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RAIL-HIGHWAY CROSSING ACCIDENT PREDICTION RESEARCH RESULTS - FY80

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PROJECT MEMORANDUM

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PREFACE

This report presents the results of research performed at the Transportation Systems Center (TSC) dealing with mathematical methods of predicting accidents at rail-highway crossings. The work consists of three parts: Part I - Revised DOT Accident Prediction Formula; Part II - Accident Prediction Formula with Accident History; Part III - Comparative Performance of three Rail-Highway Crossing Hazard Models. The study was sponsored by the U.S. Department of Transportation (DOT), (specifically, the Office of Safety of the Federal Railroad Administration and the Office of Research of Federal Highway Administration). This study supports a program which was outlined in the 1972 DOT report to Congress on safety improvement at 30,000 rail-highway crossings in the United States.

This report is part of a TSC rail-highway program under the management of Robert Coulombre. Some analysis and computer programming work for Part I was done under contract by IOCS, Cambridge, Massachusetts. Other computer programming work for all three parts was done by Steven Cultrera of TSC.

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PART I - REVISED DOT ACCIDENT PREDICTION FORMULA

1. INTRODUCTION

A new accident prediction formula has been developed using the same techniques as those used for the old formula.[1] The work was done with two purposes in mind: 1) to see if the resulting formula would be significantly different, if another set of accident data were used instead of the 1975 accident data employed for the original formula; and 2) to use a function of "exposure" for the part of the formula involving traffic volumes.

On the first point, the 1976 accident file and the August 1976 inventory were used to develop the formula. More specifically, half of the file was used to determine the coefficients, by regression and iteration, and the other half was used for testing and formula selection based on power and prediction factors.

On the second point, "exposure" is defined as the product of daily highway and daily train traffic. It was thought that exposure would be proper to use to build the volume function, because the New Hampshire formula produces good power factors in most cases. This is also true for the Peabody-Dimmick formula, which resembles an exposure formula. Another reason is that the level curves of the original hazard formula, in the C,T plane, have been observed to be nearly hyperbolic. The main benefit of an exposure function is to simplify the formula. This should add to its credibility especially for those users who are not mathematically trained and,

[1]Mengert, P., "Rail-Highway Crossing Hazard Prediction Research Results," U.S. Dept. of Transportation, Federal Railroad Administration, Washington DC, March 1980, FRA-RRS-80-02.

hence, are not comfortable with an abstract mathematical function. Having exposure represent highway and train traffic will also shorten the tables used for evaluating the formula.

In Section 2, tables are presented which are convenient to use in determining accident prediction values when only a few crossings are involved. In Section 3, characteristics of the formula are discussed, including sensitivity to the independent variables. In Section 4, the performance of the new formula is given and a comparison is made with the old formula. In Section 5, the development and derivation of the formula are discussed. Section 6 contains the conclusions.

2. FORMULA AND TABLES

The revised formula has been developed by the technique discussed in Section 5. This formula has been normalized for 1977 accidents using the May 1978 inventory. The normalization was done first by making the sum of the predicted accidents of the crossings in the groups (corresponding to the three equations) equal to the number of valid accident records in each group, respectively. Then, a further constant was applied to all three equations to make the sum of predicted accidents of all crossings equal to 12,299, the total number of crossing accidents in 1977.

The formula consists of the following three equations:

a. For Warning Device Classes 1, 2, 3, 4, (Passive Devices):

$$h = 0.00873\ e^{2X_1}$$

$$X_1 = .3839\ LOG_{10}(CT + 0.2) + .1538\ LOG_{10}\ (D + 0.2)$$

$$-0.3080\ HP + .05111\ P + .003855\ MS$$

$$-.04991\ HT + .1047\ MT$$

b. For Warning Device Classes 5, 6, 7:

$$h = 0.00434\ e^{2X_2}$$

$$X_2 = .3400\ LOG_{10}(CT + 0.2) + .05415\ LOG_{10}\ (D + 0.2)$$

$$+ .05442\ MT + .06900L + .02032\ P.$$

c. For Warning Device Class 8:

$$h = 0.00163\ e^{2X_3}$$

$$X_3 = .3588\ LOG_{10}(CT + 0.2) + .1456\ MT + .05180L.$$

h = Predicted number of accidents per year

T = Number of trains per day

C = Number of highway vehicles per day

D = Number of day thru trains per day

MT = Number of main tracks

MS = Maximum time table speed

HP = Highway paved = 1 if paved, 2 if not paved

P = Population. This is the tens digit of functional classification of road over crossing

HT = Highway type. This is the units digit of functional classification of road over crossing.

L = Number of traffic lanes.

All of the variables refer to data taken directly from the inventory.

For convenience, Table 2-1 shows which variables are in the old formula (✓) and which are in the new (revised) formula (X).

TABLE 2-1. OLD AND NEW FORMULA VARIABLES

Variable	Passive	Flashing Lights	Gates
C	X✓	X✓	X✓
T	X✓	X✓	X✓
D	X✓	X✓	
MT	X✓	X✓	X✓
HP	X✓		
MS	X		
HT	X✓	✓	
P	X✓	X✓	
L		X✓	X✓

2.1 WARNING DEVICE CLASSES 1, 2, 3, 4

Generally, it can be concluded that the old set of variables and the new set are nearly the same. Maximum time-table speed is the only new entry and only highway type has dropped out of the flashing light equation.

Tables have been prepared for these classes which are convenient for calculating hazard indexes. The equation for device classes can be written as a product of factors:

$$H = EI_{1-4} \times DT_{1-4} \times HP_{1-4} \times MT_{1-4} \times P_{1-4} \times HT_{1-4} \times MS_{1-4}$$

where

$$EI_{1-4} = .00873\ (CT + 0.2)^{.3335}$$

$$DT_{1-4} = (D + 0.2)^{.1334}$$

$$HP_{1-4} = e^{-.6160\ HP}$$

$$MT_{1-4} = e^{.2094MT}$$

$$P_{1-4} = e^{.10222P}$$

$$HT_{1-4} = e^{-.099982HT}$$

$$MS_{1-4} = e^{.007710MS}$$

Each of these factors is tabulated in Table 2-2.

2.2 WARNING DEVICE CLASSES 5, 6, 7

For warning device classes 5, 6, 7, the product form of the equation is:

$$H = EI_{5-7} \times DT_{5-7} \times MT_{5-7} \times L_{5-7} \times P_{5-7}$$

$$EI_{5-7} = .00434\ (CT + 0.2)^{.2953}$$

$$DT_{5-7} = (D + 0.2)^{.0470}$$

$$MT_{5-7} = e^{.10884MT}$$

$$L_{5-7} = e^{.13800L}$$

$$P_{5-7} = e^{.04064P}$$

Each of these factors is tabulated in Table 2-3.

TABLE 2-2. FACTORS FOR WARNING DEVICE CLASSES 1, 2, 3, 4

CT	EI_{1-4}	Day thru Trains	DT_{1-4}	Highway Paved	HP_{1-4}	Main Tracks	MT_{1-4}	Population	P_{1-4}	Highway Type	HT_{1-4}	Max Timetable Speed	MS_{1-4}
<7	<.015	0	.81	1	.54	0	1.00	0	1.00	1	.91	0-9	1.03
7-31	.02	1	1.02			1	1.23	1	1.11	2	.82	10-19	1.11
32-84	.03	2	1.11	2	.29	2	1.52	2	1.23	3	.74	20-29	1.20
85-179	.04	3	1.17			3	1.87	3	1.36	4	.67	30-39	1.30
180-328	.05	4	1.21			4	2.31	4	1.51	5	.61	40-49	1.40
329-541	.06	5	1.25			5	2.85			6	.55	50-59	1.52
542-830	.07	6	1.28			6	3.51					60-69	1.64
831-1209	.08	7	1.30			>6	>3.51					70-79	1.77
1210-1687	.09	8	1.32									80-89	1.91
1688-2278	.10	9	1.35									$\geq$90	$\geq$2.00
2279-2992	.11	10	1.36										
2993-3842	.12	11-20	1.44										
3843-4839	.13	21-30	1.54										
4840-5996	.14	31-40	1.61										
5997-7323	.15	>40	>1.61										
7324-8833	.16												
8834-10537	.17												
10538-12448	.18												
12449-14576	.19												
14577-16934	.20												
16935-19534	.21												
19535-22387	.22												
22388-25505	.23												
>25505	>.23												

TABLE 2-3. FACTORS FOR WARNING DEVICE CLASSES 5, 6, 7

CT	EI_{5-7}	Day Thru Trains	DT_{5-7}	Main Tracks	MT_{5-7}	Lanes	L_{5-7}	Population	P_{5-7}
<90	<.015	0	.93	0	1.00	1	1.15	0	1.00
90-508	.02	1	1.01	1	1.11	2	1.32	1	1.04
509-1589	.03	2	1.04	2	1.24	3	1.51	2	1.08
1590-3721	.04	3	1.06	3	1.39	4	1.74	3	1.13
3722-7342	.05	4	1.07	4	1.55	5	1.99	4	1.18
7343-12927	.06	5	1.08	5	1.72	6	2.29		
12928-20987	.07	6	1.09	6	1.92	7	2.63		
20988-33864	.08	7	1.10	>6	>1.92	8	3.02		
>33864	>.08	8	1.10			>9	>3.46		
		9	1.11						
		10	1.12						
		11-20	1.14						
		31-30	1.16						
		31-40	1.18						
		>40	>1.18						

2.3 WARNING DEVICE CLASS 8

For warning device class 8, the product form of the equation is:

$$H = EI_8 \times MT_8 \times L_8,$$

where

$$EI_8 = .00163\ (CT + 0.2)^{.3117}$$
$$MT_8 = e^{.2912\ MT}$$
$$L_8 = e^{.10360\ L}$$

Each of these factors is tabulated in Table 2-4.

TABLE 2-4. FACTORS FOR WARNING DEVICE CLASS 8

CT	EI_8	Main Tracks	MT_8	Lanes	L_8
<1650	<.015	0	1.00	1	1.11
1650-5237	.02	1	1.34	2	1.23
5238-25007	.03	2	1.79	3	1.36
25008-56004	.04	3	2.40	4	1.51
>56004	>.04	4	3.21	5	1.68
		5	4.29	6	1.86
		6	5.74	7	2.07
		>6	>5.74	8	2.29
				9	2.54
				>9	>2.54

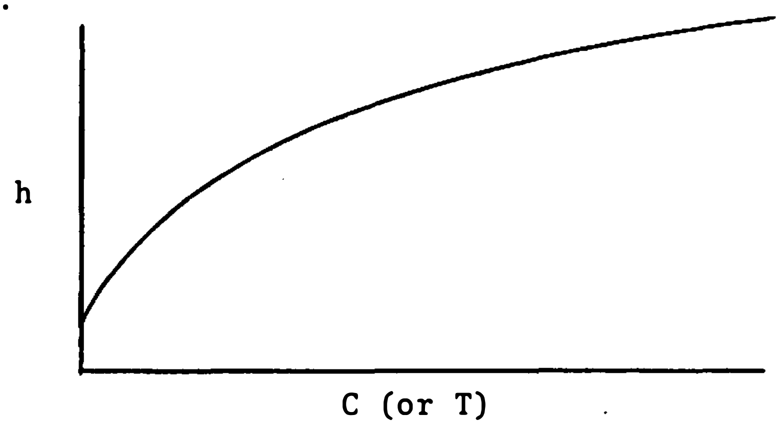
3. FORMULA CHARACTERISTICS

3.1 SENSITIVITY

It is important to determine the sensitivity of the equation for the predicted accidents per year h, to the many variables in the equations. As a function of C and T, keeping the other variables fixed, the function for h is of the form:

$$h = K\ (CT + 0.2)^{\alpha},$$

where K and α are constants. For T (or C) fixed, h is of the general form:



It increases without bound and the rate-of-change can be obtained by simple differentiation. The steepest slope is at C = 0, with the slope decreasing as C increases.

For the other variables, the sensitivity of H can be determined by simple algebra, with the results shown in Table 3-1. Thus, for example, for warning device classes 1, 2, 3, 4, if MT is increased by one, and all the other variables are held fixed, the new value of H is 1.233 times the old value. The constant 1.233, is used no matter what the values of the variables are.

TABLE 3-1. SENSITIVITY OF h TO NON-VOLUME VARIABLES

Variable	Change in h per unit Increased in Variable		
	Classes 1, 2, 3, 4	Classes 5, 6, 7	Class 8
D	$\left[\left(\frac{D + 1.2}{D + 0.2}\right)^{.1334} - 1\right] H$	$\left[\left(\frac{D + 1.2}{D + 0.2}\right)^{.0470} - 1\right] H$	-
MT	.233H	.115H	.338H
HP	-.460H	-	-
MS	.008H	-	-
F	-.095H	-	-
P	.108H	.041H	-
L	-	.148H	.109H

3.2 FORMULA GRAPHS

It is always difficult to graphically portray a multi-variable function. In this case, it is useful to consider the predicted accidents per year h as a function of the principal independent variables T and C, holding the non-volume variables fixed. When this is done, h can be viewed as a surface defined over the C, T plane. The equal level lines of the three accident prediction surfaces are shown in Figures 3-1, 3-2, and 3-3.

One of the significant results of these surfaces is that the surface for warning device classes 5, 6, 7 is everywhere less than that for warning device classes 1, 2, 3, 4, and the surface for warning device class 8 is less than that for warning device classes 5, 6, 7. This contrasts somewhat with similar surfaces for the

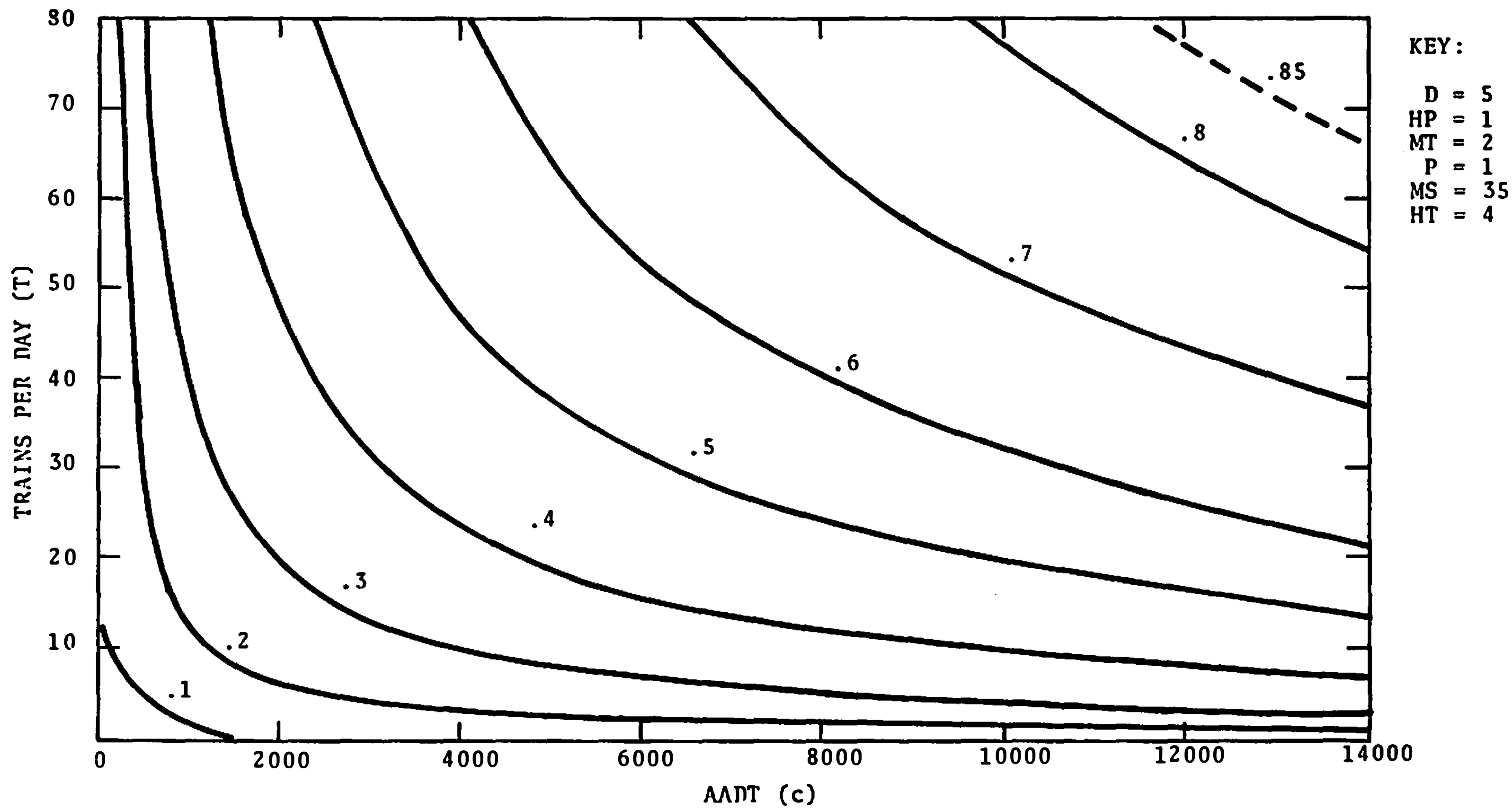


FIGURE 3-1. LINES OF EQUAL ACCIDENT PREDICTION LEVEL FOR WARNING DEVICE CLASSES 1, 2, 3, 4. NUMBERS DENOTE PREDICTED ACCIDENTS PER YEAR. h = .00870 (CT + 0.2).3335

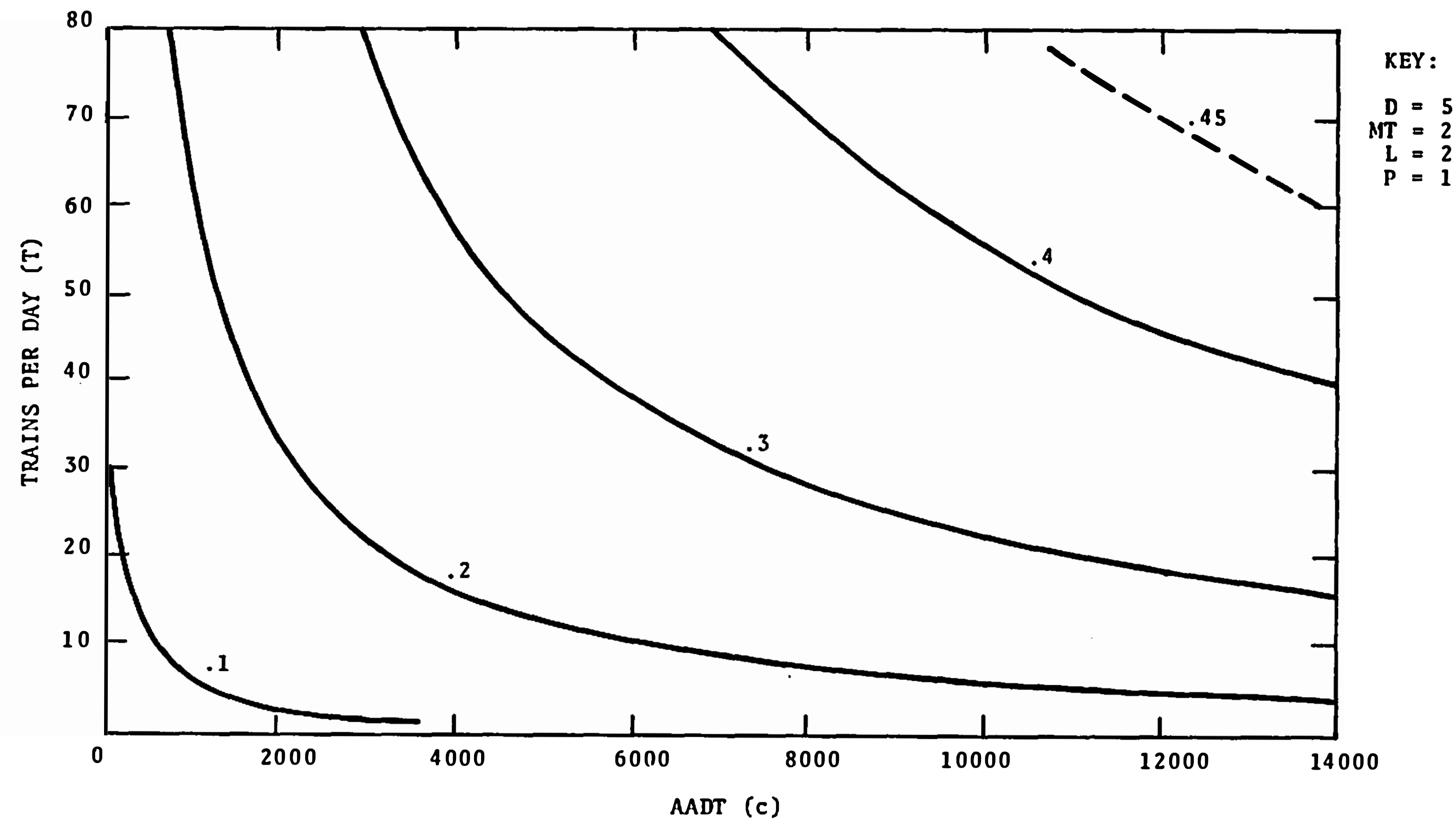


FIGURE 3.2. LINES OF EQUAL ACCIDENT PREDICTION LEVEL FOR WARNING DEVICE CLASSES 5, 6, 7. NUMBERS DENOTE PREDICTED ACCIDENTS PER YEAR DETERMINED BY EQUATION h = .00800 (CT + 0.2).2953

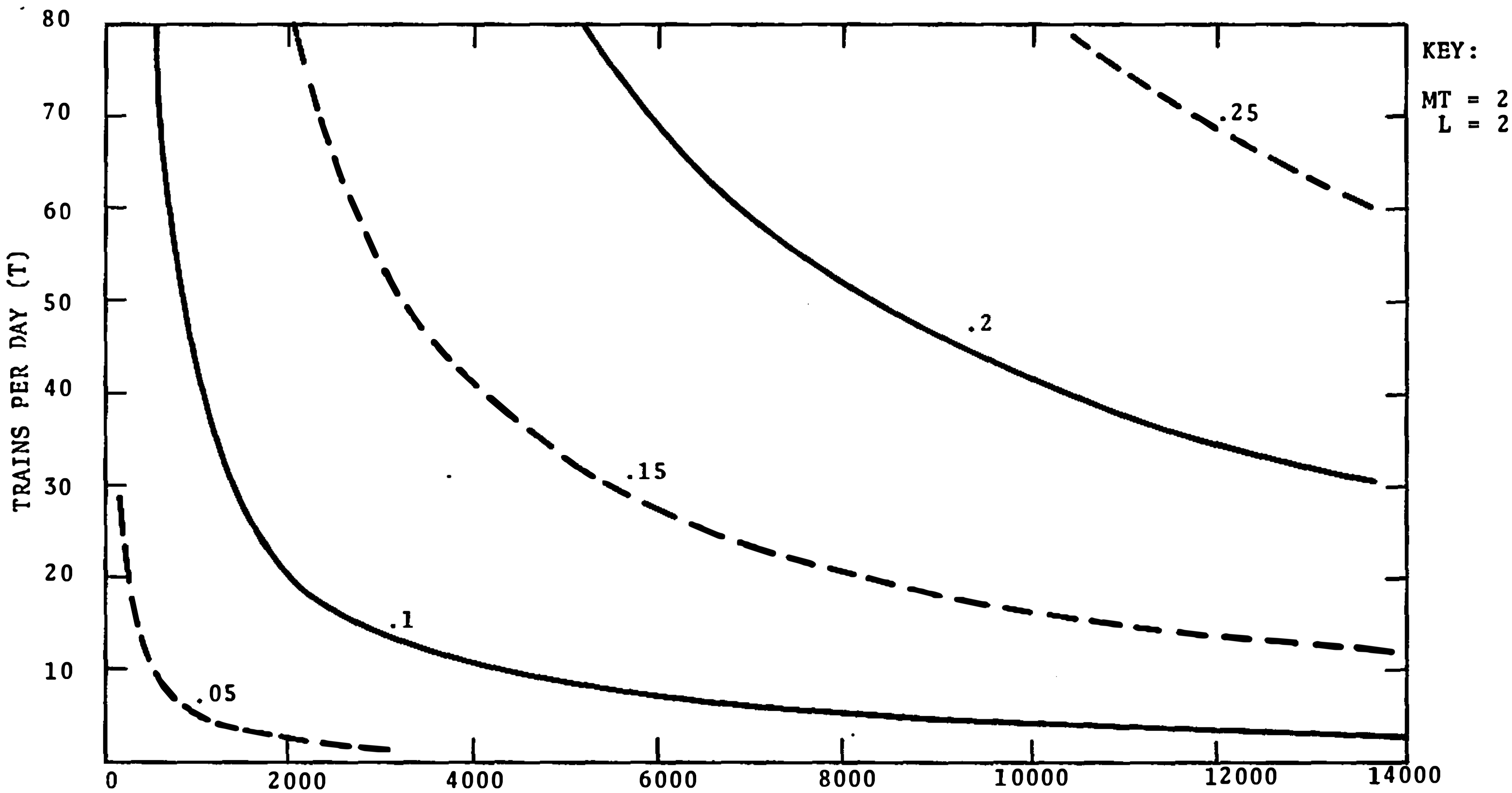


FIGURE 3-3. LINES OF EQUAL ACCIDENT PREDICTION LEVEL FOR WARNING DEVICE CLASS 8. NUMBERS DENOTE PREDICTED ACCIDENTS PER YEAR DETERMINED BY EQUATION h = .00359 (CT + 0.2).3116

previous formula[1] in that the surface for warning device classes 5, 6, 7 was higher than that for warning device classes 1, 2, 3, 4 for part of the C, T plane.

Another observation is that h, for warning device classes 1, 2, 3, 4, is monotonically increasing with T and C. This also is in contrast to the previous formula which had a flat region for the surface and even a maximum for large T and C.

For different values of the non-volume variables, the surfaces would have the same essential character. This is based on the observation from the sensitivity results that any change in the non-volume variables, other than D, will cause the surface to be changed only by a constant. The constant will be different generally for each equation, which could cause the surface for a higher warning device to be actually higher. This appears to be true only for very extreme values of the variables and for a very small number of crossings. For different values of D, the multiplier of the surface is a function of D, and hence the effect is not so simply stated. However, it is not expected that the character of the surface would change considerably.

[1]Mengert, Op. Cit., p. B-7.

4. PERFORMANCE OF REVISED FORMULA

The final performance measure of the revised formula is expressed by the power factors and prediction factors. To explain these measures, first rank the crossings by the accident prediction formula and select the X% having the highest values. Then if Y denotes the percent of accidents which occur within this selected set of crossings, and Z denotes the percent of total sum of predicted accidents represented by this selected set of crossings, then the X% power factor is Y/X and the X% prediction factor is Y/Z. Power factor expresses the ability of a formula to determine relative accident rate, and prediction factor expresses the ability of the formula to determine absolute accident rate. The power factors and prediction factors for X = 0.25%, 0.50%, 1.0%, 2.0%, 3.0%, 5.0% 10.0% for the revised formula along with the corresponding values for the old formula are given in Table 4-1.

Results are given for crossbucks (warning device class 4), for flashing lights (warning device class 7), and for gates (warning device class 8). Incorporating accident history, the formula used is given in Part II:

$$H = \frac{t_o}{t_o + t} h + \frac{t}{t_o + t} \frac{n}{t},$$

where,

H = Predicted accidents per year with accident history

h = Predicted accidents per year without accident history

t = Number of years of accident history

n = Number of accidents in t years

$$t_o = \frac{1.0}{.05 + h}$$

TABLE 4-1. PERFORMANCE OF REVISED FORMULA VS OLD FORMULA
MAY 78 INVENTORY, TEST YEAR 1978

Warning Device Class		Percent of Crossings													
		0.25		0.50		1.0		2.0		3.0		5.0		10.0	
		PwF	PdF	PwF	PdF	PwF	PdF	PwF	PdF	PwF	PdF	PwF	PdF	PwF	PdF
Crossbucks	Revised	9.03	.75	7.77	.76	7.24	.85	6.45	.92	5.68	.92	4.83	.92	3.86	.96
	Old	8.19	.66	7.17	.68	6.37	.73	5.75	.80	5.39	.85	4.64	.87	3.73	.92
Flashing Lights	Revised	5.85	1.03	6.29	1.24	5.67	1.28	4.94	1.29	4.62	1.31	3.96	1.27	3.25	1.24
	Old	4.52	.42	5.13	.55	5.58	.73	4.81	.77	4.31	.78	3.82	.82	3.25	.90
Gates	Revised	2.09	.52	3.54	.95	3.95	1.16	3.69	1.18	3.42	1.17	3.25	1.21	2.60	1.09
	Old	4.18	.56	4.42	.71	4.04	.78	3.64	.83	3.66	.93	3.37	.99	2.94	1.08
All Classes t=0	Revised	9.68	.97	8.68	1.01	7.92	1.08	6.67	1.08	6.06	1.09	5.20	1.08	4.00	1.04
	Old	10.59	.72	9.12	.76	7.70	.79	6.45	.83	5.82	.86	5.00	.90	3.90	.93
All Classes t=1,77C	Revised	18.60	1.12	14.63	1.10	11.07	1.05	8.41	1.02	7.08	1.01	5.76	1.01	4.26	1.01
	Old	18.16	.94	14.32	.94	10.83	.92	8.33	.95	7.10	.94	5.71	.94	4.20	.95
All Classes t=2,76,77C	Revised	19.37	1.03	15.54	1.04	11.96	1.01	9.07	1.00	7.63	.98	5.96	.96	4.40	.99
	Old	19.00	.91	15.43	.94	11.89	.93	9.07	.94	7.63	.94	6.02	.94	4.35	.95

Key: PwF = Power Factor, PdF = Predicted Factor, C = History Year.

Results are given for all eight classes combined, with t=0, t=1, and t=2. When t=0 (and hence n = 0), it is seen that H=h.

Overall, it appears that the revised formula performs about equally well in comparison with the old formula. In some places it is better and in other places it is worse. The difference between the formulas tends to diminish as t increases from zero, to one, to two. In fact, introducing accident history into the formula seems to overpower many other factors. Perhaps it could turn out that a very simple basic formula with accident history would be sufficiently accurate to be used in place of the more complicated basic function with accident history. The advantage of this would be simpler calculations and shorter tables. A conjecture could be offered that perhaps accident history combined only with volume variables will produce a sufficiently accurate formula because the information in the non-volume variables might be adequately represented by accident history.

These results also demonstrate the somewhat surprising fact that power factors for combined classes can be greater, for any fixed percentage of crossings, than for any of the individual classes.

An academic example which illustrates this phenomenon is the following. Consider twelve crossings: four are passive, denoted as X_1, X_2, X_3, X_4; four are flashing light crossings denoted as Y_1, Y_2, Y_3, Y_4; and four are gate crossings, denoted as Z_1, Z_2, Z_3, Z_4. These crossings are listed in rank order on the following page.

Passive			Flashing Lights			Gates		
ID	H	A	ID	H	A	ID	H	A
X_1	.52	0	Y_1	1.02	1	Z_1	.27	0
X_2	.51	0	Y_2	1.01	1	Z_2	.26	0
X_3	.49	1	Y_3	.99	1	Z_3	.24	0
X_4	.48	1	Y_4	.98	1	Z_4	.23	1

Here H denotes the predicted accidents per year and A denotes the number of accidents occurring in the given test year. The combined set of crossings, when rank ordered, are:

Combined		
ID	H	A
Y_1	1.02	1
Y_2	1.01	1
Y_3	.99	1
Y_4	.98	1
X_1	.52	0
X_2	.51	0
X_3	.49	1
X_4	.48	1
Z_1	.27	0
Z_2	.26	0
Z_3	.24	0
Z_4	.23	1

The power factors for three percentage levels for each class and for the combined class are:

Percent of Crossings	Passive (X_i)	Flashing Lights (Y_i)	Gates (Z_i)	Combined
25	0	1	0	1.7
50	0	1	0	1.14
75	.67	1	0	1.14
100	1	1	1	1

This illustrates that the combined power factors are higher than they are for the individual classes.

5. FORMULA DEVELOPMENT

The basic technique used for accident prediction formula development is the one expounded in the Mengert report.[1] To recall, the steps are:

1. Use an iterated non-linear logit regression to fit a best volume equation.
2. Check the adequacy of the volume equation with the selection regression.
3. If there is some doubt concerning which of two volume equations is better, compare them using power factors and prediction factors.
4. With a final volume equation in hand, use a selection regression to chose the non-volume variables to use.
5. Fit a best comprehensive equation using iterated non-linear regression.
6. Test best comprehensive equation using power factors and prediction factors.

5.1 DATA SETS

All the regressions referred to above were calculated on data set A, while the power factors were calculated on both data set A and data set B with important decisions being made chiefly for the results on data set B. These two data bases were constructed similarly to those described in the Mengert report.[2] Data sets A and B are disjoint and of equal size. Both are comprised of a random

[1]Mengert, P., Op. Cit.

[2]Ibid.

sample of records from the crossing inventory as of August 1976 and all linked records from the accident data for 1976. Each accident record in 1976 which could be linked with the inventory was represented by the inventory record of the crossing at which it occurred plus a field indicating an accident record. Each record sampled from the inventory was represented by the inventory record plus the accident field which in this case indicates a crossing record (i.e., "no accident"). Data sets A and B were finally broken down into 3 sub-data sets each: crossbucks, flashing lights, and gates. The records in the crossbucks' data sets were sampled uniformly at random from warning device class 4. The records in the flashing lights' data base came from class 5, while the records in the gates' data base came from class 8. The total accident and non-accident records in each sub-data set of A and B are given in Table 5-1. In it will be noticed that approximately 11% of the accident records for 1976 are not used in data sets A and B. The 11% represents the fraction for which inventory records could not be obtained by matching crossing, I.D.'s, i.e. linking was impossible due to faulty I.D.'s. The two data sets are almost exactly equal in size and in distribution of records over the categories determined by warning device class (the three nominal classes dealt with here) and accidents/non-accidents. The total number of records in the original base from which the sample data base was drawn are also shown.

The sampling ratio is seen to be different for every category. For example, for crossbucks - accident - data base A, the sampling ratio is $5639/2442 \approx 2.3$, i.e., about one record in 2.3 of the 1976 full accident data base which has a warning device class 1, 4

TABLE 5-1. COMPOSITION OF DATA SETS

COMPOSITION OF CROSSBUCKS DATA SETS			
	ACCIDENT	NON-ACCIDENT	TOTAL D.B.
1976 Total	5,639[1]	141,477[2]	
Data Set A	2,442	28,296	30,738
Data Set B	2,442	28,296	30,738

COMPOSITION OF FLASHING LIGHTS DATA SETS			
	ACCIDENT	NON-ACCIDENT	TOTAL D.B.
1976 Total	3,842[1]	33,969[2]	
Data Set A	1,747	13,588	15,335
Data Set B	1.747	13,588	15,335

COMPOSITION OF GATES DATA SETS			
	ACCIDENT	NON-ACCIDENT	TOTAL D.B.
1976 Total	929[1]	11,983[2]	
Data Set A	427	3,595	4,022
Data Set B	428	3,595	4,023

[1] Rail-Highway Grade Crossing Accidents/Incidents Bulletin (1976), p. 7.

[2] Summary Statistics of the National Railroad-Highway Inventory for Public At-Grade Crossings, June 1977, Page 3-57.

appears in data base A. For crossbucks - non-accident - data base A, the sampling ratio is 141477/28296 = 5:1.

The sampling ratios were based on experience reported in the Mengert report.[1] The basic idea was to have the same or slightly more non-accident crossings than accident crossings in the high hazard categories in all sub-data bases, to limit the number of non-accident records to allow for more efficient processing.

5.2 CROSSBUCK EQUATIONS

Formula development proceeded as follows. First the development of the crossbucks equation is discussed. It had been observed that exposure equations, i.e., accident prediction formulas based on vehicular volume times train volume have power factors almost as good as volume equations of general functional form. It therefore seemed appropriate to attempt to develop a comprehensive accident prediction formula based on an exposure equation as this would presumably perform about as well as one based on a more general volume equation and be much simpler to use. The equation, being simple, might be expected to show more stability over time.

Let X be defined by:

$$X = \log_{10}(C \cdot T + 0.2),$$

where

$$C = \text{vehicles/day},$$

and

$$T = \text{total number of trains/day}.$$

The quantity $C \cdot T$ will be called exposure while X may be referred to loosely as log (exposure). Notice that 0.2 is added to $C \cdot T$

[1]Mengert, P., Op. Cit.

before taking the logarithm in order to avoid an undefined value for X when C is 0 or T is 0. Denoting the value of the accident prediction formula by h, a general "exposure accident prediction equation" will be of the form $h = f_1 (C \cdot T)$ or $h = f_2(X)$, where f_1 and f_2 are arbitrary functions. Almost as general would be $h = e^{2y}$, where

$$Y = a_o + a_1X + a_2X^2 + A_3X^3 + \ldots + a_nX^n,$$

where n is arbitrary.

If more than the first two terms must be used, then perhaps a more general volume equation should be entertained. In any event, the simplest equation and the one to be tried first is:

$$Y = a_0 + a_1X$$

5.2.1 Crossbucks Exposure Equation

The coefficients of this equation were calibrated using the iterated non-linear regression[1] with data set A (Crossbucks subset). The results are given in Table 5-2 (refer to crossbucks case only). Note that the constant a_0 is of no generalizable significance and is determined by the relative sampling ratios of the crossing inventory and the accident data base in the working data set, i.e., data set A. It is only a_1 that is of significance. The equation is equivalent to

$$h = C_1e^{2 \log_{10}e \; a_1 \log_e (C \cdot T + 0.2)}$$

$$= C_1 (C \cdot T + 0.2)^{C_2}$$

where C_1 and C_2 are constants. $C_2 = 1$ would give essentially the New Hampshire formula. $C_2 = .18$ would give a formula very similar to the Peabody-Dimmick formula. The value C_2 determined here is approximately 0.44. As can be determined from the corresponding

[1]Mengert, P., Op. Cit.

TABLE 5-2. EXPOSURE FORMULA FOR CROSSBUCKS, FLASHING LIGHTS, AND GATES (UNNORMALIZED)*

	$h = e^{2y}$ $y = a_o + a_1 \log_{10} (C \cdot T + 0.2)$
CROSSBUCKS:	$a_0^* = -2.7151 \quad a_1 = 0.5069$
FLASHING LIGHTS:	$a_0^* = -3.4297 \quad a_1 = 0.5702$
GATES:	$a_0^* = -3.3230 \quad a_1 = 0.4760$

*Since these formulas are normalized for the sampled data base and not the national experience these values of a_0 should not be used for true normalization. See Sec. 5.5 below for normalization information.

values for other warning device classes below, the exponent should be between .4 and .5 for all warning device classes.

5.2.2 Crossbucks Volume Equation

To simplify the search for the best equation, it was decided to use this simplest exposure equation as a basis and to determine, using a selection regression, which other volume variables would potentially contribute most if included in the equation. The selection regression is for just such a purpose as described in the Mengert report.[1] It takes, as input, a basic equation specification and a list of variables to be tested and then selects those that will potentially contribute the most prediction power when

[1]Mengert, P., Op. Cit.

included with the basic equation into a comprehensive equation to be calibrated by iterative non-linear regression.

The variables used in the selection regression were:

$$\log_{10}(T + 0.2)$$

$$\log_{10}(C + 0.2)$$

$$\log_{10}(C + 0.2) \cdot \log_{10}(T + 0.2)$$

$$(\log_{10}(T + 0.2))^2$$

$$(\log 10 (C + 0.2))^2$$

The variables selected on the basis of t values greater than 4 were $\log_{10}(T + 0.2)$, $\log_{10}(C + 0.2)$ and $(\log_{10}(T + 0.2))^2$. (See Table 5-3.)

A volume equation of the form:

$$Y = a_0 + a_1 \log_{10}(T + 0.2) + a_2 \log_{10}(C + 0.2))$$
$$+ a_3(\log_{10}(T + 0.2))^2$$

was calibrated using iterative non-linear regression. The resulting coefficients are shown in Table 5-4. Again it should be pointed out that the coefficient a_0 is of no significance.

Power factors and prediction factors were produced using the simple exposure equation and the more complex volume equation. These power factors and prediction factors were determined using both data set A and data set B (both limited to subset crossbucks as the crossbucks equations are being tested). The results are in Tables 5-5 and 5-6. (Refer to Exposure Equation and Volume Equation only.)

It is seen that the power factors and prediction factors are somewhat better for the more complex volume equation than for the more simple exposure equation. The results on data set A do not

TABLE 5-3. VARIABLES CONSIDERED FOR VOLUME EQUATIONS

(Crossbucks, Flashing Lights, and Gates) (Used in a selection regression based on the exposure equation)

$\log_{10}(C + 0.2)$*
$\log_{10}(T + 0.2)$*
$\log_{10}(C + 0.2) \cdot \log_{10}(T + 0.2)$
$(\log_{10}(C + 0.2))^2$
$(\log_{10}(T + 0.2)^{2}$*,**

*Variables selected for crossbucks volume equation.
**This variable used in crossbucks case only.

TABLE 5-4. VOLUME EQUATION FOR CROSSBUCKS

$$h = e^{2y}$$

$$y = a_0 + a_1\log_{10}(C + 0.2) + a_2\log_{10}(T + 0.2) + a_3(\log_{10}(T + 0.2))^2$$

a_0^* = -2.7989

a_1 = 0.4587

a_2 = 0.8138

a_3 = -0.0611

*Note that a_0 does not refer to national experience and so must not be used as true normalization (See Section 5.5)

TABLE 5-5. POWER FACTORS AND PREDICTION FACTORS FOR CROSSBUCKS EQUATIONS ON DATA SET A

Percent of Crossings	Exposure Equations		Volume Equations		Comprehensive Equations	
	Pow. Fac.	Pre. Fac.	Pow. Fac.	Pre. Fac.	Pow. Fac.	Pre. Fac.
0.5	7.45	0.84	7.94	0.85	8.44	0.81
1.0	6.47	0.85	6.92	0.88	7.33	0.84
2.0	5.69	0.90	6.24	0.97	6.57	0.92
3.0	5.02	0.89	5.51	0.97	6.25	1.00
5.0	4.48	0.93	4.82	1.00	5.27	1.00
10.0	3.72	0.99	3.79	1.01	3.97	0.98

TABLE 5-6. POWER FACTORS AND PREDICTION FACTORS FOR CROSSBUCKS EQUATIONS ON DATA SET B

Percent of Crossings	Exposure Equations		Volume Equations		Comprehensive Equations	
	Pow. Fac.	Pre. Fac.	Pow. Fac.	Pre. Fac.	Pow. Fac.	Pre. Fac.
0.5	7.37	0.77	8.03	0.80	10.40	1.01
1.0	6.96	0.87	7.29	0.88	8.19	0.95
2.0	5.94	0.89	6.20	0.91	6.41	0.89
3.0	5.34	0.91	5.62	0.94	5.72	0.90
5.0	4.54	0.91	4.61	0.92	4.82	0.90
10.0	3.67	0.95	3.69	0.96	3.89	0.95

count as much as those on data set B, since the equations are calibrated on data set A, and the more complex equation having more degrees of freedom will perform better on that data set for that reason alone. The results on data set B, however, suggest that the volume equation is slightly superior in performance to the exposure equation. However, it was decided that the exposure equation should form the basis of the further development of the comprehensive formula. This decision was based on the following considerations.

1. The simplicity of the exposure equation will enhance its useability and perhaps its ready acceptance by the community of users.
2. A more complex TSC volume equation is already available.
3. When worked into a comprehensive equation any slight shortcomings will be partially compensated for. In other words, a comprehensive equation based on the exposure equation should be more nearly equivalent (in power factors and prediction factors) to one based on the volume equation than the basic equations are to each other.
4. When the comprehensive model is used as the basis of an accident history equation, slight differences in performance characteristics of the basic equation are nearly obliterated. This effect is dramatic and is illustrated in Part II.

   This is perhaps the most forceful reason.
5. Exposure equations were found to be as good as more complex volume equations for flashing lights and gates (see 5.3.1 and 5.4).
6. A simpler equation has a natural presumptive transferability advantage. It may be presumed that it will withstand changes in the underlying conditions to which it applies (passage of

time, specialization of region of applicability to one state, etc.) more readily because it is less specifically calibrated to the data base at hand. Because the data bases are large and the equation simple, this may not be a large effect.

For the above stated reasons, the exposure equation was used as the basis for further development of a comprehensive equation for the crossbucks case.

5.2.3 Crossbuck Case Comprehensive Equation

The variables which were considered for inclusion in the crossbucks comprehensive equation are shown in Table 5-7. A selection regression was run using the exposure equation described above as the basis. The variables that were chosen by the selection regression are shown in Table 5-8. (They are referred to there as X_1, X_2, X_3, X_4, X_5 and X_6.) All t values were over 4, indicating a strong statistical likelihood that the variables will make a significant improvement in the predictive power of the equation when combined with the basic exposure equation.

TABLE 5-7. NON-VOLUME VARIABLES CONSIDERED FOR COMPREHENSIVE EQUATIONS

(Crossbucks, Flashing Lights, and Gates) (Variables used in selection regression based on the exposure equation.)

Number of main tracks
Is highway paved?
Number of highway lanes
Population
Highway type
Day thru trains
Minimum crossing angle
Max timetable speed
Nearby intersecting highway

TABLE 5-8. COMPREHENSIVE EQUATION FOR CROSSBUCKS (UNNORMALIZED)

$$h = e^{2y}$$

$$y = a_0 + ax + a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 + a_5x_5$$

where,

$x = \log_{10}(C \cdot T + 0.2)$

$x_1 = \log_{10}(D + 0.2)$

x_2 = Highway Paved? (Y=1, N=2)

x_3 = Population (10's digit of functional classification of road)

x_4 = Max train speed

x_5 = Number of main tracks

x_6 = Highway type (units digit of functional classification of road)

a_0^* = 2.003

a = 0.3839

a_1 = 0.1538

a_2 = -0.3080

a_3 = 0.0511

a_4 = 0.0039

a_5 = 0.1047

a_6 = 0.0500

*Note that normalization should be altered as explained in Sec. 5.3.

The variables that were chosen using the selection regression were then used to form a "comprehensive equation" for the crossbucks case. The form of a comprehensive equation is

$$h = e^{2y},$$

where

$$y = aX + a_0 + a_1X_1 + a_2X_2 + \ldots$$

and where X is the log (exposure), i.e., $X = \log_{10} (C \cdot T + 0.2)$, $a, a_0, a_1, \ldots$ are constants to be calibrated and $X_1, X_2 \ldots$, are the non-volume variables, i.e., the variables, listed in Table 5-8. The constants a_0, a_1, a_2, are determined by iterated non-linear regression. The resulting equation is exhibited in Table 5-8. The power factors and prediction factors for this equation are exhibited in Tables 5-5 and 5-6 which refer to data set A and data set B respectively. The results on data set B are perhaps more reliable as the equation was calibrated on data set A. The equation exhibited in Table 5-8, is, except for normalization (see Sec. 5.5) the final comprehensive crossbucks equation developed during this phase, and its performance on a larger data set, the entire inventory and all 1978 accidents, is examined in Section 4 together with a comparison with the performance of the earlier comprehensive equation.[1]

5.3 FLASHING LIGHTS EQUATIONS

5.3.1 Flashing Lights Exposure Equations

The development was similar to the crossbucks case. First a simple exposure equation was constructed, i.e., one of the form:

$$h = e^{2y}$$

[1]Mengert, P., Op. Cit.

where

$$Y = a_1X + a_0,$$

and

$$X = \log_{10}(C \cdot T + 0.2).$$

The results are given in Table 5-2. Again notice that a_0 should not be regarded as having generalizable significance as it is determined by the sampling ratio of accident records to crossing records in the data set. Next a selection regression was run to test if any volume variables had potential to improve the equation when added to the exposure term X. The variable list for this selection regression is given in Table 5-3. The result of the selection regression was that no volume variable was selected. The variable "log trains" was the variable chosen as of most potential contribution to a voluue equation on the basis of t value but its t value was 1.15, entirely too small for consideration. Therefore, no volume equation of more complexity than the simple exposure equation was constructed. The comprehensive equation would be based on the sample exposure equation only.

5.3.2 Flashing Lights Comprehensive Equations

The list of non-volume variables to be considered for inclusion in the comprehensive equation for flashing lights is given in Table 5-7. The variables which had t values greater than 4.0 were chosen for inclusion in the comprehensive equation. With these variables, a comprehensive flashing lights equation was calibrated using iterated non-linear regression. The results are given in Table 5-9 (again no significance should be attached to the value of a_0).

TABLE 5-9. COMPREHENSIVE EQUATION FOR FLASHING LIGHTS (UNNORMALIZED)

$$h = e^{2y}$$

$$y = a_0 + ax + a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4,$$

where

$x = \log_{10}(C \cdot T + 0.2)$

$x_1 = \log_{10}(D + 0.2)$

x_2 = number of main tracks

x_3 = number of highway lanes

x_4 = population (see Sec. 2 for code),

and

$a_0^* = -2.647$

$a = 0.3400$

$a_1 = 0.0541$

$a_2 = 0.0544$

$a_3 = 0.0690$

$a_4 = 0.0203$

*Note that normalization should be altered as explained in Sec. 5.5.

The prediction factors and power factors for this equation using data set A and data set B, are shown in Tables 5-10 and 5-11 respectively. The prediction factors and power factors for the flashing lights exposure equation are given in those tables for comparison. Comparisons of the performance of the flashing lights comprehensive equation are given in Section 4. See Section 5.5 below for equation normalization.

5.4 AUTOMATIC GATES EQUATIONS

The development of an exposure equation and a comprehensive equation for gates was exactly analogous to the flashing lights case. A simple exposure equation was developed and through the use of a selection regression using volume variables, the exposure equation was deemed adequate. The exposure equation was (See Table 5-2) $h = e^{2Y}$, where $Y = a_o + A_1X$, with $X = \log_{10}(C \cdot T + 0.2)$ and in this case $a_0 = -3.323$ and $a_1 = 0.4760$ (again note that a_0 has no significance in general being a result of the sampling ratio). The power factors and prediction factors for this equation are given in Table 5-12.

Using the exposure equation as the basis, a selection regression was run, using the variables in Table 5-7.

Only the variable "number of main tracks", was selected for inclusion in the comprehensive equation. When the equation was produced on data set A and compared (using 1978 accidents) with the gate equation from the Mengert report,[1] it had somewhat poorer performance. The apparent reason was that the data set for gates was too small for good accuracy. The decision was to use both the

[1]Mengert, P., Op. Cit.

TABLE 5-10. POWER FACTORS AND PREDICTION FACTORS FOR FLASHING LIGHTS EQUATIONS ON DATA SET A

Percent of Crossings	Exposure Equations		Comprehensive Equations	
	Pow. Fac.	Pre. Fac.	Pow. Fac.	Pre. Fac.
0.5	5.04	.77	7.87	1.53
1.0	4.41	.78	6.18	1.42
2.0	4.38	.90	5.75	1.53
3.0	4.75	1.07	5.11	1.48
5.0	4.00	1.02	3.97	1.29
10.0	3.07	.96	3.23	1.24

TABLE 5-11. POWER FACTORS AND PREDICTION FACTORS FOR FLASHING LIGHTS EQUATIONS ON DATA SET B

Percent of Crossings	Exposure Equations		Comprehensive Equations	
	Pow. Fac.	Pre. Fac.	Pow. Fac.	Pre. Fac.
0.5	4.81	0.72	5.50	1.06
1.0	4.12	0.71	5.15	1.14
2.0	4.29	0.86	5.07	1.29
3.0	4.18	0.92	4.67	1.31
5.0	3.97	1.00	3.85	1.21
10.0	3.24	1.00	3.40	1.33

TABLE 5-12. POWER FACTORS AND PREDICTION FACTORS FOR GATES EXPOSURE EQUATIONS ON DATA SET B

Percent of Crossings	Exposure Equations Pow. Fac.	Pre. Fac.
0.5	1.87	0.43
1.0	2.34	0.59
2.0	3.04	0.85
3.0	3.58	1.07
5.0	3.22	1.08
10.0	2.66	1.04

These variables were the non-volume variables in the previous equation.[1] The resulting comprehensive equation is given in Table 5-13. Its performance is examined in Section 4.

TABLE 5-13. COMPREHENSIVE EQUATION FOR GATES

$$h = e^{2y}$$

$$y = a_0 + ax + a_1x_1 + a_2x_2$$

where

$x = \log_{10}(C \cdot T + 0.2)$

x_1 = number of main tracks

x_2 = number of highway lanes

and

$a_0 = -3.2096$ $\quad a_1 = .1456$

$a = .3588$ $\quad a_2 = .0518$

[1]Mengert, P., Op. Cit., p. B-7.

variable, "number of main tracks," and the variable, "number of highway lanes."

5.5 NORMALIZATION OF THE EQUATIONS

Since in each equation the value of a_0 was not appropriate to the full national sample (but was artifically controlled for efficiency in equation calibration), it was necessary to determine a_0 by normalization. The year 1977 was chosen for normalization. The value of a_0, for each comprehensive equation was chosen so that the sum of the accident prediction formula values, h, summed over all crossings in each major category (there were three major categories) would equal the total number of accidents during that year (1977) at those crossings. Incorporating those values in the equations leads to the following final comprehensive equation shown in Section 2.

Approximate normalizations for the intermediate equations developed and described in this section can be determined from the sampling ratios. The equations reported on may then be used with the a_0 values given provided they are multiplied by the normalization factors given in Table 5-14. These equations are not being proposed for general use; but for completeness, the normalizing factors are calculated. Note that the normalizing factor in each case is calculated from numbers given in Table 5-1. The rule is:

$$h_{corrected} = h_{Data\ Set\ A} r$$

where r is the normalizing factor in Table 5-14 and r is given by

$$r = \left(\frac{N_A}{N_c}\right) \Big/ \left(\frac{n_A}{n_c}\right) ,$$

where N_A = number of accidents in 1976 occurring at crossings of the given warning device class

N_C = number of records corresponding to the given warning device class in the 1976 inventory

n_A = number of accident records for the given warning device class in Data Set A

n_c = number of crossing (non-accident) records for the given warning device class in Data Set A.

TABLE 5-14. NORMALIZATION FACTORS FOR EQUATIONS DEVELOPED ON DATA SET A

$h_{corrected} = r \cdot h_{uncorrected}$
CROSSBUCKS: $r = \frac{5639}{2442} \cdot \frac{28,296}{141,477} = .462$
FLASHING LIGHTS: $r = \frac{3842}{1747} \cdot \frac{13,588}{33,969} = .880$
GATES: $r = \frac{929}{427} \cdot \frac{3595}{11,983} = .653$

6. CONCLUSIONS

The new accident prediction formula, developed with 1976 accident data instead of 1975 accident data, has essentially the same accuracy as the old, more complicated formula.[1]
Using exposure as the volume variable has proven successful. The non-volume variables are quite similar to those in the old formula. The only new variable is maximum timetable train speed, which enters into the equation for passive crossings. The only deletion is "highway type" which drops out of the flashing light equation. Adding accident history to the formula also produces the same accuracy as adding accident history to the old formula.

Although exposure with non-volume terms produces a formula with the same accuracy as the old formula, the results show that the exposure formula itself, without non-volume terms, is somewhat less accurate for crossbuck crossings than a more general volume formula by itself.

Overall, the integrity of the old formula has been corroborated. The only advantage of the new formula is simplicity.

[1]Mengert, P., Op. Cit., P. B-7.

Part II - Accident Prediction Formula with Accident History

7. INTRODUCTION

A new accident prediction formula has been developed which uses accident history as an explicit factor. With this new formula, the predicted accident rate at any given crossing is a weighted sum of the basic predicted accident rate and the observed accident rate. The "basic predicted accident rate" is the value calculated by the formula without accident history.[1] The weights are functions of this basic predicted accident rate.

Section 8, 9, and 10 give the results of the analysis and the computer calculations. This consists of a search for the optimum coefficients, the calculation of the performance of the new formula, and tables for convenient and quick calculations.

Section 5 discusses the derivation of the accident history equation. This derivation is based upon Bayesian theory, and assumes that the accident rate at a crossing is a random variable satisfying a gamma distribution. The quantity of interest, predicted accident frequency, is then the expected value of this variable, and the expectation that is conditional on the number of accidents observed in any given time period is calculated.

[1]Mengert, P., Op. Cit., p. B-7

8. SEARCH FOR OPTIMUM t_o

The accident prediction formula with accident history included as an explicit factor is derived in Section 5. The result is

$$H = \frac{t_o}{t_o + t} h + \frac{t}{t_o + t} \frac{n}{t}$$

where: h = Basic accident prediction formula which does not use accident history. Calculated values are predicted accidents per year.

t = Number of years of accident history. Need not be an integer.

n = number of accidents in t years.

H = Corrected value of predicted accidents per year with n accidents in t years.

The quantity t_o is a function of h and is of the form $t_o = A/(B+h)$. The parameters A and B were determined by an exhaustive trial-and-error search, with power factors and prediction factors being the criteria of performance. The basic accident prediction formula h, without accident history, is the one derived by Mengert[1] using 1975 accident data.

Tables 8-1 through 8-5 show the results of this search for different values of A and B. Table 8-1 gives the results for crossbuck crossings (warning device class 4), which uses the passive

[1]Mengert, Op. Cit., p. B-7

TABLE 8-1. ACCIDENT HISTORY PERFORMANCE FOR CROSSBUCK CROSSINGS. 25 PERCENT SAMPLE. AUGUST 1976 INVENTORY.

t_o		Accident Years		Percent of Crossings													
				0.25		0.50		1		2		3		5		10	
A	B	C	D	Pw.F.	Pd.F.	Pw.F.	Pd.F.	Pw.F.	Pd.F.	Pw.F.	Pd.F.	Pw.F.	Pd.F.	Pw.F.	Pd.F.	Pw.F.	Pd.F.
0.7	0.05	75	76	14.31	0.80	12.55	0.87	9.88	0.87	7.48	0.87	6.33	0.87	5.24	0.89	3.94	0.92
1.0	0.05	75	76	14.31	0.92	13.03	1.04	9.92	0.99	7.72	0.98	6.54	0.96	5.28	0.95	3.99	0.95
1.2	0.06	75	76	14.31	0.97	12.95	1.08	9.98	1.03	7.77	1.01	6.57	0.98	5.31	0.96	3.99	0.96
0.5	0.05	77	78	13.33	0.62	11.09	0.63	9.37	0.67	7.11	0.67	5.81	0.66	4.79	0.70	3.75	0.79
0.7	0.05	77	78	14.00	0.76	10.87	0.71	9.34	0.77	7.19	0.78	5.95	0.77	5.00	0.81	3.78	0.85
1.0	0.02	77	78	13.78	0.89	10.83	0.86	8.67	0.86	6.79	0.86	6.04	0.89	4.99	0.89	3.82	0.91
1.0	0.05	77	78	13.55	0.85	10.64	0.81	9.04	0.86	6.84	0.83	6.16	0.87	5.07	0.88	3.84	0.90
1.2	0.06	77	78	13.55	0.89	10.64	0.85	8.69	0.86	6.86	0.86	6.00	0.87	5.03	0.88	3.82	0.90
1.0	0.10	77	78	13.78	0.81	10.87	0.76	9.24	0.81	7.02	0.79	5.97	0.78	4.96	0.80	3.76	0.83
1.5	0.05	77	78	13.41	0.96	10.06	0.87	8.43	0.89	6.66	0.87	5.93	0.89	4.98	0.90	3.82	0.92
1.0	0.05	76	78	12.08	0.70	9.94	0.73	8.25	0.77	6.28	0.76	5.46	0.77	4.77	0.83	3.70	0.87
0.7	0.02	76	77	15.71	0.82	12.58	0.84	9.42	0.82	6.77	0.79	5.87	0.81	5.01	0.86	3.85	0.90
1.0	0.05	76,77	78	12.67	0.67	12.38	0.83	9.52	0.81	7.85	0.87	6.53	0.85	5.25	0.85	3.92	0.88
0.7	0.05	76,77	78	13.63	0.63	12.19	0.72	9.78	0.73	7.97	0.78	6.61	0.77	5.14	0.76	3.83	0.80

Legend:
Pw.F. = Power Factor
Pd.F. = Prediction Factor
C = History Years
D = Test Year
t_o = A/(B+h)

TABLE 8-2. ACCIDENT HISTORY PERFORMANCE FOR FLASHING LIGHT CROSSINGS. 50 PERCENT SAMPLE. AUGUST 1976 INVENTORY.

t_o		Accident Years		Percent of Crossings													
				0.25		0.50		1		2		3		5		10	
A	B	C	D	Pw.F.	Pd.F.	Pw.F.	Pd.F	Pw.F	Pd.F.	Pw.F.	Pd.F.	Pw.F.	Pd.F	Pw.F	Pd.F.	Pw.F	Pd.F.
0.5	0.05	77	78	16.58	0.93	11.02	0.75	8.85	0.75	6.56	0.71	5.72	0.72	4.58	0.71	3.34	0.74
0.7	0.05	77	78	16.47	1.03	10.43	0.80	8.74	0.83	6.70	0.81	5.70	0.80	4.53	0.79	3.42	0.83
1.0	0.02	77	78	15.40	1.11	10.91	0.96	9.12	1.00	6.62	0.92	5.55	0.89	4.49	0.88	3.48	0.92
1.0	0.05	77	78	15.61	1.11	10.91	0.94	9.22	0.99	6.67	0.91	5.56	0.87	4.46	0.86	3.46	0.90
1.2	0.06	77	78	14.97	1.12	10.96	1.00	9.28	1.04	6.63	0.94	5.53	0.90	4.46	0.89	3.47	0.92
1.0	0.10	77	78	15.94	1.09	10.86	0.91	8.98	0.93	6.70	0.87	5.65	0.85	4.55	0.84	3.44	0.86
1.5	0.05	77	78	14.87	1.19	11.12	1.09	8.64	1.04	6.35	0.96	5.45	0.94	4.53	0.94	3.44	0.94
1.0	0.05	76,77	78	15.40	1.06	11.66	0.96	9.17	0.92	7.30	0.92	6.38	0.93	5.01	0.89	3.65	0.89
0.7	0.05	76,77	78	14.33	0.86	11.76	0.88	9.20	0.83	7.33	0.83	6.32	0.84	5.00	0.81	3.66	0.82

Legend:
Pw.F. = Power Factor
Pd.F. = Prediction Factor
C = History Year
D = Test Year
t_o = A/(B+h)

TABLE 8-3. ACCIDENT HISTORY PERFORMANCE FOR GATE CROSSINGS. 50 PERCENT SAMPLE. AUGUST 1976 INVENTORY.

t_o		Accident Years		Percent of Crossings													
				0.25		0.50		1		2		3		5		10	
A	B	C	D	Pw.F.	Pd.F.	Pw.F.	Pd.F.	Pw.F.	Pd.F.	Pw.F.	Pd.F	Pw.F.	Pd.F.	Pw.F.	Pd.F.	Pw.F.	Pd.F.
1.0	0.02	75	76	7.78	0.83	5.95	0.80	5.26	0.89	4.98	1.06	4.23	1.02	3.52	1.01	2.96	1.09
1.0	0.03	75	76	7.78	0.81	6.18	0.83	5.03	0.84	4.75	0.99	4.20	1.00	3.59	1.01	2.95	1.06
1.2	0.06	75	76	7.78	0.84	6.18	0.85	4.92	0.83	4.69	0.98	4.27	1.01	3.64	1.02	2.84	1.02
1.5	0.05	75	76	6.86	0.84	6.18	0.95	5.26	0.99	4.75	1.09	4.31	1.12	3.55	1.07	3.00	1.14
0.5	0.05	77	78	9.97	0.71	8.64	0.74	5.90	0.61	4.57	0.59	4.21	0.63	3.55	0.65	2.65	0.67
0.7	0.05	77	78	9.97	0.80	8.64	0.85	6.23	0.75	4.53	0.68	4.13	0.72	3.60	0.76	2.68	0.77
1.0	0.02	77	78	8.97	1.00	7.48	0.97	6.23	0.97	4.65	0.88	4.21	0.92	3.32	0.87	2.76	0.95
1.0	0.05	77	78	8.97	0.93	7.64	0.93	6.15	0.89	4.61	0.81	4.24	0.86	3.44	0.84	2.77	0.90
1.2	0.06	77	78	8.97	1.00	7.31	0.95	6.15	0.95	4.65	0.87	4.32	0.92	3.55	0.91	2.78	0.93
1.0	0.10	77	78	9.97	0.93	8.47	0.93	5.90	0.78	4.57	0.73	4.29	0.79	3.64	0.81	2.67	0.78
1.5	0.05	77	78	8.97	1.15	7.48	1.10	6.15	1.05	4.44	0.91	4.21	0.99	3.26	0.91	2.79	1.00
1.0	0.05	76,77	78	9.97	0.98	7.81	0.87	6.48	0.87	5.40	0.89	4.24	0.80	3.94	0.89	2.90	0.86
0.7	0.05	76,77	78	9.97	0.83	7.97	0.76	6.56	0.75	5.02	0.72	4.35	0.71	3.90	0.78	2.89	0.76

Legend:
Pw.F. = Power Factor
Pd.F. = Prediction Factor
C = History year
D = Test Year
t_o = A/(B+h)

TABLE 8-4. ACCIDENT HISTORY PERFORMANCE.
100 PERCENT SAMPLES. MAY 1978 INVENTORY.

t_o		Accident Years		Percent of Crossings													
				0.25		0.50		1		2		3		5		10	
				Crossbuck Crossings													
A	B	C	D	Pw.F.	Pd.F.	Pw.F.	Pd.F.	Pw.F.	Pd.F.	Pw.F.	Pd.F.	Pw.F.	Pd.F.	Pw.F.	Pd.F.	Pw.F.	Pd.F.
0.7	0.05	77	78	15.56	0.80	12.58	0.82	9.76	0.81	7.60	0.82	6.32	0.82	5.29	0.86	3.96	0.89
1.0	0.05	77	78	15.25	0.92	12.12	0.91	9.74	0.92	7.45	0.90	6.45	0.91	5.29	0.92	3.98	0.93
				Flashing Light Crossings													
0.7	0.05	77	78	14.67	0.84	10.54	0.74	9.10	0.80	6.92	0.78	5.93	0.79	4.75	0.79	3.55	0.84
1.0	0.05	77	78	13.68	0.89	10.59	0.83	9.10	0.89	6.78	0.86	5.81	0.86	4.71	0.87	3.62	0.91
				Gate Crossings													
0.7	0.05	77	78	14.63	1.23	10.62	1.07	7.90	0.96	5.49	0.83	5.12	0.89	4.06	0.86	3.06	0.88
1.0	0.05	77	78	13.94	1.40	10.44	1.23	7.73	1.09	5.62	0.98	5.03	1.00	4.16	1.00	3.11	0.98

Legend:
Pw.F. = Power Factor
Pd.F. = Prediction Factor
C = History Year
D = Test Year
t_o = A/(B+h)

TABLE 8-5. ACCIDENT HISTORY PERFORMANCE. COMBINED WARNING DEVICE CLASSES. MAY 1978 INVENTORY.

t_o		Accident Years		Percent of Crossings													
				0.25		0.50		1		2		3		5		10	
A	B	C	D	Pw.F.	Pd.F.	Pw.F.	Pd.F	Pw.F.	Pd.F.	Pw.F.	Pd.F.	Pw.F.	Pd.F.	Pw.F.	Pd.F.	Pw.F.	Pd.F.
1.0	0.05	77	78	18.16	0.94	14.32	0.94	10.83	0.92	8.49	0.95	7.10	0.94	5.71	0.94	4.20	0.95
0.7	0.05	77	78	18.82	0.85	14.38	0.83	11.02	0.83	8.33	0.84	7.03	0.85	5.69	0.88	4.17	0.91

Legend:
Pw.F. = Power Factor
Pd.F. = Prediction Factor
C = History Year
D = Test Year
t_o = A/(B+h)

equation of the accident prediction formula for determining h. Because of the large amount of computer time involved in these calculations, a 25 percent sample of the crossings (with 100 percent of accidents) was used. The August 1976 inventory was used. These results are available for increments of 0.25 percent of crossings, but the 7 values of percent of crossings given in the tables are representative of all the results. This trial-and-error search is based mainly on one year of accident history, although two computations were made for two years as seen in the bottom two lines of the table.

Tables 8-2 and 8-3 are similar to Table 8-1, but the former is for flashing light crossings (Warning Device Class 7) and the latter is for gate crossings (Warning Device Class 8). The values of h are determined by the corresponding equations of the accident prediction formula. Since these sets of crossings are much smaller than the crossbuck crossings, 50 percent samples of the crossings (with 100 percent of the accidents) were used.

Table 8-4 uses the May 1978 inventory and applies to all crossbuck crossings, all flashing light crossings, and all gate crossings, respectively. This table compares A = 0.7, B = 0.05 with A = 1.0, B = 0.05.

Table 8-5 applies to the entire May 1978 inventory, consisting of all eight warning device classes. In evaluating these results, it appears that performance is relatively insensitive to values of A and B. Upon close scrutiny, it was decided that the cases A = 0.7, B = 0.05 and A = 1.0, B = 0.05 were the best, overall. This is based on a comparison of these data using a considerable amount of judgment. Tables 8-4 and 8-5 show that these two cases are quite close when using all the crossings. Perhaps A = 0.7, B = 0.05 is slightly better in power factors for small percentages of crossings, but A = 1.0, B = 0.05 is slightly better in power factors for higher percentages of crossings and is slightly better overall in prediction factors. The conclusion is that the case A = 1.0, B = 0.05 gives the best results, but not by a wide margin.

It is important to note that A = 1.0, B = 0.05 is considered the best model for all three groups of crossings. It might have been expected that different accident history models would be best - one for each of the three equations in the accident prediction formula. Of course, considerable simplicity is achieved by having to work with only one accident history model.

Comparing Table 8-4 with Table 8-5 provides an example of the interesting phenomenon of combined classes giving better power factors than any of the individual classes of crossings. This phenomenon is discussed in Section 4.

9. PERFORMANCE AS A FUNCTION OF t

The performance of the accident history accident prediction formula is given in Table 9-1 as a function of percent of crossings and number of years of accident history (t), with t_o = 1.0/(0.05 + h). The same reuslts for power factors are shown graphically in Figure 9-1. At his writing (May 1980) only four years of accident history are available. Since one year is needed for testing, the results are given for t = 0, 1, 2, 3.

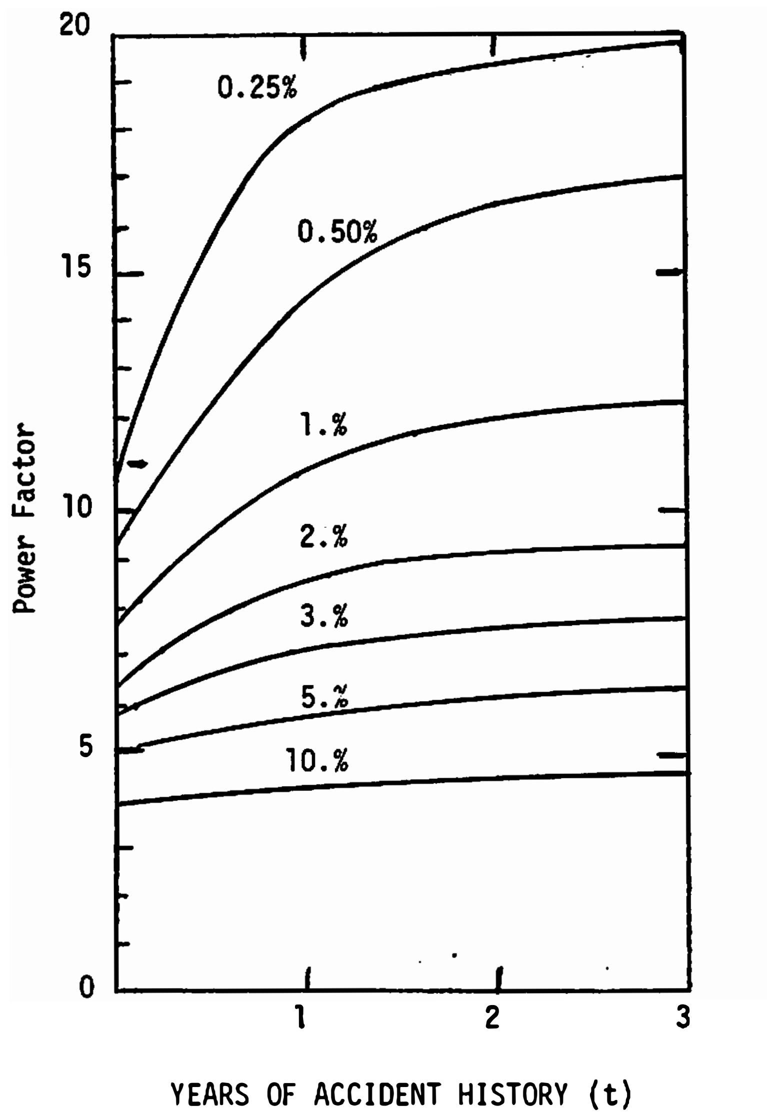
The power factors are seen to increase with t as expected. The greatest amount of improvement is from t = 0 to t = 1, with diminishing increments of improvement occurring for higher values of t. At 0.25 percent of crossings, for t = 1 the increase is 71 percent whereas for t = 3 the increase goes up only to 84 percent. For t > 3, it would appear that accident history contributes progressively smaller amounts of information in determining relative accident rates. This conjecture could be tested when future accident data becomes available. Accident history is most effective for small values of percent of crossings - which means it is most effective for the high accident rate crossings. This is seen by the increase in power factor expressed as a percentage increase over the case t = 0. The percentage increase is a monotonic decreasing function of percent of the crossings. For t = 1, at 0.25 percent of crossings the increase is 71 percent, whereas at 10 percent of crossings the increase drops to only 8 percent.

TABLE 9-1. PERFORMANCE OF ACCIDENT HISTORY AS FUNCTION OF NUMBER OF YEARS (t). MAY 78 INVENTORY. TEST YEAR - 1978

	Percent of Crossings																				
	0.25			0.50			1			2			3			5			10		
t (History Years)	Pw.F.	(%)	Pd.F.	Pw.F.	(%)	Pd.F.	Pw.F.	(%)	Pd.F.	Pw.F.	(%)	Pd.F.	Pw.F.	(%)	Pd.F.	Pw.F.	(%)	Pd.F.	Pw.F.	(%)	Pd.F.
0	10.59	(-)	0.72	9.12	(-)	0.76	7.70	(-)	0.79	6.45	(-)	0.83	5.82	(-)	0.86	5.00	(-)	0.90	3.90	(-)	0.93
1 (77)	18.16	(71)	0.94	14.32	(57)	0.94	10.83	(41)	0.92	8.49	(29)	0.95	7.10	(22)	0.94	5.71	(14)	0.94	4.20	(8)	0.95
2 (76,77)	19.00	(79)	0.91	15.43	(69)	0.94	11.89	(54)	0.93	9.07	(41)	0.94	7.63	(31)	0.94	6.02	(20)	0.94	4.35	(12)	0.95
3 (75,76,77)	19.55	(84)	0.92	15.93	(75)	0.95	12/10	(57)	0.93	9.24	(43)	0.94	7.76	(53)	0.93	6.19	(24)	0.94	4.41	)13)	0.94

Legend: Pw.F. (%) = Power Factor (Percent Increase Over t=0 Case)

Pd.F. = Prediction Factor



NOTE: The percentages above the curves are percentages of crossings

FIGURE 9-1. IMPROVEMENT DUE TO ACCIDENT HISTORY

The prediction factors show a significant improvement for t = 1 but no improvement for higher values of t. The percentage improvement of prediction factor is also greatest for small values of percent of crossings.

One caveat must be noted in presenting these power factor and prediction factor performance results. No allowance has been made for crossings having been upgraded since January 1975. In such a case, the formula should ideally use t and n that apply since the upgrade. However, the good performance obtained here is valid even though the formula is used in this less-than-perfect manner. The reason for not incorporating upgrade situations is simply due to data processing expediency. If upgrades are taken into account and the formula correctly used, an improved performance could be conjectured. It is expected that this additional performance will be determined in the near future as upgrade dates are incorporated into the TSC data files.

10. ACCIDENT HISTORY TABLES

To determine the predicted number of accidents per year for any given crossing, Tables 10-1 through 10-4 can be conveniently used instead of the formula. This is a two-dimensional table which gives the predicted accident rate H for any given value of h determined without accident history, and for any number of accidents n, with t given. These four tables are for t = 1, 2, 3, 4, respectively. If t contains a fraction, the H could be interpolated from the tables, or the formula could be used directly. In all probability, it would be sensible to use all the accident history available, and at this writing (May 1980) four years are available. However, there may be cases where less than four years of accident history is known or where it is advisable to use less than four years such as for a recent warning device upgrade.

The values of h and n are listed at the left and top of the tables, respectively. Thus, if h = 0.05 and n = 5, with t = 4, the accident prediction index is H = 0.393. Note that the following properties show the interrelationship of H, h, and n/t:

$$H < h \text{ for } h > n/t,$$

$$H > h \text{ for } h < n/t,$$

$$H = h \text{ for } h = n/t.$$

Note also that H for n = 0 is not the same as h. In fact H < h for positive h, n = 0.

TABLE 10-1. ACCIDENT PREDICTION RATE USING ACCIDENT HISTORY (t=1)

h	n: 0	1	2	3	4	5
0.00	0.000	0.048	0.095	0.143	0.190	0.238
0.01	0.009	0.066	0.123	0.179	0.236	0.292
0.02	0.019	0.084	0.150	0.215	0.280	0.346
0.03	0.028	0.102	0.176	0.250	0.324	0.398
0.04	0.037	0.119	0.202	0.284	0.367	0.450
0.05	0.045	0.136	0.227	0.318	0.409	0.500
0.06	0.054	0.153	0.252	0.351	0.450	0.550
0.07	0.063	0.170	0.277	0.384	0.491	0.598
0.08	0.071	0.186	0.301	0.416	0.531	0.646
0.09	0.079	0.202	0.325	0.447	0.570	0.693
0.10	0.087	0.217	0.348	0.478	0.609	0.739
0.20	0.160	0.360	0.560	0.760	0.960	1.160
0.30	0.222	0.481	0.741	1.000	1.259	1.519
0.40	0.276	0.586	0.897	1.207	1.517	1.828
0.50	0.323	0.677	1.032	1.387	1.742	2.097
0.60	0.364	0.758	1.152	1.545	1.939	2.333
0.70	0.400	0.829	1.257	1.686	2.114	2.543
0.80	0.432	0.892	1.351	1.811	2.270	2.730
0.90	0.462	0.949	1.436	1.923	2.410	2.897
1.00	0.488	1.000	1.512	2.024	2.537	3.049
1.10	0.512	1.047	1.581	2.116	2.651	3.186
1.20	0.533	1.089	1.644	2.200	2.756	3.311
1.30	0.553	1.128	1.702	2.277	2.851	3.426
1.40	0.571	1.163	1.755	2.347	2.939	3.531
1.50	0.588	1.196	1.804	2.412	3.020	3.627
1.60	0.604	1.226	1.849	2.472	3.094	3.717
1.70	0.618	1.255	1.891	2.527	3.164	3.800
1.80	0.632	1.281	1.930	2.579	3.228	3.877
1.90	0.644	1.305	1.966	2.627	3.288	3.949
2.00	0.656	1.328	2.000	2.672	3.344	4.016
2.10	0.667	1.349	2.032	2.714	3.397	4.079
2.20	0.677	1.369	2.062	2.754	3.446	4.138
2.30	0.687	1.388	2.090	2.791	3.493	4.194
2.40	0.696	1.406	2.116	2.826	3.536	4.246
2.50	0.704	1.423	2.141	2.859	3.577	4.296

TABLE 10-2. ACCIDENT PREDICTION RATE USING ACCIDENT HISTORY (t=2)

	n								
h	0	1	2	3	4	5	6	7	8
0.00	0.000	0.045	0.091	0.136	0.182	0.227	0.273	0.318	0.364
0.01	0.009	0.063	0.116	0.170	0.223	0.277	0.330	0.384	0.438
0.02	0.018	0.079	0.140	0.202	0.263	0.325	0.386	0.447	0.509
0.03	0.026	0.095	0.164	0.233	0.302	0.371	0.440	0.509	0.578
0.04	0.034	0.110	0.186	0.263	0.339	0.415	0.492	0.568	0.644
0.05	0.042	0.125	0.208	0.292	0.375	0.458	0.542	0.625	0.708
0.06	0.049	0.139	0.230	0.320	0.410	0.500	0.590	0.680	0.770
0.07	0.056	0.153	0.250	0.347	0.444	0.540	0.637	0.734	0.831
0.08	0.063	0.167	0.270	0.373	0.476	0.579	0.683	0.786	0.889
0.09	0.070	0.180	0.289	0.398	0.508	0.617	0.727	0.836	0.945
0.10	0.077	0.192	0.308	0.423	0.538	0.654	0.769	0.885	1.000
0.20	0.133	0.300	0.467	0.633	0.800	0.967	1.133	1.300	1.467
0.30	0.176	0.382	0.588	0.794	1.000	1.206	1.412	1.618	1.824
0.40	0.211	0.447	0.684	0.921	1.158	1.395	1.632	1.868	2.105
0.50	0.238	0.500	0.762	1.024	1.286	1.548	1.810	2.071	2.333
0.60	0.261	0.543	0.826	1.109	1.391	1.674	1.957	2.239	2.522
0.70	0.280	0.580	0.880	1.180	1.480	1.780	2.080	2.380	2.680
0.80	0.296	0.611	0.926	1.241	1.556	1.870	2.185	2.500	2.815
0.90	0.310	0.638	0.966	1.293	1.621	1.948	2.276	2.603	2.931
1.00	0.323	0.661	1.000	1.339	1.677	2.016	2.355	2.694	3.032
1.10	0.333	0.682	1.030	1.379	1.727	2.076	2.424	2.773	3.121
1.20	0.343	0.700	1.057	1.414	1.771	2.129	2.486	2.843	3.200
1.30	0.351	0.716	1.081	1.446	1.811	2.176	2.541	2.905	3.270
1.40	0.359	0.731	1.103	1.474	1.846	2.218	2.590	2.962	3.333
1.50	0.366	0.744	1.122	1.500	1.878	2.256	2.634	3.012	3.390
1.60	0.372	0.756	1.140	1.523	1.907	2.291	2.674	3.058	3.442
1.70	0.378	0.767	1.156	1.544	1.933	2.322	2.711	3.100	3.489
1.80	0.383	0.777	1.170	1.564	1.957	2.351	2.745	3.138	3.532
1.90	0.388	0.786	1.184	1.582	1.980	2.378	2.776	3.173	3.571
2.00	0.392	0.794	1.196	1.598	2.000	2.402	2.804	3.206	3.608
2.10	0.396	0.802	1.208	1.613	2.019	2.425	2.830	3.236	3.642
2.20	0.400	0.809	1.218	1.627	2.036	2.445	2.855	3.264	3.673
2.30	0.404	0.816	1.228	1.640	2.053	2.465	2.877	3.289	3.702
2.40	0.407	0.822	1.237	1.653	2.068	2.483	2.898	3.314	3.729
2.50	0.410	0.828	1.246	1.664	2.082	2.500	2.918	3.336	3.754

TABLE 10-3. ACCIDENT PREDICTION RATE USING ACCIDENT HISTORY (t-3)

	n												
h	0	1	2	3	4	5	6	7	8	9	10	11	12
0.00	0.000	0.043	0.087	0.130	0.174	0.217	0.261	0.304	0.348	0.391	0.435	0.478	0.522
0.01	0.008	0.059	0.110	0.161	0.212	0.263	0.314	0.364	0.415	0.466	0.517	0.568	0.619
0.02	0.017	0.074	0.132	0.190	0.248	0.306	0.364	0.421	0.479	0.537	0.595	0.653	0.711
0.03	0.024	0.089	0.153	0.218	0.282	0.347	0.411	0.476	0.540	0.605	0.669	0.734	0.798
0.04	0.031	0.102	0.173	0.244	0.315	0.386	0.457	0.528	0.598	0.669	0.740	0.811	0.882
0.05	0.038	0.115	0.192	0.269	0.346	0.423	0.500	0.577	0.654	0.731	0.808	0.885	0.962
0.06	0.045	0.128	0.211	0.293	0.376	0.459	0.541	0.624	0.707	0.789	0.872	0.955	1.038
0.07	0.051	0.140	0.228	0.316	0.404	0.493	0.581	0.669	0.757	0.846	0.934	1.022	1.110
0.08	0.058	0.151	0.245	0.338	0.432	0.525	0.619	0.712	0.806	0.899	0.993	1.086	1.180
0.09	0.063	0.162	0.261	0.359	0.458	0.556	0.655	0.754	0.852	0.951	1.049	1.148	1.246
0.10	0.069	0.172	0.276	0.379	0.483	0.586	0.690	0.793	0.897	1.000	1.103	1.207	1.310
0.20	0.114	0.257	0.400	0.543	0.686	0.829	0.971	1.114	1.257	1.400	1.543	1.686	1.829
0.30	0.146	0.317	0.488	0.659	0.829	1.000	1.171	1.341	1.512	1.683	1.854	2.024	2.195
0.40	0.170	0.362	0.553	0.745	0.936	1.128	1.319	1.511	1.702	1.894	2.085	2.277	2.468
0.50	0.189	0.396	0.604	0.811	1.019	1.226	1.434	1.642	1.849	2.057	2.264	2.472	2.679
0.60	0.203	0.424	0.644	0.864	1.085	1.305	1.525	1.746	1.966	2.186	2.407	2.627	2.847
0.70	0.215	0.446	0.677	0.908	1.138	1.369	1.600	1.831	2.062	2.292	2.523	2.754	2.985
0.80	0.225	0.465	0.704	0.944	1.183	1.423	1.662	1.901	2.141	2.380	2.620	2.859	3.099
0.90	0.234	0.481	0.727	0.974	1.221	1.468	1.714	1.961	2.208	2.455	2.701	2.948	3.195
1.00	0.241	0.494	0.747	1.000	1.253	1.506	1.759	2.012	2.265	2.518	2.771	3.024	3.277
1.10	0.247	0.506	0.764	1.022	1.281	1.539	1.798	2.056	2.315	2.573	2.831	3.090	3.348
1.20	0.253	0.516	0.779	1.042	1.305	1.568	1.832	2.095	2.358	2.621	2.884	3.147	3.411
1.30	0.257	0.525	0.792	1.059	1.327	1.594	1.861	2.129	2.396	2.663	2.931	3.198	3.465
1.40	0.262	0.533	0.804	1.075	1.346	1.617	1.888	2.159	2.430	2.701	2.972	3.243	3.514
1.50	0.265	0.540	0.814	1.088	1.363	1.637	1.912	2.186	2.460	2.735	3.009	3.283	3.558
1.60	0.269	0.546	0.824	1.101	1.378	1.655	1.933	2.210	2.487	2.765	3.042	3.319	3.597
1.70	0.272	0.552	0.832	1.112	1.392	1.672	1.952	2.232	2.512	2.792	3.072	3.352	3.632
1.80	0.275	0.557	0.840	1.122	1.405	1.687	1.969	2.252	2.534	2.817	3.099	3.382	3.664
1.90	0.277	0.562	0.847	1.131	1.416	1.701	1.985	2.270	2.555	2.839	3.124	3.409	3.693
2.00	0.280	0.566	0.853	1.140	1.427	1.713	2.000	2.287	2.573	2.860	3.147	3.434	3.720
2.10	0.282	0.570	0.859	1.148	1.436	1.725	2.013	2.302	2.591	2.879	3.168	3.456	3.745
2.20	0.284	0.574	0.865	1.155	1.445	1.735	2.026	2.316	2.606	2.897	3.187	3.477	3.768
2.30	0.286	0.578	0.870	1.161	1.453	1.745	2.037	2.329	2.621	2.913	3.205	3.497	3.789
2.40	0.287	0.581	0.874	1.168	1.461	1.754	2.048	2.341	2.635	2.928	3.222	3.515	3.808
2.50	0.289	0.584	0.879	1.173	1.468	1.763	2.058	2.353	2.647	2.942	3.237	3.532	3.827

TABLE 10-4. ACCIDENT PREDICTION RATE USING ACCIDENT HISTORY (t=4)

	n														
h	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
0.00	0.000	0.042	0.083	0.125	0.167	0.208	0.250	0.292	0.333	0.375	0.417	0.458	0.500	0.542	0.583
0.01	0.008	0.056	0.105	0.153	0.202	0.250	0.298	0.347	0.395	0.444	0.492	0.540	0.589	0.637	0.685
0.02	0.016	0.070	0.125	0.180	0.234	0.289	0.344	0.398	0.453	0.508	0.563	0.617	0.672	0.727	0.781
0.03	0.023	0.083	0.144	0.205	0.265	0.326	0.386	0.447	0.508	0.568	0.629	0.689	0.750	0.811	0.871
0.04	0.029	0.096	0.162	0.228	0.294	0.360	0.426	0.493	0.559	0.625	0.691	0.757	0.824	0.890	0.956
0.05	0.036	0.107	0.179	0.250	0.321	0.393	0.464	0.536	0.607	0.679	0.750	0.821	0.893	0.964	1.036
0.06	0.042	0.118	0.194	0.271	0.347	0.424	0.500	0.576	0.653	0.729	0.806	0.882	0.958	1.035	1.111
0.07	0.047	0.128	0.209	0.291	0.372	0.453	0.534	0.615	0.696	0.777	0.858	0.939	1.020	1.101	1.182
0.08	0.053	0.138	0.224	0.309	0.395	0.480	0.566	0.651	0.737	0.822	0.908	0.993	1.079	1.164	1.250
0.09	0.058	0.147	0.237	0.327	0.417	0.506	0.596	0.686	0.776	0.865	0.955	1.045	1.135	1.224	1.314
0.10	0.062	0.156	0.250	0.344	0.438	0.531	0.625	0.719	0.812	0.906	1.000	1.094	1.188	1.281	1.375
0.20	0.100	0.225	0.350	0.475	0.600	0.725	0.850	0.975	1.100	1.225	1.350	1.475	1.600	1.725	1.850
0.30	0.125	0.271	0.417	0.563	0.708	0.854	1.000	1.146	1.292	1.437	1.583	1.729	1.875	2.021	2.167
0.40	0.143	0.304	0.464	0.625	0.786	0.946	1.107	1.268	1.429	1.589	1.750	1.911	2.071	2.232	2.393
0.50	0.156	0.328	0.500	0.672	0.844	1.016	1.188	1.359	1.531	1.703	1.875	2.047	2.219	2.391	2.563
0.60	0.167	0.347	0.528	0.708	0.889	1.069	1.250	1.431	1.611	1.792	1.972	2.153	2.333	2.514	2.694
0.70	0.175	0.363	0.550	0.738	0.925	1.113	1.300	1.488	1.675	1.863	2.050	2.238	2.425	2.613	2.800
0.80	0.182	0.375	0.568	0.761	0.955	1.148	1.341	1.534	1.727	1.920	2.114	2.307	2.500	2.693	2.886
0.90	0.188	0.385	0.583	0.781	0.979	1.177	1.375	1.573	1.771	1.969	2.167	2.365	2.563	2.760	2.958
1.00	0.192	0.394	0.596	0.798	1.000	1.202	1.404	1.606	1.808	2.010	2.212	2.413	2.615	2.817	3.019
1.10	0.196	0.402	0.607	0.813	1.018	1.223	1.429	1.634	1.839	2.045	2.250	2.455	2.661	2.866	3.071
1.20	0.200	0.408	0.617	0.825	1.033	1.242	1.450	1.658	1.867	2.075	2.283	2.492	2.700	2.908	3.117
1.30	0.203	0.414	0.625	0.836	1.047	1.258	1.469	1.680	1.891	2.102	2.313	2.523	2.734	2.945	3.156
1.40	0.206	0.419	0.632	0.846	1.059	1.272	1.485	1.699	1.912	2.125	2.338	2.551	2.765	2.978	3.191
1.50	0.208	0.424	0.639	0.854	1.069	1.285	1.500	1.715	1.931	2.146	2.361	2.576	2.792	3.007	3.222
1.60	0.211	0.428	0.645	0.862	1.079	1.296	1.513	1.730	1.947	2.164	2.382	2.599	2.816	3.033	3.250
1.70	0.213	0.431	0.650	0.869	1.088	1.306	1.525	1.744	1.962	2.181	2.400	2.619	2.837	3.056	3.275
1.80	0.214	0.435	0.655	0.875	1.095	1.315	1.536	1.756	1.976	2.196	2.417	2.637	2.857	3.077	3.298
1.90	0.216	0.437	0.659	0.881	1.102	1.324	1.545	1.767	1.989	2.210	2.432	2.653	2.875	3.097	3.318
2.00	0.217	0.440	0.663	0.886	1.109	1.332	1.554	1.777	2.000	2.223	2.446	2.668	2.891	3.114	3.337
2.10	0.219	0.443	0.667	0.891	1.115	1.339	1.562	1.786	2.010	2.234	2.458	2.682	2.906	3.130	3.354
2.20	0.220	0.445	0.670	0.895	1.120	1.345	1.570	1.795	2.020	2.245	2.470	2.695	2.920	3.145	3.370
2.30	0.221	0.447	0.673	0.899	1.125	1.351	1.577	1.803	2.029	2.255	2.481	2.707	2.933	3.159	3.385
2.40	0.222	0.449	0.676	0.903	1.130	1.356	1.583	1.810	2.037	2.264	2.491	2.718	2.944	3.171	3.398
2.50	0.223	0.451	0.679	0.906	1.134	1.362	1.589	1.817	2.045	2.272	2.500	2.728	2.955	3.183	3.411

11. DERIVATION OF THE FORM OF THE ACCIDENT HISTORY EQUATION

It is well to review here and to expand upon the discussion of the accident-history-based accident prediction formula.[1] The basic form for accident-history-based hazard indexes will, as in the Mengert report, be:

$$H = h \frac{t_o}{t + t_o} + \frac{n}{t} \frac{t}{t + t_o} \qquad \text{Equation 1}$$

where:

(a) h is the non-history (or basic) predicted accident rate as developed in the Mengert report or as improved in Part I of this report. The quantity h is a function of crossing characteristics (but not of accident history) and gives the expected accident frequency conditional on crossing characteristics as reported in the crossing inventory but not conditional on previous accident history at that crossing.

(b) t is the number of years of observed accident history at the given crossing.

(c) n is the observed number of accidents in the period of t years which constitutes accident history.

(d) t_0 is a parameter, based only on crossing characteristics which characterize the weight to be given to accident

[1] Mengert, Op. Cit., pp. H-1 through H-12.

history versus the unconditional (or basic) accident prediction index, h.

(e) H is the accident-history-based accident prediction rate and like h is expressed in expected accidents per year.

Note that H is a weighted average of h, the non-history accident prediction rate, and n/t, the observed accident rate in the history period. The weighting factors

$$\frac{t_o}{t + t_o} \text{ and } \frac{t}{t + t_o}$$

add to one. The key task in developing the accident history formula is in estimating t_o, i.e., identifying and calibrating a model for t_o in terms of crossing characteristics. Before discussing the development of a model for t_o, the derivation of Equation 1 will be given.

The derivation hypothesizes that each crossing, i, has an unknown but constant accident rate ϕ_i. That means that if one could know ϕ_i then one would find that the mean number of accidents per year in the long run (over many years) would be ϕ_i. The probability of n accidents in a given year would be given by the Poisson formula:

$$P(n) = \phi^n \frac{e^{-n\phi}}{n!} \qquad \text{Equation 2}$$

and the probability of n accidents in t years would be given by

$$P(n|t) = \frac{(\phi t)^n}{n!} e^{-\phi t} \qquad \text{Equation 3}$$

These hypotentical Poisson distributions are due to the assumption of time homogeneity and known specific accident rate ϕ. In reality the specific accident rate ϕ cannot be determined from the known crossing characteristics but must itself be assumed to have random value according to a probability distribution determined by the crossing characteristics. For reasons of mathematical simplicity in addition to the assurances of accepted practice, it will be assumed that ϕ has a gamma distribution:

$$Pr(\phi<x) = \frac{b^a}{\Gamma(a)} \int_0^x \lambda^{a-1} e^{-\lambda b} d\lambda \qquad \text{Equation 4}$$

The gamma distribution is useful for describing the probability distribution of an inherently positive quantity with a unimodal distribution and with independently specifiable mean ($\mu = a/b$) and variance ($\sigma^2 = \frac{a}{b^2}$). Since analysis of ordinary accident prediction rates (non-accident history) neglects the distribution of ϕ altogether, it is not over simplified to postulate that ϕ has such a two-parameter distribution. If such a distribution is postulated for ϕ, it follows (Appendix A) from the laws of probability and from Equation 3 that the probability distribution for the number of accidents at a crossing for which a and b are known but ϕ is not known follows the negative binomial formula:

$$P(n) = \binom{a+n-1}{a-1} \left(\frac{b}{1+b}\right)^a \left(\frac{1}{1+b}\right)^n \qquad \text{Equation 5}$$

It also follows from ordinary Bayesian analysis that the distribution of ϕ, given that n accidents are observed over a period of t years as well as given the crossing characteristics through the parameters a and b, is given by the gamma distribution:

$$Pr(\phi<x|n,T) = \frac{\beta^{-\alpha}}{\Gamma(\alpha)} \int_0^x \lambda^{\alpha-1} e^{-\lambda\beta} d\lambda \qquad \text{Equation 6}$$

where $\alpha = a+n$ and $\beta = b+t$. (This is derived in Appendix A.) Now the accident prediction index, H, is by definition the expected value of ϕ given n accidents in t years i.e., $H = E(\phi|n)$. Using the conditional distribution specified in Equation 6 for ϕ, one obtains for H:

$$H = \frac{a + N}{b + t} \qquad \text{Equation 7}$$

This is because the mean of the gamma distribution with parameters a, b is a/b.

What remains is to determine a and b as functions of crossing characteristics. Note that if T=0 then n=0 of necessity (since there must be zero accidents in zero time) and that H becomes a/b in that case. But that case is just the unconditional expected accident frequency (conditional or no accidents over no time) which is just h. This leads to the equation

$$a/b = h \qquad \text{Equation 8}$$

We also change notation to $b=t_o$. This change is because a and b are standard notation for the parameters of the gamma and negative binomial distribution while t_o is a convenient notation to use.

Substituting $h = a/b$ and $t_o = b$ into Equation 7 the result comes back to Equation 1, namely:

$$H = \frac{a + n}{b + t}$$

$$= \frac{a}{b}\,\frac{b}{b + t} + \frac{n}{t}\,\frac{t}{b + t}$$

$$= h\,\frac{t_o}{t + t_o} + \frac{n}{t}\,\frac{t}{t + t_o}$$

Having derived Equation 1 it should not be noticed that t_o like h is a function of crossing characteristics. It would be sensible to make t_o an independent function of crossing characteristics, but a reasonable compromise would be to make t_o a function of h only. The decision made in the present project was to calibrate t_o as a function of h only. This decision was made primarily for simplicity so that H could be constructed from h once h had been determined. Some preliminary investigation had suggested that the performance of H as an accident prediction index was not strongly dependent on the values of t_o. Preliminary results suggested that the simplest possible model for t_o should be chosen. The simplest model would be t_o = constant but significantly better performance can be obtained making t_o a function of h.[1] Preliminary investigation suggested that t_o decreases as h increases.[2] Furthermore, it is reasonable to require that t_o have a maximum value, since otherwise crossings with extremely low values of h would result in very large values of t_o, which in turn would have no contribution to H from the term in n/t, the observed accident frequency. As a result, the form assumed for t_o was

$$t_o^{-1} = c + dh$$

[1]Mengert, Op. Cit., pp. H-1 through H-12.

[2]Ibid.

which is the simplest model consistent with:

1. t_o depends only on h
2. t_o is not constant
3. t_o^{-1} increases as h increases
4. t_o does not go to infinity as h goes to zero.

An equivalent form is:

$$t_o = \frac{r}{s + h}$$

The key task that remains is to determine the best values to use for r and s. Before doing that it is well to explain why t_o is expected to decrease as h increases. In the Mengert report this tendency had already been observed in the limited experimentation there.

Recall that the distribution of ϕ (the unknown specific hazard of a crossing) is such that its variance is equal to its mean divided by t_o, (μ=h, $\sigma^2 = \frac{h}{t_o}$) i.e.:

$$t_o^{-1} = \frac{\sigma_\phi^2}{\mu_\phi}$$

Thus, explaining why t_o^{-1} increases with increasing h is equivalent to explaining why σ_ϕ^2 increases facter than linearly with h.

An informal argument goes something as follows: Suppose ϕ is determined by two factors, one factor depending on recorded crossing characteristics (say, x) and the other factor depending on unrecorded or unknown crossing characteristics (say, y) and that ϕ = xy. Also suppose these factors are independent. Then,

$$\mu_\phi = x\mu_y = h$$

where x is the fixed factor for recorded crossing characteristics and μ_y is the mean of y which represents the unknown characteristics. Further,

$$\sigma_\phi^2 = x^2 \sigma_y^2$$

and

$$\frac{\sigma_\phi^2}{\mu_\phi} = \frac{x \sigma_y^2}{\mu_y} = h \left(\frac{\sigma_y}{\mu_y}\right)^2$$

so that σ^2/μ_ϕ increases linearly in h, the factor $\sigma_y^2/(\mu_y)^2$ being independent of recorded crossing characteristics. The idea is that σ^2 is roughly proportional to the square of the unconditional accident prediction rate. Since the unconditional accident prediction rate is primarily a function of exposure, the idea is that if the exposure is high the potential variability in ϕ is proportionately high since there are many potential accidents and whether or not they actually occur is a function of unknown crossing characteristics

If the variability in ϕ is proportional roughly to h, then σ_ϕ is proportional roughly to h and σ_ϕ^2 is proportional roughly to h^2 leading to t_o^{-1} being proportional roughly to h. In addition, we wish t_o^{-1} not to equal zero when h equals 0 or, with the same effect, that t_o^{-1} be less rapidly increasing than proportionally to h; and so the form $t_o^{-1} = c+df$ is chosen. It is, of course, necessary that c turn out to be positive when the model is calibraded for this form to be retained. That has been found to be the case, so the simple form $t_o^{-1} = c+dh$ (or equivalently $t_o = \frac{r}{s+h}$) is the form developed and discussed in this report.

The determination of the proper values of r and s in $t_o = \frac{r}{s+h}$ is now the key task in completing the development of the accident prediction formula.

It was noted in the Mengert report that a good way to calibrate a model for t_o is to use a subsample of only crossings which have had at least one accident. That is a good idea from the point of view of efficient use of the data. But since the writing of that analysis, several years of accident history have become available. As a consequence, it was decided that the best use of time and resources would be a detailed examination of the performance of alternative models on several years of actual accident experience. Using power factors and prediction factors calculated at 0.5 percent intervals, the performance of Equation 1 with $t_o = \frac{r}{s+h}$ and with various values of r and s was determined using various years as "history" years, i.e., as years to determine n and various other years as "test" years, i.e., on which to determine power factors and prediction factors. The results of these experiments are described in Section 8. The power factor and prediction factor measures, their calculation and presentation in Tables are discussed in Section 9.

12. CONCLUSIONS

The modification of the basic accident prediction formula to include accident history as an explicit factor produces a formula with much higher performance. For example, using three years of accident history, the 1000 crossings from the national inventory that have the highest predicted accident rate according to the modified formula will have 75 percent more accidents than the highest 1000 crossings identified by the non-accident history formula. This is equivalent to saying that the power factor is 75 percent higher for these crossings.

The performance of the accident history formula increases with the number of years of accident history (t). The amount of increase is greatest for t=1 with the incremental increase diminishing as t increases. The increase in permanence is higher for crossings with higher values of the predicted accident frequency.

The form of the accident history formula is a weighted sum of the basic predicted accident rate (without accident history) and the observed accident rate. The weights are not constant but are a function of the basic predicted accident rate. The weights are the same function for all three groups of crossings (passive, flashing lights, and gates).

PART III - COMPARATIVE PERFORMANCE OF THREE RAIL-HIGHWAY CROSSING HAZARD MODELS

13. INTRODUCTION

Two of the most widely used rail-highway crossing hazard models in use today are the New Hampshire (NH) model[1] and the Peabody-Dimmick (PD) model.[2] Other models use variations of the NH and PD models. With the expected wide use of the recently developed DOT model[3], possibly supplanting the NH and PD models in many cases, it is important to determine as clearly and objectively as possible how the DOT model compares in performance to these models. This memorandum provides such a comparison.

14. APPROACH

The measures of performance in comparing the three models are the "power factor" and the "performance factor" which were developed at the Transportation Systems Center (TSC).[4]

If Y denotes the percent of accidents which occur within the X% most hazardous crossings and Z denotes the percent of total hazard index represented by the X% most hazardous crossings, then the X% power factor is Y/X and the X% prediction factor is Y/Z. These performance measures are for a given hazard model and for a particular set of accident data, such as a given year's accident file. Thus, these measures will vary for different models

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[1] Federal Highway Administration, "Railroad-Highway Grade Crossing Handbook," U.S. Department of Transportation, Washington, D.C., August 1978, FHWA-TS-78-214, p. 87.

[2] Peabody, L.E., and Dimmick, T.B., "Accident Hazard at Grade Crossings," Public Roads, Vol. 22, No. 6, August 1941, pp. 123-144.

[3] Mengert, P., Op. Cit., p. B-7.

[4] Mengert, P., Op. Cit.

and will exhibit statistical variation for different accident samples.

In this analysis, four years of accident data are used: 1975, 1976, 1977 and 1978. At this writing (May 1980) these are all the accident records available that are keyed to the DOT-AAR Rail-Highway Crossing Inventory. It is thought that a test involving all the accident data would be more meaningful than if only a single year's file is used. Therefore, the test used in this analysis is against the four-year accident average for each crossing.

The NH model and the PD model are selected for comparison because they are two widely used models today and they do not require any information not in the DOT-AAR inventory. The NH model[1] is

$$
\begin{aligned}
H &= KCT \text{ for Passive Crossings} \\
&= .6KCT \text{ for Flashing Light Crossings} \\
&= .1KCT \text{ for Gate Crossings,}
\end{aligned}
$$

where H is the expected number of accidents/year. The constant K is a normalizing factor determined from the total expected number of accidents for the time period of interest. Consistent with the assumption for the DOT model, "passive crossings" consist of crossings with warning device classes 1, 2, 3, and 4 in the

[1]Federal Highway Administration, Op. Cit., p. 87.

Inventory; "flashing light" crossings consist of crossings with warning device classes 5, 6, and 7; and "gate" crossings consist of crossings with warning device class 8.

The PD model[1] consists of the formula

$$5H = 1.28 \frac{C^{.17} T^{.151}}{A} + f\ (I_u).$$

The factor "5" is present because this model produces the expected number of accidents for five years. The parameter A is a "protection coefficient" which is related to the warning device class of the crossing. Peabody-Dimmick identified thirteen types of "protection" at a crossing which results in thirteen different values for A. No coefficient is specified by them when there is no warning device or sign at a crossing. The state of Kentucky uses the PD model for which they specify nine warning device classes, including "no protection." These nine warning device classes used by Kentucky and the associated "protection coefficients" were obtained by IOCS in their recent work with Kentucky. They are:

WARNING DEVICE CLASSES USED BY KENTUCKY	PROTECTION COEFFICIENTS
No protection	1.00
Crossbucks	1.65
Bells only	1.78
Stop signs	1.86
Wigwag with bell	2.03
Flashing lights	2.18
Flashing lights with bell	2.25
Flagman	2.52
Flashing lights with gates	2.70

[1]Peabody, L.E., and Dimmick, T.B., Op. Cit., p. 125.

Since the motivation for the present analysis is to evaluate the specific model that Kentucky uses -- with the thought that this is a typical user of the PD model -- it is necessary to determine from Kentucky's coefficients the values of A that should be assigned to the eight DOT-AAR warning device classes. These values of A used in the present analysis are:

DOT-AAR WARNING DEVICE CLASS	A
1	1.00
2	1.65
3	1.86
4	1.65
5	2.52
6	2.03
7	2.22
8	2.70

The parameter $f(I_u)$ is intended to be read from a published graph[1] of $f(I_u)$ versus I_u, where $I_u=1.28C^{.17}T^{.151}/A$. Unfortunately, this graph is only defined for values of I_u up to 5.0, whereas with today's volume of train and highway traffic, I_u exceeds 5.0 for many crossings. In fact, approximately 2,500 crossings in the national inventory have values of I_u that exceed 5.0. This means that the Peabody-Dimmick model is not defined for 2,500 crossings in the national inventory and hence a comparative evaluation is hampered by this deficiency. In particular, the prediction factors cannot be obtained in any meaningful way.

[1]Peabody, L.E., and Dimmick, T.B., Op. Cit.

However, for the determination of power factors, this problem can be avoided. Since the power factor is based on the relative ranking of crossings rather than on the absolute value of H, if the ranking of crossings based on I_u is the same as that based on H, then the power factors will be the same in both cases. That this is the case is seen in the following:

Let $Y = I_u + f(I_u)$. Then

$$\frac{dy}{dI_u} = 1 + \frac{df}{dI_u} .$$

Clearly, if dy/dI_u is greater than 0 for all values of I_u, then the ranking of crossings is the same whether I_u, Y or H is used. From the graph in the Peabody-Dimmick paper, it can easily be seen that df/dI_u is greater than -0.5. Therefore, dy/dI_u is greater than 1-.5 or .5, and the assertion is proved. Thus, it is not necessary to use $f(I_u)$ in determining the power factors and hence only I_u is used in the present analysis.

15. RESULTS

The following table contains the comparative results of the three models for all warning device classes combined:

		POWER FACTORS			PREDICTION FACTORS	
Percent of Crossings	Number of Crossings	DOT	New Hampshire	Peabody-Dimmick	DOT	New Hampshire
.5	1000	9.76	8.73	1.76	.79	.20
.9	2000	8.30	7.89	2.03	.83	.25
1.8	4000	6.97	6.73	2.97	.87	.31
2.7	6000	6.24	5.90	3.38	.90	.34
3.7	8000	5.73	5.41	3.54	.91	.37
4.6	10000	4.29	5.05	3.52	.92	.39
5.5	12000	4.99	4.79	3.53	.93	.42
6.4	14000	4.72	4.51	3.50	.93	.43
7.3	16000	4.50	4.29	3.46	.94	.45
8.2	18000	4.30	4.10	3.37	.94	.47
9.1	20000	4.11	3.94	3.31	.94	.48
10.0	22000	3.95	3.78	3.23	.95	.50
15.5	34000	3.27	3.17	2.89	.96	.58

The inventory used for these calculations was that which existed in May 1978.

These results show the superiority of the DOT model over the other two. For the NH model, the power factors are reasonably high but slightly lower than for the DOT model. The prediction factors are much worse, however. This means that the hazard indexes for the high hazard crossings are too high relative to the low hazard crossings. This could be explained by the fact that a simple exposure model (as the NH model is) shows no diminishing of hazard index for high exposure, as the DOT model does.

For the PD model, the power factors are significantly lower than for the other two models. This contrasts with the good power factor performance found for individual warning device classes.[1] It is not completely clear what the reason is for the poor performance for combined classes. The performance of a model of this functional form depends on the values of the exponents (which are 0.17 and 0.151 in this case) and on the value of the coefficients in the denominator. Note that the exponents are much lower than for the NH model, which is unity for both T and C. The reason for the low power factors, of course, is that crossings which have a low accident rate are given a high hazard index. Upon examining the crossings ranked by the PD model, there appears to be a concentration of Class 1 (no warning device or sign) crossings at the top. These, it turns out, have a low accident rate. The reason Class 1 crossings appear at the top probably is due to its low coefficient value of "one." In

[1] Mengert, p., Op. Cit.

addition, it must be remembered that the PD model was developed with the accident experience and crossing traffic that existed from 1932 to 1936, and hence it is not surprising that it performs poorly on today's crossings.

The preliminary conclusion to be drawn from these poor results of the PD model is that if the functional form of this model is to be used, a better selection of values of A must be made than those used by Kentucky.

APPENDIX A

To derive Equation 6, several definitions will first be introduced.

Let:

$$Pr(\phi<\lambda) = \int_0^\lambda \rho(x)dx \qquad \text{Equation A-1}$$

(this defines $\rho(x)$)

Thus:

$$\rho(\lambda) = \frac{b^a}{\Gamma(a)} \lambda^{a-1} e^{-b\lambda} \qquad \text{Equation A-2}$$

Let $Pr(\phi<\lambda,n)$ mean "the probability that $\phi<\lambda$ and that there will be exactly n accidents in the t years of observation."

Let $Pr(n|\phi=\lambda)$(or simply $Pr(n|\lambda)$) means "the probability that there will be exactly n accidents in t years given that $\phi = \lambda$."

Let $Pr(\phi<\lambda|n)$ mean "the probability that $\phi<\lambda$ given exactly n accidents in t years."

All the above is standard notation from conditional probability.

Let:

$$Pr(\phi<\lambda,n) = \int_0^\lambda \rho(x,n)dx \qquad \text{Equation A-3}$$

(this defines $\rho(\lambda,n)$)

$$Pr(\phi<\lambda|n) = \int_0^\lambda \rho(x|n)dx \qquad \text{Equation A-4}$$

(this defines $\rho(x|n)$)

The basic formula for conditional probabilities from which Bayesian analysis derives states in general:

$$Pr(A,B) = Pr(A|B)Pr(B)$$

$$= Pr(B|A)Pr(A) \qquad \text{Equation A-5}$$

(whatever events A and B may signify). In the present case:

$$Pr(\phi<\lambda,n) = Pr(\phi<\lambda|n)\ Pr(n)$$

$$= Pr(n|\phi<\lambda)\ Pr(\phi<\lambda) \qquad \text{Equation A-6}$$

The latter is equivalent to:

$$\rho(\lambda,n) = \rho(\lambda|n)\ Pr(n)$$

$$= Pr(n|\lambda)\ \rho(\lambda) \qquad \text{Equation A-7}$$

It is known from Equation 3 that

$$Pr(n|\lambda) = \frac{(\lambda t)^n\ e^{-\lambda t}}{n!} \qquad \text{Equation A-8}$$

and from Equation A-2 that

$$\rho(\lambda) = \frac{b^a}{\Gamma(a)}\ \lambda^{a-1}\ e^{-b\lambda}$$

Since

$$\int_0^\infty \rho(\lambda|n)d\lambda = 1$$

it follows that

$$Pr(n) = \int_0^\infty Pr(n|\lambda)\ \rho(\lambda)d\lambda$$

The rest of the derivation is straightforward mathematical manipulation. From Equation A-7

$$\rho(\lambda|n) = \frac{Pr(n|\lambda)\ \rho(\lambda)}{Pr(n)}$$

$$= \frac{(\lambda t)^n\ e^{-\lambda t}}{n!}\ \frac{b^a}{\Gamma(a)}\ \lambda^{a-1}\ e^{-b\lambda}\ Pr(n)$$

$$= \lambda^{a+n-1}\ e^{-(b+t)\lambda}c$$

where c does not depend on λ.

This shows that $\rho(\lambda|n)$ is like $\rho(\lambda)$ except that a is replaced by a+n and b is replaced by b+t. This shows that the distribution of ϕ conditioned on n accidents in t years is the same as the distribution of ϕ with no accident history information except that a and b are transformed as stated.