International Journal of Thermophysics (2021) 42:34
https://doi.org/10.1007/s10765-020-02786-6

# Model for Predicting the Thermal Conductivity of Concrete

**Xiaochun Lu[1] · Fuguo Tong[1,2] · Gang Liu[1,2] · Jinkun Guan[1]**

Received: 9 November 2020 / Accepted: 16 December 2020 / Published online: 19 January 2021


**Abstract**
Thermal conductivity of concrete greatly influences the heat transfer of buildings and affected by many factors. This paper presents a prediction model for thermal conductivity of concrete by adopting the theory of Wiener bounds and considering concrete to have four components (water, air, aggregate, and cement mortar). The proposed model considers the combined effects of porosity, water saturation, and the volume fraction of aggregate on the thermal conductivity of concrete by weighting parameters of $\eta_1$, $\eta_2$, $\eta_3$, respectively. By adjusting the weighting parameters of each component, the model can consider the influence of various factors on the thermal conductivity of concrete more comprehensively. Thermal conductivity of each component and expression of weighting parameters are determined by literature and experiments. The proposed model has been verified by the measured thermal conductivity of concrete under different porosities, water content, and volume fractions of aggregate with the prediction accuracy of $\pm 12\%$. Finally, the regularity of the change in the thermal conductivity of concrete with porosity, water saturation, the volume fraction of aggregate, and temperature is analyzed.

**Keywords** Least-square method · Prediction model · Thermal conductivity of concrete · Wiener bounds

## 1 Introduction

The thermal conductivity of concrete is an important parameter to measure the ability of concrete to conduct heat and depends on the water content, porosity, temperature, and type of cement [1–3]. Given that the influence factors are complex, determining the thermal conductivity of concrete is difficult. Laboratory test is a

✉ Gang Liu
hkliugang@126.com

1 College of Hydraulic & Environment Engineering, China Three Gorges University, Yichang 443000, Hubei, China

2 Hubei Key Laboratory of Construction and Management in Hydropower Engineering, China Three Gorges University, Yichang 443000, Hubei, China

good way to obtain thermal conductivity of concrete and can be classified into two categories. Guarded hot plate and heat flow meter method, which are steady-state method developed from Fourier's law [4], are usually used to measure the homogenous materials. In addition, transient measurement techniques that make full use of the process of temperature changes over time [5–7] are also used to test the thermal conductivity of concrete. Transient plane source (TPS) method utilizes a heated plane sensor to monitor the temperature increasing process of concrete and a Hot Disk thermal analyzer to calculate the thermal conductivity of concrete [8, 9]. The precision of transient measurement methods depends on the analysis program [4]. However, such test methods remain unsuitable for practical applications because the factors affecting the thermal conductivity of concrete are too complicated [2]. Therefore, many researchers have attempted to develop a theoretical model for predicting the thermal conductivity of concrete. Prediction models that translate the experimental results into mathematical models allow researchers to obtain the characteristics that are difficult to measure experimentally [10].

Studies have provided many models for predicting the thermal conductivity of concrete [1, 3, 11–13]. Maxwell [12] proposed an effective thermal conductivity model of composite materials that can be used to calculate the thermal conductivity of concrete when concrete is simplified into a composite of aggregate and cement paste. Bruggeman [14] and Hasselman [15] developed the Maxwell model, which can consider the effects among different aggregates. Zhang et al. [16] considered that the concrete consisted of cement mortar (continuous phase) and coarse aggregate (dispersed phase) and developed a mesoscale model from Maxwell's design to predict the thermal conductivity of the damaged concrete. However, such models are only suitable for cement under total saturation or dry states. In other words, models with two-component medium fail to consider the effects of water content [1, 17]. Wang et al. [18] found that a 15% increment exists for the thermal conductivity of normal concrete when the water content increases from 0 to 10%. Given that the majority of concrete in the natural environment are unsaturated, a method for obtaining concrete thermal conductivity that considers the effects of cement paste, aggregate, and water content is necessary. Kim et al. [19] presented an empirical study that was fitted with the limited experimental data to predict the thermal conductivity of concrete. Meshgin [20] developed a multiphase and multi-scale model for PCM concrete by repeatedly using the generalized self-consistent [21] model at different scale levels. Tan et al. [22] presented a thermal conductivity model for high-permeability high-strength concrete that was built on the basis of the resistance network model of parallel aggregate, polymer slurry, and air. Jin et al. [23] proposed a three-phase fractal model for unsaturated autoclaved aerated concrete that considered the effects of water content and porosity. These concepts that regard concrete as multiphase and multi-component materials are inspiring.

In this study, concrete is considered as a continuous porous medium that consists of cement mortar, aggregate, water, and air. On the basis of this assumption and the theory of Wiener bounds, a mathematical model for normal concrete is presented with the variables of water content, porosity, volume fraction of aggregate, temperature, and thermal conductivity of cement mortar, aggregate, water, and air. Then, a series of laboratory experiments are performed to test the conductivity of each component and concrete, followed by the measurement of parameters for the thermal conductivity model. Finally, the proposed model is validated by the experimental results, and the regularity of the change in the thermal conductivity of concrete with water content and porosity is analyzed.

# 2 Derivation of the Thermal Conductivity Model of Concrete

## 2.1 Theoretical Boundaries of the Thermal Conductivity of the Porous Medium

Porous medium, such as concrete, soil, and rock, can be treated as continuous three-phase mixtures composed of solid, liquid, and gas phases. The voids of solid media are filled with liquid water and gas, and the volume fraction of each phase can be described by $\emptyset_\alpha$ (where $\alpha = s, l, g$ for solid, liquid, and gas, respectively). The total sum of the three phases is equal to 1. Theory of thermal conductivity of porous medium posits that a boundary called Wiener bound exists for the mixtures [24–26]:

$$\left[\sum \frac{\emptyset_\alpha}{\lambda_\alpha}\right]^{-1} = \lambda_m^L \le \lambda \le \lambda_m^U = \sum \emptyset_\alpha \lambda_\alpha, \tag{1}$$

where $\lambda$ is the thermal conductivity of the porous medium; $\lambda_\alpha$ is the thermal conductivity of $\alpha$ phase; $\lambda_m^L$ and $\lambda_m^U$ are the lower and upper Wiener bounds, respectively, which can be reached when all components of concrete are arranged in series and parallel (Fig. 1). The components of a porous medium in nature are not solely arranged in series or parallel; such arrangements are usually a mixture of both. Therefore, a coefficient is necessary to analyze the ratio of serial or parallel arrangements [24]. The thermal conductivity model that can express the possible arrangements of concrete between the lower and upper bounds is expressed as

$$\lambda = \eta\lambda_m^L + (1 - \eta)\lambda_m^U, \tag{2}$$

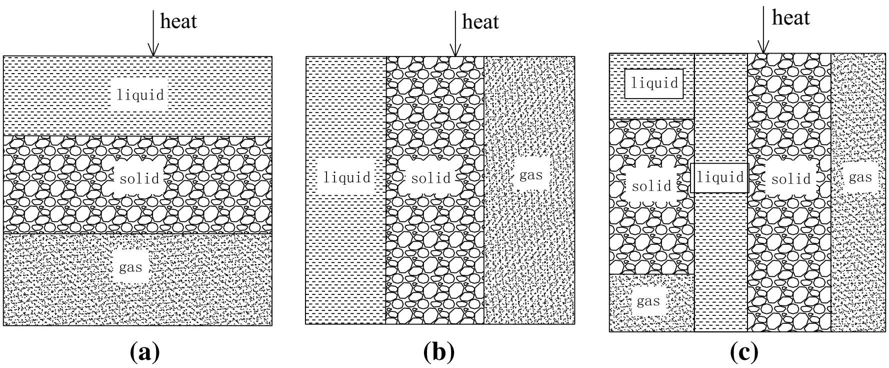


**Fig. 1** (a) Serial, (b) parallel, (c) mixture of serial and parallel arrangements

where $\eta$ is the coefficient and its value ranges from 0 to 1.

### 2.2 Derivation of the Thermal Conductivity Model of Concrete

The derivation of the thermal conductivity model of concrete is mainly divided into two steps. First, the two parts of normal concrete are solid and pore. The solid part consists of cement mortar and aggregate, whereas the pores are filled with water and gas (Fig. 2). Therefore, the thermal conductivity of concrete can be expressed as

$$\lambda = \eta_1\left[\frac{1-\emptyset}{\lambda_s}+\frac{\emptyset}{\lambda_p}\right]^{-1} + \left(1-\eta_1\right)\left[\left(1-\emptyset\right)\lambda_s + \emptyset\lambda_p\right], \tag{3}$$

where $\lambda_s$ and $\lambda_p$ are the thermal conductivity of solid and pore, respectively; $\emptyset$ is the porosity, which refers to the proportion of total pore volume to concrete volume; and $\eta_1$ is the parameter used to weight the proportion of the serial arrangement of solid and pore in the concrete [27]. The value of $\eta_1$ depends on the pore structure of concrete and is the function of porosity, which can be expressed as $0 < \eta_1\left(\emptyset\right) < 1$.

Second, the thermal conductivity of solids in concrete can be calculated using the components of cement mortar and aggregate according to the Wiener bounds as follows:

$$\lambda_s = \eta_2\left[\frac{1-\theta}{\lambda_c}+\frac{\theta}{\lambda_g}\right]^{-1} + \left(1-\eta_2\right)\left[(1-\theta)\lambda_c + \theta\lambda_g\right], \tag{4}$$

where $\lambda_c$ and $\lambda_g$ are the thermal conductivities of cement mortar and aggregate, respectively; $\theta$ is the volume ratio of cement mortar and aggregate; $\eta_2$ is the coefficient used to evaluate the proportion of cement mortar and aggregate arranged in

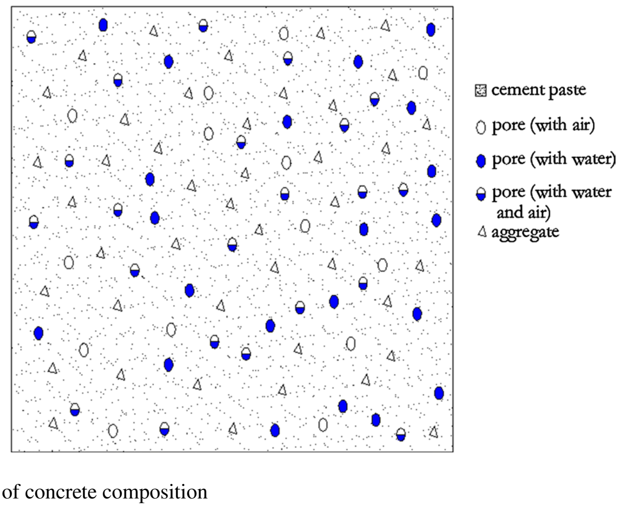


**Fig. 2** Diagram of concrete composition

series and only depends on the volume fraction of aggregate and can be defined as $0 < \eta_2(\theta) < 1$.

Similarly, the thermal conductivity of pores in concrete can be expressed by the thermal conductivity of water and air because the pores are filled with water or air. Therefore, $\lambda_p$ can be written as

$$\lambda_p = \eta_3 \left[ \frac{1 - S_r}{\lambda_a} + \frac{S_r}{\lambda_w} \right]^{-1} + \left(1 - \eta_3\right)\left[\left(1 - S_r\right)\lambda_a + S_r \lambda_w\right], \tag{5}$$

where $\lambda_a$ and $\lambda_w$ are the thermal conductivities of air and water, respectively; $S_r$ is the volume fraction of water in the pores; $\eta_3$ is the coefficient similar with the $\eta_1$ and $\eta_2$ and only related to $S_r$, $0 < \eta_3\left(S_r\right) < 1$.

By substituting Eqs. 4 and 5 into Eq. 3, the thermal conductivity of concrete is finally expressed as

$$\begin{aligned} \lambda = \eta_1 & \left\{ \frac{1 - \emptyset}{\eta_2\left[\frac{1-\theta}{\lambda_c} + \frac{\theta}{\lambda_g}\right]^{-1} + \left(1 - \eta_2\right)\left[(1-\theta)\lambda_c + \theta\lambda_g\right]} + \frac{\emptyset}{\eta_3\left[\frac{1-S_r}{\lambda_a} + \frac{S_r}{\lambda_w}\right]^{-1} + \left(1 - \eta_3\right)\left[\left(1 - S_r\right)\lambda_a + S_r\lambda_w\right]} \right\}^{-1} + \\ \left(1 - \eta_1\right) & \left\{ (1 - \emptyset)\left[ \frac{\eta_2}{\frac{1-\theta}{\lambda_c} + \frac{\theta}{\lambda_g}} + \left(1 - \eta_2\right)\left[(1-\theta)\lambda_c + \theta\lambda_g\right] \right] + \emptyset\left[ \frac{\eta_3}{\frac{1-S_r}{\lambda_a} + \frac{S_r}{\lambda_w}} + \left(1 - \eta_3\right)\left[\left(1 - S_r\right)\lambda_a + S_r\lambda_w\right] \right] \right\} \end{aligned} \tag{6}$$

The thermal conductivity model of concrete contains 10 parameters, namely, porosity; water saturation; volume fraction of aggregate; thermal conductivities of cement mortar, aggregate, water, and air; and weight parameters ($\eta_1$, $\eta_2$, and $\eta_3$). From the microcosmic angle, the interfacial thermal barrier resistance between the aggregate and the cement mortar affects the thermal conductivity of concrete [15]. The thermal conductivity of concrete with different porosities, aggregate volume fractions, and water saturations should be tested to determine the $\eta_1$, $\eta_2$, and $\eta_3$ parameters. Considering that the particle size distribution of the aggregate is not the main factor governing the thermal conductivity of concrete [37], and the aggregate particle size distribution needs to meet the continuous gradation and maximum density requirements, therefore, the influence of the change in the particle size distribution is ignored in this model [4, 28].

## 3 Experimental Test for the Thermal Conductivity of Concrete

Several parameters, such as thermal conductivity of aggregates, cement mortar, and concrete, should be tested by conducting experiments to determine the thermal conductivity model of concrete. However, some basic parameters, such as thermal conductivity of water and air, can be obtained through literature queries.

### 3.1 Thermal Conductivity of Water and Air

The thermal conductivity of water is usually affected by the temperature and pressure of the environment. According to a test performed by the International Association for the Properties of Water and Steam, the value of thermal conductivity of water under standard condition (298.15 K and 0.1 MPa) is as follows [29]:

$$\lambda_w\ (298.15,\ 0.1\ \text{MPa}) = 0.6065 \pm 0.0036\ \text{Wm}^{-1}\cdot\text{K}^{-1}$$

The thermal conductivity of water under different temperatures can be obtained using the following equation:

$$\begin{cases} \lambda_w(\text{T}) = \lambda^* * \lambda_w(298.15, 0.1MPa) \\ \lambda^* = -1.48445 + 4.122927T^* - 1.63866T^{*2} \\ T^* = T/298.15(274 \le T \le 370K) \end{cases}, \tag{7}$$

where T is the temperature (unit: K). When the temperature is 25 °C, the thermal conductivity of water is 0.6064 $\text{Wm}^{-1}\cdot\text{K}^{-1}$.

Similarly, by ignoring the water vapor in the air, the thermal conductivity of air in concrete is affected by temperature and air density as follows [24, 30]:

$$\lambda_a = \left[4.55 + 0.072T + (36.17 - 0.016T)\rho_a + (47.4 + 0.121T)\left(\rho_a\right)^2\right] \times 10^{-4}, \tag{8}$$

where $\rho_a$ is the density of air (unit: kg/m$^3$). In this study, we treat the density as a constant and neglect the effects of density change. Thus, the thermal conductivity of air under 25 °C is 0.0178 $\text{Wm}^{-1}\cdot\text{K}^{-1}$.

### 3.2 Thermal Conductivity of Cement Mortar and Aggregate

The solid phase in concrete mainly consists of cement mortar and aggregate, whereas the thermal conductivity of cement mortar depends on the type of cement and sand used; and the thermal conductivity of the aggregate depends on its mineral composition [31]. In this study, the thermal conductivity of cement mortar and aggregate is directly investigated for certain types of cement and aggregates to determine the thermal conductivity model of concrete.

The cement mortar is mixed at a ratio of water:binder:sand = 1:1.9:6.65, which is consistent with concrete (Table 1). This experiment uses Huaxin Brand cement, and its strength grade is 32.5 MPa. In accordance with the requirements of this research, we adopt a DRT-III thermal conductivity tester (Fig. 3), which is developed on the basis of the transient plane heat source technology (TPS) [8, 9].

The cement mortar and aggregate are cut into thin slices according to the requirements of the thermal conductivity test. Theoretically, the thermal conductivities of cement mortar and aggregate vary with water content, but the effects of the water content have been considered in the thermal conductivity of pore $\lambda_p$, so the effects

**Table 1** The mix design used in the experiment

| Gradation | Water–binder ratio | Sand rate (%) | Water reducing agent (%) | Air entraining agent (%) | Concrete material consumption (kg/m$^3$) | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | Water | Cement | Fly ash | Expansive agent | Sand | Stone |
| Three | 0.53 | 34 | 0.5 | 0.0111 | 108 | 123 | 82 | 0 | 718 | 1398 |

of water content on cement mortar and aggregate are not considered here again. In other words, the thermal conductivity of cement mortar and aggregate is only affected by temperature in the solving of the model. On the basis of the aforementioned simplification, the thermal conductivities of cement mortar and aggregate under the total dry state that vary with temperature can be expressed as

$$\begin{cases} \lambda_c = -0.00104(T - 298.15) + 1.520Wm^{-1}K^{-1} \\ \lambda_g = -0.00163(T - 298.15) + 2.769Wm^{-1}K^{-1} \end{cases}, \tag{9}$$

When the temperature is 25 °C, the thermal conductivities of cement mortar and aggregate are 1.520 and $2.769Wm^{-1}K^{-1}$, respectively.

### 3.3 Experimental Tests for the Thermal Conductivity of Concrete

#### 3.3.1 Concrete Thermal Conductivity Test Method Based on the Least-Square Finite Element Method (LSFEM)

The techniques commonly used to test the thermal conductivity of concrete are the steady-state and transient methods [32, 33]. The steady-state method is usually time-consuming and suitable for materials with constant thermal conductivity [4]. Given that the thermal conductivity of concrete varies with many factors, such as water content and temperature, a transient testing method based on the LSFEM [5] is used to measure the thermal conductivity of concrete. The temperature disturbance is applied to one end of the cylindrical concrete sample to monitor the internal temperature response data. The monitored temperature data with spatial and temporal differences are used as input parameters to solve the heat conduction equation. The thermal conductivity of concrete can be obtained using the LSFEM.

#### 3.3.2 Experimental Design and Materials

The data of temperature variation are crucial in measuring the thermal conductivity of concrete. On the basis of the LSFEM principle, a concrete cylinder with diameter and height of 30 and 50 cm, respectively, is used to obtain the needed data (Fig. 4). A series of high-precision temperature and humidity sensors is buried into the concrete along the axial direction. Such sensors divide the concrete cylinder into eight layers (#1-#8). The location and the type of sensors buried into the concrete are shown in Fig. 4. A temperature sensor is buried into the center of each layer, and a


**Fig. 3** DRT-III thermal conductivity tester

humidity sensor is embedded into the center of layer #5. Figure 5 shows the concrete sample and the sample with insulation. The white and the black lines in the sample are temperature and humidity sensors, respectively. The uncertainty of measurement comes from the following aspects: accuracy of data collected by temperature sensor, embedding position of the sensor, and the calculation program of the thermal conductivity. All temperature sensors are calibrated before use, and their positions are corrected after the sensor is buried to reduce the measurement error. The calculation program also uses standard materials with known thermal conductivities to perform the error calibration.

According to the thermal conductivity test in [5], the heat transfer process should be one-dimensional to obtain thermal conductivity easily. Therefore, except for the top surface, the concrete surfaces are insulated to transfer the heat in the environment into the cylinder concrete along the axial direction. Three temperature sensors are buried into the edge of layers #3, #5, and #7 to monitor the near-surface temperature which can be used to compare with the temperature in the center of the sample to evaluate whether the heat is conducted along the axial direction. The concrete samples are cured under standard conditions (temperature = 25 °C and humidity ≥ 95%) for 28 days before testing the thermal conductivity. The concrete mix design used in the experiment is shown in Table 1, which is the basis for all other schemes. For example, when testing the concrete thermal conductivity test of different aggregate volume fractions, only the variable of the aggregate volume fraction is changed.

#### 3.3.3 Test Results of Thermal Conductivity of Concrete

After curing under standard conditions, the sample is saturated and placed in an environmental test chamber which has the ability to control the environment temperature from -40 °C to 60 °C. In the testing process, the temperature of the curing

**Fig. 4** Diagram of sensor location

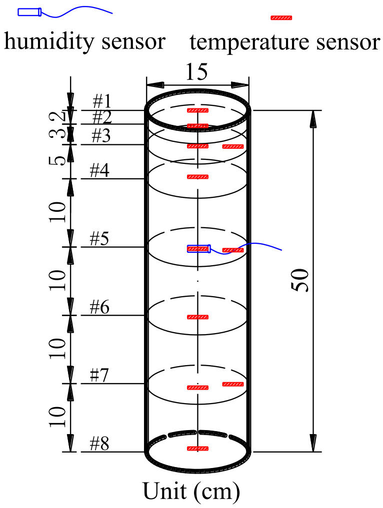


**Fig. 5** Concrete samples and the sample with insulation

environment changes periodically from -25 °C to 35 °C with an oscillation period of 2 h. The surface temperature of concrete is the same as the environment temperature and the temperature at different depths from the surface is presented in Fig. 6. Taking the data of concrete temperature as the input parameter, the thermal conductivity of concrete under different depths can be obtained by using LSFEM [5].

On the basis of LSFEM, the thermal conductivity of concrete at different depths can be obtained at each time step. Figure 7(a) shows the evolution of thermal conductivity of concrete at 0–2 cm. The results indicate that the thermal conductivity of concrete fluctuates with time and has a similar regularity with the change in temperature. The thermal conductivity of concrete can be treated as constant under a certain temperature because concrete has been properly cured to maturity. Therefore, the fluctuation of the thermal conductivity of concrete with time is mainly affected by the temperature. The relationship between the thermal conductivity and temperature of concrete is shown in Fig. 7(b). The thermal conductivity of concrete increases with the increase in temperature and a quadratic function can be used to fit the relationship of the thermal conductivity and temperature of concrete with high correlation in the range of − 25 °C to 40 °C. The thermal conductivity of concrete increases first and then decreases with the increase of temperature, and reaches the minimum near 0 °C. The reason that the thermal conductivity of concrete still increases when the temperature is below 0 °C is that the water in the pore condenses into ice, and the thermal conductivity of ice (almost 2.2 $Wm^{-1}K^{-1}$) is higher than that of water [34].

The value of concrete porosity can be obtained by the DH-200 M high-precision digital electronic gravimeter which can measure the dry density and saturated density of concrete. Water saturation in the interior of concrete is mainly determined by relative humidity, which is monitored in the experiment with a humidity sensor [35, 36]. Notably, relative humidity monitored by the sensor cannot be used directly as the water content of concrete because relative humidity indicates the water content of the air in the concrete pores. To obtain the water content of concrete, the relationship between relative humidity and water content of the concrete should be defined by experimentation. The change in the water content can be obtained by measuring the weight of concrete with a high-precision balance while monitoring relative humidity. Then, the relationship between water saturation and relative humidity in concrete is investigated under 25 °C (Fig. 8).

The thermal conductivity of concrete under different porosities (0.2, 0.3 and 0.4), water saturation (0.05, 0.56, and 0.95), and volume fractions of aggregate (0.1, 0.2, and 0.3) are tested via LSFEM. The testing group and the range of the thermal conductivity of concrete at the testing temperature are shown in Table 2.

## 4 Thermal Conductivity Model of Concrete

### 4.1 Determination of the Parameters of the Thermal Conductivity Model

In the thermal conductivity model, the three unknown parameters, namely, $\eta_1$, $\eta_2$, and $\eta_3$ are the function of porosity, the ratio of cement mortar and aggregate, and saturation, respectively. To solve those parameters, the formation of the function should be assumed first. Generally, an empirical function is often used to find the relationship between two parameters. This relationship can contain various trends by adjusting the parameter value in the function. Therefore, the function of parameters $\eta_1$, $\eta_2$, and $\eta_3$ can be expressed as follows:

$$\begin{cases} \eta_1 = a_1 \phi^{b_1} \\ \eta_2 = 1 - a_2 \theta^{b_2} \\ \eta_3 = 1 - a_3 S_r^{b_3} \end{cases} . \tag{10}$$

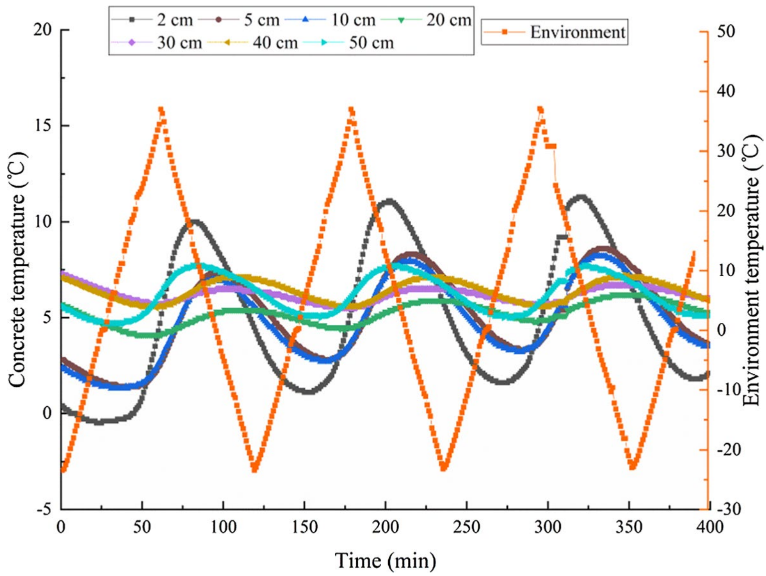


**Fig. 6** Evolution of concrete and environment temperatures

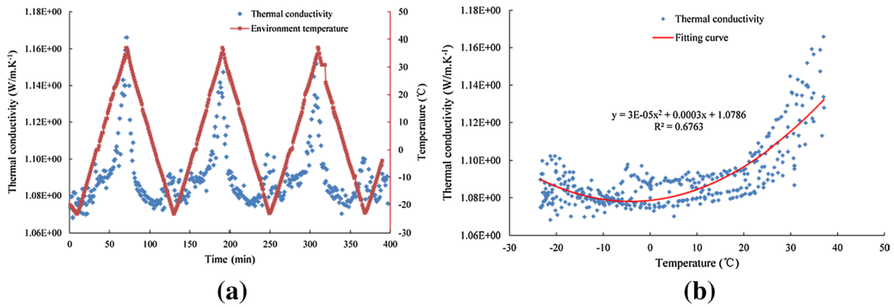


**Fig. 7** Thermal conductivity of concrete varying with (a) time and (b) temperature

Given that the thermal conductivity data provided by the experiment with a considerably more than the number of unknowns, the least-square method (LSM) is adopted to solve the values of parameters. Half of the tested data are used to solve the parameters and the other half of the experimental data are used to validate the model. The values of the parameters are listed in Table 3.

The measured and the predicted thermal conductivity results are shown in Fig. 9 to validate the fitting ability of the parameters. The measured and the predicted thermal conductivity results are considered the abscissa and ordinate axes, respectively, in this experiment. The predicted thermal conductivity values are similar to the experimental data. The discrepancy in the predicted data is less than 12%, considering that many factors affect the thermal conductivity of concrete. For example, compared with the prediction accuracy of other models in the literature, the maximum errors of Bruggeman,

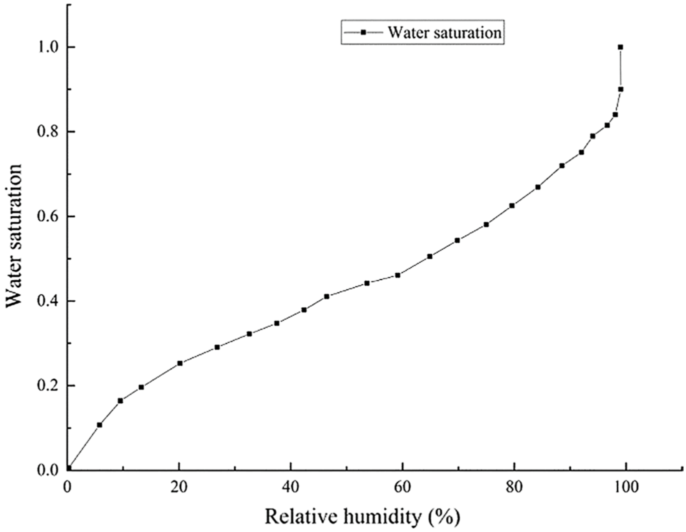


**Fig. 8** Relationship between water saturation and relative humidity in concrete

**Table 2** Testing group of the concrete

| Porosity | Volume fraction of aggregate | Saturation | Range of thermal conductivity ($Wm^{-1} \cdot K^{-1}$) |
|---|---|---|---|
| 0.2 | 0.3 | 0.95 | 1.38–1.41 |
| 0.3 | 0.3 | 0.95 | 1.16–1.28 |
| 0.4 | 0.3 | 0.95 | 1.02–1.16 |
| 0.3 | 0.1 | 0.95 | 1.01–1.10 |
| 0.3 | 0.2 | 0.95 | 1.08–1.20 |
| 0.3 | 0.3 | 0.95 | 1.16–1.28 |
| 0.2 | 0.3 | 0.05 | 0.81–0.82 |
| 0.2 | 0.3 | 0.56 | 0.91–0.96 |
| 0.2 | 0.3 | 0.95 | 1.38–1.41 |

Maxwell, Zhang's, and Serial model are 17%, 12%, 13%, and 19%, respectively [37]. Therefore, a discrepancy of 12% is deemed acceptable.

Therefore, the thermal conductivity model of concrete can be written as follows:

$$\lambda = \frac{\emptyset^{0.16}\lambda_p\lambda_s}{\left(1-\emptyset\right)\lambda_p + \phi\lambda_s} + \left(1-\emptyset^{0.16}\right)\left(1-\emptyset\right)\lambda_s + \left(\emptyset-\emptyset^{1.16}\right)\lambda_p$$

where

$$\lambda_p = \frac{\left(1-1.5*S_r^{0.829}\right)\lambda_a\lambda_w}{\left(1-S_r\right)\lambda_w + S_r\lambda_a} + 1.5*S_r^{0.829}\left(1-S_r\right)\lambda_a + 1.5*S_r^{1.829}\lambda_w$$

$$\lambda_s = \frac{\left(1-\theta^{2.756}\right)\lambda_c\lambda_g}{(1-\theta)\lambda_g + \theta\lambda_c} + \theta^{2.756}(1-\theta)\lambda_c + \theta^{3.756}\lambda_g$$

### 4.2 Discussion of the Behavior of the Thermal Conductivity Model

The thermal conductivity of concrete is a function of saturation, porosity, and the volume fraction of aggregate [4, 38]. When the volume fraction of aggregate is 0.5, the general behavior of the thermal conductivity of concrete and saturation with different porosities by using the proposed thermal conductivity model is shown in Fig. 10. For certain porosity, the thermal conductivity of concrete increases with the increase in water saturation because the thermal conductivity of water is larger than that of air under the same condition. The increase in water saturation indicates that the pores are filled with more water and less air, which is beneficial to the thermal conductivity of concrete. At the initial stage of the increasing water saturation, the increase rate of thermal conductivity is high; the increasing rate of the thermal conductivity of concrete gradually decreases at high levels of water saturation. This result indicates that the effects of water saturation are stable when the water saturation of concrete is relatively high (Sr > 0.9).

Figure 10 shows that the thermal conductivity of concrete decreases with the increase in porosity because the thermal conductivity of cement mortar and aggregate is higher than that of water and air. Increasing porosity leads to a decrease in solid content (cement mortar and aggregate). The red dotted line in Fig. 10 shows the increment of thermal conductivity of concrete (Δλ) when the water saturation increases from 0.1 to 0.95 under different porosity conditions. When the porosity is less than 0.01, the effects of saturation on the thermal conductivity of concrete are very weak because the thermal conductivity of concrete is mainly determined by the content of cement mortar and aggregate. When the porosity is about 0.2, the thermal conductivity of concrete is most sensitive to the change of water saturation, and the value of Δλ is the largest. When the porosity

**Table 3** Parameters value for $\eta_1$, $\eta_2$, and $\eta_3$

| Parameters | $a_1$ | $b_1$ | $a_2$ | $b_2$ | $a_3$ | $b_3$ |
|---|---|---|---|---|---|---|
| Value | 1.000 | 0.160 | 1.000 | 2.756 | 1.500 | 0.829 |

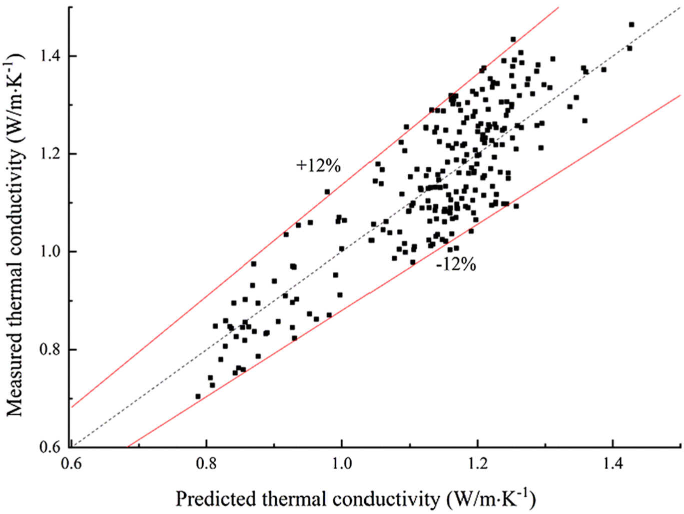


**Fig. 9** Predicted versus measured thermal conductivity of concrete

is close to 1.0, the thermal conductivity of concrete mainly depends on the thermal conductivity of water and air and the thermal conductivity of concrete is between air and water.

The volume fraction of aggregate is also the main factor that affects the thermal conductivity of concrete when water saturation is 0.5. Figure 11 shows that the thermal conductivity of concrete changes with the volume fraction of aggregate under different porosities. The thermal conductivity of concrete increases with the increase in the volume fraction of aggregate. The large volume fraction of aggregate increases the more components with high thermal conductivity in concrete and enhances the thermal conductivity of concrete. Under the same porosity and water saturation, the increase in the volume fraction of aggregate decreases the volume fraction of cement mortar, because the thermal conductivity of cement mortar is lower than that of aggregate and the thermal conductivity of concrete increases with the ratio of the increase in aggregate. Similarly with Fig. 10, the red dotted line in Fig. 11 shows the increment of thermal conductivity of concrete (Δλ) when the volume fraction of aggregate is from 0.1 to 1.0 under different porosity conditions. As the porosity increases, the effect of aggregate volume fraction on the concrete thermal conductivity is getting lower and lower. When the porosity is 1, the aggregate is no longer contained in the concrete, and the influence of the aggregate on the thermal conductivity of the concrete is reduced to zero.

Figure 12 shows that the porosity of concrete significantly affects the thermal conductivity of concrete when the volume fraction of aggregate is 0.5. The thermal conductivity of concrete decreases with the increase in porosity at a rapid or slow ratio when porosity is relatively small or large. The reason comes from that the thermal conductivity of pore $\lambda_p$ (water and air) is lower than that of solids $\lambda_s$ (cement mortar and aggregate). As a result, the thermal conductivity of concrete decreases when porosity increases. When porosity increases to 1, the "concrete" mainly

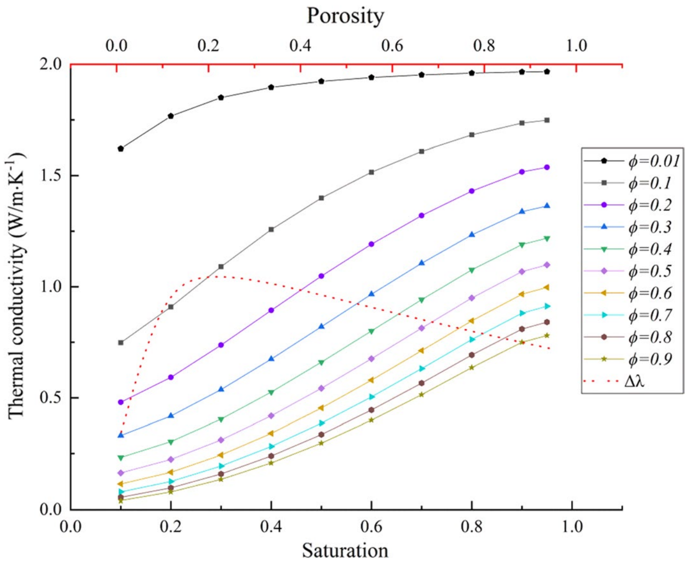


**Fig. 10** Variation in thermal conductivity of concrete with water saturation under different porosities (volume fraction of aggregate is 0.5)

consists of water and air, and the thermal conductivity of "concrete" is between that of air and water. In addition, the thermal conductivity of concrete with high water saturation under different porosities is considerably higher than that with low saturation because the thermal conductivity of water is considerably higher than that of air. The red dotted line in Fig. 12 shows the increment of thermal conductivity of concrete (Δλ) when the porosity from 0.1 to 0.9 under different water saturation conditions. When the water saturation is about 0.5, the value of the increment of thermal conductivity (Δλ) reaches the maximum. It means that the thermal conductivity of concrete is most sensitive to changes in porosity when the water saturation in the concrete pores is 0.5.

The proposed model also considers the effects of temperature. The thermal conductivity of concrete varies with temperature under different water saturation values when the ratio of aggregate volume fraction is 0.3 and porosity is 0.2 (Fig. 13). The thermal conductivity of concrete increases with the increase in temperature under different saturation values. The influence of temperature on concrete is through the change in thermal conductivity of signal components. The thermal conductivity of cement mortar and aggregate decrease with the increase in temperature, whereas the thermal conductivity of water and air increase with the increase in temperature. The total tendency of concrete increases with the increase in temperature, that is, water and air in concrete are the main factors that affect the thermal conductivity of concrete when porosity is 0.2 and the volume fraction of aggregate is 0.3. According to Eqs. 7, 8, and 9, the thermal conductivity of cement mortar and aggregate are insensitive to temperature compared with that of water.

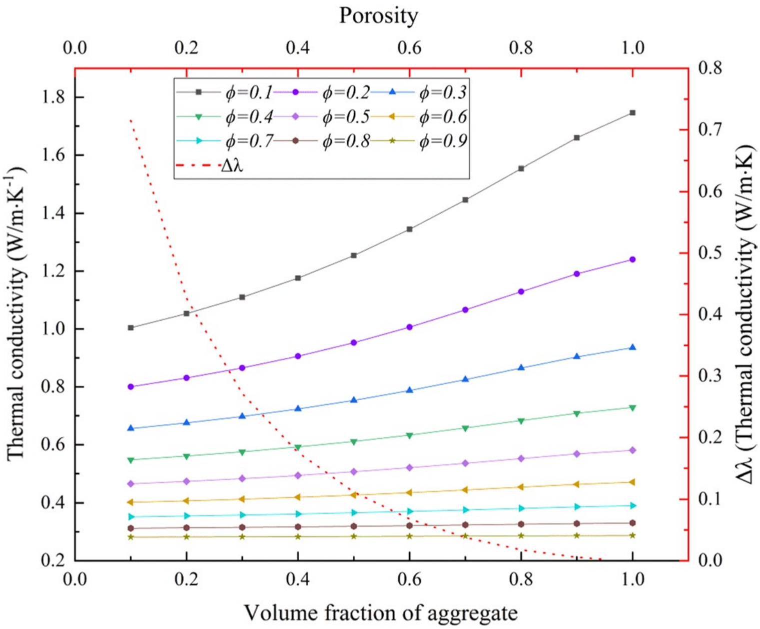


**Fig. 11** Change in thermal conductivity of concrete with the increase in the volume fraction of aggregate (water saturation is 0.5)

## 5 Conclusions

The thermal conductivity of concrete is a complex parameter that is affected by many factors. The proposed model considers the effects of porosity, water saturation, and aggregate volume fraction on the thermal conductivity of concrete. The proposed model is derived from the Wiener bounds and considers concrete with four different components, namely, cement mortar, aggregate, water, and air. Such components are organized in serial and parallel arrangements. By adjusting the $\eta_1$, $\eta_2$, and $\eta_3$ parameters and determining the weight of the contribution of each component, the thermal conductivity model of concrete can predict the maximum and the minimum thermal conductivity values of concrete. Such parameters vary for different types of concrete, and measured data are needed to solve these parameters before predicting the thermal conductivity of concrete. In this paper, a series of experiments has been carried out to test the thermal conductivity of cement mortar, aggregate, and concrete with different water–cement ratios, volume fractions, and saturations; obtain a large amount of concrete thermal conductivity data; and determine the model parameters. Notably, the accuracy of the prediction model largely relies on the experimental data, which are used to fit $\eta_1$, $\eta_2$, and $\eta_3$ parameters via the LSM.

The thermal conductivity of four components should be considered as a known quantity to solve the thermal conductivity model of concrete. Several thermal conductivity methods including TPS and LSFEM are applied to measure the thermal conductivity of cement mortar, aggregate, and concrete. LSM is utilized to

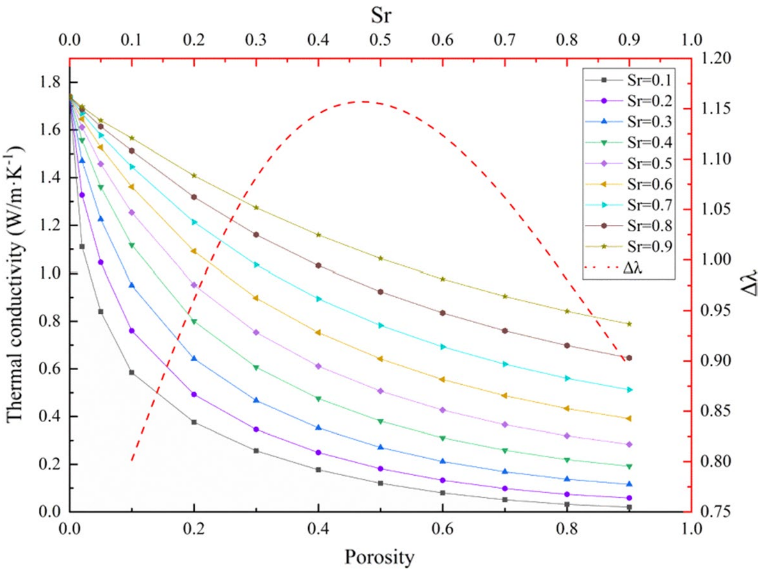


**Fig. 12** Variation in thermal conductivity of concrete with porosity under different water saturation (volume fraction of aggregate is 0.5)

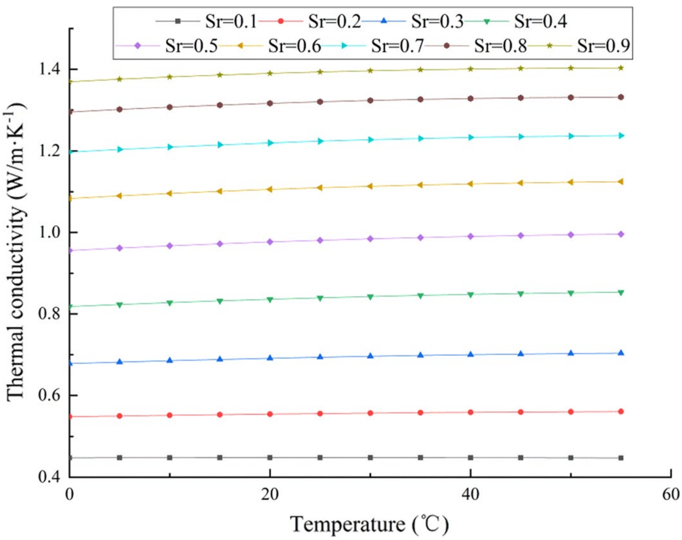


**Fig. 13** Variation in thermal conductivity of concrete with temperature under different water saturation (volume fraction of aggregate is 0.3 and porosity is 0.2)

parameterize the thermal conductivity model of concrete, and the discrepancy between the measured and predicted data is less than 12%. According to the prediction model, the effects of porosity, water saturation, and aggregate on thermal conductivity can be obtained. The results show that the thermal conductivity of concrete decreases with the increase in porosity and the decrease in water saturation and aggregate volume fraction. In general, a high ratio of components with large thermal conductivity in concrete indicates a large thermal conductivity of concrete. Porosity significantly influences the thermal conductivity of concrete because the thermal conductivity of solids (cement mortar and aggregate) is considerably larger than that of pores (water and air).

From the perspective of temperature, water and air in pores of the concrete are the main factors that affect the thermal conductivity of concrete. Compared with the components of cement mortar and aggregate, those of water and air are sensitive to temperature changes, which are dominant in the changing trend of the thermal conductivity of concrete. The thermal conductivity model of water (Eq. 7) is only suitable in the range of 274–370 K. Therefore, the proposed model cannot consider cases with temperatures below 0 °C. When the temperature decreases to 0 °C, the thermal conductivity of ice is considerably larger than that of water; this condition increases the thermal conductivity of concrete significantly [34]. Therefore, additional experimental data and a thermal conductivity model for water with wide temperature adaptability are needed to improve the applicability of the proposed model. The proposed model can be applied not only to concrete but also to porous media with multiple components by properly determine the thermal conductivity of each component.

**Acknowledgments** This research was sponsored by National Key R&D Program of China (2017YFC1501100), the National Natural Science Foundation of China (Grant Nos. 51279090, 51879145 and 51939004), and the Hubei Key Laboratory of Construction and Management in Hydropower Engineering (2020KSD11).

## Compliance with Ethical Standards

**Conflict of interest** The authors declare that there are no conflict of interest.

## References

1. D. Campbell-Allen, C.P. Thorne, The thermal conductivity of concrete. Mag. Concr. Res. **53**(43), 371–372 (1963). https://doi.org/10.1007/978-1-4615-8789-7_45
2. M.I. Khan, Factors affecting the thermal properties of concrete and applicability of its prediction models. Build. Environ. **37**(6), 607–614 (2002). https://doi.org/10.1016/S0360-1323(01)00061-0
3. L. Tinker, J.G. Cabrera, Modeling the thermal conductivity of concrete based on its measured density and porosity. Buildings V. Conference Proceedings. 91–95 (1992)
4. I. Asadi, P. Shafigh, Z.F.B.A. Hassan, N.B. Mahyuddin, Thermal conductivity of concrete—a review. J. Build. Eng. **20**, 81–93 (2018). https://doi.org/10.1016/j.jobe.2018.07.002
5. B. Li, W. Xu, F. Tong, Measuring thermal conductivity of soils based on least squares finite element method. Int. J. Heat Mass Transf. **115**, 833–841 (2017). https://doi.org/10.1016/j.ijheatmasstransfer.2017.08.056
6. O.K. Nusier, N.H. Abu-Hamdeh, Laboratory techniques to evaluate thermal conductivity for some soils. Heat Mass Transf. **39**(2), 119–123 (2003). https://doi.org/10.1007/s00231-002-0295-x

7. L. Vozár, T. Šrámková, Two data reduction methods for evaluation of thermal diffusivity from step-heating measurements. Int. J. Heat Mass Transf. **40**(7), 1647–1655 (1997). https://doi.org/10.1016/S0017-9310(96)00138-X
8. S.E. Gustafsson, Transient plane source techniques for thermal conductivity and thermal diffusivity measurements of solid materials. Rev. Sci. Instrum. **62**(3), 797–804 (1991). https://doi.org/10.1063/1.1142087
9. T. Log, S.E. Gustafsson, Transient Plane Source (TPS) technique for measuring thermal transport properties of building materials. Fire Mater. **19**(1), 43–49 (1995). https://doi.org/10.1002/fam.810190107
10. G. Pia, U. Sanna, A geometrical fractal model for the porosity and thermal conductivity of insulating concrete. Constr. Build. Mater. **44**, 551–556 (2013). https://doi.org/10.1016/j.conbuildmat.2013.03.049
11. A.D. Brailsford, K.G. Major, The thermal conductivity of aggregates of several phases, including porous materials. Br. J. Appl. Phys. **15**(3), 313 (1964). https://doi.org/10.1088/0508-3443/15/3/311
12. J.C. Maxwell, *A Treatise on Electricity and Magnetism*, vol. II. (Oxford University Press, 1937)
13. A. Simpson, A.D. Stuckes, Thermal conductivity of porous materials: I theoretical treatment of conduction processes. Build. Serv. Eng. **7**(2), 78–86 (1986). https://doi.org/10.1177/014362448600700204
14. D.A.G. Bruggeman, Calculation of Various Physics Constants in heterogenous substances I Dielectricity constants and conductivity of mixed bodies from isotropic substances. Ann. Phys. **24**(7), 636–664 (1935). https://doi.org/10.1002/andp.19354160802
15. D.P.H. Hasselman, L.F. Johnson, Effective thermal conductivity of composites with interfacial thermal barrier resistance. J. Compos. Mater. **21**(6), 508–515 (1987). https://doi.org/10.1177/002199838702100602
16. W. Zhang, H. Min, X. Gu, Y. Xi, Y. Xing, Mesoscale model for thermal conductivity of concrete. Constr. Build. Mater. **98**, 8–16 (2015). https://doi.org/10.1016/j.conbuildmat.2015.08.106
17. W.P. Zhang, Y.S. Xing, X.L. Gu, Theoretical models of effective thermal conductivity of concrete based on composite materials in mesoscale (in Chinese). Struct. Eng. **28**(2), 39–45 (2012). https://doi.org/10.15935/j.cnki.jggcs.2012.02.003
18. Y.Y. Wang, C. Ma, Y.F. Liu, D.J. Wang, J.P. Liu, Effect of moisture content on thermal conductivity of concretes (in Chinese). J. Build. Mater. **21**(4), 595–599 (2018)
19. K.H. Kim, S.E. Jeon, J.K. Kim, S. Yang, An experimental study on thermal conductivity of concrete. Cem. Concr. Res. **33**(3), 363–371 (2003). https://doi.org/10.1016/S0008-8846(02)00965-1
20. P. Meshgin, Y. Xi, Multi-scale composite models for the effective thermal conductivity of PCM-concrete. Construct. Build. Mater. **48**, 371–378 (2013). https://doi.org/10.1016/j.conbuildmat.2013.06.068
21. R.M. Christensen, *Mechanics of Composite Materials* (Dover Publications Inc, Mineola, New York, 1979)
22. Y.Z. Tan, Y.X. Liu, P.Y. Wang, Y. Zhang, A predicting model for thermal conductivity of high permeability-high strength concrete materials. Geomech. Eng. **10**(1), 49–57 (2016). https://doi.org/10.12989/gae.2016.10.1.049
23. H.Q. Jin, X.L. Yao, L.W. Fan, X. Xu, Z.T. Yu, Experimental determination and fractal modeling of the effective thermal conductivity of autoclaved aerated concrete: effects of moisture content. Int. J. Heat Mass Transf. **92**, 589–602 (2016). https://doi.org/10.1016/j.ijheatmasstransfer.2015.08.103
24. F. Tong, L. Jing, R.W. Zimmerman, An effective thermal conductivity model of geological porous media for coupled thermo-hydro-mechanical systems with multiphase flow. Int. Rock Mech. Mining Sci. **46**(8), 1358–1369 (2009). https://doi.org/10.1016/j.ijrmms.2009.04.010
25. O. Wiener, Der Abhandlungen Der Mathematisch-Physischen Klasse Der Konigl. Sachsischen Gesellshaft Der Wissenschaften. **32**, 509–604 (1912)
26. R.W. Zimmerman, Thermal conductivity of fluid-saturated rocks. J. Petrol. Sci. Eng. **3**(3), 219–227 (1989). https://doi.org/10.1016/0920-4105(89)90019-3
27. Ganjian, and Esmaiel. *The Relationship between Porosity and Thermal Conductivity of Concrete* (University of Leeds, 1990)
28. R. Demirboga, A. Kan, Thermal conductivity and shrinkage properties of modified waste polystyrene aggregate concretes. Construct. Build. Mater. **35**, 730–734 (2012). https://doi.org/10.1016/j.conbuildmat.2012.04.105

29. M.L.V. Ramires, C.A.N. Castro, Y. Nagasaka, A. Nagashima, M.J. Assael, W.A. Wakeham, Standard reference data for the thermal conductivity of water. J. Phys. Chem. Ref. Data **24**(3), 1377–1381 (1995). https://doi.org/10.1063/1.555963
30. R.T. Jacobsen, E.W. Lemmon, Viscosity and thermal conductivity equations for nitrogen, oxygen, argon, and air. Int. J. Thermophys. **25**(1), 21–69 (2003)
31. T.Z. Harmathy, Thermal Properties of Concrete at Elevated Temperatures. Journal of Materials **5**(1), 47–74 (1970)
32. A.S. Gandage, V.R. Vinayaka Rao, M.V.N. Sivakumar, A. Vasan, M. Venu, A.B. Yaswanth, Effect of perlite on thermal conductivity of self compacting concrete. Procedia Soc. Behav. Sci. **104**,188–197 (2013). https://doi.org/10.1016/j.sbspro.2013.11.111
33. V. Bindiganavile, F. Batool, N. Suresh, Effect of fly ash on thermal properties of cement based foams evaluated by transient plane heat source. Indian Concr. J. **86**(11), 7–14 (2011)
34. J.W. Gong, G.J. Cao, G.X. Chen, S.X. Li, Relationship between thermal conductivity of concrete and its saturation and temperature (in Chinese). Water Resour. Power. **35**(12), 112–115+111 (2017)
35. J. Jiang, Y. Yuan, Q. Zeng, T. Mo, Relationship of moisture content with temperature and relative humidity in concrete. Mag. Concr. Res. **65**(11), 685–692 (2013). https://doi.org/10.1680/macr.13.00211
36. Q. Yang, Inner relative humidity and degree of saturation in high-performance concrete stored in water or salt solution for 2 years. Cement Concr. Res. **29**(1), 45–53 (1999). https://doi.org/10.1016/S0008-8846(98)00174-4
37. W. Zhang, H. Wang, X. Gu, Effects of randomly distributed aggregates on thermal properties of concrete (in Chinese). J. Build. Mater. (Jianzhu Cailiao Xuebao) **20**(2), 168–73 and 197 (2017)
38. S.Y. Chung, T.S. Han, S.Y. Kim, J.H.J. Kim, K.S. Youm, J.H. Lim, Evaluation of effect of glass beads on thermal conductivity of insulating concrete using micro CT images and probability functions. Cem. Concr. Composites. **65**, 150–162 (2016). https://doi.org/10.1016/j.cemconcomp.2015.10.011