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A Mathematical Model for
Locating Exit Taxiways

THE INSTITUTE OF TRANSPORTATION
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Institute of Transportation

and Traffic Engineering, and

Department of Industrial Engineering

Special Study

A Mathematical Model for

LOCATING EXIT TAXIWAYS

A project conducted for the Operations Analysis Directorate,
Bureau of Research and Development, Federal Aviation
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By R Horonjeff
R C Grassi
R R Read
G Ahlborn

University of California

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ERRATA

Page 16: Table 3, 40 mph column, line I-3, from 38.92 to 48.92.

Table 4:

Page	Column	Line	Change From	 To
18	II -45	1-exit wave-off	27.42	27.24
18	II -45	1-exit location	6460	4195
18	II -45	3-exit location (1)	4900	2890
18	III-40	2-exit location (1)	2760	2706
18	III-55	2-exit arrival rate	65.55	65.45
18	III-55	2-exit wave-off rate	0.22	0.02
18	III-70	2-exit location (1)	(b)	(c)
18	III-70	2-exit location (2)	(b)	(c)
19	I -55	1-exit, q_1	9.999	0.999
19	I -73	3-exit acceptance rate	38.92	48.92
19	II -55	2-exit, q_1	9.851	0.851
19	II -70	3-exit wave-off rate	9.94	0.94
19	III-40	2-exit acceptance rate	86.97	76.97
20	II-2.75	2-exit location (1)	6390	5390
20	III-4	1-exit location	(b)	(c)

Page 24. Table 5, Column 1 heading, from I to i.

Preface

Concern lest the runway be a limiting factor in the traffic handling capacity of the airport-airways system prompted the Operations Research Branch of the Airways Modernization Board to enter into a contract with the University of California's Institute of Transportation and Traffic Engineering to determine the operational feasibility of high-speed exit taxiways as a possible means of increasing the acceptance rates of runways A report covering the findings of this research was submitted to the Board in the summer of 1958 The research showed that, from an operating standpoint, high-speed exit taxiways were feasible, at least for speeds up to 60 mph, but the research did not show the effectiveness of these taxiways in terms of runway acceptance The Operations Analysis Directorate, Bureau of Research and Development, Federal Aviation Agency thus extended the original contract in order that a method of determining the effectiveness of exit taxiways could be formulated This report covers this latter phase of the problem

At the University of California, Berkeley, Professors Robert Horonjeff, of the Institute staff, and R C Grassi, of the Industrial Engineering Department, were jointly responsible for the planning, analysis, and supervision of the project as a whole Dr Robert R Read, was responsible for model formulation and mathematical analysis Mr James D Cumming was in charge of developing the computational procedures for machine calculations Messrs Ernest F Bisbee and Tadao Yoshikawa assisted in the formulation of the model and in the development of the computational procedures Mr Gale Ahlborn investigated the operational feasibility of turn-ons and assisted in the analysis of the results from the operation of the model Mr Frank S Henyey performed the programming for the computer, and Mr Derek Woolfall assisted in program debugging and operated the computer

Acknowledgments

The staff is grateful to the individuals and organizations who provided information for this project Special thanks are extended to Dr. R H. Jordan, Mr Stuart Ball, and Mr W. B. Weber, Jr., of the Operation Analysis Research Directorate, Federal Aviation Agency, and to Captain J L Fleming, of the Pacific Alaska Division, Pan American World Airways Thanks are also extended to the aircraft manufacturers for providing performance information on the aircraft included in this study, and to the Federal Aviation Agency's traffic control personnel at San Francisco International Airport for information concerning traffic operating rules

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Summary

An earlier project established the feasibility of high-speed exit taxiways and set out certain guide lines for their configuration * The purpose of the present project was to formulate and test a mathematical model for determining the exit locations that will enable the runway to accept the greatest number of aircraft per hour within specified limits of percent wave-offs The model can also be used to compute, for an existing airport where runways are already fixed, the average acceptance rates and percent wave-offs for various conditions

A mathematical model was developed for a single runway used exclusively for landing The model makes it possible to determine the taxiway locations that will yield the highest runway acceptance rates, and corresponding wave-off rates, taking into account (1) number of exits, (2) exit speed, (3) aircraft arrival rates at runway threshold, (4) aircraft population (i e , a specified mixture of aircraft types), (5) pilot variability, and (6) meteorological and geographical conditions

In regard to item (3) the model was developed to take as inputs arrival rates based either on fixed intervals of time or distance or on intervals of time equal to the runway occupancy of the preceding aircraft These schemes assume a degree of control greater than being exercised today The arrivals are not treated as a statistical distribution because the information applicable to the three assumed arrival schemes was not available

Aircraft population, item (4) enters the mathematical model in the form of a statement as to the time and distance to reach exit speed, recognizing that for each aircraft these times and distances will vary from landing to landing according to some statistical disbribution, as a result of environmental changes and pilot variability, items (5) and (6) Some deceleration data were obtained in flight tests conducted at Wright-Patterson Air Force Base in connection with the previous project * But neither this information nor that available from other sources was sufficient to establish a distribution It was therefore assumed that a normal distribution would apply and that environmental changes and pilot variability could be accounted for in this statistical distribution

It was necessary to make a general assumption concerning the manner in which runway occupancy would delay the landing of subsequent aircraft The assumption made was that an aircraft not clearing the runway in time would cause the first following aircraft to be waved off (not permitted to land) but would in no case cause the second following aircraft to be waved off Some other assumptions of lesser importance also had to be made It is important that the assumptions be kept in mind when evaluating outputs from the model

To demonstrate the use of the model, optimum exit taxiway locations and average acceptance rates have been computed for (1) exit speeds of 40 and 60 mph, (2) three different aircraft populations, (3) one, two, and three exit taxiways and (4) arrival rates corresponding to various fixed intervals of (a) time and (b) distance The aircraft populations (item 2) were selected so as to be reasonably representative of I, a very large airport having practically no general aviation activity, II, an airport having a mixture of large turbo-jet transports, propeller-driven transports, and small general aviation aircraft, and III, an airport having a predominance of small general aviation aircraft

The results show that the optimum locations and the corresponding average acceptance rates are quite sensitive to aircraft population, exit speed, and number of exits Furthermore, if the number of exits and intervals of time between aircraft arriving over threshold are fixed, the optimum locations of the exits vary considerably for each aircraft population Again, the specific results should be interpreted in the light of the assumptions

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*Reference 1, page 2

A secondary objective of this project was to examine the feasibility of turn-ons as a means of increasing the capacity of a runway used both for landings and take-offs A turn-on is defined as a taxiway leading from the run-up pad to the take-off end of the runway It was thought that the possibility of such turn-ons might be examined for speeds varying from 15 mph to possibly as high as 60 mph

After reviewing the problem, it was found that the data available were so meager that it was not possible, within the time period of the contract, to formulate a mathematical analysis which would be meaningful The several reasons for this are set forth in Section 7

1. Introduction

Current forecasts indicate that the next decade will bring a rapid expansion of air transport in this country and abroad. If the predicted increases in traffic take place there is every indication that the traffic handling capacities of many airports will have to be increased to meet the expanded aircraft activity. While there are many aspects of airport design and operation which have an influence on airport capacity, one factor of importance is the aircraft acceptance rate of a runway.

Objective

The primary objective in this project was to formulate a mathematical procedure or model which could be used to determine the effectiveness of exit taxiways, effectiveness being measured in terms of acceptance rate: the average number of aircraft landings per hour which the runway can accommodate. More specifically, it was contemplated that the model would be able to show the influence of number and location of exit taxiways on the acceptance rate of a single runway used exclusively for landing. The choice of acceptance rate as a measure of effectiveness was based primarily on the fact that this index might be more meaningful to planners and designers of airports since the demand for airport facilities is normally expressed in terms of aircraft movements.

A secondary objective was to examine the feasibility of turn-ons as a means of increasing the capacity of a runway used both for landings and take-offs, a turn-on being defined as a taxiway leading to the end of the runway for use in take-offs.

Factors Affecting Exit Taxiway Location

The important factors influencing the location of exit taxiways are (1) the number of exits, (2) exit speed, (3) the manner in which aircraft arrive at runway threshold, (4) aircraft population, (5) meteorological and geographical environment at the airport, and (6) pilot variability.

The important parameters affected by items (5) and (6) are the time and distance required by a landing aircraft to reach exit speed. These two parameters are mainly dependent on the speed (relative to the ground) of the aircraft at the runway threshold. These speeds vary considerably among the different types of aircraft. The important meteorological and geographical environment at the airport affecting these parameters are airport elevation, temperature, and wind. High airport elevation or high temperatures increase the speeds over runway threshold. The higher the speed the longer the distance and time to reach exit speed. Headwinds reduce the speed of the aircraft relative to the ground and tailwinds increase it.

Pilot variability in landing techniques has an influence on the parameters, and arises both through the individual pilot's manner of reacting to the environmental conditions and through his particular flying technique.

Considerations in Developing The Model

A mathematical model could possibly be developed to accommodate all the foregoing variables as separate inputs. Such a model would, however, be extremely complex. For this reason it was felt that initially it would be better to develop a model which was simplified so as to be operationally practicable and yet reasonably descriptive of the system. Analysis of the results from such a model could then be used to determine whether refinements were justified. Another guiding consideration in the development of the model was that it should be able to accommodate, as input information, data describing various operational procedures for the control of landing aircraft for the purposes both of determining exit locations suitable to future techniques and of evaluating the techniques with respect to established locations

In devising the mathematical statements of the above variables, it was necessary to make a number of assumptions These are discussed in Section 2

Information Obtained from The Model

Although the model was developed for determining the number and location of exit taxiways and their effect on runway acceptance rate for any specific airport location, it was deemed desirable to prepare generalized solutions which would show the influence on runway acceptance rate when the following conditions were varied

1 Number of exits
2 Exit speed
3 Aircraft population (whether a mixture of pure jets, propeller driven transports, and small general aviation aircraft or some other combination)
4 Manner in which aircraft arrive at the runway threshold (separated at runway threshold either by fixed time intervals or fixed distance intervals)

For a given set of these conditions, exit locations resulting in the highest average runway acceptance rates were calculated These results are discussed in Section 4

It is important to point out that one of the principal limitations of these results arises from lack of sufficient data on the distances and times required by aircraft to decelerate from speed over threshold to speed for which a turn-off is designed, expressed as joint statistical distributions The only such landing characteristics that were available at the time this project was undertaken were for those aircraft used in the previous exit taxiway investigation [1] These were confined to (1) aircraft types KC-135 (Boeing 707), C-121 (Convair 340), C-121 (Super Constellation), L-27 (Cessna 310), F-100, B-47 and B-52, (2) the meteorological conditions which prevailed at the time the aircraft were test flown (VFR conditions, dry concrete pavements, temperatures ranging mostly between 50°F and 60°F, airport elevation 800 ft MSL), and (3) ten landings with the same pilot for each aircraft type

Another important limitation is the assumption on how aircraft arrive over runway threshold The model is able to take as input, arrivals based on (1) fixed intervals of time, (2) fixed intervals of distance, and (3) variable intervals of time (based on the runway occupancy time of each type of aircraft) Because of the limitation in contract time the model was operated only on the basis of fixed intervals of time and distance No statistical distribution was applied to the arrivals since no data applicable to the assumed schemes were available It is recognized that under current procedures arrivals at threshold do not fit any of the three schemes precisely but fall somewhere in between However, we have assumed that arrival procedures will improve in the future (greater degree of control will be excercised), consequently the model indicates what could be achieved under more controlled conditions This assumption was in part based on analytic and simulation studies of terminal-area traffic control prepared for the Air Navigation Development Board[2] which indicated that a greater degree of control in the landing is necessary if significant gains are to be achieved by the construction of high speed exit taxiways

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[1] Horonjeff, Robert, Finch, Dan M , Belmont, Daniel M , and Ahlborn, Gale Exit Taxiway Location and Design Berkeley, California Institute of Transportation and Traffic Engineering, University of California, 1958

[2] Berkowitz, Samuel M and Fritz, Edward L Analytical and Simulation Studies of Terminal-Area Air Traffic Control Indianapolis U S Civil Aeronautics Administration, Technical Development and Evaluation Center, 1955 (Technical Development Report No 251)

Form of This Report

This report is in seven sections Following this introduction, Section 2 describes the development of the model in general terms and the assumptions underlying its development Section 3 sets forth the conditions assumed for the operation of the model, and Section 4 discusses the results obtained in this "pilot" operation. Sections 5 and 6 give the explicit mathematical descriptions of the model itself (Section 5) and of the procedures for the machine computation (Section 6) Supporting mathematical material is given in the appendices Section 7 discusses the subject of turn-ons

2. Development of The Model

In order to develop a workable model, certain assumptions had to be made concerning the variables which would enter into its formulation Following are the assumptions and the considerations underlying them

Assumptions

1 Storage on the exit taxiways It was assumed that storage capacity for aircraft on the exit taxiways is large enough that acceptance rate will not be affected

2 Manner of processing arrivals It was felt that in future years, especially at high-traffic-density airports, the degree of control of arrivals would be higher than at present For this reason it was assumed that arrivals would take place in a manner approaching either (a) fixed time intervals or (b) fixed distance intervals It was realized that variations in these intervals would be bound to occur but such consideration was eliminated The main reason for this elimination was to permit the immediate development of a relatively simple model to which refinements could be added later if necessary Also, the nature of the variation cannot be determined at the present time

3 Arrival of aircraft by type The order of aircraft arrival by type (whether, for example, a jet transport follows another jet transport or follows a propeller-driven craft) was considered as random, on the grounds that present procedures are on a first-come first-served basis

4 Distribution of aircraft in the population It was assumed that the arriving population of aircraft would consist of known percentages of each type of aircraft and that these percentages would not change during the arrival period being studied

5 Runway occupancy The rule was adopted that the runway would be considered occupied from the time an aircraft was over the runway threshold until it was off the runway and that only one aircraft could occupy the runway at a time It is recognized that this rule may not correspond with the actual case, since runway occupancy, from the standpoint of practical operations, in all probability begins when the arriving aircraft is still some distance ahead of the runway threshold It is also realized that this real-life distance will vary among the different types of aircraft It is felt, however, that the rule adopted is entirely adequate to the purpose.

6. Exit speed It was assumed that an aircraft decelerates to exit velocity and then maintains exit velocity until it clears the runway It was further assumed that an aircraft cannot turn-off via an exit unless it has decelerated to exit velocity Thus, if an exit taxiway were designed for 50 mph an aircraft arriving at the point of turn-off at 51 mph would miss the exit This assumption is not entirely valid since the geometric configuration of the taxiway permits a certain amount of leeway in speed But the assumption appears usable as this leeway is probably not more than 5 mph

7 Consecutive wave-offs It was assumed that an arrival immediately following a wave-off would be accepted (in other words, that there would not be two-consecutive wave-offs) This assumption was based on a study of decelerating characteristics available for the types of aircraft included in this study and on an assumed average runway length of 10,000 ft Thus, if an accepted aircraft fails to achieve the last high-speed exit, it must go to the end of the runway but the total occupancy time will not be so great as to cause two consecutive wave-offs This assumption permits the model to be independent of the particular runway characteristics and of the particular operations chosen by the pilot to clear the runway

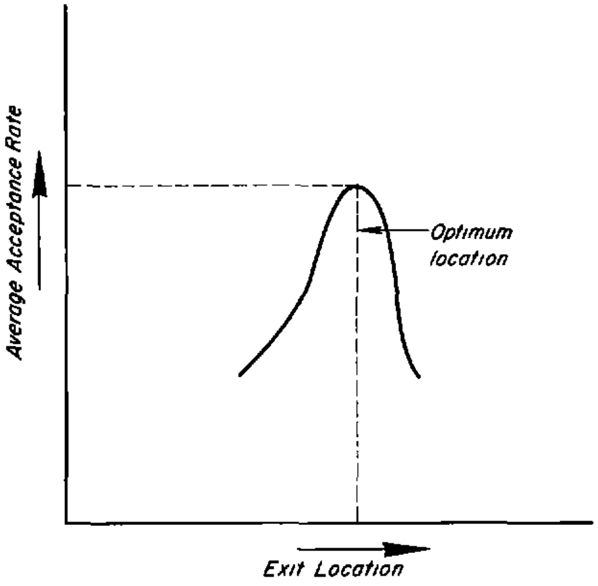

Fig 1 — Acceptance rate vs exit location for one aircraft type

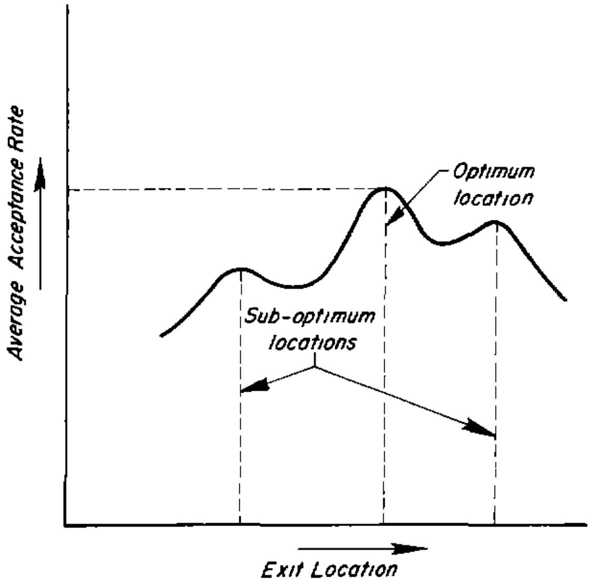

Fig 2 — Acceptance rate vs exit location for a mixture of aircraft types

8 Accidents It was assumed that no accidents would occur

9 Meteorological conditions at the airport It was assumed that there will be no change in surface temperature and wind conditions at the airport during the period of arrival being studied

Model Resume

In accordance with the foregoing a model has been developed which has the property of being closely representative of the real system while being expressed in mathematical notation and susceptible of numerical treatment The development of the model will be discussed here only in terms of its physical significance, leaving the detailed mathematical development for Section 5

With the aid of the model, the rate at which aircraft land can be related to the rate at which aircraft arrive over threshold by modifying the arrival rate by a correction factor, that is

Acceptance Rate = Arrival Rate x Correction Factor

The number and location of high-speed exit taxiways enter into the computation through the correction factor That is to say, for a given set of conditions, the number and location of the taxiways determines the average probability that a landing aircraft will cause the following aircraft to be waved off The value of the correction factor depends entirely on this probability Calling this average probability q, the correction factor is $1/(1 + q)$ With the probability q having a range from 0 to +1, the corresponding limits of the correction factor are 1 and 1/2 Thus the upper limit of the acceptance rate, obviously, is the arrival rate, and the lower limit of the acceptance rate is half the arrival rate (deriving from the assumption that only the first following aircraft can be caused to wave-off)

The probability that an aircraft will miss its "natural" exit, i e , that the aircraft will be exceeding exit design speed at the first exit for which it has a positive probability of attainment, and the probability that the aircraft will take a greater time on the runway to turn off at its "natural" exit than the interval of time between arriving aircraft will obviously depend upon the location of the exits

The probability q is a function of exit location and separation time between aircraft over threshold (reciprocal of arrival rate) The objective is, for fixed separation parameter, to find the minimum q so as to produce the maximum correction factor, thus bringing the acceptance rate as close as possible to the arrival rate The model can, for example, be operated so that the exit location corresponding to minimal q can be found

For a given arrival rate (time or distance) and a single exit, the acceptance rate may be expected to vary as a function of exit location as shown in Fig 1 It may be noted that the function is continuous with a distinct maximum If the separation time were made larger, the curve would shift to the right and down, and the maximum would become flatter and less sensitive to the exit location This figure represents a homogeneous population of aircraft The more general situation is illustrated in Fig 2, where the effects of a composite population may be seen The location of a single exit for a given separation time is in this case more complicated because the different performance characteristics of the aircraft will produce more than one local maximum Therefore sufficient calculations must be made to insure the inclusion of all local maxima in order that these maxima may be compared and the largest acceptance rate found

3. Input Data for Operation of The Model

In order to obtain a general indication of how the number and location of exit taxiways may affect the acceptance rate of a runway used exclusively for landing, solutions were obtained from the model using as inputs the information and assumptions described below

Aircraft types The only pertinent landing data available were for those aircraft included in the previous project * They covered both military and civil aircraft, but in the present project computations were made only for the civil types, these being

KC-135 (representing a Boeing 707-120 jet transport)
C-121 (Super Constellation)
C-131 (Convair 340)
L-27 (Cessna 310)

The C-121, C-121, and KC-135 represent a wide range of civil transports Similarly, the L-27 is representative of small general aviation aircraft

Aircraft populations It was decided that three types of population would be used. They were selected after reviewing forecasts as to the types of traffic the nation's airports will be handling, obtaining some notion of the percentage distribution by types through study of airline schedules and routes, and consulting with airlines as to the type of equipment which will be flown over these routes in the future In addition, existing information on the traffic pattern at New York's airports were reviewed, and observations were made at San Francisco International Airport and at Sacramento and Stockton airports The following populations were selected

I A mixture of large turbo-jets, large prop-driven transports, medium prop-driven transports and small general aviation type aircraft in the following proportions

KC-135	33%
C-121	25%
C-131	21%
L-27	21%

This population represents the type of activity that occurs at many of the nation's large airports

II. A mixture of large turbo-jets and prop-driven transports with no small general aviation type aircraft in the following proportions

KC-135	50%
C-121	30%
C-131	20%

This population represents traffic at the highest-traffic-density commercial airports, such as New York and Chicago, where small general aviation activity is practically nil

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*Reference 1, page 2

III A mixture of propeller-driven aircraft and small general aviation aircraft with no large jets in the following proportions

C-121	6%
C-131	10%
L-27	84%

This population represents the type of traffic at small city airports with some airline service, but where the traffic is mainly general aviation

It must be recognized that the landing characteristics of aircraft that the airlines expect to operate in the future do not exactly correspond with those of the aircraft included in the populations above It was concluded, however, that the landing characteristics of the above aircraft would be enough like those of their successors to yield indicative outputs from the model

One population which has not been included is a mixture of civil and military aircraft This type of activity occurs at joint-use airports such as Salt Lake City, Utah This omission as well as the omission of purely military aircraft does not infer that these populations should not be studied

Exit speeds Two exit speeds were analyzed, namely 40 and 60 mph The 60-mph speed represents a sort of upper limit of turn-off speed, and 40 mph is an intermediate speed

TABLE 1 – CONDITIONS PREVAILING DURING FLIGHT TESTS

Aircraft	Weight (lb)	Air temperature degree F	Barometer (in.)	Headwind (knots)
KC-135	156,000-179,000	51-53	28 92-28 93	calm
C-121	93,200-105,000	46-53	29 04-29 06	calm
C-131	45,990-48,190	51-52	28 92	calm
L-27	4,472-4,586	30	29 14-29 16	11-15

TABLE 2 – SUMMARY OF THE PARAMETERS OF THE JOINT DISTRIBUTIONS OF (d_i, t_i), TAKEN FROM BASIC DATA*

Item		Parameter values for the condition						
		KC-135**		C-121		C-131		L-27
Symbol	Description	60 mph	40 mph	60 mph	40 mph	60 mph	40 mph	40 mph
$E(d_i)$	Average distance required to reach exit velocity	5592	6153	3428	3762	3726	4084	2372
$\sigma(d_i)$	Standard deviation of	444 4	476 7	393 8	367 8	200 0	205.2	237 6
$E(t_i)$	Average time required to reach exit velocity	35 2	42 8	22 3	27.1	25 6	30.1	20.8
$\sigma(t_i)$	Standard deviation of	2 20	2 78	2 79	3 11	1 65	1.91	1.55
ρ_i	Correlation coefficient of the normalized variables d_i and t_i (See Section 5)	0 762	0 731	0 849	0 816	0 693	0 651	0 621
τ_i	Turn-off time	9 1	10 1	8 9	9 9	8 5	9.3	8 4

* Basic data from Appendix "D", A Research Report concerning Exit Taxiway Location and Design.

**Basic data for the KC-135 was revised by adjusting the threshold speeds to conform more closely to the Boeing 707-120 operation previously suggested in airport design literature.* However, it is assumed that the upper limit of exit speed for the L-27 is 40 mph Therefore in this analysis the exit speed for the L-27 is always 40 mph

Manner of processing arrivals The manner of processing arrivals has already been touched upon in Section 2 In the operation of the model the following arrival procedures were assumed regardless of the type of aircraft

1. Arrivals are separated at intervals of from 40 to over 100 seconds
2. Arrivals are separated at intervals of 2, 3, and 4 miles

Airport meteorological and geographical conditions The landing characteristics of the aircraft included in the previous project were used here Ten landings were made with each aircraft The tests were made at Wright-Patterson Air Force Base, Dayton, Ohio in good visibility conditions The elevation of the Base is approximately 800 ft, m s l The conditions shown in Table 1 prevailed during the test period

A summary of the pertinent test data concerned with time and distance to reach exit speeds for the above aircraft and conditions is presented in Table 2.

Number of exits One, two, and three exits were used

The analysis of the data from the operation of the model is contained in Section 4

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*Reference 2, page 2

4. Presentation and Analysis of Results

For purposes of this study, the following definitions have been adopted

Arrival rate The number of aircraft passing over the runway threshold per hour It is to be emphasized that an aircraft reaching the vicinity of the airport but not passing over the threshold on the final approach path to the runway is not regarded as an arrival

Average acceptance rate The number of landings completed per hour for a given set of conditions, including given exit locations, but not optimum locations

Maximum average acceptance rate The average acceptance rate for a given set of conditions with optimum taxiway location

Optimum taxiway location The location or locations of one or more exit taxiways which result in the highest average acceptance rate on the runway for a given set of conditions

Wave-off An arrival which does not become a completed landing

Arrival Rate vs. Acceptance Rate

The general relation between arrival and acceptance rates for a given set of conditions is shown in Fig 3 The straight line indicates the situation that would prevail on an ideal runway capable of accepting all arriving aircraft Calculated acceptance rates for any set of conditions will, at any given arrival rate, be less than the ideal This is so, even at very low arrival rates, because theoretically there will always be some chance of a wave-off inasmuch as the distances and times for aircraft to reach exit speed are assumed to follow normal distributions As a practical matter, however, the difference between calculated and ideal values is so small over a considerable range of arrival rates that the two lines may be regarded as coinciding up to some value where the calculated "real" rate begins to break sharply away from the ideal After the two lines diverge, the ordinate to the calculated line is the average acceptance rate, and the ordinate from the calculated line to the ideal line is the wave-off rate, as indicated in the figure by the example for arrival rate x_b

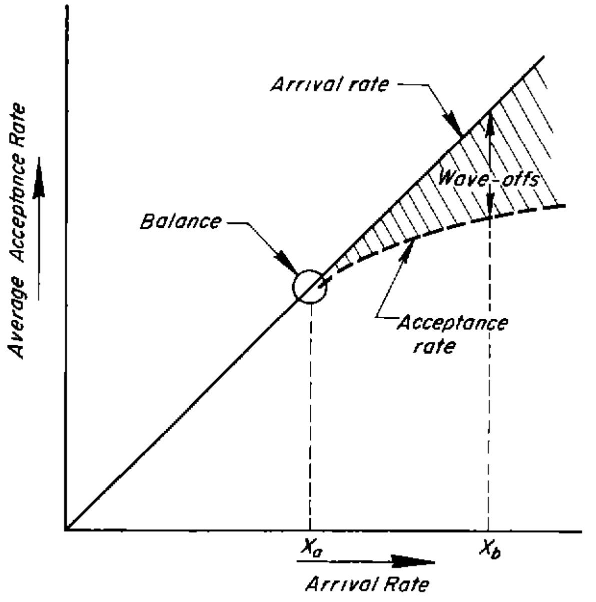


Fig 3 — General relation of average acceptance rate and arrival rate

In operating the mathematical model, it is possible, for a fixed set of conditions (to determine an acceptance rate which corresponds to a selected percent wave-off), such as the point lying above x_a

From a practical point of view, the designer would probably consider as acceptable some range of percent wave-offs between specified limits In this study the limits were arbitrarily chosen as 0 5% and 1% wave-offs, and the region between these limits is referred to as the balance region Most of the comparisons made are for values obtained in this balance region

The balance region is of special interest to the designer because it in effect represents situations where the runway is loaded to capacity That is, at lower arrival rates the runway is able to accept virtually all arrivals, but at higher arrival rates the percent wave-offs become objectionably high

Optimum Exit Locations

The model permits development of a curve similar to that in Fig 3, in which exit location is the variable, all other conditions being fixed, the curve then representing a series of optimum exit locations, each corresponding to a given arrival rate Other curves of the same sort can be developed for other given conditions The point here is that optimum location is not something that can be categorically stated There is an optimum for each condition The quantitative effects of changing conditions on optimum locations are discussed below

Obviously, the designer's problem, is to arrive at an optimum location which is the best compromise among the conditions he foresees and the practical situations he has to face Undoubtedly, minimizing wave-offs as much as practicable will always be a leading consideration, and it is for this reason that data lying in the balance region will be of special interest Comparisons made later in this section generally refer to acceptance rates in this balance region

Results

Results from the operations of the model are tabulated in Table 4 and Figs 4-6 They are computed for the three aircraft populations described on pages 7 and 8

For each population maximum average acceptance rates and the corresponding optimum locations for one, two, and three exits have been computed for two exit speeds,40 and 60 mph, and for two schemes of arrivals one based on fixed intervals of time and another on fixed intervals of distance

All of the computations are summarized in Table 4 The table shows the arrival rates, the acceptance rates, the wave-off rates, and the optimum locations of the exit taxiways In addition, the probabilities q_1, q_2, q_3, q_4 that specific aircraft in the population will cause an immediately following aircraft to be waved-off are tabulated, as well as the weighted probability q of the entire population The exit locations corresponding to acceptance rates in the balance region have been bracketed for easier identification For example, in the case of Population I, with two exits and arrivals based on fixed intervals of time, the optimum exit locations are 4, 125 ft and 6, 861 ft from runway threshold With these locations and an arrival rate of 51 43 aircraft per hour (70-sec separation) the expected maximum average acceptance rate is 51 33 aircraft, and the weighted probability, q, that any aircraft in the population will cause a wave-off of the aircraft following it is 0 002 If the arrival interval is reduced to 45 seconds the acceptance rate is larger (59 47) but q is increased to 0 345

In a number of instances the letter c appears as a notation in the columns listing the location of exit taxiways (See Population I, 70-sec interval of arrival, and 3 exits) The notation was used for the following reasons The arrival rate (51 43) is less than the arrival rate corresponding to the balance region (60 00) In other words the arrival rate is to the left of x_a (See Fig 3) If this occurs there are a number of choices of exit locations which will accept nearly all the arriving aircraft Also, it was found that if the exits were located on the basis of an arrival rate nearly in balance with the acceptance rate, the taxiways were able to accept smaller rates with negligible probability of wave-off In the example cited this would mean that if the designer located the three exits at 3550, 5660, and 6760 ft from runway threshold, the taxiways at these locations could accept nearly all aircraft arriving at rates less than 60 per hour

The data tabulated in Table 4 are presented graphically on Figs 4-6 Two scales have been placed on the abscissa of the charts One is the arrival rate in aircraft per hour and the other is the equivalent in terms of fixed time or fixed distance separation (distance given in statute miles) The computed points (from Table 4) have been joined by dotted lines This has been done merely to indicate general trends since there are, in most cases, an insufficient number

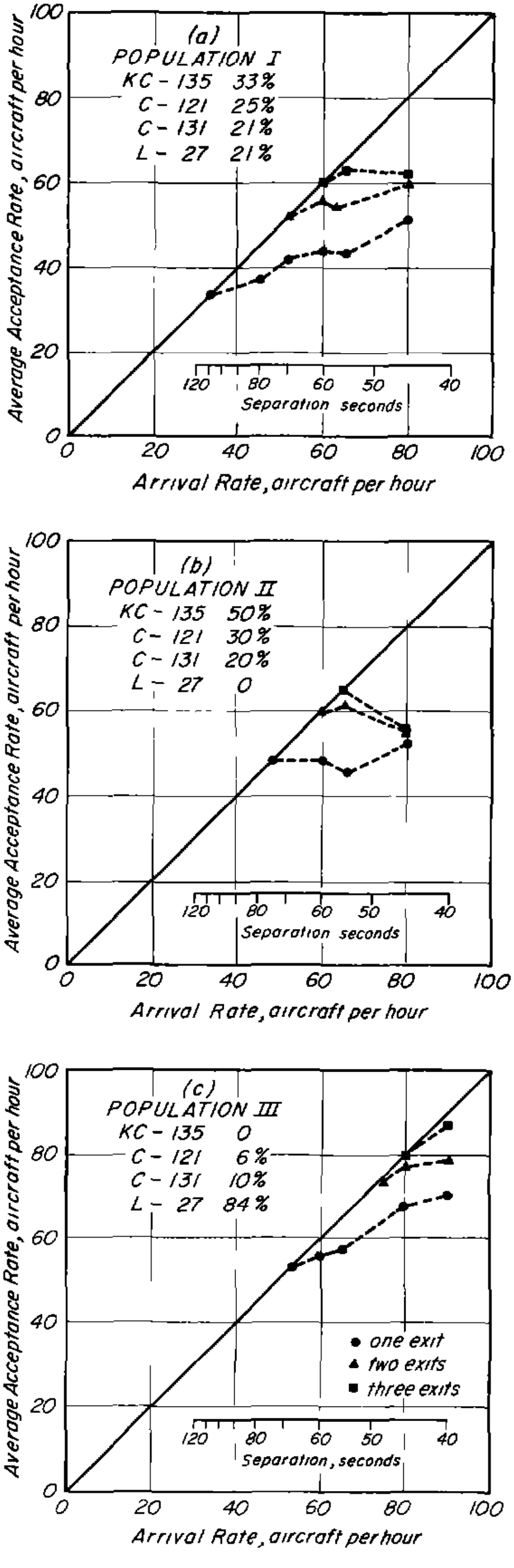


Fig 4 — Acceptance rates for fixed-time arrival separation, 60-mph exits

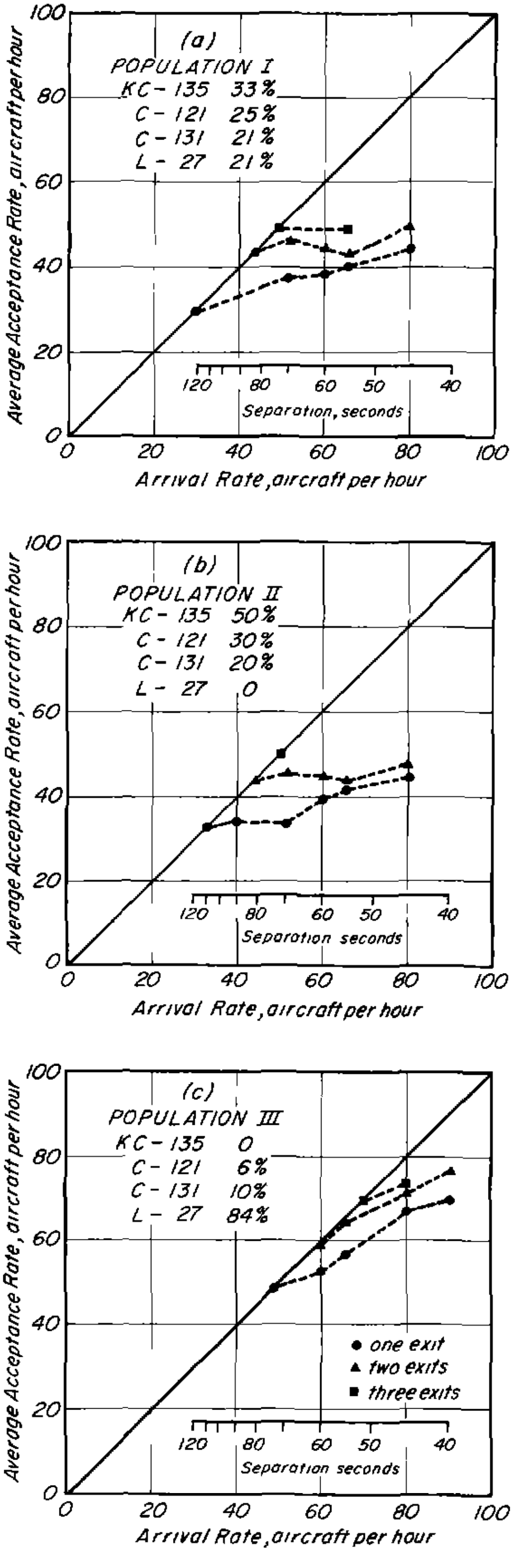


Fig 5 — Acceptance rates for fixed-time arrival separation, 40-mph exits

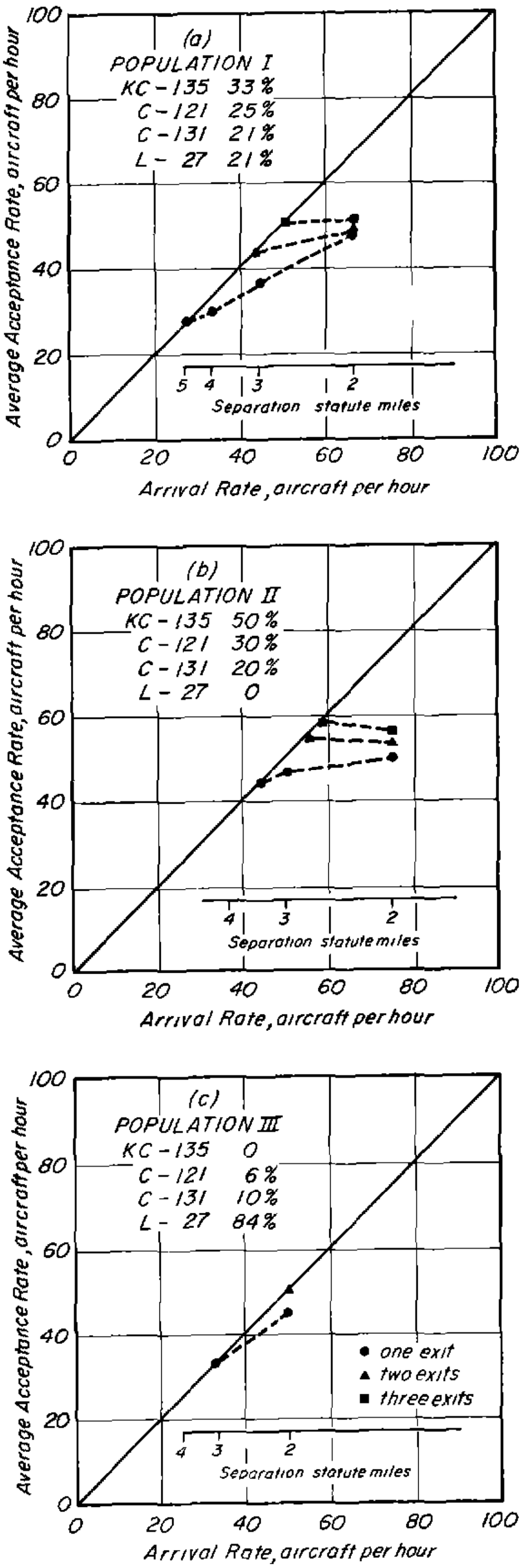


Fig 6 — Acceptance rates for fixed-distance arrival separation, 60-mph exits of computed points to obtain an accurate continuous plot of acceptance rate vs arrival rate

It will be noted that the computed points do not result in a smooth line This can be explained in a simplified way as follows According to the model there are two ways in which an aircraft can cause a wave-off First, the aircraft may miss the exit normally assigned to it because it is going too fast and takes too much time to reach the next exit, and second, the aircraft may take too much time to reach the exit normally assigned to it As the arrival separation interval is varied these two effects do not change at the same rate and hence the average acceptance rate curve cannot be expected to vary smoothly

It will also be noted that for exactly the same arrival rate the two procedures of processing arrivals (fixed time and fixed distance) yield different average acceptance rates The reason for this is that in the case of the fixed time separation it is assumed that the aircraft arrive over the threshold at fixed intervals of time regardless of their individual approach speeds Thus a separation of 60 seconds means that 60 aircraft arrive over the threshold in one hour In the case of the fixed distance separation, the intervals of time at which aircraft arrive over threshold are different and depend on the approach speeds of the individual aircraft in the population Thus a separation of 3 miles at threshold may yield on the average, 60 aircraft per hour but they are arriving at different time intervals The calculations are sensitive to this difference

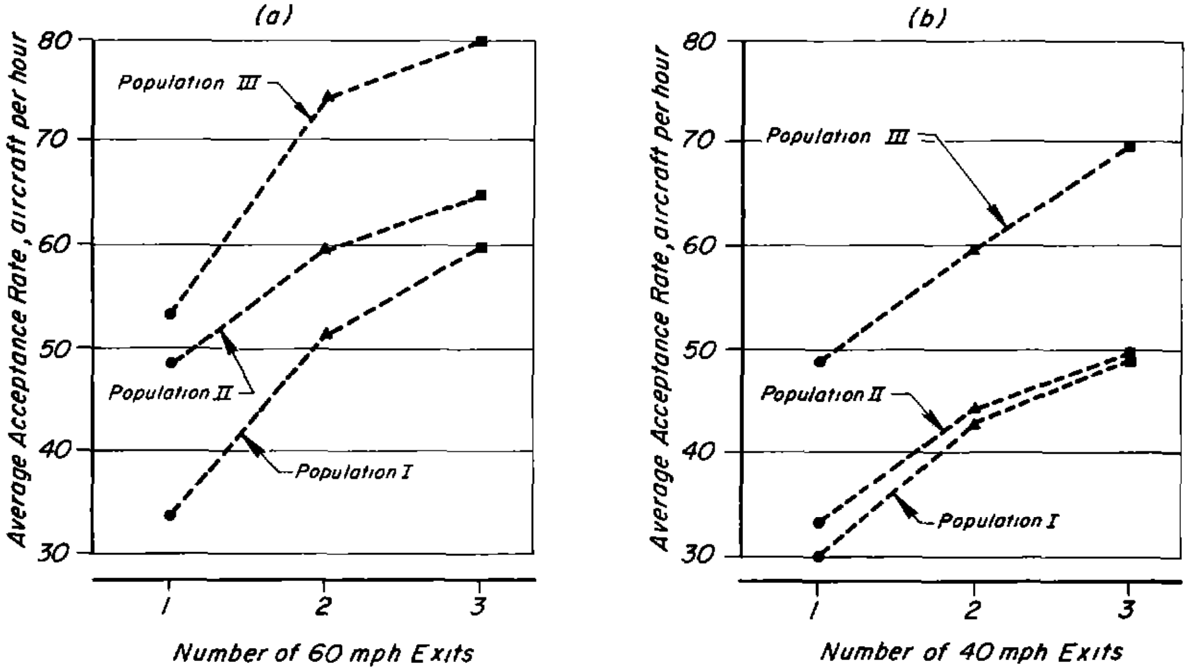
Analysis of Results

Effect of Number of Exits The effect of number of exits can best be illustrated by referring to Table 4 for Population I with arrivals based on fixed intervals of time If a near balance between arrivals and acceptance (in the region of 1% chance of wave-offs) is desired, the maximum average acceptance rate with one exit is 33 57, with two exits 51 33 and with three exits 59 87. Thus the acceptance rate is increased considerably when the number of exits is increased from one to two (about 50%) but the increase is not as large when a third exit is added (another 28%) The example just cited should not be construed as a general trend since the magnitude of the gains achieved by adding additional exits depends a great deal on the aircraft population and the exit speed This is shown graphically in Fig 7

Another use of the charts can be illustrated by reference to the previous example, (see Fig 4a) Suppose it is desirable to provide exits so the runway could accept about 60 aircraft per hour In terms of acceptance rate, two exits would practically do the job (55 73 aircraft) A third exit increases the acceptance rate to 59.87 aircraft, a gain of only 4 aircraft If the designer based his decision strictly from the viewpoint of acceptance, the addition of a third exit might be difficult to justify If, however, the percentage of wave-offs is considered, the picture is somewhat different For three exits, the percentage of wave-offs is considerably lower than for two exits This points to the fact that a prime justification for an additional taxiway may not be to increase the acceptance rate but to decrease wave-offs

Effect of Aircraft Population The effect of aircraft population is illustrated in Fig 7 The plotted points represent near balance between arrivals and acceptance and were obtained from Table 4 While the results represent a very limited study of aircraft population, it is clear that this factor has a distinct influence on acceptance rate Note that when the small general aviation aircraft (L-27) are removed from a population consisting of a mixture of large turbojet transports, and large and medium propeller driven transports (Population I), the acceptance rate of the runway is increased This is illustrated by comparing the acceptance rates for Population II with Population I for an exit speed of 60 mph (Fig 7a) Likewise if small general aviation aircraft are predominant in the population (Population III) the acceptance rates can be quite high as illustrated on this figure The reader is cautioned, however, not to consider these results as a general trend for all conditions, for in examining Fig 7b, which is the same comparison based on 40-mph exit speed, it will be noted that there is very little difference in average acceptance rates between Populations I and II

In order to show the effect that aircraft population has on exit location, a chart has been prepared (Fig 8) which shows how the optimum locations in the balance region change as the aircraft population changes This chart has been drawn in an attempt to portray pictorially the manner in which the locations shift as the aircraft population changes



Note For each population the plotted points are in the region of balance

Fig 7 — Effect of number of exits on acceptance rate (fixed-time arrival separation)

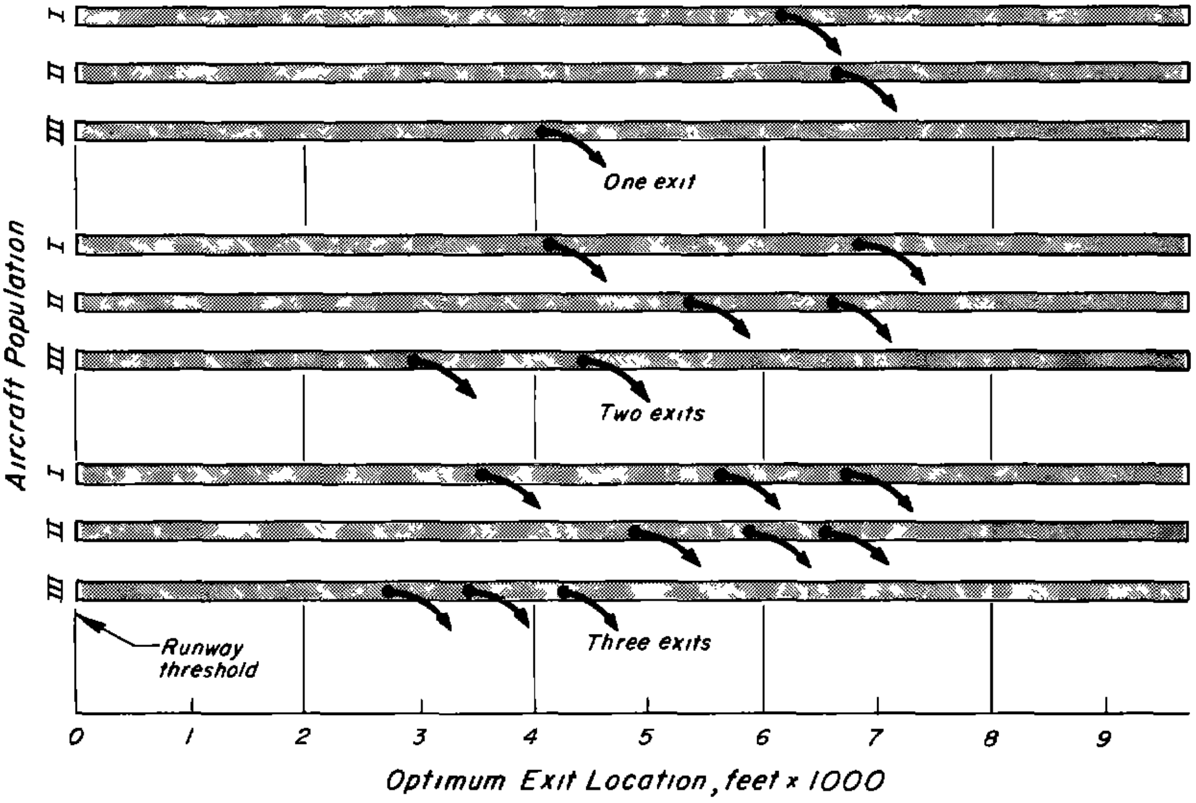


Fig 8 — Effect of aircraft population and number of exits on optimum exit location (fixed-time arrival separation)

TABLE 3 — EFFECT OF EXIT SPEED ON AVERAGE ACCEPTANCE RATE

Number of exits	Average acceptance rate*		Percent Increase
	40 mph	60 mph	
POPULATION I			
1	29 83	33.57	13
2	42 90	51.33	20
3	38 92	59.87	22
POPULATION II			
1	33 24	48 32	45
2	43 98	59.51	35
3	49 54	64 70	31
POPULATION III			
1	48 81	53 17	9
2	59 64	74 29	25
3	69 82	79 72	14

*Obtained in the region of balance

Effect of Exit Speed. The effect of exit speed can be shown quantitatively by comparing the average acceptance rates (in the balance region) for Populations I, II, and III using an arrival based on fixed time interval The data were obtained from Table 4 and are summarized in Table 3.

From the summary it is clear that the magnitude of the gains achieved from a higher exit speed are influenced materially by aircraft population

Effect of Different Procedures of Arrival. It is very difficult to find a basis for comparing the arrivals based on fixed intervals of time with those based on fixed intervals of distance because the latter is an arrival scheme which corresponds to a variable time-interval separation If a comparison is made in the balance region, then by reference to Table 4 a comparison of acceptance rates can be made for Population I, 60 mph, and 2 exits For an arrival based on fixed time interval, the average acceptance rate is 51.33 (interval of time, 70 sec),

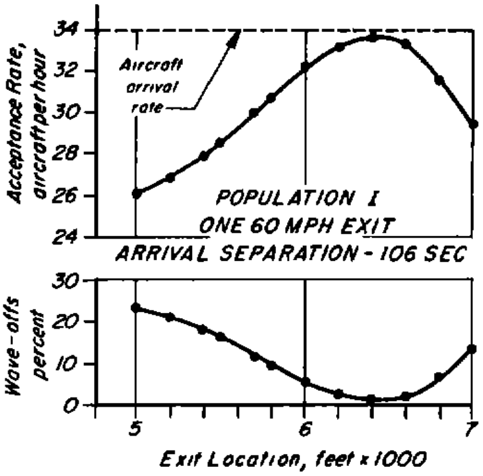


Fig 9 — Effects of exit location.

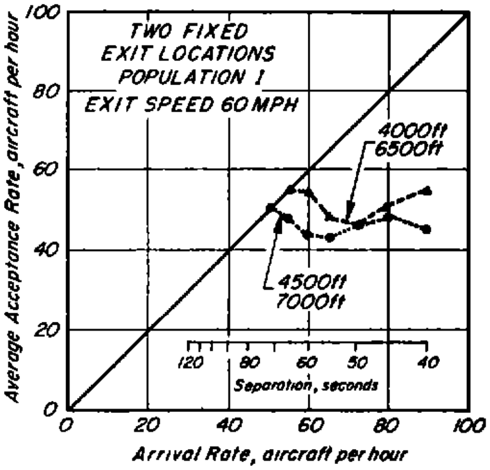


Fig 10 — Rates for two fixed locations.

and for an arrival based on a fixed distance separation, the average acceptance rate is 43 95 (interval of distance, 3 statute miles) But also note that the optimum locations of the exits are nearly the same for the two arrival separation schemes

It will also be noted from the charts that for corresponding average arrival rates the acceptance rate for arrivals based on fixed time separation are generally greater than those for the fixed distance separation

Effect of Exit Location To indicate how the analysis can be used to determine the effect of alternate locations of exit taxiways on runway acceptance rate, acceptance rates were calculated for Population I, 60-mph exit speed, one exit, arrival interval 106 sec, with the exit shifted in 200-ft intervals from 5,000 to 7,000 ft The results of these calculations are shown graphically in Fig 9 The peak of the acceptance rate graph is what has been defined in this report as the maximum average acceptance rate. The corresponding exit location has been defined as the optimum location For illustrative purposes the percentage of wave-offs for each location has also been shown

Another way the model can be operated to demonstrate the effect of exit location is shown graphically on Fig 10 In this example the location of the two exits have been placed arbitrarily (i e , not optimum) at 4,000 ft and 6,500 ft, and 4,500 ft and 7,000 ft assuming that these are alternate choices which are the subject of study at a particular airport The chart (Fig 10) shows the average acceptance rates for these locations of the two exit taxiways, as the arrival interval is varied These average acceptance rates can be compared with the maximum average acceptance rates shown in Fig 4 (for two exits, Population I). The important distinction between Figs 4 and 10 is that in the former the exit locations are not fixed and change with each arrival interval while for the latter the locations are fixed

TABLE 4 — FIXED TIME SEPARATION - 60 MPH EXITS

Item	Average rates exit locations and probabilities when the aircraft separations (in sec) are																						
	45	55	57	60	70	80	100	106	45	55	58	60	70	74	40	45	46	48	55	60	64	67	70
	POPULATION I[d]								POPULATION II[d]						POPULATION III[d]								
ONE EXIT																							
Arrival rate[a]	80 00	65 45	-	60 00	51 43	45 00	36 00	33 96	80 00	65 45	-	60 00	51 43	48 65	90 00	80 00	-	-	65 45	60 00	56 25	53 73	51 43
Acceptance rate[a]	51 48	43 70	-	43 56	41 89	37 18	32 30	33 57	52 76	45 89	-	48 39	50 18	46 32	70 42	67 67	-	-	57 87	56 19	54 83	53 17	51 23
Wave-off rate[a]	28 52	20 75	-	16 44	9 54	7 62	3 70	0 37	27 42	19 56	-	11 61	1 25	0 33	19 58	12 33	-	-	7 58	3 81	1 42	0 56	0 20
Exit location[b]	4199	5217	-	5700	6390	6985	6534	6460	6460	5268	-	5739	6405	6645	2706	2878	-	-	3444	3786	3964	4080	4190
q_1 (KC-135)*	0 999	0 801	-	0 404	0 033	0 001	0 017	0 025	0 999	0 767	-	0 370	0 034	0 009	-	-	-	-	-	-	-	-	-
q_2 (C-121)*	0 050	0 087	-	0 121	0 022	0 001	0 000	0 000	0 049	0 128	-	0 159	0 026	0 008	0 967	0 919	-	-	0 484	0 182	0 067	0 049	0 026
q_3 (C-131)*	0 009	0 009	-	0 019	0 000	0 000	0 000	0 000	0 010	0 021	-	0 035	0 000	0 000	1 000	1 000	-	-	0 921	0 382	0 117	0 038	0 010
q_4 (L-27)*	1 000	1 000	-	1 000	1 000	1 000	0 519	0 162	-	-	-	-	-	-	0 143	0 032	-	-	0 012	0 022	0 011	0 005	0 002
q*	0 554	0 498	-	0 378	0 228	0 210	0 115	0 012	0 516	0 426	-	0 240	0 025	0 007	0 278	0 182	-	-	0 131	0 068	0 026	0 011	0 004
TWO EXITS																							
Arrival rate[a]	80 00	65 45	63 16	60 00	51 43	-	-	-	80 00	65 45	62 07	60 00	51 43	-	90 00	80 00	78 26	75 00	65 55	60 00	-	-	51 43
Acceptance rate[a]	59 47	-	53 99	55 73	51 33	-	-	-	55 16	60 94	60 72	59 51	51 07	-	78 70	77 71	76 69	74 29	65 43	60 00	-	-	51 43
Wave-off rate[a]	20 53	-	9 17	4 27	0 10	-	-	-	24 84	4 51	1 35	0 49	0 36	-	11 30	2 29	1 57	0 71	0 22	0 49	-	-	0 00
Exit locations[b]	2887	-	3763	3889	4125	-	-	-	4194	5103	5280	5362	5849	-	2760	2872	2904	2970	3182	3329	-	-	(b)
	4244	-	5739	6075	6861	-	-	-	5378	6171	6440	6622	7536	-	3978	4246	4301	4440	4525	4872	-	-	(b)
q_1 (KC-135)*	0 999	-	0 375	0 141	0 005	-	-	-	0 871	0 129	0 037	0 013	0 014	-	-	-	-	-	-	-	-	-	-
q_2 (C-121)*	0 030	-	0 013	0 011	0 000	-	-	-	0 048	0 031	0 012	0 005	0 000	-	0 190	0 030	0 019	0 007	0 003	0 000	-	-	0 000
q_3 (C-131)*	0 007	-	0 093	0 061	0 001	-	-	-	0 002	0 008	0 000	0 000	0 000	-	0 121	0 006	0 003	0 000	0 000	0 000	-	-	0 000
q_4 (L-27)*	0 033	-	0 101	0 069	0 000	-	-	-	-	-	-	-	-	-	0 143	0 031	0 023	0 011	0 000	0 000	-	-	0 000
q*	0 345	-	0 170	0 077	0 002	-	-	-	0 430	0 074	0 022	0 008	0 007	-	0 144	0 029	0 020	0 010	0 000	0 000	-	-	0 000
THREE EXITS																							
Arrival rate[a]	80 00	65 45	-	60 00	51 43				80 00	65 45	-	60 00			90 00	80 00	-	-	65 45				
Acceptance rate[a]	61 45	63 15	-	59 67	51 43				55 36	64 70	-	59 98			87 11	79 72			65 45				
Wave-off rate[a]	18 55	2 30	-	0 13	0 00				24 64	0 75	-	0 02			2 89	0 28			0 00				
Exit locations[b]	2887	3420	-	3550	(c)				4900	4900	-	5124			2498	2740			2800				
	4244	5275	-	5660	(c)				4240	5910	-	6294			2981	3440			3500				
	5376	6240	-	6760	(c)				5360	6570	-	7135			3987	4270			4600				
q_1 (KC-135)*	0 871	0 096	-	0 006	0 000				0 870	0 022	-	0 001			-	-			-				
q_2 (C-121)*	0 028	0 003	-	0 000	0 000				0 029	0 003		0 000			0 155	0 016			0 000				
q_3 (C-131)*	0 003	0 011		0 001	0 000				0 006	0 000		0 000			0 135	0 004			0 000				
q_4 (L-27)*	0 033	0 009		0 001	0 000				-	-		-			0 012	0 002			0 000				
q*	0 302	0 036		0 002	0 000				0 445	0 012		0 000			0 033	0 003			0 000				

a Aircraft per hour b Feet from runway threshold. c These exits do not have an exact location. d

	KC-135	C-121	C-131	L-27
Population I is	33%	25%	21%	21%
Population II is	50%	30%	20%	0
Population III is	0	6%	10%	84%

* – Probability that a landing aircraft will cause the following aircraft to be waved off the probability that an arriving aircraft will be waved off is $\frac{q}{1 + q}$

TABLE 4 (Continued) FIXED TIME SEPARATION - 40 MPH EXITS

	Average rates exit locations and probabilities when the aircraft separations (in sec) are																														
	45	55	60	70	72	73	75	80	83	115	120	45	55	60	70	72	77	81	90	100	107	40	45	47	50	51	55	60	70	73	75
Item	POPULATION I[d]											POPULATION II[d]										POPULATION III[d]									
ONE EXIT																															
Arrival rate[a]	80 00	65 45	60 00	51 43	-	-	-	-	-	31 30	30 00	80 00	65 45	60 00	51 43	-	-	-	40 00	36 00	33 64	90 00	80 00	-	-	-	65 45	60 00	51 43	49 32	48 00
Acceptance rate[a]	44 58	41 18	39 11	37 91	-	-	-	-	-	30 80	29 83	44 92	42 14	39 53	34 28	-	-	-	34 69	34 63	33 24	70 31	67 42	-	-	-	56 69	52 78	50 13	48 81	47 75
Wave-off rate[a]	35 42	24 27	20 89	18 52	-	-	-	-	-	0 52	0 17	35 08	23 31	20 47	17 15	-	-	-	5 31	1 37	0 40	19 69	12 58	-	-	-	8 76	7 22	1 30	0 51	0 25
Exit location[b]	2891	4403	4245	4457	-	-	-	-	-	7010	$\boxed{7220}$	4110	4404	4539	4918	-	-	-	6550	6950	$\boxed{7200}$	2705	2872	-	-	-	3339	3716	4317	$\boxed{4430}$	4510
q_1 (KC-135)*	1 000	9 999	1 000	1 000	-	-		-	-	0 036	0 013	1 000	1 000	1 000	0 996	-	-	-	0 202	0 047	0 014	-	-	-	-	-	-	-	-	-	
q_2 (C-121)*	0 991	0 085	0 095	0 029	-	-	-	-	-	0 000	0 000	0 483	0 086	0 027	0 002	-	-	-	0 094	0 019	0 003	0 998	0 992	-	-	-	0 875	0 550	0 066	0 035	0 021
q_3 (C-131)*	1 000	0 139	0 223	0 035	-	-	-	-	-	0 000	0 000	0 680	0 139	0 050	0 011	-	-	-	0 117	0 051	0 022	1 000	1 000	-	-	-	1 000	0 964	0 128	0 046	0 919
q_4 (L-27)*	0 033	0 996	0 636	0 058	-	-	-	-	-	0 022	0 007	-	-	-	-	-	-	-	-	-	-	0 143	0 032	-	-	-	0 003	0 009	0 011	0 004	0 002
q*	0 795	0 590	0 534	0 367	-	-	-	-	-	0 016	0 006	0 781	0 553	0 518	0 500	-	-	-	0 153	0 040	0 012	0 280	0 187	-	-	-	0 155	0 137	0 026	0 010	0 005
TWO EXITS																															
Arrival rate[a]	80 00	65 46	60 00	51 43	-	-	48 00	45 00	43 37	-	-	80 00	65 45	60 00	51 43	-	46 75	44 44	-	-	-	90 00	80 00	-	-	-	65 45	60 00	51 43	-	-
Acceptance rate[a]	50 08	42 53	44 72	46 72	-	-	45 87	44 09	42 90	-	-	46 36	44 30	45 33	46 00	-	45 35	43 98	-	-	-	86 97	71 45	-	-	-	64 38	59 64	51 43	-	-
Wave-off rate[a]	29 92	22 92	15 28	4 71	-	-	2 13	0 91	0 47	-	-	31 64	21 15	14 67	5 43	-	1 40	0 46	-	-	-	13 03	8 55	-	-	-	1 07	0 36	0 00	-	-
Exit locations[b]	2891	4401	3739	4446	-	-	4594	4740	$\boxed{4930}$	-	-	3932	4404	4533	4916	-	5470	$\boxed{5560}$	-	-	-	2491	2872	-	-	-	3234	$\boxed{3444}$	3960	-	-
	4140	5974	4605	6659	-	-	6844	7020	$\boxed{7120}$	-	-	4310	5987	6286	6673	-	7000	$\boxed{7210}$	-	-	-	2973	4220	-	-	-	4439	$\boxed{4554}$	5010	-	-
q_1 (KC-135)*	1 000	0 852	0 999	0 236	-	-	0 126	0 059	0 033	-	-	1 000	9 651	0 613	0 231	-	0 049	0 016	-	-	-	-	-	-	-	-	-	-	-	-	-
q_2 (C-121)*	0 499	0 085	0 011	0 031	-	-	0 012	0 003	0 000	-	-	0 223	0 085	0 027	0 002	-	0 004	0 001	-	-	-	0 984	0 582	-	-	-	0 049	0 016	0 000	-	-
q_3 (C-131)*	0 647	0 130	0 031	0 021	-	-	0 001	0 000	0 000	-	-	0 438	0 130	0 045	0 011	-	0 026	0 011	-	-	-	1 000	0 576	-	-	-	0 134	0 060	0 000	-	-
q_4 (L-27)*	0 033	0 996	0 012	0 052	-	-	0 009	0 001	0 001	-	-	-	-	-	-	-	-	-	-	-	-	0 012	0 032	-	-	-	0 000	0 000	0 000	-	-
q*	0 598	0 539	0 342	0 101	-	-	0 046	0 021	0 011	-	-	0 654	0 477	0 324	0 118	-	0 031	0 010	-	-	-	0 169	0 120	-	-	-	0 017	0 006	0 000	-	-
THREE EXITS																															
Arrival rate[a]	-	65 45	-	51 43	50 00	49 37	-	-	-	-	-	-	-	-	51 43	50 00	-	-	-	-	-	-	80 00	76 60	72 00	70 59	-	-	-	-	-
Acceptance rate[a]	-	49 68	-	50 12	49 40	38 92	-	-	-	-	-	-	-	-	50 49	49 54	-	-	-	-	-	-	73 91	73 22	70 88	69 82	-	-	-	-	-
Wave-off rate[a]		15 77	-	1 31	0 60	0 45	-	-	-	-	-	-	-	-	9 94	0 46	-	-	-	-	-	-	6 09	3 38	1 12	0 77	-	-	-	-	-
Exit locations[b]		3390	-	4360	4415	$\boxed{4437}$	-	-	-	-	-	-	-	-	4842	$\boxed{4960}$	-	-	-	-	-	-	2790	2940	3008	$\boxed{3015}$	-	-	-	-	-
		4457	-	6050	6165	$\boxed{6207}$	-	-	-	-	-	-	-	-	6313	$\boxed{6376}$	-	-	-	-	-	-	3550	4022	4058	$\boxed{4075}$	-	-	-	-	-
		5980	-	6980	7135	$\boxed{7206}$	-	-	-	-	-	-	-	-	7139	$\boxed{7269}$	-	-	-	-	-	-	4240	4420	4515	$\boxed{4560}$	-	-	-	-	-
q_1 (KC-135)*		0 852	-	0 055	0 026	0 018	-	-	-	-	-	-	-	-	0 034	0 016	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
q_2 (C-121)*		0 036	-	0 012	0 006	0 004	-	-	-	-	-	-	-	-	0 001	0 002	-	-	-	-	-	-	0 339	0 131	0 036	0 023	-	-	-	-	-
q_3 (C-131)*		0 126	-	0 003	0 001	0 000	-	-		-	-	-	-	-	0 007	0 007	-	-	-	-	-	-	0 562	0 257	0 118	0 089	-	-	-	-	-
q_4 (L-27)*		0 006	-	0 019	0 008	0 005	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0 007	0 015	0 002	0 001	-	-	-	-	-
q*		0 318	-	0 026	0 012	0 008	-	-	-	-	-	-	-	-	0 019	0 009	-	-	-	-	-	-	0 082	0 046	0 016	0 011	-	-	-	-	-

---

[a] Aircraft per hour [b] Feet from runway threshold [c] These exits do not have an exact location. [d]

	KC-135	C-121	C-131	L-27
Population I is	33%	25%	21%	21%
Population II is	50%	30%	20%	0
Population III is	0	6%	10%	84%

*Probability that a landing aircraft will cause the following aircraft to be waved off the probability that an arriving aircraft will be waved off is $\frac{q}{1+q}$

TABLE 4 (Continued) FIXED DISTANCE SEPARATION - 60 MPH EXITS

Item	Average rates exit locations and probabilities when the aircraft separations (in miles) are															
	2	2 6	2 7	3	4	4 8	5	2	2 55	2 75	3	3 4	4	2	3	4
	POPULATION I[d]							POPULATION II[d]						POPULATION III[d]		
ONE EXIT																
Arrival rate[a]	66 44	-	-	44 30	33 22	27 68	26 58	75 19	-	-	50 61	44 66	37 16	49 77	33 16	24 88
Acceptance rate[a]	48 19	-	-	36 47	29 88	27 31	26 43	49 61	-	-	46 95	44 37	37 96	45 18	33 16	24 88
Wave-off rate[a]	18 25	-	-	7 83	3 34	0 37	0 15	25 38	-	-	3 66	0 29	-	4 59	0 02	0 00
Exit location[b]	4117	-	-	6161	6234	$\boxed{6430}$	6600	4151	-	-	$\boxed{6151}$	66 50	7394	3904	4394	(b)
q_1 (KC-135)*	1 000	-	-	0 100	0 074	0 030	0 012	0 994	-	-	0 104	0 009	0 000		-	-
q_2 (C-121)*	0 057	-	-	0 053	0 000	0 000	0 000	0 068	-	-	0 076	0 007	0 000	0 113	0 007	-
q_3 (C-131)*	0 026	-	-	0 012	0 000	0 000	0 000	0 019	-	-	0 016	0 000	0 000	0 187	0 000	-
q_4 (L-27)*	0 733	-	-	0 789	0 416	0 019	0 007	-	-	-	-	-	-	0 091	0 000	-
q*	0 504	-	-	0 215	0 112	0 014	0 005	0 524	-	-	0 078	0 007	0 000	0 102	0 000	-
TWO EXITS																
Arrival rate[a]	66 44	-	-	44 30	33 22	-	-	75 19	-	55 21	50 61		37 16	49 77	33 18	-
Acceptance rate[a]	48 66	-	-	43 95	-	-	-	54 17	-	54 73	50 59	-	-	49 73	33 18	-
Wave-off rate[a]	17 78		-	0 35	-	-	-	21 02	-	0 48	0 02	-	-	0 04	0 00	-
Exit locations[b]	4079	-	-	$\boxed{4077}$	(c)	-	-	4138	-	$\boxed{6390}$	5630	-	(c)	3003	(b)	-
	6046	-	-	$\boxed{6602}$	(c)	-	-	5839	-	$\boxed{6620}$	7090	-	(c)	4490	(b)	-
q_1 (KC-135)*	0 597	-	-	0 014	-	-	-	0 759	-	0 014	0 000	-	-	-	-	-
q_2 (C-121)*	0 045	-	-	0 002	-	-	-	0 060	-	0 005	0 000	-	-	0 004	-	-
q_3 (C-131)*	0 030		-	0 005	-	-	-	0 018	-	0 000	0 000	-	-	0 000	-	-
q_4 (L-27)*	0 718	-	-	0 008	-	-	-	-	-	-	-	-	-	0 001	-	-
q*	0 365	-	-	0 008	-	-	-	0 402	-	0 009	0 000	-	-	0 001	-	-
THREE EXITS																
Arrival rate[a]	66 44	51 11	49 22	44 30	-	-	-	75 19	59 54	-	50 61	-	-	-	-	-
Acceptance rate[a]	50 07	50 39	49 00	44 24	-	-	-	56 26	59 06	-	50 60	-	-	-	-	-
Wave-off rate[a]	16 37	0 72	0 22	0 06	-	-	-	18 93	0 48		0 01	-	-	-	-	-
Exit locations[b]	4030	$\boxed{3500}$	3530	3090	-	-	-	4135	$\boxed{4940}$	-	4500	-	-	-	-	-
	5640	$\boxed{5430}$	5570	5000	-	-	-	5520	$\boxed{5960}$	-	5630	-	-	-	-	-
	6500	$\boxed{6460}$	6655	6800	-	-	-	5990	$\boxed{6640}$	-	7100	-	-	-	-	-
q_1 (KC-135)*	0 489	0 035	0 011	0 004	-	-	-	0 656	0 015	-	0 001	-	-	-	-	-
q_2 (C-121)*	0 044	0 001	0 000	0 000	-	-	-	0 059	0 002	-	0 000	-	-	-	-	-
q_3 (C-121)*	0 037	0 007	0 002	0 000	-	-	-	0 018	0 000	-	0 000	-	-	-	-	-
q_4 (L-27)*	0 698	0 003	0 001	0 000	-	-	-	-	-	-	-	-	-	-	-	-
q*	0 327	0 014	0 004	0 001	-	-	-	0 349	0 008	-	0 000	-	-	-	-	-

a Aircraft per hour b Feet from runway threshold c These exits do not have an exact location d

	KC-135	C-121	C-131	L-27
Population I is	33%	25%	21%	21%
Population II is	50%	30%	20%	0
Population III is	0	6%	10%	84%

*Probability that a landing aircraft will cause the following aircraft to be waved off the probability that an arriving aircraft will be waved off is $\frac{q}{1 + q}$

5. The Mathematical Model

Since the effectiveness of the number and location of exit taxiways was to be measured in terms of average acceptance rate for a given population of aircraft and a given arrival scheme, an equation for mean acceptance rate was required. The relationship of average acceptance rate with the average arrival separation can be expressed in terms of the average probability that a landing aircraft will cause the aircraft following to be waved off.

The calculation of this average probability requires that formulae be developed which would express the runway occupancy time of the various types of aircraft in terms of the statistical properties of these types. The occupancy times also depend upon the number and location of the high-speed exits. Having developed such expressions, the exit locations for a given number of exits can be determined by maximizing the average acceptance rate.

In this section of the report the mathematical notation and formulation are presented. The expression used for average acceptance rate, the random variable representing occupancy time, and the equations whose solutions result in the exit locations are given. The derivations of the equations appear in the appendices. The assumptions listed in Section 2 serve as the basis for the development of the mathematical formulae.

Notation

q = Mean probability that a landing aircraft will cause a wave off of the following aircraft

$E(\delta)$ = Mean time separation between arrivals

A_r = Arrival rate (aircraft/hour)

A_c = Acceptance rate (aircraft/hour)

n = Number of types of aircraft

i = Index dependent on aircraft type $i = 1, 2, \ldots, n$

p_i = Proportion of the i-th type of aircraft in the population

T_i = Runway occupancy time for an aircraft of i-th type

s_i = Exit speed of an aircraft of i-th type

(d_i, t_i) = Bivariate random variable representing the distance and time an aircraft of type i takes to decelerate to exit speed from threshold

τ_i = Turn-off time of aircraft of i-th type

m = Number of exit taxiways

j = Indexes the exit taxiways $j = 1, 2, \ldots, m$

D_j = Distance from threshold to j-th exit taxiway (note $D_{j-1} < D_j$)

B_j = Half-open interval $(D_{j-1}, D_j]$, $(j = 1, \ldots, m + 1)$ $D_0 = 0$, D_{m+1} = end of runway

$I_{B_j}(d_i)$ = Defined as unity if d_i is in the interval B_j and zero otherwise

$E(\ )$ = Mathematical expectation of the enclosed quantity

$\sigma(\ )$ = Standard deviation of the enclosed quantity

$\sigma(x, y)$ = Covariance of the enclosed pair

Δ = Assigned fixed time separation

b = Assigned fixed distance separation

v_i = Average glide path ground speed of an aircraft of the i-th type, i.e., a value such that $\Delta_i = b/v_i$ is the time separation of arrivals when the succeeding aircraft is of the i-th type

The average acceptance rate can be approximated by the expression

$$E(A_c) \approx \frac{1}{E(\delta)} \cdot \frac{1}{1+q} \tag{5.1}$$

which is approximately the inverse of the average arrival separation time multiplied by a correction factor for wave-offs. The quantity $E(\delta)$ is the average arrival separation time, and q is the probability that a landing aircraft will cause the following aircraft to be waved off. Equation (5.1) is derived in detail in Appendix A.

The average arrival separation $E(\delta)$ is dependent upon the separation scheme considered. Specific average arrival separation terms (for fixed time Δ and fixed distance b respectively,) are developed in Appendix B for the two separation schemes.

The quantity q may be expressed as the weighted sum

$$q = \sum_i p_i q_i \tag{5.2}$$

where q_i is the probability that an aircraft of the i-th type will occupy the runway longer than the arrival separation time δ, i. e.

$$q_i = P_r\,\{T_i > \delta\} \tag{5.3}$$

Thus it becomes necessary to determine the runway occupancy time of an aircraft of the i-th type. The runway is considered to be occupied from the time the aircraft is over runway threshold until it has cleared the runway. Based on the assumption 6 of Section 2, T_i can be expressed as follows

$$T_i = \begin{pmatrix}\text{Time Required}\\ \text{To Decelerate}\\ \text{To Exit Speed}\end{pmatrix} + \begin{pmatrix}\text{Additional Time to Reach}\\ \text{First Available Exit Taxi-}\\ \text{way or Conventional Exit}\end{pmatrix} + \begin{pmatrix}\text{Time to Turn-}\\ \text{off Into Exit}\\ \text{Taxiway}\end{pmatrix} \tag{5.4}$$

$$= t_i + \left\{ \sum_{j=1}^{m} \left(\frac{D_j - d_i}{s_i}\right) I_{B_j}(d_i) + g_{m+1}(d_i) I_{B_{m+1}}(d_i) \right\} + \sum_{j=1}^{m} \tau_i\, I_{B_j}(d_i)$$

where $\left(\frac{D_j - d_i}{s_i}\right)$ is the additional time required to reach the j-th exit taxiway assuming the aircraft maintains the exit speed, and $g_{m+1}(d_i)$ is the additional time required for an aircraft to clear the runway in those cases that the *m-th* exit taxiway is missed. This latter time is always chosen so that two consecutive wave-offs are not possible. Note that for a single landing of an aircraft, d_i can be in only one of the sets B_j, and the indicator function $I_{B_j}(d_i)$ serves as a selector for the particular exit used. The time, τ_i (considered non-random) is the time required for an aircraft of the i-th type to complete the turn-off once it has been initiated. The detailed development of expressions for q may be found in Appendix C.

Thus, the relationship between the number and location of exit taxiways and the average acceptance rate has been established. It remains to indicate the procedure for optimizing the average acceptance rate in terms of the exit locations. The average acceptance rate is maximized by minimizing its reciprocal which is $Q = E(\delta)(1+q)$. Assuming that Q is a differentiable function of D_j, and assuming that m is fixed, the minimization will be accomplished by taking the partial derivatives of Q with respect to the variables $D_1, D_2, \ldots, D_m$, equating them to zero, and solving for the $D_1, \ldots, D_m$.

Since $E(\delta)$ is independent of locations D_j for the arrival separation schemes considered, it follows that

$$\frac{\partial Q}{\partial D_r} = E(\delta)\,\frac{\partial}{\partial D_r}(1+q) \tag{5.5}$$

and hence one need only solve the system of equations

$$\frac{\partial q}{\partial D_r} = 0 \qquad \text{for } r = 1, \cdot \cdot \quad , m \tag{5 6}$$

Upon substituting the expression for q from equation (5 2) the expression (5 6) may be written

$$\sum_i p_i \frac{\partial q_i}{\partial D_r} = 0 \quad \text{for} \quad r = 1, \quad \cdot, m \tag{5 7}$$

and by equation (5. 3) the following system results

$$\sum_i p_i \frac{\partial}{\partial D_r} \Pr \{T_i > \delta\} = 0 \qquad r = 1, \quad , m \tag{5 8}$$

Upon substituting for T_i from (5 4), the system may be rewritten as

$$\sum_i p_i \frac{\partial}{\partial D_r} \Pr \left\{ t_i + \sum_{j=1}^{m} \left(\frac{D_r - d_i}{s_i} + r_i \right) I_{B_j}(d_i) + g_{m+1}(d_i) \; I_{B_{m+1}}(d_i) > \delta \right\} = 0$$

$$\text{for} \quad r = 1, \quad , j, \quad , m \tag{5 9}$$

The system of equations (5 9) is non linear and difficult to solve, therefore machine computation procedures were used The numerical analysis and the computational procedures used are presented in Section 6 Explicit expressions for the system (5 9) are developed in Appendices C and D

For each separation scheme the optimum values $D_1, \quad , D_m$ thus determined result in specific values of q In the charts and tables presented, the listed values of q are those that correspond to optimum exit locations, given the separation parameter

6. Mathematical and Computational Methods

The computations necessary for obtaining the results presented in this report are developed in two parts. The first part is the determination of the exit locations for $m = 1, 2,$ and 3 exits, for each of the three populations, two exit speeds, and two (arrival) schemes as functions of the separating parameters Δ and b. The second part is the evaluation of the acceptance rate corresponding to each of the location solutions.

Most of the inputs necessary to accomplish these computations have been described in Section 3. Some additional remarks concerning the inputs follow. Given a population of aircraft types, the proportion p_i of aircraft of the i-th type is the probability that an arriving aircraft will be of the i-th type, and these probabilities are independent from arrival to arrival. Exit velocities are expressed in feet per second. Arrival separation times are in seconds. Various seemingly irregular values of the arrival separation parameter were used in order to search out the balance region. The fixed distance arrival separation scheme results in time separations which are not uniform but dependent upon the type of the succeeding aircraft. The glide path ground speeds used to compute these time separations appear in Table 5. The speeds listed are recommended approach speeds. Except for the L-27, they were obtained by contacting the users of these types of aircraft. For the L-27 the recommended speed was obtained from the C-310 owner's manual.

TABLE 5 – REPRESENTATIVE GLIDE PATH GROUND SPEEDS

I	Type	Speed
1	KC-135	145 knots
2	C-121	130 knots
3	C-131	110 knots
4	L-27	82.5 knots

The pair (d_i, t_i) has been assumed to have a bivariate normal distribution with parameters given in Table 2. For mathematical purposes it is desirable to make the change of variable

$$u_i = t_i - d_i/s_i$$

and work with the pair (d_i, u_i) which will also have a bivariate normal distribution with parameters, for the aircraft types and exit speeds considered, given in Table 6.

The parameters in Tables 2 and 6 were estimated from the basic data given in a previous

TABLE 6 – SUMMARY OF THE PARAMETERS OF THE JOINT DISTRIBUTION OF (d_i, u_i)

Item		Parameter values						
		i=1 (KC-135)		i=2 (C-121)		i=3 (C-131)		i=4 (L-27)
Symbol	Description	60 mph	40 mph	60 mph	40 mph	60 mph	40 mph	40 mph
$E(d_i)$	Average distance required to reach exit velocity	5592	6153	3428	3762	3726	4084	2372
$\sigma(d_i)$	Standard deviation of d_i	444.4	476.7	393.8	367.8	200.0	205.2	237.6
$E(u_i)$	Average value of $u_i = t_i - d_i/s_i$	-28.3	-62.1	-16.7	-37.0	-16.7	-39.5	-19.6
$\sigma(u_i)$	Standard deviation of u_i	3.66	6.38	2.58	4.15	1.64	7.18	3.32
ρ_i	Correlation coefficient for the pair (d_i, u_i)	-.9214	-.9548	-.8172	-.9007	-.6896	-.8408	-.9305

reference * The means, standard deviations and correlation coefficients were estimated using the appropriate functions of unbiased sample moments

Since two arrival separation schemes are considered and the equations for one parallel those of the other, the two are treated simultaneously Under the foregoing assumption of bivariate normal distribution for the pair $(d_i u_i)$, equation (5.2) may be written explicity as follows

For Scheme 1 (fixed time parameter Δ)

$$q = \sum_{i=1}^{n} p_i \sum_{j=1}^{m+1} \int_{\frac{\Delta - \tau_i - D_j/s_i - E(u_i)}{\sigma(u_i)}}^{\infty} dx \int_{\frac{D_{j-1} - E(d_i)}{\sigma(d_i)}}^{\frac{D_j - E(d_i)}{\sigma(d_i)}} dy\, \phi_i(x, y) \tag{6 1}$$

For Scheme 2 (fixed distance parameter b)

$$q = \sum_{i=1}^{n} \sum_{j=1}^{m+1} \sum_{k=1}^{n} p_i p_k \int_{\frac{\Delta_k - \tau_i - D_j/s_i - E(u_i)}{\sigma(u_i)}}^{\infty} dx \int_{\frac{D_{j-1} - E(d_i)}{\sigma(d_i)}}^{\frac{D_j - E(d_i)}{\sigma(d_i)}} dy\, \phi_i(x, y) \tag{6 2}$$

where

$$\phi_i(x, y) = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1 - \rho_i^2}} e^{-\frac{1}{(1-\rho_i^2)}(x^2 - 2\rho_i xy + y^2)}$$

$$x = \frac{u_i - E(u_i)}{\sigma(u_i)}$$

$$y = \frac{d_i - E(d_i)}{\sigma(d_i)}$$

These equations can be presented more compactly by letting

$$A_{ij} = \frac{D_j - E(d_i)}{\sigma(d_i)}$$

$$C_{ijk} = \frac{\Delta_k - \tau_i - D_j/s_i - E(u_i)}{\sigma(u_i)}$$

$$\Phi(A) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{A} e^{-\frac{1}{2}x^2}\, dx$$

$$C_{ij} = \frac{\Delta - \tau_i - D_j/s_i - E(u_i)}{\sigma(u_i)}$$

and then equations (6 1) and (6 2) can be expressed, respectively

For fixed Δ

$$q = \sum_{i=1}^{n} p_i \sum_{j=1}^{m+1} \int_{C_{ij}}^{\infty} \int_{A_{i,j-1}}^{A_{ij}} \phi_i(x, y)\, dy dx \tag{6 3}$$

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*Reference 1, page 2

For fixed b

$$q = \sum_{i=1}^{n} \sum_{j=1}^{m+1} \sum_{k=1}^{n} p_i p_k \int_{C_{ijk}}^{\infty} \int_{A_{ij-1}}^{A_{ij}} \phi_i(x, y)\, dydx \qquad (6\ 4)$$

The integral over the semi-infinite strip in equations (6 3) and (6.4) may be found by taking the difference of two semi-infinite quarter planes as follows

For fixed Δ

$$q = \sum_{i=1}^{n} \sum_{j=1}^{m+1} p_i \left\{ -\int_{C_{ij}}^{\infty} dx \int_{A_{ij}}^{\infty} dy\, \phi_i(x, y) + \int_{C_{ij}}^{\infty} dx \int_{A_{ij-1}}^{\infty} dy\, \phi_i(x, y) \right\} \qquad (6\ 5)$$

For fixed b

$$q = \sum_{i=1}^{n} \sum_{j=1}^{m+1} \sum_{k=1}^{n} p_i p_k \left\{ -\int_{C_{ijk}}^{\infty} \int_{A_{ij}}^{\infty} \phi_i(x, y)\, dydx + \int_{C_{ijk}}^{\infty} \int_{A_{ij-1}}^{\infty} \phi_i(x, y) dydx \right\} \qquad (6\ 6)$$

For computational purposes, it is desirable to transform these double integrals to a sum of single integrals by means of the transformation[3]

$$M(h, K, \rho) = 1 - \frac{1}{2}\Phi(h) - \frac{1}{2}\Phi(K) - T\left(h, \frac{K - \rho h}{h\sqrt{1-\rho^2}}\right) - T\left(K, \frac{h - \rho K}{K\sqrt{1-\rho^2}}\right) \qquad (6.7)$$

if $hK > 0$ or if $hK = 0$ and h or $K > 0$ or if both $= 0$

$$M(h, K, \rho) = \frac{1}{2} - \frac{1}{2}\Phi(h) - \frac{1}{2}\Phi(K) - T\left(h, \frac{K - \rho h}{h\sqrt{1-\rho^2}}\right) - T\left(K, \frac{h - \rho K}{K\sqrt{1-\rho^2}}\right) \qquad (6\ 8)$$

if $hK < 0$ or if $hK = 0$ and h or $K < 0$

where

$$M(h, K, \rho) = \int_{h}^{\infty} \int_{K}^{\infty} \phi_i(x, y)\, dxdy \qquad (6\ 9)$$

and

$$T(h, a) = \frac{1}{2\pi} \int_{0}^{a} \frac{e^{-\frac{1}{2}h^2(1+t^2)}}{1 + t^2}\, dt \qquad (6\ 10)$$

Then the expressions for q, (6 3) and (6 4), may be written as follows

For Scheme 1 (fixed Δ)

$$q = \sum_{i=1}^{n} \sum_{j=1}^{m+1} p_i \left\{ -M(C_{ij}, A_{ij}, \rho_i) + M(C_{ij}, A_{ij-1}, \rho_i) \right\} \qquad (6\ 11)$$

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[3] The Bivariate Normal Probability Distribution, D B Owen, Sandia Corporation, March 1957, SC 3831 (TR)

For Scheme 2 (fixed b)

$$q = \sum_{i=1}^{n} \sum_{j=1}^{m+1} \sum_{k=1}^{n} p_i p_k \left\{ -M(C_{ijk}, A_{ij}; \rho_i) + M(C_{ijk}, A_{i,j-1}, \rho_i) \right\} \qquad (6\ 12)$$

Similarly, the system of equations (5 9) whose solution gives location becomes:

For Scheme 1 (fixed Δ)

$$0 = \frac{\partial q}{\partial D_j} = \sum_{i=1}^{n} p_i \left\{ \frac{1}{\sigma(d_i)} \int_{\frac{\Delta - \tau_i - D_j/s_i - E(u_i)}{\sigma(u_i)}}^{\frac{\Delta - \tau_i - D_{j+1}/s_i - E(u_i)}{\sigma(u_i)}} dx\, \phi_i\left(x, \frac{D_j - E(d_i)}{\sigma(d_i)}\right) \right.$$

$$\left. + \frac{1}{s_i\, \sigma(u_i)} \int_{\frac{D_{j-1} - E(d_i)}{\sigma(d_i)}}^{\frac{D_j - E(d_i)}{\sigma(d_i)}} dy\, \phi_i\left(y, \frac{\Delta - \tau_i - D_j/s_i - E(u_i)}{\sigma(d_i)}\right) \right\} \qquad (6\ 13)$$

$$j = 1, \qquad , m$$

For Scheme 2 (fixed b)

$$0 = \frac{\partial q}{\partial D_j} = \sum_{i=1}^{n} \sum_{k=1}^{n} p_i p_k \left\{ \frac{1}{\sigma(d_i)} \int_{\frac{\Delta_k - \tau_i - D_j/s_i - E(u_i)}{\sigma(u_i)}}^{\frac{\Delta_k - \tau_i - D_{j+1}/s_i - E(u_i)}{\sigma(u_i)}} dx\, \phi_i\left(x, \frac{D_j - E(d_i)}{\sigma(u_i)}\right) \right.$$

$$\left. + \frac{1}{s_i\, \sigma(u_i)} \int_{\frac{D_{j-1} - E(d_i)}{\sigma(d_i)}}^{\frac{D_j - E(d_i)}{\sigma(d_i)}} dy\, \phi_i\left(\frac{\Delta_k - \tau_i - D_j/s_i - E(u_i)}{\sigma(u_i)}, y\right) \right\} \qquad (6\ 14)$$

$$j = 1, \qquad , m$$

Using the simpler notation these become

For Scheme 1 (fixed Δ)

$$0 = \frac{\partial q}{\partial D_j} = \sum_{i=1}^{n} p_i \left\{ \frac{1}{\sigma(d_i)} \int_{C_{ij}}^{C_{ij+1}} \phi_i(x, A_{ij})\, dx + \frac{1}{s_i \sigma(u_i)} \int_{A_{ij-1}}^{A_{ij}} \phi_i(y, C_{ij}) dy \right\} \qquad (6\ 15)$$

$$j = 1, \qquad , m$$

For Scheme 2 (fixed b)

$$0 = \frac{\partial q}{\partial D_j} = \sum_{i=1}^{n} \sum_{k=1}^{n} p_i p_k \left\{ \frac{1}{\sigma(d_i)} \int_{C_{ijk}}^{C_{ij+1k}} \phi_i(x, A_{ij})\, dx + \frac{1}{s_i\, \sigma(u_i)} \int_{A_{ij-1}}^{A_{ij}} \phi_i(C_{ijk}, y)\, dy \right\}$$

$$j = 1, \qquad , m \qquad (6\ 16)$$

The integral formula used in obtaining the expressions (6 15) and (6 16) is the following

$$\frac{1}{2\pi\sqrt{1 - \rho_i^2}} \int_A^B \phi_i(c, y)\, dy = \frac{1}{\sqrt{2\pi}} e^{-c^2/2} \int_{\frac{A - c\rho_i}{\sqrt{1-\rho_i^2}}}^{\frac{B - c\rho_i}{\sqrt{1-\rho_i^2}}} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2}\, du$$

$$= \frac{1}{\sqrt{2\pi}} e^{-c^2/2} \left[\Phi(B') - \Phi(A')\right] \qquad (6\ 17)$$

where

$$A' = \frac{A - c\rho_i}{\sqrt{1 - \rho_i^2}}$$

$$B' = \frac{B - c\rho_i}{\sqrt{1 - \rho_i^2}}$$

For computational purposes the system can now be expressed as·

For Scheme 1 (fixed Δ)

$$0 = \sum_{i=1}^{n} p_i \frac{1}{\sqrt{2\pi}} \left\{ \frac{e^{-A'^{2}_{ij}}}{\sigma(d_i)} \left[\Phi\left(\frac{C_{ij-1} - A_{ij}\rho_i}{\sqrt{1 - \rho_i^2}}\right) - \Phi\left(\frac{C_{ij} - A_{ij}\rho_i}{\sqrt{1 - \rho_i^2}}\right) \right] \right.$$

$$\left. + \frac{e^{-C'^{2}_{ij}/2}}{s_i\sigma(u_i)} \left[\Phi\left(\frac{A_{ij} - C_{ij}\rho_i}{\sqrt{1 - \rho_i^2}}\right) - \Phi\left(\frac{A_{ij-1} - C_{ij}\rho_i}{\sqrt{1 - \rho_i^2}}\right) \right] \right\} \qquad (6\ 18)$$

$$j = 1, \quad \cdot, m$$

For Scheme 2 (fixed b)

$$0 = \sum_{i=1}^{n} \sum_{k=1}^{n} \frac{p_i p_k}{\sqrt{2\pi}} \left\{ \frac{e^{-A'^{2}_{ij}/2}}{\sigma(d_i)} \left[\Phi\left(\frac{C_{ij-1k} - A_{ij}\rho_i}{\sqrt{1 - \rho_i^2}}\right) - \Phi\left(\frac{C_{ijk} - A_{ij}\rho_i}{\sqrt{1 - \rho_i^2}}\right) \right] \right.$$

$$\left. + \frac{e^{-C'^{2}_{ijk}/2}}{s_i\, \sigma(u_i)} \left[\Phi\left(\frac{A_{ij} - C_{ijk}\rho_i}{\sqrt{1 - \rho_i^2}}\right) - \Phi\left(\frac{A_{ij-1} - C_{ijk}\rho_i}{\sqrt{1 - \rho_i^2}}\right) \right] \right\} \qquad (6\ 19)$$

$$j = 1, \quad , m$$

These expressions contain only the univariate normal distribution function

Each of the above, i e (6 18) and (6.19), is a system of m transcendental equations The solution of such systems usually requires successive approximation methods, and various such methods were applied to the present systems depending upon the number m of exits. Initially, graphic search methods for roots were used by computing the partial derivatives and plotting. This process was suitable for one and two variables (i e , $m = 1$ and 2) but could not be efficiently used for three variables While in two variables an exhaustive search was carried out over the entire surface, seeking to determine whether some general statement might be made concerning the number of possible solutions (i e , local minima) in m-dimensions The result of this effort was that although the number of solutions can be bracketed it cannot be exactly predicted The obvious restriction $D_1 < D_2 < \quad < D_m$ offers considerable simplification in searching for roots

For dimensionality greater than 2 one of two alternative methods is used, i e the method of steepest descent, and a method of successive approximation along segments parallel to the coordinate axes, following the variable for which the function has the largest gradient

The computer used in the evaluation of these expressions was an LGP-30, 4096-word memory, digital computer with basic machine cycle measured in milliseconds The input and output is by typewriter at the rate of ten alpha-numeric characters per second with internal machine language in binary

The use of the computer chosen enabled certain exploratory calculations to be made readily during the development of the final computation forms of the equations These trials in fact influenced the final form of the computations since they gave advance information on the feasibility and the necessity of any particular line of attack A large number of calculations were made efficiently and with a minimum of laborious advance preparation of the type common to large-scale computers

Certain approximations were made in order to enable numerical solution of the equations The systems (6.18) and (6 19) are quite similar and contain two expressions which cannot be evaluated directly, namely the exponential function and the normal probability integral These functions were approximated with rational functions obtained, with minor modification, from Hastings [4] This reference also contains a plot of the error function for each approximation

The computation of (6 11) and (6 12) requires the function $T(h, a)$ of (6 10). Substituting $\frac{h^2}{2} = x$ in that equation results in

$$T(\sqrt{2x}, a) = \frac{e^{-x}}{2\pi} \int_0^a \frac{e^{-xt^2}}{1 + t^2}\, dt \tag{6 20}$$

and expanding the integrand, rearranging terms and integrating leads to the expression

$$T(\sqrt{2x}, a) = \frac{e^{-x}}{2\pi} \sum_{n=0}^{\infty} \frac{x^n}{n!} \left(\arctan a + \sum_{m=1}^{n} (-1)^m \frac{a^{2m-1}}{2m - 1}\right) \tag{6 21}$$

Equation (6 21) was found suitable for computation for values of $0 < a < 1$ and $1 < h < 4\ 75$ For other values of a and h the computations were made with the aid of the following transformations

1) $a > 1$

$$T(h, a) = \frac{1}{2}\,\Phi(h) + \frac{1}{2}\,\Phi(ah) - \Phi(h)\,\Phi(ah) - T\left(ah, \frac{1}{a}\right)$$

2) $h < 0$

$$T(-h, a) = T(h, a)$$

3) $a < 0$

$$T(h, -a) = -T(h, a)$$

4) $a = \infty$

$$T(h, \infty) = \frac{1}{2}(1 - \Phi(h))$$

5) $a = 0$

$$T(h, 0) = 0$$

6) $h = 0$

$$T(0, a) = \frac{\arctan a}{2\pi}$$

These approximations were chosen since their use is much more efficient than the use of tables of functions stored in memory

The computational approximations indicated in this section are quite good Let e_p be the magnitude of error in the final answer of the calculation of any partial derivative of type $\partial q/\partial D_r$, and let e_1, e_2, e_3 be the magnitude of error in the approximations of the probability integral, the exponential function, and the $T(h, a)$ function respectively The error e_p may be computed from the following formula

$$e_p = 2e_1(1 + 2e_2)\left(\frac{s_i\sigma(u_i) + \sigma(d_i)}{s_i\sigma(u_i)\,\sigma(d_i)}\right) \tag{6 22}$$

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[4] Approximations for Digital Computers, Cecil Hastings, Princeton University Press, 1955.

Upon substituting values which would lead to the largest possible e_p,

$$e_p \leqq (39)10^{-8} \tag{6 23}$$

Similarly if the magnitude of error in the final answer of the calculation of q is e_q then,

$$e_q = 4me_3 + e_1 , \tag{6 24}$$

and for the largest m,

$$e_q \leqq (73)10^{-6} \tag{6.25}$$

7. Runway Turn-Ons

Runway turn-ons have been proposed as a means of increasing the capacity of a runway used for both landings and take-offs by large transport aircraft The runway turn-on, of a design similar to that of a taxiway, would be placed at the end of a runway where aircraft start their take-off The thought behind this initial proposal is that aircraft could start the take-off run on the turn-on, accelerate so as to enter the runway at ground speeds higher than now used, and continue the take-off without stopping

In this evaluation it was to be assumed that aircraft are able to enter the runway at speeds ranging from normal taxi speeds to about 60 knots The scope did not include an investigation of the feasibility of turn-ons from the standpoint of aircraft operation Without actual field test of aircraft, it is not known what the maximum turn-on design speed should be, but it was felt that for the purpose of a preliminary evaluation, lacking operational data, a theoretical treatment up to 60 knots would be in order to indicate if there were any gains to be achieved up to this speed

The use of a runway turn-on appears to offer three possible advantages (1) the length of runway available is increased by the length of the turn-on if aircraft are allowed to use the turn-on for accelerating with take-off power, (2) the capacity of a runway in terms of landings and take-offs possibly could be increased by using turn-ons, and (3) the turn-on would permit a smoother operation for take-off aircraft

Operational Feasibility

As stated previously, the geometric design of a curved turn-on for entering aircraft onto the runway at speeds on the order of 30 to 60 knots would have to be investigated by field tests with aircraft Aircraft operational factors would have to be given consideration in developing a geometric design Discussions with aircraft operators and manufacturers focus attention on the following items which must be included in an evaluation of the operational feasibility of high-speed runway turn-ons

1 Using a curved turn-on with a full load of fuel in the wing tanks, there is a possibility of losing fuel through the aircraft fuel vent system
2 On a curved turn-on there could be some loss of steering ability with the nose gear when utilizing high-thrust settings on the engines Such settings do cause a definite transfer of weight from the nose gear on aircraft that have a low thrust line with respect to the center of gravity
3 Accelerating on a curved turn-on using high-thrust settings on the engines, provides very little opportunity to make proper visual checks of engine operation and to make a good compass check A precise compass check would be difficult because the aircraft would not be properly aligned with the runway until a fairly high speed was achieved, at which time the pilot would be deeply involved with the more important operational problems of performing a smooth and safe take-off
4 On a curved turn-on utilizing high-thrust settings on the engines, the loss of an engine might pose a difficult steering problem, especially with a marginal pavement condition (wet or snow-covered) This problem associated with the loss of an engine, especially an outboard engine, would apply also to a narrow, straight-in type of accelerating strip With the higher-thrust type of engines being installed on the new transport-type aircraft, safe operation on a narrow accelerating strip, with high-thrust settings and with sudden loss of an outboard engine, appears very doubtful It is felt that with a narrow, taxiway type of

turn-on the aircraft could easily be partially off the load-bearing pavement before directional control could be recovered

5 The weight of large turbo-jet transports is distributed over a large area, consequently, on a curved turn-on there might be problems in being able to have the aircraft properly develop a turn during a high-thrust acceleration

The possible problems of operating aircraft on a turn-on as outlined above demonstrate that field tests with actual aircraft are absolutely necessary before determining that the turn-ons are operationally feasible

Theoretical Considerations

A theoretical analysis of the effectiveness of turn-ons was not accomplished during the contract period The effectiveness of turn-ons depends almost entirely on traffic operating rules. More information on these rules and how they might be applied to turn-ons is necessary Detailed data on runway usage is also required

Runway usage by take-off aircraft may be divided into two operating parts as follows

Part 1 From the time that an aircraft enters and aligns with the runway until the actual take-off is started This may be further subdivided as

- a Aircraft enters the runway, aligns with the runway and comes to a full stop prior to starting the actual take-off roll.
- b Aircraft enters the runway, aligns and starts the take-off roll, all in one continuous operation (rolling take-off) Discussions with several aircraft operators concerning take-off operations on the runway indicate that some operators have procedures that require an aircraft to come to a full stop on the runway before starting a take-off For this reason the two methods of entering the runway for take-off should be considered

Part 2 From the time that an aircraft starts the take-off until it is clear of the runway

Data Required

During the conduct of this project a part of the data outlined below was being gathered at several airports in the United States under FAA sponsorship, but it was not available in time to make a real effort in formulating a mathematical model for turn-ons

The types of data needed are as follows

1 The times required by various types of aircraft to accomplish a Part 1a turn-on from a right-angle taxiway entrance and from an angled entrance Also required is the elapsed time between the time aircraft position themselves on the runway for take-off and the time when take-off power is applied These elapsed times should be free of air-traffic-control delays

2 The times required to accomplish a Part 1b turn-on.

3 The minimum practicable time intervals between departing aircraft (referenced to the application of take-off power) These times will be governed by separation rules and should reflect not only what is going on today but what may be expected in the future These minimum intervals can be considered as runway occupancy time for take-off aircraft Controlling factors limiting these times are minimum separation rules on airways, disturbance due to wing tip vortices, etc

Appendix A. Development of The Mean Acceptance Rate

It is mathematically most convenient to consider a fixed number R of arrivals over threshold and express the acceptance rate as the number of aircraft accepted by the runway divided by the time required to process (accept or wave-off) the R arrivals, thus

$$A_c = \frac{\text{number of arrivals accepted}}{\text{total time required for } R \text{ arrivals}} \qquad \text{(A 1)}$$

$$= \frac{\sum_{r=1}^{R} J_r}{\sum_{r=1}^{R} \delta_r}$$

where

J_r = A random variable defined as $J_r = 1$ if the r-th arrival is accepted, and $J_r = 0$ if the r-th arrival is waved off $r = 1, \quad , R$

δ_r = Time separation between r-th and $(r + 1)$-st arrivals

The mean value can be approximated by,

$$E(A_c) = E\left(\frac{\sum_{r=1}^{R} J_r}{\sum_{r=1}^{R} \delta_r}\right) \approx \frac{\sum_{r=1}^{R} \alpha_r}{\sum_{r=1}^{R} E(\delta_r)} \qquad \text{(A 2)}$$

where

α_r = $E(J_r)$ the probability that the r-th arriving aircraft is accepted

The accuracy of the approximation in equation (A 2) can be checked by considering the following If Y is a random variable which can be expanded in a Taylor's series about its mean, then

$$\frac{1}{Y} = \frac{1}{E(Y)} - \frac{1}{[E(Y)]^2}(Y - E(Y)) + \sum_{n=2}^{\infty} \frac{(-1)^n}{[E(Y)]^{n+1}}(Y - E(Y))^n \qquad \text{(A 3)}$$

and the ratio of two random variables $\frac{X}{Y}$ can be expressed as,

$$\frac{X}{Y} = \frac{E(X)}{E(Y)} - \frac{E(X)}{[E(Y)]^2}(Y - E(Y)) + \frac{1}{E(Y)}(X - E(X))$$

$$+ E(X)\sum_{n=2}^{\infty} \frac{(-1)^n}{[E(Y)]^{n+1}}(Y - E(Y))^n + (X - E(X))\sum_{n=2}^{\infty} \frac{(-1)^n}{(E(Y))^{n+1}}(Y - E(Y))^n \qquad \text{(A 4)}$$

and the mean value of $\frac{X}{Y}$ is equal to

$$E\left(\frac{X}{Y}\right) = \frac{E(X)}{E(Y)} + \sum_{r=2}^{\infty} \frac{(-1)^r}{[E(Y)]^{r+1}} E\,\{[Y - E(Y)]^r X\} \qquad \text{(A 5)}$$

Since the approximation used is the first term of the above series, its appropriateness can be examined by looking at the remainder term which depends upon the behavior of the ratio

$$\frac{E[(Y - E(Y))^n X]}{[E(Y)]^{n+1}} \qquad \text{(A 6)}$$

Calculations show that these quantities are negligible, being of the order of one one-hundredth and less Note that for separation Scheme 1 (fixed time separation), the approximation is exact, since the denominator is non-random

The numerator of expression (A 2), $\sum_{r=1}^{R} \alpha_r$ is evaluated as follows Note that α_r may be expressed as,

$$\alpha_r = 1 - \Pr\,\{\, J_r = 0\} \qquad \text{(A 7)}$$

From the assumption that an accepted aircraft will not cause two successive wave-offs, it follows that the probability that the r-th arrival is not accepted is equal to the probability that the $(r-1)$-st arrival is accepted and occupies the runway too long, i e.

$$\begin{aligned} \alpha_r &= 1 - \Pr\,\{J_{r-1} = 1 \text{ and the occupancy time of the } (r-1)\text{-st arrival} > \delta_{r-1}\} \\ &= 1 - \alpha_{r-1}\, q\,, \end{aligned} \qquad \text{(A 8)}$$

where

$$q = \sum_{i=1}^{n} p_i \Pr\,\{T_i > \delta_{r-1}\} \qquad \text{(A 9)}$$

For the fixed time arrival separation scheme, $\delta_{r-1} = \Delta$ for all r and hence equation (A.9) becomes,

$$q = \sum_{i=1}^{n} p_i \Pr\,\{T_i > \Delta\} \qquad \text{(A 10)}$$

For fixed distance arrival separation, the time δ_{r-1} is dependent upon the aircraft type of the r-tharrival, and thus

$$\Pr\,\{T_i > \delta_{r-1}\} = \sum_{k=1}^{n} p_k \Pr\left\{T_i > \frac{b}{v_k}\right\} \qquad \text{(A 11)}$$

so that equation (A 9) becomes, for fixed b

$$q = \sum_{i=1}^{n} \sum_{k=1}^{n} p_i p_k \Pr\left\{T_i > \frac{b}{v_k}\right\} \qquad \text{(A 12)}$$

Hence, for the two separation schemes considered, the quantity q is independent of r

Assume the first arrival is accepted, i e.,

$$\alpha_1 \equiv 1 \tag{A.13}$$

It follows that

$$\alpha_2 = 1 - q\alpha_1 = 1 - q \,,$$

$$\alpha_3 = 1 - q\alpha_2 = 1 - q(1 - q) = 1 - q + q^2 \,,$$

$$\alpha_r = 1 - q + q^2 - q^3 + \cdots + (-q)^{r-1} \,,$$

and hence

$$\alpha_r = \frac{1 + (-q)^r}{1 + q} \tag{A 14}$$

Then the numerator of (A 2) becomes

$$\sum_{r=1}^{R} \alpha_r = \frac{1}{1 + q}\left\{R - 1 - \frac{1 + (-q)^{R+1}}{1 + q}\right\} \tag{A 15}$$

Now the denominator of equation (A 2) can be evaluated as

$$E\left(\sum_{r=1}^{R} \delta_r\right) = RE(\delta_r) \tag{A 16}$$

and thus equation (A 2) becomes

$$\frac{E\left(\sum_{r=1}^{R} \alpha_r\right)}{E\left(\sum_{r=1}^{R} \delta_r\right)} = \frac{1}{E(\delta_r)} \quad \frac{1}{1 + q}\left\{\frac{R - 1}{R} + \frac{1 + (-q)^{R+1}}{R(1 + q)}\right\} \tag{A 17}$$

The mean acceptance rate equation becomes the following in the limit as $R \to \infty$

$$E(A_c) \approx \frac{1}{E(\delta_r)} \cdot \frac{1}{1 + q} \tag{A 18}$$

Therefore the mean acceptance rate is approximately the inverse of the mean separation time multiplied by a correction factor for wave-offs The approximation is very good, even for small R and moderately large q, as may be seen by examining equation (A 17)

Appendix B. Determination of Mean Separation Time

For Scheme 1 (fixed time interval separation) δ_r is constant and equal to Δ. Hence

$$E(\delta_r) = \Delta \tag{B 1}$$

For Scheme 2 (fixed distance separation) determine the time Δ_i required for the plane of the i-th type to reach threshold from the fixed distance b There exists a velocity value v_i such that

$$\Delta_i = \frac{b}{v_i} \tag{B 2}$$

Then the expected separation time between the r-th and $(r+1)$-st arrival is

$$E\delta_r = \sum_{i=1}^{n} \Delta_i p_i = b \sum_{i=1}^{n} \frac{p_i}{v_i} \tag{B 3}$$

In both cases $E(\delta_r) = E(\delta)$ is independent of r

Appendix C. Formulation of the Probability Of Causing Wave-Off

For fixed time arrival separation define a quantity q_i by

$$q_i = \Pr\{T_i > \Delta\} \tag{C 1}$$

and note that

$$\Pr\{T_i > \Delta\} = \sum_{j=1}^{m+1} \Pr\{T_i > \Delta \text{ and } d_i \in B_j\} \tag{C 2}$$

Substituting from the previously defined expression for occupancy time, equation (5 4), results in the following

$$\Pr\{T_i > \Delta\} = \sum_{j=1}^{m} \Pr\left\{t_i + \frac{D_j - d_i}{s_i} + \tau_i > \Delta \text{ and } D_{j-1} < d_i \leqq D_j\right\}$$

$$+ \Pr\{t_i + g_{m+1}(d_i) > \Delta \text{ and } D_m < d_i\} \tag{C 3}$$

It is assumed that the additional time allowed for clearing the runway if the last exit is missed, $g_{m+1}(d_i)$,is a fixed amount of time This allowed time is determined so that the assumption of no two consecutive wave-offs is not violated

Equation (C 3) may be written

$$\Pr\{T_i > \Delta\} = \sum_{j=1}^{m+1} \Pr\left\{t_i - \frac{d_i}{s_i} > \Delta - \tau_i - \frac{D_j}{s_i} \text{ and } d_i \in B_j\right\} \tag{C 4}$$

Let $u_i = t_i - \frac{d_i}{s_i}$ and equation (C 4) may be written,

$$\Pr\{T_i > \Delta\} = \sum_{j=1}^{m+1} \Pr\left\{u_i > \Delta - \tau_i - \frac{D_j}{s_i} \text{ and } d_i \in B_j\right\} \tag{C 5}$$

The joint distribution of u_i and d_i is bivariate normal with

$$E(u_i) = E(t_i) - \frac{1}{s_i} E(d_i) \tag{C 6}$$

$$\sigma^2(u_i) = \sigma^2(t_i) + \left(\frac{1}{s_i}\right)^2 \sigma^2(d_i) - \frac{2}{s_i}\sigma(t_i, d_i) \tag{C 7}$$

$$\sigma(u_i, d_i) = \sigma(t_i, d_i) - \frac{1}{s_i}\sigma^2(d_i) \tag{C 8}$$

$$\rho_i = \frac{\sigma(u_i, d_i)}{\sigma(u_i)\,\sigma(d_i)} \tag{C 9}$$

If $\phi_i(x, y)$ is the bivariate normal density function of the normalized u_i and d_i with correlation coefficient ρ_i then equation (C 5) may be expressed as,

$$\Pr\{T'_i > \Delta\} = \sum_{j=1}^{m+1} \int_{\frac{\Delta - \tau_j - D_j/s_i - E(u_i)}{\sigma(u_i)}}^{\infty} dx \int_{\frac{D_{j-1} - E(d_i)}{\sigma(d_i)}}^{\frac{D_j - E(d_i)}{\sigma(d_i)}} dy\, \phi_i(x, y) \qquad \text{(C.10)}$$

For the fixed distance arrival separation scheme, the quantity q_i may be expressed as

$$\begin{aligned} q_i &= \sum_{k=1}^{n} p_k \Pr\{T'_i > \delta_r \mid r + 1^{st} \text{ arrival is of } k\text{th type}\} \\ &= \sum_{k=1}^{n} p_k \Pr\{T'_i > \Delta_k\} \qquad \text{(C 11)} \\ &= \sum_{k=1}^{n} p_k \Pr\left\{T'_i > \frac{b}{v_k}\right\} \end{aligned}$$

The quantity $\Pr\left\{T'_i > \frac{b}{v_k}\right\}$ can be calculated from the preceding equations (i e. (C 3) and its successors) upon replacing Δ with b/v_k

Appendix D. Expressions for Minimizing the Probability Of Causing Wave-Off

In Section 5 equation (5.9) an expression is given which represents a system of equations which are to be solved for values of $D_1, \cdots, D_m$. They require the partial derivatives of q_i with respect to the D_r. If q_i is written in the form developed in Appendix C, equation (C.10), the partial derivative may be taken as follows

$$q_i = \sum_{j=1}^{m+1} \int_{\frac{\Delta - \tau_i - D_j/s_i - E(u_i)}{\sigma(u_i)}}^{\infty} dx \int_{\frac{D_{j-1} - E(d_i)}{\sigma(d_i)}}^{\frac{D_j - E(d_i)}{\sigma(d_i)}} dy\, \phi_i(x, y) \tag{D.1}$$

$$\frac{\partial q_i}{\partial D_j} = \left\{ \int_{\frac{\Delta - \tau_i - D_j/s_i - E(u_i)}{\sigma(u_i)}}^{\frac{\Delta - \tau_i - D_{j+1}/s_i - E(u_i)}{\sigma(u_i)}} dx\, \phi_i \left(x, \frac{D_j - E(d_i)}{\sigma(d_i)} \right) \frac{1}{\sigma(d_i)} \right.$$

$$\left. + \frac{1}{s_i\, \sigma(u_i)} \int_{\frac{D_{j-1} - E(d_i)}{\sigma(d_i)}}^{\frac{D_j - E(d_i)}{\sigma(d_i)}} dy\, \phi_i \left(\frac{\Delta - \tau_i - D_j/s_i - E(u_i)}{\sigma(d_i)}, y \right) \right\} \tag{D.2}$$

Equations (D.1) and (D.2) are directly applicable for the fixed time arrival separation scheme. They may be modified for the fixed distance arrival separation scheme by replacing Δ with b/v_k, multiplying the right hand side of each equation by p_k and summing over $k = 1, \cdots, n$.

SUPPLEMENT TO THE REPORT: A MATHEMATICAL MODEL FOR LOCATING EXIT TAXIWAYS

Optimum Locations of Exit Taxiways and Corresponding Acceptance Rates for Exit Speed of 15 mph

The Institute of Transportation and Traffic Engineering
and
The Department of Industrial Engineering
University of California
Berkeley, California, December 1959

Purpose

This supplement presents computations of acceptance rates and related information for an exit speed of 15 mph, which is representative of the average turn-off speed on 90-deg taxiways, the type used almost exclusively in recent years. Such computations permit comparison with the data for high-speed exit taxiways presented in the basic report and thus provide some indication of the advantages to be gained from high-speed exits.

Basis for Analysis

Computations herein are made with the mathematical model described in the basic report. Particular attention is called to Section 2 of that report, which covers the assumptions on which the model was developed.

Limitations of Results

Results presented herein are probably less representative of "real life" than those for the higher-speed exits presented in the basic report. This is because of the assumption that if an aircraft misses its normal exit, it will continue to the next exit at exit speed. While the assumption appears reasonable for high speeds, it does not appear applicable to 15 mph -- aircraft would probably maintain their higher speed or even accelerate, thus raising the average speed between exits, especially in the case of small aircraft, and especially if the second exit was some distance away.

There is also the question of whether it is reasonable to suppose that an aircraft would actually miss a 15-mph exit. An exit reasonably located for such a speed would probably not be missed by aircraft reaching it at a somewhat higher speed, because at this low absolute value the excess speed would seldom be critical.

The net result of the foregoing considerations is that the results herein probably show acceptance rates lower than those for "real life" and thus suggest advantages for high-speed exits that are greater than would actually be the case.

TABLE 1 - DATA FOR POPULATION I, 15-MPH EXITS

Item	Average rates, exit locations and probabilities when the aircraft separations (in sec.) are:										
	80	90	100	110	112	120	130	132	140	150	160
TWO EXITS											
Arrival Rate	45.00	40.00	36.00	32.73	-	30.00	27.69	-	25.71	24.00	22.50
Acceptance Rate	29.60	28.17	26.70	25.20	-	24.37	24.51	-	24.20	23.37	22.26
Wave-off Rate	15.40	11.83	9.30	7.53	-	5.63	3.18	-	1.51	0.63	0.24
Exit Locations	4615	4700	4785	4870	-	4535	4580	-	4675	4780	4895
	6685	6820	6945	7070	-	7185	7305	-	7425	7540	7655
q_1 (KC-135)	0.680	0.512	0.367	0.252	-	0.164	0.102	-	0.060	0.033	0.017
q_2 (C-121)	0.268	0.133	0.057	0.022	-	0.105	0.081	-	0.044	0.021	0.008
q_3 (C-131)	0.090	0.038	0.014	0.005	-	0.161	0.114	-	0.049	0.015	0.003
q_4 (L-27)	1.000	1.000	1.000	0.997	-	0.555	0.248	-	0.102	0.035	0.011
q	0.520	0.420	0.349	0.299	-	0.231	0.130	-	0.063	0.027	0.011
THREE EXITS											
Arrival Rate	45.00	40.00	36.00	32.73	-	30.00	27.69	27.27	25.71		
Acceptance Rate	31.24	30.21	28.60	26.67	-	25.43	27.32	27.01	25.67		
Wave-off Rate	13.76	9.79	7.40	6.06	-	4.57	0.37	0.26	0.04		
Exit Locations	4610	4700	4785	4870	-	4535	4250	4270	4310		
	6495	6550	6605	6645	-	6675	6000	6055	6245		
	6905	7120	7345	7555	-	7760	7470	7540	7810		
q_1 (KC-135)	0.438	0.221	0.095	0.034	-	0.010	0.031	0.022	0.004		
q_2 (C-121)	0.265	0.133	0.057	0.022	-	0.104	0.002	0.002	0.000		
q_3 (C-131)	0.093	0.038	0.014	0.005	-	0.161	0.000	0.000	0.000		
q_4 (L-27)	1.000	1.000	1.000	0.997	-	0.555	0.013	0.010	0.001		
q	0.441	0.324	0.259	0.227	-	0.180	0.014	0.010	0.002		
FOUR EXITS											
Arrival Rate	-			32.73	32.14	30.00					
Acceptance Rate	-			32.33	31.84	29.93					
Wave-off Rate	-			0.40	0.30	0.07					
Exit Locations	-			3685	3720	3860					
	-			5040	5110	5400					
	-			6630	6650	6740					
	-			7540	7595	7810					
q_1 (KC-135)	-			0.034	0.026	0.006					
q_2 (C-121)	-			0.003	0.001	0.000					
q_3 (C-131)	-			0.000	0.000	0.000					
q_4 (L-27)	-			0.002	0.002	0.001					
q	--			0.012	0.010	0.002					

TABLE 2 - DATA FOR POPULATION II, 15-MPH EXITS

Item	Average rates, exit locations and probabilities when the aircraft separations (in sec.) are·										
	80	90	100	106	110	118	120	130	140	149	150
TWO EXITS											
Arrival Rate	45.00	40.00	36.00	-	32.73	-	30.00	27.69	25.71	24.20	24.00
Acceptance Rate	31.29	30.69	29.91	-	28.88	-	27.67	26.35	25.06	23.93	23.80
Wave-off Rate	13.71	9.31	6.09	-	3.85	-	2.33	1.34	0.65	0.27	0.20
Exit Locations	4600	4685	4780	-	4880	-	4990	5145	5550	5685	5700
	6685	6820	6945	-	7070	-	7188	7305	7435	7625	7650
q_1 (KC-135)	0.680	0.512	0.367	-	0.252	-	0.164	0.100	0.043	0.014	0.013
q_2 (C-121)	0.260	0.128	0.057	-	0.022	-	0.008	0.003	0.014	0.008	0.007
q_3 (C-131)	0.101	0.044	0.015	-	0.004	-	0.001	0.000	0.000	0.000	0.000
q_4 (L-27)	-	-	-	-	-	-	- -	-	-	-	-
q	0.438	0.303	0.204	-	0.133	-	0.084	0.051	0.026	0.010	0.008
THREE EXITS											
Arrival Rate	45.00	40.00	36.00	-	32.73	30.50	30.00				
Acceptance Rate	34.16	34.54	33.73	-	31.94	30.22	29.78				
Wave-off Rate	10.84	5.46	2.27	-	0.79	0.28	0.22				
Exit Locations	4600	4690	4780	-	4880	4965	4990				
	6490	6545	6600	-	6640	6660	6665				
	6900	7120	7340	-	7550	7710	7750				
q_1 (KC-135)	0.438	0.221	0.095	-	0.034	0.013	0.010				
q_2 (C-121)	0.260	0.130	0.057	-	0.022	0.009	0.007				
q_3 (C-131)	0.101	0.042	0.015	-	0.004	0.001	0.001				
q_4 (L-27)	-	-	-	-	-	-	-				
q	0.317	0.158	0.067	-	0.025	0.010	0.007				
FOUR EXITS											
Arrival Rate	-	-	36.00	33.96	32.73						
Acceptance Rate	-	-	34.95	33.61	32.59						
Wave-off Rate	-	-	1.05	0.35	0.14						
Exit Locations	-	-	4780	4775	4690						
	-	-	6305	6000	5950						
	-	-	7015	6900	6910						
	-	-	7660	7670	7750						
q_1 (KC-135)	-	-	0.020	0.013	0.007						
q_2 (C-121)	-	-	0.056	0.014	0.002						
q_3 (C-131)	-	-	0.015	0.000	0.000						
q_4 (L-27)	-	-	-	-	-						
q	-	-	0.030	0.011	0.004						

Analysis of Results

The results obtained from the operation of the model are shown in Tables 1 and 2. (Comparable with Table 4 of the basic report.) Fig. 1 shows acceptance rates for fixed-time arrival separations for 2, 3, and 4 exits. Fig. 2 compares the acceptance rates obtained with 15-, 40-, and 60-mph exits. The data from which Fig. 2 was drawn are shown in Table 3.

TABLE 3 - EFFECT OF EXIT SPEED ON AVERAGE ACCEPTANCE RATE

Number of exits	Average acceptance rate* for exit of: 15 mph	40 mph**	60 mph**	Percent increase, 15 mph to 60 mph
Population I:				
2	22.26	42.90	51.33	130
3	27.01	48.92	59.87	121
Population II:				
2	23.93	43.98	59.51	149
3	30.22	49.54	64.70	115

* Obtained in the region of balance (aircraft per hour)

** From Table 3 of basic report

An interesting observation is the arrival time separation corresponding with a balance between arrival rates and acceptance rate. The data for Population I and two exits are as follows:

Exit speed, mph	Arrival interval, sec	Average acceptance rate
15	160	22.26
40	83	42.90
60	70	51.33

This clearly shows that as the exit speed is increased the arrival separation interval must be reduced if the higher acceptance capability of the high-speed exits is to be taken advantage of.

The trend in optimum locations of exits for the above example is also of interest. The data for Population I, and two exits are as follows:

Exit speed, mph	Optimum locations (2 exits), ft*
15	4895, 7655
40	4390, 7120
60	4125, 6861

* Distance from runway threshold.
In region of balance.

Thus as the exit speed is decreased, the taxiways are shifted farther and farther down the runway.

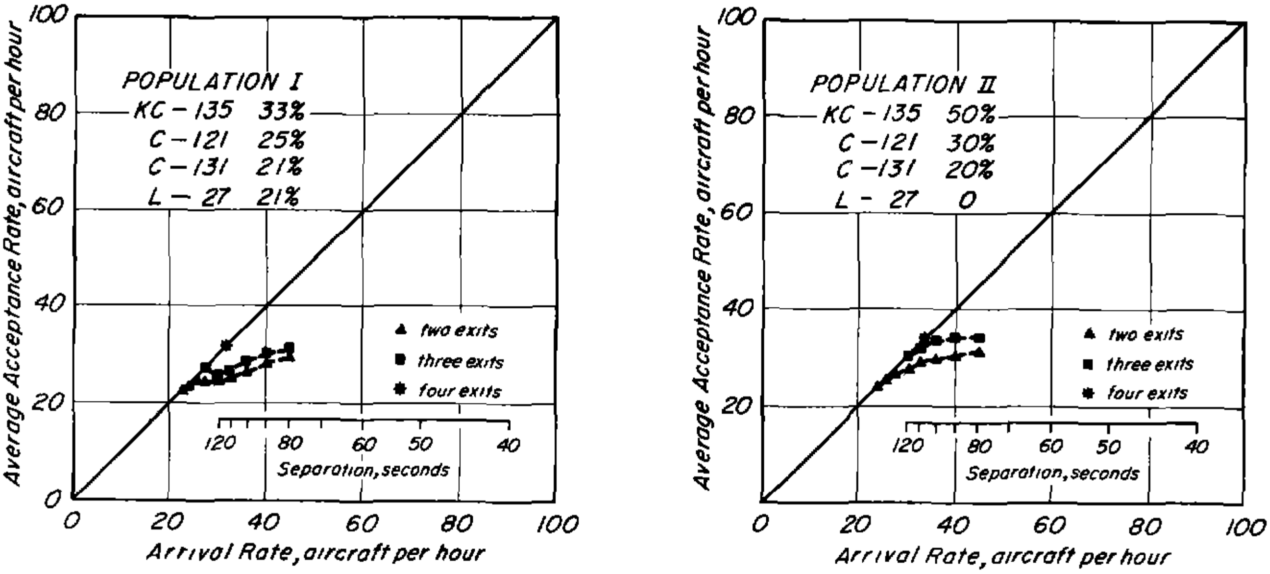

Fig 1 — Acceptance rates for fixed-time arrival separation, 15-mph exits

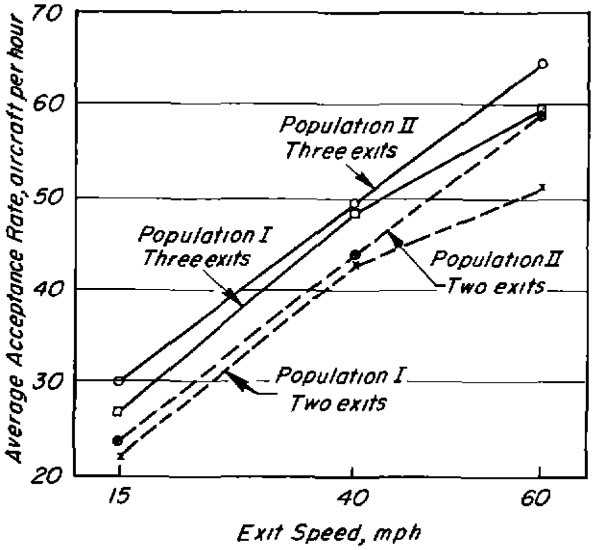

Fig 2 — Effect of exit speed on average acceptance rate (fixed-time arrival separation) Plotted points are in the region of balance