FAA-74-13

REPORT NO. FAA-RD-74-116

SIMULATION OF WAKE VORTICES DESCENDING IN A STABLY STRATIFIED ATMOSPHERE

Clinton E. Brown
Karl Kirkman

JULY 1974

FINAL REPORT

Prepared for
U.S. DEPARTMENT OF TRANSPORTATION
FEDERAL AVIATION ADMINISTRATION
SYSTEMS RESEARCH AND DEVELOPMENT SERVICE
Washington DC 20591

Technical Report Documentation Page

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
FAA-RD-74-116		

4. Title and Subtitle
SIMULATION OF WAKE VORTICES DESCENDING IN A STABLY STRATIFIED ATMOSPHERE

5. Report Date
July 1974

6. Performing Organization Code

7. Author(s)
Clinton E. Brown and Karl Kirkman

8. Performing Organization Report No.
DOT-TSC-FAA-74-13

9. Performing Organization Name and Address
HYDRONAUTICS, Incorporated*
7210 Pindell School Road
Laurel MD 20810

10. Work Unit No. (TRAIS)
FA405/R4115

11. Contract or Grant No.
DOT-TSC-694

12. Sponsoring Agency Name and Address
U.S. Department of Transportation
Federal Aviation Administration
Systems Research and Development Service
Washington DC 20591

13. Type of Report and Period Covered
Final Report
Sept. 1973-March 1974

14. Sponsoring Agency Code

15. Supplementary Notes
*Under contract to: U.S. Department of Transportation
Transportation Systems Center
Kendall Square
Cambridge MA 02142

16. Abstract

An experimental simulation of aircraft wake vortices descending in a stable atmosphere has shown that the atmospheric stability stops the downward movement and in some cases produces a subsequent rebound. The tests were carried out in a large ship model basin using a rectangular planform wing. Lift coefficients of 0.4 and 1.0 were selected and stable atmospheric conditions were obtained by temperature (density) stratification of the towing basin. Test conditions corresponding to Vaisala-Brund periods of ∞, 109, and 51 seconds were obtained. The model parameters and stability conditions covered the most extreme cases to be expected in full scale flight.

17. Key Words
Wake Vortices
Aircraft Wakes
Stratified Atmosphere
Vaisala-Brunt Period
Vortex Decay
Vortex Pair

18. Distribution Statement

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price
Unclassified	Unclassified	40	

PREFACE

The purpose of the present study is to investigate the behavior of aircraft wake vortices as they descend through a stratified atmosphere. The phenomenon is simulated in a water towing tank where the environment can be controlled. Flight test data is ambiguous as it is difficult to separate stratification atmospheric effects. There exist a number of inconsistent theoretical models describing wake descent in a stably stratified atmosphere; the present work was undertaken to give some experimental support to the modeling by indicating how vortices are affected as they descend into a stratified medium.

CONTENTS

ILLUSTRATIONS

ILLUSTRATIONS (Cont'd)

SUMMARY

An experimental simulation of aircraft wake vortices descending in a stable atmosphere has shown that the atmospheric stability stops the downward movement and in some cases produces a subsequent rebound. The tests were carried out in a large ship model basin using a rectangular planform wing. Lift coefficients of 0.4 and 1.0 were selected and stable atmospheric conditions were obtained by temperature (density) stratification of the towing basin. Test conditions corresponding to Vaisala-Brunt periods of ∞, 109 and 51 seconds were obtained. The model parameters and stability conditions covered the most extreme cases to be expected in full scale flight.

INTRODUCTION

The hazard presented to aircraft from wake vortices generated by other aircraft is under study by the Department of Transportation in an attempt to eliminate the danger or to at least locate the position of the vortices so that they may be avoided. In this latter endeavor, it is known that atmospheric conditions such as wind, turbulence, and stability can all play a role in causing deviations from the normal trajectories found in neutrally stable quiescent air. Experimental evidence in flight and various theoretical analyses of the descent of vortices in a stratified medium appear to be in disagreement. This situation led to the need for controlled laboratory experiments which might serve as a basis for selecting and improving on existing theoretical work. To meet this need a series of tests were carried out in the HYDRONAUTICS Ship Model Basin (HSMB®) using a high-aspect-ratio-rectangular wing as the generator of the wake-vortex pair to be studied. A neutral and two additional levels of the atmospheric stability were established by heating the water. The greatest stability produced corresponded to values as high or higher than those measured or anticipated in the free atmosphere. The wing was tested at several lift coefficients, positive and negative, and the behavior of the dyed-wake system was recorded by stereoscopic photography. Data obtained in these tests provide information on vortex pair trajectories and dispersion as well as the paths of selected streamlines away from the vortex cores.

DISCUSSION OF THE PROBLEM

Aircraft wings with smooth spanwise load distributions produce a wake system composed of two twin distributions of shed vorticity. These systems usually have two high-rotary-velocity centers or cores and can be hazardous to other aircraft that encounter them. In a neutrally stable quiescent atmosphere the vortex system pair is known to descend steadily for large distances before finally dispersing, Reference 1. The dispersal under these quiescent atmospheric conditions is generally assumed to occur as a result of turbulence generated from the kinetic energy of the vortex field, Reference 2. Actually, atmospheric effects or instabilities more often account for the ultimate dispersal and a recent summary, Reference 3, discusses the various influences which include:

(1) atmospheric turbulence
(2) Crow instability
(3) vortex bursting
(4) stability of the atmosphere

The present paper is concerned primarily with the latter effect. It should be noted that in the atmosphere stable conditions are found generally with rather low levels of turbulence, hence the descent of vortices in a stable atmosphere can be treated as an isolated phenomenon.

The atmosphere is in a stable stratified condition when air transferred from one level to another produces buoyancy forces tending to return the air to its original position. This condition arises when the temperature variation with height becomes more positive than the normal (negative) "adiabatic" gradient; an isothermal state being a rather stable situation. A measure of the stability can be given by the natural frequency or the reciprocal of the period of simple air mass oscillations in the system. The period (known as the Vaisala-Brunt period) is given by the expression

$$P = \left(\frac{2\pi}{N}\right)^{*} = 2\pi \left[\frac{(\gamma-1)g^2}{\gamma RT} + \frac{g}{T}\frac{dT}{dz}\right]^{-\frac{1}{2}} \qquad [1]$$

where γ is the ratio of specific heats, g the acceleration of gravity, R the gas constant, T the absolute temperature and z the vertical height (positive upwards). Under stable isothermal conditions ($dT/dz = 0$) the Vaisala-Brunt period is approximately 280 seconds at an altitude of 30,000 feet. Measurements over a fog bank in Hanford, Washington, Reference 4, have shown even more stable conditions and calculations using the measured data and Equation [1] indicate a Vaisala-Brunt period as low as 106 seconds. In this case, the second term of Equation [1] dominates.

As discussed by Lissaman, Reference 3, et al, there should be dynamic similarity between full-scale flight conditions and model test conditions when the ratio of two characteristic times is matched. The first of these is the Vaisala-Brunt period and the second is the vortex characteristic time which is formed from the wingspan divided by the vortex downward speed (in the unstratified condition) and may be expressed as

---

* Note that N was used by Lissaman (Reference 3) et al, and is the Vaisala-Brunt angular frequency.

$$T = \frac{b}{V}\left[\frac{4\pi A}{C_L}\left(\frac{b'}{b}\right)^3\right] \qquad [2]$$

where b is the wingspan, V the flight speed, A the wing aspect ratio, C_L the flight lift coefficient and (b'/b) is the ratio of rolled up vortex pair spacing to the span. This latter ratio is determined by the wing spanload distribution. When models are geometrically similar to the full scale aircraft, the values of (b'/b) will be preserved if the model is not stalled and if the model Reynolds number is suitably high (preferably greater than 10^6). The characterizing parameter is thus defined as (T/P) and is simply 2π times Lissaman's (et al) parameter (NT)*:

$$T/P = \frac{2bA(b'/b)^3\left[\frac{(\gamma-1)g^2}{\gamma RT} + \frac{g}{T}\frac{dT}{dz}\right]^{\frac{1}{2}}}{V\ C_L} \qquad [3]$$

When in a model experiment the value of (T/P) is matched with that at full scale, the velocity ratios and flow patterns will be similar. In the tests reported herein two stable atmospheric conditions were simulated; the most extreme was chosen together with the aircraft parameters to provide values of (T/P) higher than would be anticipated in flight except under most unusual conditions.

---

* Lissaman (Reference 3) et al, introduce only the vortex circulation in place of the aircraft parameters; however, their expression for the isothermal Vaisala-Brunt angular frequency is in error and should be $N = \sqrt{\gamma-1}\ g/\sqrt{\gamma RT}$ instead of $\sqrt{\gamma}\ g/\sqrt{\gamma RT}$.

Values of (T/P) for two cases of practical interest are 0.1 and 0.26. Both are obtained for an aircraft like the Boeing 747 having a wing span of about 66 meters and an aspect ratio of 7. The first value represents a high altitude cruise at $C_L = 0.4$ in an isothermal atmosphere and the second, a landing approach condition with $C_L = 1.2$ in a highly stable atmosphere as might be found just above a ground fog. The present tests encompass a range of T/P values from zero to about 2.0.

In the theoretical literature reviewed in Reference 3, analyses have been made predicting both a coming together of the vortex pair and a spreading apart. No attempt has been made in the present report to correlate the data with the several available theories. However, as will be discussed later, the gross behavior of the vortices appears to contain features of all of the prominent theories.

DESCRIPTION OF MODEL AND EQUIPMENT

Experiments were conducted in a ship model towing tank having a length of 94 meters, a width of 7.3 meters, and a depth of 3.8 meters. This facility has been used previously for measurements of trailing vortex systems behind a model of the Boeing 747, Reference 5; however, in the present tests the wing model was an untapered, unswept, 12-percent thick airfoil with a span of 0.85 meters, a chord of 0.098 meters and an aspect ratio of 8.7. It was mounted in a sting which in turn was supported by a surface piercing strut system. Provisions were included for varying the angle of attack and the depth of submergence of the wing/strut system. A sketch of the arrangement is shown in Figure 1a and a photograph of the wing used is seen in Figure 1b.

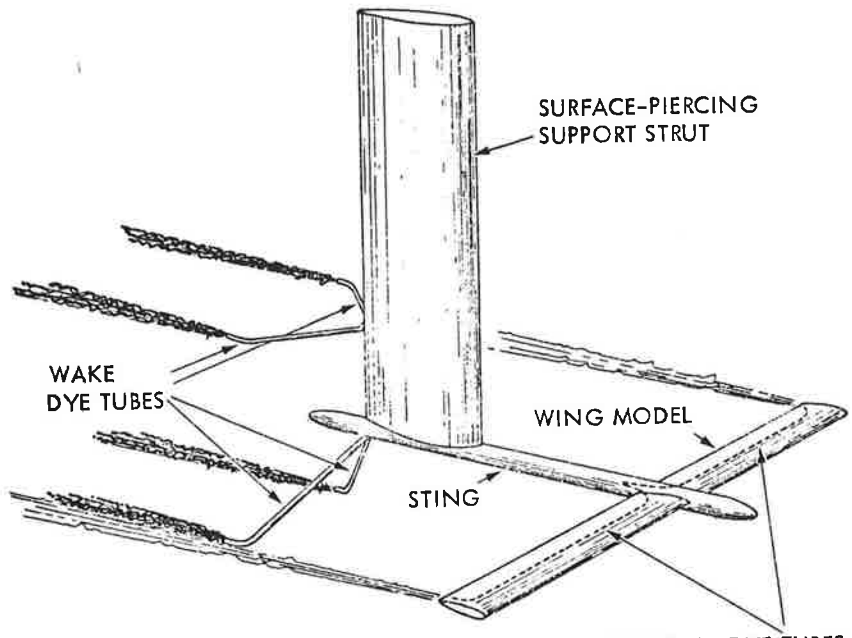


FIGURE 1a - SKETCH OF MODEL, DYE TUBES AND SUPPORT SYSTEM

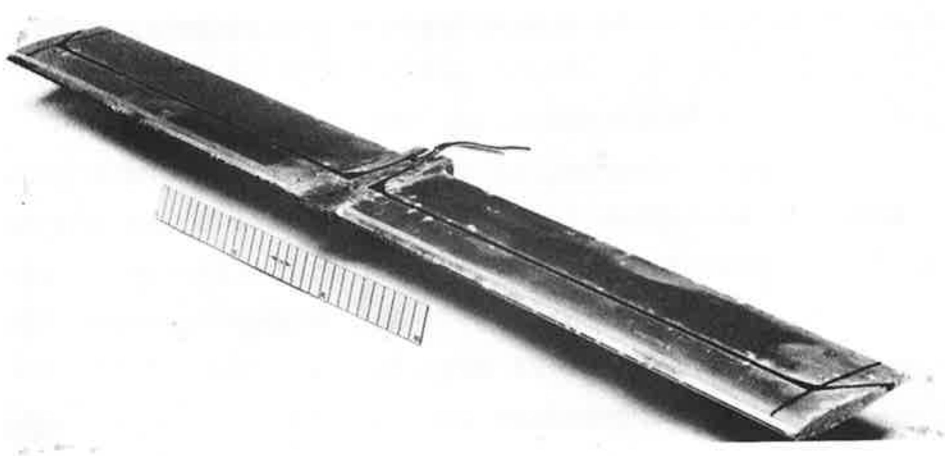


FIGURE 1b - PHOTOGRAPH OF MODEL WING

(Note: Scale shown in photograph is in centimeters)

A dye system was installed for marking the vortex paths and other streamlines in the wake flow. Twin dye tubes embedded in the wing provided a means of dyeing the tip vortices, while four adjustable external tubes attached to the support strut allowed for the emission of dye into the wake field. These tubes may be seen in Figure 1a.

Test data were obtained by a system of background grids, high intensity underwater lights, and ganged underwater cameras arranged as shown in Figure 2. This apparatus was utilized for stereoscopically viewing the location of vortices and streamlines in the vortex wake flow field. Several 35mm Nikon F cameras with FTN heads were used and all data were taken using color film and either yellow or red dye.

The tank facility includes a thermal stratification system, Reference 6, for which the important characteristics are summarized here: the system consists of a heater having a 3×10^6 BTU/hr capacity, a pair of warm-water distribution-manifolds, a single suction manifold, a recirculating pump, and a mixer (Figure 3). Output temperature of the heater can be regulated. All three manifolds are suspended from individual, precision-regulated winches, to facilitate close depth adjustment. Water is withdrawn from the HSMB at one depth and returned, after heating, at another depth. The choice of manifold depth is dictated by the portion of the temperature profile being generated. The mixer is used to carry out vertical redistribution of the temperature profile by local transport of adjacent layers through mixing of masses from the newly laid down, roughly-graduated, fluid levels; i.e., the initial sharp temperature gradients are reduced to smoother and less steep ones. Temperature gradients were measured by a vertically

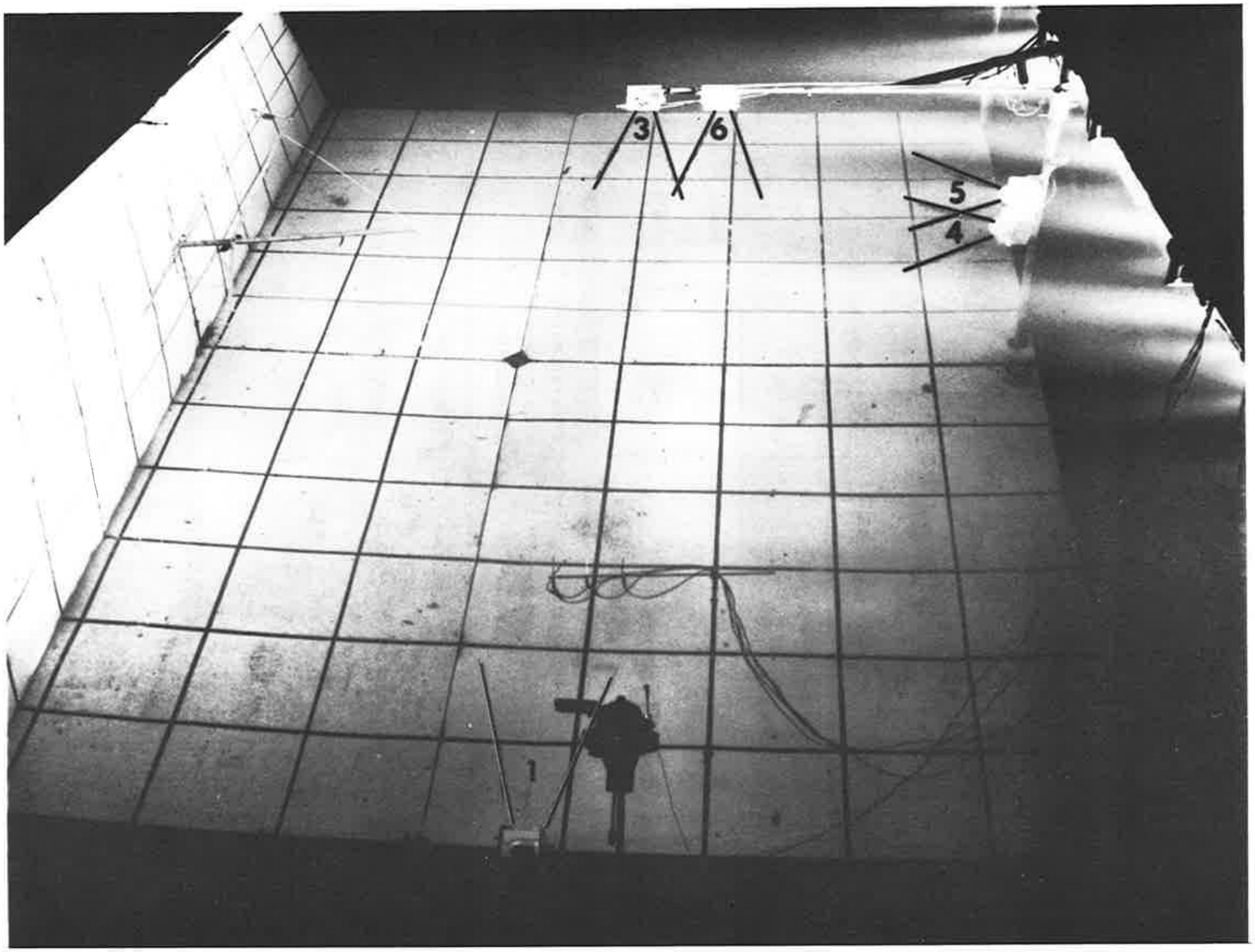


FIGURE 2 – VIEW OF STEREO GRIDS AND CAMERAS
LOOKING AFTER THE AIRCRAFT

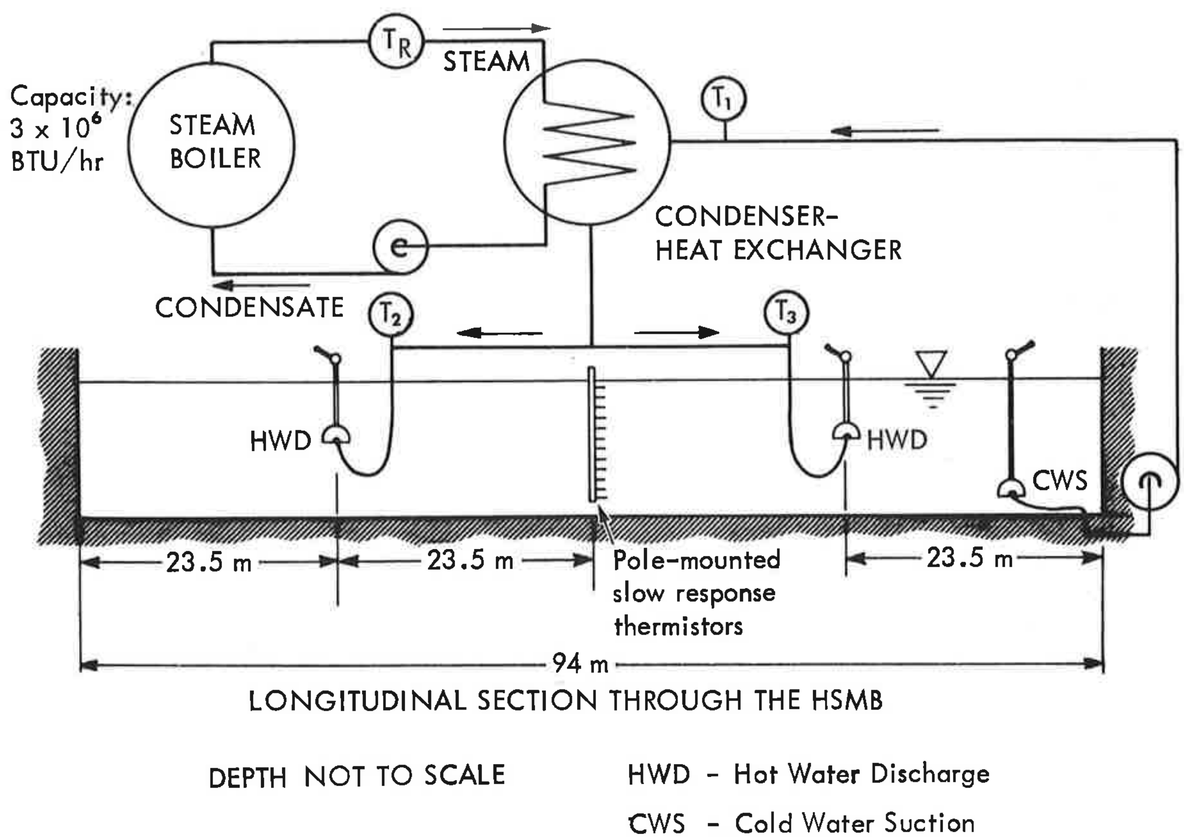


FIGURE 3 - STRATIFICATION GEAR FOR THE HSMB

traversing fast response thermistor element and recorded as a function of vertical positions by an X-Y plotter.

TEST PROGRAMS AND PROCEDURES

The matrix of test variables explored is given in Table 1. In general, tests were conducted at combinations of three lift coefficients, three thermal/density gradients, and several model speeds.

Preparations for an individual test run consisted of constructing the desired gradient in the towing basin and setting the wing to the proper depth and angle of attack. Just prior to starting the towing carriage the test section was illuminated and, with the passage of the wing through the center of the test section, a time sequence of stereoscopic underwater still photographs was initiated and continued until the dye traces were dissipated. A two-second interval between pictures was used for most of the sequences. A suitable waiting time was allowed between test runs for stabilization of the thermal gradient. The thermal gradients established in the towing basin are shown in Figure 4.

REDUCTION AND PRESENTATION OF TEST DATA

The primary objective of the tests was to determine the path and behavior of the wing tip vortices and several other stream tubes in the system. The position in the test section of any visible dye trace was obtained from the intersection of two lines each of which was drawn from the viewing camera position to the point on the wall or floor grid against which the dye trace appeared. The accuracy of this technique is mainly limited by the definition or resolution of the dye trace rather than by the optical system or photographic resolution of the grids.

TABLE 1

Summary of Model Test Program

Test No.	Run No.	V_M m/sec	C_L	Nominal Gradient	T/P (Eq.[3])
1	1	1.5	0.4	0	0
	2	1.5	0.4	0	0
	3	4.6	0.4	0	0
	4	4.6	0.4	0	0
	5	4.6	0.4	0	0
	6	4.6	0.4	0	0
	7	4.6	0.4	0	0
2	1	1.5	-0.4	0	0
	2	4.6	-0.4	0	0
3	1	1.5	1.0	0	0
	2	3.0	1.0	0	0
	3	3.0	1.0	0	0
	4	3.0	1.0	0	0
	5	3.0	1.0	0	0
	6	4.6	1.0	0	0
4	1	1.5	0.4	(1.1°C/Meter)	1.0
5	1	1.5	-0.4	(1.1°C/Meter)	1.0
6	1	1.5	1.0	(1.1°C/Meter)	0.42
	2	4.6	1.0	(1.1°C/Meter)	0.13
7	1	1.5	0.4	(4.4°C/Meter)	2.1
	2		0.4	(4.4°C/Meter)	2.1
	3		0.4	(4.4°C/Meter)	2.1
	4		0.4	(4.4°C/Meter)	2.1
	5	4.6	0.4	(4.4°C/Meter)	0.70
8	1	1.5	-0.4	(4.4°C/Meter)	2.1
9	1	1.5	1.0	(4.4°C/Meter)	0.84

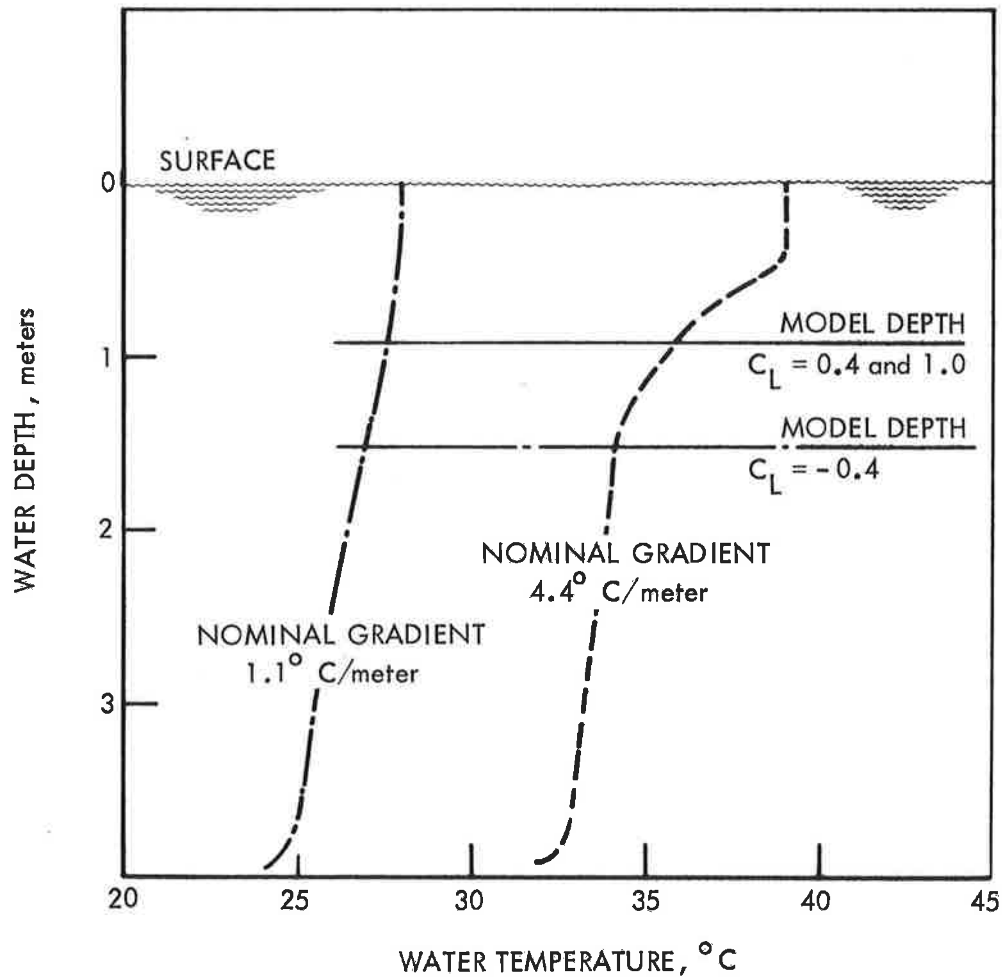


FIGURE 4 - TEMPERATURE DISTRIBUTIONS ESTABLISHED FOR THE TESTS

Actually the fuzziness of the dye markings at long times following model traverse was such that two observers viewed the photos and agreed on a position before the data point was used. In some cases, poor quality photos were obtained because of air bubble coatings on the lenses and pictures from alternate cameras were used to verify the results.

Vortex trajectories for the case of neutral stability are presented in Figures 5a, 5b, 6a and 6b, plotted in linear coordinates and in logarithmic coordinates. Data include measurements at three velocities and two lift coefficients. Some data were also obtained with the angle of attack set at negative values. Reference to Table 1 will also illustrate the variations used. There is noticeable scatter in the data which is clearly the result of turbulent eddies which had not completely decayed following a previous run. In a few cases (not plotted) one tip vortex actually failed to descend while the other descended at twice normal speed. It is presumed that under these conditions a central residual eddy must have been present in the towing basin. Much of the data represents only one tip vortex because the second vortex was not within the field of view of the stereo-camera system. Nevertheless the average curve drawn by eye through the data points is believed to be a reasonably correct representation of the motion.

In Figures 7, 8, and 9 are presented the results under various stratified conditions. The values of the test T/P ratios are shown on the figures. The mean curves of Figures 5a and 6a are drawn for comparison of the stratified and unstratified conditions. Also shown is the simple two vortex theory for the unstratified case obtained from the equation

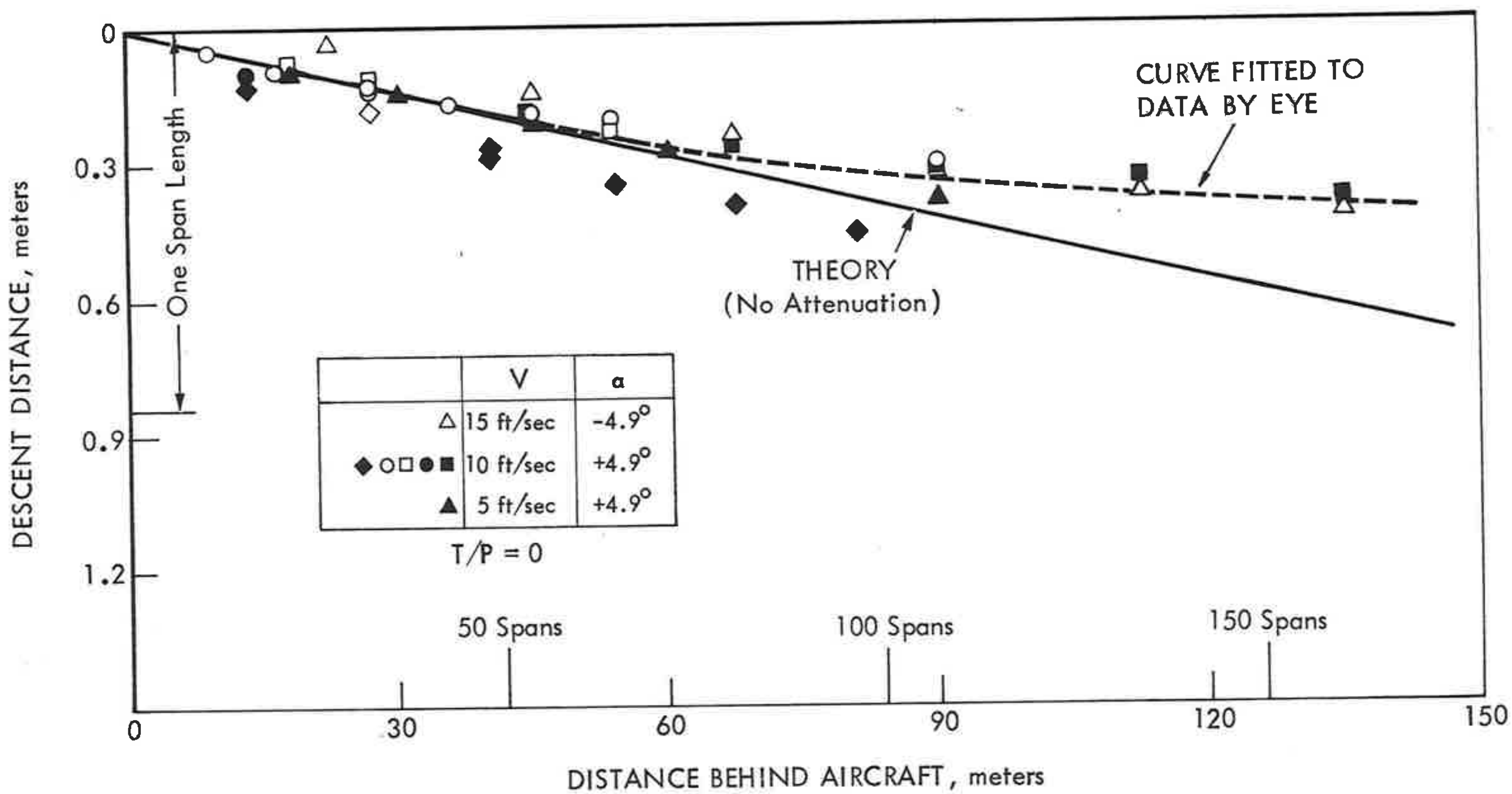


FIGURE 5a - VORTEX TRAJECTORIES FOR A RECTANGULAR WING IN AN UNSTRATIFIED ATMOSPHERE (C_L = 0.4)

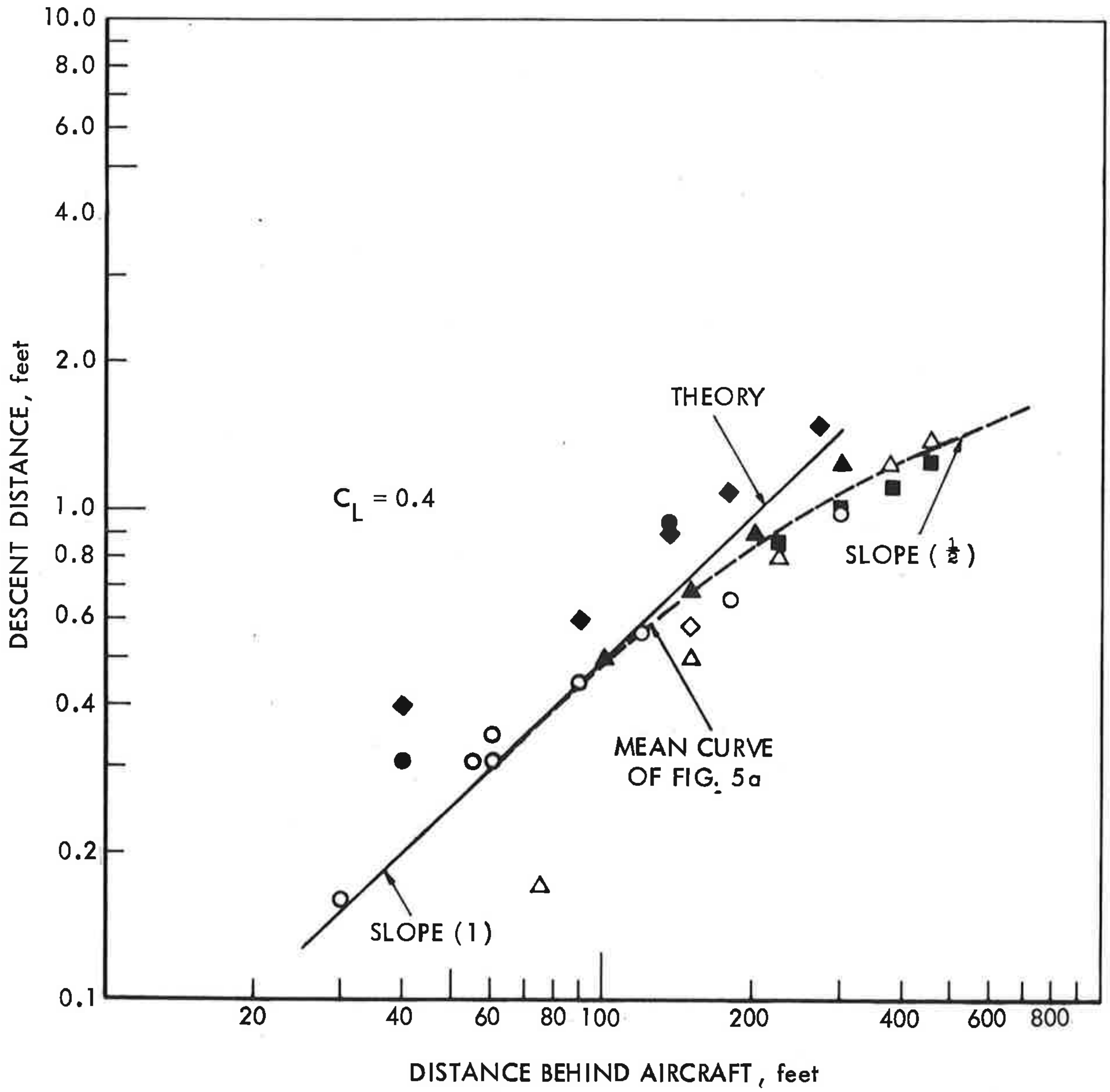


FIGURE 5b - VORTEX TRAJECTORIES FOR A RECTANGULAR WING IN AN UNSTRATIFIED ATMOSPHERE ($C_L = 0.4$)

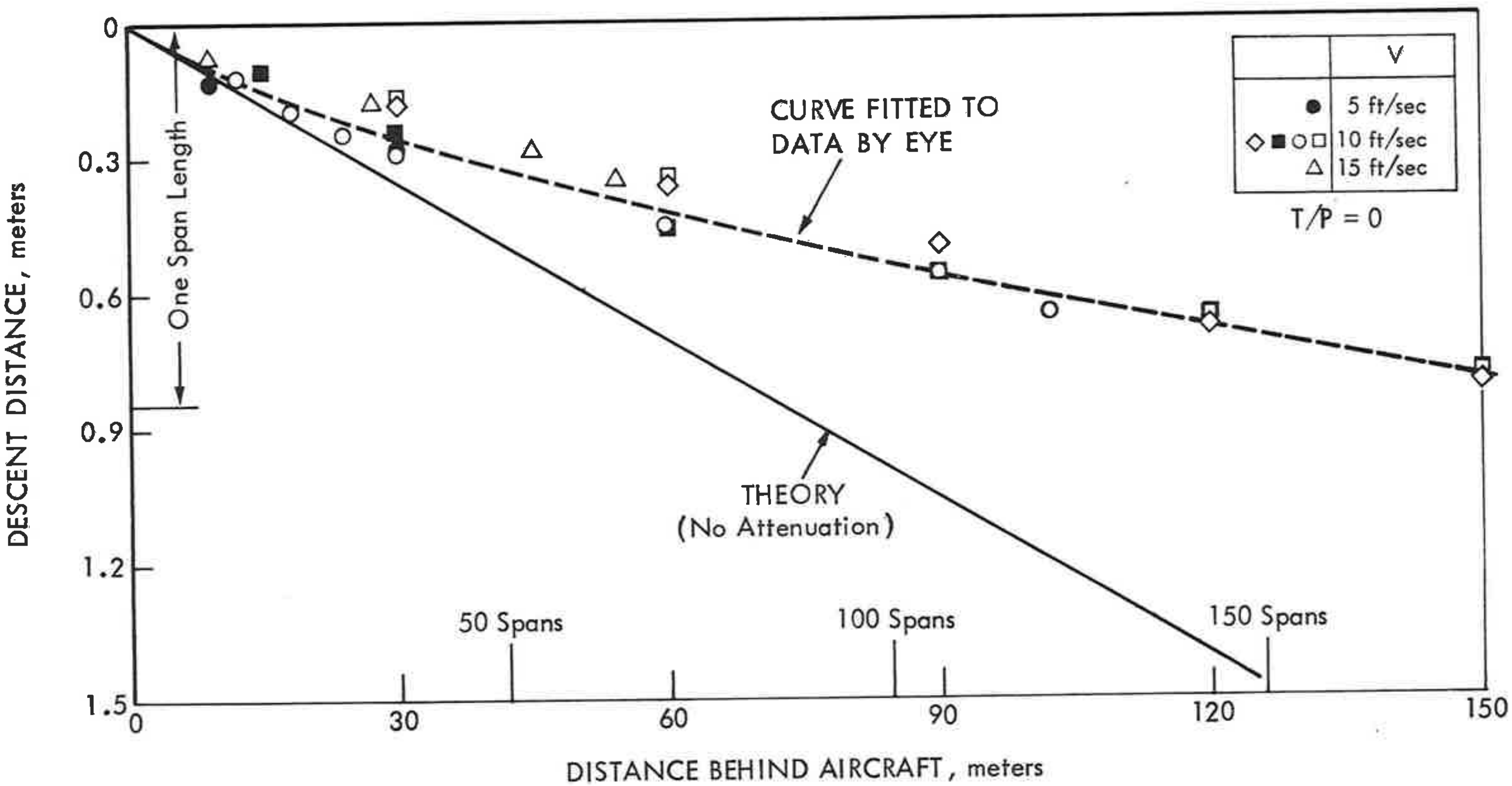


FIGURE 6a - VORTEX TRAJECTORIES FOR A RECTANGULAR WING IN AN UNSTRATIFIED ATMOSPHERE (C_L = 1.0)

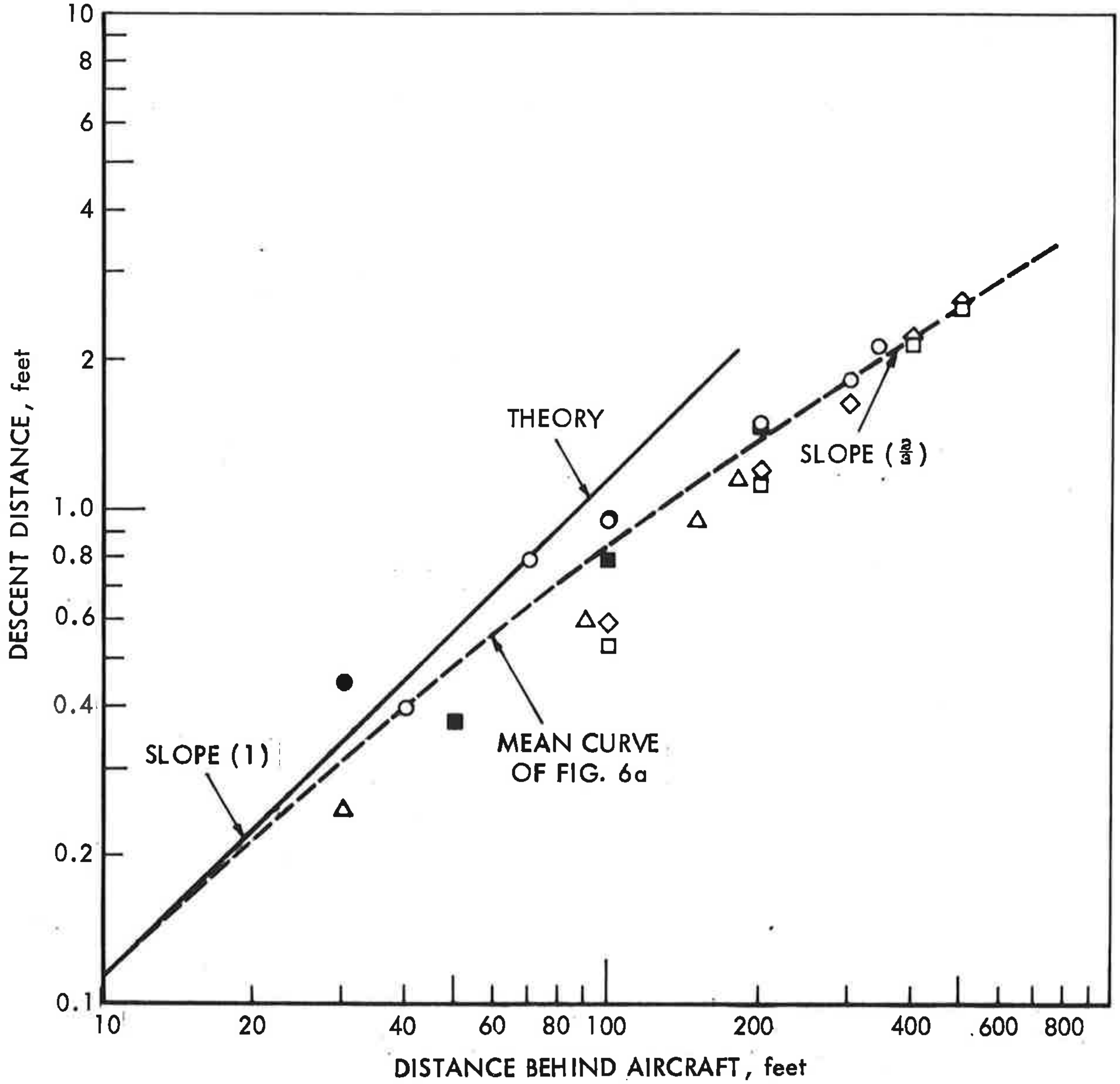


FIGURE 6b - VORTEX TRAJECTORIES FOR A RECTANGULAR WING IN AN UNSTRATIFIED ATMOSPHERE ($C_L = 1.0$)

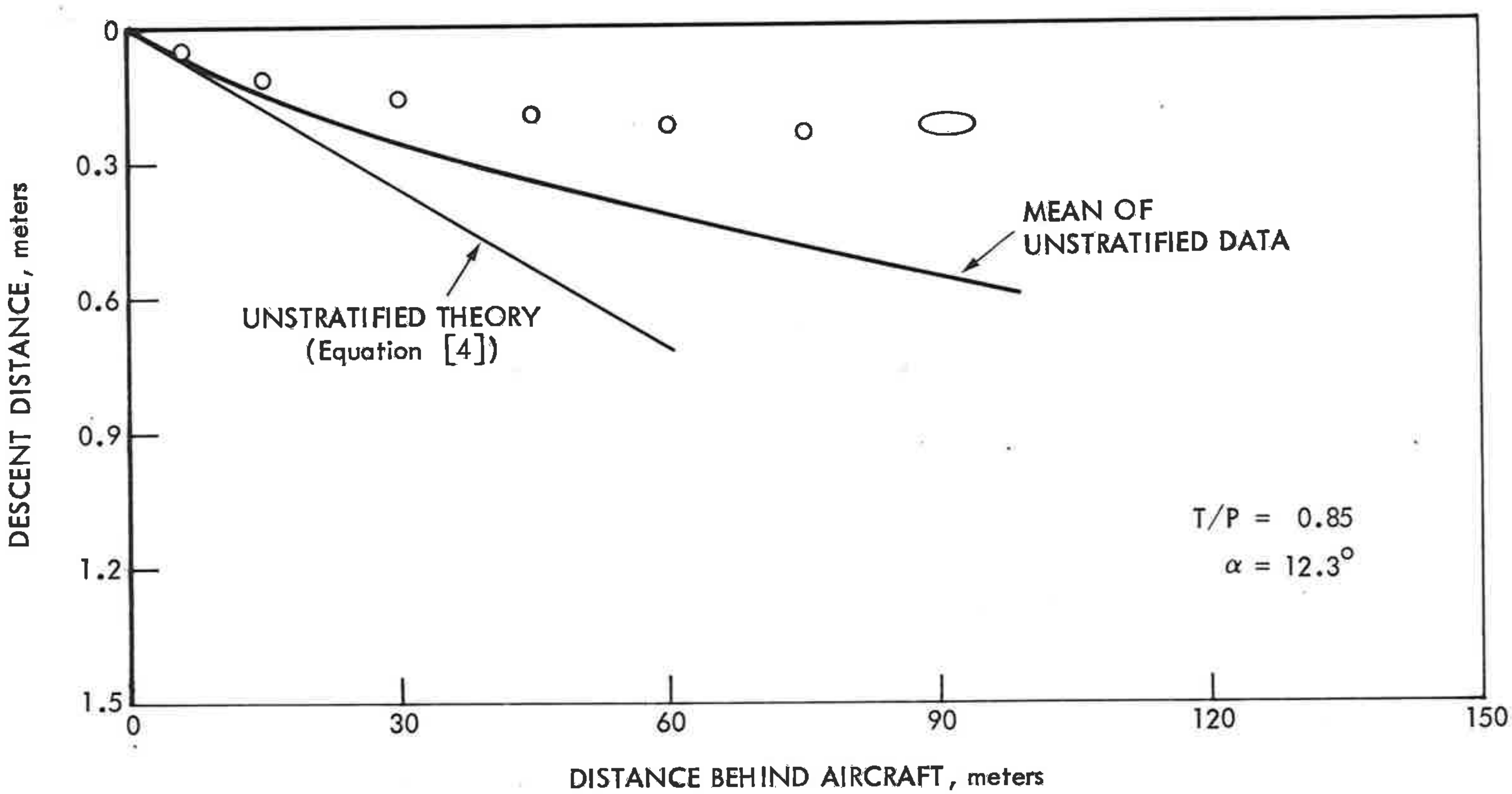


FIGURE 7 – COMPARISON VORTEX TRAJECTORIES UNDER STRATIFIED AND UNSTRATIFIED ATMOSPHERIC CONDITIONS (C_L = 1.0, T/P = 0.85)

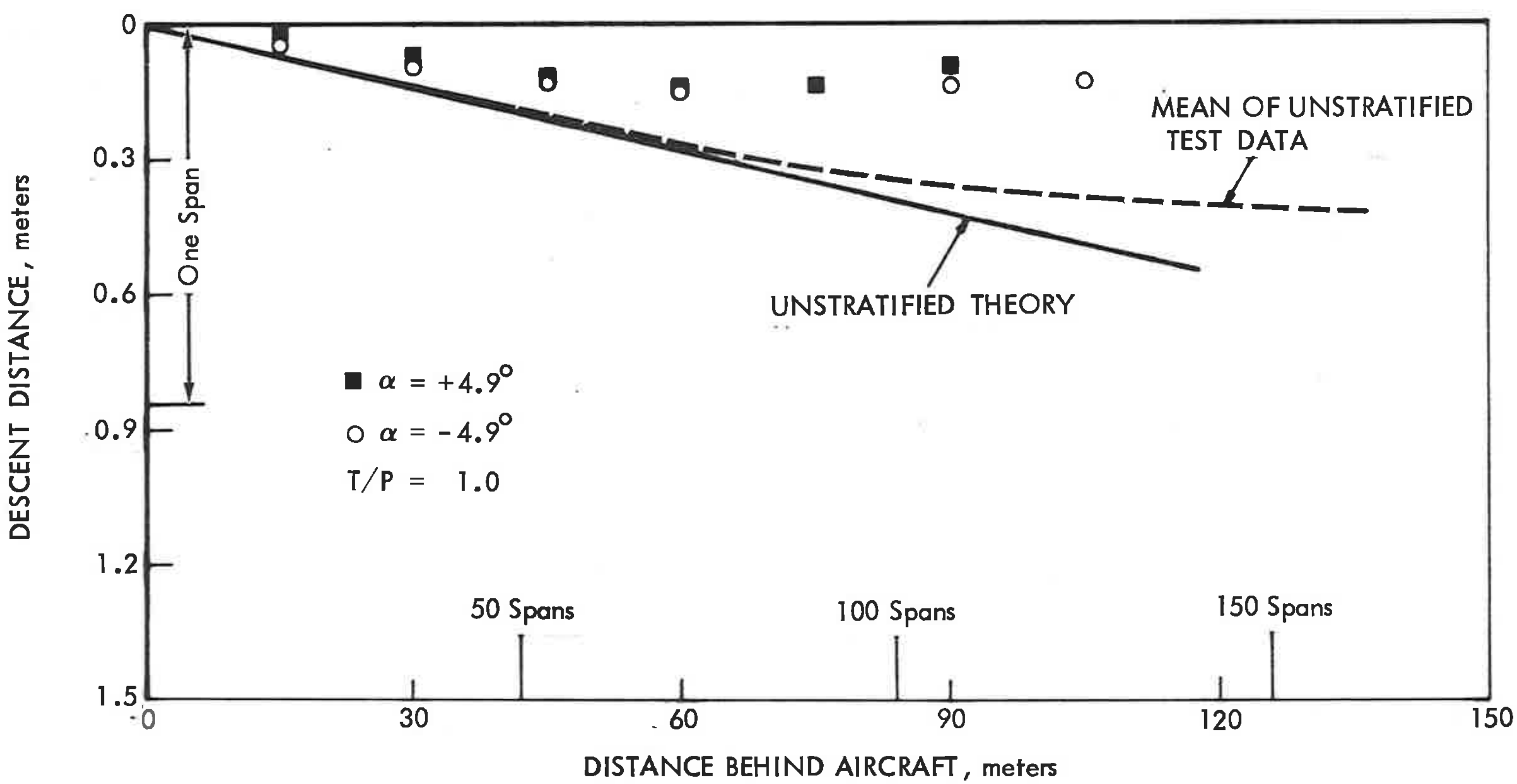


FIGURE 8 - COMPARISON OF VORTEX TRAJECTORIES UNDER STRATIFIED AND UNSTRATIFIED ATMOSPHERIC CONDITIONS ($C_L = 0.4$, T/P = 1.0)

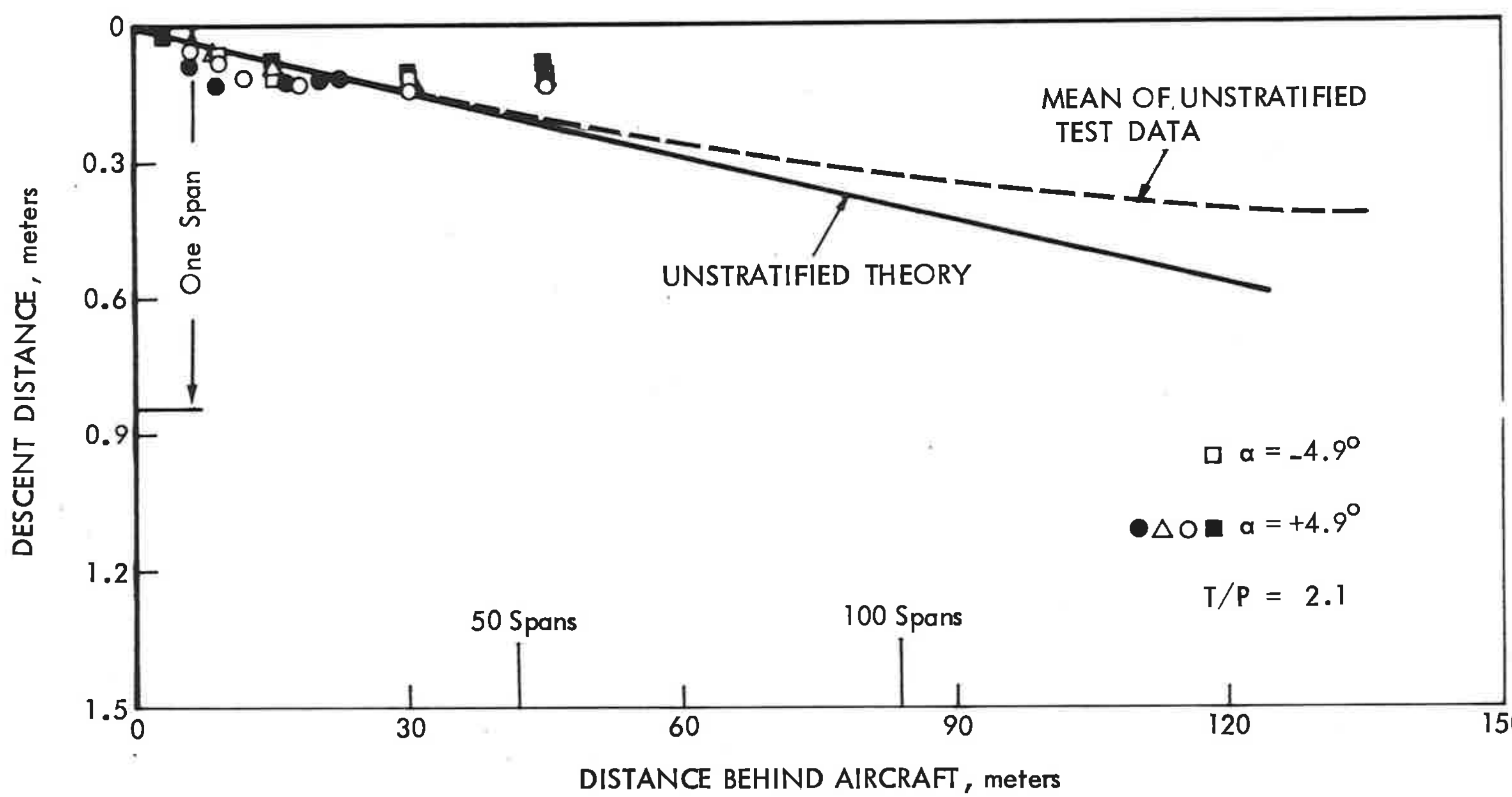


FIGURE 9 - COMPARISON OF VORTEX TRAJECTORIES UNDER HIGHLY STRATIFIED AND UNSTRATIFIED ATMOSPHERIC CONDITIONS (C_L = 0.4, T/P = 2.1)

$$\frac{\text{Descent distance}}{\text{Distance behind the aircraft}} = \left(\frac{C_L}{4\pi A}\right)\left(\frac{b}{b'}\right)^2 \qquad [4]$$

The proper value for (b/b') depends on the wing planform, twist, and camber but for the test wing is 1.12.

DISCUSSION OF RESULTS

The vortex trajectory data for zero stratification presented in Figures 5 and 6 show that the vortex pair descent speed initially agrees with simple vortex pair theory but gradually reduces with distance behind the aircraft. This behavior has been described by Maxworthy, Reference 7, and further analyzed by Lissaman, et al, Reference 3. The reduction in speed of the system is considered to result from a loss of vorticity into the "wake" of the primary vortex pair and some loss of vorticity by cancellation at the plane of symmetry. The existence of a similarity solution characterized by the descent distance being proportional to the time or distance behind the aircraft raised to some fractional power is not very evident from the log plots of Figures 5b and 6b. The data at long times or distance could conceivably be fitted by a curve with a slope of from 1/2 to 2/3, but these exponents are not among those generally obtained from similarity arguments, Reference 3. It is possible of course that the data would ultimately become asymptotic to an x^n line but if so it occurs at downstream distances beyond those of the present tests and in a region of little practical interest. During the tests, it was generally noted that vortex "bursting" would occur at short distances behind the wing. Bursting in this case might better be expressed as "fringing"; that is, the dyed core of the vortex would suddenly develop a pattern of dye in annular rings about the original core. The pattern was usually cyclic along the vortex axis and often the dyed fluid of the rings would appear to have slow motion following the aircraft. A photograph of a typical "bursting" vortex core is shown in Figure 10. The bursting provides a mechanism for rapid dispersion of vorticity which appears to precede the shedding of vorticity into the wake and the coincident slowing of the vortex descent speed. Dye pictures were not able to prove the existence of vorticity leakage into the wake, however some runs did seem to show dyed patches of fluid stretched out above the vortex pair.

Figures 7, 8 and 9 present data for values of T/P of 0.85, 1.0 and 2.1, respectively. It can be seen that in each case the atmospheric stability causes a leveling off of the vortex system and for the two weaker cases the downward velocity at no point exceeds the simple two vortex theoretical value. For the case of extreme stability (T/P = 2.1) there appears to be an increased downward speed at the beginning of the descent followed by a rapid leveling off and a slight rebound. This behavior is in agreement with the predictions of Scorer and Davenport, Reference 8, but their theoretical analysis involved the assumption that T/P was very small and neglected the effects of vorticity loss discussed above for the case of zero stratification. It is possible that the theoretically predicted increase in descent speed for small T/P values was masked in the tests by the rapid dispersion and loss of vorticity produced by the bursting phenomenon. Nevertheless, the data make it very clear that the effect of stable atmospheric stratification is to cause the vortex pair to level-off and possibly rise a bit even before ultimate disintegration of the system. After long periods (greater than 90 seconds) the traces of the original dyed core

t = 0 Seconds

t = 6 Seconds

FIGURE 10 - PHOTOGRAPHS ILLUSTRATING BURSTING PHENOMENON
$C_L = 1.0$, SPEED - 10 ft/sec

regions are found to be distributed laterally over distances as great as three spans. Figure 11 attempts to show the time history of a vortex pair under the most extreme stratified conditions. The solid lines and arrow heads indicate the path of the tip vortices and the dashed lines and the arrow heads show the path and disposition of the streamlines between the vortex pair. These streamlines were traced by means of red dye put out from two of the tubes shown in Figure 1a. As can be determined from Figure 11, the port tube (right side on the sketch) was located about 1/3 of a semi-span off the centerline and about one semi-span below the wing. The starboard dye tube was located 1/3 semi-span off the centerline and 1/3 semi-span above the wing. Initially the starboard tip vortex moved rapidly down and inward but leveled off and moved upward a bit by 30 seconds. The port tip vortex moved down and slightly outboard then leveled off also. After 30 seconds both vortices started to spread laterally with a slight upward drift in the outboard regions. The final appearance of the system was one of a laterally spread out or feathered dye pattern as can be seen in Figure 12 taken from Camera No. 1 below the vortex system looking upstream after the departing aircraft model. The streamlines as observed did not follow the usual (unstratified) circular paths about the vortex centers. Actually both dye traces moved down slightly and then remained relatively stationary but growing in size, as would be expected. The lower dye trace was seen to move upward again slightly after its initial descent but no tendency for the dye to spread out around the vortex cores is seen. A sequence of photos is presented in Figure 13 which illustrates the failure of the highly stratified case to produce the expected rotation of the streamlines. The lower set of pictures show the dye traces at time

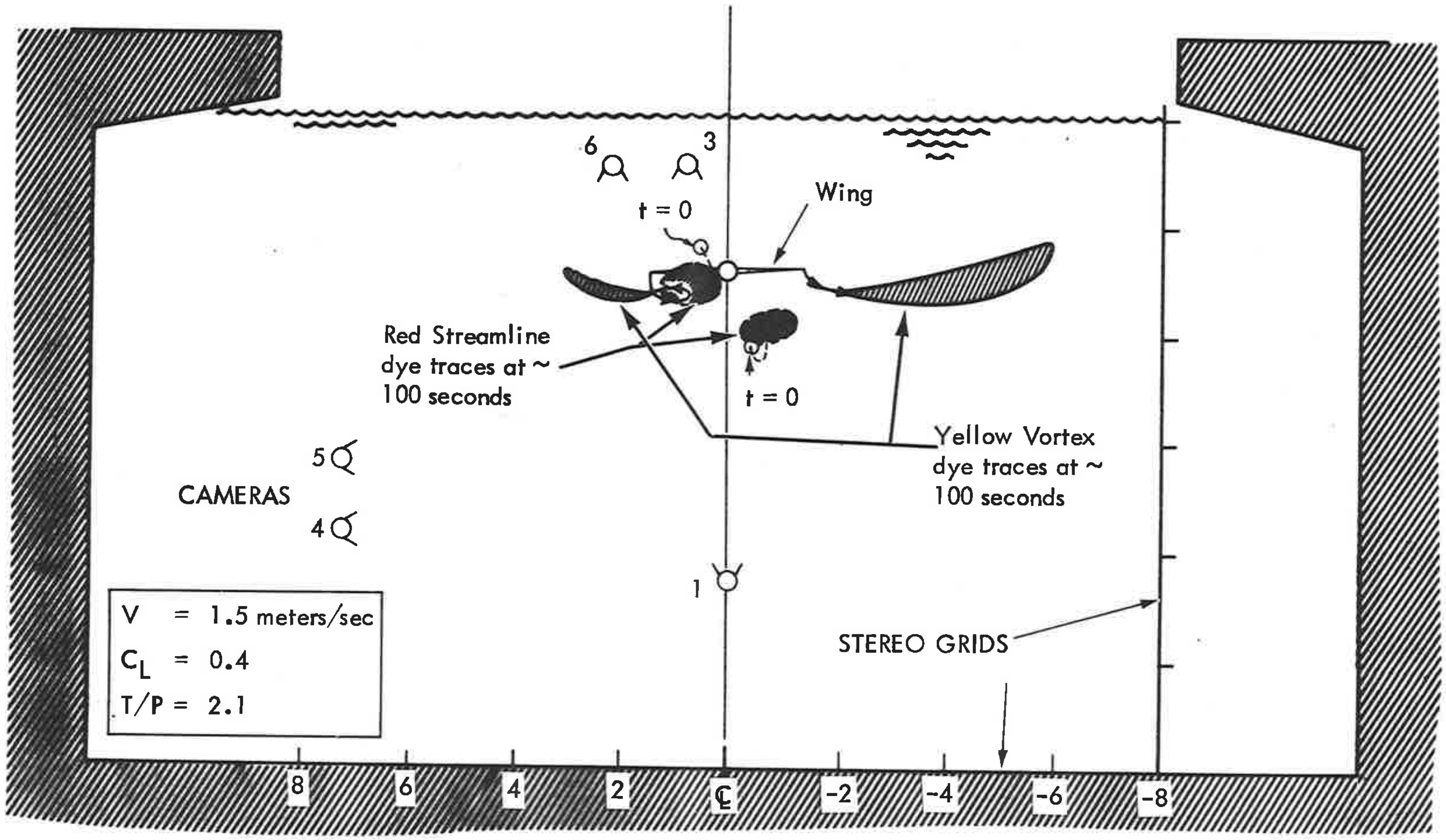
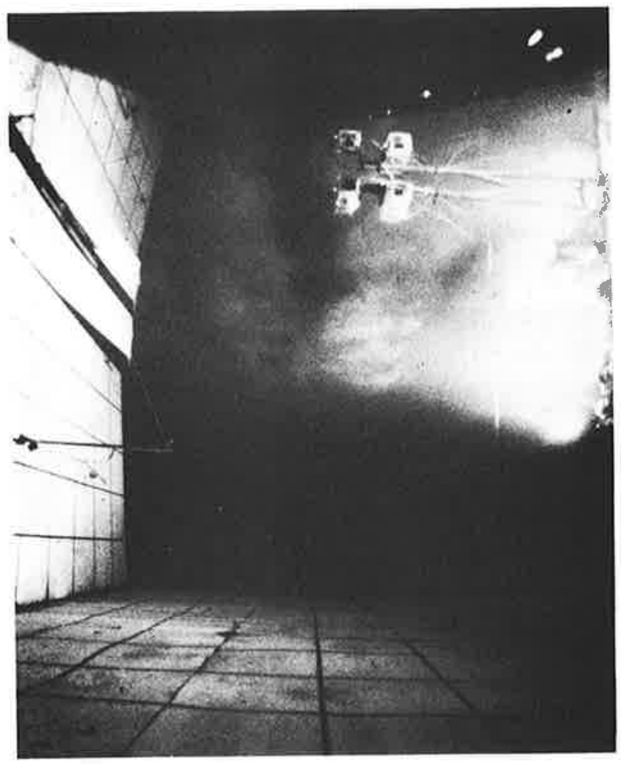


FIGURE 11 - VORTEX AND STREAMLINE TRACES UNDER HEAVY STRATIFICATION



T/P = 2.1

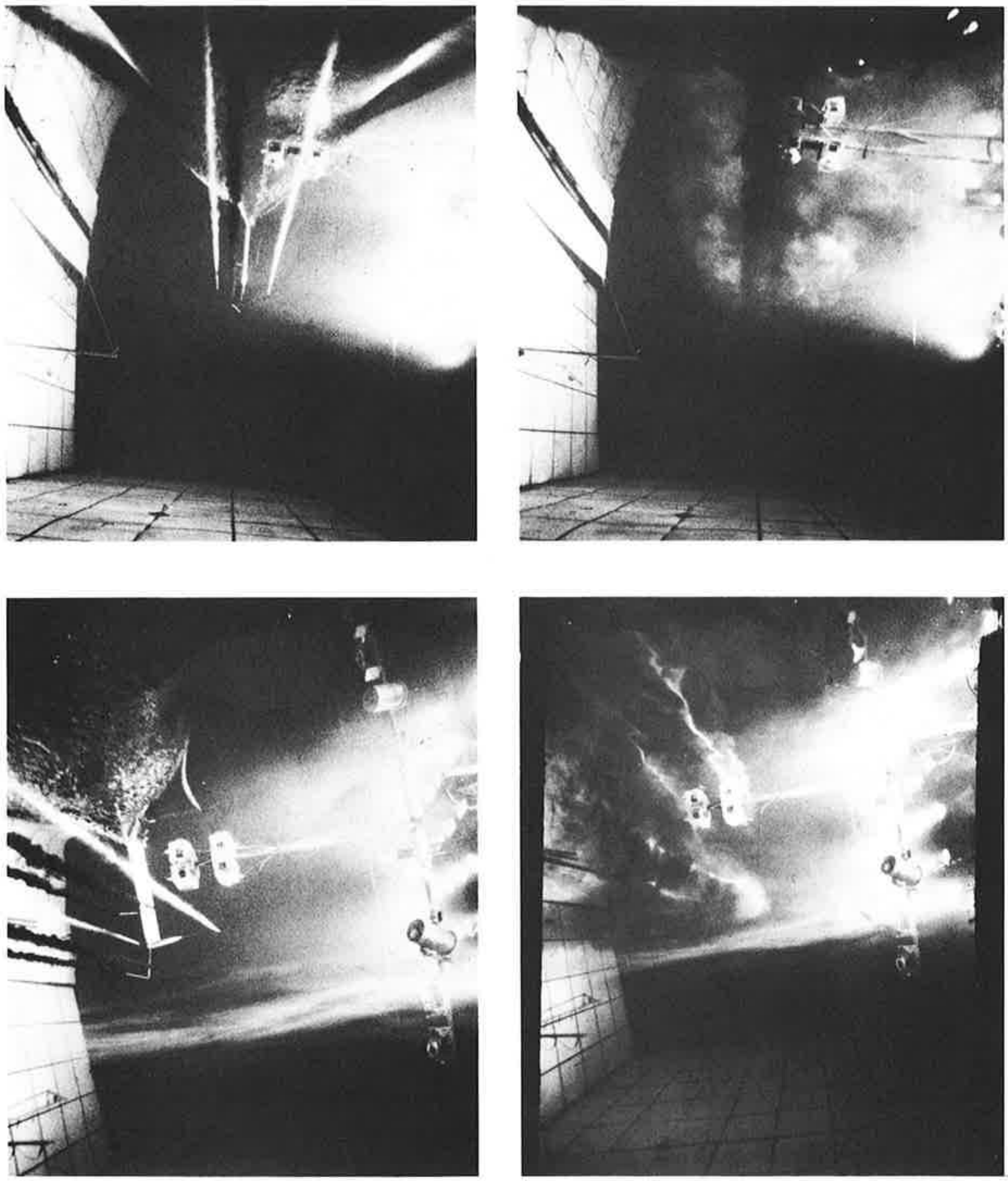
C_L = 0.4

SPEED - 5 ft/sec

FIGURE 12 - DYE PATTERNS FOR HEAVILY STRATIFIED FLOW AT 100 SECONDS

T/P = 0.84

T/P = 0.42



t = 0 t = 30 Sec.

C_L = 1.0 SPEED - 5 ft/sec

FIGURE 13 - DYE PATTERNS SHOWING EFFECT OF STRATIFICATION ON STREAMLINES NEAR THE VORTEX PAIR

zero and 30 seconds for the moderate values of T/P of 0.42. It can be seen that the dark (red) dye from the streamline tubes is moving out, up and around the vortex cores by 30 seconds. Note that the upper wavy light lines are the reflection of the vortices in the free surface. In the upper pair of photos at the higher value of the stratification parameters (T/P = 0.84) the dark dye clearly remains near the centerline of the system and is not drawn around the vortex.

CONCLUSIONS AND RECOMMENDATIONS

The data obtained show that the effect of stable stratification of the atmosphere is to cause a descending aircraft wake to level off some distance below the generating aircraft and in some cases rebound somewhat before a general lateral dispersion of the vortex system takes place. Evidence is presented to confirm theoretical predictions that at least initially the downward speed of the vortex pair may be larger than that predicted by simple theory in unstratified flow. Actual behavior of the vortex system may involve the relative magnitude of two effects, the atmospheric stability and the intensity of atmospheric turbulence. The latter appears to be responsible for vortex bursting (a rapid radial dispersion of vorticity) which in turn causes the vortex system to slow its downward speed and to dissipate. In the present tests vortex bursting was nearly always present whereas in previous tests at larger scale bursting was not commonly observed. These differences are not believed to be a Reynolds number effect but rather an indication that the turbulent energy levels of the towing basin represents a greater disturbance to the present test model than to larger models previously tested.

The test program was not originally planned to include variation in ambient turbulence levels and therefore the present results are limited in this regard. However, the data do bring out the significant interaction of atmospheric stability and turbulence and it is evident that additional study should be given to cases in which turbulent intensities are measured and varied over larger ranges than were attained in the present investigation. Further information would also be of value concerning the effect of atmospheric stability on vortex wakes at high lift coefficients for wings with realistic planforms and flap arrangements.

REFERENCES

1. Prandtl, L. "Tragflugeltheorie" I and II reprinted in Prandtl, L. and Betz, A., Vier Abhandlungen zur Hydrodynamik und Aerodynamik, Gottingen, 1927, Edwards Bros. Inc., 1943.

2. Squire, H. B., "The Growth of a Vortex in Turbulent Flow," ARC 16,666, 1954, Ministry of Supply, Aeronautical Research Council, London.

3. Lissaman, P.B.S., Crow, S. C., MacCready, P.B., Jr., Tombach, I.H. and Bate, E. R., Jr., "Aircraft Vortex Wake Descent and Decay under Real Atmospheric Effects," FAA-RD-73-120,1973.

4. Fleagle, R.E., Parrot, W. H. and Barad, M. L., "Theory and Effects of Vertical Temperature Distribution in Turbid Air," Journal of Meteorology 9, pp. 53-60, 1952.

5. Kirkman, K. L., Brown, C. E. and Goodman, Alex, "Evaluation of Effectiveness of Various Devices for Attenuation of Trailing Vortices Based on Model Tests in a Large Towing Basin," NASA CR-2202, December 1973.

6. Blaes, Viggo "Main Design Features and Prove-Out of the Thermal Stratification System Installed in the Ship Model Basin of HYDRONAUTICS, Incorporated," HYDRONAUTICS, Incorporated Technical Report 7015-4, March 1973.

7. Maxworthy, T., "The Structure and Stability of Vortex Rings," J. Fluid Mech. 55, 609-629, 1972.

8. Scorer, R. S. and Davenport, L. J., "Contrails and Aircraft Downwash," J. Fluid Mech. 43, 451-464, 1970.

REPORT OF INVENTIONS APPENDIX

After a diligent review of the work performed under this contract, no new innovation, discovery, improvement or invention was made.