IA**VSD**

Taylor & Francis
Taylor & Francis Group

Curving and running resistance of freight trains: current experience with on-track measurements

Tomáš Michálek ⬦, Martin Kohout ⬦, Jiří Šlapák ⬦, Jakub Vágner ⬦ and Jan Pulda ⬦

Faculty of Transport Engineering, University of Pardubice, Pardubice, Czech Republic

ABSTRACT

Curving resistance affects the railway vehicle running through curves. For its estimation in train dynamics calculations, different empirical formulas are used in various countries. It is practically impossible to determine its exact value because of its dependency on various – to some extent random – parameters. In the framework of the experimental research of running resistance of freight trains at the Faculty of Transport Engineering of the University of Pardubice, measurements in regular operation conditions on the Czech railway network were realised in recent years. Because some of these results can be used to quantify the curving resistance, the applied evaluation methods and the relevant results are presented in this paper. Besides that, a new general formula of running resistance for freight trains, based on the measurement results, is proposed. Attention is also paid to the effect of track quality on the vehicle running resistance.

1. Introduction

Curving resistance, as a component of track resistance, influences the total resistive force acting against the train motion on curved tracks. Besides the curve radius, many other factors affect the magnitude of the curving resistance of a specific train in a given track section. A part of these factors is given by the vehicle parameters (vehicle mass, bogie wheelbase, wheelset yaw stiffness, etc.); the second group of these factors is related to operational conditions (cant deficiency, longitudinal forces in the trainset, coefficient of friction in wheel/rail contact, etc.). It is generally known that the curving resistance from wheel/rail contact is closely related to the frictional work and angle of attack of wheelsets (see, e.g. [1,2]) and, therefore, to the wear of wheels and rails. Although the effects mentioned above can be theoretically quantified for a defined vehicle (or train) using computer simulations (see, e.g. [2,3]), especially the random nature of the operational conditions leads to significant uncertainties regarding the actual values of curving resistance.

At the Faculty of Transport Engineering of the University of Pardubice, attention was continuously paid to experimental research on the running resistance of container trains in the last decade (see, e.g. [4,5]). Relevant measurements were performed in regular

---

freight train operation conditions on the Czech railway network. Besides the operational run-down tests (see [4]), measurements with an instrumented screw coupling were also realised in 2021 and 2022. The primary aim of these measurements was an update of the train resistance formulas, which are applied by the Czech infrastructure manager for the calculation of the hauling capability of locomotives. However, a part of the experimental results can also be used to quantify the curving resistance of the investigated trains and analyse further selected aspects of the vehicle running resistance. This paper summarises the last experiences with performed measurements of the running resistance of freight trains. Besides the presentation of the observed dependency of the vehicle running resistance of a freight train on its basic parameters, special attention is paid to the methods and results of experimental determination of curving resistance of the freight trains. Besides that, the effect of track quality on vehicle running resistance is also presented as an example of evaluation of the measured data.

2. Update of general vehicle running resistance formula for freight trains

In the years 2022–2023, a project focused on updating vehicle running resistance formulas for the purposes of application in the calculation of the hauling capability of locomotives was solved at the Faculty of Transport Engineering of the University of Pardubice to order of the Czech infrastructure manager (Správa železnic). As mentioned in [4], the current Czech train resistance formulas are traditionally given in the form of *specific train resistance* r_v [N/kN] as a second-order polynomial function of speed V [km/h], i.e.:

$$r_v = \frac{R_v}{Mg} = a + bV + cV^2 \tag{1}$$

where:

- R_v [N] is the (absolute) vehicle running resistance;
- M [t] is the mass of relevant vehicles in the train;
- g is the gravitational acceleration (9.81 ms^{-2});
- the coefficients a, b, c define the specific vehicle resistance type.

This description of vehicle running resistance is based on the well-known *Davis' equation* [6] and proves some shortcomings, as discussed in [4] in more detailed.

Whereas the results presented in [4] were obtained on the basis of the evaluation of operational run-down tests, a different approach was applied to solve the project mentioned above. For the purposes of assessment of the vehicle running resistance, an instrumented screw coupling was prepared and used for measurement of tractive effort of the locomotive on the hook within a regular operation of freight trains of different categories.

2.1. Preparation of an instrumented screw coupling and on-track measurements

The tractive effort was measured indirectly through the deformation of the instrumented screw coupling parts. Coupling links are the optimal parts for instrumentation due to the highest relative deformation and a low probability of hand contact during the hanging of the first wagon and the locomotive. The strain gauge technology in the Wheatstone
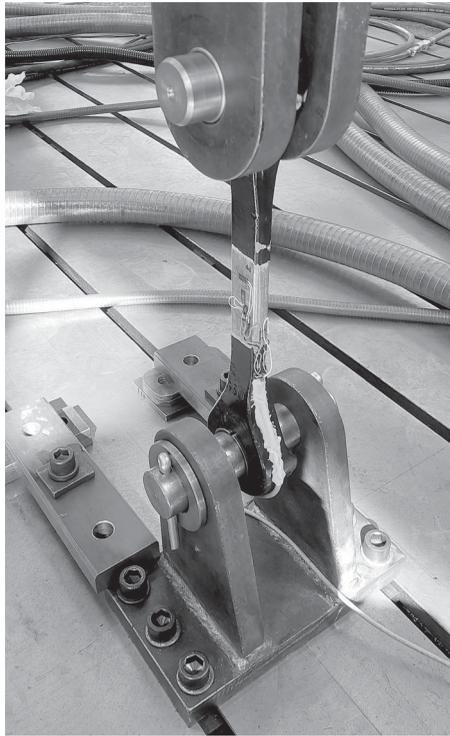
Figure 1. Instrumented link of the screw coupling during laboratory test.

full bridge connection was chosen as a sensing element. This solution enables maximal measurement sensitivity up to the highest forces, static and dynamic sensing, and full elimination of temperature effects during the measurement. In Figure 1, the testing and calibration of the instrumented screw coupling part before measurements in railway operation is presented. These tests were realised on the dynamic test stand of the Faculty of Transport Engineering of the University of Pardubice.

To be sure that the force between the locomotive and the set of wagons is transmitted through the instrumented screw coupling only, without any influence of buffer contact, the length sensors measuring the compression of individual buffers of the locomotive were used (these contact indicators can be seen in Figure 2). Even if the instrumented screw coupling was loosened during the train completion, reliable results were provided in curves with radii larger than ca. 700 m only. This fact represents one of the most important limitations of this method of running resistance investigation. Besides that, the measurement is possible only in the track sections where the locomotive exerts tractive effort (flat track and uphill; downhill only under the condition of a suitable combination of speed and track gradient).

The train speed record and the train localisation on the track were realised through an independent GNSS sensor on the locomotive, enabling synchronisation with the measured data. The measured speed was additionally compared with the speed record from the locomotive diagnostics, which was useful primarily in tunnel sections of relevant railway lines.

Figure 2. Connection of the locomotive and the first wagon by the instrumented screw coupling; the length sensors measuring compression of the locomotive buffers are also visible.

2.2. Evaluation of measurement results

During the measurements on different railway lines of the Czech railway network within a regular operation of freight trains, the following signals were recorded: tractive effort on the hook of the locomotive (signals from the instrumented screw coupling), instantaneous train speed and actual position (signals from the GNSS sensor) and compression of both locomotive buffers (signals from the length sensors). To express the vehicle running resistance of the investigated train at a particular moment, the *train equation of motion* (see [1]) was transformed in the form:

$$R_{\mathrm{v}} = F_{\mathrm{h}} - R_{\mathrm{t}} - M_{\mathrm{wag,eq}} a \tag{2}$$

where

- R_{v} [kN] is the vehicle resistance of the relevant set of wagons (without the locomotive);
- F_{h} [kN] is the measured tractive effort on the hook (in the instrumented screw coupling);
- R_{t} [kN] is the track resistance of the set of wagons;
- $M_{\mathrm{wag,eq}}$ [t] is the total inertial mass of the set of wagons (including the effects of relevant rotational masses);
- a [ms^{-2}] represents the instantaneous acceleration of the train (calculated numerically from the speed record).

The track resistance is calculated for the known position of the locomotive (i.e. the train front end) on the track respecting the train length and the train mass. For this purpose, track data (track gradient profile) provided by the infrastructure manager was used. An example of input records for evaluating the vehicle running resistance of a specific trainset is shown in Figure 3. This record shows a distinctive form of speed signal ('speed steps'). This train driving style was applied intentionally to reach a steady state of the train run at different speed values because of the minimisation of the inertial force effects of the trainset (the last term of Equation (2)) on the vehicle resistance evaluation. The vehicle running resistance was assessed only in straight-track sections for these purposes. The effect of curves on the recorded force acting in the screw coupling between the locomotive and the first wagon is also evident in Figure 3.

Concerning the proposal published in [4] (i.e. a definition of the vehicle resistance formula for freight trains as a combination of the specific expression of its constant component a and the absolute expression of the aerodynamic component C) as well as to the statistically evaluated measurement results, the updated general formula for the estimation of the vehicle running resistance of freight trains was proposed in the following form:

$$R_v = \left(a_1 + \frac{a_2}{M_{wag}/N_{wh}}\right) M_{wag} g + \tau (C_1 + C_2 l_{wag}) V^2 \tag{3}$$

where the individual quantities have the following meaning (and values):

- a_1 [N/kN] is a constant part of the specific expression of the constant component of the vehicle running resistance, representing especially the rolling resistance and the resistance in bearings; $a_1 = 0.67$ N/kN;
- a_2 [N t kN^{-1}] is a coefficient defining the hyperbolic term of the specific expression of the constant component of the vehicle running resistance, representing especially passive resistance (e.g. friction effects of imperfectly released brake blocks); $a_2 = 4$ N t kN^{-1}, i.e. a passive resistance value of approximately 40 N per wheelset;
- M_{wag} [t] is the total weight of the train (without the locomotive);
- N_{wh} [–] is the total number of wheelsets of the train (without the locomotive);
- M_{wag}/N_{wh} [t] represents the averaged axle load of the train (without the locomotive);
- τ [–] is the dimensionless *tunnel factor*, allowing to reflect an increased aerodynamic drag in tunnels (in the open landscape: $\tau = 1$);
- C_1 [N h^2 km^{-2}] is the constant (basic) part of the aerodynamic drag coefficient of the train; $C_1 = 0.38$ N h^2 km^{-2};
- C_2 [N h^2 m^{-1} km^{-2}] is the coefficient defining the linear (train length-dependent) term of the aerodynamic drag coefficient of the train; $C_2 = 0.0043$ N h^2 m^{-1} km^{-2};
- l_{wag} [m] is the total length of the train (without the locomotive);
- V [km/h] is the train speed.

It should be noted that coefficient a_1 is valid for trains with a prevailing share of non-metallic brake blocks and includes one standard deviation of the evaluated values to cover various uncertainties and deviations. However, in the case of the wagons equipped with cast iron brake blocks, a value of $a_1 = 0.95$ N/kN can be considered. From the point of view of the aerodynamic component of freight trains' vehicle running resistance, the coefficients C_1 and C_2 express the observed dependency of the relevant component on the train
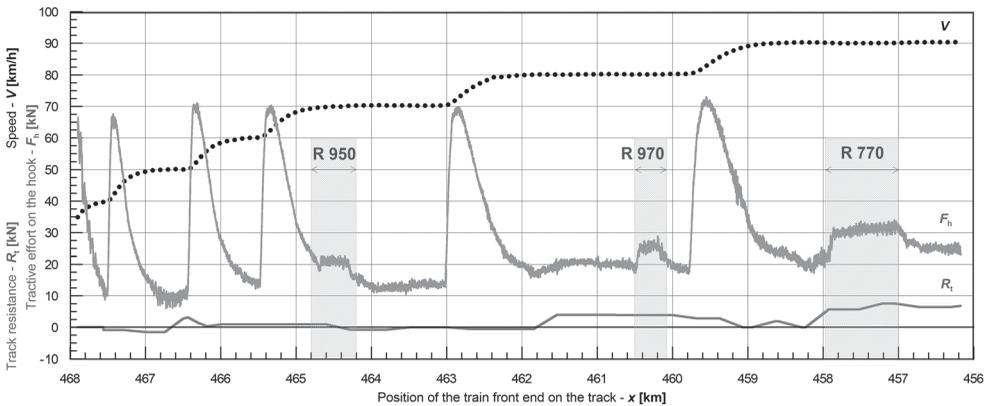
Figure 3. Example of input records of train speed, tractive effort on the hook, and calculated track resistance of an investigated train in a selected track section (curves are highlighted by the shaded bands); data for an investigated container train (16 *Sgns* type wagons, 64 wheelsets, 441.5 t, 315 m, 100% of non-metallic brake blocks).

length for the investigated trains having a total length of ca. 100 up to 630 m (without the locomotive). However, especially in the case of long container trains, the resulting aerodynamic drag coefficient can vary within a wider range depending on a specific train shape (in)homogeneity (influenced especially by the arrangement of containers on individual wagons).

If the proposed formula should be compared with some of the existing train resistance formulas, a similar concept (i.e. the combination of the specific expression of the constant component a and the absolute expression of the aerodynamic component C) can be found in principle in the German *Sauthoff's formula* (see, e.g. the overview in [4]) where:

- the constant component of $a = 1.0$ N/kN corresponds to the proposed formula if the average axle load of the wagons is 12 t. For higher axle loads (typical for loaded freight trains), the *Sauthoff's formula* provides higher (more conservative) resistance values;
- the aerodynamic component of $C = 0.0696\,(N_{wag} + 2.7)$ N h^2 km^{-2} is equivalent to the relevant term in Equation (3), but it works with number of wagons (N_{wag}) instead of their total length (l_{wag}). To provide the same aerodynamic drag value as the proposed formula, e.g. for a 600 m long train, the *Sauthoff's formula* has to calculate with 40 wagons. It means that the *Sauthoff's formula* considers relatively short wagons (up to ca. 15 m) and for longer wagons (typical for container trains) it would provide lower (underestimated) values than the proposed formula;
- a non-zero coefficient of the component with a linear speed dependency is considered. This term is neglected in the proposed formula in the form of Equation (3) but the attention to this term will be paid in Section 4 in relation to the effect of track quality.

3. Experience with estimation of curving resistance of freight trains

As already mentioned, for purposes of the general train resistance formula update, the experimental data were evaluated in straight-track sections only. However, the obtained
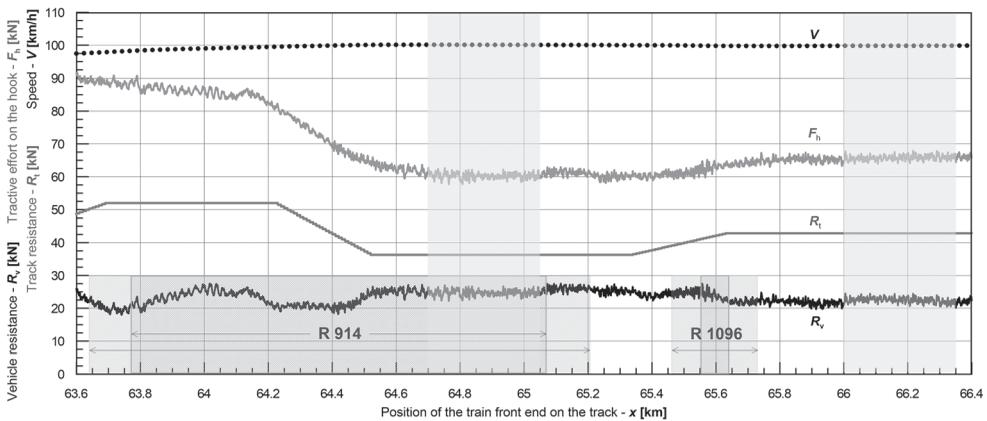
Figure 4. Evaluation of the running resistance of the investigated container train in selected sections (curve with a radius of 914 m and straight track) at the speed of 100 km/h on the basis of the measured tractive effort on the hook.

data can also be used to estimate the curving resistance of the investigated trains to a certain extent. To quantify the curving resistance of freight trains, two different types of tractive effort records were analysed within this work:

- data from the instrumented screw coupling;
- data from the on-board diagnostics (TELOC system) of an electric locomotive.

3.1. Analysis of measured data from instrumented screw coupling

An example of data evaluation is presented in Figure 4. The graph shows the signals of four basic quantities for the investigated container train (consisting of 16 four-axle 60-feet loaded *Sgns* type wagons with a total length of 315 m and a total mass of 441.5 t) in a selected 2.8 km long section of the modernised double-track railway line Beroun – Rokycany (Czech Republic). Analogically to Figure 3, the train speed V and the tractive effort on the hook F_h were measured (using the GNSS sensor and the instrumented screw coupling), and the track resistance R_t is calculated for the known train position, covering only the effect of track gradient. The track gradient in the selected section ranges from 8.37 to 12.00 ‰ (uphill). The vehicle running resistance R_v is calculated using Equation (2).

Because the curving resistance is not considered in Equation (2) as a separate component of the running resistance, the effect of the curves has to influence the calculated magnitude of the vehicle resistance R_v. In the bottom part of the graph in Figure 4, the position of curves in the selected track section is depicted:

- the curve with a radius of 914 m (and superelevation of 125 mm) begins at km 63.78 and ends at km 65.06 (or km 63.64 up to km 65.20 including transition curves);
- the curve with a radius of 1096 m (and superelevation of 85 mm) begins at km 65.56 and ends at km 65.63 (or km 65.46 up to km 65.73 including transition curves).

In this case, the data in two 350 m long subsections (km 64.70 up to km 65.05 and km 66.00 up to km 66.35 – see the grey bands in the graph in Figure 4) were used to quantify

Table 1. Calculation of running resistance of the investigated train in the selected track subsections.

Position of the train front end on the track	Curve radius R [m]	Mean value of train speed V [km/h]	Mean value of vehicle resistance R_v [kN]	Standard deviation of vehicle resistance $\sigma(R_v)$ [kN]
km 64.70–65.05	914	100.1	24.71	0.89
km 66.00–66.35	∞	99.8	22.46	0.82

the curving resistance. The first subsection is in the R914 curve; the second is in a straight track. To eliminate the influence of inertial force in Equation (2) and possible longitudinal train dynamics effects, both these subsections were chosen concerning the requirements on a constant speed of the train and constant track parameters (track gradient and curve radius) for the whole train length. The comparison of the results in these two subsections is presented in Table 1. If the mean values of vehicle running resistance in both observed subsections are compared, their difference (i.e. 2.25 kN) should quantify the curving resistance of the investigated set of wagons in the curve with a radius of 914 m.

For the purposes of quantification of the curving resistance in traction mechanics issues, its absolute value R_c [N] is usually expressed as:

$$R_c = G r_c \tag{4}$$

where

- G [kN] is the vehicle (train) weight, and
- r_c [N/kN] is the *specific curving resistance* which is commonly defined using an empirical formula, generally in the following form:

$$r_c = \frac{c_1}{R - c_2} \tag{5}$$

where

- R [m] is the curve radius, and
- c_1 [N m kN^{-1}] and c_2 [m] represent values of the relevant coefficients of the formula.

As mentioned in [2], this formula with the coefficient values of $c_1 = 650$ N m kN^{-1} and $c_2 = 55$ m is often used in the Central Europe region (Germany, Austria, Switzerland, Czech Republic, Slovakia, Hungary). It can be noted that this equation is also known as the *Röckl's formula*. It is based on the results of experiments (realised in Munich, Germany, in the 1870s) and was published in 1880 [7]. Other expressions of Equation (5) consider a zero value of the coefficient c_2 – e.g. the *Schmidt's formula* with $c_1 = 612$ N m kN^{-1} from 1927 [8]. In many different countries (USA, China, UK, Italy, Japan, etc.), modified versions of the *Schmidt's formula* are used with a 'country-specific' value of the coefficient c_1 ranging from 400 up to 700 N m kN^{-1} (see, e.g. the overview published in [2]).

The results for the investigated train in the curve with a radius of 914 m correspond to the specific curving resistance of 0.52 N/kN. For this curve radius, the *Röckl's formula* provides the value of 0.76 N/kN; in the case of the *Schmidt's formula*, it is 0.67 N/kN. Both these values are higher than the measured result. If the *Schmidt's formula* should be fitted

to the measurement result, the coefficient $c_1 = 475$ N m kN^{-1} would define the modified *Schmidt's formula*.

As mentioned in Section 2.1, the curving resistance in small-radius curves cannot be investigated in this way because of the principle of the measurement method used. The application of this method is limited by the buffer contact influencing the longitudinal force transmission between two neighbouring vehicles within the train. For a successful measurement, the force between the vehicles must be transmitted only through the (instrumented) screw coupling. Therefore, another method for estimating the specific curving resistance of trains has to be applied in tighter curves.

3.2. Analysis of TELOC data

This method, which was applied for estimating the specific curving resistance of container trains within the framework of the bachelor thesis [9], is based on evaluating the records of tractive effort, creating a part of the on-board diagnostics (TELOC) records on the TRAXX locomotives. In comparison with the method using the instrumented screw coupling (Section 3.1), the advantage of this method is that it can also be applied in small radius curves. The observed trains consisted of a locomotive type Bombardier TRAXX MS2e (Czech Class 386) and 21–24 six-axle articulated and four-axle container wagons equipped with Y 25 bogies. The total weight of these trains ranged ca. from 1000 up to 1700 t, and their total length was approximately 560 up to 650 m (including the locomotive). Besides the tractive effort record, the GNSS record (train speed and positioning on the track) was used for evaluation again. The curving resistance was investigated in the track section Brno – Blansko on the Czech part of the RFC7 corridor between Brno and Prague. Many curves with a very small radius of 250 up to 400 m occur in this particular track section.

The principle of data evaluation can be explained by using Figure 5, containing the results for a container train consisting of 10 four-axle and 13 six-axle wagons with a total weight of 1677 t and a total length of 624 m (both values including the locomotive Class 386), in a 7 km long part of the investigated track section. In the upper graph, 'measured tractive effort' (i.e. the TELOC record) is depicted and supplemented by 'calculated tractive effort'. The calculated signal is determined on the basis of the *train equation of motion*, i.e.:

$$F_w = R_t + R_v + M_{tr,eq}a, \qquad (6)$$

where:

- F_w [kN] is the calculated tractive effort on the wheels;
- R_t [kN] is the total track resistance of the train;
- R_v [kN] is the total vehicle resistance of the train;
- $M_{tr,eq}$ is the total inertial mass of the train (including the effects of rotational mass);
- a is the instantaneous train acceleration.

The calculation was performed under the following conditions:

- the curving resistance is neglected (the track resistance covers only the effect of track gradient respecting train length);
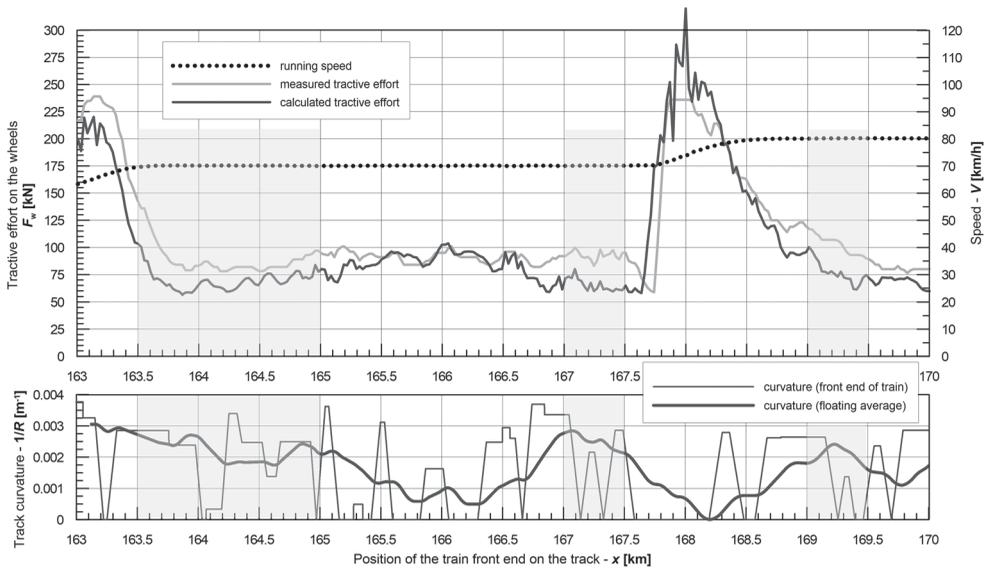
Figure 5. Comparison of the measured and calculated tractive effort for estimation of curving resistance of a container train (top) and the relevant track curvature – local and floating averaged over the train length (bottom).

- the vehicle running resistance of the whole train is characterised by the specific train resistance of the 'T4' type (see, e.g. [1,4]);
- the train acceleration is calculated numerically from the GNSS speed record (which can also be seen in the upper graph in Figure 5);
- effects of longitudinal train dynamics are neglected.

The bottom graph in Figure 5 presents the track curvature in the observed track section. The thin line represents the actual position of individual curves on the track. The thick line shows a 'floating-averaged track curvature over the train length'. The value of this quantity represents the averaged value of the track curvature for the whole train in a defined point of the track, where the train front end – i.e. the locomotive – occurs. Because the 'calculated tractive effort' neglects the effect of curving resistance, the 'measured' and the 'calculated' signal in the upper graph should be most different in such sections where the averaged track curvature reaches the highest values. If the train dynamics do not influence the force records, this difference should correspond to the curving resistance of the train:

$$R_c = F_{w,calc} - F_{w, meas}. \tag{7}$$

In the case of the presented results (see Figure 5), this assumption is confirmed in all three track subsections, where the running speed is constant and where significant values of the averaged curvature are reached (see the results in Table 2 – these subsections are highlighted by grey bands in the graphs in Figure 5). Simultaneously, the minimum differences between the 'measured' and the 'calculated' tractive effort in the subsection in km 165.5 up to km 166.4, where very low values of averaged track curvature are observed, testify the justifiability of the assumed type of the specific train resistance category ('T4').

Table 2. Estimation of the curving resistance of the investigated train and evaluation of relevant values of the coefficient c_1 of the specific curving resistance formula according to Equation (5) assuming $c_2 = 0$ m.

Position of the train front end on the track	$F_{w,calc} - F_{w,meas}$ [kN]	Mean value of track curvature	Equivalent curve radius	Coefficient c_1 [N m kN^{-1}]
km 163.5–165.0	19.7	0.00221/m	453 m	543
km 167.0–167.5	27.3	0.00252/m	397 m	660
km 169.0–169.5	25.5	0.00208/m	481 m	745

Based on the average value of the difference $F_{w,calc} - F_{w,\,meas}$ (corresponding to the average value of the curving resistance), the coefficient c_1 defining the modified *Schmidt's formula* (i.e. a formula in the form of Equation (5) considering $c_2 = 0$ m) can be estimated for each observed subsection (characterised by the averaged track curvature value $\bar{\rho}$) – see the values in Table 2. Then, the coefficient c_1 can be expressed as:

$$c_1 = \frac{R_c R}{M_{tr} g} \cong \frac{\overline{F_{w,calc} - F_{w,\,meas}}}{\bar{\rho} M_{tr} g} \tag{8}$$

where M_{tr} [t] is the total train mass and the overlined symbols represent the average values of relevant quantities.

Although the accuracy of the tractive effort signal obtained from the TELOC record can be considered questionable (it is not a measured force, but an estimation of the tractive effort on the wheels following from the mathematical model of the asynchronous traction drive) and the results show a relatively large dispersion (as it is evident from [9] as well as from the selected cases in Table 2), they provide approximately similar values of the coefficient c_1 as many commonly used empirical formulas. Higher values of the coefficient c_1 for the observed train (see the results in Table 2) can be connected with the fact that especially the four-axle container wagons (representing over 40% of the wagons in the trainset) are relatively long (*Sggnss* type wagons, 25.94 m over the buffers), which can lead to higher curving resistance (as a consequence of higher lateral components of the tractive forces acting on individual wagons in curves). Besides that, the actual curving resistance value is influenced by many operational parameters (the shape of wheel and rail profiles influencing the creepage and creep forces in wheel/rail contact, running speed influencing the cant deficiency value and, therefore, the positioning of bogies within the gauge clearance, etc.) as well as the coefficient of friction in the wheel/rail contact (and, therefore, also the weather or presence of tunnels in the relevant track section).

It should also be noted that in the case of this method, the calculated curving resistance values characterise the whole train (including the locomotive) because the calculation is based on the processing of records of the tractive effort on the wheels. In contrast, the method using the instrumented screw coupling (see Section 3.1) considers only the set of wagons. Therefore, the results of both these evaluation methods are not directly comparable if the absolute values of curving resistance are considered. A recalculation of the absolute values to the specific curving resistance is necessary.

4. Estimation of the effect of track quality on vehicle running resistance

Besides to utilisation of the measured data from the instrumented screw coupling for quantification of the curving resistance of a container train, the authors also tried to estimate
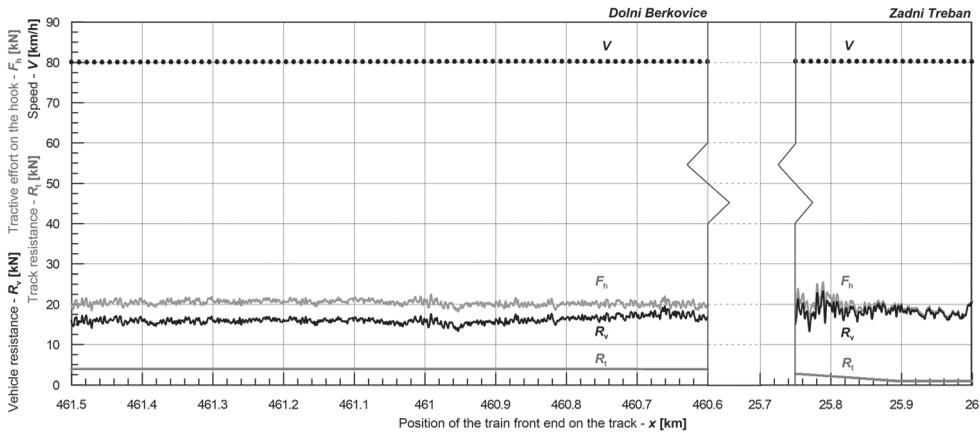
Figure 6. Evaluation of the running resistance of the investigated container train in selected sections of track with different quality of track geometry at the speed of approximately 80 km/h on the basis of measured tractive effort on the hook.

Table 3. Comparison of the statistical characteristics of the calculated vehicle running resistance of hauled vehicles of the investigated container train and the measured vertical and lateral acceleration on the locomotive bogie frame (above the right axle box of the fourth wheelset) at the running at the speed of 80 km/h in the two track section with different track geometry quality.

Track section		km 461.50–460.60	km 25.75–26.00
Type of track		Modernised	Old
Locality		Dolní Beřkovice	Zadní Třebaň
Train speed (mean value)		80.2 km/h	80.3 km/h
Vehicle running resistance	mean value	16.09 kN	18.26 kN
	std. deviation	0.82 kN	1.34 kN
Vertical acceleration on the locomotive bogie frame	minimum	$-3.23\,\mathrm{ms}^{-2}$	$-10.30\,\mathrm{ms}^{-2}$
	maximum	$2.80\,\mathrm{ms}^{-2}$	$10.61\,\mathrm{ms}^{-2}$
	RMS	$0.52\,\mathrm{ms}^{-2}$	$1.40\,\mathrm{ms}^{-2}$
Lateral acceleration on the locomotive bogie frame	minimum	$-0.63\,\mathrm{ms}^{-2}$	$-3.81\,\mathrm{ms}^{-2}$
	maximum	$0.73\,\mathrm{ms}^{-2}$	$4.26\,\mathrm{ms}^{-2}$
	RMS	$0.17\,\mathrm{ms}^{-2}$	$0.61\,\mathrm{ms}^{-2}$

the effect of track quality on the vehicle resistance. For these purposes, the measurement results in two track sections with different track quality were compared for the same container train, as investigated in Section 3.1. The observed quantities are the same as in the investigation of the curving resistance and the obtained results are presented in Figure 6. The track section in the locality Dolní Beřkovice is a part of a modernised railway line with a track speed of 160 km/h; the locality Zadní Třebáň is a part of an old railway line with a track speed of only 100 km/h.

In both observed cases, the train speed was ca. 80 km/h. The locomotive running gear was instrumented with acceleration sensors to get information about the track quality. Statistical characteristics of vertical and lateral acceleration measured on the frame of the rear bogie (above the right axle box of the fourth wheelset) calculated in both observed track sections are presented together with the mean value of speed and the evaluated values of the vehicle running resistance in Table 3. In this case, the train resistance on the old track is by 2.17 kN (i.e. by 0.50 N/kN in the specific form) higher than on the modernised railway line.

From the point of view of a potential implementation of the effect of track quality into the vehicle resistance formula, this effect could be assumed as the linearly speed-dependent component of the *Davis' formula* (see Equation (1)). This assumption corresponds to the fact that the physical base of this type of resistance lies in the energy dissipation related to the damping of the movement of vehicle parts, which is excited when running on track with irregularities and which is more significant with increasing speed. In the case of freight wagons, this energy dissipation is realised mainly by friction in *Lenoir*-type dampers, centre bowls, and side bearers, as well as through the contact between buffers. If the train is running on a high-quality track, this type of running resistance can be neglected, as in the case of the proposal of a general formula defined by Equation (3). Considering these assumptions, if the effect of poorer track quality should be implemented in Equation (3), this should be transformed into the following form:

$$R_v = \left(a_1 + \frac{a_2}{M_{\text{wag}}/N_{\text{wh}}} + bV \right) M_{\text{wag}}g + \tau (C_1 + C_2 l_{\text{wag}})V^2 \tag{9}$$

where the discussed effect can be quantified by the coefficient b [N h kN^{-1} km^{-1}]. In the case of the above-described comparison, the observed difference of the vehicle running resistance on the modernised and the old track at the train speed of 80 km/h corresponds to the value of this coefficient of $b = 0.006$ N h kN^{-1} km^{-1}. If this value should be compared, e.g. with the *Sauthoff's formula*, again, the relevant coefficient value ranges from $b = 0.0025$ N h kN^{-1} km^{-1} for passenger trains consisting of four-axle coaches to $b = 0.007$ N h kN^{-1} km^{-1} for two-axle vehicles. The measurement result is closer to the higher value (although the investigated train consisted completely of wagons with bogies), but it corresponds to the fact that the measurement was performed on the track with a poorer quality. Therefore, the higher value of this coefficient representing 'unsmooth running' of the train can be expected.

5. Conclusions

This paper summarises the latest experience of the Faculty of Transport Engineering of the University of Pardubice in the experimental research of running resistance of freight trains. Besides the update of a general formula of the vehicle running resistance based on the results of measurements performed within regular railway operation using an instrumented screw coupling (Section 2), attention is paid especially to the influence of selected aspects of the vehicle/track interaction on the running resistance – particularly to the quantification of the curving resistance (Section 3) and the estimation of the influence of track quality on the running resistance (Section 4). Based on the presented results, it is possible to state:

- the realised measurements confirmed that the constant component of the specific vehicle running resistance a can be considered as two-component, i.e. as a sum of a constant part and a hyperbolic term, indirectly dependent on the axle load;
- the constant part a_1 of the constant component of the specific vehicle running resistance of wagons braked by cast iron brake blocks shows approximately 40% higher value than in the case of wagons equipped with non-metallic brake blocks;

- the aerodynamic drag coefficient C of the trainset can be approximated by a linear dependency on the train length;
- although the actual value of the curving resistance of a specific train in a particular track section is influenced by many (to some extent random) parameters, the observed results in the form of the coefficient c_1 of the specific curving resistance, defining the modified *Schmidt's formula*, correspond to the range of typical values used in different 'country-specific' empirical formulas;
- for quantifying the influence of track quality on the vehicle running resistance, the measurement results for one train on two different tracks were evaluated. Based on these results, the general vehicle running resistance formula for freight trains is proposed in Equation (9). In this formula, the term bV represents right this type of resistance (caused mainly by the energy dissipation in suspension dampers) and is proposed as non-zero only on the old tracks. In a broader context, applying this approach can also contribute to assessing the benefits of modernisation projects of existing railway infrastructure;
- in principle, the new definition of the vehicle running resistance formula for freight trains (Equation (9)), based on the measurement results, allows an update of locomotive hauling capability calculations used by the infrastructure manager because it provides a more accurate estimation of the train resistance than the original specific vehicle resistance formulas. Increased train weight on particular railway lines is beneficial from the point of view of the efficiency of railway freight transport.

Acknowledgements

Disclosure statement

Funding

ORCID

Tomáš Michálek 🆔 http://orcid.org/0000-0001-7776-6627
Martin Kohout 🆔 http://orcid.org/0000-0001-9050-3194
Jiří Šlapák 🆔 http://orcid.org/0000-0001-9316-9320
Jakub Vágner 🆔 http://orcid.org/0000-0002-7609-6489
Jan Pulda 🆔 http://orcid.org/0009-0008-4272-9381

References

[1] Michálek T, Zelenka J. Traction Mechanics, 1st ed. Pardubice: University of Pardubice; 2024. ISBN 978-80-7560-503-0
[2] Wu Q, Wang B, Spiryagin M, et al. Curving resistance from wheel-rail interface. Veh Syst Dyn. 2022;60(3):1018–1036. doi:10.1080/00423114.2020.1843689

[3] Wu Q, Wang B, Cole C, et al. Implications of lateral coupler forces for rail vehicle curving resistance. J Comput Nonlinear Dyn. 2021;16(3):031002. doi:10.1115/1.4049530

[4] Michálek T. Modification of train resistance formulae for container trains based on operational run-down tests. Proc Inst Mech Eng F: J Rail Rapid Transit. 2018;232(6):1588–1597. doi:10.1177/0954409717738690

[5] Michálek T, Kohout M, Šlapák J, et al. Experimental measurement and simulation of running resistance of container trains. Proceedings of the 12th International Conference on Railway Bogies and Running Gears – BOGIE '22: 137–147. ISBN 978-963-9058-45-3.

[6] Rochard BP, Schmid F. A review of methods to measure and calculate train resistances. Proc Inst Mech Eng F: J Rail Rapid Transit. 2000;214:185–199. doi:10.1243/0954409001531306

[7] Röckl Av. Die Versuche der bayer. Staatseisenbahn über die Widerstände der Eisenbahnfahrzeuge bei ihrer Bewegung in den Gleisen. [in German]. Zeitschrift für Baukunde. 1880;3(4):541–562.

[8] Schmidt E C. *Freight train curve resistance on a one-degree curve and on a three-degree curve*, Bulletin No. 167. University of Illinois Engineering Experiment Station, 1927.

[9] Králík M. *Evaluation of running resistance of container trains*, Bachelor thesis. [in Czech] University of Pardubice, Faculty of Transport Engineering, 2018.