

# **Division of Engineering Research on Call Agreement 31796**

**(Task 2 – Asymmetrical Deformation of Thermoplastic Pipe Analysis)**

**Kevin White, Shad Sargand, Fouad Al Rikabi**

for the Ohio Department of Transportation  
Office of Statewide Planning and Research

and the  
United States Department of Transportation  
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*Final Technical Report*



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<b>16. Abstract</b> This study investigated asymmetric deformation (racking) of installed thermoplastic pipe. ODOT construction records were reviewed to determine the extent of racking noted within Item 611 Conduit Evaluations. Next, a rational method for assessing racking was developed within the framework of the AASHTO LRFD Bridge Design Specifications, Section 12. A spreadsheet was developed to aid in the assessment procedure. The results of the methodology were compared to test cases analyzed using the 2D culvert specific finite element software CANDE. Finally, a one-day course covering the basics of the finite element method was developed and presented to ODOT staff.			
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SI* (MODERN METRIC) CONVERSION FACTORS								
APPROXIMATE CONVERSIONS TO SI UNITS			APPROXIMATE CONVERSIONS FROM SI UNITS					
Symbol	When You Know	Multiply By	To Find	Symbol	When You Know	Multiply By	To Find	Symbol
LENGTH								
in	inches	25.4	millimeters	mm	millimeters	0.039	inches	in
ft	feet	0.305	meters	m	meters	3.28	feet	ft
yd	yards	0.914	meters	m	meters	1.09	yards	yd
mi	miles	1.61	kilometers	km	kilometers	0.621	miles	mi
AREA								
in <sup>2</sup>	square inches	645.2	square millimeters	mm <sup>2</sup>	square millimeters	0.0016	square inches	in <sup>2</sup>
ft <sup>2</sup>	square feet	0.093	square meters	m <sup>2</sup>	square meters	10.764	square feet	ft <sup>2</sup>
yd <sup>2</sup>	square yards	0.836	square meters	m <sup>2</sup>	square meters	1.195	square yards	yd <sup>2</sup>
ac	acres	0.405	hectares	ha	hectares	2.47	acres	ac
mi <sup>2</sup>	square miles	2.59	square kilometers	km <sup>2</sup>	square kilometers	0.386	square miles	mi <sup>2</sup>
VOLUME								
fl oz	fluid ounces	29.57	milliliters	mL	milliliters	0.034	fluid ounces	fl oz
gal	gallons	3.785	liters	L	liters	0.264	gallons	gal
ft <sup>3</sup>	cubic feet	0.028	cubic meters	m <sup>3</sup>	cubic meters	35.71	cubic feet	ft <sup>3</sup>
yd <sup>3</sup>	cubic yards	0.765	cubic meters	m <sup>3</sup>	cubic meters	1.307	cubic yards	yd <sup>3</sup>
NOTE: Volumes greater than 1000 L shall be shown in m <sup>3</sup> .								
MASS								
oz	ounces	28.35	grams	g	grams	0.035	ounces	oz
lb	pounds	0.454	kilograms	kg	kilograms	2.202	pounds	lb
T	short tons (2000 lb)	0.907	megagrams (or "metric ton")	Mg (or "t")	Mg (or "t") (or "metric ton")	1.103	short tons (2000 lb)	T
TEMPERATURE (exact)								
°F	Fahrenheit temperature	5(°F-32)/9 or (°F-32)/1.8	Celsius temperature	°C	°C	Celsius temperature	1.8°C + 32	Fahrenheit temperature
ILLUMINATION								
fc	foot-candles	10.76	lux	lx	lux	0.0929	foot-candles	fc
fl	foot-Lamberts	3.426	candela/m <sup>2</sup>	cd/m <sup>2</sup>	cd/m <sup>2</sup>	0.2919	foot-Lamberts	fl
FORCE and PRESSURE or STRESS								
lbf	poundforce	4.45	newtons	N	newtons	0.225	poundforce	lbf
lbf/in <sup>2</sup>	poundforce per square inch	6.89	kilopascals	kPa	kPa	0.145	poundforce per square inch	lbf/in <sup>2</sup>
or psi								or psi

\* SI is the symbol for the International Symbol of Units. Appropriate rounding should be made to comply with Section 4 of ASTM E380. (Revised September 1993)

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**Research Institute for  
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# **CHAPTER 1: INTRODUCTION**

## **1.1 Scope of Work**

The Ohio Department of Transportation (ODOT) wishes to develop a practical method for assessing asymmetrical deformation of installed thermoplastic pipes. ODOT Construction and Material Specifications (CMS) Item 611 requires such asymmetrical deformation (termed “racking”) to be evaluated by an independent Registered Engineer.

American Association of State Highway and Transportation Officials (AASHTO) Load and Resistance Factor Design (LRFD) Bridge Construction Specifications, Section 30 (2010) refer to Section 12 of the AASHTO LRFD Bridge Design Specifications (2017) for assessment of the structural suitability of installed thermoplastic pipe. However, Section 12 design procedures are all based on deflections less than 5% and are based on uniform deflection. Section 12 does not consider racking in the design method and does not consider deflections in excess of 5%. The assessment is, therefore, left wholly to the independent Registered Engineer.

Assessment of these types of distortion can be assessed using finite element modelling which can estimate the stresses and strains in the pipe wall.

The project goals are to address these deficiencies and offer guidance for the assessment of pipe distortion and racking, and to provide basic training to ODOT personnel on the use of finite element modelling (FEM). The following tasks will be utilized to accomplish these goals.

The project team will make contact with ODOT staff, both in Central Office and in District Offices to identify the types of distress commonly identified during 611 post-construction conduit inspections. In addition, we will draw on our experiences as an independent Registered Engineer and experiences of industry colleagues in identifying common conduit defects.

The project team will develop 2D FEM models using the public domain Culvert Analysis and Design (CANDE) FEM software. The models will be utilized to conduct a parametric study on the performance of distorted thermoplastic pipe.

Based on the results of the parametric study conducted as Task 2, the team will develop a Distortion Assessment Methodology. This will be a practical method for measuring thermoplastic pipe distortion and assessing the structural suitability of the distorted pipe. This will include pipe sizes with nominal diameters of 12 in to 60 in.

A short training procedure will also be developed in order to provide training of the methodology to ODOT personnel.

The project team will develop an introductory training course in the finite element method. The course will not focus on the mathematics behind the method, but rather the practical implementation of the methodology to solve soil-structure interaction problems. The training will discuss soil material models, structural element material models, and interface elements.

The training will discuss both the positive aspects of FEM as well as common pitfalls.

## **1.2 Outline of the Report**

Chapter 2 covers the literature search which aimed at review of current state of the practice in assessing non-symmetric deformation in buried thermoplastic conduits. In addition, a summary of existing ODOT 611 inspection data reviewed are provided in Chapter 2.

Chapter 3 presents the methodology utilized to develop the racking assessment tool as well as the finite element models used to assess the tool.

Chapter 4 discusses the development of the finite element method training session.

## CHAPTER 2: LITERATURE REVIEW

### 2.1 Introduction

A literature search was conducted to gather literature, specifications, and standards related to the proposed research project's main topic – assessment of asymmetric deformation in thermoplastic pipe. Past and recent publications made in relevant major technical journals and proceedings of conferences and symposia were reviewed to locate technical papers of interest. Some of the target journals and conference publications included:

- TRR (Journal of the Transportation Research Board (TRB))
- American Society of Civil Engineer (ASCE) Journals
- American Society for Testing and Materials (ASTM) Journal
- Proceedings of TRB Annual Meetings
- ASCE Conference Proceedings
- ASTM Symposium Proceedings
- ASTM Specifications

Reports issued on the topic considered in the literature search including:

- Reports Issued by Federal Highway Administration (FHWA), State Departments of Transportation, United States Department of Agriculture (USDA), and Natural Resources Conservation Service (NRCS)
- National Cooperative Highway Research Program (NCHRP) Reports and Syntheses
- Reports issued by research institutions
- Reports issued by pipe manufacturers

The team contacted Plastic Pipes Institute (PPI) member manufacturers and researched the websites of PPI member manufacturers and distributors to collect relevant information.

The results of the literature search produced very little regarding asymmetric deformation. The only germane publication is an article from the Compendium of Papers from the Transportation Research Board 94th Annual Meeting Compendium of Papers entitled “Evaluating Installation Racking in Buried Thermoplastic Conduits” (Domonell, Mailhot, & Beaver, 2015). This paper presents a methodology for assessing the flexural strain resulting from crown racking in buried arch-shaped stormwater chambers. The methodology assumes circumferential strain (thrust strain) remains essentially unchanged from the unracked condition. The method then uses field measurement tools to measure the radius of curvature of the deformed section by measuring the sagitta and chord length of the racked portion of the pipe wall. Finally, the method uses newly developed load combinations that reduce dead load factors because the shape and state of the deformed shape is measured in detail.

## CHAPTER 3: DEVELOPMENT OF RACKING ASSESSMENT TOOL

### 3.1 Assumptions and Limitations of the Methodology

Several simplifying assumptions are made to aid in the development of the racking assessment tool. These factors include:

- The deformed conduit can be reasonably estimated as an ellipse, or as a rotated ellipse.
- The deformed conduit shape is relatively stable.
- Maximum deflections do not exceed 10% to 12%. When deflections exceed this limit, the stability of the conduit ring is in question and global stability cannot be assured.
- The methodology should, insofar as practicable, be consistent with AASHTO LRFD Bridge Design Specifications, Section 12.

### 3.2 Conduit Mechanical Properties

HDPE pipes exhibit viscoelastic behavior. Viscoelastic materials tend to creep under constant stress and relax under constant strain. Stated otherwise, a conduit under a constant stress will creep (deflect). Whereas, the stress required to maintain a constant strain (deflection) will reduce with time. One interesting result of this viscoelastic response is that there is an apparent reduction in the modulus with time. This relaxation response and apparent reduction in stiffness can be seen in Figure 1 and Figure 2

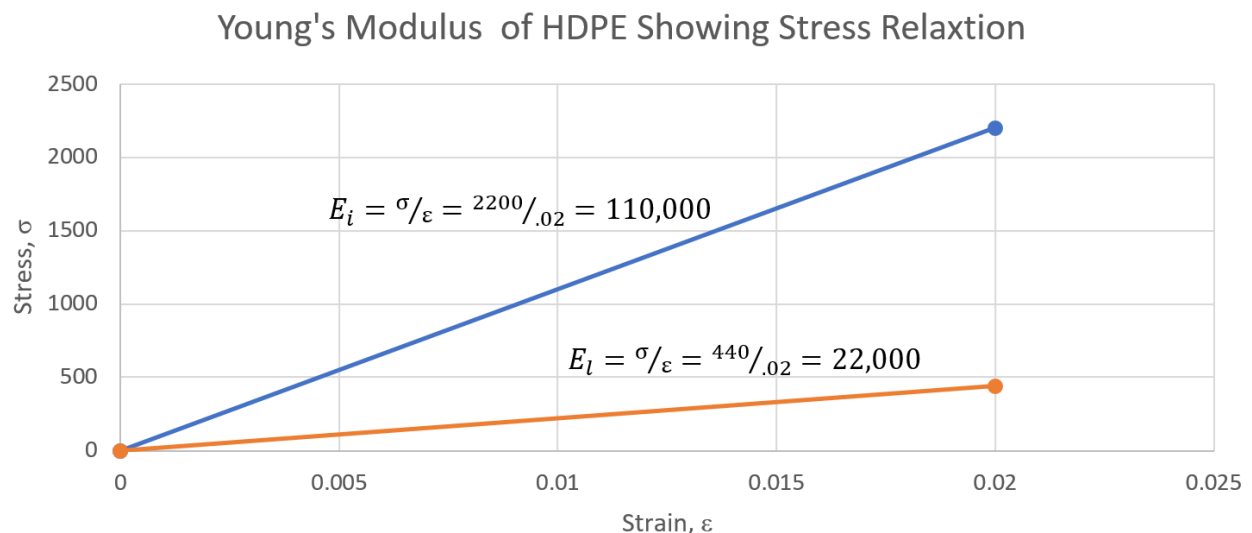
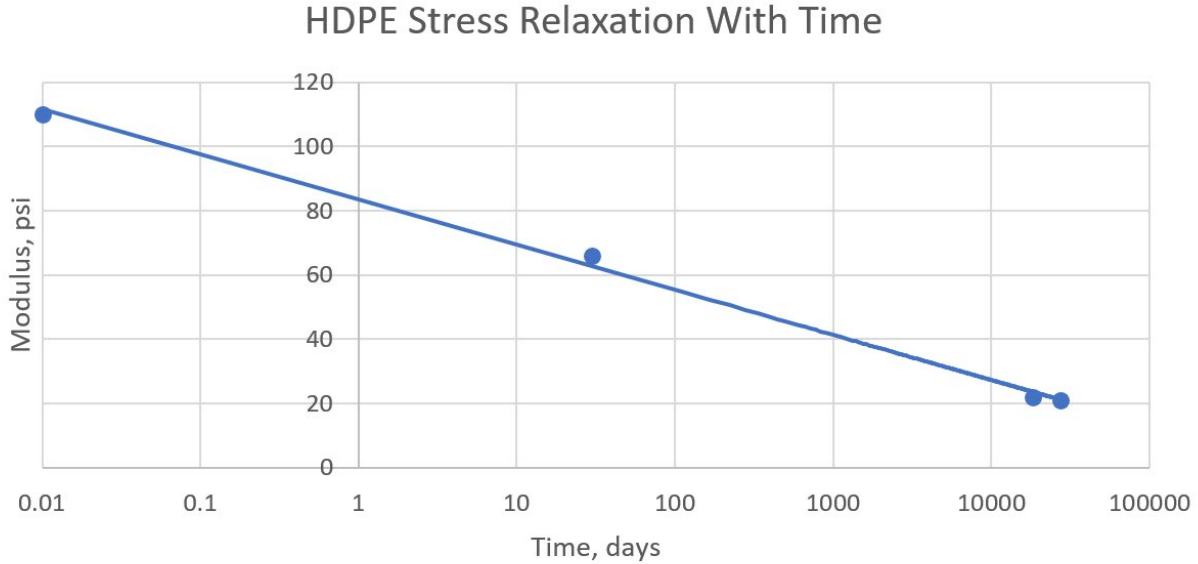


Figure 1 - Modulus of HDPE showing stress relaxation



*Figure 2 - Apparent reduction in modulus with time*

Data from NCHRP Report 870 (2018) on the relaxation of HDPE pipe under constant stress shows that after 30 days there is an apparent reduction of 60 percent in the mechanical properties of HDPE conduit meeting AASHTO M294. A best-fit log-linear equation was calculated using the AASHTO reported initial, 50- and 75-year values for the modulus of 110 ksi, 22 ksi and 21 ksi, along with a 30-day value of 66 ksi (60 percent of 110 ksi). This results in the following equation for determining the modulus as a function of time.

$$E = -6.092 \ln t + 83.419 \quad (1)$$

The flexural strength,  $f_y$ , as a function of time was calculated in a similar manner with the following resulting equation.

$$f_y = -0.147 \ln t + 2.3164 \quad (2)$$

### 3.3 Constrained Soil Modulus Estimation

Assessing the in-situ stiffness of the backfill soil around the installed conduit is a challenge. Non-destructive methods such as a cone penetrometer can be utilized. However, this may be unrealistic for conduits under pavements and can be a cost-prohibitive methodology. Considering the desire to provide a methodology consistent with AASHTO Section 12, it is necessary to determine the secant constrained soil modulus as this is a fundamental variable in the Section 12 design procedure. An equation-based methodology is proposed wherein the secant constrain soil modulus is calculated using the AASHTO modification to the Iowa

Equation for vertical deflection. The AASHTO (2017) equation expands the original Modified Iowa equation to consider both flexural defection and circumferential shortening. The equation is:

$$\Delta = \frac{K_B(D_L P_{sp} + C_L P_L) D_o}{1000(E_p I_p / R^3 + 0.061 M_s)} + \varepsilon_{sc} D \quad (3)$$

where:

$\Delta$  = Total deflection

$D_L$  = Deflection lag factor

$K_B$  = Bedding coefficient, typically 0.10

$P_{sp}$  = Soil prism pressure

$C_L$  = Live load coefficient

$P_L$  = Live load pressure

$D_o$  = Outside diameter of the conduit

$E_p$  = Modulus of the conduit material

$I_p$  = Moment of inertia of the conduit material

$R$  = Centroidal radius of the conduit

$D$  = Centroidal diameter of the conduit

$M_s$  = Secant constrained soil modulus

$\varepsilon_{sc}$  = Service compressive strain given as:

$$\varepsilon_{sc} = \frac{T_s}{1000(A_{eff} E_p)} \quad (4)$$

where:

$T_s$  = Service compressive thrust

$A_{eff}$  = Effective area of conduit wall

It is noted that service thrust is also a function of the secant constrained soil modulus which adds considerable complexity to the derivation of secant constrained soil modulus from the field measured deflection thus, an iterative solution procedure is recommended. It is also noted that the deflection lag factor is an empirical factor used to estimate the long-term settlement of the soil surrounding the conduit which results in additional long-term conduit deflection. Because



the actual field measured deflection is utilized in Equation (3), the deflection lag factor is set to unity.

As the conduit deformations exceed 7%, there is a rapid decline in the calculated constrained soil modulus to values well below what practical experience dictates as being realistic minimum values. Because of this, a minimum constrained soil modulus is set equivalent to the range of values presented for silty soils at 85 percent standard proctor density. These values are provided in Table 12.12.3.5-1 of AASHTO Section 12. The silty soil type selection as a lower bound is somewhat arbitrary and is based solely on the experience of the authors.

### 3.4 Flexural Strain

AASHTO provides an empirical approach for calculating maximum flexural strain at the outer fiber of a profile-wall conduit. The equation is given as:

$$\varepsilon_f = \gamma_{EV} D_f \left( \frac{c}{R} \right) \left( \frac{\Delta_f}{D} \right) \quad (5)$$

where:

$\varepsilon_f$  = factored flexural strain

$\gamma_{EV}$  = load factor for earth and dead load pressure

$\Delta_f$  = Vertical deflection due to flexural

$R$  = Centroidal radius of the conduit

$D$  = Centroidal diameter of the conduit

$D_f$  = Shape factor provided in AASHTO Section 12 Table 12.12.3.10.2b-1

$c$  = Distance from profile centroid to innermost or outermost fiber

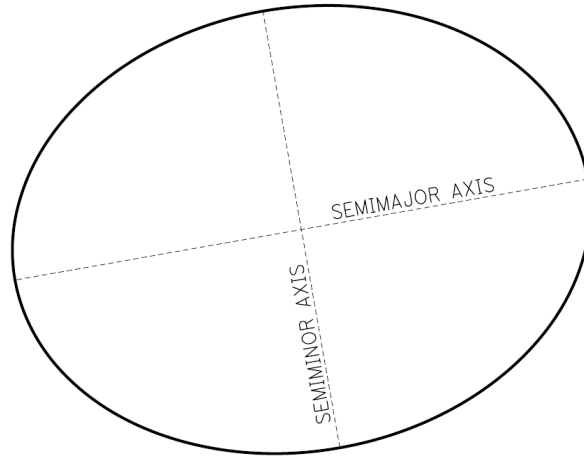
A simplified method of computing flexural strain can be determined from the deformed shape of the pipe and the change in radius of the conduit wall. Two methods for determining the change in radius are presented. The first is by assuming the deformed conduit is in the shape of an ellipse. For this methodology the changed radius is calculated as:

$$R_s = \frac{b^2}{a} \quad (6)$$

$$R_c = \frac{a^2}{b} \quad (7)$$

where:

- $R_s$  = radius of the conduit springline  
 $R_c$  = radius of the conduit crown  
 $a$  =  $\frac{1}{2}$  of the semimajor axis (see Figure 3)  
 $b$  =  $\frac{1}{2}$  of the semiminor axis (see Figure 3)



*Figure 3 - Ellipse nomenclature*

The second method uses the measured sagitta and chord length of the deformed shape to calculate the change in radius. See Figure 4 for a representation of the measurement methodology. For this methodology the changed radius is calculated as:

$$R_d = \frac{L^2}{8e} + \frac{e}{2} \quad (8)$$

where:

- $R_d$  = Changed radius of the conduit  
 $L$  = Common chord length  
 $e$  = sagitta length

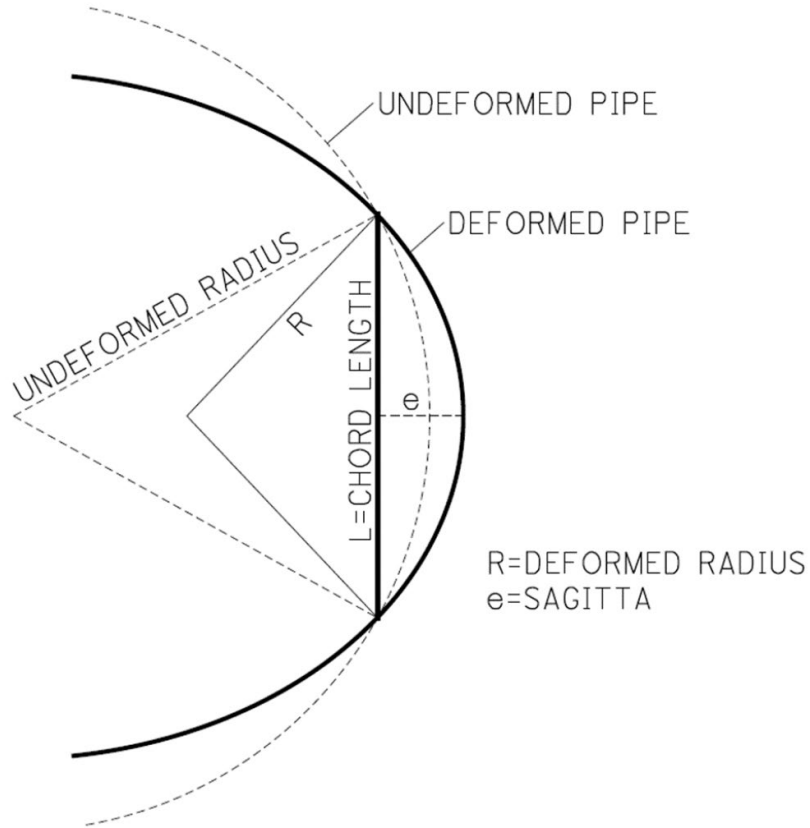


Figure 4 - Sagitta nomenclature

Once the changed radius is calculated, the equation for flexural strain can be calculated from three relationships (Waktins & Andersen, 2000). The first is the general equation for flexural stress,  $\sigma$ , given as:

$$\sigma = \frac{Mc}{I} \quad (9)$$

where:

M = Moment at the point of radius measurement

The next is the relationship between the flexural moment and the change in radius of the conduit.

$$\frac{M}{EI} = \frac{1}{R} + \frac{1}{R'} \quad (10)$$

where:

$R$  = Centroidal radius of the conduit

$R'$  = Changed centroidal radius of the conduit ( $R_c$ ,  $R_s$  or  $R_d$  from the above equations)

The final relationship is the stress/strain relationship of modulus.

Using these relationships, the flexural strain in the deflected conduit can be calculated as:

$$\varepsilon = c \left( \frac{1}{R} - \frac{1}{R'} \right) \quad (11)$$

### 3.5 Compression Strain

Compression strain in the racked conduit wall is assumed to be essentially equivalent to the compression strain in a conduit without racking. This approach is validated via finite element analysis herein as well as by the work of Domonell, *et. al.* (2015).

### 3.6 Assessment of Racking AASHTO Design Methodology

Once the estimated secant constrained soil modulus and field measured flexural strain are calculated using the methods described herein, it is possible to assess the long-term suitability of the installation using standard AASHTO design procedures. It is not necessary to check for deflection or to check the flexibility factor.

A spreadsheet has been developed to aid in the calculations. An electronic version of the spreadsheet was delivered to the ODOT Office of Hydraulic Engineering. The spreadsheet is included as Appendix A.

### 3.7 Finite Element Analysis

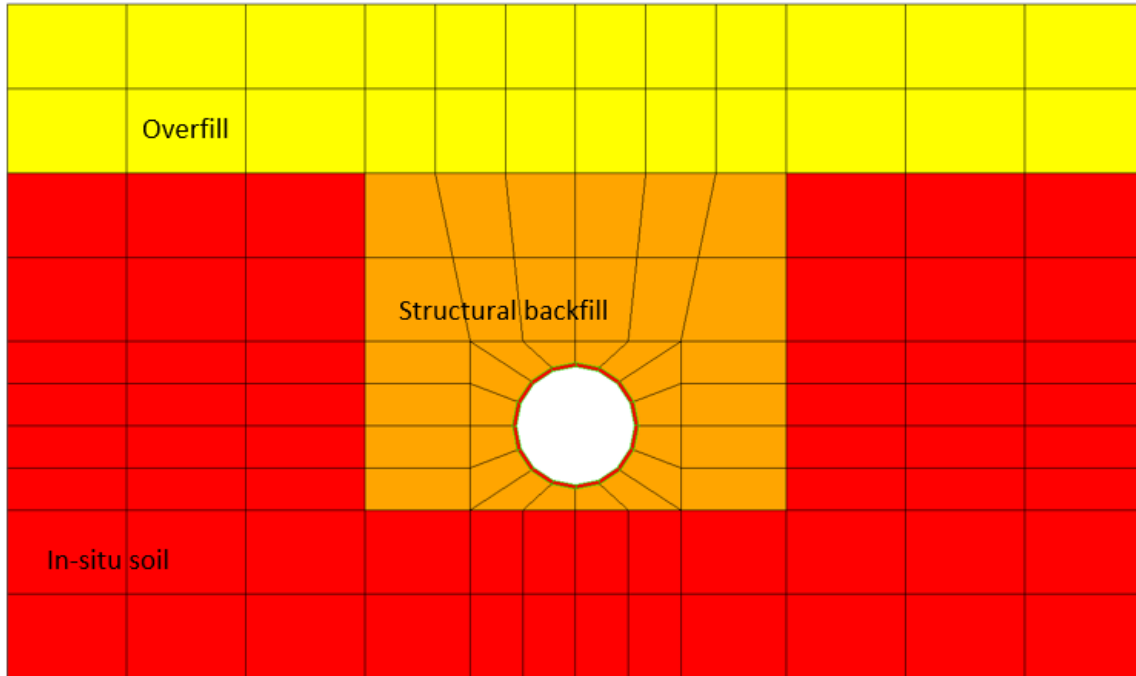
#### 3.7.1 Introduction

Finite element analysis (FEA) was performed to assess the response of a racked conduit under soil loading. The FEA was completed using the specific purpose finite element software CANDE (Culvert Analysis and Design). The response of a 36-inch conduit to racking and soils loading was determined from the CANDE output. The CANDE model utilized a 75-year apparent modulus and apparent flexural strength. The 75-year values of 21 ksi and 800 ksi, respectively, were taken from AASHTO Section 12 tabular values.

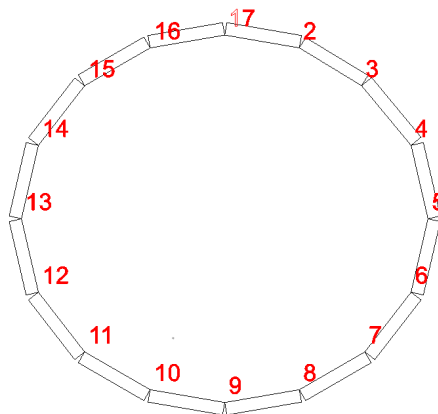
The principal of superposition was utilized for the analysis. Within the linear elastic domain, superposition is a method wherein loads applied to a system are invoked (superimposed) one at a time. The resulting total deformation is then calculated as the summation of the deformations from each individual load.

### 3.7.2 HDPE Pipe Model

The model used to analyze the effect of earth load on a 36-inch diameter HDPE pipe is shown in Figure 5. The model consists of four components, an in-situ soil trench with height and width of 8 and 10 feet, respectively, structural backfilling of 5 feet, overfill of 4 feet, and the 36-inch diameter conduit. Beam element results that follow all follow the same node numbering convention with node 1 located at the crown of the pipe and numbering then moving about the pipe in a clockwise until node 17, concurrent with node 1, is reached, as shown in Figure 6.



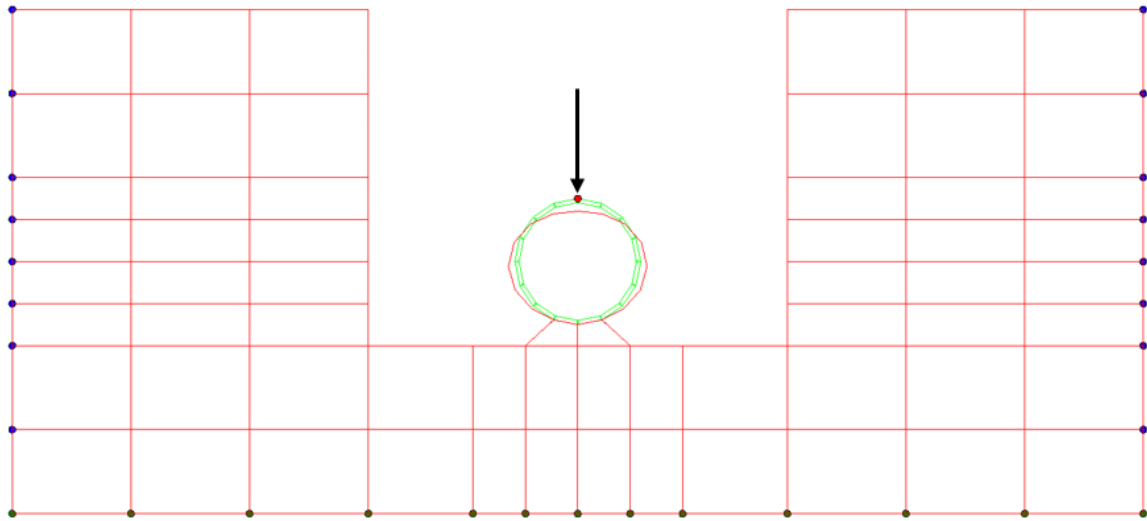
*Figure 5 - CANDE model soil zones*



*Figure 6 - CANDE Beam Element Results Numbering Convention*

### 3.7.3 Load Step 1

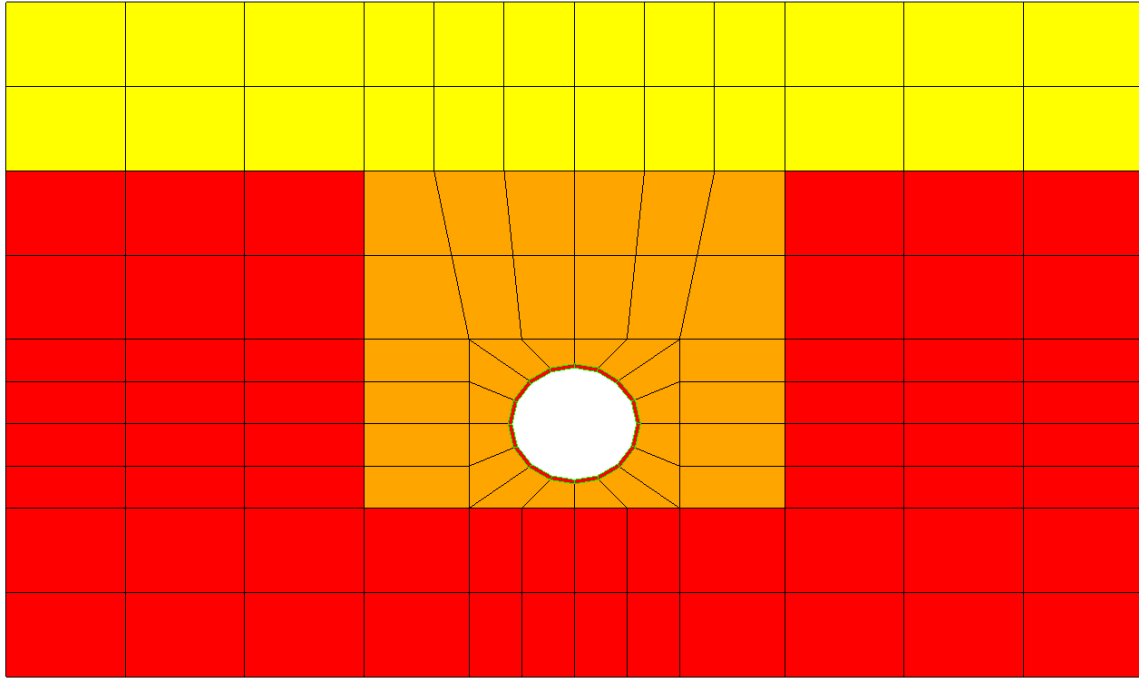
The first load step shows the in-situ soil with the conduit ring sitting above the bedding layer as depicted in Figure 7. The deformed shape was created by applying a displacement boundary condition with a value of 1.8 inches on the node at the crown of the conduit. This deformation equates to a nominal 5 percent deflection.



*Figure 7 - CANDE Load step 1*

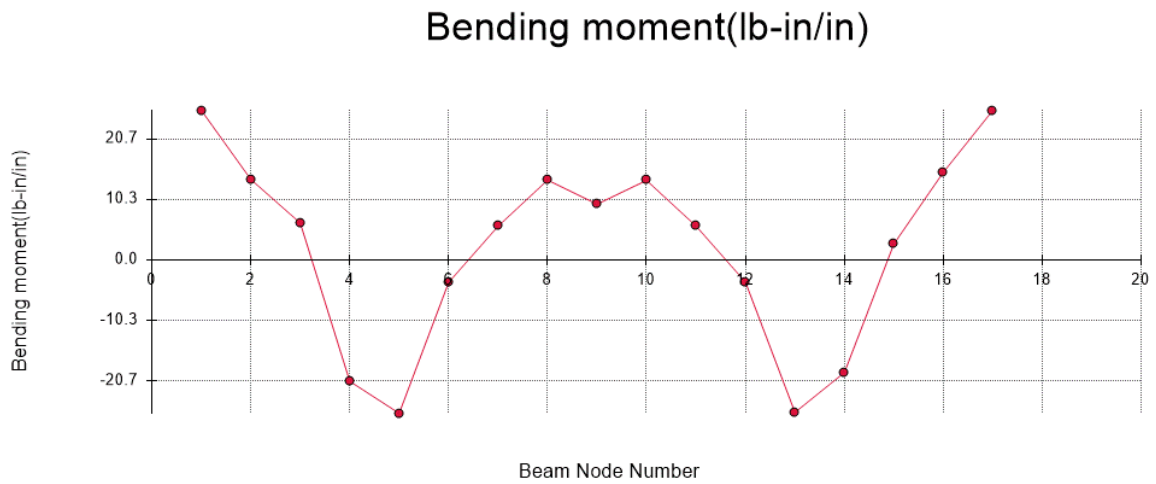
### 3.7.4 Elliptical Pipe Model

The elliptical pipe model was created by using an elliptical conduit representative of a circular conduit deflected 5% of its nominal diameter. The CANDE model is shown in Figure 8.

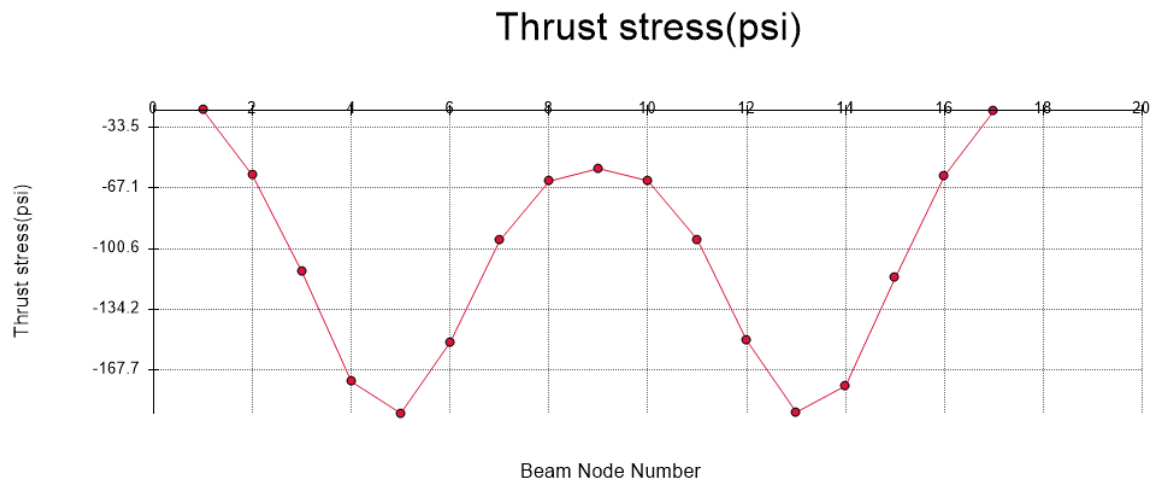


*Figure 8 - Elliptically deformed conduit model*

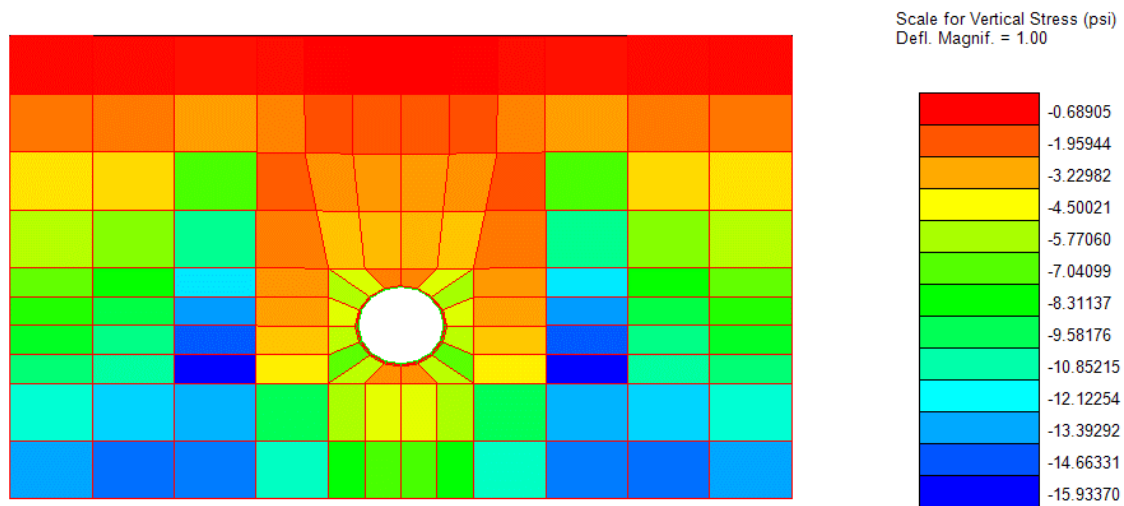
Results of the elliptical model are shown in Figure 9 through Figure 12.



*Figure 9 - Flexural moment for elliptical conduit model*

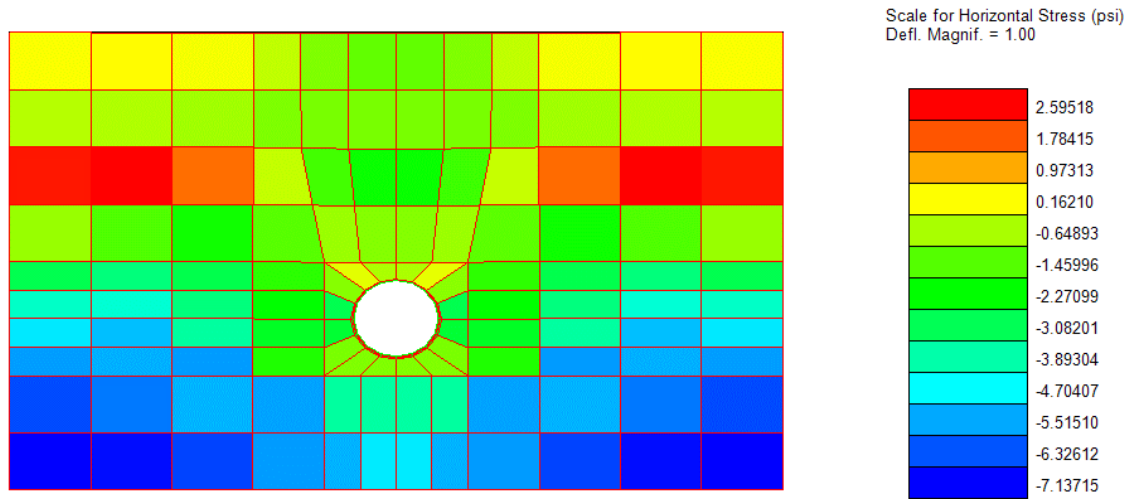


*Figure 10 - Thrust stress for elliptical conduit model*



*Figure 11 - Vertical soil stress for elliptical conduit model*

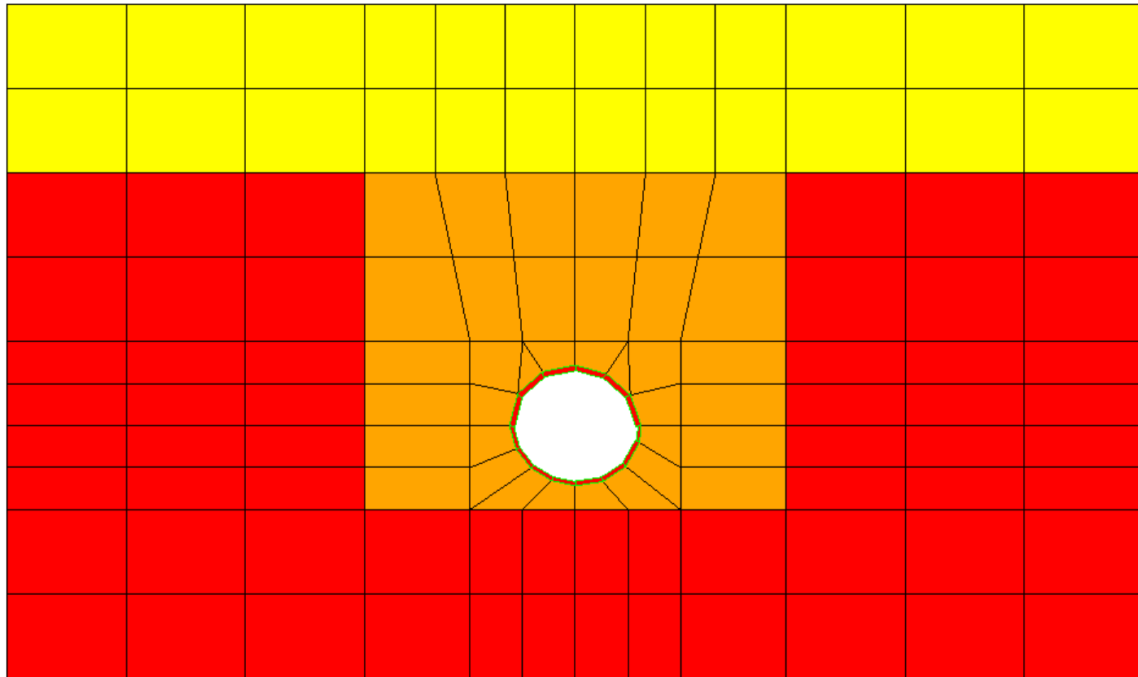




*Figure 12 - Horizontal soil stress for elliptical conduit model*

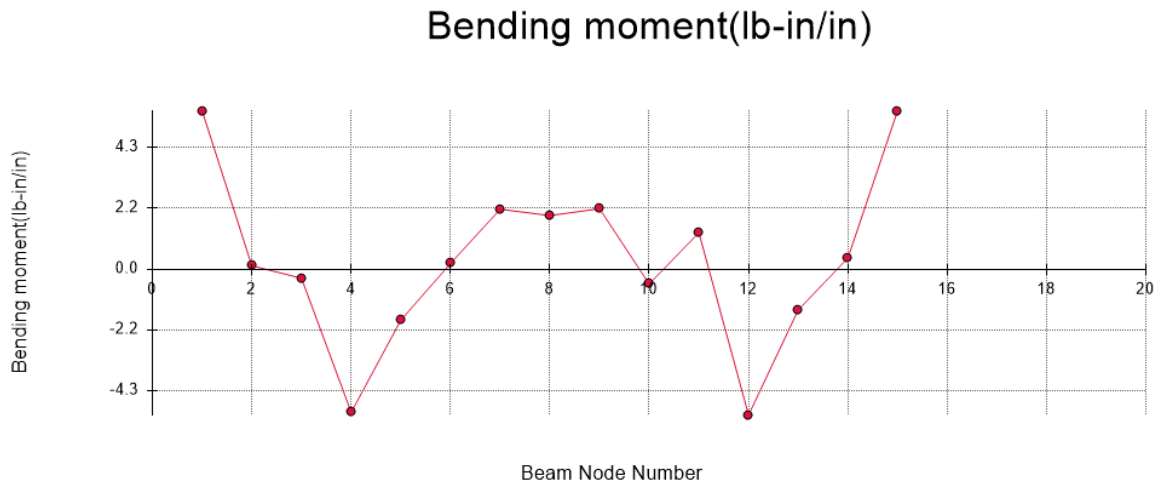
### 3.7.5 Elliptical Pipe Model Rotated Through 15°

The basic elliptical pipe model was rotated through 15° to create an idealized racked conduit. The CANDE model is shown in Figure 13.

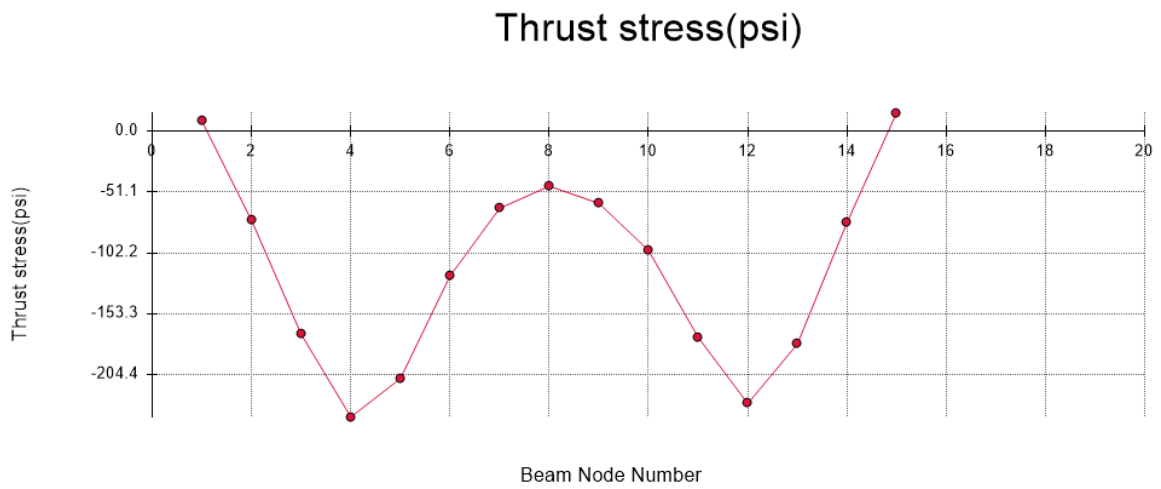


*Figure 13 - Elliptical conduit model rotated through 15°*

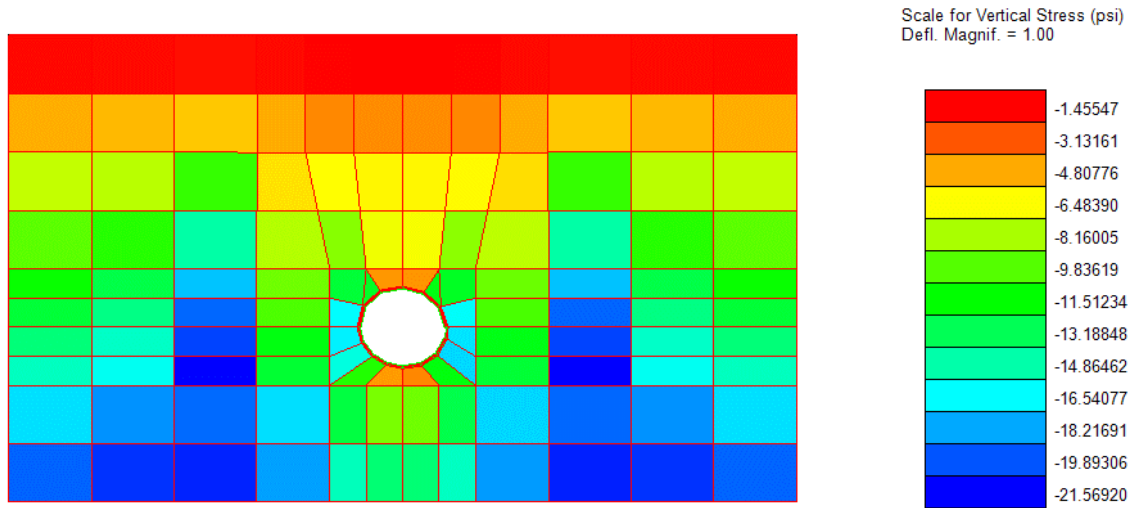
Results of the elliptical model rotated through 15° are shown in Figure 14 through Figure 17.



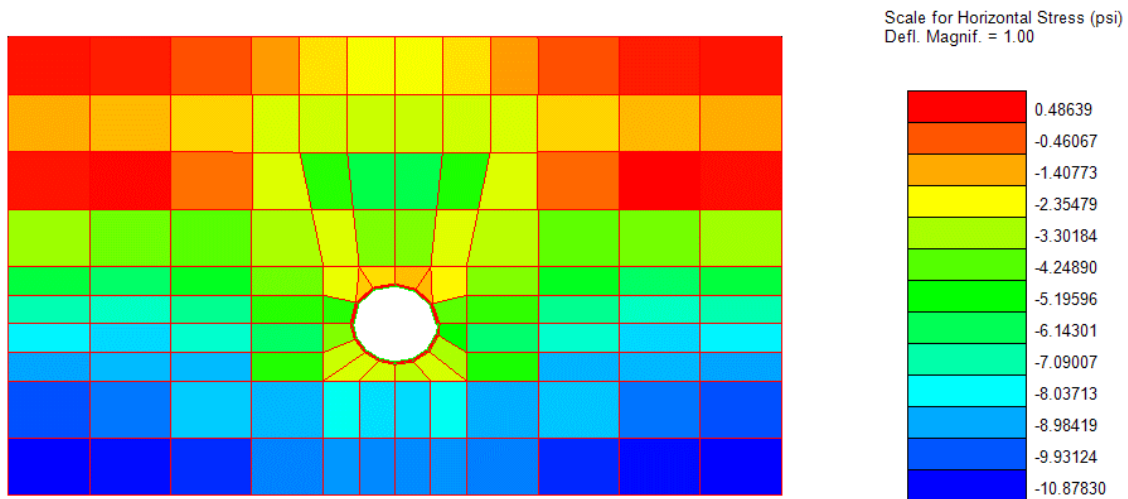
*Figure 14 - Flexural moment for 15° rotated elliptical conduit model*



*Figure 15 - Thrust stress for 15° rotated elliptical conduit model*



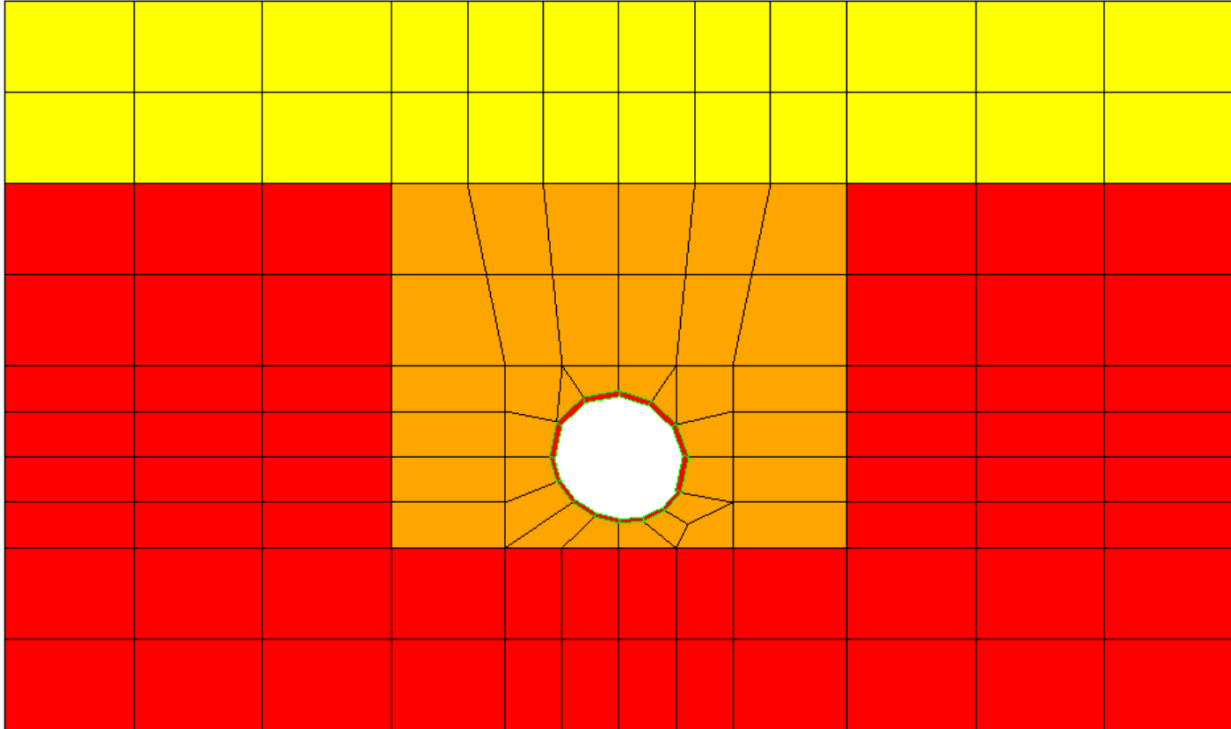
*Figure 16 - Vertical soil stress for 15° rotated elliptical conduit model*



*Figure 17 - Horizontal soil stress for 15° rotated elliptical conduit model*

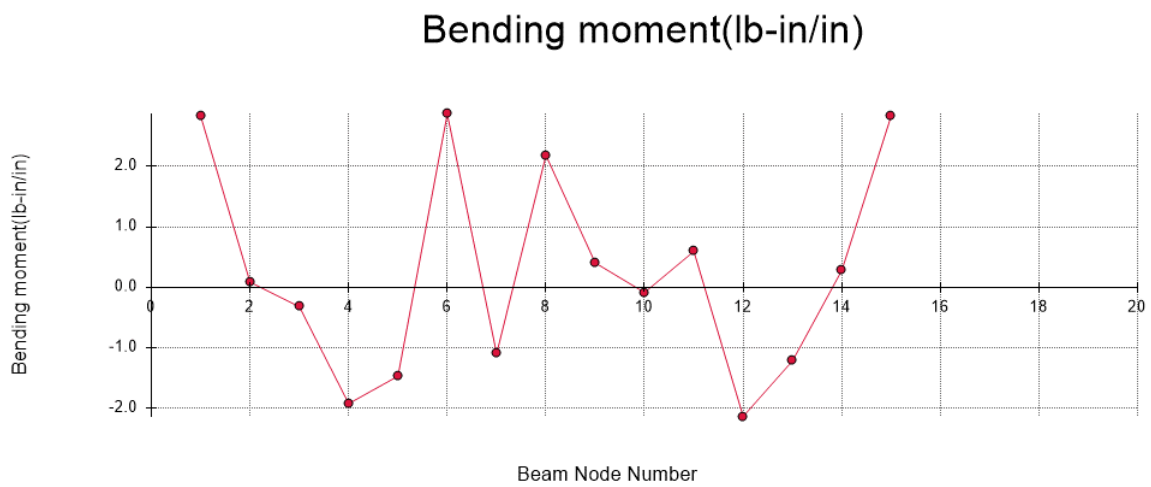
### 3.7.6 Elliptical Pipe Model Rotated Through 30°

The basic elliptical pipe model was rotated through 30° to create an idealized racked conduit. The CANDE model is shown in Figure 18.



*Figure 18 - Elliptical conduit model rotated through 30°*

Results of the elliptical model rotated through 30° are shown in Figure 19 through Figure 22.



*Figure 19 - Flexural moment for 30° rotated elliptical conduit model*

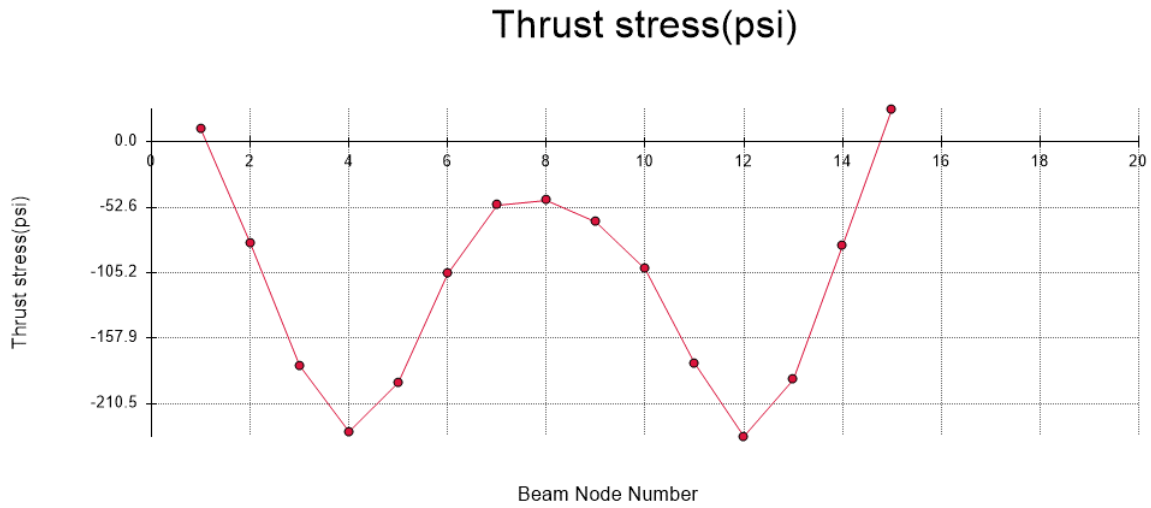


Figure 20 - Thrust stress for 30° rotated elliptical conduit model

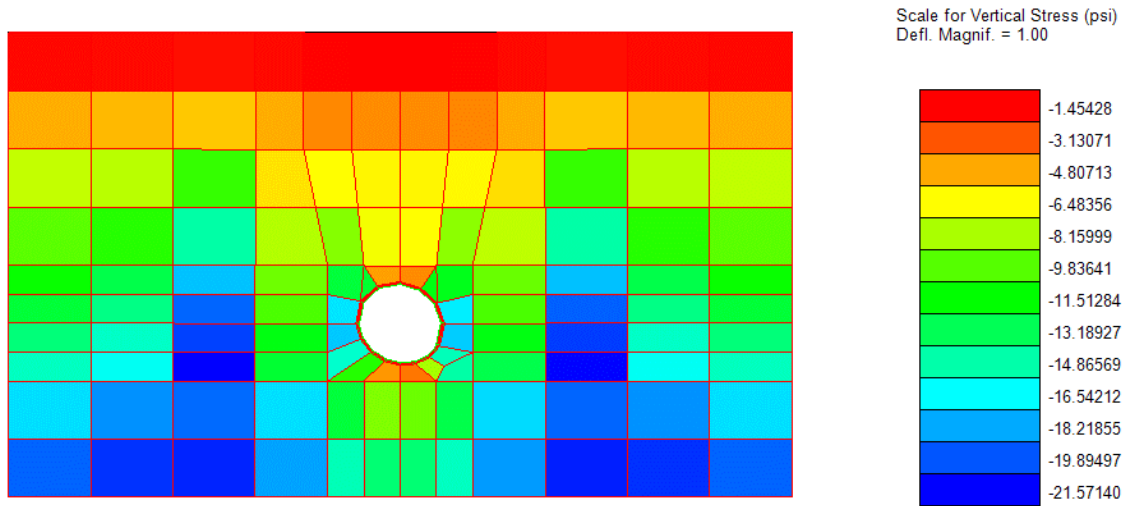
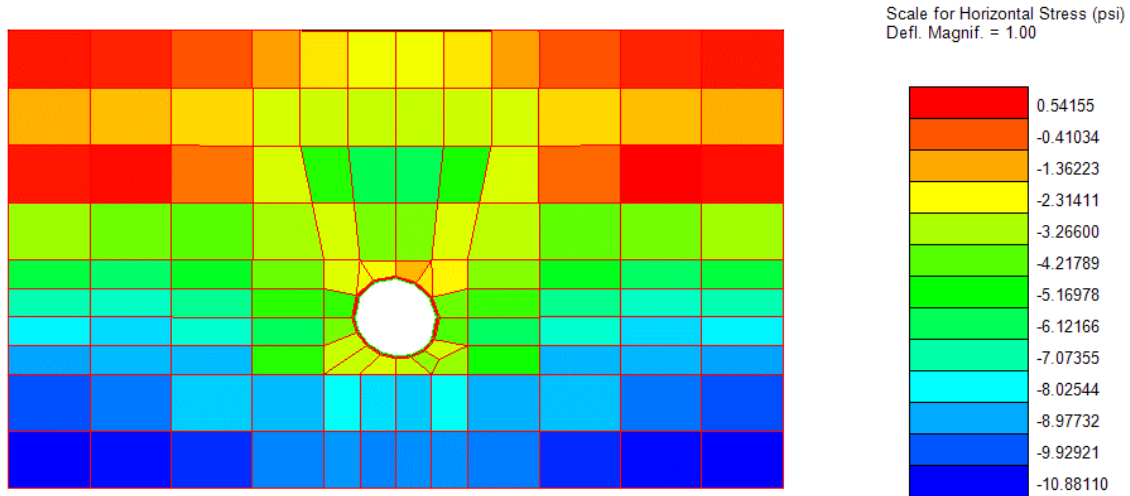


Figure 21 - Vertical soil stress for 30° rotated elliptical conduit model



*Figure 22 - Horizontal soil stress for 30° rotated elliptical conduit model*

### 3.8 Validation of the Methodology

Results derived from the methodology presented in Sections 3.3 through 3.5 are compared to the results of the finite element analyses presented in Section 3.7 to assess the suitability of the methodology.

The FEA were conducted at service state without additional load or resistance factors. For consistency, all applicable load and resistance factors have been set to unity when calculating the response of the conduit using AASHTO Section 12 methodologies. AASHTO Section 12 computes the flexural and circumferential (thrust) strains independent of one another whereas CANDE provides the combined nodal strains. For the following narrative compressive reactions are presented as negative values whereas tensile reactions are presented as positive values.

The results of the analyses are presented in Table 1. Upon inspection the tensile zone results do not correlate well between the two methods. The FEA tends to underestimate tensile strains in the conduit wall when compared to the AASHTO method. Upon further inspection the reason for the difference becomes evident. Calculation differences between the two methodologies result in substantially different wall thrust forces. As an example, the AASHTO calculated thrust force at the springline and crown are 125 lb/in and 77 lb/in, respectively. Whereas the CANDE calculated thrust forces for the 15° Ellipse are 78 lb/in and 26 lb/in, respectively. This thrust force is a compression force and tends to offset the tensile strains in the pipe wall. Another limitation in comparing CANDE to AASHTO Section 12 is that AASHTO Section 12 provides results at two discrete locations: the crown and the springline, whereas CANDE provides results at all model nodes.

*Table 1 - FEA and AASHTO Strain Results*

	Crown		Springline	
	Inner Fiber Strain (in/in)	Outer Fiber Strain (in/in)	Inner Fiber Strain (in/in)	Outer Fiber Strain (in/in)
AASHTO	-0.0025	-0.0025	-0.0041	-0.0041
Step 1	0.0121	-0.0174	-0.0108	0.0155
Ellipse	0.0096	-0.0199	-0.0149	0.0114
15° Ellipse	0.0011	-0.0181	-0.0075	0.0095
30° Ellipse	0.0012	-0.0016	-0.0108	-0.0072
Step 1 + Ellipse	0.0015	-0.0016	-0.0110	-0.0079
Step 1 + 15° Ellipse	0.0016	-0.0015	-0.0104	-0.0083
Step 1 + 30° Ellipse	0.0023	-0.0197	-0.0182	0.0023

### 3.9 Example Calculations

#### 3.9.1 Ellipse method

An example calculation using an actual conduit is provided to offer an overview of the proposed ellipse method contained within this report. A laser ring profiler obtained the deformed pipe cross-section shown in as part of an Item 611 Conduit Evaluation report. The pipe is a nominal 12-inch diameter pipe under 9 feet of cover. The installation date of the conduit was August 12, 2018 and the date of the video inspection was October 17, 2018. A portion of the laser ring report is provided in Figure 23.

For this example, a manufacturer profile was selected at random. The specific manufacturer is not identified herein since the use of such data should not be construed as an endorsement of the manufacturer. For a 12-inch pipe profile geometry values are given in Table 2.

*Table 2 - HDPE pipe profile geometry parameters*

Nominal Size (in)	Min. I.D. (in)	Max. O.D. (in)	Min. A (in <sup>2</sup> /ft)	Min. C (in)	Min. I (in <sup>4</sup> /in)	Min. PS (KSI)	Period (in)	Gross Area A <sub>g</sub> (in <sup>2</sup> /in)
12	12.2	14.4	2.340	0.429	0.029	0.050	2.0	0.195

Idealized profile geometry values are given in Table 3.

*Table 3 - Idealize profile geometry*

Nominal Size (in)	WALL THICKNESS				UNSUPPORTED LENGTH			
	Crest (in)	Web (in)	Valley (in)	Liner (in)	Crest (in)	Web (in)	Valley (in)	Liner (in)
12	0.0790	0.0960	0.1290	0.0560	0.7550	1.0420	0.3940	1.3750



*Figure 23 - Laser ring profiler cross-section*

Utilizing a CADD software, calibrated to the figure scale, a best fit ellipse is drawn over the laser ring and the semi-major and semi-minor axes are drawn. This is shown in Figure 24.





*Figure 24 - Laser ring profiler with superimposed best-fit ellipse*

From the CADD drawing the semi-major and semi-minor axes are measured as 12.3 in. and 10.0 in., respectively. The vertical deformation is taken as the difference between the pipe diameter and the measured semi-minor axis. This is calculated to be 2.2 in. These values were then input into the evaluation spreadsheet and the results determined at both the crown and springline.

The results indicate that the pipe is structurally adequate for the given height of cover with the deformed shape.

If the actual height of cover were 11 feet, the pipe would not meet the bucking capacity check at the pipe springline, and the pipe would be rejected as structurally inadequate.

### 3.9.2 Sagitta Method

An example calculation using the sagitta method is also provided. All material and physical pipe properties and installation details are as given in the previous example. A portion of the laser ring report is provided in Figure 23.

Utilizing a CADD software, calibrated to the figure scale, a representative chord estimated in a location with contract radius is drawn. The sagitta is also drawn. This is shown in Figure 25.

From the CADD drawing the chord length and sagitta are measured as 3.133 in. and 0.3089 in., respectively. The vertical deformation is taken as the difference between the pipe diameter and the measured semi-minor axis. This is calculated to be 2.2 in. These values were then input into the evaluation spreadsheet and the results determined at the pipe springline.



Figure 25 - Laser ring profiler with chord and sagitta

The results for the second example are similar to that for the first example. The pipe is structurally adequate for the given height of cover with the deformed shape.

If the actual height of cover were 11 feet, the pipe would not meet the buckling capacity check at the pipe springline, and the pipe would be rejected as structurally inadequate.

### 3.10 Conclusions and Recommendations

This study evaluated a proposed simplified methodology for assessing asymmetrical deformations (racking) in thermoplastic conduit. The methodology was developed within the framework of the existing AASHTO design procedure for buried thermoplastic conduits. At its core, the proposed methodology uses the deformed shape of the conduit to estimate flexural strains in the conduit wall. The method also uses a procedure of calculating the apparent secant constrained soil modulus based on the measured conduit deflection.

The methodology was compared with CANDE finite element models with mixed success. The compressive zones between the AASHTO and CANDE models compared quite favorably. However, the tensile zones had large differences. This could lead to unconservative results where there is a large difference in the distances from the neutral axis to the extreme inner and outer surfaces of the pipe profile. In order to ensure conservatism in the assessment methodology, consideration may be given to limiting the factored tensile flexural strain to the AASHTO limit of 5%. This ignores the considerable benefit of ring compression in reducing flexural tensile strains. In practice, tensile strain would rarely be the limit state for a design. Using AASHTO equation 12.12.3.10.2b-3 for flexural strain and the AASHTO profile limits given in Table A12-11, the vertical deflections resulting in 5% tensile flexural strain are provided in Table 2.

*Table 4 - Deflection for maximum tensile strain*

Nominal Size (in)	Deflection for 5% Tensile Strain (%)
12	14.6
15	14.2
18	15.2
24	15.5
30	16.7
36	16.7
42	16.0
48	17.5

This effort is considered an initial step to better understand the performance of thermoplastic conduits with asymmetrical deformation. Currently there is no rational method for assessing this type of deformation. Without any such method, within the language of Item 611, ODOT is left to rely solely on the interpretation of the independent engineer.

Additional research including fully instrumented field installations is recommended. Assessing the stress distribution about the conduit circumference, assessing soil stresses and assessing the resistance to ring collapse are all important topics which warrant further research.

## **CHAPTER 4: DEVELOPMENT OF FEA TRAINING SESSION**

At the request of the Ohio Department of Transportation Office of Hydraulic Engineering, a finite element analysis training course was developed. The FEM training session covered general topics on the finite element analysis as they relate to the design and analysis of buried conduits. The participants were introduced to the basics of FEA, what it is, and what it can and cannot do. Several example problems were highlighted, and the participant were given the the opportunity to develop and solve a buried pipe problem using FEA.

The goals of the course were to introduce FEA with enough detail to allow the participants to:

- Discuss the basic FEA theory
- Understand the FEA procedures necessary to develop and execute an FEA model
- Understand the limits of FEA

The training session was conducted on October 17, 2019 for ODOT staff members. The presentations utilized for the training session are included as Appendix B.

## References

- American Association of State Highway and Transportation Officials. (2010). Section 30: Thermoplastic Culverts. In *AASHTO LRFD Bridge Construction Specifications 4th Edition*. Washington, DC: American Association of State Highway and Transportation Officials.
- American Association of State Highway and Transportation Officials. (2017). Section 12: Buried Structures and Tunnel Liners. In *AASHTO LRFD Bridge Design Specifications, 7th Edition, with 2017 Interim*. Washington, DC: American Association of State Highway and Transportation Officials.
- Domonell, E., Mailhot, D., & Beaver, J. (2015). Evaluating Installation Racking in Buried Thermoplastic Conduits. *TRB 94th Annual Meeting Compendium of Papers*.
- Pluimer, M., & et al. (2018). *NCHRP Report 870: Field performance of corrugated pipe manufactured with recycled polyethylene content*. Washington, DC: Transportation Research Board of the National Academies.
- Waktins, R. K., & Andersen, L. R. (2000). *Structural Mechanics of Buried Pipes*. Boca Raton, FL: CRC Press, LLC.

## **Appendices:**

Appendix A: Racking Assessment Spreadsheet

Appendix B: FEA Training Session Presentations

## **APPENDIX 1: RACKING ASSESSMENT SPREADSHEET**





HDPE PIPE DESIGN - RACKING ASSESSMENT  
AASHTO LRFD BRIDGE DESIGN SPECIFICATIONS, EIGHTH EDITION, 2017

Design Criteria	
Nominal Pipe Size (in) 12	12
Live Load	No Load
Investigation Location	Crown
Effective Area Computation Method	Idealized Profile

Depth of Soil Cover H (ft)	11.00
Height of Water above Springline HW (ft)	0.00
Wet Soil Unit Weight (pcf)	120.0
Water Unit Weight (pcf)	62.4
Buoyant Soil Unit Weight (pcf)	57.6
Passer's Ratio of Soil to Water	0.0
Live Load (kips)	0.00
Design Live Load (KSF)	0.000
Length of Wheel Contact Area (in) L	10
Width of Wheel Contact Area (in) W	20
Wheel Spacing S	6
Initial Modulus of Elasticity (KSI) E	14
Initial Tensile Strength (KSI) F	10
Long Term Modulus of Elasticity (KSI) E	3
Long Term Tensile Strength (KSI) F	21
Long Term Tensile Strength (KSI) F	0.8
Radius to Centroid (in)	6.53
Diameter to Centroid (in)	13.06
Extreme Distance from Neutral Axis c <sub>x</sub> (in)	0.67
Pipe Stiffness (Calc) - Long Term (ksi)	0.0146

Pipe Profile	Min. I.D. (in)	Max. O.D. (in)	Min. A (in <sup>2</sup> /in)	Min. C (in)	Min. I (in <sup>4</sup> /in)	Pipe Stiffness (ksi)	Period (in)	Gross Area A <sub>g</sub> (in <sup>2</sup> /in)
AASHTO	12.20	14.40	0.195	0.43	0.029	0.050	2.00	0.195
	11.80	14.70	0.125	0.35	0.024	0.050	-	-

Resistance Factor for Soil Stiffness Φ <sub>ss</sub>	0.90
Resistance Factor for Thrust Φ <sub>tt</sub>	1.00
Resistance Factor for Global Buck Φ <sub>gb</sub>	0.70
Resistance Factor for Flexure Φ <sub>f</sub>	1.00

Racking Radius Estimation Method	
Span Length (in) L	196
Span Length (in) L	196
Span Length (in) L	196
Span Length (in) L	12.3
Semi Major Axis Length (in) a	10
Semi Minor Axis Length (in) b	7.97
Radius of Curvature r	0.0186
Total Outer Wall Unfactored Strain	0.0119
Total Inner Wall Unfactored Strain	0.0119

Date of Conduit Installation	8/12/2018
Date of Conduit Measurement	10/17/2018
Apparent Modulus of Elasticity (KSI) E	57.90
Apparent Deflection Lag Factor D <sub>o</sub>	0.79

Vertical Deflection (in) Δ <sub>v</sub>	2.20
Dead Load Modifier η <sub>dl</sub>	1.00
Dead Load Factor γ <sub>dl</sub>	1.00
Hydrostatic Load Factor γ <sub>hl</sub>	1.00
Live Load Factor γ <sub>ll</sub>	1.75
Live Load Factor γ <sub>ll</sub>	1.75
Live Load Factor γ <sub>ll</sub>	1.20
Live Load Distribution Factor LLDF	1.15
Buckling Coefficient K	4.00
Hydrostatic Uncertainty Factor K <sub>h</sub>	1.30
Installation Factor K <sub>i</sub>	1.50
Factored Compressive Strain Limit (ksi) F <sub>cl</sub>	4.10
Service Long-term Tension Strain Limit (ksi) F <sub>sl</sub>	5.00
Thrust Variation Coefficient K <sub>t</sub>	0.50
Non-Linear Calibration Factor C <sub>n</sub>	0.55
Bodding Coefficient K <sub>b</sub>	0.10
Deflection Lag Factor D <sub>o</sub>	1.5
Time Factor K <sub>st</sub>	0.3





IDEALIZED PROFILE									
Pipe Properties		WALL THICKNESS			UNSUPPORTED LENGTH			Wt Ratio	
DIAMETER (in)		Crest (in)	Valley (in)	Liner (in)	Crest (in)	Valley (in)	Liner (in)	Crest	Web
12		0.0790	0.0960	0.1290	0.7500	1.0420	1.3790	9.5570	10.8542
								3.0543	3.0543
								24.5536	24.5536

EFFECTIVE WALL AREA CALCULATION									
Slenderness Factor $\lambda$ 12.12.3.10.1b Crest	0.968								
Slenderness Factor $\lambda$ 12.12.3.10.1b Web	1.099								
Slenderness Factor $\lambda$ 12.12.3.10.1b Valley	0.673								
Effective Width Factor $\rho$ 12.12.3.10.1b Crest	0.769								
Effective Width Factor $\rho$ 12.12.3.10.1b Web	0.728								
Effective Width Factor $\rho$ 12.12.3.10.1b Valley	1.000								
Effective Width Factor $\rho$ 12.12.3.10.1b Crest	0.367								
Element Effective Width $b$ 12.12.3.10.1b Crest	0.603								
Element Effective Width $b$ 12.12.3.10.1b Web	0.758								
Element Effective Width $b$ 12.12.3.10.1b Valley	0.384								
Element Effective Width $b$ 12.12.3.10.1b Crest	0.504								
Element Effective Width $b$ 12.12.3.10.1b Web	0.012								
Element Effective Width $b$ 12.12.3.10.1b Valley	0.027								
Slab Compression Capacity $\phi P_n$ (kips)	0.049								
Effective Area of Pipe Wall $A_e$ (in <sup>2</sup> )	0.137								

LOAD CALCULATION									
Soil Prism Load (psf) - Case 1	4.453								
Soil Prism Load (psf) - Case 2	9.479								
Soil Prism Load (psf) - Case 3	9.277								
Groundwater Pressure (psf)	0.000								
Soil Prism Load $P_o$ (psf)	9.277								
Hydrostatic Pressure $P_o$ (psf)	0.000								
Calculated Constrained Soil Modulus (ksi)	0.024								
Minimum Constrained Soil Modulus (ksi)	0.369								
Constrained Soil Modulus (ksi)	0.369								
Hoop Stiffness Factor $S_1$	0.572								
Hoop Stiffness Factor $S_2$ - Initial	0.109								
Vertical Arching Factor VAP	0.852								
Impact Factor IM	1.00								
Wheel Interaction Coefficient $C_w$	3.72								
Wheel Interaction Length $L_w$ (ft)	23.4								
Live Load Application Width $W_L$ (ft)	13.5								
Rectangular Area at Depth H (ft)	274.76								
Tire Pressure at Depth H (ksi)	0.00								
Total Live Load Pressure (ksi)	0.00								
Total Live Load Pressure $P_L$ (psf)	0.00								
Live Load Distribution Coefficient $C_L$	1.00								
Live Load Distribution Coefficient $C_L$	1.23								
$F_{max}$	1.00								
$F_L$	0.89								

AXIAL THRUST CALCULATIONS									
Service Thrust Dead Loads	35.3								
Service Thrust Live Loads	0.0								
Service Thrust $T_s$ (lb/in)	35.3								
Factored Thrust Dead Loads	68.9								
Factored Thrust Live Loads	0.0								
Factored Thrust $T_u$ (lb/in)	68.9								

STRAIN CALCULATIONS AND DESIGN CHECKS									
Service Compressive Strain $\epsilon_s$ (min)	0.0122								
Factored Compressive Strain $\epsilon_{su}$ (min)	0.0239								
Compressive Strain Check	OK								
Nominal Buckling Strain Capacity $\epsilon_{su}$ (min)	0.0480								
Buckling Strain Check	OK								
Factored Tensile Strain $\epsilon_t$ (min)	0.0155								
Factored Compressive Strain $\epsilon_c$ (min)	0.0242								
Combined Strain Zone Strain Check	OK								
Combined Compressive Zone Strain Check	OK								
Allowable Interstrain Check Yes or No	YES								

## **APPENDIX 2: FEA TRAINING SESSION PRESENTATIONS**

# FINITE ELEMENT METHOD

Introduction to Finite Element Analysis

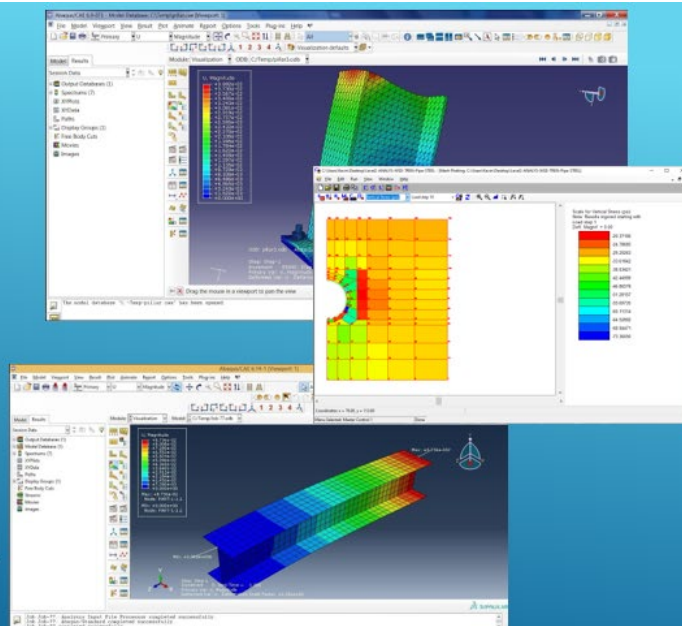
Ohio Department of Transportation

Kevin White, PE

E.L. Robinson Engineering

- ▶ Introduction
- ▶ Mathematic Overview
- ▶ FEM Software
- ▶ CANDE Tutorial

## INTRODUCTION



- ▶ The Finite Element Method (FEM) is a numerical method for simulating and analyzing engineering products and systems
- ▶ Useful for problems with complicated geometries, loadings, and material properties where analytical solutions can not be obtained
- ▶ Finite element method follows on from matrix analysis and became viable with the advent of computers. It is a computer-based analysis tool

## FINITE ELEMENT METHOD

- ▶ Typical undergraduate analysis methods
- ▶ Stress analysis for trusses, beams, and other simple structures are carried out based on dramatic simplification and idealization:
  - ▶ mass concentrated at the center of gravity
  - ▶ beam simplified as a line segment (same cross-section)
- ▶ Design is based on the calculation results of the idealized structure along with load and resistance factors based on empirical or statistical evidence

## ANALYTICAL SOLUTIONS

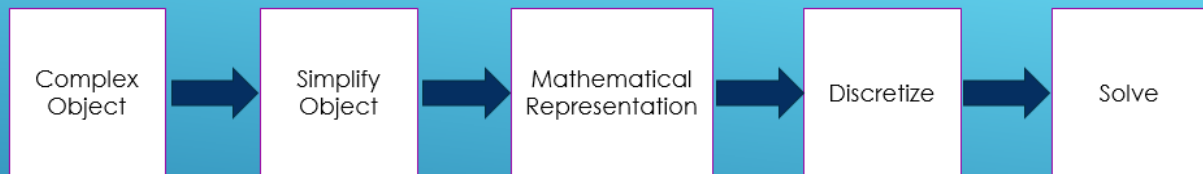
- ▶ Complex geometry
- ▶ Local accuracy is necessary.
- ▶ Understand the physical behaviors of a complex object (structural integrity, heat transfer, fluid flow, etc.)
- ▶ To predict the performance and behavior of the design
- ▶ To calculate reliability and to identify the weakness of a design accurately
  - ▶ Survivability of your iphone when dropped from a distance

## WHEN FEM?

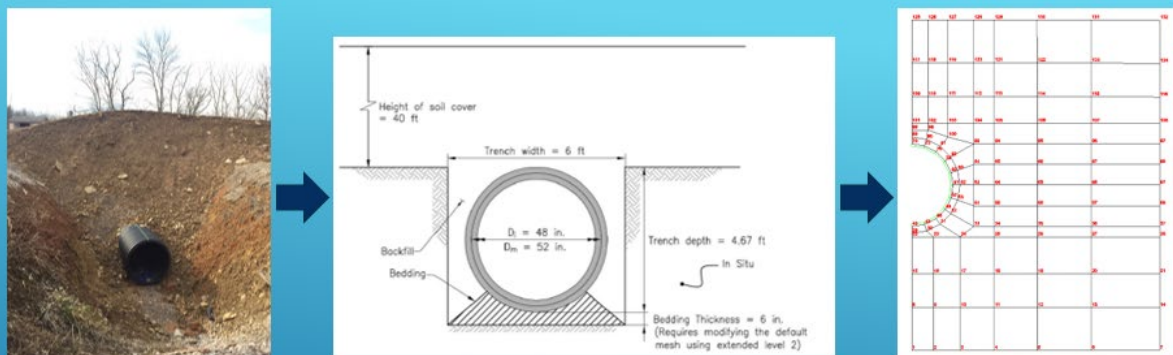
- ▶ Mechanical/Aerospace/Civil/Automotive Engineering
- ▶ Structural/Stress Analysis
  - ▶ Static/Dynamic
  - ▶ Linear/Nonlinear
  - ▶ Plastic
  - ▶ Visco-plastic/elastic
- ▶ Fluid Flow
- ▶ Heat Transfer
- ▶ Soil Mechanics
- ▶ Acoustics
- ▶ Biomechanics

## WHEN FEM?

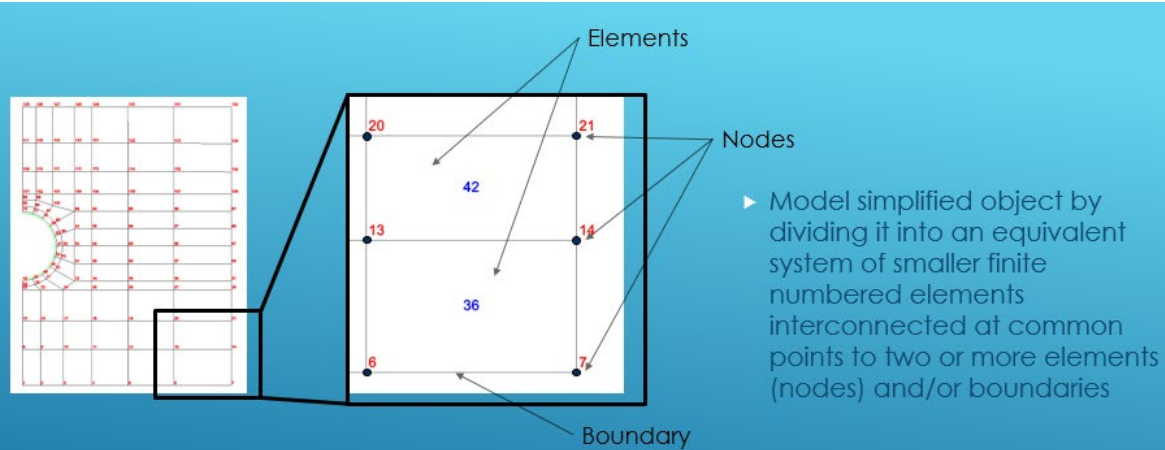




## FEM PROCESS



## DISCRETIZATION



## DISCRETIZATION

- Use matrix algebra to solve system of equations to determine unknown nodal displacements
- Use calculated nodal displacements to solve for:
  - Stress
  - Strain
  - Moment
  - Deformation
  - Rotation
  - Etc.
- ALL CALCULATED VALUES ARE ONLY AT NODES!!!

## FEM PROCESS

- ▶ Use matrix algebra to solve system of equations to determine unknown nodal displacements
- ▶ Use calculated nodal displacements to solve for:
  - ▶ Stress
  - ▶ Strain
  - ▶ Moment
  - ▶ Deformation
  - ▶ Rotation
  - ▶ Etc.
- ▶ ALL CALCULATED VALUES ARE ONLY AT NODES!!!

## FEM PROCESS



$$F = k \delta$$

$$\delta = u_2 - u_1$$

$$F = k(u_2 - u_1)$$

$$F_1 = -k(u_2 - u_1)$$

$$F_2 = k(u_2 - u_1)$$

$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

$$\{F\} = [K]\{u\}$$

## FEM PROCESS



# Finite Element Methods

Ohio Department of Transportation  
October 17-18, 2019

**Shad Sargand**

Russ Professor and Vice Director for Business Development  
Ohio Research Institute for Transportation and the Environment  
Russ College of Engineering and Technology  
Ohio University, Athens, Ohio

# Finite Element Method Overview

- Set up the problem
- Decide dimensionality, discretize problem, and create mesh
- Determine solution for each element
- Select and apply constitutive laws
- Write element equation
- Assemble equation for entire system
- Determine and apply boundary conditions
- Solve simultaneous equations
- Find solution for primary unknown, then secondary unknowns
- Interpret results



# Determine dimensionality

Is this problem best solved in one, two, or three dimensions?

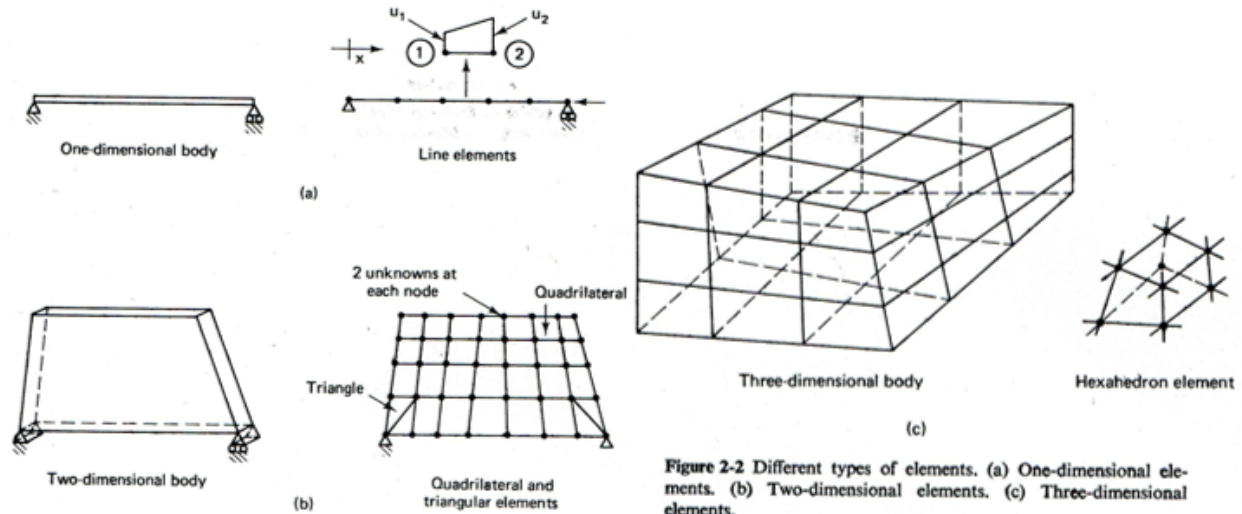
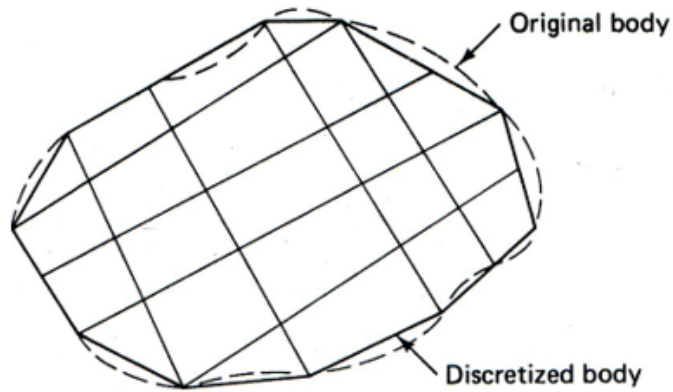


Figure 2-2 Different types of elements. (a) One-dimensional elements. (b) Two-dimensional elements. (c) Three-dimensional elements.

from C.S. Desai, 1979, *Elementary Finite Element Method*, (Prentice-Hall, Englewood Cliffs NJ).

## Discretize Problem



**Figure 2-3** Discretization for irregular boundary.

from C.S. Desai, 1979, *Elementary Finite Element Method*, (Prentice-Hall, Englewood Cliffs NJ).

## Write solution for each element

- Need a continuous function
  - Polynomial
  - Trigonometric
  - Other
- Need derivative of function





# Select and apply constitutive laws

## Material properties

- For strain:
  - Hooke's law (elastic behavior)
  - Plasticity
  - Viscoelasticity
- For thermal problems
  - Thermal conductivity
  - Coefficient of expansion
- Other problems
  - Viscosity
- Constitutive laws depend on type of material being modeled  
(steel is different than thermoplastic is different than concrete)



# Write element equation

## Various approaches

- Potential energy method
- Weighted residual method (Galerkin method)



# Potential Energy

$$\Pi_p = U + W_p$$

$$\delta \Pi_p = \delta U - \delta W_p = 0$$

$$\delta W = -\delta W_p$$

$$\frac{\partial \Pi_p}{\partial u_1} = 0$$

$$\frac{\partial \Pi_p}{\partial u_2} = 0$$

$$\vdots$$
$$\vdots$$

$$\frac{\partial \Pi_p}{\partial u_n} = 0$$



## Weighted Residual Method

$$\frac{\partial^2 u^*}{\partial x^2} - \frac{\partial u^*}{\partial t} = f(x)$$

$$Lu^* = f \text{ where } L \equiv \frac{\partial^2}{\partial x^2} - \frac{\partial}{\partial t}$$

$$u = \sum_{i=1}^n \alpha_i \varphi_i$$

$$R(x) = Lu - f$$

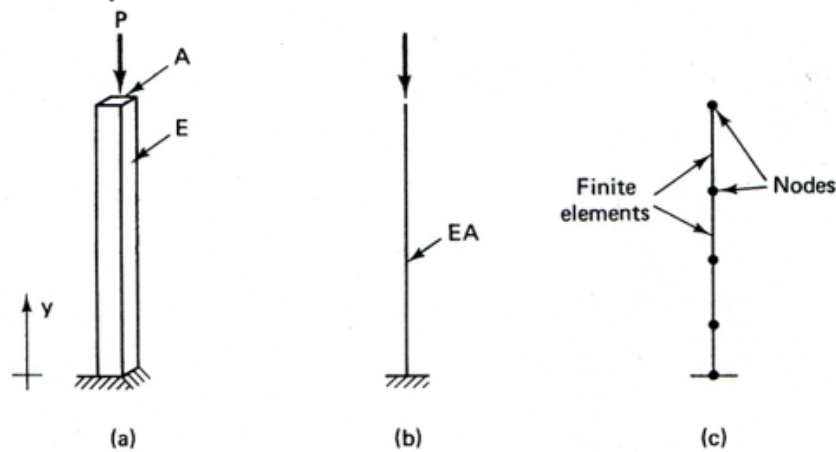
$$\int_D R(x) W_i(x) dx = 0 \quad i = 1, 2, \dots, n$$

$$[\mathbf{k}]\{\mathbf{q}\} = \{\mathbf{Q}\}$$



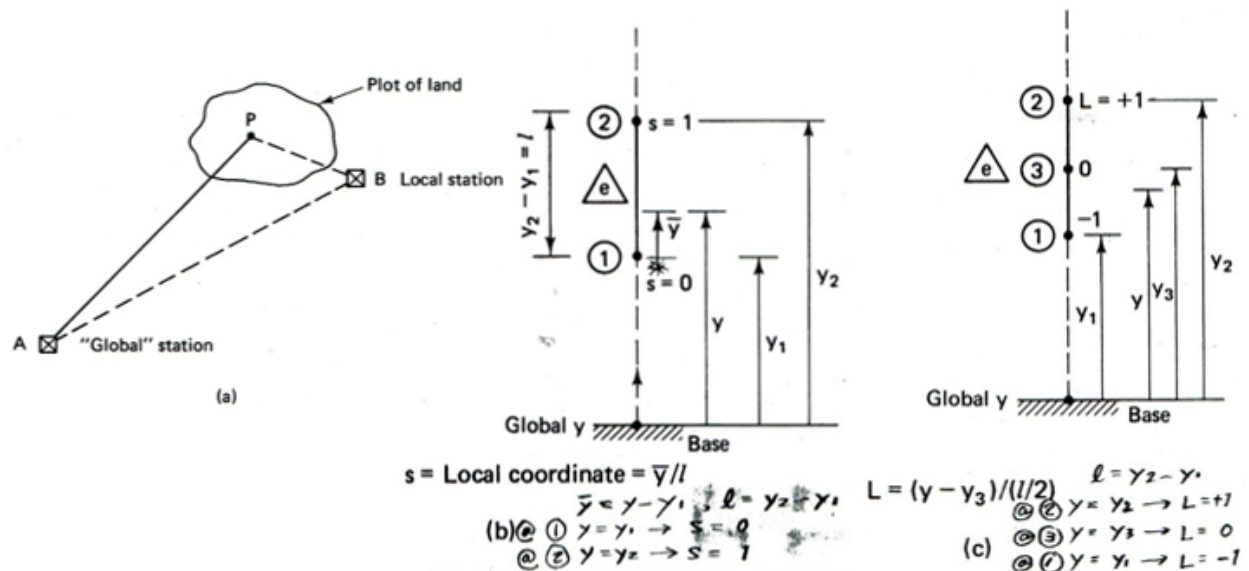
# One-Dimensional Stress Deformation

- Solve for primary result, then secondary results
- Primary result: displacement
- Secondary results: stress and strain



**Figure 3-1** Axially loaded column. (a) Actual column. (b) One-dimensional idealization. (c) Discretization.

# One-Dimensional Stress Deformation



**Figure 3-2** Global and local coordinates. (a) Concept of global and local coordinate systems. (b) Local coordinate measured from node point 1. (c) Local coordinate measured from midnode 3.

from C.S. Desai, 1979, *Elementary Finite Element Method*, (Prentice-Hall, Englewood Cliffs NJ).

# One-Dimensional Stress Deformation

$$v = \alpha_1 + \alpha_2 y$$

$$\{\mathbf{v}\} = [1 \ y] \begin{Bmatrix} \alpha_1 \\ \alpha_2 \end{Bmatrix}$$

$$\{\mathbf{v}\} = [\Phi] \{\alpha\}$$

$$v_1 = \alpha_1 + \alpha_2 y_1$$

$$v_2 = \alpha_1 + \alpha_2 y_2$$

$$\begin{Bmatrix} v_1 \\ v_2 \end{Bmatrix} = \begin{bmatrix} 1 & y_1 \\ 1 & y_2 \end{bmatrix} \begin{Bmatrix} \alpha_1 \\ \alpha_2 \end{Bmatrix}$$

$$\{\mathbf{q}\} = [\mathbf{A}] \{\alpha\}$$



# One-Dimensional Stress Deformation

$$\begin{matrix} \begin{Bmatrix} \alpha_1 \\ \alpha_2 \end{Bmatrix} = \frac{1}{l} \begin{bmatrix} y_2 & -y_1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} v_1 \\ v_2 \end{Bmatrix} \\ (2 \times 1) \qquad (2 \times 2) \quad (2 \times 1) \end{matrix}$$

$$\alpha_1 = \frac{y_2 v_1 - y_1 v_2}{l}$$

$$\alpha_2 = \frac{-v_1 + v_2}{l}$$





# One-Dimensional Stress Deformation

$$v = \frac{1}{2}(1 - L)v_1 + \frac{1}{2}(1 + L)v_2$$

$$= N_1 v_1 + N_2 v_2$$

$$= [N_1 \ N_2] \begin{Bmatrix} v_1 \\ v_2 \end{Bmatrix}$$

$$= [N]\{q\}$$



# One-Dimensional Stress Deformation

$$\Pi_p = \iiint_V \frac{1}{2} \sigma_y \epsilon_y dV - \iiint_V \bar{Y} v dV - \iint_{S_1} \bar{T}_y v dS - \sum_{i=1}^M P_{ii} v_i$$

$$\frac{AE}{l} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} v_1 \\ v_2 \end{Bmatrix} = \frac{Al\bar{Y}}{2} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} + \frac{\bar{T}_y l}{2} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} + \begin{Bmatrix} P_{1l} \\ P_{2l} \end{Bmatrix}$$

$$[\mathbf{k}]\{\mathbf{q}\} = \{\mathbf{Q}\}$$

$$[\mathbf{k}] = \frac{Al}{2} \int_{-1}^1 [\mathbf{B}]^T E [\mathbf{B}] dL$$

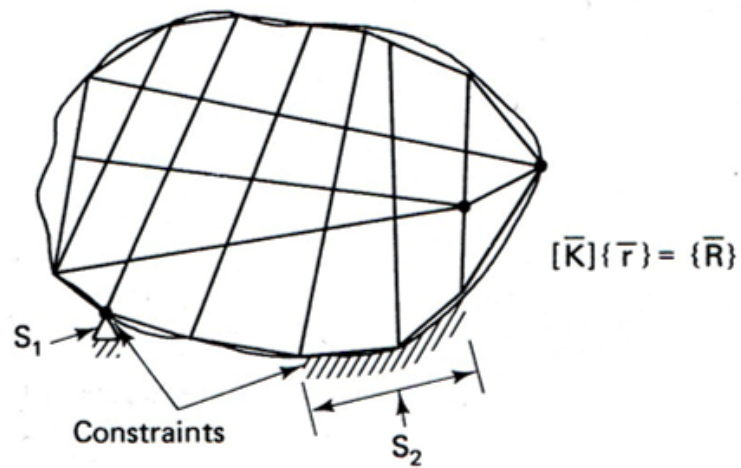
$$\{\mathbf{Q}\} = \frac{Al}{2} \int_{-1}^1 [\mathbf{N}]^T \bar{Y} dL + \frac{l}{2} \int_{-1}^1 [\mathbf{N}]^T \bar{T}_y dL + \{P_{il}\}$$

# Integrate elements into larger system

- Assemble equation for entire problem



## Apply boundary conditions



Body with constraints.

from C.S. Desai, 1979, *Elementary Finite Element Method*, (Prentice-Hall, Englewood Cliffs NJ).

# Solve simultaneous equations

- Solve for primary unknown
- Find secondary unknown(s)



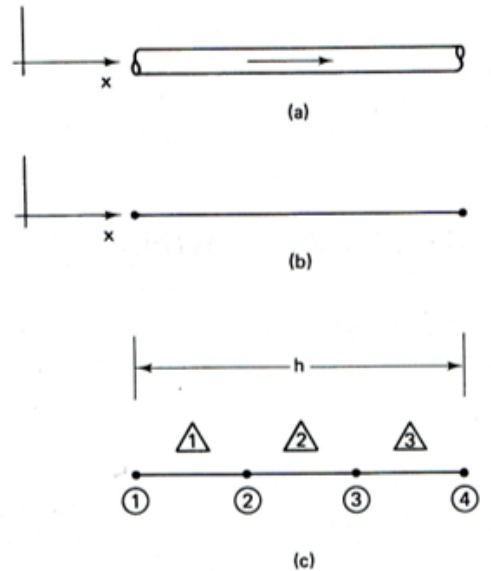
# Interpret Results



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# One-Dimensional Flow



**Figure 4-1** Idealization for flow in pipe. (a) Flow through pipe. (b) One-dimensional idealization. (c) Discretization in three elements.

from C.S. Desai, 1979, *Elementary Finite Element Method*, (Prentice-Hall, Englewood Cliffs NJ).

# One-Dimensional Flow

$$k_x \frac{\partial^2 \varphi^*}{\partial x^2} = f(x) = \bar{q}(x)$$

$$\varphi = \alpha_1 + \alpha_2 x$$

$$\varphi = N_1 \varphi_1 + N_2 \varphi_2 = [\mathbf{N}] \{\boldsymbol{\varphi}_n\}$$

$$g_x = \frac{\partial \varphi}{\partial x}$$

$$v_x = -k_x \frac{\partial \varphi}{\partial x} = -k_x g_x$$





# One-Dimensional Temperature

## Uncoupled problem

$$\epsilon_{y_0} = \alpha' T$$

$$\epsilon_{y_n} = \epsilon_y - \epsilon_{y_0}$$

$$\sigma_y = E \epsilon_{y_n} = E(\epsilon_y - \epsilon_{y_0})$$



# One-Dimensional Temperature

## Uncoupled problem

$$\Pi_p = \frac{A}{2} \int_{y_1}^{y_2} \sigma_y \epsilon_{y_n} dy - A \int_{y_1}^{y_2} \bar{Y}_y v dy - \int_{y_1}^{y_2} \bar{T}_y v dy - \sum P_{il} v_i$$

$$\begin{aligned} U' &= \frac{A}{2} \int_{y_1}^{y_2} E(\epsilon_y - \epsilon_{y_0})(\epsilon_y - \epsilon_{y_0}) dy \\ &= \frac{A}{2} \int_{y_1}^{y_2} E \epsilon_y^2 dy - A \int_{y_1}^{y_2} E \epsilon_y \epsilon_{y_0} dy + \frac{A}{2} \int_{y_1}^{y_2} E \epsilon_{y_0}^2 dy \\ &= U_1 + U_2 \end{aligned}$$



## One-Dimensional Temperature

$$\begin{aligned}U_2 &= A \int_{y_1}^{y_2} [v_1 \quad v_2] \frac{1}{l} \begin{Bmatrix} -1 \\ 1 \end{Bmatrix} E \epsilon_{y_0} dy \\&= A \{\mathbf{q}\}^T \int_{y_1}^{y_2} [\mathbf{B}]^T [\mathbf{C}] \{\epsilon_{y_0}\} dy \\&= \frac{AE \epsilon_{y_0}}{l} \int_{y_1}^{y_2} (-v_1 + v_2) dy \\&= \frac{AE \epsilon_{y_0}}{l} \frac{l}{2} \int_{-1}^1 (-v_1 + v_2) dL \\&= \frac{AE \epsilon_{y_0}}{2} [(-v_1 L)|_{-1}^1 + (v_2 L)|_{-1}^1] \\&= AE \epsilon_{y_0} (-v_1 + v_2)\end{aligned}$$

# One-Dimensional Temperature

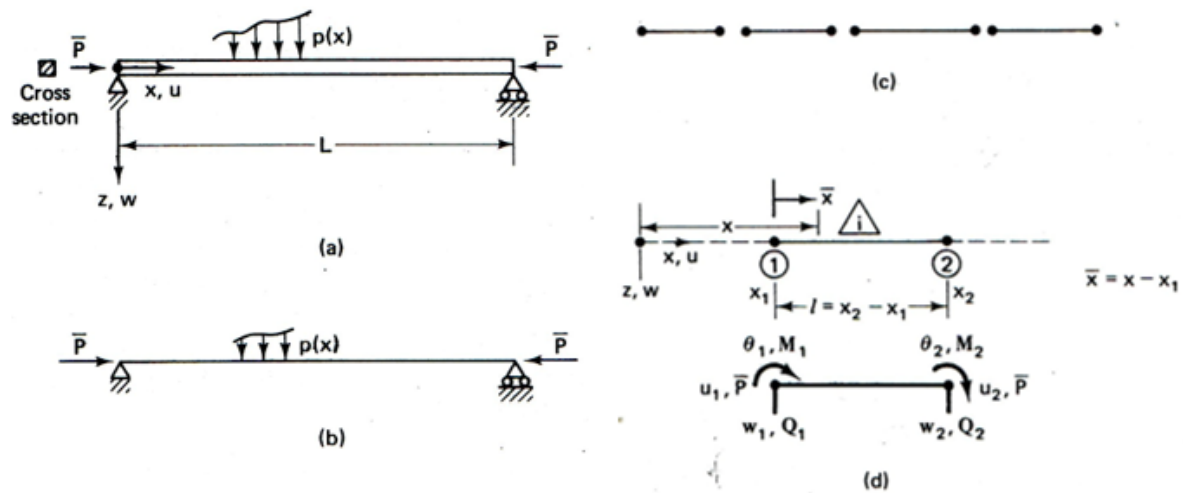
$$\{\mathbf{Q}_0\} = AE\epsilon_{y_0} \begin{Bmatrix} -1 \\ 1 \end{Bmatrix} = A\sigma_{y_0} \begin{Bmatrix} -1 \\ 1 \end{Bmatrix}$$

$$\{\mathbf{Q}_0\} = A \int_{y_1}^{y_2} [\mathbf{B}]^T [\mathbf{C}] \{\epsilon_{y_0}\} dy$$

$$[\mathbf{k}]\{\mathbf{q}\} = \{\mathbf{Q}\} + \{\mathbf{Q}_0\}$$



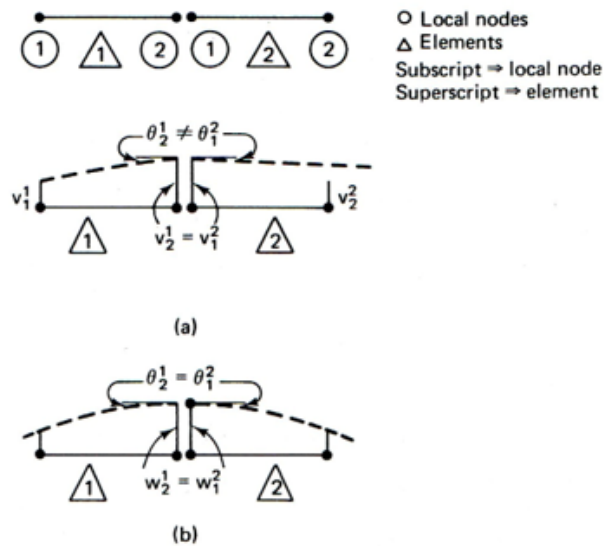
# Beam Bending and Beam-Column



**Figure 7-1** Beam bending and beam-column. (a) Beam with transverse and axial loads. (b) One-dimensional idealization. (c) Discretized beam. (d) Generic element.

from C.S. Desai, 1979, *Elementary Finite Element Method*, (Prentice-Hall, Englewood Cliffs NJ).

# Beam Bending and Beam-Column



**Figure 7-2** Requirements of interelement compatibility. (a) Inter-element compatibility for axial deformation (Chapter 3). (b) Inter-element compatibility for beam bending.

from C.S. Desai, 1979, *Elementary Finite Element Method*, (Prentice-Hall, Englewood Cliffs NJ).

# Beam Bending and Beam-Column

$$F \frac{d^4 w^*}{dx^4} = p(x)$$

$$w(x) = N_1 w_1 + N_2 \theta_1 + N_3 w_2 + N_4 \theta_2$$

$$w(x) = [\mathbf{N}]\{\mathbf{q}\}$$



# Beam Bending and Beam-Column

$$N_1 = 1 - 3s^2 + 2s^3$$

$$N_2 = ls(1 - 2s + s^2)$$

$$N_3 = s^2(3 - 2s)$$

$$N_4 = ls^2(s - 1)$$

$$w(x) = \alpha_1 + \alpha_2 x + \alpha_3 x^2 + \alpha_4 x^3$$

$$\Pi_p = \int_{x_1}^{x_2} \frac{1}{2} F(w'')^2 dx - \int_{x_1}^{x_2} p w dx$$

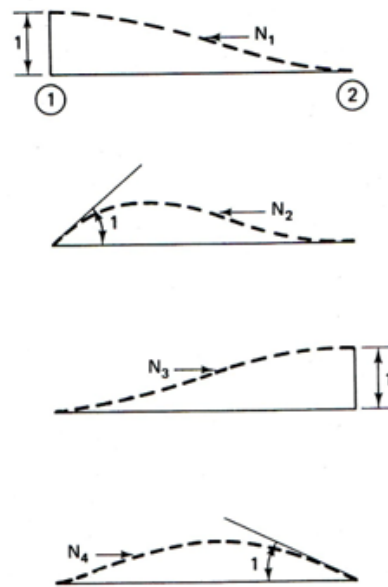


Figure 7-3 Plots of  $N_i$ ,  $i = 1, 2, 3, 4$ ,

from C.S. Desai, 1979, *Elementary Finite Element Method*, (Prentice-Hall, Englewood Cliffs NJ).



## Beam Bending and Beam-Column

$$[\mathbf{k}] = \frac{F}{l^3} \begin{bmatrix} 12 & 6l & -12 & 6l \\ & 4l^2 & -6l & 2l^2 \\ \text{symmetric} & & 12 & -6l \\ & & & 4l^2 \end{bmatrix}$$

$$\{\mathbf{Q}\} = \frac{l}{20} \begin{Bmatrix} 7p_1 + 3p_2 \\ \frac{l}{3}(3p_1 + 2p_2) \\ 3p_1 + 7p_2 \\ -\frac{l}{3}(2p_1 + 3p_2) \end{Bmatrix}$$

# Two-Dimensional Stress-Deformation Analysis

$$\{\sigma\} = \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}$$

$$\{\epsilon\} = \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

$$\{\sigma\} = [C]\{\epsilon\} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \{\epsilon\}$$

# Two-Dimensional Stress-Deformation Analysis

## Plane strain approximation

$$\{\sigma\} = \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} \quad \{\epsilon\} = \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

$$\sigma_z = \nu(\sigma_x + \sigma_y)$$

$$\{\sigma\} = [C]\{\epsilon\} = \frac{E}{(1-\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix} \{\epsilon\}$$

$$\{\epsilon\} = \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \end{Bmatrix}$$

# Two-Dimensional Stress-Deformation Analysis

## Axisymmetric approximation

$$\{\sigma\} = \begin{Bmatrix} \sigma_r \\ \sigma_\theta \\ \sigma_z \\ \tau_{rz} \end{Bmatrix} \quad \{\epsilon\} = \begin{Bmatrix} \epsilon_r \\ \epsilon_\theta \\ \epsilon_z \\ \gamma_{rz} \end{Bmatrix}$$

$$\{\sigma\} = [C]\{\epsilon\} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & 0 \\ \nu & 1-\nu & \nu & 0 \\ \nu & \nu & 1-\nu & 0 \\ 0 & 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix} \{\epsilon\}$$

$$\{\epsilon\} = \begin{Bmatrix} \epsilon_r \\ \epsilon_\theta \\ \epsilon_z \\ \gamma_{rz} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u}{\partial r} \\ \frac{u}{r} \\ \frac{\partial w}{\partial z} \\ \frac{\partial u}{\partial z} + \frac{\partial w}{\partial r} \end{Bmatrix}$$



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# Two-Dimensional Stress-Deformation Analysis

## Finite element formulation

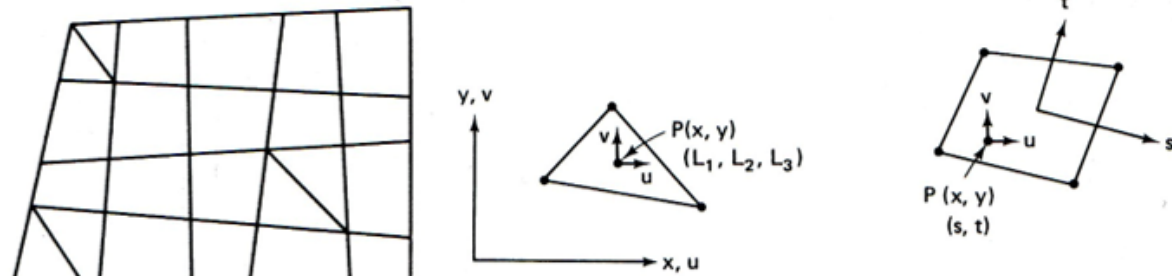


Figure 13-4 Discretization with triangular and quadrilateral elements.

$$\begin{aligned}u(x, y) &= \alpha_1 + \alpha_2 x + \alpha_3 y + \alpha_4 xy \\v(x, y) &= \beta_1 + \beta_2 x + \beta_3 y + \beta_4 xy\end{aligned}$$

from C.S. Desai, 1979, *Elementary Finite Element Method*, (Prentice-Hall, Englewood Cliffs NJ).

# Two-Dimensional Stress-Deformation Analysis

$$N_1 = \frac{1}{4}(1 - s)(1 - t)$$

$$N_2 = \frac{1}{4}(1 + s)(1 - t)$$

$$N_3 = \frac{1}{4}(1 + s)(1 + t)$$

$$N_4 = \frac{1}{4}(1 - s)(1 + t)$$



# Two-Dimensional Stress-Deformation Analysis

$$x = \sum_{i=1}^4 N_i x_i \quad i = 1, 2, 3, 4$$

$$y = \sum_{i=1}^4 N_i y_i \quad i = 1, 2, 3, 4$$

$$\begin{matrix} \begin{Bmatrix} x \\ y \end{Bmatrix} \\ (2 \times 1) \end{matrix} = \begin{bmatrix} [\mathbf{N}] & [\mathbf{0}] \\ [\mathbf{0}] & [\mathbf{N}] \end{bmatrix} \begin{Bmatrix} \{x_n\} \\ \{y_n\} \end{Bmatrix} \begin{matrix} (2 \times 8) & (8 \times 1) \end{matrix}$$

# Two-Dimensional Stress-Deformation Analysis

$$[\mathbf{k}] \simeq h \sum_{i=1}^N [\mathbf{B}(s_i, t_i)]^T [\mathbf{C}] [\mathbf{B}(s_i, t_i)] |J(s_i, t_i)| W_i$$

$$\{\mathbf{Q}_1\} = h \sum_{i=1}^N [N(s_i, t_i)]^T \{\bar{\mathbf{X}}\} W_i$$





# Construction sequence FEM

## Initial set up – embankment

- When simulating the stresses on an embankment, it is important to model the construction process.
- Create a mesh for the existing system.
- Compute the initial stresses due to the weight
  - In many cases, if you assume a reasonable modulus, you will have a realistic level of stress
- After calculating the initial stresses, set the initial strains and displacements to 0.
  - These developed over an essentially infinite time and have no impact on the final result.



# Construction sequence FEM

## Adding layers to embankment

- Model adding a layer of soil by adding a layer(s) of elements to the mesh
  - Simulate with nonlinear nodal forces
  - Use the nonlinear constituent laws to compute stresses, strains, and displacements.
  - Adjust mesh.
  - Update constituent laws/material properties in response to new conditions
    - strains and displacements are no longer 0, so stress-strain relationship will be different
- Add additional layers of soil as above until full embankment is constructed
  - Material properties are adjusted with each layer



# Construction sequence FEM

## Simulating excavation

- Excavation is the reverse of embankment
  - Create initial mesh
  - Remove a layer of soil from the mesh
  - Adjust material properties following nonlinear constitutive relations
  - Iterate by removing layers and adjusting properties until excavation is complete



# Interface elements

- Interface elements are crucial in soil-structure interaction problems
- Interface elements are located where one material is adjacent to another
  - For example pipe (e.g. thermoplastic) and soil
- Interface elements are nonlinear
  - Allow separation of materials
  - Allow slippage





# Create for Good.



# CANDE 2007 TUTORIAL

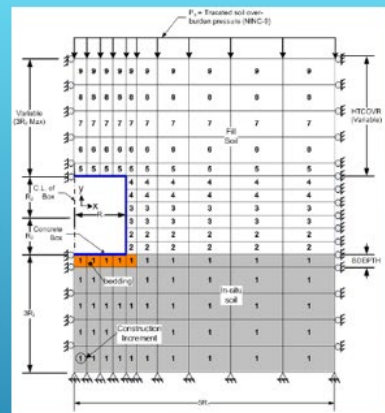
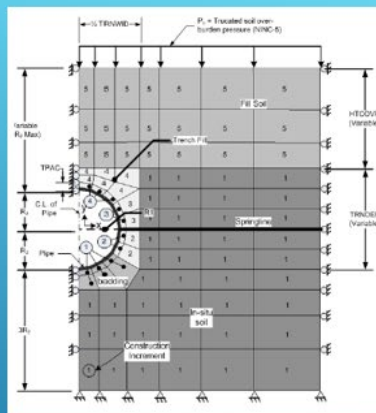
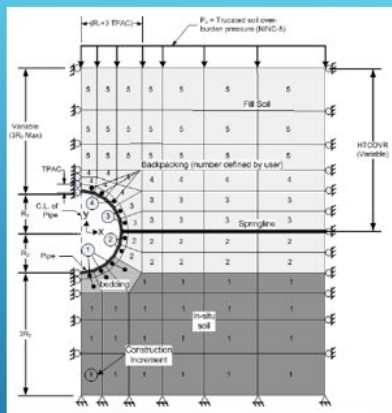
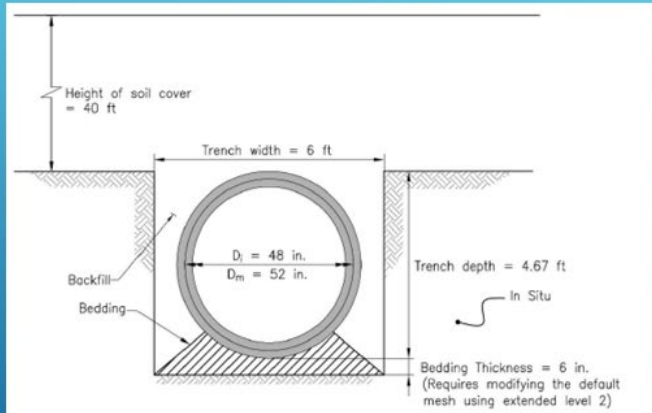
Introduction to Finite Element Analysis

Ohio Department of Transportation

- ▶ Analyze a 48-inch corrugated steel pipe
  - ▶ Service load
  - ▶ Level 2 analysis
    - ▶ Modified automatic meshing
      - ▶ 6-inch bedding
      - ▶ Haunch zone material
  - ▶ Trench installation

## STATEMENT OF THE PROBLEM

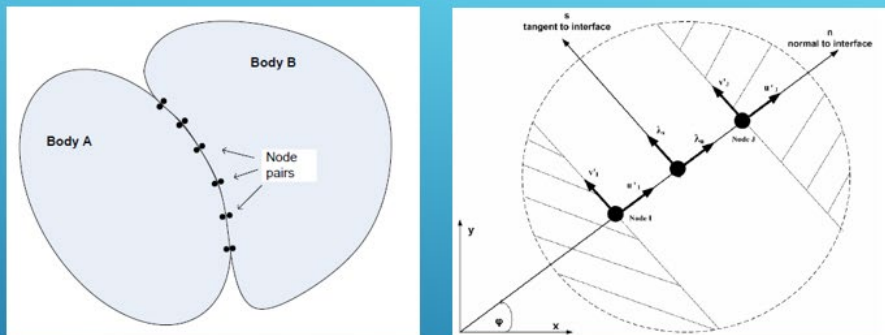
## PROBLEM DETAILS



## CANDE AUTOMATED MESH

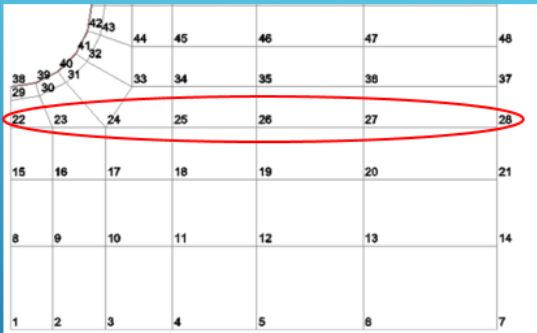
- ▶ Analysis method
- ▶ Analyze at service load
- ▶ Level 2 solution
- ▶ Canned pipe mesh
- ▶ Trench mesh
- ▶ Pipe-soil interface elements
- ▶ Modified level 2 mesh
  - ▶ 7 nodes will have changed coordinates
  - ▶ 2 elements with new material properties

## STATEMENT OF THE PROBLEM



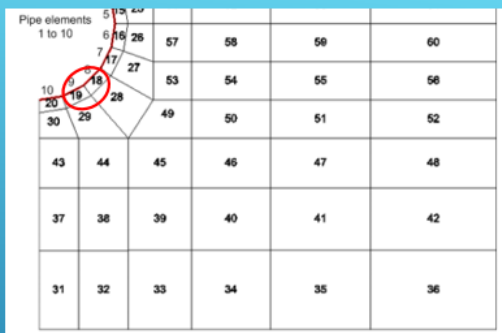
## INTERFACE ELEMENTS





Node Numbers	X-coordinate	Y-coordinate
22	0	-30
23	12.47	-30
24	28.19	-30
25	47.99	-30
26	72.95	-30
27	104.38	-30
28	144	-30

## NODE CHANGES



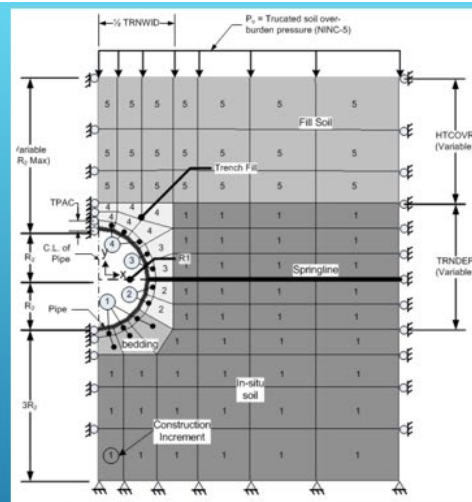
Element Number	Node I	Node J	Node K	Node L	Material	Const. Step
18	49	46	31	32	5	1
19	46	43	30	31	5	1

## ELEMENT CHANGES

- ▶ 2 2/3" X 1/2" corrugation profile
  - ▶ Young's modulus = 29,000,000 psi
  - ▶ Yield stress = 33,000 psi
  - ▶ Poisson's ratio = 0.3
  - ▶ Area of pipe wall = 0.0806 in<sup>2</sup>/in
  - ▶ Moment of inertia of pipe wall = 0.0019 in<sup>4</sup>/in
  - ▶ Section modulus of pipe wall = 0.00667 in<sup>3</sup>/in
- ▶ Soil properties
  - ▶ Good cohesive soil for in situ material
  - ▶ Good granular soil for bedding and backfill
  - ▶ Good cohesive soil for overfill
- ▶ Small deformation analysis

## STATEMENT OF THE PROBLEM

- ▶ 48-inch pipe diameter
- ▶ Trench width of 1.5\*D = 6 ft
- ▶ Trench depth = 4.67 ft
- ▶ Height of soil = 40 ft (above trench)
- ▶ Density of soil above mesh = 120 pcf
- ▶ 10 load steps
  - ▶ 5 inherent in canned mesh
  - ▶ 5 for boundary loads



## STATEMENT OF THE PROBLEM

**Main Input Control Parameters**

### Control Information

Type of analysis

☒ Analysis

☐ Design

Method of analysis/design

☐ LRFD

☒ Service

Solution level

☐ Elasticity (Level 1)

☒ FEM-auto mesh (Level 2)

☐ FEM-user mesh (Level 3)

☒ Use the auto-generate option for the interface elements

1  Number of pipe element groups (Level 3 only)

New input file

Heading for output

Level 2 Specific

Canned mesh type

☒ Pipe mesh

☐ Box mesh

☐ Arch mesh

Soil mesh pattern

☐ Embankment

☒ Trench

☐ Homogenous

Interface elements (pipe only)

☒ Pipe-soil

☐ Trench-insitu

☐ None

☒ MOD-Make changes to the basic mesh

7  Number of nodes to change

2  Number of elements to change

0  Number of new loading/boundary conditions

## CANDE 2007 Input Wizard

[Welcome to the CANDE input Wizard!](#)

You will enter some basic information about your model and CANDE will prepare a starter input document that you can customize for your particular model. After you complete the input for each screen in the Input Wizard, press the 'Next' button until you have reached the end. Once completed, press the 'Finish' button to enter the CANDE input menus. [Control Information](#)

On the control information screen, enter key information regarding the type of model, method of analysis, etc.

**Main Input Control Parameters**

### Pipe Material 1

Pipe material type

☐ Aluminum

☐ Basic

☐ Concrete

☐ Plastic

☒ Steel

1  Number of connected beam elements

Concrete specific input

Reinforcement shape

☒ Standard

☐ Elliptical

☐ Arbitrary

☐ Boxes

Plastic specific input

Wall section type

☒ Smooth (design and analysis)

☐ General (analysis only)

☐ Profile (analysis only)

Steel specific input

Joint slip

☒ No

☐ Yes

☐ Yes, show trace

Vary joint travel length

☒ Same lengths

☐ Different lengths

1  Number of joints

## CANDE 2007 Input Wizard

[Pipe Material Information](#)

Enter information on this screen related to the Pipe Material chosen. For Level 1 and 2 type models, only one pipe material is entered.

For Level 3 models, this screen will be repeated N times, where N is the "Number of pipe element groups" entered on the "Control Information" screen.

As you change your input on this screen input will be enabled or disabled depending on the applicability for the material chosen.

Enter the soil material information

### Soil Properties

	Soil Material Model	Select 'canned' or 'User' soil parameters (Soil models 3, 4, and 5 only)
► Soil 1-in situ	4-Overburden dependent	Canned
Soil 2-bedding	4-Overburden dependent	Canned
Soil 3-backfill	4-Overburden dependent	Canned
Soil 4-overfill	4-Overburden dependent	Canned

## CANDE 2007 Input Wizard

[Soil Properties Information](#)  
Enter information on this screen related to the Soil Properties. This screen is only applicable for Levels 2 and 3.  
For Level 2 models, the number of soil models is predetermined by CANDE.  
For Level 3 models, the number of soil models is input on the "Level 3 Information" screen.  
Set the Soil Material Model type along with information related to the type chosen. Specific soil names and properties will input on the main CANDE input screens once the input wizard has completed the initial preparation of the

Input complete

## Input Complete!

### Click 'Finish'

## CANDE 2007 Input Wizard

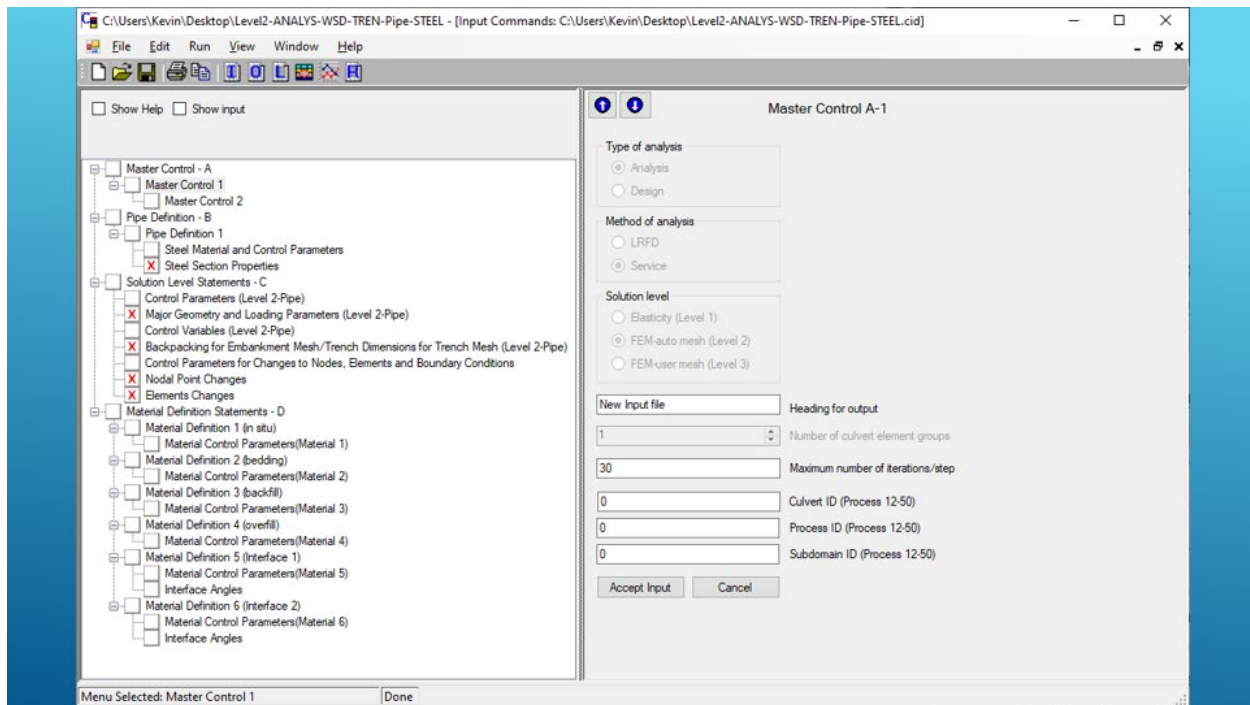
[You have completed your input document with the CANDE input Wizard!](#)  
Click on the 'Finish' button to enter the CANDE input screens.

[Completing your input](#)

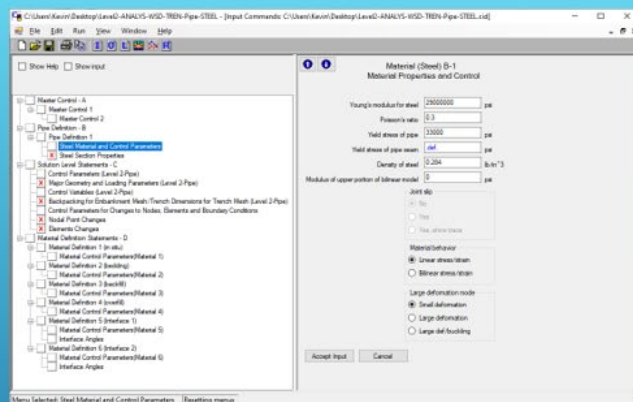
- ☐ Solution Level Statements - C
- ☐ Control Parameters (Level 2-Box)
- ☒ Control Variables and Installation Dimensions (L
- ☐ Control Parameters for Changes to Nodes, Eler

Review 'undefined' v

<< Prev   Next >>   Finish   Cancel   Press 'F1' for help

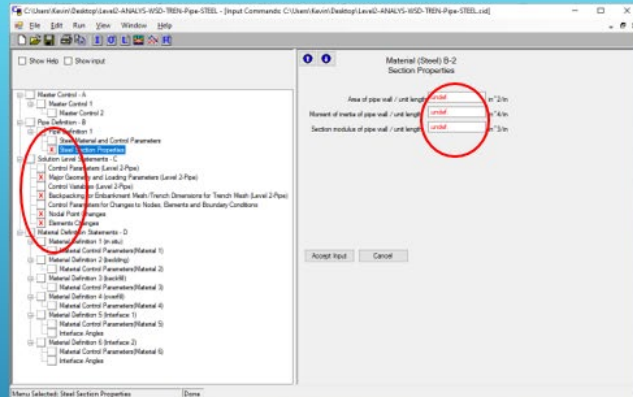


- ▶ Left pane is the control panel
- ▶ Right pane is the input panel
- ▶ Note “.def” in input panel
  - ▶ Default value – must verify appropriateness

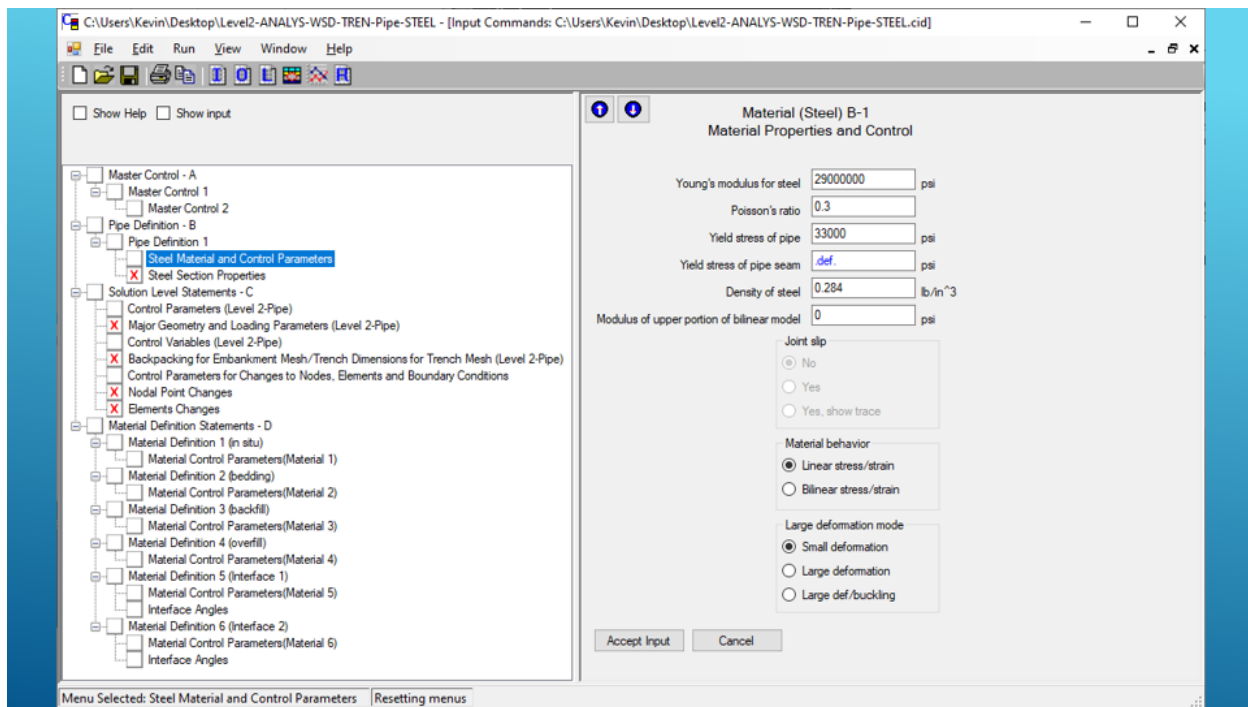


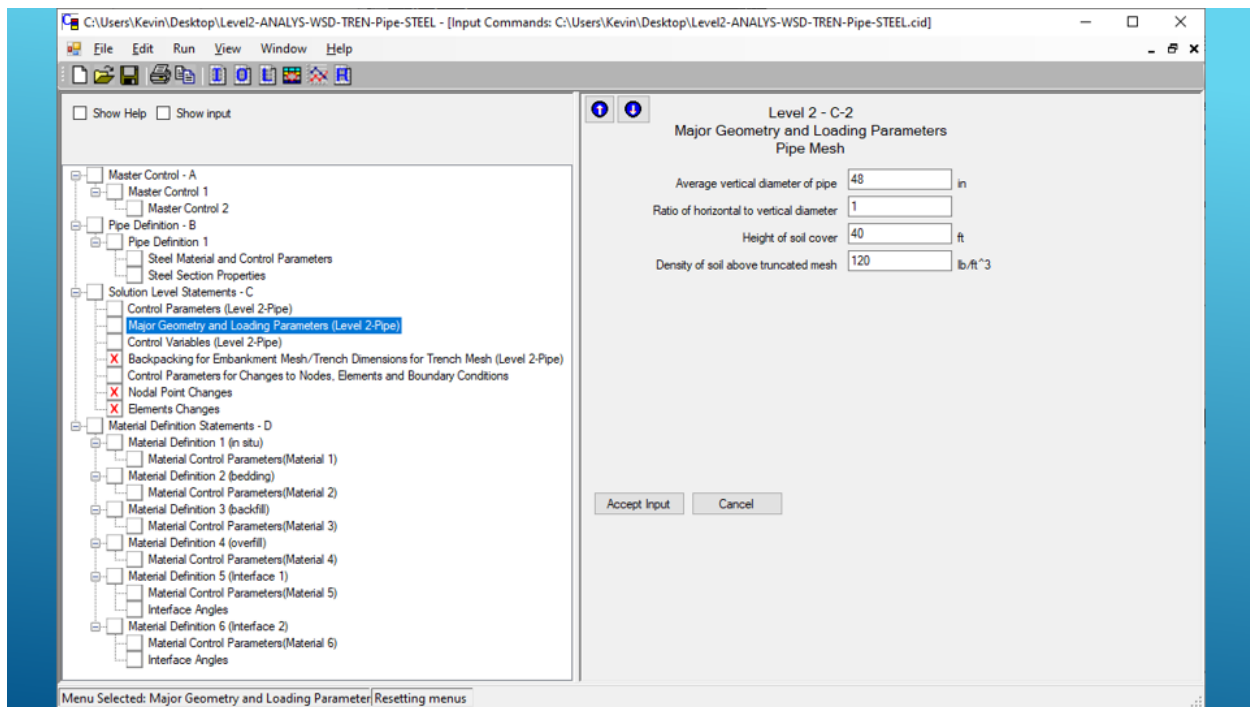
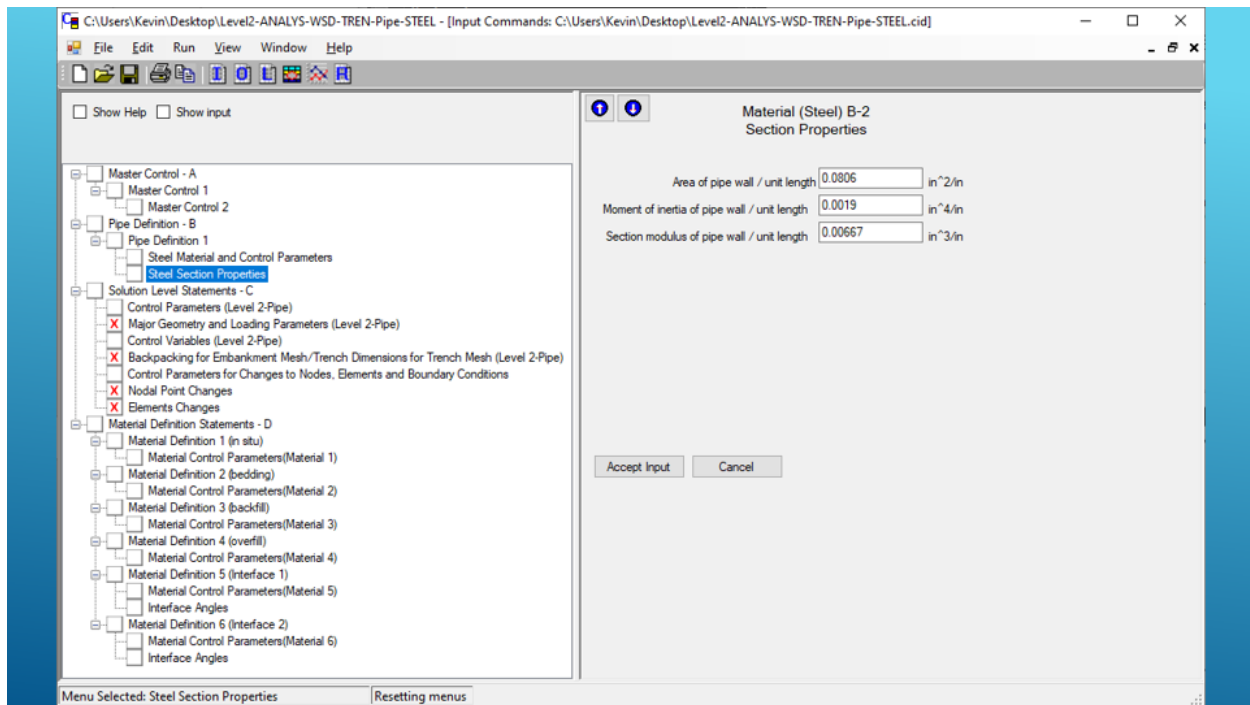
## CANDE INTERFACE

- ▶ Left pane is the control panel
- ▶ Right pane is the input panel
- ▶ Note "X" in control panel
  - ▶ All undefined parameters must be reconciled

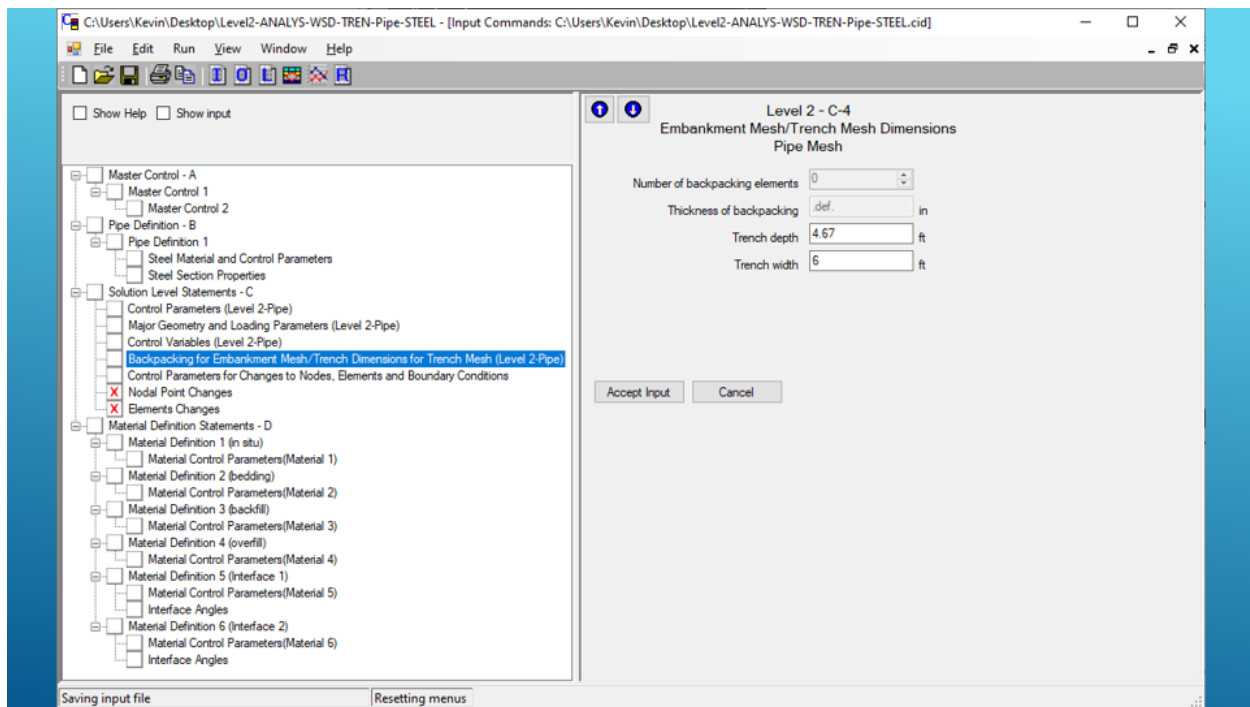
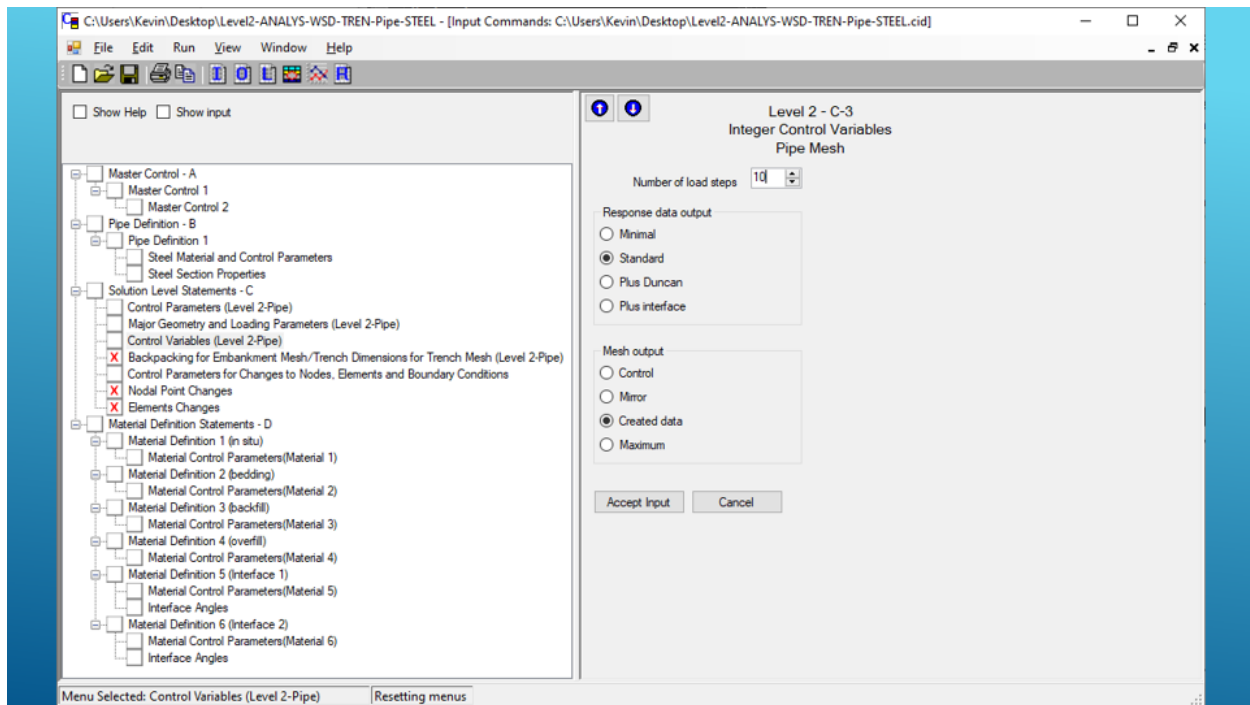


## CANDE INTERFACE

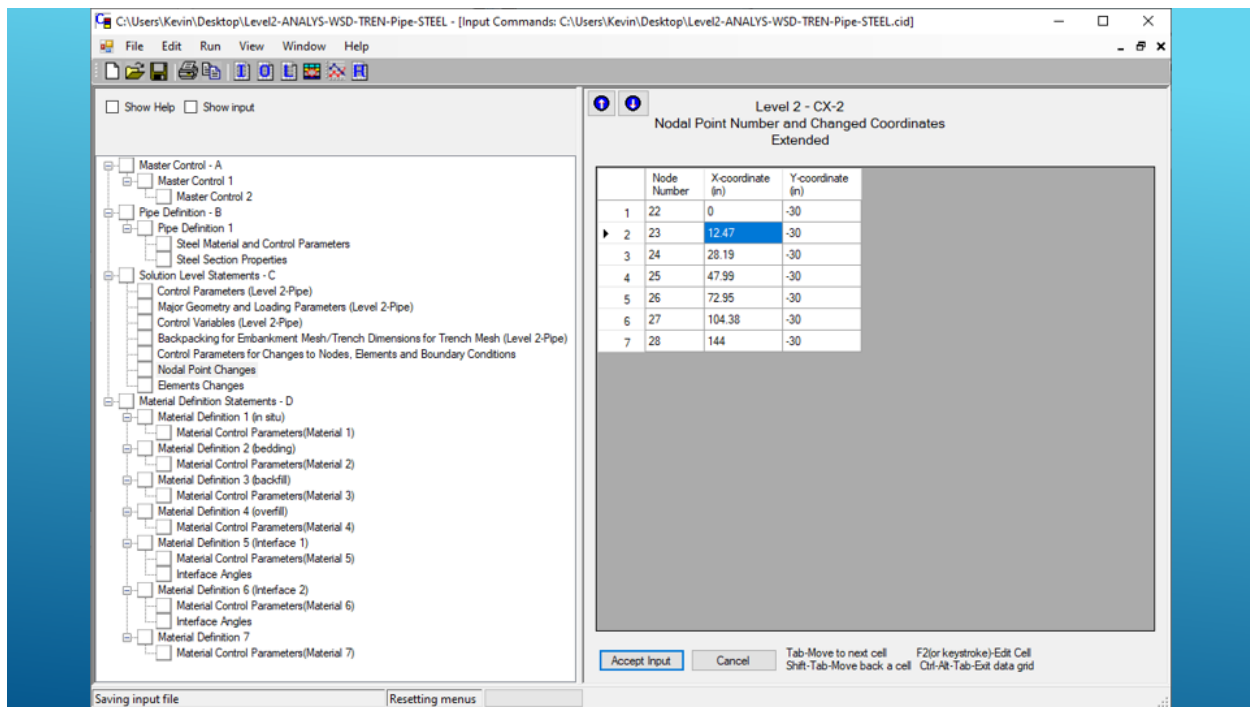
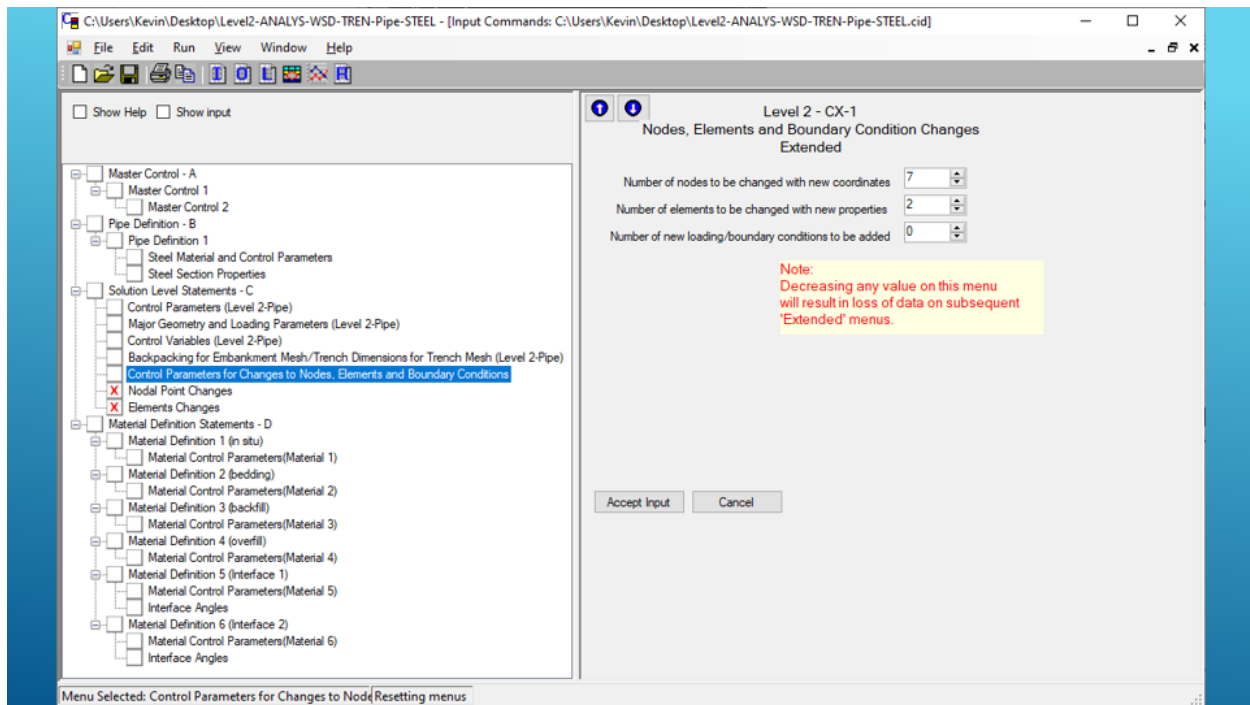


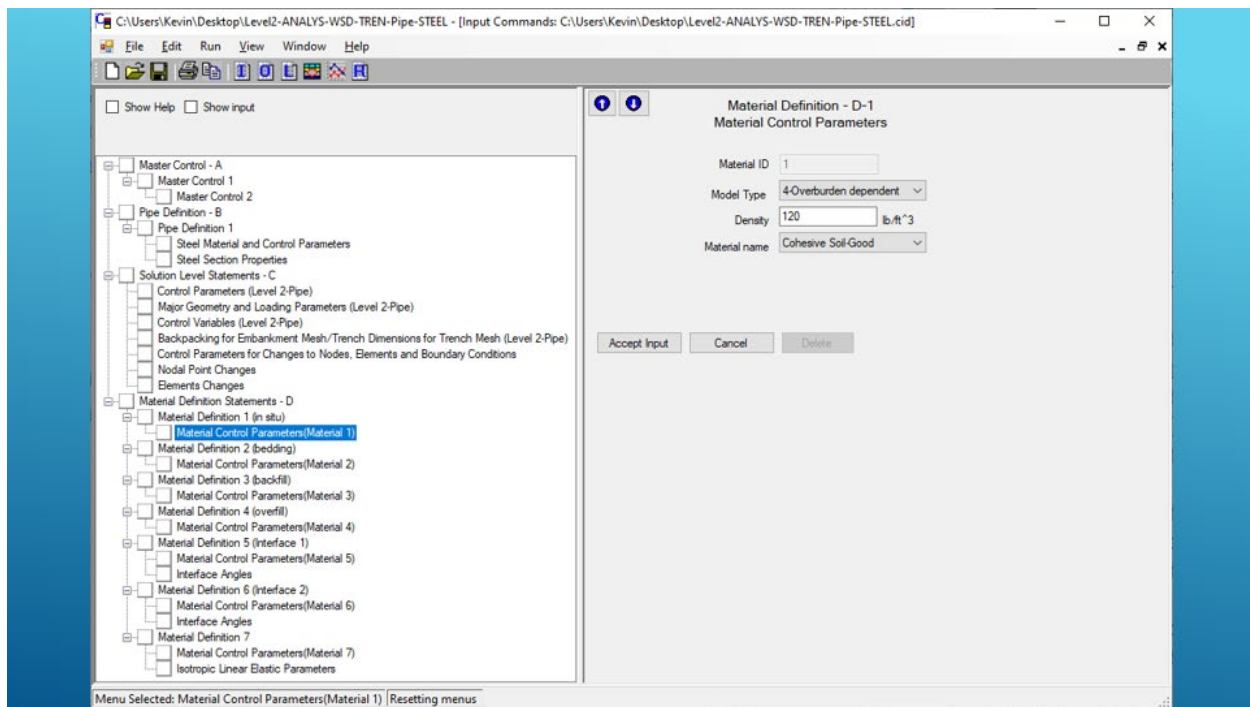
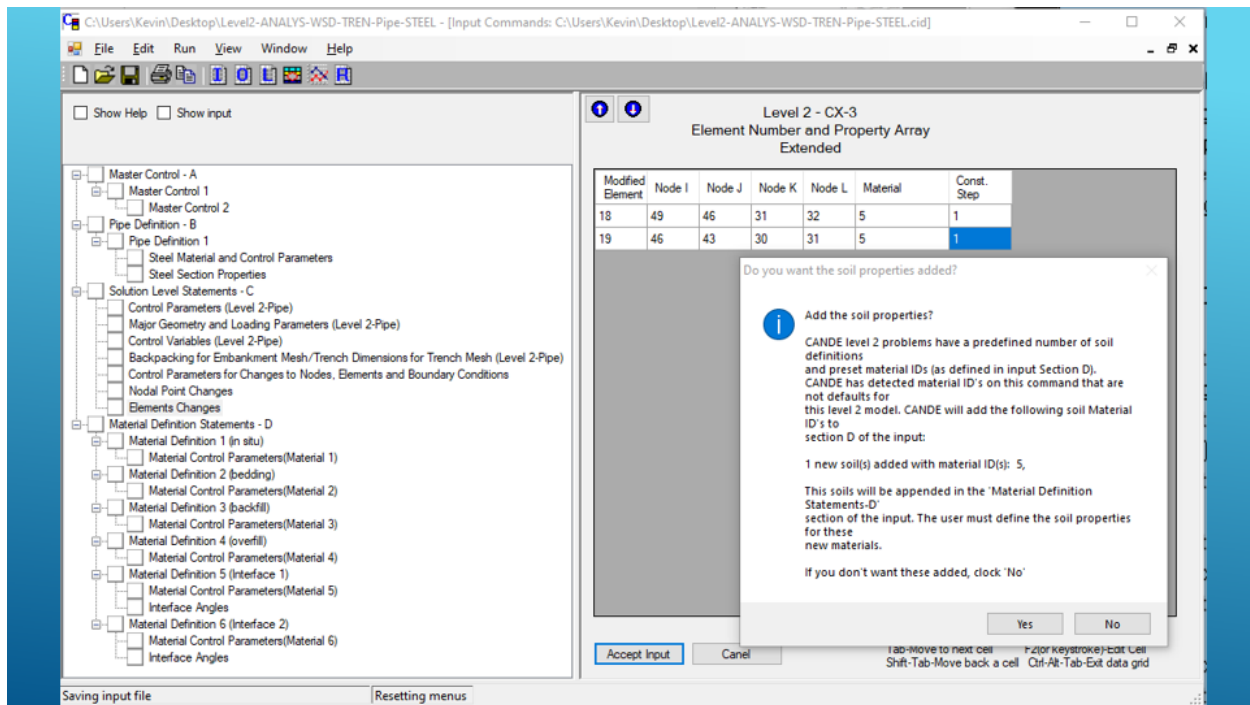


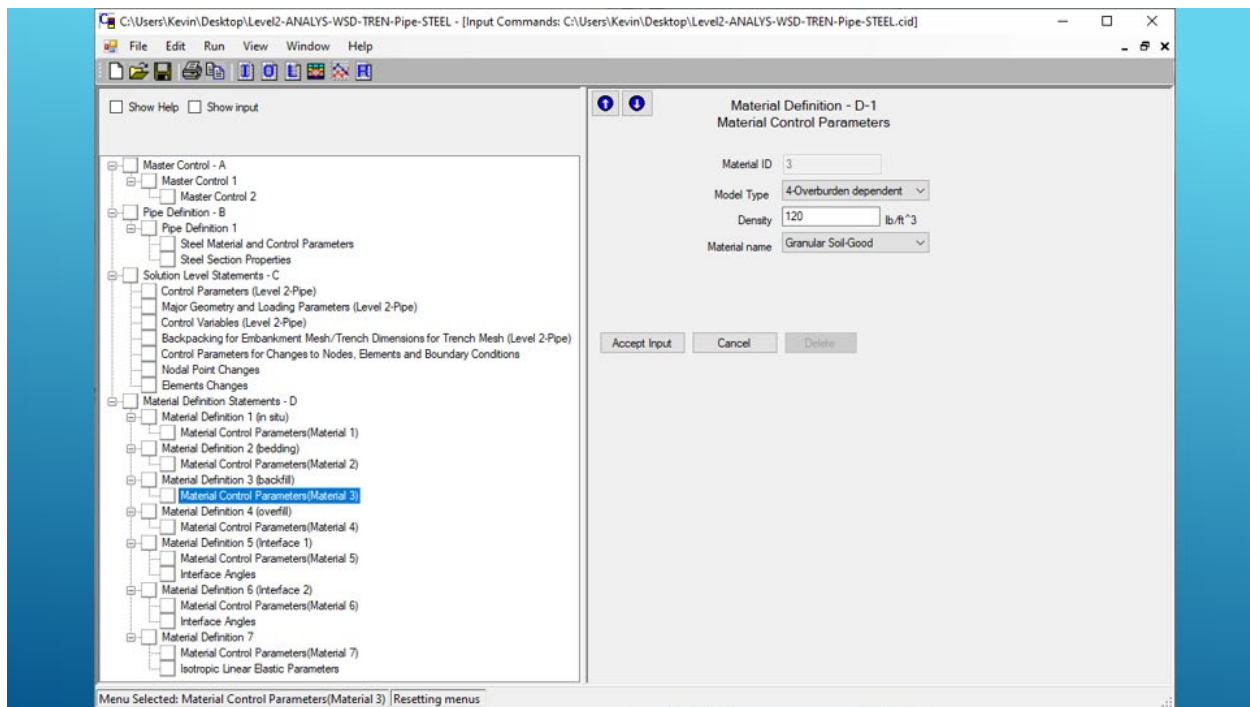
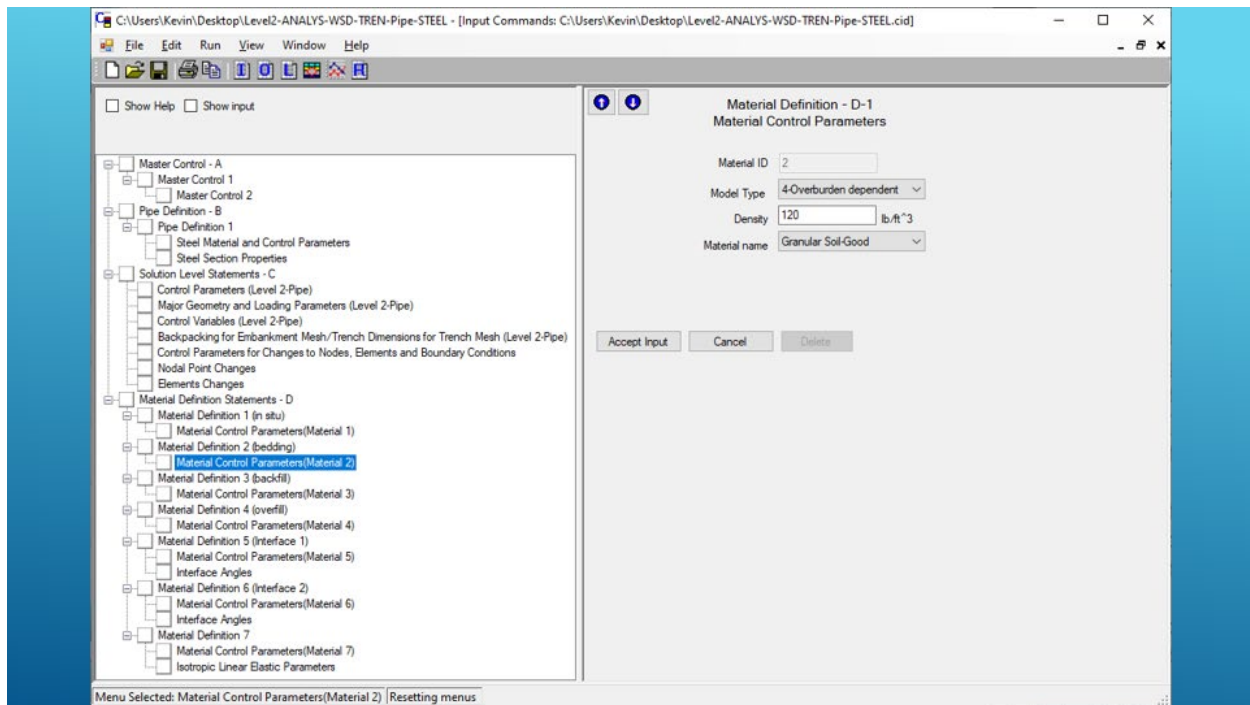


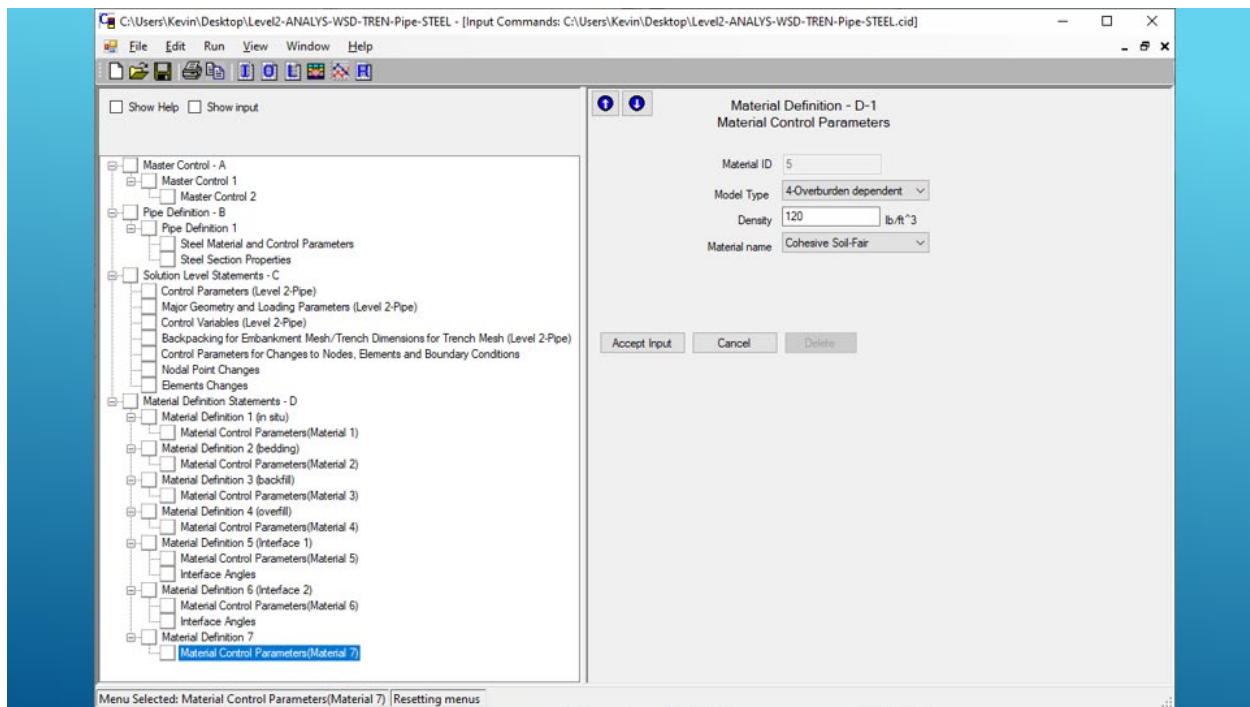
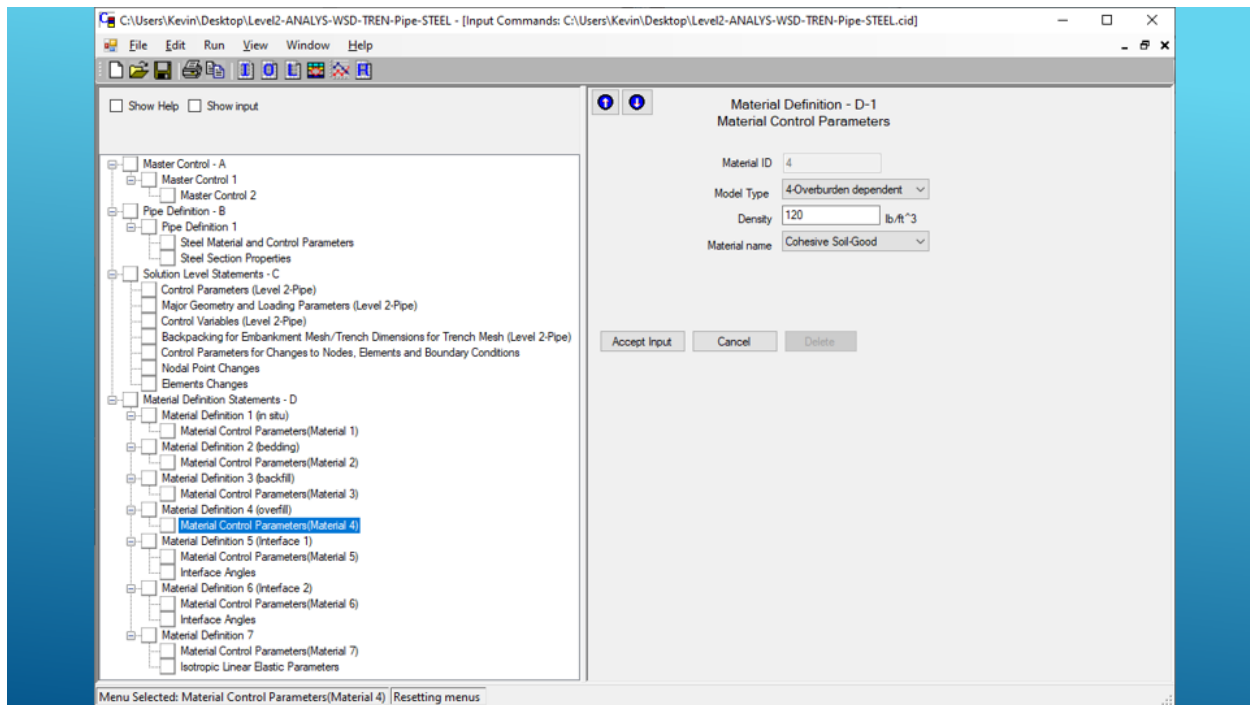


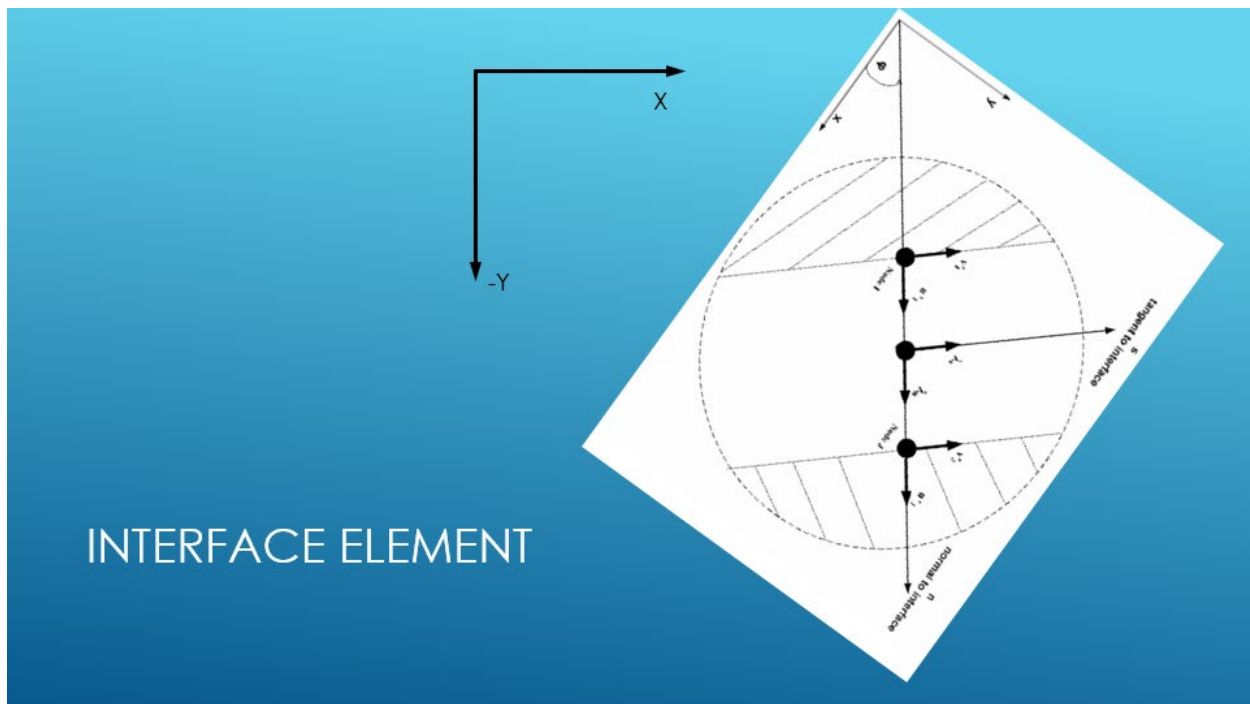
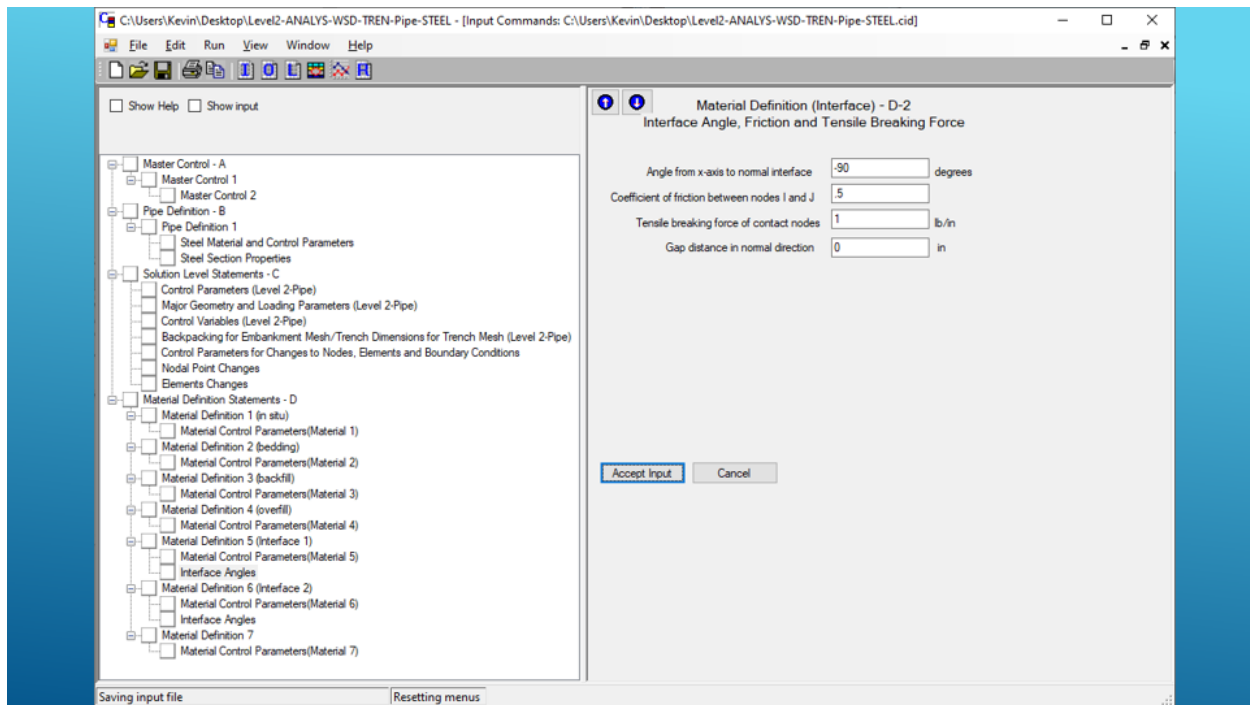


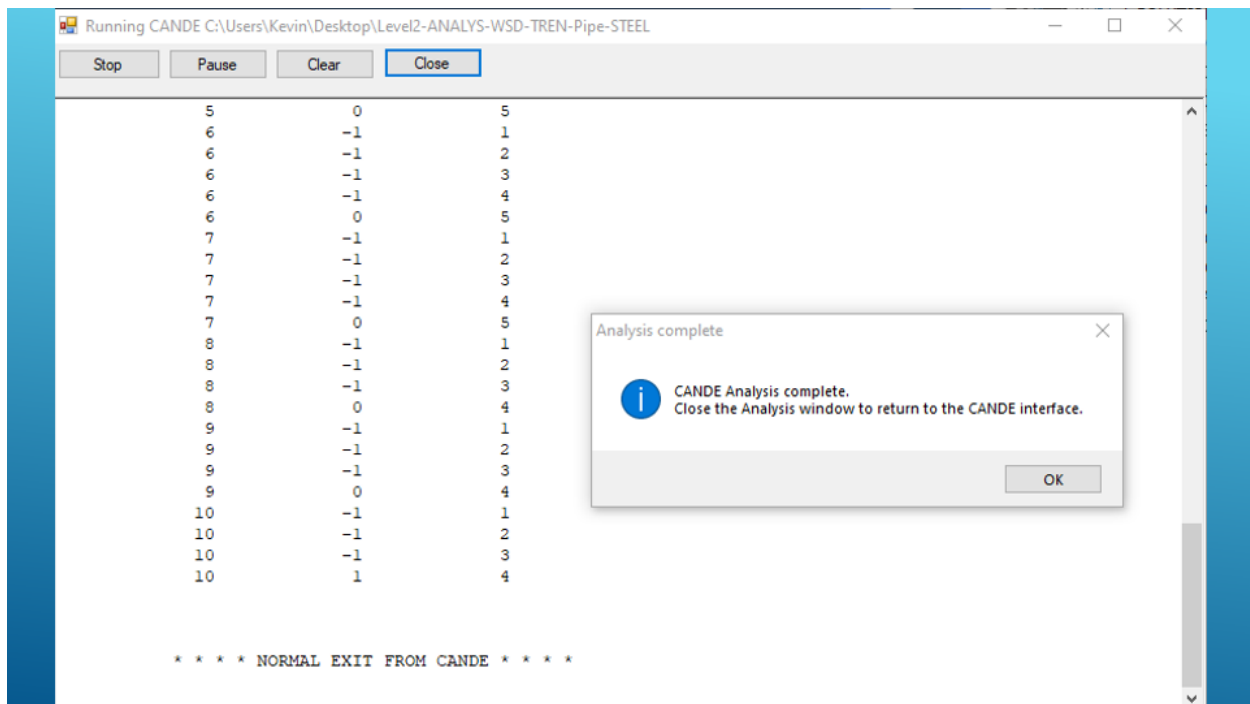
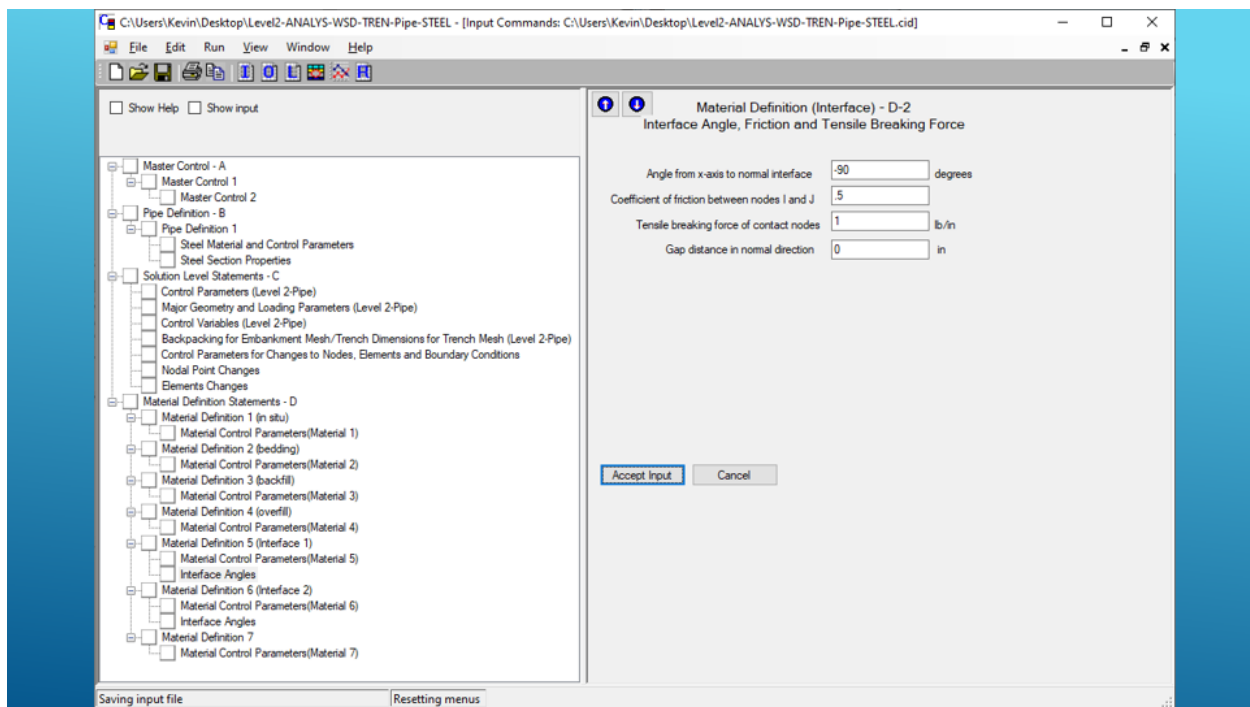






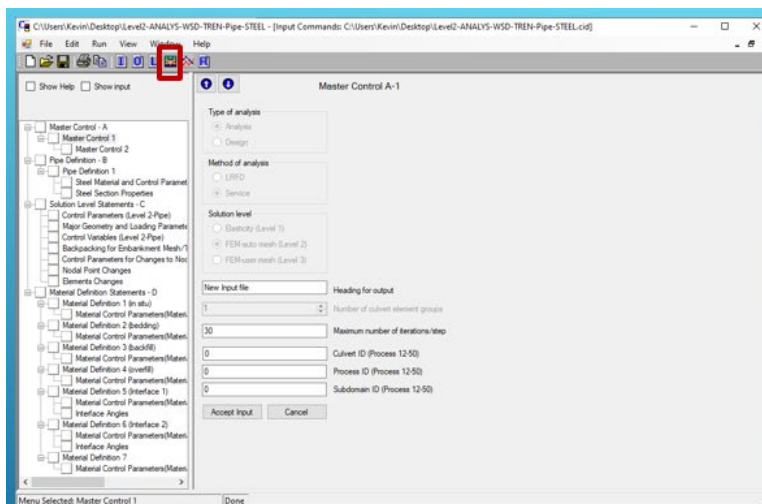




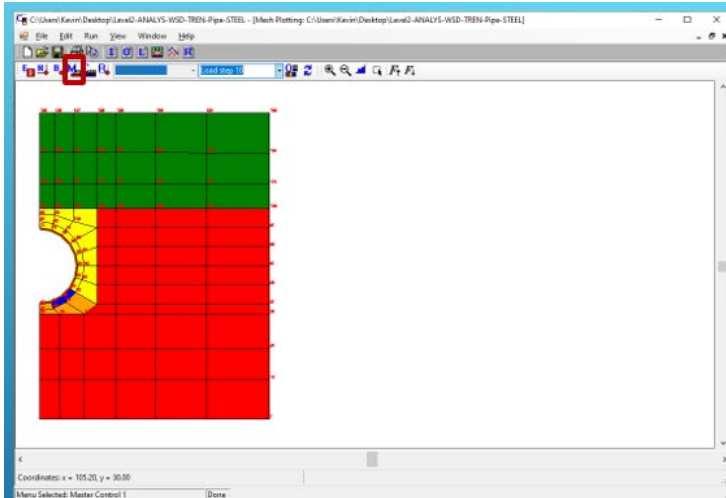




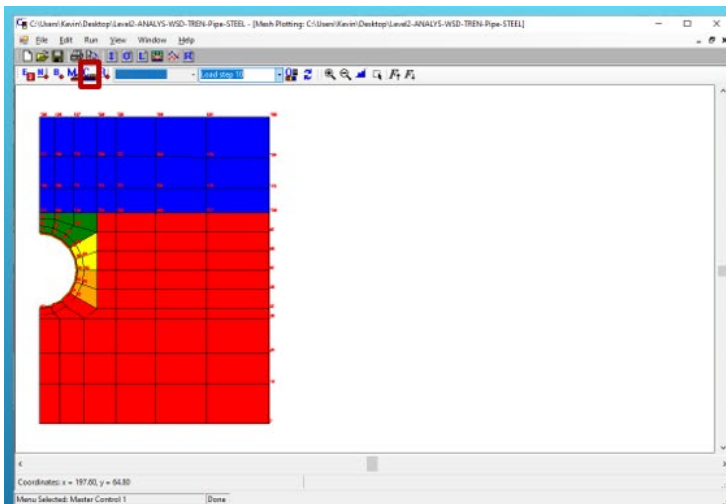
## POST PROCESSING - RESULTS



## VIEW MESH PLOT

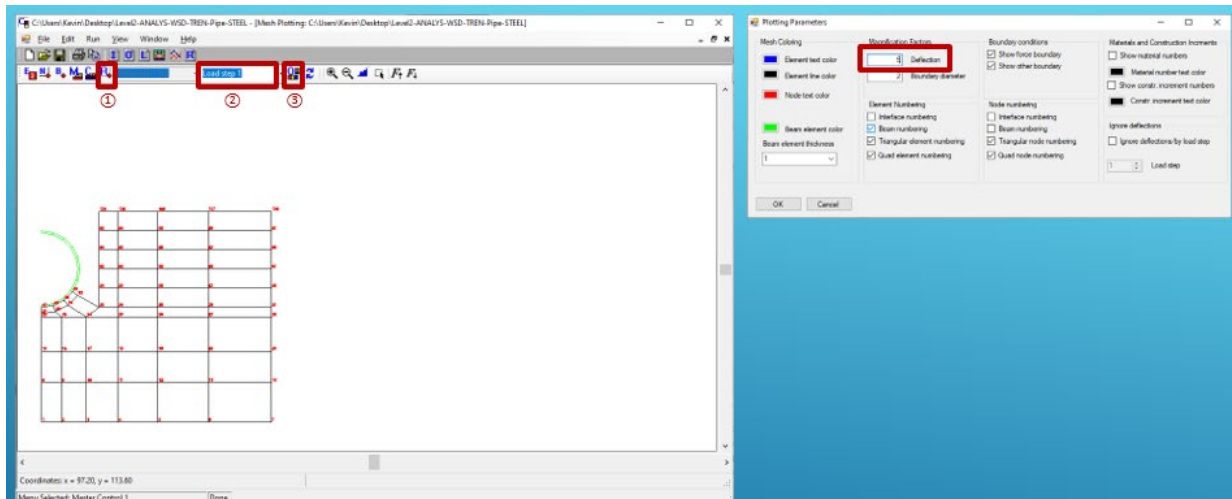


MESH PLOT - MATERIALS

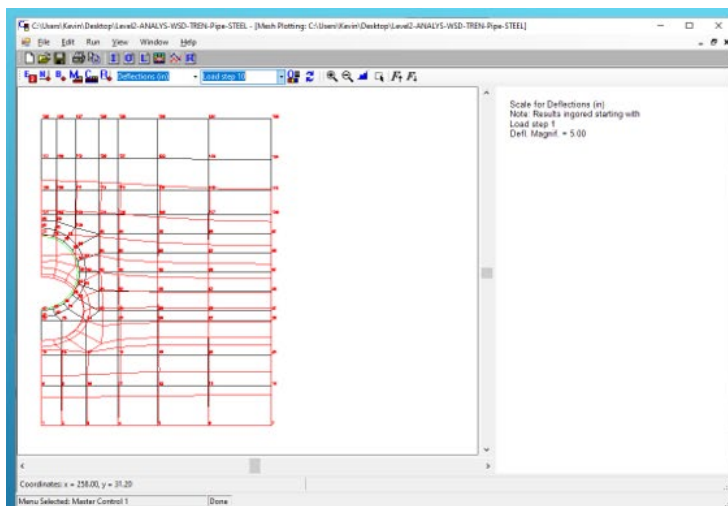


MESH PLOT – CONSTRUCTION INCREMENT

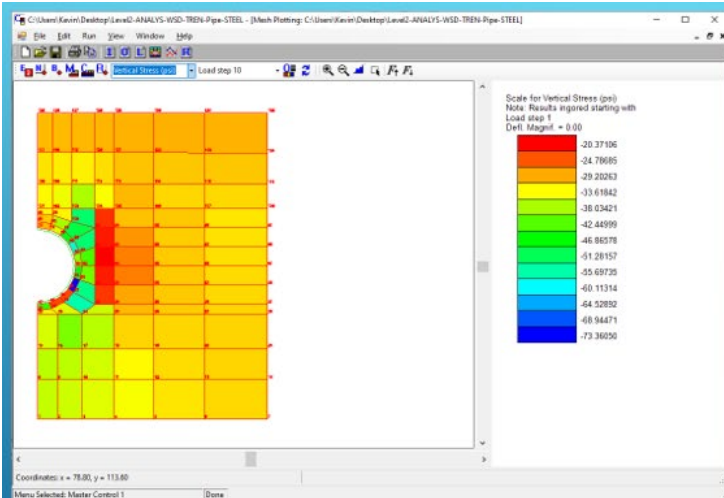




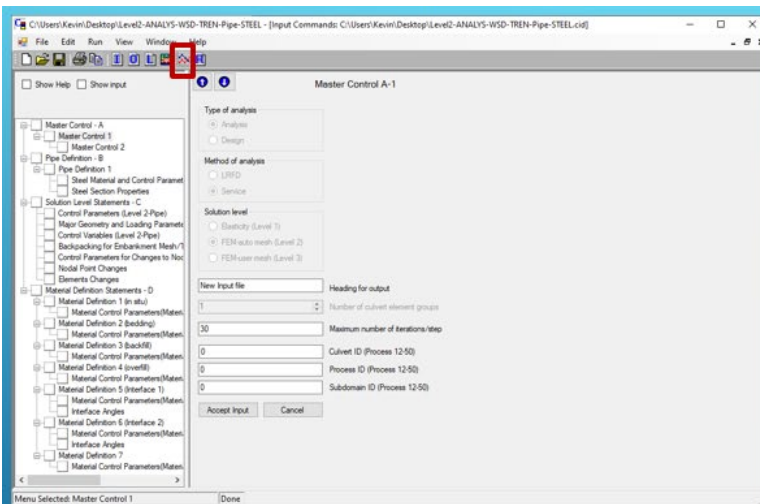
## MESH PLOT – PLOTTING PARAMETERS



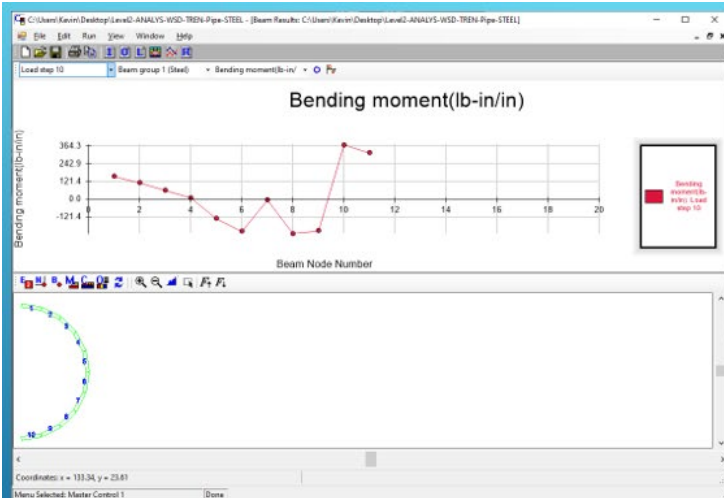
## MESH PLOT - DEFORMATIONS



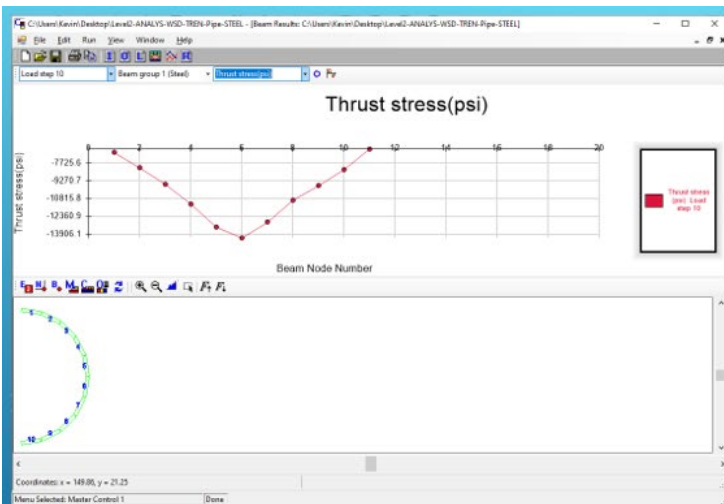
## MESH PLOT – VERTICAL STRESS



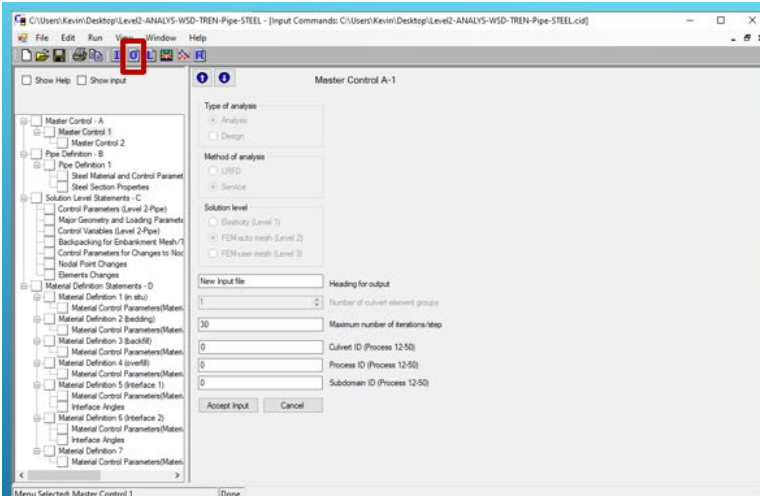
## VIEW CANDE GRAPHS



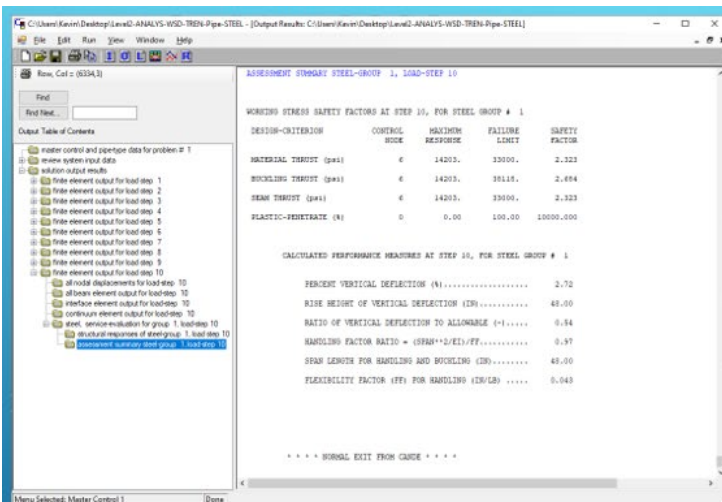
## CANDE GRAPHS – BENDING MOMENT



## CANDE GRAPHS – THRUST STRESS



## VIEW CANDE OUTPUT



## CANDE OUTPUT