FAA TECHNICAL CENTER LETTER REPORT

POST FLIGHT POSITION COMPUTATIONS AND AN ACCURACY ANALYSIS OF A PORTABLE TRACKING SYSTEM USED FOR HELICOPTER FLIGHT TESTS

by

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Abstract

This report documents the measured position accuracy of a portable tracking system and the details of the post flight data computations. The system was assembled by personnel at the FAA Technical Center using a combination of off-the-shelf hardware and in-house designed hardware and software. The system will be used to provide an accurate position reference for remote base helicopter Area Navigation (RNAV) non-precision approach flight tests. The measured accuracy of the portable tracking system was 61.4 meters circular error, 99.5% confidence. Based on the accuracy results, the system is suitable for use as a position reference for remote base helicopter RNAV non-precision approach flight tests.

Key Words

Range Instrumentation
Position Reference
Tracking System
Kalman Filter
RNAV Approach

Period Covered

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INTRODUCTION

The Federal Aviation Administration Technical Center is examining various types of navigation systems to meet the requirements of the post 1990 time period. In order to collect test data to determine compliance with the requirements specified in FAA Advisory Circular (AC) 90-45A (reference 1) and to examine the technical/operational issues specified in the Federal Radionavigation Plan (FRP) (reference 2), a computer controlled data collection system with navigation system interfaces was developed by Technical Center personnel (reference 3). Also, a portable ranging system was integrated into the data collection system, and post-flight processing techniques were developed to provide an accurate position reference for remote base helicopter RNAV non-precision approach flight tests.

The purpose of this report is to document the details of the post flight position computations and to document the measured position accuracy of the portable tracking system, which was flight tested at the Technical Center in a Sikorsky twin turbine CH-53 helicopter (N39).

DISCUSSION

Portable Tracking System

The portable tracking system consists of two major elements: the portable ranging system (PRS) and the post processing software (a Kalman Filter) which reconstructs the approach trajectory. The operation of the PRS and the underlying theory of the post flight trajectory estimation software is discussed at length in the following sections. Mathematical analyses are relegated to the appendices.

PORTABLE RANGING SYSTEM

Operational Description

The portable ranging system is an off-the-shelf item manufactured by Motorola, Inc. known as the Mini-Ranger* III System. The MRS III consists of a receiver-transmitter unit with antenna, a range console, and four reference stations with antennas. These units operate on the basic principle of pulsed radar ranging systems. The receiver transmitter interrogates and waits for replies from each reference station, while the range console times the transmission delay. The range console converts the measured time delay to a range measurement, which is available in a parallel BCD format for use by peripneral equipment. Range measurement accuracy is claimed to be approximately 3 meters in a static situation (See Reference 4). Dynamic accuracy will be discussed in the section entitled Flight Test. The MRS-III included a four code commutation option permitting the range console to interrogate four reference stations, in groups of two, on alternate measurement cycles. The original commutation board purchased from Motorola yielded a range measurement cycle time of 500 milliseconds. A four code commutation board was designed and fabricated at the FAA Technical Center to reduce measurement cycle time to 200 milliseconds

*Motorola Inc, registered trademark

During the measurement cycle, the receiver-transmitter sends pulse coded interrogations to two reference stations. These transmissions are decoded by the interrogated reference station, which responds with a coded transmission. When five sequential interrogations produce five replies from a reference station, the averaged time count is stored in data interface latches as the range to the reference station. Range measurements are taken from a group of two reference stations (codes 1 and 2) for 200 milliseconds and then from the second group of two reference stations (codes 3 and 4).

The four range measurements are recorded once every 200 milliseconds through the interface in the aircraft systems coupler and stored on a floppy disk or tape recorder under control of the data collection software resident in the data collection computer. The program tests the contents of the real time clock for an elapsed time of 200 milliseconds thus determining start of a data collection period. On four consecutive data collection periods, four sets of MRS-III range data are collected and temporarily stored in a buffer memory. During the fifth 200 millisecond period, range data, current time from the airborne time code generator and all other airborne data are collected and stored in a buffer memory. After the fifth data collection period is completed, data is permanently recorded and the data collection cycle is reinitiated. It is important to note that range measurement cycles, and data collection cycles are asynchronous with each other. Ramifications of this will be clarified in the next section.

Geographic position of the reference stations was determined by two JMR-4 land survey sets. The survey sets, which used the Navy Transit satellite to determine position, were placed at chosen sites for 36 to 48 hours. The doppler surveyors recorded information from the satellites on magnetic tape cartridges. Data was subsequently processed on the FAA Technical Center°s Honeywell Model 66/60 computer to provide estimated position of the ground sites. The 3-dimensional accuracy provided by the JMR-4 survey system is on the order of 5 meters, world wide.[Ref. 5] In most instances, post processing yielded estimated standard deviations of the errors in the estimated ground site position which were on the order of one meter in each axis.

RANGE DATA PROCESSING

Printouts of MRS III range measurements recorded during developmental flight tests indicated that certain types of problems (exhibited as range errors) occur which can be eliminated or reduced by post flight processing raw range measurements prior to updating the Kalman filter. Intermittent multipath, loss of signal, wild value range measurements, and time skew between the four range measurements are typical problems which fall in this category.

Time skew between the range updates is a result of the asynchronous operation between the four code commutation board, and the data collection interface. The asynchronous range measurement and data collection cycles cause the updated range measurements to occur randomly throughout the recorded data. Thus, it is necessary to search backward in time for the most recent range measurements and extrapolate each forward to the update time (airborne time during fifth collection period). Range data are searched and time occurrence is estimated based on a nominal 200 millisecond spacing of recorded values. Range selection is accomplished by comparing measured ranges to predicted ranges computed from extrapolated position coordinates from the Kalman filter, and substituting the predicted range if tolerances functionally dependent on ground speed are exceeded. Signal loss is indicated by two identical sequential range measurements, and is corrected by inserting a predicted range. Finally, the range measurements are filtered by an alpha-beta tracking filter which extrapolates the ranges forward in time with a filtered range rate term (See Reference 6). Beta is set to .7 to allow for adequate dynamic response. Initial values for the range tracking filter are set to zero. Figure 1 is a schematic representation of the range processing technique. Intermittent multipath effects are reduced by this technique. Continued multipath reception must be eliminated by choosing ground sites to provide unobstructed line of sight along the approach path.

OPTIMAL TRAJECTORY ESTIMATION

Kalman Filter

All measurements necessary to reconstruct approaching helicopter trajectories were recorded by the airborne data collection system. Eight measurements consisting of four ranges to known ground reference positions, barometric pressure altitude, barometric altitude rate, inertially derived track angle and ground speed and Kalman filter theory provide an optimal linear filtering technique for estimating the state vectors (three dimensional local cartesian position and velocity vectors) from noisy measurements (References 7 and 8).

A Kalman filter was developed (in-house at the Technical Center) in the form of post flight processing software which provides a minimum error (linear mean square) estimate of position and velocity vectors for the assumed models.

Development of a specific filter required a dynamic system model of helicopter motion, a measurement model which related recorded data to states in the dynamic system model, a method of determining initial state vectors, and statistical knowledge of random processes associated with each model (Reference 9).

Dynamic System Model

Treating motion in three uncoupled coordinate axes simplifies the discrete dynamic system model. Final specification of the discrete dynamic system model requires a measure (variance) of random accelerations.

A discrete system model consisting of double integrators driven by zero mean uncorrelated random acceleration is assumed for each cartesian coordinate axis. Assumed square root values of acceleration variances are 9.8m/sec/sec in X, Y axes and 12.1 m/sec/sec in the Z axis.

Measurement Model

Choosing a right hand cartesian coordinate system with x and y axes aligned to north, east directions and origin located at some fixed arbitrary point on the earth's surface leads to a simplified measurement model. Surveyed geodetic coordinates of four reference stations are converted to local coordinates (flat earth approximations are employed) and entered into four simultaneous three dimensional nonlinear range equations. Formulating derivatives of each range equation with respect to state vectors yields linearized, position dependent weighting functions (direction cosines) for a model of range measurement.

Barometric pressure altitude is corrected for earth curvature and local pressure datum and modeled as a direct measurement of coordinate z. Barometric pressure altitude rate is modeled as a direct measurement of Z velocity. Inertial ground speed is resolved into X and Y velocities by trigonometric functions (sine, cosine) of inertial track angle. Each measurement is treated as containing unbiased, uncorrelated additive noise. An assumed total error budget is presented in Table 1.

Initialization

Initial position vectors are estimated by solving four simultaneous range equations in three unknowns (X, Y, Z). The over-determined (more equations than unknowns) nonlinear set of equations are linearized and solved by Newton's iterative method. employing pseudoinverses of direction cosine matrices recomputed at each iteration step (Reference 10). Iteration begins with a guess of the initial position coordinates (X=0, Y=0, Z=barometric altitude), and terminates upon completion of five iterations, or sooner if a computed two dimensional residual position error is less than 305 meters. Upon successful determination of an initial position vector, current measured values of X, Y, Z velocities become initial velocity vector estimates. Errors in initial state estimates are assumed to be unbiased and uncorrelated random variables. An assumed error budget of initialization errors is presented in Table 2.

Software Implementation

The previously described Kalman filter has been implemented in double precision FORTRAN and imbedded in the data analysis program which provides the position error data contained in this report. This implementation is a modified version of the filter described in reference 1. The newer version includes earth curvature correction and provides a flag which indicates an initialization settling period is in effect. During the 20 second settling time, the position data is excluded from the error analysis.

PORTABLE TRACKING SYSTEM PERFORMANCE TESTS

A series of flight tests was conducted at the FAA Technical Center for the purpose of evaluating the accuracy performance of the portable tracking system. Early tests consisted of low approaches to runway 4 and runway 13 at Atlantic City Airport with four reference stations placed at surveyed points at opposite ends of each runway. The results indicted that the system could provide adequate accuracy with additional hardware/software modifications. These improvements were implemented and a second series of tests was designed to simulate conditions typical of remote base operations. These tests were conducted as full dress rehearsals of operational procedures, thus affording all personnel the opportunity to become familiar with assigned responsibilities.

Developmental Tests

Early tests consisted of low approaches to runway 4-22 and runway 13-31 (Atlantic City Airport) with four reference stations placed at surveyed NAVAID test bed sites at opposite ends of each runway.

Results indicated feasibility of the system, but hardware modifications and additional post-processing software were necessary to achieve satisfactory performance. Hardware modifications included a re-designed four code commutation board for range measurements (previously discussed) and a custom designed omnidirectional antenna with a displaced elevation gain beamwidth. This antenna improves reception of signals from reference stations when high elevation angles are encountered by an approaching helicopter. Inverted mounting of the antenna (a use which the original equipment antenna design did not anticipate) on the fuselage underneath the helicopter necessitated the change.

One reception problem remains, and cannot be readily eliminated. Occasionally, pitch and roll attitudes of the helicopter may cause shielding of the antenna; loss of line-of-sight will cause a multipath measurement or a loss of range measurement update. A software technique which greatly reduces the effects of multipath and short gaps in range measurements has been discussed in Range Processing.

In the developmental test results, the best value of measured accuracy for any dynamic range measurement, ignoring means, is 26.1 meters rms. The minimum measured system radial distance error, ignoring means, is 28.0 meters rms, a very promising result.

Operational Tests

The operational tests were designed to simulate conditions typical of remote base operations. These tests were conducted as full dress rehearsals of operational procedures (which were intended to test suitability of various RNAV systems for non-precision helicopter approaches), thus affording all personnel adequate opportunity to become familiar with assigned responsibilities.

Operational Test Procedures

Several weeks before flight tests additional ground reference station sites were surveyed with JMR-4 satellite survey sets. Ground sites were chosen to create baseline geometry similar to situations encountered at remote airfields. Line-of-sight is a very important consideration, and will often dictate baseline geometry; a desire to stay within boundaries of small airfields restricts baseline lengths to a maximum of 2500 meters in most cases. Two basic patterns were selected; a diamond-shaped array and a "T"-shaped array with no baseline greater than 2500 meters. Several hours prior to scheduled flights, a briefing was held for all members of the test team to discuss objectives and procedures for the flight. Coordination with Air Traffic Control and FAA Technical Center Range Instrumentation was accomplished and final preparations were begun. While the helicopter ground crew readied N-39 for flight, the four reference stations were placed at surveyed ground sites. A series of approaches to runway 13 was flown with four reference stations placed at satellite surveyed sites which formed a "T" when viewed from the approach end of runway 13.

A second series of approaches to runway 4 was flown with the four reference stations placed at satellite surveyed sites which formed a diamond-shaped configuration when viewed from the approach end of runway 4. The FAA Technical Center°s modified Nike-Hercules Target Tracking Radar (TTR) (Reference 11) tracked the helicopter and recorded its position at a 10 Hz rate. Data was collected to perform a preliminary performance evaluation of the developed system.

DATA PROCESSING

Nike-Hercules tracking tapes were processed on the FAA Technical Center°s Honeywell Model 66/60 computer to transform azimuth, elevation and range measurements at the radar site into the local X, Y, and Z coordinates. Processed radar data and airborne data were time merged (+/- 50 millisecond skew tolerance) on a PDP 11-34 Minicomputer and stored on magnetic tape. Data from merge data tapes were subsequently transferred to RK-05 magnetic disks for faster access by data analysis software.

DATA ANALYSIS

Printouts of processed radar data, airborne data, and merged data were generated and visually examined to determine start and stop times which were entered into a data analysis program. This program searched for the start time, initialized the Kalman filter algorithm, processed range data, generated filtered estimates of helicopter position, computed errors (differences) between filtered estimates and position reference (Nike-Hercules tracking data), and accumulated the number of data points, sums of errors and sums of squared errors. When a stop time was encountered the data analysis program printed means, standard deviations (sigma) and root-mean-square (RMS) values of the circular errors and terminated operation. All analysis software was written in FORTRAN, using double precision computations, and was run on a PDP 11/34 minicomputer.

Several sources of errors which were not eliminated are included in the computation of error measures. Radar position and radar time are considered as truth, and any errors due to radar accuracy or time skew between radar time and airborne measurement update time are lumped into error of the portable tracking system. Wild point editing of errors which occur between start and stop times is not performed. Effects of extrapolation across gaps in merge data due to radar data editing, which takes place during processing on the Honeywell model 66/60 computer, are included in error accumulations when occurring between start and stop times. Degraded system errors due to loss of one or more range measurements are included in error accumulations. Errors in surveyed coordinates of reference stations are included in error accumulations. This stringent philosophy of total system position error is adopted since the portable tracking system must provide an adequate position reference, independently, at remote airfields. For the purpose of this analysis, an adequate position reference will enable the reconstruction of low approach trajectories of helicopters from a distance of 10 nautical miles, line-of-sight, through missed approach procedures or to touchdown, and meet a specified criteria of 61.4 meters total circular error (95% confidence level). This criteria is based on one-tenth of the allowable RNAV System crosstrack error for non-precision approaches (AC 90-45A, reference 1).

NUMERICAL RESULTS

Preliminary results of the operational tests were reported in reference 3. Table 3 is a reprint of these results. These statistics summarize data taken in continuous segments from approaches, segmentation of data occurs due to extended gaps in either MRS-III range updates (all four beacons out of line-of-sight) or Nike Hercules tracking data (radar data below elevation cutoff angles are eliminated by Honeywell Model 66/60 computer programs). Ranges out to approximately 7 nautical miles, missed approach/go around maneuvers, and outbound legs where the helicopter is as much as 2 nautical miles from runway center are represented in the results. These results showed that the system was on the verge of meeting the specified criteria of 61.4 meters total circular error (95% confidence level). In fact, an empirical count of errors inside and outside this bound showed the system to be within 61.4 meters 98% of the time (2062 out of 2106).

Printouts of error data were examined more closely, and it was determined that a settling time of 20 iterations of the Kalman filter was necessary to eliminate transient behavior due to initialization errors. Secondly, an earth curvature correction model was incorporated into the Kalman filter software. A third change was the adjustment by 8 meters of the coordinates of one beacon for the second series of approaches to runway 4.

The error in the beacon coordinate occurred because the survey stake was lost when grass surrounding the airfield was cut. The beacon was placed as close to the spot as possible (at the time this was thought to be adequate). Errors which resulted prompted the investigation which lead to the analysis in Appendix D.

This instance clearly shows the need for accurate survey information of the ground beacon sites.

The data were reprocessed with these changes to the analysis software with very satisfactory results. The numerical results presented in Table 2 show the system meets the 61.4 meter criteria by all measurements considered. Finally, a count of the errors falling inside and outside the 61.4 meter circle shows 1877 of 1886 or 99.5% of errors fall inside the specified limit.

QUALITATIVE ANALYSIS

Two plots of position errors which are typical of system performance are presented in Appendix E. Figures E1 and E2 represent before and after results of changing the X-coordinate of one baseline beacon by 8 meters (see Appendix D).

The angular error in the y direction is virtually eliminated by the correction, without affecting the x error significantly. Several error phenomena are evident in the plots in Figure E.2 and deserve explanation. The small amplitude ramping which is most noticeable in the 2D position error is a result of using a line frequency clock to time the airborne data collection cycle. Each record is accurately time tagged, however, the update rate is about 980 milliseconds. Radar data is recorded and time tagged every 100 milliseconds. The merge skew tolerance is 50 milliseconds so once in every 5 merge points the airborne data and the radar data cycle through a +/- 50 millisecond skew window in 20 millisecond increments. The ramp error is largely dependent on velocity. At approach speeds of 100 kts (51.5 m/s) the ramp error amplitude would be +2.6 meters (0.5 times 51.5) which compares favorably with the error plotted in figure E.2. The slope of the ramp error in X changes sign from negative to positive as the transition is made from inbound to outbound leg, as expected. For figure E.2, inbound velocity is in the positive direction, therefore, more negative error values occur when airborne data lags radar data, and more positive error values occur when airborne data leads radar data. For outbound legs, effects are just the opposite, hence the sign reversal of the ramp slope.

Large position errors which appear in the first 20 seconds of figure E.2 are the result of initialization errors and the subsequent transient response.

The noticeable excursion in Y error (lateral axis) in E.2 between 1 and 2 minutes is due to loss of update from one beacon on the lateral baseline. The large error excursions between 3.5 and 4.5 minutes ocurred during missed aproach procedures and transition to an outbound leg (180' course change). This error is due to high dynamics maneuvering while flying in close proximity to the beacon cluster, as well as momentary loss of range updates due to antenna shielding during the transition maneuvers. Loss of update does not always occur during these maneuvers, but errors identical to those in figure E.2 occurred regularly during such maneuvers. Thus, lagging dynamic response of the position determination filtering scheme is the primary cause of these exursions with loss of range updates occasionally contributing to the errors.

SUMMARY

This report discusses the details of post flight position determination software used in a portable tracking system. Flight tests of the portable tracking system are described, and results of an accuracy analysis of the flight test data are presented. Based on the data analysis, the measured accuracy of the portable tracking system is 61.4 meters circular error, 99.5% confidence.

CONCLUSIONS

Based on the accuracy analysis results, the portable tracking system described in this report is suitable for use as position reference for remote base helicopter RNAV non-precision approach flight tests.

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FIGURE 1
RANGE PROCESSING SCHEMATIC

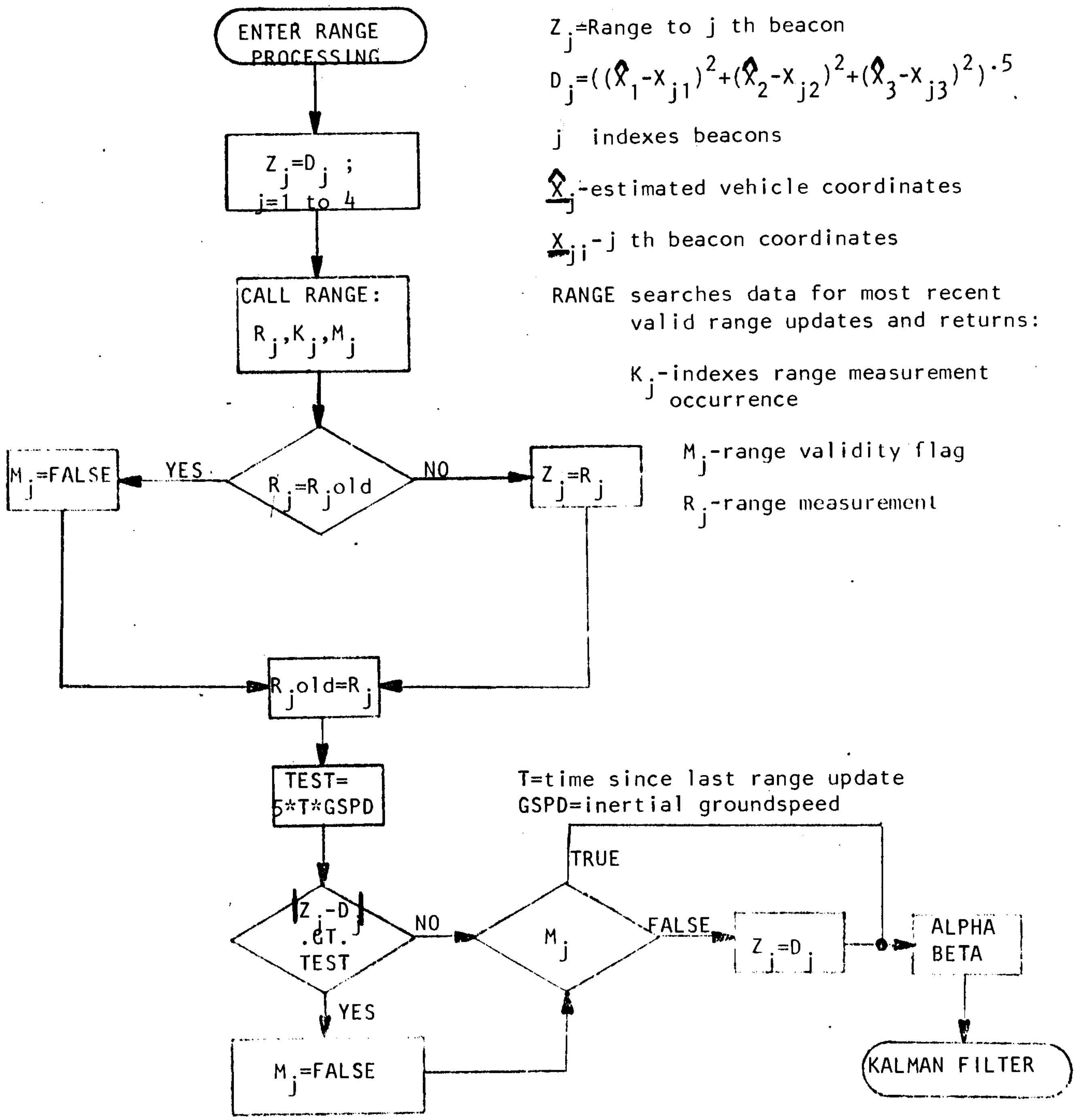


Table 1. Total Measurement Error Budget

Measurement	One Sigma
Range	5 0m
Barometric Altitude	7.6m
Barometric Altitude Rate	3 0m/s
X, Y Velocity	2.0m/s

Table 2. Initial Estimate Error Budget

Estimate	One Sigma
Position (X or Y)	305m
Position (Z)	7.6m
Velocity(X,Y Z)	3 0m

Table 3 Portable Tracking System
Circular Error Statistics (meters)
Reprinted From Reference 3

Segment	Samples	Mean	2 Sigma	2 RMS	Mean + 2 Sigma
Runway 13 "T" Ground Site Geometry					
1	162	22.3	20.0	48.8	42.3
2	169	25.6	49.0#	70.7**	74.6*
3	216	26.8	27.9	60.3	54.7
4	166	27.0	24.1	59.2	51.1
5	93	22.3	14.2	46.8	36.5
Runway 4 "Diamond" Ground Site Geometry					
6	111	29.7+	18.2	62.1*	47.9
7	255	27.5	26.0	60.7	53.5
8	287	28.7	34.7	67.1	63.4*
9	275	26.4	24.9	58.2	51.3
10	289	27.8	26.4	61.5	54.2
11	83	23.2	28.6	54.3	51.,8
Worst Case		29.7+	49.0#	70.7**	78.7***

*Fails 61.4 meter criterion
*Worst Case Mean
+Worst Case
#Worst Case 2 Sigma
**Worst Case 2 RMS
***Worst Case Mean + Worst Case 2 Sigma

Table 4 Portable Tracking System
Circular Error Statistics (meters)
Final Results

<u>Runway 13 "T" Ground Site Geometry</u>

Segment	Samples	Mean	2 Sigma	2 RMS	Mean + 2 Sigma
1	142	22.1	13.6	46.3	35.7
2	149	21.4	11.0	44.3	55.2
3	196	26.8	28.4#	60.7**	55.2
4	146	27.9+	23.4	60.5	51.3
5	73	22.3	15.4	47.2	37.7
<u>Runway 4 "Diamond" Ground Site Geometry</u>					
6	91	10.1	18.4	27.3	28.5
7	235	10.4	23.8	31.6	34.2
8	267	12.1	21.7	32.5	33.8
9	255	9.4	17.0	25.3	26.4
10	269	11.9	11.5	26.4	23.4
11	63	12.8	4.6	26.1	17.4
Worst Case		27.9+	28.4#	60.7**	56.3***

+Worst Case Mean
#Worst Case 2 Sigma
**Worst Case 2 RMS
***Worst Case Mean + Worst Case 2 Sigma

APPENDIX A - KALMAN FILTER EQUATIONS

The mathematical structure of the Kalman filter as developed here is contained in Table A.1. The following discussion concerns the details of the computation of the elements of the vectors and arrays contained in Table A.1.

$$\underline{X}(k) = (x\ y\ z\ \dot{x}\ \dot{y}\ \dot{z})^T \qquad (1)$$

is the cartesian position and velocity vector.

The state transition matrix is:

$$D = \begin{bmatrix} 1 & 0 & 0 & T & 0 & 0 \\ 0 & 1 & 0 & 0 & T & 0 \\ 0 & 0 & 1 & 0 & 0 & T \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad (2)$$

where T is the time between measurements.

The dynamic noise model q is 3 random accelerations,

$$\underline{q} = (a_x\ a_y\ a_z)^T \qquad (3)$$

The noise input matrix is

$$G = \begin{bmatrix} T^2/2 & 0 & 0 & T & 0 & 0 \\ 0 & T^2/2 & 0 & 0 & T & 0 \\ 0 & 0 & T^2/2 & 0 & 0 & 0 \end{bmatrix} \qquad (4)$$

TABLE A.1 DISCRETE KALMAN FILTER MODEL

System Model: $\underline{X}(k) = D\,\underline{X}(k-1) + G\,\underline{q}(k)$ (5)

Non linear Measurement Model: $\underline{Z}(k) = \underline{h}(\underline{X}(k)) + \underline{r}(k)$ (6)

Noise Models: $Q = E(\underline{q}\,\underline{q}^T),\quad \underline{q} \sim N(0,Q)$ (7)

$R = E(\underline{r}\,\underline{r}^T),\quad \underline{r} \sim N(0,R)$ (8)

$E(\underline{q}(k)\,\underline{r}(j)) = 0$, all j,k (9)

Initial Conditions: $E(\underline{X}(0)) = \hat{\underline{X}}_0$ (10)

$E\left((\underline{X}(0)-\hat{\underline{X}}_0)(\underline{X}(0)-\hat{\underline{X}}_0)^T\right) = P_0$ (11)

State Estimate Extrapolation: $\hat{\underline{X}}(k^-) = D\,\hat{\underline{X}}(k-1)$ (12)

Error Covariance Extrapolation $P(k^-) = D\,P(k-1)\,D^T + Q'$ (13)

$Q' = GQG^T$ (14)

Kalman Gain Matrix:

$K(k) = P(k^-)H^T(k)\left(H(k)P(k^-)H^T(k) + R\right)^{-1}$ (15)

State Estimate Update: $\hat{\underline{X}}(k) = \hat{\underline{X}}(k^-) + K(k)\left(\underline{Z}(k) - \underline{h}(\hat{\underline{X}}(k^-))\right)$ (16)

Error Covariance Update $P(k) = (I^* - K(k)H(k))P(k^-)$ (17)

Linearized Measurement Model $\underline{Z}(k) = H(k)\underline{X}(k) + \underline{r}(k)$ (18)

$$H(k) = \frac{\partial}{\partial \underline{X}}\,\underline{h}(\underline{X}(k))\Big|_{\underline{X}=\hat{\underline{X}}(k^-)}$$

*I is an identity matrix

The measurement vector is

$$\underline{Z}(k) = (r_1\ r_2\ r_3\ r_4\ -h_B\ \dot{x}\ \dot{y}\ -\dot{h}_B)^T \qquad (20)$$

where

ri = range to ith beacon (20a)

h_B = barometric altitude (corrected) (20b)

x,y = x,y velocities (20c)

$\dot{h}_B$ = barometric altitude rate (20d)

The non-linear measurement model is

$$\underline{h}(\underline{x}) = (d_1\ d_2\ d_3\ d_4\ z\ \dot{x}\ \dot{y}\ \dot{z})^T \qquad (21)$$

where

$$d_i = \left((x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2\right)^{1/2} \qquad (22)$$

xi, yi, zi - coordinates of ith beacon

The measurement noise model is

$$\underline{r}(k) = (\Delta r\ \Delta r\ \Delta r\ \Delta r\ \Delta B\ \Delta\dot{x}\ \Delta\dot{y}\ \Delta\dot{z})^T \qquad (23)$$

where

Δ r is range measurement error (23a)

Δ B is altitude measurement error (23b)

Δ $(\dot{x},\dot{y})$ is speed measurement error (23c)

Δ $\dot{z}$ is altitude rate measurement error (23d)

The system dynamic model noise covariance matrix is

$$Q = \begin{bmatrix} \sigma_{a_x}^2 & 0 & 0 \\ 0 & \sigma_{a_y}^2 & 0 \\ 0 & 0 & \sigma_{a_z}^2 \end{bmatrix} \qquad (24)$$

where

$$\sigma_{a_x} = \sigma_{a_y} = 9.8 \text{ m/sec}^2 \qquad (25)$$

$$\sigma_{a_z} = 12.1 \text{ m/sec}^2 \qquad (26)$$

The measurement noise covariance matrix is

$$R = \begin{bmatrix} \sigma_r^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_r^2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_r^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_r^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_h^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_{\dot{x}}^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{\dot{y}}^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{\dot{z}}^2 \end{bmatrix} \qquad (27)$$

where

$$\sigma_r = 5 \text{ m} \qquad (28)$$

$$\sigma_h = 7.6 \text{ m} \qquad (29)$$

$$\sigma_{\dot{x}} = \sigma_{\dot{y}} = 2 \text{ m/sec} \qquad (30)$$

$$\sigma_{\dot{h}} = 3 \text{ m/sec} \qquad (31)$$

The initial state error covariance matrix is

$$P_o = \begin{bmatrix} \sigma_{x_o}^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_{y_o}^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_{z_o}^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_{\dot{x}_o}^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_{\dot{y}_o}^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_{\dot{z}_o}^2 \end{bmatrix} \qquad (32)$$

where

$\sigma_{xo} = \sigma_{yo}$ = 305 meters, (33)

σ_{zo} = 7.6 meters (34)

$\sigma_{\dot{x}o} = \sigma_{\dot{y}o} = \sigma_{\dot{z}o}$ = 3.0 meters/sec (35)

The initialization procedure is covered in a separate appendix.

The extrapolated dynamic model error covariance matrix is

$$Q' = G\,Q\,G^T = \begin{bmatrix} \frac{T^4 \sigma_{a_x}^2}{4} & 0 & 0 & \frac{T^3 \sigma_{a_x}^2}{2} & 0 & 0 \\ 0 & \frac{T^4 \sigma_{a_y}^2}{4} & 0 & 0 & \frac{T^3 \sigma_{a_y}^2}{2} & 0 \\ 0 & 0 & \frac{T^4 \sigma_{a_z}^2}{4} & 0 & 0 & \frac{T^3 \sigma_{a_z}^2}{2} \\ \frac{T^3 \sigma_{a_x}^3}{2} & 0 & 0 & T^2 \sigma_{a_x}^2 & 0 & 0 \\ 0 & \frac{T^3 \sigma_{a_y}^2}{2} & 0 & 0 & T^2 \sigma_{a_y}^2 & 0 \\ 0 & 0 & \frac{T^3 \sigma_{a_z}^2}{2} & 0 & 0 & T^2 \sigma_{a_z}^2 \end{bmatrix} \quad (36)$$

The linearized measurement matrix is

$$H(k) = \frac{\partial}{\partial \underline{X}} \underline{h}(\underline{X}(k)) \Big|_{\underline{X}(k) = \hat{\underline{X}}(k^-)}$$

$$H(k) = \begin{bmatrix} \frac{2(x-x_1)}{d_1} & \frac{2(y-y_1)}{d_1} & \frac{2(z-z_1)}{d_1} & 0 & 0 & 0 \\ \frac{2(x-x_2)}{d_2} & \frac{2(y-y_2)}{d_2} & \frac{2(z-z_2)}{d_2} & 0 & 0 & 0 \\ \frac{2(x-x_3)}{d_3} & \frac{2(y-y_3)}{d_3} & \frac{2(z-z_3)}{d_3} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (37)$$

$$x = \hat{x}(k^-) \quad (38)$$

$$y = \hat{y}(k^-) \quad (39)$$

$$z = \hat{z}(k^-) \quad (40)$$

$$d_i = \left((\hat{x}(k^-) - x_i)^2 + (\hat{y}(k^-) - y_i)^2 + (\hat{z}(k^-) - z_i)^2 \right)^{1/2} \quad (41)$$

APPENDIX B INITIALIZATION COMPUTATION

The mathematics underlying the initialization computation are presented in this appendix. The initial position of the aircraft is computed by iterating to a solution of 4 simultaneous distance equations in 3 unknowns (the coordinates to be determined).

Problem Statement:

Given 4 measured ranges to known beacon locations, find the initial cartesian coordinates of the aircraft.

Ri = ith measured range (42)

Di = ith computed range (43)

Xi, Yi, Zi = ith beacon coordinates (44)

X, Y, Z = aircraft position (to be determined) (45)

$$D_i = \left((X-X_i)^2 + (Y-Y_i)^2 + (Z-Z_i)^2 \right)^{1/2} \quad (46)$$

Solve the problem by Newton's Technique (reference 10)

Define

$$F_i(X, Y, Z) = R_i^2 - D_i^2 = R_i^2 - (X-X_i)^2 - (Y-Y_i)^2 - (Z-Z_i)^2 = 0 \quad (47)$$

Linearize Fi (x,y,z) about an estimated position (x,y,z):

$$F_i(X,Y,Z) = F_i(\hat{X},\hat{Y},\hat{Z}) + \frac{\partial F_i}{\partial X}(X-\hat{X}) + \frac{\partial F_i}{\partial Y}(Y-\hat{Y}) + \frac{\partial F_i}{\partial Z}(Z-\hat{Z}) \quad (48)$$

Define:

$$\Delta X = (X-\hat{X}), \quad \Delta Y = (Y-\hat{Y}), \quad \Delta Z = (Z-\hat{Z}) \quad (49)$$

Evaluating partial derivatives at the estimated position yields.

$$\left.\frac{\partial F_i}{\partial X}\right|_{X=\hat{X}} = -2(\hat{X}-X_i) \quad (50)$$

$$\left.\frac{\partial F_i}{\partial Y}\right|_{Y=\hat{Y}} = -2(\hat{Y}-Y_i) \quad (51)$$

$$\left.\frac{\partial F_i}{\partial Z}\right|_{Z=\hat{Z}} = -2(\hat{Z}-Z_i) \quad (52)$$

Next Define:

$$A = 2\begin{bmatrix} \hat{x}-x_1 & \hat{y}-y_1 & \hat{z}-z_1 \\ \hat{x}-x_2 & \hat{y}-y_2 & \hat{z}-z_2 \\ \hat{x}-x_3 & \hat{y}-y_3 & \hat{z}-z_3 \\ \hat{x}-x_4 & \hat{y}-y_4 & \hat{z}-z_4 \end{bmatrix} \quad (53)$$

$$\underline{\Delta} = (\Delta x \quad \Delta y \quad \Delta z)^T \quad (54)$$

$$\underline{F} = \begin{bmatrix} F_1(\hat{x}, \hat{y}, \hat{z}) \\ F_2(\hat{x}, \hat{y}, \hat{z}) \\ F_3(\hat{x}, \hat{y}, \hat{z}) \\ F_4(\hat{x}, \hat{y}, \hat{z}) \end{bmatrix} \quad (55)$$

Then

$$\underline{F} = [A]\,\underline{\Delta} \quad (56)$$

Since A is nonsquare and the measurements are noisy, the $(\Delta x, \Delta y, \Delta z)^T$ vector will be found by a least squares approach.

$$[A^{\#}] = [(A^T A)^{-1} A^T] \quad (57)$$

$$\underline{\Delta} = [A^{\#}]\,\underline{F} \quad (58)$$

This result can be used to iterate to a solution for x y,z. The procedure follows:

Initial guess: $\hat{x} = 0 \qquad \hat{y} = 0 \qquad \hat{z} = \text{baro. alt.}$

Step 1. Compute the Di.

Step 2. Compute $Fi = Ri^2 - Di^2$

Step 3. Compute $\underline{\Delta} = (A\#)\ \underline{F}$

Step 4. Compute new estimate of x,y,z

$$\hat{x} = x + \Delta x \qquad (59)$$

$$\hat{y} = y + \Delta y \qquad (60)$$

$$\hat{z} = z + \Delta z \qquad (61)$$

Step 5. Test for covergence. If converged, proceed to Step 6. If more than 5 iterations have occurred, get new measurements and begin again. If not converged, iterate from Step 1 with the new estimates.

Step 6. Use x,y,z to initialize the Kalman filter.

This is an application of Newton's technique for solving non-linear equations. The functions are quadratic in 3 unknowns, hence 3 iterations are necessary (theoretically) to converge to the solution. Five iterations have been allowed in the software to accommodate numerical problems which might inhibit the rate of convergence.

APPENDIX C EARTH CURVATURE ALTITUDE CORRECTION

The geometric relationship between height above a tangent plane (z) to the earth, and height above sea level (h) is shown in figure C.1.

Rs = Slant Range to origin of tangent plane

Re = radius of earth

Z = height above tangent plane

h = height (barometric) above sea level

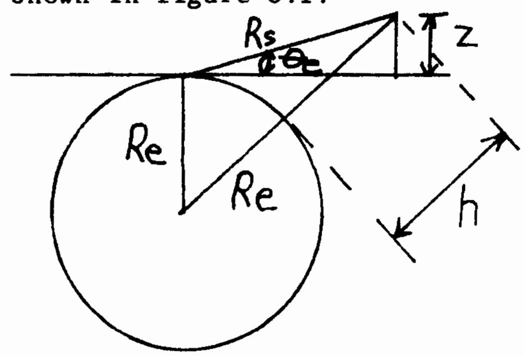


Figure C.1

From the law of Cosines, the following relationship is derived.

$$(Re+h)^2 = Re^2 + Rs^2 - 2\,Re\,Rs\,\cos(90+\theta_e) \qquad (62)$$

$$Rs^2 = X^2 + Y^2 + Z^2 \qquad (63)$$

$$\cos(90+\theta_e) = -\sin(\theta_e) = \frac{-Z}{Rs} \qquad (64)$$

Substituting 64 into 62 yields

$$(Re+h)^2 = Re^2 + R_h^2 + 2\,Z\,Re \qquad (65)$$

Multiplying out 65 yields

$$Re^2 + 2hRe + h^2 = Re^2 + Rs^2 + 2\,Z\,Re \quad (66)$$

Solving for Z in 66 yields

$$Z = h + \frac{h^2}{2Re} - \frac{Rs^2}{2Re}$$

After substituting (63) into (67) and rearranging terms, the result is:

$$Z = h - \frac{X^2 + Y^2}{2Re} + \frac{h^2 - Z^2}{2Re} \qquad (68)$$

For approaching aircraft, the third term on the right hand side of equation 68 is negligible, thus dropping the term yields the final result.

$$Z \approx h - \frac{X^2 + Y^2}{2Re} \qquad (69)$$

Table C.1 shows numerical values for the correction term at various distances from the origin of the tangent plane.

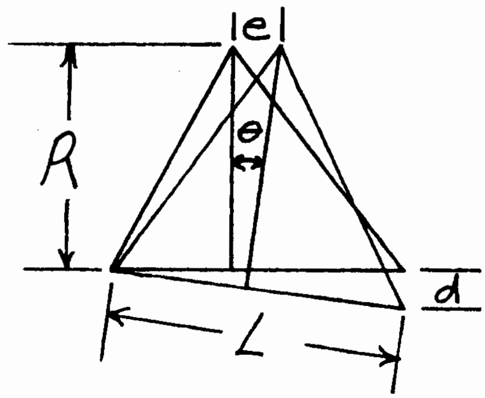
Table C.1 Altitude Correction Factors

Distance (nm)	Correction (ft)
10	-88
6	-32
2	-3.5
1	- .9

APPENDIX D BASELINE BEACON POSITIONING ERROR ANALYSIS

The following mathematical analysis points out the need for accurate surveys of beacon sites in order to eliminate errors due to angular mis-alignment of baseline geometry. Consider the situation shown in figure D.1.

Figure D.1 Baseline Mis-Alignment Geometry



A small error d has occurred in the position of one baseline beacon, resulting in the lateral position error e of the aircraft at a range of R from the baseline. The angular error relationship is

$$e = R \operatorname{Tan}(\theta) \qquad (70)$$

$$\operatorname{Tan}(\theta) = \frac{d}{L} \qquad (71)$$

Substituting 71 into 70 yields

$$e = \frac{Rd}{L} \qquad (72)$$

At six miles from a baseline one mile long, a 5 meter position error results in 30 meters lateral position error.

The situation described actually occurred, and a correction was made based on the result in equation 72. The correction factor becomes,

$$d = \frac{eL}{R} \qquad (73)$$

Figure E.1 shows the lateral error in the second trace. The error is about 48 meters at a distance of 6 nm. The baseline length is approximately 1 nm. The correction factor becomes

$$d \cong \frac{48(1)}{6} = 8 \text{ meters} \qquad (74)$$

The x coordinate of one baseline beacon was adjusted by this amount, with dramatic results (see figure E.2).

APPENDIX E GRAPHICAL DATA

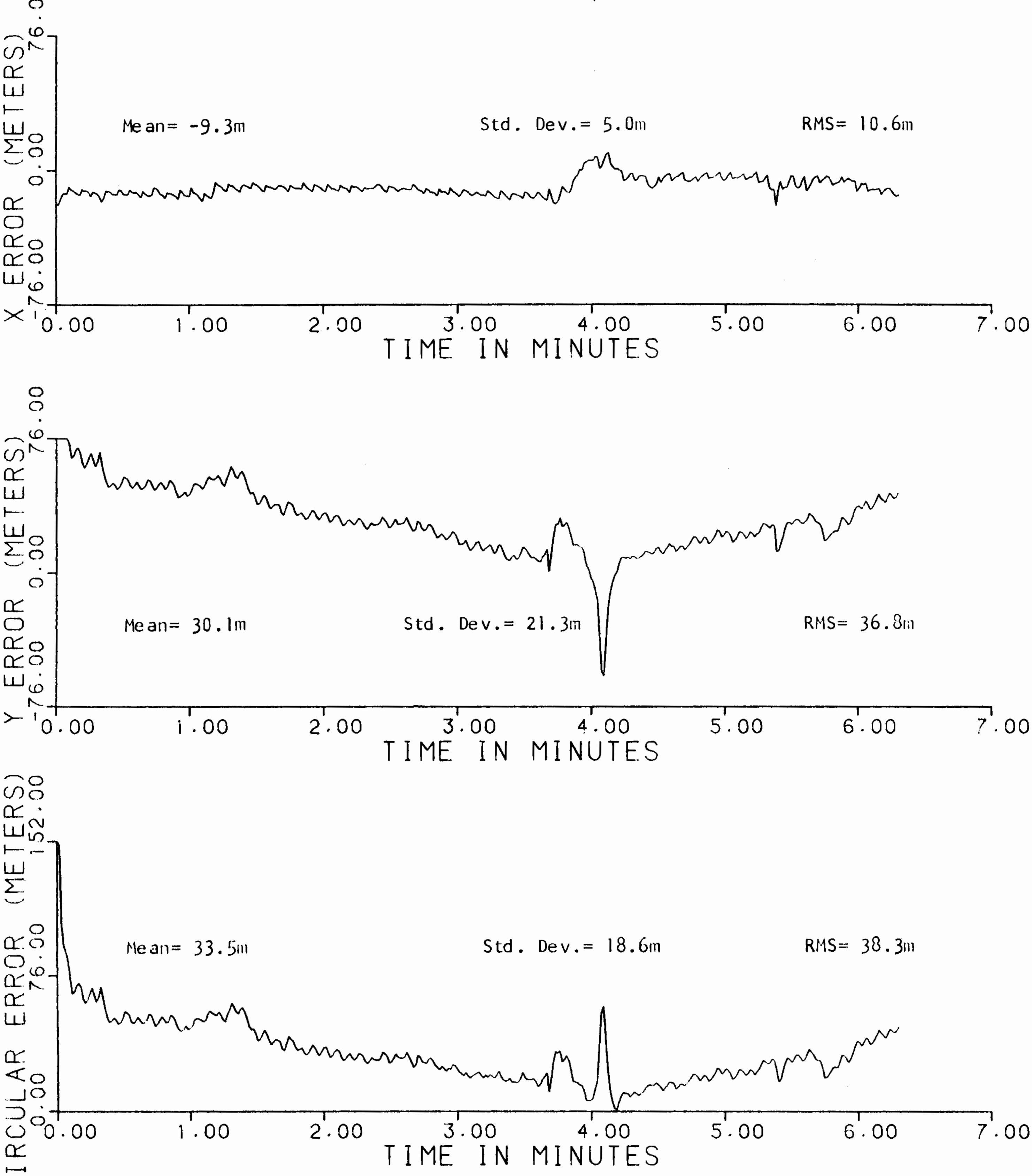
The data presented in this Appendix is slightly different from that presented elsewhere in the following respect. The radar tracking data for the approaches to runway 4 was reprocessed on the Honeywell 66/60 computer with a less stringent radar elevation cutoff angle data validity criteria in order to provide a long data segment for qualtitative analysis.

The lessening of the radar data validity criteria may introduce some doubts. However, it is argued that the lateral 2D position statistics incorporated in the plots are comparable to results based on radar data processed with more stringent validity criteria; and qualitatively, the error traces are typical of the system performance.

This longer segment presented here brackets the segment presented as run 8 elsewhere in this report. Since the horizontal axis is time since start of processing, it is important to note that these plots include a 6 nm final approach, a missed approach procedure transitioning to an outbound leg 5 nm in length.

FIGURE E.1

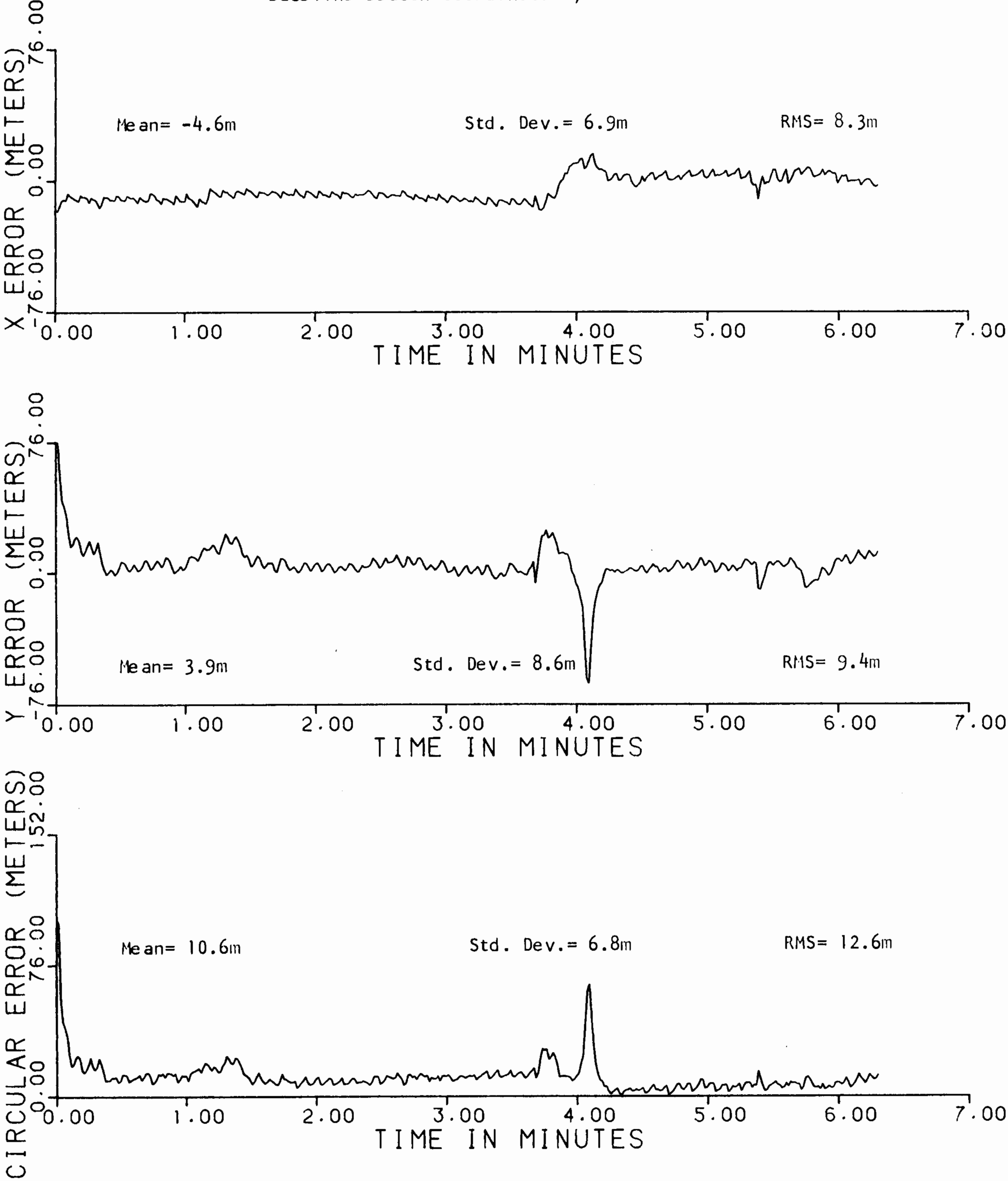
Position Errors Prior to Correcting
Baseline Beacon Coordinate By 8 Meters



Data Processed By FAA Technical Center
Atlantic City, N.J. 08405

FIGURE E.2

Position Errors After Correcting
Baseline Beacon Coordinate by 8 Meters



Data Processed By FAA Technical Center

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