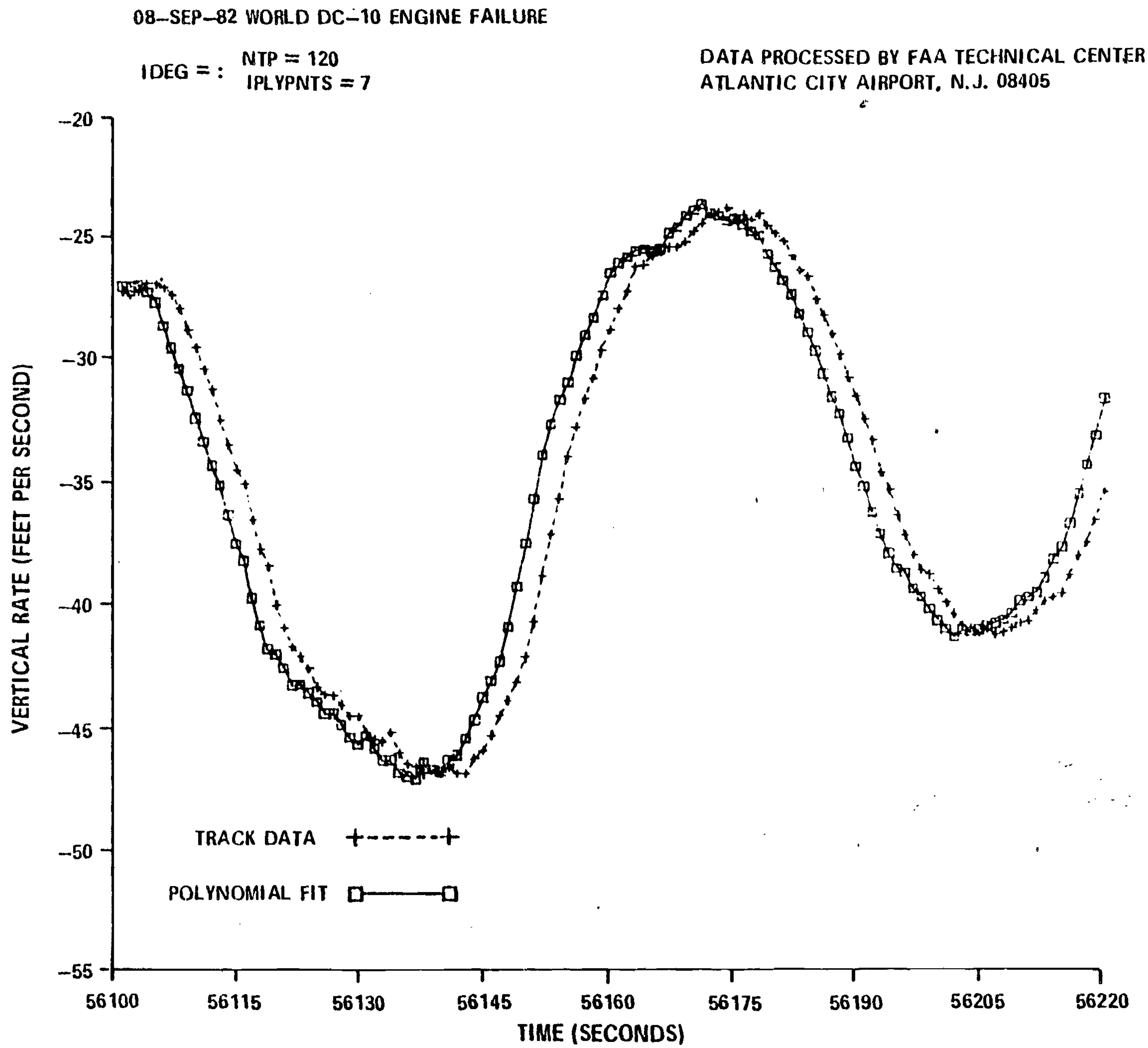

FIGURE 4. ALPHA-BETA TRACKED RATE COMPARED TO GRAHAM FIT "TRUE RATE"

The TCAS resolution logic requires the projection of vertical position based on the current vertical rate estimate. A good method of evaluating tracker performance is to compare the vertical position projection at time t with the actual vertical position at time t+32 seconds. Figure 5 presents the cumulative distribution of the projection error. The projection is within 100 feet of the true position 86 percent of the time. Only 3 percent of the time did the error exceed 150 feet. The improvement in the projection with alpha-beta tracking of DADC altitude data can be seen by comparing figure 5 with figure 6. Figure 6 presents the error in the projection that resulted when the original altitude data were quantized into Mode C reports and tracked using the nonlinear tracker. Nonlinear tracking of Mode C data provided considerably poorer performance than was obtained with alpha-beta tracking of the DADC output.

CONCLUSIONS

As a result of literature research and numerical testing, the parametric values used should be $\alpha = 0.50$ and $\beta = 0.15$. Numerical analysis revealed that alpha must exceed 0.45 in order to obtain the desired settling time and noise reduction characteristics. The observed peak error in the velocity estimate are slightly greater than the theoretical peak error when $\alpha \geq 0.45$. However, the peak errors fall in the 3 to 5 ft/sec. range, and the TCAS collision avoidance logic can tolerate this error magnitude. Since there is little additional reduction in the observed settling times for $\alpha > 0.50$ with $\beta \geq 0.13$, $\alpha = 0.50$ should be the selected value.

In terms of variance reduction and observed peak error, $\beta = 0.15$ should provide sufficent performance with $\alpha = 0.50$. This value of beta is somewhat smaller than the value that satisfies the Benedict and Bordner condition ($\beta = 0.167$). However, the smaller value of beta resulted in a smaller observed peak error (2.93 ft/sec versus 3.33 ft/sec.).

The only remaining task is to obtain sufficient DADC altitude data to insure the resulting variance reduction ratio, $V_s = 0.404$, is sufficient. If not adequate or if the error in the DADC altitude is autocorrelated, slight adjustment to the selected parameters may be necessary.

08-SEP-82 WORLD DC-10 ENGINE FAILURE
TIME PERIOD = 56101.0 to 56220.0
NPT6 = 0 NTP = 120
IDEG = : IPLYPNTS = 7

DATA PROCESSED BY FAA TECHNICAL CENTER
ATLANTIC CITY AIRPORT, N.J. 08405

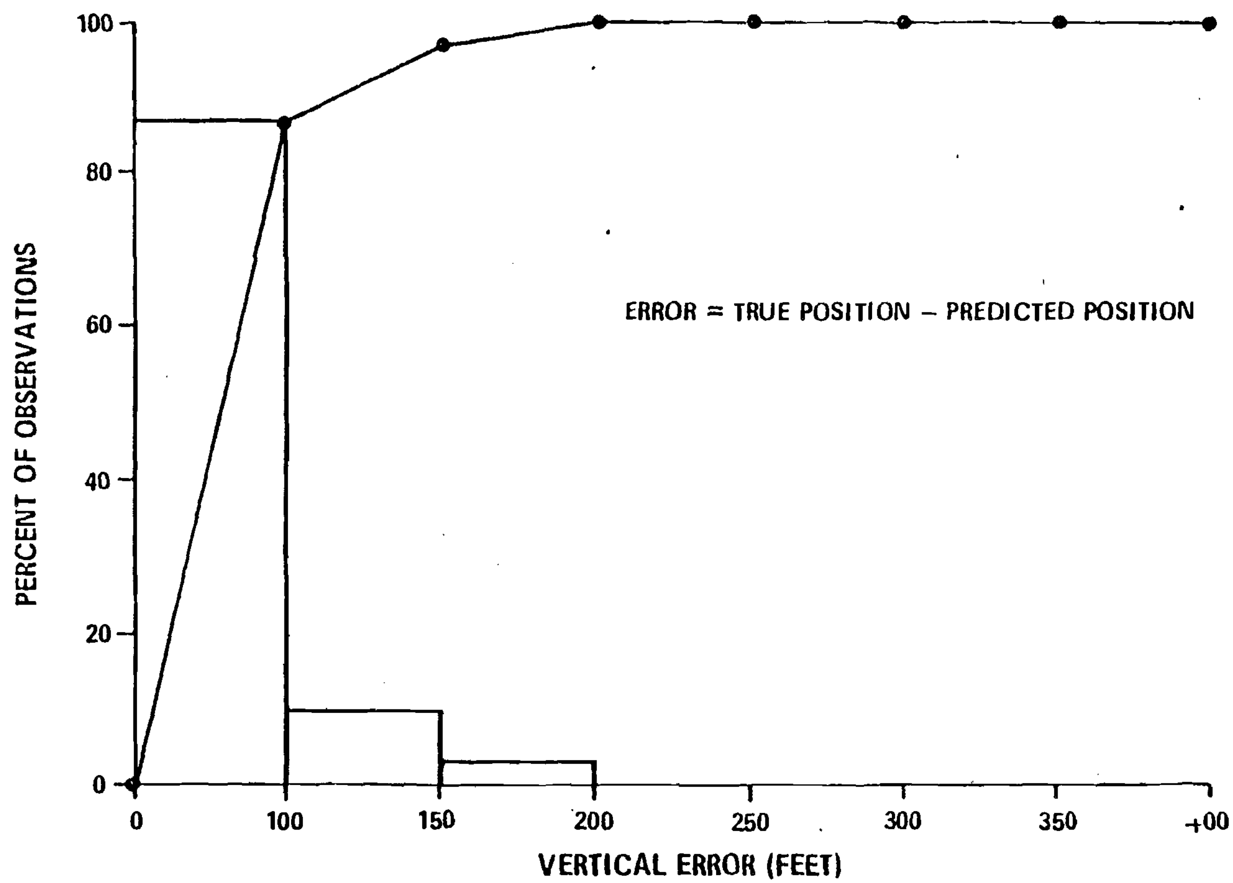

FIGURE 5. CUMULATIVE DISTRIBUTION OF ERROR IN PREDICTED VERTICAL POSITION (ALPHA-BETA TRACKING)

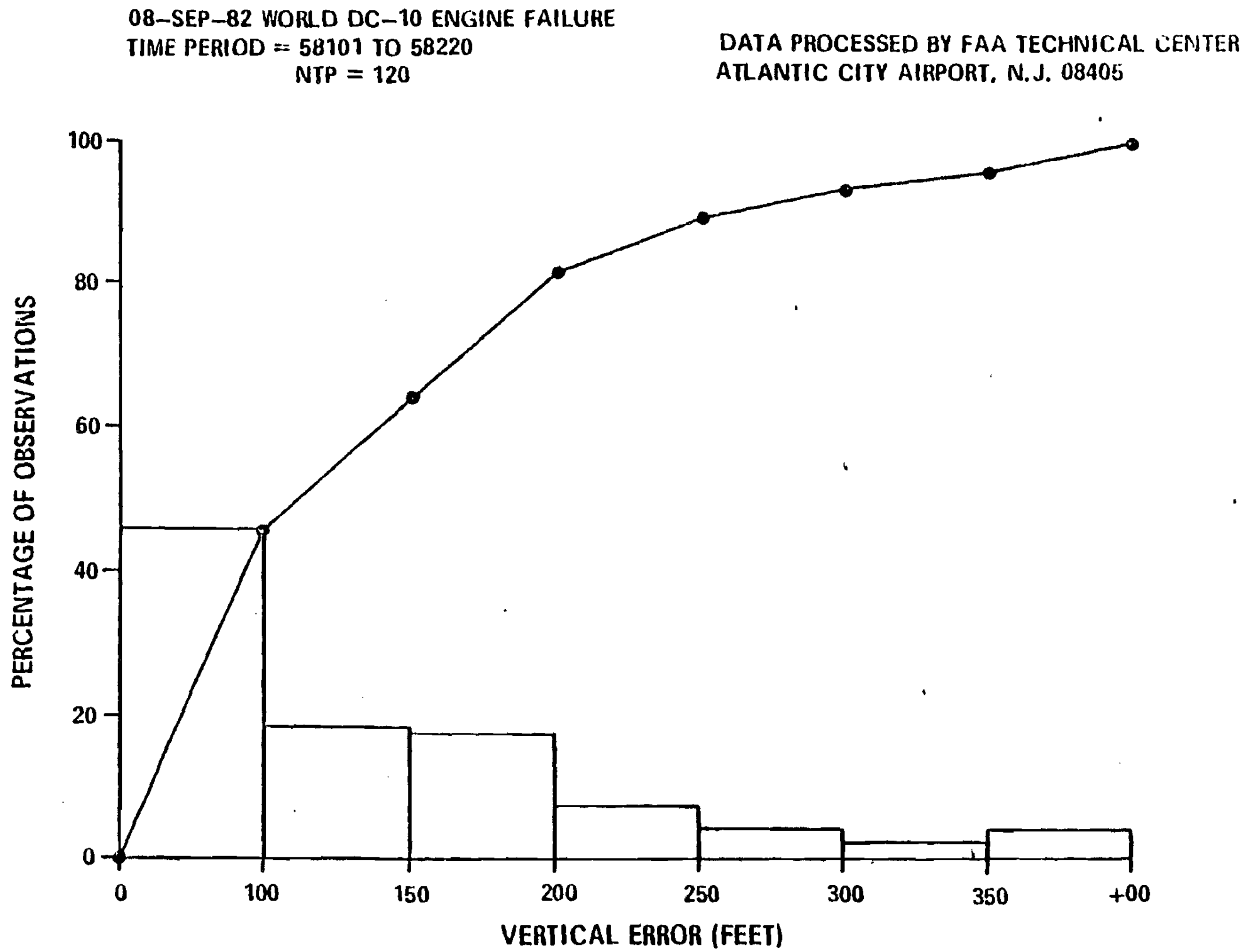

FIGURE 6. CUMULATIVE DISTRIBUTION OF ERROR IN PREDICTED VERTICAL POSITION (NON-LINEAR MODE C TRACKING)

REFERENCES

1. Sperry Digital Air Data Computer Specification, IT4031800, May, 1976.

2. Andrews, J. A., An Improved Technique for Altitude Tracking of Aircraft, Lincoln Laboratory, FAA Report RD 82-14, March 1981.

3. Billmann, B. R., Analysis of a Nonlinear Altitude Tracking Method, FAA Technical Center, FAA Report RD 81-15A, February, 1982.

4. Lefferts, R. E., A Parametric Study of Altitude Tracking Performance for the En Route Conflict Alert Function, FAA Technical Center, FAA Report CT 82-51, Publication Pending.

5. Benedict, T. R., and Bordner, G. W., Sythesis of an Optimal Set of Radar Track-While-Scan Smoothing Equations, IRE Trans. on Automatic Control, Vol. AC-7, pp 27-32, July, 1962.

6. Navarro, I. A., Description of an Alpha Beta Tracking Filter with Minimal Maximum Transient Error Response, Physics Laboratory National Defense Research Organization, The Hague, Netherlands, March, 1977.

7. Bauxbaum, P. J., and Reber, R., Evaluation Guide for Radar Tracking Algorithms with Applications to Ballistic Missile Defense Volume 1, General Research Corp., September, 1971.

8. Kincard, Cheney D., Numerical Mathematics and Computing, Brooks Collier Publishing Co. Monterey, Ca., 1980