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A Parametric Study of Altitude Tracking Performance for the En Route Conflict Alert Function

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16. Abstract

The altitude tracking algorithm presently used for en route air traffic control is a fixed-parameter $\alpha-\beta$ filter. Altitude tracking data are required to support the operation of the Conflict Alert algorithm which detects potentially hazardous conditions between two aircraft. A mathematical theory is developed to evaluate the average warning time before a collision provided by the Conflict Alert. It was found that the average warning time is not a useful measure of tracking performance since the warning time is primarily determined by other factors, such as the prediction time used by Conflict Alert and the initial separation of the targets. A more significant measure of altitude tracking performance is the variance of the warning time. The variations in warning time will measure any performance degradation resulting from the practical implementation of the tracker; e.g., the accuracy of the time measurement, the computational precision used and the quantization errors in the altitude measurements. It is possible that a substantial loss in warning time will be observed if the altitude tracker suffers performance degradation due to finite precision effects which are well-known defects of recursive digital filters.

17. Key Words
Conflict Alert, Collision Avoidance, Alpha-Beta Filters, Radar Tracking, Track-While-Scan Radar, Air Traffic Control, Digital Filtering, Limit Cycles, Altitude Tracking.

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EXECUTIVE SUMMARY

The fundamental basis of operation of all the safety-related automation features is the prediction of the position of an aircraft sufficiently far into the future to allow intervention by a controller in situations in which this is warranted. The future altitude position estimates used by the En Route Conflict Alert function are obtained via numerical differentiation of the Mode C data performed by the altitude tracker. The objective of this study is to determine the relationship between the performance of the altitude tracking algorithm and that of the Conflict Alert function. Since altitude tracking has been found to be a particularly difficult function to perform adequately for the safety-related automation features, it is important to quantify this relationship for specification in the new Advanced Automation System.

An analytical model was developed to evaluate the average warning time before a collision. This model was used to perform a sensitivity analysis for the various factors which influence the performance of the Conflict Alert function. As a result of the simplicity of the computations required, it was possible to identify general relationships between the various factors of importance which would be too time consuming to evaluate using the standard National Airspace System simulation program available. Identifying the factors of greatest importance is very useful in generating system-level performance requirements that guarantee altitude tracking performance sufficient to support the automation features.

The results of this study, obtained under the assumption of statistically independent errors, showed that the average warning time is not a useful measure of tracking performance since the warning time is primarily determined by other factors, such as the prediction time used by Conflict Alert and the flight time. The variance of the warning time might be used as a measure of tracking performance; however, in practical situations it is possible that factors other than the tracking performance may have a greater impact on the warning time variance. As noted previously, all of these conclusions require the assumption of statistically independent errors which may not be valid because of finite precision effects which are known to be associated with recursive digital filters. In a properly designed system there are well known techniques which can be used to guarantee the absence of any deleterious effects due to finite precision computations and which will guarantee the statistical independence of the tracking errors. In the present system, however, certain modifications are required to improve the tracking algorithm performance (e.g., time correction) in order to assure statistically independent errors.

The results of this study have shown that the use of warning time as a system-level performance requirement is not a sufficiently sensitive enough measure to guarantee an acceptable level of performance from the altitude tracker. A system-level performance requirement to obtain satisfactory altitude tracking performance will probably require a limitation on the probability of an alert for trajectories which do not violate a specified separation threshold. A specification of this form would reduce the excessive number of alerts currently being generated. It was also found in this study that the altitude separation threshold used by Conflict Alert can be reduced with no significant impact on warning time, thus providing an obvious means to reduce the number of alerts.

1. INTRODUCTION

The emphasis on improving safety in aviation has led to the addition of various automation features in the computer programs used for air traffic control. The intended use of these automation features is to alert the air traffic controller to potentially hazardous situations as they occur. The principal basis of operation of all these features is the prediction of the future positions of the aircraft under observation using the estimated velocities provided by the tracking algorithm. An integral part of the prediction process is the estimation of the future altitude of the aircraft.

Despite the fact that the need for accurate altitude information was recognized early in the development of the automation features (reference 1), operational problems have been encountered repeatedly in the area of altitude tracking (references 2-5). In particular, the lack of time correction in the vertical tracker has been known to lead to errors of several thousand feet in the predicted altitude which is obviously unacceptable when using separation standards of 1,000 and 2,000 feet (references 6 and 8). The primary emphasis in this report will be on the impact of vertical tracking on the En Route Conflict Alert automation function (references 7 to 9). Although there are other automation features which use altitude tracking data, such as the Mode C intruder algorithm (reference 10) and the En Route Minimum Safe Altitude Warning system (references 3 and 9), the emphasis in this study on the Conflict Alert function is justified since this feature has the most stringent requirements with regard to altitude accuracy. All three of the automation features discussed above make use of the same altitude tracking data (current position and velocity) and vary the prediction time for different purposes.

The objective of this study is to provide a quantitative parametric study of the impact of vertical tracking on the performance of the En Route Conflict Alert algorithm. Such a study is appropriate at this time because of the potential replacement of the en route computer system and because vertical tracking was not examined in the previous parametric analysis of Conflict Alert (reference 11). In addition, no analysis has been performed to determine the tracking accuracy required to support conflict detection (reference 4). As a result, it is useful to compare the optimum theoretical performance of the vertical tracking algorithm with that actually achieved to provide baselines on the attainable performance and thereby develop criteria which may be used for purposes of functional specification and testing. Emphasis will be placed on trajectories which result in collisions. There are presently no operational field performance data available with which to characterize the nature of the scenarios which result in predictions of separation violations, but no collisions.

2. MATHEMATICAL ANALYSIS

The altitude tracking algorithm considered in this report is an $\alpha-\beta$ filter. The $\alpha-\beta$ tracking filter is a widely used technique for performing the operation of numerical differentiation to obtain velocity estimates from noisy position measurements. The simplicity of the algorithm and the limited computational requirements have resulted in the use of this filter in many practical situations and, as a consequence, extensive analytical studies have been conducted of the $\alpha-\beta$ filter (e.g., references 6 and 12-19). The characteristics and mathematical performance of the $\alpha-\beta$ tracking filter are described in the following sections.

2.1 DEFINITION OF THE $\alpha-\beta$ TRACKING FILTER.

The $\alpha-\beta$ tracking filter is a recursive algorithm which performs the operations of position smoothing, position prediction, and numerical differentiation for velocity estimation. It is specified by the equations,

$$
\begin{aligned}
Z_s(k) &= Z_p(k) + \alpha(Z_m(k) - Z_p(k)) \\
Z_v(k) &= Z_v(k-1) + (\beta/T)(Z_m(k) - Z_p(k)) \qquad (1) \\
Z_p(k+1) &= Z_s(k) + TZ_v(k)
\end{aligned}
$$

where $Z_s(k)$ = smoothed position at the k^{th} time epoch

$Z_v(k)$ = velocity estimate

$Z_p(k)$ = predicted position

$Z_m(k)$ = measurement position

T = sampling period (assumed constant)

α,β = smoothing constants.

For the purposes of the tracking algorithm, it is only necessary to predict the future position of the target one time interval into the future. However, for the purposes of advanced air traffic control functions, it is necessary to make position predictions much further into the future so that an extended time-interval position prediction will be defined as

$$Z_p(k,T') = Z_s(k) + T' Z_v(k) \qquad (2)$$

in which the time interval T' is arbitrary. The accuracy of the extended time-interval position prediction is dependent on the accuracy of the tracking filter outputs, Z_s and Z_v, and also on the degree to which the actual flightpath follows the constant velocity, straight-line assumption inherent in (2).

The algorithm as defined by (1) assumes that all computations and measurements are coincident with the epoch times. In an asynchronous environment, however, data may be received at any time between the operations of the tracking algorithm. In such cases, it is convenient to assume a reference time for the smoothing and prediction process which may not necessarily be the time of operation of the tracking algorithm or the time of receipt of the measurement datum. In the case of the en route portion of the National Airspace System, the tracking function operates at a fixed rate, not necessarily that of the sensor, with the computational time taken as the midpoint of the tracking cycle operation (see reference 9).

The smoothing and prediction process is assumed to use the center of the tracking cycle as the reference time, thus predicting from the center of the present cycle. Since measurement data may not be received at the reference time used by the tracking algorithm, the estimated velocity from the previous cycle may be used to move the data point, either forward or backward in time, to make it appear as though the measurement datum was received in synchronism at the center of the cycle. This process is known as time correction and has been extensively analyzed in previous reports (references 6, 12 and 14).

In this case, the smoothing equations are

$$\begin{aligned} Z_s(k) &= Z_p(k) + \alpha(Z_m(k) + \Delta T(k)\, Z_v(k-1) - Z_p(k)) \\ Z_v(k) &= Z_v(k-1) + (\beta/T)\,(Z_m(k) + \Delta T(k)\, Z_v(k-1) - Z_p(k)) \end{aligned} \qquad (3)$$

where

$$\Delta T(k) = kT - T_m(k),$$

with $T_m(k)$ being the actual time at which the position measurement was made.

Using time correction, it is possible for the tracking algorithm to operate at a fixed cyclic rate and with measurement data which are obtained asynchronously. The multiple sensor environment of the en route air traffic control system meets the conditions just described. It should be noted that if measurements are obtained asynchronously without time correction, then this is equivalent to the introduction of an error equal to the difference between the measured true positions at the time the measurement should have been made if the requirement for synchronism between the data source and the tracking algorithm had been fulfilled.

Omission of the time-correction process will introduce an additional source of error into the tracking algorithm which is unnecessary if the time of receipt of the measured position is known. There is also some question as to the value of T which should be used in (3) and this will be discussed in section 2.3.1.

2.2 TRANSIENT ANALYSIS OF $\alpha-\beta$ ALTITUDE TRACKING ALGORITHMS.

The technique used to calculate the transient response of the altitude tracking algorithm is based on the standard Z-transform approach used for sampled data systems (e.g., reference 19). Since it is not the purpose of this study to provide a detailed explanation of the Z-transform approach, only a brief outline of the approach will be provided with the important results given in tabular form in tables 3 and 4.

2.2.1 Z-Transform Analysis of $\alpha-\beta$ Filter.

The equations for the $\alpha-\beta$ tracking filter with time correction can be rewritten for time epoch k as

$$Z_s(k) = Z_p(k) + \alpha(Z_m(k) + \Delta T(k)\, Z_v(k-1) - Z_p(k))$$

$$Z_v(k) = Z_v(k-1) + (\beta/T)\,(Z_m(k) + \Delta T(k)\ \ Z_v(k-1) - Z_p(k)) \qquad (4)$$

$$Z_p(k) = Z_s(k-1) + TZ_v(k-1)$$

For the present, it will be assumed that $\Delta T(k)$ is a fixed constant with no dependence on k. Taking the Z-transform of the tracking filter yields the following:

$$Z_s(z) = Z_p(z) + \alpha(Z_m(z) + \Delta T z^{-1} Z_v(z) - Z_p(z))$$

$$Z_v(z) = z^{-1}Z_v(z) + (\beta/T)(Z_m(z) + \Delta T z^{-1} Z_v(z) - Z_p(z)) \qquad (5)$$

$$Z_p(z) = z^{-1}Z_s(z) + Tz^{-1}Z_v(z)$$

where $Z_s(z)$, $Z_v(z)$, $Z_p(z)$ and $Z_m(z)$ are the Z-transforms of the respective time domain sequences. Since the $\alpha-\beta$ tracking filter is a one input and three output filter, there are three transfer functions which are used to define these relationships,

$$Z_s(z) = H_1(z)Z_m(z)$$

$$Z_v(z) = H_2(z)Z_m(z) \qquad (6)$$

$$Z_p(z) = H_3(z)Z_m(z)$$

Dividing through (5) by $Z_m(z)$ and recognizing the appropriate transfer functions yields a set of three simultaneous linear equations which define the system transfer functions:

$$
\begin{aligned}
&-H_1(z) - TH_2(z) + H_3(z) = 0 \\
&H_1(z) - \alpha\Delta T z^{-1} H_2(z) - (1 - \alpha)H_3(z) = \alpha \\
&(z - 1 - \beta\Delta T/T)H_2(z) + (z\beta/T)H_3(z) = \beta z/T
\end{aligned}
\tag{7}
$$

When the time-correction factor is omitted (i.e., $\Delta T(k) \equiv 0$), the resulting equations reduce to those obtained in a previous analysis (reference 19). Solving these equations simultaneously yields the system transfer functions,

$$
\begin{aligned}
H_1(z) &= z(\alpha z + \beta - \alpha)/\Delta \\
H_2(z) &= (\beta/T)z(z - 1)/\Delta \\
H_3(z) &= (z(\alpha + \beta) - \alpha)/\Delta
\end{aligned}
\tag{8}
$$

where

$$\Delta = z^2 - z(2 - \alpha - \beta + \beta\Delta T/T) + (1 + \beta\Delta T/T - \alpha) \tag{9}$$

The response for the extended time-interval position prediction, (2), can be written as the sum of the responses to $Z_s(k)$ and $T'Z_v(k)$. The response of the $\alpha-\beta$ tracking filter can be determined by multiplying the Z-transform of the input signal, $Z_m(z)$, by the appropriate transfer function and then taking the inverse Z-transform to give the filter output in the time domain.

2.2.2 Time-Domain Response of the $\alpha-\beta$ Tracking Filter to Piecewise Linear Signals.

Examination of many plots of measured altitude versus time for field data shows that most altitude changes can be represented by a constant velocity ramp. Consequently, a typical altitude trajectory could be represented by a piecewise linear signal (a generalized input consisting of impulses, steps, and ramps). Practical trajectories can be represented by the sum of these three signals and, by superposition, the response to the sum is the sum of the responses to the individual signals. Hence, it is only necessary to find the response of the $\alpha-\beta$ filter to these three signals. In addition, since the predicted altitude is the sum of the smoothed position and a weighted velocity contribution, only the position and velocity responses need to be calculated. Since there are three input signals and two filter responses, there are six output responses to be calculated, but only one will be derived in detail since the same procedure is followed in the other cases.

Since the data provided to the altitude tracker contain various errors and other errors are introduced within the filter itself due to computational inaccuracies, it is necessary to characterize the filter performance in statistical terms. If it is assumed that the errors are additive and that the filter is linear, then by superposition the response to the signal and the response to the noise can be considered separately. If it is also assumed that the errors are unbiased, then the response to the signal will be the mean value of the filter output. The response to the noise can then be characterized as the variance of the errors about the mean. Since the time-correction factor is, in fact, a random variable, the computation of the mean response to the signal will require some assumption as to the mean value of $\Delta T(k)$. It will be assumed in this study, as it has been in previous work (references 6, 12 and 14), that $\Delta T(k)$ is uniformly distributed about the epoch times so that $E(\Delta T(k)) = 0$. Consequently, for the computation of the signal response (mean value) ΔT in (9) will be zero. The effect of time correction on the filter performance will be modeled as an additional additive noise source with appropriate statistical characteristics (see additional comments in section 3.4).

The three signals of interest are given in table 1 along with the appropriate transforms.

TABLE 1. STANDARD INPUT SIGNALS AND ASSOCIATED Z-TRANSFORMS

Unit Function	Representation in Time Domain	Z-Transform
Impulse	$\delta(k) = \begin{cases} 0 & k \neq 0 \\ 1 & k = 0 \end{cases}$	1
Step	$u(k) = \begin{cases} 0 & k < 0 \\ 1 & k \geqslant 0 \end{cases}$	$z/(z-1)$
Ramp	$r(k) = ku(k)$	$z/(z-1)^2$

The Z-transform of the output of the $\alpha-\beta$ tracking filter can be found by multiplying the Z-transform of the input signal under consideration by the appropriate system transfer function given by (8). Since the same techniques will be used to find the response to each of the standard signals, only the velocity response to the ramp function will be calculated in detail.

The Z-transform of the velocity response to a ramp function is given by

$$Z_V(z) = \beta T^{-1} z^2 / \left\{ (z-1)(z^2 - z(2 - \alpha - \beta) + 1 - \alpha) \right\} \qquad (10)$$

where the inverse transform of $Z_V(z)$ is the time domain response to the ramp input. The simplest approach to finding the inverse transform is to make a partial fraction expansion of (10) and to identify the individual terms.

In this case

$$Z_v(z) = \beta T^{-1}\left[a_o + a_1/(z-1) + (a_2 z + a_3)/(z^2 - z(2 - \alpha - \beta) + 1 - \alpha)\right] \quad (11)$$

where

$$a_o = 0$$

$$a_1 = \beta^{-1}$$

$$a_2 = (\beta - 1)/\beta$$

$$a_3 = (1 - \alpha)/\beta$$

or

$$Z_v(z) = T^{-1}\left[\frac{1}{z-1} - (1-\beta)z^{-1}\left(\frac{z(z-(1-\alpha)/(1-\beta))}{z^2-z(2-\alpha-\beta)+1-\alpha}\right)\right] \quad (12)$$

Further expansion of the quadratic factor in (12) could be made using the appropriate factors for the real or imaginary roots; however, in some cases it is useful to retain the quadratic factor as it stands. The transform pairs in table 2 can then be useful in the case where the response is a damped sinusoid (as used, e.g., in reference 18). By addition and subtraction of appropriate quantities (12) becomes

$$Z_v(z) = T^{-1}\left[\frac{1}{z-1} - (1 - \beta)z^{-1}\left(\frac{z(z - (1 - (\alpha + \beta)/2))}{z^2 - z(2 - \alpha - \beta) + 1 - \alpha}\right.\right.$$
$$\left.\left. - \frac{z\,((1 - \alpha)/(1 - \beta) - 1 + (\alpha - \beta)/2)}{z^2 - z(2 - \alpha - \beta) + 1 - \alpha}\right)\right] \quad (13)$$

TABLE 2. Z-TRANSFORMS OF DAMPED SINUSOIDAL WAVEFORMS

Time-Domain		Z-Domain
$e^{-at}\sin \omega t$	$\longleftrightarrow$	$\dfrac{z\,e^{-aT}\sin \omega T}{z^2 - 2ze^{-aT}\cos \omega T + e^{-2aT}}$
$e^{-at}\cos \omega t$	$\longleftrightarrow$	$\dfrac{z(z-e^{-aT}\cos \omega T)}{z^2 - 2ze^{-aT}\cos \omega T + e^{-2aT}}$

which can be written

$$Z_v(z) = (zT)^{-1}\left[\frac{z}{z-1} - (1-\beta)\left\{\frac{z(z-e^{-aT}\cos\omega T)}{z^2 - 2ze^{-aT}\cos\omega T + e^{-2aT}} - \frac{mze^{-aT}\sin\omega T}{z^2 - 2ze^{-aT}\cos\omega T + e^{-2aT}}\right\}\right] \quad (14)$$

where

$$a = -\ln(1-\alpha)/(2T) \quad (15)$$

$$\omega = T^{-1}\arccos\left((1-(\alpha+\beta)/2)/(1-\alpha)^{\frac{1}{2}}\right) \quad (16)$$

$$m = \frac{((\beta-\alpha)/(1-\beta) + (\alpha+\beta)/2}{(\beta - ((\alpha+\beta)/2)^2)^{\frac{1}{2}}} \quad (17)$$

Recognizing the z^{-1} factor as a unit delay, the time-domain velocity response of the $\alpha-\beta$ tracking filter is

$$h_v(t) = u(t-T)T^{-1}\left[1 - (1-\beta)\, e^{-a(t-T)}(\cos\omega(t-T) - m\sin\omega(t-T))\right] \quad (18)$$

where T is the sampling interval.

The resulting response (i.e., (18)) is seen to be a delayed unit-step, which is the steady-state response, and an exponentially decaying oscillatory waveform which is the transient solution. Since a target changing altitude at a constant velocity would have an input signal of $r(k) = kvT$, and the steady-state response of (18) is a step of amplitude v, the correct answer, and the transient response is the error in the estimated velocity in this case. Thus the $\alpha-\beta$ tracking filter provides an unbiased estimate of the target velocity after the transient response has died.

The other responses for the $\alpha-\beta$ tracking filter can be derived in a similar manner. The results are summarized in tables 3 and 4. The filter response for the predicted position is given by

$$h_p(t) = h_s(t) + T_p h_v(t) \quad (19)$$

where T_p is the prediction time.

TABLE 3. TRANSIENT RESPONSE OF THE $\alpha-\beta$ TRACKING FILTER

Input	Smoothed Position	Velocity
Impulse	$h_s(t)=\alpha e^{-at}(\cos\omega t-g_i \sin\omega t)$	$h_v(t)=\frac{\beta}{T}[e^{-at}(\cos\omega t-m_i \sin\omega t)]$
Step	$h_s(t)=u(t)-(1-\alpha)[e^{-at}(\cos\omega t -g_s \sin\omega t)]$	$h_v(t)=\frac{\beta}{T}[e^{-at}(\cos\omega t-k_s \sin\omega t)]$
Ramp	$h_s(t)=r(t)-(1-\alpha)u(t-T)[e^{-a(t-T)} (\cos\omega(t-T) -g_r \sin\omega(t-T))]$	$h_v(t)=u(t-T)[1-(1-\beta)\{e^{-a(t-T)} (\cos\omega(t-T)-m_r \sin\omega(t-T))\}]$

TABLE 4. PARAMETERS OF TRANSIENT RESPONSE

$a= -\ln(1-\alpha)/(2T)$

$\omega=T^{-1}\arccos((1-(\alpha+\beta)/2)/(1-\alpha)^{\frac{1}{2}})$

$g_i=((\alpha+\beta)/2-\beta/\alpha)/\Delta$

$m_i=(\alpha+\beta)/(2\Delta)$

$g_s=m_i$

$m_s=((\alpha+\beta)/2-1)/\Delta$

$g_r=m_s$

$m_r=\left[\frac{\beta-\alpha}{1-\beta} + \frac{(\alpha+\beta)}{2}\right]/\Delta$

$\Delta= (\beta- (\alpha+\beta)^2/4)^{\frac{1}{2}}$

These results apply only to the case where the system is stable and the response is oscillatory. The restrictions on the α and β parameters required to result in a stable system are discussed elsewhere (see references 15 to 19). For practical tracking filters, the parameter values are well within the stable region. The additional restriction required to ensure an oscillatory response is given by the $\alpha-\beta$ relationship which results in critical damping (i.e., the point at which the roots of the characteristic equation change from real to imaginary). The point of critical damping occurs when the denominator of the factors in table 4 becomes zero or

$$\beta_c= 2 - \alpha - 2\sqrt{1 - \alpha} \tag{20}$$

which is the same as that derived elsewhere (reference 16), and for the tracking filter response to be oscillatory, it is required that $\beta > \beta_c$. For computational purposes β must be slightly larger than β_c (by a factor of 1.001 for typical single precision arithmetic) in order to obtain accurate numerical results. As in the case of the restrictions required for stability, most $\alpha-\beta$ tracking filters are operated in the region where the response is slightly oscillatory.

2.3 STATISTICAL PERFORMANCE OF THE $\alpha-\beta$ TRACKING FILTER.

The input to the tracking filter consists of the sum of the signal plus a random noise component. Using the principal of superposition, the responses of the tracking filter to the signal and noise can be considered separately. If the mean of the noise component is zero, then the filter response to the signals of the previous section will be the mean value of the filter output. The filter response to the noise requires the determination or characterization of the errors about the mean in the filter output.

2.3.1 Variance Reduction Ratios for $\alpha-\beta$ Tracking Filters Using Time Correction.

The statistical performance of the $\alpha-\beta$ tracking filter is usually expressed in terms of the variance reduction ratios. These are the ratios of the error variances at the output of the filter to the variance of the errors at the input of the filter. The variance reduction ratios describe the performance of the tracking filter in a steady-state situation in which all transients have decayed.

As stated previously, the transient error for constant velocity targets will eventually decay to zero for the tracking filter regardless of whether or not time correction is used. Computation of the variance reduction ratios will be facilitated if the tracking algorithm equations are expressed in the matrix form:

$$\begin{bmatrix} Z_s(k) \\ Z_v(k) \end{bmatrix} = \begin{bmatrix} 1-\alpha & T(1+\alpha\Delta T(k)/T-\alpha) \\ -\beta/T & (1+\beta\Delta T(k)/T-\beta) \end{bmatrix} \begin{bmatrix} Z_s(k-1) \\ Z_v(k-1) \end{bmatrix} + \begin{bmatrix} \alpha \\ \beta/T \end{bmatrix} Z_m(k) \qquad (21)$$

or

$$Z(k) = A(T,\Delta T)Z(k-1) + B(T)\ (u(k) + w(k)) \qquad (22)$$

where

$$Z(k) = \begin{bmatrix} Z_s(k) \\ \\ Z_v(k) \end{bmatrix}$$

$$A(T,\Delta T) = \begin{bmatrix} 1-\alpha & T(1+\alpha\Delta T(k)/T-\alpha) \\ \\ -\beta/T & (1+\beta\Delta T(k)/T-\beta) \end{bmatrix}$$

$$B(T) = \begin{bmatrix} \alpha \\ \\ \beta/T \end{bmatrix}$$

and the measurement datum, $Z_m(k)$, is expressed as the sum of a true deterministic component, u(k), and a random error component, w(k), with variance σ_w^2 which will be assumed to be white stationary noise representing the measurement error.

The noise response of the filter is obtained in terms of the covariance matrix for the errors at the filter output, and this response is given by (reference 20)

$$P(k + 1) = A(T,\Delta T)P(k)A'(T,\Delta T) + B(T)\ \sigma_w^2\ B'(T), \qquad (23)$$

where σ_w^2 is the variance of the input noise. All of the coefficients in (23) are constant with the exception of $\Delta T(k)$ which is the random time-correction factor. Cantrell has shown that in the case where matrices A and B are random variables which are identically distributed and independent from sample to sample, that the covariance matrix is given by (reference 21).

$$\overline{P(k + 1)} = \overline{A(T,\Delta T)P(k)A'(T,\Delta T)} + \overline{B(T)\sigma_w^2\ B'(T)} \qquad (24)$$

where the bar denotes the expected value (averaged over the random variable of interest, in this case $\Delta T(k)$).

To solve for the variance reduction rates, $A(T, \Delta T)$ and $B(T)$ are used in (24) with the resulting equations, then being averaged over ΔT. By performing the required operations and noting that in the steady-state case

$$P(k + 1) = P(k), \tag{25}$$

then (24) becomes, after some rearranging, and assuming that $E(\Delta T(k)) = 0$,

$$\begin{bmatrix} \alpha(2-\alpha) & -2T(1-\alpha)^2 & -T^2(1-2\alpha+\alpha^2(1+\sigma^2_{\Delta T}/T^2)) \\ \beta(1-\alpha)/T & 2\beta-2\alpha\beta+\alpha & -T(1-\alpha-\beta+\alpha\beta(1+\sigma^2_{\Delta T}/T^2)) \\ -(\beta/T)^2 & 2\beta(1-\beta)/T & 2\beta-\beta^2(1+\sigma^2_{\Delta T}/T^2) \end{bmatrix} \begin{bmatrix} P_{ss} \\ P_{vs} \\ P_{vv} \end{bmatrix} = \begin{bmatrix} \alpha^2 \\ \alpha\beta/T \\ (\beta/T)^2 \end{bmatrix} \sigma^2_w \tag{26}$$

where

$\sigma^2_{\Delta T} = E(\Delta T^2(k))$

P_{ss} = steady-state variance of the smoothed position, $Z_s(k)$

P_{vs} = steady-state covariance of $Z_v(k)$ and $Z_s(k)$

and P_{vv} = steady-state variance of $Z_v(k)$.

Solving these equations simultaneously gives

$$\begin{aligned} K_s &= P_{ss}/\sigma^2_w = (2\alpha^2 - 3\alpha\beta + 2\beta)/\Delta \\ K_{vs} &= P_{vs}/\sigma^2_w = \beta(2\alpha-\beta)/(T\Delta) \\ K_v &= P_{vv}/\sigma^2_w = 2(\beta/T)^2/\Delta \end{aligned} \tag{27}$$

with

$$\Delta = \alpha(4-2\alpha-\beta) - 2\sigma^2_{\Delta T}(\beta/T)^2 \tag{28}$$

where K_s, K_{vs}, and K_v are the normalized variance reduction ratios with respect to the input noise. In the case where $\sigma^2_{\Delta T} = 0$, these equations reduce to the results found elsewhere (references 19 and 21).

In the case of the predicted position, $Z_p(k, T')$ given by (2), the variance reduction ratio can be expressed in terms of the variance and covariance reduction ratios as

$$K_p(T') = K_s + 2T'K_{vs} + (T')^2 K_v \tag{29}$$

It has been assumed in the analysis just given that the smoothing interval used for velocity estimation is a constant equal to the sampling or rotation period of the sensor. While this will be correct on the average, it may seem incongruous to use the time-correction process and yet at the same time assume a constant smoothing interval. For this reason, (1) and (3) will be modified to use the actual time interval between the center of the tracking cycles, i.e.,

$$Z_v(k) = Z_v(k-1) + (\beta/T_i(k))(Z_m(k) + \Delta T(k)Z_v(k-1) - Z_p(k)) \tag{30}$$

with

$$T_i(k) = T(k) - T(j) \tag{31}$$

where T(j) is the time of the center of the tracking cycle during which the last datum was processed for this track. In this case, the smoothing interval, $T_i(k)$ is also a random variable. Following the same procedure as used previously and assuming that $T_i(k)$ is statistically independent of all other random variables, the variance reduction ratios in this case are defined by the equation,

$$\begin{bmatrix} 2\alpha-\alpha^2 & -2(1-\alpha)^2E(T_i(k)) & -(1-\alpha)^2E(T_i^2(k))-\alpha^2E(\Delta T^2(k)) \\ \beta(1-\alpha)E(1/T_i(k)) & 2\beta-2\alpha\beta+\alpha & -(1-\alpha)(1-\beta)E(T_i(k))-\alpha\beta E(\Delta T^2(k))E(1/T_i(k)) \\ -\beta^2E(1/T_i^2(k)) & 2\beta(1-\beta)E(1/T_i(k)) & 2\beta-\beta^2-\beta^2E(\Delta T^2(k))E(1/T^2(k)) \end{bmatrix} \begin{bmatrix} K_s \\ K_{vs} \\ K_v \end{bmatrix} = \begin{bmatrix} \alpha^2 \\ \alpha\beta E(1/T_i(k)) \\ \beta^2E(1/T_i^2(k)) \end{bmatrix} \tag{32}$$

where E(·) are the appropriate expected values. In the case where the smoothing interval is a constant, (32) reduces to (26); and while in the case where time correction is not used (i.e., $\sigma_{\Delta T}^2 = E(\Delta T^2(k)) = 0$), (32) reduces to the results given elsewhere (reference 21) for a random update interval filter. If the tracking filter operates at a fixed rate, then $T_i(k)$ will be a discrete random variable taking on only a limited number of values equal to integer multiples of the tracking filter period.

Unfortunately, the solutions to (32) do not reduce to any simplified form as in the case of (27) and it is necessary to solve (32) numerically.

2.3.2 Application of the Asynchronous Variance Reduction Ratios to Altitude Tracking.

The equations derived above can be applied to the analysis of an $\alpha-\beta$ tracking filter as used for the en route altitude tracking function (reference 9). Since time correction is not used in all cases, the performance of the tracking filter must be evaluated both with and without time correction. If the time correction procedure is not performed in an asynchronous situation, then this is equivalent to the introduction of an error equal to the difference between the measured position and the true position at the time at which the filter assumes the measurement to have been made. If the target is moving at a constant true velocity Z_V, then the error which is introduced is equal to $\Delta T Z_V$ so that the errors at the input to the filter can be considered as two additive errors as illustrated in figure 1. The error ΔZ_q will be assumed to arise as a consequence of the quantization of the true position, Z_T. It will also be assumed that these errors are statistically independent. The case in which time correction is used is illustrated conceptually in figure 2. As seen in this figure, the time-correction process is a feedback loop in which the estimated velocity is multiplied by ΔT to form a corrected input. A second noise source is also needed in this case to account for the fact that time is also quantized so that instead of the error being $\Delta T Z_V$, it is now $\Delta T_q Z_V$ where ΔT_q is the time-quantization unit.

The performance of the tracking filter, in the case where the only errors are those discussed above, can be written in terms of the appropriate variances and variance reduction ratios. For example, in the case of the variance of the velocity errors, the filter performance without time correction is

$$P_{vv} = K'_v(\sigma^2_{\Delta z} + Z^2_V \sigma^2_{\Delta T}), \tag{33}$$

where

$$K'_v = 2(\beta/T)^2/\alpha(4-2\alpha-\beta)$$
$$\sigma^2_{\Delta z} = \text{variance of measurement quantization errors,} \tag{34}$$

and in the case in which time correction is used

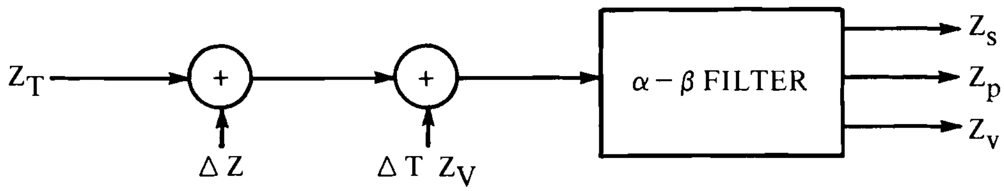

FIGURE 1. ILLUSTRATION OF INPUT DATA ERRORS FOR ASYNCHRONOUS FILTER WITHOUT TIME CORRECTION

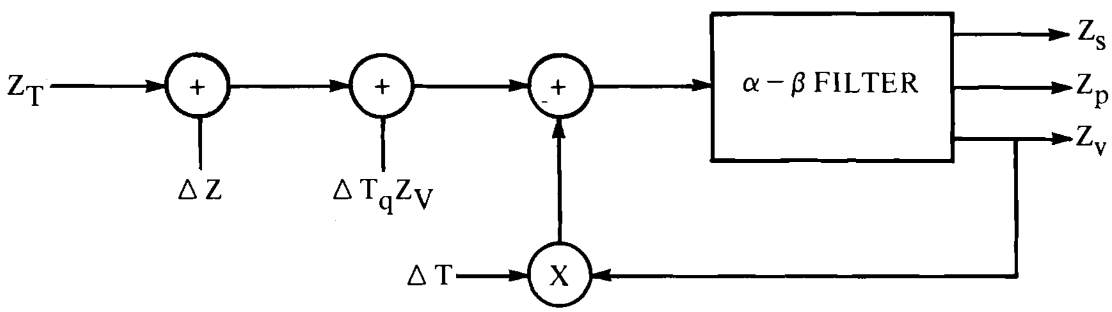

FIGURE 2. ILLUSTRATION OF INPUT DATA ERRORS FOR ASYNCHRONOUS FILTER WITH TIME CORRECTION

$$P_{vv} = K_v(\sigma^2_{\Delta z} + Z^2_V \sigma^2_{\Delta T_q}), \tag{35}$$

where K_V is given by (27) or (32) and

$\sigma^2_{\Delta T_q}$ = variance of time-quantization errors

Similar results can be specified for the variances of the smoothed and predicted positions and for the covariance between the smoothed position and the velocity. If it is assumed that all the measurement errors and the time-correction factor, ΔT, are uniformly distributed with a mean of zero, then the error variances are

$$\sigma^2_{\Delta Z} = \Delta Z^2_q/12$$
$$\sigma^2_{\Delta T} = \Delta T^2/12 \tag{36}$$
and
$$\sigma^2_{\Delta T_q} = \Delta T^2_q/12,$$

where now ΔZ_q, ΔT, and ΔT_q are the widths of the intervals in which these quantities are contained. Much of the material in section 2.3 was extracted from references 6 and 14 which contain additional material on time correction, as well as numerical results on the performance of tracking algorithms in which time correction is used. In particular, it is shown there that the errors in the predicted altitude can be in excess of several thousand feet if the time-correction process is not used.

2.4 APPLICATION TO CONFLICT PREDICTION.

The response of the vertical tracking algorithm, given in sections 2.2 and 2.3, was in terms of the mean and variance of the smoothed position and velocity. Using the assumption of Gaussian statistics, which is considered to be justified by the additive nature of the process, the equations describing the performance of the tracking algorithm can be used to compute the probabilistic performance of the algorithms using the tracking data. The performance can be calculated on the basis of a scan-by-scan history throughout the duration of a particular scenario or on the basis of a few statistics which summarize the performance for the scenario under consideration. For reasons which will become obvious later, it will be necessary to limit the number of performance statistics used to characterize the results in each scenario.

The automation feature of primary interest in this study is the En Route Conflict Alert algorithm (references 7 to 9). The practical implementation of the Conflict Alert function makes use of various sources of information, such as controller inputs, adaptation, and flight plan information in addition to the Mode C altitude data provided by the radar. In addition, there are various parameters which affect the performance of the Conflict Alert function. In many cases, the use of a substantial number of parameters and various sources of data serve only to confuse the results and obfuscate the system performance features due to tracking. As a result, it is necessary to eliminate much of the operational and procedural detail which exists in the Conflict Alert function in order to identify the system performance which is tracking dependent. For this purpose, a simplified Conflict Alert function will be examined in which no information other than the Mode C radar data is assumed to be available. No distinction will be made between targets using Visual Flight Rules and those using Instrument Flight Rules (as in references 10 and 22).

2.4.1 Description of Simplified Conflict Alert Function.

The simplified version of the Conflict Alert algorithm is described below. The present study is restricted to vertical tracking only, although there are corresponding requirements for horizontal tracking. Since any conflict must involve at least two aircraft, some means must be available to select aircraft pairs for evaluation, but this will not be considered. The complete functional specifications for the En Route Conflict Alert algorithm are given in sections 13 and 14 of reference 9.

For the purposes of the present study, the collision avoidance algorithm will be simplified to use only the position and velocity information based on the Mode C data. Using the tracking data, two types of conflicts or separation violations can be identified for a particular pair of targets: a current violation based on the current position or a predicted violation based on a future position. For operational and theoretical purposes, no distinction is made between the two types of alerts; however, the conditions leading to each type of alert must be explicitly defined for mathematical computation of system performance.

Let Z_i and V_i be the position and velocity, respectively, of the ith target and similarly for Z_j and V_j. If Z_T is the vertical separation threshold required for a separation violation, then the i-j conflict pair is defined to be in a current altitude separation violation at epoch k if

$$|\Delta Z(k)| < Z_T \tag{37}$$

where

$$\Delta Z(k) = Z_i(k) - Z_j(k). \tag{38}$$

For a conflict pair not in a current separation violation, a hazardous situation may exist if the pair under consideration will violate, or will appear to violate, the separation standard a short time in the future. Therefore, a time interval T_T will be defined and it is desired to know if the conflict pair will violate the separation threshold Z_T within this interval.

For a predicted altitude separation violation to occur, the requirements are somewhat more complicated. In this case, the targets must be converging in altitude

$$\Delta Z(k)\Delta V(k) < 0 \tag{39}$$

where

$$\Delta V(k) = V_i(k) - V_j(k) \tag{40}$$

and the time to violation is computed at

$$t_1 = \min(\tau_1,\tau_2) \tag{41}$$

$$t_2 = \max(\tau_1,\tau_2) \tag{42}$$

where

$$\tau_1 = (Z_T - \Delta Z(k))/\Delta V(k) \tag{43}$$

$$\tau_2 = (-Z_T - \Delta Z(k))/\Delta V(k). \tag{44}$$

If the tracks are converging so slowly that the predicted separation violation will not occur within a specified time, then no violation is said to occur so the restrictions

$$0 \leq t_1 \leq t_2 \leq T_T \tag{45}$$

must be imposed. Equations (41) - (44) are formulated so that it does not matter whether $Z_i > Z_j$ or $Z_j > Z_i$. The times calculated are for the beginning and end of a violation of the separation standard, not for a collision, and this is illustrated in figure 3. A more complete illustration of these concepts in an operational context is given elsewhere (reference 8). For operational use, the separation standards Z_T and T_T may change under various conditions throughout a particular scenario, but for the purposes of this report, the same parameter values will be used throughout the entire scenario. In the case when horizontal tracking is also considered, the time interval in which a predicted altitude violation will occur must also overlap the interval in which a predicted horizontal separation violation occurs in order for a separation violation to be reported to the controller.

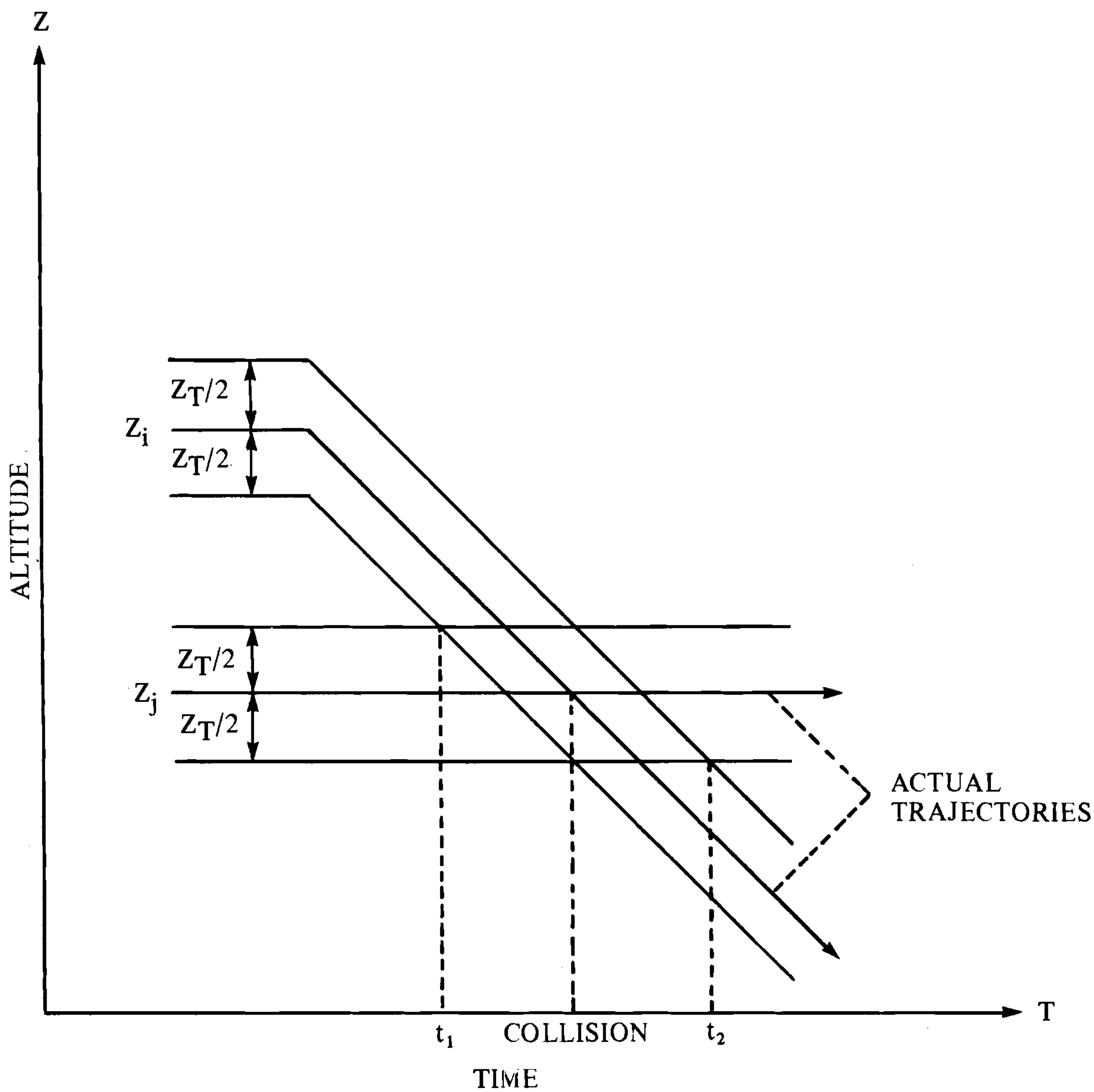

FIGURE 3. ILLUSTRATION OF TIME TO VIOLATION COMPUTATIONS

2.4.2 Mathematical Calculation of Performance Statistics for Conflict Alert.

Since ΔZ and ΔV are random variables with means and variances as determined from the results in sections 2.2 and 2.3, respectively, the probability of a conflict declaration can be computed on a scan-by-scan basis by integrating the joint density function of ΔZ and ΔV over the region of the ΔZ- ΔV plane in which each particular type of alert is defined. Such a phase-plane analysis has been used previously to compute the probability associated with hazardous situations (reference 23). For the current alert, the probability is easily calculated from the definition (37) as

$$P_c \doteq \int_{-Z_T}^{Z_T} f(\Delta Z) d\Delta Z \qquad (46)$$

where the dependence on k will always be implied but not always explicitly denoted. Under the assumption of independent Gaussian distributions for Z_i and Z_j, (46) becomes

$$P_c = \tfrac{1}{2} \left[\text{erf}((Z_T - u_d)/\sqrt{2}\sigma_d) - \text{erf}((-Z_T - u_d)/\sqrt{2}\sigma_d)\right] \qquad (47)$$

where erf (x) is the standard error function (reference 24),

$$u_d(k) = u_i(k) - u_j(k) = E(Z_i(k)) - E(Z_j(k)) \qquad (48)$$

$$\sigma_d^2(k) = \sigma_i^2(k) + \sigma_j^2(k) \qquad (49)$$

and E(Z(k)) is the mean value of the smoothed position as computed using the results in table 3 for the target maneuver of interest. The variances of the current positions are given by P_{ss} using the appropriate variance reduction ratios ((27) or (32)) depending on whether or not time correction is used. An exactly similar equation can be defined for a predicted conflict by using a fixed prediction time. In this case, $u_d(k)$ would be calculated using (19) and the variance would be obtained from (29), but this does not take into account the fact that a conflict may actually occur between the current time and the predicted time. In effect, even though no conflict is indicated at either the current or predicted time, a conflict may occur at some intermediate point.

Consequently, in order to compute the probability of an altitude separation violation which includes the possibility of both a current alert and a predicted alert at some time up to T_T in the future, it is necessary to integrate over the appropriate regions in the ΔZ - ΔV plane. The limits of integration will be determined by finding the regions in the ΔZ - ΔV plane in which the time intervals computed from (41) to (44) result in solutions obeying the restrictions imposed by (45). For $\tau_1 = \tau_2 = 0$, it is found that $\Delta Z = \pm Z_T$, which is exactly equivalent to a current alert, and for $\tau_1 = \tau_2 = T_T$

$$\Delta Z = \pm Z_T - T_T \Delta V. \tag{50}$$

The restrictions on ΔZ and ΔV which are imposed by (45) are plotted in figure 4. Note that by (39), predicted alerts can only be generated by the contributions to the alert probability in the second and fourth quadrants. The regions to the left of the boundary line $\Delta Z = Z_T - T_T \Delta V$ in the fourth quadrant is the region in which a predicted alert will occur at some time less than T_T in the case where $Z_i > Z_j$; and similarly for a predicted alert in the second quadrant where $Z_i < Z_j$. Note that there is an overlap of the regions in which predicted and current alerts are defined so that the probabilities of these events are not mutually exclusive.

From the discussion above and figure 4, the probability of an alert which includes both the current and predicted alert definitions is

$$P_A(k) = \int_{-\infty}^{\infty} \int_{-Z_T}^{\max(Z_T, Z_T - T_T \Delta V)} f(\Delta Z, \Delta V) d\Delta Z \, d\Delta V \tag{51}$$

for the case $Z_i > Z_j$; and

$$P_A(k) = \int_{-\infty}^{\infty} \int_{\min(-Z_T, - Z_T - T_T \Delta V)}^{Z_T} f(\Delta Z, \Delta V) \, d\Delta Z \, d\Delta V \tag{52}$$

for $Z_j > Z_i$. The function $f(\Delta Z, \Delta V)$ is the joint probability density function for the differences in the smoothed positions and velocities. Evaluation of the double integral can be simplified using the properties of the Gaussian distribution to rewrite the joint density function as the product of two funtions (reference 25)

$$f(\Delta Z, \Delta V) = f(\Delta V) f(\Delta Z / \Delta V) \tag{53}$$

in which the variables of integration are separable and where the moments of the conditional density function are

$$u' = E(\Delta Z / \Delta V) = u_d + r\sigma_d(\Delta V - u_{\Delta V}) / \sigma_{\Delta V} \tag{54}$$

$$\sigma' = \sigma_d \sqrt{1 - r^2} \tag{55}$$

where $\sigma_{\Delta V}$ is the standard deviation of the difference in the velocities and r is the correlation coefficient between the smoothed position and velocity; i.e., $K_{VS}/\sqrt{K_S K_V}$. Since the variables in the density function are separable, (51) can be written

$$P_A(k) = \tfrac{1}{2}\int_{-\infty}^{\infty} \frac{e^{-(\Delta V - u_{\Delta V})^2/2\sigma^2_{\Delta V}}}{\sqrt{2\pi\,\sigma^2_{\Delta V}}} \Big[\mathrm{erf}((\max(Z_T,\ Z_T - T_T\Delta V) - u')/\sqrt{2}\sigma') - \mathrm{erf}((-Z_T - u')/\sqrt{2}\sigma')\Big]\, d\Delta V \tag{56}$$

where $u_{\Delta V}$ is the difference in the mean value of the velocity estimates obtained from table 3 for the type of maneuver under consideration. An equivalent procedure can be used for the evaluation of (52). The numerical evaluation of (56) was performed using the Gaussian quadrature technique (reference 26). For this purpose, a 16-point quadrature formula was used twice to cover the region from $u-5\sigma$ to u and from u to $u+5\sigma$, which gave sufficient numerical accuracy for the purposes of this study.

The equations just given can be used to compute the probability of an alert on a scan-by-scan basis throughout any scenario. All of the factors of interest are included in this equation; namely, the mean and variance of the tracking filter output (which are dependent on the smoothing parameters), the separation threshold Z_T, and the time interval T_T. For every scenario and parameter set it is desired to evaluate, there will be a time series, $P_A(k)$, which characterizes the performance of the Conflict Alert algorithm for the duration of the scenario. The comparison of the system performance for a large number of scenario and parameter combinations would be rather tedious using the $P_A(k)$ sequence. In addition, the $P_A(k)$ sequence is not particularly satisfying in terms of an operationally meaningful performance measure. Consequently, a simpler and more meaningful performance statistic would be desirable to summarize the performance of the Conflict Alert algorithms. The statistic which has been selected for this purpose is the mean value of the time interval from the declaration of an alert until the actual collision occurs (in this study, only scenarios in which a collision actually occurs will be considered).

The sequence of probabilities, $P_A(k)$, can be used to compute the mean warning time, but a new sequence, P(k), must be defined which is the probability that an alert is first detected at epoch k. Obviously, as k increases to the point at which the collision occurs, $P_A(k)$ will approach one (assuming the algorithm is properly designed) but this does not indicate the warning time in this situation. The probability that an alert will first be detected at epoch k is the product of the probability of detection at k times the probability that it was not detected in a previous epoch; i.e.,

$$P(k) = P_A(k) \prod_{i=1}^{k-1} (1 - P_A(i)) \tag{57}$$

under the assumption that the underlying events are statistically independent from epoch-to-epoch. This is a very important assumption which will be discussed in more detail in a later section. The mean warning time, T_W, is given by

$$T_W = E(T(k)) = \sum_k T(k)P(k) \tag{58}$$

where T(k) is the time from epoch k until the collision occurs. Another statistic which may be of interest is the variance of the warning time

$$\sigma^2_W = E(T^2(k) - E^2(T(k))) \tag{59}$$

but since a discrete time system is being considered, it might be more meaningful to consider the probability of a unit time interval change in system performance.

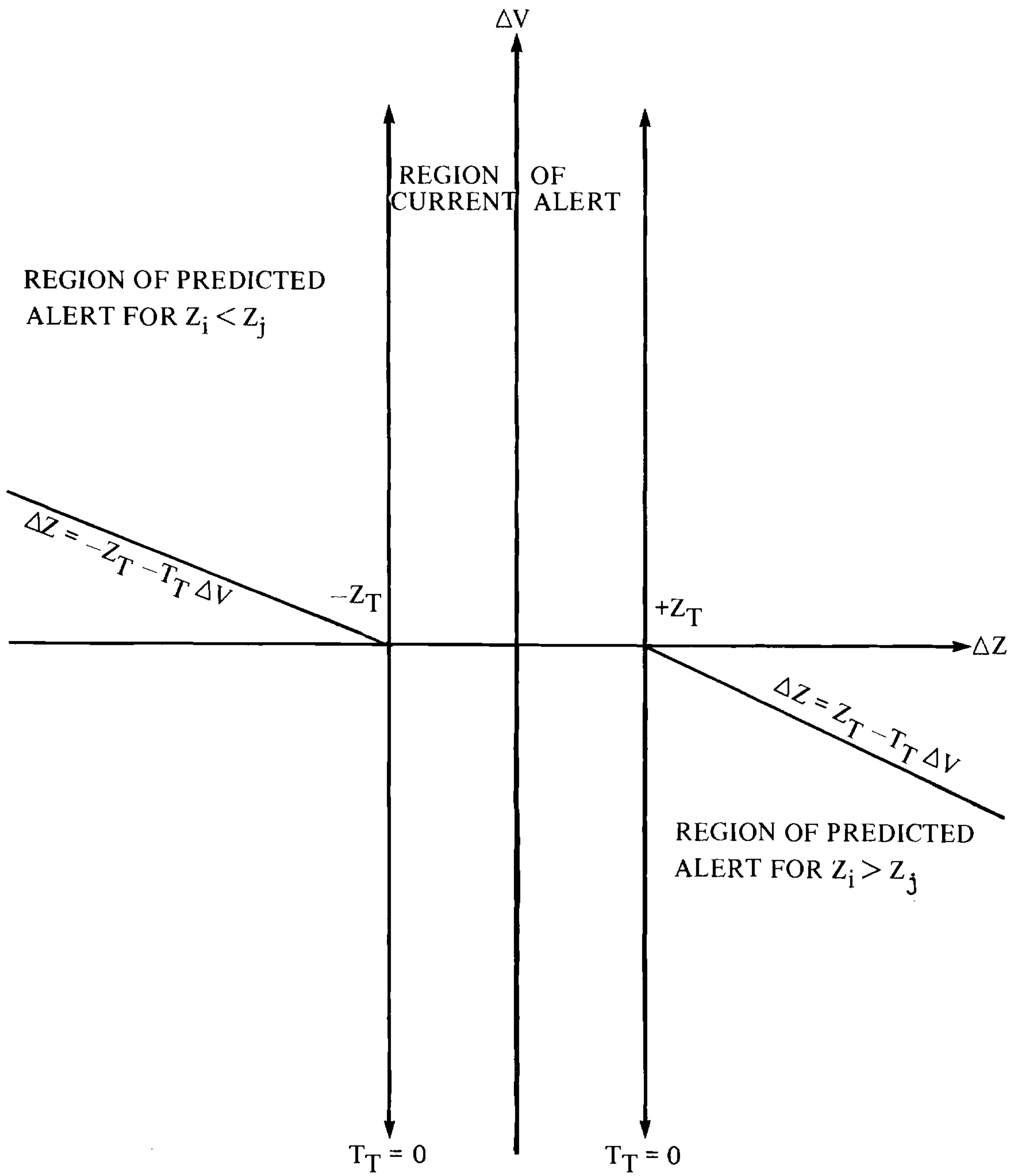

FIGURE 4. ILLUSTRATION OF INTEGRATION LIMITS FOR PROBABILITY OF ALERT GENERATION

3. NUMERICAL RESULTS

The theory developed in section 2 will now be applied to determine the sensitivity of the performance of the Conflict Alert algorithm to variations in the performance of the altitude tracker.

3.1 PERFORMANCE OF THE $\alpha-\beta$ TRACKING FILTER

The transient and steady-state performance of the $\alpha-\beta$ tracking filter was derived in sections 2.2 and 2.3, respectively. For illustrative purposes, numerical results will be presented to demonstrate the performance of the $\alpha-\beta$ filters in a typical altitude tracking application. For this purpose, the most realistic situation of importance is the response to ramp function which would correspond to an aircraft changing altitude at a constant rate as illustrated in figure 5. The function Z(t) represents a piecewise linear signal for which the response of the $\alpha-\beta$ filter can be determined from the results given in tables 3 and 4. The response of the $\alpha-\beta$ filter to Z(t) in figure 5 would actually be the sum of two responses: the first would be the response to a constant velocity ramp starting at t_1 and the second would be the response to a ramp of equal magnitude, but opposite sign, starting at t_2. The second input ramp would thus cancel the first resulting in level flight starting at t_2.

Naturally, Z(t) represents an idealized, but realistic, version of a typical maneuver which would be observed in an air traffic control environment. In practice, the actual input signal to the tracking algorithm (the Mode C data provided by the Air Traffic Control Radar Beacon System (reference 27)) will be corrupted by quantization and measurement errors. In addition, even if the pilot intends to fly at a constant vertical rate, the aircraft control system will not be capable of doing this exactly because of such external factors as random wind gusts. A realistic trajectory will also show a very slight acceleration at the start and end of a maneuver (the response of the airframe to the maneuver command), but this will only be present for a very short period of time and could not be estimated accurately with the data rate and quantization level presently used. In one study, reference 28, the acceleration phase at the start of a constant velocity maneuver was specified to last for only 5 s which would obviously be undetectable with a 10 s data rate typical of en route sensors.

The analytical solution to the response of the $\alpha-\beta$ tracking filter (without considering time correction) is given in table 3 by $h_s(t)$ and $h_v(t)$ for the smoothed position and velocity, respectively. Examining the form of the response, it is easily

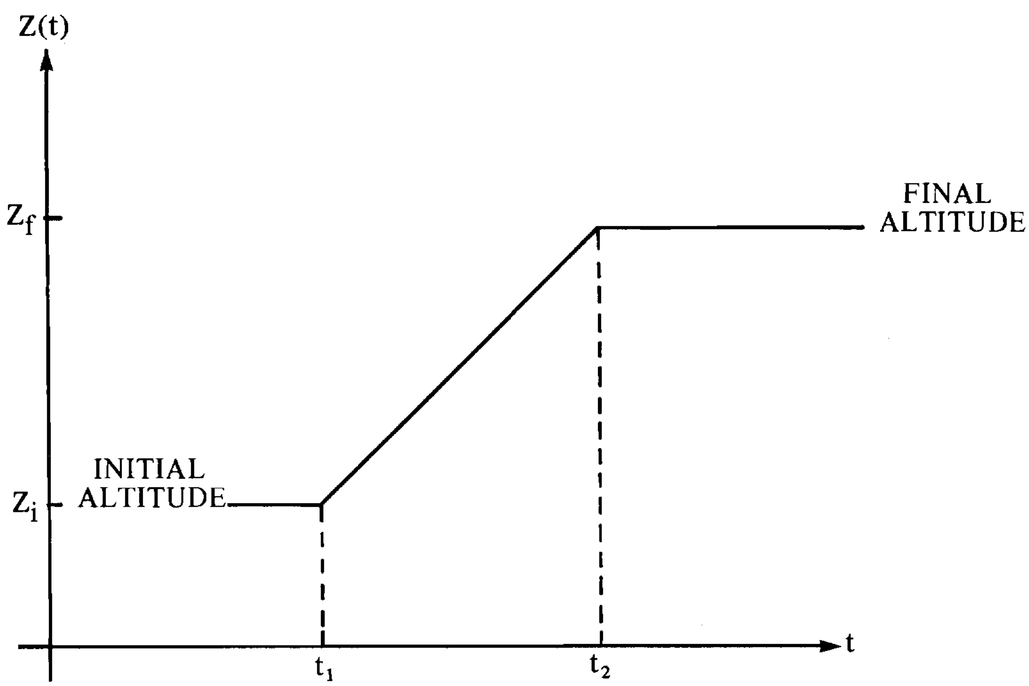

FIGURE 5. ILLUSTRATION OF TYPICAL ALTITUDE TRAJECTORY USED AS INPUT TO TRACKING FILTER

seen that the smoothed position consists of two components. The first is the desired response (i.e., a ramp) while the remaining terms constitute a transient component which decays exponentially. In this case, the transient term is also the error term since the $\alpha-\beta$ filter will follow a ramp with a zero steady-state error in the mean value. In a similar manner, the transient solution for the velocity response is also the sum of a true plus, an error term.

The $\alpha-\beta$ filter responses given in table 3 are all normalized with respect to the input. In the case of a ramp input, the normalization is with respect to the velocity. The actual error in the smoothed position is determined by multiplying the normalized error by target movement in one scan; i.e., by vT where v is the true velocity and T is the scan time. Similarly, the normalized velocity error must be multiplied by the true velocity to obtain the actual velocity error. In the case of the smoothed position, the normalized error is given by

$$e_s(\tau) = \begin{cases} (1-\alpha)e^{-a\tau}(\cos \omega\tau - g_r \sin \omega\tau) & \tau \geq 0 \\ 0 & \tau < 0 \end{cases} \tag{60}$$

and for the velocity

$$e_v(\tau) = \begin{cases} (1-\beta)e^{-a\tau}(\cos \omega\tau - m_r \sin \omega\tau) & \tau \geq 0 \\ 0 & \tau < 0 \end{cases} \tag{61}$$

where τ is time normalized with respect to T. The origin for τ is taken as the first scan after the start of the maneuver (i.e., $\tau = (t-T)/T$ where t is real time).

The solutions for the $\alpha-\beta$ tracking filter are functions of both α and β with α and β being chosen independently within certain restricted ranges required for stability. In many practical situations, the α and β smoothing parameters are chosen according to the optimal relationship derived by Benedict and Bordner (reference 15)

$$\beta = \alpha^2/(2 - \alpha) \tag{62}$$

so that only one parameter value must be chosen. It can be verified that the pairs related by (62) are within the region of an oscillatory response α specified by (20). Using this simplification, the normalized errors were calculated for various values of α and these results are plotted in figures 6 and 7. Since the system under consideration is a discrete time system, the errors given by (60) and (61) are only observed at integer values of τ. The range of a used in figures 6 and 7 covers the

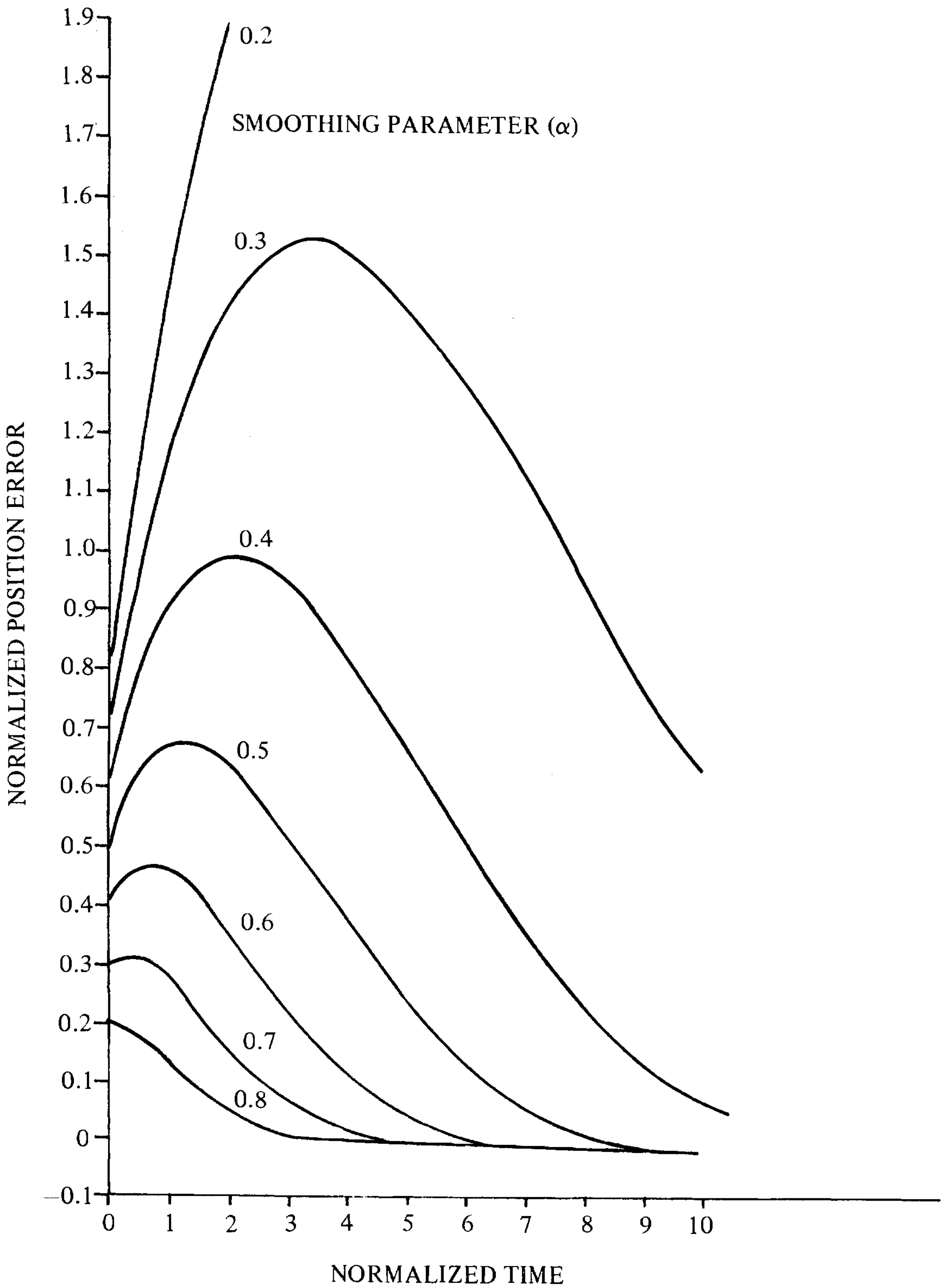

FIGURE 6. NORMALIZED POSITION ERROR OF $\alpha-\beta$ TRACKING FILTER

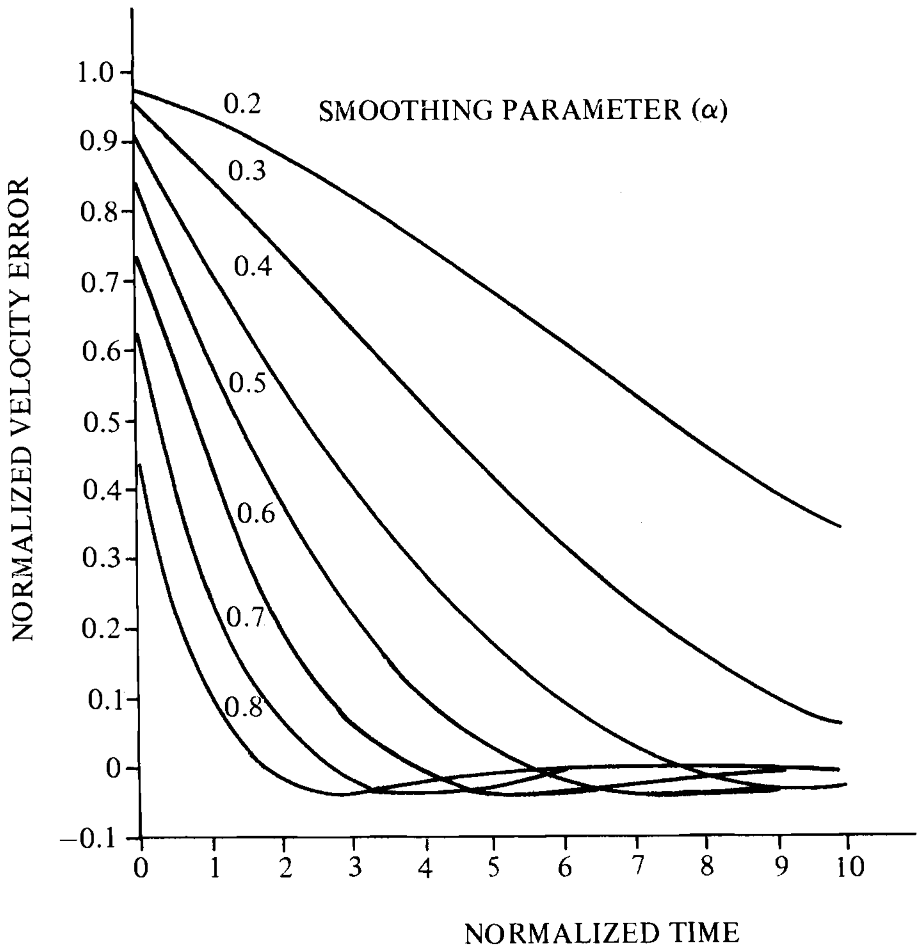

FIGURE 7. NORMALIZED VELOCITY ERROR OF $\alpha-\beta$ TRACKING FILTER

range likely to be encountered in practice with values of 0.4 (references 4 and 7) and 0.6 (references 4, 5, and 29) having been selected in the past.

Determining the point at which the transient error becomes negligible can only be done using the actual error. For example, for a target velocity of 2,000 ft/min and a 12 s scan time, the unit value of the normalized error $e_s(t)$ is 400 feet. One criterion for determining the point at which the transient error is negligible would be to select the time at which the transient error is less than one-half the quantization interval of the input data. Using this criterion, the transient error would have to be less than 50 feet or 0.125 in the case of the normalized error for a velocity of 2,000 ft/min. For the presently used value of α (0.6), the transient error in the smoothed position is negligible 5 scans (normalized time plus 1) after the start of the maneuver and 10 scans for a value of 0.4. If the target velocity were increased, then the response time would also increase as specified by the point at which the normalized error corresponds to an actual error of 50 feet or less. However, the operational significance of such errors is very difficult to determine.

The normalized velocity errors in figure 7 must be multiplied by the true velocity to determine the actual error. The vertical velocity is used in the Conflict Alert algorithm for the prediction of the future position of a target. In the case of the velocity, the product of the prediction time and the velocity error can be specified to be less than one-half of the vertical separation threshold in order to determine a criterion for the response time of the velocity. Using this approach will give results similar to those obtained in the case of the smoothed position. While the response times (or time to reach a steady-state condition) may seem rather large in many situations, the actual impact of the transient response is not that significant.

The other aspect of tracking performance, which is of interest, is the response to the errors resulting from quantization of the input signals. The response in this case is usually described in terms of the variances of the errors in the filter outputs about the mean values. The noise response of the altitude tracker was discussed in section 2.3.

To illustrate the differences in altitude tracking performance under various conditions, the error in the altitude as predicted 2 minutes into the future was calculated. The error in the predicted altitude, as specified by three standard deviations, is plotted in figure 8 for a total of four cases: fixed and variable smoothing intervals and both with and without time correction. It has been assumed in these calculations that ΔZ = 100 ft, ΔT = 6 s, ΔT_q = 0.5 s and T = 10 s with the corresponding variances as given by (36). In the case of

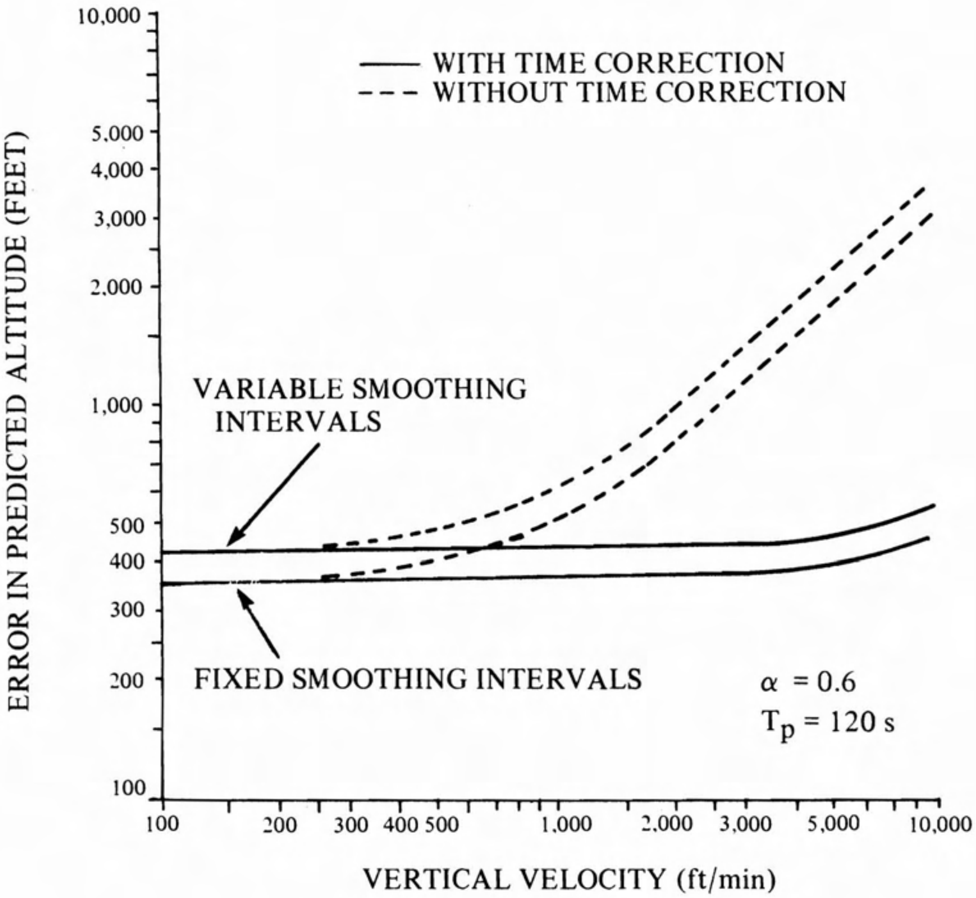

FIGURE 8. COMPARISON OF ASYNCHRONOUS FILTER PERFORMANCE FOR FIXED AND VARIABLE SMOOTHING INTERVALS

variable smoothing intervals, it was assumed that only 6 and 12 s intervals were used and that Pr(6) = 1/3 and Pr(12) = 2/3 so that the mean value of the smoothing interval is 10 s, as used in the fixed interval case. Only one value of the smoothing parameter was selected for illustration, but the same general trends are seen in all cases; namely, the altitude prediction errors increase substantially as the velocity increases if time correction is not used. The reason for this is because the timing errors became substantially larger than the input quantization errors as the velocity increases. The results for the variable smoothing intervals, which are actually more realistic, follow the same general trend as for the fixed smoothing intervals, but are slightly larger (by about 20% in this case).

The analysis given above includes only the most obvious error sources which are specified in terms of quantization errors. In practice, however, there are additional errors which have not been considered. For example, it has been found that the standard deviation of the errors in the reported altitude is actually two to four times larger than that indicated by quantization alone (reference 30) and that the timing errors in the measured time of receipt may extend to 0.8 s (reference 14). In addition, since the use of the predicted altitude is to predict the vertical separation between two targets, the errors in figure 8 would have to be multiplied by $\sqrt{2}$ to give the equivalent error in the predicted separation of two targets at the same vertical velocity.

The most important aspect of the performance of practical digital filters which has not yet been considered is the use of finite-precision arithmetic. It is well known that under certain conditions finite-precision digital filters produce large amplitude oscillatory error response at the filter output, usually referred to as limit-cycles (see, e.g., references 31-33). One of the causes of limit-cycle oscillations, which are very difficult to analyze, is generally considered to be quantization error sequences which are highly correlated as a result of quantizer nonlinearities in recursive filters. Clearly, the case of an asynchronous digital filter without time correction is, in reality, a digital filter with a sequence of highly correlated input errors since the error at one epoch will determine the error at the next, assuming the sensor and tracking filter operate at fixed rates. Thus, the performance of an asynchronous digital filter which is subject to limit-cycle oscillations may be substantially worse than that indicated in figure 8, which is based on the assumption of statistically independent quantization errors.

While the same phenomena may be encountered in a digital filter using time correction, it would not be expected that the input error correlation would be as potentially significant as in the asynchronous case and that the limit-cycle behavior, if any exist, for the time-corrected filter would be inconsequential as compared to that for the asynchronous filter. The reason for this is the fact that for the filter with time correction, the errors at the input are predominately due to altitude measurement quantization errors (rather than timing errors) throughout most of the vertical velocity range of interest. Thus, the time-correction process improves the performance of the asynchronous digital filter by not only reducing the amplitude of the input errors but also, and perhaps more importantly, by destroying the correlation between the input errors. However, limit-cycles can also exist because of correlation in the altitude measurement quantization errors and time correction will not eliminate this difficulty.

3.2 PRELIMINARY CONFLICT ALERT PERFORMANCE CONSIDERATIONS.

Before presenting the parametric performance results for the Conflict Alert algorithms, one example will be given of the detailed performance data for a single scenario. In addition, since the performance of the Conflict Alert algorithm is limited by the scenario under consideration, it is important to separate constraints imposed by external factors from limitations due to the performance of the tracking algorithm. Since the objective in this study is to examine the relationship between tracking and Conflict Alert, the impact of any external scenario dependent factors must be determined. The theoretical warning time in various scenarios was calculated to determine the impact of the scenario dependent parameters. These results will also provide performance bounds on the Conflict Alert algorithm and a check on the reasonableness of the numerical calculations used to compute the warning time.

3.2.1 Scan-by-Scan Performance of Conflict Alert.

The most fundamental performance statistic of the Conflict Alert algorithm in a particular scenario is the scan-by-scan history of the probability of an alert (given by (51) or (52)). This probability sequence can then be used to calculate the mean value of the warning time via (57) and (58). The ideal shape of the plot of $P_A(k)$ versus time is easily determined from a conceptual viewpoint. As illustrated in figure 9, in an ideal situation, with no measurement or computation errors and no transient errors, the probability of an alert would jump from zero to one instantaneously at the point t_i where the T_T - projection of the velocity was within Z_T of another target. Likewise, at the point, t_f, at which the two targets are separating and the separation becomes greater than Z_T, the probability of alert should drop instantaneously to zero. Also shown in figure 9 are two examples of what might be observed for

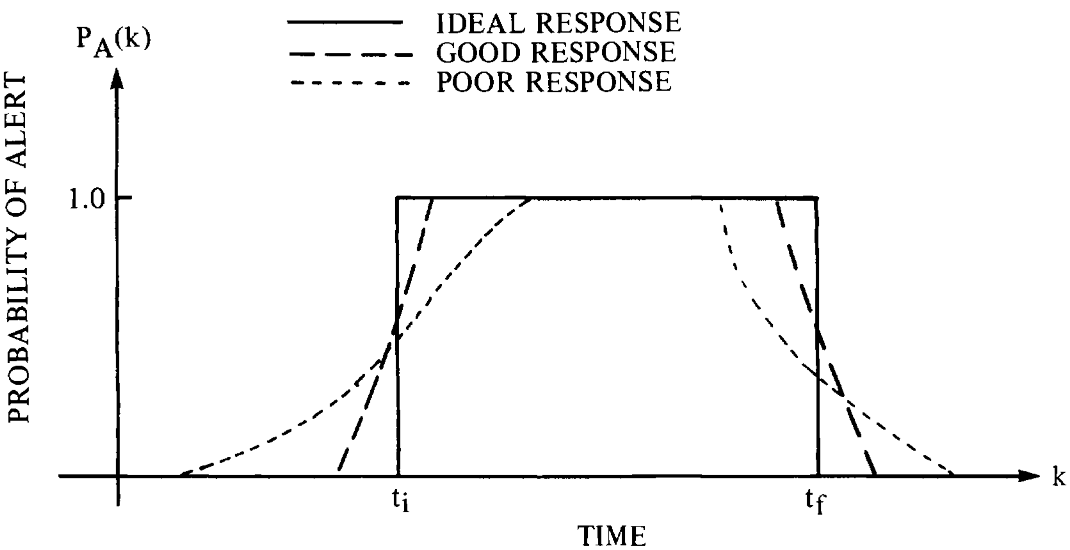

FIGURE 9. CONCEPTUAL ILLUSTRATION OF IDEAL VERSUS PRACTICAL ALGORITHM PERFORMANCE

practical trackers with errors. A poor tracker will have a slow rise in the probability of alert leading to both premature and late alerts. Similar results were expected in the case of the horizontal tracker and examples of such plots show this to be the case (reference 11).

In order to illustrate these concepts, a particular scenario was selected which is representative of situations in which midair collisions have occurred. This scenario is illustrated in figure 10 and consists of two targets in which the higher target descends onto the lower target at a fixed rate. Although many other scenarios could be defined with trajectories leading to a collision, the selected scenario is believed to be more realistic because only one target is actually maneuvering. The scenario is characterized by only two parameters: the initial separation in altitude, ΔZ, and the vertical velocity V.

A sample of the scan-by-scan probability of alert is given in figure 11 for a standard set of parameters. The tracking errors assumed in this case are the same as those used in the previous section. The results are plotted both with and without time correction, but, in general, very similar results were obtained in each case. For the purpose of this study, the "turn-off" characteristics of the algorithm as targets separate will not be considered (for scenarios in which the targets actually collide this characteristic would not be realistic in any case).

One point which must be emphasized in connection with all the results to be presented in this study is that it is assumed that the measurement and computational errors are statistically independent between epochs. The assumption of statistical independence is a critical assumption which will be discussed in greater detail in a later section. The results for the probability of alert in the case of horizontal tracking (reference 11) show a much wider variation in the results than those given in figure 11.

Another illustration of the results in the example scenario just discussed is given in figure 12 in terms of the trajectory of the ΔZ - ΔV mean values in the phase-plane representation. Note that in this case, the transient error in velocity has decayed to an almost undetectable level well before the trajectory enters the region of integration for a predicted alert. In a subsequent section, it will be demonstrated that the transient error phase only impacts the warning time results when the transient portion of the trajectory exists at the boundary of the predicted alert region.

A portion of the scan-by-scan results for scans 10 to 15 of the sample scenario are given in table 1. Note that at scan 12, the mean (from figure 11) is just on the boundary of the predicted alert region so it would be expected that approximately one-half of the probability density function would be within the alert

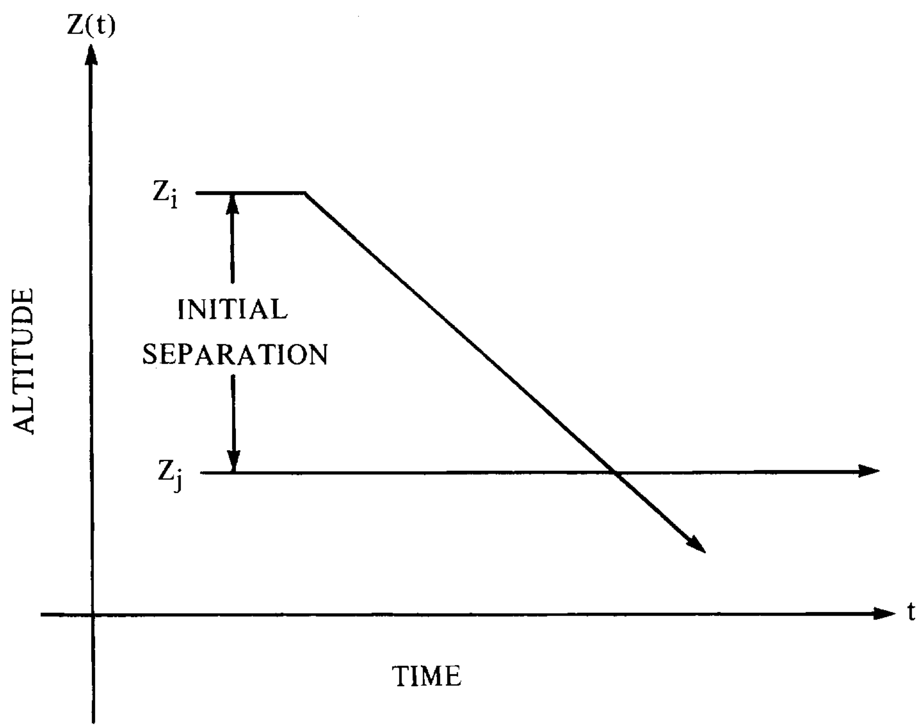

FIGURE 10. STANDARD SCENARIO FOR WARNING TIME COMPUTATIONS AND SIMULATIONS

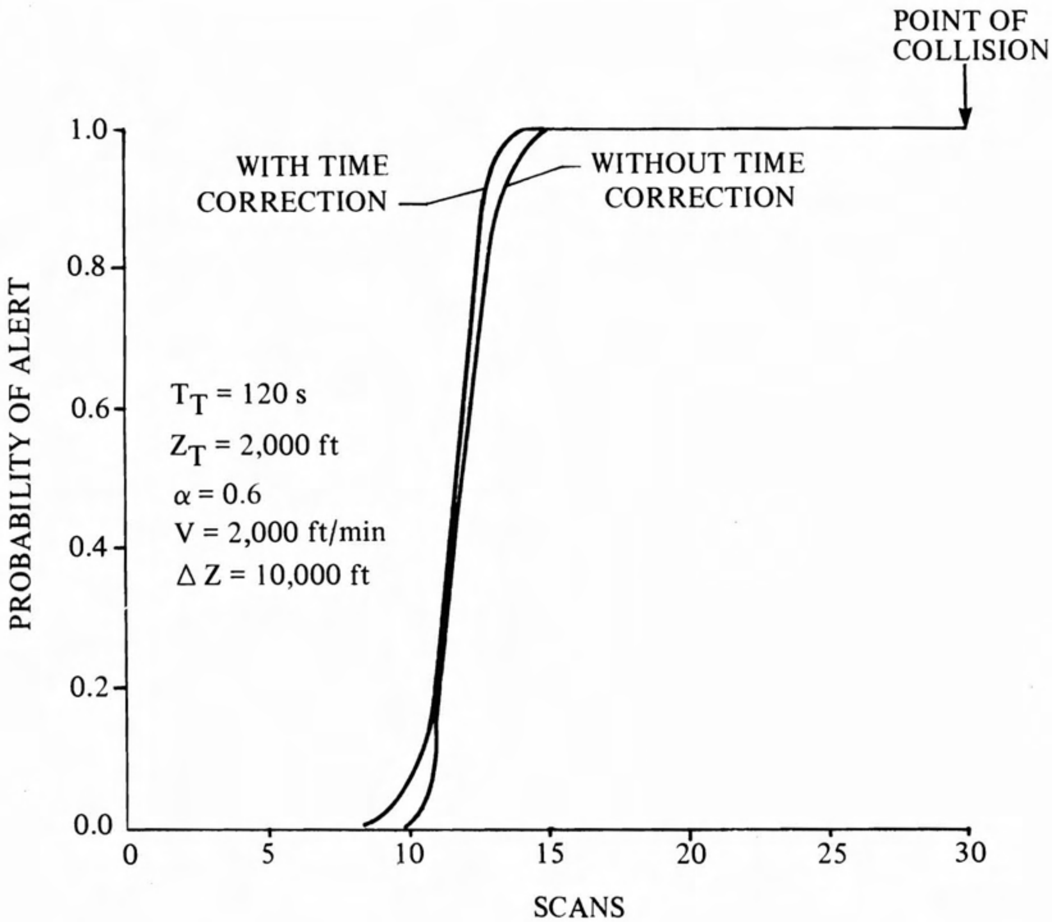

FIGURE 11. ILLUSTRATION OF SCAN-BY-SCAN PERFORMANCE OF CONFLICT ALERT FOR SAMPLE SCENARIO

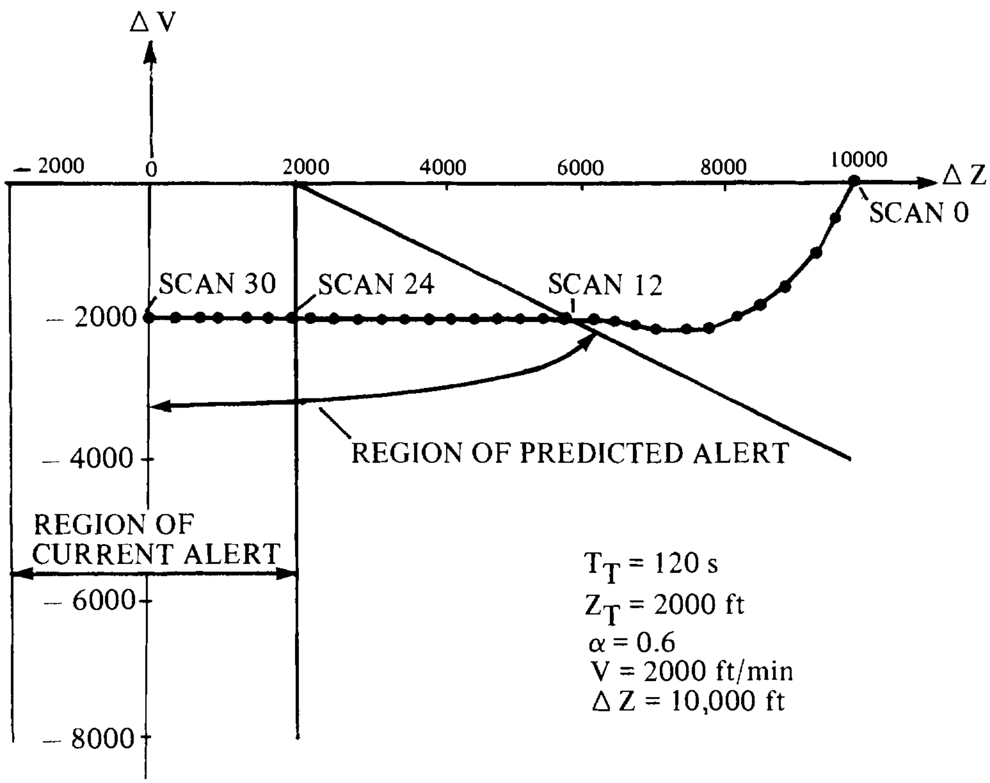

FIGURE 12. ILLUSTRATION OF PHASE-PLANE TRAJECTORY FOR SAMPLE SCENARIO

generation region and the results in table 1 show this to be the case since $P_A(12) = 1/2$ in both cases. The means and standard deviations of the warning times are also given in table 5. The results illustrate the small differences in warning time due to the use of time correction; however, since the scan-to-scan errors are statistically independent, it would be expected that the difference in the mean values would be small and that the major impact would be observed in the standard deviation. In a previous study, the error in the calculation of the time to reach a specified altitude, equivalent to a time to separation violation calculation, was estimated to be on the order of 1 to 2 scans (reference 6) and the results in table 5 confirm this. Further discussion of the impact of time correction on the warning time results is given in sections 3.3 and 3.4.

TABLE 5. SELECTED SCAN-BY-SCAN RESULTS FOR SAMPLE SCENARIO

	PROBABILITY OF ALERT	
Scan	With Time Correction	Without Time Correction
10	0.001	0.038
11	0.058	0.186
12	0.502	0.501
13	0.945	0.824
14	0.999	0.974
15	1.000	0.998
Warning Time: Mean	175.65 s	178.12 s
Standard Deviation	6.48 s	9.73 s

3.2.2 Theoretical Warning Times Derived from Conflict Alert Parameters.

The performance statistic of fundamental importance in the evaluation of the Conflict Alert algorithm is the warning time before a collision. It is important to recognize that certain aspects of the Conflict Alert system performance are determined solely by constraints imposed by the scenario and not by the performance of the tracking algorithm. For this reason, the warning time constrained imposed in the case of the standard scenario given in figure 10 will be calculated.

The ideal warning time is determined solely by the parameters ΔZ, V, T_T, and Z_T. It is independent of the performance of the tracking algorithm. The manner in which Conflict Alert works, as described in section 2.4.1, is to predict the future position of each target at time T_T and to determine if any other targets are present with an altitude separation less than Z_T. It is clear that the maximum warning time, t_w, which can be achieved is equal to the prediction time plus the time required to travel through the separation threshold or

$$t_w = T_T + Z_T/V \qquad (63)$$

under the assumption that the targets are sufficiently far apart at the start of the maneuver. In order for (63) to be valid

$$(\Delta Z - Z_T - VT)/V \geq T_T \qquad (64)$$

or, equivalently, the time to travel before the start of a current violation must be at least one epoch greater than the prediction time. If the predicted position at the first scan of the maneuver is already in violation of the separation threshold, then the maximum warning is

$$t_w = \Delta Z/V - T \qquad (65)$$

where T is the epoch time and it is assumed that the maximum warning time is one scan less than the time required to travel the initial separation since it takes at least one epoch for the maneuver to be detected (i.e., the maneuver starts at epoch 0 but is not detected until epoch 1).

The warning times as given by (63) and (65) are given in figures 13 and 14 for the case in which the standard Conflict Alert parameters where used (T_T = 120 s and Z_T = 2,000 feet). Different parameter values may be used depending on the particular operational situation as described in reference 9. The maximum warning time and the minimum initial separation required to achieve the maximum are given in figure 15 for a range of altitude separation threshold parameters. The range of velocities was chosen to cover the range likely to be observed in practice. A recent study found that most vertical velocities were less than 4,000 fpm with only 5 percent exceeding 5,000 fpm, but occasional velocities of 10,000 fpm and higher were observed (reference 3).

For velocities on the order of a few thousand feet per minute and initial separations on the order of several thousand feet, it is seen that in many cases observed in practice, the theoretical warning time will be less than the prediction interval (T_T)

used in the Conflict Alert algorithm. Since no allowance has been made for the transient response of the tracking filter, it is obvious that the warning time in practical cases must be less than that given in figures 13 and 15. It is also seen that for low vertical velocities, less than 2,000 fpm but especially those on the order of a few hundred feet per minute, the warning time is substantially greater than the prediction time. At the lower velocities, there can also be a substantial difference in warning time as the vertical separation threshold, Z_T, varies. The variation in warning time under changing conditions was also noted in the previous study of horizontal tracking and, as a result, it was suggested that variable (i.e., scenario dependent) prediction times and separation thresholds be used in order to get a more uniform warning time performance (reference 11). No attempt has yet been made to examine the possibility of scenario dependent parameters since the operational data necessary to conduct such an investigation is not available.

The warning time before a collision is an obvious measure of the protection provided by the Conflict Alert algorithm for targets which actually collide. Another similar measure of performance which might be used is the warning time provided before the actual separation violation rather than before the collision. In this case, the warning time to separation violation for the standard scenario is given by

$$t_W' = \min\left\{(\Delta Z - Z_T - VT)/V,\ T_T\right\} \tag{66}$$

which is less than or equal to the prediction time, T_T, and is obviously less than the warning time before the collision. When the travel time required to reach a separation violation is less than the prediction time, the travel time limits the warning time in this case. For the standard Conflict Alert parameters (T_T = 120 s, Z_T = 2,000 feet) at a velocity of 2,000 fpm, t_W' would be given by the results in figure 13 shifted down by 60 s. Equivalently shorter warning times would be observed for the other velocities. Although the warning time to separation violation is also a valid measure of performance, for both colliding and noncolliding targets, it will not be considered further in this report since t_W' can be computed from t_W by a simple shift of the reference time to eliminate the time to travel through the separation standard.

3.3 PARAMETRIC STUDY OF WARNING TIMES GENERATED BY THE EN ROUTE CONFLICT ALERT ALGORITHM

The warning times calculated for various scenarios will be given in the following sections. The performance of the Conflict Alert algorithm will be expressed in terms of the actual mean warning times calculated from (58) for each set of scenario parameters and will be compared to the ideal results obtained from (63) to

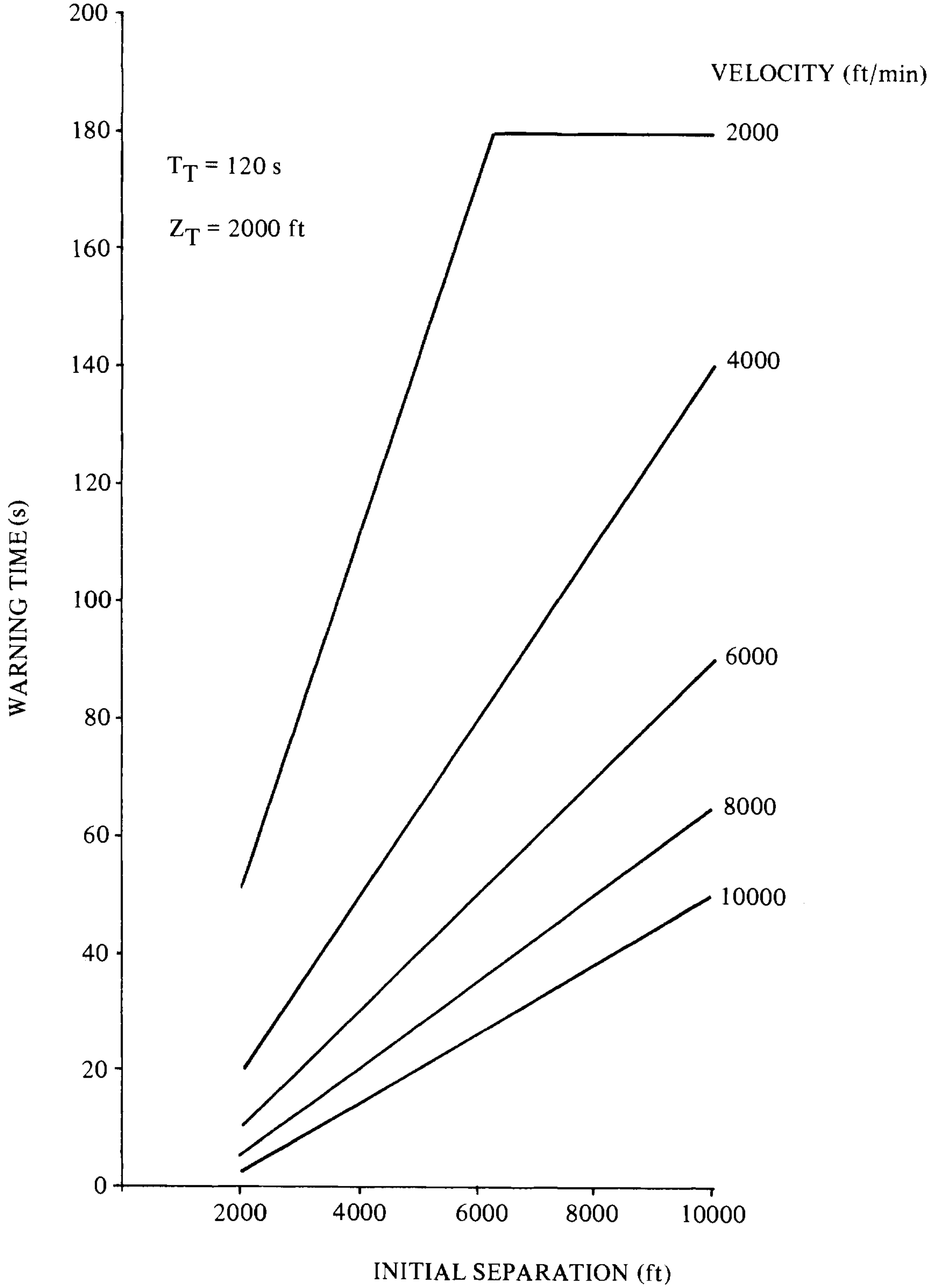

FIGURE 13. THEORETICAL WARNING TIME VERSUS INITIAL SEPARATION

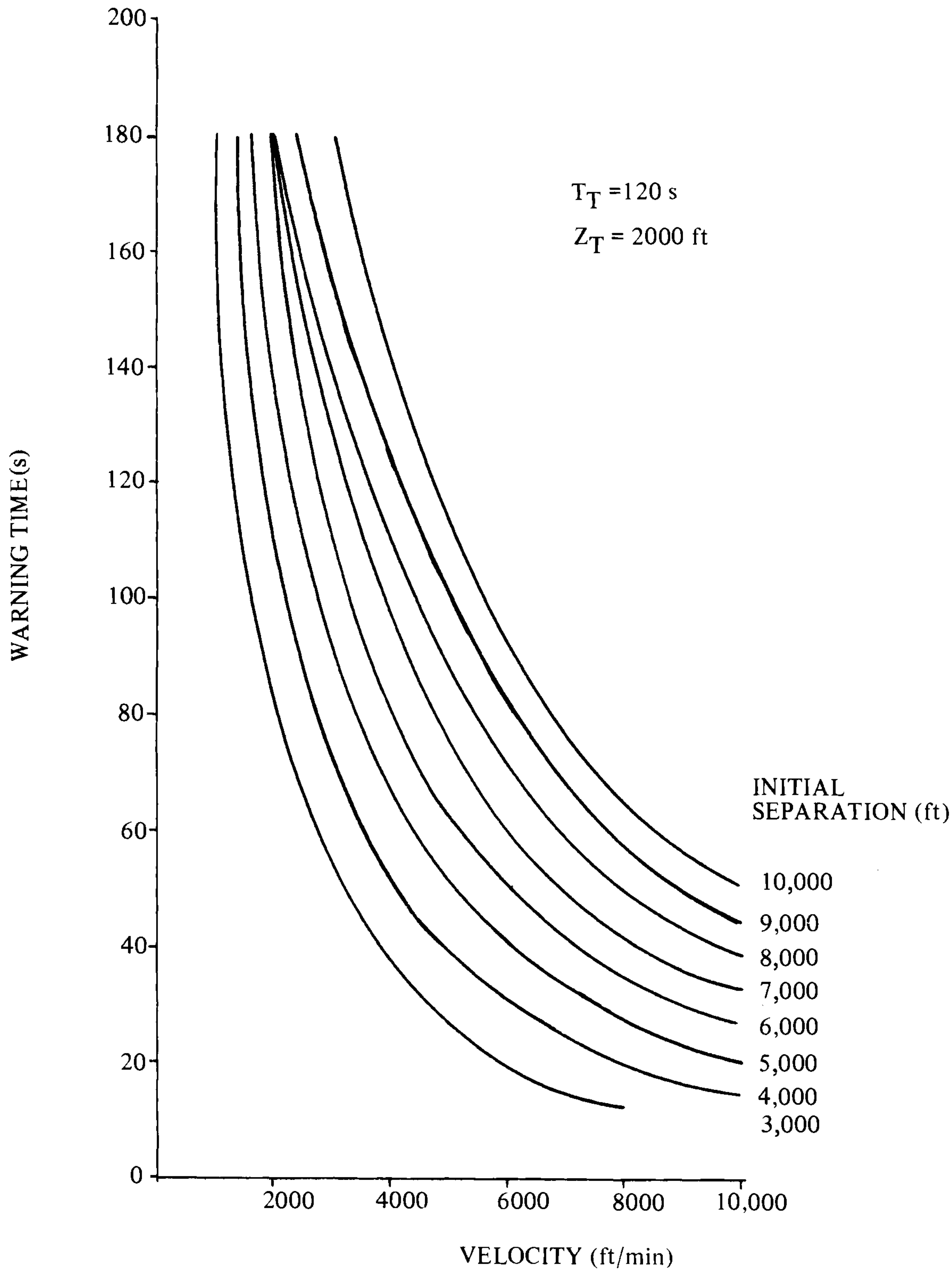

FIGURE 14. THEORETICAL WARNING TIME VERSUS VELOCITY

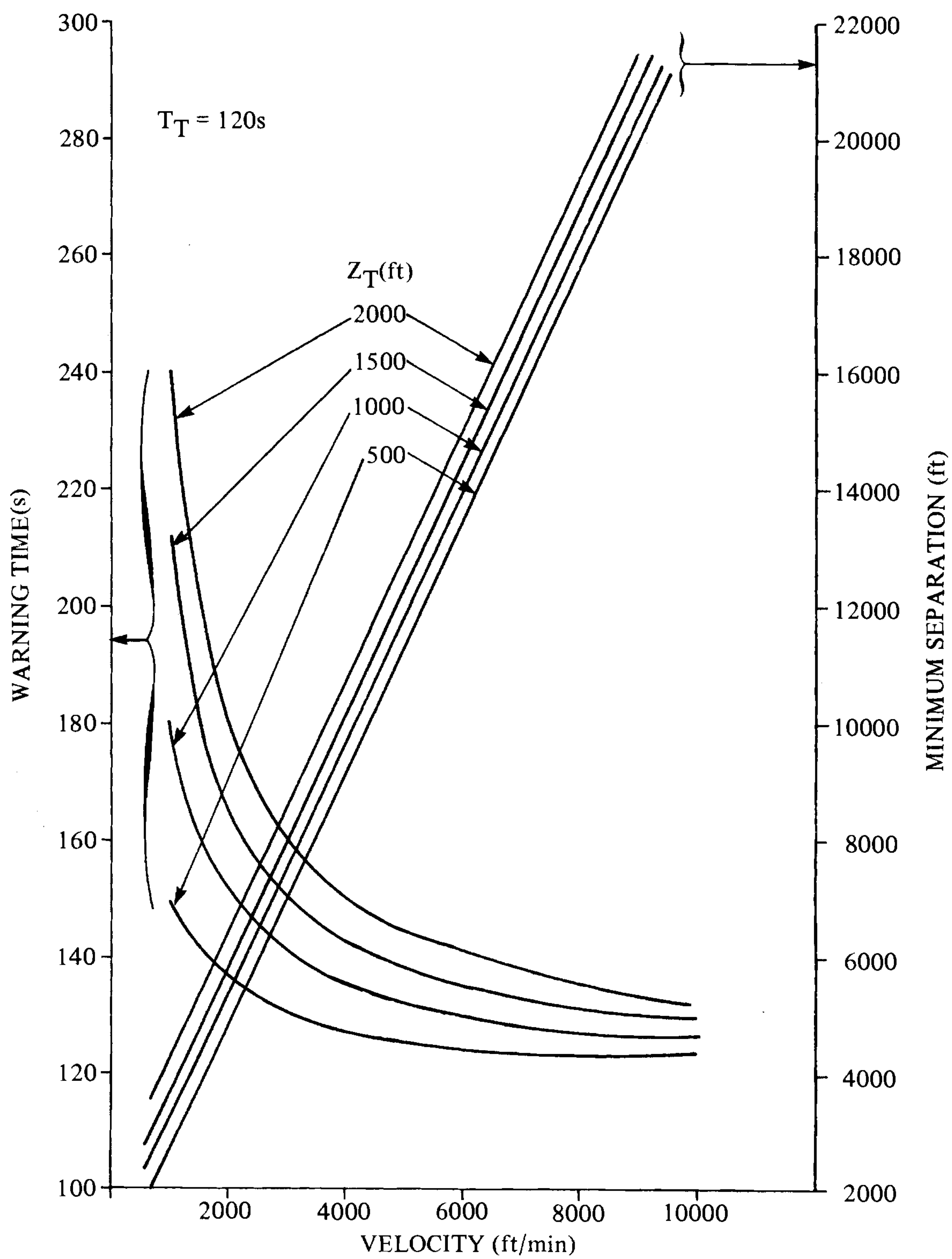

FIGURE 15. THEORETICAL MAXIMUM WARNING TIME AND MIMINUM INITIAL SEPARATION REQUIRED

(65). In some cases, the variance of the warning times (σ_w^2 from (59)) will also be examined. Using the probability of alert sequence, $P_A(k)$, for making performance comparisons would be rather difficult considering the number of parameter combinations which are possible.

The various parameters of interest are given in table 6 and can be divided into two groups. Clearly, the major parameters of interest are those concerned directly with the performance of the Conflict Alert function (Z_T, T_T, V, ΔZ and α), while the minor variations of interest are concerned with the measurement errors and data rate (ΔZ_q, ΔT, ΔTq, and T). In all cases, the velocity smoothing parameter (β) will be directly related to the position smoothing parameter (α) by (62) so that it is only necessary to consider one variable associated with the smoothing parameters. The presence or absence of time correction with fixed or variable smoothing intervals provides another set of variations.

TABLE 6. LIST OF PARAMETERS WHICH AFFECT CONFLICT ALERT

Source of Variation	Parameter	Values
Conflict Alert	Z_T	500, 1,000, 1,500, 2,000* feet
	T_T	120* s
Scenario	ΔZ	
	V	
Tracking	α	0.2 to 0.8 (0.6*)
	Time Correction	With* or Without
	Smoothing Interval	Fixed or Variable*
Measurement Errors*	ΔZ_q	100 ft
and Timing	ΔT	6.0 s
	ΔT_q	0.5 s
	E(T)	10.0 s
		(Pr(6)=1/3, Pr(12)=2/3)

*Standard Values (see text)

Since there are 11 factors in table 6 which may be considered in the evaluation of Conflict Alert, it is obvious that the

the evaluation of Conflict Alert, it is obvious that the presentation of the results will be difficult. Thus, the presentation of the results in terms of warning time would be forced by practical considerations even if it were desired to use the time series representation of the results $P_A(k)$. Many of the variables listed in table 6 are of minor significance, since there is either no substantial effect on performance or no reasonable prospect of change, so little attempt will be made to determine the sensitivity of the results to these variables. For practical purposes, a set of standard conditions will be defined and unless otherewise stated, it will be assumed that the standard conditions apply. The standard scenario is defined by the two parameters ΔZ and V. In all cases, it will also be assumed that time correction is being used although it is not currently used in the operational vertical tracking filter. The justification for this choice will be discussed in section 3.4. It is necessary to make such initial simplifications in order to reduce the problem to a manageable proportion.

One factor which should be considered in evaluating the warning time results presented in this study is that all of the results were obtained independently of the performance of the horizontal tracker. In order for an alert to be generated in the operational environment, it is necessary to have both a vertical and a horizontal separation violation in which the time interval of violation overlap (reference 9). Thus, the warning times presented in this study must be considered optimistic because the simultaneous occurrence of both situations may not occur until substantially later than the vertical violation derived warning time reported in this study. In such cases, the operational warning times may be much less than those derived from (58) or the theoretical results. The simultaneous consideration of both horizontal and vertical performance would be considerably more complicated because of the additional scenario dependent factors required to describe the horizontal encounter (speed, heading, range to the sensor, etc., for two targets).

3.3.1 Parametric Study of Warning Times for Standard Values.

The ideal warning times given by (63) and (65) and the actual warning time, including the effect of the transient response of the altitude tracker, given by (58) are given in figures 16 and 17. These correspond to the results in figures 13 and 14, respectively. For the range of parameter values given in figures 16 and 17, the results obtained for the actual warning times are within about 20 seconds of those derived from theoretical considerations. In the case of the results in figure 16 for a velocity of 2,000 fpm, the maximum difference occurs at the point where the theoretical warning time becomes limited by the prediction time, i.e., the breakpoint or "knee" of the curve. If the data for other velocities are plotted for larger initial separations, a similar effect is found.

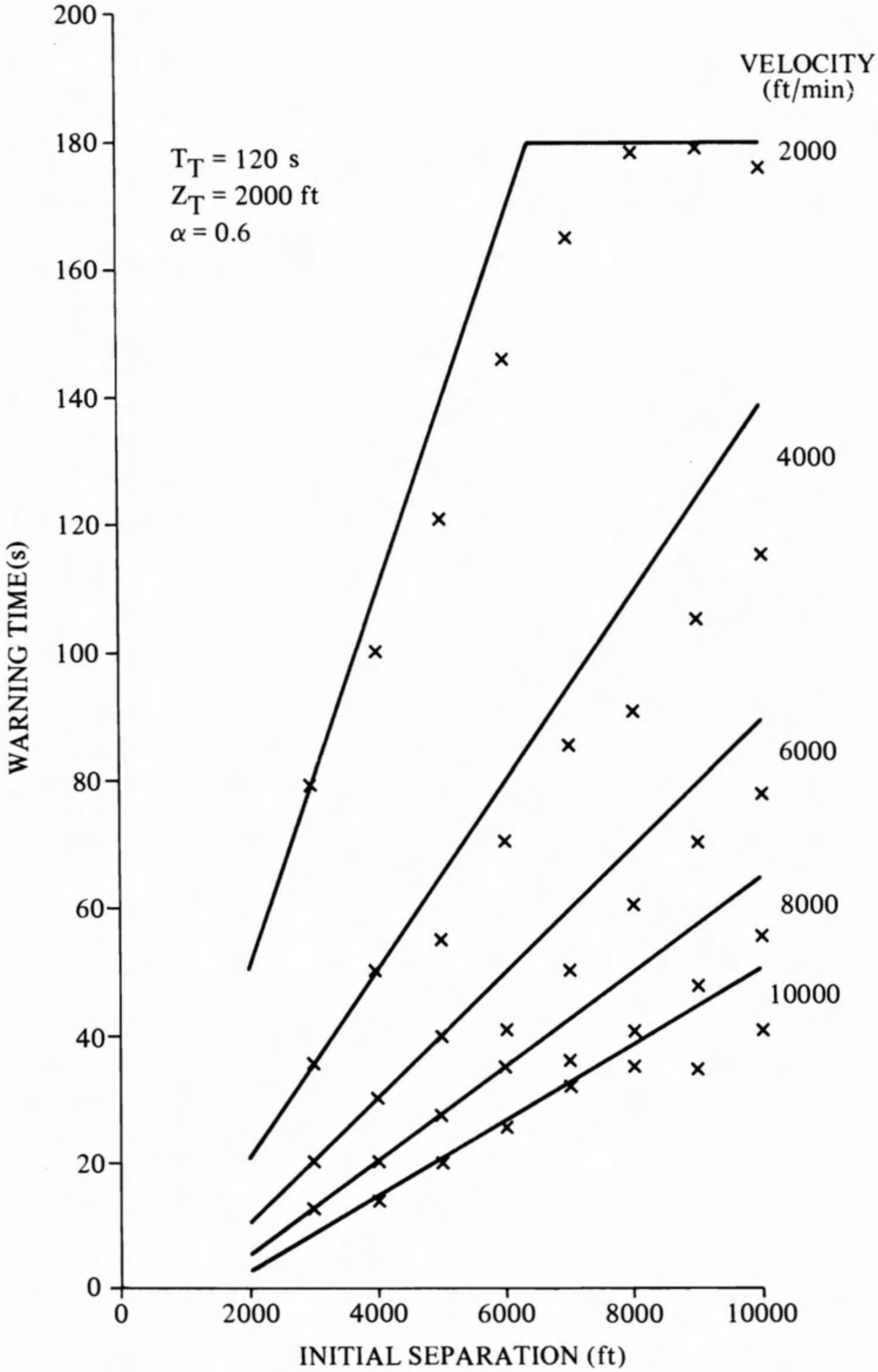

FIGURE 16. THEORETICAL AND ACTUAL MEAN WARNING TIME VERSUS SEPARATION

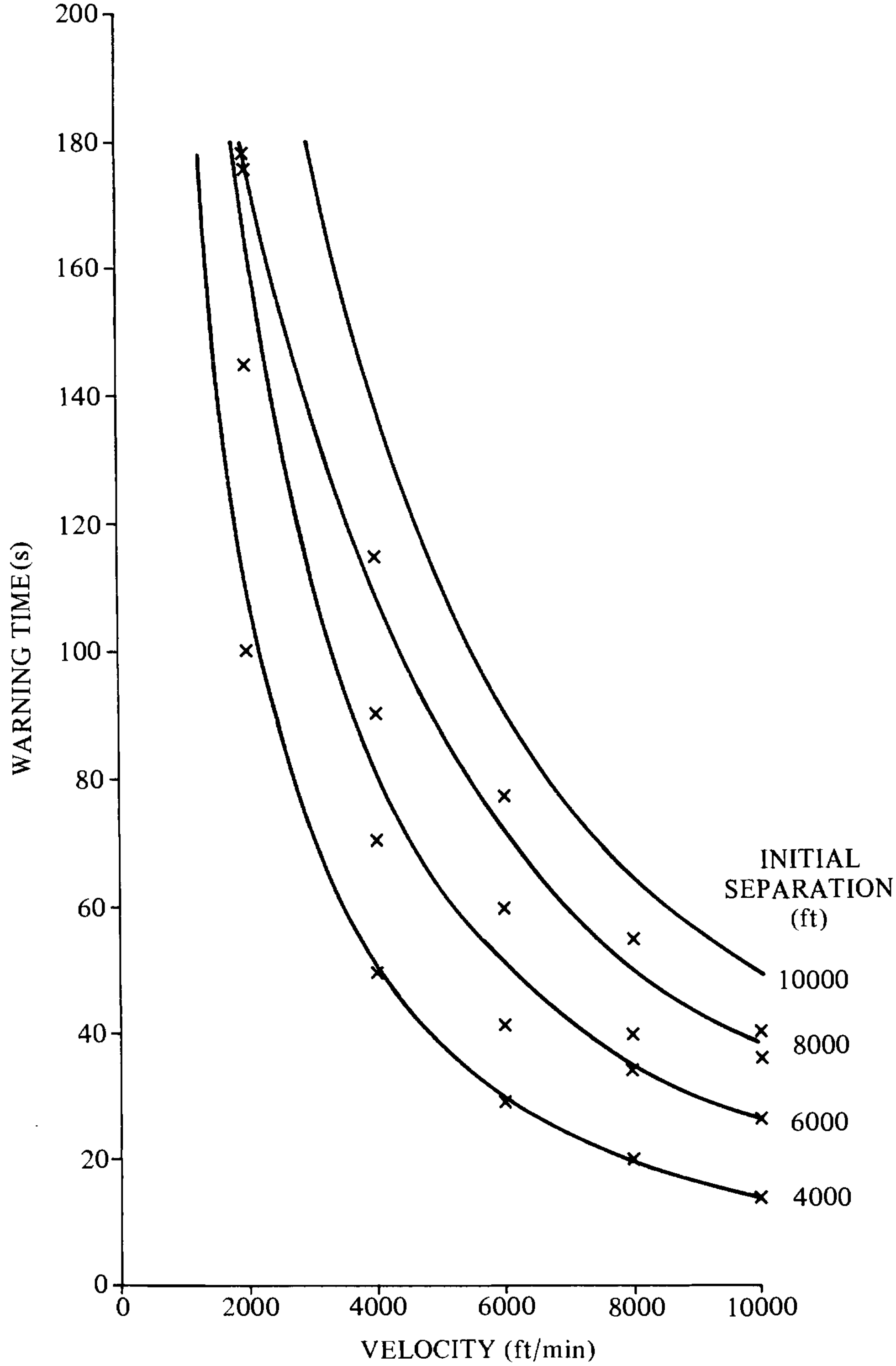

FIGURE 17. THEORETICAL AND ACTUAL MEAN WARNING TIME VERSUS VELOCITY

The identification of the point at which the largest difference in performance is observed is important to minimize the amount of data being considered. At the knee of the curve, the transient response has the greatest effect since the actual velocity must be known in order for an alert to be detected. For larger separations, the transient has decayed to a point of negligible impact while for smaller initial separations even a vertical velocity estimate with a substantial error is sufficient for the projected position to be in conflict. Thus, for many scenarios, the transient response has very little impact on the warning time generated by the Conflict Alert algorithm.

When the warning time results are expressed as a function of velocity, as in figure 17, similar agreement is found between the theoretical and actual warning times. In general, the deviation between the theoretical results and the actual warning time increases as the initial separation increases up to the point at which the warning time is limited by the prediction time. Also, the deviation is approximately the same for all velocities at a fixed initial separation.

When considering the differences between the mean warning time in various scenarios, it must be remembered that the sensor is assumed to operate at a fixed scan time of 10 s while the tracker operates at a different rate. Since the standard deviations of the warning times are, in many case, significantly less than the sensor or tracking epochs, the changes in the mean warning times will tend to jump by multiples of the scan time. In the operational program, however, events can occur on a time scale less than the epoch time and which are not considered at all in the theoretical analysis. For example, in the analytical solution, it was assumed that the start of a maneuver was exactly synchronized with the epoch times, whereas in practice this need not be the case. Consequently, nothing which occurs on a time scale of less than one epoch time has been considered and any differences in the theoretical warning time on the order of one epoch time should be discounted as insignificant since it is unlikely that such differences could be observed in an operational environment. However, differences in performance on the order of two epochs (20 s or more) should be at least detectable in a field environment, although it is unlikely that differences on this order of magnitude would be considered of practical significance.

3.3.2 Parametric Study of Warning Time Under Nonstandard Conditions

The results presented in the previous section demonstrated that the average warning time provided by the Conflict Alert algorithm is determined to a great extent by factors other than tracking. It was shown that under the standard conditions used previously, the actual warning time was always within 30 s of the theoretical warning time, and in most cases, it was actually much closer. For completeness, it is of interest to determine if a similar relationship between theory and practice holds for nonstandard conditions. However, due to the large number of parameters on which the performance is dependent, it will only be possible to perform a limited evaluation of the impact of the various parameters on the performance.

Certain parameters can be dismissed from consideration because of the relatively inconsequential differences in performance. The quantization parameters $\Delta T_q, \Delta T$ and ΔZ_q are in this category. The values given in table 6 used to determine the variances specified by (36) correspond only to the contribution of the basic quantization level of these variables. In addition to quantization, other errors present have not been included, so the actual variances of these errors may be considerably larger than that indicated by quantization alone. When these errors were increased to the levels reported in other studies (references 14 and 30), the mean warning times were changed by only a few seconds, but there were no obvious trends in these changes. The standard deviations of the warning times were also increased by several seconds with the larger increases occurring for the higher values of α. Again, however, the performance differences were of little significance. As a result, it was concluded that based on the linear theory used in this study, these three parameters were of little significance.

In the choice of the smoothing interval, the use of a variable smoothing interval is more consistent with reality than the fixed intervals. As a result, no studies were made with the fixed interval tracker. It would be expected, however, that as with the measurement errors, the changes in performance due to the choice of smoothing intervals will also be of negligible significance. The last factor considered in connection with the tracker is the question of time correction. All of the detailed results presented in both this section and the previous section are for the case in which time correction is used despite the fact that it is not yet used in the present operational system. It has been recommended that time correction be used in the altitude tracking algorithm (reference 6), but the recognition that time correction must be used has not yet been perceived. In comparing the mean warning times with and without time correction, it was found that in most cases the differences in the means were less than 1 s with occasional differences as high as 6 s; however, neither procedure was uniformly better in all cases. The larger differences were again observed for the larger values of the smoothing parameter α. The standard deviations of the warning times were mostly in the range of 1 to 4 s with time correction and 2 to 10 s without time correction. The differences in the standard deviations with and without time correction were less than 3 s in most cases with occasional differences for 6 s for large values of α. An obvious trend in this case was that the standard deviations with time correction were less than the standard deviations without time correction. None of these differences were considered to be operationally significant. However, all of these results are dependent on the assumption of statistical independence, which has been noted several times previously and this critical assumption will be discussed in section 3.4.

Since the significance of the prediction time used by Conflict Alert, T_T, can be inferred directly from the theoretical calculations given previously, no attempt was made to vary this parameter. The value of this parameter was chosen on the basis of providing sufficient warning time in hazardous situations to allow intervention by a controller when this is warranted. An average warning time of at least T_T will be obtained except in situations in which the theoretical warning time is limited by the flying time between the two targets. In scenarios where the warning time is not limited by flying time, any change in T_T will produce an approximately equivalent change in the mean warning time.

In a situation involving many variables, the reduction of the problem to a few key variables is frequently the most important step in understanding the critical design and performance parameters which most influence the system performance. The multivariable problem has now been reduced to a total of four variables (Z_T, ΔZ, V and α). In the case of variation with the separation threshold Z_T, some selected results are plotted in figure 18 along with the applicable idealized results. In most cases, the actual mean warning time results are within one to two epochs (10 to 20 s) of the theoretical results. For initial separations less than the minimum separation for the maximum warning time (see figure 15), the results are almost independent of the threshold being used. For larger initial separations, the loss in warning time as the threshold is reduced is almost exactly equal to the flight time for the reduction. For a velocity of 2,000 fpm, each 500-foot reduction in the vertical separation threshold used by Conflict Alert is equal to a loss in the mean warning time of 15 seconds; however, such a loss occurs only for large initial separations for which the reduction in warning time is of no significant consequence. Thus, the selection of the vertical threshold in Conflict Alert is of little significance as far as the mean warning time is concerned. In situations in which the warning time is impacted, the overall system performance will continue to be satisfactory while for the cases in which little warning is given, the warning time is limited by flight time and will not be changed by the use of a different separation threshold value. Note that the vertical separation threshold being discussed in this report is that used by the Conflict Alert function and does not necessarily have to be the same as that used by the controller. Indeed, there is considerable justification for using a vertical separation for Conflict Alert which is less than that used by

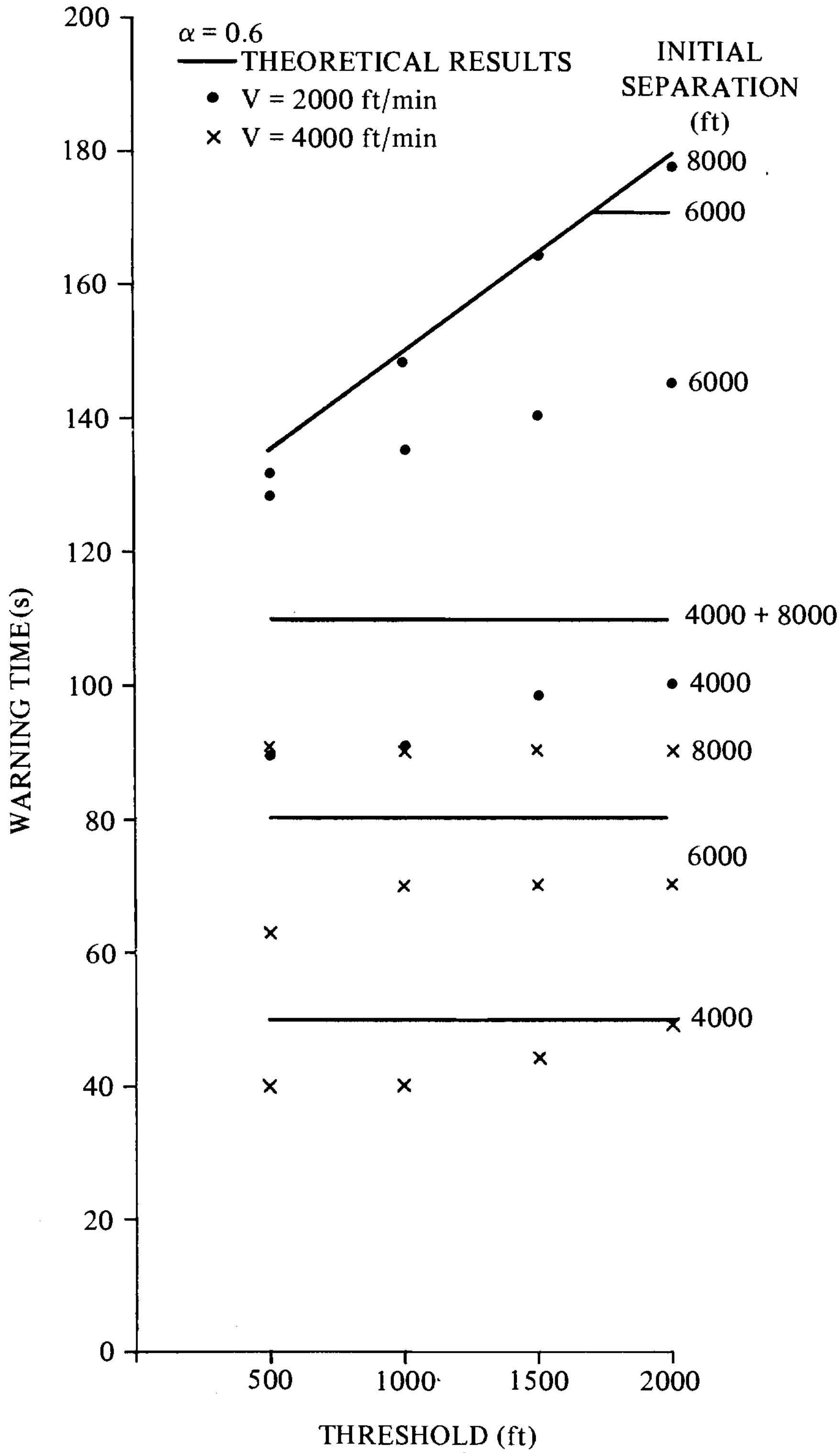

FIGURE 18. WARNING TIME AS A FUNCTION OF VERTICAL SEPARATION THRESHOLD

the controller in order to reduce the number of false alerts.

The next parameter which will be considered is the smoothing parameter, α. The results in this case are given in figures 19 to 21. The curves in figures 20 and 21 are empirical approximations to the data points plotted. In the case of large initial separations; i.e., well beyond the region where flight time limits warning time, illustrated in figure 19, it is seen that the mean warning time is not a function of the smoothing parameter. In almost all cases illustrated, the actual warning time is within 10 s of the theoretical warning time.

In figure 20, the mean warning time is plotted as a function of the smoothing parameter for several initial separations. For the range of smoothing parameters which has been used in practice (0.4 to 0.6), the differences in the mean warning for initial separations, where the results are limited by flight time, are on the order of 10 s. The results for the mean warning time at the minimum initial separation defined in figure 15, where logically the most substantive variation with a would occur, are given in figure 21. The difference in performance over the range of practical interest (0.4 to 0.6) is approximately 20 s even at the initial separation at which the greatest difference in mean warning time is expected. From figure 21, the performance difference is seen to be independent of velocity. Over the range in a of 0.3 to 0.7, the change in the mean warning time is approximately 10 s per 0.1-change in α and this is for scenarios in which the greatest change is expected. The largest differences between the actual mean warning times and the theoretical values are approximately 30 s or less for $\alpha = 0.6$ and 50 s or less for $\alpha = 0.4$.

The last item of interest is the sensitivity of the mean warning time to the parameters defining the scenario. In the case of the scenario parameters, ΔZ and V, the results are of a somewhat different nature since in some cases the warning time is not a function of the scenario parameters. In other cases, the warning time is limited by the flight time $\Delta Z/V$, so it is a function of the ratio of the two parameters. A third case is the situation defined by the minimum initial separation given in figure 15. Thus, the ΔZ-V plane is divided into three regions depending on which factors limit the warning times and these relationships are illustrated conceptually in figure 22. Suppose it is desired to determine the warning time in a particular scenario. Depending on the accuracy required, the actual mean warning time is approximately equal to the theoretical value within about 10 s except in regions close to the initial separations defined when the equality in (64) holds, i.e., the minimal initial separation of figure 15. The largest differences between theory and practice are found close to the minimum initial separation line. For $\alpha = 0.6$, the actual mean warning times are approximately 30 s less than the theoretical values in the region from about 500 feet above to 2,000 feet below the minimum initial separation

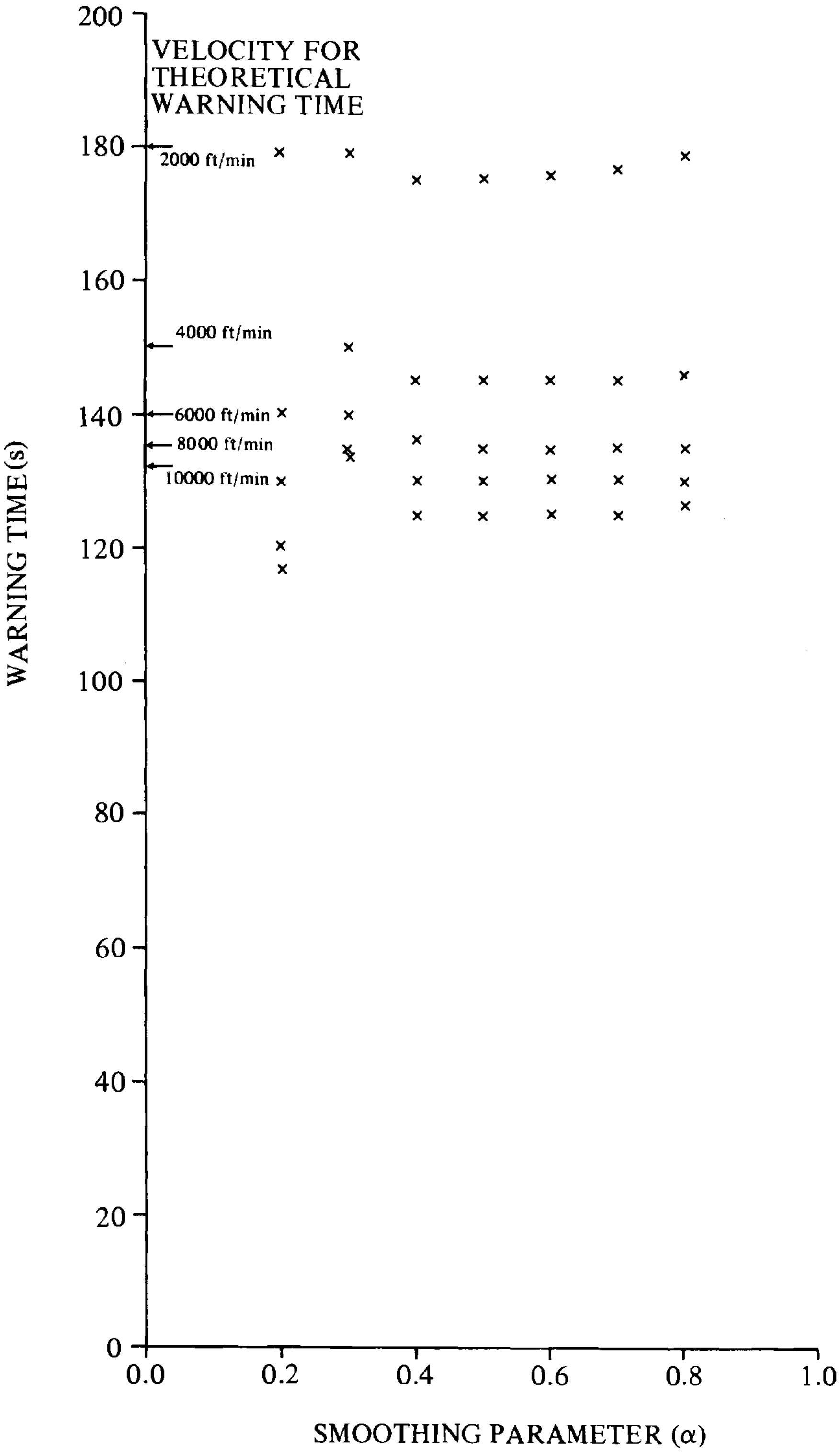

FIGURE 19. WARNING TIME VERSUS SMOOTHING PARAMETER FOR LARGE INITIAL SEPARATIONS

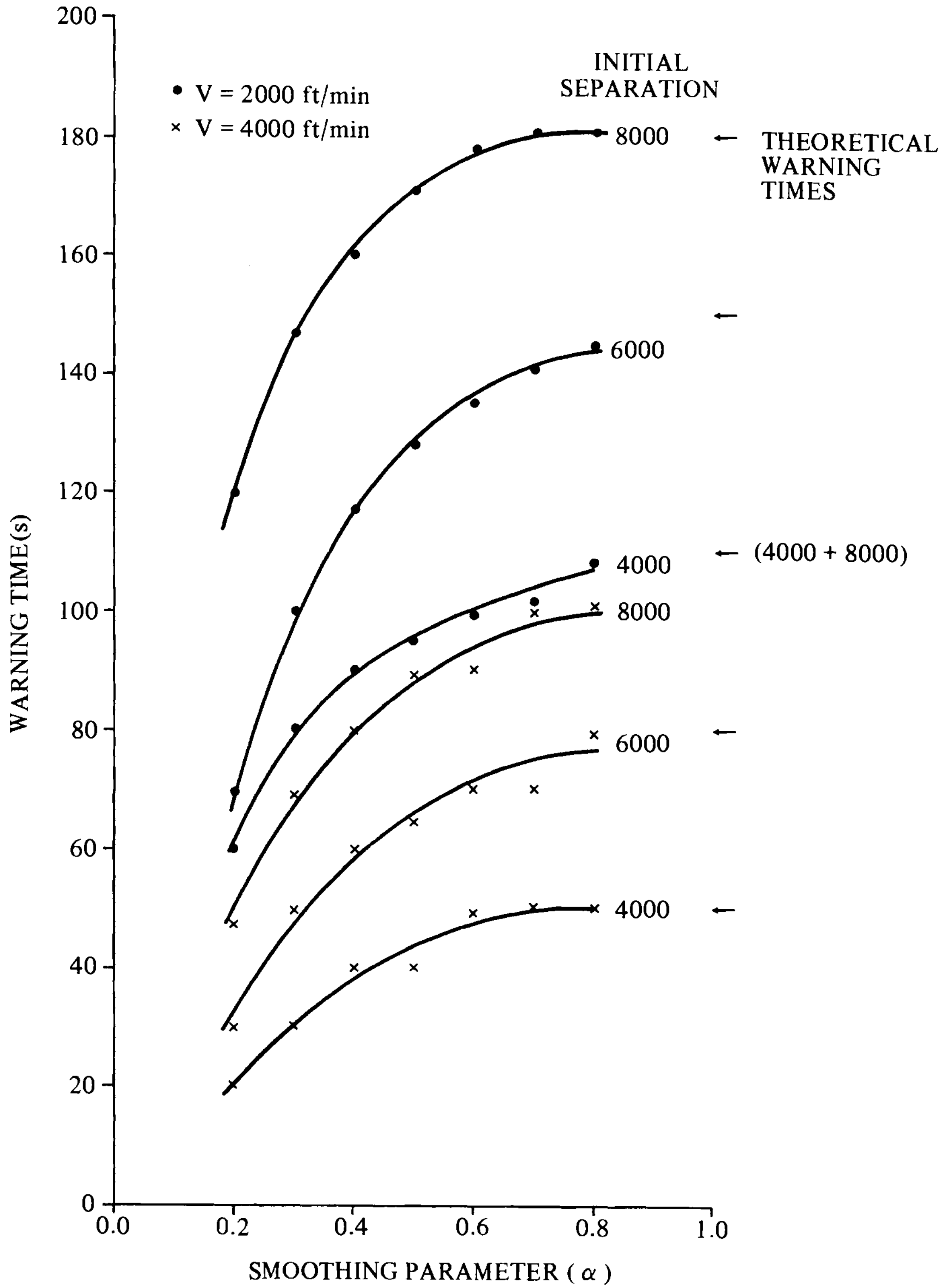

FIGURE 20. WARNING TIME VERSUS SMOOTHING PARAMETER

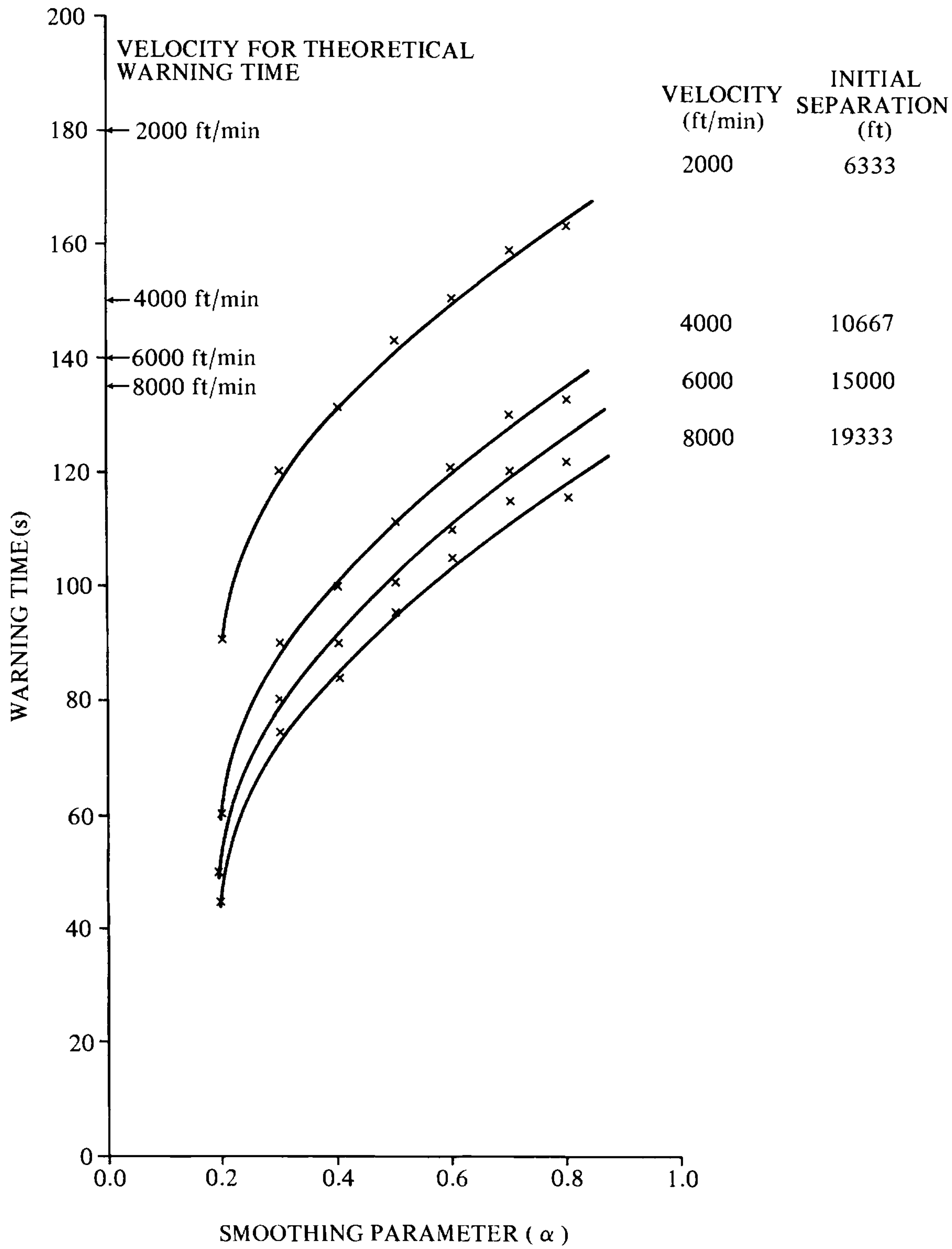

FIGURE 21. WARNING TIME VERSUS SMOOTHING PARAMETER FOR MAXIMUM WARNING TIME SEPARATION

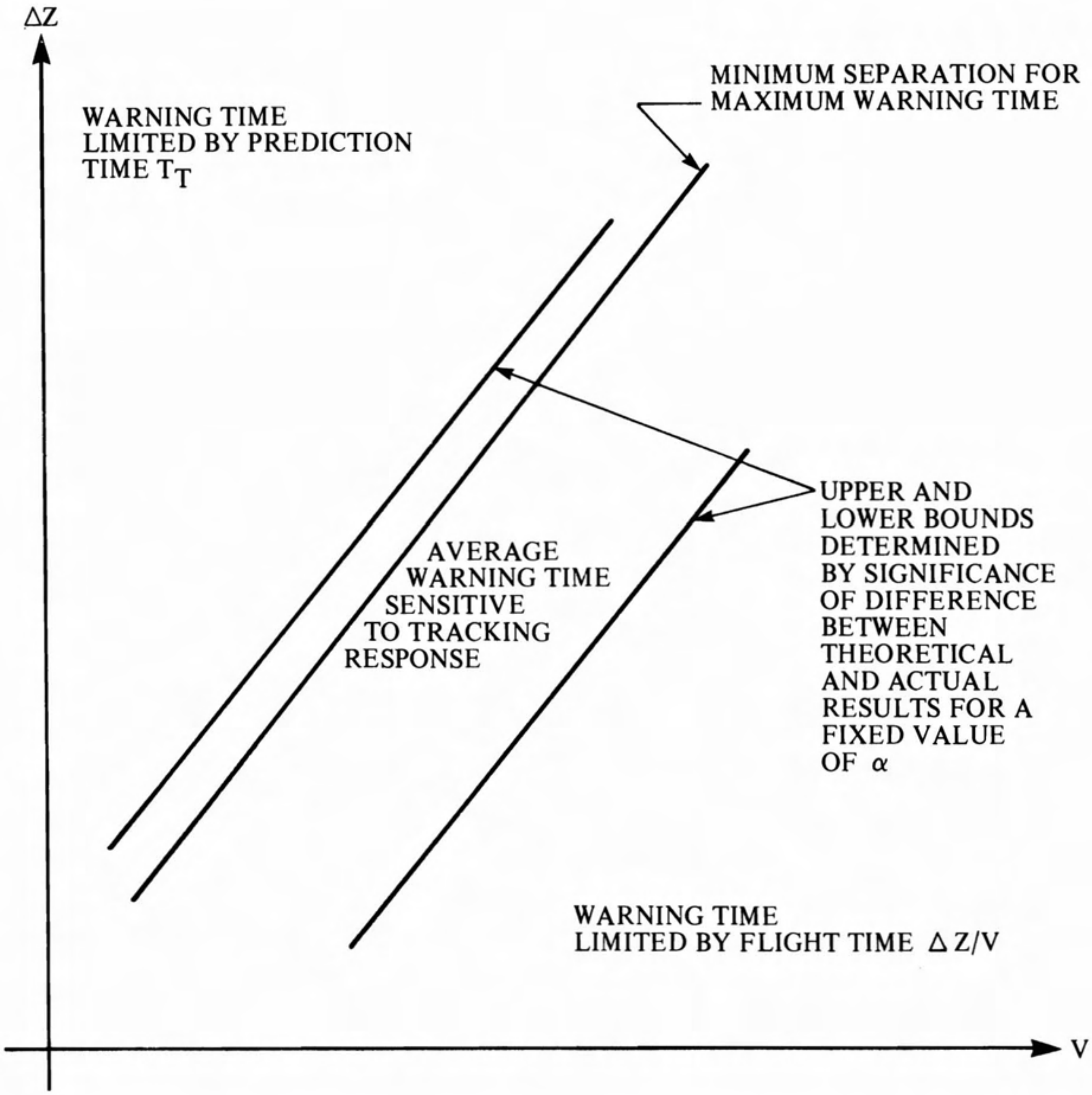

FIGURE 22. CONCEPTUAL ILLUSTRATION OF AVERAGE WARNING TIME LIMITATIONS FOR THE STANDARD SCENARIO

line. For lower values of α and tighter requirements on the accuracy of the results, the boundary lines defining the region in which the mean warning time is sensitive to the transient performance of the filter would become further apart.

It has been demonstrated that in many cases, the mean warning time is not a function of tracking performance. In such situations, the standard deviation in the warning time is the only significant measure of tracking performance since the mean warning time is determined by other factors. However, as discussed in the following section, there are other factors which increase the standard deviation in the operational environment so that the standard deviation is not useful for tracking evaluation. A summary of the various aspects of the performance of the Conflict Alert algorithm, as measured by the mean warning time, is given in table 7.

TABLE 7. EFFECTS OF VARIOUS PARAMETER VARIATIONS OF THE MEAN WARNING TIME

Source of Variation	Effect
Measurement Errors	Standard deviation increases with error and mean warning time changes slightly, but both changes are insignificant.
Time Correction	Mean warning time differs little between filter with and without time correction. Effect of time correction depends on correlation which could not be examined.
T_T	Warning time varies directly with T_T beyond separation where flight time limits warning time.
Z_T	Insignificant change in mean warning time when limited by flight time, otherwise change equal to change in Z_T divided by velocity.
α	Insignificant change in mean warning time unless close to minimal separation defined by (63) when change is 10 s for each 0.1-change in α.

3.4 SIGNIFICANCE OF THE ASSUMPTION OF THE STATISTICAL INDEPENDENCE OF TRACKING ERRORS.

It has been stated repeatedly throughout this study that the measurement, timing, and arithmetic errors are all assumed to be statistically independent from epoch to epoch. This assumption is the singularly most critical one with regard to possible deviations between theory and practice. Consequently, it is essential to determine if and under what circumstances the statistical independence assumption is valid.

In all of the mean warning time results presented in the previous sections, the data actually presented was for the case in which time correction was used in the altitude tracker. Although the differences between the results with and without time correction were negligible when the errors are statistically independent, this may not be true in practice. The available evidence indicates that the statistical independence assumption is not valid.

When time correction is not used, the statistical independence assumption can be shown to be invalid by considering sequential pairs of time-correction factors. If the assumption of independence were valid, then the knowledge of one time-correction factor should yield no information about the next time-correction factor. Consider a timing error of $\Delta T(k)$ as is illustrated in figure 23. If the sensor actually supplies data 10 s apart, then there are two possible ways in which a tracker operating on a 6 s epoch time can process this data: either the data are processed in adjacent tracking cycles or the data are processed in the second tracking cycle following receipt of the data. The two possibilities just discussed are illustrated in figure 23. In other words, if $\Delta T(k)$ is the timing error due to the lack of time correction in epoch k, then the next timing error will either be in the following epoch $\Delta T(k+1)$, or in the second epoch, $\Delta T(k+2)$, depending on the value of $\Delta T(k)$.

It is easily shown that the timing errors in the first case are related by

$$- \Delta T(k) + T + \Delta T(k+1) = T_s \tag{67}$$

while the errors in the second case are related by

$$- \Delta T(k) + 2T + \Delta T(k+2) = T_s \tag{68}$$

where T is the time between the center of the tracking cycles (6 s), T_s is the time between the data points (10 s), and $\Delta T(k)$ is the timing error (or time-correction factor) measured with respect to the center of the tracking cycle in which the data are received. The relationships (67) and (68) are plotted in figure 24. It was assumed previously that in order to result in an average smoothing interval of 10 s, the first case would occur with a probability of 1/3 and the second case would occur with a probability of 2/3. Over a period of many tracking

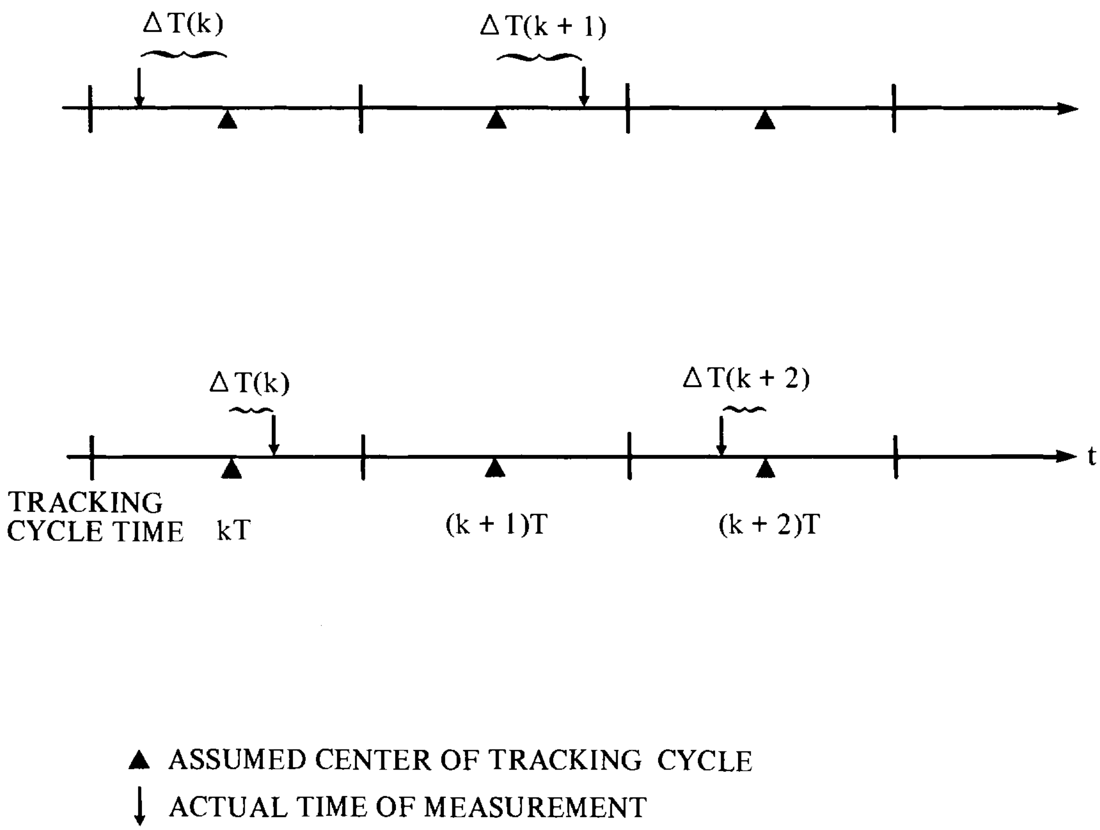

FIGURE 23. ILLUSTRATION OF RELATIONSHIPS BETWEEN SEQUENTIAL TIME-CORRECTION FACTORS

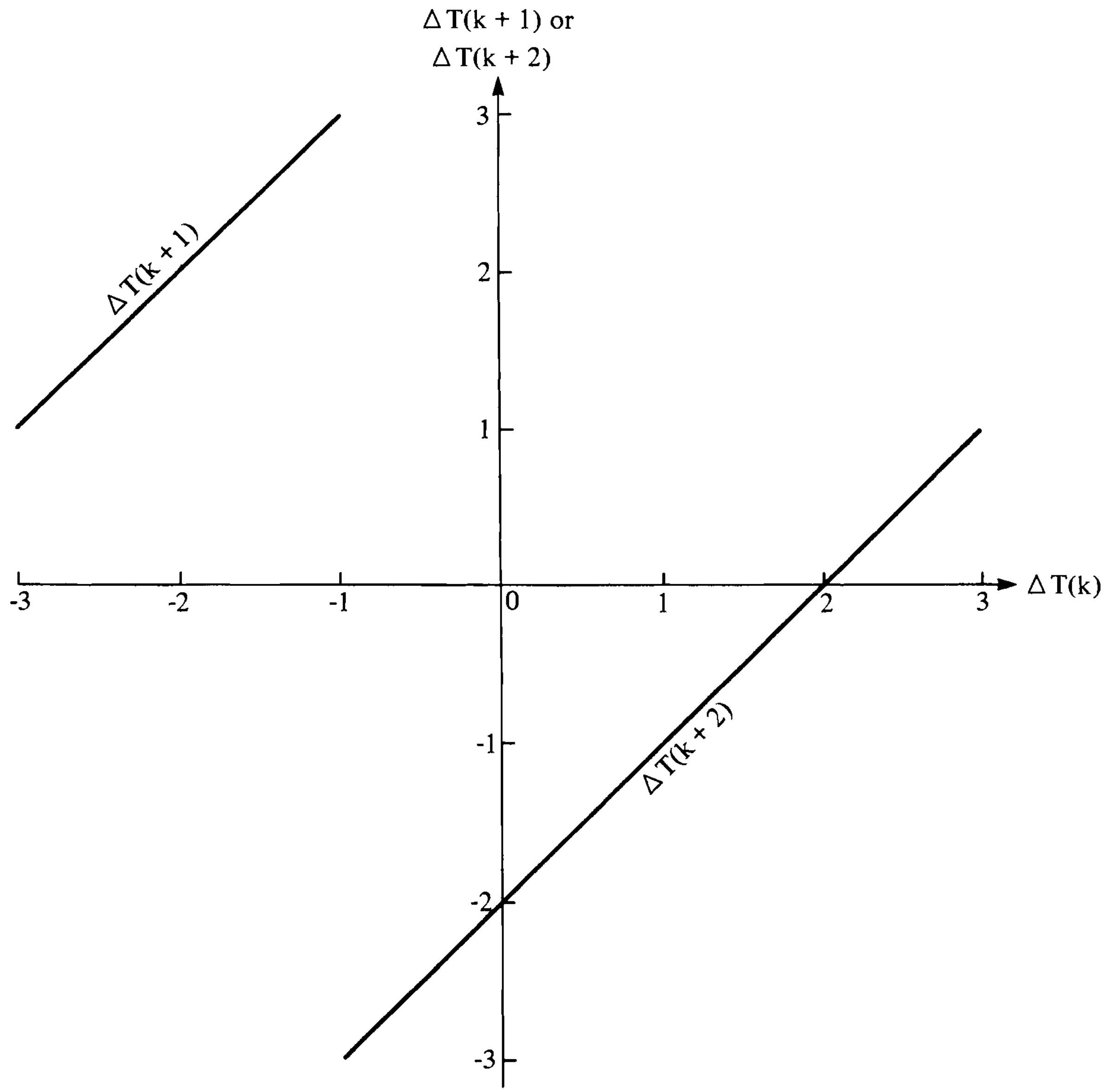

FIGURE 24. ILLUSTRATION OF LINEAR DEPENDENCE OF TIME-CORRECTION FACTORS

occur with a probability of 1/3 and the second case would occur with a probability of 2/3. Over a period of many tracking cycles, the timing errors will alternate back and forth between the two relationships resulting in an oscillatory error sequence.

Obviously, if the lack of time correction results in timing errors which are linearly related, then the tracking errors cannot be assumed to be statistically independent in the case where time correction is not used. In addition, there is some evidence to indicate that the actual altitude measurement errors are correlated (reference 34). Presently, there is insufficient data available to characterize the nature of the correlation, but it is obvious that the correlation would be nonstationary (i.e., time dependent) because of the oscillatory nature of the timing errors. Even if it were possible to characterize the nature of the correlation, the expansion of the theory given in section 2.4.2 to include the effect of such correlation is not considered to be feasible. The reason for the selection of the numerical result based on a tracker with time correction is now apparent. Since the operational implementation of the tracking algorithm does not currently meet the assumption of statistical independence, because of the lack of time correction, the available theory is inapplicable in this case. Correlation can never improve the system performance and will result in an increase in the variance of the warning time as illustrated conceptually in figure 9. However, just because one of the assumptions used to obtain the theoretical results is invalid, (in this case, statistical independence of the tracking errors) it does not mean that the differences are of any practical significance. The question to be answered at this point is whether or not the lack of statistical independence would result in any significantly different prediction of operational system performance.

In the case of digital filters implemented with finite-precision arithmetic, the answer to this question is already known. In effect, the lack of time correction is merely a highly quantized measurement of time, but it is a well-known fact that recursive digital filters implemented with finite-precision arithmetic can suffer extreme degradations in performance due to unwanted oscillatory responses. As discussed at the end of section 3.1, such error responses are referred to as limit-cycles (see e.g., references 31 - 33) and are a consequence of quantization error sequences which are highly correlated. The fact that oscillatory error responses are found in the altitude tracker is not surprising. Other attempts at designing altitude trackers for air traffic control automation features have also encountered similar problems resulting in operational deficiencies due to finite-precision implementation of recursive algorithms (the operational nature of these problems is described in references 35 - 37).

In the technical literature dealing with limit-cycles, most of which appears in the IEEE Transactions on Acoustics, Speech and Signal Processing, Circuit Theory, and Automatic Control, the only factors considered are the quantization and round-off or truncation in the arithmetic operations. In some cases, the oscillations are so large in amplitude that they overflow the finite word length registers used in the implementation of the digital filter and thereby override the input signal (reference 33). In the case of the altitude tracker, in which both the input data and time of measurement are quantized for use in a recursive digital filter, it would be anticipated that even worse degradations in performance would be observed. Thus, not only are the errors resulting from the limit-cycle phenomena potentially large in ampliltude, but since the errors are oscillatory, they are obviously not statistically independent. If the data being used as input to the tracking algorithm is subject to additional error inducing processing, such as the square root and stereographic projection approximations (references 38 and 39), then this will increase the possibility of large amplitude oscillatory errors.

The nonlinear nature of limit-cycles is such that several modes of oscillation can exit each with a different amplitude and frequency. A subject of extensive research effort at present is the development of implementation rules for digital filters which yield limit-cycle free performance (see, e.g., reference 40). Unfortunately, the theory used to predict the presence or absence of limit-cycles is not only complicated, but also very specific to both the algorithm and arithmetic operations used for implementation of the digital filter. The altitude tracker is a dual input (amplitude and time) dual output (position and velocity) digital filter which is more difficult to analyze than the simple single input-single output digital filter considered in most theoretical studies.

Once the pervasive impact of limit cycles on the performance of digital filters is recognized, the next question is: How should the performance of the altitude tracker be measured to determine the operational impact on the Conflict Alert function? Clearly, the mean value of the warning time is not a useful measure of the performance of the altitude tracker, since in most scenarios, the warning time is determined by factors other than tracking. Even if the tracker output did not contain significant oscillatory errors, it would be expected that some alerts would be early and some late so the net result could very well be little or no change in the mean warning time even for a very poor altitude tracker. In any case, the mean warning time is not particularly sensitive to tracking performance and would not be a useful measure of tracking performance.

Since the mean warning time is not a useful measure of tracking performance, the standard deviation of the warning time could then be considered a measure of tracking performance. The standard deviations calculated in this study are so small that it is highly probable that in an operational environment, other factors, such as the asynchronous operation of the tracker with respect to the sensor and various display filters, would produce more variation in the results than that found in this study. Since limit cycles may only occur under certain circumstances, it would be necessary to consider a large number of scenarios in order to determine if any anomalous tracking performance will be found. When the interaction with the horizontal tracker is considered, the number of scenarios in which the standard deviation would have to be computed could easily become unmanageable. Even if a substantial effort were made to simulate a large number of scenarios, the simulation program would have to be accurate to the level of the least significant bit since it is phenomena at this level which cause limit cycles.

The simulation program presently available does not meet this accuracy requirement, so any results obtained using this program are questionable. Obviously, it is not possible to evaluate the standard deviation in warning time using operational data since no two scenarios are identical and the trajectories will not result in a collision. If simulation data were used, and degradations in tracking performance due to limit cycles were found in this relatively "clean" environment, then it is fairly certain that such degradations would be found in the operational environment. Based on these considerations, the following conclusion is made. It is not possible to test the performance of the altitude tracker in any way other than through the use of both simulation and operational data in which the actual altitude tracking performance is examined for sequentially correlated and/or oscillatory sequences of errors.

In evaluating the significance of sequential correlation in the performance of the altitude tracker, the ultimate use of the altitude tracking data must be considered; namely, safety. In the case of a separation violation declared by Conflict Alert, one can consider the alert to be either true or false. If an alert is false, then safety will not be compromised by the performance of the altitude tracker. Although false alerts may be annoying, they do not lead to a reduction in safety unless the number of false alerts is so large the controller ignores the alerts. Thus, false alerts may lead to a general lack of confidence in the performance of Conflict Alert and, thereby, effect safety indirectly. In the case of a true alert, safety is certainly not reduced if the alert is generated before it would have been in the absence of limit cycles. The reverse situation is clearly the only case in which safety is reduced; namely, a delay in the generation of an alert in a situation in which an alert should be generated.

The crucial item of importance, as far as safety is concerned, is the fact that the amplitude of limit cycles may be sufficient to create the appearance of vertical separation in situations in which none exists. It is very likely that the amplitude and duration of the limit cycles will be sufficient to prevent the generation of a predicted alert, either through failure of the convergence check (39) or errors in the predicted altitude, in which case, the warning time before a collision may be far less than the design value.

Therefore, the most significant direct measure of tracking performance is the scan-to-scan statistical correlation, or lack thereof, between errors in the output of the tracking algorithm. The lack of correlation is an indication that the tracking algorithm implementation does not suffer from degradation due to limit cycles. As mentioned in section 3.1, it is quite likely that the time-correction process will improve the performance of the altitude tracker by not only reducing the magnitude of the error, but also by reducing the correlation in the tracking errors. The lack of time correction creates exactly the type of system performance which is unacceptable for use in a safety-related application. It is almost certain that under the proper conditions the lack of time correction will produce error sequences which mask the predicted alert. As a result, there may be no warning of a vertical separation violation and little warning of a collision. The penalty for an improper implementation of a digital filter, in this case, is severe. Failure to correct an obvious safety-related system deficiency is ludicrous in view of the well-known propensity for serious degradation in filter performance due to limit cycles.

4. SUMMARY AND CONCLUSIONS.

This report provides a parametric study of the performance of the altitude tracking algorithm used to support the En Route Conflict Alert function. The altitude tracking algorithm is a fixed-parameter $\alpha-\beta$ tracking filter which is a widely used technique for obtaining velocity estimates from noisy position measurements. As a result of the usefulness and simplicity of the $\alpha-\beta$ filter, extensive analytical studies have been conducted of the performance of this algorithm. One problem which has been identified in connection with the en route tracking algorithm, is the lack of time correction, which means that the time receipt of the altitude data is not used in the tracking algorithm.

In sub-sections 2.1 to 2.3, a mathematical theory is developed to predict the performance of the altitude tracker. This theory includes various practical aspects of implementation, such as the asynchronous operation of the tracking filter and the sensor, and the quantization levels used for measurement of the data. In section 2.4, the mathematical theory is extended to provide a means to evaluate the average warning time to a collision which will be provided by the Conflict Alert algorithm for vertical maneuvers. When all of the operational and procedural modifications within the operational program are removed, the operation of the Conflict Alert algorithm is relatively simple to evaluate. The scope of the effort reported in this study is limited to system performance in the vertical plane. In an operational situation, there would be an interaction between the horizontal and vertical algorithms which had not been considered. However, the interaction could only have the effect of reducing the warning times reported in this study.

The mathematical analysis was developed to evaluate the performance both with and without, as is presently the case, the time-correction process. One particularly important point which must be considered is that in all of the mathematical analysis, it was assumed that all of the random errors are statistically independent from scan to scan. However, the assumption of statistical independence is not generally true and an extensive discussion is given in section 3.4 to explain the consequences of this assumption not being met and the conditions under which the assumption will be met. The actual filter performance will be considerably worse than when the random errors are statistically independent.

The numerical results obtained in this study are presented in section 3. In addition to the warning time to collision, as calculated for Conflict Alert, the ideal warning time which could be obtained if the vertical velocity was known exactly was used as a basis for comparison. Numerical results for the position and velocity errors of a target transitioning from level flight to a constant velocity maneuver are given in section 3.1. Before presenting the numerical warning time results, some additional comments on comparison of the Conflict Alert performance and the theoretical warning times are given in section 3.2. The parametric study of warning times for a standard scenario are given in section 3.3 for a wide variety of conditions. The standard scenario consists of two targets in level flight in which the higher target executes a constant rate descent onto the lower target. Such a scenario is representative of cases in which near midair collisons involving vertical transitions have occurred in the En Route environment, but collisions have actually occurred in the terminal environment. Since there are many factors involved in determining the warning time, the number of interactions between factors is too large to consider all possible interactions.

The results for the average warning time to collision showed that under most conditions the tracking performance has very little impact on the warning time. The reason for this is the fact that the warning time is determined primarily by factors other than tracking, such as the prediciton time used by Conflict Alert and the flight time determined by the initial separation and velocity. Significant deviations from the ideal performance were only found for the limited case where the true velocity projection just resulted in a violation, but the velocity calculated by the tracking filter did not. In other words, tracking performance was only significant in determining warning time in cases where the projection time was approximately the same as the flight time. Thus, it was concluded that the average warning time to a collison is not a useful measure for evaluation of the altitude tracker. A summary of the impact of the various factors on the average warning time is given in table 7. One particularly important result that was found was that the change in the average warning time as a function of the change in the vertical separation threshold used by Conflict Alert is insignificant. Thus, the separation threshold could be reduced to minimize false alerts with no significant loss in warning time.

It was emphasized repeatedly throughout this study that the errors in the tracking filter output are assumed to be statistically independent on a scan-to-scan basis. However, there are three reasons why this may not be true. First, the measurement errors in the input data may be correlated. Second, the errors introduced by the lack of time correction are not independent since the error at one scan will determine the error at the next. Third, the use of finite-precision arithmetic for implementation of a recursive filtering algorithm is known to result in oscillatory error sequences in many cases. The assumption of statistical independence is considered to be the most critical of all the assumptions made in this study with regard to the potential deviations between theory and practice. Thus, the impact of correlated error sequences must be carefully examined. The technical literature in the area of digital filtering contains many papers describing the impact of finite-precision arithmetic on the performance of recursive digital filters. One fact that must be recognized is that such filters are subject to a well-known error phenomena referred to as limit-cycle oscillations caused by quantization and other nonlinearities in the feedback structure of the filter. It is also known that in some cases, the magnitude of these oscillatory error sequences is so large as to override the input signal causing an overflow in the registers used for implementation of the filter. In view of this, it is essential to avoid the limit-cycle phenomena at all costs.

In the case of the en route altitude tracker, in which both the input data and the time of measurement are coarsely quantized, it would be anticipated that extreme degradation in performance would be observed if, in fact, the filter begins to oscillate. Not only are the errors resulting from limit cycles potentially very large in amplitude, but since the errors are oscillatory, they are obviously not statistically independent.

The significance of limit cycles, from a safety viewpoint, is that the amplitude of these oscillations may very well be sufficient to create the appearance of vertical separation in the predicted position when, in fact, none exists. The amplitude and duration of the limit cycles may be sufficient to prevent the generation of a predicted alert either through failure of the convergence/divergence check (equation 39) or by the actual error in the predicted position. If this is the case, then the warning time provided before a collision will be much less than anticipated. In fact, the loss in warning time can be equivalent to no warning whatsoever before a separation violation and, depending on the velocity of the target, perhaps only 10 to 20 seconds warning before a collision. Naturally, the failure of the convergence/divergence check can also create the opposite effect in which false separation violations are declared for cases in which the targets involved are actually diverging.

The use of time correction in the en route altitude tracker will improve the performance by not only reducing the magnitude of the errors, but also by eliminating the conditions leading to limit-cycle oscillations. As a result of the lack of time correction, there may be no warning of a hazardous condition until an actual separation violation occurs. Since no attempt was made in operational testing to find either the correlation in the altitude errors or to identify whether or not oscillatory error sequences occur, there is no justification to believe that the altitude tracking performance is satisfactory. To accept the possibility of limit-cycle oscillations in a safety-related automation feature such as Conflict Alert is totally unrealistic. In conclusion, the present en route altitude tracker must be considered as not effective unless the time-correction process is added. Even if limit cycles are not found in a finite sample of operational data, this does not guarantee they do not exist, only that they did not exist in the sample that was examined. Since the tracking modifications required for implementation of time correction are trivial, one should not accept even the remote possibility that the Conflict Alert algorithm will fail to detect a separation violation as a result of limit-cycle oscillations.

5. RECOMMENDATIONS

As a result of the findings in this report, the following recommendations are made:

1. The en route altitude tracking algorithm is not effective and must be modified to include time correction in order to avoid the loss in warning time due to the potential of oscillatory error sequences known to be associated with recursive digital filters.

2. An evaluation of the altitude tracker itself should be conducted using both simulated and operational data to confirm that the warning time which should be provided by Conflict Alert is not lost. However, it is not sufficient to use Conflict Alert output alone for the evaluation of the altitude tracker.

3. Since the mean warning time is not a valid measure of altitude tracking performance, the statistical correlation between sequential tracking errors should be used to determine if the filter implementation is effective. In addition, the filter output should be examined for any indication of oscillatory error sequences.

4. The position smoothing parameter a should be varied over the range 0.4 to 0.6 to find the point at which the performance is best according to the criterion just given.

5. The vertical separation threshold used by Conflict Alert can be reduced to minimize false alarms with no significant loss in warning time.

6. A similar study should be conducted using parameters appropriate for the terminal area in order to determine the theoretical performance of Conflict Alert. Such a study will provide a definitive basis for performance evaluation. The oscillatory error sequences associated with recursive digital filters are known to be present in the terminal area altitude tracker (reference 41).

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