Proceedings of the 2008 ASME International Mechanical Engineering Congress & Exposition
IMECE08
Oct 31-Nov 6, 2008, Boston, Massachusetts, USA

IMECE2008-66430

A KINEMATIC/KINETIC HYBRID AIRPLANE SIMULATOR MODEL

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ABSTRACT

A kinematics-based flight model, for normal flight regimes, currently uses precise flight data to achieve a high level of aircraft realism. However, it was desired to further increase the model's accuracy, without a substantial increase in program complexity, by determining the vertical velocity and vertical acceleration using EUROCONTROL's Base of Aircraft DAta (BADA) model [1]. BADA is a well-known aircraft performance database model maintained and developed by EUROCONTROL Experimental Centre in France.

The hybrid model uses the BADA algorithm to determine the vertical velocity and gives original results for determining the vertical acceleration. The approximate accuracy of these vertical parameters was checked by comparing them with pre-existing test distributions [2] and an in-house flight simulator application. The hybrid model uses kinematic algorithms for all other functions and parameters. To obtain specific results, C code was written to access text data from BADA's collection of approximately one hundred airplanes. Accessing this database causes an increase in overall program execution time that was deemed acceptable due to the infrequency of changing plane types. Also, by examining many airplane trajectories obtained from different BADA airplanes, we determined that the model is accurate enough to uniquely represent many different types of aircraft.

INTRODUCTION

Complex aircraft flight simulations that account for many forces and that solve associated dynamical equations provide an accurate model of aircraft motion. Since our aircraft only fly in normal flight regimes (i.e., within the flight envelope, non-emergency, standard operation, and non-turbulent conditions) and exhibit smooth trajectories, a much less complex approach will still obtain high accuracy. The model proposed here applies to these simplified conditions, is primarily kinematics based, and accurately describes the airplane's smooth center of mass movement. In addition, the model does not require detailed proprietary aircraft manufacturer's data, is mathematically concise, and hence provides for rapid development and execution.

As a result, this kinematic model is particularly well suited for use in a study where there is no turbulence, where fast computation is important, and where code adaptability is advantageous. It is an especially good fit for Monte Carlo studies, because it is the simplest realistic mathematical flight model and, hence, executes much faster than more complicated models. Because the kinematic model is much simpler than other models, the code can be altered more quickly to adjust for various external effects, such as changes in environmental factors like wind.

The kinematic model, however, does have the drawback that most parameters must be input by hand and can't be derived physically. Many of these parameter values, such as vertical acceleration, are difficult to accurately acquire. The BADA model helps to determine the key parameters of vertical velocity and vertical acceleration during climb, cruise, and descent. The BADA model provides the equation for the vertical velocity and its application to a few major flight conditions. However, the vertical velocity under all flight conditions will be determined by additional computations. In addition, the BADA model does not specify the vertical acceleration, but this will be derived mathematically for all flight conditions. In addition, the vertical velocity and acceleration are determined for the regions below and above the tropopause (i.e., in the troposphere and the stratosphere).

With the vertical velocity and vertical acceleration specified for each airplane type the entire trajectories are uniquely specified. This is critical for our study because it is necessary to determine when aircraft are within close proximity to each other.

KINEMATIC MODEL BACKGROUND

High-fidelity flight models are frequently desired for simulation to achieve the highest level of performance. However, due to a lack of detailed proprietary data for many types of aircraft and a primary concern with normal flight regimes and in order to accelerate development, a more mathematically concise and computationally efficient aircraft model has been developed [3]. The model is point-based and describes the movement of the aircraft's center of mass. This mostly kinematics-based model currently uses detailed maneuvering, takeoff, climb, and approach collected data and provides an acceptable level of fidelity, especially in normal flight regimes.

BADA MODEL BACKGROUND

BADA is an aircraft performance database and model. It is maintained and developed by EUROCONTROL Experimental Centre. The BADA database specifies a comprehensive set of parameters, for nearly 100 well known aircraft, that describes the drag, thrust, and physical extent. Additional variable values required by the model must be provided by the user. Using an energy balance for the airplane, the BADA model can be adapted to accurately determine the vertical velocity during climb, cruise, and descent. By extending the BADA model, the vertical acceleration can be determined. Since the vertical velocity and the vertical acceleration still depend on the indicated airspeed (IAS), the model resulting from merging BADA with the kinematic model is still primarily kinematic.

AIRCRAFT STATE

Because the kinematic equations are called separately at each time step a number of independent inputs must be determined to ensure the proper position is calculated. The inputs include items such as the aircraft's state, its commanded state, and simulator clock time.

The aircraft's state vector includes the following: current position (latitude, longitude, and altitude), magnetic heading, IAS, bank angle, roll rate, vertical velocity, and vertical acceleration. The vertical velocity and vertical acceleration need to be specified only when the model is operating without using BADA input. When incorporating the BADA model they are no longer independent and are mostly functions of altitude, flight phase, the physical atmospheric model, and the functional relation between true air speed (TAS) and IAS. Note that the IAS is considered constant for certain phases of flight (e.g., approach) and hence the state vector does not include a linear acceleration term that changes the IAS.

The commanded (as opposed to current) aircraft state must also be passed to the kinematic model during each run. These parameters are commanded by the pilot and can include the bank angle, heading, and vertical velocity.

Vertical speeds are calculated from either the kinematic equations or from the BADA model. For example, the initial vertical speed in a stochastic run is calculated from the kinematic equations such that the airplane, starting at a certain height and distance, will land exactly on the runway threshold. In this case, the airplane is not vertically turning, i.e., its vertical velocity is commanded constant, and the vertical velocity and vertical acceleration are uncoupled. This vertical velocity is different from that calculated using the BADA model. For general flying while using the BADA model and the kinematic model the vertical velocity and the vertical acceleration are coupled. And in the case of the BADA model the vertical acceleration is the first-order time-derivative of the vertical velocity. Thus, the kinematic and BADA models are used independently to determine the vertical velocity and the vertical acceleration such that the kinematic model is used for initial approaches and for commanded vertical turning and the BADA model is used for all other flight phases.

The various aircraft properties (IAS, bank angle, roll rate, thrust, drag, mass, etc.) ensure that different aircraft move differently through spacetime given the same initial conditions. Through the provided aircraft data, and the BADA database, different aircraft will have different trajectories based upon their predefined properties. Although the actual equations of motion are essentially generic, the differences in these parameters provide for a dynamic aircraft simulator.

Initial values are generated to launch the aircraft model and the required initial values include latitude, longitude, altitude, bank angle (typically zero initially), IAS, vertical velocity, and heading. To incorporate BADA, users must specify the flight phase: climbing, cruising, descending, approaching, and landing.

COORDINATE SYSTEM

The World Geodetic System 1984 (WGS-84) [4] is used to describe the aircraft's position. That is, the two-dimensional surface of an ellipsoid is parameterized by the geodetic latitude and longitude angles. Height, or altitude, is described by the usual convention perpendicular to earth's surface and is reported in feet above mean sea level. These coordinates were partly chosen to utilize real-world positions because airplane data typically includes reference to latitude and longitude. In addition, it may be desirable to travel long distances between landings and latitude and longitude are particularly well-suited coordinates for this.

The kinematic model velocities are independently specified for motion parallel to the geodetic radius direction and motion parallel to earth's surface. For example, IAS applies tangentially to earth's surface and the vertical velocity is defined perpendicular to earth's surface. Thus the motion of a descending airplane, for example, is described by a one-dimensional path of decreasing altitude and a two-dimensional path along earth's surface with changing latitude and longitude. The two paths are independently specified but are always coupled, in that they are plotted simultaneously.

Since almost all aviation data is given using magnetic headings, the aircraft model also processes its data in this format. However, cartography is generally performed using true north. Therefore, since the latitude and longitude grid is true by construction, all magnetic information is converted to true using the known local magnetic variation just prior to calculation of the aircraft's updated latitude and longitude. This must be calculated at each time step.

Selection of a time step is made sufficiently small to produce accurate latitude and longitude. This time step Δt is used to obtain the differential change in distance and, ultimately, the aircraft's subsequent position.

KINEMATIC ASSUMPTIONS

The kinematic model is a physical abstraction in that it does not explicitly incorporate much of the specific physical forces that act on a real aircraft and hence cause the plane to fly (say according to non-compressible fluid mechanics, for example). Indeed, in general the model functions without forces and since forces are believed necessary for accurate functioning then all the appropriate forces are assumed implicitly present. Many of the assumptions must be made because the simulator functions with only the supplied data. For example, aircraft turn data, in the form of bank angle and roll rate are the only turn data available, so the calculation of turns must be accomplished using this data only. Thus, assumptions are made to simplify the flight model; however, these can result in some limitations on the use of the kinematic model as a flight simulator.

Engine data, provided through BADA, is only obtained for determining vertical velocity and vertical acceleration in the climbing, cruising, and descending phases. This implies that for all other configurations thrust is assumed to implicitly change in order to produce the desired effect. Thus the kinematic method for changing the thrust is left to the pilot/autopilot "black box," but it is done in such a way to produce the desired result. For example, in simple kinematic motion the horizontal and vertical components of motion are decoupled. To achieve this, it has to be assumed that while turning but remaining level, adequate (implicit) thrust is maintained to keep the aircraft level. However, when incorporating the BADA model some of these assumptions can be removed due to the explicit introduction of thrust.

VERTICAL VELOCITY

We would like to use the BADA model to achieve accurate vertical velocities and accelerations. The following Eq.(1) represents the energy balance for the airplane, and as described by EUROCONTROL in their BADA manual it "equates the rate of work done by forces acting on the aircraft to the rate of increase in potential and kinetic energy":

$$(T - D)V_{TAS} = mg\frac{dh}{dt} + mV_{TAS}\frac{dV_{TAS}}{dt} \qquad (1)$$

T is engine thrust, D is airplane drag, V_{TAS} is the true air speed (TAS), m is the total airplane mass, g is the acceleration of the airplane toward earth caused by gravity, h is the perpendicular height above earth, and t is Galilean time measured in an earth centered inertial frame. By using the chain rule $\frac{d}{dt} = \frac{d}{dh}\frac{dh}{dt}$, we can now solve Eq.(1) for the vertical velocity:

$$V_v = \frac{dh}{dt} = \frac{(T - D)V_{TAS}}{mg}\, fmach \qquad (2)$$

where *fmach* is given by:

$$fmach = \left(1 + \frac{V_{TAS}}{g}\frac{dV_{TAS}}{dh}\right)^{-1} \qquad (3)$$

We now need to determine the various elements of the vertical velocity. The BADA model treats the thrust, T, as a polynomial in the height, h:

$$(T_{\max climb})_{ISA} = C_1\left(1 - \frac{h}{C_2} + C_3 h^2\right) \qquad (4)$$

The constants C_1, C_2, C_3 all have the appropriate units. The drag depends on the drag coefficient, C_D, in the following manner:

$$D = \frac{C_D \rho V_{TAS}^2 S}{2} \qquad (5)$$

S represents the airplane surface area and ρ is the density of air. The drag coefficient is itself dependent upon the coefficient of lift, C_L. See p.C17 in [1] for specific values. But V_{TAS} is computed differently depending on whether the airplane altitude is below or at the tropopause (tropospheric) or above the tropopause (stratospheric).

The approximate tropopause height, in meters, is determined from $h_{trop} = 11{,}000 + \frac{\Delta T_{ISA}}{0.0065}$, where ΔT_{ISA} is given by the difference (see Eq.(A-2)) between the actual ground temperature in Kelvins, T_0, and the international standard atmosphere (ISA) ground temperature, $(T_0)_{ISA}$.

For tropospheric flying the relationship between the TAS and the calibrated airspeed (CAS) is approximately:

$$V_{TAS} = \frac{V_{CAS}}{\sqrt{\left(1 - \frac{Bh}{T_0}\right)^{4.25864}}} \tag{6}$$

where B is the lapse rate 0.0065 K/m and T_0 is the ground temperature in Kelvins. We make the approximation, based on our previous kinematic model work, that V_{CAS} is equal to V_{IAS}. We are almost in a position to calculate V_v for the atmosphere since we now only need $\frac{dV_{TAS}}{dh}$. This is a straightforward calculation because V_{TAS} is an explicit function of h. The result is given by Eq.(A-3). We now have all the information needed to calculate V_v for the troposphere by substituting Eqs.(4), (5), (6), and (A-3) into Eq.(2). For stratospheric flying, we have a different expression for V_{TAS} given by Eq.(A-4) where P is the pressure, P_{ISA} is the ISA pressure, ρ is the density, ρ_{ISA} is the ISA density, and $\mu = (\gamma-1)/\gamma$ using the isentropic expansion coefficient, γ, for air. For the stratosphere, the BADA model assumes exponential behavior for pressure, $P = P_{trop} e^{\frac{-g}{RT_{trop}}(h - h_{trop})}$, and also for density, $\rho = \rho_{trop} e^{\frac{-g}{RT_{trop}}(h - h_{trop})}$. R is the universal gas constant, and P_{trop} and ρ_{trop} are constant values. Since $\frac{P}{\rho} = \frac{P_{trop}}{\rho_{trop}}$ and $\frac{\rho_{ISA}}{P_{ISA}}$ are constant then the only dependence on h comes from the factor $\frac{P_{ISA}}{P} \equiv Ae^{bh}$ where A and b are the constants $A = \frac{\rho_{ISA} RT_{ISA}}{P_{trop} e^{\frac{gh_{trop}}{RT_{trop}}}}$ and $b = \frac{g}{RT_{trop}}$. If, in addition we further simplify by defining the following two constants, $a_1 = \frac{2}{\mu}\frac{P}{\rho} = \frac{2}{\mu}\frac{P_{trop}}{\rho_{trop}}$ and $b_1 = \left[\left(1 + \frac{\mu}{2}\frac{\rho_{ISA}}{P_{ISA}} V_{IAS}^2\right)^{\frac{1}{\mu}} - 1\right]$ then V_{TAS}. can be written more compactly as:

$$V_{TAS} = \sqrt{a_1}\left\{\left(1 + b_1 A e^{bh}\right)^{\mu} - 1\right\}^{\frac{1}{2}} \tag{7}$$

Now, since V_{TAS} is only an explicit function of the altitude, h, we can easily find that $\frac{dV_{TAS}}{dh}$ is given by Eq.(A-5). We now have all the necessary components to calculate V_v in the stratosphere by substituting Eqs.(4), (5), (7), and (A-5) into Eq.(2).

VERTICAL ACCELERATION

To determine the vertical acceleration we take the first-order time-derivative of the vertical velocity defined by Eq.(2). By differentiating the vertical velocity, and applying the chain rule, $\frac{d}{dt} = \frac{d}{dh}\frac{dh}{dt}$, to Eq.(2), the following equation for the vertical acceleration results:

$$a_v = \frac{d^2 h}{dt^2} = I - J \tag{8}$$

where I and J are defined by:

$$I = \frac{V_v}{mg}\left(\frac{dT}{dh}V_{TAS} + (T - D)\frac{dV_{TAS}}{dh}\right) fmach \tag{9}$$

$$J = \frac{V_v}{mg}(T - D)\frac{V_{TAS}}{g}\left(\left(\frac{dV_{TAS}}{dh}\right)^2 + V_{TAS}\frac{d^2 V_{TAS}}{dh^2}\right) fmach^2 \tag{10}$$

The full vertical acceleration equation, without separation into two terms is given by Eq.(A-6) for the reader's convenience.

From Eq.(8) we now see that it is possible to calculate a_v once $\frac{d^2 V_{TAS}}{dh^2}$ and $\frac{dT}{dh}$ are found. The thrust is defined by Eq.(4) and its first-order derivative with respect to h can easily be found. Since we know V_{TAS} as a function of h for both tropospheric and stratospheric conditions, it is straightforward, if sometimes lengthy, to find its second-order derivative with respect to *h*. This result for the troposphere, however, is easily given by Eq.(A-7). The result for the second-order derivative of V_{TAS} with respect to *h* in the stratosphere is given by Eqs.(A-8), (A-9), and (A-10). This is seen to have a complex dependence on *h*.

The vertical acceleration, a_v, in the troposphere can now be found by substituting Eqs.(4), (5), (6), (A-3), and (A-7) and the result for the tropospheric V_v into Eq.(8). Similarly, the vertical acceleration in the stratosphere can now be found by substituting Eqs.(4), (5), (7), (A-5), and (A-8) and the result for the stratospheric V_v into Eq.(8).

INTEGRATING BADA INTO THE KINEMATIC MODEL

The previous mathematical work consisted of lengthy algebra and differentiation. However, we now have general equations for the vertical velocity and vertical acceleration. The BADA model uses approximations to Eq.(3) based on the Mach number. For example, *fmach* is set to one for a constant Mach number in the stratosphere. Our results do not need to specify any approximations and thus should be easier to adapt to given physical simulations with varying airplane altitudes or with non-constant Mach numbers.

There are two main areas in our kinematic model that concern vertical velocity and vertical acceleration. The first, airplane logic, contains expressions for the vertical velocity and the differential altitude. We determine the well-known relations between height, velocity, and acceleration by integrating a constant acceleration, a_v, twice with respect to time. Integrating once we find the vertical velocity as a function of time:

$$V(t) = \int a_v dt = a_v \int dt = a_v t + V_v \qquad (11)$$

where, V_v, is a constant of integration representing the initial vertical velocity for the time t = 0. Now integrating a second time we find $h(t) = \int (a_v t + V_v) dt = \frac{1}{2} a_v t^2 + V_v t + h_0$, where h_0 is the height at time t = 0. In our computations, however, we are interested in determining the change in altitude, $\Delta h = h(\Delta t) - h(0)$, due to the change in Δt, the time step, and given the vertical acceleration, a_v, and the vertical velocity, V_v:

$$\Delta h = \frac{1}{2} a_v (\Delta t)^2 + V_v \Delta t \qquad (12)$$

The second area of kinematic code concerning the vertical velocity and vertical acceleration is the vertical velocity turning code. The user initiates a vertical turn by commanding a vertical velocity. The turning code then determines if a vertical turn is desired and is sufficiently different in magnitude from the current vertical velocity. If the turn is large enough then a change in the vertical acceleration of correct sign is commanded. This new acceleration, now given by a_v in Eq.(11) generates a new *V(t)* via Eq.(11). In addition, the V_v variable in Eq.(12) is then updated to *V(t)*. In this manner the airplane's speed approaches the desired speed and the airplane's altitude adjusts accordingly. When the airplane speed, *V(t)*, is close to the desired target speed, the airplane begins to rollout from its vertical turn. This is achieved by altering the vertical acceleration sign and/or vertical acceleration magnitude.

The easiest way to merge the BADA generated vertical velocity and vertical acceleration with the corresponding kinematic generated values is to simply flip/flop them by turning off the commanded values while the user is interested in receiving the BADA values and turning off the BADA values when the user is interested in the kinematic commanded values. In this way the values for a_v and V_v can take on both BADA and kinematic values but only at different times. To achieve the BADA dominated flip/flop in our kinematic code, it was necessary to command the vertical velocity to be the BADA velocity. In this way the vertical velocity turning code is effectively disabled since the vertical velocity equals the commanded vertical velocity. For kinematic operation only, it is simple to disable BADA.

GRAPHICAL COMPARISON

To compare the BADA results for vertical velocity and vertical acceleration we use test data that model a passenger jet airplane. A Johnson SB probability density function (PDF), given by Eq.(A-1) as *f(x)*, models the vertical velocity and vertical acceleration using the parameter values found in Tab. 1.

TABLE 1. JOHNSON PARAMETERS

Johnson SB Parameter	Vertical Velocity PDF	Vertical Acceleration PDF
γ	2.592	0.225
δ	2.185	0.927
λ	4526.482	394.848
ε	1400.0	88.072
minimum	1442.8	123.6
maximum	3108.9	462.6

For modeling the vertical velocity in the climbing phase the x variate represents the vertical velocity as shown in Fig.1. To model the vertical acceleration during take-off phase the x variate now represents the vertical acceleration as shown in Fig.2. Our kinematic model currently only uses a constant IAS. Thus we chose 200 knots (103 m/s) for the approximate IAS of the airplane during all various phases of flight.

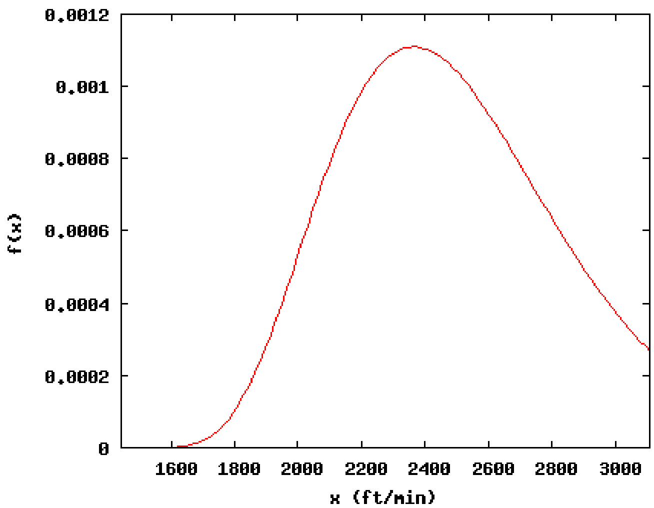

FIGURE 1: CLIMB VERTICAL VELOCITY PDF

BADA data files were provided through the John A. Volpe National Transportation Systems Center that contained the flight characteristics (thrust, drag, lift, etc.) of many different types of aircraft. A program was written to read the data files and input this information directly to the kinematic program. Then by incorporating the BADA model into the kinematic program code, data were obtained for the altitude, vertical velocity, and the vertical acceleration during the climbing, cruising, and descending phases for the passenger jet (see Fig. 3, 4, and 5).

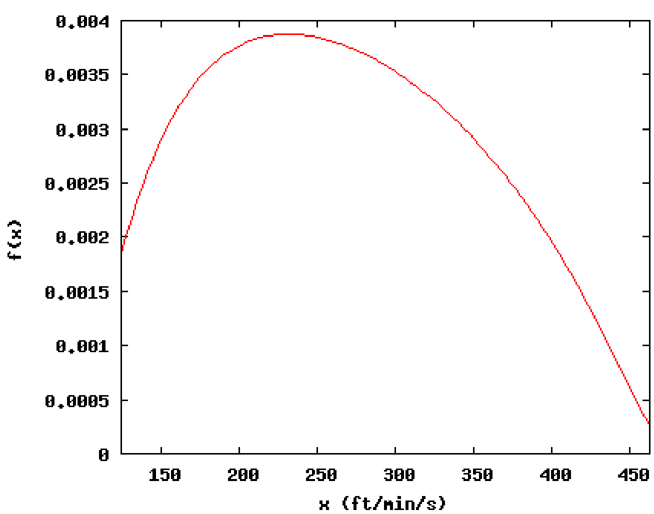

FIGURE 2: TAKE-OFF VERTICAL ACCELERATION PDF

For the climbing phase shown in Fig.4, the airplane achieves a vertical velocity of approximately 5000 ft/min (1524 m/min) after take-off phase and generally decreases gradually to 0 ft/min at reaching cruise phase or level flight. This gives an average of 2500 ft/min (762 m/min) that is close to the average shown in Fig.1. For the cruising phase the airplane has 0 ft/min as expected. For the descending phase the airplane starts with -3000 ft/min (-914 m/min) approximately at initial descent and increases to approximately 0 ft/min just prior to landing. For the climbing phase shown in Fig.5, the airplane initially decelerates near -5 ft/min/s (-1.5 m/min/s) after take-off phase, decelerates slightly more, and then generally increases to 0 ft/min/s upon reaching level flight. For cruising the airplane has 0 ft/min/s as expected. For the descending phase the aircraft accelerates slightly at about 12 ft/min/s (3.7 m/min/s) after initial descent and decreases to about 0 ft/min/s just prior to landing.

Thus we see that the test values for vertical velocity during the climb and the BADA values for the vertical velocity during climb are in reasonably good agreement. Also, the vertical velocity magnitude for the descent phase is found to be less than the climb phase as expected [5]. The test values for the vertical acceleration during take-off phase can be checked approximately using a rough calculation of the average vertical acceleration: $a_{avg} = \frac{V_2 - V_1}{t_2 - t_1}$. The first point represents the airplane the moment it leaves the runway and the second point represents the airplane when it begins its climbing phase. From our vertical velocity data we have approximately that $V_1 = 0$, $V_2 = 5000$ ft/min, $t_1 = -15$ s, and we take $t_2 = 0$ s giving 15 s as a typical acceleration time[1]. This yields $a_{avg} = 333$ ft/min/s (101 m/min/s) but varies slightly depending on the variation of t_2. This acceleration, a_{avg}, is in good agreement with the variation of take-off accelerations found in Fig.2. The BADA data, however, applies to the subsequent climb phase where the plane is very gently decelerating at near constant vertical velocity. Thus it will eventually level off at its cruise altitude. The BADA data also applies to the reverse situation where the plane is descending, and so it must very gently accelerate upwards to gradually reduce the vertical velocity magnitude. Our BADA acceleration data supports this scenario because of its small magnitude (climb and descent accelerations being an order of magnitude smaller than the take-off acceleration) and its direction.

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[1] Conversation with Dr. Yan Zhang, 2008, John A. Volpe National Transportation Systems Center, Cambridge, MA.

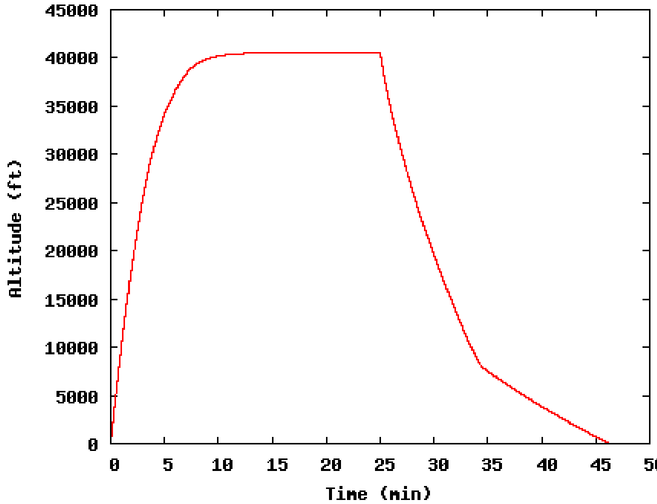

FIGURE 3: ALTITUDE VS. TIME

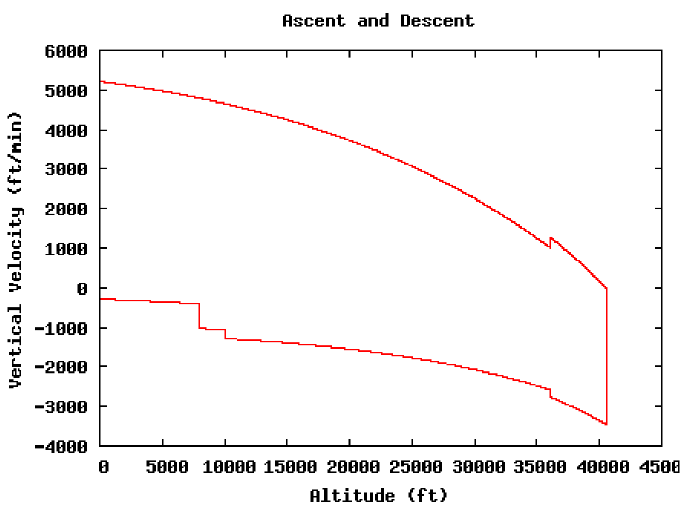

FIGURE 4: VERTICAL VELOCITY VS. ALTITUDE

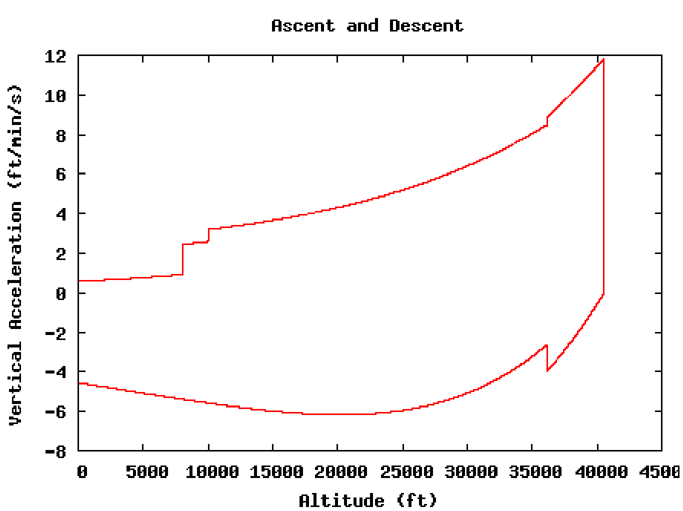

FIGURE 5: VERTICAL ACCELERATION VS. ALTITUDE

The BADA model apparently exhibits some abrupt changes in velocity and acceleration although the path in Fig.3 appears relatively smooth. Most of these anomalies are simply due to the BADA thrust algorithm for descents that changes abruptly at the altitudes of 10,000 ft (3048 m) and 8,000 ft (2438 m). However, there is also an abrupt velocity and acceleration change at the tropopause (here at 36,000 ft (10,973 m)) that is an artifact of using different atmospheric models below and above the tropopause. Since there is no visible indication of the tropopause in Fig.3, the effect on the airplane's trajectory can probably be considered negligible for most modeling purposes. The model provided by BADA for the TAS and atmosphere was tried for both above and below the tropopause and yielded the same results as the model described herein (which uses a simple standard atmospheric model below the tropopause and the model provided by BADA for the TAS and atmosphere above the tropopause). Thus, only applications needing to cross the tropopause and requiring exceptional accuracy may require methods for obtaining a smoother transition across the tropopause boundary. Of course, for flights below the tropopause this is not an issue.

In Fig.6 we show the spacetime plots for a few airplane trajectories. Each trajectory was obtained, from the kinematic program using BADA data, for a different type of large airline passenger jet. It is apparent that potentially significant differences exist depending on the flight phase. In particular, many airplanes obtain widely different ceilings and have different trajectories for the climbing phase and for the descent phase. In these situations, it is seen that the trajectories of two initially separated airplanes sometimes converge and that, the opposite effect, two initially nearby trajectories, sometimes diverge from each other. Both of these trajectory behaviors are exhibited in the descent phase.

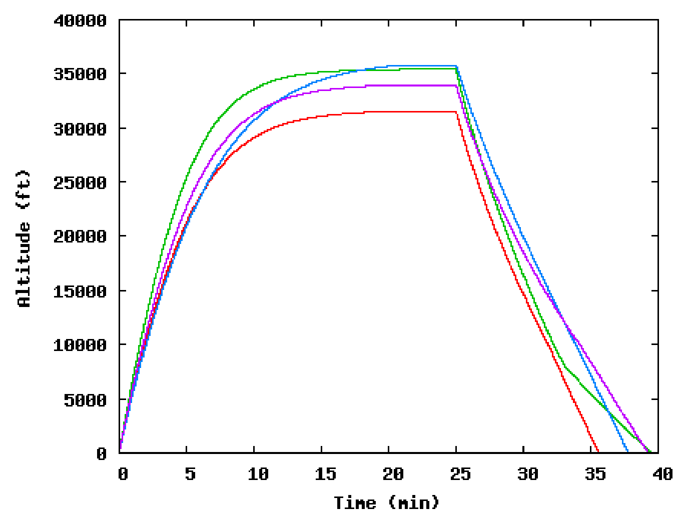

FIGURE 6: ALTITUDE VS. TIME

In our current results, airplanes only sometimes fly in close proximity to each other. It is possible that incorporation of the BADA data may cause closely separated airplanes to become more separated or cause more widely separated airplanes to become less separated. In either case the statistics will be affected depending on the fleet mix of airplanes. Thus, for our current study that models descents, we would expect more realistic results and potentially significantly different results by incorporating the BADA model.

CONCLUSION

The capability and accuracy of our kinematic model was significantly increased by the inclusion of the BADA model. And this was accomplished without a substantial increase in program complexity and with an acceptable program execution time.

The BADA model gives the user the capability to generate the general result for vertical velocity and specializes this to simplified computations for a few major flight conditions. Our results built upon and extended the BADA model and gave an explicit accurate derivation of the vertical velocity and vertical acceleration for general flight conditions. In addition, the vertical velocity and vertical acceleration were determined for both the troposphere and the stratosphere. The accuracy of the vertical velocity and vertical acceleration were approximately checked by comparing with test distributions, in-house simulator data, and technical report data.

Many airplane trajectories (such as those represented in Fig.6) were visually examined. These trajectories were obtained from the kinematic/kinetic program using the calculated vertical velocity Eq.(2), the vertical acceleration Eq.(8), and BADA data. The trajectory separations were generally different enough to uniquely represent many different types of aircraft for the climbing, cruising, and descent phases of flight. Although it is difficult to quantify, it is possible that these flight trajectories, having varying relative proximity, could have a significant effect upon Monte Carlo results.

ACKNOWLEDGMENTS

This paper is based on work supported under RITA/VNTSC #FA8G ED972. Any opinions, findings, and conclusions or recommendations expressed in this paper are those of the authors.

NOMENCLATURE

A – dimensionless constant
B – temperature lapse rate
D – drag force
E – part one of second-order derivative for V_{TAS}
F – part two of second-order derivative for V_{TAS}
I – part one of vertical acceleration
J – part two of vertical acceleration
P – local pressure at airplane
R – gas constant for air
S – surface area of airplane
V – vertical velocity as function of time
b – constant inverse length
fmach – dimensionless function of Mach number
g – gravitational acceleration
h – altitude of airplane
m – mass of airplane
t – time
Δh – change in altitude for a single step
Δt – change in time for a single step
γ – isentropic expansion coefficient
γ – Johnson parameter, dimensionless
δ – Johnson parameter, dimensionless
ε – Johnson parameter, dimension of variate
λ – Johnson parameter, dimension of variate
ρ – density of air
a_1 – constant velocity squared
a_{avg} – average vertical acceleration
a_v – vertical acceleration
b_1 – dimensionless constant
C_1 – thrust coefficient, force
C_2 – thrust coefficient, length
C_3 – thrust coefficient, inverse length squared
C_D – drag coefficient
C_L – lift coefficient
h_0 – initial height for each iteration
h_{trop} – height of troposphere
P_{ISA} – ISA pressure
P_{trop} – pressure at troposphere
T_0 – local ground temperature
$T_{max\ climb}$ – maximum thrust force
$(T_0)_{ISA}$ – ISA ground temperature
T_{trop} – temperature at troposphere
V_v – vertical velocity
V_1 – vertical velocity at time t_1
V_2 – vertical velocity at time t_2
V_{CAS} – calibrated airspeed
V_{TAS} – true airspeed
ΔT_{ISA} – ISA temperature difference
ρ_{ISA} – ISA air density

REFERENCES

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APPENDIX A

LIST OF EQUATIONS

Johnson SB Probability Density Function:

A-1: $f(x)=\frac{\delta\lambda}{(x-\varepsilon)(\lambda-x+\varepsilon)\sqrt{2\pi}}\exp\left\{-\frac{1}{2}\left[\gamma+\delta\ln\frac{x-\varepsilon}{\lambda-x+\varepsilon}\right]^2\right\}$

Ground/ISA Temperature

A-2: $T_0=(T_0)_{ISA}+\Delta T_{ISA}$

First-order derivative of tropospheric TAS:

A-3: $\frac{dV_{TAS}}{dh}=2.129\left(1-\frac{Bh}{T_0}\right)^{-3.129}\frac{BV_{IAS}}{T_0}$

TAS for Stratosphere:

A-4: $V_{TAS}=\sqrt{\frac{2}{\mu}\frac{P}{\rho}\left\{\left(1+\frac{P_{ISA}}{P}\left[\left(1+\frac{\mu}{2}\frac{\rho_{ISA}}{P_{ISA}}V_{CAS}^2\right)^{1/\mu}-1\right]\right)^{\mu}-1\right\}}$

First-order derivative of stratospheric TAS:

A-5: $\frac{dV_{TAS}}{dh}=\sqrt{a_1}\left[\frac{1}{2}\frac{\mu\left(1+b_1Ae^{bh}\right)^{\mu-1}b_1Abe^{bh}}{\sqrt{\left(1+b_1Ae^{bh}\right)^{\mu}-1}}\right]$

Vertical Acceleration:

A-6: $a_v=\frac{d^2h}{dt^2}=\frac{V_v}{mg}\left[\left(\frac{dT}{dh}V_{TAS}+(T-D)\frac{dV_{TAS}}{dh}\right)fmach-(T-D)\frac{V_{TAS}}{g}\left(\left(\frac{dV_{TAS}}{dh}\right)^2+V_{TAS}\frac{d^2V_{TAS}}{dh^2}\right)fmach^2\right]$

Second-order derivative of tropospheric TAS:

A-7: $\frac{d^2V_{TAS}}{dh^2}=6.662\left(1-\frac{Bh}{T_0}\right)^{-4.129}\left(\frac{B}{T_0}\right)^2V_{IAS}$

Second-order derivative of stratospheric TAS:

A-8: $\frac{d^2V_{TAS}}{dh^2}=E+F$

A-9: $E=\sqrt{a_1}\left[\frac{-1}{4}\left(\left(1+b_1Ae^{bh}\right)^{\mu}-1\right)^{\frac{-3}{2}}\left(\mu\left(1+b_1Ae^{bh}\right)^{\mu-1}b_1Abe^{bh}\right)^2\right]$

A-10: F = $\sqrt{a_1}\left[\frac{1}{2}\left(\left(1+b_1Ae^{bh}\right)^{\mu}-1\right)^{\frac{-1}{2}}\left\{\mu(\mu-1)\left(1+b_1Ae^{bh}\right)^{\mu-2}\left(b_1Abe^{bh}\right)^2+\mu\left(1+b_1Ae^{bh}\right)^{\mu-1}b_1Ab^2e^{bh}\right\}\right]$