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STOCHASTIC ANALYSIS OF
FUTURE VEHICLE POPULATIONS

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16. Abstract

The purpose of this study was to build a stochastic model of future vehicle populations. Such a model can be used to investigate the uncertainties inherent in Future Vehicle Populations.

The model, which is called the Future Automobile Population Stochastic Model (FAPS Model), consists of two major components:

(1) Model of New Car Sales. The model of new car sales is the model of automobile demand developed by Wharton Econometric Forecasting Associates, revised to incorporate the new vehicle survival model that was developed.

(2) A Procedure for Specifying Future Planned and Unplanned Events. This procedure, which specifies the Future Values of exogenous parameters of the model, incorporates the uncertainty of these parameters into the model.

A computer program of the FAPS Model was written and is documented in the report.

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PREFACE

This is a final report of a study entitled "Stochastic Analysis of Future Vehicle Populations," prepared for the Transportation Systems Center of the U.S. Department of Transportation. The purpose was to build a stochastic model to predict future vehicle populations by age and by type. That model is referred to in this report as the FAPS Model (Future Automobile Population Stochastic Model).

This study was accomplished through the cooperation of many individuals. Their contributions are gratefully acknowledged.

Professor Stephen M. Pollock of the Department of Industrial and Operations Engineering provided invaluable input by his review of the technical material and final report. A number of Highway Safety Research staff, including Kent B. Joscelyn, Barbara C. Richardson, and James Haney, provided assistance in preparation and review of this document. Finally, the authors wish to acknowledge the help of Charles Phillips, of TSC for his useful comments and suggestions.

METRIC CONVERSION FACTORS

Approximate Conversions to Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
		LENGTH		
in	inches	2.5	centimeters	cm
ft	feet	30	centimeters	cm
yd	yards	0.9	meters	m
mi	miles	1.6	kilometers	km
		AREA		
in²	square inches	6.5	square centimeters	cm²
ft²	square feet	0.09	square meters	m²
yd²	square yards	0.8	square meters	m²
mi²	square miles	2.6	square kilometers	km²
	acres	0.4	hectares	ha
		MASS (weight)		
oz	ounces	28	grams	g
lb	pounds	0.45	kilograms	kg
	short tons (2000 lb)	0.9	tonnes	t
		VOLUME		
tsp	teaspoons	5	milliliters	ml
Tbsp	tablespoons	15	milliliters	ml
fl oz	fluid ounces	30	milliliters	ml
c	cups	0.24	liters	l
pt	pints	0.47	liters	l
qt	quarts	0.95	liters	l
gal	gallons	3.8	liters	l
ft³	cubic feet	0.03	cubic meters	m³
yd³	cubic yards	0.76	cubic meters	m³
		TEMPERATURE (exact)		
°F	Fahrenheit temperature	5/9 (after subtracting 32)	Celsius temperature	°C

Approximate Conversions from Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
		LENGTH		
mm	millimeters	0.04	inches	in
cm	centimeters	0.4	inches	in
m	meters	3.3	feet	ft
m	meters	1.1	yards	yd
km	kilometers	0.6	miles	mi
		AREA		
cm²	square centimeters	0.16	square inches	in²
m²	square meters	1.2	square yards	yd²
km²	square kilometers	0.4	square miles	mi²
ha	hectares (10,000 m²)	2.5	acres	
		MASS (weight)		
g	grams	0.035	ounces	oz
kg	kilograms	2.2	pounds	lb
t	tonnes (1000 kg)	1.1	short tons	
		VOLUME		
ml	milliliters	0.03	fluid ounces	fl oz
l	liters	2.1	pints	pt
l	liters	1.06	quarts	qt
l	liters	0.26	gallons	gal
m³	cubic meters	35	cubic feet	ft³
m³	cubic meters	1.3	cubic yards	yd³
		TEMPERATURE (exact)		
°C	Celsius temperature	9/5 (then add 32)	Fahrenheit temperature	°F

°F -40   0   32   40   80   98.6   120   160   200   212 °F
°C -40   -20   0   20   37   40   60   80   100 °C

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Project Objective

The principal objectives of this study were to describe a method of investigating the uncertainties inherent in future vehicle populations and to build a new model of vehicle survival. To accomplish this, a stochastic model containing a new model of new vehicle survival was constructed for the forecasting of domestic vehicle populations. Such a model may be used to investigate the uncertainties in future populations created by, for example, the application of new strategies by private industry and government to promote new designs for safer, cleaner, and more fuel-efficient vehicles. The uncertain elements in these populations are the number of vehicles by age and type.

Summary of Approach

Uncertainty in forecasts of future vehicle populations are caused, in large part, by the uncertainties in the occurrence times and levels of impacts of future unplanned events, and by the uncertainties involved in describing the impacts of future planned and unplanned events. A planned event is a controllable event or strategy, such as the EPCA Standards. An unplanned event is inherently non-controllable. An oil embargo, a technological breakthrough, or a recession are examples of unplanned events.

To predict the distributions of future vehicle populations with respect to future planned and unplanned events, a stochastic model was constructed. This model, which is called the Future Automobile Populations Stochastic Model (FAPS Model), consists of three major components: a procedure for specifying future planned and unplanned events, a model of vehicle survival, and a model of new car sales.

The procedure for specifying future planned and unplanned

events outlined in this report describes a method for generating the future values and the probabilities of the occurrence of these values of the model's exogenous variables. The procedure is based on an interview process developed at the Stanford Research Institute. Such a procedure, which has not been applied in analyzing future vehicle populations, is an appropriate procedure for incorporating the uncertainties associated with future planned and unplanned events into the model.

The other components of the FAPS Model--the model of vehicle survival and the model of new car sales--contain the equations necessary to predict the distributions, by age and type, of future vehicle populations for given values of the exogenous variables. The model of vehicle survival predicts how long vehicles, by type, survive on the road. The model of vehicle survival developed in this study is a new model very much different from any existing models of vehicle survival. The model of new car sales predicts how many new cars, by type, will be placed annually on the road. The model of new car sales used in this study is the model of automobile demand developed by Wharton Econometric Forecasting Associates, revised to incorporate the new vehicle survival model.

Tests of the accuracy of the models of vehicle survival and new car sales to forecast were performed in this study. However, the procedure for specifying planned and unplanned events has not been tested. A computer program of the FAPS Model has been developed for future testing and application of the model.

1. INTRODUCTION

1.1 BACKGROUND AND SIGNIFICANCE

Since energy conservation has become a national goal, much concern has been focused on the fuel efficiency of automobiles. Automobiles, which are a major consumer of gasoline, account for nearly 35 percent of the petroleum consumed in this country. An important tool in analyzing future vehicle energy consumption is a model of the composition of future vehicle populations by age and type. Such a model must have two major components: (1) a new car sales forecaster that predicts the sales of cars by type; and (2) a vehicle survival model that predicts the scrappage of cars by age and type. Whereas adequate models of new car sales for forecasting purposes have been developed, the same is not true for vehicle survival models. It was thus a goal of this study to build a model of future vehicle populations which contains a comprehensive vehicle survival model.

Another problem with existing models of future vehicle populations is that they fail to predict the uncertainties inherent in their forecasts. In recent years, major unpredictable events have limited the utility of industry and government forecasts. For example, vehicle fuel supply estimates based on traditional models of future vehicle populations have been confounded by the oil embargo, the abnormally cold weather in the Winter of 1977, and (on the brighter side) the production of lighter and more fuel-efficient automobiles. Events and developments of this kind occur at uncertain times and their impacts are likewise uncertain. By explicitly taking into account these uncertainties in forecasts, it may be possible for industry and government planners to better assess the effects of policies and the risks associated with them. It was thus another goal of this study to construct a stochastic model of future vehicle populations which includes the uncertainties in its forecasts.

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1.2 OBJECTIVES OF THE STUDY

The principal objectives of this study were to describe a method of investigating the uncertainties inherent in future vehicle populations and to build a new model of vehicle survival. To accomplish this, a stochastic model containing a new vehicle survival model was constructed to provide distributions, by age and type, of future vehicle populations. This model predicts these distributions with respect to the impacts of two types of events: planned and unplanned events.

A planned event is a controllable event or strategy. Examples of such events are industry and government programs to promote new designs for safer, cleaner, more fuel-efficient vehicles.

An unplanned event is inherently non-controllable, one for which descriptions, occurrence times, and levels of impact are unpredictable. For example, such events may include oil (or other economic) embargoes, technological breakthroughs, and consumer reactions. Because these developments contain random components, it is desirable to have the distribution of their impacts incorporated in the model.

The specific objectives of this study were to:

- perform a literature survey;
- describe a procedure for specifying future planned and unplanned events;
- construct a forecasting model of vehicle survival;
- construct a model of future vehicle populations by age and type, using the new vehicle survival model, a revised model of new car sales, and the procedure for specifying future planned and unplanned events; and
- develop a computer program for using the model to produce forecasts of vehicle populations.

The stochastic model of future vehicle populations uses a revised version of the automobile demand model developed by Wharton Econometric Forecasting Associates (21) to forecast new car sales.

1.3 REPORT ORGANIZATION

The report has four major parts plus a series of appendices. The first part, Sections 1. and 2., introduce the stochastic model and discuss existing models of future vehicle populations. The next four sections, 3. through 6., describe the FAPS model. The third part of the report, Section 7., examines the predictive ability of the model. The last section concludes the report with a summary and recommendations.

Documentation for the computer program of the stochastic model is contained in Appendix A. Appendix B contains a comprehensive review of the literature, and Appendix C describes the Weibull distribution on which the model of vehicle survival developed in this report is based.

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2. REVIEW OF EXISTING MODELS

Since automobile manufacturing has become a major industry in the United States, many mathematical models of vehicle demand and survival have been developed. Over the years, these models have increased in sophistication. The earlier models attempted to explain the relationships between automobile demand and economic variables, particularly disposable income and new car purchase price. These early models assumed constant survival rates for each type of vehicle, and are usually simple one- or two-equation models.

The more recent models are, in general, very complicated. The predominant approach relies upon the stock-adjustment process within a dynamic econometric model. The fundamental assumption is that gross expenditure on a commodity (measured in units sold) can be calculated from the difference between desired stock and stock already available as a result of prior purchases, plus the need to replace old stock which has worn out. In this section, several of the more important models of vehicle demand and survival are briefly outlined. A more extensive discussion of these models is presented in Appendix B.

The earliest stock-adjustment model of vehicle demand was developed by Gregory Chow in the 1950s (4). This model assumes that: (1) desired total stock of vehicles, in dollars, is a linear function of disposable income and the average price of a vehicle; (2) new car purchases can be calculated as the sum of the desired change in stock and the depreciation of the old stock; and (3) within a one-year period, consumers generally do not adjust their stock of vehicles to the desired level but achieve only a fraction of the change to the equilibrium level.

From these assumptions, per capita car purchases are expressed as a linear function of disposable income, average car price, and the stock of vehicles on the road in the previous year.

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Realizing that the demand for automobiles may depend on more than price and income, several authors in the years following Chow's work have built stock-adjustment models which include additional variables. In 1961, Daniel Suits adjusted the average retail price of cars by the number of months' duration of an average credit contract (23). Hamburger included interest rates (13), and Saul Hymans (15) included male unemployment rate and a consumer sentiment index.

In recent years, modelers have included many more variables in their new car sales equations. Chase Econometrics included the new variables: gasoline prices, unemployment rates, an index of credit rationing, and dummy variables for auto strikes and cancellation and reinstatement of investment tax credits (3). These variables, plus annual vehicle miles travelled, are included in recent models by EEA (8), Sweeney (10), Faucett (16), and Wharton (21).

The development of vehicle survival models has been much less extensive. Models reported in the literature assume that vehicles face the same survival process regardless of type. Only a few of the models assume that the survival process changes over time-- namely, the models by Walker (24), Faucett (16), and Wharton (21). Walker fits the logarithm of the vehicle survival rates expressed as an odds to a function of vehicle age, rate of turnover in automobile ownership, and the level of used car prices to relative costs of representative car repair services. Faucett assumes that survival rates of the first eight years of a vehicle's life are constant from year to year at the average levels, based on fifteen years of vehicle survival data. For nine-year-old and older vehicles, an adjustment to the average rates is calculated based on current employment rates and average new car price. Wharton goes one step further than Faucett with an adjustment to the survival rates of all ages of vehicles, based on new car sales, scrap metal

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prices, vehicle miles travelled, and other variables. These models
are limited in that they attempt to involve only the economic
determinants of vehicle survival and not the physical characteris-
tics of vehicles. A goal of this study was to build a more
comprehensive vehicle survival model.

As mentioned, in the recent models vehicle demand and survi-
val have been used for forecasting purposes. Equations are included
in the models so that vehicle demand, fleet composition, and fleet
gasoline consumption can be examined. To produce forecasts,
modelers formulate scenarios that are put into the models to gener-
ate expected values of the future variables. No existing models
generate probability distributions of the forecast variables, nor
do they estimate variances of these estimates (except in the simple
one-equation models). Another goal of this study was to build a
model which will generate probability distributions of the forecast
variables.

3. STRUCTURE OF THE FUTURE AUTOMOBILE POPULATION STOCHASTIC MODEL (FAPS MODEL)

The FAPS Model developed in this study is made up of three major components:

1) Model of New Car Sales

2) Model of Vehicle Survival

3) Procedure for Specifying Future Planned and Unplanned Events

3.1 MODEL OF NEW CAR SALES

The first major component of the FAPS Model is a forecaster of new car sales. As mentioned in the introduction, the model of new car sales is a revised version of the Wharton Model of automoble demand. The Wharton Model is a very large econometric model designed to forecast the long-run size and composition of the U.S. automobile demand and stock. It is one of the more recent models of this type to have been constructed, and it has been recently used by several government agencies to evaluate proposed automobile-related policies.

The Wharton Model segments the vehicle population into five size categories. In addition, it distinguishes between domestic- and foreign-made cars. The five size categories are based on wheelbase and price as follows:

1) Subcompact--up to 100 inch wheelbase

2) Compact--100 to 110 inch wheelbase

3) Mid-size--110 to 118 inch wheelbase

4) Full-size--over 118 inch wheelbase

5) Luxury--all "high" priced cars.

An overview of the Wharton Model is presented in the next section.

The Wharton Model contains a vehicle survival model which assumes that the scrappage rates for a given age of vehicle are the

same regardless of the vehicle's size-class. Since a new and more comprehensive vehicle survival model was developed in this study, the Wharton Model was modified to remove all equations that are used to predict vehicle scrappage. The new model of vehicle survival was incorporated into the modified Wharton Model to generate scrappage forecasts. Details on how this was done are presented in Section 5.3.

3.2 MODEL OF VEHICLE SURVIVAL

The second major component of the FAPS Model is the new model of vehicle survival. This new model is very much different from existing vehicle survival models. It is based on the modified Weibull model, which has three parameters with the following survival distribution:

$$S(t) = pe^{-(a/b)t^b} + 1 - p$$

where:

$S(t)$ is the probability a vehicle survives to age t

a,b are Weibull parameters

p is the probability that a vehicle fails.

The modified Weibull distributions accurately model the failure process of automobiles. The modified Weibull Model also takes into account the fact that a certain proportion of vehicles is never scrapped. (These cars are probably preserved as "historical" vehicles.) A full explanation of the vehicle survival model is presented in Section 5.0.

3.3 PROCEDURE FOR SPECIFYING FUTURE PLANNED AND UNPLANNED EVENTS

The third major component of the FAPS Model is a procedure for specifying future planned and unplanned events. The specification procedure is a two-step process. The first step is to represent future events, planned or unplanned, in terms of the

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exogenous variables or parameters of the FAPS Model. The second step is to describe the unpredictable nature of unplanned events.

There are uncertainties associated with the specifications in both steps of the procedure. In the first step, there may be an uncertain relationship between an event and exogenous parameters of the model. For example, the new social security tax, which is a planned event, will have a major but uncertain influence on real disposable income as well as on many of the other exogenous parameters in the model. In the second step, unplanned events may have an uncertain impact, and the occurrence time of an unplanned event is certainly uncertain. The specification procedure describes the uncertainties in each of these events in terms of subjective probabilities--that is, probabilities derived in an interview process with an "expert" on the event in question.

The probabilities generated in the specification procedure are used to determine the probability distribution of the forecast values of the FAPS Model. This is achieved by assigning the probabilities associated with the exogenous parameters for each possible sequence of planned and unplanned events to the model forecast values which are generated by those exogenous parameters. In other words, forecasts by the model will be generated for each alternative future--that is, each sequence of future planned and unplanned events--and the probability that these forecasts will occur will be equal to the probability of the occurrence of the alternative future.

A full explanation of the specification procedure of future planned and unplanned events is presented in Section 6.0.

4. AN OVERVIEW OF THE WHARTON MODEL

The model of new car sales used in this study is a modified version of the model of automobile demand developed by Wharton E.F.A. (21). The modification to the Wharton Model entailed the removal of all its equations of vehicle scrappage. These equations were replaced by the vehicle survival model described in the next section. This section presents the general structure of the original Wharton Model. Emphasis is placed on how new car sales, total and by size-class, are predicted.

The Wharton Model contains five basic sets of statistically derived relationships:

- Desired stock
- Desired stock by size-class
- New registrations
- New registrations by size-class
- Scrappage

There are many other equations in the model. However, they may be regarded as subordinate to the five basic sets. The relationships among these equations are illustrated in a flowchart of the Wharton Model (Figure 1).

Desired shares (percentage of vehicle stock by size-class) and desired stock are critical concepts in the model. The desired stock or share may be interpreted to be a long-run "steady-state" level that would exist if all factors affecting automobile demand were held constant. New car sales and scrappage are determined by the "gap" between the desired stock and the actual, previous year-end stock. Similarly, the shares by size-class of new car sales are expressed as a function of the difference between the desired shares and actual shares. The mechanism linking desired levels to actual levels is called a stock-adjustment process.
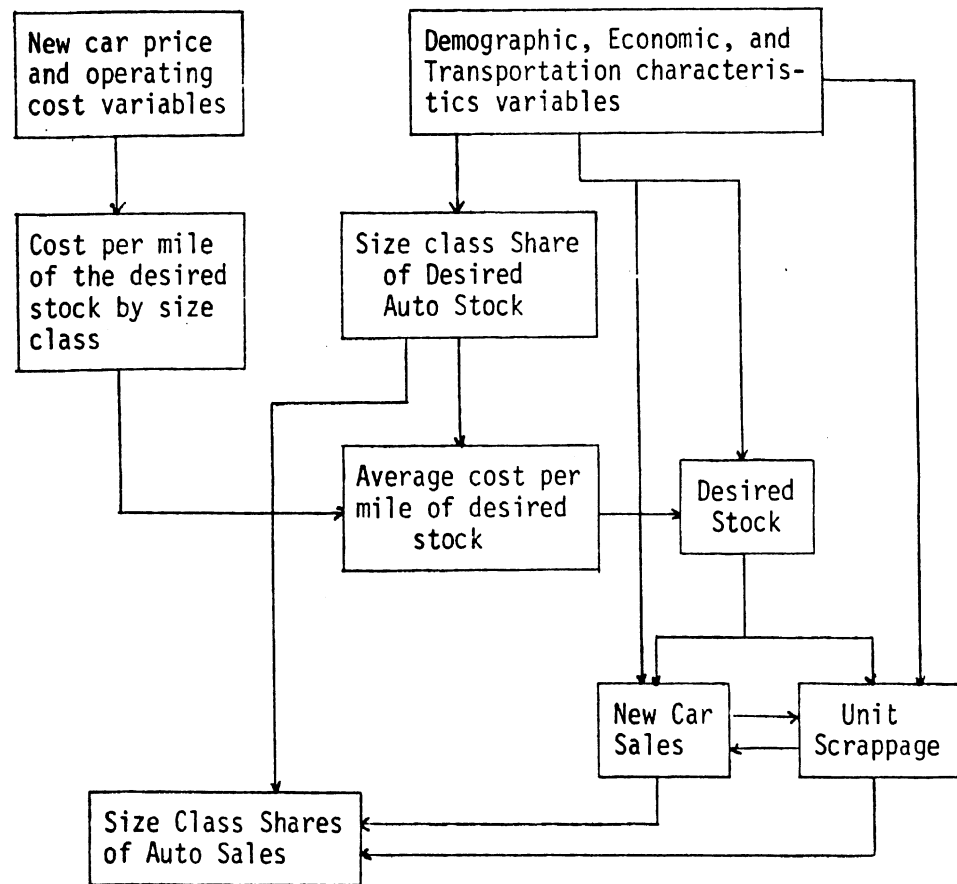
FIGURE 1. Flowchart of Wharton Model

The model operates as follows. From the appropriate exogenous inputs a capitalized cost per mile for each size-class of vehicle is computed. The capitalized cost per mile is essentially the present value of all costs associated with the purchase, sale, and operation of a car (a ten-year lifetime and a lifetime mileage of 100,000 is assumed). This variable, along with the ratio of family income to automobile costs, income distribution, and various demographic factors, is used to determine desired stock values for five size-classes of vehicles: subcompact, compact, mid-size, full-size, and luxury. Equation (1), the desired mid-size share of desired stock, is an example of a desired share equation in the Wharton Model.

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Wharton Model Equation for
Desired Mid-Size Share of Desired Stock (1)

$$\ln\left(\frac{\text{Desired Mid-size Share}}{1\text{-Desired Mid-size Share}}\right) = \underset{(.39)}{.211} - \underset{(4.57)}{1.98} \ln (\text{CPMM/T-M})$$

$$- \underset{(1.3)}{.161} \ln (\text{YDI/FM/CT}) + \underset{(4.73)}{.786} \ln (\text{FM3+4/FM})$$

$\bar{R}^2 = .683$ S.E. = .08   t-statistics in parentheses

where:

    desired mid-share = desired mid-size share of desired stock

CPMM/T-M   = cost per mile for mid-size cars over desired share weighted cost per mile for all other classes.

YDI/FM/CT   = disposable income over number of family units over fixed weighted cost per mile (where weights are the desired share values for the five size-classes of cars for 1972).

FM3+4/FM   = number of 3 and 4 member families over number of family units.

Source: Page 3-38 Wharton Model report (21).

The most important factor in determining desired shares for each class (except for luxury class) is relative cost-per-mile-- that is, the capitalized cost per mile of that class divided by the share-weighted capitalized cost per mile for all other classes. As this relative cost term increases, the desired share decreases. A second important factor is the ratio of family income to average capitalized cost per mile. As this term decreases, the desired share of larger car decreases and the desired share of smaller cars increases.

The desired shares are used to compute an average capitalized cost per mile. This average, along with income per family, income

distribution, various demographic variables, and non-auto-related transportation indicators, is used to determine the desired stock per family. (See equation [2].) As expected, increased income and number of licensed drivers per family have a strong positive impact on desired stock. On the other hand, an increased percentage of families earning over $15,000 has a strong negative impact on desired stock--i.e., as families become wealthier, the rate at which they increase their stock of cars decreases. The average capitalized cost-per-mile term has a somewhat weak negative effect on desired stock per family.

<div align="center">

Wharton Model Equation for
Desired Stock Per Family Unit   (2)

</div>

$$\ln \text{(Desired Stock Per Family)} = -1.91 + .563 \ln \text{(RDIP4/FM)}$$
$$(2.40) \quad (3.13)$$

$$- .101 \ln \text{(PER15+)}/(100\text{-PER15+}) - .200 \ln \text{(CPMTTCAP/PC)}$$
$$(1.92) \qquad\qquad\qquad\qquad (.84)$$

$$+ .421 \ln \text{(LD/FM)} - .054 \ln \text{(MTWNA/FM)} + .099 \text{(NPMET/100)}$$
$$(3.07) \qquad\qquad (1.48) \qquad\qquad\qquad (1.61)$$

$\bar{R}^2 = .461$ S.E. = .06   t-statistics in parentheses

where:

RDPP4	= real permanent income per family unit
PER15+	= percentage of families earning $15,000 or more in 1970 dollars
CPMTTCAP/PC	= desired share weighted cost per mile over consumer price index
LD/FM	= number of licensed drivers over number of family units
MTWNA/FM	= number of persons not using an automobile to drive to work over number of family units

NPMET = percentage of population living in SMSAs

Source: Page 3-37 of Wharton Model report (21).

Total desired stock is used to forecast new car sales and scrappage. (See equation [3] and equation [4].) As expected, new car sales respond positively to desired level of stock, and scrappage responds negatively. In addition, family income relative to past trends and the percentage increase in new car price are important positive and negative factors, respectively, in the new car sales equation. Important factors in the scrappage equation are miles driven per vehicle and average age of the stock, both of which have a positive impact.

Wharton Model Equation for New Car Sales (3)

$$\ln\left(\frac{\text{New Car Sales}}{\text{Stock}_{-1} - \text{Scrap}}\right) = \underset{(35.2)}{-2.915} \quad \underset{(9.90)}{+3.79} \ln\left(\frac{\text{Desired Stock}}{\text{Stock}_{-1} - \text{Scrap}}\right)$$

$$\underset{(8.39)}{+6.039} \ln\left(\frac{Y_{d/FM}}{Y/FM}\right) \underset{(3.45)}{-1.267} \ln\left(\frac{\text{Price}}{\text{Price}_{-1}}\right) \underset{(2.49)}{-.255} \text{ Dummy}$$

$$\bar{R}^2 = .864 \qquad S.E. = .05$$

where:

New Car Sales = total yearly new car sales

Stock_{-1} = last year's end of year total number of vehicles on the road

Scrap = total yearly scrappage (equation [8] in Section 4.0)

$Y_{d/FM}$ = real disposable income per family unit

Y/FM = permanent family income

Price = average new car price

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Price_{-1} = last year's average new car price

Dummy = strike dummy variable

Source: Page 3-44 of Wharton Model report (21).

Wharton Equation for Total Vehicle Scrappage (4)

$$\ln\left(\frac{\text{Scrap} - \text{Given Scrap}}{\text{Stock}_{-1} + \text{New Car Sales}}\right) = \underset{(7.99)}{-6.98}$$

$$\underset{(4.50)}{-3.83}\ln\left(\frac{\text{Desired Stock}}{\text{Stock}_{-1} + \text{New Car Sales}}\right) \underset{(5.32)}{+2.91}\ln(\text{Avg. Age of Stock})$$

$$\underset{(2.20)}{-.15}\ln\left(\frac{\text{Price of Old Used Car}}{\text{Scrap Metal Price}}\right) \underset{(4.33)}{-.338}\ln(\text{Unemployment Rate})$$

$$\underset{(2.42)}{+2.23}\ln\left(\frac{\text{VMT/k}}{\text{VMT/k}(-1)}\right) \underset{(3.60)}{+4.20}\ln\left(\frac{\text{VMT/k}(-1)}{\text{VMT/k}(-2)}\right) \underset{(2.86)}{+3.45}\ln\left(\frac{\text{VMT/k}(-2)}{\text{VMT/k}(-3)}\right)$$

$$\bar{R}^2 = .923 \qquad \text{S.E.} = .046 \qquad \text{D.W.} = 2.60$$

Fit Period: 1954-1974

t-statistics in parenthesis.

where:

Scrap = total yearly scrappage

Given Scrap = number of vehicles that are over 21 years old on the road

New Car Sales = total yearly new car sales (Equation [3])

Stock_{-1} = last year's end of year total number of vehicles on the road

Average Age of Stock = average age of the vehicle stock 0 to 20 years of age

VMT/k = vehicle miles travelled divided by total mid-year stock.

Source: Page 3-44 of Wharton Model report (21).

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The new car sales and scrappage equations are closely related. New car sales depend on scrappage and likewise, scrappage depends on new car sales. Thus, if scrappage increases, new car sales increase, and vice versa.

As mentioned previously, the new car sales in a size-class is a function of the desired stock share. The size-class share of new car sales responds directly to changes in the desired stock share of that size-class, and the strength of that response depends on the difference between the existing stock share (after scrappage) and the desired stock share. The Wharton Model has an equation for each of the five size-class shares of new car sales. (Equation [5], the mid-size share of new car sales, is an example.)

<div align="center">

Wharton Model Equation for   (5)
Mid-Size Share of New Car Sales

</div>

$$\ln\left(\frac{\text{New Mid-size Share}}{1 - \text{New Mid-size Share}}\right) = \ln\left(\frac{\text{Desired Mid-size Share}}{1 - \text{Desired Mid-size Share}}\right)$$

$$\begin{array}{cc} -.002 & - .873 \\ (.66) & (82.94) \end{array} \left[\ln\left(\frac{\text{Actual Mid-size Share}}{1 - \text{Actual Mid-size Share}}\right)\right.$$

$$\left. - \ln\left(\frac{\text{Desired Mid-size Share}}{1 - \text{Desired Mid-size Share}}\right)\right]$$

$$\overline{R}^2 = .997 \qquad \text{S.E.} = .01 \qquad \text{t-statistics in parentheses}$$

where:

    new mid-size share   = mid-size share of new car sales

    desired mid-size share = desired mid-size share of desired stock
                                      (See Equation [1])

    actual mid-size share = actual mid-size share of stock after
                                        scrappage

Source: Page 3-46 of Wharton Model report (21).

<div align="center">16</div>

5. MODEL OF VEHICLE SURVIVAL

The first component of the FAPS Model--the Wharton model of new car sales--was presented in the previous section. In this section, the second component--the model of vehicle survival--is discussed.

It is important that a model of vehicle survival involve two sets of hypotheses: (1) hypotheses about the relationship of the technology used in, and engineering built into, vehicles and the survival rates of vehicles; and (2) hypotheses about the relationship of government policies, economic conditions, travel patterns, etc. and decisions to scrap a vehicle. The first set of hypotheses are technological; the second, behavioral. They are both important in identifying how changes in the age distributions of vehicles cause changes in the fuel economy, safety mix, and total emissions of future fleets. For example, if future vehicles were constructed so that the wear-out times of small fuel-efficient vehicles became much different from those of large less-fuel-efficient vehicles, substantial changes in the fleet fuel efficiency of older-vintage vehicles may result. This could have a substantial effect on total future fuel consumption.

The relationships derived from the two sets of hypotheses lead to a probability distribution of vehicle survival times for each type and vintage vehicle. The use of probabilities makes it possible to represent the natural differences between vehicles of the same type and vintage. The procedure to generate the probability distributions is to choose an appropriate survival distribution based on the technological hypotheses and then transform this distribution into a time-dependent distribution based on the behavioral hypotheses. In other words, a survival distribution which describes the wear-out process of a vehicle is chosen. The parameters of this distribution are then adjusted yearly in the forecast for changes in policy, economic conditions, travel patterns, and other factors.

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5.1 TECHNOLOGICAL HYPOTHESES

This section discusses several sets of technological hypotheses. Examination of those hypotheses resulted in the modified Weibull model of vehicle survival described below.

Technological hypotheses are examined by using data on vehicle scrappage. A good way of investigating the relationships between the survival of a vehicle and the technology employed in its construction is to use data on yearly vehicle scrappage by vintage and by model. A table of vehicle scrappage by vintage and name plate is available.* Certain name plates correspond to specific types (i.e., size categories) of vehicles, for example, Lincoln, Cadillac, and Imperial. Other name plates correspond to general size categories of vehicles. For example, Mercuries and Oldsmobiles are generally intermediate to large vehicles. Even for name plates that correspond to a broad range of types of vehicles, presumably a fairly consistent engineering process is used in the manufacturing of these vehicles. Thus, an analysis of these name plate data should reveal some characteristics of engineering-related survival patterns of the corresponding vehicles.

Aggregate scrappage data over all vehicle types, which have exclusively been used in previous studies, are not adequate to examine the technological and engineering-related wear-out processes of vehicles unless all makes of vehicles wear out consistently.

A motor vehicle can be thought of as a system of major elements--that is, a body, transmission, chassis, motor, etc. Each of these elements is exposed to wear and may fail with time--the body rusts out; the bearings and gears wear out; the vehicle is in a major accident and the entire front-end needs replacing, etc. When a major failure occurs, a decision is made to either scrap or

---

*These tables are derived from the R. L. Polk and Co. tables "United States Summary, Passenger Cars in Operation as of July 1," presented in Ward's Automotive Yearbook from 1956 to 1977.

repair the vehicle. Decisions to scrap a vehicle are also made by automobile dealers when they decide that the cost of replacing worn-out parts of a trade-in vehicle is too high. Under these conditions, the decision to scrap a vehicle is made because one or more components of the vehicle has nearly failed. Although information about life characteristics of major components of vehicles by type and vintage is not available, under certain assumptions a life distribution (survival distribution) can be postulated. Four simple assumptions are considered:

Assumption I: The vehicle is scrapped when its shortest-lived major element fails.

Assumption II: The vehicle is scrapped when the longest-lived major element fails. This condition may be called the "last straw."

Assumption III: The vehicle is scrapped only when a catastrophic failure occurs--e.g., a major accident.

Assumption IV: The vehicle is scrapped after a series of moderate to major failures through its life. These failures may include accidents, engine component failures, brake failures, etc. The lifetime of the vehicle can then be thought of as the sum of interfailure times--that is, the times between failures.

Assumptions I and IV seem the most reasonable, and Assumption II, the least. It also seems reasonable that Assumption III is applicable to some proportion of the automobile population.

Assumption I leads to a Weibull lifetime probability distribution (36,38). (See Appendix C for a discussion of the Weibull distribution.) The Weibull distribution is the two-parameter distribution:

$$S(t) = \text{Prob}(L > t) = e^{-\frac{a}{b}t^b} \tag{6}$$

where:

   $S(t)$ = probability a vehicle survives to age t

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L   = lifetime random variable

a,b   = parameters

The mean and variance of the Weibull distribution are:

$$E[L] = (\frac{b}{a})^{\frac{1}{b}} \Gamma(1 + \frac{1}{b}) \hspace{3cm} (7)$$

$$VAR(L) = (\frac{b}{a})^{\frac{2}{b}} \left[\Gamma(1 + \frac{2}{b}) - \Gamma^2(1 + \frac{1}{b}) \right]$$

The instantaneous scrappage rate, which is defined as the proportion of vehicles scrapped in a time interval (t, t+dt) among those vehicles surviving up to time t, associated with the Weibull distribution is:

$$h(t) = at^{b-1}$$

The scrappage rate of Lincolns* for the first 9 to 10 years (see Figure 2) does seem to increase according to this type of function, which implies that Assumption I may be reasonable.

Assumption III leads to an exponential distribution for the lifetime of a vehicle. The exponential distribution is a one-parameter distribution:

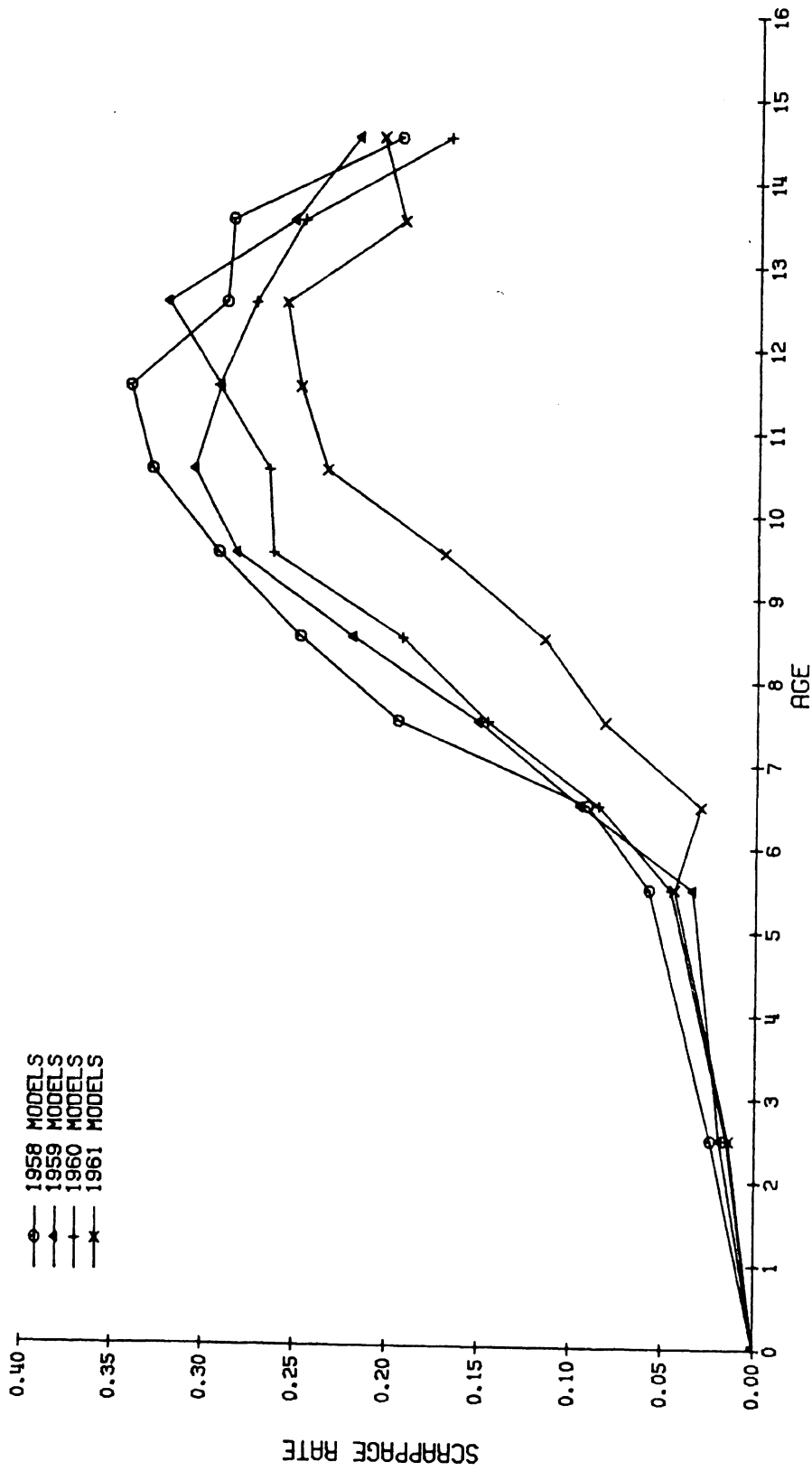$$S(t) = e^{-ct}$$

where:

  c = parameter.

_____

    *The following analyses are based on scrappage data of Lincolns because these data include only one type of vehicle, namely a large luxury vehicle.

SCRAPPAGE RATES OF 1958 TO 1961 LINCOLNS
FIGURE 2

The mean and variance of the exponential distribution are:

$$E[L] = \frac{1}{c} \qquad\qquad VAR(L) = \left(\frac{1}{c}\right)^2$$

We may combine Assumptions I and III, by assuming that a proportion, p, of the population of vehicles of a certain type is specified. This is the fraction of vehicles whose failure process can be represented by a Weibull distribution. The remainder fail according to the exponential distribution. This leads to a survival distribution:

$$S(t) = pe^{-\frac{a}{b}t^b} + (1-p)e^{-ct} \qquad\qquad (8)$$

Maximum likelihood estimates for the parameters of this survival distribution were calculated for 1958 to 1961 Lincoln data. Goodness-of-fit tests indicate equation (8) is a reasonable functional term (see Table 1). The parameters of the exponential part of the distribution is extremely small. This indicates that these cars are rarely used or almost always repaired at any cost. The proportion of Lincolns which fall into this category is quite large, 6 to 10 percent.

TABLE 1

MAXIMUM LIKELIHOOD ESTIMATES OF SURVIVAL DISTRIBUTIONS
OF THE WEIBULL-EXPONENTIAL MODEL AND CHI-SQUARE TEST RESULTS

Lincoln Model/Car	a	b	c	p	Chi-Square (100s of units)	Degrees of Freedom
1958	.0004	4.1	.000001	.94	10.9	6
1959	.0004	4.0	.000001	.93	3.1	6
1960	.0004	3.9	.000001	.91	1.5	6
1961	.0004	3.8	.000001	.91	9.1	6

The instantaneous scrappage rate associated with a failure process represented by equation (8) is:

$$h(t) = \frac{pat^{b-1}e^{-\frac{a}{b}t^b} + (1-p)ce^{-ct}}{pe^{-\frac{a}{b}t^b} + (1-p)e^{-ct}} \qquad (9)$$

A comparison of this function (with parameter values from Table 1) and observed scrappage rates for 1960 Lincolns is shown in Figure 3. The two curves are very close for the first 10 years, in which time more than 50 percent of the 1960 Lincolns sold were scrapped, and the curves are fairly close after nine years. The largest error occurs at around 11 to 12 years, when only 20 percent of the 1960 Lincolns sold are remaining in the population. A similar comparison is presented for 1959 Lincolns in Figure 4.

Assumption I may also be modeled with a minimum-extreme-value distribution. This distribution is a two-parameter distribution, specified by the following equation:

$$S(t) = e^{-e^{b(t-a)}}$$
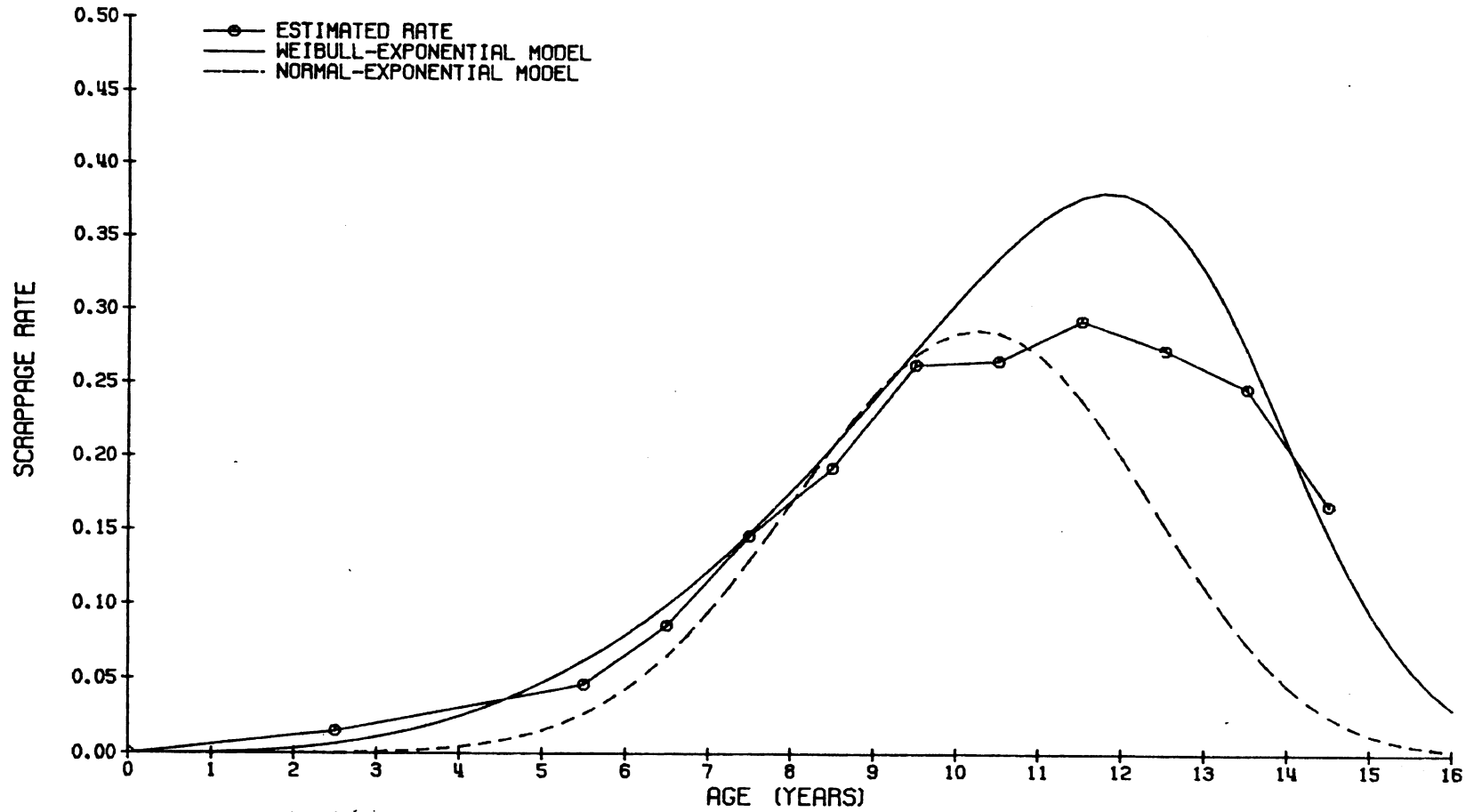
where:

a,b are parameters, and:

$$E[L] = a - \frac{.57722}{b}$$

$$VAR[L] = \frac{1.64493}{b^2}$$

The minimum extreme value distribution can also be combined with the exponential distribution to form the minimum extreme value-exponential distribution:

COMPARISON OF SCRAPPAGE RATES
FOR 1960 LINCOLNS
FIGURE 3

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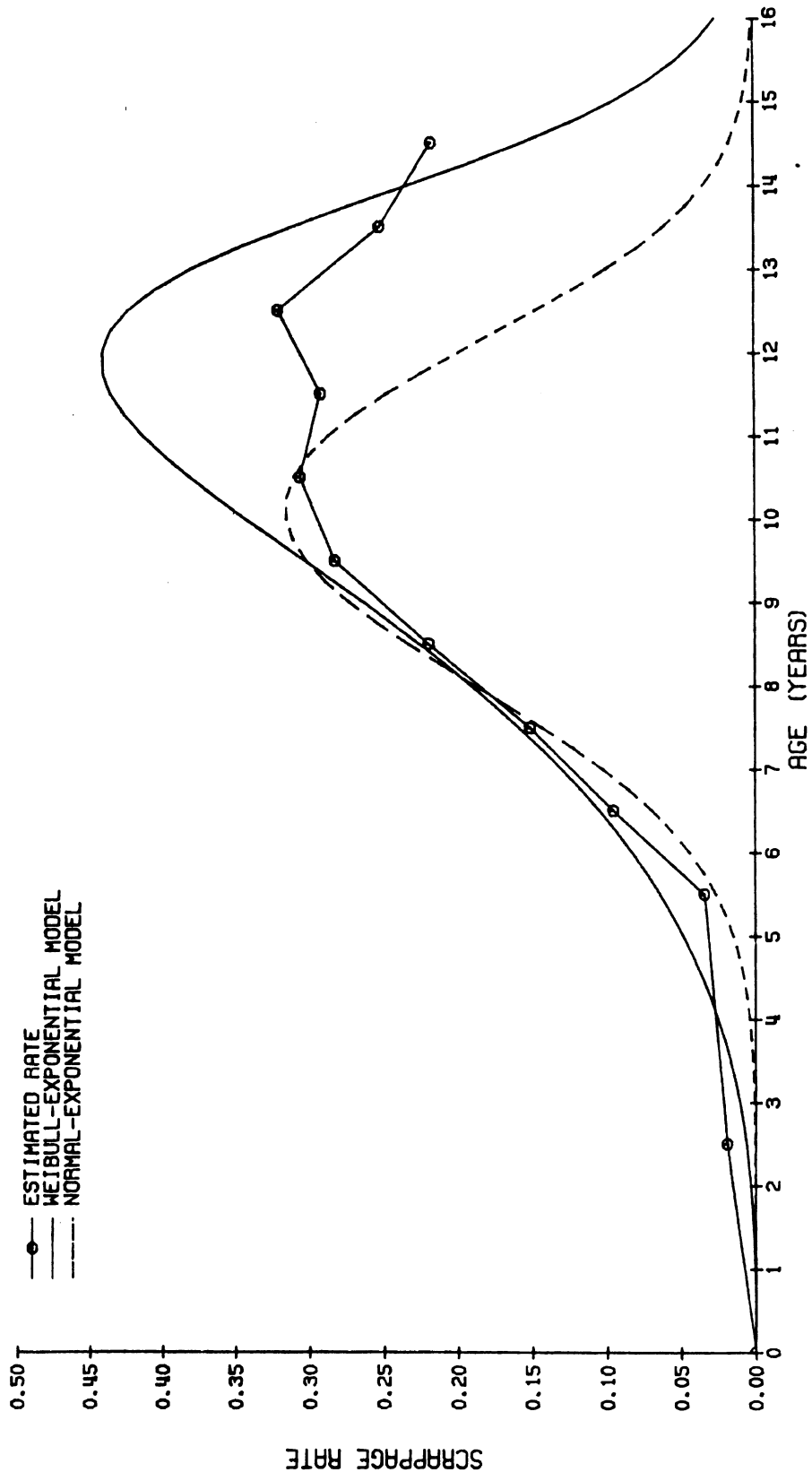
COMPARISON OF SCRAPPAGE RATES
FOR 1959 LINCOLNS
FIGURE 4

$$S(t) = pe^{-e^{b(t-a)}} + (1-p)e^{-ct}$$

Maximum likelihood estimates for the parameters of the distribution were calculated for the 1960 Lincoln data (see Table 2). The results for this distribution fit the data much less closely than the Weibull-exponential model.

TABLE 2

MAXIMUM LIKELIHOOD ESTIMATES OF SURVIVAL DISTRIBUTIONS AND
COMPARISON OF CHI-SQUARE TEST RESULTS FOR 1960 LINCOLN DATA

Distribution	a	b	c	p	Chi-Square (100s of units)
Weibull-Exp.	.0005	3.9	.000001	.91	1.5
Min. Ext. Val-Exp	10.2	.39	.000001	.92	13.3
Max. Ext. Val-Exp	8.1	.40	.000001	.92	10.0
Gamma-Exp	.72	6.8	.000001	.94	15.0
Normal-Exp	9.08	2.7	.000001	.92	0.9

Degrees of freedom is 6 for all tests.

Assumption II leads to a maximum-extreme-value lifetime distribution (36,38). This distribution, which is similar in form to the minimum-extreme-value distribution, is also a two-parameter distribution:

$$S(t) = 1-e^{-e^{-b(t-a)}}$$

where:

a,b are parameters, and

$$E[L] = a + \frac{.57722}{b}$$

$$VAR(L) = \frac{1.64493}{b^2}$$

Maximum likelihood estimators were calculated for the parameters of the maximum-extreme-value-exponential distribution (see Table 2).

As expected, this distribution did not fit the data as well as the Weibull-exponential model.

Assumption IV states that the lifetime of a vehicle is made up of the sum of interfailure times. If the failures occur randomly over a vehicle's life, which seems highly unlikely, then the survival distribution of this vehicle would be approximately Gamma distributed. The Gamma survival distribution is specified by:

$$S(t) = \int_t^\infty \frac{a(a\tau)^{b-1}e^{-a}}{\Gamma(b)} \, d\tau$$

The estimated probability-weighted Gamma-exponential distribution did not fit the data well (see Table 2). However, if the number of failures in a vehicle lifetime is large, then it follows from the central limit theorem that the sum of interfailure times tends toward a normal distribution:

$$S(t) = \int_t^\infty \frac{1}{b\sqrt{2\pi}} e^{-\left(\frac{\tau-a}{b}\right)^2} \, d\tau$$

Maximum likelihood estimators for the parameters of the normal-exponential distribution were calculated for the 1958 to 1961 Lincoln data (see Table 3). The normal-exponential distribution is:

$$S(t) = \int_t^\infty \frac{p}{b\sqrt{2\pi}} e^{-\left(\frac{\tau-a}{b}\right)^2} \, d\tau + (1-p)e^{-ct} \tag{10}$$

The instantaneous scrappage rate associated with the normal-exponential distribution is:

$$h(t) = \frac{\frac{p}{b\sqrt{2\pi}} e^{-\left(\frac{t-a}{b}\right)^2} + (1-p)ce^{-ct}}{\int_t^\infty \frac{p}{b\sqrt{2\pi}} e^{-\left(\frac{\tau-a}{b}\right)^2} \, d\tau + (1-p)e^{-ct}} \tag{11}$$

For each year, the normal-exponential model fit the data as well as or better than the Weibull-exponential model. For comparative purposes, the scrappage rates based on the normal-exponential model are also presented in Figures 3 and 4.

A graphic comparison of all models discussed in this section is presented in Figure 5.

TABLE 3

MAXIMUM LIKELIHOOD ESTIMATES OF SURVIVAL DISTRIBUTIONS OF
NORMAL-EXPONENTIAL MODEL AND CHI-SQUARE TEST RESULTS
(a = MEAN AND b = STANDARD DEVIATION OF NORMAL DISTRIBUTION)

Lincoln Model/Year	a	b	c	p	Chi-Square (100s of units)	Degrees of Freedom
1958	8.6	2.7	.000001	.94	4.5	6
1959	9.0	2.6	.000001	.93	1.8	6
1960	9.1	2.7	.000001	.92	0.9	6
1961	10.1	3.1	.000001	.91	9.2	6

It is not surprising that the normal-exponential model, equation (10), and Weibull-exponential model, equation (8), perform almost equally well. Assumption I, which is modeled by the Weibull distribution, may be thought of as a subset of Assumption IV, which is modeled by the normal distribution. The difference between the assumptions is that in the first one a vehicle is scrapped because of the first occurrence of any major vehicle component failure or a major accident. The fourth assumption implies that the vehicle may be scrapped at this first occurrence of a major failure or the second occurrence of a major failure or at the occurrence of a non-major component failure, as long as more than a few failures have occurred in the vehicle's lifetime. In addition, Assumption I includes cases when vehicles are scrapped due to a major failure or accident during the first several years of a vehicle's lifetime-- that is, when only a few or no moderate failures have occurred--

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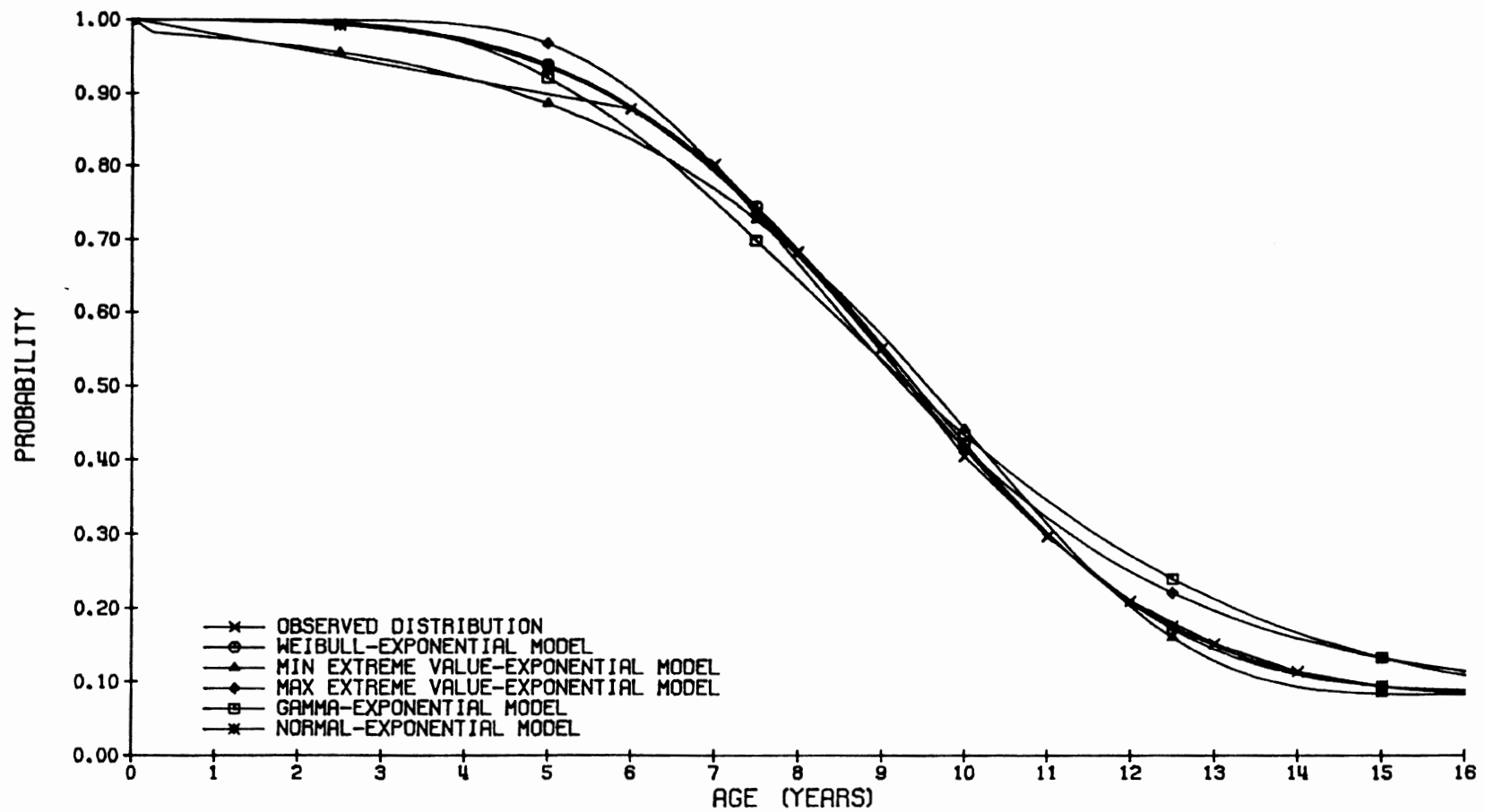
SURVIVAL DISTRIBUTIONS FOR 1960 LINCOLNS
COMPARISON OF MODELS
FIGURE 5

whereas Assumption IV, modeled by the normal distribution, does not include such cases.

A reexamination of the scrappage rates drawn in Figures 3 and 4 illustrates the differences between the models based on the Weibull-exponential distribution, equation (9), and the normal-exponential distribution, equation (11). The scrappage rates for the Weibull-exponential model match the observed rates for the first nine years of vehicle life more closely than the normal-exponential model. This indicates that the Weibull distribution models early scrappage better than the normal distribution. During these first nine years of vehicle life, approximately 45 percent of the Lincolns originally sold are scrapped. The normal-exponential model fits the data well during the ninth through twelfth years when approximately 35 percent of the Lincolns originally sold are scrapped. Because the theoretical scrappage rates of the Weibull exponential model match the observed rates for a greater number of years of vehicle life, and because this model fits the data better during the early years of vehicle life when a vehicle is driven most extensively, the survival model will be based on the Weibull exponential distribution.

5.1.1 Modified Weibull Model

A modified version of the Weibull-exponential model is used as the model of vehicle survival in this study. Since the rate parameter, c, of the exponential portion of the distribution, equation (8), is so close to zero (the c value, .000001, implies that about .0001 percent of surviving Lincolns which fail according to Assumption III will be scrapped per year), a value of c is equal to zero is assumed. This modified distribution, which is called the modified Weibull distribution in this study, has only three parameters with the following survival distribution:

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$$S(t) = pe^{-(a/b)t^b} + 1 - p \qquad (12)$$

where: a,b are Weibull parameters and 1-p is the probability that a vehicle is "never" scrapped. Note the mean and variance are undefined for equation (12). This distribution has the scrappage rate:

$$h(t) = \frac{pat^{b-1}e^{-(a/b)t^b}}{S(t)}$$

The Lincoln survival parameters were reestimated for the modified Weibull model. The results, which are presented in Table 4, are almost identical to those for the Weibull-exponential model.

TABLE 4

MAXIMUM LIKELIHOOD ESTIMATES OF LINCOLN SURVIVAL
DISTRIBUTIONS OF THE MODIFIED WEIBULL MODEL

Lincoln Model/Car	a	b	p	Chi-Square (100s of units)	Degrees of Freedom
1958	.00041	4.08	.94	10.90	7
1959	.00042	4.00	.93	3.20	7
1960	.00049	3.90	.92	1.54	7
1961	.00043	3.76	.89	9.53	7
1962	.00045	3.75	.85	15.97	7

Maximum likelihood estimates of parameters of the modified Weibull model were also calculated for other nameplate data. The parameter estimates are presented in Table 5, and theoretical scrappage rates and survival distributions for several nameplates are illustrated in Figures 6 and 7.

The parameter estimates vary from nameplate to nameplate. The graphs of scrappage rates and survival distribution illustrate this variance. Cadillacs which have the lowest Weibull parameters survive on the road for the most part longer than any of the other nameplates. The expected lifetime of those Cadillacs which fail according to a Weibull process, equation (7), is 10.64 years.

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TABLE 5

MAXIMUM LIKELIHOOD ESTIMATES OF SURVIVAL DISTRIBUTIONS OF THE MODIFIED WEIBULL MODEL

Nameplate/Model	a	b	p	Chi-Square [1]	Degrees of Freedom
Cadillacs:					
1956	.00029	3.93	.93	4.59	7
1958	.00029	3.92	.94	3.09	7
1960	.00031	3.82	.93	5.07	7
Oldsmobiles:					
1956	.00030	4.06	.95	4.18	7
1957	.00029	4.13	.96	6.49	7
1958	.00028	4.20	.96	7.01	7
1959	.00029	4.10	.96	8.64	7
1960	.00029	4.05	.94	7.87	7
1961	.00029	4.10	.94	5.90	7
1962	.00027	4.15	.91	8.03	6
1963	.00031	4.10	.88	4.86	5
Plymouths:					
1956	.00030	4.10	.95	5.25	7
1957	.00031	4.24	.97	13.47 [2]	5
1958	.00033	4.22	.97	14.45 [2]	6
1959	.00034	4.21	.95	9.64	6
1960	.00036	4.06	.95	4.63	6
1961	.00039	4.01	.94	5.46	6
1962	.00042	3.92	.88	2.04	5
Chevrolet:					
1960	.00031	4.00	.93	1.71	7
Miscellaneous: [3]					
1956	.00034	3.64	.88	4.71	5
1957	.00042	3.80	.84	6.39	6
1958	.00048	3.82	.88	10.86 [2]	5
1959	.00055	3.82	.88	20.28 [2]	4
1960	.00053	3.83	.86	14.17 [2]	4
1961	.00057	3.74	.84	7.81	6
1962	.00063	3.70	.80	3.78	6
Mid-size Cars: [4]					
1958	.00035	4.04	.95	5.94	7
1960	.00035	4.00	.93	2.09	6
1962	.00036	3.96	.87	3.74	6
Full-Size Cars: [5]					
1958	.00036	4.08	.95	2.82	7
1960	.00034	4.00	.94	2.54	7
1962	.00034	4.02	.89	5.64	7

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[1] The chi-square is in terms of: 1000s of units for Cadillacs, Oldsmobiles, Plymouths, Miscellaneous; 10,000s of units for Chevrolets, Mid-size Cars, and Full-size Cars.

[2] Rejection of hypothesis at five percent level of significance.

[3] Miscellaneous cars are mostly imported vehicles.

[4] The Mid-size Car category is the sum of Chevrolets, Fords, and Plymouths.

[5] The Full-size Car category is the sum of Buicks, Chryslers, DeSotos, Dodges, Mercuries, Oldsmobiles, and Pontiacs.
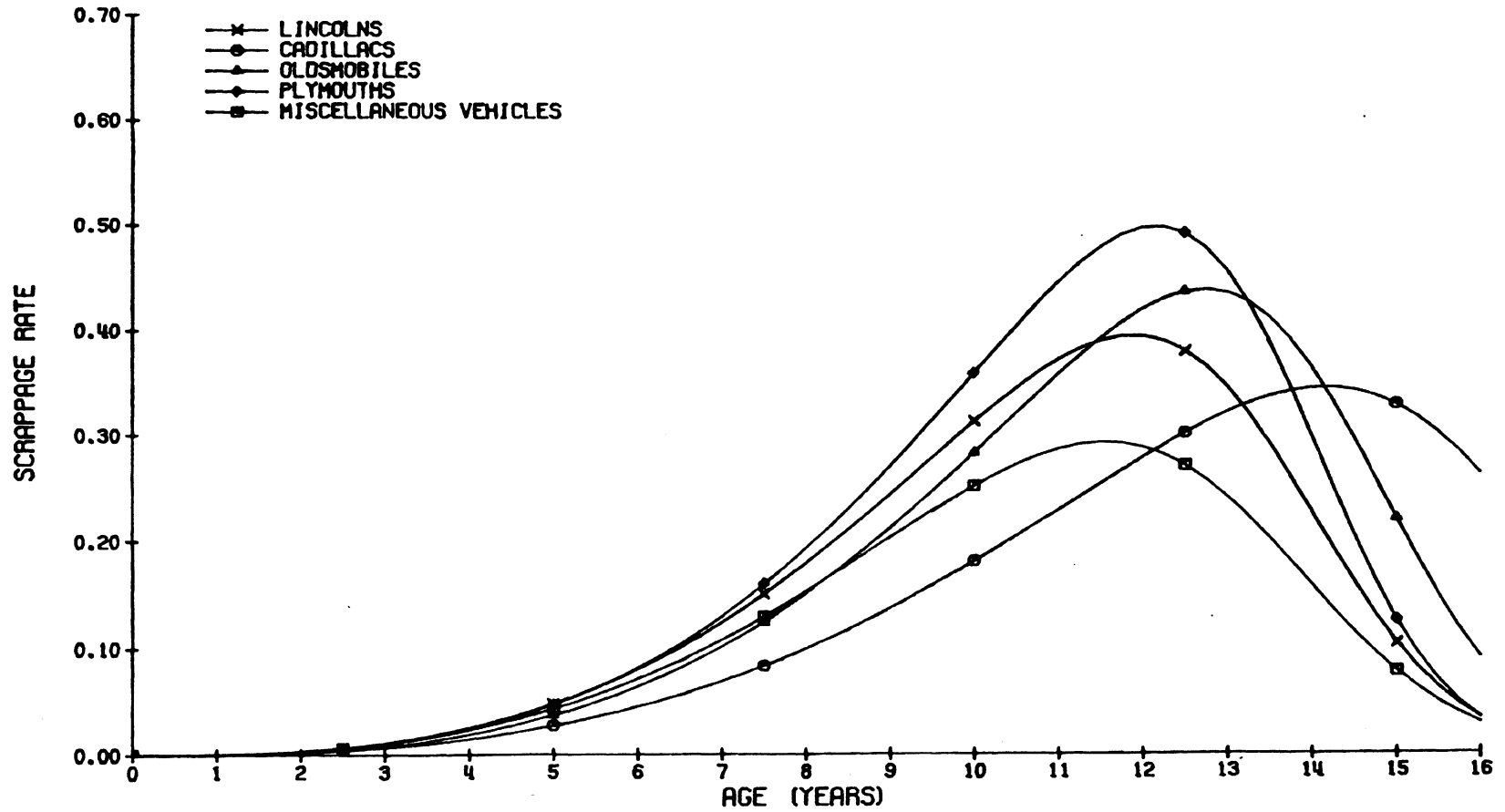
SCRAPPAGE RATES FOR 1960 VINTAGES
FIGURE 6

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SURVIVAL DISTRIBUTIONS
FOR 1960 VINTAGES
FIGURE 7

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The corresponding expected lifetimes for miscellaneous, mid-size, and full-size cars are 9.2, 9.5, and 9.5 years. This low scrappage rate may be because Cadillacs are constructed better, maintained better, or driven less than other cars. The survival parameters of the miscellaneous vehicles, which are mostly comprised of imports such as Volkswagons and other small vehicles, have a high "a" value, but a low "b" value. The high "a" value accounts for slightly higher scrappage rates of miscellaneous vehicles in 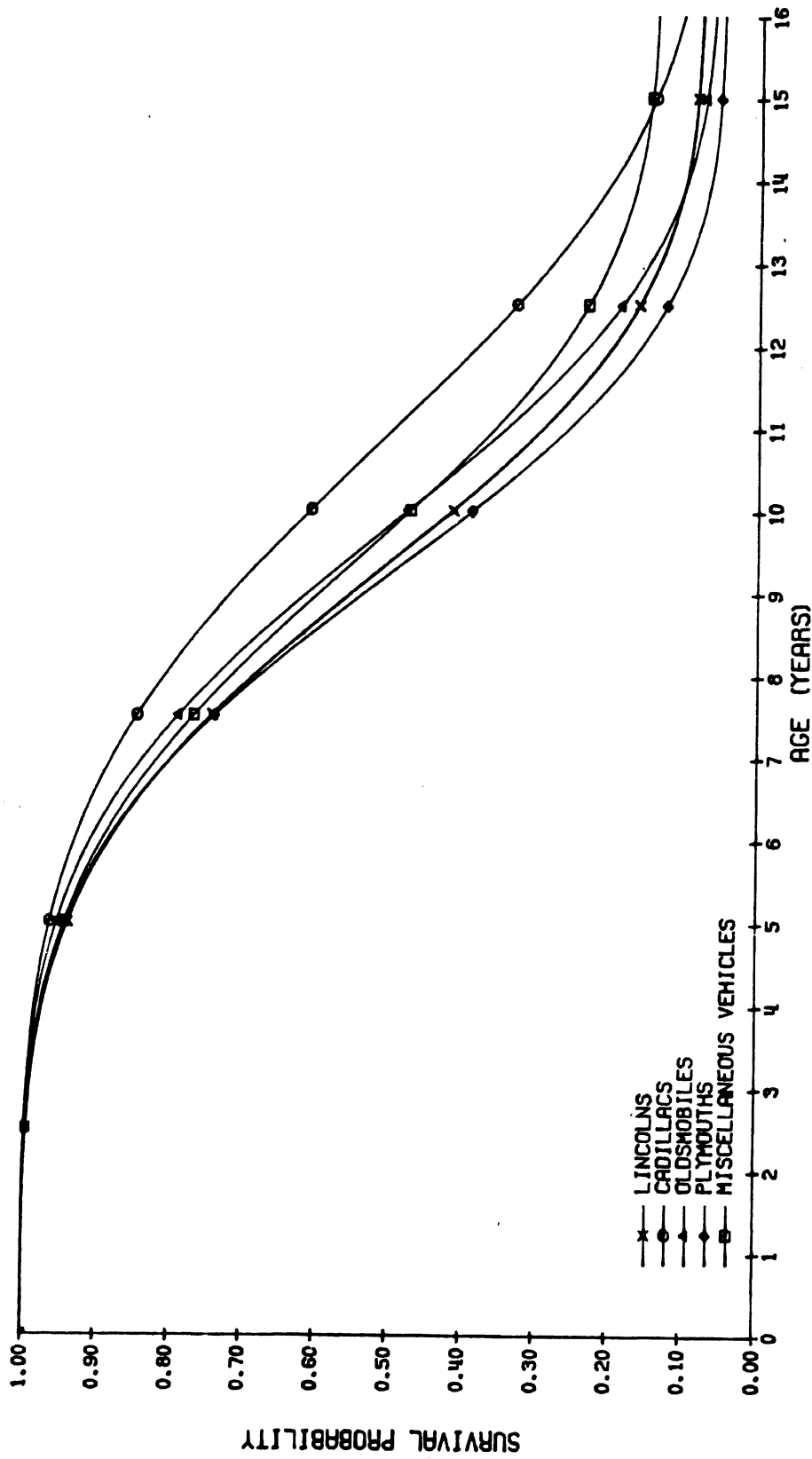their first few years of life. (At age two, a miscellaneous vehicle has a scrappage rate = .0032; Plymouth's rate = .0028; and Cadillac's rate = .0020.) These slightly higher scrappage rates can be explained by the higher probability of a small car being totaled in a collision over a large car. However, since small cars are, in general, relatively inexpensive to repair, then if a major failure occurs, these cars would probably be repaired a greater percentage of the time than large cars. This may explain why the scrappage rate of miscellaneous cars is generally lower than other cars after their first several years of vehicle life.

5.1.2 Vehicle Survival Model Parameters

Whereas the parameter estimates in Table 5 do vary a fair amount from nameplate to nameplate, the parameters, especially the Weibull parameters, remain fairly constant from year to year. Thus, the survival parameters for the vehicle survival model are assumed to be constant through time, and they are set to the values listed in Table 6. These parameter values were based on the values in Table 5. The parameters for subcompact and compact were based on the parameter estimates for miscellaneous vehicles, because the registration data of miscellaneous vehicles is the only set available composed predominantly of subcompacts and compacts. Note that because of the lack of data, these parameters are assumed for both domestic and foreign-made subcompacts and compacts even though the miscellaneous vehicle category is predominantly foreign-made cars.

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TABLE 6
MODIFIED WEIBULL PARAMETERS

Size-Class	a	b	p
Subcompact	.00060	3.7	.85
Compact	.00060	3.7	.85
Mid-Size	.00036	4.0	.95
Full-Size	.00034	4.0	.95
Luxury	.00030	3.9	.94

The parameters for luxury vehicles are based on the estimates for Cadillacs, which make up the largest share of luxury vehicles.

5.2 BEHAVIORAL HYPOTHESES

This section examines three behavioral hypotheses of vehicle scrappage. The variables that seem to cause fluctuations in yearly total scrappage are new and used car prices, unemployment rates, scrap metal prices, vehicle miles travelled, and the average age of the vehicle population. These variables presumably affect yearly vehicle scrappage rates differently for different types of vehicles. Thus, ideally, behavioral hypotheses should be constructed for each type of vehicle. Those hypotheses could be constructed, but the extensive analysis necessary for validating them lies beyond the scope of this study. Instead, several simple behavioral hypotheses were formulated. These hypotheses assume that the fluctuations in yearly scrappage by type are similar to the fluctuations in total yearly scrappage.

This first behavioral hypothesis tested in the model is the one used in the original Wharton model (see Appendix B). This hypothesis states that scrappage is a function of desired stock, actual stock, new car sales, average age of stock, unemployment rate, changes in vehicle travel, and the price of used cars and scrap metal. The Wharton equation for scrappage is:

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$$\ln\left(\frac{\text{Scrap - Given Scrap}}{\text{Stock}_{-1} + \text{New Car Sales}}\right) = \begin{array}{c}-6.98 \\ (7.99)\end{array}$$

$$\begin{array}{c}-3.83 \\ (4.50)\end{array} \ln\left(\frac{\text{Desired Stock}}{\text{Stock}_{-1} + \text{New Car Sales}}\right) \begin{array}{c}+2.91 \\ (5.32)\end{array} \ln(\text{Avg. age of stock})$$

$$\begin{array}{c}-.15 \\ (2.20)\end{array} \ln\left(\frac{\text{Price of Old Used Car}}{\text{Scrap Metal Price}}\right) \begin{array}{c}-.338 \\ (4.33)\end{array} \ln(\text{Unemployment rate})$$

$$\begin{array}{c}+2.23 \\ (2.42)\end{array} \ln\left(\frac{\text{VMT/k}}{\text{VMT/k}(-1)}\right) \begin{array}{c}+4.20 \\ (3.60)\end{array} \ln\left(\frac{\text{VMT/k}(-1)}{\text{VMT/k}(-2)}\right) \begin{array}{c}+3.45 \\ (3.45)\end{array} \ln\left(\frac{\text{VMT/k}(-2)}{\text{VMT/k}(-3)}\right)$$

$$\bar{R}^2 = .923 \qquad\qquad \text{S.E.} = .046 \qquad\qquad \text{D.W.} = 2.60$$

Fit Period: 1954-1974

t-statistics in parenthesis.

Where: Scrap = total yearly scrappage

   Given Scrap = number of vehicles that are over 21 years old on the road

New Car Sales = total yearly new car sales (equation 4)

   Stock_{-1} = last year's end of year total number of vehicles on the road

Average age
   of stock = average age of the vehicle stock 0 to 20 years of age.

   VMT/k = vehicle miles travelled divided by total mid-year stock.

    The parameters for equation (13) were estimated assuming that the survival probabilities by age in a given year are the same for all vehicles regardless of a vehicle's size or type. If the vehicle survival model outlined in the previous sections, equation (12) with the parameters specified in Table 6, is used

to calculate the survival probabilities, the given scrappage and average age of stock variables assume different values. Thus, the Wharton scrappage equation using these modified variables was reestimated with the following results:

<u>Form 2 Equation</u>   (14)

$$\ln \left(\frac{\text{Scrap} - \text{Given Scrap}}{\text{Stock}_{-1} + \text{New Car Sales}} \right) = \underset{(-6.17)}{-17.453} \quad \underset{(-3.39)}{-6.417} \ln \left(\frac{\text{Desired Stock}}{\text{Stock}_{-1} + \text{New Car Sales}} \right)$$

$$\underset{(5.33)}{+8.906} \ln (\text{Avg. age of stock}) \quad \underset{(2.76)}{-0.440} \ln (\text{Unemployment rate})$$

$$\underset{(-.16)}{-0.286} \ln \left(\frac{\text{VMT/k}}{\text{VMT/k}(-1)} \right) + \underset{(2.56)}{6.146} \ln \left(\frac{\text{VMT/k}(-1)}{\text{VMT/k}(-2)} \right) + \underset{(2.36)}{6.242} \ln \left(\frac{\text{VMT/k}(-2)}{\text{VMT/k}(-3)} \right)$$

$$\overline{R}^2 = .883 \qquad \text{S.E.} = .11 \qquad \text{D.W.} = 1.42$$

The price of used cars/scrap metal price term became insignificant in the reestimation, and therefore it was dropped from the equation.

A third form of the scrappage equation, which has a different structure from the first two equations, is tested in the model. This third equation attempts to incorporate the technological hypotheses. Instead of describing scrappage as a ratio to actual stock, this equation describes scrappage as being directly proportional to expected scrappage. Expected scrappage is the total number of vehicles that would be scrapped based on the technological hypotheses. The expected scrappage is calculated by multiplying the scrappage probabilities determined by the technological hypotheses of the vehicle survival model times the age distribution of vehicles by size-class. The specific equation for

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expected scrappage is discussed in Section 5.3. (See
equation (16) in Section 5.3.) The third form of the scrappage
equation is:

$$\ln\left(\frac{\text{Scrap} - \text{Given Scrap}}{\text{Expected Scrap} - \text{Given Scrap}}\right) \underset{(-2.40)}{-4.467} \underset{(5.59)}{-6.997} \ln\left(\frac{\text{Desired Stock}}{\text{Stock}_{-1} + \text{New Car Sales}}\right) \tag{15}$$

$$\underset{(2.48)}{+2.741} \ln(\text{Avg. age of stock}) \underset{(-5.14)}{-.541} \ln(\text{Unemployment rate})$$

$$\underset{(.67)}{-.77} \ln\left(\frac{\text{VMT/k}}{\text{VMT/k}(-1)}\right) + \underset{(4.01)}{6.351} \ln\left(\frac{\text{VMT/k}(-1)}{\text{VMT/k}(-2)}\right) + \underset{(3.50)}{6.093} \ln\left(\frac{\text{VMT/k}(-2)}{\text{VMT/k}(-3)}\right)$$

$$\bar{R}^2 = .946 \qquad\qquad \text{S.E.} = .0517 \qquad \text{D.W.} = 1.68$$

It is not surprising that the coefficients of this equation are
very much the same as the second form equation. As in equation
(13), increases in desired stock and unemployment tend to produce
decreases in yearly scrappage. Also, increases in the average of
stock and vehicle miles travelled tend to produce increases in
scrappage.

Whereas the regression fits for the second and third forms of
the equations are fairly good, these regression results should
improve with a better specification of the modified Weibull para-
meters. These parameters greatly influence the values of expected
and given scrappage and the average age of stock.

Each form of the behavioral equation for scrappage (13), (14),
and (15), is included in the computer program of the FAPS Model.
The computer program prompts the user for the form of the equation
to be used in a forecast simulation. (See Appendix A for details.)

5.3 THE VEHICLE SURVIVAL MODEL

Previous sections described the equations and parameters that make up the vehicle survival model. This section discusses details on how these equations and parameters are used to predict the scrappage of vehicles by age and size-class. It also presents details on how the vehicle survival model is incorporated into the Wharton Model.

The vehicle survival model predicts, by age and size-class, the number of cars scrapped in a year. This scrappage is calculated in two steps. The first step is to calculate the expected scrappage of cars by age and size-class. Expected scrappage is calculated using the modified Weibull Model of vehicle survival, equation (11), discussed in Section 5.1.1, as follows:

$$\text{Expected Scrap } (i,t) = \text{PROB}(i,t\text{-}1) \cdot \text{STOCK } (i,t\text{-}1) \qquad (16)$$

where:

Expected Scrap (i,t) = expected scrappage of cars of age $t-1$ to age t and size-class i during the year.

PROB$(i,t-1)$ = probability that a car of size-class i is scrapped during its tth year in operation.

STOCK$(i,t-1)$ = the stock of cars age $t-1$ to age t and size-class i in operation at the beginning of the year. (Note: STOCK$(i,0)$ are the new car sales of size-class i.)

$t = 1,2,...,21$

i = subcompact, compact, mid-size, full-size, luxury.

PROB $(i,t-1)$ is determined by the modified Weibull model as follows:

PROB$(i,t-1)$ = Probability of scrappage at age $t-1$ to t given survival to age $t-1$.

$\qquad$ = Pr[scrappage at age $(t-1,t)$|survive to age $t-1$]

$\qquad$ = Pr[survive to age $t-1$|survive to age $t-1$]

$\qquad\quad$ - Pr[survive to age t|survive to age $t-1$]

$\qquad = 1.0 - \dfrac{\text{Pr[survive to age } t]}{\text{Pr[survive to age } t\text{-}1]}$

$\qquad = 1.0 - S(t)/S(t\text{-}1)$

where:

S(t) is the vehicle survival distribution defined in equation (11) in Section 5.1.1.

All cars over 21 years old are assumed to be scrapped at the end of their 20th year.

To determine actual scrappage, expected scrappage is adjusted based on the results of one of the three behavioral equations of scrappage, (13), (14), and (15), discussed in Section 5.2. The adjustment procedure is a somewhat simplistic one that can be improved in future work on the model.

The adjustment procedure operates as follows: The total expected scrappage (EXP SCRAP) is calculated by summing the expected scrappage for all ages and size-classes of cars. This sum includes the given scrappage of cars which is the number of cars over 21 years old (GIVEN SCRAP). A total yearly scrappage (SCRAP) is determined by one of the behavioral equations (13), (14), (15), in Section 5.2. The scrappage adjustment (ADJ) is then calculated by:

$$ADJ = (SCRAP - GIVEN\ SCRAP)/(EXP\ SCRAP - GIVEN\ SCRAP)$$

and scrappage by age and type is calculated by:

$$SCRAP(i,t) = EXPECTED\ SCRAP(i,t) \cdot ADJ$$

Note that the behavioral hypotheses are assumed to affect the scrappage of cars similarly, regardless of age and size-class. This is clearly a simplification, but this adjustment process does approximate the behavioral adjustments to scrappage and guarantees that the scrappage estimates are consistent with other predicted variables in the model.

Once the scrappage by age and size-class is determined, the age distributions of vehicles by size-class can also be calculated, as follows:

$$STOCK(i,t) = STOCK(i,t-1) - SCRAP(i,t)$$

The number of vehicles by size-class is determined by summing STOCK(i,t) over age, t, for each size-class, i. The total number of vehicles is then simply calculated by summing the number of vehicles by size-class.

The estimates for total yearly scrappage, number of vehicles by size-class, and total number of vehicles are put into the revised Wharton Model to calculate total new car sales and the new car share estimates for the five size-classes of vehicles.

6. PROCEDURE FOR SPECIFYING FUTURE PLANNED AND UNPLANNED EVENTS

This section discusses the third component of the FAPS Model, the procedure for specifying future planned and unplanned events. The ability of any model to forecast accurately is highly dependent upon how well the planned and unplanned events are specified. As discussed in the introduction, planned events are the programs and policies the user of the model wishes to evaluate, and unplanned events are those non-controllable developments that may affect the states of the model. A clear specification of these events involves: (1) a representation that can be interpreted and used by the model; and (2) a method of describing the unpredictable nature of unplanned events. Procedures to handle these specification problems are described in this section.

6.1 GENERAL DISCUSSION

The approach to the specification of planned and unplanned events in this model is quite different from the general approach to exercising of models of long-term vehicle demand and composition for policy analysis. The commonly used approach is to develop an expected future, given present policy. This expected future is called the baseline scenario. A baseline scenario is usually developed by using, for example, projections by the U.S. Bureau of Census and econometric models.

A proposed policy, such as a proposed federal regulation, can be evaluated by modifying the baseline scenario to reflect the conditions of that policy. Forecast simulation results for the policy-modified scenario are compared with the baseline results. The relative merits of the proposed policy can be examined in this way. For example, the simulated results, given an excise tax on new car purchase price based on vehicle fuel economy, may indicate a five percent reduction in automobile gasoline consumption in

1985 as compared with the baseline assumptions. Other proposed policies can be compared in this manner, and this approach can indicate the "best" policy.

A drawback of the baseline scenario approach is that it does not indicate the range or probability distribution of forecast values. Probability distributions of forecast values are important because the cost measure or utility associated with each possible value of a forecast variable may not be simply related to the fore- cast variable. Thus, the expected cost of a policy given a probability distribution of a forecast will not, in general, be equal to the cost associated with the baseline forecast. Using the above example, an excise tax may cause a significant drop in gaso- line consumption only if the country has become deeply aware of and committed to conservation. The excise tax may have no effect otherwise--i.e., if drivers continue to consume ever-increasing amounts of gasoline. The expected drop in gasoline consumption (calculated by: the decrease in gasoline consumption given drivers are conservation minded in 1985 times the probability that drivers will be conservation minded in 1985 plus the increase in gasoline consumption given that drivers are not conservation minded in 1985 times the probability that drivers will not be conservation minded in 1985) may still be five percent in 1985. However, the policy would be ineffective where the potential costs are the greatest-- i.e., when gasoline consumption is excessively high--and thus the potential for a gasoline supply shortfall is the greatest. The expected cost associated with this policy may then be much higher than the cost associated with a five percent reduction in gasoline consumption. If this were the case, the effectiveness of an excise tax policy would have to be seriously questioned.

Another drawback of the baseline scenario approach is that it usually does not take into account the occurrence of major unplanned events. This is so because major unplanned events, such

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as embargoes, war, or severe recessions, are not likely to happen in any given year. The approach to the specification of planned and unplanned events in this model is designed to generate probability distributions and to include major unplanned events. For these reasons, the approach is expected to be more useful than the baseline scenario approach.

6.2 EVENT REPRESENTATION PROCEDURE

The first procedure in the specification of future planned and unplanned events relies on the relationship between the events and the exogenous parameters of the model. The procedure is to base the values of the parameters on the occurrence of the planned and unplanned events. As an example of how this may be done, consider the recently enacted federal standard that all automobile manufacturers must attain an average of 27.5 miles per gallon for the fleet of vehicles they manufacture in 1985. If average new car price is a parameter of the model, then this standard may be represented, in part, by an increase in the 1985 price of vehicles due to more expensive materials that must be used in the vehicle manufacturing to achieve the fuel economy goal. (Note that this representation of the standard is only an example.) The method of deriving the relationships between the parameters and planned and unplanned events is to initially analyze their historical relationships. Where there are no historical precedents, a subjective estimation of the relationship will be used.

In defining the relationship between the parameters and planned and unplanned events, it may not be possible to assign a set of fixed parameters corresponding to an event. An event, especially one without historical precedent, may be better represented as a probability-weighted set of parameters. In the above example, there is a probability that the automobile manufacturers may not have to resort to more expensive materials to achieve the

fuel economy goal. Therefore, there is a certain probability, p_1, that the average real price of 1985 vehicles will not increase, a probability, p_2, that the average real price will increase by x_2, a probability, p_3, that the price will increase by x_3, and so on. Under this uncertainty, the representation of this event in terms of average real vehicle price would be no increase in the 1985 price with probability, p_1, and an increase equal to x_2 in price with probability, p_2, an increase of x_3 with probability, p_3, etc.

6.3 UNPREDICTABILITY OF UNPLANNED EVENTS

The second procedure to be discussed is the method of describing the unpredictability of unplanned events. The future is uncertain and may unfold in an infinite number of directions. A procedure is therefore necessary to select a plausible and manage-able set of alternative futures. The procedure commonly used in existing vehicle-related models is to generate a series of possible alternative futures called scenarios. A set of parameters based on each scenario is generated and independently put into the model, producing a series of results (i.e., expected values of the states of the model) that a decision-maker must evaluate. A good decision-maker will weigh each scenario by the probability of its occurring, and then use this information to derive a crude probability distribution of the expected values of the states of the model.

The procedure can be improved in two ways. First, the distribution of the states given a scenario should be weighted instead of weighting the expected values. This can be done if the variance of the forecast values is generated by the model. Second, a more formal procedure, similar to those used in building decision trees in decision analysis, can be used to generate scenarios-- that is, to build scenario trees. Scenario trees may be quite

complicated to generate, but their use may pay off in a more realis-
tic description of alternative futures.

6.3.1 Scenario Trees

The major feature of scenario trees is that they may branch
into two or more probability-weighted scenarios at any time during
the model's time horizon. At these time points, each branch would
correspond to an event specified by the scenario. As an example,
consider two scenarios of economic conditions in 1980. One
scenario states that a recession will occur (call this event 1),
and the other scenario states that the economy will experience
normal growth (call this event 2). If the probability that event
i occurs is p, then the scenario tree would have two branches in
1980. The first branch would consist of the set of parameter
values corresponding to event 1, and the results of the model with
this parameter input would occur with probability p. Likewise,
the results of the model generated on the second branch would
occur with probability 1-p.

The scenario tree only approximates an expert's assessment
of how the model's parameters may behave in the future. Indeed,
the decision-maker may view most parameters as stochastic
processes which may take a continuum of values over time. Even
if such a stochastic process could be expressed mathematically,
it could require more effort to incorporate into the model than
would be worthwhile. The scenario tree approach can be easily
implemented, and this is one of the primary reasons for using it.

A scenario tree may include planned and unplanned variables.
This is highly desireable. A planned event is represented on
the tree as occurring with probability 1, and an unplanned event
is represented as occurring at one or more time points under a
set of probability conditions. The implementation of a program,

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which is a planned event, may be conditional on non-controllable events. An example is a program which requires the modification of a manufacturing facility. The timing of this modification may depend on interest rates, the strength of the market, etc. A conditional event may be represented on the scenario tree as a branch with a weight of probability 1 if this event branches from a scenario where the conditions are met.

An example of a simple scenario tree containing planned and unplanned events is presented in Figure 15.

In using the model described in this report, a scenario tree should be constructed for each of a series of potential alternative programs and policy decisions to be evaluated. The construction of a set of plausible decision trees will probably be the most difficult process in using the model.

6.3.2 Scenario Tree Formation

The scenario trees should be constructed through interviews with "experts" in automobile design and economic forecasting. An expert is defined here to mean anyone with special knowledge about future automobiles and/or general economic forecasting. The interview process must be well thought out so that accurate probability assessments of future events can be determined. The interview process is based on a method developed by Spetzler and Von Helstein at the Stanford Research Institute (34), and it is briefly outlined here. The interview is divided into six phases:

1) Motivating
2) Structuring
3) Conditioning
4) Pretesting
5) Encoding
6) Verifying
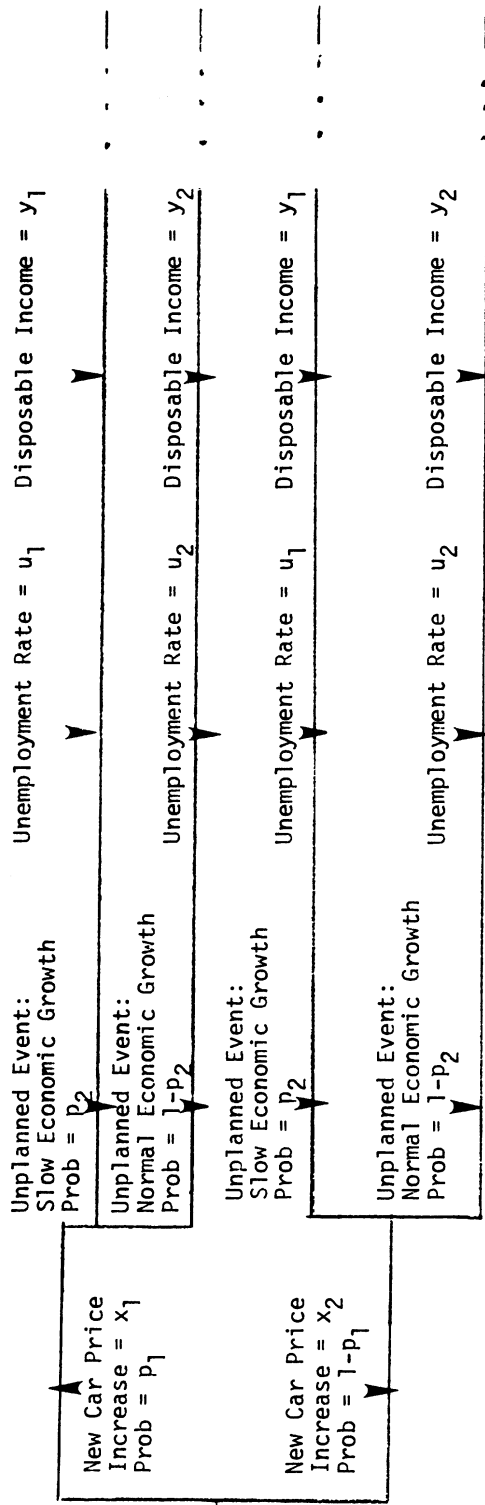
FIGURE 15 EXAMPLE SCENARIO TREE

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The first phase, motivating, is to familiarize and explain the importance of scenario trees in a forecasting effort. In particular, an explanation with examples should be presented on the advantages of the scenario tree approach over the baseline forecasting approach.

The second phase, structuring, involves two steps. First, the expert is probed in order to enumerate a list of reasonable planned and unplanned events into the future that may affect automobile demand and scrappage. At this step, the events are expressed in whatever terms are most familiar to the expert. The second step is the establishment of an event representation procedure. In this step, the expert is asked to think about how the list of events can be translated into the parameters of the model. The expert is asked to consider which parameters may have uncertain relationships with events and to consider the degree of uncertainty. Since certain parameters have a great deal more impact on the forecasts of the model than other parameters, the expert is guided to give additional thought to the values of these parameters.

Once the scenario approach is structured, the next two phases are conducted to prepare the expert for the encoding phase. The first of these phases is the conditioning phase. The conditioning phase is aimed at exploring how the expert goes about making probability assessments. In particular, the expert is asked to specify the basis for his judgement and to indicate what information is being taken into account in making his assessments.

The purpose of this phase is to discover and head off any biases that are associated with modes of judgment. These biases include:

Availability bias. This bias occurs when an event which is clearly in mind may cause the expert to assign a higher

probability to it than is reasonable. This bias may occur when an
event is similar to a recent event or when the expert can easily
visualize the event.

Anchoring bias. This bias occurs when probability assess-
ments by the expert are formulated by adjustment to the.most
readily available piece of information about the event. This bias
often occurs when the initial response about an event serves as
a basis for later responses.

Representativeness bias. This bias occurs when an expert,
who is asked to give a probability assessment given some
evidence, will think of a probability that makes the evidence
more likely. This bias is also known as sterotyping.

Conjunctive event bias. This bias occurs when an event is
the result of a series of events. The probability of the occur-
rence of such an event tends to be overestimated.

Disjunctive event bias. This bias occurs when an event is
dependent on one event's occurring out of a group of events. The
probability of the occurrence of this kind of event tends to be
underestimated.

Bias of unstated assumption. This bias occurs when the
expert's assessments are conditional on unstated assumptions.
The resulting probability distribution, in this case, may not
properly reflect the expert's total uncertainty.

The last step before the encoding phase is a pre-test. The
pre-test serves to familiarize the expert with the encoding pro-
cess. In this phase, the expert is asked to form a scenario
tree of events for some historical period, say the late 1950s.
The pre-test should give a good indication of any biases that may
arise. These biases can then be corrected for in the encoding
phase by structuring the problem further or by modifying the
encoding phase interview technique.

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In the last two phases, the actual encoding of the scenario tree is conducted, and the results are shown graphically as distribution plots to the expert to verify that they conform to the expert's beliefs.

The model computer program, which is discussed in Appendix A, is designed to accept scenario trees as input.

7. HISTORICAL SIMULATIONS CONDUCTED WITH THE FUTURE AUTOMOBILE POPULATION STOCHASTIC MODEL (FAPS MODEL)

Simulations over the historical period, 1960 to 1974, were generated using each form of the behavioral equation, (13), (14), (15), for yearly scrappage discussed in Section 5.2. The results for seven important variables--new car sales, scrappage, and new car shares of the five size-classes--(Figures 8-14) are discussed in this section.

A simulation over an historical period is very much like a forecast simulation. In both, the model is initialized before the first forecast period. The model then predicts the values of all the dependent variables for the first forecast period, and it uses these predicted results to predict the values of the dependent variables for the second forecast period. In an historical simulation the actual values of certain exogenous variables are known, and these values serve as the scenario. The results of a simulation generated over an historical period can be compared with actual historical values, and examined to see how "close" they are to actual values.

The results of the historical simulation for new car sales (Figure 8) do match the actual values quite well. Increases and decreases in actual sales are predicted in the simulation results. Similarly, the scrappage results (Figure 9) also match the actual values, although not as well as those for new car sales.

The results of the simulation for the new car shares do point out some problems in the forecasting ability of this segment of the model as it is presently constructed. (Note that the share results were almost identical for each version of the model.) The results for compact and luxury shares (Figures 11 and 14) are fairly close to the actual values. The results for compact shares, in particular, predict the increases and decreases in the actual

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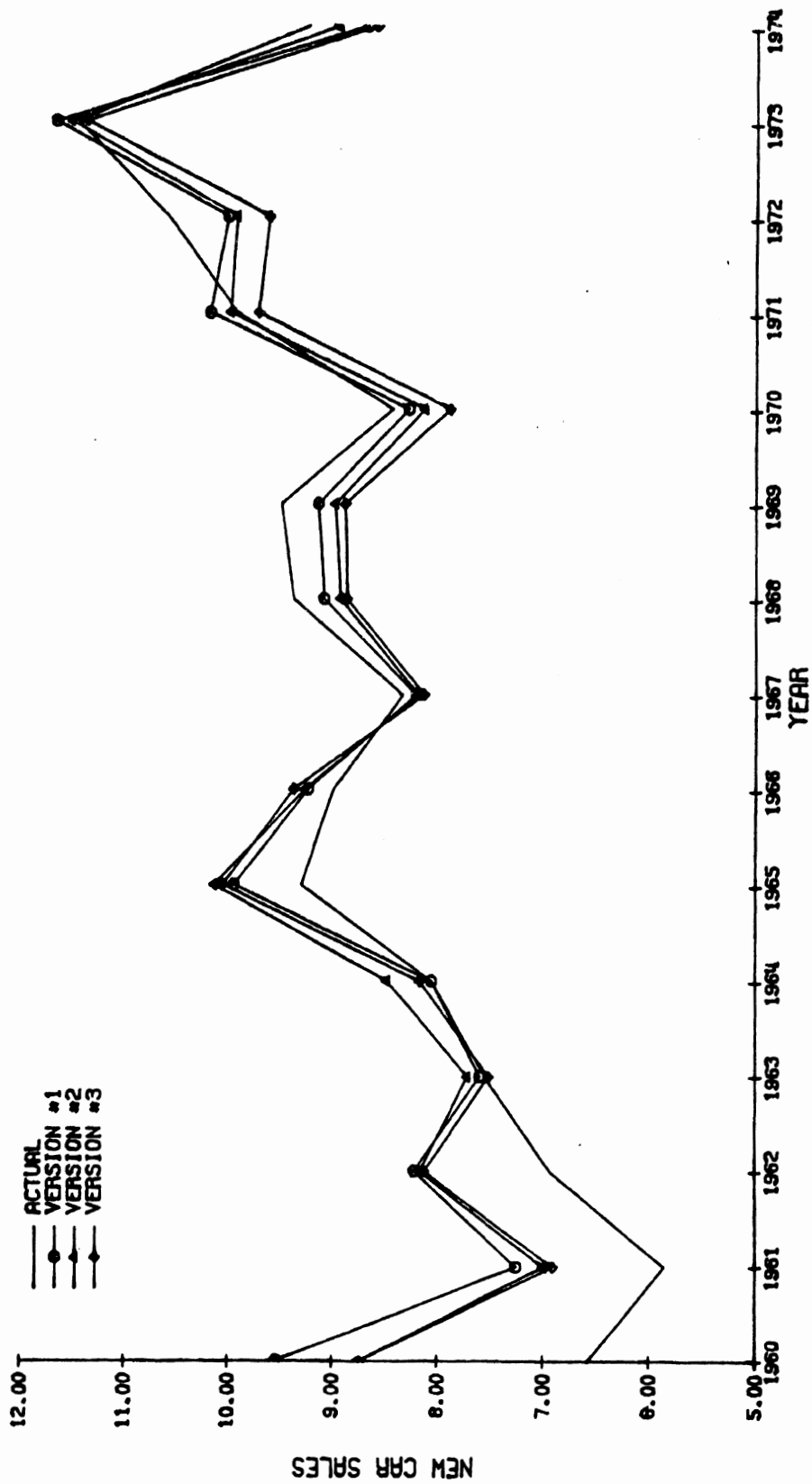
DYNAMIC SIMULATION
OF NEW CAR SALES
FIGURE 8
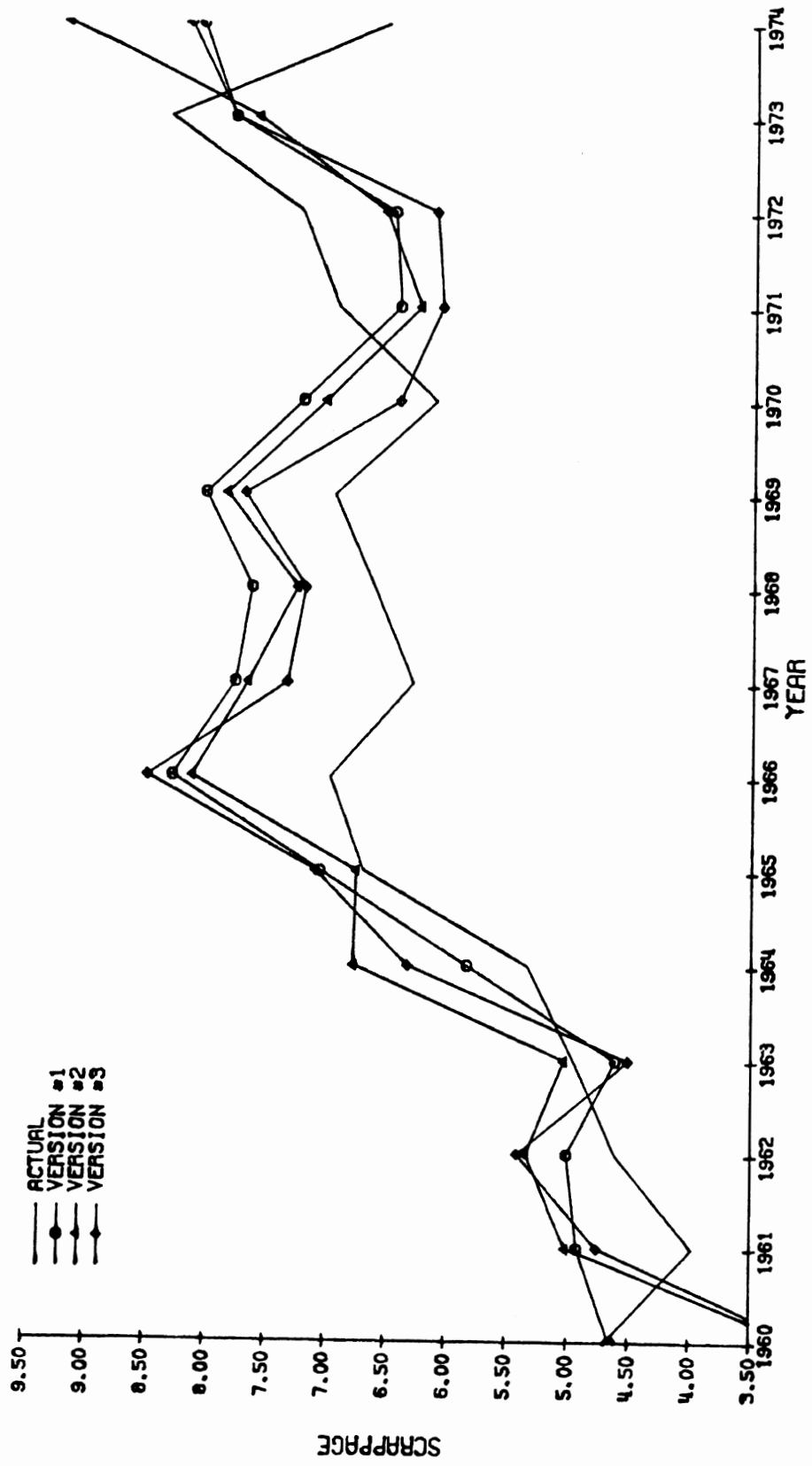
DYNAMIC SIMULATION OF SCRAPPAGE
FIGURE 9
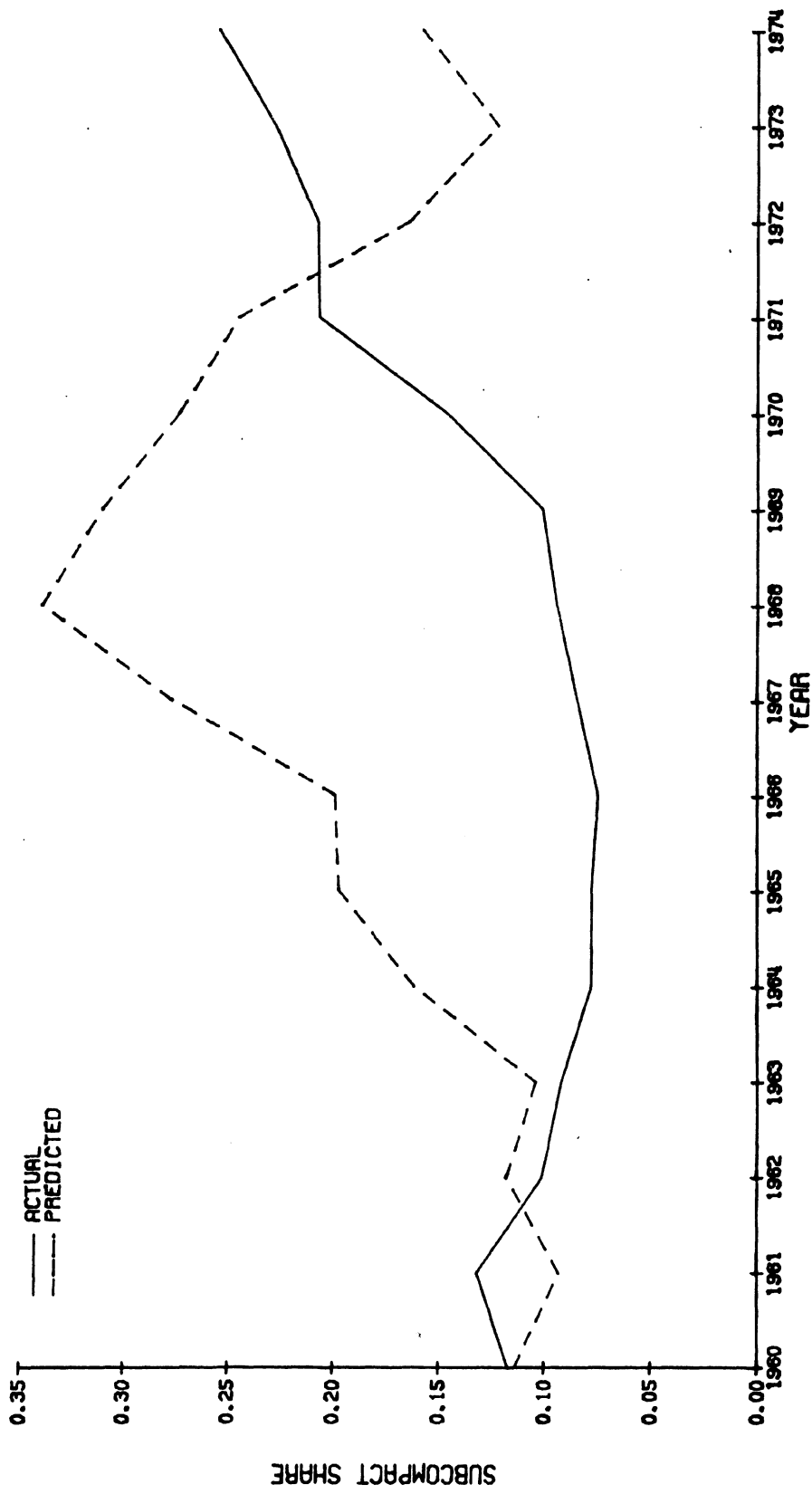
DYNAMIC SIMULATION OF SUBCOMPACT SHARE
OF NEW CAR SALES
FIGURE 10

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DYNAMIC SIMULATION OF COMPACT SHARE
OF NEW CAR SALES
FIGURE 11

DYNAMIC SIMULATION OF MID-SIZE SHARE
OF NEW CAR SALES
FIGURE 12

DYNAMIC SIMULATION OF FULL-SIZE SHARE
OF NEW CAR SALES
FIGURE 13
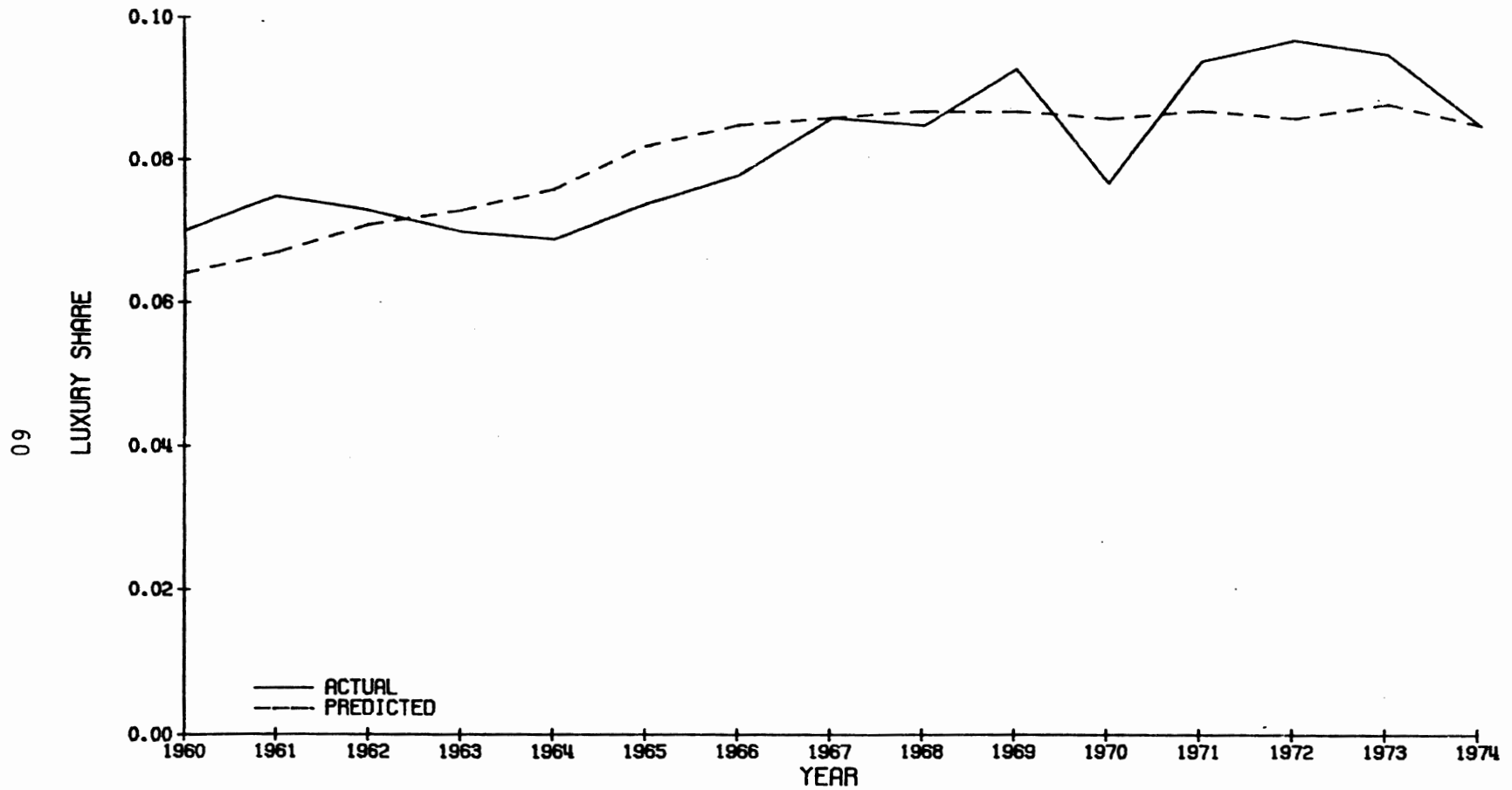
DYNAMIC SIMULATION OF LUXURY SHARE
OF NEW CAR SALES
FIGURE 14

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shares fairly well. The results for the sub-compact shares (Figure 10) are poor. The results for mid-size and full-size shares (Figures 12 and 13) are fairly close after t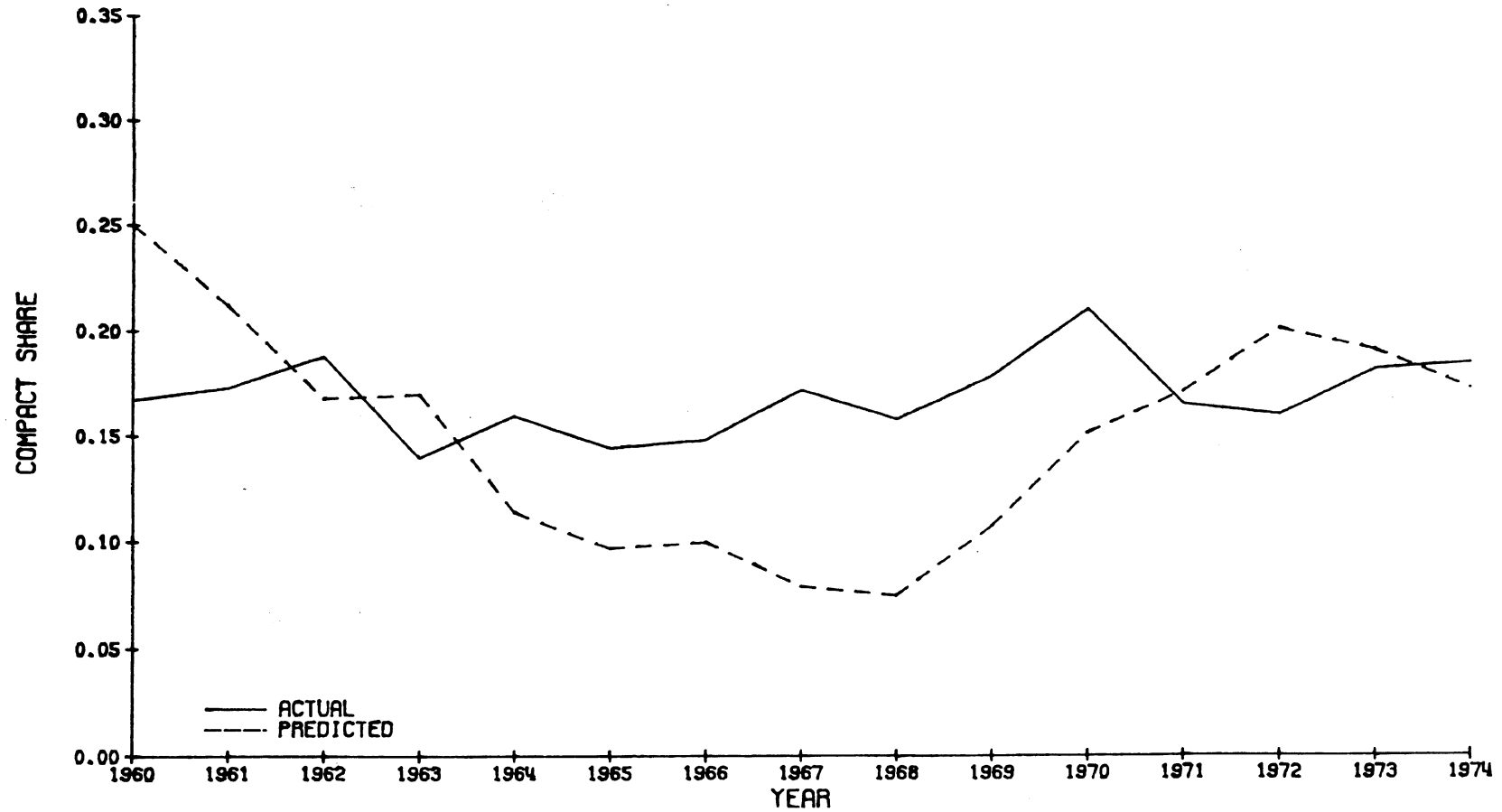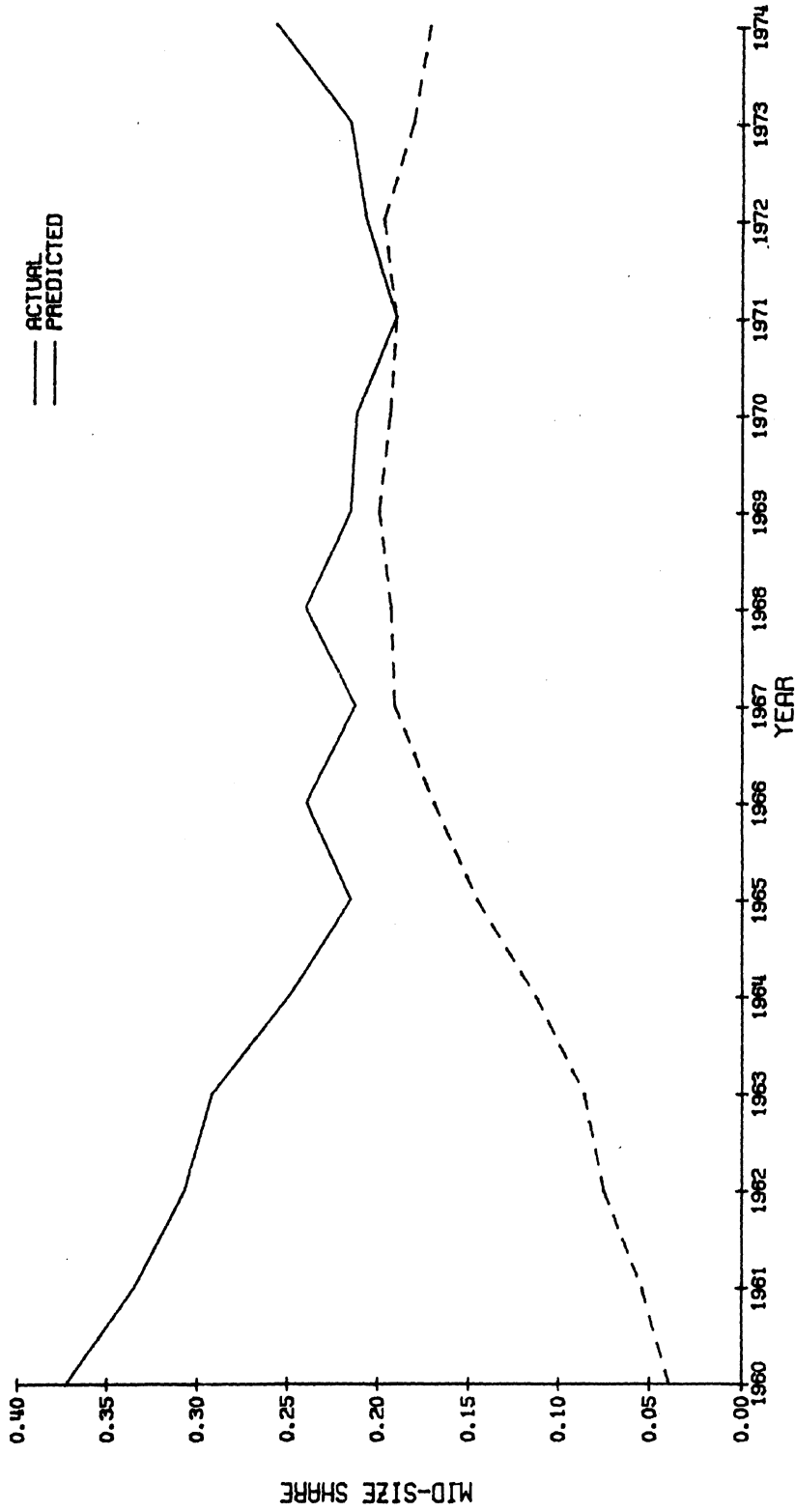he first several years of the simulation period. However, neither share results predicts the changes in the actual shares correctly for the last several years of the simulation period.

The share predictions do depend on the specification of the vehicle survival model. Each of the new car sales shares by size-class equations have the following form:

$$\ln(\text{new car share}) = \ln(\text{desired share of total stock}) + c \cdot \ln(\text{desired share of total stock} - \text{actual share of total stock})$$

where:

c is an estimated coefficient.

If the actual share of total stock which is calculated using the vehicle survival model is predicted poorly, the new car shares will also be predicted poorly.

In conclusion, as the model is presently constructed, the historical predictions for total new car sales and scrappage are fairly close to their historical values. However, the predictions for the new car shares are not as good and should be used carefully.

8. SUMMARY AND RECOMMENDATIONS

The primary objective of the study was to assemble the components of a stochastic model of future vehicle populations (FAPS Model) and write a computer program of the model. The three components of the model are:

1) Automobile Demand Model. This model of stochastic vehicle populations uses several major parts of the model of automobile demand developed by Wharton E.F.A. (21). These parts include the equations for new car sales and the new car shares by size-class.

2) Vehicle Survival Model. This new model of vehicle survival, which is very much different from any existing vehicle survival model, was constructed. This vehicle survival model, which is based on the modified Weibull distribution, explains the difference in the lifetime distributions of car by size-class.

3) Procedure for Specifying Planned and Unplanned Events. The procedure outlined in this report for generating the future values of the model's exogenous variables--that is, the specification of future planned and unplanned events--is based on an interview process developed at Stanford Research Institute (34). Such a procedure, which has not been applied in analyzing future vehicle populations, is an appropriate procedure for incorporating the uncertainty of future values of exogenous variables into the model.

The three components of the model have been coded into an interactive computer program which is outlined in Appendix A.

There are two recommendations for further work. First, the procedure for specifying planned and unplanned events should be

fully tested. Whereas the procedure has been described, its application may be very complex and may require use of some simplifying assumptions in work required to verify its usefulness.

Second, it is recommended that further work be done to improve the vehicle survival model. One of the major weaknesses of all models of vehicle populations evaluated in this study was in estimating vehicle scrappage. While we believe that our vehicle survival model is a significant improvement over any of the models evaluated, there is still room for further refinement.

APPENDIX A

Future Automobile Population Stochastic Model (FAPS Model)
Computer Program Documentation

The computer program of the FAPS Model is designed to:

1) Accept scenario trees as input

2) Exercise the three versions of the model (corresponding to forms 1, 2 and 3 of the scrappage equation discussed in Section 5.0).

3) Produce forecasts up to the year 2000.

The program does not print the forecast results. Instead, the program generates an output file which can be easily analyzed by the statistical analysis package, MIDAS, at The University of Michigan. The MIDAS package then can be used to print the forecast results or manipulate the data so that histograms of forecast results may be generated.

The computer program is designed for conversational use. The following series of prompts are issued upon execution:

1) ENTER FIRST, LAST YEAR OF FORECAST

The forecast period is entered as XXXX,YYYY. Presently, all forecasts must begin with 1975.
Example: 1975,2000

2) ENTER VERSION NO

A "1," "2," or "3," which correspond to the form or the scrappage equation to be used, is entered.

3) ENTER NON-STOCHASTIC EXOGENOUS VARIABLES
VAR,VALUES

Those exogenous variables which are non-stochastic (i.e., their annual values remain constant no matter what path through the scenario tree is exercised) are entered with the variable index first and variable values separated by commas. If the number of values entered is less than the number of forecast periods, the last entry value is assumed for the

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remaining periods. A blank entry line terminates the prompts for non-stochastic exogenous variables.
Example for a five year forecast:
57,56.197,57..108,58.117,59.105,60.102   (U.S. families)
124,5.50   (maximum passbook savings)

4) ENTER TREE CARDS
NODE,YEAR,BRANCH,PROB

Each node of the scenario tree is entered on three cards.
The first card defines a scenario node. At a node, the values
of the stochastic exogenous variables are set. The NODE entry
is the number of the node; YEAR is the year in the forecast to
which the node corresponds; BRANCH is the node from which NODE
is connected; and PROB is the probability of the scenario
branching to NODE given BRANCH. The nodes must be numbered
according to the following rules:

1) All nodes have unique numbers

2) A node in year t must have a higher number than any node
   in a previous year

3) Node numbers are set such that the lowest unassigned node
   number is assigned to that node which is linked to the
   lowest unassigned branch. An unassigned branch is a
   linkage of two nodes where the node of the lesser year is
   assigned and the node of the latter year is not. The
   value of an unassigned branch is equal to the assigned
   node number of the branch.

4) The node corresponding to year previous to the forecast
   period is set to zero.

After the node definition card is entered, a variable list
of the stochastic exogenous variables is entered followed by
the values of these variables on a separate line. A set of
three cards for each node is entered until the entire
scenario tree has been specified. A blank entry line signals
the program that all node information has been specified.
Example for a four year forecast:
In this example, personal income (Variable 58) and unemploy-
ment (Variable 393) are the only stochastic exogenous
variables considered. The scenario tree cards are:

1,1975,0,1.0
58,393
1249.7,8.50
2,1976,2,1.0
58,393
1376.1,7.70

```
3,1977,2,.5
58,393
1520,7.2
4,1977,2,.5
58,393
1600,6.8
5,1978,3,.4
58,393
1650,6.0
6,1978,3,.6
58,393
1700,5.5
7,1978,4,1.0
1650,6.0
```

The scenario tree corresponding to this input is:


```
1974      1975      1976      1977      1978
```
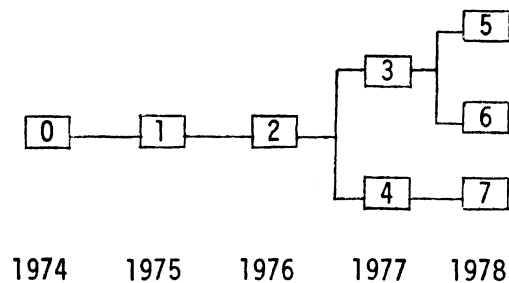
5) ENTER ADJUSTMENTS
    VAR,VALUES

Forecast adjustment values for a variable are entered with
the variable index first and adjustment values separated
by commas. Forecast adjustments are constants added to
variable forecasts to: (1) align forecasts with currently
available data, (2) adjust forecasts for data revisions,
and (3) adjust data for trends not accounted for in the
forecast variable's equation. If the number of values
entered is less than the number of forecast periods, the
last entry is assumed for the remaining periods. A blank
entry terminates input.

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Sample Run of Computer Program

```
ENTER FIRST,LAST YEAR OF FORECAST
?1975,1978

ENTER VERSION NO
?1

ENTER NON-STOCHASTIC EXOGENOUS VARIABLES
VAR,VALUES
?$continue with -base return        ← File -base contains the non-stochastic
?$endfile                             exogenous variables listed in Volume III
                                      of Wharton Model Report (21).
ENTER TREE CARDS
NODE,YEAR,BRANCH,PROB
?$continue with -t return           ← File -t contains the tree cards listed
?$endfile                             on previous page.

ENTER ADJUSTMENTS
VAR,VALUES
?$continue with -adjust return      ← File -adjust contains the adjustment
?$endfile                             constants listed in Volume I of
#r stat:midas                         Wharton Model Report (21).


M I D A S
STATISTICAL RESEARCH LABORATORY
UNIVERSITY OF MICHIGAN
14:06:59
MAR 10, 1978




COMMAND
?read fi=-out c=1-12 v=1-695 l=* fo=single  ← Output of model
                                              program is in file -out

READ OBSERVATIONS 1-12
VARIABLES BY CASE

12 CASES READ FOR 695 VARIABLES
```

```
COMMAND
?wr * v=200,202,203,279,299
```

WRITE OBSERVATIONS
VARIABLES BY CASE

200. SOLUTION	202. YEAR	203. PROBABIL	279. SALES	299. SCRAPPAG
1.0000	1975.0	.30000	8.7406	7.0088
1.0000	1976.0	.30000	11.429	8.4236
1.0000	1977.0	.30000	12.651	10.093
1.0000	1978.0	.30000	11.099	8.8625
2.0000	1975.0	.20000	8.7406	7.0088
2.0000	1976.0	.20000	11.429	8.4236
2.0000	1977.0	.20000	12.651	10.093
2.0000	1978.0	.20000	13.258	9.4641
3.0000	1975.0	.50000	8.7406	7.0088
3.0000	1976.0	.50000	11.429	8.4236
3.0000	1977.0	.50000	17.005	11.147
3.0000	1978.0	.50000	8.9975	9.0710

12 CASES WRITTEN FOR 5 VARIABLES

Three solutions corresponding to the three paths through the scenario tree have been generated.

Solution 1 has a probability of .3 of occurring; Solution 2, .2; and Solution 3, .5. The total new car sales and total scrappage variables are in million of units.

COMMAND
?wr * v=200,202,203,53,398

WRITE OBSERVATIONS
VARIABLES BY CASE

200. SOLUTION	202. YEAR	203. PROBABIL	53. INCOME	398. UNEMPLOY
1.0000	1975.0	.30000	1249.7	8.5000
1.0000	1976.0	.30000	1376.1	7.7000
1.0000	1977.0	.30000	1520.0	7.2000
1.0000	1978.0	.30000	1650.0	6.0000
2.0000	1975.0	.20000	1249.7	8.5000
2.0000	1976.0	.20000	1376.1	7.7000
2.0000	1977.0	.20000	1520.0	7.2000
2.0000	1978.0	.20000	1700.0	5.5000
3.0000	1975.0	.50000	1249.7	8.5000
3.0000	1976.0	.50000	1376.1	7.7000
3.0000	1977.0	.50000	1600.0	6.8000
3.0000	1978.0	.50000	1650.0	6.0000

12 CASES WRITTEN FOR 5 VARIABLES

This output displays the stochastic exogenous variables specified by the tree cards.

APPENDIX B
Literature Survey

B.1 Summary

Since automobile manufacturing has become a major industry
in the United States, many mathematical models of vehicle demand
and survival have been developed. The earlier models reported
in the literature are vehicle survival models, with the first
vehicle demand models appearing in the literature about ten
years later. Over the years these types of models have increased
in sophistication and many additional types of models have been
developed. Included in the list of model types are:

- Short-run sales
- Marketing
- Pricing
- Market Shares
- Fleet

In recent years, with the growing concern over effects
of the domestic fleet on the environment and future fuel supplies,
new types of models have been developed, including:

- Accident
- Air Pollution
- Air Quality
- Energy Consumption
- Fuel Consumption
- Fuel Economy
- Safety

These recent models, in general, are designed to be used as
input for fleet distribution of vehicles by type and age, generated
by vehicle demand and survival models.

In the survey, the principal types of models reviewed are long-run vehicle demand models and vehicle survival models. These two types of models will be the major components of the stochastic model of future vehicle populations to be constructed in this study. The demand and survival models are discussed in section B.2 and B.3, respectively. In addition, those models which can utilize the output of the vehicle demand and survival models are briefly discussed in B.4. These models are called application models.

The last section of the review contains a bibliography. This bibliography is divided into four parts:

- Vehicle demand and survival models
- Application models
- Scenario trees and probability assessment procedures
- Stochastic modeling theory

This bibliography contains the reference material which is being used to build the stochastic model of future vehicle populations.

B.2 Demand Models

The predominant approach used in modeling automobile demand is the stock-adjustment model. Both of the two major models being exercised by the Federal Government today--the Faucett Model and the Wharton Model--are stock-adjustment models. Stock-adjustment models are forms of dynamic econometric models. Their theoretical basis is that gross expenditure on a commodity, measured in units sold, is calculated from the difference between desired stock and stock already available as a result of prior purchases, and the need to replace the old stock which has worn out.

The stock-adjustment approach to modeling automobile demand was first applied separately by Chow and Nerlove in the mid-1950s. Since their applications, this approach has been employed in an increasingly sophisticated manner by researchers. In this survey, the development of this modeling approach is discussed by focusing on a few of the stock-adjustment automobile demand models. Also, because of their importance in defining the automobile demand problem, two studies earlier than Chow's are discussed.

Early Models

Two important models of automobile demand were constructed in the 1930s. In each, an interesting formulation of a demand function is derived. Demand is split into two components: new ownership demand and replacement demand. In the earlier work by deWolff, replacement demand was estimated from yearly age distributions of vehicles on the road, based on registration data.[7] Disposable income was found to correlate well with the estimated replacement demand. New ownership demand was approximated by subtracting the replacement demand from total new car sales. deWolff found that new ownership demand could be modeled as a function of the price index of new cars and a purchasing power indicator as represented by corporate profits.

The second study, by Roos and von Szeliski, was published in 1939.[20] This study is similar to deWolff's, in that automobile demand

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was separated into new owner demand and replacement demand. However, a more sophisticated theory of demand was developed. The theory was based on five assumptions:

1) The demand for automobiles is a derived demand since the primary demand is for a transportation service.

2) Consumers are continuously adjusting the number of cars in operation toward some particular level called "the maximum ownership level."

3) The maximum ownership level is continuously changing according to the economic status of consumers and such variables as car durability and price.

4) Consumers are continuously adjusting the quality of the car population toward an optimum level of replacement.

5) The rate at which the car population is adjusted depends on general and economic conditions.

It is interesting to note that these assumptions are very similar to those underlying stock-adjustment models.

Based on these assumptions, Roos and vonSzeliski hypothesized that new owner sales are a function of the price of cars, an income factor, the capacity of sales outlets, the trade-in price ratio, and the difference between the maximum ownership level and existing stock of cars. The maximum ownership level is hypothesized to depend on an income factor, the price of cars, and the durability of cars as measured by average vehicle life. Replacement sales are assumed to depend on the rate of used car scrappage, the price of cars, the trade-in ratio, and per capita income. The three hypothesized relationships were incorporated into a single relationship for new car sales. Estimates of income and price elasticities using different data sets were calculated. Both the income and price elasticities were found to be quite high (income: 1.38 to 3.51; price: -1.0 to -2.0).

Stock-Adjustment Models

The earliest stock-adjustment model of vehicle demand was developed by Gregory Chow in the 1950's (4). In his model, Chow first develops a model of the desired (equilibrium) total stock of vehicles, in dollars. In this model the desired stock is assumed to be a linear function of economic and demographic conditions. Using his expression for desired stock, Chow reasons that new car purchases can be calculated as the sum of the desired change in stock (a stock-adjustment term) and the depreciation of the old stock. However, Chow argues that within a one year period, consumers in general do not adjust their stock of vehicles to the desired or equilibrium level. An assumption is made that consumers will achieve only a fraction of the change to the equilibrium level.

The lag in adjustment to the equilibrium is explained more fully in an article by Marc Nerlove that appeared about the same time as Chow's work.[19] Nerlove states that the lag in adjustment may result for a variety of reasons. One reason given is that a consumer may be unwilling to change his stock because of anticipation of changes in the current levels of economic variables. Another is that a consumer's budget may be committed to other items such as installment payments or life insurance premiums.

Based on the assumptions outlined, Chow developed his model of new car purchases as follows:

$$X_t = a + bp_t + cI_t$$

where: X_t = desired stock in year t

p = average price of a vehicle

I_t = disposable income

a,b,c = constants

then: $X_{new} = k(X_t - X_{t-1}) + (1 - d)X_{t-1} + v$

$$= ka + kbp_t + kcI_t + (1 - d - k)X_{t-1} + v$$

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where: X_{new} = per capita new car purchases, in units

k = stock-adjustment factor

d = depreciation rate

v = stochastic error

The regression results are:

$$X_{new} = .0779 - .0201p_t - .2310X_{t-1} + .0117I_d$$
$$\phantom{X_{new} = .0779} (.0026)\quad (.0472)\quad\quad (.0011)$$

R^2 = .858   Standard errors in parentheses

The stock-adjustment factor, k, is estimated to be around .59 and the depreciation rate, .74.

Nerlove developed his model in a manner similar to Chow's model. Using Chow's notation, Nerlove assumes that the long-run equilibrium stock can be expressed by:

$$X_t = a + bp_t + cI_t$$

then: $$X_{new_t} = k[X_t - (1-d)X_{t-1}] + (1-k)X_{new_{t-1}}$$

or: $$X_{new_t} = kad - kbp_t - kb\,(1-d)p_{t-1} + kcI_t$$
$$- kc\,(1-d)I_{t-1} + (1-b)X_{new_t} + v$$

In estimating parameters for this equation based on 1921-53 data, Nerlove found the depreciation rate equal to approximately .45 and the stock-adjustment factor equal to about .73. His estimate for the price elasticity is about -.9 and the estimate for the income elasticity is 2.8.

Realizing that the demand for automobiles may depend on more than price and income, several authors built models which include

additional variables in the years following the publication of Chow and Nerlove's models. In 1961, Daniel Suits included a credit term by adjusting the average retail price of cars by the number of months' duration of an average credit contract.(23) Hamburger, in his model, included interest rates and found an interest rate elasticity of -.85. (13) In another model by Saul Hymans, male unemployment rate and a consumer sentiment index were included.(15) .

These models improved the state of the art of vehicle demand modeling, and they provided the groundwork on which the recent, large models have been constructed.

Recent Models

In recent years, many models of new car demand have been developed for forecasting and policy analysis purposes. Of these models, two are being exercised fairly extensively as aids to federal policymaking. These two models are the Faucett Model and the Wharton Model. These two models will be extensively discussed in this section. In addition, brief descriptions of other important recent models are presented.

In 1974, Chase Econometric Associates built a model to assess the probable energy and economic inpacts of six vehicle tax and regulatory assumptions for the Council on Environmental Quality.(3) The primary purpose of the study was to predict gasoline consumption by passenger vehicles for 1974 to 1986. Basic to the model are equations for annual new car sales and market shares by five size classes—subcompacts, compacts, intermediates, standards, and luxury cars. Included in the sales equation is a term for the relative price of gas and oil, which in previous studies had not been significant. The elasticity for this price is -.82 of mean values. The other important factors included in the model are disposable income, relative new car price, unemployment rate, stock of cars, and a credit index.

Also in 1974, Rand Corporation built a new car sales model as part of an annual multi-equation forecasting model.(25) Their equation

for new car sales is a linear function of used car price, the ratio of this year's permanent income per household to last year's permanent income per household, new car prices, and a dummy strike variable. A direct relationship between new car sales and gasoline prices could not be found. However, the equation for used car prices does depend on gasoline prices, so that the model of new car sales does respond to gasoline prices through changes in used car prices. Also, affecting new car sales through used car price is last year's stock of cars per household. Thus, this model does include a stock-adjustment term.

A different approach was taken in the new car sales model developed in 1975 by Energy and Environmental Analysis, Inc. (EEA).[8] EEA found that new car sales increase with positive changes in vehicle miles travelled from one year to the next and decrease when the ratio of new car price to used car price increases. An equation for total vehicle miles per household was estimated to input into the sales equation. This equation indicates that vehicle miles travelled increase as the unemployment rate increases, increase as real disposable income increases, and decrease as the gasoline cost per vehicle mile traveled increases.

James Sweeney also found that vehicle miles travelled is a significant factor affecting new car sales.[10] Sweeney includes this variable, the unemployment rate, and disposable income into a standard stock-adjustment model of new car sales.

Faucett Model

The Faucett Model was written in 1976 under the sponsorship of the Federal Energy Administration.[16] Its objective is to model the effects of alternative fuel economy policies on future gasoline consumption, vehicle miles travelled, new car sales, fleet size, and fleet composition, through the year 2000.

The automobile demand estimator used in the model is a short-run stock-adjustment model which is a variation of the classic stock-

adjustment model used by previous researchers. Adjustment in this model occurs due to a gap between a "target" stock of automobiles, O_t^*, and the "existing" stock of automobiles as of the end of the current period. In the previous stock-adjustment models, the gap is usually defined as the difference between the target stock and the existing stock at the end of the _previous_ period.

The target stock is hypothesized to be related to household income. To derive the target stock, first, automobile ownership per household is related to household income based on the 1970 Consumer Buying Indicators by the following formula:

Auto ownership per household = $0.01786(\text{income})^{.4743}$

This particular form of the relationship was chosen because it reflects the tapering off of additional automobile ownership with income. The target ownership is then computed by multiplying automobile ownership per household estimated for each of six income groups times the number of households in each group, and then summing over all groups.

Once the target automobile ownership is calculated, the gap between target and actual ownership is computed as the difference between target ownership and the stock of cars on hand at the beginning of the year less the number of cars scrapped over the year. The gap term is combined with a generalized price term to estimate the following formula for new car sales:

$$N_t = 286721.3[O_t^* - (\text{Autos}_t - D_t)]^{.2178}(X_t)^{-.17039}$$

where: N_t = Total annual new car sales in year t

O_t^* = Target ownership of automobiles in year t

Autos_t = The stock of automobiles at the beginning of year t

D_t = Scrappage of vehicles over course of year t

X_t = Index of generalized price with 1967=1

The generalized price term includes new car price and a discounted, perceived lifetime price of gasoline for a new car.

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Wharton Model

The Wharton Automobile Demand Model is a very large econometric model designed to forecast the long-run size and composition of the U.S. automobile demand and stock to the year 2000.[21] The model contains about eighty statistically estimated relationships plus some three hundred associated identities. Included in the model forcast outputs are total stock, vehicle miles travelled, the composition of stock by five size classes, yearly demand for new cars by size class, yearly scrappage of cars by vintage and size class, new car prices by size class, and used car prices by size class and vintage.

The Wharton automobile demand estimator is based on a stock-adjustment process similar to the one used in the Faucett Model. New registrations are determined by the gap between a desired, equilibrium stock and the actual stock. The Wharton Model, however, differs from previous models in two major aspects. First, the theoretical concepts used in deriving the desired stock are new and innovative. Second, the yearly scrappage of vehicles is simultaneously determined with new car sales, and thus scrappage is affected by the stock-adjustment process.

The desired stock of automobiles is estimated from state cross-sectional data at one point in time as opposed to time-series data as in earlier studies. Wharton argues that in order to analyze the characteristics of the consumer's decision-making the choice and technology available in vehicles must be held constant. In addition, demographic variations (across states) in desired demand may be analyzed.

Wharton decided to use 1972 cross-sectional data for their estimation of desired stock. They argue that in 1972, desired stock by state was approximately equal to the actual stock. In this year, the economy was fairly stable, with moderate unemployment and inflation; pollution controls had yet to make an impact; and smaller domestic cars had been present in the market for several years.

The desired stock equation is estimated in terms of desired units per family, as follows:

$$\ln(X_t^*/FM) = -1.910 + .563 \ln(Y/FM) - 0.100 \ln(Y15/100-Y15)$$
$$(2.40) \quad (3.13) \quad\quad (1.92)$$

$$- .1995 \ln(CPM/CPI) + .4212 \ln(LD/FM)$$
$$(.84) \quad\quad\quad (3.07)$$

$$- .0537 \ln(MTWNA/FM) + .099 \ln(NPMET/100)$$
$$(1.48) \quad\quad\quad (1.61)$$

$\overline{R}^2 = .461$ t-statistics in parentheses

where:

X_t^*/FM	=	Desired stock per family unit
Y/FM	=	Permanent real disposable income per family
Y15	=	Percentage of families earning $15,000 or more in 1970 dollars
CPM	=	Desired share weighted cost per mile
CPI	=	Consumer Price Index (1972 = 1.0)
LD/FM	=	Number of licensed driver divided by number of family units
MTWNA/FM	=	Number of persons not using an automobile to travel to work divided by the number of family units
NPMET	=	Percentage of population living in SMSA's

As may be expected, the income and number of licensed driver terms have the strongest positive effect on desired stock, with elasticities of .56 and .42, respectively. The negative sign of the term, percentage of families which earn more than $15,000, expressed as an "odds," reflects income saturation--i.e., above a certain income level, families increase their stock of cars at a lower rate with increases in income than below this level.

The desired share weighted cost per mile term is a measure of combined purchase and operating costs and appropriately has a negative

sign. It is computed by first calculating a cost per mile for each size class of vehicle. This cost includes the purchase price and discounted finance and operating costs. The fleet cost per mile is then computed as a weighted average of these size class costs by the desired shares of each class. Formulas for the desired shares are estimated based on 1971 and 1972 state cross sectional data. With this formulation, the desired stock equation responds to changes in the composition of desired stock. However, it should be noted that the t-statistic indicates that the coefficient of the cost-per-mile term is not significantly different from zero. Even though the term is statistically weak, its magnitude and sign seem to justify its inclusion in the model.

With an estimate of desired stock, the new car registration and scrappage equations can be estimated. These equations are expressed as rates as follows:

$$\ln\left(\frac{Xnew}{X_{t-1} - S_t}\right) = 3.79 \ln\left(\frac{X_t^*}{X_{t-1} - S_t}\right) - .255D$$

$$\qquad\qquad\qquad (9.90) \qquad\qquad\qquad (2.49)$$

$$+ 6.039 \ln\left(\frac{Y_d/FM}{Y/FM}\right) - 1.267 \ln\left(\frac{P_t}{P_{t-1}}\right) - 2.915$$

$$\qquad\qquad (8.30) \qquad\qquad\qquad (3.45) \qquad\qquad (35.2)$$

$$\overline{R}^2 = .864$$

and total auto scrappage:

$$\ln\left(\frac{S_t - S20^t}{X_t + Xnew}\right) = -6.98 - 3.828 \ln\left(\frac{X_t^*}{X_t + Xnew}\right) + 2.911A$$

$$\qquad\qquad (7.99) \quad (4.50) \qquad\qquad\qquad (5.32)$$

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$$- .145 \ln\left(\frac{PU}{PS}\right) - .338 \ln U + \sum_{i=0}^{2} \alpha i \ln\left(\frac{VMT}{VMT\,(-1)}\right)_{-i}$$

$$(2.20) \qquad\qquad (4.33)$$

$\alpha o = 2.23 \qquad \alpha_1 = 4.20 \qquad \alpha_2 = 3.45$

$\quad (2.42) \qquad\qquad (3.60) \qquad\qquad (2.86)$

$\overline{R}^2 = .923$

where:
Xnew	=	New car registration
X_t	=	Year-end stocks in opertion in year t
S_t	=	Total auto scrappage
$X_t{}^*$	=	Desired stock in year t
Y_d/FM	=	Real disposable income per family
Y/FM	=	Permanent family income
P_t	=	Average new car price in year t
D	=	Dummy strike variable
$S20^t$	=	Vehicles over 20 years of age
A	=	Average age of stock
PU	=	Average price of used cars
PS	=	Scrap metal price
U	=	Unemployment
VMT	=	Vehicle miles travelled this year
VMT(-1)	=	Vehicle miles travelled last year

In both equations the rate of new car sales and scrappage are functions of the gap between desired and actual stock, and both new sales and scrappage respond strongly (positively and negatively, respectively) to changes in desired stock.

New car registrations also, as expected, fall with sharp increases in new car prices. Similar to the cost-per-mile variable in the desired stock equation, the price variables are calculated based on an averaging of prices by the shares of actual new stock. In an interesting formulation, the actual new shares equations are modeled as a stock-

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adjustment process toward the desired shares. Thus, the new car registrations will respond to changes in the composition of new car registrations through the price variables.

The Wharton Model is a very complex model. However, it is well-thought out and includes many logical, dynamic interactions.

B.3 Vehicle Survival/Scrappage Models

The study of survival of automobiles dates back to the 1920's to an article by Clare Griffin (12). In this article Griffin demonstrates how the "standard" methods of constructing life tables of persons can be applied to constructing life tables of automobiles. His procedure is simply to normalize the registration data by model year to that model's production figures. Thus, if x cars were produced in 1920 and y of these cars remain in 1923, then the fraction $y \div x$ remain in 1923. For each series of model year data, Griffin fitted the following equation:

$$\log L_{m,t} = \log K + t \cdot \log S + c^t \log G$$

where: $L_{m,t}$ = fraction of survivors of model year m in year m + t.

k, S, c, G are constants.

The three components on the right-hand side of the equation represent, according to Griffin, the three causes of automobile retirement: (1) accidents, which occur at a fairly constant rate from year to year, (2) deterioration, which is age-specific, and, (3) obsolescense, which is also age-specific.

Griffin's model of automobile survival was partially incorporated into Roos and von Szeliski's replacement automobile demand model (20). In particular, they used Griffin's model to calculate "theoretical" scrappage rates for vehicles of vintage 1919 to 1926. For vehicles of vintage after 1926, they adjusted Griffin's rates by the actual scrappage data to derive the theoretical scrappage. From this information, they generated a time series (1919 to 1938) of yearly theoretical aggregate scrappage which was included in their equation for replacement rates.

The majority of the vehicle survival/scrappage models since Griffin have not been very sophisticated. Usually, average

survival rates are calculated based on several years of registration data. This method assumes fixed survival rates, and likewise, fixed scrappage rates. Some other models assume a constant survival rate. These models, therefore, assume that vehicle survival is an exponential or geometric function of age. In the last ten years, a few innovative models of vehicle survival have been constructed. Some of these models will be discussed in the remainder of this review.

In 1968, Franklin Walker published a vehicle scrappage model based on the logistic function (24). He found that the relationship between the average scrappage rates, based on new and used car registrations compiled by R. L. Polk Company, and the age of vehicle can be represented by the logistic function:

$$M_t = 1/(L+Be^{-kt}) + E_t$$

where: M_t is the mean scrappage rate of vehicles of age t.

L, B are parameters.

E_t is a random error term.

Walker used this scrappage equation to investigate the fluctuations in annual aggregate vehicle scrappage from the expected annual scrappage. He constructed a model that relates these fluctuations to (1) the rate of turnover in automobile ownerships, for which the ratio of new-car to total registrations is used as a surrogate, and (2) the level of used car prices to relative costs of representative car repair services. His results show that almost two-thirds of the variations of the actual scrappage from its trend is explained by these two factors.

In 1974, Chase Econometrics published a vehicle survival model that is quite different from previous models (3). This model assumes that the survival rates of a model year of vehicles, as calculated by dividing current year registrations for a model year by the original registrations of that model year's production run, is related to the age of the model year car by the following function:

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$$\frac{R_{my,t}}{R_{my}} = ae^{bT^3}$$

where: $R_{my,t}$ = current registrations for the model year (my) in year t.

R_{my} = original registrations.

T = t-my = age of the model year car.

a,b = model year parameters.

Chase calculated separate model year car survival functions for 1951 to 1968 model year vehicles. However, no attempt is made in this model, as in Walker's model, to adjust total scrappage by exogenous factors, or to explain the differences in the parameters of the survival functions for different model year vehicles.

In addition, neither Chase nor previous investigators had looked at the effect of exogenous factors on the scrappage rates of vehicles of various ages. In the Marketing and Mobility Panel Report of the Motor Vehicle Goals Study (16), the influence of economic factors, specifically new car prices and unemployment rate, on scrappage rates of older cars is investigated. The arguments outlined are that as new car prices rise, new cars become less attractive, creating a tendency to postpone new car purchases and to hold onto existing stock. Also, new car prices are assumed to be positively related to used car prices, and a rise in new car prices causes a rise in used car prices. This changes a consumer's decision when to scrap or repair a vehicle.

The unemployment rate is included in the model as an indicator of cyclical macroeconomic conditions. Thus, it is expected as unemployment increases, scrappage would decrease.

The specific scrappage model developed assumes the scrappage rates for each age of the first eight years of a vehicle's life are fixed at the average levels based on the 1957 to 1974 model year Polk registration data. These average scrappage rates increase from .2 percent for a one-year-old vehicle to 15.7 percent for an eight-

year-old vehicle. For nine-year-old and older vehicles, an adjustment to the scrappage rate is calculated based on the following formula:

$$SPG_t = 0.40675 - 0.78433(P_n)_t - 0.015519U_t$$
$$(9.856) \quad (-1.911) \quad\quad (-3.052)$$

$R^2 = .6587$ degrees in freedom = 10

$(P_n)_t$ = new car real price index U_t = unemployment rate in
          in year t   year t.

(parenthetical numbers are t-statistics based on the regression estimates). SPG_t is divided by the average scrappage rate for nine-year or older vehicles and multiplied by the average scrappage rate for a vehicle of a specific age to determine the adjusted scrappage rate.

This model, originally developed by Jack Faucett Associates, assumes, as do all the previous models, that each size category of vehicles (market class) has the same average scrappage rates. However, the adjustment procedure is applied separately for each market class of vehicle in their forecasting model. This allows unusual average price changes in a single class to be reflected in the scrappage of that class of vehicles alone.

Understanding the importance of accounting for yearly variations in the scrappage rates, Wharton Econometric Associates constructed a complex scrappage model with annual adjustments (21). Unlike the Faucett procedure, in the Wharton model the adjustment factors are applied to the scrappage rates of all ages of vehicles. The derivation of these adjustment factors is quite interesting.

Wharton first assumes that all vehicles are scrapped by the end of their 20th year. The average scrappage rates (q_i, i=0, ..., 20) for the first 20 years of vehicle life are calculated from the 1953 to 1974 Polk registration data of vehicles in operation. The adjustment factors are calculated numerically by a recursive procedure beginning with 1953. To begin the recursive

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procedure, the scrappage rates for the 20 years prior to 1953 are assumed to be constant and equal to the average rates. The adjustment factor for 1953 is then:

$$a_{1953} = \frac{\text{Actual total 1953 scrappage}}{\text{Theoretical total 1953 scrappage}}$$

where the theoretical total 1953 scrappage equals:

$$q_0 N_{1953} + \sum_{i=1}^{20} q_i \prod_{j=0}^{i-1} (1-q_j) \; N_{1953-i}$$

where: N_k = number of new registrations in model year k.

q_i = average scrappage rate of vehicle of age i years.

the calculation for the adjustment factor for 1954 is:

$$a_{1954} = \frac{\text{Actual total 1954 scrappage}}{\text{Theoretical total 1954 scrappage}}$$

where the 1954 theoretical scrappage equals:

$$q_0 N_{1954} + \sum_{i=1}^{20} q_i (1-a_{1953} q_{i-1}) N_{1953}$$

$$+ \sum_{i=2}^{20} q_i (1-a_{1953} q_{i-1}) \prod_{j=0}^{i-2} (1-q_j) \; N_{1954-i}$$

The procedure is continued until all adjustment factors through 1974 are calculated.

Wharton notes in their documentation that for 1955 through 1957, when new car demand was strong, scrappage rates were higher than average. The adjustment factor drops sharply in the 1958 recession period and climbs slowly through 1965, then fluctuates around one through 1973 when new car demand was strong. The adjustment factor falls significantly below one in 1974, a recession year.

The forecast of adjustment factors in the Wharton model is generated by the same recursive formula. However, the actual total scrappage in the formula is substituted by the estimated total scrappage generated in the Wharton Automobile Demand Model. This total scrappage equation is a function of new car registrations, scrap metal prices, vehicle miles travelled, and other variables. Thus, the adjustment factor would reflect these variables also.

B.4 Application Models

Application models are those which utilize the
output of vehicle demand and survival models to investigate
automobile-related inpacts on safety, air quality, fuel consump-
tion, etc. Quite often an application model is built as a sub-
model to a vehicle demand and survival model. For example, the
Chase (3), EEA (8), Faucett (16), and Rand (25) models discussed
in section 2.3 have submodels to compute fleet fuel consumption.

The application models are usually accounting models based
on a set of simplifying assumptions. As an example of how a
typical fuel consumption model is constructed, the Faucett model
is discussed here.

Historical data, the output of the Faucett vehicle demand,
survival, and market share models are used to generate age dis-
tribution of the fleet of vehicles by three size categories.
Using these age distributions, vehicle miles travelled by
vehicle age is determined by an application of the age/VMT
relationship reported by the Federal Highway Administration,
as follows:

$$VMT\ (AGE) = \sum_{sc=1}^{3} FLEET\ (SC,AGE) * 17.9729$$

$$-9.57481 * LOG_{10}\ (AGE)$$

Where: VMT(AGE)= vehicle miles travelled in a year for vehicles
of age, AGE.

SC = size class

FLEET(SC,AGE)= Number of cars in the fleet of size class,
SC and age, AGE.

Note that an assumption is made that each size class of vehicle
is driven the same average miles per year. The fuel economy of each
age of vehicle is computed by dividing the number of vehicles of

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that age by the sum of the division of the number of vehicles in each size class of that age by the fuel economy for that size class and vintage of vehicle. This is simply an averaging procedure. Note that the fuel economy of a particular vehicle is assumed not to change with age. The total fuel consumption is then calculated by summing the total miles travelled divided by the fuel economy for each age of vehicle.

Many models with similar outputs as the application models have been constructed but are not listed in the bibliography. This is because these models do not require age distributions of vehicles as inputs.

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APPENDIX C

Notes on the Weibull Distribution

The Weibull distribution was proposed in 1939 by Waloddi Weibull, a professor at the Royal Institute of Technology in Sweden, as an appropriate distribution to describe the life length of metals.* He formulated the distribution as follows:

Assume that we have a chain consisting of n links, and if any link breaks (fails) then we say the chain as a whole fails. Accordingly, the probability of nonfailure of the chain, $(1-P_n)$, is equal to the probability of the simultaneous nonfailure of all the links. Thus:

$$(1-P_n) = (1-P)^n$$

where:

P is the probability of failure of any link of the chain. Considering only the class of failure distributions of the form:

$$P = F(x) = 1-e^{-q(x)}$$

where:

x is a load level applied to a link.
then the probability of nonfailure of the chain is:

$$(1-P_n) = (e^{-q(x)})^n = e^{-nq(x)}$$

The simplest form of $q(x)$ such that $F(x)$ is a positive, nondecreasing function, vanishing at a value x_u, which is not necessarily equal to zero, is:

$$q(x) = c(x-x_u)^b$$

and thus:

$$F(x) = 1-e^{-c(x-x_u)^b} \qquad x \leq x_u$$

The formula for nonfailure of the chain becomes:

$$(1-P_n) = e^{-nc(x-x_u)^b} \qquad x \leq x_u$$

If x_u is set to zero, and nc is set to a, then:

$$(1-P_n) = S(x) = e^{-ax^b} \qquad (i)$$

which is the Weibull survival distribution (equation [6]) discussed in Section 5.1

It is interesting that Weibull proposed this distribution without recognizing that it is an extreme-value distribution. The extreme-value random variable corresponding to the Weibull distribution is the weakest and/or shortest-lived component of a system of components. Gumbell, in his book, <u>The Statistics of Extremes</u> (38), derives the Weibull distribution by applying a logarithmic transformation to the random variable of the minimum extreme-value distribution, as follows:

The density function for the minimum extreme value distribution is:

$$f(x) = be^{-b(x-c)} \; e^{-e^{-b(x-c)}}$$

If we make the change of variable $x = \ln(1/t)$ and set $c = \ln(1/d)$, then:

$$f(t) = \frac{b}{t} e^{-b(\ln(\frac{1}{t}) - \ln(\frac{1}{d}))} \; e^{-e^{-b(\ln(\frac{1}{t}) - \ln(\frac{1}{d}))}}$$

$$= b_t^{b-1} (\frac{1}{d})^b \; e^{-(\frac{t}{d})^b}$$

If we let:

$$(\frac{1}{d})^b = a$$

then:

$$f(t) = bat^{b-1} e^{-at^b}$$

which is the Weibull density function corresponding to the survival distribution (i).

Thus, as argued in Section 5.1, if a car is thought of as being composed of several critical components such that the car is scrapped when one of these components fails, then the lifetime distribution of the car tends toward a Weibull. This is because, as presented above, a Weibull distribution models the lifetime of the shortest-lived component in a system of components.

APPENDIX D
REPORT OF INVENTIONS

The objective of this study was to investigate the uncertainties of predicting future automobile populations. To this end, two components were added to an existing model of new car sales. The first was a procedure of specifying uncertain future events and their effect on new car sales and other variables affecting the automobile fleet. The second was an improved model of vehicle survival from year to year. Both components were innovative in application, but not entirely new inventions. As discussed in the report, the procedure of quantifying uncertainty is based on a method developed and used by the Stanford Research Institute. The model of vehicle survival depends on the Weibull distribution of statistics proposed by Waloddi Weibull of Sweden.

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