CPBS

Center for Pedestrian and Bicyclist Safety

A Tier-1 University Transportation Center

Multiple-vehicle Trajectory Planning Framework Considering Vulnerable Road Users

July 2024

A Report From the
Center for Pedestrian and Bicyclist Safety

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About the Center for Pedestrian and Bicyclist Safety (CPBS)

The Center for Pedestrian and Bicyclist Safety (CPBS) is a consortium of universities committed to eliminating pedestrian and bicyclist fatalities and injuries through cutting-edge research, workforce development, technology transfer, and education. Consortium members include: The University of New Mexico; San Diego State University; The University of California Berkeley; The University of Tennessee Knoxville; and The University of Wisconsin Milwaukee. More information can be found at: https://pedbikesafety.org

Disclaimer

Acknowledgments

This study was funded, partially or entirely, by a grant from the Center for Pedestrian and Bicyclist Safety (CPBS), supported by the U.S. Department of Transportation (USDOT) through the University Transportation Centers program. The authors would like to thank CPBS and the USDOT for their support of university-based research in transportation, and especially for the funding provided in support of this project. We would like to express our gratitude to the University of Wisconsin-Milwaukee Police Department for providing real-world surveillance camera data. Special thanks to Prof. Tian Zhao from the UWM Department of Computer Science, as well as his students Anisha Tasnim and Agam Damaraju, for their assistance in processing the video data and extracting vehicle trajectories.

TECHNICAL DOCUMENTATION

1. Project No.	2. Government Accession No.	3. Recipient's Catalog No.
23UWM07		

4. Title and Subtitle	5. Report Date
Multiple-vehicle Trajectory Planning Framework Considering Vulnerable Road Users	July 2024
	6. Performing Organization Code
	N/A
7. Author(s)	**8. Performing Organization Report No.**
Di Shen https://orcid.org/0009-0001-9611-2984 Xiaowei Shi https://orcid.org/0000-0002-7288-3186 Xiao Qin https://orcid.org/0000-0002-2299-1656	N/A
9. Performing Organization Name and Address	**10. Work Unit No. (TRAIS)**
Center for Pedestrian and Bicyclist Safety Centennial Engineering Center 3020 The University of New Mexico Albuquerque, NM 87131	**11. Contract or Grant No.** 69A3552348336
12. Sponsoring Agency Name and Address	**13. Type of Report and Period Covered**
United States of America Department of Transportation Office of Research, Development, and Technology (RD&T)	Final Report – June 2023 to May 2024
	14. Sponsoring Agency Code
	USDOT OST-R

15. Supplementary Notes

Report accessible via the CPBS website https://pedbikesafety.org and DOI https://doi.org/10.21949/2cdw-te21

16. Abstract

Connected and Automated Vehicles (CAVs) promise enhanced traffic safety through precise, sensor-based control, but their effectiveness depends on accurately predicting and responding to the interactions between CAVs and Vulnerable Road Users (VRUs). This project addresses three main technical questions: modeling the multi-agent decision-making process for CAVs, incorporate and anticipate VRUs' responses to CAVs in modeling, and solving these challenges in real-time. Utilizing game theory and a discrete-time Markov sequential game framework, the interactions between CAVs and VRUs are modeled to maximize utility while accounting for the dynamic nature of VRU behavior. To achieve real-time implementation, efficient heuristics algorithms are developed, combining customized dynamic programming and adaptive algorithms. Numerical simulations validate the proposed model, highlighting its effectiveness in managing interactions in a two-lane traffic scenario with a crosswalk. The findings underscore the potential of this approach to significantly improve the operational safety of CAVs in mixed traffic environments.

17. Key Words	18. Distribution Statement
Vulnerable road users; Autonomous vehicles; Connected vehicles; Vehicle design; Trajectory control; Traffic safety	No restrictions. This document is available through the National Technical Information Service, Springfield, VA 22161.

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price
Unclassified	Unclassified	28	

Form DOT F 1700.7 (8-72) **Reproduction of completed page authorized.**

SI* (MODERN METRIC) CONVERSION FACTORS

APPROXIMATE CONVERSIONS TO SI UNITS

Symbol	When You Know	Multiply By	To Find	Symbol
		LENGTH		
in	inches	25.4	millimeters	mm
ft	feet	0.305	meters	m
yd	yards	0.914	meters	m
mi	miles	1.61	kilometers	km
		AREA		
in^2	square inches	645.2	square millimeters	mm^2
ft^2	square feet	0.093	square meters	m^2
yd^2	square yard	0.836	square meters	m^2
ac	acres	0.405	hectares	ha
mi^2	square miles	2.59	square kilometers	km^2
		VOLUME		
fl oz	fluid ounces	29.57	milliliters	mL
gal	gallons	3.785	liters	L
ft^3	cubic feet	0.028	cubic meters	m^3
yd^3	cubic yards	0.765	cubic meters	m^3
		NOTE: volumes greater than 1000 L shall be shown in m^3		
		MASS		
oz	ounces	28.35	grams	g
lb	pounds	0.454	kilograms	kg
T	short tons (2000 lb)	0.907	megagrams (or "metric ton")	Mg (or "t")
		TEMPERATURE (exact degrees)		
^oF	Fahrenheit	5 (F-32)/9 or (F-32)/1.8	Celsius	^oC
		ILLUMINATION		
fc	foot-candles	10.76	lux	lx
fl	foot-Lamberts	3.426	candela/m^2	cd/m^2
		FORCE and PRESSURE or STRESS		
lbf	poundforce	4.45	newtons	N
lbf/in^2	poundforce per square inch	6.89	kilopascals	kPa

APPROXIMATE CONVERSIONS FROM SI UNITS

Symbol	When You Know	Multiply By	To Find	Symbol
		LENGTH		
mm	millimeters	0.039	inches	in
m	meters	3.28	feet	ft
m	meters	1.09	yards	yd
km	kilometers	0.621	miles	mi
		AREA		
mm^2	square millimeters	0.0016	square inches	in^2
m^2	square meters	10.764	square feet	ft^2
m^2	square meters	1.195	square yards	yd^2
ha	hectares	2.47	acres	ac
km^2	square kilometers	0.386	square miles	mi^2
		VOLUME		
mL	milliliters	0.034	fluid ounces	fl oz
L	liters	0.264	gallons	gal
m^3	cubic meters	35.314	cubic feet	ft^3
m^3	cubic meters	1.307	cubic yards	yd^3
		MASS		
g	grams	0.035	ounces	oz
kg	kilograms	2.202	pounds	lb
Mg (or "t")	megagrams (or "metric ton")	1.103	short tons (2000 lb)	T
		TEMPERATURE (exact degrees)		
^oC	Celsius	1.8C+32	Fahrenheit	^oF
		ILLUMINATION		
lx	lux	0.0929	foot-candles	fc
cd/m^2	candela/m^2	0.2919	foot-Lamberts	fl
		FORCE and PRESSURE or STRESS		
N	newtons	0.225	poundforce	lbf
kPa	kilopascals	0.145	poundforce per square inch	lbf/in^2

Multiple-vehicle Trajectory Planning Framework Considering Vulnerable Road Users

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Acronyms, Abbreviations, and Symbols

CAV	Connected Automated Vehicle
VRU	Vulnerable Road Users
CPBS	Center for Pedestrian and Bicyclist Safety
RL	Reinforcement Learning

Abstract

Connected and Automated Vehicles (CAVs) promise enhanced traffic safety through precise, sensor-based control, but their effectiveness depends on accurately predicting and responding to the interactions between CAVs and Vulnerable Road Users (VRUs). This project addresses three main technical questions: modeling the multi-agent decision-making process for CAVs, incorporate and anticipate VRUs' responses to CAVs in modeling, and solving these challenges in real-time. Utilizing game theory and a discrete-time Markov sequential game framework, the interactions between CAVs and VRUs are modeled to maximize utility while accounting for the dynamic nature of VRU behavior. To achieve real-time implementation, efficient heuristics algorithms are developed, combining customized dynamic programming and adaptive algorithms. Numerical simulations validate the proposed model, highlighting its effectiveness in managing interactions in a two-lane traffic scenario with a crosswalk. The findings underscore the potential of this approach to significantly improve the operational safety of CAVs in mixed traffic environments.

Chapter 1. Introduction

In recent years, the explosive increase in the number of vehicles and conflicts between vehicles and Vulnerable Road Users (VRUs) on the roads have triggered a series of traffic problems. To create a safe and efficient transportation environment, it is essential to consider both vehicles and VRUs. VRUs typically include pedestrians and cyclists, who are considered vulnerable groups. According to the World Health Organization, approximately 1.3 million individuals die annually from road traffic crashes, with pedestrians and cyclists alone accounting for 24% of these deaths. Human-driven vehicles are a primary factor in these fatalities (Kevin A. Harkin et al., 2024).

The advent of Connected and Automated Vehicles (CAVs) holds significant potential to reduce traffic accidents, enhance the efficiency of transportation systems, and improve quality of life through advanced sensor-based controls and real-time decision-making capabilities. However, the traffic environment is very complex, and CAVs need to plan paths in real-time to avoid collisions with dynamic and static obstacles, especially with highly uncertain VRUs who have their own motion properties (Y Li et al., 2020, G F Li et al., 2020). Integrating VRUs into the CAV operational framework presents a complex challenge due to their unpredictable behaviors. This report delves into the intricacies of multiple-vehicle trajectory planning in environments with VRUs, focusing on a two-lane traffic scenario with a crosswalk.

Chapter 2. Literature Review

Researchers have used various methods to address conflicts between CAVs and VRUs. Some studies have utilized deep reinforcement learning (RL) (Shai Shalev-Shwartz et al., 2016, Alex Kuefler et al., 2017, Y Zhang et al., 2018). One common characteristic of RL approaches is their policy-oriented nature. Offline training is often done through exploration/exploitation, a strategy that is usually very slow. The heavy computational burden makes it difficult to tackle inverse problems and mechanism design problems under this framework. Moreover, since RL embodies specific information from the training context, the trained optimal policy may struggle to generalize to new contexts.

Chae et al. (2017) successfully applied a new deep RL strategy in traffic systems to prevent collisions using a pedestrian crash avoidance mitigation (PCAM) system. However, the CAV agent in this study was limited to braking actions, neglecting other factors such as controlled acceleration. Many researchers have attempted to propose new deep RL-based systems, but these often fail to account for random pedestrian behavior (Deshpande et al., 2020, Papini et al., 2021). Deshpande et al. (2021) introduced multi-objective deep RL for interactions between CAVs and pedestrians, but this focused more on CAV navigation than pedestrian crossing decisions. In this line of research, surrounding agents are often treated as passive backgrounds, and their strategic behavior and coordination are not explicitly endogenized.

Typically, interactions between CAVs and VRUs can be simplified into three parts: (a) a pedestrian waits on the sidewalk for the right moment to initiate a crossing; (b) the pedestrian follows a certain trajectory based on the characteristics of the approaching vehicle and environmental conditions; (c) the approaching vehicle anticipates pedestrian behavior and reacts accordingly to ensure a safe and comfortable interaction for both the pedestrian and passengers (Arash et al., 2022). Many studies choose to have CAVs stop when crossing pedestrians are recognized, leading to inefficient traffic flow. Path planning for intelligent vehicles often focuses on avoiding static obstacles, with dynamic obstacle avoidance, especially for crossing pedestrians, being less explored (J K Wang et al., 2020, H Y Guo et al., 2020). Therefore, it is crucial to adopt a strategy that considers interactions between CAVs and VRUs.

Game theory is an effective strategy for modeling such traffic behaviors. It addresses issues of interaction and coordination among smart vehicles, smart infrastructure, and other players (VRUs). The theory can solve three classes of problems: (a) endogenizing decision-making using optimization given preferences and game rules, allowing for various CAV behaviors such as coasting, distance keeping, yielding, merging, lane-changing, passing, entering, and exiting, while also considering VRU behaviors (Cunningham et al., 2015; Galceran et al., 2017; Mehta et al., 2018); (b) deriving and quantifying human driving behavioral models given observed action sequences in real traffic situations, known as inverse problems, combined with machine learning (Payam Naerejad et al., 2022; Hsu et al., 2018); and (c) optimizing traffic rules to benefit society. Rong Rui et al. (2022) proposed an evolutionary game model considering the benefits of AVs, pedestrians, and traffic managers, exploring the impact of policy intervention. The game analysis results indicate that strategies of the three agents are closely associated with pedestrians' trust in driverless technology, human-vehicle communication, and government regulation. In regulated scenarios, increasing AV supervision threats can facilitate equilibrium towards pedestrians crossing and AVs yielding.

Additionally, some studies based on game theory combined with Nash Equilibrium have been developed. Musbbir et al. (2023) and Kunming Li et al. (2023) created frameworks for CAVs to interact safely with pedestrians using a game-theoretic approach, building on stochastic games to find Nash Equilibrium. However, Nash Equilibrium is not perfect, and dynamic programming in this nonlinear setting is challenging to implement in real-time. To better model human driving behaviors with bounded rationality, we aim to simplify the solution concept by replacing the Nash equilibrium condition with a heuristic and adaptive optimization using finite look-ahead anticipation. This new algorithm significantly improves computational efficiency.

In conclusion, the primary objective of this research is to develop a robust trajectory planning framework that accounts for the interactions between Connected and Automated Vehicles (CAVs) and Vulnerable Road Users (VRUs). The framework is based on a multi-agent decision-making model using game theory and a discrete-time Markov sequential game. This theoretical foundation enables the modeling of interactions in a way that maximizes utility for each agent, whether a vehicle or a pedestrian, while maintaining safety and efficiency.

One of the key innovations in this study is the game theory-based adaptive algorithm that allows CAVs to dynamically adjust their trajectories in response to the behaviors of VRUs. By considering VRUs' responses to CAV actions, the algorithm ensures that the planned trajectories are not only safe but also adaptable to varying traffic densities and patterns. This adaptability is crucial for real-world implementations, where traffic conditions can change rapidly. The research methodology involves numerical simulations to validate the proposed framework. Two specific traffic scenarios are examined: one with higher vehicle density and another with higher VRU density. These simulations demonstrate how traffic patterns evolve under different conditions, such as whether pedestrians wait for vehicles to pass, or vehicles stop at crosswalks to allow pedestrians to cross. The results illustrate the effectiveness of the multi-agent modeling approach in improving traffic efficiency and safety.

This study is structured as follows: Section 3 elaborates on the methodology, Section 4 presents numerical experiments to illustrate the validity of the established models, and Section 5 concludes the study.

Chapter 3. Methodology

3.1 Notation

To facilitate the understanding this paper, a summary of the notation has been compiled and presented in Table 1.

Table 1. Notation List

Parameters	Description
t	time step in simulation.
T	total time steps of simulation.
τ	time step in path planning.
h	time horizon of path planning for action optimization: h=8.
$\mathcal{T}$	Studied time horizon in a discrete time interval, $t \in \{1,2,\cdots,\mathbb{T}\}$.
j, j'	agent index.
s_t	list of agent states at time t.
s_t^{in}	list of incoming agent states at time t.
s_t'	list of agent states at time t in path planning.
n	number of agents for given states list.
$a_{t,j}$	the action of agent j at time t.
a_t	list of actions for all agents at time t.
a_t^*	list of optimal actions for all agents at time t.
ϕ_1	speed reward of one car.
ϕ_2	collision penalty between a pair of cars and is non-positive by definition.
ϕ_3	collision penalty (to car) between a car and a pedestrian and is non-positive by definition.
ϕ_4	deviation from middle-of-lane penalty for a car and is non-positive by definition.
ϕ_5	collision penalty (to pedestrian) between a pedestrian and a car and is non-positive by definition.
θ_1	shape parameters of speed reward.
θ_2	shape parameters of car-car collision penalty.
θ_3	shape parameters of car-pedestrian collision shape parameters of car-car collision penalty.
θ_4	shape parameters of mid-lane deviation penalty.
θ_5	shape parameters of pedestrian-car collision penalty.

Parameters	Description
$\phi_2^{(0)}$	threshold collision penalty for safe entry of incoming vehicles.
$\delta^{(0)}$	threshold value for determining an exit path.
ω	weight parameter of each reward and penalty term (a vector: $\omega = [1, \omega\_1, \omega\_2, ...]$).
f	state evolution using Newtonian mechanics: $s\_\{t+1\} = f(s\_t, a\_t)$.
f_j	state evolution from agent j's forecast perspective: $s_{t+1} = f_j\left(s_t, a_t \middle\| \text{predicted\_path}_{j' \neq j}\right)$.
$U_{t,j}$	utility function of agent j within the planning horizon starting at t.

3.2 Problem Statement

The existence of Connected and Automated Vehicles (CAVs) is expected to significantly reduce the number of crashes that occur in our daily lives. However, achieving this level of safety presents a critical challenge: the real-time planning of CAV trajectories while considering the presence of Vulnerable Road Users (VRUs) in the environment. While avoiding collisions is often feasible, it frequently results in autonomous vehicles getting stuck to ensure safety, thereby sacrificing traffic efficiency. To maintain both safety and efficiency, we must integrate VRUs into the planning framework, despite the complexity this adds. The unpredictable behavior of VRUs introduces a level of uncertainty that must be accounted for in the planning process. Additionally, vehicle trajectory planning is inherently time sensitive. The planned trajectories must be generated in real-time to effectively prevent crashes and ensure smooth traffic flow. Any delay in response or miscalculation could result in collisions, underscoring the importance of efficiency and precision in the planning algorithms. Addressing these challenges requires sophisticated algorithms capable of dynamic adaptation to real-time data, ensuring that CAVs can navigate safely and efficiently even in the presence of unpredictable VRUs.

To tackle this issue, we consider a two-lane traffic scenario featuring a crosswalk, as illustrated in Figure 1. This setup allows us to simulate real-world conditions where CAVs must navigate while accommodating the unpredictable behaviors of pedestrians. Our analysis includes two types of agents: vehicles and pedestrians. For simplicity, we assume there are no collisions among pedestrians. The colors in Figure 1 encode the velocity of each agent, with rectangles representing vehicles and polygons representing pedestrians.

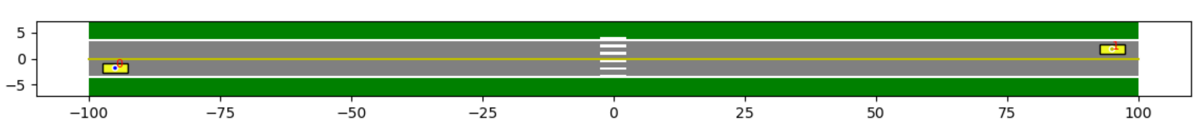

Figure 1. Problem illustration

Beyond achieving a collision-free environment as the primary objective, we aim to observe the dynamic responses of CAVs in various traffic scenarios, such as differing traffic flux, to test the proposed algorithms' ability to handle complex interactions and achieve traffic efficiency. This involves assessing the adaptive trajectory planning algorithms under different conditions to ensure that they can manage both vehicle and pedestrian movements effectively. Based on our experiments, we have identified two scenarios to test and illustrate the algorithm framework.

In the first setting (Figure 2), we will simulate a scenario with a high density of vehicles approaching a crosswalk. The goal is to observe how the adaptive trajectory planning algorithm handles the initial flow of vehicles while ensuring the safety of pedestrians waiting to cross. As the simulation progresses, if pedestrians have responded to AVs, then we expect the algorithm must be able to predict the intents and pass by with safety distance to maintain the traffic flow efficiency. Meanwhile CAVs should still guarantee collision avoidance and the capability of waiting at crosswalks.

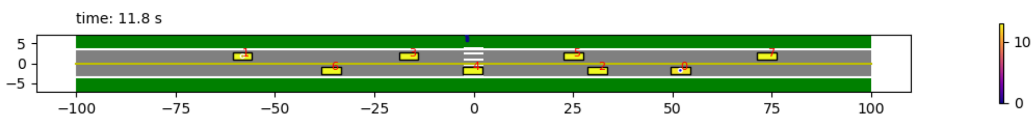

Figure 2. A high density of vehicles approaching a crosswalk

The second setting (Figure 3) focuses on a scenario with a higher density of pedestrians crossing the road. Here, the objective is to test the responsiveness of the vehicles' adaptive planning in the face of a constant flow of crossing activity created by groups of pedestrians. This setup challenges the algorithm to manage the vehicles' speed and stopping behavior to allow pedestrian groups to cross safely, thereby avoiding collisions and maintaining smooth traffic flow despite the high density of VRUs.

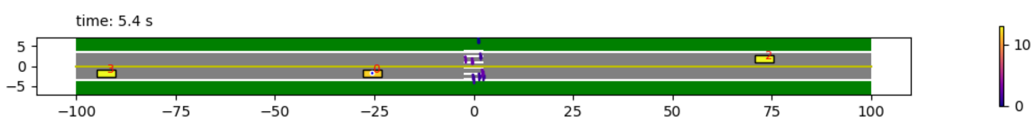

Figure 3. A higher density of pedestrians crossing the road

Through these experiments, we aim to illustrate the robustness and adaptability of the proposed algorithms in achieving both safety and efficiency in a complex, real-world traffic environment.

3.3 Simulation Pseudo Code

The state of each vehicle or pedestrian is defined by six variables: $[x, y, v, \alpha, length, width]$. Here, x, y represents the Cartesian coordinates of the agent's location. The velocity, v, is described by its magnitude, while α denotes the direction. The dimensions of each agent are quantified by its length and width.

From the ego agent's perspective, the predicted state evolution within the planning horizon follows these principles:

1. Each agent knows its intended destination but lacks knowledge of other agents' destinations.
2. The ego agent extrapolates its location and velocity by assuming constant action.
3. Other agents' locations and velocities are extrapolated by assuming zero action.

The simulation process begins by setting up the simulation environment. This involves defining the layout of the roads, crosswalks, and any relevant traffic rules. Once the environment is ready, the initial positions and states of all vehicles and pedestrians are generated randomly following traffic rules. The simulation runs through a series of time steps, which is discretized into steps of 0.2 seconds at the level of maneuver planning, ensuring a detailed temporal resolution for our simulations. During each time step, the simulation will perform:

1. Gather Current States: The algorithm retrieves the current positions and states of all vehicles and pedestrians.
2. Handle New Agents: It random generates and checks for any new vehicles or pedestrians entering the simulation.
3. Determine Optimal Actions: This is the main contribution of this work to provide a motion planning algorithm then calculates the best possible action for each vehicle. This involves finding the optimal speed and direction that maximizes a utility function, which takes into account the vehicle’s current state and its goals.
4. Update Agent States: Based on the calculated actions, the algorithm updates the positions and states of all agents. This step simulates the movement and interactions of the vehicles and pedestrians.
5. Check Boundaries: Finally, the algorithm checks if any agents have moved out of the defined simulation area. If they have, these agents are removed from the list of current agents to keep the simulation focused on the relevant interactions.

```
initialize the simulation environment
initialize the states for all agents s_0
for t = 0 : T − 1, do
    s_t ← get_current_agent_states()
    if s_t^in is not empty:
        for s_{t,j}^in in s_t^in, do
            if s_{t,j}^in is pedestrian: s_t.append(s_{t,j}^in)
            if s_{t,j}^in is vehicle:
                φ_2^min = min(φ_2(s_{t,j}^in, {s_t, s_{t,j'≠j}^in}))
                if φ_2^(0) < φ_2^min < 0: s_t.append(s_{t,j}^in)
        end for

    # find optimal action for each vehicle
    a_t^* ← empty_list()
    for s_{t,j} in s_t, do        # this loop is done in parallel
        a_{t,j}^* ← argmax_{a_{t,j}} U_{t,j}(a_{t,j}|s_t, h)     #(Optimization, details in next page)
        a_t^*.append(a_{t,j}^*)
    end for
    s_{t+1} = f(s_t, a_t^*)
    # check if agent is out of bound/scope
    for s_{t+1,j} in s_{t+1}, do
        if s_{t+1,j} is not in scope:
            s_{t+1}.remove(s_{t+1,j})
    end for
end for
```

3.4 Optimization Pseudo Code

The aim of the optimization algorithm is to find the best action for each vehicle j at a given time t, maximizing its utility function over a specified planning horizon h. The algorithm begins by defining the possible actions a vehicle can take, such as acceleration and changes in steering angle. It then initializes the vehicle's state, which includes its position, velocity, and other relevant parameters. For each possible action, the algorithm sets this action for the vehicle being optimized while keeping the actions of other vehicles at zero. This approach isolates the impact of the chosen action on the vehicle's performance. Next, the algorithm simulates the vehicle's trajectory over the planning horizon. At each step in the horizon, it updates the vehicle's state based on the chosen action and calculates various rewards and penalties. The utility(objective) function for the vehicle is defined as the weighted sum of these penalties, considering the worst-case scenario at each step in the horizon. The algorithm then selects the action that maximizes this utility function.

For Vehicle j at time t

action = (acceleration, change of steering angle)

$s_0' = s_t; \quad s_{0,j}' = s_{t,j}$

for a^{temp} in action_list, **do**:

$a_j^{\text{temp}} = a^{\text{temp}}$

$a_i^{temp} \underset{i \neq j}{=} (0, 0)$

$a^{\text{temp}} = [a_0^{\text{temp}}, a_1^{\text{temp}}, ..., a_{n-1}^{\text{temp}}]$

for τ in 0:h-1, **do**:

$s_{\tau+1}' = f(s_\tau', a^{\text{temp}})$

if $\tau == 0$: $\phi_{1,(t+1,j)} \leftarrow \phi_1(s_{1,j}'|\theta_1)$

$\phi_{2,(t+\tau+1,j)}^{\min} \leftarrow \min_{j'}(\phi_2(s_{\tau+1,j}', s_{\tau+1,j' \neq j}', |\theta_2))$ # largest collision penalty btw car j and other cars

$\phi_{3,(t+\tau+1,j)}^{\min} \leftarrow \min_{j'}(\phi_3(s_{\tau+1,j}', s_{\tau+1,j' \neq j}', |\theta_3))$ # largest collision penalty btw car j and pedestrians

$\phi_{4,(t+\tau+1,j)}^{\min} \leftarrow \min_{j'}(\phi_4(s_{\tau+1,j}', s_{\tau+1,j' \neq j}', |\theta_4))$ # largest middle-of-lane penalty for car j

end for

The utility function is defined as the worst case of all penalties

$U_{t,j}(a^{\text{temp}}) = \omega_1 \phi_1(s_{t+1,j}') + \underset{\tau}{\text{argmin}}(\ \omega_2 \phi_{2,(t+\tau,j)}^{\min} + \omega_3 \phi_{3,(t+\tau,j)}^{\min} + \omega_4 \phi_{4,(t+\tau,j)}^{\min}\ )$

end for

$a_{t,j} = \underset{a^{\text{temp}}}{\text{argmax}}\ U_{t,j}(a^{\text{temp}})$

Similar optimization problem is performed to drive the movements of each pedestrian with gaussian distributed random noise added. The only difference is the utility(objective) function is formulated differently for pedestrians and is simpler than vehicles', where we just want to ensure the pedestrian have reasonable maneuvers in simulations and create challenging test cases for CAVs.

For Pedestrian j at time t
action = (instant speed, direction)
$s_0^{'} = s_t$; $s_{0,j}^{'} = s_{t,j}$
for a^{temp} in action_list, **do:**
  $a_j^{\text{temp}} = a^{\text{temp}}$
  $a_i^{temp} \underset{i \neq j}{=} (0, 0)$
  $a^{\text{temp}} = [a_0^{\text{temp}}, a_1^{\text{temp}}, ..., a_{n-1}^{\text{temp}}]$
  for τ in 0:h-1, **do:**
    $s_{\tau+1}^{'} = f(s_{t+\tau}^{'}, a^{\text{temp}})$
    if $\tau == 0$: $\phi_{1,(t+1,j)} \leftarrow \phi_1(s'_{1,j}|\theta_1)$
    $\phi_{5,(t+\tau+1,j)}^{\text{min}} \leftarrow \min_{j'}(\phi_5(s'_{\tau+1,j}, s'_{\tau+1,j' \neq j}, |\theta_5))$ # largest collision penalty btw pedestrian j and cars
  end for
  $U_{t,j}(a^{\text{temp}}) = \omega_1 \phi_1(s'_{t+1,j}) + \underset{\tau}{\text{argmin}}(\ \omega_5 \phi_{5,(t+\tau,j)}^{\text{min}}\ )$
end for
$a_{t,j} = \underset{a^{\text{temp}}}{\text{argmax}}\ U_{t,j}(a^{\text{temp}})$

Below are mathematical forms of penalties or reward used in the optimization problem, parameters are manually tuned to show pattern differences and ensure desired behaviors in simulations qualitatively. Quantitative calibration for these parameters is proposed in future work as a second stage of this project.

Speed Reward $\boldsymbol{\phi_1}$
This term encourages agents to maintain a desired speed.

$$\phi_1 = \theta_1 (v_j - v_{\text{desired}})^2$$

where:
- $v_{\text{desired}} = 13\ m/s\ for\ car, and\ 2m/s\ for\ pedestrian$
- $\theta_1 = 0.2$

Collision Penalty Between Cars $\boldsymbol{\phi_2}$
This term penalizes collisions between cars.

$$\phi_2 = \theta_2 \max_j \left[\exp\left(-\frac{\left(x_{ij} - \frac{l_i}{2}\right)^2}{2\sigma_x^2} \right) \cdot \exp\left(-\frac{\left(y_{ij} - \frac{w_i}{2}\right)^2}{2\sigma_y^2} \right) \right]$$

where:
- x_{ij} and $y_{ij}$are the relative positions between car_i and car_j
- σ_x and σ_y are the safe distances in the x and y directions, respectively following formula
- $\sigma_i = \frac{\text{safe\_d\_i}}{\sqrt{2 \ln \frac{1}{\text{safe\_ratio}}}}$, safe_ratio=0.05

- $safe\_dx\_i = \left(\frac{v_{\mathrm{i}}}{v_{\mathrm{desired}}}\right)^3 \cdot 15 + 1; safe\_dy\_i = \left(\frac{v_{\mathrm{i}}}{v_{\mathrm{desired}}}\right)^3 \cdot 0.6 + 0.15$
- $\theta_2 = 2$

Collision Penalty Between Car and Pedestrian $\boldsymbol{\phi_3}$
This term penalizes collisions between cars and pedestrians.

$$\phi_3 = \theta_3 \max_j (v+1) \cdot \sigma(|x_{ij}| - a_x, -b_x) \cdot \sigma(|y_{ij}| - a_y, -b_y)$$

where:

-
- $a_x = \frac{l + \mathrm{safe\_d\_x}}{2},\ b_x = \frac{9.2}{\mathrm{safe\_d\_x}}$
- $a_y = \frac{w + \mathrm{safe\_d\_y}}{2},\ b_y = \frac{9.2}{\mathrm{safe\_d\_y}}$
- safe_d_x=10, safe_d_y=2, $\theta_3 = 6$

Mid-Lane Deviation Penalty $\boldsymbol{\phi_4}$
This term penalizes deviation from the center of the lane for cars.

$$\phi_4 = \theta_4\, Dist(car_i\,, lane\ center), \theta_4 = 0.05$$

Collision Penalty for Pedestrians $\boldsymbol{\phi_5}$
This term penalizes pedestrians for collisions with cars. The functional form maintains the same as ϕ_3, with different safety distance and parameters θ_5. safe_d_x=7, safe_d_y=1, $\theta_5 = 10$

Chapter 4. Numerical Experiments

This section presents the results of numerical experiments designed to validate the proposed trajectory planning framework for Connected and Automated Vehicles (CAVs) in environments with Vulnerable Road Users (VRUs). The experiments focus on two primary scenarios: one with a higher density of vehicles and another with a higher density of VRUs. These scenarios demonstrate the effectiveness of the game theory-based adaptive algorithm in managing interactions between vehicles and VRUs, ensuring safety and efficiency in mixed traffic conditions.

The numerical experiments were conducted in a simulation environment with kinematical models and sufficient randomness. At this level of planning, high-fidelity vehicle models in simulation are not necessary for replicating urban traffic scenarios and patterns. The environment features a two-lane traffic system with a crosswalk, where the interactions between vehicles and pedestrians are closely monitored.

Before delving into the details, it’s important to note that the functional form, environment geometry, and parameters used in the two experiments are identical. However, the traffic patterns differ significantly, influenced by varying levels of traffic flux, resulting in distinct outcomes.

4.1 Experiment with larger vehicle density

In this experiment, we simulate a scenario with a higher density of vehicles approaching a crosswalk. The key observations and results are as follows:

1. **Initial Conditions:** The simulation begins with a high number of vehicles approaching the crosswalk. Pedestrians are initially waiting at the crosswalk.
2. **Pedestrian Behavior:** Pedestrians wait for the vehicles to pass before starting to cross. This behavior minimizes the risk of collisions at the expense of pedestrian waiting time.
3. **Vehicle Behavior:** As pedestrians begin to cross after a few vehicles have passed, the subsequent vehicles (e.g., car 6) decelerate to allow pedestrians to cross safely. This deceleration is a direct result of the adaptive trajectory planning algorithm.
4. **Safety and Efficiency:** The simulation results highlight that the adaptive algorithm effectively balances safety and efficiency. Vehicles slow down only when necessary, and pedestrians can cross without excessive delay.

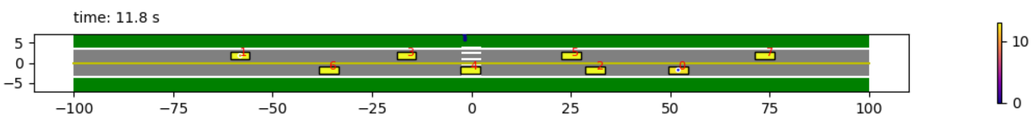

Figure 4. (11.8s) The simulation shows a higher vehicle density compared to pedestrian density. A single pedestrian is waiting on the side of the crosswalk, preparing to cross the road.

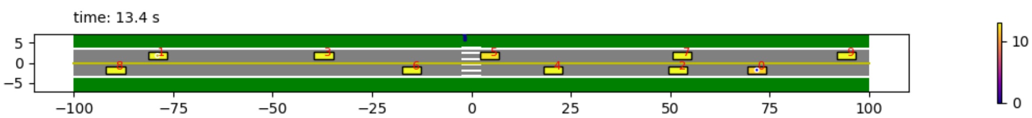

Figure 5. (13.4s) Vehicle 5 passes by without decelerating because the pedestrian shows no intent to cross and remains safely on the pavement.

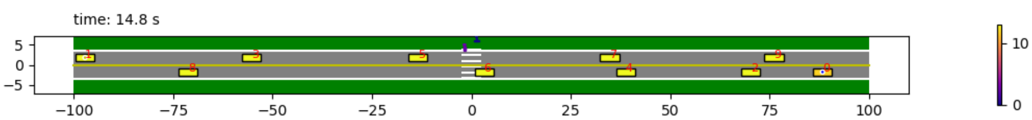

Figure 6. (14.8s) Once Vehicle 5 has passed, the pedestrian begins to cross, anticipating a large enough gap created by the approaching Vehicles 6 and 8 from the opposite direction.

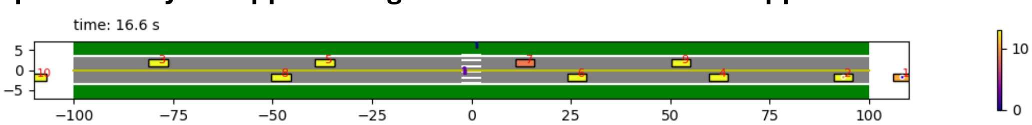

Figure 7. (16.6s) As the pedestrian is crossing, Vehicle 7 noticeably slows down to ensure the pedestrian's safety.

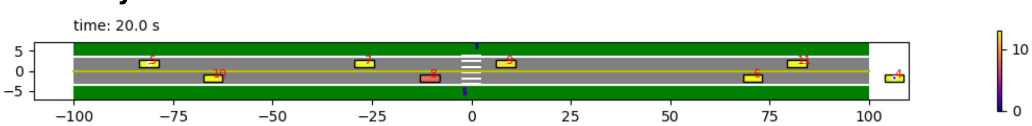

Figure 8. (20.0s) Although the pedestrian has mostly crossed the road, they are still close to the traffic. Vehicle 8 slows down to further ensure safety.

4.2 Experiment with larger VRU density

This experiment examines a scenario with a higher density of pedestrians (VRUs) crossing the road. The key observations and results are:

1. **Initial Conditions:** The simulation starts with a large number of pedestrians waiting to cross at the crosswalk, with a moderate flow of vehicles approaching.
2. **Pedestrian Behavior:** Pedestrians begin to cross in groups, creating a constant flow of crossing activity. This situation tests the responsiveness of the vehicles' adaptive planning.
3. **Vehicle Behavior:** Vehicles exhibit adaptive behaviors such as slowing down and stopping to allow groups of pedestrians to cross (e.g., car 0). The algorithm ensures vehicles yield appropriately, avoiding collisions.
4. **Traffic Flow:** Despite the high density of VRUs, the adaptive algorithm manages to maintain a smooth flow of traffic. Vehicles resume their paths promptly after pedestrians cross, minimizing overall traffic disruption.

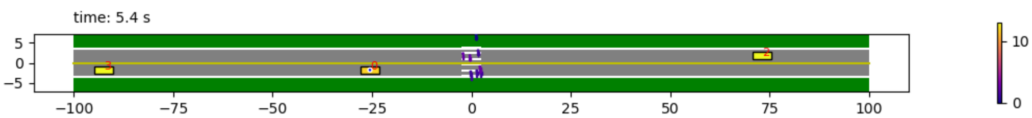

Figure 9. (5.4s) The simulation depicts a higher pedestrian density compared to vehicle density.

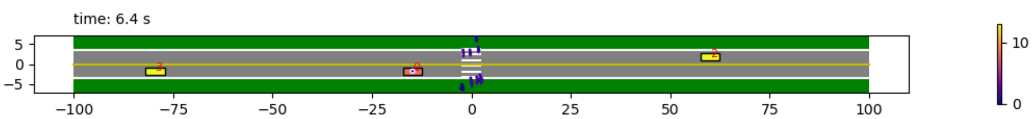

Figure 10. (6.4s) Vehicle 0 begins to brake as it approaches the crosswalk, observing the presence of pedestrians.

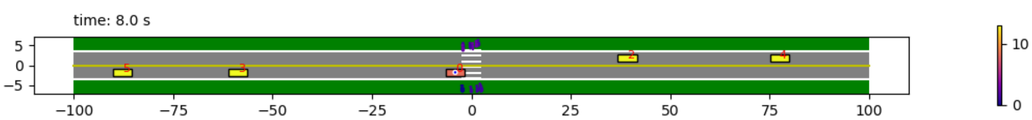

Figure 11. (8.0s) The crosswalk is cleared of pedestrians, allowing Vehicle 0 to pass by slowly and safely.

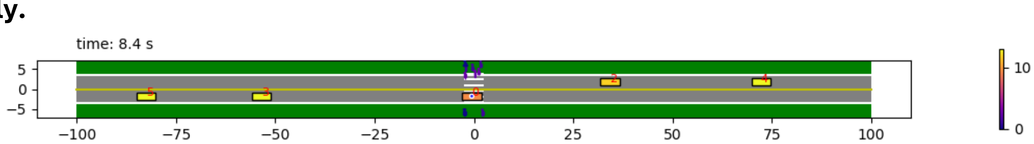

Figure 12. (8.4s) With all pedestrians now far from the road, Vehicle 0 starts to accelerate.

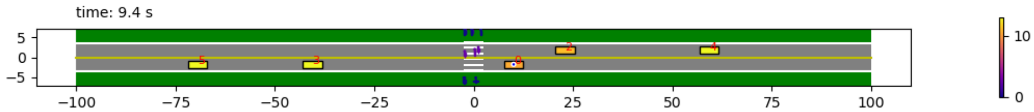

Figure 13. (9.4s) After Vehicle 0 has passed, a new group of pedestrians begins to cross the road, prompting Vehicle 2 to start slowing down.

Chapter 5. Conclusions

This study contributes to the growing body of knowledge aimed at integrating CAVs into urban traffic systems, paving the way for safer and more efficient transportation networks. In experiments above, we embarked on a journey to explore the intricate dynamics between Connected and Automated Vehicles (CAVs) and Vulnerable Road Users (VRUs), utilizing a sophisticated framework that blends game theory with adaptive algorithms. This innovative approach aimed to enhance the real-time traffic management capabilities of CAVs, ensuring both safety and efficiency in urban environments.

We conducted two distinct simulation experiments, each designed to test the robustness of our framework under varying traffic conditions. Despite using the same set of parameters, the traffic patterns differed significantly between the two experiments, highlighting the framework's adaptability and effectiveness.

The first experiment focused on a scenario with a higher density of vehicles approaching a crosswalk. Initially, pedestrians waited at the crosswalk as vehicles passed. As the simulation progressed, the adaptive trajectory planning algorithm allowed pedestrians to cross safely once the initial batch of vehicles had passed. Vehicles, in turn, decelerated appropriately to accommodate the crossing pedestrians. This scenario underscored the framework's ability to balance pedestrian safety with traffic flow efficiency, as vehicles only slowed down when necessary, minimizing delays.

The second experiment, on the other hand, dealt with a higher density of pedestrians crossing the road. Here, groups of pedestrians created a constant flow of crossing activity, challenging the responsiveness of the vehicles' adaptive planning. The results were impressive: vehicles exhibited adaptive behaviors such as slowing down and stopping to allow groups of pedestrians to cross, ensuring no collisions occurred. Despite the high density of VRUs, the traffic flow remained smooth, with vehicles resuming their paths promptly after pedestrians crossed.

These experiments vividly demonstrate the strength of our framework. By successfully modeling the complex interactions between CAVs and VRUs, our approach provides a robust solution for real-time traffic management. The adaptive algorithms effectively handle varying traffic densities and patterns, enhancing both safety and efficiency. This adaptability is crucial for the integration of CAVs into urban traffic systems, paving the way for smarter and more efficient transportation networks.

Moreover, the framework's reliance on game theory and discrete-time Markov sequential games ensures that the CAVs can anticipate and respond to the dynamic behaviors of VRUs. This predictive capability is essential for maintaining safety in unpredictable urban environments.

In conclusion, the study not only validates the effectiveness of the proposed framework but also highlights its potential to revolutionize urban traffic management. By facilitating safe and efficient interactions between CAVs and VRUs, this framework sets a new standard for smart transportation systems, contributing significantly to the future of autonomous urban mobility.

Chapter 6. Future Work

Building on the promising results of this project, our future work will focus on enhancing the trajectory planning framework by incorporating real traffic data. The next phase will transition from simulated data to real-world traffic data to improve the trajectory planning framework for Connected and Automated Vehicles (CAVs). This involves several key steps to enhance the model's accuracy and applicability in real traffic environments.

First, we will integrate real traffic data into our framework. Video recordings from urban traffic environments will be utilized to detect and track objects, including vehicles and Vulnerable Road Users (VRUs), using advanced computer vision techniques. This real-time object detection and location tracking will provide a more accurate representation of traffic dynamics, capturing the nuanced behaviors of road users. Thanks to the support from the University of Wisconsin-Milwaukee Police Department, we have access to real-time streaming data from multiple surveillance cameras at UWM campus. We will use this data to regress and fine-tune the parameters of our proposed algorithm. By analyzing actual movement patterns and interactions between vehicles and VRUs, we can calibrate the model parameters to better reflect real-world dynamics.

Once the parameters have been adjusted, we will enhance the adaptive algorithm to incorporate these new insights. This refinement will involve updating the game theory-based decision-making processes to improve the algorithm's ability to predict and respond to the behaviors of VRUs and other vehicles in a dynamic environment. The goal is to ensure that CAVs can safely and efficiently navigate real traffic scenarios, even in the presence of unpredictable road users. The enhanced framework will then be validated through a series of numerical simulations based on real traffic data. These simulations will help verify the robustness and accuracy of the trajectory planning algorithm under various traffic conditions.

Finally, we will showcase the enhanced algorithm's capability by comparing the simulated traffic behaviors with actual traffic data. This comparison will highlight the algorithm's effectiveness in replicating real traffic patterns and managing interactions between different road users. The outcomes will include visual demonstrations and performance metrics that illustrate the improved safety and efficiency of the proposed approach.

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