

# FAA TECHNICAL CENTER LETTER REPORT

FEDERAL AVIATION ADMINISTRATION

MAR 22 Rec'd

BENDIX ALPHA-BETA TRACKING FILTER TEST

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by

WILLIAM ELWELL

U. S. DEPARTMENT OF TRANSPORTATION  
FEDERAL AVIATION ADMINISTRATION  
TECHNICAL CENTER  
Atlantic City Airport, N.J. 08405

**PURPOSE:**

THE PURPOSE OF THIS PROGRAM IS TO GRAPH THE IMPULSE AND RAMP RESPONSES OF THE BENDIX ALPHA-BETA TRACKING FILTER. THE PROGRAM SIMULATES THE OPERATION OF THE BENDIX ALPHA-BETA FILTER BY USING THE FOLLOWING SET OF DIFFERENCE EQUATIONS:

$$\begin{aligned} X_{BK} &= X_{PK} + A * (X_K - X_{PK}) \\ X_{DKB} &= X_{DKB1} + (B/T) * (X_K - X_{PK}) \\ X_{PK1} &= X_{BK} + T * (X_{DKB}) \end{aligned}$$

**WHERE**

T= SAMPLING INTERVAL  
A= ALPHA  
B= BETA  
X<sub>K</sub>= KTH MEASUREMENT OF POSITION  
X<sub>BK</sub>= SMOOTHED ESTIMATE OF POSITION AT THE KTH TIME INDEX  
X<sub>DKB</sub>= SMOOTHED VELOCITY AT THE KTH TIME INDEX  
X<sub>DKB1</sub>= SMOOTHED VELOCITY AT THE K-1 TIME INDEX  
X<sub>PK</sub>= PREDICTED VALUE OF POSITION AT THE KTH TIME INDEX  
X<sub>PK1</sub>= PREDICTED VALUE OF POSITION AT THE K+1 TIME INDEX

THE INPUT TO THE FILTER IS APPLIED TO X<sub>K</sub>. OUTPUTS ARE OBTAINED FROM X<sub>PK</sub>, X<sub>BK</sub>, AND X<sub>DKB</sub>. THESE OUTPUTS ARE PLOTTED AGAINST TIME FOR 40 INTERVALS.

TO OBTAIN AN IMPULSE RESPONSE, A PULSE OF AMPLITUDE 1/T AND WIDTH T IS APPLIED TO THE INPUT AT THE FIRST SAMPLING.

THE RAMP RESPONSE IS OBTAINED BY FEEDING THE FILTER AN INPUT OF ZERO AT THE FIRST SAMPLING. THE INPUT LEVEL IS THEN RAISED BY THE VALUE OF T EACH SAMPLING FOR 10 PERIODS. THE INPUT IS THEN HELD AT ZERO FOR THE REMAINING SAMPLES.

**OPERATION:**

TO OPERATE THIS PROGRAM THE FOLLOWING STEPS MUST BE PERFORMED.

1. PUSH CNL INT ON THE COMPUTER PANEL  
THE COMPUTER WILL THEN READ  
\*\*\*CONSOLE INTERRUPT  
DP05 CC
2. TYPE \$IMPRMP  
THIS WILL START THE PROGRAM  
THE FOLLOWING WILL THEN APPEAR ON THE SCREEN  
ENTER VALUES FOR ALPHA, BETA, AND T
3. ENTER THE DESIRED VALUES, EACH SEPERATED BY A COMMA  
A, B, T (CR)

THE PROGRAM WILL THEN PRODUCE A GRAPH ON THE TEKTRONIX 4010 TERMINAL. THE COMPUTER WILL PRINT TYPE (CR) TO CONTINUE. WHEN A CARRAGE RETURN IS DEPRESSED A NEW GRAPH WILL APPEAR. THERE ARE A TOTAL OF SIX GRAPHS, ONE WILL APPEAR AFTER EACH (CR). AFTER THE LAST CARRAGE RETURN THE SCREEN WILL BE ERASED AND THE COMPUTER WILL EXIT THE PROGRAM.

THE GRAPHS WILL APPEAR IN THE FOLLOWING ORDER:

1. IMPULSE RESPONSE, SMOOTHED POSITION
2. IMPULSE RESPONSE, SMOOTHED VELOCITY
3. IMPULSE RESPONSE, PREDICTED POSITION
4. RAMP RESPONSE, SMOOTHED POSITION
5. RAMP RESPONSE, SMOOTHED VELOCITY
6. RAMP RESPONSE, PREDICTED POSITION

SELECTION OF ALPHA AND BETA:

THE SELECTED VALUES OF ALPHA AND BETA ARE LISTED ON AN ENCLOSED LETTER FROM BENDIX. FOR MORE INFORMATION ABOUT THE METHODS USED TO CALCULATE ALPHA AND BETA ONE MAY REFER TO THE BENDIX TECHNICAL NOTE BCD-TN-81-033, FB-467-012 JUNE 1981, ALPHA-BETA TRACKING ERRORS FOR ORBITING TARGETS.

END

To Tom Potthast

From A. I. Sinsky

Subject Expected Smoothing of Alpha-Beta Filter in TCAS Simulation

The following table lists the expected standard deviation reduction ratios for the optimum alpha-beta values selected by Steve Guarino. The smoothed position and velocity values are listed.

Update Rate Periods (sec)	Alpha	Beta	$\sigma_{\dot{x}}/\sigma_x$	$\sigma_{\ddot{x}}/\sigma_x$
1	.250	.066	.491	.101
2	.370	.175	.613	.116
3	.465	.300	.690	.125
4	.530	.431	.745	.132
5	.580	.565	.788	.139
6	.620	.685	.820	.142
7	.645	.855	.861	.158
8	.665	.910	.872	.149

Example: If angle measurement standard deviation is  $1^\circ$ , then for update rate of three seconds, the standard deviation in smoothed angle is  $0.69 \times 1^\circ = .69^\circ$ . The standard deviation in angle rate is  $0.125 \times 1^\circ = .125^\circ/\text{sec}$ .

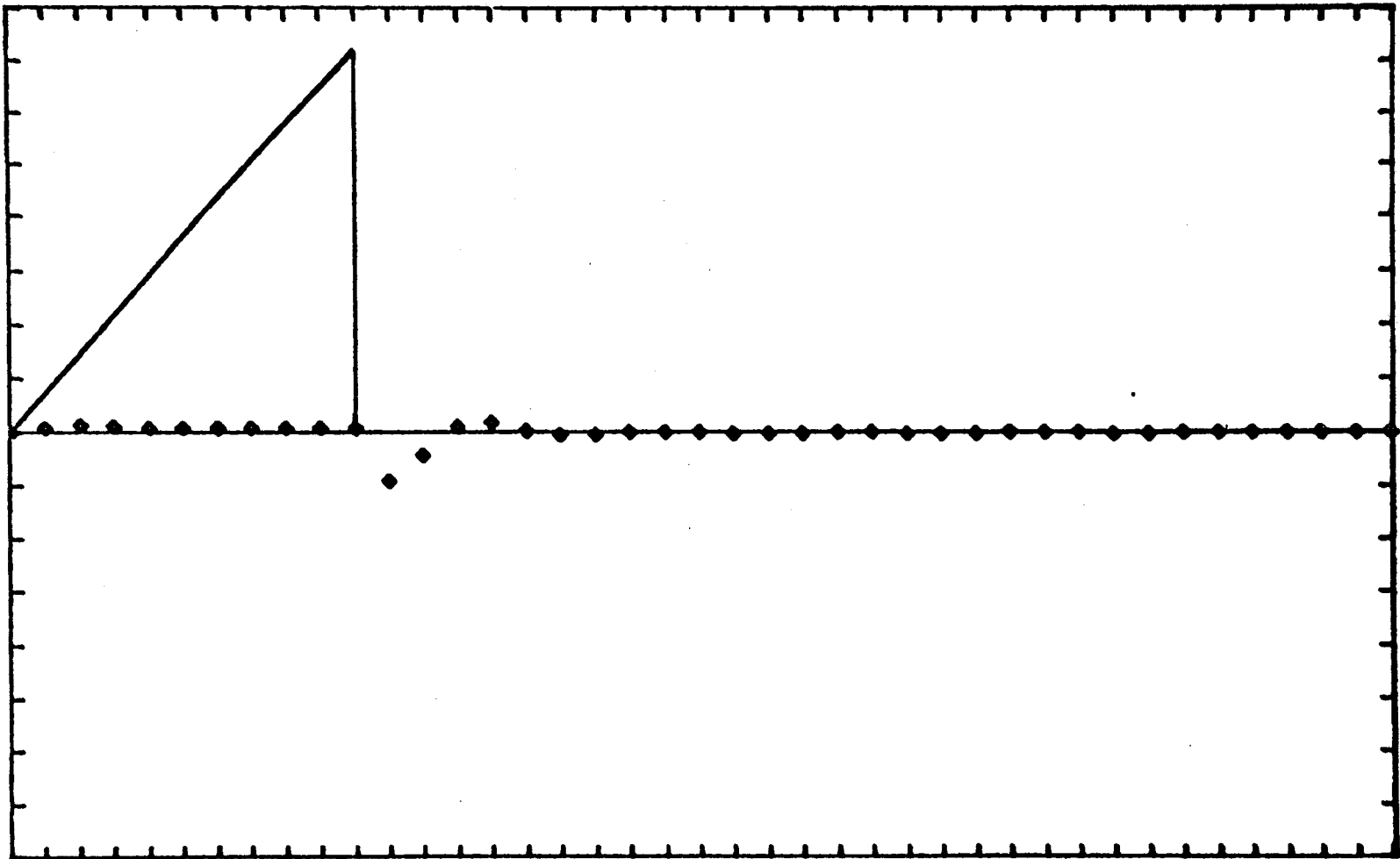
Reference: Page 4 of TN, FB-467-012



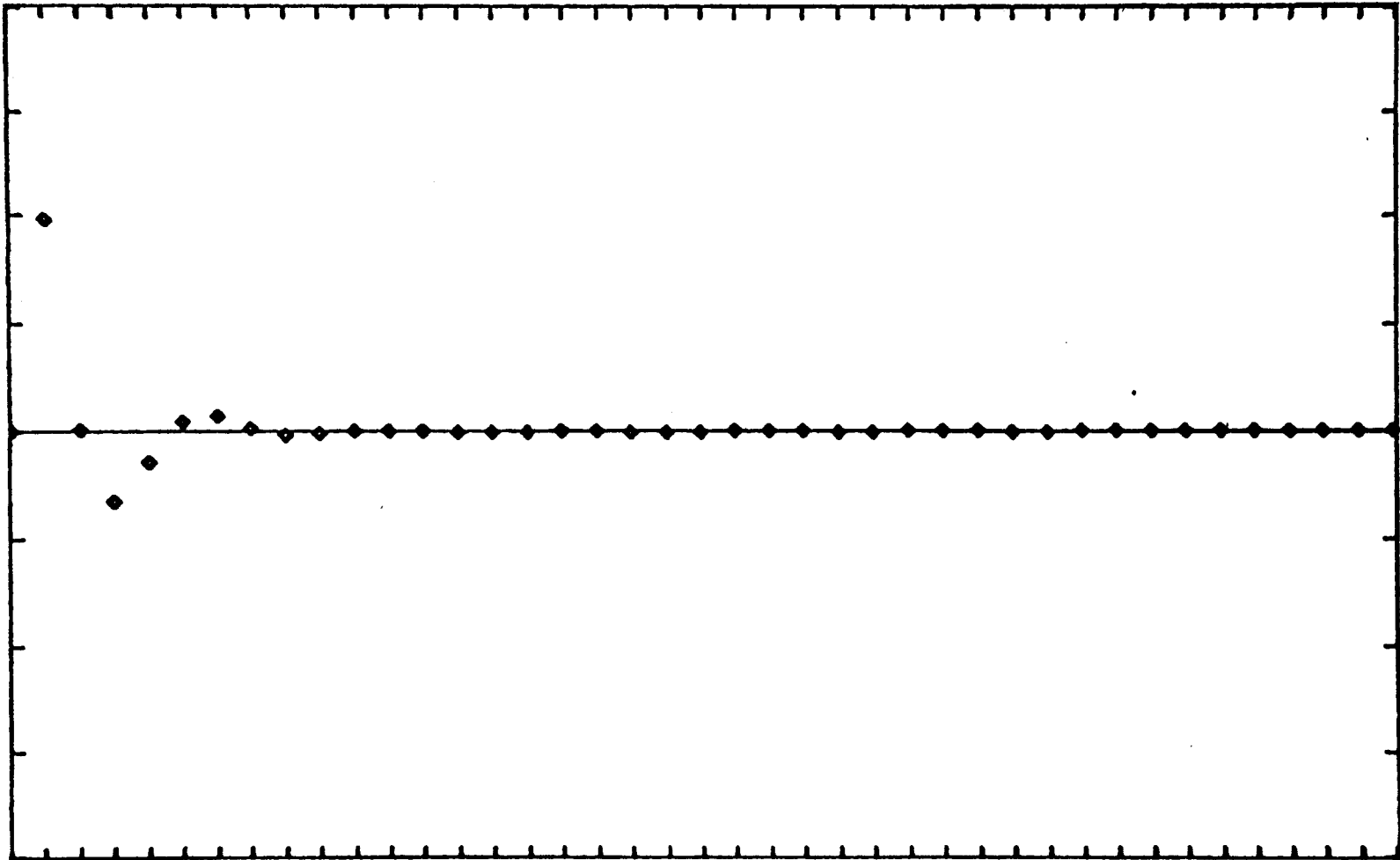
A. I. Sinsky

/kgm

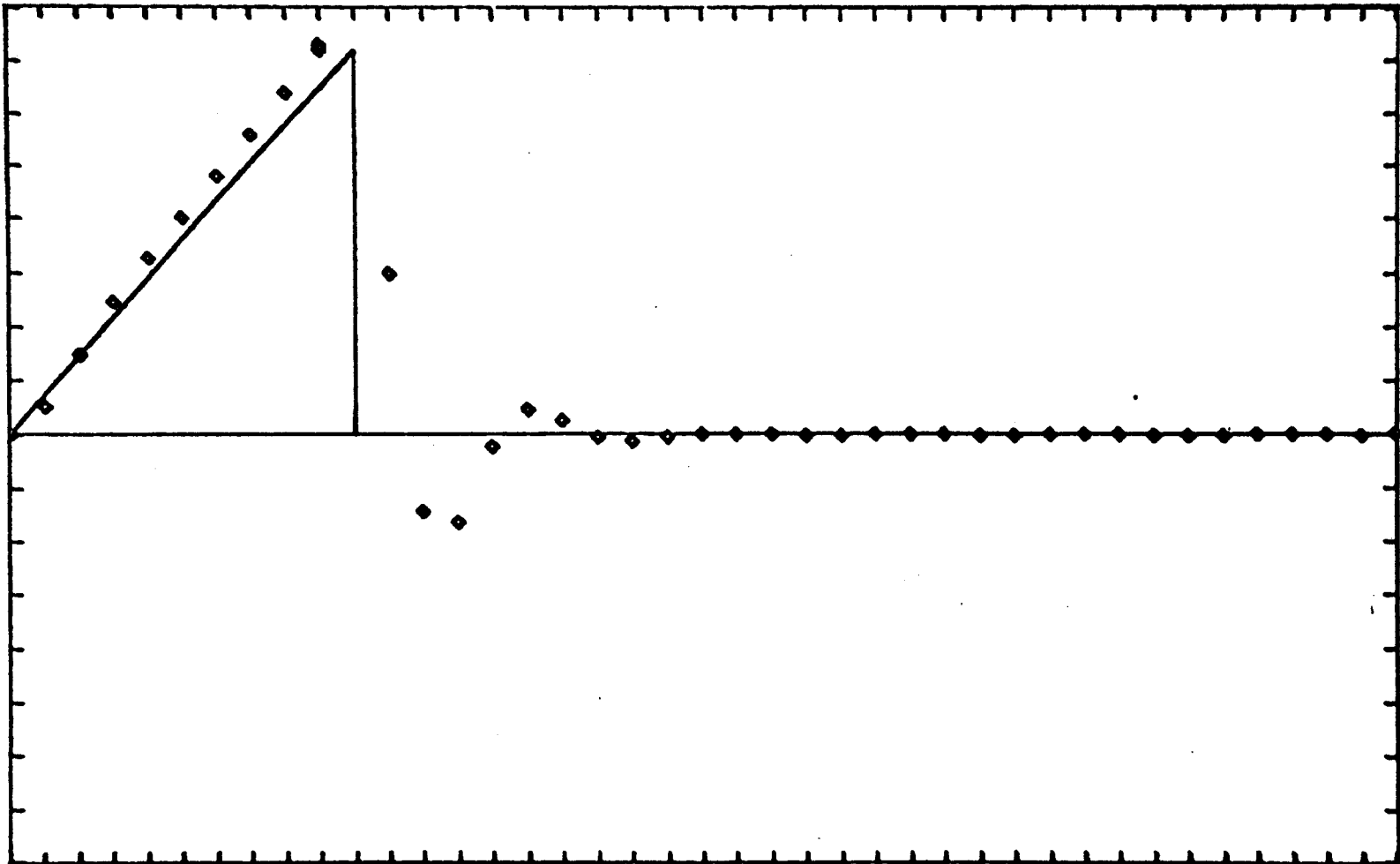
cc: T. L. Gabriele  
S. Guarino  
A. J. Spuria  
J. E. Reed  
D. C. Hecht



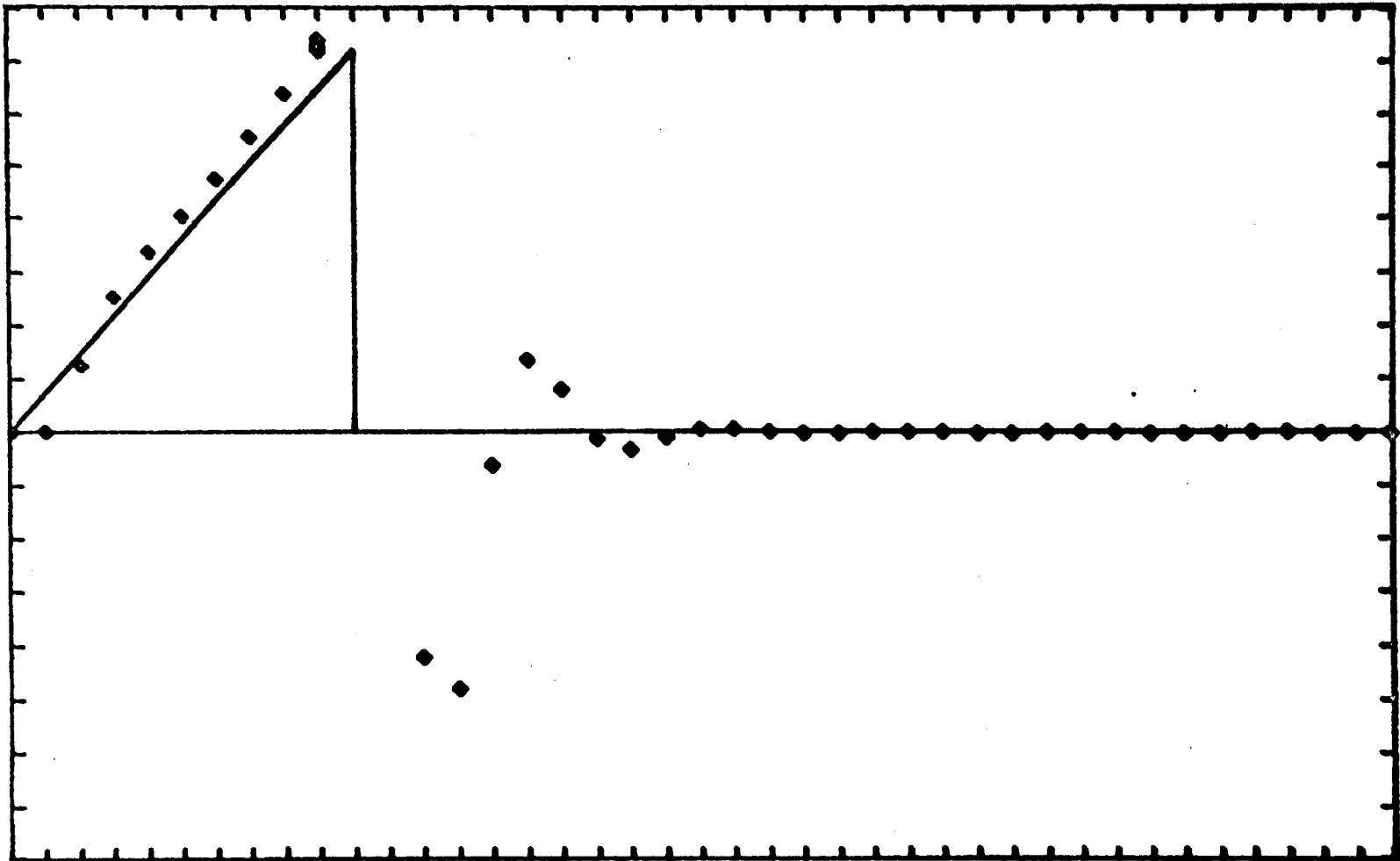
RAMP RESPONSE OF THE BENDIX ALPHA-BETA FILTER  
ALPHA= 0.66 BETA= 0.91 T= 8.00 SECONDS SFY=10.00  
PREDICTED POSITION  $XP(K)$   
PROCESSED BY FAA TECHNICAL CENTER ACT-100J



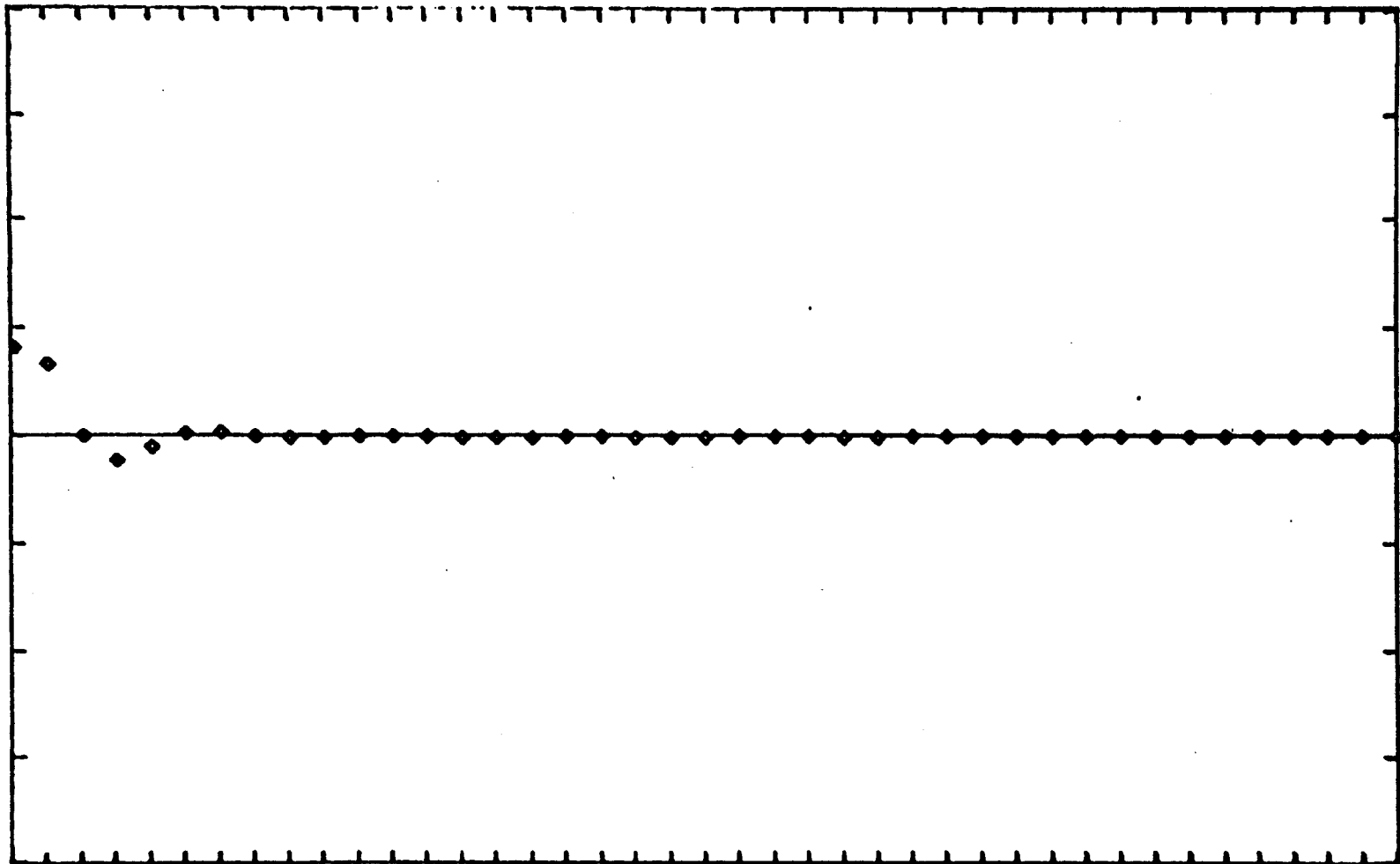
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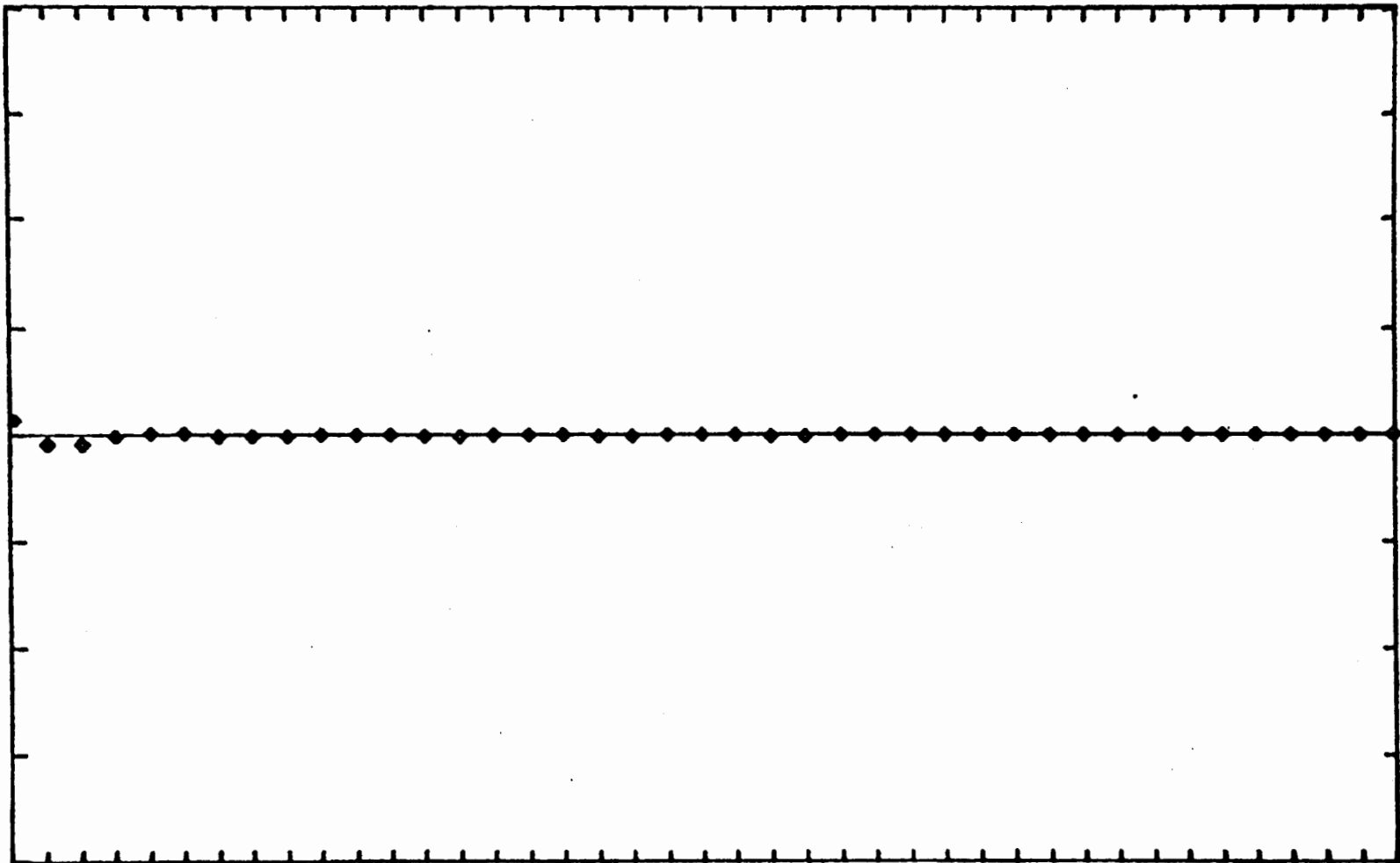
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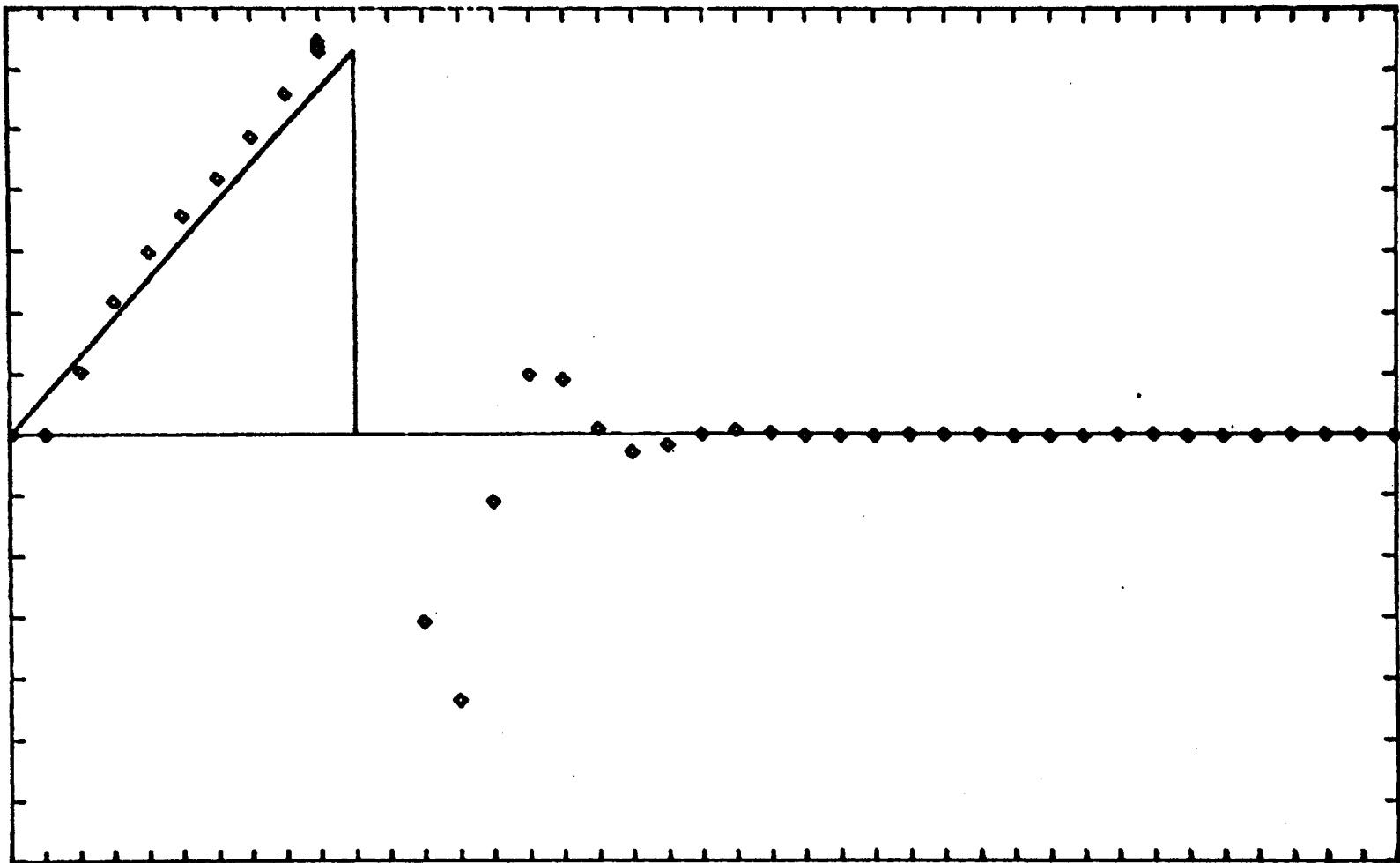
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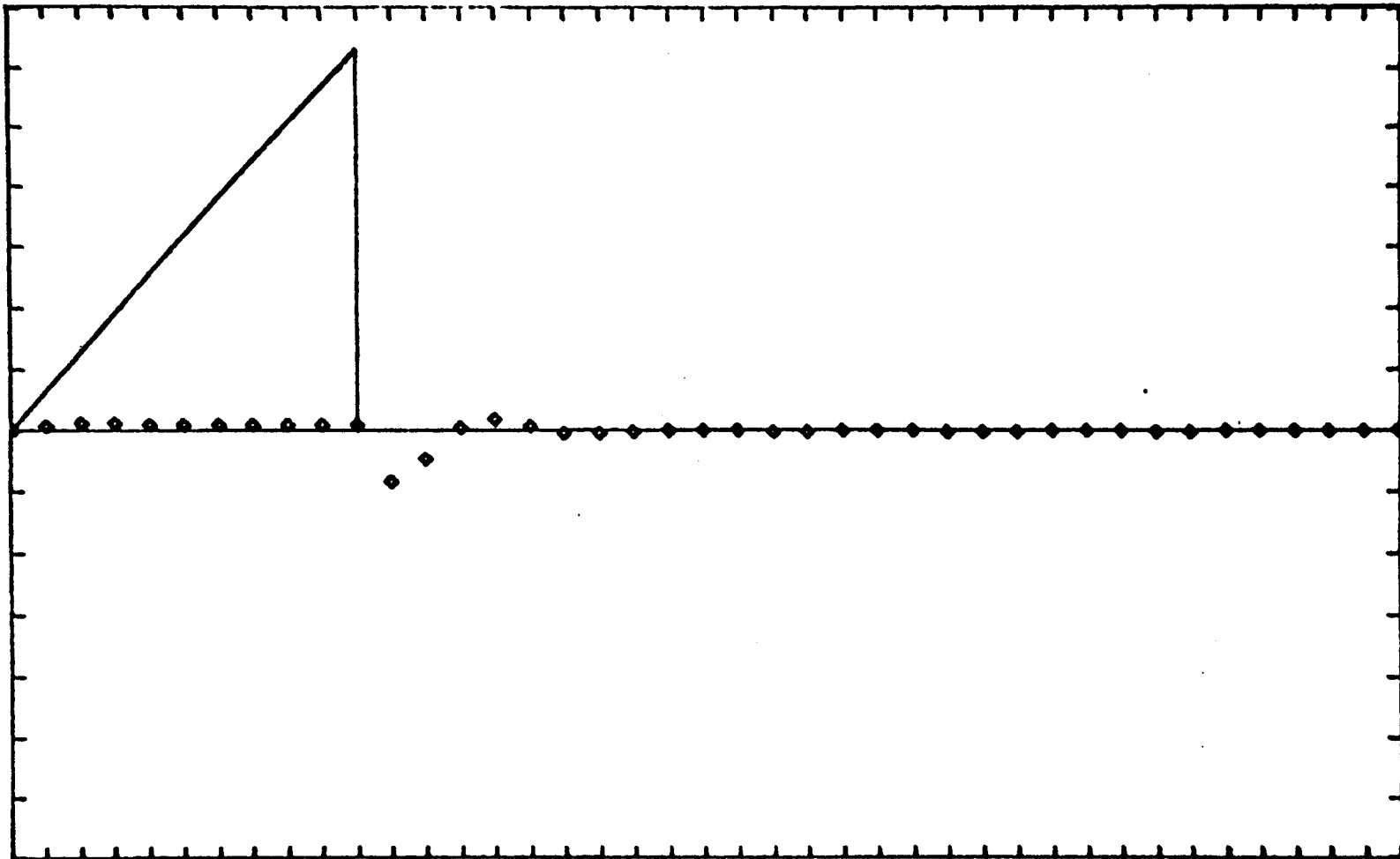
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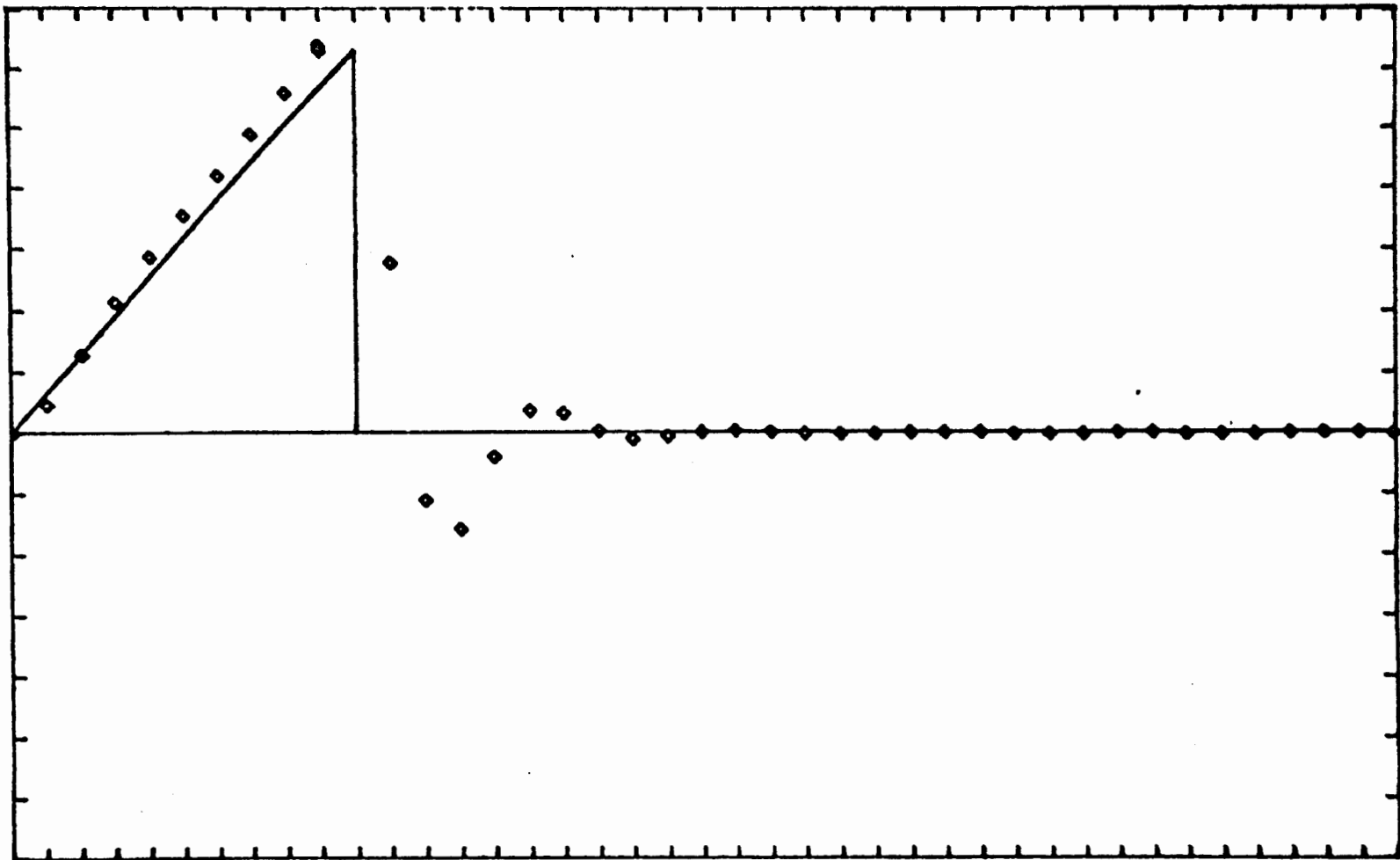
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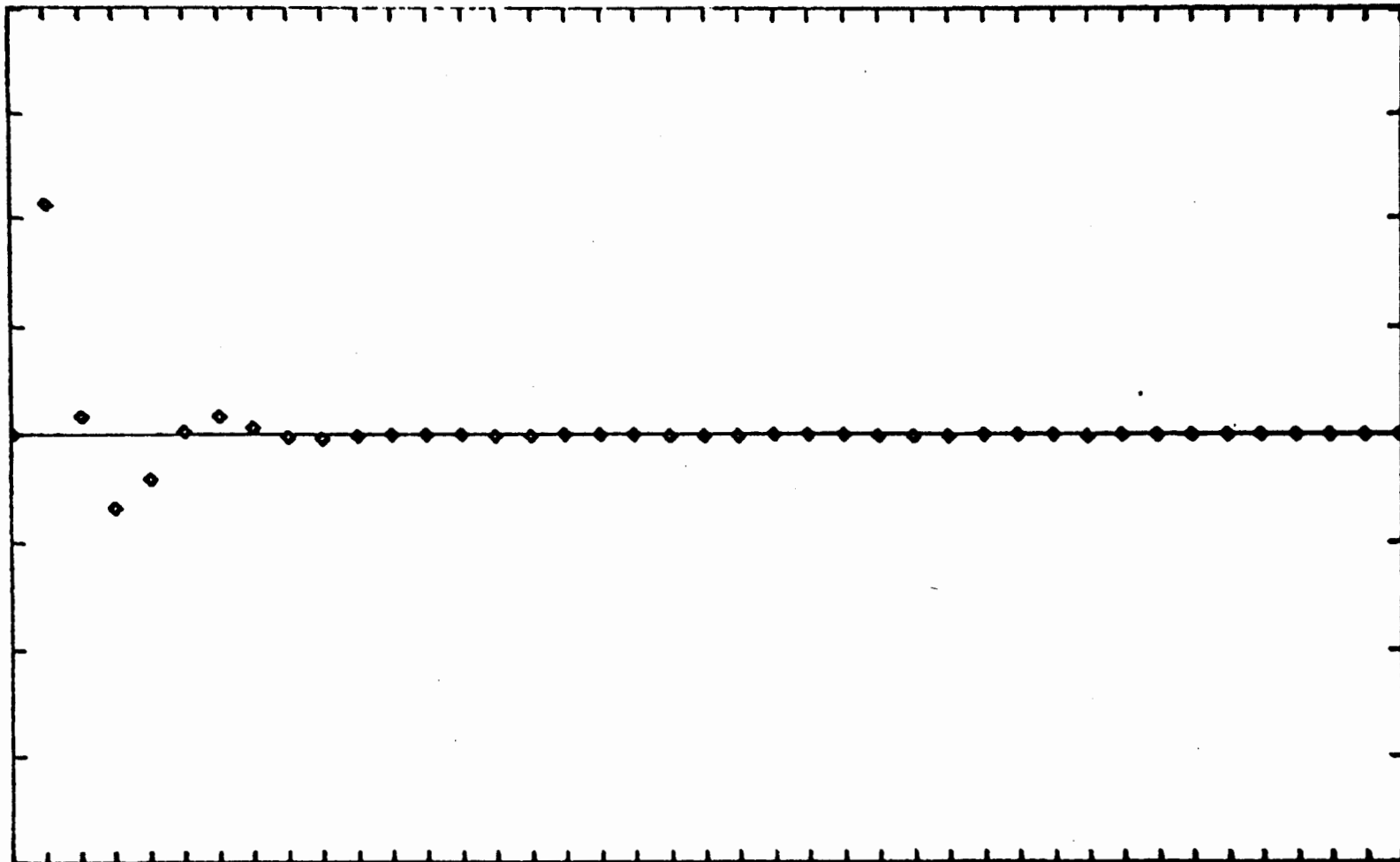
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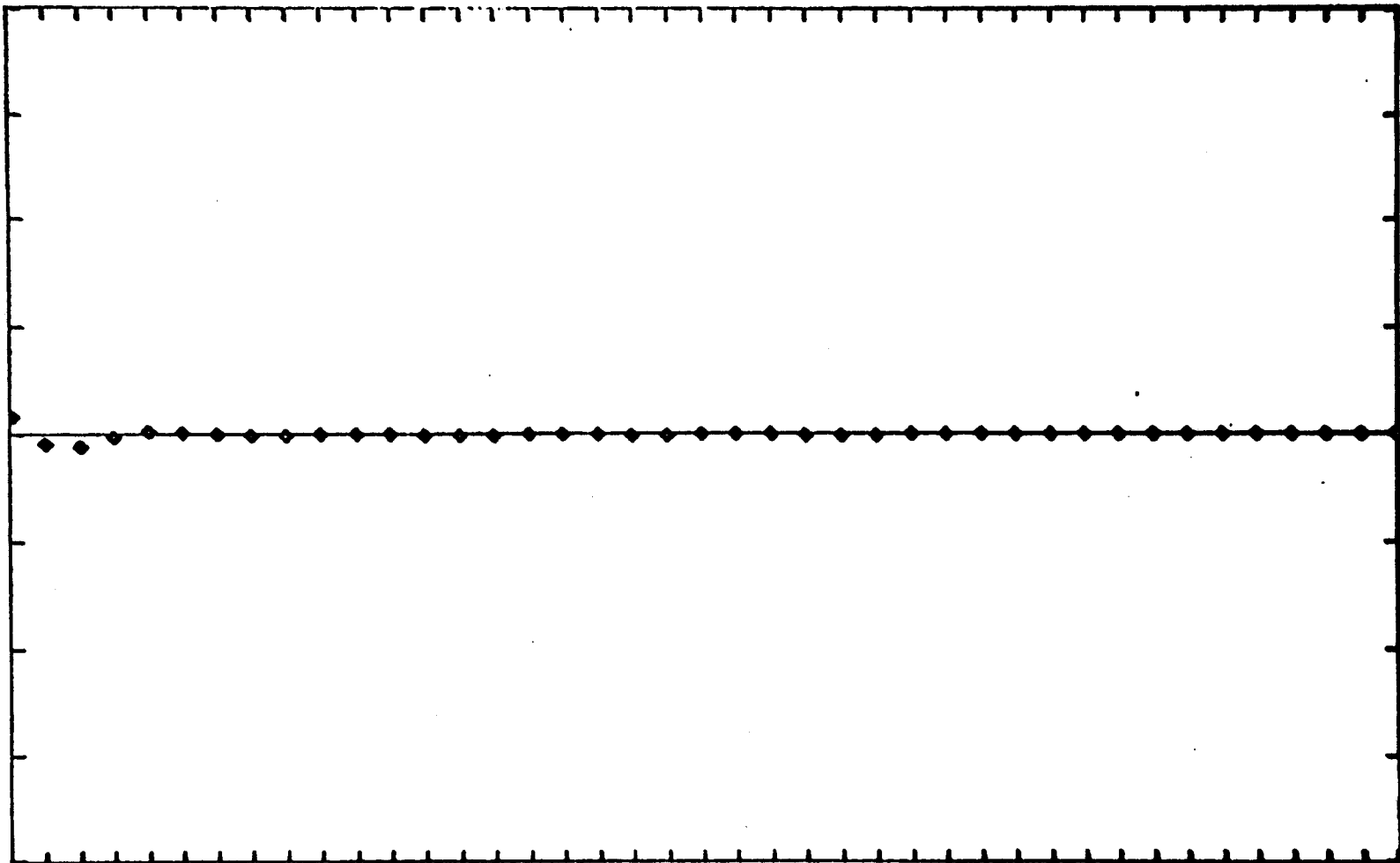
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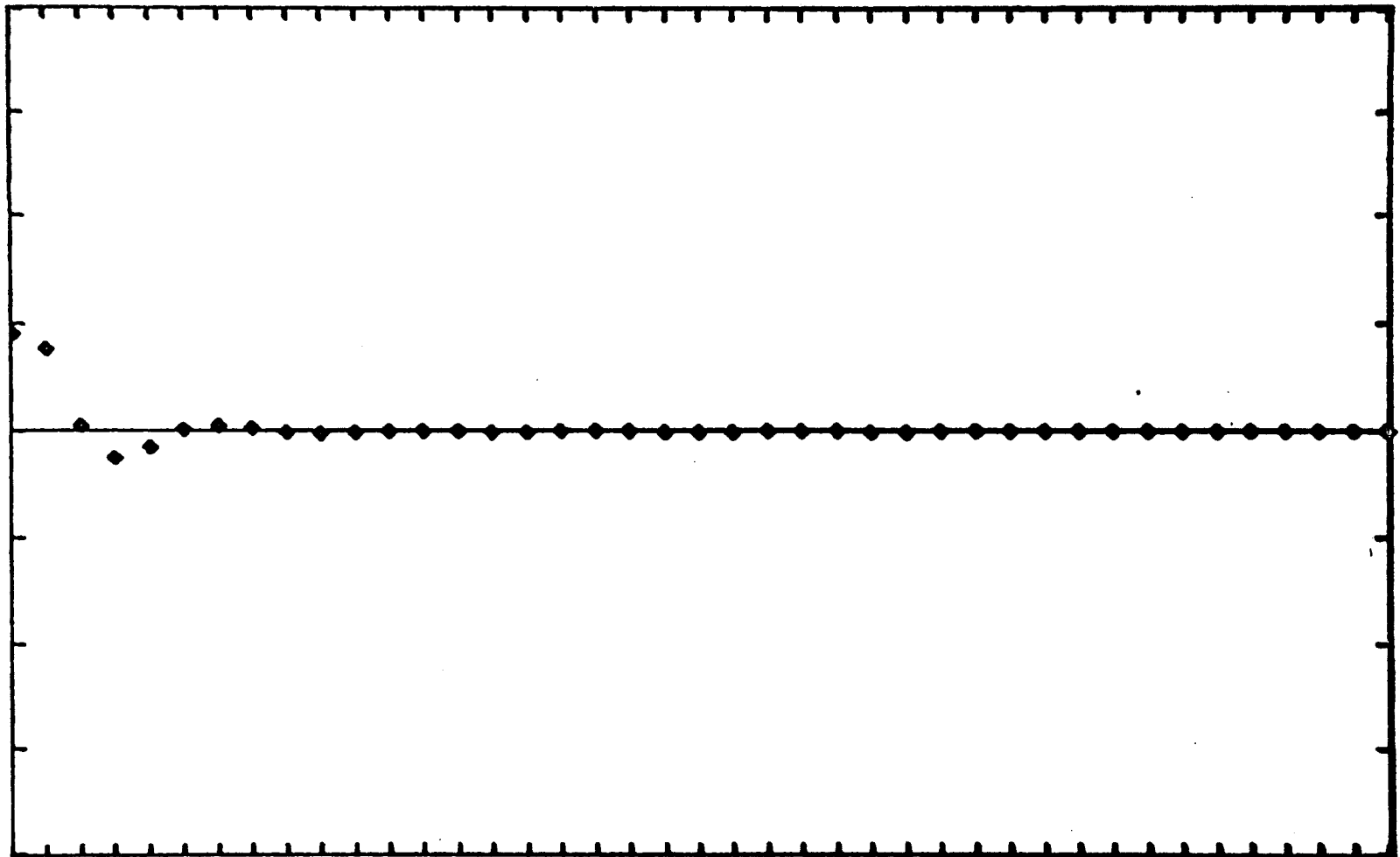
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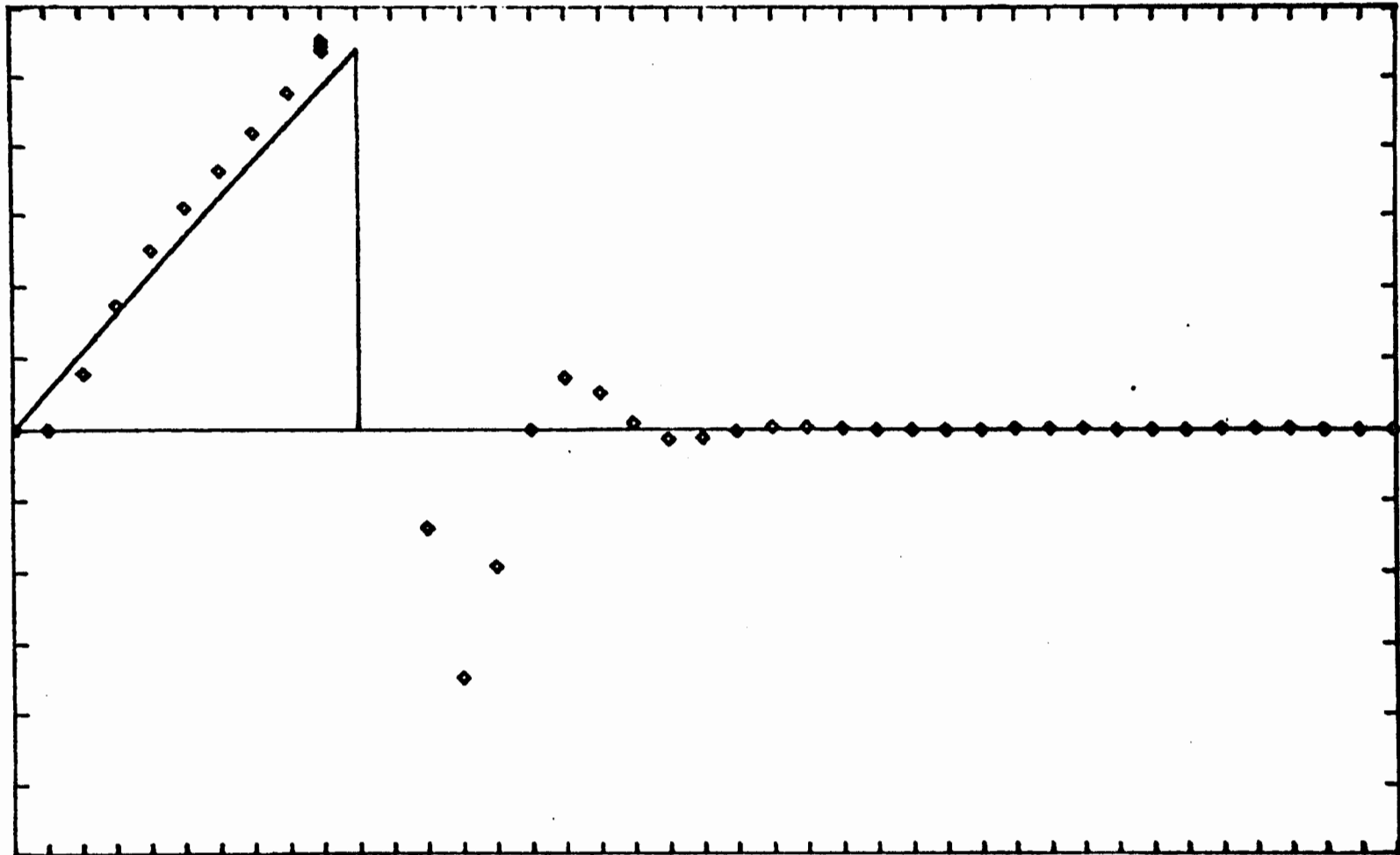
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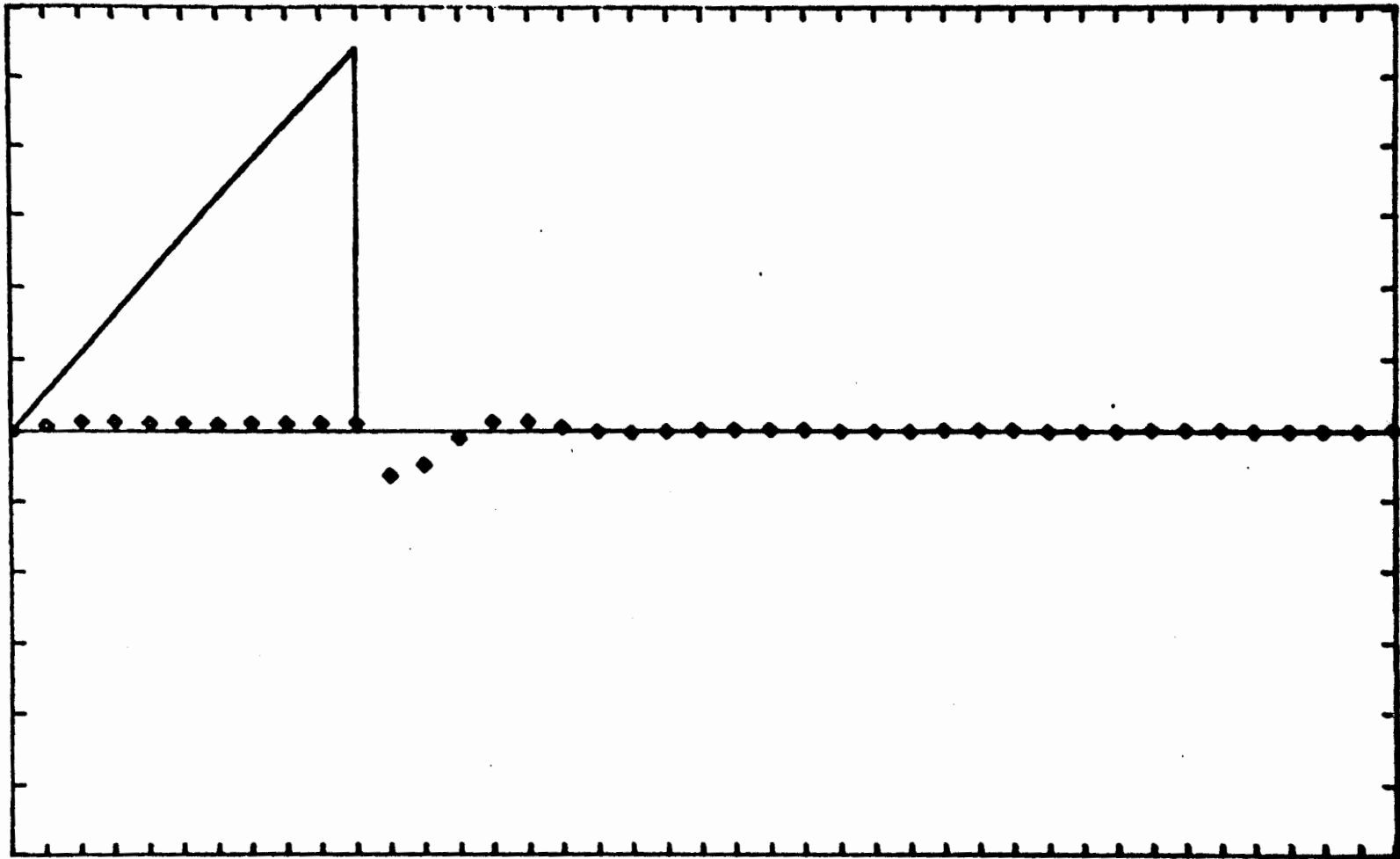
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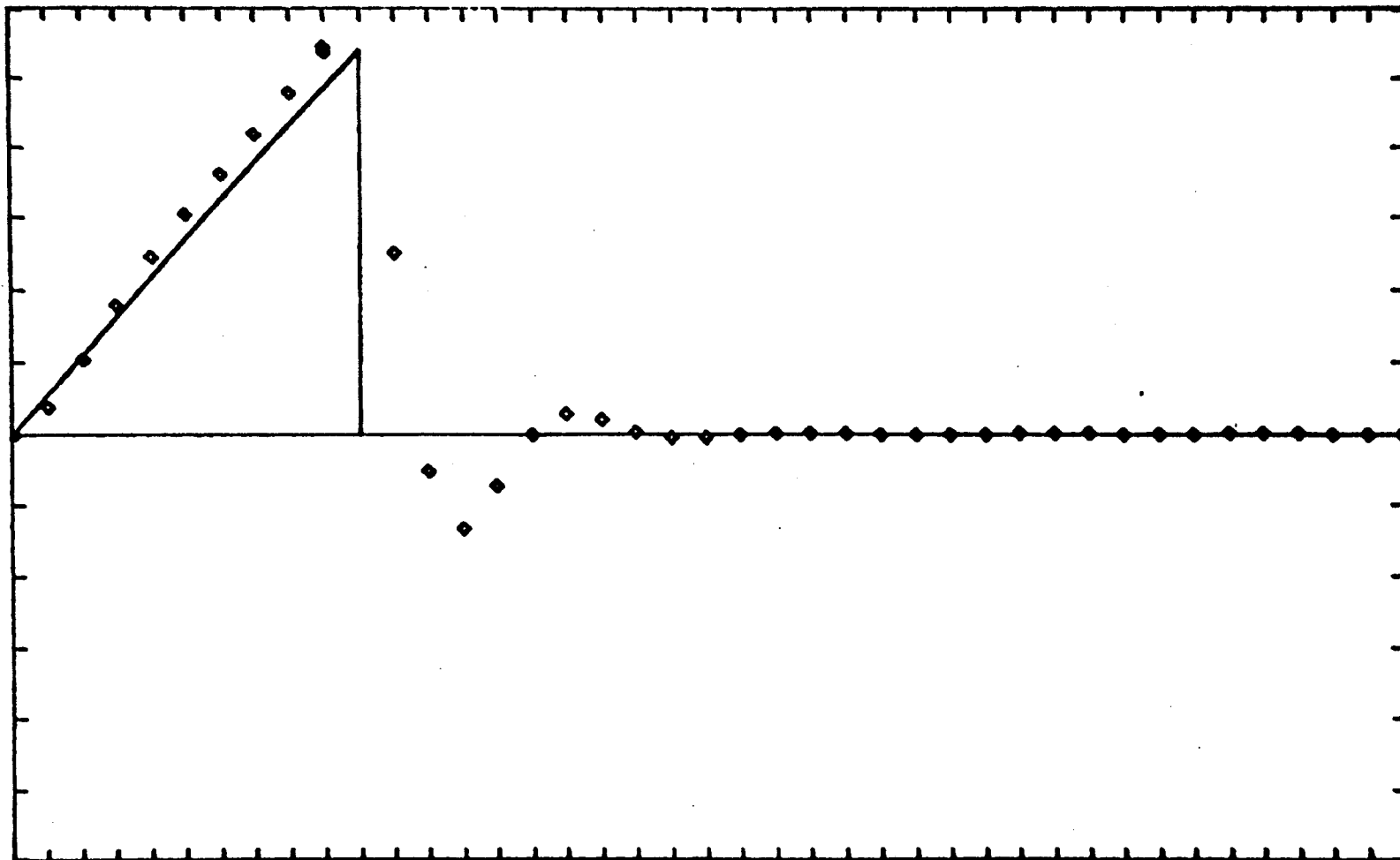
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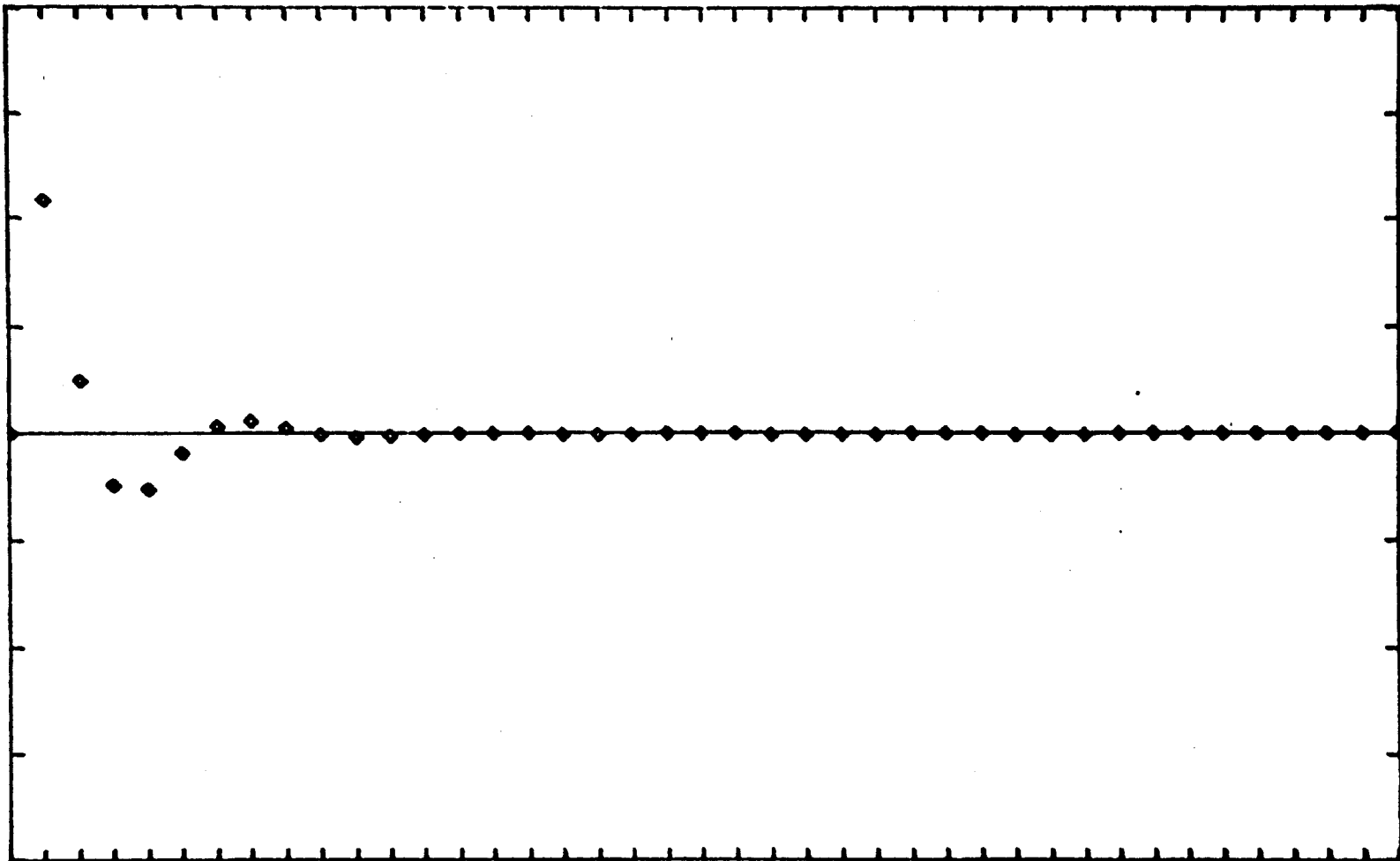
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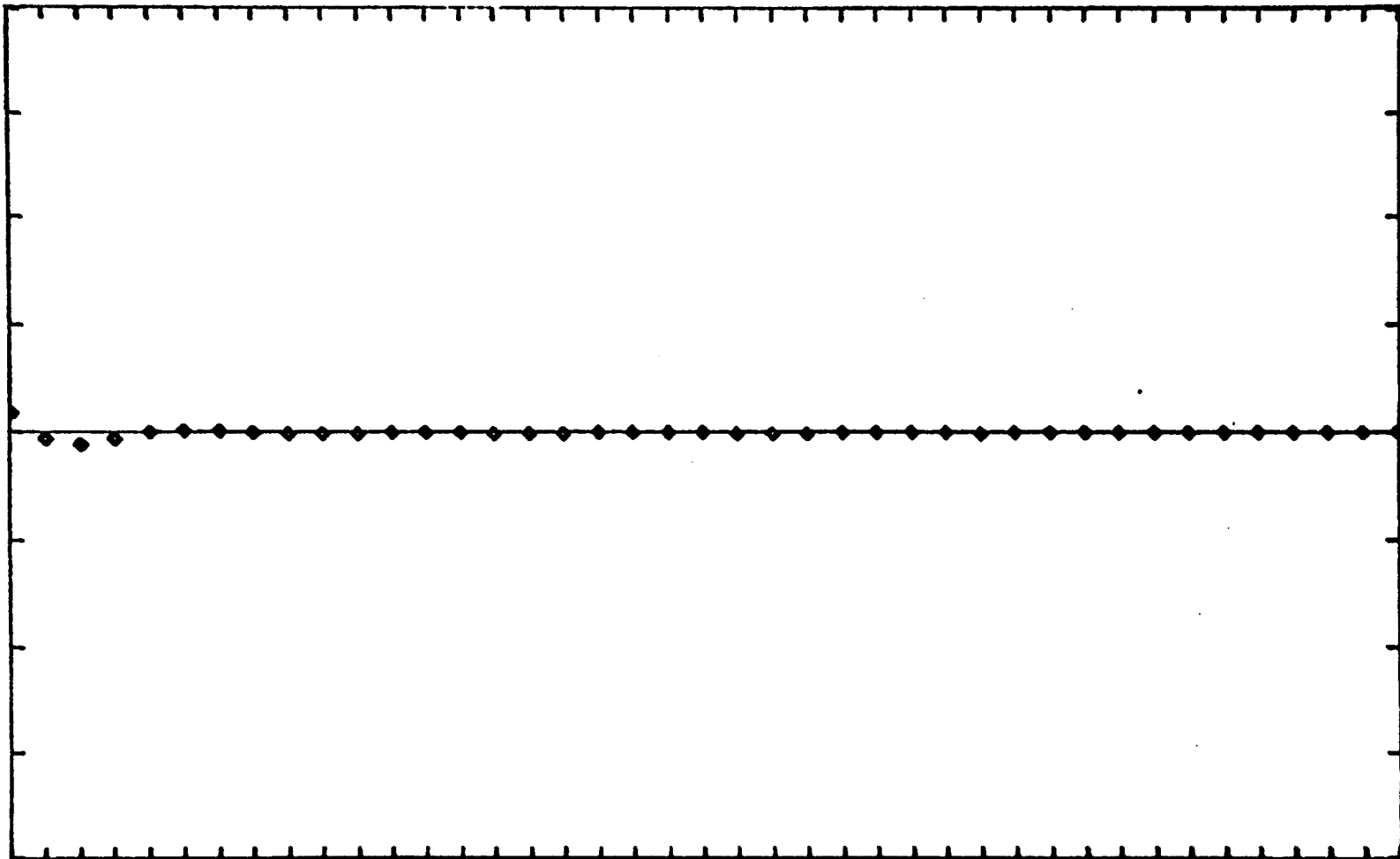
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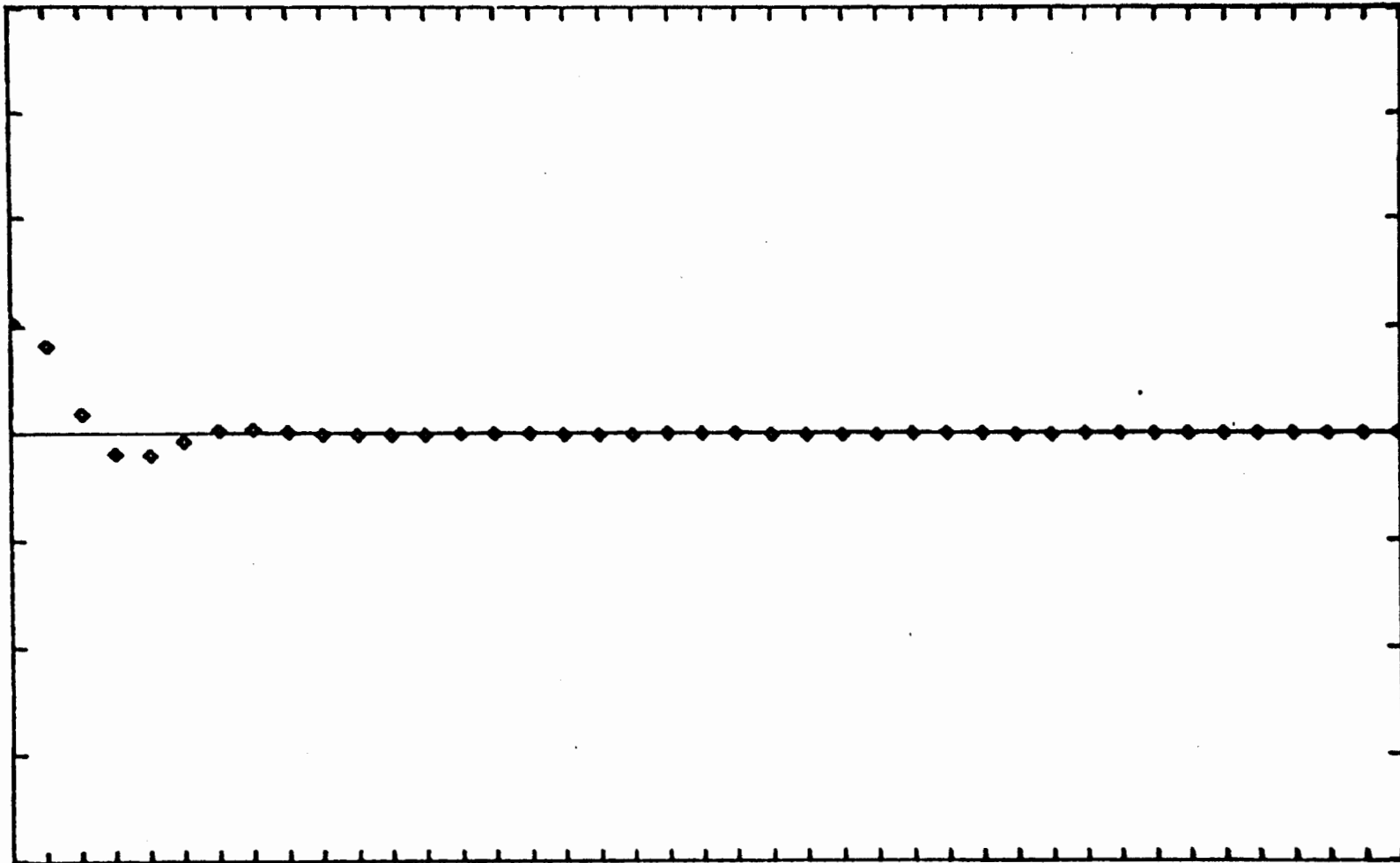
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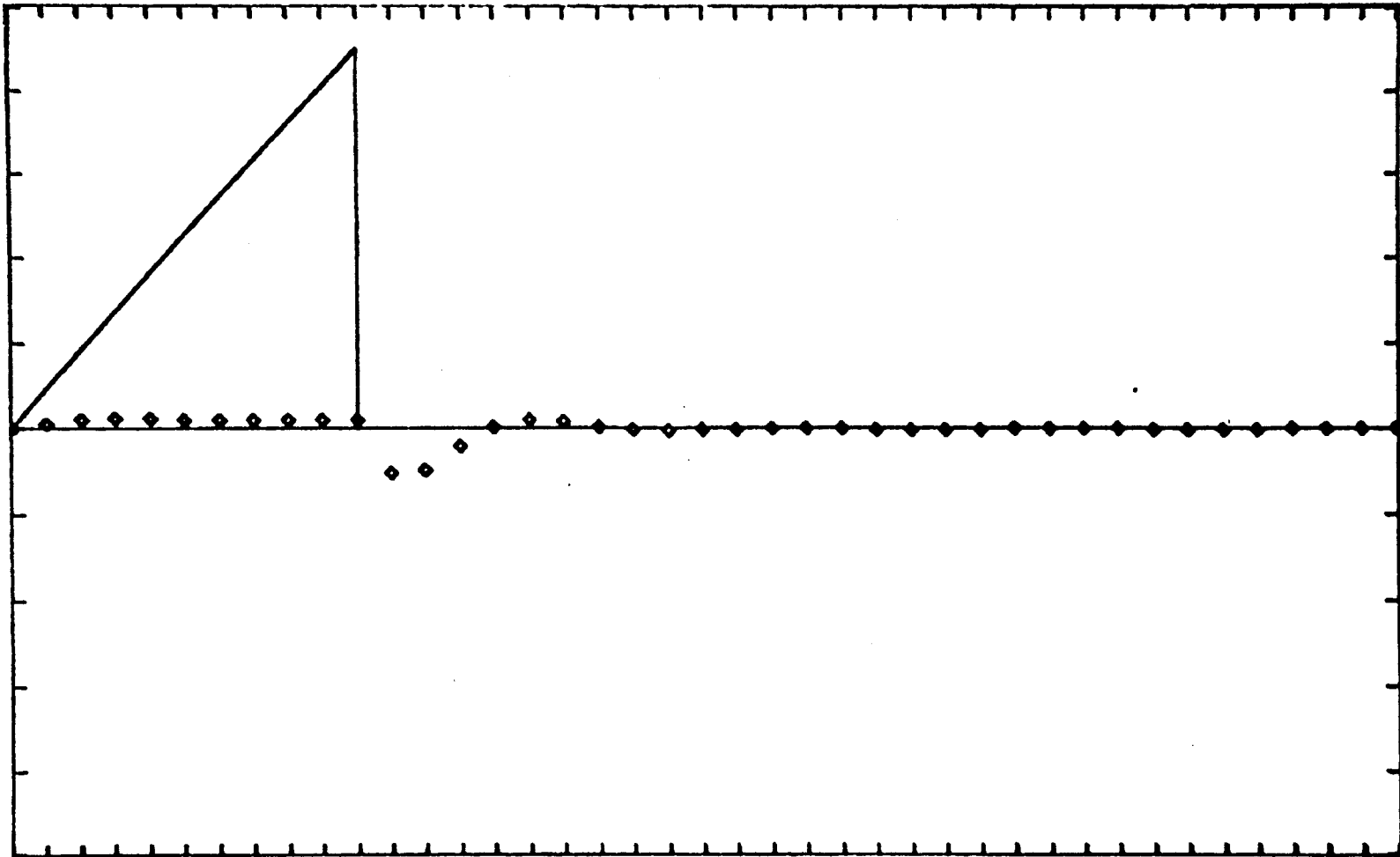
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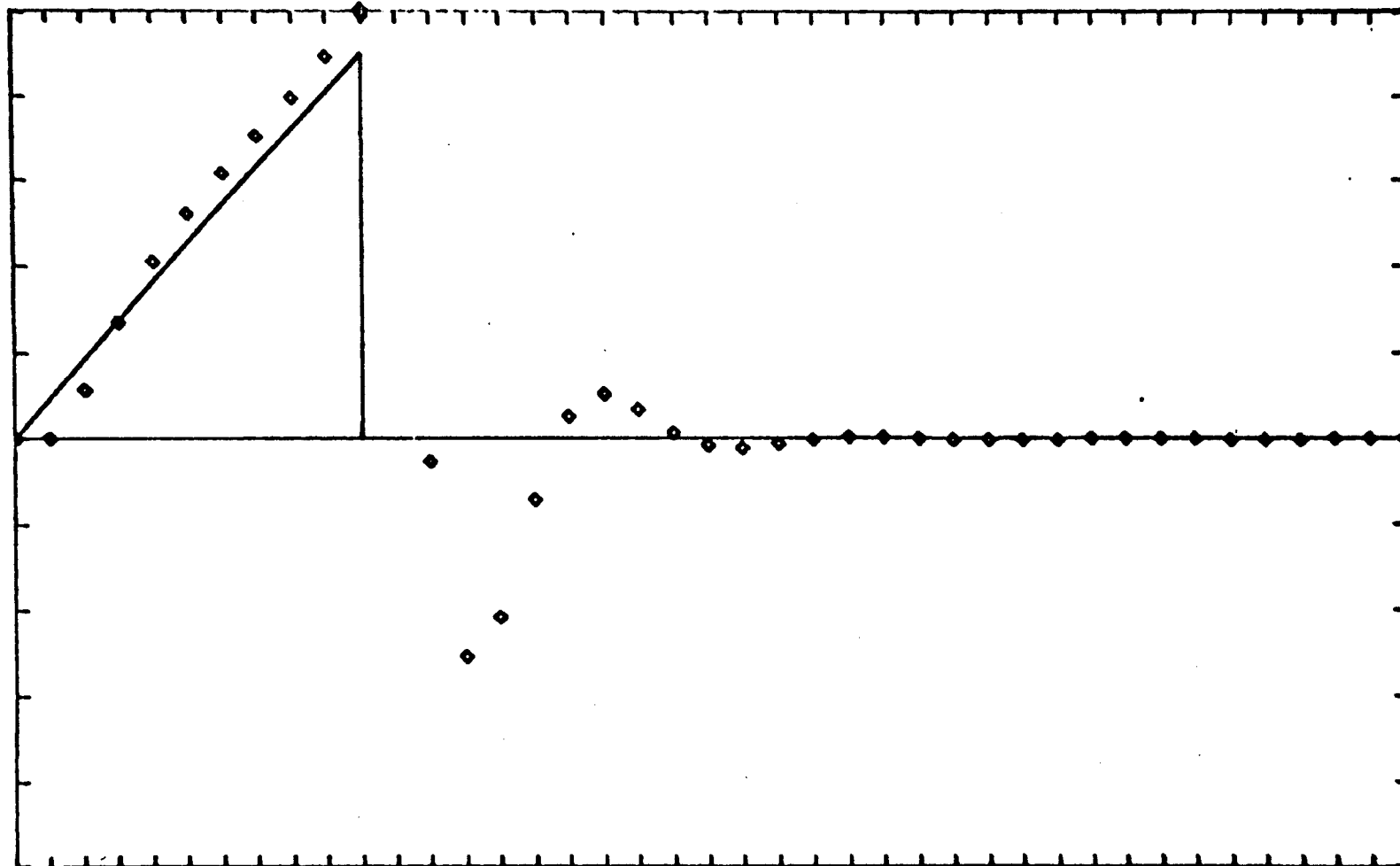
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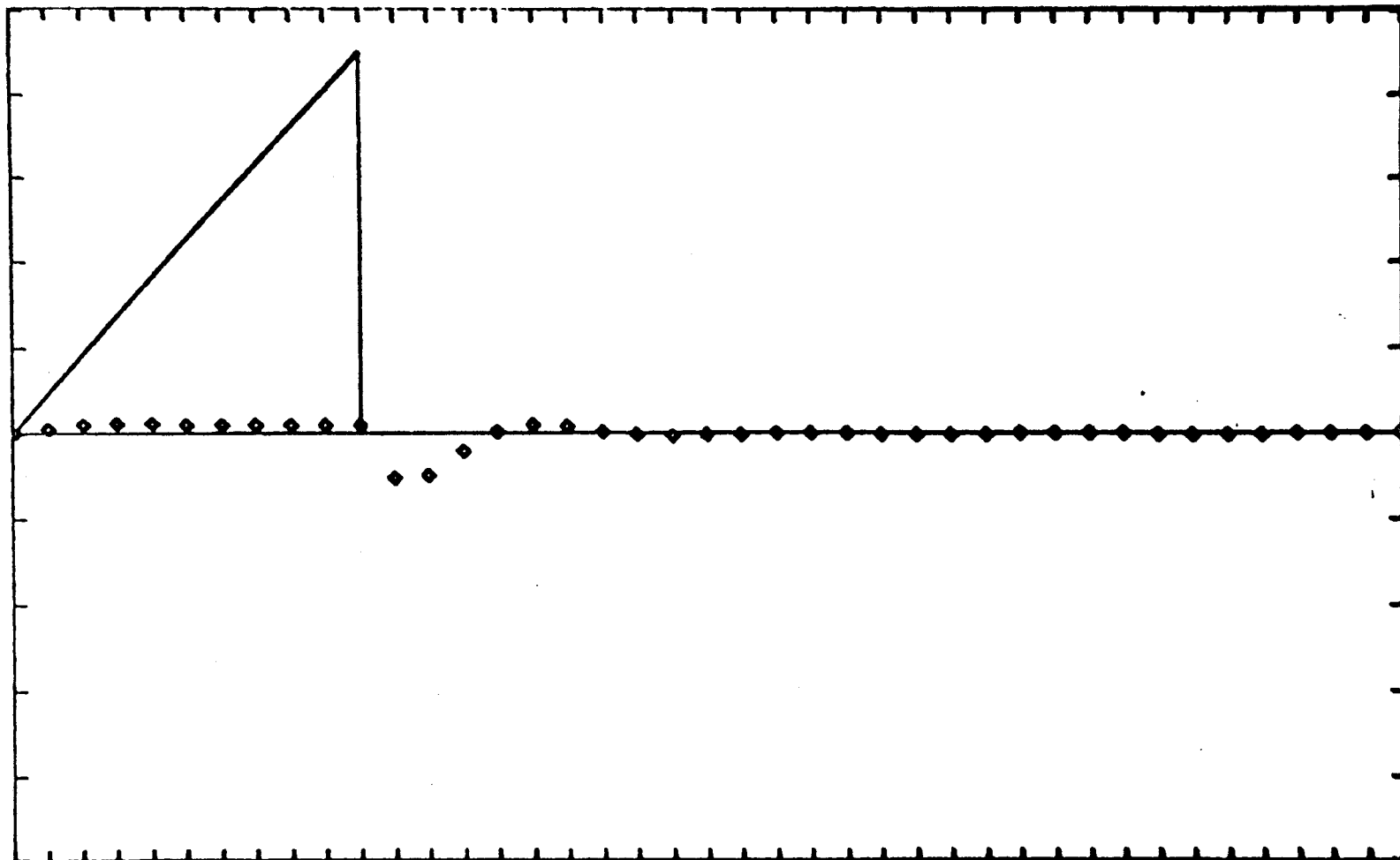
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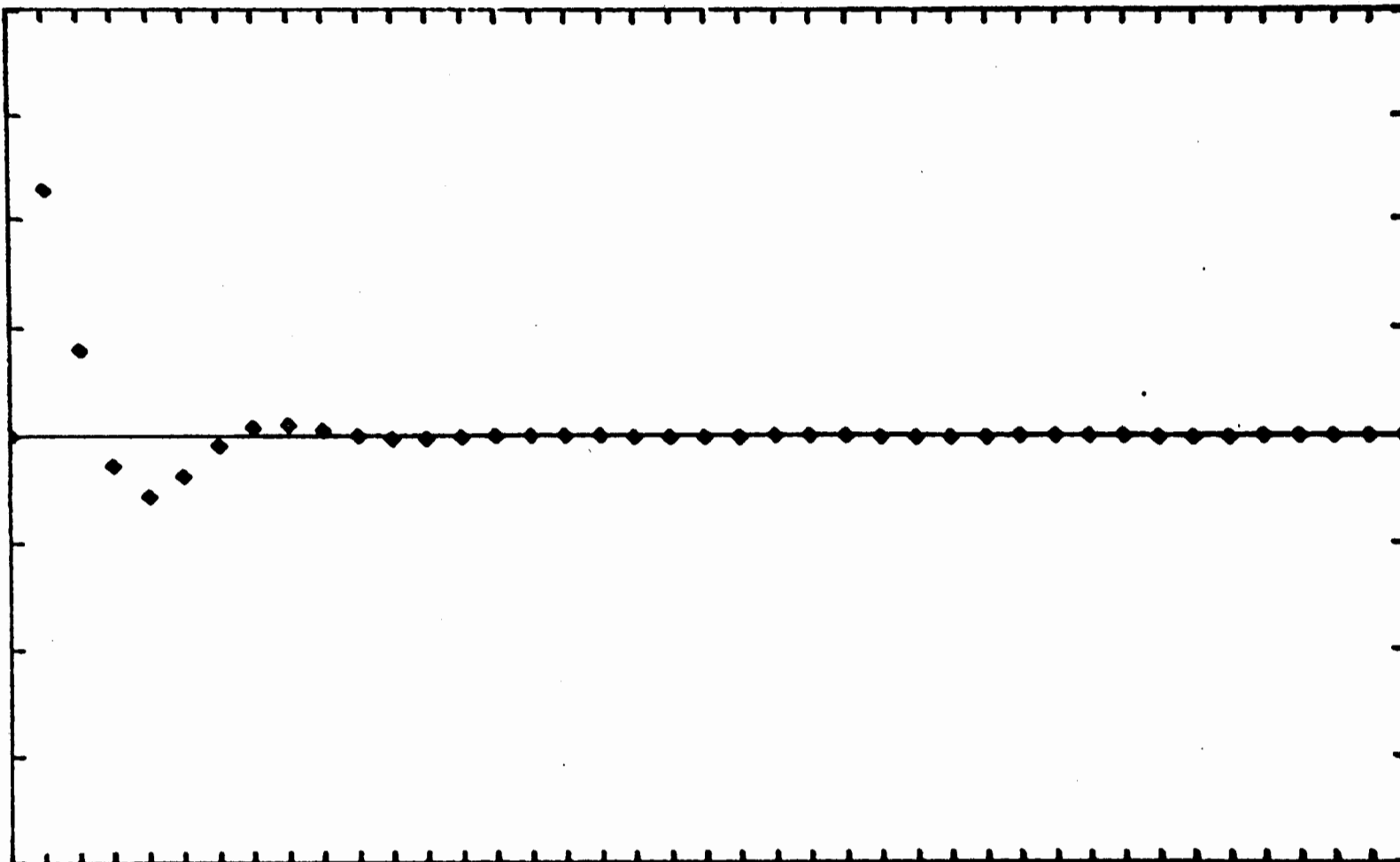
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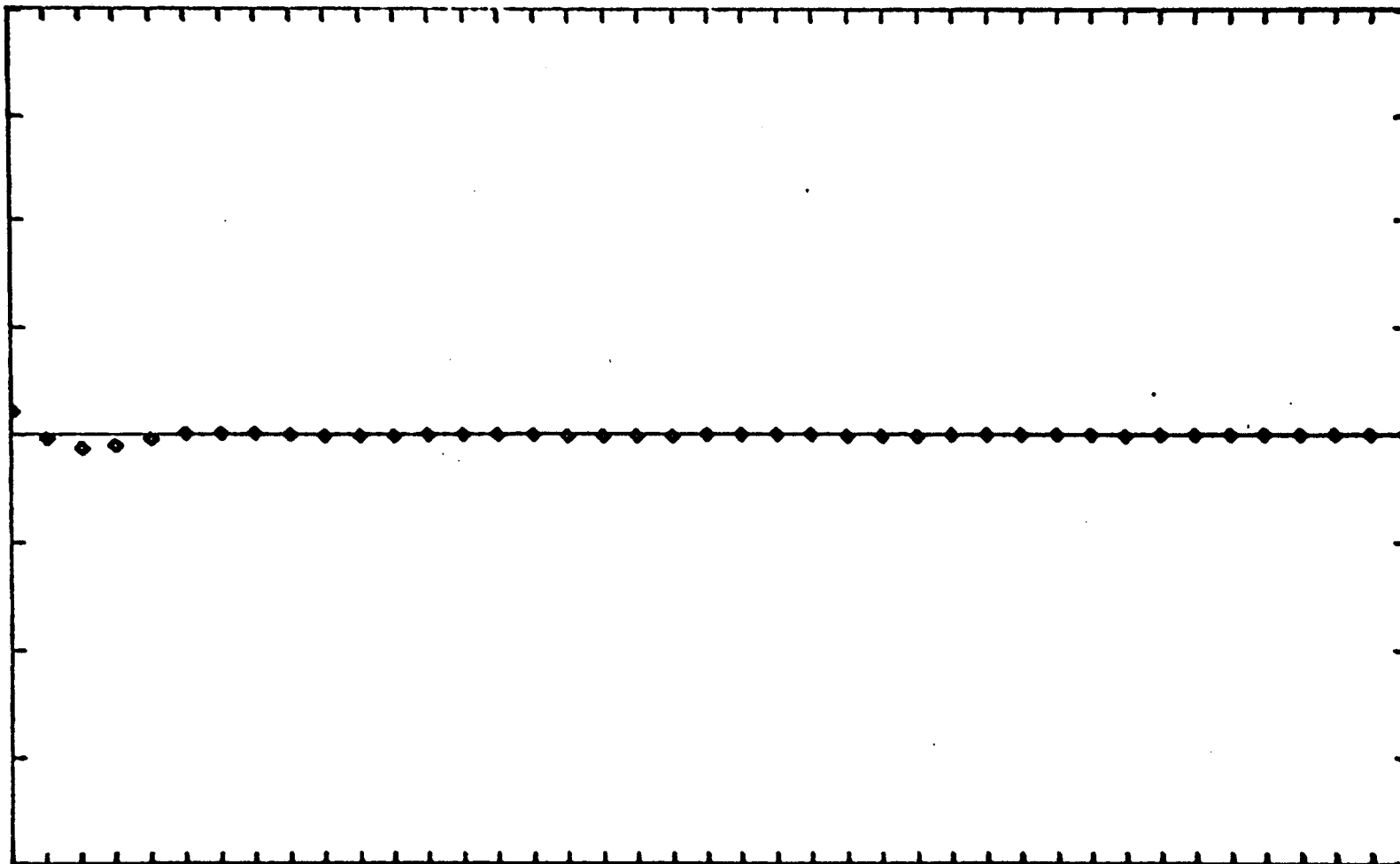
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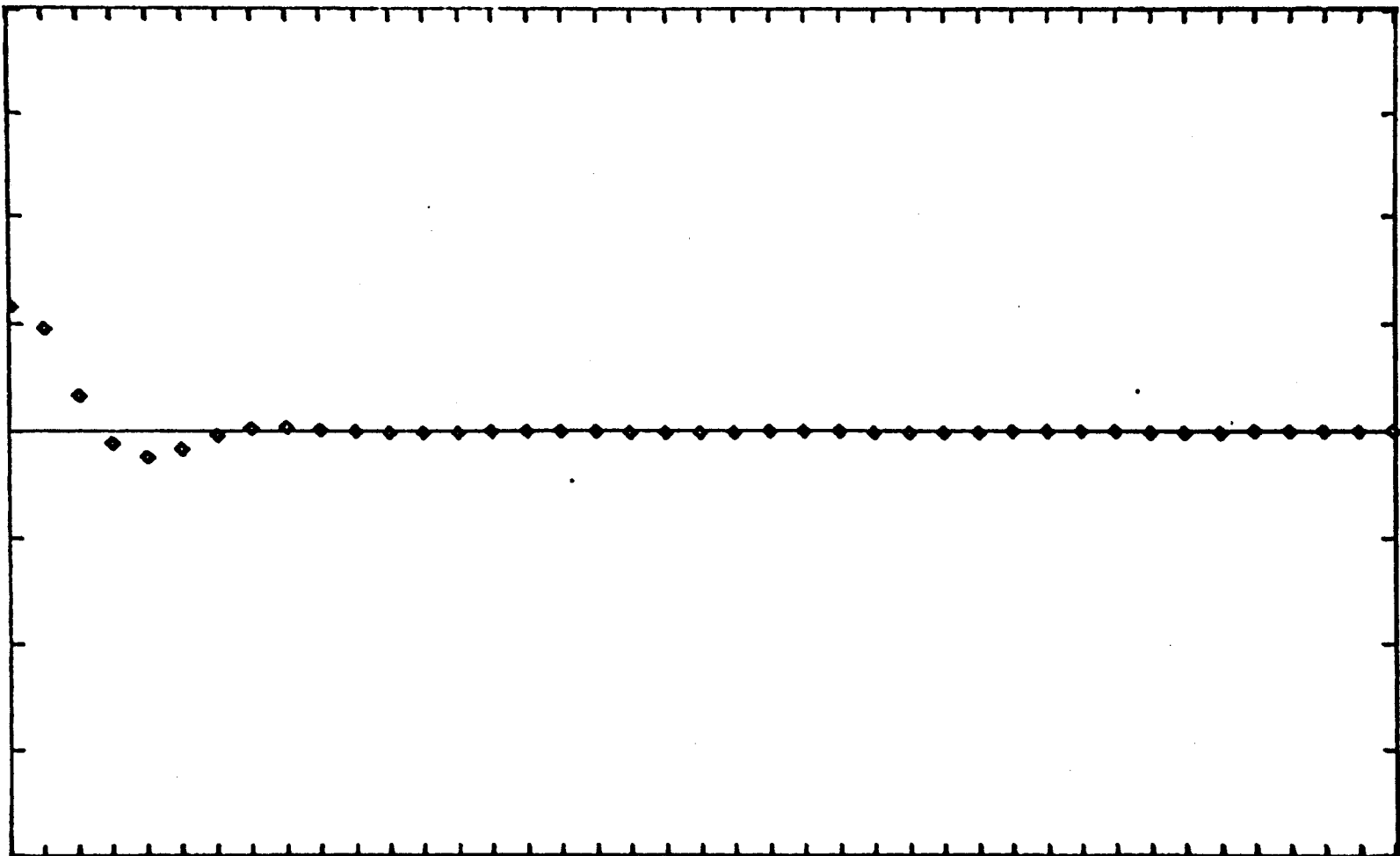
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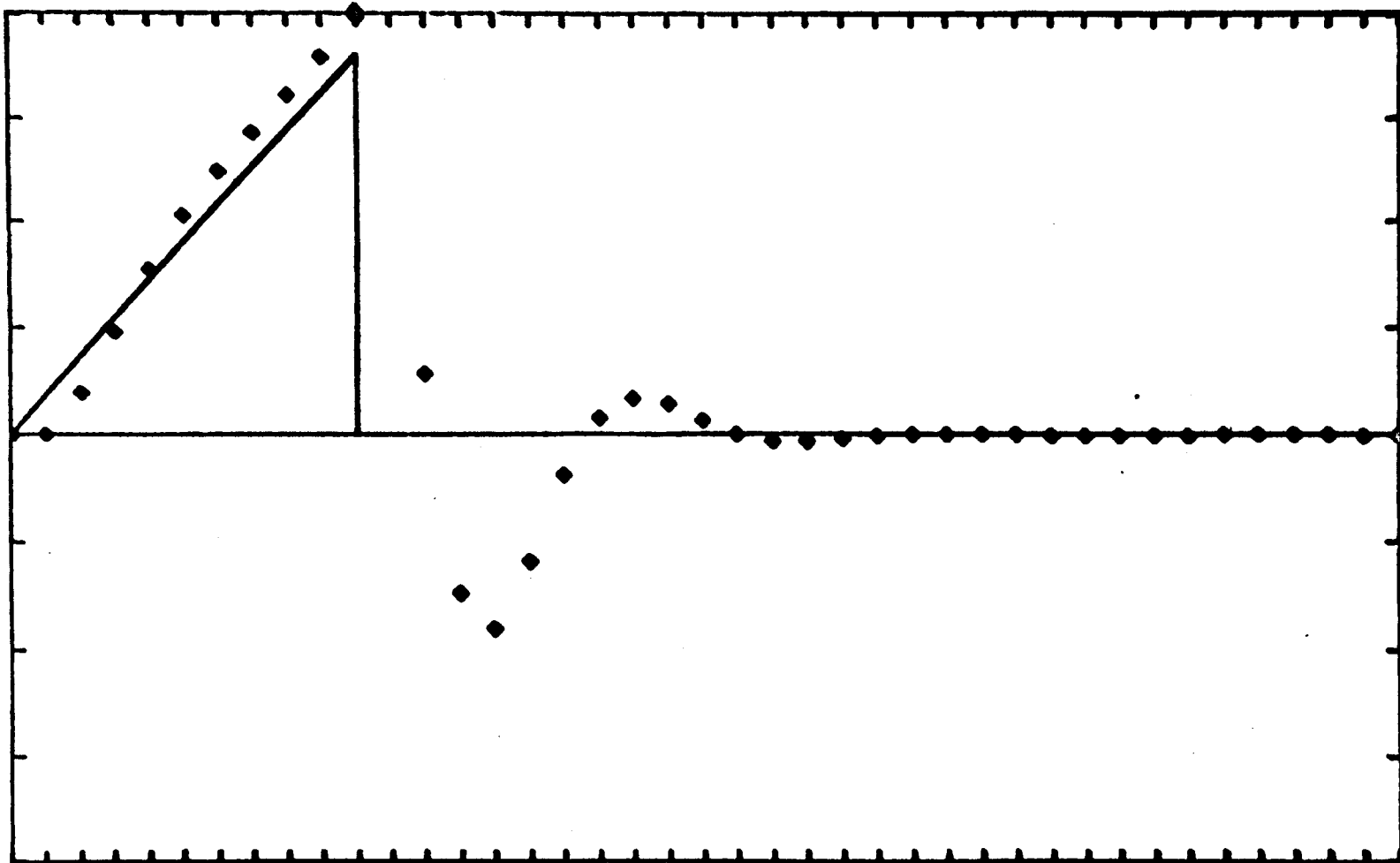
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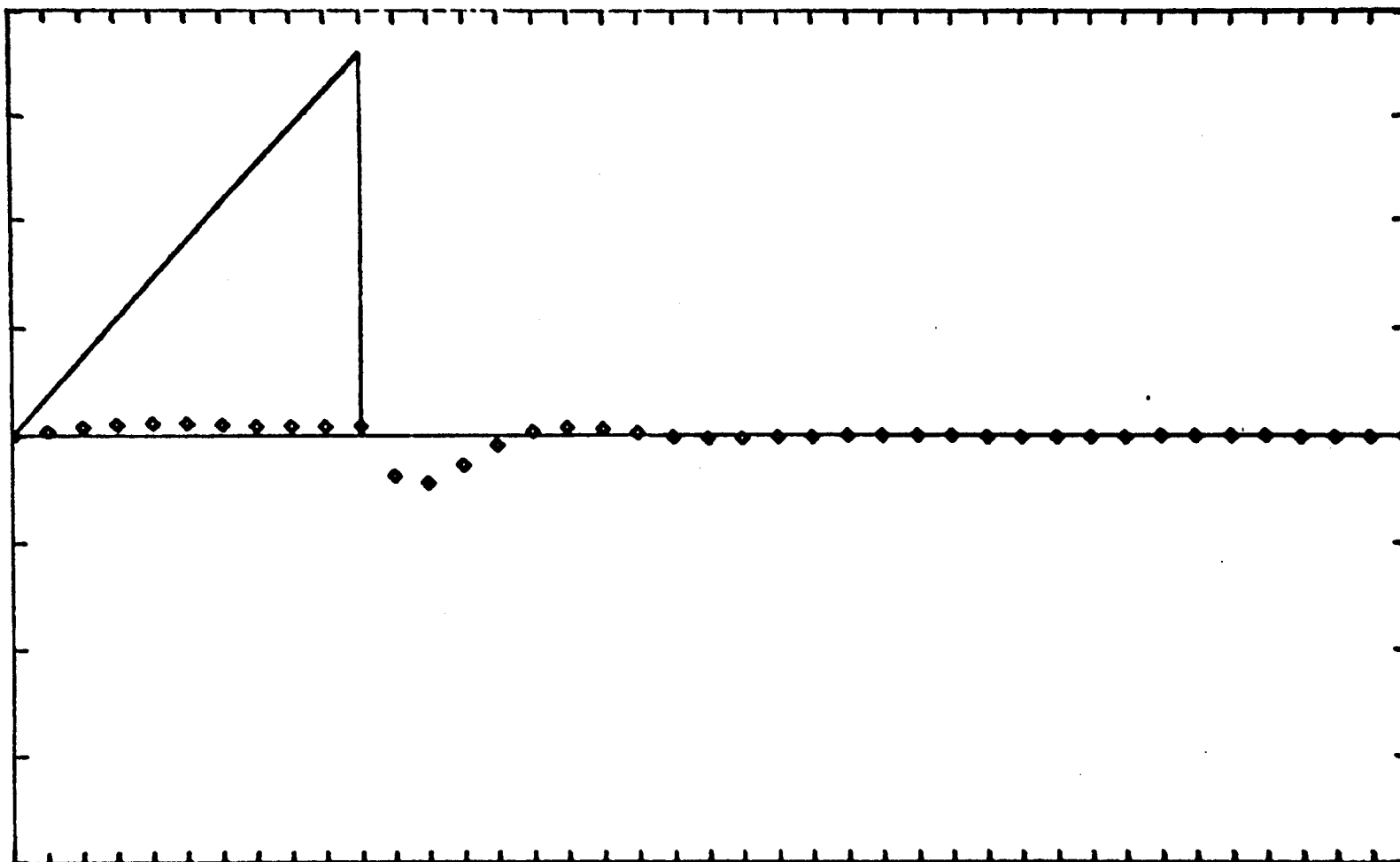
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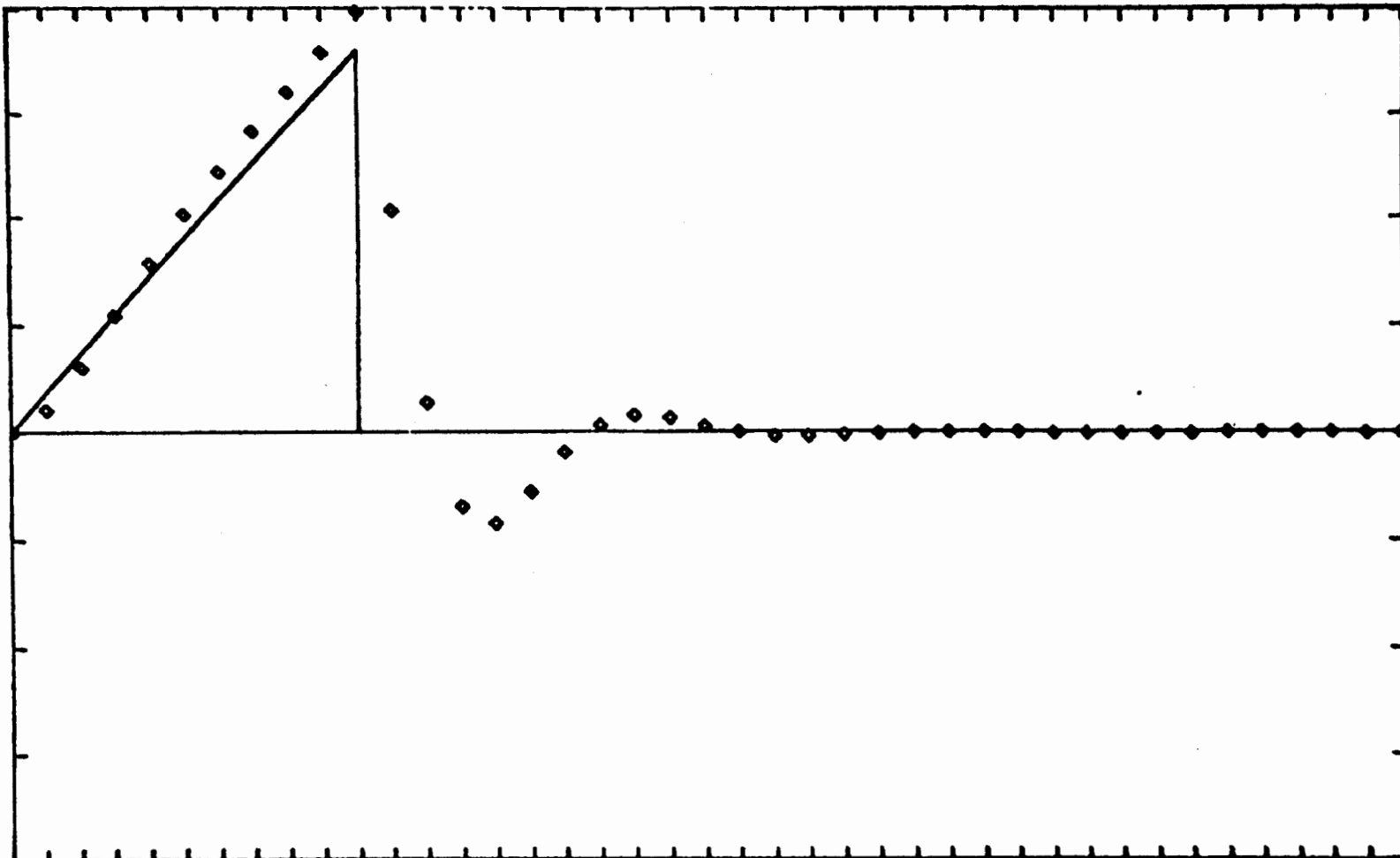
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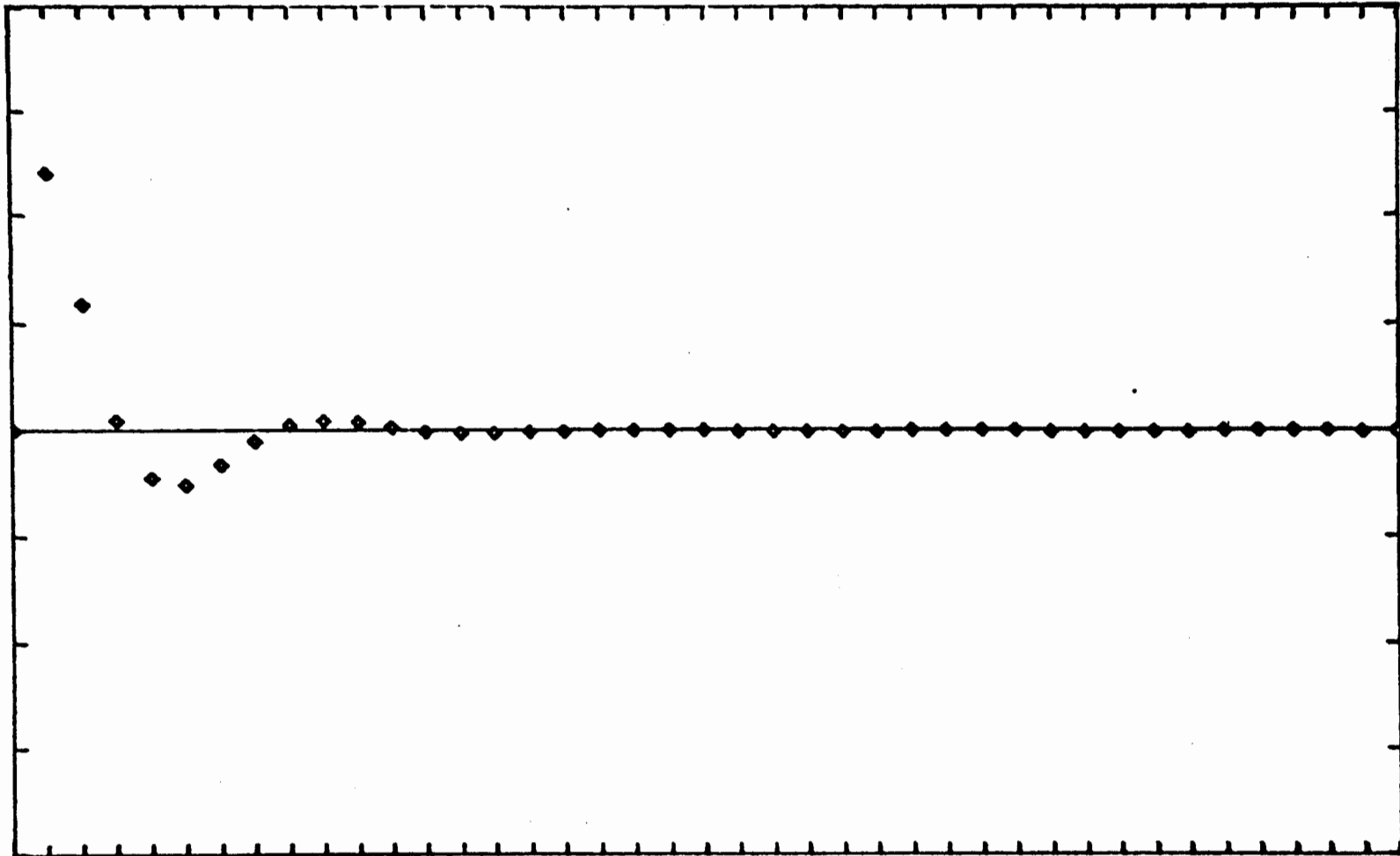
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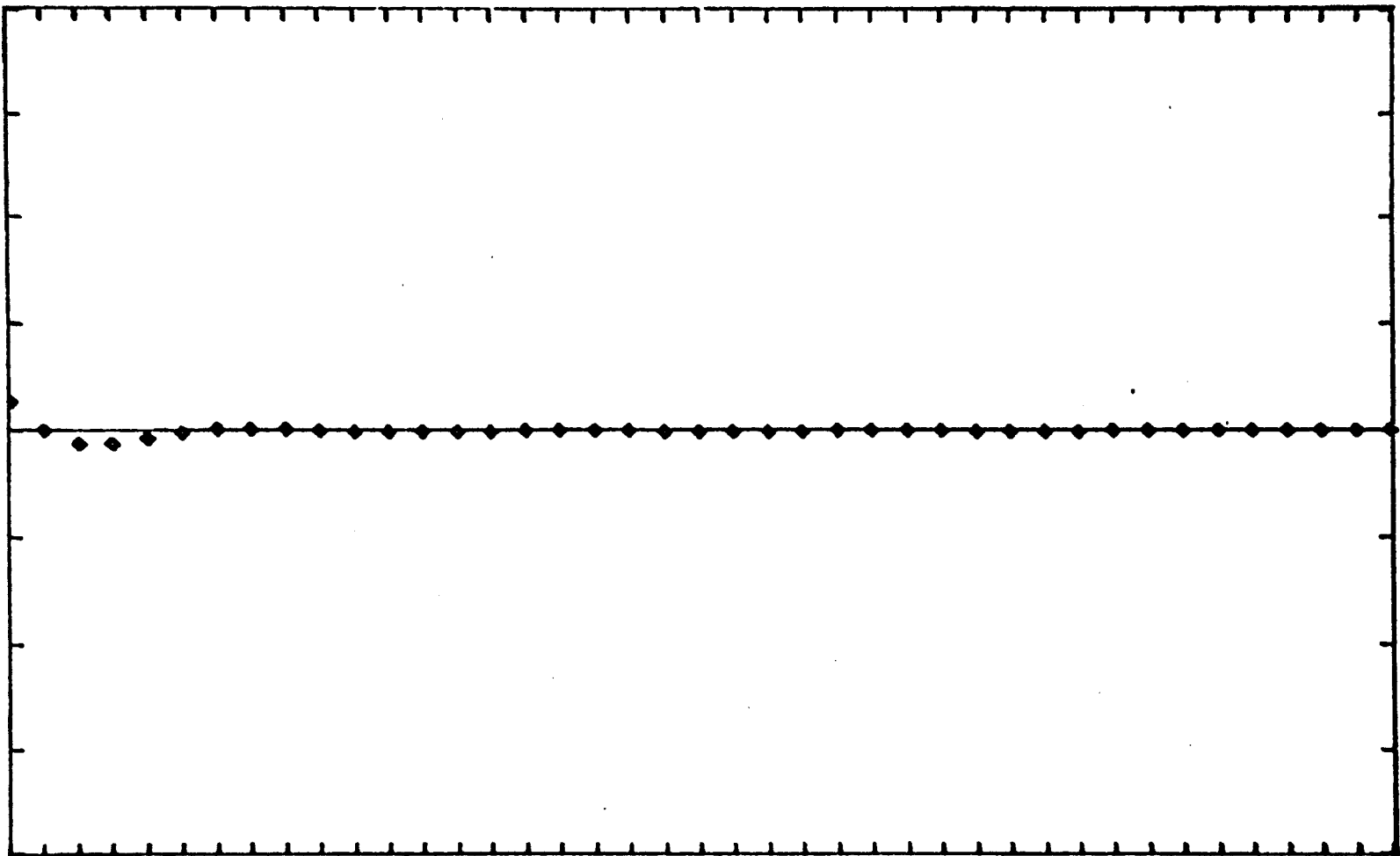
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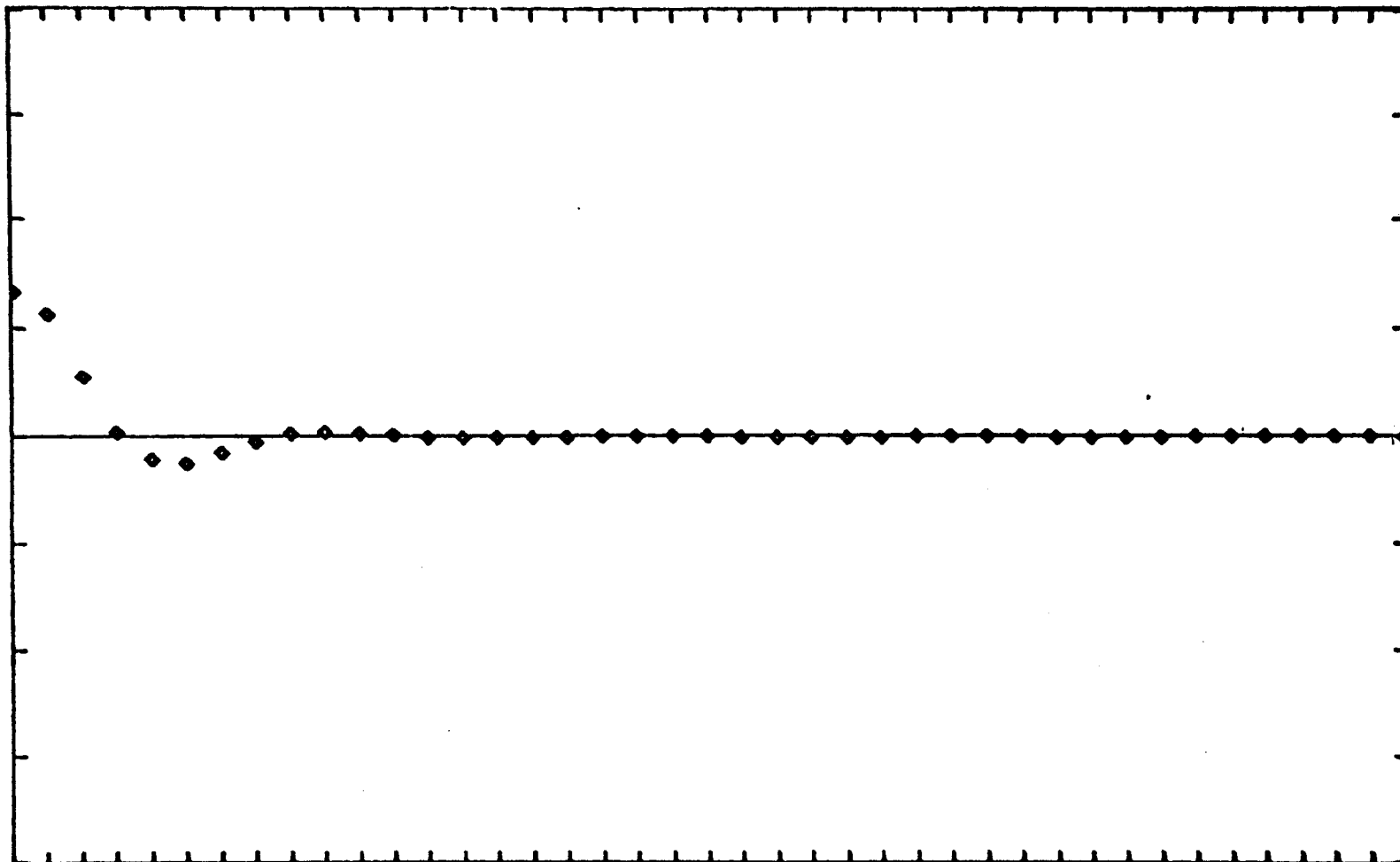
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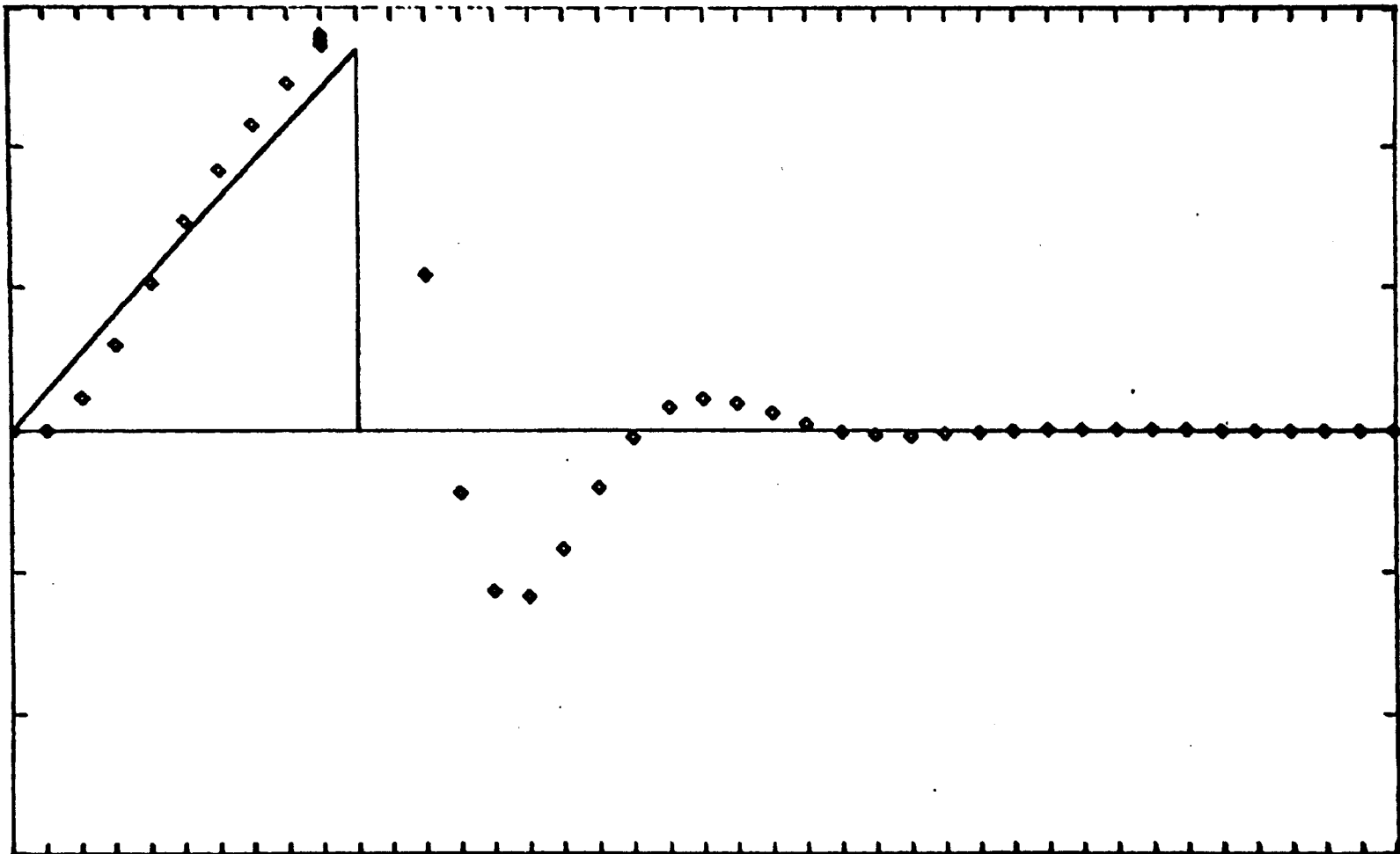
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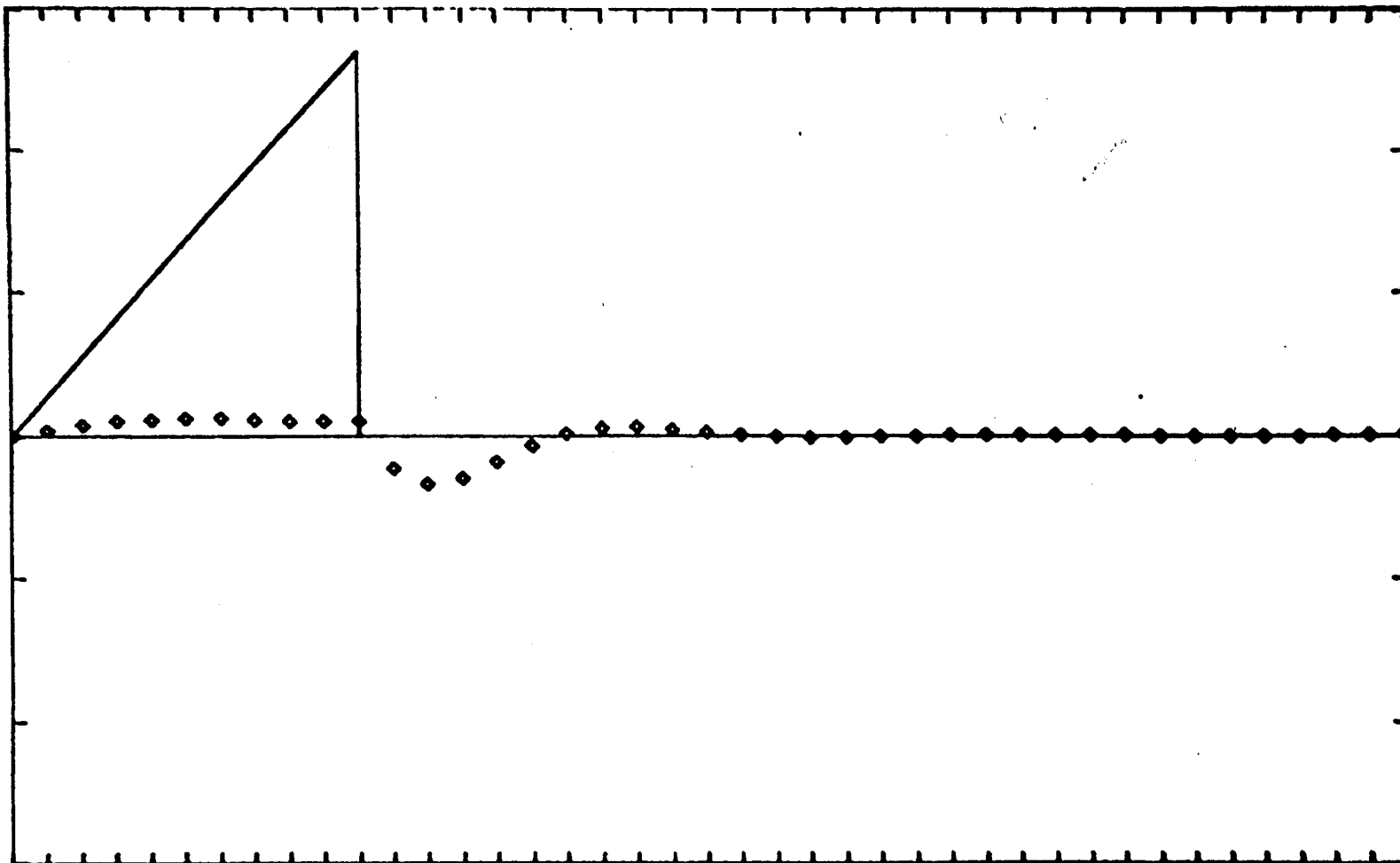
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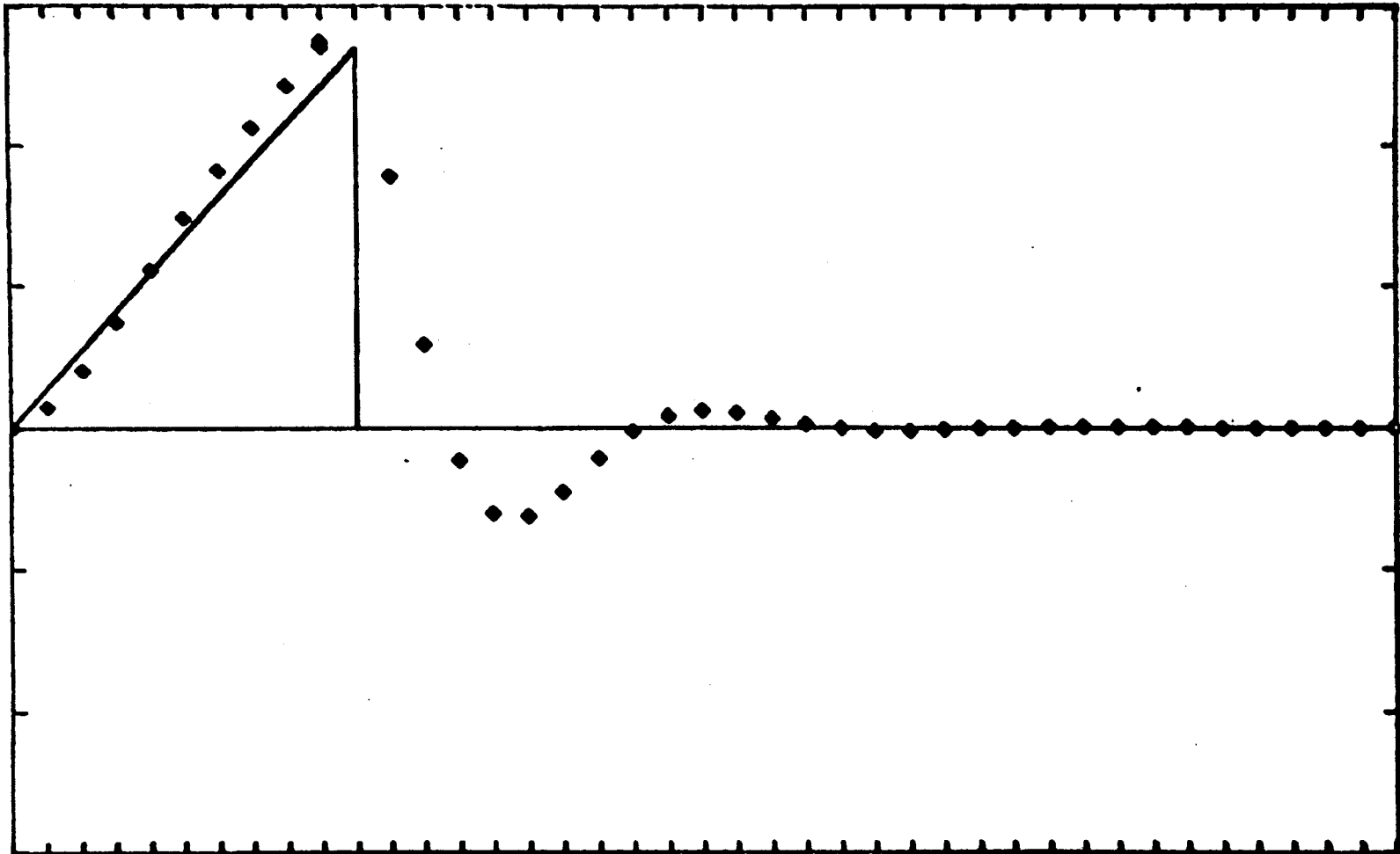
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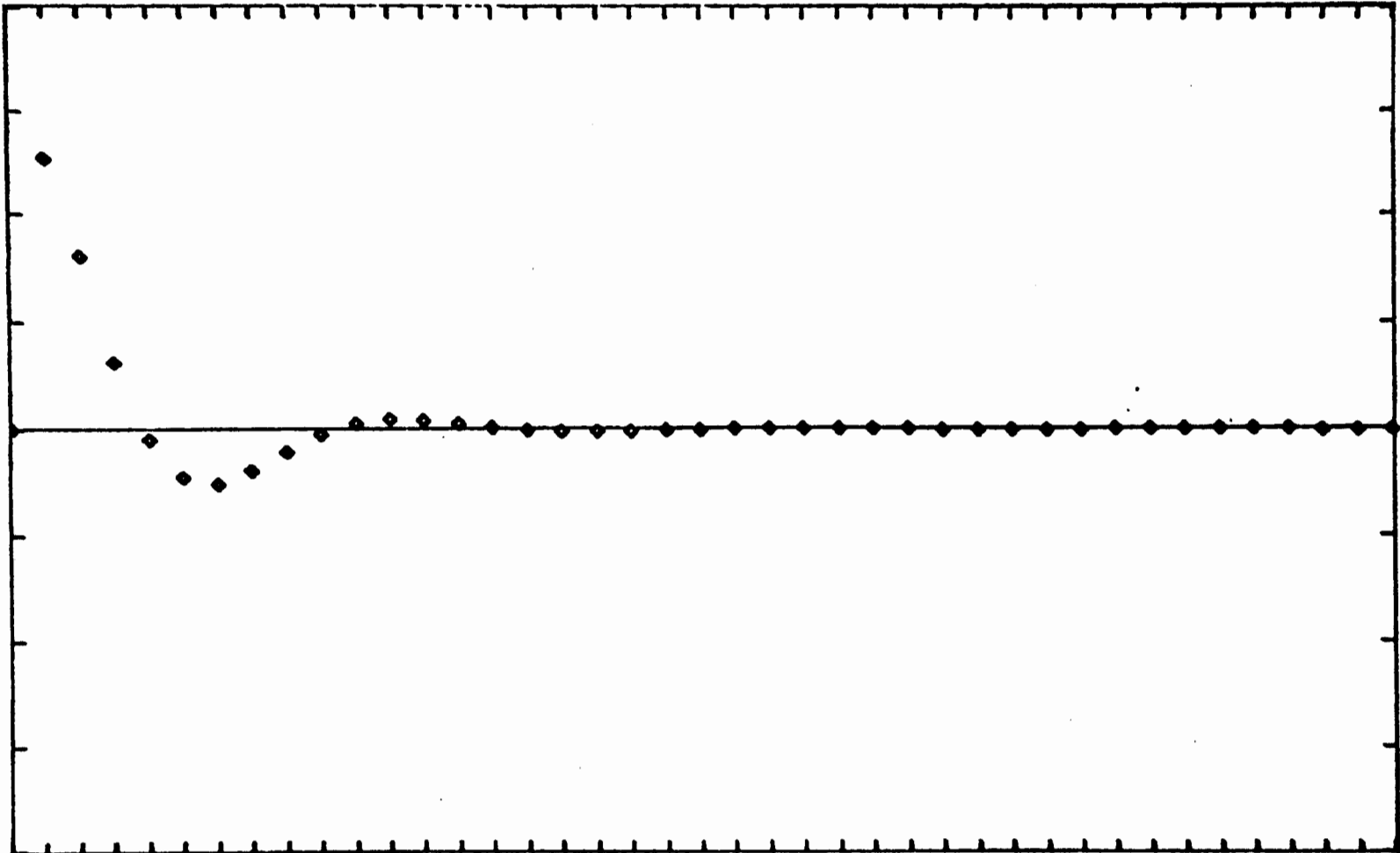
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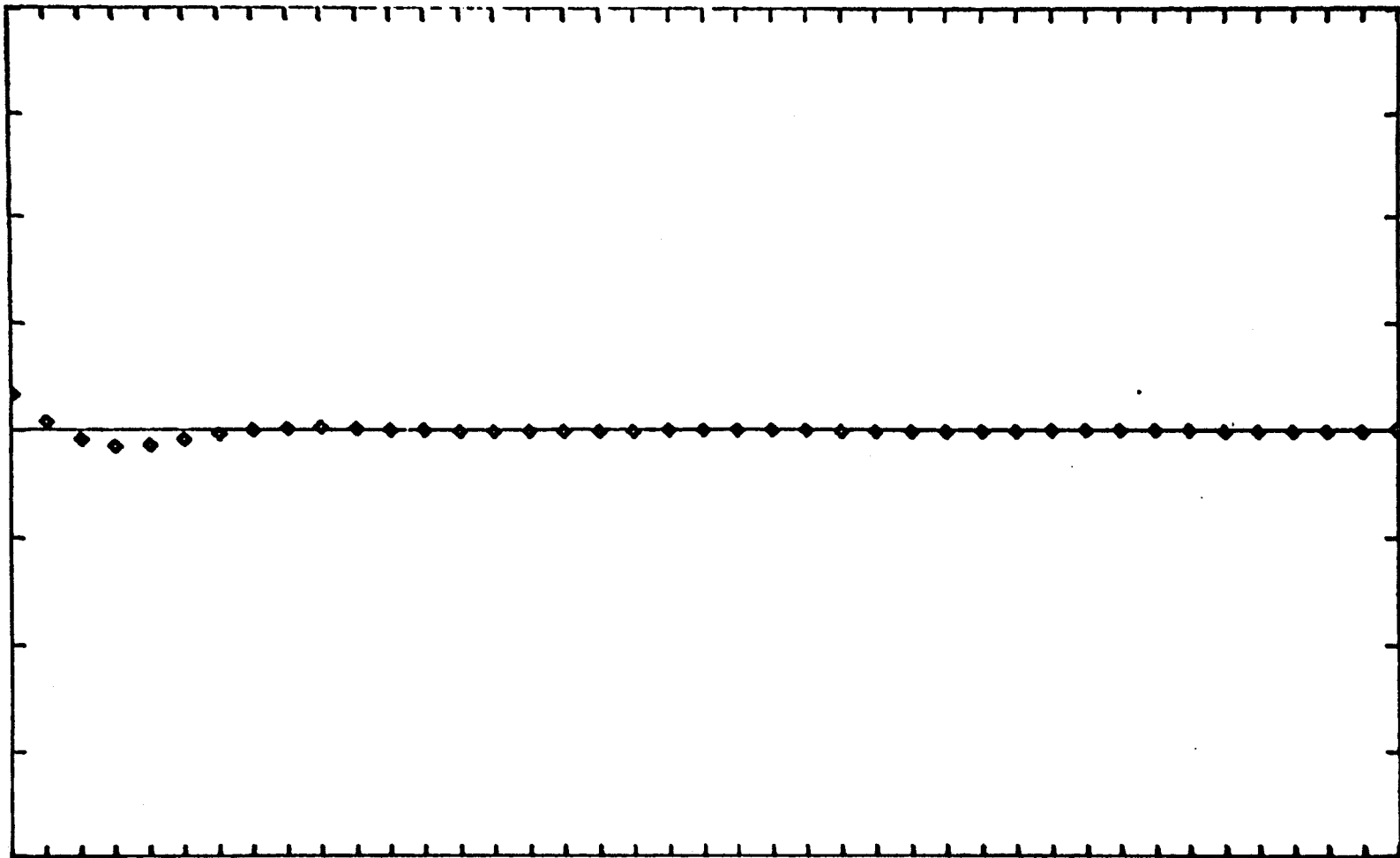
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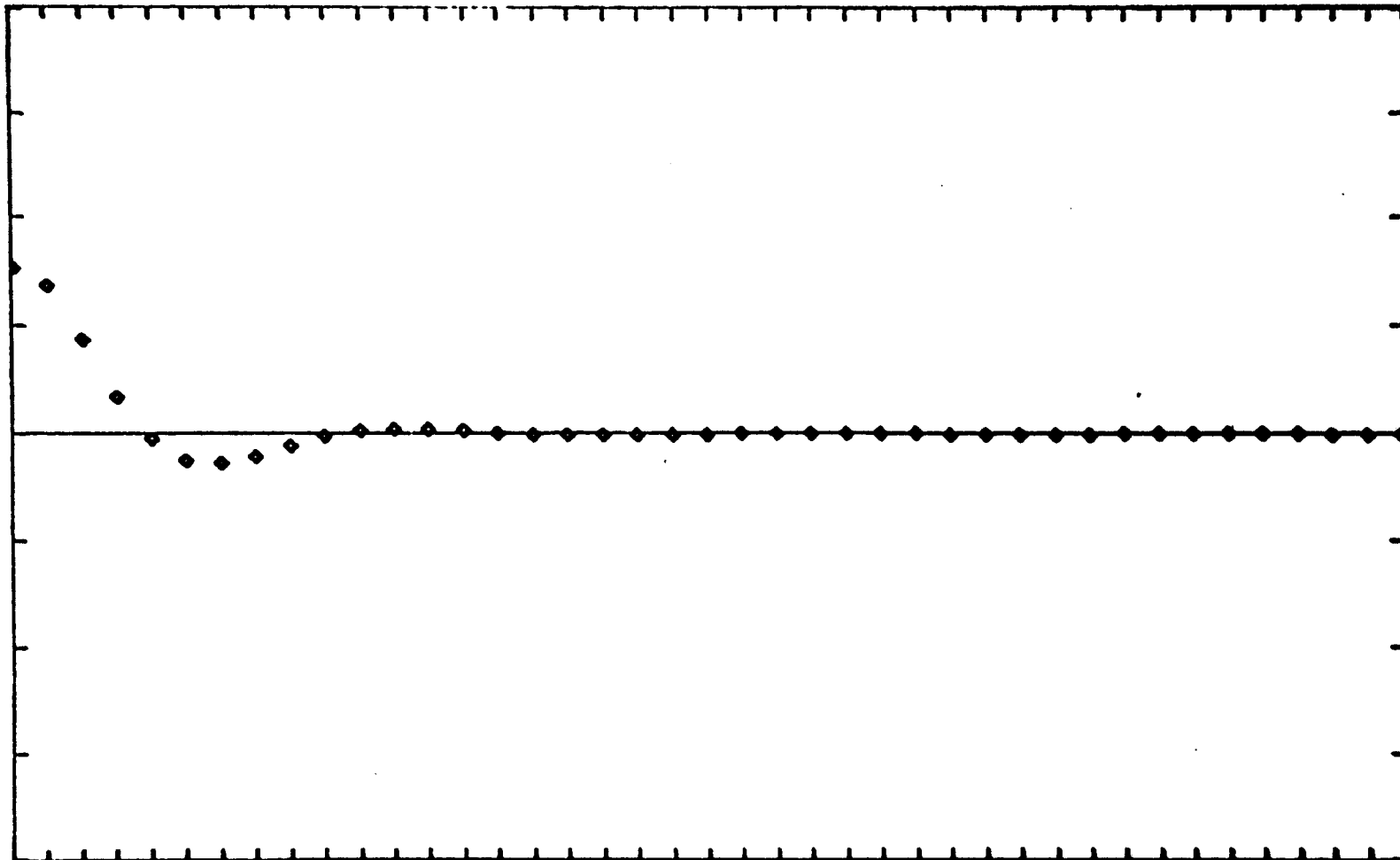
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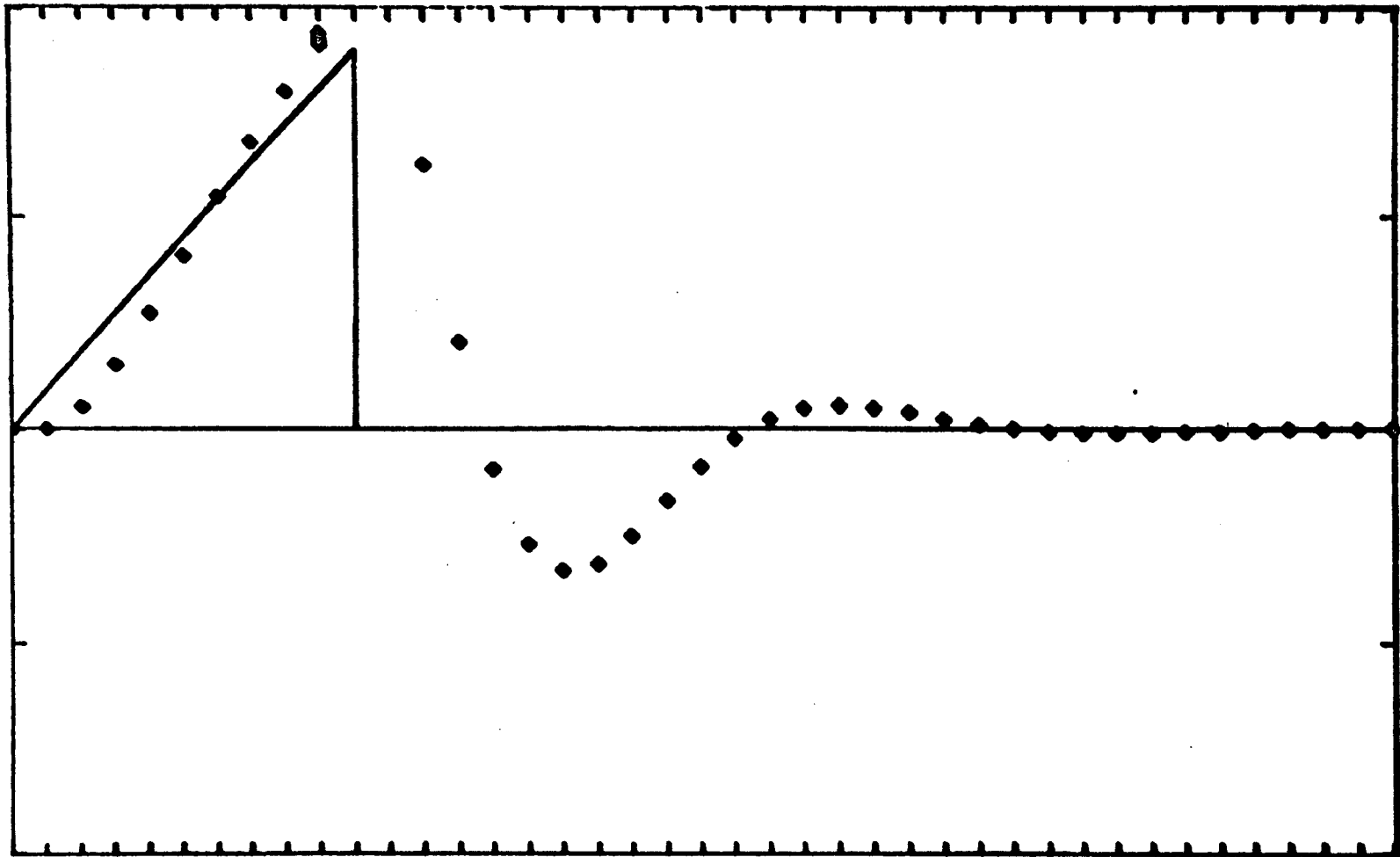
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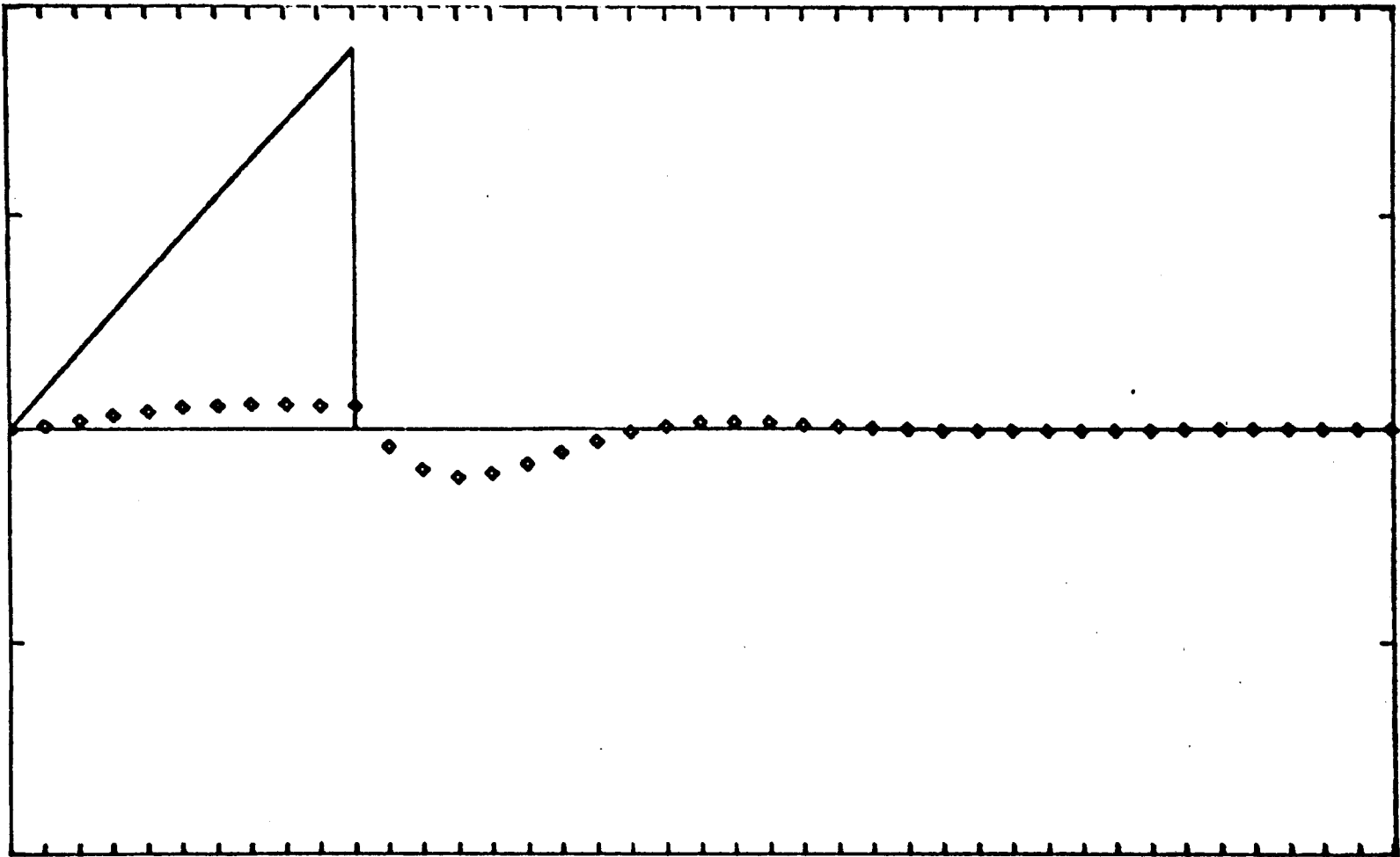
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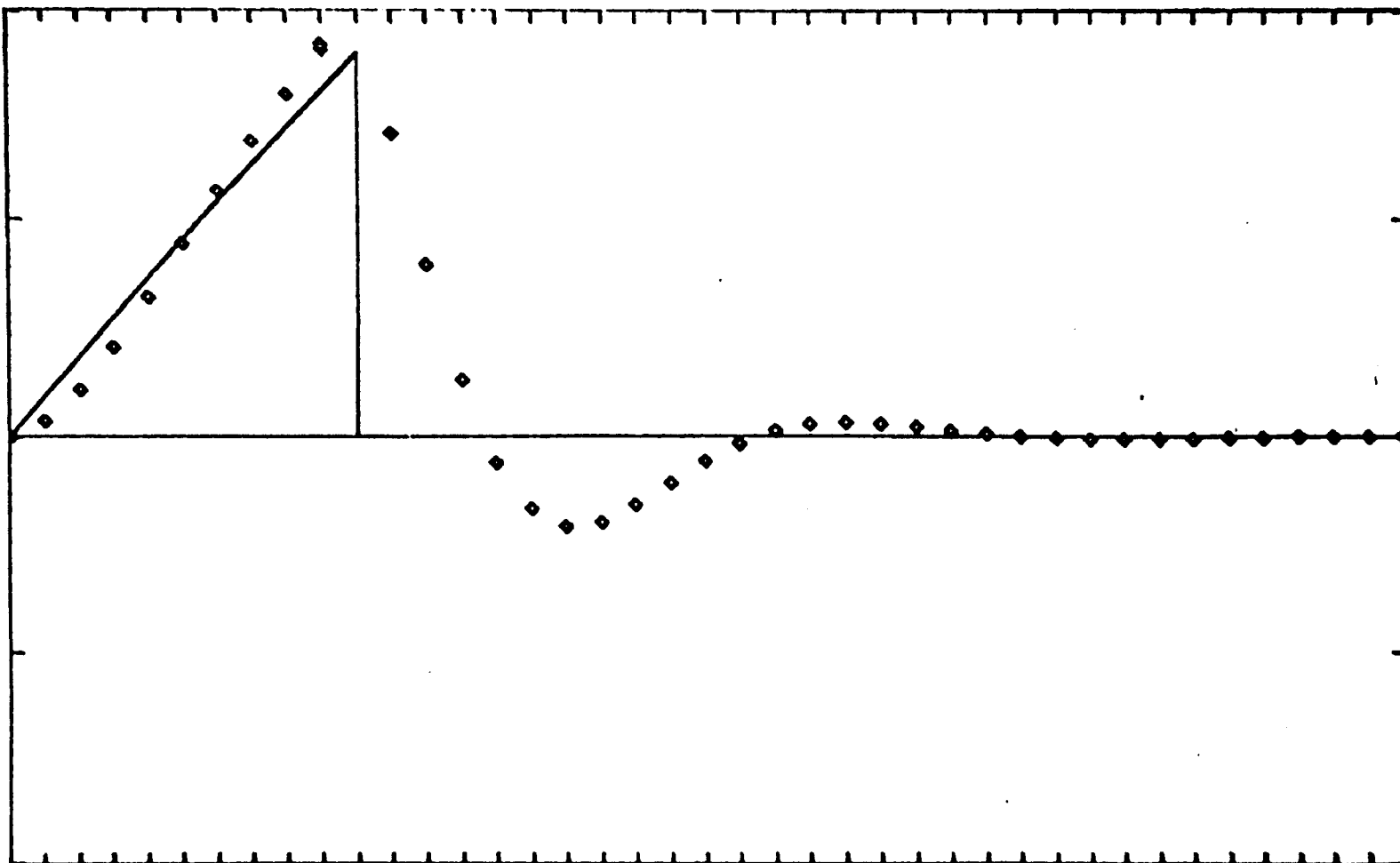
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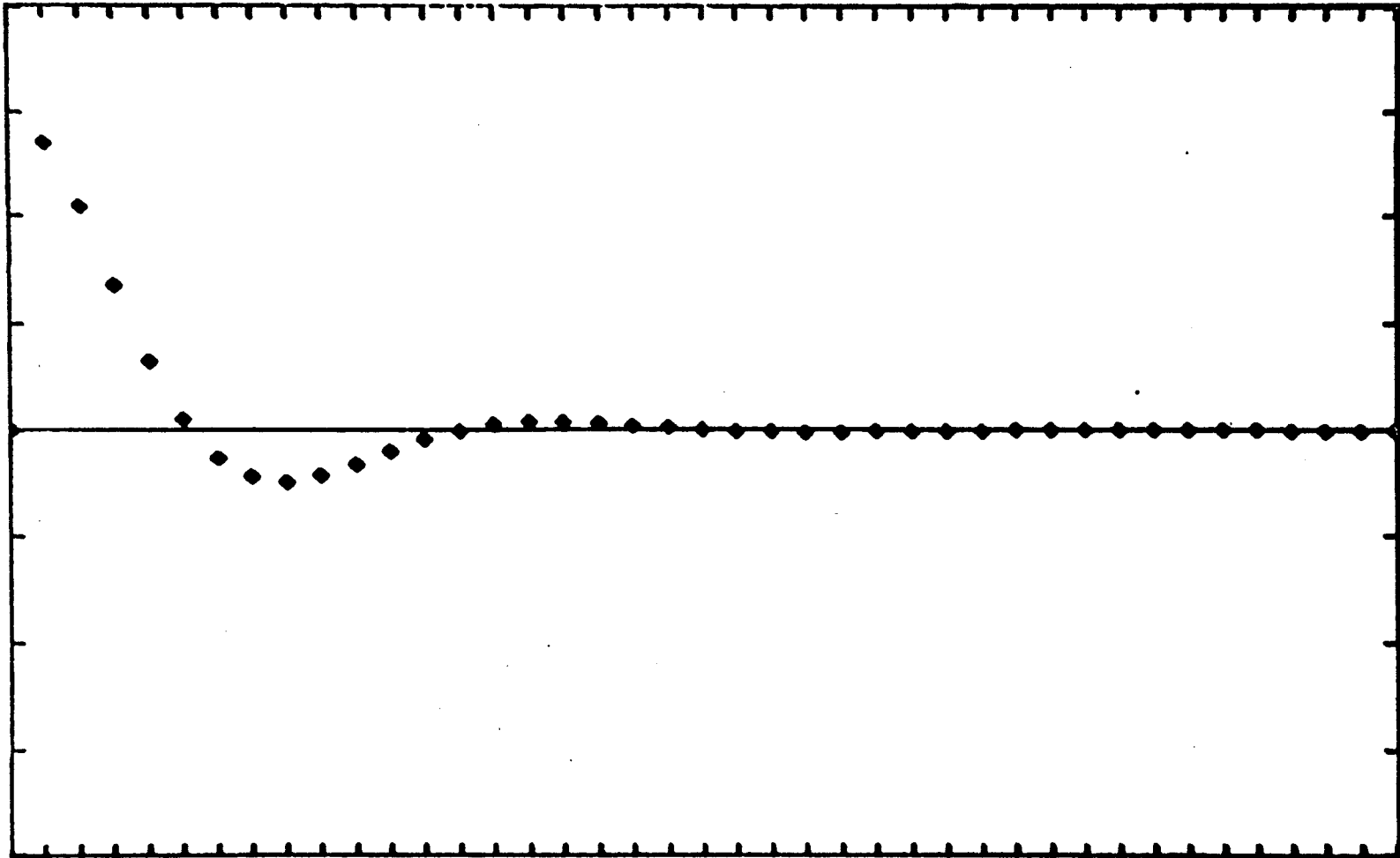
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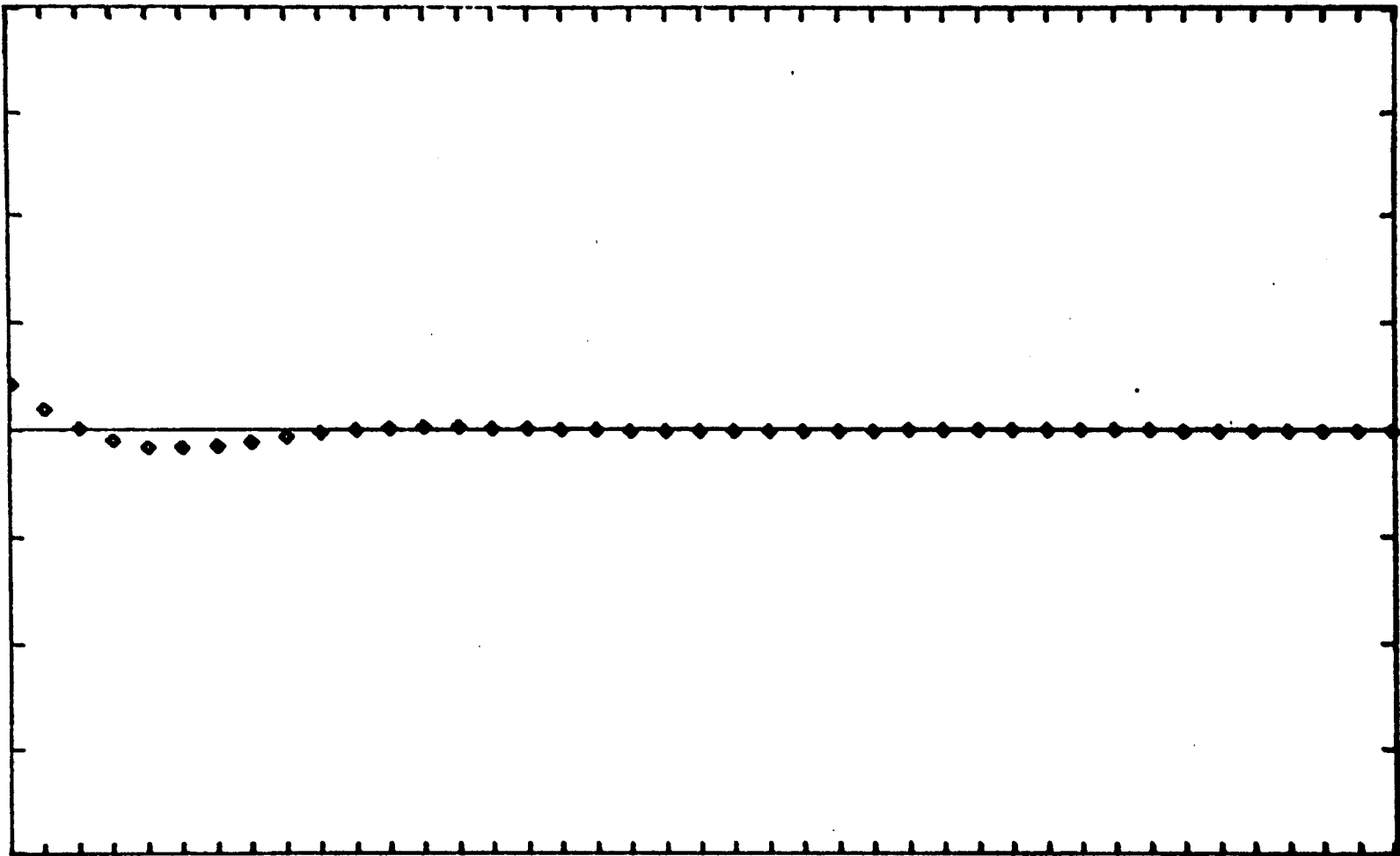
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ALPHA= 0.37 BETA= 0.17 T= 2.00 SECONDS SFY=10.00  
PREDICTED POSITION XP(K)  
PROCESSED BY FAA TECHNICAL CENTER ACT-100J



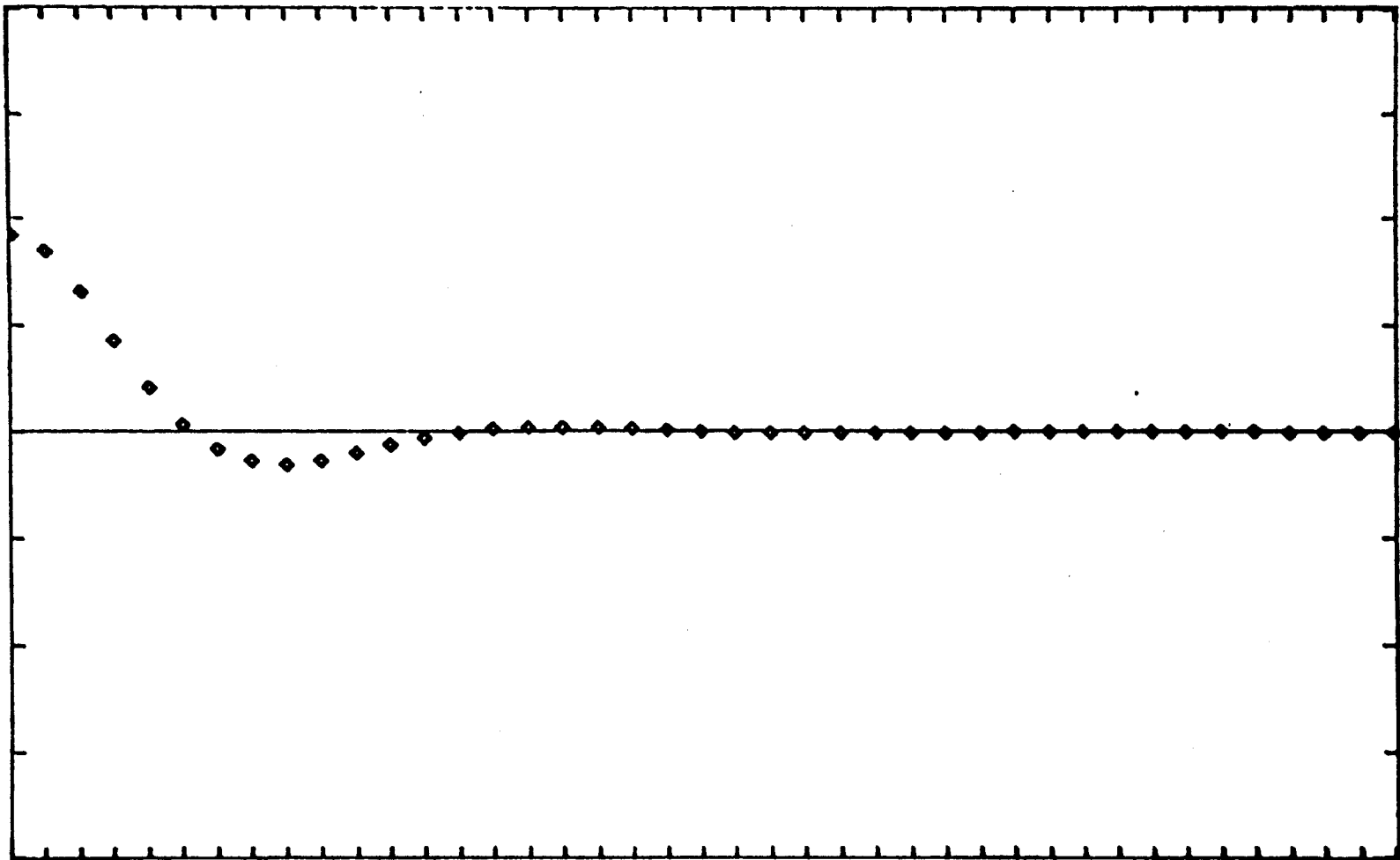
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ALPHA= 0.37 BETA= 0.17 T= 2.00 SECONDS SFY=10.00  
SMOOTHED VELOCITY XD(KB)  
PROCESSED BY FAA TECHNICAL CENTER ACT-100J



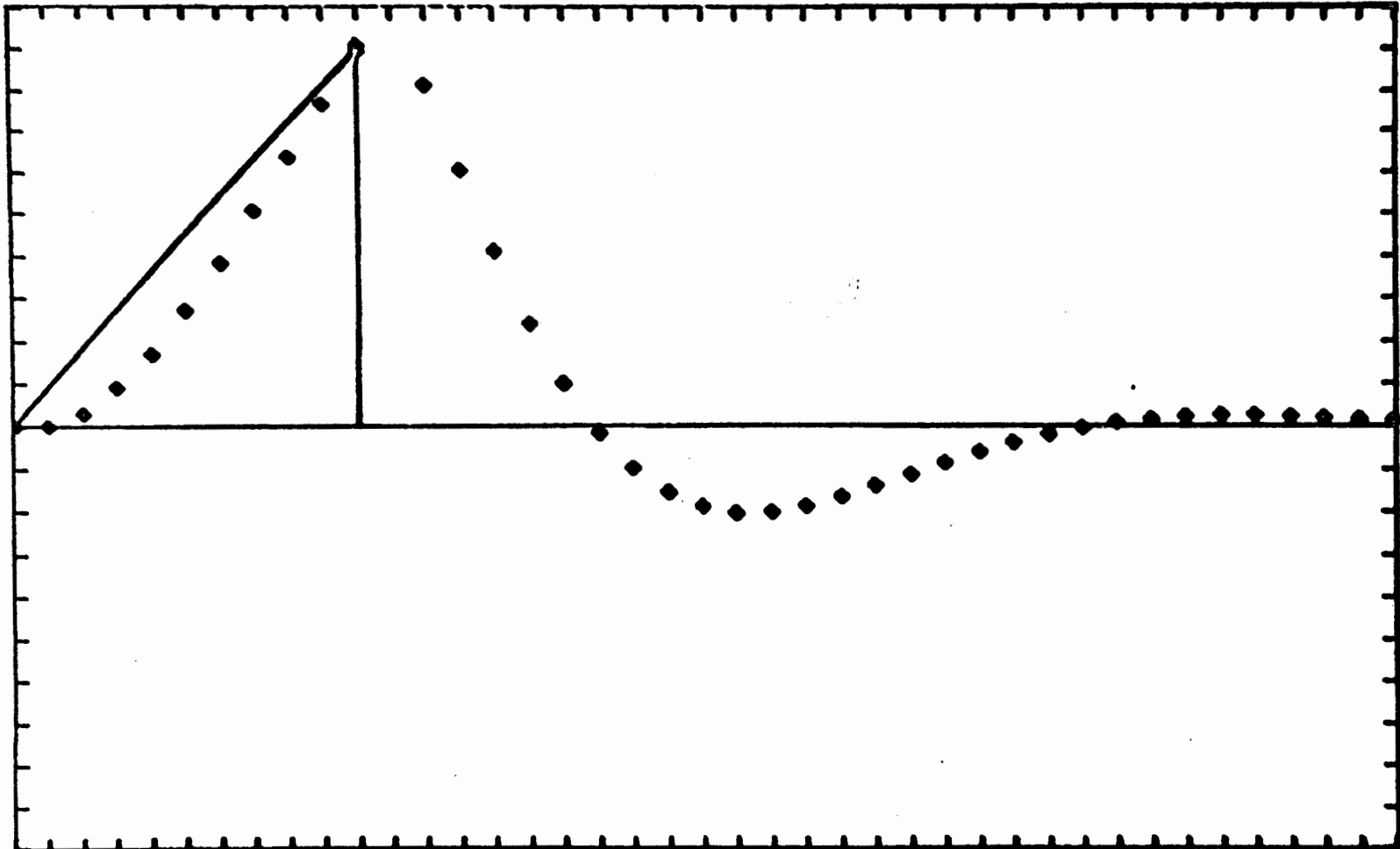
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ALPHA= 0.37 BETA= 0.17 T= 2.0 SECONDS SFY= 0.10  
PREDICTED POSITION XP(K)  
PROCESSED BY FAA TECHNICAL CENTER ACT-100J



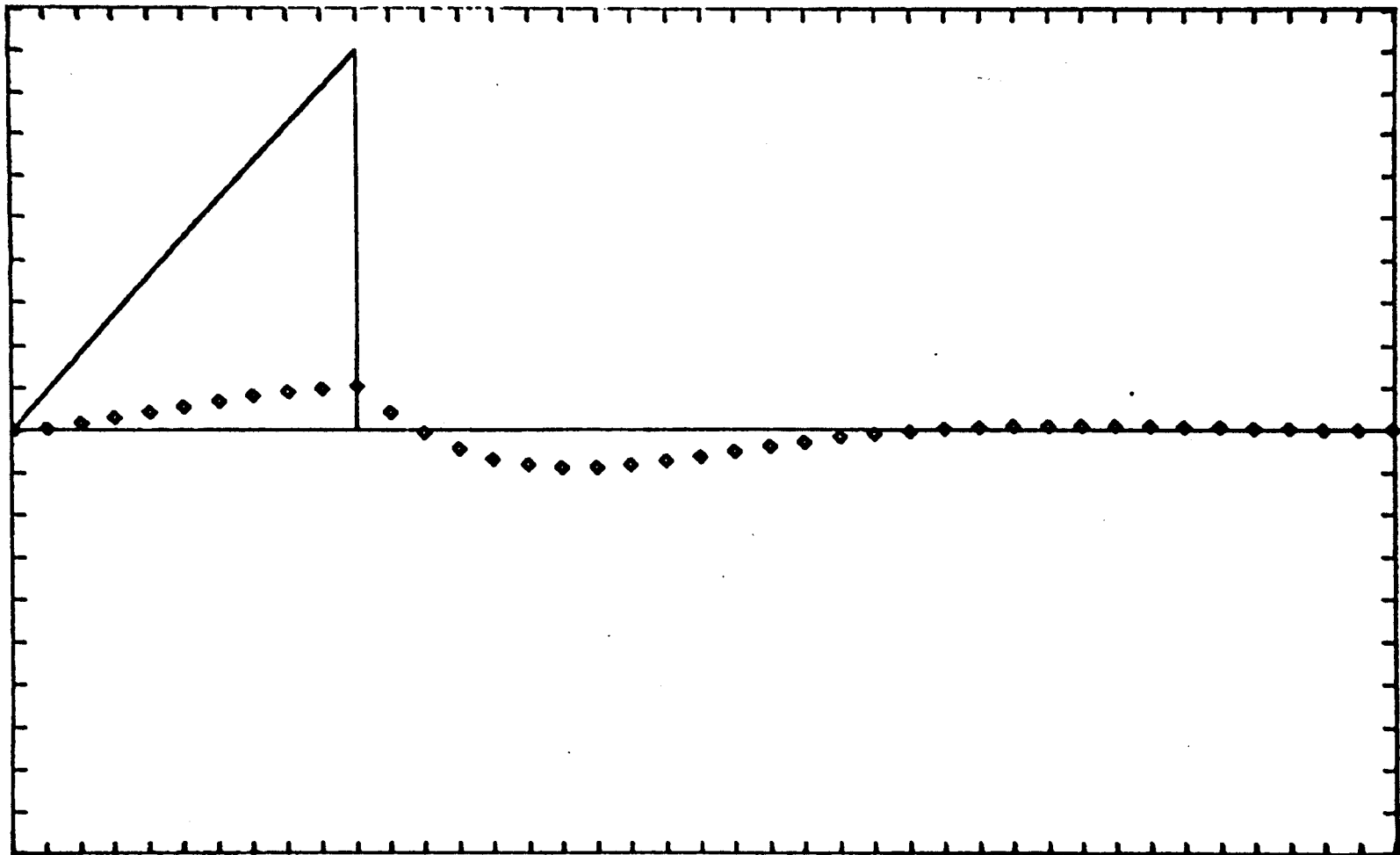
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ALPHA= 0.37 BETA= 0.17 T= 2.0 SECONDS SFY= 0.10  
SMOOTHED VELOCITY XD(KB)  
PROCESSED BY FAA TECHNICAL CENTER ACT-100J



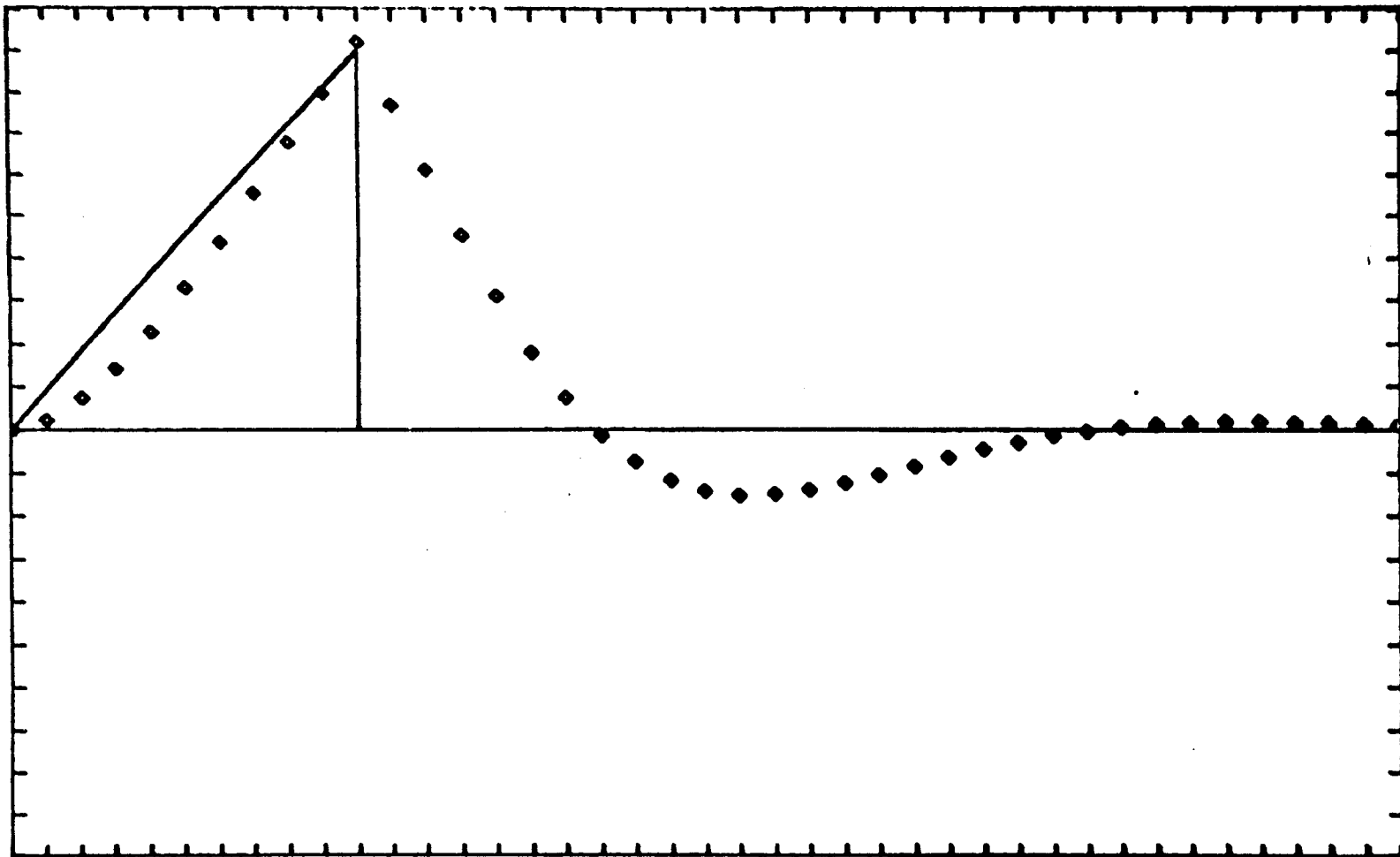
IMPULSE RESPONSE OF THE BENDIX ALPHA-BETA FILTER  
ALPHA= 0.37 BETA= 0.17 T= 2.0 SECONDS SFY= 0.10  
SMOOTHED POSITION  $X(k)$   
PROCESSED BY FAA TECHNICAL CENTER ACT-100J



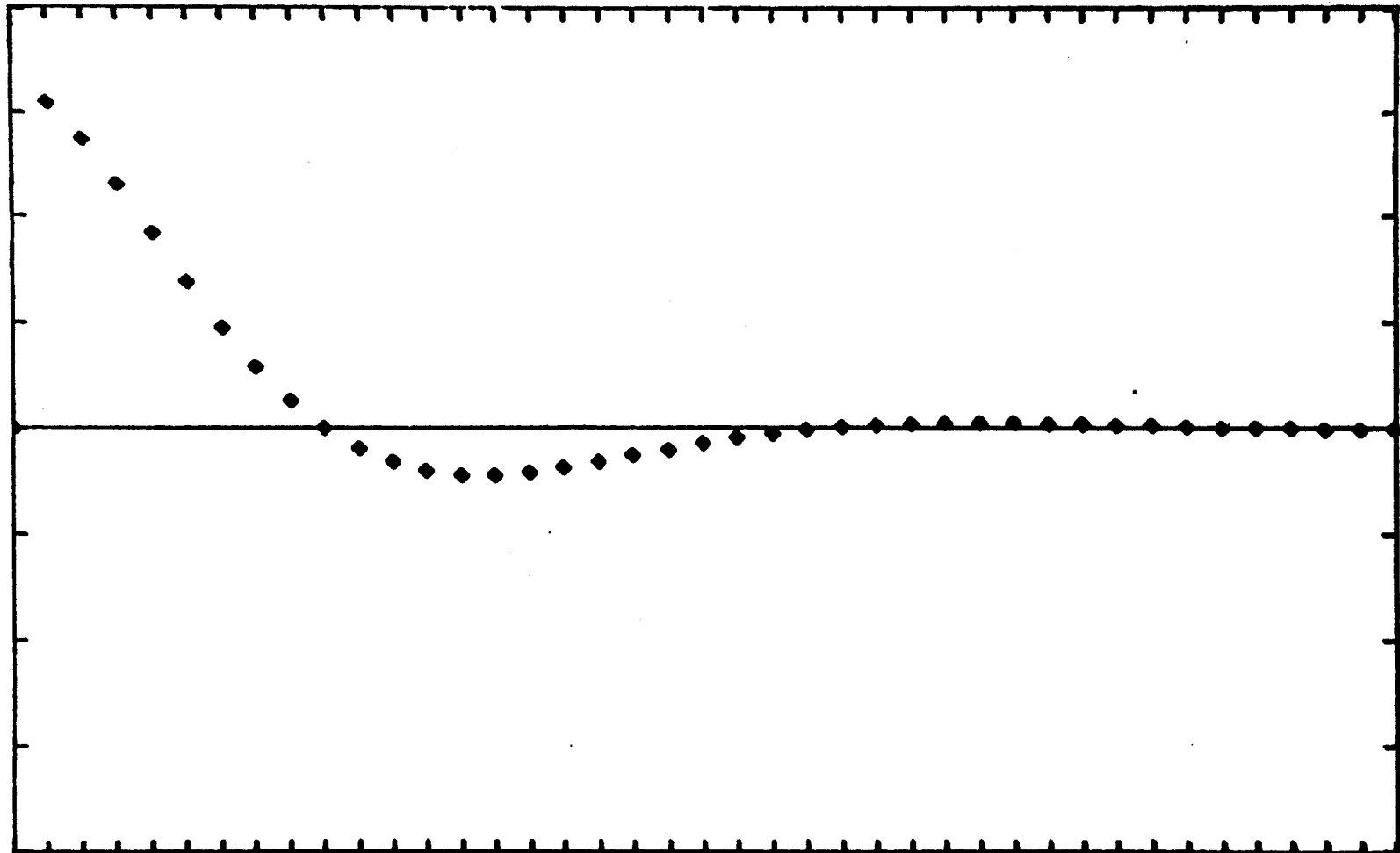
RAMP RESPONSE OF THE BENDIX ALPHA-BETA FILTER  
ALPHA= 0.25 BETA= 0.06 T= 1.00 SECONDS SFY= 1.00  
SMOOTHED POSITION X(K)  
PROCESSED BY FAA TECHNICAL CENTER ACT-100J



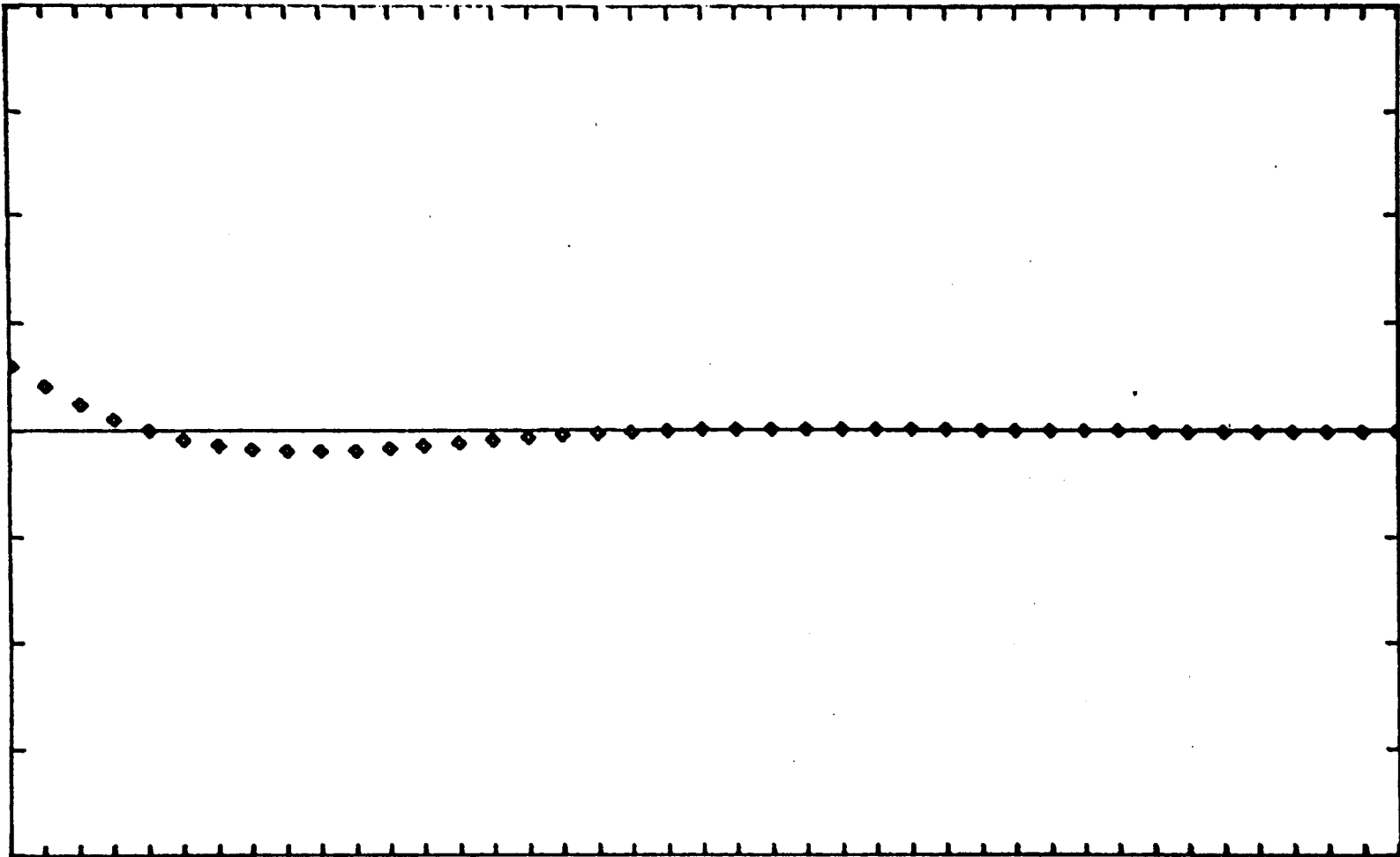
RAMP RESPONSE OF THE BENDIX ALPHA-BETA FILTER  
ALPHA= 0.25 BETA= 0.06 T= 1.00 SECONDS SFY= 1.00  
PREDICTED POSITION XP(K)  
PROCESSED BY FAA TECHNICAL CENTER ACT-100J



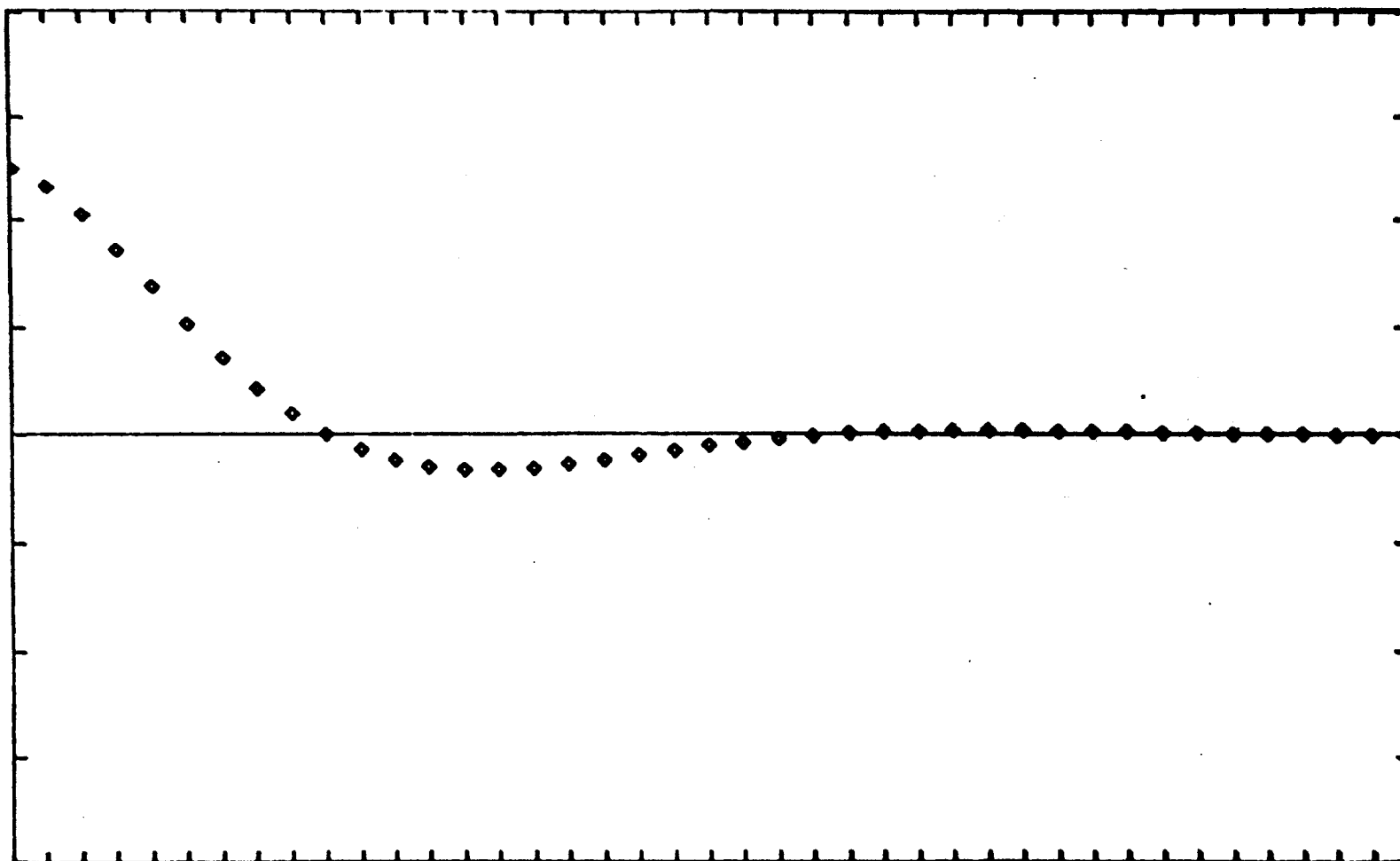
RAMP RESPONSE OF THE BENDIX ALPHA-BETA FILTER  
ALPHA= 0.25 BETA= 0.06 T= 1.00 SECONDS SFY= 1.00  
SMOOTHED VELOCITY XD(KB)  
PROCESSED BY FAA TECHNICAL CENTER ACT-100J



IMPULSE RESPONSE OF THE BENDIX ALPHA-BETA FILTER  
ALPHA= 0.25 BETA= 0.06 T= 1.0 SECONDS SFY= 0.10  
PREDICTED POSITION  $x_p(k)$   
PROCESSED BY FAA TECHNICAL CENTER ACT-100J



IMPULSE RESPONSE OF THE BENDIX ALPHA-BETA FILTER  
ALPHA= 0.25 BETA= 0.06 T= 1.0 SECONDS SFY= 0.10  
SMOOTHED VELOCITY XD(KB)  
PROCESSED BY FAA TECHNICAL CENTER ACT-100J



IMPULSE RESPONSE OF THE BENDIX ALPHA-BETA FILTER  
ALPHA= 0.25 BETA= 0.06 T= 1.0 SECONDS SFY= 0.10  
SMOOTHED POSITION X(K)  
PROCESSED BY FAA TECHNICAL CENTER ACT-100J