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DEVELOPMENT OF MULTIVARIATE EXPOSURE AND FATAL ACCIDENT INVOLVEMENT RATES FOR 1977
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October 1985

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## EXECUTIVE SUMMARY

The need for multivariate accident involvement rates is often encounted in accident analysis. The FARS (Fatal Accident Reporting System) files contain records of fatal involvements characterized by many variables while NPTS (National Personal Transportation Survey) contains reports of trip records similary characterized by many variables. When the classificatory variables available in both data bases are examined the following are indentified as of the most interest in accident analysis:

Driver Age<br>Driver Sex<br>Vehicle Weight<br>Vehicle Age<br>Land Use (urban/rural)<br>Season (winter/summer)<br>Time of Day<br>Number of Occupants

The fatal involvement data (from FARS) and the VMT data (from NPTS) were separately classified by these variables (each limited to two or three levels). Missing data was accounted for. Missing weight data was estimated, where possible, based on make, model, and/or other vehicle characteristics.

Log-linear models were fit to the classified data to improve the accuracy and statistical stability. Esimates of standard errors were produced by sample splitting techniques. Specifically, random repeated replications were used. Standard errors were calculated for the fatal involvement estimates, for VMT estimates and for fatal involvement rates (the ratio of fatal involvements to VMT). Tables 3.2, 3.4 and 3.5 give multivariate estimate of fatal involvement, VMT, and fatal involvement rates respectively from log-linear models while Tables 4.1, 4.2, and 4.3 give the respective standard errors.

The fatal involvement rate estimates were used to study the question of whether small cars or large cars are more involved in fatal accidents controlling for driver and amount and type of driving. It was concluded that small cars are
overinvolved in fatal accidents overall, but in urban situations especially with younger drivers and perhaps with newer cars, the results suggested that smaller cars are less involved than larger cars in fatal accidents. Note that a fatal collision between a small car and a large car counts as a fatal involvement for both cars regardless of which vehicle had a fatality in it. Consequently, a lower fatal involvement rate for small cars in some situation does not imply that smaller cars are "safer" in those situations. A lower fatal involement rate for a type of car means a lower tendency to be involved in fatal accidents and does not at all necessarily mean fewer fatalities to occupants of the car.

## DEVELOPMENT OF MULTI-VARIATE PATAL ACCIOENT INVOLVEMENT RATES

### 1.0 INTRODUCTION

The objective of this study was to develop multi-variate fatal accident involvement rates. Multi-variate rates are necessary to understand the influence of a factor while controlling for other factors. For example, in deciding whether older cars are more frequently involved in fatal accidents per mile than newer cars, it is necessary to compare older cars with newer cars when driven by similar drivers and when driven in similar circumstances. If the comparison is made without regard to the driver or driving situation, the effect measured could be simply differences in driver or situation.

We have used accident involvement rate measured per vehicle mile of travel (VMT) because VMT is the most appropriate general measure of exposure to accident situations. Fatal accident involvements are used rather than fatalities because we hope to measure risk associated with active participants, the drivers rather than passengers, the passive victim. Further, involvement in a fatal accident rather than involvement in a vehicle whose driver or passenger died is a more appropriate measure of this risk. So, we count as a fatal involvement, any driver or vehicle involved in a fatal accident whether or not a fatality occured in that vehicle.

The most serious obstacle in developing multi-variate involvement rates is obtaining reliable multi-variate exposure (VMT) data. The best source of such data is the 1977 National Personal Transportation Survey (NPTS). The NPTS is a statistical survey in which a collection of households are selected and interviewed. This survey was a stratified, multi-stage cluster design in which 17,000 households were interviewed and their trip patterns were documented for the previous day.* The survey was designed to represent all household trips in the U.S. in 1977. The NPTS is described and the general results are presented in a series of reports by the Federal Highway Administration.

[^0]A serious difficulty in using the NPTS data has been the ambiguity in constructing the weights by which the sample trip is scaled up to U.S. total VMT. "User's Guide to the SAS Version of the 1977 NPTS" by R. Bair, who also participated in this study, documents a new Statistical Analysis System (SAS) file containing the NPTS data and presents some illustrative examples which show how to calculate weighting factors for households, trips, and vehicles.

The fatal involvement data are taken from the 1977 Fatal Accident Reporting System (FARS) which contains data on every fatal motor vehicle accident in 1977.

Table 1.1 presents the dimensions used in creating the multi-variate fatal involvement rates. These eight dimensions represent all of the variables which are common to both NPTS and FARS which might affect accident involvement rates. The levels or categories in each dimension are also identified in Table 1.1. Section 2.0 documents how the FARS and NPTS data were classified into these dimensions and levels.

The levels were chosen to minimize the total number of categories into which the data would be classified. The eight dimensions and associated levels shown in Table 1.1 lead to 576 separate cells. Each VMT in NPTS and fatal accident involvement in FARS must be classified into one of these cells. The levels were minimized to keep the resulting cell counts as high as possible because the size of the cell count influences the reliability of the estimated fatal involvement rate. Log linear models were fit to both the FARS and NPTS data to further improve the reliability of the cell estimates. This process is described in Section 3.0 along with the resulting smoothed fatal involvement rates.

The standard error of each cell in the FARS fatal involvement array, the NPTS VMT array and in the fatal involvement rate array is presented in Section 4.0, along with the methods used to estimate these standard errors. The standard error is needed in order to judge the significance of observed differences in fatal accident involvement rates.

Finally, Section 5.0 presents an example of how these fatal accident involvement rates and the associated standard errors can be used to assess the relative fatal involvement rates of small and large cars.

## Table 1.1 Dimensions and Levels of Multivariate Ratal Involvement Rates

| Dimensions | Levels |
| :---: | :---: |
| Driver Age | LE 25 |
|  | 26-55 |
|  | GE-56 |
| Driver Sex | Male |
|  | Female |
| Vehicle Age | LE 5 |
|  | GE 6 |
| Vehicle Weight | LE 3000 lbs. |
|  | GT 3000 lbs. |
| - |  |
| Number of Occupants | 1 |
|  | GE 2 |
| Time of Day | Late Night |
|  | Rush Hour |
|  | Other |
| Land Use | Urban |
|  | Rural |
| Season | Summer |
|  | Winter |

### 2.0 CLASSIFICATION OF NPTS AND FARS DATA

This section describes the procedures for classifying of NPTS and FARS data for inclusion in the fatal accident involvement rate analysis. The NPTS and FARS data bases were examined to determine the extent of overlap between variables in the two data bases. Only variables found on both data bases are useful in developing fatal involvement rates. Eight common variables or dimensions were identified for this analysis. The eight dimensions are listed in Table 1.1. Detailed definitions of the levels for FARS and NPTS appear in Table 2.1. Observations in each of the data bases are distributed to the resulting 576 element array based on these definitions.

In order to assure that the fatal accident rates derived from the NPTS and FARS data are as accurate as possible care was taken to ensure that both FARS and NPTS arrays counted vehicles of precisely the same types: passenger cars including station wagons. Further, the definitions of the dimensions and levels in NPTS and FARS were matched as closely as possible because any mismatch could strongly distort the fatal involvement rates. Finally, missing data was carefully accounted for and missing values in either data set were reduced to very low levels by estimating the missing values from other information known about the vehicle or driver.

The remainder of this section describes the steps taken to develop the mutlivariate FARS and NPTS arrays, accounts for the missing data, and explains the methods used to estimate the missing values.

### 2.1 FARS/NPTS Working File Development

All fatal involvement data in this analysis originates from the 1977 FARS file. VMT data were extracted from the 1977 NPTS file. Subsets of the FARS and NPTS data files were constructed for this analysis. Development of the FARS working file from the original 1977 FARS is summarized in Figure 2.1. The NPTS working file development is summarized in Figure 2.2

### 2.1.1 FARS Working File Development

The 1977 FARS file is divided into three segments: the person level file with 111,108 records; the vehicle/driver level file containing 61,254 records; and the

Definitions of Variables Used in Fatal Accident Involvement Rate Analysis

## Table 2.1

$$
\begin{aligned}
& \text { NPTS Code } \\
& \text { Interview Month }= \\
& 4,5,6,7,8,9 \\
& \text { Interview Month = } \\
& 10,11,12,1,2,3 \\
& \text { Urban VMT } \\
& \text { Rural VMT } \\
& \text { Model Year = 73-78 } \\
& \text { Model Year = earlier } \\
& \text { than 73 } \\
& \text { Total \# of persons in } \\
& \text { in the vehicle = } 1 \\
& \text { Total \# of persons in } \\
& \text { and Occupants LE } \\
& \text { 96 vehicle GT } 1 \\
& \text { (day-week=2-6) and } \\
& \text { (600 LE time of travel } \\
& \text { or (1530 LE time LE } \\
& 859) \text { or (1530 LE } \\
& \text { time of travel LE 1829) } \\
& \text { Where TIME } \\
& \text { TRAVEL = time trip } \\
& \text { started plus length of } \\
& \text { time } \\
& \text { destination. }
\end{aligned}
$$

FARS Code
Month $=4,5,6$,
$7,8,9$
Month $=10,11,12$
$1,2,3$
Land Use $=1$
Land Use $=2$
Mod-year $=73-78$
Mod-year less than 73
Occupants = 1
Occupants GT 1
LE 1829)
(day-week=2-6) and
(600 LE time LE 8:59)
(1

| Summer | April, May, June, <br> July, August, September |
| :--- | :--- |
| Winter | October, November, December, <br> January, February, March |
| Urban | Urban Area |
| Rural | Non-urban Area |
| New | greater than 5 years old |
| Old | Driver Only |
| One | Driver with passenger(s) |
| More than one less |  |
| one |  |
| Rush Hour | M-F 6:00-8:59 a.m. |

SEASON
VEHICLE AGE

## LAND USE

TIME OF DAY

OCCUPANTS
5

Rush Hour
3:30-6:29 p.m.

More than one one
One
Table 2.1

| Late Night | 9:30 p.m. - 5:59 a.m. | Time GE 2130 or (0 LE Time LE 559) | Time of Travel GE 2130 or (0 LE Time of Travel LE 559) |
| :---: | :---: | :---: | :---: |
| Other | all else | all other times | all other times |
| Male | Male | Sex $=1$ | Sex $=1$ |
| Female | Female | Sex $=2$ | Sex $=2$ |
| Young | Less than 26 years old | Age LE 25 | Age LE 25 |
| Middle-aged | 26-55 years old | (Age GE 26 and Age LE 55) | (Age GE 26 and Age LE 55) |
| Older | Over 55 years old | Age GT 55 | Age GT 55 |
| Light <br> Heavy | LE 3000 pounds GT 3000 pounds | VIN_WGT LE 3000 <br> VIN_WGT GT 3000 | Shipping Wt LE 3000 Shipping Wt GT 3000 |

DRIVER AGE
VEHICLE WEIGHT

## Figure 2.1

## Development of PARS Working File: Summary of 1977 Record Distribution FARS Working File



## Figure 2.2

## NPTS Working File Development

|  | Total Trips | Total VMT $\left(* 10^{9}\right)$ |
| :---: | :---: | :---: |
| NPTS segment 5 | 96,974 |  |
| Trips in houschold vehicles | 67,299 | 880.5 |
| Trips in household vehicle with driver and vehicle records matching | 65,435 | 807.3 |
| Total trips in passenger cars with known values for all dimensions excepting vehicle weight | 55,594 | 672.2 |

accident level tile, consisting of 42,211 records. The FARS working file was developed as follows:

1. Only person level records in which the person was identified as a vehicle driver were retained. There were $\mathbf{6 0 , 0 4 9}$ records identified as driver records in the person level file. The remaining 51,059 records in the person level FARS file were dropped from the subsequent analysis.
2. . Only vehicle records with "body type of automobile" were retained from the vehicle/driver level file. This subgroup, amounting to 38,696 vehicles was designated as passenger cars. Truck, motorcycle and moped body types, numbering 22,558 records, were dropped from further consideration.
3. The driver and passenger car subfiles were merged. As a result, 38,419 records remained. Drivers of vehicles other than passenger cars, and vehicles with no driver (e.g. parked cars) had no match during this merge step.
4. The accident level file was merged with the person/vehicle level file of Step 3. At this point, the FARS working file contained $\mathbf{3 8 , 4 1 9}$ records.
5. If data for any of the eight dimensions, with the exception of vehicle weight are missing on a record, it is not possible to assign the record to one of our 576 cells. An additional 431 records were dropped as a result of this step leaving 37,988 fatal involvements. Some of these records had unknown values for more than one of the dimensions. The total number of instances of unknown values for each of the dimensions other than vehicle weight follows.

Number of records with unknown values for:
sex - 5
age - 83
model year - 213
no. of occupants - 172
time of day - 84
land use - 91
season - 0
Section 2.3 discusses the methods used to assign vehicle weight to records with unknown weight.

### 2.1.2 NPTS Working File Development

The 1977 NPTS contains detailed information on 96,974 travel day trips. The total number of trips made in a household vehicle amounts to 67,299 trips with 880.5*10 ${ }^{9}$ VMT using the "total distance to destination" field also known as the reported distance. This total compares well with the total of $880,163,000,000$ reported VMT which is listed on page 29 of the 1977 NPTS User's Guide. Since trips had to be disaggregated into urban and rural VMT components for the subsequent analysis, mapped VMT rather than reported.VMT is used in the working file. The total mapped mileage for household vehicles is $838.0 * 10^{9}$ VMT. The ratio of reported to mapped VMT which these calculations yield is 1.051. This ratio compares favorably to the ratio of 1.056 deriveable from the figures contained in FHWA report No. 8, Urban/Rural Split of Travel (page 7) from the FHWA series of reports on the 1977 NPTS.

The total of all trips in a household vehicle for which both a driver and a vehicle record are present equals 65,435 trips and $807.3 * 10^{9}$ VMT. Certain groups of trips other than those with missing vehicle weights were discarded from the analysis because not all the desired dimensions are present on those records. They account for $14.9 * 10^{9}$ VMT and are distributed as follows:

$$
\begin{array}{r}
\text { missing time of day }: 5.0 * 10^{9} \mathrm{VMT} \\
\text { no vehicle age }: 1.8 * 10^{9} \mathrm{VMT} \\
\text { NPTS vehicle type }=7,8,9,12,999: 4.1 * 10^{9} \mathrm{VMT} \\
\text { unable to assign weight }: \\
4.1 * 10^{9} \mathrm{VMT}
\end{array}
$$

At this stage of its development, the NPTS working file contains 64,167 trips with 792.4* $10^{9}$ mapped VMT. The distribution of these trips and VMT to vehicle type categories is summarized in Table 2.2. Since the remainder of this analysis

## Table 2.2

## Distribution of Trips to Vehicle Type Categories

|  | Total Records | Mapped VMT |
| :---: | :---: | :---: |
|  | 64,167 | 792.4*109 |
| Vehicle Type | \% Records | \% Mapped VMT |
| 1 | 76.55 | 75.25 |
| 2 | 10.09 | 9.58 |
| 3 | 1.94 | 2.36 |
| 4 | . 62 | . 79 |
| 5 | 9.70 | 10.66 |
| 6 | . 58 | . 78 |
| 10 | . 51 | . 57 |
| 11 | . 01 | . 01 |
|  | 100.00 | 100.00 |

focuses on passenger cars only, the total of trips and VMT for passenger cars only, namely vehicle types 1 and 2 , amounts to 55,594 trips and $672.2 * 10^{9}$ VMT. All of the desired dimensions with the exception of vehicle weight are well defined in this group of trips. Unknown vehicle weights are present on 8783 trips which represent $109.9 * 10^{9}$ VMT. A full description of the NPTS weight assignment process follows in section 2.3.2.

### 2.2 Definition of Variables in FARS/NPTS Working Files

Variables in FARS and NPTS were compared to ensure consistency between the two data sets. Some of the dimensions, such as age, sex, time of day/day of week, and number of occupants are easy to construct in a comparable way for the two data sets. Two definitions - vehicle weight and land use - are not obvious, however.

Land Use: The FARS and NPTS working files both use the FHWA classification of roadways to determine whether an area is rural or urban. A fatal accident such as those reported in FARS occurs at a specific location. That location can be categorized as urban or rural based on the FHWA classifications. However, it is much more difficult to categorize entire trips which cross several land use types. Trips in the NPTS file were disaggregated into urban and rural VMT components based on the mileage mapping that was performed by FHWA and included in the NPTS data tape. As noted in the section of this chapter which deals with development of the NPTS working file, the VMT calculated using mapped mileage is somewhat lower than the VMT total using reported mileage in NPTS. The total difference between reported VMT and mapped VMT is $42.5 * 10^{9}$ VMT. That total difference is distributed as follows: 1,564 records and $28.5 * 10^{9}$ VMT are associated with trips which have reported VMT but no mapped VMT; $4.7 * 10^{9}$ VMT is the excess of reported VMT over mapped VMT for the 2,537 records in which a portion of the trip is off the map; $0.9 * 10^{9}$ VMT is the excess of reported VMT over mapped VMT for the 1,881 records which are all off the map; and $8.4 * 10^{9}$ VMT represents the excess of reported VMT over mapped VMT for all other trips. The equations for calculating VMT are the same whether reported or mapped mileage is chosen. The product of the time inflation factor and household trip weight is applied to each mileage and the result is summed over all records.

Vehicle Weight: The vehicle weight field reported in the FARS data base is vehicle shipping weight. The vehicle identification number (VIN) is recorded on the FARS data collection forms. However, to protect privacy rights, the VIN is not recorded in the FARS data base. Instead, the VIN is used as input to the VINA computer program available from the R. L. Polk Company. VINA in turn derives a series of useful characteristics from the VIN including vehicle make, model, and shipping weight. In the case of NPTS, it was necessary to use shipping weight to ensure that the same weight definitions are being used in both working files.

A comparison of vehicle weights for comparable vehicles was performed for selected makes and models of autos present in both FARS and NPTS. The results are detailed in Table 2.3. Specific vehicles were included in the comparison if the vehicle make and model was represented on at least 30 records in each data base. Mean weights for each make/model in the comparison were computed separately for FARS and NPTS. While there are differences in the average shipping weights between the data bases, the differences are not large enough or one-sided enough to cause any of these make/model vehicles to be incorrectly classified as large or small vehicles. The simple average of the weight differences is only 14 pounds. As a result, there does not appear to be systematic difference between the reporting of vehicle weights in the two data bases.

### 2.3 Vehicle Weight Assignment Methods

In both FARS and NPTS data bases, a substantial number of vehicle records contained unknown values for vehicle weights. A good deal of effort was expended during this study to assign weights to these vehicle records. The objective was to maximize the size of the FARS and NPTS working files which could be utilized as input to the multivariate analysis. If groups of records with unknown weights were deleted from the FARS and NPTS working files, it could result in misleading results in the multivariate analysis. Generally, weights were assigned when other records with known vehicle weights and identical model year, make and model information were available. Details of methods used to assign weights to records appear below.

## Table 2.3

Comparison of Vehicle Weights for Selected Model Automobiles: FARS and NPTS

|  |  | FARS <br> Mean <br> Weight | (Sample <br> Size) | NPTS <br> Mean <br> Weight | (Sample <br> Size) | Difference <br> (lbs.) |
| :--- | :--- | :--- | :--- | :--- | :--- | ---: |
| Make | Model | Cordoba | 4130.0 | $(52)$ | 4000.0 | $(34)$ |
| Chrylser | Aspen | 3237.8 | $(32)$ | 3234.4 | $(32)$ | 130.0 |
| Dodge | Volare | 3270.4 | $(37)$ | 3239.6 | $(36)$ | 3.4 |
| Plymouth | LTD | 4341.2 | $(84)$ | 4300.0 | $(57)$ | 49.8 |
| Ford | Granada | 3136.5 | $(103)$ | 3302.0 | $(98)$ | -165.5 |
|  | Century | 3655.1 | $(61)$ | 3829.1 | $(51)$ | -174.0 |
| Buick | Malibu | 3706.7 | $(63)$ | 3787.0 | $(46)$ | -80.3 |
| Chevrolet | Nova | 3218.2 | $(90)$ | 3342.2 | $(64)$ | -124.0 |
|  | Camaro | 3468.0 | $(66)$ | 3464.3 | $(42)$ | 3.7 |
|  | Chevette | 1923.1 | $(56)$ | 1900.0 | $(37)$ | 23.1 |
|  | Cutlass | 3707.6 | $(118)$ | 3794.8 | $(96)$ | 87.2 |
| Oldsmobile | Grand Prix | 4044.2 | $(72)$ | 4000.0 | $(45)$ | 44.2 |

Total of Differences $=\mathbf{- 1 7 1 . 2}$ lbs.
Average Difference $=-14.27 \mathrm{lbs}$.

### 2.3.1 FARS Vehicle Weight Assignment Algorithm

The FARS weight assignment process involves four steps. First, fatal involvement records with known vehicle weights were assigned to one of two categories: light (if vehicle weight was less than or equal to 3000 pounds) or heavy (if vehicle weight was greater than 3000 pounds). There are a total of 37,988 records in the PARS working file. In this step, 27,803 fatal involvement records were classified into one of the two weight categories. The remaining 10,185 fatal involvement records did not have a known vehicle weight.

Second, both the vehicles with known weights and those without known weights were grouped together by common make, model, and model year. Within each group of records with these common characteristics, the number of vehicles classified in the light category and those classified in the heavy category were totaled. Those vehicles with unknown weights were then assigned to the two weight categories based on the proportion of the vehicles with known weights which were in each weight category. 4,839 records were assigned to vehicle weight categories during this step.

The third step was similar to the second step. Records were grouped by common make and model year.* Those vehicles with unknown weight which had not been assigned to the light or heavy categories, as a result of step two, were assigned to a category based on the proportion of those vehicles within the same group which were already categorized as light or heavy. Another 4,367 records were assigned weight categories during the third step.

After these three steps, 979 vehicles remained unclassified by vehicle weight. In this final step, all records were again regrouped by make and age of vehicle. Two age groups were used. Those vehicles less than or equal to five model years and those vehicles older than five model years. Another 312 records were assigned to weight categories based on the age/make combination.

After the fourth step, 667 records were unassigned. The final 1977 FARS fatal accident involvement array was constructed from the remaining 37,321 FARS involvement records.

[^1]
### 2.3.2 NPTS Vehicle Weight Assignments

The NPTS vehicle weight assignment process is summarized in Table 2.4. NPTS contains not one, but three distinct vehicle weights. Curb weight, shipping weight, and inertial weight are all reported in NPTS. Since FARS makes use of shipping weight, all NPTS weights are converted to shipping weight so that valid comparisons can be made in this analysis. In the event that shipping weight is reported in NPTS, that weight is used directly. If there is no shipping weight in NPTS, but curb weight is reported, then curb weight is converted to shipping weight using the relationship: shipping weight = curb weight -100 . If neither shipping weight nor curb weight is reported, but inertial weight is reported in NPTS, then inertial weight is converted to shipping weight using the relationship: shipping weight $=$ inertial weight -400 . The relationships among shipping weight, curb weight and inertial weight were derived from an analysis of the differences in the mean shipping curb and inertial weights for vehicles in which at least two weights were reported. The average difference between curb weight and shipping weight was 123 pounds. Since NPTS reports weight only to the nearest hundred pounds, the average difference was rounded down to 100 pounds for the purpose of imputing weights. The average difference between inertial weight and curb weight was 300 pounds, hence the difference between inertial weight and estimated shipping weight of $\mathbf{4 0 0}$ pounds.

The number of trips in which shipping weight is given is 23,271 with $282.2 * 10^{9}$ VMT. The number of trips in which shipping weight was derived from curb weight is 23,473 with $279.5^{*} 10^{9}$ VMT. The number of trips in which shipping weight was derived from inertial weight is 67 with $0.636 * 10^{9}$ VMT. The total number of passenger car trips with known vehicle weight is 46,811 . They account for $562.3^{*} 10^{9}$ VMT. This leaves 8,783 passenger car trips and 109.9*10 ${ }^{9}$ VMT to be assigned to weight categories.

The final step requires that the remaining records, which have no reported weight, be assigned to one of the two vehicle type categories. VMT is assigned to each vehicle weight category based on the vehicle age, make, number of cylinders, and weight information for the 46,811 vehicles with known weights.

Table 2.4

## NPTS Weight Assignment Process

|  | Total Trip Records |  | Total VMT(109) |
| :---: | :---: | :---: | :---: |
| all passenger cars | $\underline{55,594}$ | $\underline{672.2}$ |  |
| shipping weight known | 23,271 | 282.2 |  |
| shipping weight derived <br> from inertial weight | 67 | 0.636 |  |
| shipping weight derived <br> from curb weight | 23,473 | 279.5 |  |
| Total known weights | 46,811 | 8,783 |  |
| Total assigned weights | 8, | 109.9 |  |

The process of imputing VMT to vehicle weight categories makes use of a lookup table in which vehicles with known weight and vehicle type are grouped into a matrix.* The matrix is based on vehicle make, number of cylinders, and model year. Three cylinder types are used: 4 cylinders, 6 cylinders, and 8 cylinders. Four model year groupings are used: 1971 and earlier; 1972 and 1973; 1974 and 1975; and 1976 and later. The mapped VMT for each of the passenger cars with known weight is assigned to one of the matrix cells based on make, model year, and number of cylinders. For passenger cars with unknown weight, the mapped VMT is assigned to vehicle weight categories based on the proportion of vehicles with the same make, model year, and number of cylinders with known vehicle weights. E.g., if 40 percent of the 4 cylinder 1980 Chevrolets with known weight are in the less than 3,000 pound category, then a 4 cylinder 1980 Chevrolet with unknown weight would have 40 percent of its mapped VMT assigned to the lower weight category and 60 percent of its mapped VMT assigned to the higher weight category.

Of the 8,783 records and $109.9 * 10^{9}$ VMT with unknown weights, 3,840 records with 53.045*10 ${ }^{9}$ VMT are perfectly categorized by vehicle weight. A perfect categorization, in this sense, means all vehicles with known weight for a particular make, model year and cylinder count fall into the same weight category, either the lower or the higher category. In the course of the VMT assignment process, the proportion of vehicles in each weight category which have known weight and the same make, model year, and number of cylinders is computed. The smaller of the two proportions is called Pmin. In the case of perfect assignment, $\mathrm{Pmin}=0$. Two additional quantities were calculated for all records with assigned weights:

$$
\begin{aligned}
& \text { VMT*Pmin }=7.426 * 10^{9} \\
& \text { VMT* }(\operatorname{Pmin})^{2}=1.933 * 10^{9}
\end{aligned}
$$

These results show that the Pmin values tend to be low. This means that very little VMT was actually split between vehicle weight categories, and the vehicle make, model year, and number of cylinders is quite good at discriminating between heavy and light vehicles which lends confidence in the weight assignment results.

[^2]After the trip records with unknown vehicle weight had been assigned as far as possible by the above described procedure to the light and heavy vehicle category, the percent distribution of VMT by vehicle weight was: heavy: 68.3 percent; light, 31.7 percent (see Table 3.5 described in next section). The same distribution was calculated for only the records with known VMT with the resulting distribution: heavy: 70.1; light: 29.8.

The difference is due to the fact that the records with unknown vehicle weight, 17 percent of all VMT, when assigned to the heavy and light categories had a different. distribution than the records of known vehicle weight. It can be calculated that the records with missing vehicle weight were assigned in the proportion of about 60 percent to 40 percent heavy to light.

This higher percentage of light vehicles in the records with missing weight information leads to the increase from 29.8 percent light in the records with known weight to 31.7 percent light in the overall VMT. Note that a shift of only one percent of the VMT from light to heavy would change 31.7 percent light to 29.7 percent light.

### 2.4 Preliminary FARS/NPTS Frequencies

Table 2.5 summarizes the univariate frequencies of all the dimensions and levels of the FARS and NPTS working files. The total counts for fatal involvements in Table 2.5 do not exactly agree with the total of fatal involvements in the FARS working file $(37,321)$ because of rounding errors. However, the general results of these univariate frequencies are reasonable and provide confidence in use of the FARS and NPTS final working files as input to the multi-variate analysis.

Since the Polk tapes record vehicle registration for 1977 a special test was run to see whether the NPTS counts of vehicles in Classes 1 and 2 in the light and heavy categories agreed with the Polk registration counts. The results reported in Table 2.6 are quite satisfactory.
Patal Involvements
VMT $\left(10^{9}\right)$ Table 2.5
Comparison of Prequencies in FARS/NPTS Patal Involvements Rate Analysis $29^{\circ} 9 S$
$L 9^{\circ} 9 \varepsilon$
$\varepsilon 9^{\circ} 80$ 64.02
39.76 39.76

55.06
56.09
 تロ 909
409
409 웅 ~ +ic + 33.5
66.5 45.5
17.0
37.5

Total Fatal
Involvements 16,801
14,908
5,600
 19,563
17,746
16,573
20,736 16,573
20,736

 O 16,962
6,338
14,009 19,793
17,516
54.8
45.2
23.0
62.2
14.7
65.1
34.9
52.9
42.7

62.8
37.2

57.7
42.3

31.7
68.3
52.9
42.7

62.8
37.2

57.7
42.3

31.7
68.3
52.9
42.7

62.8
37.2

57.7
42.3

31.7
68.3
52.9
42.7

62.8
37.2

57.7
42.3

31.7
68.3
56.8
33.3
9.9
6,338
14,009
86
16,962 -

\% Total VMT


Total VMT
$\left(\times 10^{9}\right)$
54.8
417.9
9.0
437.0
234.7
355.3
316.4
421.6
250.1
387.6
284.1
381.2
223.7
66.8
367.8
303.9
- $\begin{array}{ll} & \\ 154.8 & 23.0 \\ 417.9 & 62.2 \\ 99.0 & 14.7\end{array}$
Total VMT
$\left(\times 10^{9}\right)$
437.0
234.7
355.3
316.4
421.6
250.1
387.6
284.1
212.6
459.1
381.2
223.7
66.8
367.8
303.9 Driver Age
$=25$ yeurs
$26-55$
55
Driver Sex
Male
Female
Season
Summer
Winter
Land Use
Urban
Rural
Vehiele Age
LE 5 years old
GT 5 years old
Vehicle Weight
LE 3000 lbs.
GT 3000 lbs.
Time of Day
Other
Rush Hour
Late Night
Occupants
Driver Only
Driver \& Passengers
$\begin{aligned} & \infty \\ & \infty \\ & \infty \\ & \infty \\ & \infty\end{aligned}$
$\begin{array}{r}44.50 \\ 28.33 \\ 209.72\end{array}$


## Table 2.6

## Comparison of NPTS with Polk on Cars and Stationwagons by Weight

|  | Total Vehicles/106 | Precent* <br> Heavy | Percent* <br> Light | Percent With <br> Unknown Weight |
| :--- | :---: | :---: | :---: | :---: |
| Polk 1978** | 100.7 | 69.6 | 30.5 | 16.9 |
| NPTS | 95.5 | 69.4 | 30.6 | 19.9 |

*Percentages for Heavy and Light are of vehicles with known weights.
**Polk 1978 covers registrations in 1977.

### 3.0 IMPROVING THE ACCURACY OF THE FATAL INVOLVEMENT RATES USING LOG-LINEAR MODELS

### 3.1 Introduction

When the FARS fatal accident involvement counts and the NPTS VMT totals are distributed over the multidimensional tables with 576 cells each, many of the cells are "noisy," i.e. their contents (in counts or VMT) are subject to sampling error.* In the extreme, some of the cells are empty (three empty cells are observed in the case of NPTS (see Table 3.3) and 1 in the case of FARS (see Table 3.1). Further evidence of the noisiness of the data is presented in Sections 3.3 and 4.4. Because of the noisiness of the raw data, accurate estimates of fatal involvement rates cannot be obtained by simply dividing the raw FARS cell counts by the raw NPTS cell VMT sums. It is desirable to first "smooth" the FARS and NPTS tables in order to obtain more accurate cell estimates.

Probably the best available method to accomplish this smoothing is through the application of log-linear models.** Log-linear models were used to produce all the estimates contained in this study. A substantial increase in accuracy resulted from this smoothing process (see Sections 3.3 and 4.4 for the evidence).

### 3.2 The Construction of the Log-Linear Models

This section deals with the general strategy for selecting a log-linear model specification used for this study, more details are given in Appendix A.

[^3]In using the classical maximum likelihood approach to fitting hierarchical* loglinear models, the fitted model parameters are determined by certain margins of the data matrix. The selection of the proper margins (which is equivalent to the selection of what interactions to include in the model) is discussed in this section. The reason why the modelled cell estimates are more statistically stable (i.e. have less sampling variance) than the raw cell counts is that the margins of a data matrix have greater (relative) statistical stability than the raw data matrix itself (because they are more aggregate and have cells with larger sample sizes).

The basic approach to model selection is to find the hierarchical log-linear model with highest possible number of (residual) degrees of freedom** and the lowest possible chi square. These two optimization goals are at odds and a tradeoff must be sought.

The primary objective for making this tradeoff should ideally be accuracy. In this case, the tradeoff between high degrees of freedom and low chi square can be viewed in terms of the components of inaccuracy: bias and variance. Bias results from fitting a parsimonious model (one of limited complexity) to a set of data generated by a more complex process.*** Thus, bias is the result of a too parsimonious model, and in general, a less parsimonious, more complex model has lower bias. Variance (sampling variance), on the other hand, is lower for a more parsimonious model. Total (mean squared) error is determined by the two components, bias and variance. The minimum total error occurs at a model of intermediate complexity where neither bias nor variance is extremely large.
*All log-linear models considered in this study are hierarchical. (This is usual, e.g. Reference 2 is devoted exclusively to hierarchical log-linear models.) A hierarchical model is one which contains all lower order interactions associated with any high order interaction it contains. Interactions are terms in a model which depend on a specified combination of variables, e.g. the 1-3-4 interaction depends simultaneously on variables 1, 3, and 4 but not on other variables. More detailed definitions of the terms used here are given in References 1 and 2.
**The degrees of freedom associated with a model are the residual degrees of freedom in the data. This equals the number of data cells minus the number of independent parameters in the model (see Reference 2, p. 114). Thus maximum degrees of freedom is equivalent to minimum model complexity.
***Bias may also be thought of as model misspecification error.

However, to determine the optimum level of complexity, both the bias and the variance must be estimated.

In this study, the bias was not estimated although the variance was. Instead, the complexity of the model is set somewhat arbitrarily at a rather high level with the expectation that the bias will be negligible. If the bias is negligible then all the error is due to the variance and so the smoothed data is necessarily more accurate than the unsmoothed data.* The total error in the cell estimates would then be well characterized by the standard error determined from the variance. Evidence in Appendix A suggests that the models were not underfit (i.e. that the bias is small); furthermore in Section 4.4 it is observed that a very sizeable reduction in the standard error of the cell estimates was obtained by the loglinear model smoothing process. From this it is concluded that the smoothing process substantially increases the accuracy of the cell estimates.

## The Model Specification Process

A brief description is given in this subsection of the process used to attempt to find the model with minimum chi square for a given number of degrees of freedom; or maximum number of degrees of freedom (residual) for a given chi square. Basically the strategy is to find a model such that no interaction (in the model) can be replaced by another (not in the model) with the same number of degrees of freedom without increasing the ratio of chi square to degrees of freedom. Before outlining the process in more detail, it should be pointed out that the assumptions needed in log-linear modelling in the classical sense are not satisfied in either the FARS or NPTS case. In the FARS case, the classical assumptions will be invoked as approximately true while in the NPTS case a factor is developed to make the classical chi square statistic approximately valid. The assumptions made and the justifications for them are discussed in Appendix C. In this section and in Appendix A, it will be assumed that both data sets (transformed as necessary by a factor in the NPTS case) are suitable for the application of classical log-linear modelling techniques.
*Since the $\log$-linear modelling process can only decrease the variance.

The ideal goal is to find a model of a certain degree of complexity such that no other model of that degree of complexity (i.e. with the same number of degrees of freedom) has a lower chi square. However, the chi square value for a model can only be obtained by constructing that model and this is expensive. Therefore, the ideal procedure is infeasible due to the prohibitively large number of possible hierarchical models.

In order to describe a compromise procedure which is feasible, some terminology is needed. The increase in the chi square value due to dropping a certain interaction from a certain model or the decrease due to adding a certain interaction to that model will be denoted by $\Delta X^{2}$.* The (absolute value of the) change in degrees of freedom will be denoted by $\Delta \mathrm{DF}$. The ratio $\Delta \mathrm{x}^{2} / \Delta \mathrm{DF}$ is the quantity of interest.

The compromise goal is to seek a model such that, the value of $\Delta \mathrm{X}^{2} \Delta \mathrm{DF}$ for any interaction in the model is greater than the value of $\Delta X^{2 /} \Delta D F$ for any interaction not in the model.** It can be seen that if $\Delta X^{2}$ did not depend on what model it referred to, this procedure would lead to a global rather than a local minimum in chi square for the resulting number of degrees of freedom.

Even this compromise goal is quite ambitious for a matrix as complex as the FARS and NPTS matrices in this study. Before describing the basic steps in searching for a (compromise) optimum model, it is necessary to say some more about how to compute $\Delta X^{2}$. There are basically three ways of estimating $\Delta x^{2}$ for interactions (with respect to a given model); these are, in order of decreasing accuracy and decreasing computational cost as follows:
*In words " $\Delta x^{2}$ " is "delta chi square."
**A necesssary exception to this in the case of some lower order interactions "implied by" higher order interactions is discussed later in this section (in a footnote on "dilution").

1. Calculate the value of chi square for the model with the interaction in and for the model with the interaction out. The difference is the most accurate value of $\Delta x^{2}$.
2. Estimate $\Delta X^{2} / \Delta D F$ according to the procedure described in Appendix $A$ based on the standardized effects for the model in question. This approximation is essentially exact if the number of degrees of freedom ( $\triangle D F$ ) (independent parameters) in the interaction is 1 and less accurate if the degrees of freedom is greater than 1.
3. Use the standardized effects in the same manner as just described (under item 2) but use the standardized effects for a more complex model which contains the given model as a submodel. This procedure allows $\Delta X^{2} / \Delta D F$ to be estimated with limited accuracy for a great many interactions at once. This is the least accurate estimate but least costly computationally.

Now a series of models are constructed together with the standardized effect estimates for the interactions in the models. Interactions are added or deleted from intermediate models, each on the basis of whether its $\Delta X^{2} / \Delta D F^{*}$ value is above or below a given threshold (the threshold varies at various stages; see Appendix A).

The final model corresponds to a fixed threshold (3.0) and has the property that no interaction in the model has a $\Lambda X^{2} / \Delta D F$ (determined in the most accurate manner which involves fitting a separate model corresponding to deleting each interaction) less than 3.0 while no interaction not in the model has an estimated

* In the case of high order interactions for which some implied (by the property of being hierarchical) lower order interactions do not have the required $\Delta \mathrm{X}^{2} / \Delta \mathrm{DF}$ the $\triangle X^{2}$ and $\triangle D F$ values are calculated for the higher order and implied lower order interactions together. The resulting $\Delta \mathrm{X}^{2} / \Delta \mathrm{DF}$ is then said to be the appropriate "diluted" value pertaining to the higher order interaction. Example: Suppose the threshold for $\Delta X^{2} / \triangle D F$ is 3.0 , that the $1-2-3$ interaction has $\Delta X^{2}=$ 4.0 and $\triangle D F=1$ but the $1-2$ interaction has $\Lambda X^{2}=1$ with $\triangle D F=1$ while the 1-3 and 2-3 interactions have $\Delta X^{2} / \triangle D F>3.0$ and are thus in on their own. Then the diluted $\Delta \mathrm{X}^{2} / \angle \mathrm{DF}$ for the $1-2-3$ interaction is $(4.0+1.0) /(1+1)=2.5$. Thus the $1-$ $2-3$ interaction is not entered if the threshold is 3.0 because it would "force in" the weak 1-2 interaction.
$\Delta \mathrm{X}^{2} \not \swarrow \mathrm{DF}$ as great as (or greater than) 3.0. (The $\Delta \mathrm{X}^{2} \not \Perp \mathrm{DF}$ of 5 way and higher interactions have not been estimated). The $\Delta^{2} X^{2} \Delta D F$ estimates for most of the interactions not in the model are determined by their standardized effects in intermediate models. However, if a $\Delta \mathrm{X}^{2} \Delta \mathrm{DF}$ estimate is close enough to 3.0 to warrant further consideration, it is reestimated more accurately to determine whether it belongs in the final model.

The resulting model is believed to be a good approximation to, and may well be a model which achieves, the compromise goal described earlier in this subsection. There are, however, far too many interactions not in the model to test each using the most accurate method. Furthermore, such a procedure would be wasteful, since if a $\Delta X^{2} / \Delta D F$ estimated from standardized effects is small the accurate $\Delta \mathrm{X}^{2} / \Delta \mathrm{DF}$ cannot be very large.

### 3.3 Results of Smoothing

In this section, the results of specifying and fitting log-linear models to the FARS and NPTS data according to the procedures described in Section 3.2 and Appendix A are presented along with the raw data matrices. The ratios of the smoothed cell estimates, the estimated fatal involvement rates, are also presented. Section 4 presents the relative standard errors of the smoothed cell estimates and of the resulting fatal involvement rates.

Table 3.1 shows the raw* fatal involvement data from FARS. It contains the 576 cell counts derived from the 1977 FARS data base as described in Section 2. Note that the fatal involvement counts are not integers because some involvements were fractionally assigned to more than one cell because of incomplete information.

Table 3.2 shows the results of applying log-linear smoothing to the raw data in Table 3.1. The results are the best estimates of fatal involvements for each cell in a typical year like 1977.
*The term "raw data" as used here refers to the data after going through the processes described in Chapter 2 (including vehicle weight estimation in some cases) but before the log-linear modelling process.

Table 3.3 shows the raw ${ }^{*}$ NPTS data as described in Section 2 while Table 3.4 shows the results of log-linear model smoothing of these data. The cell entries in Table 3.4 represent the most accurate estimates of VMT derived from the NPTS data by log-linear model smoothing.

Table 3.5 shows the estimated fatal involvement rates obtained by dividing the 1977 smoothed FARS involvements (Table 3.2) by the corresponding 1977 smoothed NPTS VMT estimate (Table 3.4). The smoothed VMT estimates and the fatal involvement rates made possible by this exposure data are the primary objectives of this project. The next chapter develops the standard errors of these estimates.

The following observations are made on these tables:

1. The raw and fitted data differ substantially, indicating that the smoothing has made a considerable difference in the estimates. When this is coupled with the prior observation (Section 3.1) that the models were not substantially underfit, it indicates a substantial advantage to the smoothing process (further evidence of this will be seen in Section 4 when the standard errors are calculated).
2. The fatality rates show a striking variation from cell to cell (the largest rate is over 200 times the smallest). The extent to which this is just a noisy fluctuation will be discussed in Section 4 where the relative standard errors are given.

[^4]SEASUA: SUMMER



TABLE 3. $\bar{z}$ SMUUTHEU ESTIHATES UF Fital IHVULVEPFATS (FARS, IS77)

SEASUN: INTEK


$\begin{array}{lll}29.732 & 30.054 & 31.612\end{array}$

SEASUM: SUAAER




SEASUA: SUhmek


SEASUA: almtek

SEASUH: SUYHER



| fenale | $\begin{aligned} & \text { LE } 25 \\ & 26-55 \\ & \text { GE } 56 \end{aligned}$ | $\begin{aligned} & 84.62 \\ & 79.64 \\ & 61.87 \end{aligned}$ | $\begin{aligned} & 68.23 \\ & 54.24 \\ & 47.49 \end{aligned}$ | $\begin{array}{r} 155.87 \\ 90.10 \\ 123.44 \end{array}$ | $\begin{array}{r} 155.55 \\ 79.12 \\ 97.78 \end{array}$ | $\begin{aligned} & 207.93 \\ & 200.20 \\ & 234.08 \end{aligned}$ | $\begin{aligned} & 163.71 \\ & 96.67 \\ & 196.61 \end{aligned}$ | $\begin{aligned} & 307.26 \\ & 283.73 \\ & 519.71 \end{aligned}$ | $\begin{aligned} & 256.10 \\ & 151.12 \\ & 385.17 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| MALE | LE 25 | 32.10 | 51.15 | 86.61 | 10z. 21 | 63.81 | 102.94 | 226.13 | i33.23 |
|  | 26-55 | 14.27 | 10.16 | 32.02 | 27.97 | 25.20 | 24.79 | 66.70 | 43.32 |
|  | GE 56 | 19.58 | 22.32 | $6 \mathrm{C}$. | 42.64 | 42.18 | 30.49 | 170.26 | $6 \leq .03$ |
| PEmale | LE 25 | 31.75 | 37.20 | 53.60 | 55.49 | 66.34 | 75.88 | 144.31 | 124.60 |
|  | 26-55 | 16.23 | 12.51 | 27.75 | 19.68 | 42.72 | 23.33 | 83.82 | 36.05 |
|  | GE 56 | 49.25 | 36.64 | 119.76 | 69.23 | 87.03 | 74.66 | 268.48 | 145.60 |
| hale | LE 25 | 32.53 | 38.02 | 87.76 | 75.97 | 60.89 | 95.72 | 289.18 | zte. 89 |
|  | 26-55 | 9.49 | 12.15 | 21.28 | 21.03 | 20.90 | 23.26 | 55.34 | 40.65 |
|  | GE 56 | 22.10 | 27.31 | 68.38 | 52.18 | 73.19 | 57.39 | 295.50 | 122.38 |
| PRMALE | LE 25 | 37.74 | 32.77 | 63.70 | 48.48 |  |  |  |  |
|  | 26-55 | 14.11 | 12.42 | 24.12 | 19.54 | 46.34 | 28.91 | 90.92 | 44.68 |
|  | GE 56 | 52.08 | 44.76 | 126.59 | 80.22 | 141.50 | 133.39 | 436.82 | 25S.44 |
| male |  | 25i. 43 | 316.77 | 674.30 | t33.01 | 425.09 | 542.25 | 1519.91 | 1226.51 |
|  | 26-55 | 73.47 | 84.96 | 105.68 | 147.05 | 96.14 | 96.11 | 254.50 | 167.93 |
|  | CE 56 | 37.49 | 44.03 | 115.98 | 91.77 | 115.73 | 94.04 | 467.10 | 200.49 |
| female | LE 25 | 200.74 | 256.34 | 338.44 | 233.19 | 356.73 | 271.27 | 776.08 | 146.25 |
|  | 26-55 | 72.87 | 47.94 | 124.56 | 75.45 | 141.43 | 65.94 | 277.48 | 101.91 |
|  | CE 36 | 78.76 | 58.38 | 191.65 | 104.67 | 199.76 | 162.21 | 616.73 | . 315.11 |

### 4.0 VARIANCE ESTIMATES

Sampling variance remains after the log-linear modelling smoothing process. In order to make judgments or decisions based on the smoothed fatal involvement rate data, it is necessary to have estimates of this inaccuracy. A sampling variance is needed for each (smoothed) cell estimate in both the FARS and NPTS matrices and from these a sampling variance can be estimated for each cell for the ratio (fatal involvements per VMT).

These variances are for cell estimates based on log-linear models. As these quantities are the result of highly complex interactive and non-linear transformations of the original data, it is difficult to estimate their variances. In the case of NPTS, the job is made even more difficult because of the complex stratified multistage cluster sampling plan on which it is based.

These problems are overcome by "computer intensive" resampling methods such as the jackknife, the bootstrap or half sampling procedures (see Reference 3 for a general discussion). The methods used for this project were of the half sampling type. The basic principles of using half samples for variance calculations will be descibed first very briefly and then some of the specifics of the FARS and NPTS cases will be dealt with.

### 4.1 Variance Calculations Using Half Sample Techniques

A large sample can be split into half samples (two mutually exclusive and exhaustive sub-samples) in very many different ways. For example, a sample of 1000 individuals can be split into exactly equal half samples in over $10^{300}$ ways. The theory of sample splitting techniques (such as jackknife, bootstrap, and half sampling) establishes that the sampling variance of a sample determined quantity can be estimated from its variance over a limited random sample of appropriately determined half samples. (Note that only one half of the sample pair consisting of the half sample and its complement might be used in a given variance calculation.)

Basically, the procedure for estimating the cell variance consists of the following steps:

1. Form N half samples using appropriate procedures.* Appropriate procedures for the FARS and NPTS cases are outlined below.
2. Determine an estimate for each parameter (for which a variance is desired) on each of the N half samples. In the present case the parameters of interest will be the cell estimates determined by a $\log -$ linear model. This will consequently involve fitting the log-linear model to each half sample.
3. Determine the variance of each parameter over the N half samples. If the parameter is scaled to be sample size invariant** then this variance will be a good estimate of the variance of the parameter as estimated from the whole sample, i.e. its sampling variance.

These considerations lead to useful variance estimates in the present context. The accuracy of the estimate is limited primarily by how many half samples are used. The technique is "computer intensive" and can be quite costly if too many half samples are used. A reasonable trade-off between cost and accuracy can be made, however.

The considerations in the trade-off are the following:

1. The number of half samples, $N$, is essentially equivalent, from the point of veiw of accuracy, to the degrees of freedom in an ordinary variance estimate.
*The procedures to be used will result in half samples which are approximately half the size of the full sample.
**In the case of a quantity, such as a cell estimate, which is not sample size invariant but proportional to sample size, multiply the half sample estimated variance by $4\left(\mathbf{2}^{2}\right)$ to scale to a valid variance for a full-sized sample.
2. A complete model estimation procedure (in the present case the fitting of a complex log-linear model) must be performed on each of the N half samples.

It is well known that if the sample size is greater than about 30 , then the confidence interval based on a variance from that sample is only slightly wider than if the variance were based on an infinitely large sample. A 75 percent confidence interval based on a sample of size 20 is about 6 percent wider than one based on an infinite sample. This is probably adequate in most cases. Since the calculation of the variance using 20 half samples is quite feasible, the use of 20 half samples for variance determination is reasonable.

### 4.2 Construction of Half Samples for the FARS Data

As previously noted, the FARS data used consisted of a matrix of counts of fatal accident involvements by driver, vehicle and environmental characteristics. As noted in Appendix C, it is usual in log-linear modelling to assume that the cell counts can be considered to be independently and Poisson distributed. Both these conditions are violated to some extent by the nature of accident involvement data but the data are treated in this study as if they have the required properties. This assumption is discussed in Appendix C. Based on this assumption, the following method for producing half samples will lead to valid variance estimates for the FARS case.

For each cell, $i$, in the fatal involvement matrix $M_{i}(i=1, \ldots, 576)$ form a random number binomially distributed with $n=M_{i}$ and $p=q=\frac{1}{2}$. Denote this number by $X_{1 i}$ and repeat the process $N$ times forming $X_{2 i} \ldots, X_{3 i} \ldots, X_{N i}$. The numbers $X_{j i}$ $i=1, \ldots .576$ then form the $j$ th the half sample.*

[^5]Using $N=10$ half samples, the sampling variance of cell estimates can then be calculated in the manner indicated in Section 4.1.

### 4.3 Construction of Half Samples for the NPTS Data

The preferred sample splitting technique for constructing variances for samples taken according to complex statistical designs is based on the use of repeated replications* for constructing the required half samples. This technique is useful when there are many strata (say 30) in the sampling scheme. It is described in References 3, 4, and 5.

The procedure calls for dividing the observations in each stratum into two "halfstrata ${ }^{n}$ in such a way that each "hall-stratum" has the same sampling characteristics as the whole stratum. Among other things, this means that clusters in the whole stratum are not split in developing "half-strata" and as a consequence the "half-strata" may not contain precisely hall of the observations from the whole stratum.

Once a set of half strata has been constructed, one pair of half strata for each stratum, the half samples are constructed as follows. The hall strata for each stratum are labelled " 1 " and " 2 " arbitrarily. Then for each half sample to be constructed a specification of " 1 " or "2" for each stratum is given and the half sample is constructed by including the designated half from each stratum. This procedure is repeated for each half sample needed. The list of ${ }^{\prime \prime} 1$ 's" and " 2 's" needed for the construction of each hall sample can be generated randomly (with each selection being made independently with equal probabilities for " 1 " and "2") or according to the principles of "balanced repeated replications" or "partially balanced repeated replications." These matters are discussed in Appendix E where it is concluded that a random choice for each half stratum designation is adequate and is to be preferred for simplicity.

[^6]In the case of the NPTS data there are two types of strata: self-representing (SR) and non-self-representing (NSR).* The SR strata are handled in the standard manner (i.e. split into two half strata preserving the clusters intact, etc.) but the NSR strata are paired into pseudo strata. Thus, each half stratum is either hall of the sample from a SR Primary Sampling Unit (PSU) or the whole sample from one of a pair of NSR PSU's. The two half strata may be said to comprise a pseudo stratum whether it is an actual stratum (SR PSU) or a pseudo stratum (represented by a pair of NSR PSU's).

As there are 156 SR strata and 220 NSR strata in the NPTS data there are (ideally) $156+220 / 2=266$ pseudo strata (PSU's) for the purpose of selecting half sample replications. (A different number of pseudo strata was actually used as will be explained below.)

A designation of what part of the sample each household serial number comes from is not part of the NPTS user tapes. For this project the Bureau of the Census decided to provide directly a designation of a numbered pseudo stratum and half stratum for each household serial number. The numbered pseudo strata were not identified geographically in keeping with the policy of restricting information which might affect privacy. This privacy consideration limited the amount of information supplied (as opposed to identifying clusters, etc. in order to permit half stratum construction) while providing the necessary information for the construction of balanced repeated replications. Further limiting the information supplied, Census determined that even to designate a numbered pseudo stratum and half stratum would violate their privacy restrictions in the case of 14 PSU's. Consequently, these strata were all lumped together into one psuedo stratum. Census estimated that this lumping would lead to underestimating the variance by 2 to 3 percent. This seems to be a high estimate of the error since the lumped pseudo stratum in question accounted for

[^7]only 26 percent of the total number of households. Such a small effect on the variance is quite acceptable for the present needs.

The tape from Census listed all the household serial numbers in the NPTS sample and for each gave a pseudo stratum number from 1 to 253 (266-14 + 1) and a hall stratum designation of 1 or 2 . This enabled the half samples to be constructed in the manner outlined previously in this section.

### 4.4 Final Calculations of Variance and Results

Once the set of half samples has been constructed, the actual variance calculation is the same for the FARS and NPTS cases. The procedure is as follows: fit the model as finally specified for the given data set (FARS or NPTS) to each half sample. Form the variance of the (smoothed) cell estimate for each cell over the collection of half samples and multiply the variance by four (since cell estimates are not sample size invariant).

A relative variance is then constructed by dividing by the square of the corresponding cell estimate. Once the relative cell variance for the FARS and NPTS cell estimates are constructed, the relative variance for their ratio is easily estimated by taking the sum of the FARS and NPTS relative variances for the cell in question. Approximate $95 \%$ intervals are constructed using relative variances as follows:
$\mathrm{X}_{\mathrm{E}} \boldsymbol{e}^{-\lambda^{d}} \leq \mathrm{X}_{\mathrm{T}} \leq e^{2 \sigma} \mathrm{X}_{\mathrm{E}} \quad \Gamma$ where $\mathrm{X}_{\mathrm{T}}$ is the true value $\mathrm{X}_{\mathrm{E}}$ the estimated value and $\sigma$ the square root of the corresponding relative variance.*

[^8]Table 4.1 gives the relative standard errors* for use with the FARS cell estimates while Table 4.2 gives the corresponding values for the NPTS cell estimates and Table 4.3 gives the relative standard errors for the ratios (fatal involvement rates). (Tables 4.1, 4.2 and 4.3 give relative standard errors corresponding to Tables 3.2, 3.4 and 3.5 respectively.)

The use of these Tables is illustrated by calculating a confidence interval for a fatal involvement rate using Table 3.5 and Table 4.3. The fatal involvement rate for males, 25 years old or less, on urban roads, in light cars, less than 5 years old, alone in the car, during the "other" time period (i.e. not late night or rush hour) in the "summer" season is estimated by the top left entry in Table 3.5, i.e. 37.2 fatal involvements per billion VMT. The corresponding estimated relative standard error is . 142418 . Thus, the 95 percent confidence interval for the true value of this fatal invovlement rate is

$$
37.2 \mathrm{e}^{-2} \times .142 \leq \mathrm{R} \leq 37.2 \mathrm{e}^{+2 \times .142} \text { or } 28.0 \leq \mathrm{R} \leq 49.4
$$

The following comments are based on the tables. First, it will be noted that relative standard errors for the rates are rather large. For example, the upper limit of the confidence interval calculated above, 49.4 is nearly 1.8 times the lower limit, 28.0. This is fairly typical. However, the ratios of fatal involvement rates which are typically found in Table 3.5 are often much larger than two to one (the largest was observed in Section 3.3 to be over 200 to 1). Consequently many comparisons observable in Table 3.5 cannot be ascribed to noise alone. In each case where a comparison of two rates in Table 3.5 is to be made, a relative standard error for the ratio of the rates can be constructed by forming the square root of the sum of the squares of the relative standard errors of the two rates to be compared. This standard error should ideally be corrected by a covariance term. However, in the absence of any explicit estimates of the covariances it is suggested that:

[^9](a) If the cells being compared are near each other (e.g. differ in only one variable) the covariance correction may often reduce the standard error of the ratio.
(b) Even if the covariance correction increases the estimated standard error of the ratio it can never increase it by more than a factor of 1.414 (i.e. $\sqrt{2}$ ).

An example of these considerations is as follows. The fatal involvement rate in the top left corner of Table 3.5 is 37.2 while the rate in the top right corner is 204.35; the ratio of these is $204.35 / 37.2=5.5$. The relative standard error for this ratio may be estimated to be less than $1.414 \sqrt{(.142)^{2}+(.152)^{2}}=.29$. Therefore the true value of the ratio of the rates lies between $5.5 \mathrm{e}-.29 \times 2$ and 5.5e . 29 XX 2 with high confidence ( 95 percent). The interval explicitly is 3.08 to 9.82. It may be reemphasized that since the standard error estimate is an upperbound, the interval is probably considerably wider than needed for 95 percent confidence.

Another observation is based on Table 4.1 with reference to Table 3.2. The observation is that the sampling variances represented in Table 4.1 are greatly reduced from the sampling variances of the raw FARS counts. If the raw counts are assumed to be Poisson distributed (see Appendix $C$ for a discussion of this assumption) then the relative standard error of each cell count should be equal to the reciprocal of the square root of the mean value. The mean value is best estimated by the smoothed cell estimate. The relative standard error to compare this with is the relative standard error of the smoothed estimate in Table 4.1. For example, if the top left entry in Table 3.2 is considered, an estimate of $1 . / \sqrt{66.5}=.12$ is obtained for the relative standard error for the count in this cell. This may be compared to the relative standard error for the smoothed estimate obtained from Table 4.1 namely .0463. Other similar comparisons may be made:
(a) Top right hand cell. Relative standard error of raw count is estimated as $1 / \sqrt{243.8} \geq .064$ (from Table 3.2). Relative standard error of smoothed estimate from Table 4.1 is $\mathbf{. 0 3 3 1}$.
(b) Bottom left corner cell. Compare $1 / \sqrt{1.214}=.91$ from Table 3.2 to . 156 from Table 4.1.
(c) Bottom right. $1 / \sqrt{5.404}=.43$ vs. . 135 .

In general the standard error seems to have been reduced by a factor of at least 2 in most cases. This demonstrates the effectiveness of the log-linear smoothing process.
SEASUM: SUMmeR
LAMD USE:
table a.l rflative stahdary errors of the smothe d fatal iavclucinetit estiyates mathed fatal


VEHICLE AGE:


.082681 .079874 .069033 .066057 .078079 .075065 .055147 .060683 .060 .061064 .069974 .066851 .067544 .062375 .064532 .065961 $.042883 .036454 \quad .033489 \quad .027075 \quad .041983 \quad .036419 \quad .042306 \quad .034463$ $.048137 \quad .025534 \quad .045142 \quad .036296 \quad .047050 \quad .034868 \quad .043358 \quad .032260$ \begin{tabular}{l}
5 <br>
5 <br>
0 <br>
0 <br>
0 <br>
8 <br>
0 <br>
0 <br>
0 <br>
0 <br>
0 <br>
0 <br>
0 <br>
0 <br>
0 <br>
\hline

 $.074723 \quad .054922 \quad .069314 \quad .057106 \quad .064323 \quad .050999 \quad .078006 \quad .060864$ $.152776 \quad .131180$. 139283 . 116429 . 136790 . 115205 . 127941 . 109040 

.048530 \& .040217 \& .047053 \& .040842 \& .045524 \& .037224 \& .040455 <br>
\hline$-040-0.025041$
\end{tabular}



 $.060418 \quad .058025 \quad .062048 \quad .054330 \quad .044542 \quad .048964 \quad .042439 \quad .043484$




 $\begin{array}{r}87 \\ 09 \\ 09 \\ 08 \\ \hline 8\end{array}$ | 8 |
| :--- |
| 웅 |

 .068852
.120651
.118346
.126062
TABLE 4.2 RELATIVE STANDARD ERRORS OF THE SMOOTHED VMT ESTIMATES ESTIMATE)

$\begin{array}{rr}\text { LE } & 25 \\ 26 & 55 \\ \text { GE } & 56\end{array}$

| DCCUFANTS | time | lann use: VEHICLE AGE: VEHICLE WEISHT: |  | URRAN |  |  |  | 1 RURAI. |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | $\begin{aligned} & 5 \text { YRS } \\ & 1 \text { I.IGHT } \end{aligned}$ | OR LESS <br> I HEAUY | $\begin{array}{cc} \mathbf{I} & \text { OUER } \\ \text { I } & \text { LIGHT } \end{array}$ | 5 YRS <br> I HEAUY | $\begin{array}{lc} 1 & 5 \text { YRS } \\ 1 & \text { IIBHT } \end{array}$ | OR L.ESS 1 heauy | $\begin{aligned} & \text { I GUER } \\ & \text { I IGHT } \end{aligned}$ | yRE <br> I HEAUY |
|  |  | SEX | AGE | ${ }^{\mathbf{I}}$ | I | 1 | I | I | 1 | 1 | I |
| ONE | OTHER | malee | $\begin{aligned} & \text { LE } 25 \\ & 26-55 \\ & \text { GE } 56 \end{aligned}$ | 0.082390 0.091977 0.239025 | $\begin{aligned} & 0.099597 \\ & 0.061227 \\ & 0.100306 \end{aligned}$ | $\begin{aligned} & 0.101086 \\ & 0.090119 \\ & 0.141078 \end{aligned}$ | $\begin{aligned} & 0.119920 \\ & 0.068458 \\ & 0.105755 \end{aligned}$ | $\begin{aligned} & 0.147831 \\ & 0.122594 \\ & 0.275769 \end{aligned}$ | $\begin{aligned} & 0.139581 \\ & 0.06890 .3 \\ & 0.135611 \end{aligned}$ | $\begin{aligned} & 0.183346 \\ & 0.130321 \\ & 0.234536 \end{aligned}$ | $\begin{aligned} & 0.142241 \\ & 0.073378 \\ & 0.164081 \end{aligned}$ |
|  |  | FEMALE | $\begin{aligned} & \text { LE } 25 \\ & 26-55 \\ & \text { GE S6 } \end{aligned}$ | $\begin{aligned} & 0.100120 \\ & 0.087170 \\ & 0.168771 \end{aligned}$ | $\begin{aligned} & 0.154309 \\ & 0.076190 \\ & 0.123747 \end{aligned}$ | $\begin{aligned} & 0.096662 \\ & 0.100606 \\ & 0.151086 \end{aligned}$ | $\begin{aligned} & 0.064793 \\ & 0.057404 \\ & 0.161172 \end{aligned}$ | $\begin{aligned} & 0.190291 \\ & 0.098490 \\ & 0.206401 \end{aligned}$ | $\begin{aligned} & 0.128661 \\ & 0.106656 \\ & 0.172850 \end{aligned}$ | $\begin{aligned} & 0.214926 \\ & 0.145163 \\ & 0.298623 \end{aligned}$ | $\begin{aligned} & 0.073709 \\ & 0.116764 \\ & 0.184150 \end{aligned}$ |
|  | RUSH | male | $\begin{aligned} & \text { IE } 25 \\ & 26-55 \\ & \text { GE. } 56 \end{aligned}$ | $\begin{aligned} & 0.104887 \\ & 0.098225 \\ & 0.220540 \end{aligned}$ | $\begin{aligned} & 0.105912 \\ & 0.068429 \\ & 0.119363 \end{aligned}$ | $\begin{aligned} & 0.133249 \\ & 0.084877 \\ & 0.170717 \end{aligned}$ | $\begin{aligned} & 0.109368 \\ & 0.057911 \\ & 0.122039 \end{aligned}$ | $\begin{aligned} & 0.140194 \\ & 0.135699 \\ & 0.235888 \end{aligned}$ | $\begin{aligned} & 0.123403 \\ & 0.092824 \\ & 0.165877 \end{aligned}$ | $\begin{aligned} & 0.143327 \\ & 0.147840 \\ & 0.213479 \end{aligned}$ | $\begin{aligned} & 0.105804 \\ & 0.072042 \\ & 0.163236 \end{aligned}$ |
|  |  | FEMALE | $\begin{aligned} & \text { LE } 25 \\ & 26-55 \\ & \text { GE } 56 \end{aligned}$ | $\begin{aligned} & 0.116629 \\ & 0.072943 \\ & 0.122735 \end{aligned}$ | $\begin{aligned} & 0.152934 \\ & 0.070683 \\ & 0.125172 \end{aligned}$ | $\begin{aligned} & 0.115470 \\ & 0.097825 \\ & 0.147199 \end{aligned}$ | $\begin{aligned} & 0.068841 \\ & 0.049682 \\ & 0.175359 \end{aligned}$ | $\begin{aligned} & 0.193563 \\ & 0.100703 \\ & 0.199895 \end{aligned}$ | $\begin{aligned} & 0.113802 \\ & 0.118075 \\ & 0.186006 \end{aligned}$ | $\begin{aligned} & 0.222441 \\ & 0.152054 \\ & 0.293847 \end{aligned}$ | $\begin{aligned} & 0.079393 \\ & 0.103993 \\ & 0.187200 \end{aligned}$ |
|  | LATE | MALE | $\begin{aligned} & \text { LE } 25 \\ & 26-55 \end{aligned}$ | $\begin{aligned} & 0.148684 \\ & 0.112022 \end{aligned}$ | $\begin{aligned} & 0.089584 \\ & 0.092415 \end{aligned}$ | $\begin{aligned} & 0.142971 \\ & 0.116665 \end{aligned}$ | $\begin{aligned} & 0.131713 \\ & 0.093298 \end{aligned}$ | $\begin{aligned} & 0.137986 \\ & 0.100190 \end{aligned}$ | $\begin{aligned} & 0.133054 \\ & 0.097822 \end{aligned}$ | $\begin{aligned} & 0.212290 \\ & 0.156155 \end{aligned}$ | $\begin{aligned} & 0.157528 \\ & 0.118313 \end{aligned}$ |
|  |  |  | GE 56 | 0.288593 | 0.151760 | 0.250828 | 0.145616 | 0.262382 | 0.222345 | 0.265907 | 0.272277 |
|  |  | Femal.e | LE 25 | 0.137629 | 0.145754 | 0.122209 | 0.093403 | 0.232532 | 0.144013 | 0.264106 | 0.078136 |
|  |  |  | 26-55 | 0.099317 | 0.693296 | 0.091577 | 0.067140 | 0.139213 | 0.130901 | 0.154372 | 0.140975 |
|  |  |  | GE 56 | 0.251111 | 0.164784 | 0.281337 | 0.143188 | 0.261610 | 0.214952 | 0.388618 | 0.239969 |
| MORE THAN OME | OTHER | MALE | LE 25 | 0.134957 | 0.119626 | 0.109101 | 0.095429 | 0.170581 | 0.184566 | 0.129908 | 0.153974 |
|  |  |  | 26-55 | 0.118093 | 0.062289 | 0.110580 | 0.096871 | 0.241137 | 0.139359 | 0.151752 | 0.127594 |
|  |  |  | GE 56 | 0.206840 | 0.111093 | 0.159239 | 0.123693 | 0.196984 | 0.117245 | 0.250884 | 0.165192 |
|  |  | female | I.E 25 | 0.170375 | 0.197444 | 0.125985 | 0.124387 | 0.222533 | 0.174821 | 0.213830 | 0.124940 |
|  |  |  | 26-55 | 0.121118 | 0.088774 | 0.124606 | 0.090430 | 0.150295 | 0.132118 | 0.158719 | 0.102589 |
|  |  |  | GE 56 | 0.184124 | 0.162038 | 0.181717 | 0.201640 | 0.319773 | 0.262372 | 0.294224 | 0.208996 |
|  | RUSH | MALE |  |  |  |  |  |  |  |  |  |
|  |  |  | $26-55$ | 0.122147 | 0.045465 | 0.119560 | 0.092167 | 0.230994 | 0.114158 | 0.178214 | $0.134774$ |
|  |  |  | GE 56 | 0.227442 | 0.142322 | 0.171565 | 0.141527 | 0.149828 | 0.140597 | 0.258513 | 0.191200 |
|  |  | female | LE 25 | 0.179859 | 0.196680 | 0.147205 | 0.126514 | 0.189405 | 0.152305 | 0.189868 | 0.169974 |
|  |  |  | 26-55 | 0.115989 | 0.076478 | 0.119760 | 0.075962 | 0.140370 | 0.131070 | 0.159800 | 0.125106 |
|  |  |  | GE 56 | 0.183911 | 0.149601 | 0.255336 | 0.210274 | 0.285476 | 0.254791 | 0.325250 | 0.204442 |
|  | LATE | Mal.e | $\begin{aligned} & \text { LE } 25 \\ & 26-55 \end{aligned}$ | $\begin{aligned} & 0.195256 \\ & 0.127377 \end{aligned}$ | $\begin{aligned} & 0.151441 \\ & 0.101512 \end{aligned}$ | $\begin{aligned} & 0.145076 \\ & 0.115775 \end{aligned}$ | $\begin{aligned} & 0.088522 \\ & 0.094850 \end{aligned}$ | $\begin{aligned} & 0.208438 \\ & 0.213607 \end{aligned}$ | $\begin{aligned} & 0.210546 \\ & 0.137528 \end{aligned}$ | $\begin{aligned} & 0.134457 \\ & 0.166194 \end{aligned}$ | $\begin{aligned} & 0.136886 \\ & 0.153777 \end{aligned}$ |
|  |  |  | GE 56 | 0.330158 | 0.134102 | 0.252086 | 0.145131 | 0.256928 | 0.159996 | 0.201615 | 0.220489 |
|  |  | FEMALE | 1525 | 0.216777 | 0.218321 | 0.187401 | 0.120195 | 0.272489 | 0.177666 | 0.264150 | 0.122548 |
|  |  |  | 26-55 | 0.139900 | 0.095804 | 0.161953 | 0.119028 | 0.186673 | 0.159210 | 0.214168 | 0.185757 |
|  |  |  | GE 56 | 0.244788 | 0.192843 | 0.341467 | 0.180829 | 0.450711 | 0.316852 | 0.445790 | 0.259331 |


table 4.3 (CONTINIED). RELATIVI: :STANDARD ERRORS OF TIIE FATAI. INVOI.VEMIENT RATES (Estimated standard Error-cell esimate)


### 5.0 SMALL CAR/LARGE CAR FATAL ACCIDENT INVOLVEMENT RATE EXPOSURE STUDY

This section provides an example of the data and methods developed in this report. This section compares the fatal involvement rates of light cars with heavy cars. The measure to be used for this comparison is the ratio of the fatal involvement rates for light cars ( $3,000 \mathrm{lbs}$. or less) to that for heavy cars (over $3,000 \mathrm{lbs}$.$) . The multivariate fatal involvement rates shown in Table 3.5$ permit this comparison to be made while controlling for the seven variables other than vehicle weight in Table 1.1. Further, they permit the dependence of this ratio on these other variables to be determined.

The ratios of fatal involvement rates are given in Table 5.1. The table gives the ratio of the fatal involvement rate for light vehicles to that for heavy vehicles for each combination of levels of the other seven variables. There are 288 cells in this table, which has the same format as Table 3.5. Beside each fatal involvement rate ratio is a sequence of seven small numbers indicating the level of each of the seven variables (these are of help in some detailed studies of the table, but are not of any interest in this report). Table 5.2 gives the relative standard errors of the fatal involvement rate ratios given in Table 5. These are calculated using the sample splitting techniques described in Section 4.*

An examination of Table 5.1 shows that the largest fatal involvement rate ratio is just under 3.0 while the smallest is just over .56. Based on the relative standard errors in Table 5.2, most comparisons in Table 5.1 unlike those in Table 3.5 are not significant. The fatal involvement rate ratio changes considerably from cell to cell but not by a large enough amount to make an analysis easy.
*The relative variance of the ratio of corresponding light and heavy cells is calculated separately for the numerator (FARS) and the denominator (NPTS). The relative variance of the latal involvement rate ratio itself is equal to the sum of these relative variances. The relative standard error is the square root of the relative variance. As noted in Section 4, it can be interpreted as the ratio of the standard error of the estimate to the estimate itself (in this case fatal involvement rate ratio). The details of calculating relative variances using FARS and NPTS are given in Section 4.

Primary Measure of the Effect of Car Size: Ratio of Fatal. Invoivement




In spite of the considerable variability of the fatal involvement rate ratio from cell to cell in Table 5.1 and the considerable noisiness in the individual cells, it is desirable to have a summary of the information it contains. In particular, the answers to simple questions are sought:

1. Overall, can we say that light or heavy cars tend to have higher fatal involvement rates in similar situations?
2. On which variables does the ratio of fatal involvement rates (light/heavy) depend most strongly? Do these variables have a consistent effect?

Any attempt at a simple analysis quickly reveals that the presence of higher order interactions makes it hard to determine simple effects.

The chief tool to be used in this analysis is the weighted average over cells of the logarithm of the ratio of fatal involvement rate (FIR) of light cars to that of heavy cars. The weighting factor is estimated fatalities in each cell (as given in Table 3.2).

This may be illustrated by calculating the overall comparison of light to heavy vehicles averaged over all cells.

Consider
$\sum_{\text {ceils }} \log ($ FIR (light)/FIR (heavy))* Fatal Involvements $=\log R$

$$
\sum_{\text {cells }} \text { Fatal Involvements }
$$

Here FIR (light) denotes the fatal involvement rate for light vehicles for a particular combination of the seven variables (i.e., a particular cell). FIR (light)/FIR (heavy) denotes the ratio of fatal involvement rates light to heavy. The factor Fatal Involvements is a weighting factor (it is specific to the cell, it is the estimated fatal involvements in Table 3.2). The sum over cells is over all cells in this case. Since the average of the logarithm of the FIR ratio is being calculated, the anti-logarithm is taken at the end to get $R$, fatal involvement -
weighted average ratio. When this formula is applied to Table 5.1, the result is $\mathbf{R}=1.059$. On the average, overall, light cars have a 5.9 percent higher fatal involvement rate than heavy cars. This average is obtained while controlling for the other seven factors:
a. Driver Age
b. Driver Sex
c. Vehicle Age
d. Time of Day
e. Season
f. Urban/Rural
g. Number of Occupants

These factors are controlled for in the sense that FIR ratios are obtained for specific values of these variables and then averaged. The effect of any tendency for either light or heavy cars to be used more in particular circumstances identifiable by these variables (e.g., by young drivers late at night) is eliminated (to the extent that the categories for each variable are fine enough).

To find the fatal involvement rate ratio without controlling for these variables, the overall fatal involvement rate for small cars (total fatalities divided by total VMT) is divided by the rate for large cars. This leads to the estimate that small cars have a fatal involvement rate which is 8.9 percent* higher than for large cars. Since this is more unfavorable to small cars than the ratio controlling for the other factors (i.e., 8.9 percent to 5.9 percent), it may be concluded that the circumstances for small cars (their drivers, environment, etc.) lead to their getting into more latal accidents than just the specific characteristics of the cars themselves.

Before calculating how the fatal invovlement rate ratio is influenced by certain factors, it should be pointed out that the estimated overall average fatal involvement rate ratio 1.059 , although it shows that small cars get into more fatal accidents than large cars, is really quite close to 1 . This may seem a little
*This number is derived from the "Vehicle Weight" row of Table 2.5 as follows: (Fatal Involvements LE 3000 lbs )/(VMT LE 3000 lbs ) : (Fatal Involvements GT $3000 \mathrm{lbs}) /($ VMT GT 3000 lbs$)=(12,504 / 212.6) /(24,805 / 459.1)=1.089$
paradoxical since small cars afford less occupant protection than large cars. However, a large component of the crashworthiness advantage that large cars have is nullified in the FIR ratio measures. Specifically all fatal accidents between small and large cars are counted against both the small car involved and the large car involved regardless of which vehicle(s) a fatality occurred in. Thus, large cars may be "safer" for their occupants but are only slightly better in terms of their fatal involvement rate.

The next three subsections deal with the selection of the most important factors to examine for a simpler representation of the dependences of the FIR ratio (fatal involvement rate ratio) than in Table 5.1. Basically the criteria are strength and consistency of effect. There are three techniques to be used to find out which factors have the strongest, most consistent effect:

1. Examining individual cells
2. Examining the log-linear model representing the FIR ratio
3. Examining weighted averages over groups of cells.

### 5.1 Examining Single Cells

The chief idea here is to look for cells that have FIR ratios significantly higher or significantly lower than the mean. To see if the FIR ratio is significantly large or small, the weighted mean of the log of the FIR ratio is subtracted from the log of the FIR ratio for each cell. The result is divided by the standard error of the $\log$ of the FIR ratio (for the particular cell).* If the result is larger than 2.326, or smaller than -2.326, the FIR ratio for that cell is labelled as "significant". The number 2.326 was chosen somewhat arbitrarily; a standard normal random variable (zero mean, unit standard deviation) has a probability of . 01 of being larger than 2.326. If the true value of all the FIR ratios is equal to the mean value, then sample cell estimates (such as those produced here) would be more than 2.326 standard errors larger than the mean approximately one percent of the time, i.e., out of 288 cells, the expected number is around 3 (approximately 2.88). There were actually 19 FIR ratios over 2.326 standard errors greater than the mean. Clearly there is a very high probability that the true FIR ratio for most of these 19 cells is different from the mean.
*These standard errors are given in Table 5.2.

If the 19 cells which are significantly less than the mean (by this definition) are each marked by a row of asterisks as in Table 5.3, the pattern they make shows that certain levels of some variables are more stongly and consistently associated with small ratio cells than others. The conditions most associated with the small ratio cells from Table 5.3 appear to be:

1. Young drivers (and to a lesser extent middle-aged drivers)
2. Urban driving
3. Only one occupant in vehicle
and to a lesser extent:
4. "Other" time period
5. Male drivers

If the same exercise is carried out on cells which have FIR ratios significantly higher than the mean, the 38 cells marked in Table 5.4 are identified. The conditions most associated with high FIR ratio cells appear to be:

1. Rural driving
2. Older and middle-aged drivers
and to a less extent:
3. More than one occupant in the vehicle
4. Vehicle over five years old

As a result of this analysis, the most promising single variables are Urban/Rural and Driver Age; both of which have strong effect and which have a consistent effect in that when you reverse the variable, the FIR ratio reverses. It appears that this effect may be somewhat independent of the other variables.

### 5.2 Examination of the Log-Linear Model

The log-linear model for the FIR ratio can be calculated from the $\log$-linear model estimating the FARS fatal involvements by cell and the log-linear model estimating the NPTS VMT total by cell (both are described in Section 1.0). The fatal involvement rate, FIR, is the ratio of the FARS cell estimate to the NPTS cell estimate. Thus, $\log$ FIR $=\log$ FARS $-\log$ NPTS, which means that the terms in the log-linear model for the fatal involvement rate are obtained by
FIR Ratios（Light／Heavy）Significantly Less than Mean
VEHICLE AGE：
UEHICIE UEIGHT：
UREAN

L. b/ 1
SEASON: WINTER

UEHICLE AGE：
クロリ＇
SEASON：SUMMER

5,48
FIR Ratios (Light/Heavy) Significantly Greater than Mean (continued)

subtracting the terms in the NPTS log-linear model from those in the FARS loglinear model.

The logarithm of the FIR ratio is then obtained by subtracting the log of the fatal involvement rate for heavy vehicles from that for light vehicles: $\log ($ FIR ratio $)=\log$ FIR (light) $-\log$ FIR (heavy)

Now the terms which do not involve vehicle weight cancel between $\log$ FIR (light) and log FIR (heavy) while those terms which do involve weight are in FIR (heavy) with the same magnitude but opposite sign as the corresponding terms in FIR (light). Thus $\log$ (FIR ratio) $=2 * \log$ FIR (retricted to terms involving vehicle weight), i.e., the log-linear model for the FIR ratio consists of the terms in the log-linear model of the fatal involvement rate which involve vehicle weight multiplied by a factor of 2.

The resulting terms are listed in Table 5.5. This table lists all coefficients in the log-linear model which are not redundantly determined by other coefficients. For each two-level variable in an interaction, the coefficient appropriate to the lower level (level 1) is listed and the coefficient appropriate to the upper level is obtained by multiplying by -1 . For each three-level variable, the coefficients appropriate to the lowest level is listed first. Thus, the coefficient -.1584 in the 5-7 interaction corresponds to level 1 of both variable 5 and variable 7. The coefficient . 0442 in the 3-6 interaction corresponds to level 1 for both variables 3 and 6. The coefficient . 0196 in the 5-7 interaction corresponds to level 2 of variable 5 and level 3 of variable 7

The purpose of Table 5.5 is to give an idea of the relative magnitude of various lower order and higher order effects. Clearly, Driver Age and Urban/Rural are among the most influential single effects. Number of Occupants and Car Age are also influential. Note that there are strong interactions involving driver age, (2-7, 3-7, 5-7...) but these do not seem to completely overwhelm the main efect. Other variables such as Driver Sex and Time of Day have strong simple effects but enter into even stronger interactions. Even the strongest main effect can be swamped by a combination of high order interactions. This is consistent with the general picture that no single variable has a highly-consistent effect.

Table 5.5
Lof-Linear Model foe PIR Ratio


### 5.3 Examination of Groups of Cells

The analysis to this point suggests that four variables have a strong relatively consistent effect:

## 1. Driver Age <br> 2. Urban/Rural <br> 3. Number of Occupants <br> 4. Vehicle Age

The evidence for the variable Vehicle Age is not as strong as for the first three.

The last analysis will confirm these observations. The idea of this analysis is that if one fixes an important variable at a critical value, the average FIR ratio will be significantly different from before the variable is fixed and this will be true even when other variables are fixed at various levels. This can best be seen by a concrete example.

A particular cell is chosen (as the "target" cell) such as the cell with the largest (or smallest) FIR ratio. This determines a level for each variable. Now an ordering of the variables is chosen and in turn each variable is fixed at the chosen level and a weighted average of the FIR ratio over cells is taken. The weighted average is first taken over all cells, then over only those cells corresponding to the chosen level of the first variable,* then only those cells corresponding to the chosen levels of the first two variables, etc. until the final average is over just one cell, the one selected as the target cell (and so no average is needed).

The results of such a process will be referred to somewhat arbitrarily as a "tree". Table 5.6 shows three such "trees". This table is best explained by indicating the information given in some typical lines. The second line of data shows that the first variable to be fixed was variable 7, it was fixed to level 3 (older drivers).

[^10]Analysis by Groups of Cells


This level corresponded to 96 cells $(96=288 / 3)$. The total number of estimated fatal involvements for these 96 cells was 5602 (this gives an idea of the statistical stability of an average of these cells); the standard error of the mean of the log of the FIR ratio over these 96 cells was .0355,* the weighted average (with fatal involvements as weighting factor) of the logarithm of the PIR ratio over these 96 cells was .2989 ; the anti-logarithm of .2989 is 1.348 . The next line contains similar information about an average with the next variable (variable number 3) fixed to level number 2 (older cars). This left 48 cells and the weighted average of the logarithm of the FIR ratio over these 48 cells was .4311 , etc.

Certain lines contain an asterisk after the logarithm of the FIR ratio. This indicates that this quantity changed by more than .1 from the previous line. This indicates that the variable first fixed on that line appears to be relatively important. The variables so identified in the three trees in this table (all three end at the cell with the largest FIR ratio 2.977) and in the tree in Table 5.7 (which ends at the smallest FIR ratio .5844) are as follows:

1. Driver Age, Car Age, Number of Occupants, Urban/Rural, Driver Sex
2. Driver Age, Number of Occupants, Car Age, Urban/Rural
3. Number of Occupants, Driver Age
4. Number of Occupants, Urban/Rural, Driver Age, Time Period
[^11]Table 5.7
One Tree Rading in Cell with Smallest Ratio


Further analyses of this type of confirmed that the strongest and most consistent variables were:

1. Driver Age
2. Urban/Rural
3. Number of Occupants
and possibly,
4. Age of Car

### 5.4 Analysis Using Three Variables

Certain variables have an inconsistent effect (Driver Sex, Time of Day) or a weak effect (Season). But four variables have been identified as having a strong, consistent effect (although the fourth, Vehicle Age, is not as strong as the others). This section is concerned with characterizing how the fatal involvement rate ratio explicitly depends on three out of these four variables. The variable, Number of Occupants, will not be included since it is felt to be largely a crashworthiness connected factor: small cars are less crashworthy than large cars, therefore, more occupants in the car does more to make small cars more likley to have a fatal accident than it does to make large cars more likely.

The analysis comes down to three variables for explicit consideration:

1. Driver Age
2. Urban/Rural
3. Vehicle Age

The other five variables (including Number of Occupants) are controlled for. This is done taking weighted averages (weighting factor equal to fatal involvements) over groups of cells corresponding to specific levels for the three specified variables. The result is given in Table 5.8. For each combination of levels for the three variables the weighted average (over all cells with those characteristics) of the logarithm of the FIR ratio is given in rows labelled "(2)" (as indicated by the legend). The anti-logarithm of this number is given in the rows labelled " $(1)^{\prime \prime}$. For example, looking at the top left corner of the table, young drivers in urban areas in newer cars are estimated to have only 82 percent

## Analysis by Groups of Cells

Table 5.8
Bramination of the Most Important Pactors
Affecting the FIR Ratio

|  | Young Drivers |  | Middle Age Drivers |  | Old Drivers |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Newer Cars | Older Cars | Newer Cars | Older Cars | Newer Cars | Older Cars |
| Urban (1) | . 8200 | . 8936 | . 9703 | . 9825 | . 9440 | 1.1652 |
| (2) | -. 1984 | -. 1125 | -. 0301 | -. 0382 | -. 0576 | . 1529 |
| (3) | -. 3386 | -. 1503 | -. 1137 | . 0767 | . 0257 | . 2140 |
| (4) | 2753 | 4518 | 3270 | 3518 | 1000 | 1456 |
| (5) | . 0383 | . 0505 | . 0324 | . 0439 | . 0291 | . 0419 |
| Rural (1) | . 8133 | 1.0631 | 1.9686 | 1.406 | 1.280 | 1.932 |
| (2) | -. 2067 | . 0612 | . 1797 | . 341 | . 2467 | . 6586 |
| (3) | -. 0779 | . 1104 | . 1493 | . 3376 | . 2884 | . 4747 |
| (4) | 3710 | 5825 | 3712 | 4350 | 1366 | 1780 |
| (5) | . 0330 | . 0489 | . 0486 | . 0515 | . 0356 | . 0625 |

(1) Ratio
(2) Log ratio
(3) Estimated $\log$ ratio - main effects model
(4) Total fatal involvements in cell
(5) Standard Error (over cells) of log ratio
(.82) as many fatal accidents (per VMT) in small cars as in large cars. On the other hand, older drivers in older cars in rural areas are estimated to have 93 percent more (1.932) fatal accidents in small cars than in large cars. The rows labelled ${ }^{n}(4)^{n}$ and ${ }^{n}(5)^{n}$ are for use in assessing the statistical stability of the estimates in the same way as described for Table 5.6 (they give total fatal involvements in the group and the standard deviation over cells withn the group of the logarithm of the FIR ratio). This table, although based on only three variables, is still too complex for easy comprehension. The dependence of the FIR ratio on these variables would be simple to describe and understand if it were according to a simple main effects model with no interactions. More specifically, assume a simple log-linear model -- that the FIR ratio is determined by three factors, one for each variable, i.e, the $\log$ of the FIR ratio is the sum of three terms each depending on only one of the variables. The results of fitting such a model (using linear modelling techniques on the log of the PIR ratio*) are shown in the rows labelled "(3)". The entries in the rows labelled " $(3)^{\prime \prime}$ are to be compared to the corresponding entries in the rows labelled " $(2)^{\prime \prime}$ (the logarithm of the FIR ratio) to see how well the model fits. The fit, although not exceptionally good, is satisfactory and indicates that the model estimates the FIR ratio in all conditions with at least "ball park" accuracy. When the model estimates high, the ratio is high, when the model estimates low, the ratio is low, when the model is between, so is the ratio. This analysis suggests that there is value in looking at the effect of the variables singly.

Table 5.9 shows the results of averaging over all variables but one when the variable not averaged over is each of the three selected variables in turn. This table is similar in content to the previous tables (Table 5.6 and Table 5.8). The column labelled "Log Ratio" gives the weighted average of the logarithm of the FIR ratio over all cells corresponding to the variable level (condition) specified in the last column. The first column gives the anti-logarithm of the second column; it gives the best estimate of the aggregate FIR ratio for the specified condition, controlling for the other variables.

[^12]Analysis by Groups of Cells

> Table 5.9 Adpregate Bffects of Three Main Variables

| Ratio | Log Ratio | M of Cells | Total Fatal <br> Lnvolvements | SE <br> (over cells) | Variable |
| ---: | :---: | :---: | :---: | :--- | :--- |
| .934 | -.0679 | 144 | 16578 | .0185 | urban |
| 1.170 | .1566 | 144 | 20743 | .0277 | rural |
| .971 | -.0294 | 144 | 15811 | .0213 | newer cars |
| 1.128 | .1204 | 144 | 21510 | .0274 | older cars |
| .916 | -.0872 | 96 | 16806 | .0254 | young drivers |
| 1.137 | .1284 | 96 | 14913 | .0278 | middle age drivel |
| 1.348 | .2989 | 96 | 5602 | .0355 | old drivers |

The columns labelled "Total Fatal Involvements" and "S E (over cells)" are as usual for assessing statistical stability (ratio-based on more than 10,000 fatal involvements, with a "standard error over cells" of less than .03 should be statistically very stable). The conclusions to be drawn from Table 5.9 are these:
(1) In urban driving, small cars have about six to seven percent fewer fatal involvements per VMT in similar situations than do large cars;
(2) In rural driving, the comparison favors large cars by about 16 percent;
(3) For young drivers, small cars have about 9 prcent fewer fatal involvements (again, per VMT and controlling for other factors);
(4) For older drivers, small cars are 30 pecent worse than large cars according to this measure (fatalities per VMT controlling for other factors); and
(5) For newer cars, small cars may be about three percent better than large cars (this may not be statistically significant).

## APPENDIX A

## Development of PARS and NPTS Log-Linear Models

The log-linear models for the FARS and NPTS data were constructed using the LOGLIN computer package developed at the Harvard School of Public Health. In general, the method used for constructing the models is to start with the three sets of (1) all 28 two-way interactions (I-J), (2) all 56 three-way interactions (I-JK ), and (3) all 70 four-way interactions ( $\mathrm{I}-\mathrm{J}-\mathrm{K}-\mathrm{L}$ ), select significant interactions from these sets to create a more parsimonious model, and continue a process of eliminating the weaker interactions and adding in stronger interactions until a satisfactory model is developed.

Interactions are eliminated (or added) on the basis of thresholds for $\Delta X^{2} / \Delta d f$ (those interactions with $\Delta X^{2} / \Delta \mathrm{df}$ lower than the threshold are dropped). Note, however, that higher order interactions draw in implied lower order interactions if the latter are not already in the model (for example, interaction I-J-K includes $\mathrm{I}, \mathrm{J}, \mathrm{K}, \mathrm{I}-\mathrm{J}, \mathrm{J}-\mathrm{K}, \mathrm{I}-\mathrm{K})$. Therefore, when eliminating interactions on the basis of $\Delta X^{2} / \Delta d f$ for a higher order interaction, provisions must be made for the dilution of strength ( $\Delta \mathrm{X}^{2} \| \mathrm{df}$ ) by implied lower order interactions not includable on the basis of their own $\Delta x^{2} / \Delta d f$.

In developing these models, $\Delta X^{2} \| \mathrm{df}$ for each interaction was approximated using standardized effect estimates (effect coefficients divided by their standard error)* for the interactions in the final model. The results of this comparison indicate the approximations are exact for interactions of one degree of freedom but rough estimates of $\Delta \mathrm{X}^{2} \Delta \mathrm{df}$ for interactions with multiple degrees of freedom.

The following is a description of the steps used to develop the FARS and NPTS models. For both the FARS and NPTS data:
*The estimate used was $\Delta X^{2} \Delta \Delta d f=\left(\Sigma_{i=1}^{K} E_{j}^{2}\right) / K$ where $E_{i}$ is the value of the $i$ th standardized effect coefficient and $K$ is the number of such coefficients for the given interactions.

1. Calculate the standardized effects estimates for models based on the sets of all 28 two-way interactions (1-2, 1-3, 1-4,... 7-8), all 56 three-way interactions ( $1-2-3,1-2-4, \ldots, 6-7-8$ ) and all 70 four-way interactions (1-2-3-4, 1-2-3-5, ..., 5-6-7-8). Table A. 1 summarizes the results of fitting the models.

Note 1: A similar run of all 8 one-way ineractions (1, 2, 3, 4, 5, 6, 7, 8) was not done, as it was decided all would be included in the final model (all one-way interactions were strong at each stage of model development).

Note 2: All four-way interactions were fit to the NPTS data (originally scaled by $\mathbf{1 0}^{5}$ ), then the data was scaled such that the resulting $\mathrm{X}^{2}=\mathrm{df}$ :
$\frac{X^{2}}{\text { scaling factor }}=d f$
$118493=263$
scaling factor
scaling factor $\mathbf{= 4 5 0 . 5 4}$
The scaled data was then used to develop the model, including the initial runs of all two, three, and four-way interactions.
2. Construct a model consisting of all interactions from these three runs with an estimated $\mathrm{X}^{2} /$ df greater than 2.5.

Example calculation:
Standardized effect estimate, interaction 4-5, FARS data, four-way run.

|  | DIM.5* |  |  |
| :--- | :---: | :---: | ---: |
| DIM.4* | (1) | (2) | $(3)$ |
| $(1)$ | -10.31 | 7.81 | .65 |
| $(2)$ | 10.31 | -7.81 | -.65 |

*The variables are associated with numbers in Table A-7. Variable 5 is "Time" and variable 4 is "Occupants."

TABLE A.1: Results of Fitting the FARS and NPTS Data to the Models Consisting of All Two, Three and Four-Way Interactions

FARS:

|  | X $^{2}$ | df | $P^{*}$ |
| :--- | :--- | :--- | :--- |
| two ways | 907.29 | 522 | $.33 \times 10^{-22}$ |
| three ways | 462.69 | 418 | .065 |
| four ways | 270.41 | 263 | .36 |

NPTS (data scaled by 105 , divided by 450.54 ):

|  | X $^{2}$ | df | P |
| :--- | :--- | :--- | :--- |
|  |  |  |  |
| two ways | 840.05 | 522 | $.29 \times 10^{-16}$ |
| three ways | $\mathbf{4 7 3 . 6 5}$ | 418 | .031 |
| four ways | 263.00 | 263 | .49 |

```
Estimated \(\Delta \mathrm{X}^{2} / \Delta \mathrm{df}=\)
\(\left((-10.31)^{2}+(7.81)^{2}+(.65)^{2}+(10.31)^{2}+(-7.81)^{2}+(.65)^{2}\right) / 6\)
\(=55.90\)
```

Note: At this step, the weakening of the standardized effects estimates by weak implied lower order interactions was ignored. The ignoring of dilution effects at an early stage is a conservative convenience as it retains more interactions than necessary.
3. The models constructed from the strong interactions were fitted to the two data sets. The results are shown in Table A. 2.

* $P$ is the probability of getting a chi square value as large as or larger than that reported (in column labelled $\mathrm{X}^{2}$ ) if the data arose from a process in which the specified model were accurate, i.e. according to a binomial (or poisson) distribution over cells with each cell probability proportional to the model predicted frequency.

TABLE A.2: Fit Resulting from Strong Interactions in Two, Three and Four-Way Russ

|  | $\underline{X}^{2}$ | $\frac{\text { df }}{}$ | $\frac{P}{42}$ |
| :--- | :--- | :--- | ---: |
| FARS | 414.05 | 429 | .69 |
| NPTS | 390.98 | 396 | .56 |

4. The estimated $\Delta X^{2} / \Delta d f$ for the interactions in these models were calculated. From these results, new models were constructed using a 2.0 threshold, including the dilution of $\Delta \mathrm{X}^{2}$ by implied weak lower order interactions.
5. These new models were fitted to the data sets. The results are shown in Table A.3:

TABLE A.3: Results of Fitting Models Developed Using a 2.0 Threshold for $\mathrm{X}^{2} / \mathrm{di}$

|  | $\frac{\mathrm{X} 2}{}$ | df | $\frac{\mathrm{P}}{\mathrm{I}}$ |
| :--- | :--- | :--- | :--- |
| FARS | 490.20 | 480 | .36 |
| NPTS | 497.29 | 463 | .13 |

6. Step 4 was repeated using a 2.5 threshold for $X^{2} /$ df on the latest models. The results are shown in Table A.4.

## TABLE A.4: Results of Fitting Models Developed Using a 2.5 Threshold for $\mathrm{X}^{2 /} \mathrm{df}$

|  | $\underline{X 2}$ | $\frac{\text { df }}{}$ | $\underline{P}$ |
| :--- | :--- | :--- | :--- |
| FARS | 494.14 | 482 | .34 |
| NPTS | 544.66 | 485 | .031 |

7. Step 4 was repeated using a 3.0 threshold for $\Delta x^{2} / \Delta d f$ on the latest models. The results are shown in Table A.5.

Note: For the NPTS model, there was no change (all interactions included in the previous run at the 2.5 threshold had estimated $\Delta X^{2} / \Delta \mathrm{df} \quad 3.0$ )

TABLE A. 5 Results of Fitting Models Developed Using a 3.0 Threshold for $\mathrm{X}^{2 /}$ di

|  | X2 | df | $\mathbf{P}$ |
| :--- | :--- | :--- | :--- |
| FARS | $\mathbf{4 9 8 . 0 1}$ | $\mathbf{4 8 3}$ | .31 |
| NPTS | 544.66 | 485 | .031 |

8. At this step, the models were examined and the original runs of all two, three and four-way interactions were searched for potentially significant interactions to be 'readded' to the model for a 'second chance.' In the NPTS model, all but 3 two-way interactions were already included, hence these three were retried. Likewise with the FARS model, all but 4 were included, hence these were retried. Additionally, any three or four-way interactions with an estimated $\Delta \mathrm{X}^{2} / \Delta \mathrm{df}$ greater than 2.5 , not diluted, were retested (five and eight respectively for the-FARS data and three and nine respectively for the NPTS data).

These interactions were added to the respective models, one at a time, and the actual $\Delta X^{2}\langle\mathrm{df}$ was calculated, including implied lower order interactions. With the NPTS model, one two-way, one three-way and one four-way were significant ( $\Delta X^{2} / \Delta d f>3.0$ ), hence added to model. With the FARS model, none were significant.
9. Next, for interactions in each model that had estimated $X^{2} /$ df (diluted) close to 3.0 , the actual $\Delta X^{2} \angle \Delta \mathrm{~d}$ was calculated. Those less than 3.0 were eliminated (this included one three-way from the NPTS model and a oneway and a four-way from the FARS model).
10. These final models were fitted to the data and the actual $\Delta X^{2} / \Delta d i f$ values were calculated. No higher order interactions, including dilution, were less than 3.0. Table A. 6 shows the results of fitting these final models to the FARS and NPTS data. Table A. 7 lists the interactions included in each model. (Note that Table A. 7 refers to the interactions by number and also indicates which variables correspond to each number.)

## TABLB A. 6 Results of Fitting Final Models to FARS and NPTS Data

|  | X2 | d | $\underline{P}$ |
| :--- | :--- | :--- | :--- |
| FARS | $\mathbf{5 0 8 . 2 4}$ | $\mathbf{4 8 9}$ | .26 |
| NPTS | 514.88 | 479 | .12 |

Note that the large $P$ values with the given number of degrees of freedom strongly indicate that the models were not underfit.* The $P$ value is the probability of getting so large a chi square if there were no underfit bias. Even a small bias will tend to make $\mathbf{P}$ quite small with so many degrees of freedom. As noted in Section 3.1, it was intended that the models not be underfit in order that any inaccuracy be due to sampling error and not to bias.
*Of course the strength of this indication is limited by the fact that the $\mathrm{X}^{2}$ value on which $P$ is based is artificially constructed in the case of the NPTS data.

TABLE A. 7 Interactions in Pinal FARS and NPTS Models (Implied Lower Order Interactions Not Listed Separately)

| FARS | NPTS |
| :---: | :---: |
| 3-4-6-7 | 3-4-6-7 |
| 2-6-7, 6-7-8 | 2-6-7-8 |
| 2-3-7 | 2-3-4-7 |
| 2-3-6 | 2-3-6 |
| 1-2-7 | 1-2-7 Commonality |
| 4-5-6 | 4-5-6 |
| 3-7-8 | 3-7-8 |
| 1-3 | 1-3 |
| 1-4 | 1-4 |
| 2-4-5 | 5-7-8 |
| 4-5-7 | 2-5-7 |
| 1-2-5 | 2-4-6 |
| 1-5-6 | 2-3-8 |
| 5-6-8 | 1-6-8 |
| 5-6-7 | 3-4-8 |
|  | 3-6-8 |
|  | 1-6-7 |
|  |  |
| Variable Number: | Variable Name: |
| 1 | Season |
| 2 | Urban/Rural |
| 3 | Model Year |
| 4 | Occupants (one/more than one) |
| 5 | Time |
| 6 | Sex of Driver |
| 7 | Age of Driver |
| 8 | Weight of Vehicle |

## APPENDIX B

## Empirical Test of Using Standardized Effect Estimates as a Prozy for $\Delta x^{2} / \angle \mathrm{df}$

The following tables show the actual $\Delta \mathrm{X}^{2} / \Delta \mathrm{df}$ and the estimated $\Delta \mathrm{X}^{2} / \Delta \mathrm{df}$ for the interactions in the final FARS and NPTS models (except the implied one- and two-way interactions). The estimated $\Delta X^{2} \not \Delta$ df were computed from the standardized effects estimates from the final models as described in Appendix A. The actual $\Delta x^{2} / \Delta \mathrm{df}$ were calculated by eliminating one interaction at a time from the model.and calculating the resulting fit.

The implied two-way interactions were not tested because virtually all were included in the models, either on their own or because they were in several higher order interactions. Therefore, if a three-way interaction was eliminated, all two-way interactions were retained so only the three-way interaction was tested. The three-way interactions implied in the four-way interactions were tested however. They give an indication of the accuracy of the estimates for implied lower order interactions. (Not all implied three-ways were tested in the NPTS model because there were so many.) In the case of the implied three-way interactions which were tested, the actual $\Delta \mathrm{X}^{2} / \triangle \mathrm{df}$ is based on the difference between the chi square without the four-way interactions but all implied threeways retained and the chi square without the four-way and three-way being tested. In other words, the base model in the case of these interactions is the full model minus the four-way interaction itself.

TABLE B. 1
Results of Test on FARS Model

| INTERACTIONS |  |  | $\triangle \mathrm{X}^{2}$ |  |
| :---: | :---: | :---: | :---: | :---: |
| ELIMINATED** | $\mathrm{x}^{2}$ | df | ESTIMATED | ACTUAL |
| None | 508.24 | 489 | -- | -- |
| 3-4-6-7 | 525.17 | 491 | 8.71 | 8.47 |
| *3-4-6-7,3-4-6 | 525.17 | 492 | 1.69 | 0 |
| *3-4-6-7,3-4-7 | 549.69 | 493 | 3.51 | 12.26 |
| *3-4-6-7,3-6-7 | 551.19 | 493 | 14.23 | 13.01 |
| *3-4-6-7,4-6-7 | 554.75 | 493 | 14.65 | 14.79 |
| 2-6-7 | 515.97 | 491 | 3.95 | 3.87 |
| 6-7-8 | 524.08 | 491 | 5.90 | 7.92 |
| 2-3-7 | 519.37 | 491 | 4.79 | 5.57 |
| 2-3-6 | 520.97 | 490 | 12.74 | 12.73 |
| 1-2-7 | 519.67 | 491 | 4.33 | 5.72 |
| 4-5-6 | 518.39 | 491 | 5.38 | 5.08 |
| 3-7-8 | 555.03 | 491 | 25.95 | 23.40 |
| 2-4-5 | 556.68 | 491 | 20.92 | 24.22 |
| 4-5-7 | 587.10 | 493 | 14.94 | 19.72 |
| 1-2-5 | 517.68 | 491 | 5.56 | 4.72 |
| 1-5-6 | 517.62 | 491 | 5.23 | 4.69 |
| 5-6-8 | 518.04 | 491 | 4.77 | 4.90 |
| 5-6-7 | 524.45 | 493 | 3.87 | 4.05 |
| 1-3 | 517.35 | 490 | 9.12 | 9.11 |
| 1-4 | 525.29 | 490 | 17.06 | 17.05 |

* Actual $\angle \mathrm{X}^{2} \not \triangle \mathrm{df}$ as compared to fit with the higher order four-way eliminated. **Note: The variables are referred to by the numbers associated with them in Table A-7.


## TABLE B. 2

## Results of Test on NPTS Model

| INTERACTIONS ELIMINATED* | $\mathrm{x}^{2}$ | df | $\Delta x^{2} / \Delta d f$ |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  |  | ESTIMATED | ACTUAL |
| None | 514.88 | 479 | --- | --- |
| 2-3-4-7 | 525.79 | 481 | 4.54 | 5.46 |
| -2-3-4-7,2-3-4 | 526.91 | 482 | 4.28 | 1.12 |
| *2-3-4-7,2-4-7 | 531.08 | 483 | 3.19 | 2.65 |
| 2-6-7-8 | 523.45 | 481 | 5.08 | 4.29 |
| - $2-6-7-8,2-6-7$ | 526.13 | 483 | 3.41 | 1.34 |
| *2-6-7-8,6-7-8 | 545.12 | 483 | 10.95 | 10.84 |
| *2-6-7-8,2-7-8 | 530.67 | 483 | 2.5 | 7.22 |
| 3-4-6-7 | 522.49 | 481 | 4.01 | 3.81 |
| *3-4-6-7,3-4-6 | 527.89 | 482 | 12.39 | 5.4 |
| *3-4-6-7,3-6-7 | 532.42 | 483 | 3.37 | 4.97 |
| *3-4-6-7,4-6-7 | 582.91 | 483 | 29.34 | 27.51 |
| 5-7-8 | 538.93 | 483 | 3.12 | 6.01 |
| 3-4-8 | 535.20 | 480 | 20.25 | 20.32 |
| 1-6-8 | 531.72 | 480 | 16.81 | 16.84 |
| 1-2-7 | 525.90 | 481 | 5.48 | 5.51 |
| 2-3-6 | 51.8 .75 | 480 | 3.88 | 3.87 |
| 2-3-8 | 518.92 | 480 | 4.04 | 4.04 |
| 2-4-6 | 525.31 | 480 | 10.43 | 10.43 |
| 2-5-7 | 527.37 | 483 | 3.94 | 3.12 |
| 3-7-8 | 522.72 | 481 | 3.63 | 3.92 |
| 1-6-7 | 526.54 | 481 | 5.97 | 5.83 |
| 4-5-6 | 543.85 | 481 | 9.19 | 14.49 |
| 3-6-8 | 520.21 | 480 | 5.34 | 5.33 |
| 1-4 | 548.47 | 480 | 33.52 | 33.59 |
| 1-3 | 543.62 | 480 | 28.73 | 28.74 |

*Acutual $\Delta \mathrm{X}^{2} / \Delta \mathrm{df}$ as compared to fit with the higher order four-way eliminated. **Note: The variables are referred to by the numbers associated with them in Table A-7.

The results of these tests show that the estimates are exact for interactions with one degree of freedom but rough approximations for interactions with multiple degrees of freedom. Even so, the estimates appear to give valid indications of the actual $\mathrm{AX}^{2} / \Delta \mathrm{df}$ for interactions with multiple degrees of freedom. Of the 17 interactions with multiple degrees of freedom in the FARS model, the estimates overstated the actual $\Omega \mathrm{X}^{2} / \Delta \mathrm{d} \boldsymbol{f}$ eight times, and understated it nine times. Only three estimates were off by more than $33 \%$. With the 17 in the NPTS model (with multiple degrees of freedom), 10 estimates were overstated and 7 understated. Only six estimates were off by more than one-third.

Since there was no systematic bias, this indicates the reasonableness of the method, as does the tendency for the estimates to be close to the actual $\Delta X^{2} / \Delta D F$.

## APPENDIX C <br> Log Linear Modelling Assumptions

This appendix considers the assumptions involved in applying classical log-linear modelling techniques to the NPTS and FARS data sets used in this study which do not strictly satisfy the classical requirements for applying log-linear models.

The FARS case will be treated first. The classical conditions are that the cell counts can be taken as independently distributed and Poisson distributed. These conditions are expected to hold for accidents classified by a set of variables but in the case of accident involvements (as in the FARS data matrix), each two-car accident leads to two involvements which contribute to two (possibly different) cells. This leads to the cell counts not being independent and not being Poisson distributed. However, if there are very many cells and no one cell has an appreciable probability to contain any one randomly chosen involvement, then both conditions should hold approximately. This is to say that in the case of very many cells none containing an appreciable fraction of all involvements, the cell counts should be approximately independent and Poisson distributed and this approximation should approach perfection as the fraction in the largest cell approaches zero. Since there are 576 cells in the present case and the cell containing the most involvements contains only about 2 percent of the total, it appears to be justified to treat the cell counts as satisfying the classical conditions for log-linear modelling. Furthermore, about half the involvements are single car and may be assumed independent and Poisson (per cell) from the outset. Thus, in the FARS case the classical requirements are approximately met and it is assumed that the degree of approximation is sufficient for the present purposes.

In the NPTS case the classical requirements for log-linear modelling are not satisfied to the degree needed for direct application of the method. The cell data are sums of VMT, a continuous measure, rather than counts as required. The use of log-linear models in this case is discussed in Reference 1 (Section 3.2.4). It is concluded there that the classical approach works well except that the chi square measure used for model selection is not valid. It is suggested that the chi square statistic may be approximately valid when properly scaled. The
scaling factor involved can be applied instead to the raw data allowing it to be treated as true (Poisson distributed) count data from that point on. In order for the chi square statistic on the scaled data to be valid some rather strong conditions on the original data are needed. However, if the scaled chi square is all that is available for model selection then it seems appropriate to use it for this purpose. That is the current circumstance and so a scaled chi square is used for model selection in the case of the NPTS data.

An estimate of the proper scale factor is obtained in a different manner from that suggested in Reference 1. For this study the scale factor was obtained by making use of the fact that for high enough order models the expected chi square and the (residual) degrees of freedom are equal. Since (for the NPTS data) a hierarchical model fitting all fourth order interactions (as well as all lower order interactions) has a residual degrees of freedom of 263 , it is to be expected that its chi square value should be approximately equal to 263. (For the FARS data the value of $\mathrm{X}^{2}$ was 270 for all four-way interactions, very close to 263.) The data values (cell VMT sums) were scaled so that the usual chi square calculation produced a value of 263 for this model (all four factor interactions). The resulting scaled data behaved very much (in terms of chi square values for various models) like the true count data in the FARS matrix and this is taken as further evidence that the procedure of scaling the data and then using it as though it were count data is a satisfactory means of applying log-linear modelling techniques to the NPTS data.

## APPENDIX D

It is noted in Appendix A that the log linear model fit to the VMT matrix does not appear to be under fit. This is based on the $\mathrm{X}^{2}$ and degrees of freedom for the model.*

A more parsimonious (i.e. simpler) model is expected to have a lower standard error of estimate but also a higher bias. But, if the original model is substantialy overfit, then a more parsimonious model would be expected to have a lower overall error (consisting of the components bias and standard error). The question arises whether a more parsimonious model might have lower standard errors of estimates and also have a low enough bias that the overall accuracy would be increased. A more parsimonious model would also have the advantage of simplicity.

By producing a more parsimonious model and examining both the change in cell estimates and the change in standard errors, a better idea of whether the original model could have its error reduced by simplifying could be obtained.

A more parsimonious model was produced for these reasons. The terms in the more parsimonious model, its chi square value and its degrees of freedom are given in Table D.1. It was developed by techniques similar to those described for the original model in Section 3.2 and Appendix A. The final threshold value for $\Delta X^{2} / \Delta D_{1}=$ was 6 , twice as high was for the original model.
*The $\mathrm{X}^{2}$ value had to be calculated on the basis of a scaling of uncertain validity, but the assertion that the model was not underfit is supported by the fact that the standard errors of terms in the log-linear model for the VMT estimates were on the average about 2 times as large when calculated using sample splitting techniques as when calculated from the standard. error estimates of the LOGLIN program (based on the scaling described in Appendix A). Since LOGLIN underestimated the variance this suggests that there would have been a tendency to overfit. In the case of the fatal involvement model, the two estimates of the standard error were consistently much closer (usually within 20 percent of each other). These observations are based on samples of 10 model parameters in both cases.

An examir estimated cell variances for the two models (see Section 4) and of the difne cell estimates will give some indications of how the accuracy of the moous model compares to that of the original model. Define four sums as fc
$S_{1}=$ ariances for the original model.
$\mathrm{S}_{2}=$ ariances for the new model.
$\mathbf{S}_{\mathbf{3}}=e$, the reduction in variance obtained by the new model.
$\mathrm{S}_{4}$ =he squares of the differences in cell estimates.

Thus, $S_{4}$ th of bias although the difference is not due to bias alone but has a componss as well.

The valuantities are as \{ollows:

1. $\sum_{\text {cel }} \mathrm{S}_{1}$
2. $\sum_{\text {ceI }}=S_{2}$
3. $\sum_{\text {cel }} 7.7=S_{3}$
4. $\left.\left.\sum_{\text {cel }} \mathrm{ate}\right)_{\mathrm{A}}-(\text { cell estimate })_{\mathrm{B}}\right)^{2}=11.7=\mathrm{S}_{4}$
where tl refers to the original model and the subscript B refers to the second rious model. Thus, the first number, 21.1, is an aggregate measure of the stability or noise in the first model and 13.4 is the corresponding measurend model. There is clearly a substantial improvement in this regard.

The fou.7, is the sum of two components. One is the sum of the squares of the das, i.e., a measure of increased bias of the second model over the first. Tonent is a measure of the noise in the difference in cell estimates betweerdels. Unfortunately, the latter quantity cannot be directly estimatumbers given here and so the measure of the bias cannot be estimal Clearly 11.7 is an upper bound on the increased bias sum of squares not a very tight bound. These quantities could be estimated by returni samples to get estimates of the variances in the differences betwees. In the absense of this (not done to save time and money), one
may try to get another estimate of this variance. Let $\mathrm{C}_{\mathrm{A}}$ denote a cell estimate of model $A$ and $C_{B}$ the corresponding estimate of model $B$. Then:

$$
\sigma_{( }^{2}-(a)=\sigma_{A}^{2}+T^{2}-2 \pi
$$

Now $\sigma_{A B}$ is the covariance of the two estimates. It seems reasonable to assume that

$$
0 \leq \sigma_{A B} \leq \sigma_{B}^{2}
$$

If this is so

$$
\sigma_{\left(c_{A}-c_{B}\right)}^{2} \geq \sigma_{A}^{2}-\sigma_{B}^{2}
$$

$$
\begin{aligned}
& \text { Since* }
\end{aligned}
$$

$$
\begin{aligned}
& \approx 11.7-7,7=40
\end{aligned}
$$

Thus, 4.0 is the corresponding upper bound estimate of the increase in bias sum of squares. This is fairly small compared with the variance sum of squares of the second model, 13.4. The bias in the first model having nearly twice as many independent parameters (576-479 = 97 vs. 576-523 = 53) should be smaller than the change in bias. Therefore, this is more evidence that the first model had low bias.

Although these arguments are not rigorous, it appears that the second model may have a slightly smaller total aggregate error sum of squares, $13.44+4.0 \quad 21.1$, but this is offset by the fact that the first model has a more accurate estimate of its total error, namely the statistical standard error in Table $4 . \dot{2}$ since the bias is thought to be small.

In summary, the second model may be used when simplicity is desired and it probably is slightly more accurate. However, the first model should be used if it is important to have a good estimate of the accuracy.

The first model is used for all the analyses in the rest of this report. Tables for the second (more parsimonious) model corresponding to Tables 3.4, 3.5, 4.2, and 4.3 for the original model are given in this Appendix as Table D.2, D.3, D.4, and 0.5 respectively.

* ${ }^{\mathrm{HE}}()^{\mathrm{I}}$ denotes expected value. The expected value of a quantity summed over cells is well estimated by the observed value.

As already noted, the more parsimonious model is thought to be slightly more accurate than the original model but all standard errors referring to cell estimates or derived quantities (including fatal involvement rates) are less reliable indicators of accuracy than the corresponding standard errors for the original model.

## Table D.1: Second Model for VMT

```
DF=523
X2 = 675.84*
```


## Effects

168
246
347
348
456
467
678
13
14
25
28
36
57
*Scaled as indicated in Appendix A.
Table D．2：Smoothed Estimates of Vehicle
Miles Traveled（New Model）（Billions）
I ANJ JSE：


0.7041080 .070914 $\begin{array}{ll}1.266961 & 2.780252 \\ .258597 & 0.940746\end{array}$ $0.755147 \quad 0.963462 \quad 0.5952970 .663979$ 0.59529710 .66351 0.574667 73372 0.5931950 .733725 2.640842
0.594941 － 402798 1.489662
0.300721
 $\begin{array}{llllllll}0.810926 & 0.909728 & 0.705510 & 0.691807 & 0.269862 & 0.381880 & 0.234782 & 0.290403\end{array}$ $\begin{array}{llllllll}1.095835 & 2.181021 & 0.840246 & 1.461750 & 0.364675 & 0.915535 & 0.279619 & 0.613604 \\ 0.103534 & 0.341605 & 0.095042 & 0.274101 & 0.034454 & 0.143397 & 0.031628 & 0.115060\end{array}$


 $1.6196141 .905515 \quad 1.071660 \quad 1.5603541 .307116 \quad 1.9398560 .864888 \quad 1.588474$
 $\begin{array}{llll}0.713461 & 0.954650 & 0.427756 & 0.708328 \\ 1.194893 & 4.052577 & 0.572638 & 2.403518\end{array}$ $\begin{array}{llll}1.194893 & 4.052577 & 0.572638 & 2.403518 \\ 0.181117 & 0.697011 & 0.074340 & 0.354053\end{array}$


$0.311831 \quad 0.139724 \quad 0.231372$

 | 0.415541 | 0.486894 | 0.274953 | 0.400336 | 0.276815 | 0.410814 | 0.183162 | 0.336400 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | $\begin{array}{llllllll}0.415541 & 0.486894 & 0.274953 & 0.400336 & 0.276815 & 0.410814 & 0.183162 & 0.336400\end{array}$ $0.0992550 .3399910 .04450 日 \quad 0.1905970 .0654530 .2656920 .0296490 .160158$

$0.192311 \quad 0.203997 \quad 0.115300 \quad 0.1513610 .1017780 .136185 \quad 0.0610210 .101046$




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0.921981
233048
0.2340053
0.444410
$\vec{m}$
$\overrightarrow{8}$
$\vdots$
$\vdots$
0
1.848999
0.462934
0.338903
1.2996548
0.127161
$\begin{array}{llll}0.669430 & 1.397304 & 0.354060 & 0.914592 \\ 0.098255 & 0.339991 & 0.04450 日 & 0.190597\end{array}$

258161
$\qquad$
s. $3 /$
I.ANV USE:

> Table D. 3: Fatal Involvement Rates (Smoothed Based on New Model)
(Fatal Involvements Per Billion V

Table D.3: Fatal Involvement Rates (Smoothed Based on New Model)
(Fatal Involvements Per Billion VMT) I ANH USE: (continued)

Table D.4: Relative Standard Errors of Smoothed VMT Estimates (New Model)

| Ocmuranis | TIMF | I ANI USE: UEHICIE AGF: UFHICLE WEIGHT: |  | URFAN |  |  |  | 1 gIJRAI |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | $\begin{aligned} & 5 \mathrm{YRS} \\ & 1 \text { IIGHt } \end{aligned}$ | OR L.ESS <br> I heauy | $\begin{array}{cc} \text { I } & \text { OVER } \\ \text { I } & \text { I.IGHT } \end{array}$ | 5 YRS <br> 1 HFAUY | $\begin{array}{ll} 1 & 5 \mathrm{YRS} \\ 1 & 1.16 H T \end{array}$ | OR IFSS <br> I HEAUY | $\begin{array}{lc} \text { I } & \text { OUER } 5 \\ 1 & 1 \mathrm{GHI} \end{array}$ | YRS <br> I HEAUY |
|  |  |  |  | 1 | I | 1 | 1 | 1 | $t$ | 1 | 1 |
| ONE | OIHER | hat E | $\begin{aligned} & \text { IF } 25 \\ & 26-55 \\ & \text { GE SK } \end{aligned}$ | $\begin{aligned} & 0.092955 \\ & 0.09509 ? \\ & 0.149210 \end{aligned}$ | $\begin{aligned} & 0.04817: 8 \\ & 0.049770 \\ & 0.091577 \end{aligned}$ | $\begin{aligned} & 0.069647 \\ & 0.05448 ? 2 \\ & 0.147314 \end{aligned}$ | $\begin{aligned} & 0.071928 \\ & 0.047134 \\ & 0.109381 \end{aligned}$ | $\begin{aligned} & 0.069185 \\ & 0.069020 \\ & 0.157475 \end{aligned}$ | $\begin{aligned} & 0.073102 \\ & 0.049806 \\ & 0.092607 \end{aligned}$ | $\begin{aligned} & 0.078919 \\ & 0.056591 \\ & 0.155681 \end{aligned}$ | $\begin{aligned} & 0.093762 \\ & 0.064106 \\ & 0.107706 \end{aligned}$ |
|  |  | FEMAI E | $\begin{aligned} & \text { LE } 25 \\ & 26-55 \\ & \text { GE } 56 \end{aligned}$ | $\begin{aligned} & 0.080941 \\ & 0.108577 \\ & 0.163695 \end{aligned}$ | $\begin{aligned} & 0.096926 \\ & 0.069170 \\ & 0.098150 \end{aligned}$ | 0.079085 <br> 0.070908 <br> 0.119764 | $\begin{aligned} & 0.098308 \\ & 0.060774 \\ & 0.1: 0350 \end{aligned}$ | $\begin{aligned} & 0.081919 \\ & 0.086543 \\ & 0.172220 \end{aligned}$ | $\begin{aligned} & 0.085734 \\ & 0.083478 \\ & 0.111644 \end{aligned}$ | $\begin{aligned} & 0.092393 \\ & 0.064984 \\ & 0.121830 \end{aligned}$ | $\begin{aligned} & 0.079561 \\ & 0.0786130 \\ & 0.116140 \end{aligned}$ |
|  | Filsh | MAIE | IE 25 26-55 GE S6 | $\begin{aligned} & 0.0981: 30 \\ & 0.099165 \\ & 0.158887 \end{aligned}$ | $\begin{aligned} & 0.080641 \\ & 0.059435 \\ & 0.090785 \end{aligned}$ | $\begin{aligned} & 0.072483 \\ & 0.056091 \\ & 0.16 .3968 \end{aligned}$ | $\begin{aligned} & 0.087090 \\ & 0.060448 \\ & 0.116621 \end{aligned}$ | $\begin{aligned} & 0.042099 \\ & 0.09 \mathrm{~A} 100 \\ & 0.167144 \end{aligned}$ | $\begin{aligned} & 0.067959 \\ & 0.075105 \\ & 0.092247 \end{aligned}$ | $\begin{aligned} & 0.091666 \\ & 0.080713 \\ & 0.171417 \end{aligned}$ | $\begin{aligned} & 0.088054 \\ & 9.060609 \\ & 0.118472 \end{aligned}$ |
|  |  | frmale | $\begin{aligned} & 1 E 25 \\ & 26-55 \\ & 6 E 56 \end{aligned}$ | $\begin{aligned} & 0.100603 \\ & 0.106442 \\ & 0.138150 \end{aligned}$ | $\begin{aligned} & 0.105986 \\ & 0.069684 \\ & 0.096476 \end{aligned}$ | $\begin{aligned} & 0.102567 \\ & 0.074274 \\ & 0.111435 \end{aligned}$ | $\begin{aligned} & 0.109980 \\ & 0.067692 \\ & 0.127969 \end{aligned}$ | $\begin{aligned} & 0.105116 \\ & 0.093955 \\ & 0.125959 \end{aligned}$ | $\begin{aligned} & 0.092941 \\ & 0.084092 \\ & 0.107505 \end{aligned}$ | $\begin{aligned} & 0.110878 \\ & 0.070392 \\ & 0.097257 \end{aligned}$ | $\begin{aligned} & 0.082702 \\ & 0.075155 \\ & 0.123381 \end{aligned}$ |
|  | 1 ATE: | MAIE | $\begin{aligned} & \text { 1E } 25 \\ & 26-55 \\ & \text { liE } 56 \end{aligned}$ | $\begin{aligned} & 0.123457 \\ & 0.117677 \\ & 0.172313 \end{aligned}$ | $\begin{aligned} & 0.093145 \\ & 0.075782 \\ & 0.134668 \end{aligned}$ | $\begin{aligned} & 0.101972 \\ & 0.061672 \\ & 0.165649 \end{aligned}$ | $\begin{aligned} & 0.095013 \\ & 0.071431 \\ & 0.153632 \end{aligned}$ | $\begin{aligned} & 0.107591 \\ & 0.106778 \\ & 0.188448 \end{aligned}$ | $\begin{aligned} & 0.108150 \\ & 0.092318 \\ & 0.125362 \end{aligned}$ | $\begin{aligned} & 0.115916 \\ & 0.094638 \\ & 0.183639 \end{aligned}$ | 0.117808 0.0938 BO 0.13698 B |
|  |  | FEMAIE | $\begin{aligned} & \text { IE } 25 \\ & \text { :16-5: } \\ & \text { if } 56 \end{aligned}$ | $\begin{aligned} & 0.095222 \\ & 0.124427 \\ & 0.186485 \end{aligned}$ | $\begin{aligned} & 0.114631 \\ & 0.085260 \\ & 0.144439 \end{aligned}$ | $\begin{aligned} & 0.109872 \\ & 0.092750 \\ & 0.161869 \end{aligned}$ | $\begin{aligned} & 0.127668 \\ & 0.081313 \\ & 0.175414 \end{aligned}$ | $\begin{aligned} & 0.100212 \\ & 0.111222 \\ & 0.181951 \end{aligned}$ | $\begin{aligned} & 0.106971 \\ & 0.110351 \\ & 0.134910 \end{aligned}$ | $\begin{aligned} & 0.11 .3287 \\ & 0.083156 \\ & 0.146347 \end{aligned}$ | $\begin{aligned} & 0.097461 \\ & 0.091682 \\ & 0.144217 \end{aligned}$ |
| MOEE 1 HAN ONF | OTHFE | HAIE: | $\begin{aligned} & \text { IE } 35 \\ & \text { 2B } 55 \\ & \text { GF Sín } \end{aligned}$ | $\begin{aligned} & 0.091734 \\ & 0.092051 \\ & 0.131413 \end{aligned}$ | $\begin{aligned} & 0.083138 \\ & 0.049711 \\ & 0.109810 \end{aligned}$ | $\begin{aligned} & 0.096 .390 \\ & 0.094097 \\ & 0.14020 .3 \end{aligned}$ | $\begin{aligned} & 0.059743 \\ & 0.081954 \\ & 0.107979 \end{aligned}$ | $\begin{aligned} & 0.108210 \\ & 0.101326 \\ & 0.132673 \end{aligned}$ | $\begin{aligned} & 0.096152 \\ & 0.067304 \\ & 0.111357 \end{aligned}$ | $\begin{aligned} & 0.084645 \\ & 0.084782 \\ & 0.130593 \end{aligned}$ | $\begin{aligned} & 0.082706 \\ & 0.077256 \\ & 0.100070 \end{aligned}$ |
|  |  | FEMAIE | $\begin{aligned} & 1 F 25 \\ & 26.55 \end{aligned}$ $\text { GE: } 56$ | $\begin{aligned} & 0.096876 \\ & 0.08764 .3 \\ & 0.227510 \end{aligned}$ | $\begin{aligned} & 0.123716 \\ & 0.048547 \\ & 0.141970 \end{aligned}$ | $\begin{aligned} & 0.094044 \\ & 0.0884 .30 \\ & 0.233654 \end{aligned}$ | $\begin{aligned} & 0.105210 \\ & 0.081410 \\ & 0.150834 \end{aligned}$ | $\begin{aligned} & 0.111208 \\ & 0.089158 \\ & 0.229670 \end{aligned}$ | $\begin{aligned} & 0.122770 \\ & 0.075 i 382 \\ & 0.161900 \end{aligned}$ | $\begin{aligned} & 0.099610 \\ & 0.098760 \\ & 0.240262 \end{aligned}$ | $\begin{aligned} & 0.085779 \\ & 0.09729 H \\ & 0.163013 \end{aligned}$ |
|  | KIISH | MAIF | $\begin{aligned} & 1525 \\ & 26-55 \\ & \text { iE } 56 \end{aligned}$ | $\begin{aligned} & 0.122722 \\ & 0.091964 \\ & 0.150166 \end{aligned}$ | $\begin{aligned} & 0.119207 \\ & 0.059677 \\ & 0.1: 6207 \end{aligned}$ | $\begin{aligned} & 0.172331 \\ & 0.124068 \\ & 0.165893 \end{aligned}$ | $\begin{aligned} & 0.084791 \\ & 0.101611 \\ & 0.13066 .3 \end{aligned}$ | $\begin{aligned} & 0.141923 \\ & 0.107510 \\ & 0.144401 \end{aligned}$ | $\begin{aligned} & 0.1121 .33 \\ & 0.062066 \\ & 0.123955 \end{aligned}$ | $\begin{aligned} & 0.116532 \\ & 0.120859 \\ & 0.153356 \end{aligned}$ | $\begin{aligned} & 0.088906 \\ & 0.097738 \\ & 0.127613 \end{aligned}$ |
|  |  | fFMAIF | $\begin{aligned} & 1 F=35 \\ & 26-555 \\ & 6 E .56 \end{aligned}$ | $\begin{aligned} & 0.097176 \\ & 0.078966 \\ & 0.211164 \end{aligned}$ | $\begin{aligned} & 0.142271 \\ & 0.064577 \\ & 0.129210 \end{aligned}$ | $\begin{aligned} & 0.097364 \\ & 0.07601 .3 \\ & 0.21150 .5 \end{aligned}$ | $\begin{aligned} & 0.130823 \\ & 0.099434 \\ & 0.132996 \end{aligned}$ | $\begin{aligned} & 0.096119 \\ & 0.084374 \\ & 0.194614 \end{aligned}$ | $\begin{aligned} & 0.13 .3338 \\ & 0.082070 \\ & 0.142273 \end{aligned}$ | $\begin{aligned} & 0.071657 \\ & 0.0133970 \\ & 0.197655 \end{aligned}$ | $\begin{aligned} & 0.109911 \\ & 0.11168 \mathrm{~B} \\ & 0.138930 \end{aligned}$ |
|  | LATE | MAIE | $\begin{aligned} & 15 \\ & 26-55 \\ & 26 \end{aligned}$ | $\begin{aligned} & 0.105407 \\ & 0.095473 \end{aligned}$ $0.164805$ | 0.1 .4116 .3 <br> 0.080217 <br> 0.139932 | $\begin{aligned} & 0.107721 \\ & 0.109043 \\ & 0.188158 \end{aligned}$ | $\begin{aligned} & 0.089416 \\ & 0.095767 \\ & 0.145471 \end{aligned}$ | $\begin{aligned} & 0.117668 \\ & 0.120851 \\ & 0.168592 \end{aligned}$ | $\begin{aligned} & 0.125118 \\ & 0.093721 \\ & 0.115769 \end{aligned}$ | $\begin{aligned} & 0.081661 \\ & 0.104365 \\ & 0.179078 \end{aligned}$ | $\begin{aligned} & 0.095207 \\ & 0.106521 \\ & 0.116444 \end{aligned}$ |
|  |  | FFMAIE | $\begin{aligned} & 1 F 25 \\ & 26-56 \\ & \text { (iF } 56 \end{aligned}$ | $\begin{aligned} & 0.140233 \\ & 0.146389 \\ & 0.274015 \end{aligned}$ | 0.177590 <br> 0.113906 <br> 0.18946 .3 | $\begin{aligned} & 0.14 .5736 \\ & 0.12905: 55 \\ & 0.347030 \end{aligned}$ | $\begin{aligned} & 0.156717 \\ & 0.125847 \\ & 0.217059 \end{aligned}$ | $\begin{aligned} & 0.18014 日 \\ & 0.176642 \\ & 0.747344 \end{aligned}$ | $\begin{aligned} & 0.192089 \\ & 0.155400 \\ & 0.208731 \end{aligned}$ | 0.145646 <br> 0.146448 <br> 0.304599 | $\begin{aligned} & 0.158829 \\ & 0.161204 \\ & 0.220940 \end{aligned}$ |


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Table D.S: Relative Standard Errors of Fatal Involvement Rates (Based on New Model)

Table D.5: Relative Standard Errors of Fatal
Involvement Rates (Based on New Model) (cont inued)


## APPENDIX B

## Balanced, Partially Balanced, and Random Repeated Replications

When estimating variances for data collected according to a complex sampling plan, the method of repeated replications is recommended and if posible balanced repeated replications should be used. Balanced repeated replications were introduced in Reference 4. In that paper McCarthy estimated the relative variance of the variance estimate (variance of the variance estimate divided by the square of its mean) as

$$
\frac{2(L-1)}{k L}+\frac{B+1}{2 L}
$$

when repeated replications according to random patterns of " 1 "s and " 2 " s (i.e. random choices of half strata for each replication) are used.

Here $L$ represents the number of strata, $k$ the number of replications used and $B$ is a certain kurtosis which is therefore positive and might be expected to be around 3 or somewhat larger.

When partially balanced repeated replications are used the formula becomes
$2(L-k) /(k L)+(B+1) /(2 L)$
so long as $k \quad L$. When $k=L$ then "partially balanced" becomes "balanced" and so the formula for balanced repeated replications is
( $\mathrm{B}+1$ )/(2L).
This is the minimum relative variance obtainable using repeated replications. Since $L=253$ in the present study and only $k=20$ replications were to be produced either partially balanced or random repeated replications had to be used. The ratio of relative variances between random and partially balanced repeated replications is estimated by
$(2(L-1) / k L+(B+1) / 2 L)-(2(L-k) /(k L)+(B+1) /(2 L))$

This is easily seen to be less than (L-1)/L-k).

Substituting 20 for $k$ and 253 for $L$, we estimate that the advantage for using partially balanced over random repeated replications is less than a factor of 1.28. In other words, using random rather than balanced repeated replications is
estimated to result in a variance estimate with a relative variance at most 8 percent larger than that resulting from using partially balanced repeated replications.

Because the advantage in accuracy of the variance estimate for using partially balanced repeated replications is so slight, it was decided to use random repeated replications which were somewhat simpler to produce.

## REFERENCES

1. "Analytical Methods in Multivariate Highway Safety Exposure Data Estimation" by P. Mengert and E. Roberts, DOT-HS806-494 (1984).
2. Discrete Multivariate Analysis by Y.M.M.Bishop, S.E.Fienberg, and P.W. Holland, MIT Press (1975).
3. The Jacknife, the Bootstrap, and the Other Resampling Plans by B. Efron, Society for Industrial and Applied Mathematics, Monograph No. 38, (1982).
4. "Replication: An Approach to the Analysis of Data from Complex Surveys" by P.J. McCarthy, National Center for Health Statistics, Series 2, No. 14, (1966).
5. "Balanced Repeated Replications for Standard Errors" by L. Kish and M.R. Frankel, J. American Statistical Association, V. 65, No. 331, pp. 1071-1094 (1970).

[^0]:    *Longer trips were also identified for the prior week, but those trips were not used in this study because they would over-represent long trips in the aggregate if they were included.

[^1]:    *Vehicles with model year earlier than 1967 were classified as 1967.

[^2]:    *Model was not used in the weight assignment process since those records which had a model designated also had a weight designated.

[^3]:    * Note that even in the case of FARS the data should be considered a sample (even though FARS records all fatal accidents in the year) since what is of interest is the data for the year 1977 as representative of the fatal accidents in other years.
    **A short discussion of the use of the standard maximum likelihood method for fitting log-linear models to count data and a discussion of the formal application of the method to non-count data is found in Reference 1. (A more complete discussion of log linear modelling is found in Reference 2).

[^4]:    *See previous footnote.

[^5]:    * A justification for this procedure for developing half samples procedes along the following lines: If the underlying population giving rise to the FARS data could be divided (say spatially or temporally) into two subsets identical in the statistical properties of the accident involvements they generate, then assuming each FARS cell count is Poisson distributed, the distribution of each total cell over each of the two halves would be binimonial as described. The half samples generated as described in the text are thus statistically identical (under the independence and Poisson assumptions) to those that would be generated by half populations.

[^6]:    *The term "pseudo replications" also applies. The term "balanced repeated replications" refers to a special case of repeated replications and will be mentioned below and discussed in Appendix E.

[^7]:    * A self-representing stratum coincides with an important PSU. A non-selfrepresenting stratum consists of several PSU's represented by one PSU. The half samples are constructed using trip records derived from the NPTS data. The basic requirement is a knowledge of which stratum and half stratum each trip record (as identified by household serial number) belongs to.

[^8]:    *Since these relative variances were calculated (originally) by dividing a regular variance by the square of a mean value, they underestimate the variance of the logarithm slightly. This bias ranges trom negligible ( $<1 \%$ ) when $\sigma=.25$ (usual case) to approximately $6 \%$ when $\sigma=.6$ (worst case). Thus the confidence interval is at most $6 \%$ in error (due to this effect) and usually much less.

[^9]:    *The relative standard error is the standard error divided by the cell estimate or equivalently, by the last paragraph, the standard error in the logarithm of the cell estimate.

[^10]:    *Here "first variable" is in the sense of some arbitrarily chosen ordering; this is not necessarily "variable 1 " as referred to consistently in this report.

[^11]:    *This "standard error" is calculated as follows. Let $L_{i}$ be the $\log$ of the FIR ratio for the ith cell and let $W_{i}$ be the weight (this is, of course, the estimated fatalities for this cell), then $L=\sum W_{i} L_{i} / \sum_{i} W_{i}$, just as defined earlier, is the weighted mean over cells, then, $\sigma_{\mathrm{L}}{ }^{2}=\Sigma_{i}\left(\mathrm{~L}_{\mathrm{i}}-\mathrm{L}\right)^{2} \mathrm{~W}_{\mathrm{i}} / \Sigma_{i} \mathrm{~W}_{\mathrm{i}}$. The standard error in $L$ is estimated as $\sigma / \sqrt{N}$. Here $N$ is the total number of cells being used to find L . This process is the usual process for calculating a standard error for a mean. Even through the conditions for applying this process are not present, it is still of some value. The standard error in L gives an estimate of the instability of $L$ due to the fact that it varies from cell to cell, both because of changing circumstances (from cell to cell) and because of sampling error. If the cell estimates were independent then the standard error in $L$ would be an in upperbound on the sampling error in L. Since the cell estimates are not independent (because of the log-linear modelling process) we no longer have an upper-bound in sampling error in this quantity. However, it may be used in conjunction with total fatalities to estimate the statistical stability of L .

[^12]:    *This differs from the log-linear model fitting algorithm for count data, but is suitable for this jurpose.

