# Optimizing Airport Capacity Utilization in Air Traffic Flow Management Subject to Constraints at Arrival and Departure Fixes

Eugene P. Gilbo

***Abstract*—This paper formulates a new approach for improvement of air traffic flow management at airports, which leads to more efficient utilization of existing airport capacity to alleviate the consequences of congestion. A new model is presented, which first considers the runways and arrival and departure fixes jointly as a single system resource, and second considers arrivals and departures simultaneously as two interdependent processes. The model takes into account the interaction between runway capacity and capacities of fixes to optimize the traffic flow through the airport system. The effects are achieved by dynamic time-dependent allocation of airport capacity and flows between arrivals and departures coordinated with the operational constraints at runways and arrival and departure fixes as well as with dynamics of traffic demand and weather. Numerical examples illustrate the potential benefits of the approach.**

***Index Terms*—Airport capacity, air traffic flow management, delay, optimization, queue.**

## I. Introduction

INABILITY of airport and airspace capacity to meet the growing air traffic demand is a major cause of congestion and extremely costly delays. Severe congestion during peak periods when traffic demand exceeds available capacity became the everyday reality in the United States and Western and Central Europe, as well as in some parts of the Pacific Rim. According to a Federal Aviation Administration (FAA) report [1], in 1991 23 major U.S. airports experienced more than 20 000 h of annual aircraft flight delays each. The average airline operating cost of 1-h delay is $1600, which implies an average annual loss of $740 million for the 23 airports. The projected growth of the traffic demand will make the situation worse in the near future if no actions are undertaken for capacity improvements. For example, by 2002 the number of airports with more than 20 000 h of annual delays is projected to increase from 23 to 33 if the capacity is kept on the current level. The total annual airline losses for these airports (in today's cost of delays) would be more than $1 billion.

Europe faces similar if not more acute problems. In 1990, due to airport and airspace congestion, 23.8% of international departures within Europe were delayed by more than 15 min [2]. The situation in Europe is especially complicated since its airspace structure is distributed over a dozen independent countries.

It is clear that the phenomenon of growing traffic demand should be met by a concomitant improvement in airport capacity. The FAA conducts extensive analysis and coordinates several projects to attack the problem.

Possible measures for increasing airport capacity are discussed in [1] and [3]. The long-term programs include construction of new airports and expansion of runway systems at existing airports. The short-term programs consider new operational methods in traffic flow management and capacity utilization as potentially effective measures for improving the existing capacity resources. Recent analysis showed [4] that optimization of the present airport system by the operational and technological measures might result in increasing current traffic flow by up to 50%.

This paper considers operational measures for increasing traffic flow at airports. The work reported in the paper has been conducted in the scope of the Advanced Traffic Management System (ATMS), the FAA research and development program that explores, prototypes, and evaluates new concepts in air traffic management automation. The ATMS products are implemented in the operational real-time Enhanced Traffic Management System (ETMS), an automated system which supports the strategic management of air traffic in the United States. The ETMS has been installed and used in all FAA ARTCC's (Air Route Traffic Control Centers) and TRACON's (Terminal Radar Approach Control Facilities).

Congestion problems occur at an airport whenever traffic demand exceeds the available capacity. Currently the ETMS Monitor/Alert functionality identifies congested periods by comparing traffic demand and capacity for each 15-min interval. Traffic managers strategically control the traffic and resolve the congestion problems by delaying some flights with a ground delay program so that the flow at the airport system meets but does not exceed the available capacity.

In this paper, we consider a strategic traffic flow management (TFM) problem at airports on a 15-min aggregation level operating with the predicted traffic demand, traffic flow, and capacity per 15 min for several hours in advance; flight-by-flight considerations are beyond the scope of this paper.

Manuscript received January 20, 1995; revised November 1, 1996. Recommended by Associate Editor, D. W. Repperger. This work was sponsored by the Federal Aviation Administration and was conducted in the scope of the Advanced Traffic Management System project at the John A. Volpe National Transportation Systems Center, Cambridge, MA.

The author is with the Volpe National Transportation Systems Center, Cambridge, MA 02142 USA.

Publisher Item Identifier S 1063-6536(97)06206-4.

In [5], a new operational approach to the optimization of traffic flow at airports was proposed. The key element of the approach is consideration of airport arrival and departure capacities as interdependent variables whose values depend on arrival/departure ratio in the total airport operations. In contrast to the conventional representation of airport capacity by two separate constants (one for arrival capacity and the other for departure capacity) the airport capacity is represented in [5] by an arrival–departure capacity curve, which determines a set of paired values "arrival capacity–departure capacity" in the entire range of arrival–departure ratios.

The method, presented in [5], is based on the joint consideration of the arrival and departure processes at the airport and on the optimal time-dependent allocation of arrival and departure capacities during an assigned time period. The allocation reflects the dynamics of arrival and departure demand and weather. In other words, the optimization procedure mutually matches available capacity and traffic demand. The method, however, was applied only to runway capacity. It did not consider the restricted capacity in the near-terminal airspace, in particular, the capacities of arrival and departure fixes.

This paper presents a new optimization model which considers the airport (runways) and arrival and departure fix capacities jointly as a single system resource. The incoming flow passes through the arrival fixes before landing, and the outgoing flow passes through the departure fixes after leaving the runways. The model takes into account the interaction between runway capacity and capacity of fixes to optimize the traffic flow through the airport system.

In general, the total capacity of fixes is greater than the airport runways' capacity. Therefore, one might think that in case of congestion, the runway capacity, not the capacity of fixes, limits the maximum throughput at the airport system. This is true when the traffic demand is distributed more or less evenly over the fixes. However, extensive analysis of real traffic at major airports showed that traffic demand, especially arrival traffic, is not always evenly distributed over fixes [6]. There are time periods when some fixes are overloaded while others have very small demand. For example, at Chicago O'Hare International Airport, the demand over arrival fixes is often imbalanced because the traffic comes in waves during the day, first westbound and then eastbound, due to the time difference between the east and west coasts. It may happen that during these periods the fixes, not runways, create a bottleneck at the airport system and limit the total traffic.

During periods of congestion it is very important to properly coordinate and fully utilize runways and fixes.

The optimization model presented in this paper can be used by traffic managers and controllers as an automated support tool for suggesting optimal strategic decisions on flow management at airports during periods of congestion. In particular, for a given time period, runway configuration, weather forecast, and predicted arrival and departure demand for runways and fixes (input data), one can determine an optimal strategy for managing arrival/departure traffic at an airport (output), i.e., how many flights can be accepted (arrivals) and released

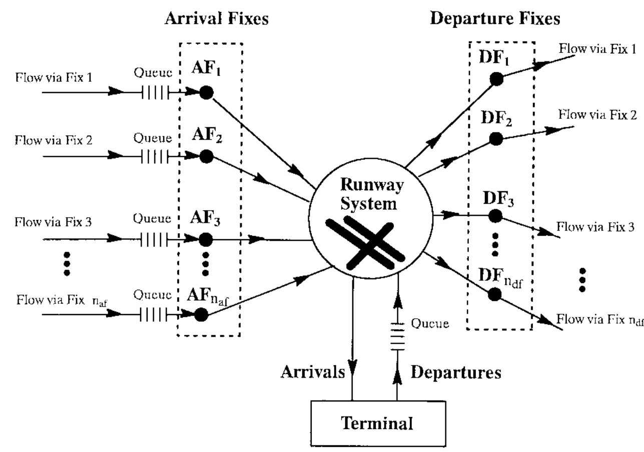


Fig. 1. The arrival–departure scheme of an airport and its fixes.

(departures) during congested periods at the airport, how to distribute the arrival and departure flow over the fixes at each 15-min interval, and how many flights are to be delayed and for how long.

To estimate the efficiency of optimal solutions provided by the model, extensive numerical calculations have been performed at the Volpe National Transportation Systems Center [7]. In this paper, we reproduce a fragment of these calculations as illustrative examples. In particular, the effects are illustrated in the examples calculated for a congested 3-h period at the Chicago O'Hare International airport (ORD).

This paper has been organized as follows. Section II describes a general scheme of arrival–departure system of a single airport. A mathematical optimization model is presented in Section III. Section IV contains numerical examples.

## II. Arrival–Departure System of a Single Airport

A simplified operational scheme of a single airport system that reflects the arrival and departure processes at the airport and its fixes is shown in Fig. 1.

The system comprises $n_{af}$ arrival fixes AF, $n_{df}$ departure fixes DF, and a runway system. There are two separate sets of arrival and departure fixes located in the near-terminal airspace area (50–70 km off the airport) so that the arrival fixes serve only arrival flow, and the departure fixes serve only departure flow. The runway system on the ground serves both arrival and departure flows.

The arrival flights are assigned to specific arrival fixes, and, before landing, they should pass the fixes. After leaving runways, the arriving flights follow the taxiways to the gates at the terminal. The departure flights, after leaving the gates, are headed for the runways, and, after leaving runways, go through the departure fixes. The departing flights are also assigned to the specific fixes.

The arrival queues are formed before the fixes (see Fig. 1). This means that the flights which passed through the fixes, must be accepted at the runways. If there is an arrival queue, a certain amount of flights should be delayed. Some of them are to be delayed in the air and some of them on the ground at the departure airports. The departure queue is formed before

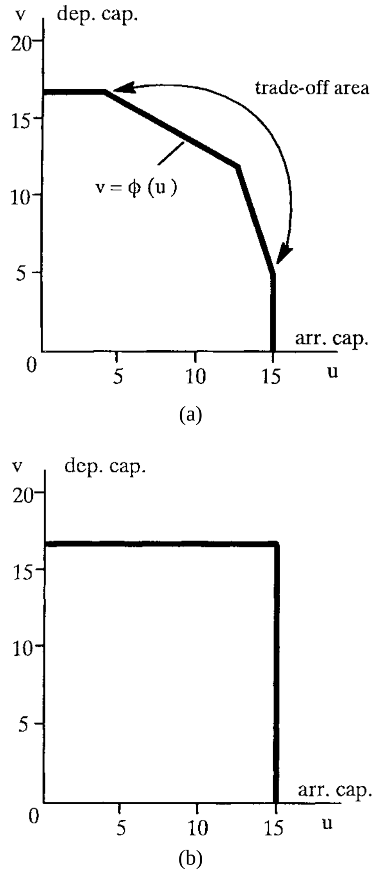


Fig. 2. Airport arrival–departure capacity curves.

the runway system, and flights can be delayed either at their gates or on the taxiway.

The arrival and departure fixes have constant capacities (service rates), which show the maximum number of flights that can cross a fix in a 15-min interval (or other interval). These capacities determine the operational constraints in the near-terminal airspace.

The operational limits on the ground (runways) are characterized by arrival capacity and departure capacity. These capacities are generally variable and interdependent.

There are a number of major airports with runway configurations that practice the tradeoff between arrival and departure capacities. For these configurations the arrival capacity $u$ and departure capacity $v$ are interdependent and can be represented by a functional relationship $v = \phi(u)$. Generally, the function is a piecewise linear convex one. Graphical representation of the function on the "arrival capacity–departure capacity" plane is called the airport capacity curve [5], [8]–[10]. Fig. 2(a) illustrates a 15-min capacity curve with the tradeoff area. The representation of airport runway capacity through the capacity curves is a key factor in the optimization model.

For a runway configuration, which is not able to perform the tradeoff, the capacity curve degenerates into a rectangle [Fig. 2(b)]. There is no tradeoff area, and the runway configuration has constant arrival and departure capacities regardless of the arrival–departure ratio. In Fig. 2(b), the arrival and departure capacities are equal to 15 and 17 flights per 15 min, respectively.

The traffic demands for the airport and fixes are given by the predicted number of arriving and departing flights per each 15-min interval of the time period of interest.

An optimization model for managing arrival and departure traffic at a single airport system is now presented.

## III. Mathematical Model of a Single Airport System

### A. Notation

| | |
|---|---|
| $T$ | Time period of interest, consisting of $N$ discrete-time intervals of length $\Delta$ (e.g., $\Delta = 15$ min); $T = N\Delta$. |
| $I = \{1, 2, \cdots, N\}$ | A set of time intervals. |
| $\Phi = \|!\{\phi^{(1)}(u), \phi^{(2)}(u), \cdots, \phi^{(M)}(u)\}$ | A set of $M$ airport capacity curves that represent the operational limits for all runway configurations under various weather conditions. |
| $\phi_i(u)$ | An arrival–departure capacity curve that determines the airport operational limits at the $i$th time interval; $\phi_i(u) \in \Phi$ |
| $n_{af}$ | Number of arrival fixes. |
| $n_{df}$ | Number of departure fixes. |
| $J = \{1, \cdots, n_{af}\}$ | A set of arrival fixes. |
| $K = \{1, \cdots, n_{df}\}$ | A set of departure fixes. |
| $F_{Ai}^j$ | Capacity of the $j$th arrival fix corresponding to the $i$th interval at the airport, $i \in I, j \in J$. |
| $F_{Di}^k$ | Capacity of the $k$th departure fix corresponding to the $i$th interval at the airport, $i \in I, k \in K$. |
| $a_i^j$ | Arrival demand through the $j$th fix for the $i$th time interval at the airport, $i \in I, j \in J$. |
| $d_i^k$ | Departure demand through the $k$th fix for the $i$th time interval at the airport, $i \in I, k \in K$. |
| $X_i^j$ | Queue at the $j$th arrival fix for the beginning of the $i$th time interval at the airport, $i \in I, j \in J$. |
| $X_i$ | Total airport arrival queue at the beginning of the $i$th time interval, $i = 1, 2, \cdots, N+1$. |
| $Y_i^k$ | A fraction of the departure queue at the airport at the beginning of the $i$th time interval, caused by the $k$th departure fix, $i \in I, k \in K$. |
| $Y_i$ | Total airport departure queue at the beginning of the $i$th time interval, $i = 1, 2, \cdots, N+1$. |
| $u_i$ | Airport (runways) arrival capacity at the $i$th time interval, $i \in I$. |
| $v_i = \phi_i(u_i)$ | Airport (runways) departure capacity at the $i$th time interval, $i \in I$. |

$w_i^j$ Flow through the $j$th arrival fix for the $i$th time interval at airport, $i \in I, j \in J$.

$z_i^k$ Flow through the $k$th departure fix for the $i$th time interval at airport, $i \in I, k \in K$.

## B. Assumptions and Simplifications

In this paper, a deterministic single airport model is considered. It is assumed that the following input data are given:

- the time period $T$ for which the traffic management problem is to be solved;
- the airport capacity curves for each time interval of the period in accordance with a predicted schedule of runway configurations and weather forecast;
- the number of arrival and departure fixes and their capacities;
- predicted arrival and departure demand for the airport and the arrival and departure fixes at each time interval.

There are several assumptions and simplifications connected with the arrival and departure fixes.

- All the flights assigned to the arrival fixes land at the same destination airport and there are no other flights following through the arrival fixes to other airports.
- All the flights assigned to the departure fixes are originated from the same airport and there are no other flights crossing the departure fixes which are originated from other airports.
- A flight, which is assigned to a specific arrival or departure fix, must fly through the fix and cannot be reassigned to another fix.
- All demands and flows through the fixes are related to specific time intervals at the airport.

The latter makes it easy to match the demand and capacities of the fixes to the demand and capacities of the airport for each time interval and hence to keep the demands and flows through the fixes and the runways consistent.

For example, if $a_i^j$ is the arrival demand at the fix $j$ for the time interval $i$ at the airport then the total demand $a_i$ at the airport for the time interval $i$ is equal to the sum of demands at all fixes

$$a_i = \sum_{j=1}^{n_{af}} a_i^j$$

where $n_{af}$ is a number of arrival fixes.

Similarly, if $d_i^k$ is the departure demand through the fix $j$ for the time interval $i$ at the airport then the total demand $d_i$ at the airport for the time interval $i$ is equal to the sum of demands at all fixes

$$d_i = \sum_{k=1}^{n_{df}} d_i^k$$

where $n_{df}$ is the number of departure fixes.

Similar simplification has been also applied to the traffic flows $w_i^j$ and $z_i^k$ through the fixes.

## C. Dynamics of Arrival–Departure Processes at the Airport System

The following equations and inequalities determine the dynamics of arrival and departure processes at the airport system.

1) Flow balance at the arrival fixes

$$X_{i+1}^j = X_i^j + a_i^j - w_i^j, \qquad i \in \boldsymbol{i}, \quad j \in \boldsymbol{J} \tag{1}$$

with the given initial conditions $X_1^j$. $X_{N+1}^j$ is an outstanding queue at the end of time period $T$, i.e., number of flights assigned to arrival fix $j$ that are delayed beyond the period $T$.

According to these equations, the number of flights in a queue at the $j$th fix at the beginning of the $(i+1)$th interval is equal to the difference between the demand at the $i$th interval (which includes the "inherited" queue from the previous slots and the original demand for the slot) and the number of aircraft left the fix during the $i$th interval.

2) The nonnegativity conditions for the queues (1)

$$w_i^j \leqq X_i^j + a_i^j, \qquad i \in \boldsymbol{I}, \quad j \in \boldsymbol{J}. \tag{2}$$

3) At each time interval, the total arriving flow (from all arrival fixes) can not exceed the runway arrival capacity

$$\sum_{j=1}^{n_{af}} w_i^j \leqq u_i, \qquad i \in \boldsymbol{I}. \tag{3}$$

4) Flow balance for departure fixes

$$Y_{i+1}^k = Y_i^k + d_i^k - z_i^k, \qquad i \in \boldsymbol{I}, \quad k \in \boldsymbol{K} \tag{4}$$

with the given initial conditions $Y_1^k$. $Y_{N+1}^k$ is an outstanding queue at the end of time period $T$, i.e., number of flights assigned to departure fix $k$ that are delayed beyond the period $T$.

5) The nonnegativity of the queues (4)

$$z_i^k \leqq Y_i^k + d_i^k, \qquad i \in \boldsymbol{I}, \quad k \in \boldsymbol{K}. \tag{5}$$

6) At each time interval, the total departing flow (through all departure fixes) cannot exceed the runway departure capacity

$$\sum_{k=1}^{n_{df}} z_i^k \leqq \phi_i(u_i), \qquad i \in \boldsymbol{I}. \tag{6}$$

7) At each time interval, the flows through the fixes cannot exceed the fix capacities

$$w_i^j \leqq F_{Ai}^j, \qquad i \in \boldsymbol{I}, \quad j \in \boldsymbol{J} \tag{7}$$

$$z_i^k \leqq F_{Di}^k, \qquad i \in \boldsymbol{I}, \quad k \in \boldsymbol{K}. \tag{8}$$

8) Constraints for runway arrival capacities at each time interval

$$0 \leqq u_i \leqq U_i, \qquad i \in \boldsymbol{I} \tag{9}$$

where $U_i$ is the upper bound for the arrival capacity at the $i$th interval.

9) The total airport arrival and departure queues at the beginning of the $(i+1)$th interval are obtained by summation of queues at arrival and departure fixes, respectively,

$$X_{i+1} = \sum_{j=1}^{n_{af}} X_{i+1}^j, \qquad i \in \boldsymbol{I} \tag{10}$$

$$Y_{i+1} = \sum_{k=1}^{n_{df}} Y_{i+1}^k, \qquad i \in \boldsymbol{I}. \tag{11}$$

10) The nonnegativity and integrality conditions

$$u_i, w_i^j, z_i^k \text{ are nonnegative and integer} \quad i \in \boldsymbol{I}, j \in \boldsymbol{J}, k \in \boldsymbol{K}. \tag{12}$$

### D. Optimization Model

First of all we formulate an optimization criterion. One of the conventional measures of quality of air traffic management is the total aircraft flight delay time, which is calculated as a sum of delay times of all flights considered. The amount of delay substantially depends on how well the available capacity is utilized to meet the traffic demand, especially during the congested periods. Therefore a meaningful criterion of optimality could be the minimization of total aircraft flight delay time. In case of discrete time, timing accuracy of each flight is within the range of the time discreteness. In particular, with 15-min discreteness, the delay time can only be expressed through the number of 15-min blocks.

In turn, the total number of 15-min blocks in the total aircraft flight delay time can be expressed through the queues at the end of each 15-min interval of the time period $T$. A simple analysis of propagation of queues at the end of each 15-min interval over a period $T$ shows that, if all the flights have been assigned within the considered time period $T$, i.e., there is no outstanding flights left unserved by the end of the period, then the total number of 15-min blocks in the total aircraft flight delay time is equal to the sum of queues at the end of each 15-min interval over a period of time $T$ (we will call it the cumulative queue). Hence, the total aircraft flight delay time is equal to the cumulative queue multiplied by 15 min. In this case, minimization of total delay is equivalent to minimization of the cumulative queue.

The queues at the end of each 15-min interval are easily calculated as the difference between demand and capacity (the queue is equal to zero if demand is less or equal to capacity). A queue shows the number of flights that cannot be served at a time interval and should be delayed to some later intervals.

According to 2.1 notation, cumulative arrival and departure queues at the airport over a period of time $T$ are, respectively,

$$\sum_{i=1}^{N} X_{i+1} \quad \text{and} \quad \sum_{i=1}^{N} Y_{i+1}. \tag{13}$$

The queues $X_{i+1}$ and $Y_{i+1}$ can be expressed through demands and capacities by using (1), (4), (10), and (11).

As an optimality criteria, we will consider the minimum of a linear function of cumulative arrival and departure queues at the airport over a period $T$

$$\underset{u,w,z}{\text{minimize}} \left[ A \sum_{i=1}^{N} X_{i+1} + B \sum_{i=1}^{N} Y_{i+1} \right] \tag{14}$$

where $A$ and $B$ are nonnegative weight coefficients; $u, w,$ and $z$ denote the sets of decision variables, the airport arrival capacities $\{u_i\}$, and flows $\{w_i^j\}$ and $\{z_i^k\}$ through the arrival and departure fixes, respectively.

If at the end of time period $T$ there are no arrival and departure queues ($X_{N+1} = 0$ and $Y_{N+1} = 0$) then (14) minimizes also a weighted sum of total arrival and departure aircraft flight delays. Generally, there can be outstanding queues at the end of period $T$, and (14) includes both intermediate and outstanding queues.

The coefficients $A$ and $B$ in the objective function (14) can have various meanings. For example, $A$ and $B$ can denote an average cost of a unit of time of delay for arrivals and departures, respectively. In this case, (14) minimizes an average cost of total arrival and departure delays for the set of flights considered.

Another application of coefficients $A$ and $B$ is to use them as control parameters of the model. By varying their values it is possible to vary relative impact of arrival and departure queues or delays in the objective function (14), which in turn can affect the optimal strategies of managing traffic flow and allocation of arrival and departure delays at the airport. It is convenient to normalize the coefficients by dividing (14) by $(A+B)$. Then instead of (14), we can write

$$\underset{u,w,z}{\text{minimize}} \sum_{i=1}^{N} [\alpha X_{i+1} + (1-\alpha) Y_{i+1}] \tag{15}$$

where $\alpha = A/(A+B), 0 \le \alpha \le 1$.

The normalization made it possible to reduce number of parameters from two ($A$ and $B$) to one ($\alpha$).

Coefficient $\alpha$ varies from zero to one. While increasing the weight $\alpha$ for cumulative arrival queue in (15), the weight $(1-\alpha)$ for cumulative departure queue decreases and vice versa, so that varying $\alpha$ we can increase or decrease an impact of arrival or departure component in the objective function. Therefore, it is possible to interpret the coefficient $\alpha$ as a tradeoff parameter between arrivals and departures. It can be also associated with the priority rate for arrivals. In extreme cases of $\alpha = 1$ or $\alpha = 0$, we give a full priority to arrivals or departures, respectively, optimizing only arrival or only departure operations. In case of $\alpha = 0.5$, we assume equal priority for arrivals and departures (or give no priority to any of the two operations), and minimize the sum of cumulative arrival and departure queues or the sum of total aircraft flight delays for all arrival and departure flights at the airport over a period $T$. Thus the coefficient $\alpha$ may be used as a policy parameter that reflects the operational priorities at the airport.

There is another application of the coefficient $\alpha$. It is well known (see, e.g., [10]) that in the real world, the maximum arrival capacity is usually less than the maximum departure capacity and thus the airport capacity curves are asymmetric.

If the difference between maximum arrival and departure capacities is significant, then even for equal priority for arrivals and departures [$\alpha = 0.5$ in (15)] the allocation of airport operations for arrivals and departures can be more favorable to departures. The effect of asymmetry can be compensated by increasing parameter $\alpha$ above 0.5.

In (15), coefficient $\alpha$ is constant for all time intervals over a period $T$. In a more general case, the coefficient $\alpha$ can be time-dependent, i.e., $\alpha = \alpha_i, i = 1, 2, \cdots, N$. It may be connected with changing operational policies at the airport for some time segments of a period $T$, and assigning various arrival priority rates at various time intervals may reflect the changes. The possibility to vary the parameter $\alpha$ makes the model more realistic and more flexible in providing alternative solutions. In this case, criterion (15) transforms to

$$\underset{u,w,z}{\text{minimize}} \sum_{i=1}^{N} [\alpha_i X_{i+1} + (1 - \alpha_i) Y_{i+1}] \tag{16}$$

where $0 \le \alpha_i \le 1$.

The criterion (16) can be further modified as follows:

$$\underset{u,w,z}{\text{minimize}} \sum_{i=1}^{N} \gamma_i [\alpha_i X_{i+1} + (1 - \alpha_i) Y_{i+1}] \tag{17}$$

with additional parameter $\gamma_i, i = 1, 2, \cdots, N$.

The parameter $\gamma_i$ can be introduced to reflect relative importance of or difference in values of various time intervals. For example, it can be connected with the reliability in predicting the traffic and/or airport capacity. Generally, for more distant time intervals, that are farther into the future, the reliability of the forecast decreases. Therefore for those intervals the smaller values of $\gamma_i$ can be assigned.

Criteria (14)–(16) are the special cases of (17) and can be easily obtained from (17) by the corresponding assignment of coefficients $\alpha_i$ and $\gamma_i$.

For all versions of optimality criteria, the optimization is achieved by controlling arrival and departure flows through the fixes and runways at each time interval through the proper allocation of arrival and departure resources.

The decision variables comprise:

- $N$ airport arrival capacities $u_i$ $(i = 1, 2, \cdots, N)$;
- $(N * n_{af})$ flows $w_i^j$ through arrival fixes $(i = 1, 2, \cdots, N; j = 1, 2, \cdots, n_{af})$;
- $(N * n_{df})$ flows $z_i^k$ through departure fixes $(i = 1, 2, \cdots, N; k = 1, 2, \cdots, n_{df})$.

There are $N * (n_{af} + n_{df} + 1)$ decision variables altogether.

Now we can formulate the following optimization problem: *determine the optimal values of airport arrival capacities and the flows through the arrival and departure fixes which satisfy the optimality criterion (17) [or any other from (14)–(16)], subject to (1) through (12).*

After the optimal values of the airport arrival capacities $u_i$ have been determined the corresponding departure capacities $v_i$ are determined through the airport capacity curves

$$v_i = \text{trunc}\, \phi_i(u_i), \qquad i = 1, 2, \cdots, N. \tag{18}$$

There are various methods to obtain the optimal solutions. All numerical results presented in this paper were derived using the integer linear program techniques.

The decision variables are present in the optimization criteria (14)–(17) implicitly. Keeping in mind that the criteria (14)–(16) are the the special cases of (17), let us transform the optimization problem (17), subject to (1)–(12), to another form with the decision variables represented explicitly in both the optimization criteria and the constraints. The transformation is very useful methodologically, because it helps establish the equivalence between the minimization of queues and maximization of flows. The duality relations can be also useful for computational purposes.

Using the recurrent relationships (1) and (4), the queues at the arrival and departure fixes can be expressed through the decision variables and through the original demand and initial conditions as follows:

$$X_{i+1}^j = X_1^j + \sum_{p=1}^{i} a_p^j - \sum_{p=1}^{i} w_p^j, \qquad i \in \boldsymbol{I}, \quad j \in \boldsymbol{J} \tag{19}$$

$$Y_{i+1}^k = Y_1^k + \sum_{p=1}^{i} d_p^k - \sum_{p=1}^{i} z_p^k, \qquad i \in \boldsymbol{I}, \quad k \in \boldsymbol{K}. \tag{20}$$

Then, instead of inequalities (2) and (5) the following non-negativity conditions for the queues can be obtained directly from (19) and (20):

$$\sum_{p=1}^{i} w_p^j \leqq X_1^j + \sum_{p=1}^{i} a_p^j, \qquad i \in \boldsymbol{I}, \quad j \in \boldsymbol{J} \tag{21}$$

$$\sum_{p=1}^{i} z_p^k \leqq Y_1^k + \sum_{p=1}^{i} d_p^k, \qquad i \in \boldsymbol{I}, \quad k \in \boldsymbol{K}. \tag{22}$$

After a series of transformations in the criterion (17) using (10), (11), (19) and (20), and taking into account the expressions (3), (6)–(9), (12), (19) and (20), the optimization problem is formulated as follows:

$$\underset{u,w,z}{\text{maximize}} \sum_{i=1}^{N} \gamma_i \left[ \sum_{p=1}^{i} \left( \alpha_i \sum_{j=1}^{n_{af}} w_p^j + (1 - \alpha_i) \sum_{k=1}^{n_{df}} z_p^k \right) \right] \tag{23}$$

subject to

$$\sum_{p=1}^{i} w_p^j \leqq X_1^j + \sum_{p=1}^{i} a_p^j, \qquad i \in \boldsymbol{I}, \quad j \in \boldsymbol{J} \tag{24}$$

$$\sum_{j=1}^{n_{af}} w_i^j \leqq u_i, \qquad i \in \boldsymbol{I} \tag{25}$$

$$\sum_{p=1}^{i} z_p^k \leqq Y_1^k + \sum_{p=1}^{i} d_p^k, \qquad i \in \boldsymbol{I}, \quad k \in \boldsymbol{K} \tag{26}$$

$$\sum_{k=1}^{n_{df}} z_i^k \leqq \phi_i(u_i), \qquad i \in \boldsymbol{I} \tag{27}$$

$$w_i^j \leqq F_{Ai}^j, \qquad i \in \boldsymbol{I}, \quad j \in \boldsymbol{J} \tag{28}$$

$$z_i^k \leqq F_{Di}^k, \qquad i \in \boldsymbol{I}, \quad k \in \boldsymbol{K} \tag{29}$$

$$u_i \leqq U_i, \qquad i \in \boldsymbol{I} \tag{30}$$

$$u_i, w_i^j, z_i^k \text{ are nonnegative and integer} \quad i \in \boldsymbol{I}, j \in \boldsymbol{J}, k \in \boldsymbol{K} \tag{31}$$

where $X_1^j, Y_1^k, F_{Ai}^j, F_{Di}^k, U_i$ are given nonnegative constants and $\phi_i(u)$ are given nonnegative functions, $i \in \boldsymbol{I}, j \in \boldsymbol{J}, k \in \boldsymbol{K}$.

The optimization problem (23)–(31) is equivalent to (1)–(12), (17). It means that the problem (1)–(12), (17) to minimize a weighted sum of arrival and departure queues at the airport is equivalent to the problem (23)–(31) to maximize the weighted sum of arrival and departure flows at the airport.

In the case of constant weight coefficients $\gamma_i$ and $\alpha_i$ (i.e., $\gamma_i = \gamma, \alpha_i = \alpha$) for the entire period of time considered, the optimization criteria (23) is transformed to

$$\underset{u,w,z}{\text{maximize}} \sum_{i=1}^{N} (N-i+1) \left( \alpha \sum_{j=1}^{n_{af}} w_i^j + (1-\alpha) \sum_{k=1}^{n_{df}} z_i^k \right). \tag{32}$$

Criterion (32) corresponds to criterion (15) which minimizes a weighted sum of cumulative arrival and departure queues (or a weighted sum of total arrival and departure delays) over a period $T$.

## IV. Numerical Examples

The presented optimization model has been developed in the scope of the FAA Advanced Traffic Management System (ATMS). To assess its potential benefits, extensive numerical experiments have been performed for several major U.S. airports using the real data [7].

In this section, we describe several examples calculated for the Chicago O'Hare International Airport (ORD), one of the busiest airports. Heavy traffic was predicted over the 3-h period on February 12, 1993 from 16:45 to 19:45 local time. During this period, four arrival fixes and four departure fixes were supposed to be used for the incoming and outgoing flows, respectively. The airport has six runways that are used in different combinations or runway configurations. Some of the configurations allow the arrival/departure tradeoff within certain limits and some of them do not. In this section, we suppose that during the 3-h period, a runway configuration with the tradeoff capability will be used.

The airport capacity curves for VFR and IFR operational conditions are shown in Fig. 3. The coordinates of vertices of the curves show some capacity values (the first number corresponds to the arrival capacity). For example, the coordinates of vertices of the VFR curve (17, 30), (24, 24), and (28, 15) show that under the maximum departure capacity of 30 flights per 15 min, the arrival capacity is equal to 17 flights per 15 min. Under the maximum arrival capacity of 28 flights per 15 min, the departure capacity is 15 flights per 15 min. For a 50/50 arrival–departure mix, the airport capacities for arrivals and departures are identical and equal to 24 flights per 15 min. According to Fig. 3, the IFR capacities are approximately 30% less than VFR capacities.

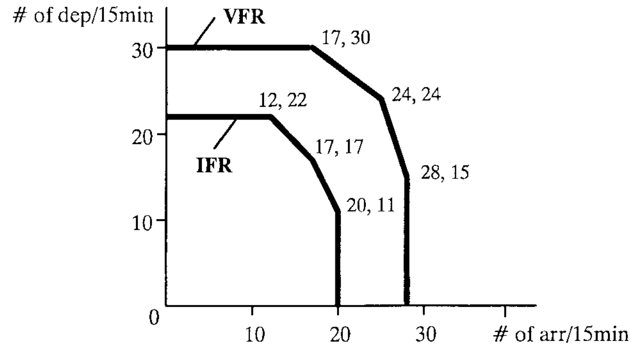


Fig. 3. Airport capacity curves for ORD.

Capacities of the fixes are assumed to be the same for arrival and departure fixes and are equal to ten flights per 15 min for each fix.

Table I shows the predicted arrival and departure demand at the airport distributed through the fixes for each 15-min interval of the 3-h period.

As we can see from the table, the demands for arrivals and departures are distributed nonuniformly over the 3-h period (see columns for the airport demands). The highly congested intervals are alternated with the relatively quiet ones.

The first 30 min, from 16:45 to 17:15, are extremely congested for both arrivals and departures. The arrival and departure demands for this half hour are 64 (26 + 38) and 68 (36 + 32) flights, respectively, which substantially exceed the airport capacity. For the next half hour, there is still a high arrival demand (71 flights) and relatively low departure demand (24 flights). The following 45 min are characterized by low demands (33 arrivals and 34 departures). The demands increase at the next 45 min (85 arrivals and 89 departures). The last half hour is relatively calm with 25 arrival and 14 departure flights in demand.

Below we present some computational results of the optimization problem (32), subject to (1) through (12), for the demand data presented in Table I with the following values of the parameters: $N = 12$ (12 intervals of 15-min each in the 3-h period), $n_{af} = n_{df} = 4$ (four arrival and four departure fixes).

The results include the optimal strategies of managing the arrival and departure flows calculated separately for two values of parameter $\alpha$ (0.5 and 0.7) and for two weather scenarios, which were forecasted for the 3-h period. The weather was taken into account in the optimization model by using the VFR and IFR capacity curves from Fig. 3 at the corresponding time segments. For each strategy, the arrival and departure queues were calculated. To illustrate the propagation of the queues at the airport and fixes over a 3-h period, the numerical results are shown in separate tables.

### A. VFR Weather Conditions

*Case 1: Arrival Priority Rate $\alpha = 0.5$:* The optimal solution for this case is shown in Table II(a). The table contains the optimal allocation of arrival and departure flows at the airport and the distribution of the flows through the fixes at

TABLE I

| TIME | ARRIVAL DEMAND | | | | | DEPARTURE DEMAND | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | FIXES | | | | AIRPORT (TOTAL) | FIXES | | | | AIRPORT (TOTAL) |
| | 1 | 2 | 3 | 4 | | 1 | 2 | 3 | 4 | |
| 16:45–17:00 | 10 | 11 | 1 | 4 | 26 | 9 | 9 | 9 | 9 | 36 |
| 17:00–17:15 | 13 | 14 | 5 | 6 | 38 | 8 | 8 | 8 | 8 | 32 |
| 17:15–17:30 | 15 | 12 | 7 | 8 | 42 | 2 | 2 | 2 | 3 | 9 |
| 17:30–17:45 | 9 | 15 | 2 | 3 | 29 | 4 | 4 | 4 | 3 | 15 |
| 17:45–18:00 | 2 | 2 | 2 | 0 | 6 | 2 | 2 | 2 | 1 | 7 |
| 18:00–18:15 | 1 | 1 | 5 | 6 | 13 | 1 | 3 | 3 | 3 | 10 |
| 18:15–18:30 | 4 | 0 | 4 | 6 | 14 | 4 | 4 | 4 | 5 | 17 |
| 18:30–18:45 | 2 | 3 | 7 | 8 | 20 | 9 | 8 | 8 | 8 | 33 |
| 18:45–19:00 | 5 | 2 | 14 | 19 | 40 | 8 | 8 | 10 | 8 | 34 |
| 19:00–19:15 | 2 | 2 | 12 | 9 | 25 | 5 | 7 | 5 | 5 | 22 |
| 19:15–19:30 | 3 | 6 | 2 | 2 | 13 | 3 | 3 | 3 | 4 | 13 |
| 19:30–19:45 | 2 | 6 | 0 | 4 | 12 | 1 | 0 | 0 | 0 | 1 |
| TOTAL | 68 | 74 | 61 | 75 | 278 | 56 | 58 | 58 | 57 | 229 |

each 15-min interval. The weather conditions are expressed in terms of the operational category in the OP. CAT. column.

Optimal values of airport capacities are shown in two right-hand columns. As we can see from the table, the optimal airport capacities are not constant over the period of time considered. They vary to best satisfy the original demand by trading off the arrival and departure operations at each 15-min interval.

The queue values at the airport and at the fixes at the end of each 15-min interval are presented in Table II(b).

Table II(b) shows that the original demand has been satisfied within the 3-h time frame: there are neither arrival nor departure queues at the end of the last 15-min interval. Cumulative arrival and departure queues at the end of the 3-h period are 143 and 77 flights, respectively. It also means that the total arrival delay and the total departure delay are, respectively, equal to 143 and 77 15-min intervals.

*Case 2: Arrival Priority Rate $\alpha = 0.7$:* Let us increase the arrival priority rate from 0.5 to 0.7 to get a new optimal strategy for managing the flows that is more favorable to arrivals.

For $\alpha = 0.7$, the optimal values of airport capacities and the flows through the fixes and the airport are presented in Table III(a). The corresponding queues are shown in Table III(b).

Increasing the value of parameter $\alpha$ from 0.5 to 0.7 changed the allocation of arrival and departure capacities at the airport at each 15-min interval and, as a result, changed the allocation of arrival and departure flows at runways and the distribution of flows through the fixes. The arrival operations have been improved at the expense of departures.

Although, according to Tables II(a) and III(a), the cumulative arrival capacity increased insignificantly (from 281 to 286), the arrival queues, and, hence, the total arrival delay, decreased significantly [see Tables II(b) and III(b)]. The total arrival delay was reduced from 143 to 94 15-min intervals (more than 34%); the maximum arrival queue at the airport was reduced from 37 to 26. This effect was achieved due to the rational allocation of arrival capacities at each 15-min interval without dramatic increase in the total (cumulative) arrival capacity.

At the same time, the cumulative departure capacity is decreased from 279 to 256, the total departure delay increased from 77 to 185 15-min intervals, and the maximum departure queue increased from 20 to 32. Nevertheless, the whole departure demand as well as arrival demand is satisfied so that there is neither arrival nor departure flights left unserved within the 3-h period.

Other strategies of the utilization of runways and fix capacities can be obtained by varying parameter $\alpha$. This would allow a traffic manager to generate several alternative strategies and choose the best of them.

### B. Changeable Weather

Consider another weather scenario. Suppose that according to the weather forecast the IFR conditions are predicted for the first hour of the 3-h period, and the VFR conditions for the remaining 2 h.

*Case 3: Changeable Weather, Arrival Priority Rate $\alpha = 0.5$:* The optimal values of airport capacities and the flows through the fixes and the airport for $\alpha = 0.5$ are presented in Table IV(a). The corresponding queues are shown in Table IV(b).

Tables IV(a) and IV(b) reflect the effect of reduced airport capacity during the first hour on the overall optimal strategy of managing traffic through the runways and fixes.

The reduction resulted in a significant increase of the arrival and departure queues at the end of the first hour in comparison with the VFR conditions. The arrival queue increased from

TABLE II
(a) Optimal Solution for ORD (VFR, $\alpha = 0.5$). (b) Queues at ORD (VFR, $\alpha = 0.5$)

| TIME | OP. CAT. | ARRIVAL FLOW | | | | | DEPARTURE FLOW | | | | | AIRPORT CAPACITY | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | FIXES | | | | AIRPORT (TOTAL) | FIXES | | | | AIRPORT (TOTAL) | | |
| | | 1 | 2 | 3 | 4 | | 1 | 2 | 3 | 4 | | ARR | DEP |
| 16:45–17:00 | VFR | 9 | 10 | 1 | 4 | 24 | 6 | 6 | 6 | 6 | 24 | 24 | 24 |
| 17:00–17:15 | | 10 | 10 | 3 | 1 | 24 | 6 | 6 | 6 | 6 | 24 | 24 | 24 |
| 17:15–17:30 | | 8 | 10 | 2 | 4 | 24 | 6 | 6 | 6 | 6 | 24 | 24 | 24 |
| 17:30–17:45 | | 8 | 10 | 3 | 5 | 26 | 5 | 5 | 5 | 4 | 19 | 26 | 19 |
| 17:45–18:00 | | 10 | 10 | 4 | 4 | 28 | 2 | 2 | 2 | 2 | 8 | 28 | 15 |
| 18:00–18:15 | | 5 | 5 | 9 | 9 | 28 | 1 | 3 | 3 | 3 | 10 | 28 | 15 |
| 18:15–18:30 | | 4 | 0 | 4 | 6 | 14 | 4 | 4 | 4 | 5 | 17 | 17 | 30 |
| 18:30–18:45 | | 2 | 3 | 7 | 8 | 20 | 7 | 7 | 7 | 6 | 27 | 20 | 27 |
| 18:45–19:00 | | 3 | 1 | 10 | 10 | 24 | 6 | 5 | 7 | 6 | 24 | 24 | 24 |
| 19:00–19:15 | | 3 | 1 | 10 | 10 | 24 | 6 | 6 | 6 | 6 | 24 | 24 | 24 |
| 19:15–19:30 | | 2 | 5 | 7 | 10 | 24 | 6 | 6 | 6 | 6 | 24 | 24 | 24 |
| 19:30–19:45 | | 4 | 9 | 1 | 4 | 18 | 1 | 2 | 0 | 1 | 4 | 18 | 29 |
| TOTAL | | 68 | 74 | 61 | 75 | 278 | 56 | 58 | 58 | 57 | 229 | 281 | 279 |

(a)

| TIME | ARRIVAL QUEUES | | | | | DEPARTURE QUEUES | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | FIXES | | | | AIRPORT (TOTAL) | FIXES | | | | AIRPORT (TOTAL) |
| | 1 | 2 | 3 | 4 | | 1 | 2 | 3 | 4 | |
| 16:45–17:00 | 1 | 1 | 0 | 0 | 2 | 3 | 3 | 3 | 3 | 12 |
| 17:00–17:15 | 4 | 5 | 2 | 5 | 16 | 5 | 5 | 5 | 5 | 20 |
| 17:15–17:30 | 11 | 7 | 7 | 9 | 34 | 1 | 1 | 1 | 2 | 5 |
| 17:30–17:45 | 12 | 12 | 6 | 7 | 37 | 0 | 0 | 0 | 1 | 1 |
| 17:45–18:00 | 4 | 4 | 4 | 3 | 15 | 0 | 0 | 0 | 0 | 0 |
| 18:00–18:15 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 18:15–18:30 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 18:30–18:45 | 0 | 0 | 0 | 0 | 0 | 2 | 1 | 1 | 2 | 6 |
| 18:45–19:00 | 2 | 1 | 4 | 9 | 16 | 4 | 4 | 4 | 4 | 16 |
| 19:00–19:15 | 1 | 2 | 6 | 8 | 17 | 3 | 5 | 3 | 3 | 14 |
| 19:15–19:30 | 2 | 3 | 1 | 0 | 6 | 0 | 2 | 0 | 1 | 3 |
| 19:30–19:45 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| TOTAL | 37 | 35 | 39 | 41 | 143 | 18 | 21 | 17 | 21 | 77 |

(b)

37 to 67 flights, and departure queue increased from 1 to 24 flights [see Tables II(b) and IV(b)].

Significant reduction in the airport capacity during the first hour affected the total airport operations for the 3-h period. Because of the reduction, total arrival and departure queues and delays increased dramatically. Moreover, the arrival demand was not completely satisfied within the 3-h period, and at the end of the period eight arrival flights left unserved [see Table IV(b)]. At the same time the departure demand was completely satisfied, and there is no outstanding departure queue at the end of the 3-h period.

If the outstanding arrival queue of eight flights is not satisfactory for a traffic manager, it is possible to obtain the alternative strategies which are more favorable to arrivals by increasing parameter $a$. The quantitative effect of increasing the arrival priority rate from 0.5 to 0.7 to improve the arrival operations is illustrated in Tables V(a) and V(b).

The comparison of optimal solutions for $\alpha = 0.5$ and $\alpha = 0.7$ from Tables IV(a) and V(a) shows that during the first hour under the IFR conditions, the optimal arrival capacity increased from 68 to 80 flights/h, and the departure capacity decreased from 68 to 44 flights/h. As a result, by the end of

TABLE III
(a) OPTIMAL SOLUTION FOR ORD (VFR, $\alpha = 0.7$). (b) QUEUES AT ORD (VFR, $\alpha = 0.7$)

| TIME | OP. CAT. | ARRIVAL FLOW | | | | | DEPARTURE FLOW | | | | | AIRPORT CAPACITY | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | FIXES | | | | AIRPORT (TOTAL) | FIXES | | | | AIRPORT (TOTAL) | | |
| | | 1 | 2 | 3 | 4 | | 1 | 2 | 3 | 4 | | ARR | DEP |
| 16:45–17:00 | VFR | 10 | 10 | 1 | 4 | 25 | 5 | 5 | 5 | 6 | 21 | 25 | 21 |
| 17:00–17:15 | | 10 | 10 | 4 | 4 | 28 | 4 | 4 | 4 | 3 | 15 | 28 | 15 |
| 17:15–17:30 | | 10 | 10 | 3 | 5 | 28 | 3 | 4 | 4 | 4 | 15 | 28 | 15 |
| 17:30–17:45 | | 10 | 10 | 3 | 5 | 28 | 4 | 4 | 4 | 3 | 15 | 28 | 15 |
| 17:45–18:00 | | 9 | 10 | 6 | 3 | 28 | 4 | 4 | 4 | 3 | 15 | 28 | 15 |
| 18:00–18:15 | | 1 | 5 | 5 | 6 | 17 | 6 | 7 | 7 | 7 | 27 | 17 | 30 |
| 18:15–18:30 | | 4 | 0 | 4 | 6 | 14 | 4 | 4 | 4 | 6 | 18 | 17 | 30 |
| 18:30–18:45 | | 2 | 3 | 7 | 8 | 20 | 7 | 7 | 7 | 6 | 27 | 20 | 27 |
| 18:45–19:00 | | 5 | 2 | 10 | 10 | 27 | 4 | 4 | 5 | 4 | 17 | 27 | 17 |
| 19:00–19:15 | | 2 | 2 | 10 | 10 | 24 | 6 | 6 | 6 | 6 | 24 | 24 | 24 |
| 19:15–19:30 | | 3 | 6 | 8 | 10 | 27 | 4 | 4 | 4 | 5 | 17 | 27 | 17 |
| 19:30–19:45 | | 2 | 6 | 0 | 4 | 12 | 5 | 5 | 4 | 4 | 18 | 17 | 30 |
| TOTAL | | 68 | 74 | 61 | 75 | 278 | 56 | 58 | 58 | 57 | 229 | 286 | 256 |

(a)

| TIME | ARRIVAL QUEUES | | | | | DEPARTURE QUEUES | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | FIXES | | | | AIRPORT (TOTAL) | FIXES | | | | AIRPORT (TOTAL) |
| | 1 | 2 | 3 | 4 | | 1 | 2 | 3 | 4 | |
| 16:45–17:00 | 0 | 1 | 0 | 0 | 1 | 4 | 4 | 4 | 3 | 15 |
| 17:00–17:15 | 3 | 5 | 1 | 2 | 11 | 8 | 8 | 8 | 8 | 32 |
| 17:15–17:30 | 8 | 7 | 5 | 5 | 25 | 7 | 6 | 6 | 7 | 26 |
| 17:30–17:45 | 7 | 12 | 4 | 3 | 26 | 7 | 6 | 6 | 7 | 26 |
| 17:45–18:00 | 0 | 4 | 0 | 0 | 4 | 5 | 4 | 4 | 5 | 18 |
| 18:00–18:15 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| 18:15–18:30 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 18:30–18:45 | 0 | 0 | 0 | 0 | 0 | 2 | 1 | 1 | 2 | 6 |
| 18:45–19:00 | 0 | 0 | 4 | 9 | 13 | 6 | 5 | 6 | 6 | 23 |
| 19:00–19:15 | 0 | 0 | 6 | 8 | 14 | 5 | 6 | 5 | 5 | 21 |
| 19:15–19:30 | 0 | 0 | 0 | 0 | 0 | 4 | 5 | 4 | 4 | 17 |
| 19:30–19:45 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| TOTAL | 18 | 29 | 20 | 27 | 94 | 48 | 45 | 44 | 48 | 185 |

(b)

the first hour the arrival queue decreased from 67 to 55 flights, but the departure queue increased from 24 to 48 flights [see Tables IV(b) and V(b)].

Increasing the arrival priority rate from 0.5 to 0.7 provided the optimal capacity allocation which improved the overall arrival operations during the 3-h period. At the end of the period the total arrival demand was completely satisfied, the cumulative arrival queue decreased from 386 to 257 flights and the total arrival delay decreased from at least 386 to 257 15-min intervals. This improvement, however, was achieved at the expense of the departure operations. Departure demand was not completely satisfied within the 3-h period, and at the end of the period, the outstanding departure queue increased from zero to seven flights. Additionally, the cumulative departure queue and total departure delay increased significantly.

### C. Effect of Fix Constraints on Utilization of Airport Capacity

In this section we illustrate the effect of a finite capacity of near-terminal airspace, in particular, the limited capacity of

TABLE IV
(a) OPTIMAL SOLUTION FOR ORD (VFR AND IFR, $\alpha = 0.5$). (b) QUEUES AT ORD (VFR AND IFR, $\alpha = 0.5$)

| TIME | OP. CAT. | ARRIVAL FLOW | | | | | DEPARTURE FLOW | | | | | AIRPORT CAPACITY | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | FIXES | | | | AIRPORT (TOTAL) | FIXES | | | | AIRPORT (TOTAL) | | |
| | | 1 | 2 | 3 | 4 | | 1 | 2 | 3 | 4 | | ARR | DEP |
| 16:45–17:00 | IFR | 7 | 9 | 0 | 1 | 17 | 4 | 4 | 4 | 5 | 17 | 17 | 17 |
| 17:00–17:15 | | 7 | 7 | 2 | 1 | 17 | 4 | 4 | 5 | 4 | 17 | 17 | 17 |
| 17:15–17:30 | | 6 | 4 | 3 | 4 | 17 | 5 | 5 | 3 | 4 | 17 | 17 | 17 |
| 17:30–17:45 | | 5 | 9 | 1 | 2 | 17 | 4 | 4 | 5 | 4 | 17 | 17 | 17 |
| 17:45–18:00 | VFR | 10 | 10 | 2 | 2 | 24 | 6 | 6 | 6 | 6 | 24 | 24 | 24 |
| 18:00–18:15 | | 7 | 6 | 4 | 10 | 27 | 3 | 5 | 5 | 4 | 17 | 27 | 17 |
| 18:15–18:30 | | 5 | 3 | 9 | 10 | 27 | 4 | 4 | 4 | 5 | 17 | 27 | 17 |
| 18:30–18:45 | | 2 | 3 | 9 | 10 | 24 | 6 | 6 | 6 | 6 | 24 | 24 | 24 |
| 18:45–19:00 | | 4 | 2 | 8 | 10 | 24 | 6 | 6 | 7 | 5 | 24 | 24 | 24 |
| 19:00–19:15 | | 3 | 2 | 10 | 9 | 24 | 6 | 6 | 6 | 6 | 24 | 24 | 24 |
| 19:15–19:30 | | 4 | 7 | 7 | 6 | 24 | 6 | 6 | 6 | 6 | 24 | 24 | 24 |
| 19:30–19:45 | | 6 | 10 | 4 | 8 | 28 | 2 | 2 | 1 | 2 | 7 | 28 | 15 |
| TOTAL | | 66 | 72 | 59 | 73 | 270 | 56 | 58 | 58 | 57 | 229 | 270 | 237 |

(a)

| TIME | ARRIVAL QUEUES | | | | | DEPARTURE QUEUES | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | FIXES | | | | AIRPORT (TOTAL) | FIXES | | | | AIRPORT (TOTAL) |
| | 1 | 2 | 3 | 4 | | 1 | 2 | 3 | 4 | |
| 16:45–17:00 | 3 | 2 | 1 | 3 | 9 | 5 | 5 | 5 | 4 | 19 |
| 17:00–17:15 | 9 | 9 | 4 | 8 | 30 | 9 | 9 | 8 | 8 | 34 |
| 17:15–17:30 | 18 | 17 | 8 | 12 | 55 | 6 | 6 | 7 | 7 | 26 |
| 17:30–17:45 | 22 | 23 | 9 | 13 | 67 | 6 | 6 | 6 | 6 | 24 |
| 17:45–18:00 | 14 | 15 | 9 | 11 | 49 | 2 | 2 | 2 | 1 | 7 |
| 18:00–18:15 | 8 | 10 | 10 | 7 | 35 | 0 | 0 | 0 | 0 | 0 |
| 18:15–18:30 | 7 | 7 | 5 | 3 | 22 | 0 | 0 | 0 | 0 | 0 |
| 18:30–18:45 | 7 | 7 | 3 | 1 | 18 | 3 | 2 | 2 | 2 | 9 |
| 18:45–19:00 | 8 | 7 | 9 | 10 | 34 | 5 | 4 | 5 | 5 | 19 |
| 19:00–19:15 | 7 | 7 | 11 | 10 | 35 | 4 | 5 | 4 | 4 | 17 |
| 19:15–19:30 | 6 | 6 | 6 | 6 | 24 | 1 | 2 | 1 | 2 | 6 |
| 19:30–19:45 | 2 | 2 | 2 | 2 | 8 | 0 | 0 | 0 | 0 | 0 |
| TOTAL | 111 | 112 | 77 | 86 | 386 | 41 | 41 | 40 | 39 | 161 |

(b)

arrival and departure fixes, on the utilization of the runways capacity.

The effect is illustrated in the scope of the above examples by comparison of the optimal allocation of arrival and departure traffic flows at the airport and delays under VFR conditions in two cases: 1) limited capacity of fixes (ten flights per 15 min for each fix) and 2) unlimited capacity of fixes.

Table VI shows the optimal values of total airport traffic flows and queues at each 15-min interval calculated under limited and unlimited capacities of fixes for $\alpha = 0.7$.

In this table, the values that are different in both cases are shown by the bold font.

The difference in optimal results for the first 15-min interval can be easily explained, if we calculate the maximum flow through the fixes, using demand data from Table I. Maximum arrival flows through the fixes with unlimited and limited (ten flights per 15 min) capacities are equal to 26 and 25 flights, respectively. Both values are within the limits of runway arrival capacity. However, because of fix constraints, the original demand of 26 arrival flights could not be completely satisfied. Maximum flow through departure fixes is the same in

TABLE V
(a) OPTIMAL SOLUTION FOR ORD (VFR AND IFR, $\alpha = 0.7$). (b) QUEUES AT ORD (VFR AND IFR, $\alpha = 0.7$)

| TIME | OP. CAT. | ARRIVAL FLOW | | | | | DEPARTURE FLOW | | | | | AIRPORT CAPACITY | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | FIXES | | | | AIRPORT (TOTAL) | FIXES | | | | AIRPORT (TOTAL) | | |
| | | 1 | 2 | 3 | 4 | | 1 | 2 | 3 | 4 | | ARR | DEP |
| 16:45–17:00 | IFR | 8 | 10 | 0 | 2 | 20 | 3 | 3 | 3 | 2 | 11 | 20 | 11 |
| 17:00–17:15 | IFR | 8 | 9 | 1 | 2 | 20 | 3 | 3 | 2 | 3 | 11 | 20 | 11 |
| 17:15–17:30 | IFR | 8 | 4 | 3 | 5 | 20 | 2 | 2 | 3 | 4 | 11 | 20 | 11 |
| 17:30–17:45 | IFR | 4 | 10 | 3 | 3 | 20 | 3 | 3 | 3 | 2 | 11 | 20 | 11 |
| 17:45–18:00 | VFR | 10 | 10 | 4 | 4 | 28 | 4 | 4 | 4 | 3 | 15 | 28 | 15 |
| 18:00–18:15 | VFR | 7 | 7 | 6 | 6 | 26 | 4 | 5 | 5 | 5 | 19 | 26 | 19 |
| 18:15–18:30 | VFR | 4 | 5 | 7 | 8 | 24 | 5 | 6 | 6 | 7 | 24 | 24 | 24 |
| 18:30–18:45 | VFR | 4 | 1 | 9 | 10 | 24 | 6 | 6 | 6 | 6 | 24 | 24 | 24 |
| 18:45–19:00 | VFR | 5 | 2 | 7 | 10 | 24 | 6 | 5 | 7 | 6 | 24 | 24 | 24 |
| 19:00–19:15 | VFR | 3 | 2 | 10 | 9 | 24 | 6 | 7 | 6 | 5 | 24 | 24 | 24 |
| 19:15–19:30 | VFR | 2 | 5 | 8 | 9 | 24 | 6 | 6 | 6 | 6 | 24 | 24 | 24 |
| 19:30–19:45 | VFR | 5 | 9 | 3 | 7 | 24 | 6 | 6 | 6 | 6 | 24 | 24 | 24 |
| TOTAL | | 68 | 74 | 61 | 75 | 278 | 54 | 56 | 57 | 55 | 222 | 278 | 223 |

(a)

| TIME | ARRIVAL QUEUES | | | | | DEPARTURE QUEUES | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | FIXES | | | | AIRPORT (TOTAL) | FIXES | | | | AIRPORT (TOTAL) |
| | 1 | 2 | 3 | 4 | | 1 | 2 | 3 | 4 | |
| 16:45–17:00 | 2 | 1 | 1 | 2 | 6 | 6 | 6 | 6 | 7 | 25 |
| 17:00–17:15 | 7 | 6 | 5 | 6 | 24 | 11 | 11 | 12 | 12 | 46 |
| 17:15–17:30 | 14 | 14 | 9 | 9 | 46 | 11 | 11 | 11 | 11 | 44 |
| 17:30–17:45 | 19 | 19 | 8 | 9 | 55 | 12 | 12 | 12 | 12 | 48 |
| 17:45–18:00 | 11 | 11 | 6 | 5 | 33 | 10 | 10 | 10 | 10 | 40 |
| 18:00–18:15 | 5 | 5 | 5 | 5 | 20 | 7 | 8 | 8 | 8 | 31 |
| 18:15–18:30 | 5 | 0 | 2 | 3 | 10 | 6 | 6 | 6 | 6 | 24 |
| 18:30–18:45 | 3 | 2 | 0 | 1 | 6 | 9 | 8 | 8 | 8 | 33 |
| 18:45–19:00 | 3 | 2 | 7 | 10 | 22 | 11 | 11 | 11 | 10 | 43 |
| 19:00–19:15 | 2 | 2 | 9 | 10 | 23 | 10 | 11 | 10 | 10 | 41 |
| 19:15–19:30 | 3 | 3 | 3 | 3 | 12 | 7 | 8 | 7 | 8 | 30 |
| 19:30–19:45 | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 1 | 2 | 7 |
| TOTAL | 74 | 65 | 55 | 63 | 257 | 102 | 104 | 102 | 104 | 412 |

(b)

both cases and equal to 36 flights, which exceed the runway departure capacity. Reduction in arrival flow from 26 to 25 flights was compensated for by increasing departure flow from 19 to 21 flights due to the tradeoff between runway arrival and departure capacities. Similar situations affected the optimal solutions for some of the subsequent intervals as shown in the remainder of Table VI.

The difference in optimal allocation of airport capacity and its utilization for the limited and unlimited capacity of fixes resulted in different quality of managing the arrival and departure traffic. The quantitative effect is illustrated in Table VII, where the total arrival and departure delays are shown. In case of unlimited capacity of fixes, the total arrival and departure delay times are equal to 85 and 203 15-min intervals, respectively. Under the limited capacity of fixes, the optimal solution provides greater total arrival delay of 94 intervals. At the same time the total departure delay is reduced from 203 to 185 intervals. The optimization procedure automatically reallocates the airport arrival and departure resources because of the fixes constraints.

TABLE VI
OPTIMAL ALLOATION OF ARRIVAL AND DEPARTURE FLOWS AND QUEUES ($\alpha = 0.7$)

| TIME | UNLIMITED CAPACITY OF FIXES | | | | LIMITED CAPACITY OF FIXES | | | |
|---|---|---|---|---|---|---|---|---|
| | TRAFFIC FLOW | | QUEUES | | TRAFFIC FLOW | | QUEUES | |
| | ARR | DEP | ARR | DEP | ARR | DEP | ARR | DEP |
| 16:45–17:00 | **26** | **19** | **0** | **17** | **25** | **21** | **1** | **15** |
| 17:00–17:15 | 28 | 15 | **10** | **34** | 28 | 15 | **11** | **32** |
| 17:15–17:30 | 28 | 15 | **24** | **28** | 28 | 15 | **25** | **26** |
| 17:30–17:45 | 28 | 15 | **25** | **28** | 28 | 15 | **26** | **26** |
| 17:45–18:00 | 28 | 15 | **3** | **20** | 28 | 15 | **4** | **18** |
| 18:00–18:15 | **16** | **30** | 0 | **0** | **17** | **27** | 0 | **1** |
| 18:15–18:30 | 14 | **17** | 0 | 0 | 14 | **18** | 0 | 0 |
| 18:30–18:45 | 20 | 27 | 0 | 6 | 20 | 27 | 0 | 6 |
| 18:45–19:00 | **28** | **15** | **12** | **25** | **27** | **17** | **13** | **23** |
| 19:00–19:15 | **26** | **19** | **11** | **28** | **24** | **24** | **14** | **21** |
| 19:15–19:30 | **24** | **24** | 0 | 17 | **27** | **17** | 0 | 17 |
| 19:30–19:45 | 12 | 18 | 0 | 0 | 12 | 18 | 0 | 0 |
| TOTAL | 278 | 229 | **85** | **203** | 278 | 229 | **94** | **185** |

For equal arrival and departure priorities ($\alpha = 0.5$), the optimal allocation of airport capacity proved to be the same for the limited and unlimited capacity of fixes. In this case the capacity of fixes of ten flights per 15 min was not restrictive for the utilization of runway capacity. The optimal values of arrival and departure flows and the airport capacities are presented in Table II(a).

These examples illustrate the abilities of the proposed model to determine the optimal strategies for utilization of the operational resources at the airport and near-terminal airspace in accordance with the dynamics of traffic demands and weather. They also illustrate how these resources interact to provide the optimal traffic flow at airports.

TABLE VII
TOTAL DELAY TIMES FOR $\alpha = 0.7$

| | | Unlimited capacity of fixes | Limited capacity of fixes |
|---|---|---|---|
| Total delay time (number of 15-minute intervals) | Arrival | 85 | 94 |
| | Departure | 203 | 185 |

## V. CONCLUSIONS

In this paper, a problem has been formulated to optimize the utilization of airport runways and near-terminal airspace capacities to improve the efficiency of managing arrival and departure traffic at airports. Runways and arrival and departure fixes were considered as an integrated unit and a single system resource.

It has been shown that the limited capacity of fixes and imbalance in distribution of demand over the fixes with some overloaded and some underloaded fixes can significantly affect the utilization of airport capacity. Neglecting the fix constraints in these cases can result in overly optimistic, nonrealizable scenarios of managing traffic at the airport. The optimization model presented automatically finds the best strategies for utilization of runways and near-terminal airspace resources during congested periods. The model allocates these resources between arrivals and departures so that no available slots are lost.

## ACKNOWLEDGMENT

The author thanks his colleagues at the Volpe Center for their time and attention helping to refine this paper. Special thanks to L. McCabe, R. Oiesen, E. Roberts, and R. D. Wright. The author thanks the Associate Editor D. Repperger and three anonymous reviewers for their valuable and insightful comments and suggestions, which were useful in the revision of this paper.

## REFERENCES

[1] "1993 aviation system capacity plan," FAA, Office of System Capacity and Requirements, Washington, D.C., Rep. DOT/FAA/ASC-93-1.
[2] G. Huttig, W. Busch, and N. Gronak, "Growing demand and capacity of airports," *Transportation Res.-A*, vol. 28-A, no. 6, pp. 501–509, 1994.
[3] S. G. Hamzawi, "Lack of airport capacity: Exploration of alternative solutions," *Transportation Res.-A*, vol. 26-A, no. 1, pp. 47–58, 1992.
[4] "Airport/airway system capacity needs," Airport Assoc. Council Int.—North America, Washington, D.C., 1992.
[5] E. P. Gilbo, "Airport capacity: Representation, estimation, optimization," *IEEE Trans. Contr. Syst. Technol.*, vol. 1, pp. 144–154, 1993.
[6] E. P. Gilbo and L. P. Parkin, "Fix load analysis," Volpe Center, Cambridge, MA, Interim Tech. Rep., Oct. 1994.

[7] E. P. Gilbo, “Optimization of airport capacity and arrival–departure flow through the runway-fix system,” Volpe Center, Cambridge, MA, Interim Tech. Rep., Feb. 1994.
[8] G. F. Newell, “Airport capacity and delays,” *Transportation Sci.*, vol. 3, no. 3, pp. 201–241, 1979.
[9] W. J. Swedish, “Upgraded FAA airfield capacity model,” The MITRE Corp., Rep. FAA-EM-81-1, vols. 1 and 2, 1981.
[10] I. Frolow and J. H. Sinnott, “National airspace system demand and capacity modeling,” *Proc. IEEE*, vol. 77, no. 11, pp. 1618–1624, 1989.

**Eugene P. Gilbo** received the M.S. degree in mechanical engineering and the Ph.D. and Doctor of Sciences degrees in electrical engineering and applied mathematics from St. Petersburg (formerly Leningrad) Polytechnic Institute in 1960, 1965, and 1975, respectively.

After immigrating to the United States from the Soviet Union in 1988, He was with Unisys Corporation, Cambridge, MA, until 1995. In 1995, he joined the Volpe National Transportation Systems Center of the U.S. Department of Transportation, Cambridge, MA. He is the author or coauthor of more than 80 scientific publications including the monograph with I. B. Chelpanov, *Signal Processing Based on Order Statistics* [Moscow: Soviet Radio Publishing House, 1975 (in Russian)]. His major research interests include system optimization, scheduling, and robustness in statistics and control. His current research concentrates on the optimization of air traffic flow management strategies with an emphasis on airport operations.

Dr. Gilbo is a member of the Institute for Operations Research and Management Science (INFORMS).