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Enhanced Network Models for Multimodal Resiliency

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16. Abstract
This report investigates how "operating capacity" can be meaningfully defined in multimodal maritime freight systems. Shipping channels and ports are complex systems that interact deeply, and the capacities of individual components may differ from the overall capacity of these systems. Accurately determining the operating capacity of a port aids stakeholders in making informed decisions about large-scale infrastructure investments and resource allocation. We present a novel queuing theory-based operating capacity model for computing capacities for waterway, import, and export processes. Our proposed model estimates the operating capacity of a port system by accounting for the interactions between waterways, terminals, and landside infrastructure without the need for a simulation. However, when used with a simulation, our model can compare capacities across different scenarios, thereby helping compare investment alternatives. We demonstrate the utility of the proposed method using data for the Port of Houston. The results from our study suggest that the proposed model is a viable method for estimating port operating capacity.

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EXECUTIVE SUMMARY

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1 INTRODUCTION

An estimated 90% of global trade is conducted via water (*1*), highlighting the role of maritime systems in the global supply chain. Given this critical role, the ability to evaluate the current and future operating capacity of port systems is needed to inform large-scale investment decisions and long-term infrastructure expansion plans (*2*). Unlike roads, no equivalent version of the Highway Capacity Manual (*3*) exists to define "capacity" for maritime systems. Past studies have attempted to define the capacity of a port based on individual components (*2*) or using a simulation (*4*). However, both of these methods have limitations. Bellsolà Olba et al. (*5*) emphasized that network capacity should not be measured by defining capacities for individual components and identifying the most limited capacity, as there might be interdependencies between components. Additionally, using simulation to estimate capacity requires highly accurate representations of port functions, which can be time-consuming to model, and the input data can often be inaccessible.

Studies have used different metrics to quantify port performance, including throughput, vessel density, waterway congestion, travel time, and dwell time. However, these parameters indicate level of service rather than capacity. There exists a need to define a metric for the operating capacity of multimodal maritime transportation networks in a manner that incorporates the behavior of both waterside and landside systems and relates to the freight-moving ability of the integrated system. This metric would have value for stakeholders interested in understanding how infrastructural investments, such as channel expansion or terminal construction projects, may influence the overall capacity of an integrated port system. A metric capable of predicting the long-term outcomes of these initiatives and drawing comparisons between different project alternatives over an extended period would provide value to port planners and stakeholders.

This report proposes a queuing theory-based capacity model for multimodal port freight systems. We define the *operating capacity* of a port as the amount of cargo per hour that can be processed over an extended period under specific port conditions. Queuing theory-based models to estimate system capacity have been successfully applied in other domains, such as healthcare (*6, 7*) and logistics (*8*). These studies estimate system capacity by analyzing arrival rates and mean queue times or lengths using simulations or empirical data (*9*). However, the queuing models do not directly apply to port operations. We develop queuing models to suit port operations and compute the operating capacity of the port. Our proposed model incorporates long-term behavior while accounting for level-of-service parameters, such as vessel dwell times, truck turn times, and yard utilization.

Contributions

The key contributions of this report are as follows:

1. We propose a queuing theory-based model for modeling long-term port operations, which can be calibrated using real-world data.
2. We propose a definition of long-term operating capacity for multimodal port systems and use our queuing model to estimate long-term port network operating capacity.
3. We showcase the effectiveness and extensibility of the proposed model through a case study of the Port of Houston.

The remainder of the report is structured as follows: Section 2 provides a brief review of relevant literature. Next, Section 3 details the queuing models for each of the subsystems. Section 4 compares the proposed techniques' performance on empirical data for the Port of Houston. Finally, Section 5 summarizes our findings and suggests potential extensions to the research.

2 LITERATURE REVIEW

Defining capacity in a multimodal port network is challenging due to the complex interactions among multiple resources and parameters (*10*). Consequently, there is no universally accepted definition of long-term port operating capacity (*4*). Several studies have proposed capacity definitions for specific port components. Liu and Li (*11*) used arrangement theory to calculate the carrying capacity of navigable waterways by determining the number of ships that can be safely accommodated on the water. Fan and Cao (*12*) defined anchorage capacity as "the maximum number of vessels that can be accommodated by the anchorage over a period of time." Using automatic identification system (AIS) vessel tracking data, Liu et al. (*13*) defined waterway capacity as "the ratio of the spatiotemporal resources of the port waterway to the spatiotemporal resources occupied by a single ship generally sailing in and out of the port waterway." However, in real-world scenarios, vessels are not often tightly packed, and waterway capacities may be constrained by other factors beyond channel dimensions.

Static and dynamic capacity concepts have been used to model capacities of individual port components (*14*). Static capacity is based on channel dimensions or resource availability, while dynamic capacity is based on the volume a component can handle over time. Interaction functions have been used to mathematically model the interactions between components (*15*). However, calculating network capacities based on the capacities of individual components may not sufficiently capture the interdependent effects present among system components (*5*). Aggregate port performance metrics, such as congestion level (*16*) and turnaround time (*17*), are often used to account for these interdependencies. However, these performance measures do not directly correlate with port capacity as it is commonly defined in transportation networks.

The Bureau of Transportation Statistics (*18*) defines port capacity as a "measure of the maximum throughput in tons, twenty-foot equivalent unit (TEU), or other units that a port and its terminals can handle over a given period." Calculating an exact capacity for the entire port network is challenging due to the extensive interactions between systems and modes. Therefore, most capacity models focus on simulating numerous scenarios to derive insights on capacity and identify potential bottlenecks (*19*). For instance, Huang et al. (*20*) evaluated anchorage capacity by simulating a realistic mix of vessels and analyzing anchorage utilization. They defined anchorage capacity as "the mean utilized area when a new vessel cannot be accommodated, weighted by the time period from the rejection of the vessel to the acceptance of the next vessel." O'Halloran et al. (*21*) used query-and-simulate loops to determine waterway capacity by establishing the minimum channel dimensions required for safe sailing. Recently, researchers have developed the concept of port network traffic capacity (PNTC), which incorporates the interdependencies among port subsystems (*5*). Bellsolà Olba et al. (*4*) defined PNTC as "the maximum average vessel flow that can be handled by a port, with its specific infrastructure layout, vessel fleet, traffic composition, and demand, satisfying the required safety and service level." The authors ran multiple simulations with varying demands to identify stable and unstable conditions and determine PNTC. However, these models strongly depend on the underlying simulation model and how "stable conditions" are defined.

Apart from determining capacity, understanding the operation of ports has been of significant research interest. Several studies (*22–25*) used queuing theory to model port operations. Groenveld (*26*) developed a queuing model with only one queue at the anchorage and berths as servers. Zrnić et al. (*27*) used an anchorage-ship-berth link as a multiple server queuing system. Mrnjavac and Zenzerovic (*28*) described how a container terminal could be simulated using known arrival and service rates. They also analyzed the effect of increasing the arrival rate with a fixed number of berths on port operations. Queuing theory and simulations have also been used to determine the optimal number of berths in a seaport (*29*) and the impact of disruptions (*30*). Souf-Aljen et al. (*31*) used a simulation model and AIS data for a container terminal to estimate port throughput. Bugaric and Petrovic (*32*) analyzed how to increase terminal capacity without making large investments. Networks of ports have also been analyzed using closed Jackson network queuing models (*33*).

Queuing models have been employed in a wide variety of domains to determine system capacities, including production (*34*), healthcare (*6, 35*), rail (*36*), and aerial vehicles (*37*). As observed by Lantz and Rosén (*9*), capacity notions typically stem from two types of approaches: engineering-based or time-based, both of which have inherent limitations. Engineering-based capacity measurements are sensitive to input values of variables, while time-based studies can introduce behavioral biases, such as the Hawthorne effect (*38*). Queuing theory-based capacity computations (*9*) help mitigate these challenges while still considering all interdependencies. Zenzerović and Vilke (*39*) provided evidence that queuing models can be used to optimize capacities in container terminals, while Lee et al. (*40*) proposed a queuing model explicitly designed for the unique characteristics of container terminals to estimate capacity. However, a research gap exists in determining the long-term operating capacity of a multimodal port system using queuing models and assessing the viability of such models in predicting the operating capacity of real-world systems. This research aims to fill this gap by proposing an operating capacity formulation by applying queuing models to the entire port system.

3 PORT QUEUING MODEL

Applying queuing models requires knowledge of arrival processes and queuing disciplines. Researchers have demonstrated that vessel arrival rates at anchorages and terminals often follow a Poisson arrival process (*41*). Chen and Yang (*42*) and Liu et al. (*43*) employed Poisson processes to model truck arrivals at container terminal gates. Wang et al. (*44*) investigated various arrival rules for queuing models, noting that the first-come-first-serve (FCFS) rule is the most common and fundamental for ships entering and leaving ports, although alternative rules, such as prioritizing large-tonnage ships (large-ton-ship first service or LSFS), are also utilized. Their findings indicate no significant difference in waterway throughput between FCFS and LSFS rules, while other rules, such as prioritizing ships based on tides, resulted in a 10–25% increase in throughput.

Queuing models require a well-defined abstraction of the processes and services that are found in real-world operations. To outline our multimodal port queuing model, we first explain the life cycle of a vessel in a port and introduce the notations used in our model. We then propose queuing models that represent the waterway, terminal import, and terminal export processes. A vessel's port life cycle begins with it entering the anchorage area and queuing until it can proceed to its destination terminal. In narrow waterways, a set of restrictions may govern navigation, such as combined beam, combined draft, and/or daylight. Once a berth is available and channel restrictions are met, the vessel requests resources (pilots and tugboats) to navigate to its terminal berth. Let L_a^i denote the average number of ships waiting in the anchorage for vessel type i, where i can be either container, break-bulk, or liquid bulk vessels, and W_a represent the average time vessels of all types spend waiting in the anchorage queue.

We employ a multiclass $M/M/1$ queuing model with equal priorities for the ship channel operation. Each class represents a cargo type, and since all vessels pass through the same channel, they are serviced at the same rate. However, each class of cargo arrives at a different arrival rate λ_a^i, following a Poisson process. We further assume that service times to enter the channel follow an exponential distribution with a mean service rate μ_a, and departing vessels are prioritized over arriving vessels in the waterway channel. Consequently, we assume queues only form at the anchorage.

Once a vessel arrives at its terminal berth, cargo is transferred to the terminal yard for storage. The cargo is then queued in the yard and awaits transfer to landside transport modes such as trucks or trains. Let L_c^I represent the average amount of cargo in the yard due to imports and W_c^I represent the mean cargo waiting time in the yard for the entire import process. We assume that vessels arrive at the terminal using a Poisson process with a mean interarrival rate λ_c^I. Each arrival consists of a batch of cargo, where the amount of cargo in each batch is a random variable X with first and second moments $E[X]$ and $E[X^2]$, respectively. The entire import process is assumed to have exponentially distributed service times with a mean service rate μ_c^I. We use an $M^{[X]}/M/1$ queuing model to represent the import process.

Concurrently with imports, the export process occurs. For exports, a truck arrives at the terminal's gate and queues until it can be serviced by the gate. We assume trucks arrive following a Poisson process with a mean interarrival rate λ_c^E. Once a truck enters the gate, it waits in the holding area until its cargo can be offloaded and stored in the terminal yard. Finally, we assume cargo from the yard is loaded onto the ship in a process independent of the cargo offloading. For the export process, queues form at three locations: the truck gate, the holding area, and the terminal yard. We model this using a tandem queue of three processes. We assume truck arrivals at the terminal gate follow a Poisson process with an interarrival rate λ_c^E, and each gate has exponentially distributed service times with a mean μ_c^G. Let W_c^G denote the mean waiting time before a gate serves a truck, and L_c^G is the mean length of the queue of trucks at the gate. Suppose there are S_c gates. We employ an $M/M/S_c$ queue to model gate behavior. Since the gates follow an $M/M/S_c$ model, the arrival rate of trucks in the holding area (λ_c^H) is the same as that of the arrival rate at the gate. The process is modeled as an $M/M/1$ queue, with service times exponentially distributed with mean μ_c^H. Let L_c^H denote the mean number of trucks in the holding area and the mean turn time be W_c^H. Finally, we assume exponentially distributed service times with a mean μ_c^E for processing cargo in the yard. We model the export yard processes using an $M/M/1$ queuing model. Let W_c^E denote the time cargo spends in the yard before being loaded onto the ship, and L_c^E the average amount of cargo in the yard due to exports.

Each queue follows FCFS behavior throughout the system, and service rates are assumed to be independent of arrival rates. Additionally, all other assumptions for the respective queuing models apply. The length of the cargo queue due to import and export combined is denoted by L_c. The yard utilization, Y_c, is the percentage of the yard storage space that is occupied by cargo at any moment. Further details on each of the queuing models are described below. The notations used in this study are summarized in Table 1, and Figure 1 illustrates the queuing network.

Symbol	Meaning
λ_a^i	Vessel arrival rate to the system (at anchorage) for type i
L_a^i	Mean length of queue of vessels of type i at the anchorage
μ_a	Service rate of the waterway when vessels arrive at arrival rate λ_a
W_a	Mean waiting time at anchorage for all vessel types
λ_c^I	Cargo batch arrival rate via vessel
L_c^I	Mean queue of import cargo in the terminal yard berth
μ_c^I	Rate of processing import cargo
W_c^I	Mean import waiting time of cargo at the yard
X	Random variable indicating batch size
$E[X]$	First moment of X
$E[X^2]$	Second moment of X
λ_c^G	Cargo arrival rate via truck
λ_c^H	Export cargo arrival rate at the holding area
λ_c^E	Export cargo arrival rate at the yard
L_c^G	Mean queue of export cargo to be processed at the port gate
L_c^H	Mean queue of export cargo in the holding area
L_c^E	Mean queue of export cargo in the terminal yard
μ_c^G	Rate of servicing cargo at the terminal gates
μ_c^H	Rate of processing export cargo in the holding area
μ_c^E	Rate of processing export cargo in the terminal yard
W_c^G	Mean export waiting time of cargo at the terminal gate
W_c^H	Mean export waiting time of cargo at the holding area
W_c^E	Mean export waiting time of cargo at the yard
S_c	Total number of truck gates
L_c	Mean total amount of cargo in the terminal yard

TABLE 1: Notations

3.1 The anchorage queuing model

We model the anchorage area using a multiclass $M/M/1$ queuing model, where the "server" represents an imaginary gate to the waterway channel. Each vessel requests the server for channel entry, and entry is granted, provided the destination berth is available and no navigation restrictions are violated. In this way, the service time distribution captures all channel restrictions and resource availability affects vessels experience throughout the analysis period. The mean waiting time for a multiclass $M/M/1$ queue with equal priorities (*45*) can be computed using Equation (1). The mean queue length for each class can be determined by applying Little's law (*46*), as shown in Equation (2). Empirical data or simulation output can provide the waiting times and queue lengths for each ship class at the anchorage.

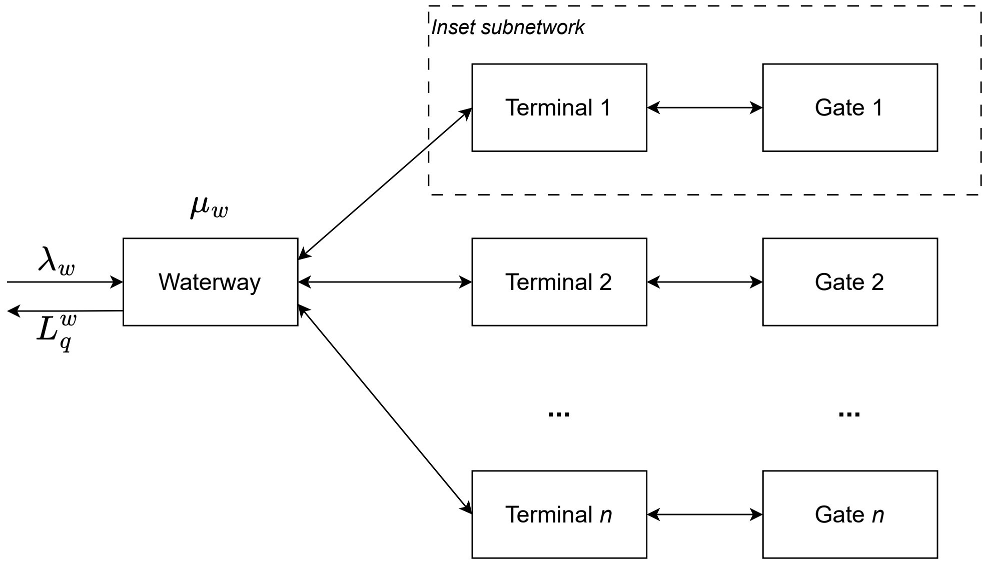


FIGURE 1: Structure of port queuing model

$$W_a = \frac{\sum_i \frac{\lambda_i}{\mu_a^2}}{1 - \sum \frac{\lambda_i}{\mu_a}} \tag{1}$$

$$L_a^i = \frac{\lambda_a^i \left(\sum_j \frac{\lambda_a^j}{\mu_a^2} \right)}{1 - \sum_j \frac{\lambda_a^j}{\mu_a}} \tag{2}$$

The equations (1) and (2) only apply under stable queue formation, or when the traffic intensity $\rho_a = \sum_i \lambda_a^i / \mu_a < 1$. Using the observed queue lengths of each class, $\hat{L}_a^i$, we apply a simple optimization model (3) to compute the service rate, μ_a. We compare the observed waiting times with those calculated using Equation (1) to validate the computed service rate. *The waterway operating capacity is defined as the service rate that maintains the desired quality of service (QOS), characterized by the queue length and waiting times over an extended period of stable queue formation.*

$$\begin{aligned} \min_{\mu} \quad & \sum_i \left(\frac{\lambda_a^i \left(\sum_j \frac{\lambda_a^j}{\mu^2} \right)}{1 - \sum_j \frac{\lambda_a^j}{\mu}} - \hat{L}_a^i \right)^2 \\ \text{subject to} \quad & \sum_i \frac{\lambda_a^i}{\mu} < 1 \end{aligned} \tag{3}$$

3.2 The terminal queuing model

We model terminal imports using a batch Markov arrival process (*47*), specifically $M^{[X]}/M/1$. We assume cargo arrives in batches, where the interarrival times between different batches follow an exponential distribution with mean λ_c^I. The amount of cargo arriving is considered a batch at each arrival instance, with the batch size represented by the random variable X with first and second moments $E[X]$ and $E[X^2]$, respectively.

The condition for the formed queue to be stable is given by Equation (4). The arrival rate λ_c^I can be observed empirically from archival AIS data for a terminal. The import dwell time (W_c^I) and the moments of the batch size can be derived from historical port data. Alternatively, all these parameters can also be computed through simulation. Equation (5) is then solved numerically to compute the cargo processing rate for imports subject to the stability constraint that $\rho_c^I < 1$. *The operating capacity of the import process is defined as the service rate that maintains the desired import cargo dwell over an extended period of stable queue formation.*

$$\rho_c^I = \frac{\lambda_c^I E[X]}{\mu_c^I} \tag{4}$$

$$W_c^I = \frac{\frac{\rho_c^I}{1-\rho_c^I}\left[\frac{E(X)+E(X^2)}{2E(X)}\right]}{\lambda_c^I E[X]} - \frac{1}{\mu_c^I} \tag{5}$$

$$L_c^I = \frac{\rho_c^I}{1-\rho_c^I}\left[\frac{E(X)+E(X^2)}{2E(X)}\right] - \rho_c^I \tag{6}$$

We employ a tri-tandem $M/M/S_c - M/M/1 - M/M/1$ queuing model for export processes. Specifically, the gate processes are modeled as an $M/M/S_c$ system, while the holding area and yard processes are each modeled as an $M/M/1$ system. By applying Burke's theorem (*48*), we assume cargo arrives at the terminal for export following a Poisson process. Empirical observations or simulations can provide the waiting times in the yard queue (W_c^E) and the cargo arrival rate at the gate λ_c^G. The service rate for exports can then be estimated using Equation 7 as derived by (*9*). *The operating capacity of the export processes is defined as the service rate that maintains the desired export dwell time over an extended period of stable queue formation.* The computed service rates for the import and export processes are validated by estimating the amount of cargo queued in the terminal using Equations (6) and (8). To validate the model, we compare the total average cargo stored in the terminal, considering both import and export processes ($L_c = L_c^E + L_c^I$), with the observed yard utilization.

$$\mu_c^E = \frac{\lambda_C^E}{2} + \sqrt{\left(\frac{\lambda_c^E}{2}\right)^2 + \frac{\lambda_c^E}{W_c^E}} \tag{7}$$

$$L_c^E = \frac{(\lambda_c^E)^2}{\mu_c^E(\mu_c^E - \lambda_C^E)} \tag{8}$$

Although service rates at the gate and holding area for trucks can be determined using gate wait times and truck turn times, these rates are not computed in this study due to the lack of data needed to validate the model. Nonetheless, the model can be validated if the gate and holding area queue lengths are known. Table 2 summarises the various queuing models described in this section.

Model	**Model 1 (Anchorage)**	**Model 2 (Import)**	**Model 3 (Export)**		
Queuing Model	Multiclass $M/M/1$	$M^{[X]}/M/1$	Tandem $M/M/S_c - M/M/1 - M/M/1$		
Location	Anchorage	Yard	Gate	Holding area	Yard
Model Input – 1 (Q.O.S)	Queue length at anchorage (L_a^i)	Import dwell time in yard (W_c^I)	Wait time in queue (W_c^G)	Turn time (W_c^H)	Export dwell time in yard (W_c^E)
Model Input – 2 (Arrival)	Arrival rate of classes (λ_a^i)	Arrival rate and batch dist. (λ_c^I, X)	Arrival rate of trucks (λ_c^G)	Arrival rate at holding (λ_c^H)	Arrival rate of cargo (λ_c^E)
Model Output	Rate of service (μ_a)	Rate of service for import (μ_c^I)	Rate of service at gate (μ_c^G)	Rate of service at holding (μ_c^H)	Rate of service for export (μ_c^E)
Validation	Wait time in anchorage (W_a)	Yard Utilization (L_c)	Queue length at gate (L_c^G)	Queue length at holding (L_c^H)	Yard Utilization (L_c)

TABLE 2: Summary of queuing models

4 RESULTS AND DISCUSSIONS

We tested our models using data from the Port of Houston, which includes the Houston Ship Channel and over 200 terminals handling container, liquid, break-bulk, and general cargo. The Port of Houston is one of the largest in the US, ranking first in foreign waterborne tonnage and fifth in total TEUs among US container ports (*49*). The high complexities and interdependencies make the Port of Houston an ideal candidate for validating the models proposed in this study. We obtained various input parameters for our model from archival AIS data, published reports from the Port of Houston, and relevant literature. To demonstrate the application of the models, we analyzed the Houston anchorage area and the Barbours Cut Container Terminal (BCT) on a quarterly basis from the last quarter of 2021 through the last quarter of 2023 to ensure stable queue formations. For a queue to be stable, it must satisfy the stability condition ($\rho < 1$) as described in Section 3, which would mean the mean and variance of the performance indicators (queue length and waiting time) should not change significantly over time. Consequently, a very short time scale (on the order of days) would be insufficient to ensure stable queues, as the mean values of these performance indicators fluctuate daily. Conversely, a large time scale (on the order of years) would not provide adequate instances to compare operating capacity effectively. A three-month period strikes a balance between the two time-scales.

To evaluate our proposed model for the waterway, we analyzed archival AIS data to determine input values for different cargo classes at the anchorage. The AIS-derived values for vessel arrival rates, mean anchorage wait times and anchorage queue lengths are shown in Table 3. The computed mean service rate was approximately 0.8 vessels processed by the ship channel per hour. It is important to note that the mean service rate does not reflect the maximum number of vessels that the channel can accommodate at any given time. Instead, it indicates the mean inbound number of vessels that the channel can handle over an extended period. For example, the computed μ_a values in Table 3 represent the mean number of inbound vessels that can be processed across each three-month period, including all night, weekend, holiday, and weather or fog disruption periods, and reflect the presence of channel restrictions and any resource constraints. Thus, capacity values derived from this queuing model should only be compared against capacity values from the same queuing model for the same location under different conditions to assess various capacity scenarios. We observed that the channel's operating capacity remained relatively stable over the analysis period, although the mean waiting time varied from 32.97 hours to 48.55 hours, a variation of about 47%. Our model predicted the waiting time within close margins to the observed waiting times from archival AIS data, with relative errors ranging from 0.35% in the fourth quarter of 2022 to 16.78% in the fourth quarter of 2023. Table 3 summarizes the calculated service rate and yard queue length for the waterway.

Year	Quarter	Cargo type	λ_a^i	L_a^i	μ_a	W_a (Calculated)	W_a (Actual)	Relative error (%)
2021	Q4	Container	0.09	3.77	0.8	41	39.67	3.35
		Break-bulk	0.17	4.87				
		Liquid	0.52	21.87				
2022	Q1	Container	0.09	6.03	0.77	41.89	44.3	-5.44
		Break-bulk	0.17	6.89				
		Liquid	0.49	20.24				
	Q2	Container	0.09	7.11	0.81	43.47	44.38	-2.04
		Break-bulk	0.17	4.27				
		Liquid	0.53	23.74				
	Q3	Container	0.1	9.41	0.79	44.59	48.55	-8.16
		Break-bulk	0.16	5.34				
		Liquid	0.52	22.68				
	Q4	Container	0.1	5.79	0.81	45.2	45.04	0.35
		Break-bulk	0.17	5.71				
		Liquid	0.52	23.93				
2023	Q1	Container	0.09	2.97	0.8	49.68	44.97	10.47
		Break-bulk	0.15	4.13				
		Liquid	0.54	28.05				
	Q2	Container	0.1	1.1	0.8	37.15	32.97	12.68
		Break-bulk	0.15	3.84				
		Liquid	0.52	20.41				
	Q3	Container	0.1	2.1	0.81	45.54	40.82	11.56
		Break-bulk	0.14	3.82				
		Liquid	0.55	26.18				
	Q4	Container	0.09	1.25	0.81	45.19	38.7	16.78
		Break-bulk	0.13	2.35				
		Liquid	0.57	26.96				

TABLE 3: Anchorage results ***(λ_a^i: Arrival rate per hour; L_a^i: Queue length; μ_a: Service rate per hour; W_a (Calculated): Calculated mean wait time in hours; W_a (Actual): Actual mean wait time in hours)***

To demonstrate the terminal import and terminal export queuing models, we examined the BCT terminal in the Port of Houston from the fourth quarter of 2021 to the fourth quarter of 2023. Arrival rates were computed from archival AIS data (*50*). Import and export container dwell times were obtained from Port of Houston terminal reports (*51*). The first and second moments of batch size were predicted from the mean and variance of import container distribution using data from BCT terminal reports (*52*). Table 4 summarizes the calculated service rate and yard queue length for the import processes. The export model was analyzed during the same period. The truck arrival rates were estimated from the Port of Houston Lynx portal (*53*), which has a database of all gate transactions for imports and exports at BCT. The container arrival rates were assumed to be the same as truck arrival rates, assuming each truck carries one container. Table 5 summarizes the export processes' calculated service rate and yard queue length. The operating capacity for import and export processes is about 53.77 and 72.43 containers per hour. As in the case of the waterway, the values should be interpreted as the long-term operating capacity for import and export processes and not the maximum rate at which imports and exports could happen in one instant.

To validate our proposed import and export models, we estimated the yard capacity using BCT yard inventory data from the Lynx database (*53*) and yard utilization information (*51*) as of July 2024. The estimated capacity was found to be 25,208 containers. Our estimate aligns closely with the latest value in the literature by Huynh and Hutson (*54*), where the yard capacity was estimated to be about 23,000 containers. To validate our terminal model, we calculated the total queue length for imports and exports and compared it with yard utilization data obtained from the BCT terminal reports (*51*), averaged over each three-month period. The validation results are summarized in Table 6. The predicted and actual utilization of the yard space was close, with an error margin of 0.86% to 17.79%, except for the first four quarters, indicating the validity of our models.

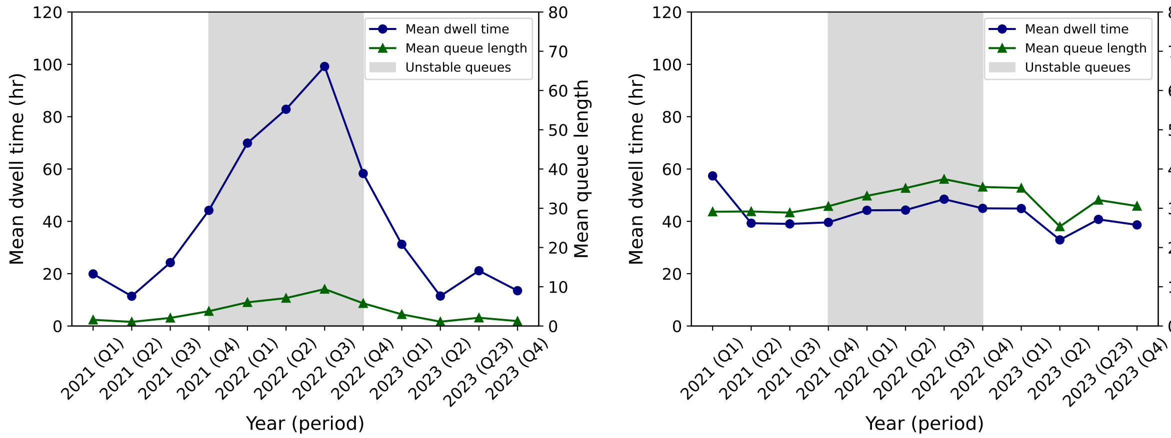
The terminal model predictions show high errors during the first four quarters of the analysis period from 2021 Q4 to 2022 Q3. We attribute this error to unstable queues and high container dwell times at the Barbours Cut container terminal during this period (*55, 56*). This period coincides with a peak in container dwell times at the anchorage, as observed in Figure 2(a). As discussed in Section 3, our capacity prediction models are only applicable for long-term periods of stable queue formation. Hence, the results from these periods are likely not valid. The anchorage model, however, made accurate predictions for all the time periods, including that of COVID-19. This could be attributed to the fact that, despite the high dwell times and queue lengths for container vessels, the overall dwell times and queue lengths remained stable, as observed in Figure 2(b). The overall vessel mix of the Houston ship channel comprises about 70% tankers, which did not experience similar congestion levels as a significant cause of congestion in container dwell times was related to truck and chassis availability, issues that did not impact tankers.

Year	Quarter	λ_c^I	$E[X]$	$E[X^2]$	W_c^I	μ_c^I	L_c^I
2021*	Q4	0.045			7.90	41.61	7262
2022*	Q1	0.054			6.83	49.79	7534
2022*	Q2	0.055			6.13	51.08	6887
2022*	Q3	0.057			6.30	52.67	7335
2022	Q4	0.056	851.2	10.66×10^5	5.11	52.78	5845
2023	Q1	0.057			4.46	54.38	5193
2023	Q2	0.063			3.41	61.29	4388
2023	Q3	0.065			3.32	63.2	4408
2023	Q4	0.058			3.37	57.12	3993

TABLE 4: Terminal imports model results ***(λ_c^I: Arrival rate per hour; $E[X]$: First moment of batch size; $E[X^2]$: Second moment of batch size; W_c^I: Import dwell time in yard in days; μ_c^I: Calculated service rate per hour; L_c^I: Calculated yard queue length due to import; *: Unstable queues in container terminal during COVID-19 period. Model predictions not applicable)***

5 CONCLUSION

We propose a queuing theory-based model to estimate operating capacities for multimodal port processes. The models were tested using real-world archival AIS and port data from the Port of Houston. The results indicate that queuing models are viable for determining the operating capacity of port networks. An advantage of queuing models is that they allow for the modeling of the entire system instead of individual components. Additionally, they provide a way to include port



(a) Container vessels at anchorage **(b) All vessels at anchorage**

FIGURE 2: Mean dwell times and mean queue length at anchorage for Port of Houston as obtained from archival AIS data. The shaded region shows the period where our container terminal model predictions had high relative errors.

Year	Quarter	λ_c^E	W_c^E	μ_c^E	L_c^E
2021*	Q4	60.47	10.40	60.48	15093
2022*	Q1	68.66	10.13	68.67	16698
2022*	Q2	73.75	10.87	73.75	19233
2022*	Q3	76.15	10.55	76.15	19281
2022	Q4	70.65	9.73	70.65	16498
2023	Q1	75.70	9.05	75.70	16442
2023	Q2	72.35	8.08	72.35	14029
2023	Q3	80.43	8.11	80.43	15647
2023	Q4	73.65	8.56	73.66	15130

TABLE 5: Terminal export model results (λ_c^E: Container arrival rate per hour; W_c^E: Export dwell time in yard in days; μ_c^E: Calculated service rate per hour; L_c^E: Calculated yard queue length due to export, *: Unstable queues in container terminal during COVID-19 period. Model predictions not applicable)

performance measures in developing the capacity model. The model requires minimal data that can be easily obtained, making it extendable to any port without the need to develop a detailed simulation. However, when combined with simulation, the model can predict throughput and operating capacity for different conditions, thereby aiding decision-making and bottleneck analyses.

Nevertheless, our proposed model has several limitations. Queuing theory-based models miss several lower-level details, and information is inevitably lost in aggregation. Furthermore, truck arrivals in this study are assumed to follow a homogeneous Poisson process, whereas trucks generally do not arrive at night. A non-homogeneous Poisson process might be more suitable for modeling truck arrivals, which remains a direction for future research. Due to a lack of available data, we could not validate our models in detail on the terminal side. These models should be

Year	Quarter	Calculated				Observed	Relative error (%)
		L_c^I	L_c^E	L_c	Y_c (%)	Y_c (%)	
2021*	Q4	7262	15093	22356	88.69	53.03	67.23
2022*	Q1	7534	16698	24233	96.13	57.53	67.09
2022*	Q2	6887	19233	26121	103.6	54.83	88.98
2022*	Q3	7335	19281	26617	105.5	87.97	20.03
2022	Q4	5845	16498	22344	88.64	81.80	8.36
2023	Q1	5193	16442	21635	85.83	72.87	17.79
2023	Q2	4388	14029	18418	73.07	73.70	-0.86
2023	Q3	4408	15647	20056	79.56	82.33	-3.37
2023	Q4	3993	15130	19123	75.86	77.57	-2.20

TABLE 6: Terminal model validation (L_c^I: Calculated yard queue length due to import, L_c^E: Calculated yard queue length due to export, L_c: Calculated total yard queue length due to import; Y_c: Yard utilisation; *: Unstable queues in container terminal during COVID-19 period. Model predictions not applicable)

tested with real data or a simulation to ensure their validity and extensibility. An interesting future direction is to compare queuing-based models with the existing simulation-based PNTC definition proposed in (*4*). Another promising direction is to examine how input parameter uncertainty would affect capacity computation, as vessel data are often associated with significant uncertainties.

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