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# Simulation Modeling of Longitudinal Passing of Same- Direction Traffic on Two Adjacent Parallel Routes

Ben W. Miller

December 1982

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<p>16. Abstract</p> <p>Estimation of collision risk is an important technique used to assess the safety of current and proposed air traffic control route-separation standards. The collision risk model separates this risk into independent lateral, vertical, and longitudinal risks.</p> <p>This paper considers risks only for aircraft in level flight in the same direction on the same flight level on two adjacent parallel routes. Under this assumption, an important factor in the risk estimate is the proportion of time passing (<math>P_x</math>), previously estimated by the streaming aircraft model. Another factor, which should be considered separately in some cases, is the average frequency of passings (<math>N_x</math>).</p> <p>A simulation model was written to model traffic on two parallel route segments. The model randomly determined aircraft arrival times and ground speeds on each of the two route segments and then identified those aircraft with longitudinal passings. Statistics were accumulated for each simulation run, and linear regression was then used to estimate the parameters <math>P_x</math> and <math>N_x</math>.</p> <p>The streaming aircraft model estimate for <math>P_x</math> was verified by these simulations. This estimate defines <math>P_x</math> in terms of the average density of arrivals and the average aircraft speed. In addition, an estimate was obtained for <math>N_x</math> in terms of the average density of arrivals and the standard deviation of the distribution of aircraft speeds.</p>			
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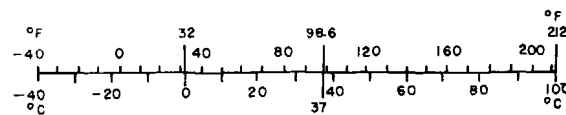
## METRIC CONVERSION FACTORS

### Approximate Conversions to Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
<b>LENGTH</b>				
in	inches	2.5	centimeters	cm
ft	feet	30	centimeters	cm
yd	yards	0.9	meters	m
mi	miles	1.6	kilometers	km
<b>AREA</b>				
in <sup>2</sup>	square inches	6.5	square centimeters	cm <sup>2</sup>
ft <sup>2</sup>	square feet	0.09	square meters	m <sup>2</sup>
yd <sup>2</sup>	square yards	0.8	square meters	m <sup>2</sup>
mi <sup>2</sup>	square miles	2.6	square kilometers	km <sup>2</sup>
	acres	0.4	hectares	ha
<b>MASS (weight)</b>				
oz	ounces	28	grams	g
lb	pounds	0.45	kilograms	kg
	short tons (2000 lb)	0.9	tonnes	t
<b>VOLUME</b>				
tsp	teaspoons	5	milliliters	ml
Tbsp	tablespoons	15	milliliters	ml
fl oz	fluid ounces	30	milliliters	ml
c	cups	0.24	liters	l
pt	pints	0.47	liters	l
qt	quarts	0.95	liters	l
gal	gallons	3.8	liters	l
ft <sup>3</sup>	cubic feet	0.03	cubic meters	m <sup>3</sup>
yd <sup>3</sup>	cubic yards	0.76	cubic meters	m <sup>3</sup>
<b>TEMPERATURE (exact)</b>				
°F	Fahrenheit temperature	5/9 (after subtracting 32)	Celsius temperature	°C

### Approximate Conversions from Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
<b>LENGTH</b>				
mm	millimeters	0.04	inches	in
cm	centimeters	0.4	inches	in
m	meters	3.3	feet	ft
m	meters	1.1	yards	yd
km	kilometers	0.6	miles	mi
<b>AREA</b>				
cm <sup>2</sup>	square centimeters	0.16	square inches	in <sup>2</sup>
m <sup>2</sup>	square meters	1.2	square yards	yd <sup>2</sup>
km <sup>2</sup>	square kilometers	0.4	square miles	mi <sup>2</sup>
ha	hectares (10,000 m <sup>2</sup> )	2.5	acres	
<b>MASS (weight)</b>				
g	grams	0.035	ounces	oz
kg	kilograms	2.2	pounds	lb
t	tonnes (1000 kg)	1.1	short tons	
<b>VOLUME</b>				
ml	milliliters	0.03	fluid ounces	fl oz
l	liters	2.1	pints	pt
l	liters	1.06	quarts	qt
l	liters	0.26	gallons	gal
m <sup>3</sup>	cubic meters	35	cubic feet	ft <sup>3</sup>
m <sup>3</sup>	cubic meters	1.3	cubic yards	yd <sup>3</sup>
<b>TEMPERATURE (exact)</b>				
°C	Celsius temperature	9/5 (then add 32)	Fahrenheit temperature	°F



\*1 in = 2.54 exact. For other exact conversions and more detailed tables, see NBS Misc. Publ. 286, Units of Weights and Measures, Price \$2.25, SD Catalog No. C13.11-286.

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## EXECUTIVE SUMMARY

Estimation of collision risk is an important technique used to assess the safety of current and proposed air traffic control route-separation standards. The collision risk model separates this risk into independent lateral, vertical, and longitudinal risks.

This paper considers risks only for aircraft in level flight in the same direction on the same flight level on two adjacent parallel routes. Under this assumption, an important factor in the risk estimate is the proportion of time passing ( $P_x$ ), previously estimated by the streaming aircraft model. Another factor, which should be considered separately in some cases, is the average frequency of passings ( $N_x$ ).

A simulation model was written to model traffic on two parallel route segments. The model randomly determined aircraft arrival times and ground speeds on each of the two route segments and then identified those aircraft with longitudinal passings. Statistics were accumulated for each simulation run, and linear regression was then used to estimate the parameters  $P_x$  and  $N_x$ .

The streaming aircraft model estimate for  $P_x$  was verified by these simulations. This estimate defines  $P_x$  in terms of the average density of arrivals and the average aircraft speed.

In addition, an estimate was obtained for  $N_x$  in terms of the average density of arrivals and the standard deviation of the distribution of aircraft speeds.



## 1.0 INTRODUCTION

1.1 Estimation of collision risks is an important technique used to assess the safety of current and proposed air traffic control route-separation standards. These risks cannot usually be estimated directly by measuring the frequency of collisions, since actual midair collisions are such rare events that it would not be practical to collect enough data for direct estimation. Instead, mathematical models are used to relate the collision risk, which cannot be directly measured, to various factors which can be estimated from observed data.

1.2 One approach to this problem is a collision risk model, proposed by Marks and Reich (references 1 through 8), which estimates the risk of midair collision between aircraft separated in the lateral, longitudinal, and vertical dimensions by purely strategic separation standards. The major assumption of this model is that a collision requires three independent events to occur simultaneously (lateral overlap, longitudinal overlap, and vertical overlap). Probabilities of lateral overlap, longitudinal overlap, and vertical overlap can then be estimated separately and combined to obtain an estimate for collision risk. This method has been applied to determine route spacing requirements in the North Atlantic and Central East Pacific regions (references 9, 10, and 11). A description of the mathematical derivation of the model, as well as its use in one case, is given by Busch, Colamosca, and Vander Veer (reference 11). Some problems in estimating parameters and testing hypotheses for this model are described by Busch, Colamosca, Hunter, and Polhemus (reference 12).

1.3 Under the basic independence assumption, it is possible to consider lateral, vertical, and longitudinal collision risks separately. It is also useful to consider risks for different route configurations separately. This paper considers risks only for aircraft in level flight in the same direction on the same flight level on two adjacent parallel routes. The risk equation for this case is given in paragraph 2.3.3. With minor changes, the results of this paper can also be applied to the case of same direction aircraft on two adjacent flight levels on the same route.

1.4 For fixed probabilities of lateral and vertical displacement, which can be measured but which cannot usually be controlled, the most important contributor to collision risk is the probability of lateral passing ( $P_x$ ). This probability has been estimated in previous work from the average traffic density and the average aircraft speed by the streaming aircraft model (references 13 and 14).

1.5 Since the streaming aircraft model does not consider any variation in aircraft speeds, there have been some questions about its validity. A simulation model was developed in order to verify the streaming aircraft model under various conditions of variability in aircraft speeds.

1.6 The average times per longitudinal, lateral, and vertical passing have usually been represented by constants estimated from the data (reference 11). This approach is valid under the assumption of constant aircraft speeds on each route, but it cannot be used to compare risks under different aircraft speed assumptions. Average times per lateral and vertical passings are assumed to be relatively insensitive to changes in relative aircraft speeds, and these parameters continue to be represented by constants estimated from data. However the simulation model is used to estimate the average time per longitudinal passing, and hence the average frequency of longitudinal passings, directly.

## 2.0 DESCRIPTION OF THE COLLISION RISK MODEL

### 2.1 GEOMETRY OF THE TRACK SYSTEM

2.1.1 The track system described by this Collision Risk Model is assumed to consist of parallel tracks or routes. Any two adjacent routes are separated laterally by a lateral separation standard ( $S_y$ ). Aircraft on each route are segregated into discrete flight levels separated by a vertical separation standard ( $S_z$ ). Aircraft on the same flight level on the same route segment all fly in the same direction, and they are separated upon entry into the track system by a time difference sufficient to ensure a longitudinal separation standard ( $S_x$ ). This defines a rectangular track system. A composite track system may be defined by overlapping two rectangular track systems offset from each other by one half of a separation standard in both the lateral and vertical dimensions.

2.1.2 Each aircraft is assumed to fly on only one route and one flight level, and the average flow of aircraft is assumed constant for all flight levels. Thus, the lateral collision risk can be estimated from a model consisting of one flight level on each of two adjacent parallel routes.

2.1.3 Imposition of these separation standards is assumed to be the only method of separation used. No allowance is made for the benefits of radar surveillance of aircraft positions by the air traffic control system. Separation standards are viewed as the only control parameters manipulated by the air traffic control system to ensure safety.

### 2.2 ASSUMPTIONS USED BY THE COLLISION RISK MODEL

The Collision Risk Model is derived based upon the following assumptions. It is necessary to make these assumptions in order to simplify the system sufficiently so that a mathematical model can be derived. The derived mathematical model is then assumed to apply to the real system being modeled.

2.2.1 Potential collisions are assumed to occur only between aircraft whose planned flightpaths bring their intended positions close to one separation standard in one dimension while their planned separation is no larger than one separation standard in the other two dimensions. There is assumed to be no risk of collision between two aircraft with longitudinal, lateral, or vertical planned separation greater than one separation standard.

2.2.2 It is assumed that no more than two aircraft are involved in any one collision.

2.2.3 The flying errors of an aircraft in the longitudinal, lateral, and vertical dimensions are assumed to be independent of each other.

2.2.4 The flying errors of different aircraft are assumed to be independent of each other.

2.2.5 For the purpose of this model, it is assumed that a collision does not prevent the aircraft involved from completing the remainder of their flights. This assumption makes the mathematical derivation of the model much simpler and, since the probability of any one aircraft being involved in two separate collisions even under this assumption is extremely small relative to the probability of collision, this assumption does not materially affect the results of the model.

2.2.6 A typical track system aircraft is modeled as a slab of dimensions equal to the aircraft's metallic wingspan, length, and height, and the slab is oriented with zero-degree bank, pitch, and yaw angles with respect to track. All track system aircraft are assumed to be the same size.

2.2.7 The collision risks for planned longitudinal, lateral, vertical, and composite separations are assumed to be independent of each other.

2.2.8 In clearing aircraft on flightpaths, the air traffic control system is assumed to commit no errors which would make a midair collision a highly probable event (such as clearing two aircraft on the same route and flight level in opposite directions).

2.2.9 It is assumed that no collision avoidance maneuvers are initiated as the result of visual or instrument contact between aircraft.

2.2.10 The navigational performance of an aircraft in the track system is assumed to be the same in all parts of the track system and not to be a function of flight time or distance along track.

2.2.11 All aircraft speeds are derived from the same probability distribution, and speeds of different aircraft are assumed to be independent. The speed differences are assumed to be small enough, however, that lateral and vertical closing speeds can be assumed to be the same for all aircraft.

### 2.3 SAME-DIRECTION LATERAL COLLISION RISK EQUATION

2.3.1 The following variables are used to define the lateral collision risk for aircraft flying in the same direction on the same flight level on two adjacent parallel routes.

2.3.1.1 The probability of lateral overlap of aircraft on adjacent parallel routes given a planned separation of one lateral separation standard is represented by  $P_y(S_y)$  .

2.3.1.2 The probability of vertical overlap of aircraft on the same flight level is represented by  $P_z(0)$  .

2.3.1.3 The proportion of time spent in longitudinal passing is represented by  $P_x$  . (See paragraph 3.2.)

2.3.1.4 The average time per longitudinal passing (in hours) is represented by  $t_x$  . (See paragraph 3.5.) This is approximated by dividing twice the aircraft length by the average longitudinal closing speed.

2.3.1.5 The average time per lateral passing (in hours) is represented by  $t_y$  . This is calculated by dividing twice the aircraft wingspan by the average lateral closing speed.

2.3.1.6 The average time per vertical passing (in hours) is represented by  $t_z$  . This is calculated by dividing twice the aircraft height by the average vertical closing speed.

2.3.1.7 The average frequency of longitudinal passings (per hour) is represented by  $N_x$  . (See paragraph 3.6.) This is calculated by dividing the proportion of time spent in longitudinal passing ( $P_x$ ) by the average time per longitudinal passing ( $t_x$ ).

2.3.2 The expected number of accidents in ten million track system flying hours due to the loss of all planned lateral separation by same-direction aircraft is equal to the sum of the following three separate numbers.

2.3.2.1 The expected number of accidents caused by collision between the front of one aircraft and the rear of another aircraft is equal to

$$10,000,000 * P_y(S_y) * P_z(0) * P_x * ( 1 / t_x ) ,$$

or equivalently to

$$10,000,000 * P_y(S_y) * P_z(0) * N_x .$$

2.3.2.2 The expected number of accidents caused by collision between the sides of two aircraft is equal to

$$10,000,000 * P_y(S_y) * P_z(0) * P_x * ( 1 / t_y ) .$$

2.3.2.3 The expected number of accidents caused by collision between the top of one aircraft and the bottom of another aircraft is equal to

$$10,000,000 * P_y(S_y) * P_z(0) * P_x * ( 1 / t_z ) .$$

2.3.3 The expected number of accidents in ten million track system flying hours due to the loss of all planned lateral separation by same-direction aircraft is therefore equal to

$$10,000,000 * P_y(S_y) * P_z(0) * P_x * ( 1 / t_x + 1 / t_y + 1 / t_z ) ,$$

or equivalently to

$$10,000,000 * P_y(S_y) * P_z(0) * ( N_x + P_x * ( 1 / t_y + 1 / t_z ) ) .$$

### 3.0 ANALYTIC ESTIMATES OF LONGITUDINAL PASSINGS

#### 3.1 ASSUMPTIONS USED BY THE STREAMING AIRCRAFT MODEL

Analytic estimates of longitudinal passing ( $P_x$ ) are obtained from the streaming aircraft model (references 13 and 14). This model is based on the following assumptions. A major purpose of the simulation modeling is to determine the dependence of the results of the streaming aircraft model on these assumptions.

3.1.1 Arrivals on the two parallel routes are assumed to be independent of each other. Arrivals on each route are assumed to follow a Poisson distribution with an average arrival density of  $N_1$  aircraft per hour on route one and  $N_2$  aircraft per hour on route two.

3.1.2 Ground speeds are assumed to be constant on each route. All aircraft on route one are assumed to have a ground speed of  $V_1$  knots and all aircraft on route two are assumed to have a ground speed of  $V_2$  knots.

3.1.3 All aircraft on both routes are assumed to have the same aircraft length of  $M$  nautical miles.

3.1.4 Each of the two route segments is assumed to be  $L$  nautical miles in length.

3.1.5 A steady-state time period of  $H$  hours is assumed by the model. This time period is assumed to be long enough relative to the route length that transitional effects at the beginning and end of the time period can be ignored.

#### 3.2 DERIVATION OF THE STREAMING AIRCRAFT MODEL FOR ESTIMATING $P_x$

3.2.1 An aircraft with a ground speed of  $V$  knots requires  $L / V$  hours to complete a route segment of length  $L$  nautical miles. Thus, each aircraft on route one requires  $L / V_1$  hours and each aircraft on route two requires  $L / V_2$  hours to complete the route segment.

3.2.2 A total of  $H * N_1$  aircraft will enter route one and a total of  $H * N_2$  aircraft will enter route two during the given steady-state period of  $H$  hours.

3.2.3 The total flying time on each route segment can be calculated by multiplying the number of aircraft on that route segment (paragraph 3.2.2) by the time for each aircraft to complete the route segment (paragraph 3.2.1). Thus, there are a total of

$$( H * L * N1 ) / V1$$

aircraft flying hours on route one and a total of

$$( H * L * N2 ) / V2$$

aircraft flying hours on route two. The total flying time on both route segments is therefore

$$H * L * ( N1 / V1 + N2 / V2 )$$

hours.

3.2.4 The number of aircraft on each route at any one instant of time can be obtained by multiplying the route segment flying time (paragraph 3.2.1) by the arrival density (paragraph 3.1.1). This gives

$$( L * N1 ) / V1$$

aircraft on route one and

$$( L * N2 ) / V2$$

aircraft on route two at any one instant of time.

3.2.5 An aircraft on route two is longitudinally passing an aircraft on route one if and only if its center is located within an interval of length  $2 * M$  nautical miles centered on the center of the other aircraft. At any instant of time, there are

$$( L * N1 ) / V1$$

of these intervals on route one (paragraph 3.2.4) with a total length of

$$( 2 * M * L * N1 ) / V1$$

nautical miles out of a total route segment length of  $L$  nautical miles. Thus, the probability that an arbitrary aircraft on route two is longitudinally passing at an arbitrary instant of time is

$$( 2 * M * N1 ) / V1 .$$

3.2.6 The total passing time for all aircraft on route two can be calculated by multiplying the aircraft flying time on route two (paragraph 3.2.3) by the probability of longitudinal passing for an arbitrary instant of time (paragraph 3.2.5). This gives a total longitudinal passing time of

$$( 2 * M * H * L * N1 * N2 ) / ( V1 * V2 )$$

hours for all aircraft on route two. Since each passing requires exactly the same amount of time for an aircraft on route one as for an aircraft on route two, the total longitudinal passing time for all aircraft is

$$( 4 * M * H * L * N1 * N2 ) / ( V1 * V2 )$$

hours.

3.2.7 The proportion of time passing (Px) can be computed by dividing the hours passing (paragraph 3.2.6) by the total system flying hours (paragraph 3.2.3). The proportion of time passing (Px) is therefore equal to

$$( 4 * M * N1 * N2 ) / ( N1 * V2 + N2 * V1 ) .$$

3.2.8 If the arrival densities for both routes are equal (i.e.,  $N1 = N2 = N$ ) and the ground speeds for both routes are also equal (i.e.,  $V1 = V2 = V$ ), then the proportion of time passing (Px) is given by

$$( 2 * M * N ) / V .$$

### 3.3 ESTIMATION OF THE PROPORTION OF TIME PASSING (Px)

Before running the simulation model, the analytic streaming aircraft model was used to investigate the effects of changes in the average aircraft arrival density on the proportion Px (table 3-1). The calculations were made for a constant aircraft ground speed of 450 knots and for average aircraft arrival densities varying from 24 to 1/8 aircraft per hour. For each density, the proportion Px was calculated using the analytic streaming aircraft model (paragraph 3.2.8). The average duration tx of 0.44444 hours was calculated by dividing the route segment length of 200 nautical miles by the constant aircraft ground speed of 450 knots, and the frequency Nx was then calculated by dividing the proportion Px by the average duration tx.

### 3.4 ESTIMATION OF THE AVERAGE RELATIVE GROUND SPEED

3.4.1 The average relative ground speed is defined to be the average difference in absolute value between the ground speed of an arbitrary aircraft on route one and the ground speed of an arbitrary aircraft on route two. This calculation is based on the following assumptions.

3.4.1.1 Ground speeds on each route are assumed to follow a Gaussian distribution.

3.4.1.2 The ground speed distributions for the two routes are assumed to be independent.

3.4.1.3 Both distributions are assumed to have the same mean (equal to V knots) and the same standard deviation (equal to S knots).

3.4.2 The differences in ground speeds (route one minus route two) follow a Gaussian distribution with mean zero and standard deviation equal to S knots multiplied by the square root of two.

3.4.3 The average relative ground speed is the mean absolute deviation of the distribution of differences in ground speeds. For a Gaussian distribution, this is equal to the standard deviation of ground speed differences (paragraph 3.4.2) multiplied by the square root of the quotient of two divided by pi.

3.4.4 The average relative ground speed is thus equal to

$$( 2 / \text{SQRT}(\text{PI}) ) * S$$

knots, or approximately to

$$1.1283792 * S$$

knots.

### 3.5 ESTIMATION OF THE AVERAGE TIME PER PASSING (tx)

3.5.1 An estimate for the average time per passing (tx) is given by dividing the length of a single passing ( 2 \* M nautical miles, where M is the aircraft length) by the average relative ground speed (paragraph 3.4.4).

3.5.2 This gives an estimate for the average time per passing (tx) equal to

$$\text{SQRT}(\text{PI}) * ( M / S )$$

hours, or approximately to

$$1.7724539 * ( M / S )$$

hours.

3.5.3 There are two major problems with using the above estimate for the average time per passing. These two problems lead to errors in opposite directions, however, and their net effect does not appear to be very large.

3.5.3.1 The reciprocal of the average relative ground speed is used to estimate the average time per passing. This is not accurate since the reciprocal of the average of a set of numbers is not in general equal to the average of the set of reciprocals of those numbers.

3.5.3.2 The average time per passing should be based only on those pairs of aircraft (one on each route) which actually pass. The average relative ground speed, however, is based on all pairs of aircraft (one on each route). This would be valid only if the probability that two aircraft pass were independent of their relative ground speed. There is reason to believe that this is not true; in fact, it is reasonable to assume that aircraft pairs with a large relative ground speed are more likely to pass than aircraft pairs with a small relative ground speed.

3.5.4 Further investigation is necessary in order to obtain a better analytic estimate of the average time per passing ( $t_x$ ) not subject to these errors. Until this is done, these estimates will be used only for comparison with the simulation results.

### 3.6 ESTIMATION OF THE AVERAGE FREQUENCY OF PASSINGS ( $N_x$ )

3.6.1 The average frequency of passings ( $N_x$ ) can be calculated by dividing the proportion of time passing ( $P_x$ ) (paragraph 3.2.8) by the average time per passing ( $t_x$ ) (paragraph 3.5.2).

3.6.2 This gives an estimate for the average frequency of passings ( $N_x$ ) equal to

$$( 2 / \text{SQRT}(\text{PI}) ) * N * ( S / V )$$

passings per flying hour, or approximately to

$$1.1283792 * N * ( S / V )$$

passings per flying hour.

3.6.3 Since this estimate is based on the previous estimate for the average time per passing, the problems with that estimate (paragraph 3.5.3) affect this one also. However, this estimate can also be used for comparison with the simulation results.

Table 3-1  
STREAMING AIRCRAFT MODEL RESULTS FOR VARYING AIRCRAFT ARRIVAL DENSITIES

Average Density (A/C per Hour)	Ground Speed Mean (Knots)	Ground Speed Std Dev (Knots)	Total Flight Time (Hours)	Px Proportion of Time Passing	Nx Frequency of Passings (per Hour)	tx Average Duration (Hours)
24	450	0	8,889	2.67E-03	0.006	0.44444
22	450	0	8,889	2.44E-03	0.006	0.44444
20	450	0	8,889	2.22E-03	0.005	0.44444
18	450	0	8,889	2.00E-03	0.005	0.44444
16	450	0	8,889	1.78E-03	0.004	0.44444
14	450	0	8,889	1.56E-03	0.004	0.44444
12	450	0	8,889	1.33E-03	0.003	0.44444
10	450	0	8,889	1.11E-03	0.003	0.44444
8	450	0	8,889	8.89E-04	0.002	0.44444
6	450	0	8,889	6.67E-04	0.002	0.44444
5	450	0	8,889	5.56E-04	0.001	0.44444
4	450	0	8,889	4.44E-04	0.001	0.44444
3	450	0	8,889	3.33E-04	0.001	0.44444
2 1/2	450	0	8,889	2.78E-04	0.001	0.44444
2	450	0	8,889	2.22E-04	0.001	0.44444
1 1/2	450	0	8,889	1.67E-04	0.000	0.44444
1	450	0	8,889	1.11E-04	0.000	0.44444
3/4	450	0	8,889	8.33E-05	0.000	0.44444
1/2	450	0	8,889	5.56E-05	0.000	0.44444
3/8	450	0	8,889	4.17E-05	0.000	0.44444
1/4	450	0	8,889	2.78E-05	0.000	0.44444
1/8	450	0	8,889	1.39E-05	0.000	0.44444

## 4.0 SIMULATION MODELING OF LONGITUDINAL PASSING

### 4.1 DESCRIPTION OF THE SIMULATION MODEL

4.1.1 A simulation model was written to model traffic on two parallel route segments. The model randomly determined aircraft arrival times and ground speeds on each of the two route segments and then identified those aircraft with longitudinal passings. The total number of longitudinal passings and the total duration of all longitudinal passings were calculated, as well as the total flight time in the system. From these totals were calculated the proportion of time passing ( $P_x$ ), the frequency of passings ( $N_x$ ), and the average duration of the passings ( $t_x$ ).

4.1.2 Two parallel route segments were modeled. Arrival times and ground speeds on the two route segments were assumed to be independent of each other, and the two route segments were assumed to have the same probability distributions.

4.1.3 Only one flight level was modeled on each of the two parallel route segments. This was assumed to be representative of all flight levels, and each flight level was assumed to be independent of all others.

4.1.4 Scheduled arrival times on each segment were assumed to follow a Poisson distribution with a given mean aircraft arrival density. Equivalently, scheduled interarrival times on each segment were assumed to follow an exponential distribution with a given mean.

4.1.5 Aircraft ground speeds were assumed to follow a truncated normal distribution with a given mean and standard deviation or a combination of two truncated normal distributions with given means and standard deviations and with a given proportion allocated to each distribution. These truncated normal distributions were defined by eliminating the tails beyond three standard deviations on each side of the mean from a normal distribution.

4.1.6 Each aircraft is delayed prior to entry if its scheduled arrival time would result in insufficient longitudinal separation with the previous aircraft on the same route segment. Radar separation minimums of five nautical miles were required at both the beginning and the end of the route segment (and thus also at all points in between). An aircraft was assumed to be held on its previous segment until the earliest time at which the required separation could be guaranteed for the entire route segment.

4.1.7 The length of each aircraft was assumed to be 0.025 nautical miles. This parameter was assumed to be constant and it was not varied in any of the model runs.

4.1.8 The length of each of the two parallel route segments was assumed to be 200 nautical miles, except in one set of simulation runs (paragraph 4.2.4) where this length was varied to see if segment length had any effect on the model results.

4.1.9 The model was run for the number of system hours necessary to obtain an expected value of 10,000 aircraft on each route segment. This required time interval was calculated by dividing the desired 10,000 aircraft by the given average aircraft arrival density for each individual simulation run. It should be noted that the actual number of aircraft simulated varied slightly around this nominal value. Sufficient aircraft flights were generated both before and after the data collection time interval so that there were no starting or ending transient effects in the data. This long time interval was used to represent a number of shorter steady-state time intervals in which traffic was assumed to follow a single probability distribution.

#### 4.2 SIMULATION RUNS WITH VARIED SPEED DISTRIBUTIONS

4.2.1 The simulation model was then run to investigate the effects of changes in the average aircraft arrival density and in the aircraft ground speed standard deviation on the proportion  $P_x$ , the frequency  $N_x$ , and the average duration  $t_x$  (tables 4-1 through 4-8). Aircraft ground speeds were assumed to follow a truncated normal distribution with a mean of 450 knots. A separate simulation run was made for each combination of average aircraft arrival density (with values of 16, 8, 4, 2, 1, 1/2, 1/4, and 1/8 aircraft per hour) and aircraft ground speed standard deviation (varying from 5 knots to 75 knots at intervals of 5 knots). In addition, the streaming aircraft model was used analytically for the limiting case of a ground speed standard deviation of 0 knots (paragraph 3.3).

4.2.2 The simulation model was next used to investigate the effects of changes in the mean aircraft ground speed on the proportion  $P_x$ , the frequency  $N_x$ , and the average duration  $t_x$  (tables 4-9 and 4-10). Aircraft ground speeds were assumed to follow a truncated normal distribution with a standard deviation of 30 knots. A separate simulation run was made for each value of ground speed mean (varying from 200 knots to 600 knots at intervals of 50 knots) for each of two average aircraft arrival densities (8 and 2 aircraft per hour).

4.2.3 The next set of simulation runs were made to investigate the effects of a nonnormal distribution of aircraft ground speeds on the proportion  $P_x$ , the frequency  $N_x$ , and the average duration  $t_x$  (tables 4-11 and 4-12). These runs were also used to investigate further the effects of changes in the aircraft ground speed mean and standard deviation. The aircraft population was assumed to consist of two distinct subpopulations with separate truncated normal distributions of ground speed--a subpopulation of standard aircraft with a mean of 450 knots and a standard deviation of 30 knots and a subpopulation of slow aircraft with a mean of 300 knots and a standard deviation of 50 knots. For each of two average aircraft arrival densities (8 and 2 aircraft per hour), the model was run for proportions of slow aircraft varying from 0 to 24 percent of the total population. Note that the ground speed means and standard deviations in the tables are those calculated for the total population.

4.2.4 Next the effects of route segment length were investigated (table 4-13). This was done for a single average aircraft arrival density (8 aircraft per hour) and a single distribution of aircraft ground speeds (a truncated normal distribution with a mean of 450 knots and a standard deviation of 30 knots). The model was run for route segment lengths varying from 25 to 400 nautical miles.

4.2.5 The model was then run to investigate the effect of jointly varying the aircraft arrival density, the aircraft ground speed mean, and the aircraft ground speed standard deviation (tables 4-14 and 4-15). These simulation runs were also used to investigate the inherent variability in the simulation model by comparing the sets of simulation runs with the same parameters. Eight separate simulation runs were made for each combination of values of aircraft arrival density (1 and 8 aircraft per hour), aircraft ground speed mean (300 knots and 450 knots), and aircraft ground speed standard deviation (30 knots and 50 knots), and an additional six simulation runs were made for one of these combinations. Aircraft ground speeds for each run were assumed to follow a truncated normal distribution with the given mean and standard deviation.

#### 4.3 SIMULATION RUNS WITH HIGH-ALTITUDE JET ROUTE SPEED DISTRIBUTIONS

4.3.1 Ground speed data from the high-altitude VOR-defined jet route study (references 15 and 16) was analyzed in an effort to fit a probability distribution to this actual data. The data was segregated into ten groups based upon five aircraft type classifications (commercial jet, military jet, general aviation jet, multi-engine turboprop, and multi-engine piston) and two directions of flight (eastbound and westbound). The ground speed data from each of these ten groups appeared to be independent of other variables and to be reasonably close to following normal distributions. Also the eastbound and the westbound distributions for the same aircraft type classification appeared to have the same variance and thus to differ only in the location of the mean. The eastbound and the westbound distributions for each aircraft type classification were thus combined into a single distribution with pooled mean and variance. A minimum value was also obtained from the data for each aircraft type classification. The simulation model was run with aircraft ground speed assumed to follow distributions composed of combinations of five truncated normal distributions with given means and standard deviations and with given minimum values used for truncation (table 4-16).

4.3.2 The percentage breakdown into these five aircraft type classifications was computed from the high-altitude VOR-defined jet route study for each of three air route traffic control centers and for the combined population from all three centers (table 4-16). The simulation model was run for a population composed entirely of commercial jets (table 4-17), for a population using the Cleveland Air Route Traffic Control Center (ARTCC) population breakdown (table 4-18), for a population using the Memphis ARTCC population breakdown (table 4-19), for a population using the Albuquerque ARTCC population breakdown (table 4-20), and for a population using the combined three-ARTCC population breakdown (table 4-21).

4.3.3 For the commercial jet aircraft ground speed distribution, the simulation model was run for average aircraft arrival densities of 1/8, 1/2, 2, 4, and 8 aircraft per hour (table 4-17). For the Cleveland, Memphis, and Albuquerque ARTCC aircraft ground speed distributions, the simulation model was run for average aircraft arrival densities of 1/8, 1/2, 2, and 8 aircraft per hour (tables 4-18, 4-19, and 4-20). For the combined three-ARTCC aircraft ground speed distribution, the simulation model was run for average aircraft arrival densities of 1/8, 1/4, 1/2, 1, 1 1/2, 2, 3, 4, 5, 6, 8, and 10 aircraft per hour (table 4-21).

Table 4-1  
SIMULATION RESULTS FOR VARIOUS GROUND SPEED STANDARD DEVIATIONS  
(Average Arrival Density: 16 Aircraft per Hour)

Average Density (A/C per Hour)	Ground Speed Mean (Knots)	Ground Speed Std Dev (Knots)	Total Flight Time (Hours)	Px Proportion of Time Passing	Nx Frequency of Passings (per Hour)	tx Average Duration (Hours)
16	450	0	8,889	1.78E-03	0.004	0.444444
16	450	5	8,945	1.79E-03	0.195	0.00918
16	450	10	8,913	1.76E-03	0.400	0.00441
16	450	15	8,954	1.71E-03	0.597	0.00286
16	450	20	8,926	1.79E-03	0.801	0.00224
16	450	25	8,912	1.75E-03	1.008	0.00173
16	450	30	8,989	1.77E-03	1.214	0.00146
16	450	35	8,963	1.81E-03	1.406	0.00128
16	450	40	9,059	1.89E-03	1.654	0.00114
16	450	45	9,040	1.79E-03	1.787	0.00100
16	450	50	9,021	1.76E-03	1.998	0.00088
16	450	55	9,009	1.83E-03	2.236	0.00082
16	450	60	9,036	1.76E-03	2.452	0.00072
16	450	65	9,087	1.77E-03	2.621	0.00068
16	450	70	9,097	1.81E-03	2.831	0.00064
16	450	75	9,172	1.79E-03	3.050	0.00059

Table 4-2  
SIMULATION RESULTS FOR VARYING GROUND SPEED STANDARD DEVIATIONS  
(Average Arrival Density: 8 Aircraft per Hour)

Average Density (A/C per Hour)	Ground Speed Mean (Knots)	Ground Speed Std Dev (Knots)	Total Flight Time (Hours)	Px Proportion of Time Passing	Nx Frequency of Passings (per Hour)	tx Average Duration (Hours)
8	450	0	8,889	8.89E-04	0.002	0.444444
8	450	5	8,899	8.66E-04	0.105	0.00823
8	450	10	8,934	9.27E-04	0.196	0.00473
8	450	15	8,869	8.50E-04	0.307	0.00277
8	450	20	8,916	9.51E-04	0.404	0.00235
8	450	25	8,965	8.91E-04	0.501	0.00178
8	450	30	8,915	9.10E-04	0.615	0.00148
8	450	35	8,983	8.55E-04	0.715	0.00120
8	450	40	9,010	8.78E-04	0.786	0.00112
8	450	45	8,990	8.95E-04	0.907	0.00099
8	450	50	9,044	8.87E-04	1.030	0.00086
8	450	55	9,038	9.14E-04	1.097	0.00083
8	450	60	9,104	9.01E-04	1.213	0.00074
8	450	65	9,096	8.89E-04	1.305	0.00068
8	450	70	9,012	8.98E-04	1.443	0.00062
8	450	75	9,206	9.70E-04	1.545	0.00063

Table 4-3  
SIMULATION RESULTS FOR VARYING GROUND SPEED STANDARD DEVIATIONS  
(Average Arrival Density: 4 Aircraft per Hour)

Average Density (A/C per Hour)	Ground Speed Mean (Knots)	Ground Speed Std Dev (Knots)	Total Flight Time (Hours)	Px Proportion of Time Passing	Nx Frequency of Passings (per Hour)	tx Average Duration (Hours)
4	450	0	8,889	4.44E-04	0.001	0.44444
4	450	5	8,906	3.82E-04	0.052	0.00740
4	450	10	8,904	4.51E-04	0.103	0.00437
4	450	15	8,909	3.98E-04	0.148	0.00269
4	450	20	8,879	4.32E-04	0.200	0.00216
4	450	25	8,926	4.19E-04	0.246	0.00170
4	450	30	8,979	4.41E-04	0.311	0.00142
4	450	35	8,914	4.12E-04	0.343	0.00120
4	450	40	9,011	4.09E-04	0.402	0.00102
4	450	45	9,030	4.11E-04	0.442	0.00093
4	450	50	9,052	4.27E-04	0.512	0.00083
4	450	55	9,066	4.36E-04	0.544	0.00080
4	450	60	9,105	4.53E-04	0.627	0.00072
4	450	65	8,980	4.59E-04	0.672	0.00068
4	450	70	9,131	4.52E-04	0.726	0.00062
4	450	75	9,050	4.41E-04	0.787	0.00056

Table 4-4  
SIMULATION RESULTS FOR VARYING GROUND SPEED STANDARD DEVIATIONS  
(Average Arrival Density: 2 Aircraft per Hour)

Average Density (A/C per Hour)	Ground Speed Mean (Knots)	Ground Speed Std Dev (Knots)	Total Flight Time (Hours)	Px Proportion of Time Passing	Nx Frequency of Passings (per Hour)	tx Average Duration (Hours)
2	450	0	8,889	2.22E-04	0.001	0.44444
2	450	5	8,861	2.36E-04	0.027	0.00863
2	450	10	8,942	2.28E-04	0.051	0.00442
2	450	15	8,870	2.18E-04	0.080	0.00274
2	450	20	8,919	2.58E-04	0.104	0.00249
2	450	25	8,929	2.49E-04	0.127	0.00196
2	450	30	8,917	2.25E-04	0.147	0.00153
2	450	35	8,957	2.31E-04	0.180	0.00128
2	450	40	8,979	2.31E-04	0.214	0.00108
2	450	45	8,920	2.34E-04	0.236	0.00099
2	450	50	8,972	2.23E-04	0.258	0.00087
2	450	55	9,078	2.24E-04	0.288	0.00078
2	450	60	9,106	2.30E-04	0.315	0.00073
2	450	65	9,137	2.41E-04	0.347	0.00069
2	450	70	9,237	2.22E-04	0.351	0.00063
2	450	75	9,051	2.18E-04	0.395	0.00055

Table 4-5  
SIMULATION RESULTS FOR VARYING GROUND SPEED STANDARD DEVIATIONS  
(Average Arrival Density: 1 Aircraft per Hour)

Average Density (A/C per Hour)	Ground Speed Mean (Knots)	Ground Speed Std Dev (Knots)	Total Flight Time (Hours)	Px Proportion of Time Passing	Nx Frequency of Passings (per Hour)	tx Average Duration (Hours)
1	450	0	8,889	1.11E-04	0.000	0.44444
1	450	5	8,903	1.22E-04	0.011	0.01069
1	450	10	8,944	8.77E-05	0.022	0.00396
1	450	15	8,912	1.04E-04	0.038	0.00273
1	450	20	8,879	1.01E-04	0.049	0.00207
1	450	25	8,930	9.83E-05	0.057	0.00174
1	450	30	8,829	1.03E-04	0.075	0.00137
1	450	35	8,916	1.13E-04	0.087	0.00130
1	450	40	8,933	1.03E-04	0.102	0.00101
1	450	45	9,032	1.04E-04	0.109	0.00096
1	450	50	9,054	1.12E-04	0.129	0.00087
1	450	55	9,041	1.07E-04	0.130	0.00082
1	450	60	9,027	1.07E-04	0.150	0.88872
1	450	65	9,137	1.11E-04	0.169	0.00066
1	450	70	9,134	1.04E-04	0.172	0.00060
1	450	75	9,171	1.12E-04	0.190	0.00059

Table 4-6  
SIMULATION RESULTS FOR VARYING GROUND SPEED STANDARD DEVIATIONS  
(Average Arrival Density: 1/2 Aircraft per Hour)

Average Density (A/C per Hour)	Ground Speed Mean (Knots)	Ground Speed Std Dev (Knots)	Total Flight Time (Hours)	Px Proportion of Time Passing	Nx Frequency of Passings (per Hour)	tx Average Duration (Hours)
1/2	450	0	8,889	5.56E-05	0.000	0.44444
1/2	450	5	8,940	5.67E-05	0.006	0.01014
1/2	450	10	8,934	4.35E-05	0.012	0.00354
1/2	450	15	8,912	5.42E-05	0.022	0.00249
1/2	450	20	8,929	3.62E-05	0.020	0.00177
1/2	450	25	8,967	5.23E-05	0.029	0.00179
1/2	450	30	8,987	5.17E-05	0.036	0.00143
1/2	450	35	8,914	6.25E-05	0.047	0.00133
1/2	450	40	8,968	6.32E-05	0.048	0.00131
1/2	450	45	8,987	6.45E-05	0.056	0.00115
1/2	450	50	8,973	5.83E-05	0.065	0.00090
1/2	450	55	9,041	5.45E-05	0.070	0.00078
1/2	450	60	9,002	6.05E-05	0.071	0.00085
1/2	450	65	9,096	6.30E-05	0.086	0.00073
1/2	450	70	9,134	5.50E-05	0.088	0.00063
1/2	450	75	9,128	5.34E-05	0.105	0.00051

Table 4-7  
SIMULATION RESULTS FOR VARYING GROUND SPEED STANDARD DEVIATIONS  
(Average Arrival Density: 1/4 Aircraft per Hour)

Average Density (A/C per Hour)	Ground Speed Mean (Knots)	Ground Speed Std Dev (Knots)	Total Flight Time (Hours)	Px Proportion of Time Passing	Nx Frequency of Passings (per Hour)	tx Average Duration (Hours)
1/4	450	0	8,889	2.78E-05	0.000	0.44444
1/4	450	5	8,941	2.28E-05	0.003	0.00785
1/4	450	10	8,907	2.44E-05	0.006	0.00402
1/4	450	15	8,913	2.17E-05	0.009	0.00254
1/4	450	20	8,958	2.81E-05	0.011	0.00257
1/4	450	25	8,905	2.56E-05	0.014	0.00181
1/4	450	30	8,884	2.44E-05	0.019	0.00129
1/4	450	35	8,995	2.59E-05	0.019	0.00137
1/4	450	40	8,919	3.24E-05	0.030	0.00109
1/4	450	45	8,936	2.60E-05	0.029	0.00090
1/4	450	50	9,022	3.03E-05	0.035	0.00086
1/4	450	55	9,042	2.47E-05	0.038	0.00065
1/4	450	60	9,066	2.78E-05	0.039	0.00071
1/4	450	65	9,085	2.30E-05	0.040	0.00058
1/4	450	70	9,170	2.89E-05	0.047	0.00061
1/4	450	75	9,098	2.63E-05	0.047	0.00055

Table 4-8  
SIMULATION RESULTS FOR VARYING GROUND SPEED STANDARD DEVIATIONS  
(Average Arrival Density: 1/8 Aircraft per Hour)

Average Density (A/C per Hour)	Ground Speed Mean (Knots)	Ground Speed Std Dev (Knots)	Total Flight Time (Hours)	Px Proportion of Time Passing	Nx Frequency of Passings (per Hour)	tx Average Duration (Hours)
1/8	450	0	8,889	1.39E-05	0.000	0.44444
1/8	450	5	8,940	8.61E-06	0.002	0.00550
1/8	450	10	8,907	1.52E-05	0.003	0.00482
1/8	450	15	8,949	9.95E-06	0.005	0.00193
1/8	450	20	9,014	1.71E-05	0.006	0.00285
1/8	450	25	8,888	1.45E-05	0.009	0.00154
1/8	450	30	8,980	1.19E-05	0.011	0.00111
1/8	450	35	8,988	1.87E-05	0.013	0.00142
1/8	450	40	9,012	1.35E-05	0.014	0.00094
1/8	450	45	8,994	1.45E-05	0.013	0.00112
1/8	450	50	9,054	1.19E-05	0.017	0.00072
1/8	450	55	8,999	2.46E-05	0.018	0.00138
1/8	450	60	9,070	1.42E-05	0.020	0.00072
1/8	450	65	9,057	2.37E-05	0.021	0.00116
1/8	450	70	9,135	1.54E-05	0.024	0.00064
1/8	450	75	9,128	2.21E-05	0.025	0.00090

Table 4-9  
SIMULATION RESULTS FOR VARYING GROUND SPEED MEANS  
(Average Arrival Density: 8 Aircraft per Hour)

Average Density (A/C per Hour)	Ground Speed Mean (Knots)	Ground Speed Std Dev (Knots)	Total Flight Time (Hours)	Px Proportion of Time Passing	Nx Frequency of Passings (per Hour)	tx Average Duration (Hours)
8	200	30	20,284	2.05E-03	1.346	0.00152
8	250	30	16,236	1.57E-03	1.059	0.00148
8	300	30	13,412	1.31E-03	0.893	0.00147
8	350	30	11,428	1.13E-03	0.752	0.00150
8	400	30	9,956	1.21E-03	0.652	0.00185
8	450	30	8,887	1.00E-03	0.599	0.00167
8	500	30	8,038	7.79E-04	0.543	0.00143
8	550	30	7,311	6.93E-04	0.491	0.00141
8	600	30	6,717	6.25E-04	0.457	0.00137

Table 4-10  
SIMULATION RESULTS FOR VARYING GROUND SPEED MEANS  
(Average Arrival Density: 2 Aircraft per Hour)

Average Density (A/C per Hour)	Ground Speed Mean (Knots)	Ground Speed Std Dev (Knots)	Total Flight Time (Hours)	Px Proportion of Time Passing	Nx Frequency of Passings (per Hour)	tx Average Duration (Hours)
2	200	30	20,375	5.03E-04	0.340	0.00148
2	250	30	16,214	3.97E-04	0.273	0.00145
2	300	30	13,444	3.30E-04	0.229	0.00144
2	350	30	11,595	2.79E-04	0.198	0.00141
2	400	30	10,118	2.71E-04	0.174	0.00156
2	450	30	8,918	2.21E-04	0.155	0.00143
2	500	30	7,976	1.79E-04	0.129	0.00139
2	550	30	7,262	1.76E-04	0.124	0.00142
2	600	30	6,640	1.71E-04	0.120	0.00143

Table 4-11  
SIMULATION RESULTS FOR VARYING PERCENTAGES OF SLOW AIRCRAFT  
(Average Arrival Density: 8 Aircraft per Hour)

Average Density (A/C per Hour)	Percent Slow Aircraft		Ground Speed Mean (Knots)	Ground Speed Std Dev (Knots)	Total Flight Time (Hours)	Px Proportion of Time Passing	Nx Frequency of Passings (per Hour)	tx Average Duration (Hours)
8	0 %		449.9	29.84	8,846	9.19E-04	0.619	0.00148
8	0.5 %		449.1	31.52	8,994	8.86E-04	0.633	0.00140
8	1 %		448.3	32.82	9,028	9.23E-04	0.665	0.00139
8	2 %		447.0	36.59	8,936	8.86E-04	0.732	0.00121
8	4 %		443.8	42.69	9,104	9.20E-04	0.849	0.00108
8	6 %		441.2	47.50	9,176	9.44E-04	0.951	0.00099
8	8 %		437.7	51.78	9,228	9.52E-04	1.075	0.00089
8	10 %		435.3	55.71	9,425	9.49E-04	1.208	0.00079
8	12 %		431.6	59.01	9,532	9.44E-04	1.289	0.00073
8	14 %		429.0	62.19	9,525	9.60E-04	1.391	0.00069
8	16 %		426.0	64.70	9,760	9.90E-04	1.489	0.00066
8	18 %		423.1	66.86	9,832	9.16E-04	1.534	0.00060
8	20 %		420.0	69.16	9,954	9.54E-04	1.653	0.00058
8	22 %		416.7	71.22	9,977	9.64E-04	1.700	0.00057
8	24 %		413.6	73.15	10,097	1.00E-03	1.796	0.00056

Table 4-12  
SIMULATION RESULTS FOR VARYING PERCENTAGES OF SLOW AIRCRAFT  
(Average Arrival Density: 2 Aircraft per Hour)

Average Density (A/C per Hour)	Percent Slow Aircraft		Ground Speed Mean (Knots)	Ground Speed Std Dev (Knots)	Total Flight Time (Hours)	Px Proportion of Time Passing	Nx Frequency of Passings (per Hour)	tx Average Duration (Hours)
2	0 %		450.1	29.75	8,864	2.15E-04	0.151	0.00142
2	0.5 %		449.1	31.52	8,997	2.59E-04	0.159	0.00163
2	1 %		448.6	33.40	9,093	2.02E-04	0.166	0.00122
2	2 %		447.1	36.77	9,142	2.09E-04	0.187	0.00112
2	4 %		444.2	41.96	9,027	2.26E-04	0.200	0.00113
2	6 %		440.9	47.92	9,282	2.31E-04	0.255	0.00091
2	8 %		437.8	51.78	9,232	2.56E-04	0.272	0.00094
2	10 %		434.8	55.55	9,327	2.00E-04	0.287	0.00070
2	12 %		431.2	59.97	9,405	2.07E-04	0.313	0.00066
2	14 %		428.9	62.01	9,517	2.41E-04	0.337	0.00071
2	16 %		426.3	63.94	9,812	2.65E-04	0.367	0.00072
2	18 %		423.1	67.03	9,700	2.30E-04	0.378	0.00061
2	20 %		420.0	68.94	10,014	2.37E-04	0.406	0.00058
2	22 %		417.0	70.82	10,108	2.41E-04	0.425	0.00057
2	24 %		413.7	73.43	10,155	2.50E-04	0.450	0.00056

Table 4-13  
SIMULATION RESULTS FOR VARYING ROUTE SEGMENT LENGTHS  
(Average Arrival Density: 8 Aircraft per Hour)

Average Density (A/C per Hour)	Route Segment Length (NM)	Ground Speed Mean (Knots)	Ground Speed Std Dev (Knots)	Total Flight Time (Hours)	Px Proportion of Time Passing	Nx Frequency of Passings (per Hour)	tx Average Duration (Hours)
8	25	450	30	1,114	8.43E-04	0.576	0.00146
8	50	450	30	2,231	9.44E-04	0.629	0.00150
8	100	450	30	4,518	9.04E-04	0.612	0.00148
8	150	450	30	6,722	9.37E-04	0.612	0.00153
8	200	450	30	8,880	8.59E-04	0.583	0.00147
8	250	450	30	11,081	8.92E-04	0.585	0.00153
8	300	450	30	13,282	9.21E-04	0.595	0.00155
8	350	450	30	15,837	8.67E-04	0.611	0.00142
8	400	450	30	17,722	8.54E-04	0.581	0.00147

Table 4-14  
 REPEATED SIMULATION RESULTS FOR TWO GROUND SPEED MEANS AND STANDARD DEVIATIONS  
 (Average Arrival Density: 1 Aircraft per Hour)

Average Density (A/C per Hour)	Ground Speed Mean (Knots)	Ground Speed Std Dev (Knots)	Total Flight Time (Hours)	Px Proportion of Time Passing	Nx Frequency of Passings (per Hour)	tx Average Duration (Hours)
1	300	30	13,392	1.752E-04	0.1151	0.001521
1	300	30	13,654	1.743E-04	0.1147	0.001520
1	300	30	13,298	1.584E-04	0.1101	0.001439
1	300	30	13,383	1.796E-04	0.1151	0.001560
1	300	30	13,407	1.416E-04	0.1059	0.001337
1	300	30	13,475	1.676E-04	0.1124	0.001491
1	300	30	13,647	1.863E-04	0.1155	0.001613
1	300	30	13,500	1.606E-04	0.1083	0.001483
1	300	50	13,698	1.753E-04	0.2024	0.000866
1	300	50	13,733	1.625E-04	0.1898	0.000856
1	300	50	13,929	1.638E-04	0.1907	0.000859
1	300	50	13,754	1.643E-04	0.1889	0.000870
1	300	50	13,773	1.700E-04	0.1924	0.000884
1	300	50	13,664	1.699E-04	0.1882	0.000902
1	300	50	13,616	1.671E-04	0.1879	0.000889
1	300	50	13,907	1.727E-04	0.1973	0.000875
1	450	30	8,914	1.172E-04	0.0756	0.001550
1	450	30	8,889	1.172E-04	0.0763	0.001537
1	450	30	8,873	1.119E-04	0.0821	0.001364
1	450	30	8,869	1.200E-04	0.0792	0.001516
1	450	30	8,955	1.064E-04	0.0701	0.001518
1	450	30	9,049	9.846E-05	0.0745	0.001322
1	450	30	8,994	1.187E-04	0.0794	0.001496
1	450	30	8,933	9.034E-05	0.0674	0.001341
1	450	50	8,935	1.088E-04	0.1222	0.000890
1	450	50	9,003	1.067E-04	0.1195	0.000893
1	450	50	9,029	1.188E-04	0.1318	0.000902
1	450	50	8,909	1.053E-04	0.1284	0.000820
1	450	50	9,060	9.581E-05	0.1205	0.000795
1	450	50	9,005	1.324E-04	0.1344	0.000985
1	450	50	9,032	1.070E-04	0.1214	0.000881
1	450	50	8,961	1.087E-04	0.1283	0.000847

Table 4-15  
 REPEATED SIMULATION RESULTS FOR TWO GROUND SPEED MEANS AND STANDARD DEVIATIONS  
 (Average Arrival Density: 8 Aircraft per Hour)

Average Density (A/C per Hour)	Ground Speed Mean (Knots)	Ground Speed Std Dev (Knots)	Total Flight Time (Hours)	Px Proportion of Time Passing	Nx Frequency of Passings (per Hour)	tx Average Duration (Hours)
8	300	30	13,362	1.363E-03	0.8917	0.001529
8	300	30	13,403	1.271E-03	0.8862	0.001435
8	300	30	13,401	1.307E-03	0.8787	0.001488
8	300	30	13,468	1.287E-03	0.8866	0.001451
8	300	30	13,376	1.339E-03	0.8997	0.001488
8	300	30	13,474	1.300E-03	0.9026	0.001441
8	300	30	13,542	1.336E-03	0.9016	0.001482
8	300	30	13,466	1.307E-03	0.9000	0.001452
8	300	50	13,825	1.332E-03	1.5265	0.000872
8	300	50	13,621	1.328E-03	1.4925	0.000890
8	300	50	13,602	1.358E-03	1.5289	0.000888
8	300	50	13,743	1.358E-03	1.5001	0.000905
8	300	50	13,727	1.289E-03	1.4936	0.000863
8	300	50	13,820	1.373E-03	1.5475	0.000887
8	300	50	13,658	1.417E-03	1.5188	0.000933
8	300	50	13,655	1.350E-03	1.5443	0.000874
8	450	30	9,047	9.067E-04	0.6026	0.001505
8	450	30	8,887	1.003E-03	0.5993	0.001674
8	450	30	9,046	9.094E-04	0.5905	0.001540
8	450	30	8,865	1.033E-03	0.5773	0.001790
8	450	30	8,930	8.961E-04	0.5982	0.001498
8	450	30	8,913	8.575E-04	0.5962	0.001438
8	450	30	8,920	8.504E-04	0.5865	0.001450
8	450	30	9,033	8.936E-04	0.5994	0.001491
8	450	30	8,856	8.808E-04	0.5935	0.001484
8	450	30	8,884	8.262E-04	0.5943	0.001390
8	450	30	8,944	8.873E-04	0.6042	0.001469
8	450	30	9,064	8.852E-04	0.6077	0.001457
8	450	30	8,873	8.560E-04	0.5890	0.001453
8	450	30	8,975	8.701E-04	0.6046	0.001439
8	450	50	8,958	8.151E-04	0.9812	0.000831
8	450	50	9,002	9.112E-04	1.0038	0.000908
8	450	50	9,026	9.068E-04	1.0153	0.000893
8	450	50	9,118	8.816E-04	1.0068	0.000876
8	450	50	9,021	8.822E-04	1.0034	0.000879
8	450	50	8,998	8.726E-04	1.0133	0.000861
8	450	50	9,008	9.083E-04	1.0309	0.000881
8	450	50	8,932	8.697E-04	0.9707	0.000896

Table 4-16  
 DISTRIBUTIONS OF AIRCRAFT GROUND SPEED  
 (High-Altitude VOR-Defined Jet Route Study)

	Commercial Jet	Military Jet	Gen. Avn. Jet	Multi-Eng. Turboprop	Multi-Eng. Piston
POPULATION STATISTICS:					
Mean	466.71 k	452.74 k	423.55 k	294.42 k	249.57 k
Standard Deviation	31.13 k	56.57 k	43.13 k	49.40 k	58.52 k
Minimum	200.00 k	200.00 k	200.00 k	150.00 k	140.00 k

DISTRIBUTION PERCENTAGES:

	Commercial Jet	Military Jet	Gen. Avn. Jet	Multi-Eng. Turboprop	Multi-Eng. Piston
Commercial Jet	100.00 %	0.00 %	0.00 %	0.00 %	0.00 %
Cleveland ARTCC	86.72 %	0.62 %	11.94 %	0.68 %	0.04 %
Memphis ARTCC	78.50 %	2.31 %	16.70 %	1.37 %	1.12 %
Albuquerque ARTCC	68.80 %	14.24 %	14.67 %	2.29 %	0.00 %
Combined Three-ARTCC	79.66 %	5.31 %	13.54 %	1.30 %	0.19 %

Table 4-17  
 SIMULATION RESULTS FOR VARYING AVERAGE ARRIVAL DENSITIES  
 (Commercial Jet Ground Speed Distribution)

Average Density (A/C per Hour)	Ground Speed Mean (Knots)	Ground Speed Std Dev (Knots)	Total Flight Time (Hours)	Px Proportion of Time Passing	Nx Frequency of Passings (per Hour)	tx Average Duration (Hours)
1/8	466.9	30.98	8,624	1.171E-05	0.0090	0.001295
1/2	466.7	31.07	8,636	5.269E-05	0.0389	0.001354
2	466.9	31.08	8,650	2.051E-04	0.1482	0.001384
4	467.0	31.33	8,624	4.043E-04	0.3036	0.001332
8	466.7	30.89	8,684	8.603E-04	0.5990	0.001436

Table 4-18  
SIMULATION RESULTS FOR VARYING AVERAGE ARRIVAL DENSITIES  
(Cleveland ARTCC Ground Speed Distribution)

Average Density (A/C per Hour)	Ground Speed Mean (Knots)	Ground Speed Std Dev (Knots)	Total Flight Time (Hours)	Px Proportion of Time Passing	Nx Frequency of Passings (per Hour)	tx Average Duration (Hours)
1/8	460.2	38.58	8,774	1.356E-05	0.0134	0.001008
1/2	459.7	38.78	8,733	5.428E-05	0.0486	0.001118
2	460.5	38.42	8,812	2.258E-04	0.1872	0.001206
8	460.4	38.44	8,681	8.445E-04	0.7398	0.001142

Table 4-19  
SIMULATION RESULTS FOR VARYING AVERAGE ARRIVAL DENSITIES  
(Memphis ARTCC Ground Speed Distribution)

Average Density (A/C per Hour)	Ground Speed Mean (Knots)	Ground Speed Std Dev (Knots)	Total Flight Time (Hours)	Px Proportion of Time Passing	Nx Frequency of Passings (per Hour)	tx Average Duration (Hours)
1/8	454.2	47.71	8,903	1.741E-05	0.0186	0.000934
1/2	454.3	48.82	8,889	5.546E-05	0.0664	0.000836
2	454.4	48.71	8,978	2.299E-04	0.2482	0.000926
8	454.6	47.81	8,984	8.618E-04	0.9450	0.000912

Table 4-20  
SIMULATION RESULTS FOR VARYING AVERAGE ARRIVAL DENSITIES  
(Albuquerque ARTCC Ground Speed Distribution)

Average Density (A/C per Hour)	Ground Speed Mean (Knots)	Ground Speed Std Dev (Knots)	Total Flight Time (Hours)	Px Proportion of Time Passing	Nx Frequency of Passings (per Hour)	tx Average Duration (Hours)
1/8	454.3	47.81	8,944	1.621E-05	0.0168	0.000967
1/2	454.1	48.06	8,828	5.290E-05	0.0553	0.000957
2	454.1	47.89	9,051	2.169E-04	0.2400	0.000904
8	454.1	47.20	8,882	8.514E-04	0.9086	0.000937

Table 4-21  
SIMULATION RESULTS FOR VARYING AVERAGE ARRIVAL DENSITIES  
(Three-ARTCC Combined Ground Speed Distribution)

Average Density (A/C per Hour)	Ground Speed Mean (Knots)	Ground Speed Std Dev (Knots)	Total Flight Time (Hours)	Px Proportion of Time Passing	Nx Frequency of Passings (per Hour)	tx Average Duration (Hours)
1/8	458.1	43.65	8,860	1.140E-05	0.0144	0.000789
1/4	457.7	42.73	8,843	2.906E-05	0.0267	0.001089
1/2	457.5	43.27	8,847	5.550E-05	0.0588	0.000944
1	457.6	43.75	8,873	1.083E-04	0.1113	0.000973
1	458.1	43.65	8,860	1.219E-04	0.1140	0.001069
1	457.5	43.03	8,850	1.011E-04	0.1028	0.000984
1 1/2	458.0	42.58	8,884	1.656E-04	0.1616	0.001024
2	457.0	43.25	8,809	2.625E-04	0.2273	0.001155
3	457.3	43.73	8,885	2.956E-04	0.3232	0.000914
4	457.7	42.73	8,841	4.424E-04	0.4386	0.001009
5	458.1	43.66	8,857	5.471E-04	0.5307	0.001031
6	457.0	43.24	8,814	6.442E-04	0.6403	0.001006
8	458.1	43.65	8,856	8.513E-04	0.8636	0.000986
10	457.0	43.26	8,805	1.158E-03	1.0905	0.001062
10	457.3	43.32	8,737	1.051E-03	1.0454	0.001005
10	457.2	43.38	8,836	1.139E-03	1.0533	0.001081

## 5.0 RESULTS OF SIMULATION MODELING

### 5.1 SIMPLE CORRELATIONS

5.1.1 Simple correlation coefficients were computed for the proportion of time passing ( $P_x$ ), the average time per passing ( $t_x$ ), and the average frequency of passings ( $N_x$ ) with the average aircraft arrival density ( $N$ ), with the aircraft ground speed mean ( $V$ ), and with the aircraft ground speed standard deviation ( $S$ ) (table 5-1). These correlation coefficients were calculated using the Statistical Package for the Social Sciences (SPSS) computer program (reference 17). The correlations of  $P_x$  with  $N$  and with  $V$ , the correlation of  $t_x$  with  $S$ , and the correlations of  $N_x$  with  $N$ , with  $V$ , and with  $S$  are significant at the 1 percent confidence level; the correlation of  $P_x$  with  $S$  and the correlations of  $t_x$  with  $N$  and with  $V$  are not significant even at the 10 percent confidence level.

5.1.2 Additional correlation coefficients were computed for  $P_x$ ,  $t_x$ , and  $N_x$  with  $1/N$ , with  $1/V$ , with  $1/S$ , with  $N*V$ , with  $N/V$ , with  $N*S$ , and with  $N*(S/V)$  (table 5-1). The strongest correlations found are  $P_x$  with  $N/V$ ,  $t_x$  with  $1/S$ , and  $N_x$  with  $N*(S/V)$ . This agrees with the analytic results previously obtained (paragraphs 3.2.8, 3.5.2, and 3.6.2).

5.1.3 Correlation coefficients were computed for  $P_x$ ,  $t_x$ , and  $N_x$  with the route segment length using a set of simulation runs planned for this purpose (table 4-13). None of the calculated simple correlation coefficients ( $-0.2778$  for  $P_x$ ,  $-0.1066$  for  $t_x$ , or  $-0.2779$  for  $N_x$ ) are significant even at the 20 percent confidence level.

### 5.2 INHERENT VARIABILITY OF THE SIMULATION MODEL

5.2.1 In order to estimate the inherent variability of the simulation model, repeated simulation runs were made with the same parameters (paragraph 4.2.5). These simulation runs were made for two values each of average aircraft arrival density (1 and 8 aircraft per hour), aircraft ground speed mean (300 knots and 450 knots), and aircraft ground speed standard deviation (30 knots and 50 knots) (tables 4-14 and 4-15). Since proportion of time passing ( $P_x$ ) is independent of aircraft ground speed standard deviation (paragraph 5.1.1), the runs with different aircraft ground speed standard deviations were combined for estimating a standard deviation of the proportion of time passing ( $P_x$ ). Similarly, the simulation runs with differences in both average aircraft arrival density and aircraft ground speed mean were combined for estimating a standard deviation for average time of passing ( $t_x$ ).

5.2.2 Variances and standard deviations were computed for each set of simulation runs (tables 5-2, 5-3, and 5-4). Variances for proportion of time passing ( $P_x$ ) and average frequency of passings ( $N_x$ ) for different aircraft ground speed means were then pooled, since these variances appear to be independent of these differences.

5.2.3 It should be noted that the variance of  $P_x$  is not independent of average aircraft arrival density, the variance of  $t_x$  is not independent of aircraft ground speed standard deviation, and the variance of  $N_x$  is not independent of either average aircraft arrival density or aircraft ground speed standard deviation. The hypothesis of homogeneity of variances does not hold, and care should be used in interpreting linear regression results based on this data.

### 5.3 LINEAR REGRESSIONS

5.3.1 Linear regressions were calculated for  $P_x$  on  $(M*N)/V$ , for  $t_x$  on  $M/S$ , and for  $N_x$  on  $N*(S/V)$ . The choice of these regressions was based on analytic calculations (paragraphs 3.2.8, 3.5.2, and 3.6.2) and on simple correlation coefficients (paragraph 5.1.2). Each of these linear regressions was done for the five different subsets of simulation runs described in paragraph 4.2.1 (tables 4-1 through 4-8), in paragraph 4.2.2 (tables 4-9 and 4-10) (except for  $t_x$ ), in paragraph 4.2.3 (tables 4-11 and 4-12), in paragraph 4.2.5 (tables 4-14 and 4-15), and paragraph 4.3 (tables 4-17 through 4-21) and for the set of all simulation runs.

5.3.2 The linear regressions were calculated using the Statistical Package for the Social Sciences (SPSS) computer program (reference 17). The following information is given for each linear regression -- the number of simulation runs, the slope of the regression line, the intercept of the regression line, the proportion of total variation explained by the regression (R squared), and the standard error of estimate of the regression (tables 5-5, 5-6, and 5-7). In addition, the slopes and intercepts derived analytically (paragraphs 3.2.8, 3.5.2, and 3.6.2) are given for comparison purposes.

5.3.3 Each of these regressions explains a substantial proportion of the total variation (R squared). A comparison of the regression standard errors of estimate with the inherent simulation variability (paragraph 5.2) shows that all three of these regressions provide excellent fits to the simulation data.

5.3.4 All of the regression lines have intercepts close to zero.

5.3.5 The standard errors of the slopes of the regression lines were calculated, and ninety-five percent confidence limits were derived (tables 5-8, 5-9, and 5-10) using the Student t distribution (reference 18). The slope of the regression line for  $P_x$  agrees with the value of 2 from the streaming aircraft model (paragraph 3.2.8). The slopes of the regression lines for  $t_x$  and  $N_x$ , however, appear to differ slightly from those from the analytic model (paragraphs 3.5.2 and 3.6.2). This slight difference is not unexpected, because of some problems with the analytic derivation noted above (paragraphs 3.5.3 and 3.6.3).

Table 5-1  
SIMPLE CORRELATION COEFFICIENTS

Description	Independent Variable for Correlation	Px Proportion of Time Passing	Nx Frequency of Passings (per Hour)	tx Average Duration (Hours)
Average Arrival Density	N	0.9666	0.8047	0.0028
Ground Speed Mean	V	-0.2195	-0.1826	0.0724
Ground Speed Std. Dev.	S	-0.0102	0.3621	-0.6800
	1/N	-0.5083	-0.4328	0.0476
	1/V	0.2256	0.1720	-0.0667
	1/S	-0.0317	-0.2494	0.9779
	N*V	0.9086	0.7559	0.0132
	N/V	0.9985	0.8274	-0.0109
	N*S	0.8002	0.9789	-0.2386
	N*(S/V)	0.8389	0.9991	-0.2396

Table 5-2  
 INHERENT VARIABILITY OF PROPORTION OF TIME PASSING (Px)

Average Arrival Density	Ground Speed Mean	Number of Runs	Px Variance	Px Standard Deviation
1	300 k	16	0.00000000010304	0.000010151
1	450 k	16	0.00000000010939	0.000010459
1	Pooled	32	0.00000000010622	0.000010306
8	300 k	16	0.000000000143246	0.000037848
8	450 k	22	0.000000000237823	0.000048767
8	Pooled	38	0.000000000198416	0.000044544

Table 5-3  
 INHERENT VARIABILITY OF AVERAGE TIME PER PASSING (tx)

Ground Speed Std. Dev.	Number of Runs	tx Variance	tx Standard Deviation
30 k	38	0.00000000749428	0.000086570
50 k	32	0.00000000108853	0.000032993

Table 5-4  
 INHERENT VARIABILITY OF AVERAGE FREQUENCY OF PASSINGS (Nx)

Average Arrival Density	Ground Speed Std. Dev.	Ground Speed Mean	Number of Runs	Nx Variance	Nx Standard Deviation
1	30 k	300 k	8	0.00001330	0.003646
1	30 k	450 k	8	0.00002393	0.004892
1	30 k	Pooled	16	0.00001862	0.004315
1	50 k	300 k	8	0.00002617	0.005116
1	50 k	450 k	8	0.00003180	0.005639
1	50 k	Pooled	16	0.00002899	0.005384
8	30 k	300 k	8	0.00007897	0.008887
8	30 k	450 k	14	0.00006726	0.008201
8	30 k	Pooled	22	0.00007136	0.008447
8	50 k	300 k	8	0.00047301	0.021749
8	50 k	450 k	8	0.00036682	0.019153
8	50 k	Pooled	16	0.00041992	0.020492

Table 5-5  
 LINEAR REGRESSIONS OF PROPORTION OF TIME PASSING (Px) ON (M\*N)/V

Description	Table Numbers	Number of Runs	Regression Slope	Regression Intercept	R Squared	Standard Error of Estimate
N, S	4-1 to 4-8	120	2.0109022	-0.0000018	0.99858	0.0000220
N, V	4-9 to 4-10	18	2.0438388	-0.0000042	0.98850	0.0000610
Slow	4-11 to 4-12	30	2.0508682	-0.0000050	0.99685	0.0000207
N, V, S	4-14 to 4-15	70	1.9995477	0.0000007	0.99565	0.0000332
ARTCCs	4-17 to 4-21	33	1.9954190	-0.0000000	0.99685	0.0000215
All Simulation Runs		280	2.0123567	-0.0000011	0.99688	0.0000291
Analytic Estimate			2.0000000	0.0000000		

Table 5-6  
 LINEAR REGRESSIONS OF AVERAGE TIME PER PASSING (tx) ON M/S

Description	Table Numbers	Number of Runs	Regression Slope	Regression Intercept	R Squared	Standard Error of Estimate
N, S	4-1 to 4-8	120	1.6762071	0.0000520	0.95343	0.0004437
Slow	4-11 to 4-12	30	1.8987829	-0.0000786	0.96195	0.0000652
N, V, S	4-14 to 4-15	70	1.8173149	-0.0000289	0.95348	0.0000676
ARTCCs	4-17 to 4-21	33	1.5575602	0.0001086	0.83007	0.0000665
All Simulation Runs		280	1.6788936	0.0000537	0.95623	0.0002950
Analytic Estimate			1.7724539	0.0000000		

Table 5-7  
 LINEAR REGRESSIONS OF AVERAGE FREQUENCY OF PASSINGS (Nx) ON N\*(S/V)

Description	Table Numbers	Number of Runs	Regression Slope	Regression Intercept	R Squared	Standard Error of Estimate
N, S	4-1 to 4-8	120	1.1397809	0.0004029	0.99977	0.0096303
N, V	4-9 to 4-10	18	1.1063527	0.0057884	0.99955	0.0078371
Slow	4-11 to 4-12	30	1.2313096	-0.0263630	0.99530	0.0374972
N, V, S	4-14 to 4-15	70	1.1333107	-0.0025538	0.99933	0.0125280
ARTCCs	4-17 to 4-21	33	1.1160249	0.0036625	0.99936	0.0093652
All Simulation Runs		280	1.1465425	-0.0022062	0.99823	0.0229715
Analytic Estimate			1.1283792	0.0000000		

Table 5-8  
SLOPES OF LINEAR REGRESSIONS OF PROPORTION OF TIME PASSING (Px) ON (M\*N)/V

Descrip- tion	Table Numbers	Number of Runs	Regression Slope	Standard Error of Slope	Lower 95 % Confidence Limit	Upper 95 % Confidence Limit
N, S	4-1 to 4-8	120	2.0109022	0.0069867	1.9970685	2.0247359
N, V	4-9 to 4-10	18	2.0438388	0.0551189	1.9269867	2.1606909
Slow	4-11 to 4-12	30	2.0508682	0.0217761	2.0062707	2.0954657
N, V, S	4-14 to 4-15	70	1.9995477	0.0160319	1.9675801	2.0315153
ARTCCs	4-17 to 4-21	33	1.9954190	0.0201502	1.9543126	2.0365254
All Simulation Runs		280	2.0123567	0.0067540	1.9991189	2.0255945
Analytic Estimate			2.0000000			

Table 5-9  
SLOPES OF LINEAR REGRESSIONS OF AVERAGE TIME PER PASSING (tx) ON M/S

Descrip- tion	Table Numbers	Number of Runs	Regression Slope	Standard Error of Slope	Lower 95 % Confidence Limit	Upper 95 % Confidence Limit
N, S	4-1 to 4-8	120	1.6762071	0.0341034	1.6086824	1.7437318
Slow	4-11 to 4-12	30	1.8987829	0.0713667	1.7526239	2.0449419
N, V, S	4-14 to 4-15	70	1.8173149	0.0486802	1.7202466	1.9143832
ARTCCs	4-17 to 4-21	33	1.5575602	0.1265738	1.2993496	1.8157708
All Simulation Runs		280	1.6788936	0.0215427	1.6366699	1.7211173
Analytic Estimate			1.7724539			

Table 5-10  
SLOPES OF LINEAR REGRESSIONS OF AVERAGE FREQUENCY OF PASSINGS (Nx) ON N\*(S/V)

Descrip- tion	Table Numbers	Number of Runs	Regression Slope	Standard Error of Slope	Lower 95 % Confidence Limit	Upper 95 % Confidence Limit
N, S	4-1 to 4-8	120	1.1397809	0.0015789	1.1366547	1.1429071
N, V	4-9 to 4-10	18	1.1063527	0.0058983	1.0938483	1.1188571
Slow	4-11 to 4-12	30	1.2313096	0.0159823	1.1985778	1.2640414
N, V, S	4-14 to 4-15	70	1.1333107	0.0035604	1.1262113	1.1404101
ARTCCs	4-17 to 4-21	33	1.1160249	0.0050904	1.1056405	1.1264093
All Simulation Runs		280	1.1465425	0.0028959	1.1408665	1.1522185
Analytic Estimate			1.1283792			

## 6.0 CONCLUSIONS AND RECOMMENDATIONS

### 6.1 SIMULATION ESTIMATES FOR $P_x$ , $t_x$ , AND $N_x$

6.1.1 Linear regression estimates have been made for the proportion of time passing ( $P_x$ ), the average time per passing ( $t_x$ ), and the average frequency of passings ( $N_x$ ). These estimates are based on 280 separate simulation runs of traffic on parallel route segments, each simulation run consisting of approximately 10,000 aircraft per route segment.

6.1.2 The following variables are used in the regression equations to estimate  $P_x$ ,  $t_x$ , and  $N_x$ .

6.1.2.1 The average aircraft arrival density (per hour) is represented by  $N$ . In the simulation runs, it assumed values between 1/8 and 16 aircraft per hour.

6.1.2.2 The mean of the aircraft ground speed distribution (in knots) is represented by  $V$ . In the simulation runs, it assumed values between 200 knots and 600 knots.

6.1.2.3 The standard deviation of the aircraft ground speed distribution (in knots) is represented by  $S$ . In the simulation runs, it assumed values between 0 knots and 75 knots.

6.1.2.4 The aircraft length (in nautical miles) is represented by  $M$ . In the simulation runs, it assumed the constant value of 0.025 nautical miles.

6.1.3 The proportion of time passing ( $P_x$ ) is approximately equal to

$$( 2 * M * N ) / V .$$

This agrees with the streaming aircraft model.

6.1.4 The average time per passing ( $t_x$ ) is approximately equal to

$$( 1.68 * M ) / S$$

hours.

6.1.5 The average frequency of passings ( $N_x$ ) is approximately equal to

$$1.15 * N * ( S / V )$$

aircraft per hour.

6.1.6 These estimates appear to be independent of the lengths of the two route segments and of the exact distributional form of the aircraft ground speed distributions.

## 6.2 EXAMPLE OF RISK ESTIMATION

6.2.1 These estimates for  $P_x$ ,  $t_x$ , and  $N_x$  can be used with the same-direction lateral collision risk formula (paragraph 2.3.2) and estimates for the other parameters to estimate the same-direction lateral collision risk.

6.2.2 The following parameter values are assumed for use in this example calculation.

6.2.2.1 The probability of vertical overlap ( $P_z(0)$ ) is given the value 0.25.

6.2.2.2 The aircraft length ( $M$ ) is given the value 0.025 nautical miles.

6.2.2.3 The aircraft wingspan is given the value 0.033 nautical miles, and the lateral closing speed is given the value 53 knots. Therefore, the average time per lateral passing ( $t_y$ ) is given the value 0.00062 hour, and its reciprocal ( $1 / t_y$ ) is given the value 1606 passings per hour.

6.2.2.4 The aircraft height is given the value 0.0085 nautical miles, and the vertical closing speed is given the value 1 knot. Therefore, the average time per vertical passing ( $t_z$ ) is given the value 0.0085 hour, and its reciprocal ( $1 / t_z$ ) is given the value 118 passings per hour.

6.2.3 These values are substituted into the same-direction lateral risk equation (paragraph 2.3.3). For this example, the expected number of accidents in ten million track system flying hours due to the loss of all planned lateral separation by same-direction aircraft is equal to

$$10,000,000 * P_y(S_y) * 0.25 * ((1.15*N*(S/V))+((2*0.025*N)/V))*(1606+118))$$

or equivalently to

$$10,000,000 * P_y(S_y) * ( N / V ) * ( 0.2875 * S + 21.55 ) .$$

6.2.4 Note in this case that the term containing the aircraft speed standard deviation dominates the risk only if the standard deviation ( $S$ ) is at least 75 knots and that this term is relatively less important for smaller values of the speed standard deviation. For example, increasing this standard deviation from 30 knots to 60 knots increases the total risk by less than thirty percent in this case.

## 6.3 RECOMMENDATIONS FOR RISK ESTIMATION

6.3.1 The streaming aircraft model should continue to be used to estimate the proportion of time passing ( $P_x$ ). The results of the streaming aircraft model have been shown by these simulations to hold without the assumption of constant aircraft velocities, and they can now be used without making this unrealistic velocity assumption.

6.3.2 In most cases, an estimate for average time per passing ( $t_x$ ) would be acceptable for risk estimation in individual cases. However the regression equation for average frequency of passings ( $N_x$ ) should be used whenever comparisons are made between different policies which could be expected to result in different aircraft speed distributions.

#### 6.4 RECOMMENDATIONS FOR FURTHER WORK

6.4.1 Data for the study of occupancy is available at the Federal Aviation Administration Technical Center from a data collection to determine lateral pathkeeping of aircraft flying VOR-defined jet routes (references 15 and 16). Additional data will be available at the Technical Center from a planned data collection to determine lateral pathkeeping of aircraft flying low-altitude VOR-defined airways (reference 19). Each of these data collections includes flight progress strip data and radar tracking data on aircraft flying on selected routes. Several different analyses should be done with this actual flight data.

6.4.1.1 The arrival times over a point from this data should be analyzed to check the assumptions used in the analytic and simulation models and to estimate arrival densities. Statistical tests should be made for the presence of a Poisson arrival process, for homogeneity of the arrival process (from hour to hour, from day to day, and from week to week), for dependence of arrivals on adjacent routes, and for dependence of arrivals on different altitudes of the same route. This analysis should result in a better arrival distribution for further analytic and simulation modeling.

6.4.1.2 The aircraft velocities from this data should also be analyzed to check the assumptions used in the models and to estimate average velocities and average relative velocities. Statistical tests should be made for dependence between consecutive flights on the same route and altitude and for dependence among simultaneous flights on different routes or altitudes. A better velocity distribution should result from this analysis.

6.4.1.3 This data should also be analyzed to identify all passings on the selected routes and then to count their number and to sum their durations directly. Estimates of occupancy obtained from this direct counting process should then be compared with the estimates obtained from the analytic and simulation models.

6.4.2 Sufficient simulation runs have been made to verify the streaming aircraft model estimate for the long-term expected value of  $P_x$ . Neither the streaming aircraft model nor the simulation results in this paper, however, provide any information on the variability of hourly or daily  $P_x$  values about this expected value. Additional simulation runs should be made to generate repeated estimates of  $P_x$  for short time intervals (such as one hour, eight hours, or one day) under a fixed set of arrival and velocity distributions. The variability of these estimates of  $P_x$  should then be analyzed, and a distribution should be fitted to these  $P_x$  values.

6.4.3 The simulation model should be expanded to consider other assumptions, such as dependent arrival loading on the two routes, different arrival densities on the two routes, dependent arrival loading on more than one altitude on each route, and variation in arrival densities over time.

6.4.4 An attempt should be made to derive a valid analytical expression to estimate the average time per passing ( $t_x$ ), or equivalently the average frequency of passings ( $N_x$ ), under suitable assumptions.

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                  **same-direction traffic on two**  
                  **adjacent parallel routes /**  
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