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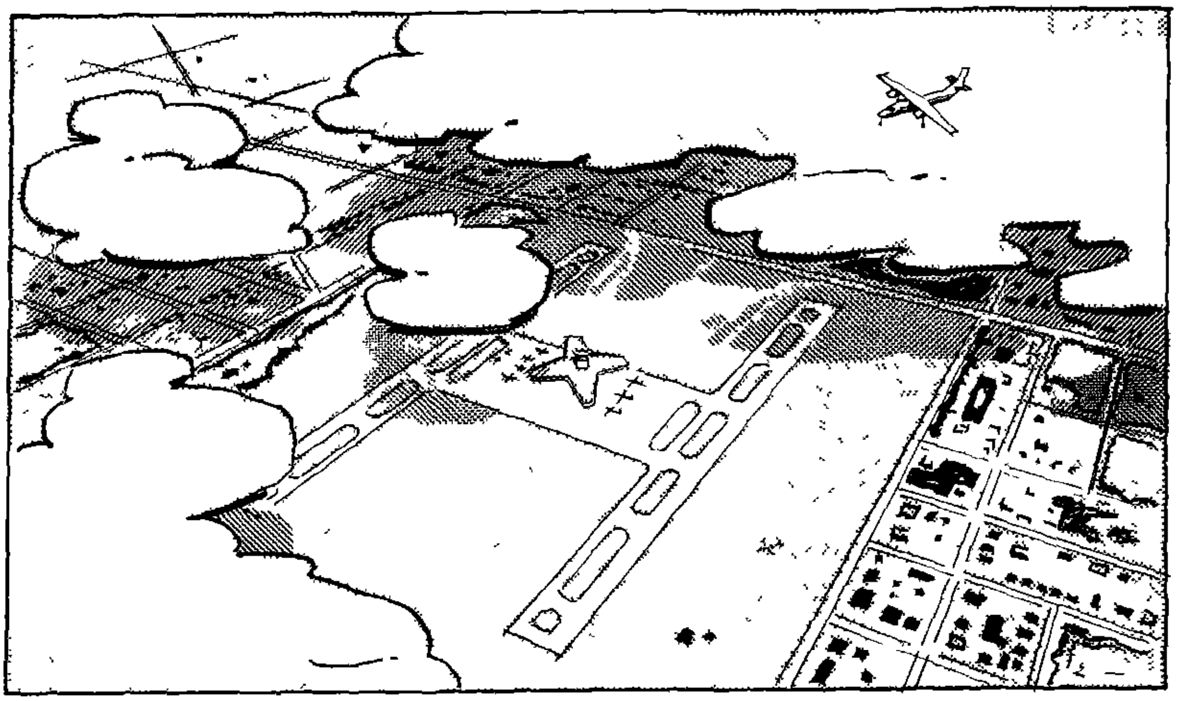

AIRPORT RUNWAY AND TAXIWAY DESIGN

REPORT NO 7601-1
JULY 1960

Prepared by

AIRBORNE INSTRUMENTS LABORATORY
A Division of CUTLER-HAMMER, Inc
DEER PARK, LONG ISLAND, NEW YORK

AIRPORT RUNWAY AND TAXIWAY DESIGN

by

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This report has been approved by the Director, Bureau of Research and Development, Federal Aviation Agency Since this is a technical information report the contents do not necessarily reflect the official FAA policy in all respects, and is intended only for distribution within the Federal Aviation Agency

James L Anast
Director
Bureau of Research and Development
Federal Aviation Agency

REPORT NO 7601-1
Contract FAA/BRD-136
July 1960

Prepared in Cooperation with
PORTER & O'BRIEN

by

AIRBORNE INSTRUMENTS LABORATORY
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ERRATA

Please make the following changes in Report No 7601-1

Page	Line	
A-1	2	Change "Sections VII and X" to "Sections VIII and XII"
A-9	29	Change "Appendix C" to "Supplement II"
A-24	14	Change "Section X" to "Section XII"
A-43	6	Change "Appendix B" to "Supplement III"
C-7	13	Change "Figures 9-10, 9-11, and 9-12" to "Figures 9-12 through 9-20"
D-1	4	Change "Section IX" to "Section X"

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FAA NEWS
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FAA- #4

FOR IMMEDIATE RELEASE WOrth 7-3801

SPECIAL STUDY ON AIRPORT CAPACITY AND GROUND TRAFFIC CONTROL PROBLEMS CONCLUDED BY FAA

A year-long study to provide a technique that permits forecasting airport capacity and analyzing ground traffic control problems has been concluded under auspices of the Federal Aviation Agency.

James L. Anast, Director of the FAA Bureau of Research and Development, today reported that as a result of the extensive project, airport owners and airline operators will have sufficient tools with which to develop economic analyses.

"They not only offer a basis for which business-like decisions can be made about airport improvements," he stated, "but traffic control management can analyze the effect on acceptance rate of procedural changes or changing aircraft populations."

Conducted by Airborne Instruments Laboratory at Deer Park, L. I., New York, in cooperation with Porter and O'Brien, consulting engineers, the study covers observations made at 14 airports of all classes, sizes and shapes throughout the country.

At each airport the researchers observed operations and interviewed controllers, operators and airport managers. Detailed measurements were made in some instances by recording over-threshold, off-runway, clear to take-off, start-roll and other pertinent data.

This information formed a statistical basis for forecasting actual airport capacity.

The project team was able to document what is called the "pressure factor" -- meaning that as the load on an airport increases, the efficiency of pilots and controllers improve. This actual data makes it possible to develop the spacing factors that are inputs to the mathematical models.

(more)

Charts and figures used in connection with the study show the average delay resulting from a given movement rate during visual flight rule (VFR) conditions.

They also indicate the effect of jet transports on airport capacity, as well as that of runway turn-off design capacity. On one type of runway the capacity was found to vary from 36 to 54 movements per hour, depending on the turn-off layout used.

The report likewise cites that the delay resulting from various operating rates can be used to determine whether an improvement can be made economically.

One figure shows that operating costs for aircraft vary from $15 a minute for the large jet transports to as low as 25 cents per minute for small single-engine craft.

By applying such costs to the delay values forecast for various operating rates and comparing them with the expense of building runways and taxiways, additional economics of these construction costs can be further evaluated.

The lengthy volume contains a number of conclusions and recommendations dealing with runway and taxiway design that will be of special interest to both airport and airline officials.

As soon as possible, copies of the report will be printed by the Government for public distribution

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For Further Information:
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ACKNOWLEDGMENTS

This study extended from July 1959 to July 1960. During that time numerous individuals have helped us obtain data, reviewed our efforts, and lent their assistance. These persons include many within the Federal Aviation Agency Bureau of Research and Development, air controllers at 14 airports, airport and airline officials at these airports, pilots, airport engineers, and other contractors. We appreciate their assistance in this work, which we believe is a necessary contribution to the orderly development of a most vital but much neglected fundamental of aviation--the airport.

FOREWORD

This report contains the results of a study program on airport runway and taxiway design. The complete report is published under the title:

"Airport Runway and Taxiway Design"

That portion of principal concern to airport planners has been issued separately under the title and subtitle:

"Airport Runway and Taxiway Design Excerpts on Typical Configurations--Capacities--Evaluation of Designs"

Additional reference sources can be found in the bibliography that is issued as Supplement I to this report:

"Bibliography of Source Material on Airport Design and Applicable Methods of Theoretical Analysis"

The two other supplements are

II. "Use of Exit Taxiways at New York International Airport"

III. "Take-Off Performance of Selected Piston-Engine Aircraft in Routine Operation"

ABSTRACT

Airport operations at numerous civil airports have been observed and measured. The data have been analyzed to identify the elements important to airport capacity and that cause delay to operations, and to identify and evaluate aircraft spacing intervals.

A particularly interesting result of the field observations was the ability to document the so-called "pressure factor"--that factor that evidences itself in higher efficiency at higher operating rates. Controllers and pilots, sensitive to the tempo of the operation, reduce the spacings and react more promptly to traffic control. Thus, as the airport operating rate increases, the delay build-up is less pronounced than the increase in the operating rate would indicate.

It has been found useful to study airports by analyzing the delay to operations that results from various movement rates rather than maximum capacity ratings since, from a practical standpoint, operations at maximum capacity are not observable. Mathematical formulas have been devised and tested that use observed operating data in relating delay to movement rates for runway operations, and analyzing taxiways and holding aprons. These formulas have been used in typical airport operating problems to demonstrate and analyze airport configurations. These studies have demonstrated the importance and effect upon operating rates of various aircraft populations, runway lengths, runway turn-off layouts, runway layouts with respect to other runways, and the ratio of arrivals to departures.

The multiplicity of airport designs around the country have been examined to determine which are typical. The mathematical formulas developed have been applied to airport configurations to determine which are typical and will best satisfy general airport needs. A procedure has been suggested for making economic analyses of proposed airport improvements using the delay predictions obtained with the mathematical formulas. In this procedure, an economic value is placed on delay. The change in delay between the configurations being compared is then determined and converted into dollars.

A guide for the application of the mathematical formulas is provided that indicates how the operational elements are selected and how computations are made. The mathematical formulas can be applied to many airport problems, including runway operating rates with VFR and IFR, taxiway intersection, runway crossing, aircraft holding areas, and the staging of airport construction.

I. INTRODUCTION

The most recent official airport design guide is the CAA booklet, "Airport Design."* Although portions of this booklet have been revised from time to time by the issuance of airport engineering bulletins, there is a need to update information on configurations to adapt airport design to new aircraft types and to incorporate features that have been found desirable in airport construction. This report presents methods of analyzing airport improvements that can provide a logical basis for evaluating proposed changes in design.

In addition to the need for re-examination of airport configurations, a method is needed for accurately assessing the capacity of airport runways. The report on "Modernizing the National System of Aviation Facilities"** presents typical airport configurations and places capacity ratings on each of these designs. The report states, however, that "determination, simulation, and flight tests should be extended to determine more accurately maximum runway capacity and a minimum spacing of aircraft to the runway. It is vital that we determine ultimate runway capacity to more accurately plan for long-range development of adequate airport facilities on a schedule which will meet the projected increase in traffic.

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* "Airport Design," U. S. Department of Commerce, U. S. Government Printing Office, January 1949.

** "Modernizing the National System of Aviation Facilities," Office of Aviation Facilities Planning, The White House, U. S. Government Printing Office, May 1957.

At least five years headway must be provided in planning to construct a new airport and equip it for operation."

In the past, airport construction and improvements for runways and taxiways could not easily be evaluated from an economic standpoint for lack of a reasonable means of analysis.

Many different airport layouts exist around the country. This fact, together with the growth of air traffic and the increase in operations at these airports, requires more uniform design criteria and more precise economic measures for airport improvements. In this way we can measure more accurately the improvement in delay or capacity obtainable with a given airport design and judge whether or not improvements should proceed, based on their economic benefit.

The results of this study can be used to meet these needs in airport planning and design.

II. SUMMARY

The contract required that we investigate previous work done in airport design, obtain an up-to-date picture of actual current operations and practices, apply new mathematical and analytical techniques to airport operations, and make practical application of these factors to the development of design criteria and airport configurations.

A. PREVIOUS WORK

In reviewing the previous work done in this field, we have prepared a bibliography that will be issued as a supplement: "Bibliography of Source Material on Airport Design and Applicable Methods of Theoretical Analysis."

B. OBSERVATION OF AIRPORT OPERATIONS

We have analyzed applicable data available from the earlier FAA terminal area studies and have visited 14 airports to observe operations at first hand. At several airports, we recorded detailed data on VFR runway operations. To obtain valid operating data, and evaluate it with minimum manpower, requires the use of economical but effective techniques that are described in this report. No IFR data were taken under this contract, but data available from allied work done by AIL have been used in this report. Analysis of the data obtained from the airport visits gave a good insight into the requirements for the design of the mathematical models.

C. MATHEMATICAL TECHNIQUES

Prior to the airport visits, and while these visits were under way, mathematical models or formulas were developed that could be used in forecasting operating rates of runways and taxiways with associated delays. Development of these models was the responsibility of Dr. H. P. Galliher, Deputy Director, Operations Research Center, Massachusetts Institute of Technology, who served as consultant to AIL on this project. An original mathematical model was devised covering delays under priority-type operations (typical in runway operations). This mathematical model, together with an available queuing model, was found to have many applications in analyzing airport problems.

As our observations progressed, these models were tested to determine their applicability. This testing ensured that the three mathematical models that were developed represented operating conditions and thus made accurate prediction of operating rates practical. This procedure--checking analytical work against actual observed data while the study progressed--proved a productive method for arriving at valid conclusions.

D. AIRPORT DESIGN CRITERIA

Minimum clearance standards had previously been determined and published in airport design and airport engineering bulletins. It was desired that these bulletins be reviewed using new techniques devised as part of this contract. The most logical approach was the probability analysis of accident data on civil airports. These data were analyzed and the physical dimensions of aircraft were reviewed to determine desirable minimum clearance standards.

E. TYPICAL CONFIGURATIONS

The configurations existing at airports around the country were grouped into categories using the mathematical models together with the observed data. The configurations were individually analyzed to compare their relative merits. Following this analysis, typical configurations were developed to satisfy assumed population and traffic volume characteristics. These configurations can be useful in deciding on the configuration best suited to a particular site.

A procedure was developed for making economic evaluations of airport runway and taxiway improvements. This is, we feel, a basic necessity in establishing airport design on a more rational long-term basis.

F. APPLICATION OF MATHEMATICAL MODELS

The inputs to the mathematical models are described, together with preliminary data, to aid in selecting values for the elements. Thus, if the airport designer wishes to evaluate a specific configuration, he can use the procedure, data, and techniques in this report to develop the detailed analysis.

The airport planner and airport designer is thus provided with a tool with which he can analyze the operational merits of airport design, and then make an economic evaluation of that design.

III. CONCLUSIONS

A. PRACTICAL CAPACITIES

It does not appear that there will be any major improvement in runway operations or design (beyond design criteria now available) that will substantially increase VFR runway capacity, though IFR runway capacity can still be increased. Consequently, the most effective way to increase airport capacity is to construct parallel runways.

Parallel runways of unequal length, designed so that the shorter runway is suitable only for light aircraft, reach maximum use when the air traffic population is such that the number of light-aircraft movements is considerably greater than the number of heavy-aircraft movements--usually about 2 to 1, but dependent upon the total aircraft population.

The need to control taxiing aircraft crossing active runways, so often required in parallel-runway operations, increases the workload of the "local" controller and thereby reduces airport capacity. Because of this problem, not more than two parallel runways should be built on the same side of the aircraft terminal ramp.

When airport site restrictions do not permit use of parallel runways, intersecting runways--if wind conditions permit their use--can provide a substantial increase in capacity. The maximum increase in capacity with intersecting runways, when the intersection is near the downwind end of the runway, would be perhaps 65 percent. This could diminish to about single runway capacity if the intersection occurs near the upwind end of the runway.

Runway operations of large jet aircraft like the Boeing 707 considerably reduce available runway capacity. This is due to the particular aircraft characteristics, such as longer engine warm-up on the runway and longer runways required for both landing and take-off.

The periodic revision of the standards for airport runways to increase their length frequently causes changes in airport master plans that are detrimental to their operational efficiency. Standards of runway length should be stabilized to permit orderly planning, to obtain maximum operational efficiency, and to avoid making essential airport sites obsolete.

Optimum runway turn-offs can increase runway capacity perhaps 50 percent over minimum turn-off designs.

Runway turn-on designs need accommodate only normal taxi speeds, but they should not require an aircraft to turn downwind on the runway to get to the end of the runway.

Although basic design criteria can be developed that have general application, there is no single airport design that is optimum and that will satisfy all desired conditions. In addition, even for a given type of airport, such as the continental airport, there is no "best" design because of local site and airspace problems.

With modern jet aircraft, the cost of taxiway delays or delays in waiting for a gate, approaches the cost of delays in the air. Consequently, efficiency is essential at all operational areas of the airport as well as on the runways.

B. EFFECTIVENESS OF MATHEMATICAL FORMULAS

Using delay predictions for airport operations enables the economic analysis of specific airport designs. Techniques have been suggested for accomplishing this economic analysis.

Analytical models have been developed that can be used to relate operating rates on runways and taxiways to the resulting delay in a realistic manner. The results of these analyses are considered superior to those obtained in the past by any other means.

The analytical models can also be used to develop basic airport design criteria from factual data.

A technique to estimate delay during a peak or overloaded period has been developed and is described.

The operating efficiency of pilots and controllers during landings and take-offs increases as the landing rate increases. This "pressure factor" must be recognized in predicting airport capacities.

Additional data that are needed for adequate evaluation of runway configurations include measurement of the intervals between successive landings on a runway equipped with high-speed turn-offs at high traffic loads when pilots are well acquainted with the layout of the high-speed turnoff. The data available thus far indicates that there is good reason to doubt that the time saved on the runway will be fully used to reduce the spacing between arrivals and thus increase runway capacity to maximum potential.

The input times for departures (ready to go) were found to have a Poisson distribution. The output times for departures (start roll) were found to meet the conditions of Poisson input with Erlang service.

The theoretical variations of the average delay of a sample queuing operation appear so large that it seems unlikely that airport operations rigidly conform to any simple queuing model.

Delay predicted by steady-state models, when applied to airport operations with heavy traffic, must be carefully interpreted since it may take hundreds or even thousands of operations to achieve steady-state conditions.

The analytical models can be improved in two ways.

1. More comprehensive data on actual airport operations, particularly IFR, would improve the quality of the inputs.
2. Further testing and modification with respect to intersecting and parallel runways would make their use more universal.

IV. ANALYSIS OF TYPICAL RUNWAY CONFIGURATIONS AND FACTORS AFFECTING AIRPORT CAPACITY

The many runway configurations in existence or proposed have been examined, and representative configurations have been studied to determine the relative merits of each from the standpoint of runway capacity. The runway configurations studied are shown in Figure 4-1.

To make it possible to compare one configuration with another, common aircraft populations have been assumed for continental and trunk airports as follows.

Class	Type	Percentage of Aircraft Population: Continental	Percentage of Aircraft Population: Trunk
A	Large turbo jet aircraft	20	-
B	Four-engine propeller transports, including turboprops, and heavy two-engine transports (CV440)	30	30
C	Two-engine transports (8000 to 36,000 pounds)	20	30
D	Two-engine aircraft (2800 to 8000 pounds) and high-performance single-engine aircraft	10	20
E	Single-engine aircraft below 2800 pounds	20	20

The runway length chosen for analysis for the continental airport population was 9000 feet because jet aircraft were included in the analysis. For the trunk airport population, a 7000-foot runway has been assumed.

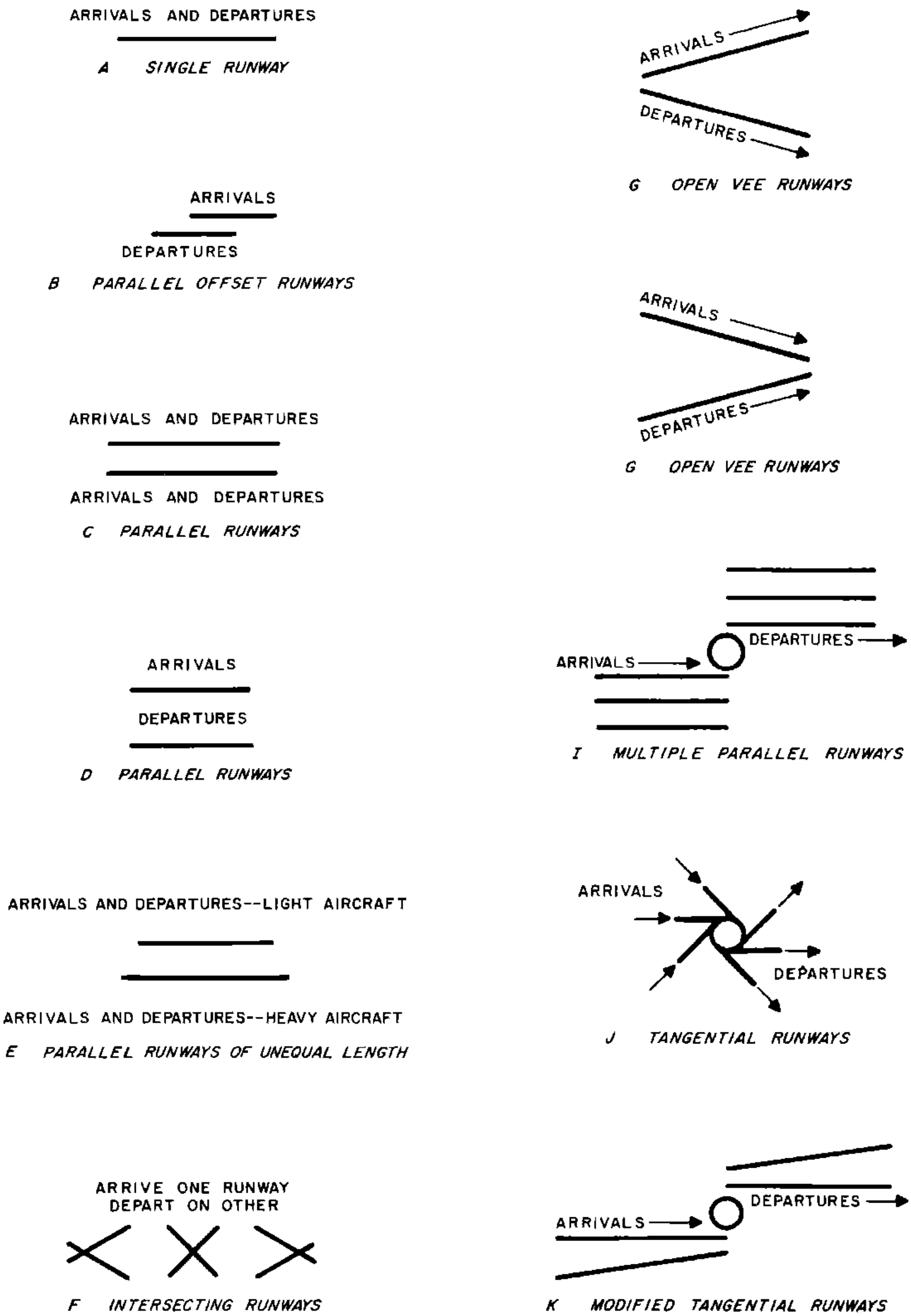

FIGURE 4-1 TYPICAL RUNWAY CONFIGURATIONS

The runway layouts have been analyzed from the standpoint of capacity by developing a curve of the operating rates of landings and take-offs and the corresponding delay to departures that will result for a particular operating rate. The techniques used to relate the operating rate to the corresponding delay are described in Section VIII, which also discusses the meaning of the predicted delay so that it can be interpreted properly when analyzing an airport configuration. In these configurations, where one layout is compared with another, we have selected one value for the delay and compared configurations using that value.

We have selected the 6-minute average delay value as our basis for comparing the configurations in the remainder of this chapter. The meaning of the average delay is discussed in detail in Section VIII. Briefly, however, the 6-minute delay value is the result of considering such factors as the distribution of delay, this average includes aircraft that have no delay and aircraft that will occasionally encounter a 30-minute delay. The economic value of the delay to aircraft is appreciable. In the continental airport population that is used extensively in this report, the cost of the delay averaged over the operating aircraft types is $5.55 per minute. In a normal VFR operation this could total $750 per hour per runway in delay costs. Also, an airport operating at a rate that produces a 6-minute average delay is operating at a relatively high utilization, temporary overloads will soon saturate the operation and result in abnormal delays. Thus, a 6-minute delay represents a somewhat conservative average figure.

The delay curves presented represent what might be called the situation on an average peak day. We believe that the results are on the conservative side, but this is desirable. If the analysis is accomplished for an average peak day, there will be days when traffic volume will exceed that on which the analysis is based, and a conservative rating is therefore desirable.

The delay curves utilize the results of extensive field observations. The field observations included periods when landing and departure rates were so high that wave-offs and turbulence due to slipstream were evident. The capacity ratings indicated here have been selected to avoid these conditions, if the rates shown here are exceeded to any extent for a lengthy period, wave-offs will probably occur and slipstream turbulence will be evident on occasion.

The delay curves presented can be used with confidence if it is applied specifically to the configuration and aircraft population for which it has been prepared. When practical operating rates for other configurations or other populations are desired, they should be recomputed. The following pages will show the pronounced effect these two items have on runway capacity.

In normal VFR operation, little delay to arrivals occurs, and in general they are given priority to use the runway. In the delay curves shown, this normal mode of operation has been assumed and only the delay to departures has been predicted.

In many of the configurations studied, a runway using right-angle turn-offs has been the basis for comparison. The reason for this is that a runway layout with right-angle turn-offs gives a relatively high operating rate for minimum delay. This does not mean that a runway with high-speed turn-offs would not provide higher operating rates, because analysis shows that high-speed turn-offs do increase the operating rate. However, the field data available has been based almost completely on right-angle turn-off operations.

It has not been possible to obtain adequate observations of the operation of high-speed turn-offs in the field. Thus, the selection of a runway with right-angle turn-offs as a basis for comparison was a matter of using the best data available, rather than indicating a preference

A. EFFECT OF AIRCRAFT POPULATION

Figure 4-2 indicates the effect that aircraft population has on VFR airport capacity. Note that the basic difference in population is that the 1962 population includes 20 percent large jet aircraft, whereas the 1958 population includes only piston-engine aircraft. Significantly, the jet aircraft reduce the runway capacity, in actual movements the reduction is from 54 to 46 movements per hour--a reduction of 15 percent. In some of the succeeding graphs, a greater proportion of light aircraft will be included, and it will be shown that the capacity is substantially increased over that available with heavier aircraft. Thus, it is essential to carefully describe the population for an airport in developing planning information on airport capacity.

Of interest here is the airport situation where weekend traffic may be substantially different from the routine traffic during the week. Some airports may have a high proportion of airline-type traffic and a low volume of light aircraft during the week. The weekend traffic, however, may involve a substantial portion of light aircraft and possibly military jet aircraft. The airport capacity may be quite different for the two conditions. If this is the case and will be so over a long-range period, it would appear advisable to analyze airport capacity for the different conditions to determine the most critical periods of operation.

ARRIVAL RATE (λ_1) - DEPARTURE RATE (λ_2)

VFR SPACING

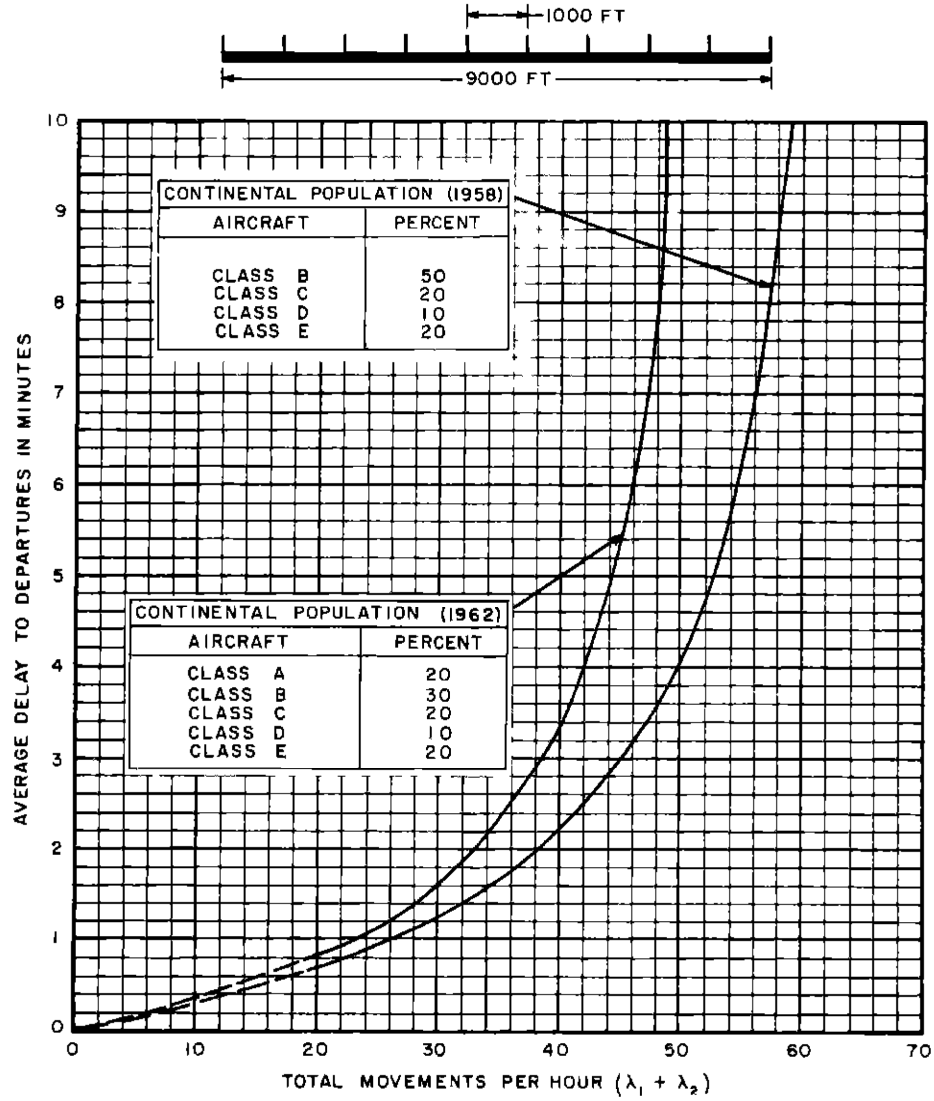

FIGURE 4-2 EFFECT OF JET TRANSPORTS ON DELAY

B. EFFECT OF ARRIVAL-TO-DEPARTURE RATIO

Figure 4-3 analyzes a continental airport runway and shows the effect of various arrival-to-departure ratios during VFR operations. It can be seen that for the aircraft population chosen, the highest practical operating rate is obtained when the departure rate is high and the arrival rate is low. Conversely, the lowest practical operating rate results when the landing rate is high and the departure rate is low. The reduction is from 50 movements per hour to 43 movements per hour--a decrease of 14 percent.

Therefore, the airport arrival and departure distribution must be studied to determine this ratio if an accurate prediction of airport capacity is to be made.

C. EFFECT OF TURN-OFF LAYOUT

Figure 4-4 indicates the effect of the spacing of turn-offs on VFR capacity. Layouts 1, 2, and 3 assume right-angle turn-offs at the spacings shown. Layouts 4 and 5 assume high-speed turn-offs as shown* with layout 5 including a turn-off for light aircraft.

The relative practical operating rate for each of these layouts is

Turn-Offs	Movements per Hour
3000-foot	35
1500-foot	42
1000-foot	46
3 high-speed	48 to 53

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* R. Horonjeff, G Ahlborn, and R. Read, "Exit Taxiway Locations," Institute of Transportation and Traffic Engineering, University of California, 1960.

ARRIVAL RATE (λ_1) - DEPARTURE RATE (λ_2)

VFR SPACING

CONTINENTAL AIRPORT POPULATION

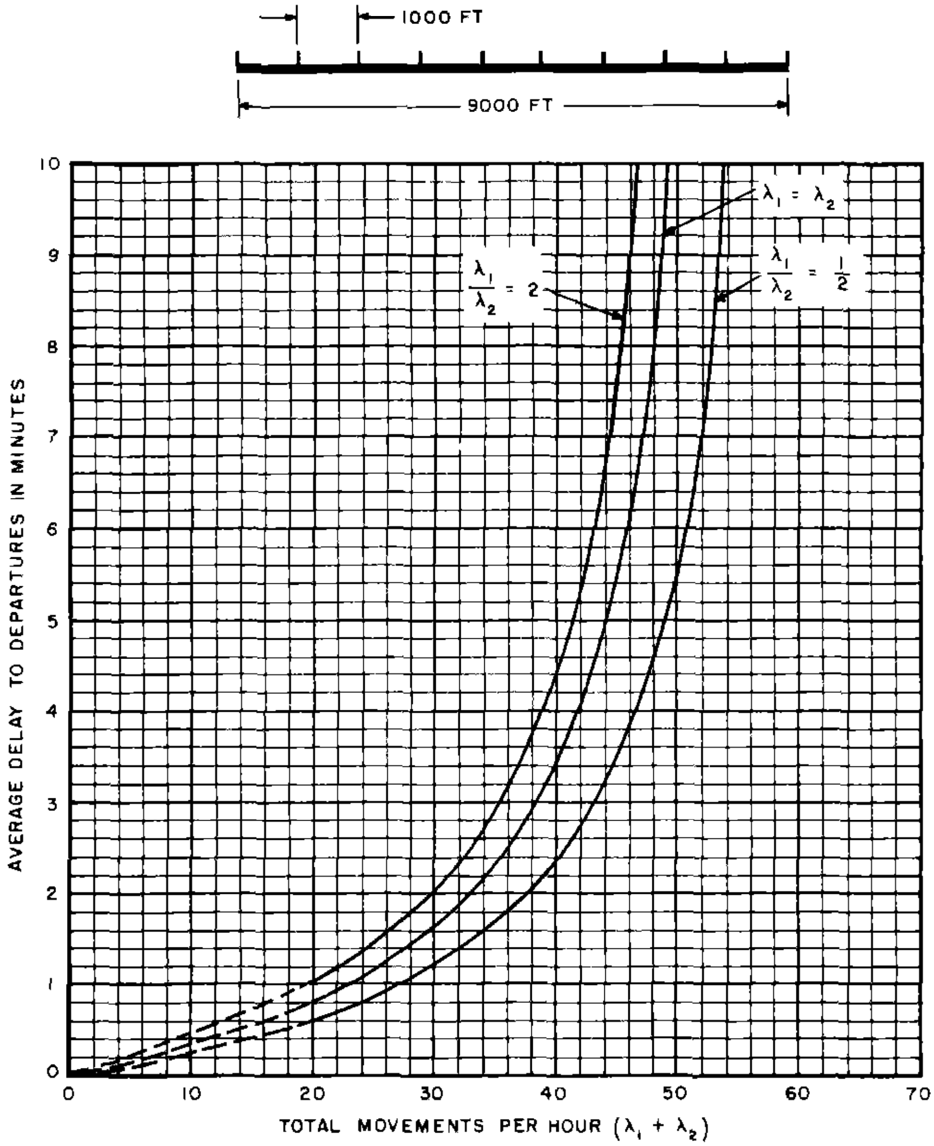

FIGURE 4-3 EFFECT OF ARRIVAL-TO-DEPARTURE RATIO

ARRIVAL RATE (λ_1) - DEPARTURE RATE (λ_2)

VFR SPACING

CONTINENTAL AIRPORT POPULATION

CURVES 5 AND 5A INDICATE POSSIBLE SPACING CHANGE WITH HIGH-SPEED TURN-OFFS

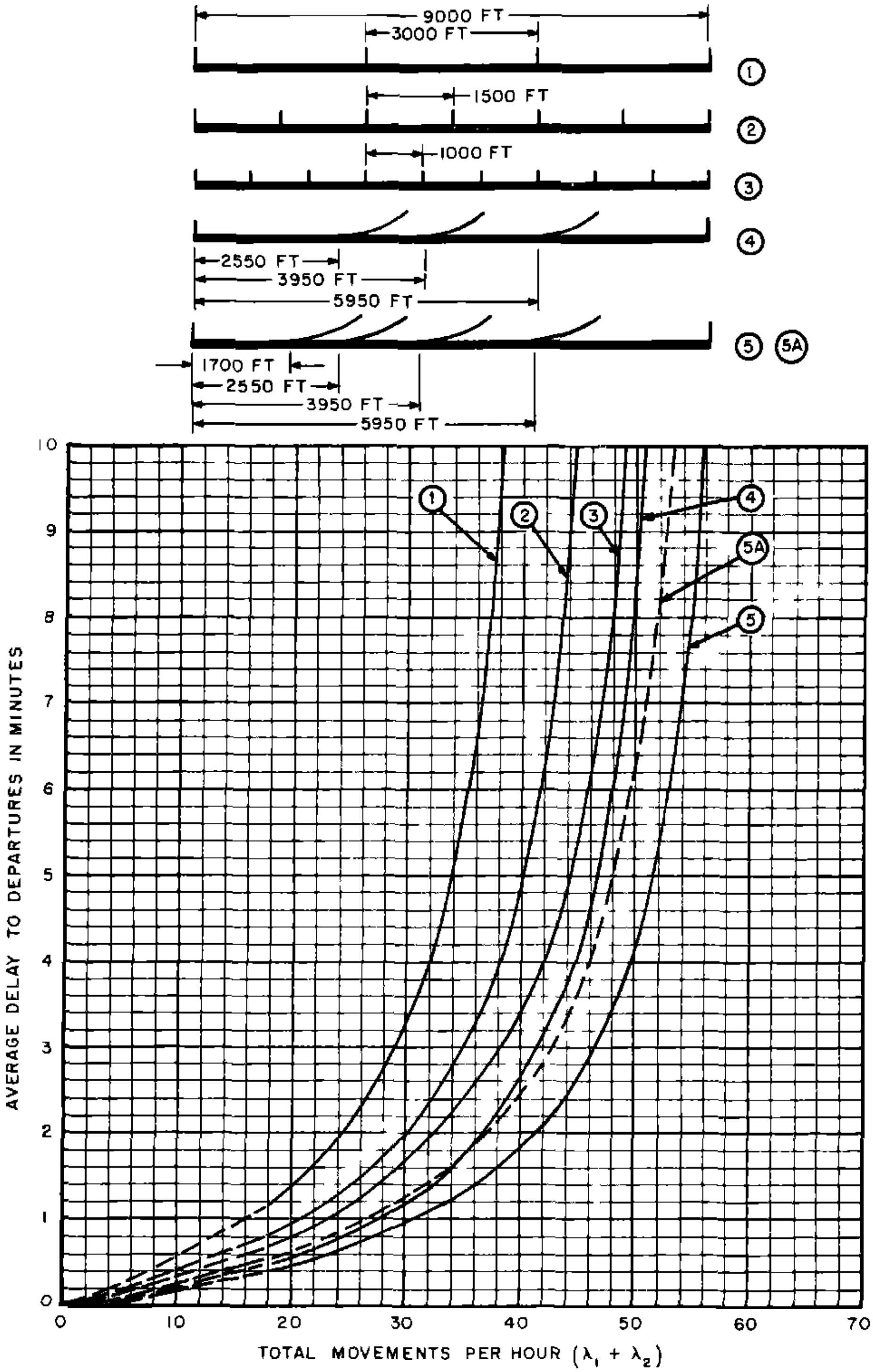

FIGURE 4-4 EFFECT OF TURN-OFF LAYOUT

The preceding analysis is based on our observation that the average time between arrivals for a given population at high movement rates remains relatively constant unless the runway turn-off layout is very poor. Thus, in the analysis of right-angle turn-offs, arrival times are the same except for the 3000-foot spacing when the average arrival time is increased

The capacity of the layouts with high-speed turn-offs is given as a range. Note that layout 5 includes a turn-off for light aircraft. Field observations are needed of spacings between arrivals on a runway equipped for high-speed turn-offs when the runway is operating at a high rate. It is not yet certain that the reduced runway time available with high-speed turn-offs will be used in closing up the spacing between landing aircraft. There is no doubt that this shortened runway time can be used in the mixed operation for take-offs, but as indicated previously our observations make us doubt that full advantage of this feature will be taken on landings, particularly at the higher operating rates. For example, the higher rates that we have observed at airports like LaGuardia have already reduced spacing in the air to the point where wave-offs occurred and light aircraft were being subjected to undesirable slipstream effects. In such cases the spacing in the air will probably not be reduced any further. It is interesting that using our data and assuming that the landing interval is decreased results in a slight increase in delay--compare curve 5A with curve 5. Again, it is emphasized that additional field experience is required.

D PARALLEL RUNWAYS OF UNEQUAL LENGTHS (CONTINENTAL AIRPORT)

"Modernizing the National System of Aviation Facilities,"* suggests the use of parallel runways with a short

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* "Modernizing the National System of Aviation Facilities," Office of Aviation Facilities Planning, The White House, U. S. Government Printing Office, May 1957.

runway suitable for light aircraft and a longer runway suitable for heavy aircraft. Much interest has been shown in this configuration; its analysis for VFR operations is shown in Figure 4-5. It is interesting to note the marked effect of runway lengths and aircraft population on runway capacity. The 9000-foot runway in this analysis has mixed operations, but only with large jet aircraft and four-engine turboprop or piston-engine aircraft. This runway reaches its practical operating rate at 6-minute delays with a landing and departure rate of only 35 movements per hour.

The shorter, 5000-foot runway for the twin-engine transports and light aircraft reaches its practical operating capacity at 65 movements per hour, or almost twice the operating rate of the long runway with heavy aircraft. Thus, a parallel runway layout would have optimum usage when the light aircraft comprise about two-thirds of the total airport traffic. The analysis is made on the basis that the runways are operating independently and with their own traffic patterns which do not interfere with one another. This should generally be practical.

E. PARALLEL RUNWAYS OF EQUAL LENGTHS (CONTINENTAL AIRPORT)

Figure 4-6 is a composite of various configurations for VFR conditions. A basic curve 1 shows mixed operation on a runway with 1000-foot turn-offs and shows its capacity as being 46 movements per hour at a 6-minute delay point.

Curve 2, a composite of total runway capacity, was obtained by totaling the movements shown on the two curves of Figure 4-5 for a 9000-foot and a 5000-foot parallel runway layout. The composite has been developed by finding the average delay of the corresponding total movement rate of the two runways. Thus, at 30 movements per hour the delay is 3.6 minutes on the long runway compared with 0.65 minute

ARRIVAL RATE (λ_1) = DEPARTURE RATE (λ_2)

VFR SPACING

CONTINENTAL AIRPORT POPULATION

RUNWAYS ASSUMED TO OPERATE INDEPENDENTLY

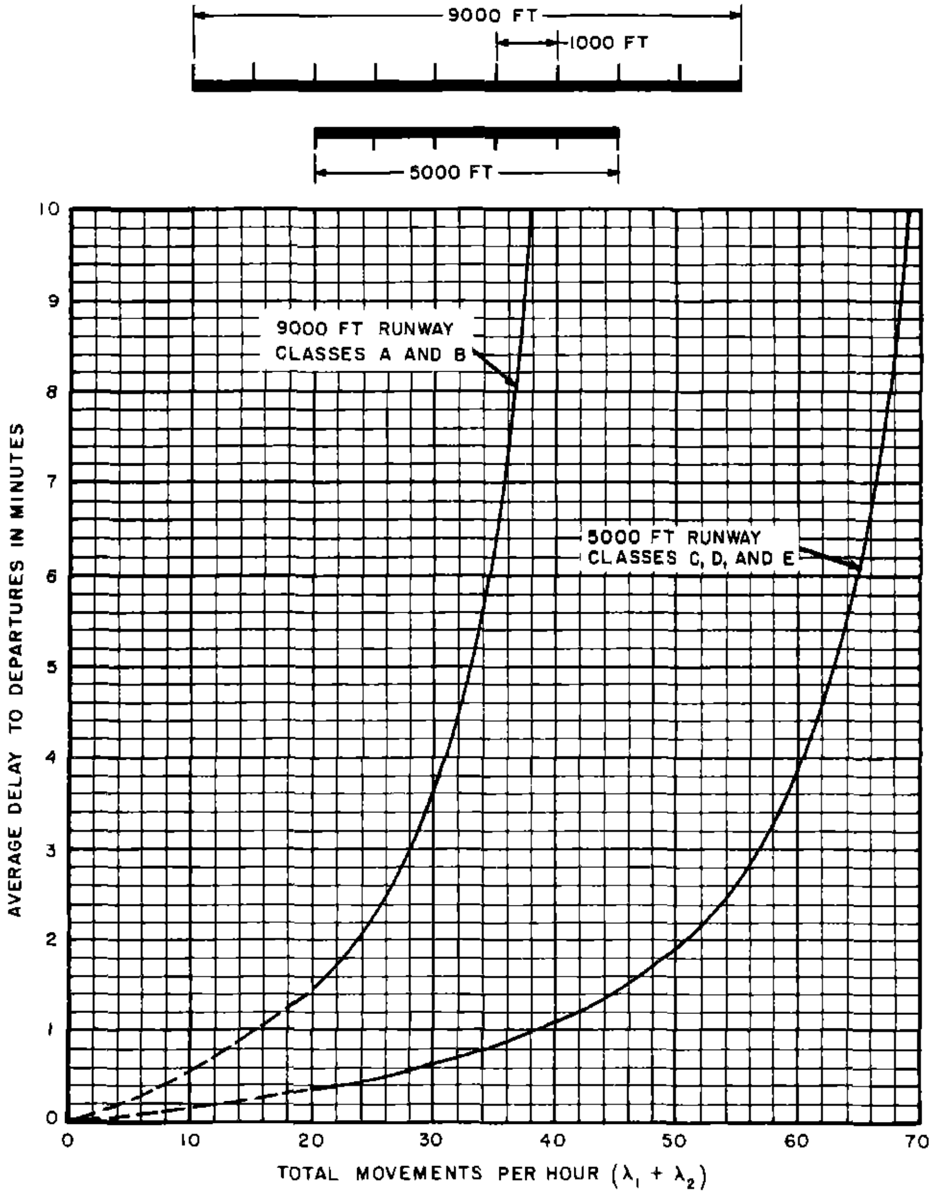

FIGURE 4-5 ANALYSIS OF PARALLEL RUNWAYS OF UNEQUAL LENGTH (CONTINENTAL POPULATION)

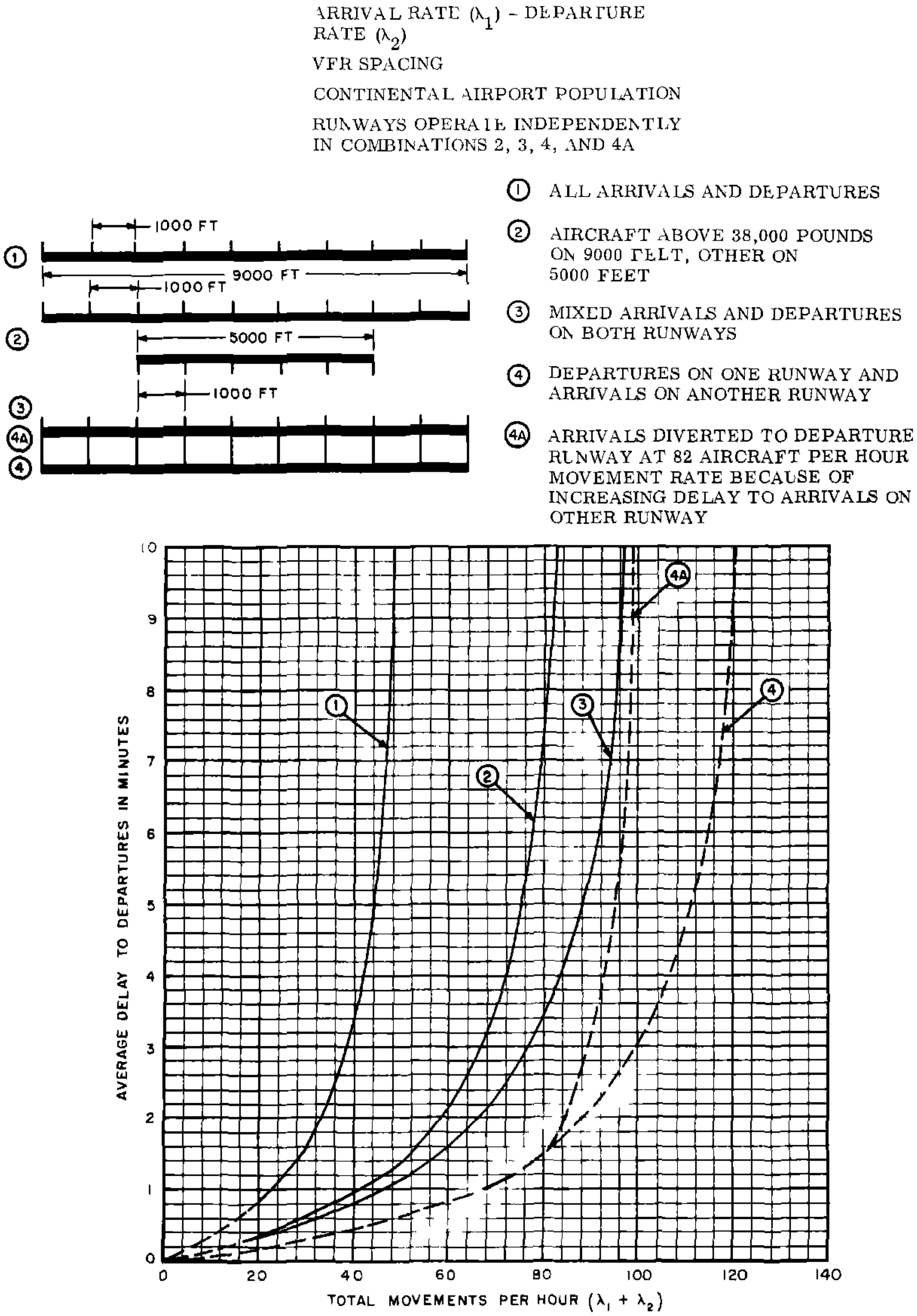

FIGURE 4-6 ANALYSIS OF PARALLEL RUNWAYS

on the short runway. The average delay for the total movement rate of the two runways of 60 movements per hour is

$$\frac{3.6 + 0.65}{2} = 2\ 12 \text{ minutes.}$$

The curve is quite hypothetical because it does not make optimum use of the short runway that is lightly loaded. Further, operations on the long runway will be conducted with a high average delay compared with that on the short runway. A much better use of this configuration would be at an airport where the population was about one-third Classes A and B traffic, and two-thirds Classes C, D, and E traffic. For this combination, curve 2 would move to the right of its position shown in Figure 4-6.

Curve 3 forecasts the movement rate that can be expected on parallel runways 9000 feet in length, and separated by about 700 feet, so that in VFR weather they can be operated independently of one another with mixed operations on both runways. This combination is quite practical and will become more common in the future.

Curve 3 indicates that 92 movements per hour can be accomplished with a 6-minute delay. With such operating rates, it is important to carefully analyze the airspace situation at the airport involved. A feeding system to the airport must be available that will provide aircraft at this rate over an extended period. Further, the runway crossing problem should be evaluated (Section VIII) as this may limit capacity.

Like curve 3, curve 4 considers two runways, except that now they are operating with landings only on one and departures only on the other. The high movement rates shown, when compared with curve 3, indicate that this is an optimum situation. However, this curve can be misinterpreted. With other curves, it has been assumed that delays to arrivals are practically nonexistent. In curve 4, delay to arrivals does occur because of the very high movement rate. Some idea of the runway capacity with arrival delay limited, as in the other curves, is given by curve 4A, which starts at the point where the arrival delay reaches an average of 1 minute, the landing rate on the landing-only runway also remains at this level, and excess arrivals are handled on the departures-only runway. In everyday operation it would be difficult to conduct an operation that would result in the condition shown by curve 4A because the controller could not easily accomplish the diversion just as indicated. However, it does indicate that the practical capacity will probably lie well below curve 4 and toward curve 4A.

Our field observations indicate that the mixed operation shown in curve 3, for propeller aircraft, is an optimum operating situation, particularly where the runways have enough separation for independent operation and yet are close enough to permit a "common queue" for arrivals and departures. This study does not assume a common queue because there is no mathematical technique to analyze this complex situation. In such a case, either runway can be used for arrival or departure and delay is minimized. However, the aircraft population used in this study includes jet aircraft--which, because of engine run-up on the runway, appear to be best handled on a take-off only runway. Thus the choice of a mode of operation for parallel runways will be influenced by the aircraft population.

F. PARALLEL RUNWAYS (TRUNK AIRPORT)

In Figure 4-7, the aircraft population assumed is suitable for operation at a trunk airport (as shown in the beginning of this section) and includes only Classes B, C,

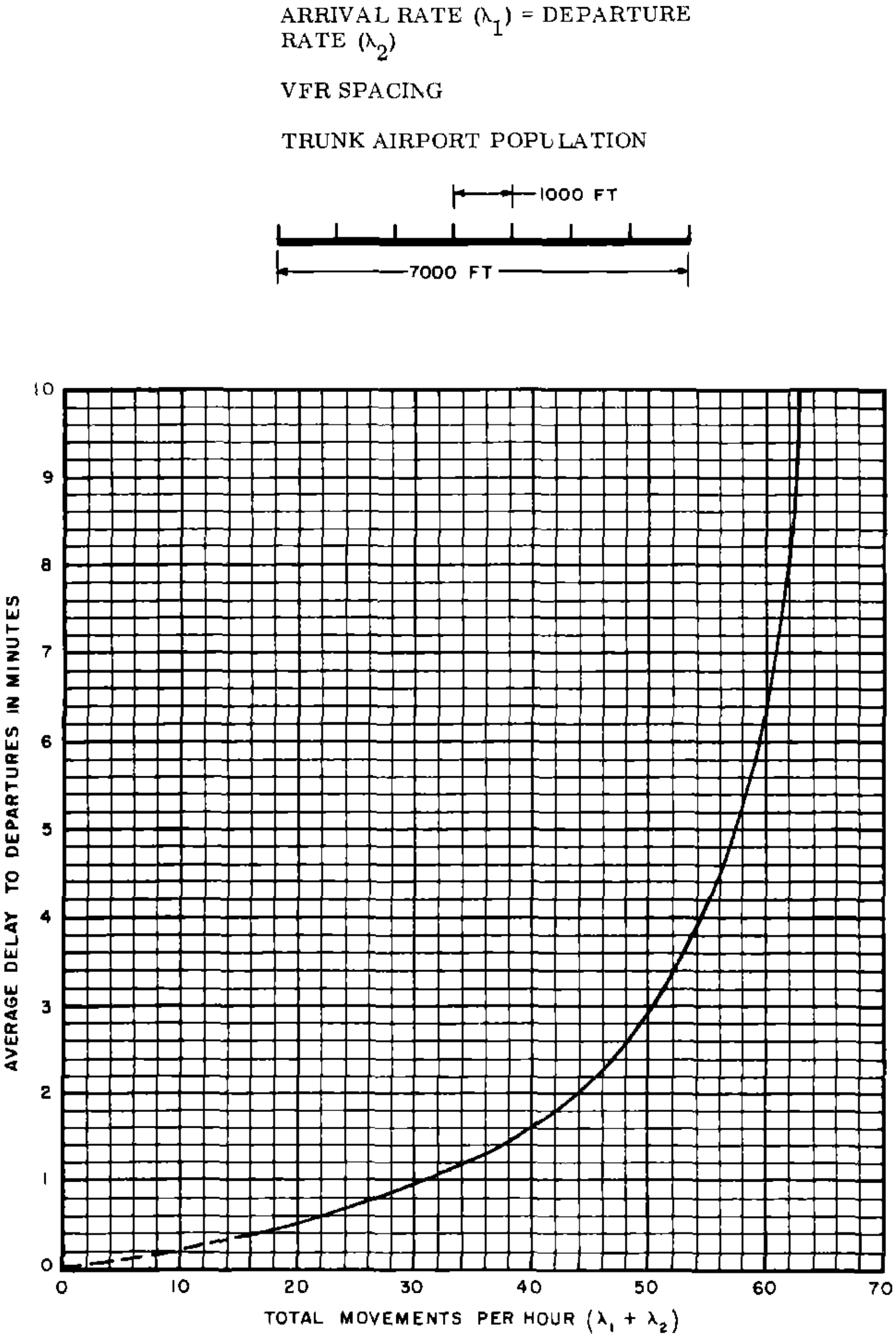

FIGURE 4-7 ANALYSIS OF 7000-FOOT SINGLE RUNWAY

D, and E aircraft. The practical VFR capacity for a 6-minute average delay is shown to be 59 movements per hour

In Figure 4-8, the trunk aircraft population has been divided so that the larger aircraft are assigned to the 7000-foot runway, and the smaller aircraft to the 4000-foot runway. The VFR operating rates for a 6-minute delay are

Runway (feet)	Movements per Hour
7000	56
4000	91

The analysis of this runway layout is similar to that in Figure 4-5. For optimum use of this layout, the percentage of aircraft in a population that can use the shorter runway should be substantially higher (almost 2 to 1) than the percentage of aircraft that must use the longer runway.

G. INTERSECTING RUNWAYS

Figure 4-9 shows how intersecting runways can increase VFR airport capacity If wind conditions permit a cross-runway operation, substantial benefits can be realized that approach the operating rates of parallel runways (Figure 4-6). However, if winds are variable, airport capacity will vary with the direction of operation because of the effect of the intersection location on capacity. Note that the position of the intersection causes the operating rate at a 6-minute delay to vary from 78 to 47 movements per hour--the latter being about that of single-runway operation.

H. OPEN-VEE RUNWAYS

The open-vee runway layout has often been suggested. Under optimum wind conditions, and when approach and departure paths do not intersect, the open-vee layout provides a capacity

ARRIVAL RATE (λ_1) = DEPARTURE RATE (λ_2)

VFR SPACING

TRUNK AIRPORT POPULATION

RUNWAYS OPERATE INDEPENDENTLY

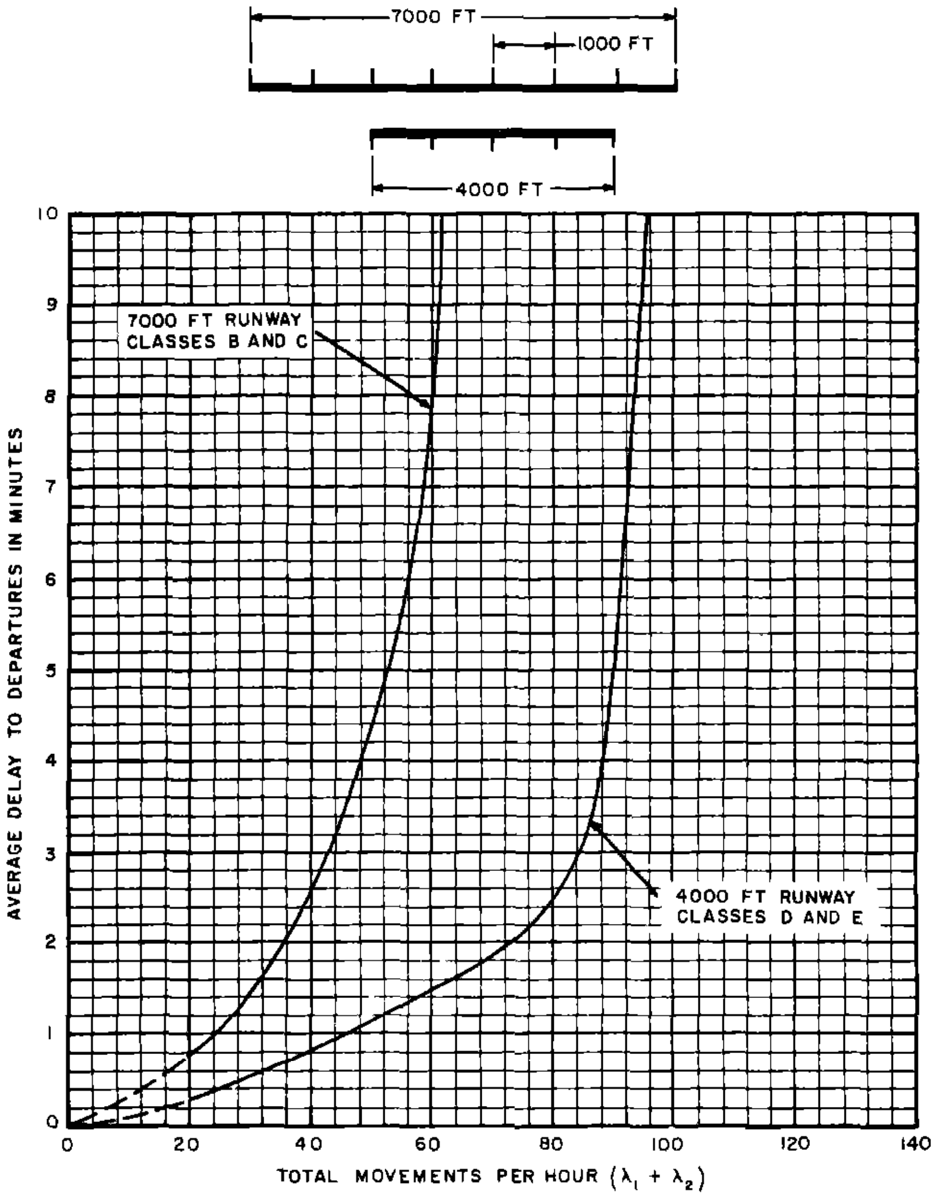

FIGURE 4-8 ANALYSIS OF PARALLEL RUNWAYS OF UNEQUAL LENGTH (TRUNK POPULATION)

ARRIVAL RATE (λ_1) - DEPARTURE RATE (λ_2)

VFR SPACING

CONTINENTAL AIRPORT POPULATION

9000-FOOT RUNWAYS WITH 1000-FOOT TURN-OFFS

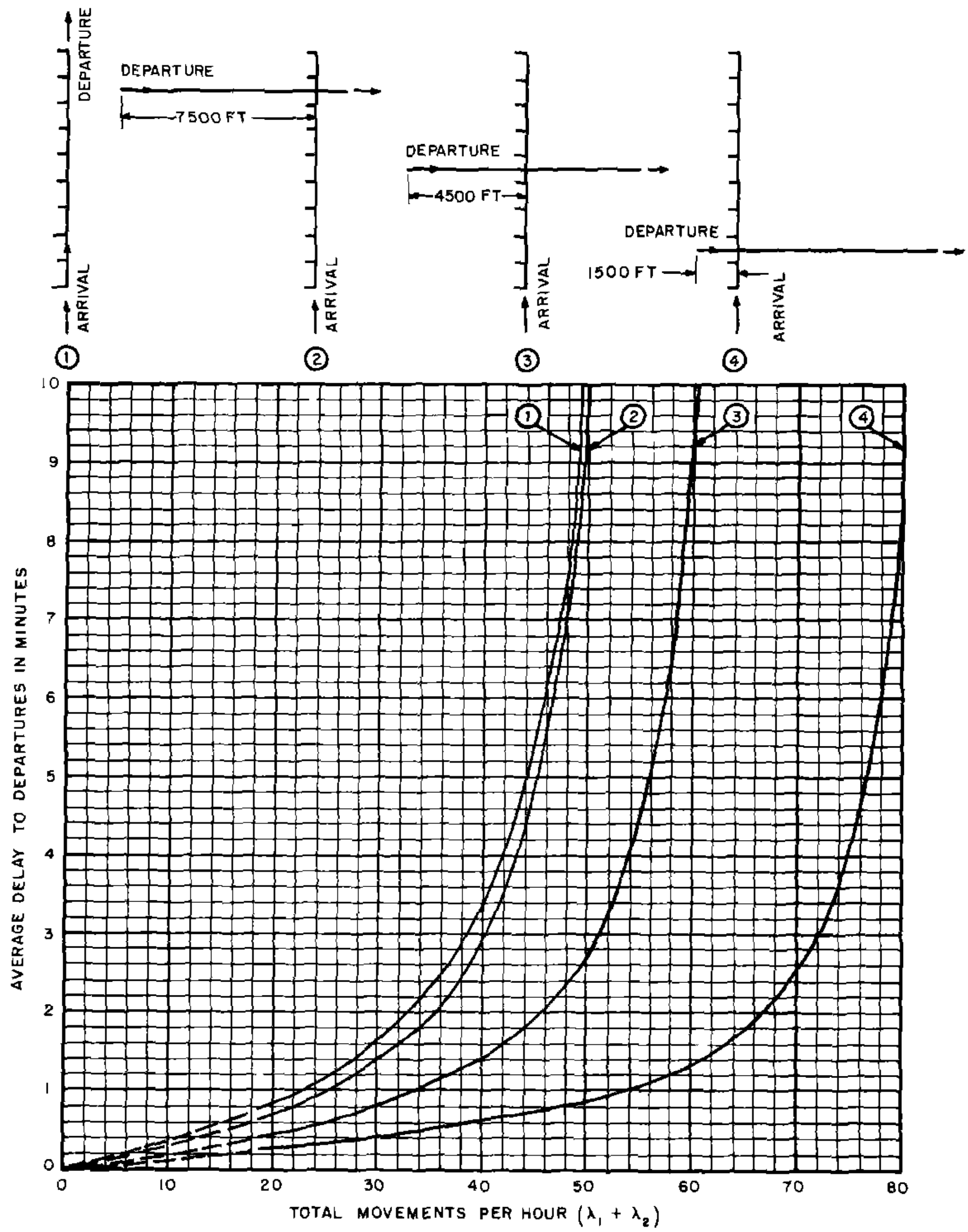

FIGURE 4-9 ANALYSIS OF INTERSECTING RUNWAYS

similar to that for a parallel runway, and possibly more because the departure path and missed approach path diverge. However, one runway must be used for landings only and the other for departures only to avoid conflicting approach paths on both runways. When the wind indicates use of intersecting paths, the capacity is reduced (Figure 4-10). The hourly VFR operating rates at a 6-minute delay are.

	Movements per Hour
Optimum conditions	108*
Crossing conditions	65 to 79**

In the crossing operation, the separation provided between a missed approach and departures is conservative and may be more than is actually necessary.

The lower rating curve assumes that a take-off must be at the upwind end of the runway before a landing can be cleared, the higher rating curve assumes that a landing can be cleared when the take-off has proceeded halfway down the runway. The actual operating rules will no doubt depend on the distance to the projected intersection point.

I. OTHER CONFIGURATIONS

The possibility of using a layout such as number 9 in Figure 4-1 has been analyzed from a runway crossing standpoint. The number of parallel runways on one side of the

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* Note This is based on letting arrival delay on the arrivals-only runway build up to a 3-minute average delay--a somewhat questionable procedure. At 2- and 1-minute delays, the operating rates are 100 and 82 movements, respectively.

** A spread is shown that reflects the effect of varying the operating rules for a possible missed approach.

ARRIVAL RATE (λ_1) = DEPARTURE RATE (λ_2)

VFR SPACING

CONTINENTAL AIRPORT POPULATION

SHADED AREA OF CURVE 2 INDICATES VARIATION RESULTING FROM DEGREE OF DEPENDENCE BETWEEN RUNWAYS

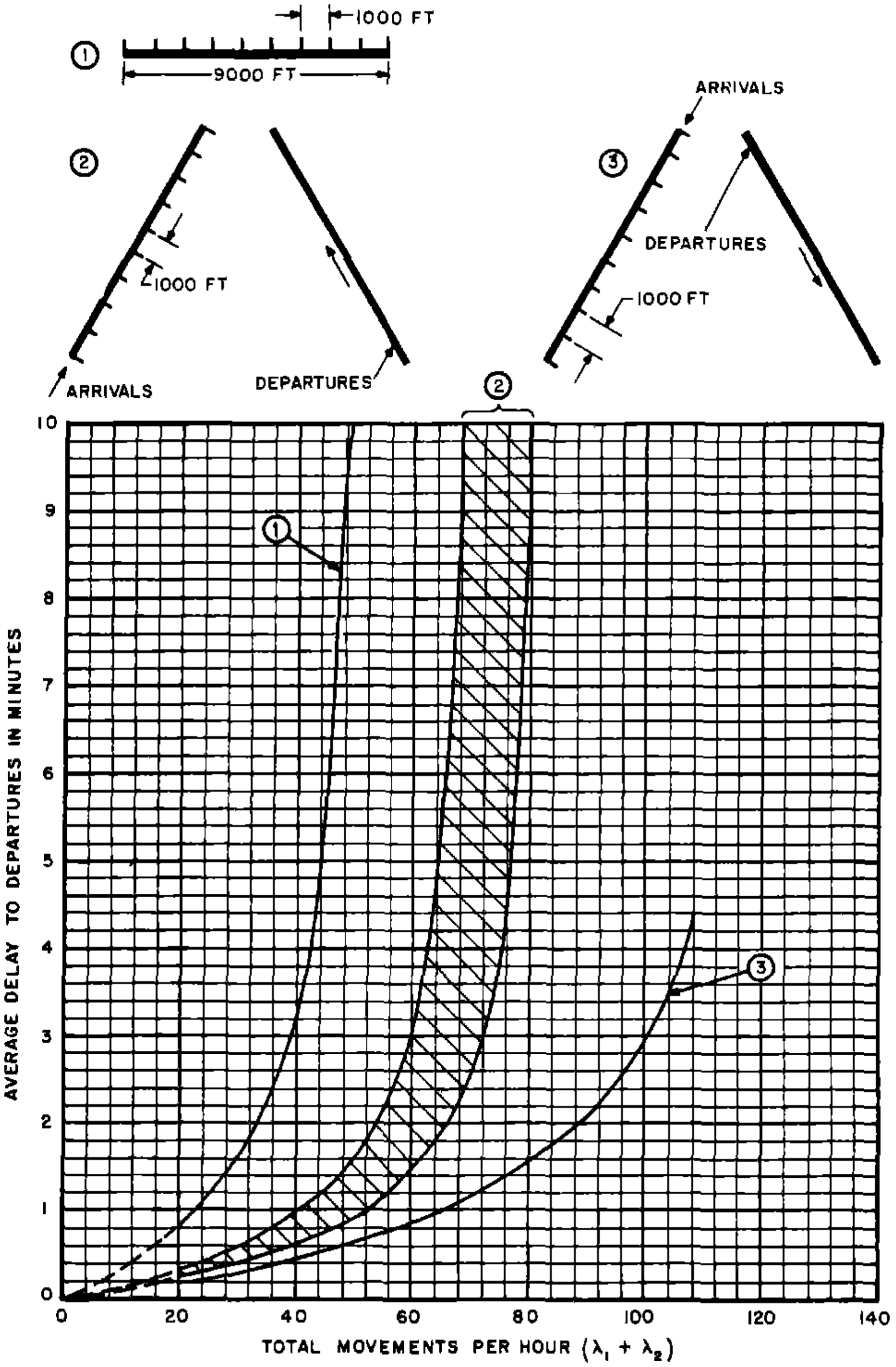

FIGURE 4-10 ANALYSIS OF OPEN-VEE RUNWAYS

terminal should not exceed two because the ground control problem and crossing delay become too great with three or more parallel runways.

Tangential configuration 10 in Figure 4-1 uses all the airspace surrounding the airport. Independent approaches and departures are theoretically possible, but it is not known how much coordination must be accomplished because of converging paths of missed approaches and departures. The mathematical techniques used in this study can approximate capacity if assumptions are made regarding the operational uncertainties. A large capital investment is required for land for the multiple-runway layout and the necessary airspace reservation.

Configuration 11 in Figure 4-1 is a theoretical configuration that is similar to the tangential, with two pairs of nearly parallel runways offset and on opposite sides of the terminal area. These adjacent runways converge near the terminal area at an angle of 10 to 20 degrees, with their ends separated by possibly 2000 feet. Landings are on coverging paths, and take-offs on diverging paths. This configuration has been proposed with the suggestion that, since the convergence on approach is so small, procedures can be established to make simultaneous approaches possible. If so, the configuration would have the capacity of two independent landing runways, and two independent departure runways in VFR and IFR.

J. IFR OPERATION

In current operations, runway capacities are substantially less during IFR than during VFR. This results primarily from the IFR spacing criteria and the long, straight-in approaches that are necessary. It may also be partially due to a change in aircraft population since the volume of light aircraft will diminish.

The same mathematical techniques used in the previous capacity analyses apply to IFR operations since the spacing factors used can be adjusted for the IFR condition However, there is not sufficient observed data available to determine the actual spacings that occur in routine IFR operations. Without such observed data, capacity analyses will not be too practical Theoretical spacings to meet IFR criteria can be computed, but we must know the actual spacings achieved as compared with the theoretical spacings before attempting to relate operating rate to delay.

From other work accomplished by AIL, limited IFR actual spacing data are available. These have been applied to a 6500-foot runway to show the variation between VFR and IFR operations The operating rates for a 6-minute average departure delay are

	Hourly Operating Rate	Percentage of Aircraft Population: Transport	Percentage of Aircraft Population: General Aviation
VFR	56	40	60
IFR	36	82	18

For comparison, a parallel runway of the same length was assumed with about 700 feet separation, requiring coordination between arrivals and departures in IFR With arrivals on one runway and departures on the other runway·

Hourly operating rate with 6-minute average departure delay in IFR 52

Parallel runways with a separation adequate to permit independent operations (Section V) would have about twice the capacity of a single runway, depending on whether landings and take-offs are segregated or mixed.

The difference in operating rates during VFR and IFR will decrease as IFR control techniques are improved. An analysis of airport capacity should consider this disparity as well as the change in aircraft population and demand.

An item of airport layout that will affect IFR airport capacity is the wind coverage afforded by the runway with the instrument landing system (ILS). To keep the airport open the maximum amount of time may require the capability of straight-in instrument approach in two directions (180 degrees apart). Otherwise, the airport IFR capacity will be reduced substantially or the airport will be closed whenever the wind does not permit use of the one ILS approach.

Selecting an airport site free from air traffic complexities is a basic requirement in an airport design. This factor can become so predominant that it may make it impossible to operate an airport at any reasonable IFR rate. Well-known examples can be cited, such as the Washington National Airport, Bolling Field, and Anacostia Naval Air Station complexes. These can operate satisfactorily in VFR weather by proper allocation of airspace and approach directions. In IFR, however, the whole complex has the capacity of but one airport. Another example is the Newark-Teterboro complex in New Jersey. These airports can also operate to maximum capacity in VFR weather without interference with each other. Yet they seriously handicap one another in IFR conditions, even though the airports are about 12 miles apart. Their location, with respect to other New York air traffic problems of arriving and departing aircraft, is such that it is unlikely that the IFR capacity can be improved considerably.

K. TAXIWAYS, PARKING, AND HOLDING AREAS

The taxiway system between runway, terminal, and hangar areas generally has little effect on runway capacity. However, any unnecessary time used in taxiing is delay to an aircraft, just as much as that incurred when waiting to land or take off. Consequently, the taxiway circulation system must be reasonably efficient.

The design for run-up aprons at the ends of the take-off runway should be studied to make certain that they are large enough to accommodate (1) the queue build-up due to the variations in engine run-up and cockpit check times, and (2) the aircraft awaiting IFR take-off clearance prior to departing. At a busy airport, several aircraft may be waiting and they should be positioned so that other aircraft can bypass them for take-off. The capacity of the runway is affected by this factor only when take-off aircraft are unable to reach the take-off runway because aircraft positioned ahead but not "ready to go" are blocking access to the take-off runway.

The aircraft loading position layout should have adequate apron space to reasonably match the capacity of the airport runway layout. Waiting to obtain gate space is again a delay to air operations that is as important to the passenger as any other delay Furthermore, a delay awaiting a gate often will cause taxiway congestion that may affect airport capacity.

L. FACTORS TO CONSIDER IN DETERMINING AIRPORT DESIGN AND CAPACITY

In selecting and evaluating airport designs, the airport planner and designer must consider many factors. It is difficult to develop a precise relative rating of the importance of these factors because their effect on capacity is not always direct or measurable. Further, their importance will vary from site to site. However, in the following discussion, the factors have been listed in the order of importance indicated by the preceding analysis

1. Airspace air traffic considerations,
2. Number of parallel runways on the airport,
3. The layout of the runways within the airport,
4. The ceiling and visibility conditions,
5. The traffic control rules and procedures,
6. The navigational aids provided for instrument weather operations,
7. The exit taxiway layout for the runways,
8 Pilot and controller "pressure factor,"
9. The mixture of traffic as to types of aircraft,
10. The hourly distribution of arrivals and departures (discussed more completely in Sections VII and VIII),
11. The variability of wind conditions,
12. Holding apron capacity and bypass provisions at the runway ends,
13. The capacity of the gate positions,
14. The layout of the taxiway system between runways and gates.

Note: Items 2, 3, 6, 7, 9, 11, 12, 13, and 14 are of direct concern in airport surface design.

V. RECOMMENDED DESIGN CRITERIA

A. BASIC CRITERIA

The design criteria given in Table 5-I are based on the definitions in TSO N-6b* that establish the four types of service.

1. Local --Airports to serve on local service routes providing service in the short-haul category, normally not exceeding 500 miles
2. Trunk.--Airports to serve on airline trunk routes and engage in intermediate length hauls, normally not exceeding 1000 miles.
3. Continental.--Airports to serve on long nonstop flights, exclusive of coast to coast, normally entirely within the continental United States These airports serve nonstop flights up to 2000 miles.
4. Intercontinental.--Airports to serve the longest-range nonstop flights in the transcontinental, transoceanic, and intercontinental categories.

These appear to be very logical classifications to fit traffic flows and air carrier operations.

Local service is being provided in the United States at about 400 airports, trunk service at about 90, continental service at about 40; and intercontinental service at about 20 Traffic volume as well as aircraft size will usually increase with the type of service from local to intercontinental However, the addition of general aviation aircraft will influence

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* "Runway Strength and Dimensional Standards for Air Carrier Operations," TSO N-6b, Civil Aeronautics Administration, 3 October 1958.

TABLE 5-I

RECOMMENDED DESIGN CRITERIA*

Airport	Basic Runway Length	Run-way Width	Taxi-way Width	Runway Centerline to Taxiway Centerline	Taxiway Centerline to Taxiway Centerline	Runway Centerline to Runway Centerline	Taxiway Center-line to Apron	Taxiway Centerline to Obstruction	Runway Centerline to Building Line
Local	5000**	100	50	250	200	500	150**	100	500**
Trunk	6000	150	75	350	250**	500	200**	125**	750
Conti-nental	7500	150	75	450**	300	700	250**	150**	750
Intercon-tinental	9500**	150	75	450	300**	700	250**	150**	750

* These criteria are based on our review of aircraft dimensions, accident probabilities, observations of airport operations, and present design criteria and are recommended for future use. All dimensions are given in feet.

** These dimensions differ from those in "Airport Engineering Data Sheet," Item No. 24, Federal Aviation Agency, 17 August 1959.

this volume greatly, particularly at the trunk and continental airports. In the few instances where multiple airports exist for air carrier use, they influence traffic volume by segregating types of traffic.

The smallest airport for regular air carrier service is the local airport. These serve cities of 15,000 to 75,000 inhabitants with 1500 to 10,000 scheduled operations of twin-engine aircraft annually. Normally, the total operations at these airports include sufficient general aviation aircraft to make the total annual traffic 15,000 to 75,000 operations. At present, only about 10 percent have FAA control towers. Although the general aviation traffic may include occasional four-engine aircraft, the operating conditions are usually such that scheduled service determines airport design. Most of the scheduled service is now provided by aircraft such as the Douglas DC-3, Fairchild F-27, and Convair types. Only the Fairchild F-27 twin-engine turboprop was designed for this service. The Douglas DC-3, which is still the most common aircraft, is slowly being replaced by the Fairchild F-27 and Convair twin-engine aircraft. Some of the Convair aircraft are being refitted with turboprop engines.

The basic runway length for local service is considered to be 5000 feet. Many of the 400 locations are shown in the National Airport Plan* as requiring trunk airports because TSO N-6b gives a maximum length of 4200 feet for local service (except on ILS runways).

Because of the low traffic volume and small income from such airports, improvements must be kept to the minimum

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* "National Airport Plan - 1959," Bureau of Facilities, Airports Division, Federal Aviation Agency, 1959.

required for safety in this type of operation. A runway width of 100 feet and a taxiway width of 50 feet have proved adequate. Since it has been shown in paragraph D, Section IX that there is practically no chance of collision between an aircraft operating on the runway and one on the taxiway with this traffic volume, a minimum spacing of 250 feet between the centerline of the taxiway and the runway is recommended. This spacing does not permit the use of high-speed exits, which generally cannot be economically justified by the volume of traffic.

Trunk service will be provided largely by four-engine turboprop aircraft and short-range jet aircraft. These aircraft indicate an increase in taxiway and runway width, with a corresponding increase in minimum taxiway to runway spacing to 350 feet. Where the traffic volume justifies high-speed exits, this minimum must be increased to 450 or 750 feet. Usually, it is necessary to make a 180-degree turn from runway to parallel taxiway, this would require a separation of 450 feet for a 40-mph exit, and 750 feet for a 60-mph exit.

Continental and intercontinental airports will handle the same basic types of large jet and, ultimately, supersonic aircraft The only difference will be that maximum flight distances require longer runways for intercontinental traffic. The volume of traffic at these airports can generally be expected to warrant high-speed exits (see Section IV for their effect on capacity) with taxiway to runway separations of 450 feet for 40-mph exits and 750 feet for 60-mph exits.

Other clearances are based on aircraft having these maximum wingspreads.

	Feet
Local	125
Trunk	150
Continental and intercontinental. .	175

Overall maximum lengths for aircraft are assumed as

	Feet
Local	125
Trunk	150
Continental and intercontinental	200

Because of the probable future use of all runways in low-visibility conditions, it is desirable to use the same clearances for all runways. A minimum distance to the building line of 500 feet seems adequate for local service, the current 750 feet is adequate for all larger airports.

In addition to making the runway length for local service 5000 feet, it is desirable to establish a standard runway length for all types of service. Our airport visits indicated the undesirable results of continually increasing runway lengths. Master plans prepared on the basis of TSO N-6 or TSO N-6a must be distorted to accommodate the new runway lengths of TSO N-6b (or to meet airline requirements). This generally results in reducing the operational efficiency of the master plan layout. Therefore, we have used 9500 feet as the basic runway length. From reported performance data, it appears that the correction of 7 percent for each 1000 feet above sea level is adequate for current aircraft types. The correction of 20 percent for each 1 percent of runway gradient is high and should be reviewed.

B. SEPARATION OF PARALLEL RUNWAYS

The probability analysis of accident data (paragraph F following) and actual operating experience indicates that a spacing of 500 feet between parallel runways is adequate for VFR operations on local and trunk airports with aircraft that operate from 5000- and 6000-foot runways. With the higher volume of traffic and the larger aircraft at continental and intercontinental airports, a minimum of 700 feet is recommended for VFR operations. A spacing of 1000 feet will permit a better design of high-speed exit taxiways and increase operating time only slightly. Greater spacing tends to lower efficiency for VFR operations since: (1) landing aircraft cannot be readily directed to either runway on final approach, and (2) departing aircraft cannot be directed to either runway from the holding aprons.

Since the rate of landings under IFR conditions is often the controlling factor for heavily scheduled operations, the separation distance for simultaneous landings becomes very important. Research on actual simultaneous landing operations on parallel runways in IFR conditions is limited. However, FAA has made a probability analysis of flight paths of actual operations. This analysis indicates minimum spacings of 3800 feet for entry to glide slopes at the same altitude, and 2700 feet for entry to glide slopes separated by 500 feet vertically. These figures are tentative, they have yet to be validated by actual flight testing.

Many problems of operations are best solved by having parallel runways on opposite sides of the building area. For a major airport, this requires a separation of at least 5000 feet to give adequate space for taxiways and other terminal activities

Until actual operations establish a better figure, it is recommended that 5000 feet be used as the minimum separation of parallel runways for simultaneous instrument landings when the runways can be located on opposite sides of the terminal area

Some advantage may be gained in instrument operations by having parallel runways separated sufficiently to permit simultaneous or independent landings on one runway and take-offs on the other (see Section IV for this analysis).

With no restrictions on departures because of missed approach turns on the landing runway, this can be accomplished with a spacing of about 3000 feet. At spacings of 700 to 1000 feet, departures must be coordinated with landings--that is, departures must not be cleared until the landing aircraft is committed to landing. This solution may be justified where the operating rate is critically high and runways on opposite sides of the terminal or diverging departures are impracticable. Diverging runways may give a better solution where parallel runways separated by at least 5000 feet are not feasible.

C. HIGH-SPEED EXIT TAXIWAYS

For some time, the FAA has been engaged in research on high-speed exit taxiways to reduce the runway occupancy time of landing aircraft and thereby increase the airport acceptance rate or capacity. Under a research contract, the University of California has developed criteria for the configuration of high-speed exits and the locations along the runway for various aircraft populations.* This study has not extended to the incorporation of exits into the taxiway system and airport configurations.

Figure 5-1 is a deceleration chart based on laws of physics that can be used to determine the relationship between velocity, deceleration, and displacement. Figure 5-2 shows the basic configuration for a 30-degree exit developed from the University of California study.

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* R Horonjeff, D. Finch, D. Belmont, and G Ahlborn, "A Research Project Concerning Exit Taxiway Location and Design," Institute of Transportation and Traffic Engineering, University of California, August 1958.

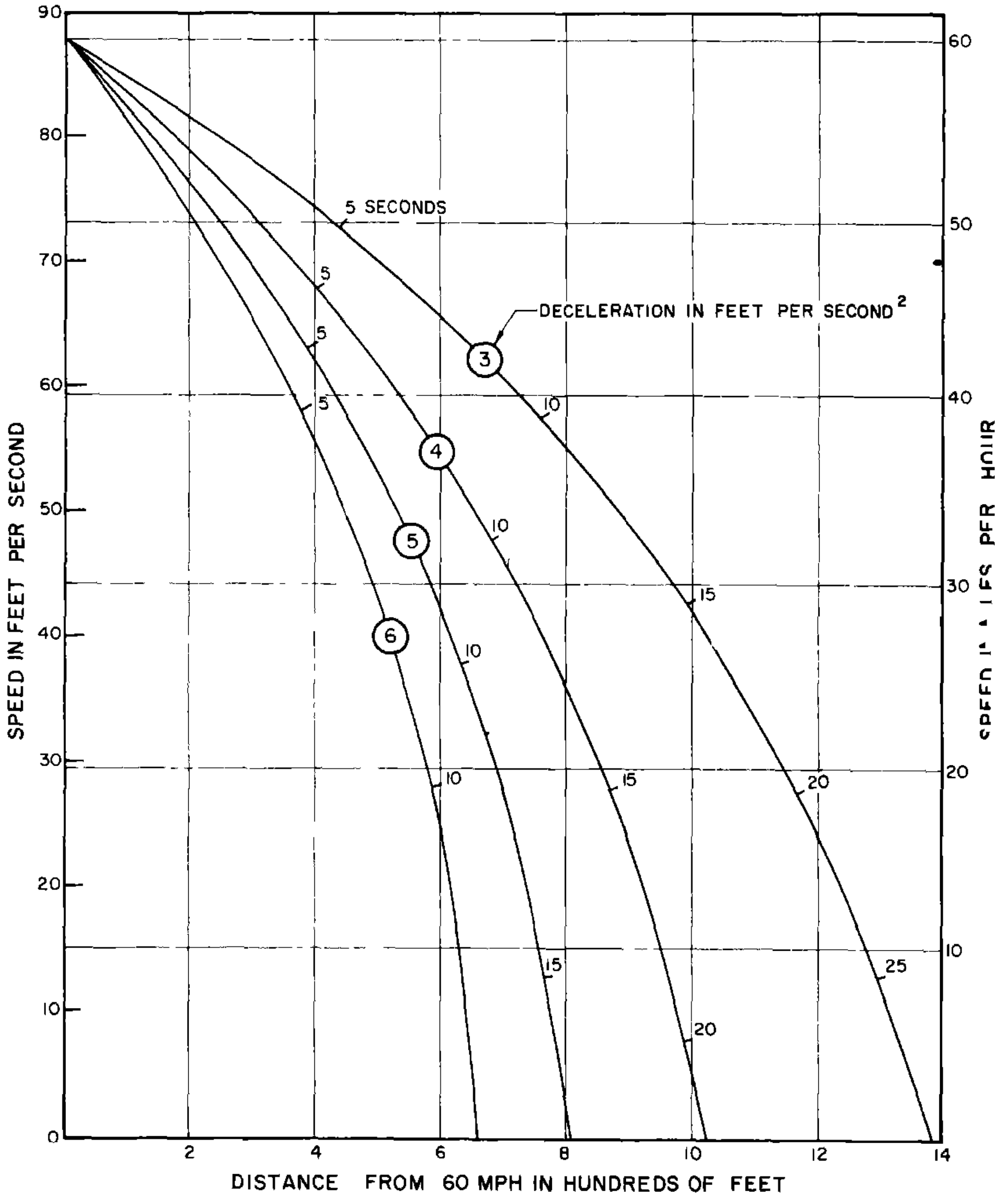

FIGURE 5-1 DECELERATION CHART

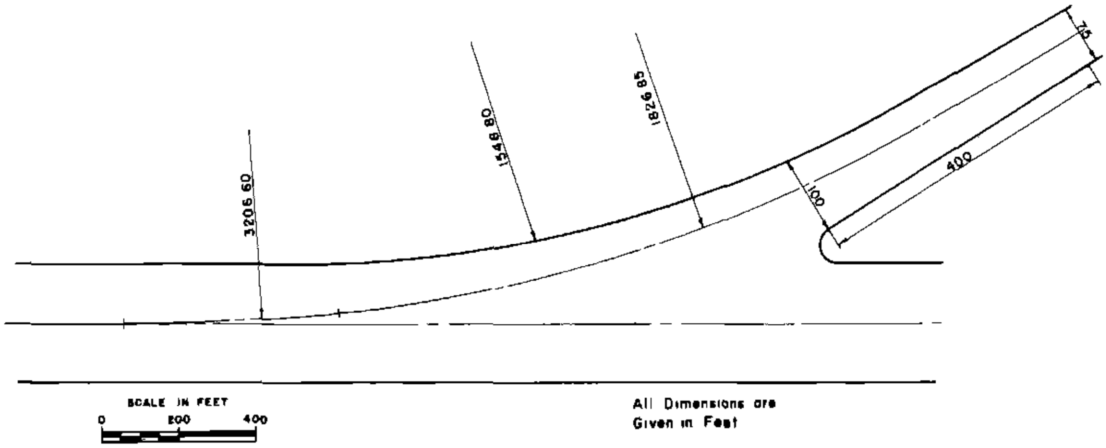

FIGURE 5-2 30-DEGREE HIGH-SPEED TURN-OFF (FAA)

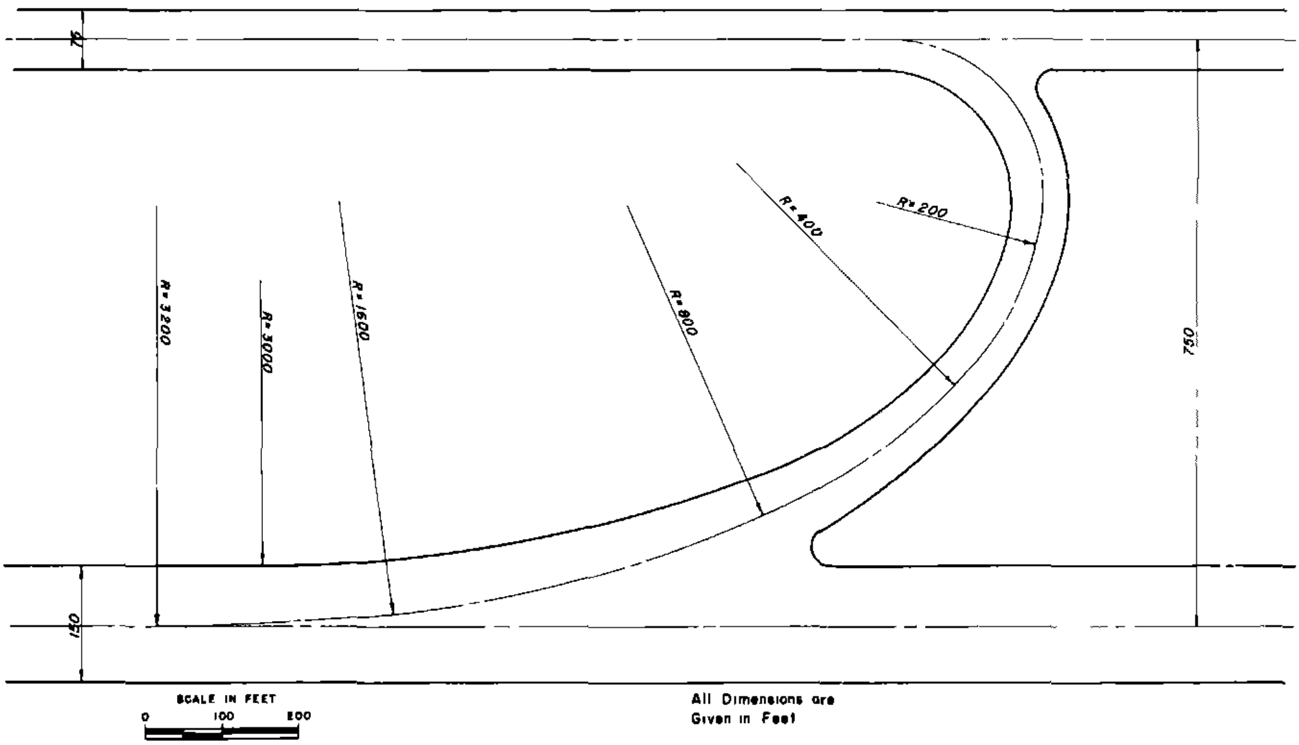

FIGURE 5-3 MULTICENTERED CURVE (EXIT TAXIWAY)

For many airport configurations where high-speed taxiways may be advantageous, the relative locations of the runways, taxiway systems, and terminal will require the aircraft landing in one direction to turn and taxi on a parallel taxiway in the opposite direction to reach the terminal area. There is the advantage of decreased taxi distance (and taxi time) if the distance from the beginning of the exit from the runway centerline to a point on the parallel taxiway opposite this beginning measured along the taxiway can be kept to a minimum. This can be accomplished most efficiently by the use of a continuous multicentered curve that approximates a cycloid curve. Such a design requires braking on the curve so that the centrifugal force at any point does not exceed a safe and comfortable value. Figure 5-3 shows a design for a 60-mph exit with a constant deceleration value of 3.3 feet per second^2 and a lateral force of 0.14 g, which appear to be reasonable design figures.

Because of the foreshortened appearance of such a curve and a preference by pilots to do heavy braking only on a straight section, the exit taxiway design shown in Figure 5-4 may be preferable--even though a longer route is required and two separate turns must be made. This design permits a small (20-degree) turn with little or no braking and a 750-foot straight portion where the aircraft can be decelerated from 60 to 20 mph at a rate of 4.5 feet per second^2. If moderate braking is done on the turn, the maximum rate of deceleration can be reduced.

The turn of 20 degrees is more satisfactory than a greater angle since the pilot has a much better view when starting the turn and a greater length of straight taxiway with a spacing of 750 feet between the runway and parallel taxiway centerlines. The width of the exit throat and the amount of paving required is about the same for a 20-degree

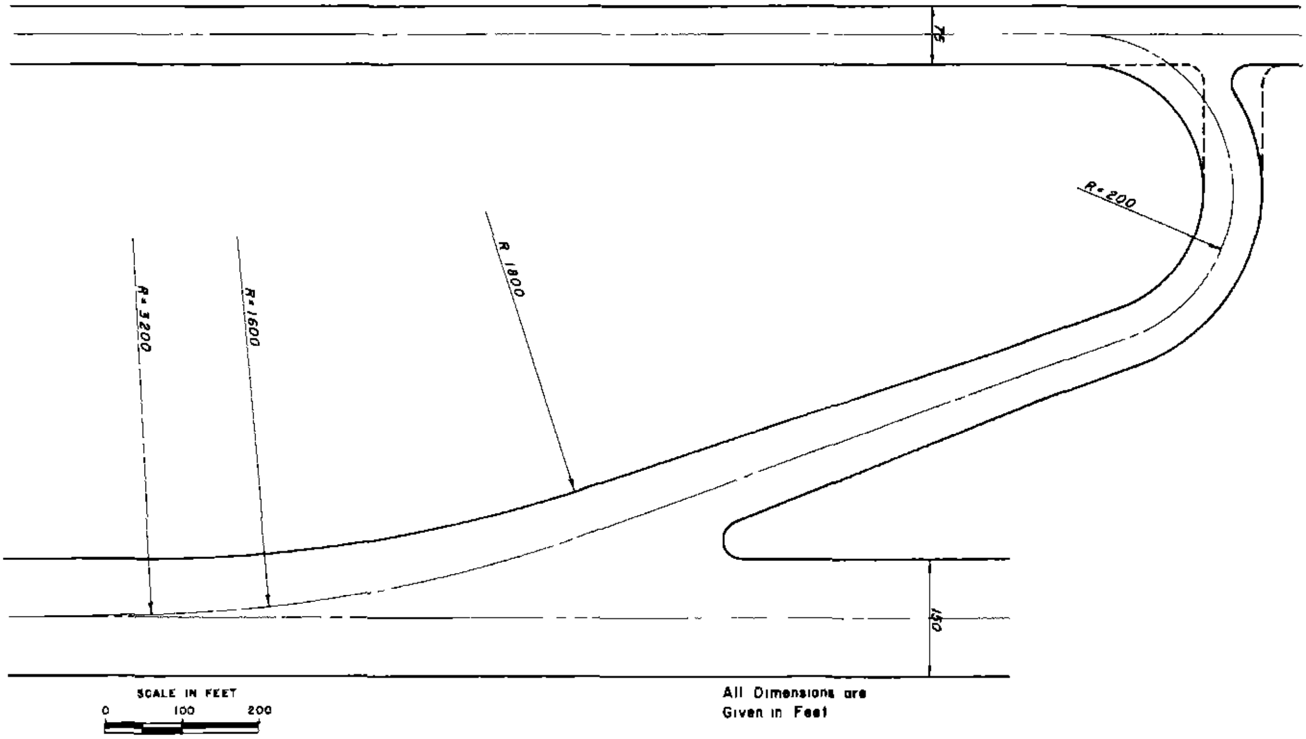

FIGURE 5-4 750-FOOT CENTERLINE-TO-CENTERLINE HIGH-SPEED EXIT (TAXI IN OPPOSITE DIRECTION TO LANDING)

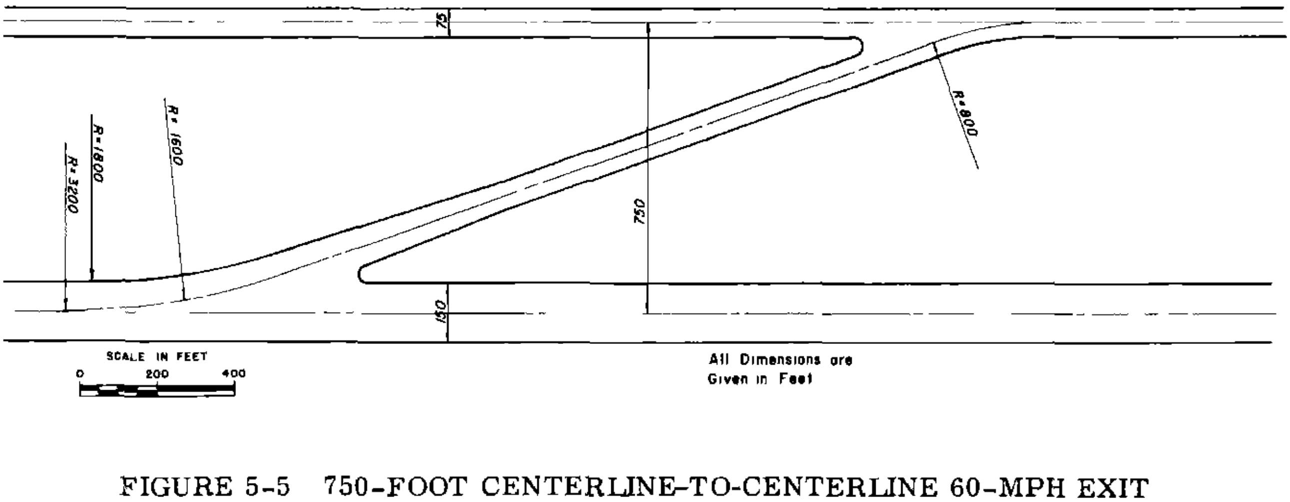

FIGURE 5-5 750-FOOT CENTERLINE-TO-CENTERLINE 60-MPH EXIT (TAXI IN DIRECTION OF LANDING) turn as for a 30-degree turn. Widening of the taxiway at the throat of the straight portion is provided on the inside of the turn, which is normal practice in highway design because of the tendency of vehicles to cut inside a turn. Additional widening is provided on the outside of the turn to reduce the optical effect when turning from a 150-foot runway to a 75-foot taxiway. Figures 5-5 and 5-6 show the variations available to cover other departure directions from the exit taxiway.

High-speed exits are generally not feasible where the distance between the runway and parallel taxiway centerlines is less than 450 feet unless the aircraft is able to continue in the same direction. Suggested layouts for 40-mph exits are shown in Figures 5-7, 5-8, and 5-9 for spacings of 250, 350, and 450 feet, respectively.

Using the procedures outlined in Section VII, the desirability of constructing high-speed exits can be determined by economic analysis.

D. HOLDING APRONS

A holding apron is provided at the take-off end of the runway to permit two or more aircraft to complete engine run-up checks or await en route flight-plan clearance while permitting a following aircraft that is ready to go to pass and enter the runway. Obviously, the need for these aprons increases with the volume of traffic and particularly air carrier traffic. Usually, these facilities are not required unless the air carrier traffic exceeds 20,000 operations or the total traffic exceeds 75,000 operations annually.

The number of positions required on the apron will vary with the types of aircraft as well as the volume and hourly distribution. Four-engine piston aircraft require a long engine check time, turbine engines require none. Airway congestion may require more time for clearances at certain

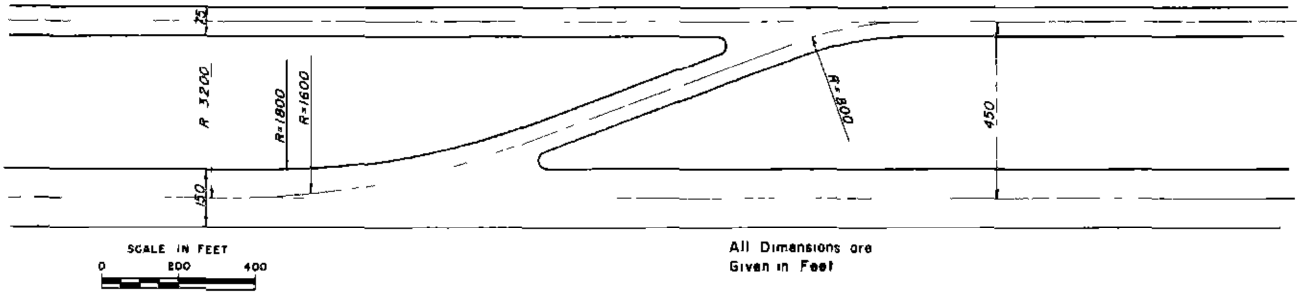

FIGURE 5-6 450-FOOT CENTERLINE–TO–CENTERLINE 60-MPH EXIT

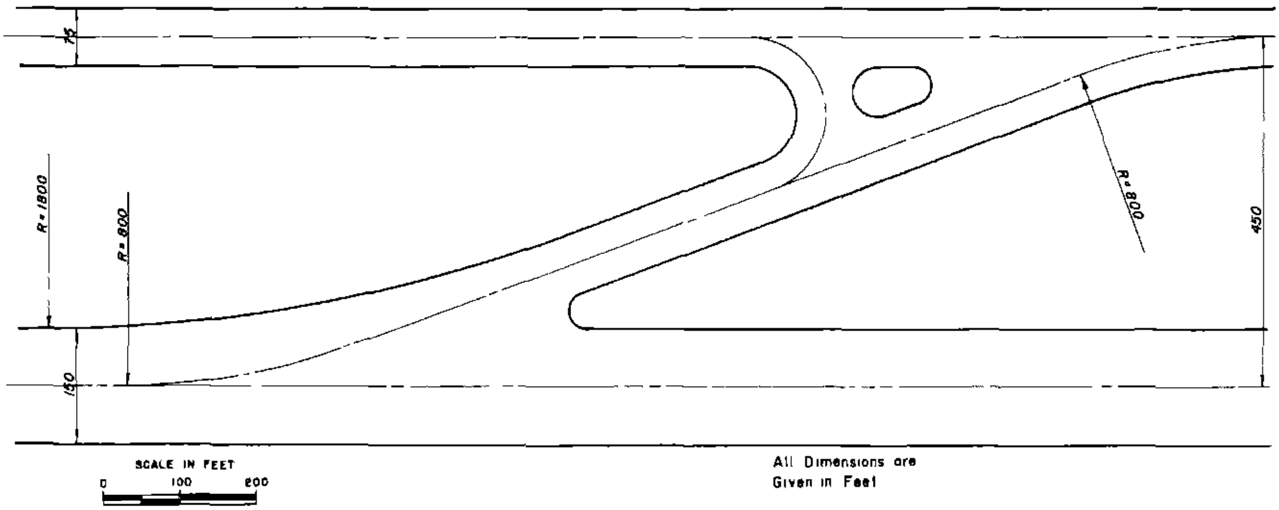

FIGURE 5-7 450-FOOT CENTERLINE–TO–CENTERLINE 40-MPH EXIT

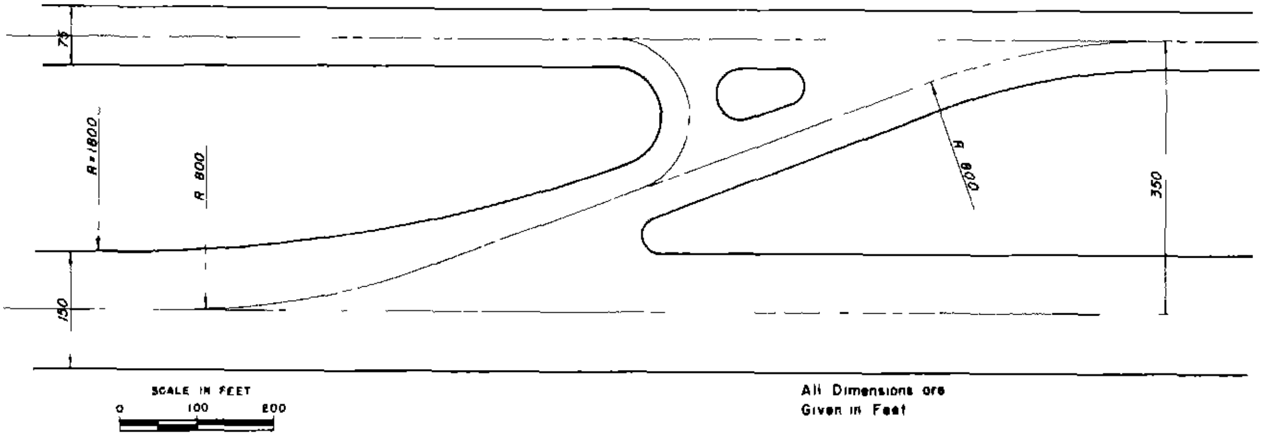

FIGURE 5-8 350-FOOT CENTERLINE-TO-CENTERLINE 40-MPH EXIT

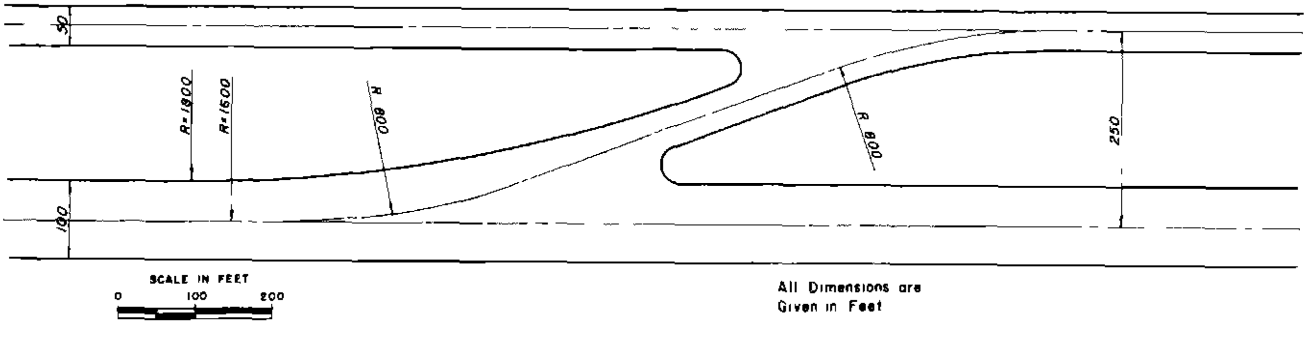

FIGURE 5-9 250-FOOT CENTERLINE-TO-CENTERLINE 40-MPH EXIT airports. This is an increasing need because of the tendency to use IFR flight plans in VFR weather. The need at a particular location can be estimated by survey or by use of the mathematical solution presented in later paragraphs, after the pertinent input data are determined. Normally, two to four spaces for holding aircraft are sufficient, thus making it possible for any one of three to five aircraft to enter the runway without interference.

The holding apron will be most efficient if the aircraft can be positioned with a minimum of maneuvering and can go into take-off position quickly. Aircraft in warm-up positions should direct their engine blast clear of other aircraft and the bypass taxiway. Suggested layouts for warm-up aprons are shown in Figures 5-10, 5-11, 5-12, and 5-13.

In addition to the holding apron, major airports need room along the taxiway for aircraft to wait for take-off in a queue. At many existing airports, the queue blocks other taxiways or apron space and delays other operations. This is most critical where take-off positions are adjacent to the terminal or maintenance area.

Since local service airports will have limited air carrier traffic and since close spacing of parallel taxiways does not permit a normal apron, only a bypass taxiway should be provided to permit small aircraft to bypass multi-engine aircraft. Such a bypass can also be used to prevent delays in flight training at these airports.

The mathematical solution that follows is used to determine the number of aircraft that can be accommodated in a holding apron. If it is understood that sufficient capacity will be provided on the holding apron to prevent delays--such as blocking by an aircraft ahead requiring engine warm-up time,

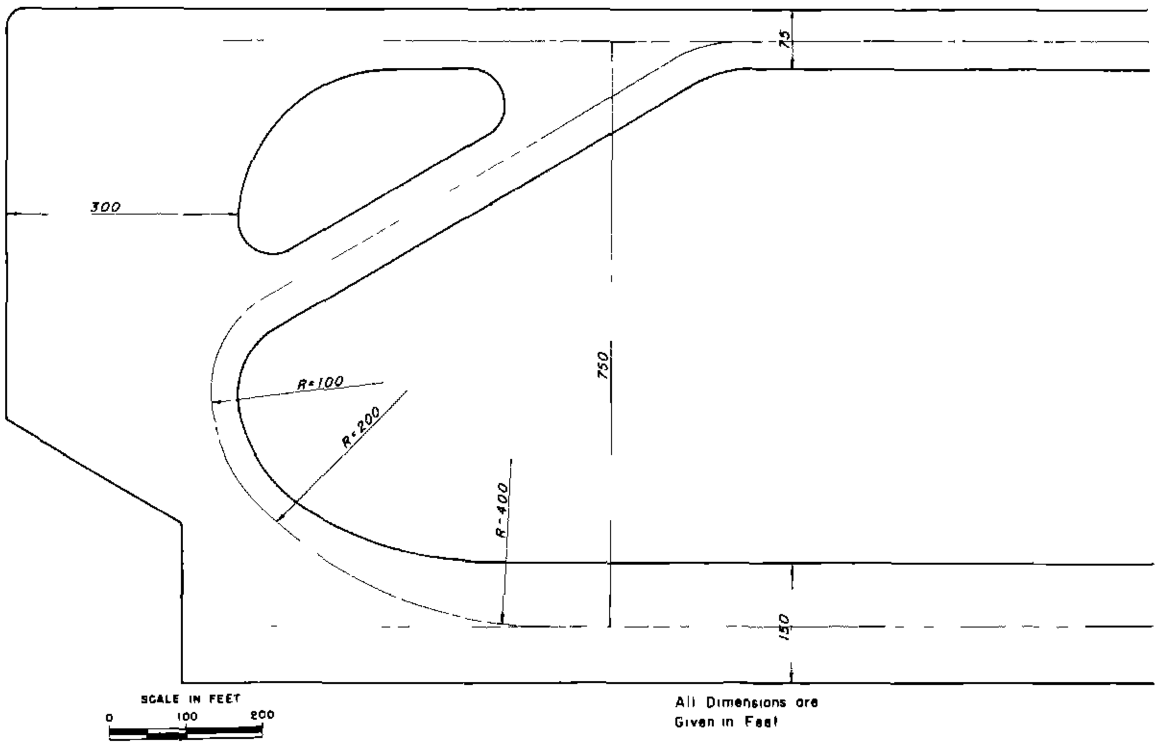

FIGURE 5-10 INTERCONTINENTAL HOLDING APRON

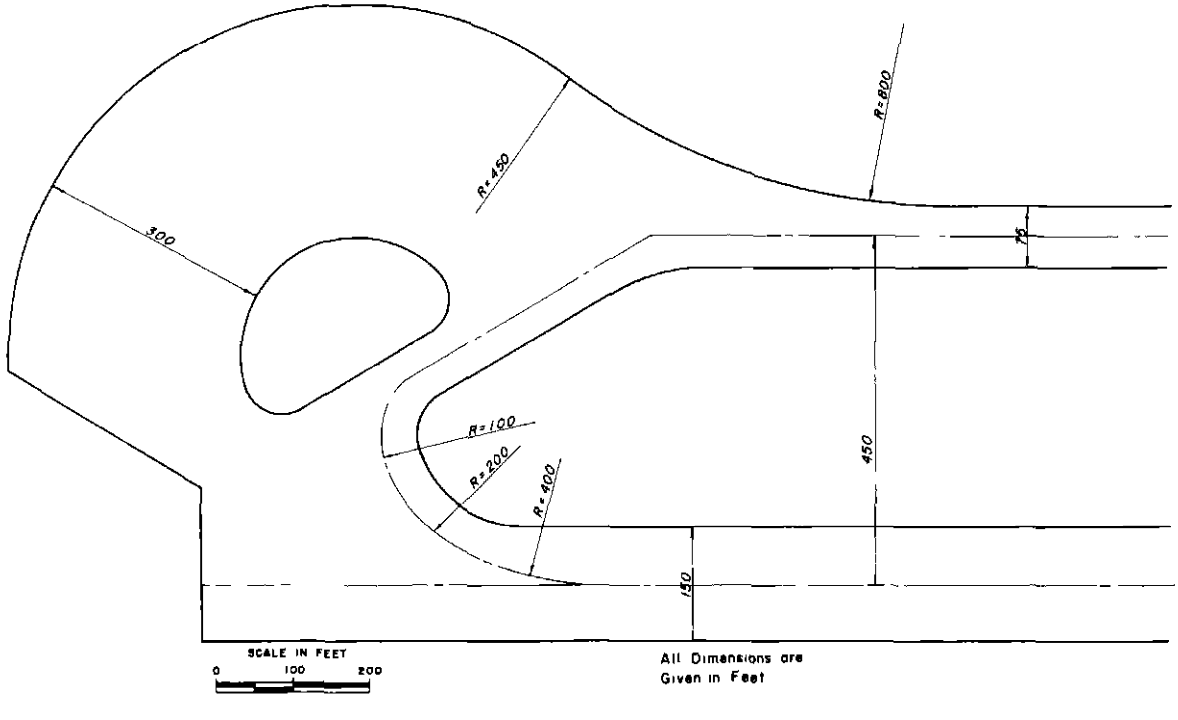

FIGURE 5-11 450-FOOT CENTERLINE-TO-CENTERLINE CONTINENTAL HOLDING APRON

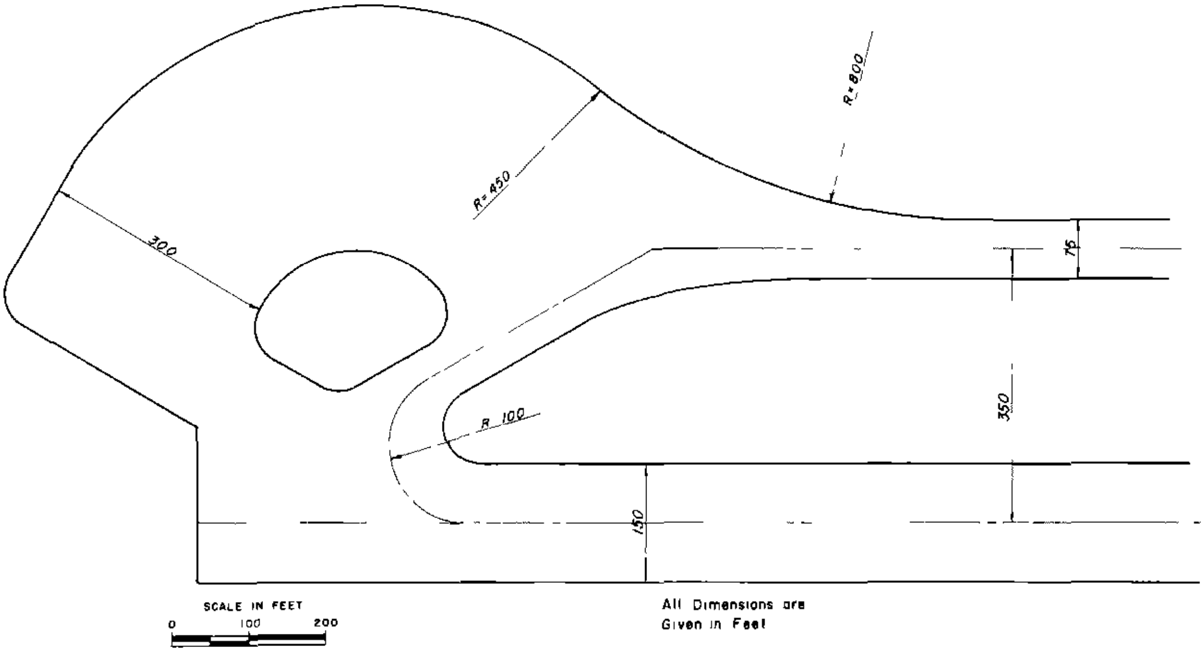

FIGURE 5-12 350-FOOT CENTERLINE-TO-CENTERLINE TRUNK HOLDING APRON

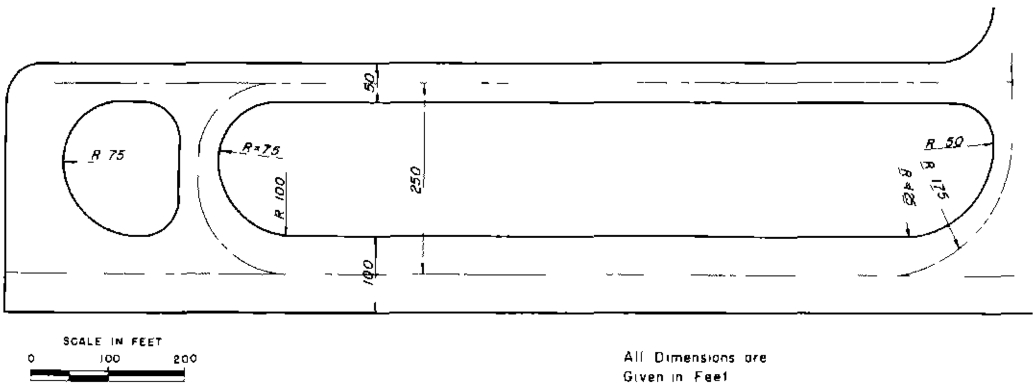

FIGURE 5-13 250-FOOT CENTERLINE-TO-CENTERLINE LOCAL HOLDING APRON or possibly waiting to obtain an IFR clearance--then the amount of capacity required can be readily computed.

A conservative approach is to determine the average departure movement rate λ_W, at its peak during the day, of those aircraft that require time only to warm up engines (or await clearance). The average warm-up or waiting time T_W for such aircraft is then determined. Thus, assuming sufficient capacity, the probability that the number of aircraft engaged in warm-up at any moment will be n is

$$p(n) = \frac{\rho_W^n e^{-\rho_W}}{n!}$$

where $\rho_W = \lambda_W T_W$.

This probability is the Poisson formula, tables using this formula can be found in many statistical books.* A plot of these probabilities for various values of ρ_W is given in Figure 5-14. The required capacity can be determined by choosing n large enough that the probability of more than n aircraft being engaged in warm-up is quite small--about 1 percent. This value can then be converted into area required by multiplying by the area required per aircraft.

E. HIGH-SPEED TAKE-OFF TAXIWAYS

It has been suggested that operations might be expedited by high-speed entrance taxiways such as those used to connect "alert" hangars of the Air Defense Command to the take-off end of the runway. Since none of these are available for use, an evaluation must be made by theoretical

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* See for example, R. S. Burrington and D. C. May, Jr., "Handbook of Probability and Statistics with Tables," Handbook Publishers, Inc , Sandusky, Ohio, 1953.

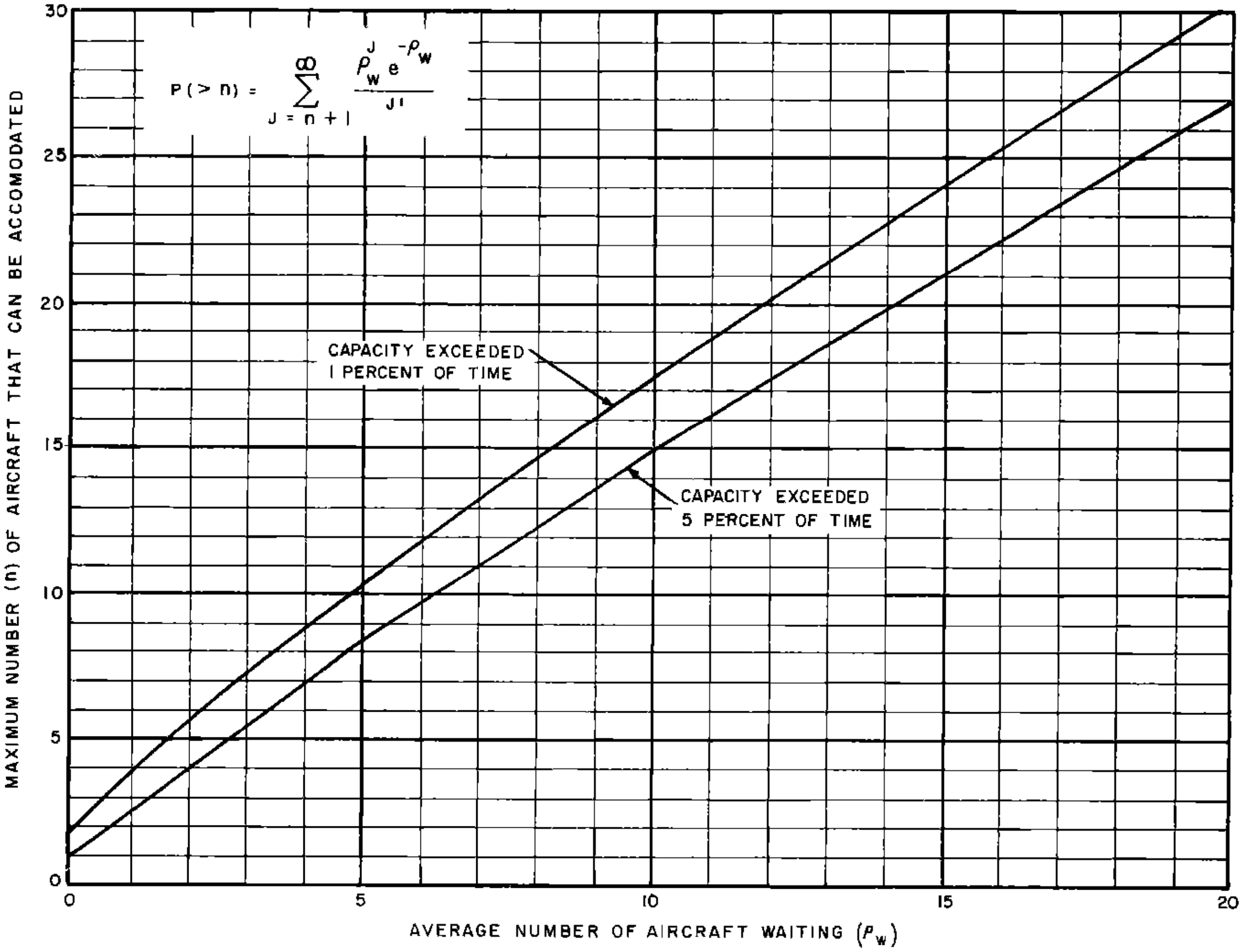


FIGURE 5-14 PROBABILITY OF WAITING IN TAKE-OFF QUEUE

analysis of the steps involved. The effect on operations would be to clear aircraft for take-off from a position on the entrance taxiway rather than from a position on the runway.

If a departure is to follow a departure, there is usually sufficient time for the succeeding aircraft to move on the runway into take-off position and be ready to go as soon as the first aircraft has cleared the boundary. A similar situation exists for a departure following a landing on the same runway. The taxiway access to the runway end must, however, be laid out to permit an easy turn into take-off position and not require backtracking downwind before turning upwind. Where intersecting runways are used for segregated operations, the departing aircraft can be in position while a preceding aircraft is landing. The only long wait (relatively) in take-off position is with turbojet aircraft, particularly those using water injection. The procedure is to run up to full power and balance the engines before releasing the brakes; this requires 20 to 40 seconds. A high-speed entrance could help this situation only if the pilots were willing to make the necessary turns with full power; this does not seem desirable. It is hoped that more turbojet experience, the elimination of water injection, and the use of engines with more take-off thrust will reduce or eliminate this run-up time on the runway.

F. ANALYSIS OF ACCIDENT PROBABILITY WITH RESPECT TO RUNWAY AND TAXIWAY SEPARATION

If an accident should occur either during landing or take-off, it is possible that the damaged aircraft will leave the runway. It was important to find the probability that a damaged aircraft would affect operations on a parallel taxiway or runway as part of the criteria for determining separation.

The analysis included in Appendix B indicates that the probability of collision between landing and taxiing aircraft, or between landing aircraft on adjacent runways, is very low for all separations analyzed--and some of these separations are less than separations in use today. Table 5-II summarizes this analysis.

TABLE 5-II

ANALYSIS OF RUNWAY AND TAXIWAY SEPARATIONS

Runway-Taxiway or Runway-Runway Separations (feet)	I	II	III
200	209	282	71
225	239	323	81
250	254	342	86
275	280	377	95
300	377	508	127
325	399	539	135
350	439	592	148
375	472	634	159
400	574	772	193
425	628	846	212
450	694	935	234
475	737	988	247
500	940	1270	318
750	1312	1588	397
1000	1474	1970	493

Column I is the probability in years that the damaged aircraft may collide with an aircraft on a parallel taxiway when the airport is operating at about 250,000 operations per year.

Column II is the probability in years that the damaged aircraft may collide with an aircraft on a parallel runway when the airport is operating at about 250,000 operations per year.

Column III is the same as Column II but for 400,000 operations per year.

VI. TYPICAL AIRPORT DESIGN CONFIGURATIONS

A. INTRODUCTION

The airport provides the connection between ground vehicles and airborne traffic. Its purpose is to effect this interchange of persons and property with the minimum delay, expense, and inconvenience consistent with safety. In this study, we are concerned with the operations from the completion of the loading of the departing aircraft to the start of unloading of the arriving aircraft--the block-to-block phase as affected by airport design. The en route portion of the trip is considered only with respect to its effect on the timing of arrivals to the airport area.

To focus the study on the individual airports, it is best to consider the separate aircraft operating segments at the airport approach, landing, taxiing to gate position, taxiing from gate to runway, run-up, and take-off. We are, therefore, concerned with the efficiency of the airport in accommodating these movements.

The design of the airport can affect the time required for all operations in this cycle. Since any unnecessary time spent in these operations represents additional costs to the aircraft operator and his customers, investments required to reduce these losses can be compared and the best solution determined.

As discussed in Section IV, there are so many variables--aircraft types, traffic volumes, weather, and topography--that each solution can cover only one airport. However, many of the principles and basic design criteria will apply to all airports.

The first operation at the airport is the approach--from the time the aircraft leaves the en route path until it crosses the threshold. Multiple directions of runways appear to use all airspace around the airport and permit more direct approaches. However, potential conflicts at intersections, in the air or on the ground, complicate traffic procedures when the rate of operations is high (even in VFR conditions). This is often accentuated by varying preferences of pilots for a particular runway. In IFR operations, the air route patterns and the limitations of navigational aids restrict the number of feasible directions. Greatest efficiency is usually obtained at high operating rates when all approaches and departures are parallel. Complete navigational aids for instrument landing are usually available in only one direction, or one direction and the reverse heading on the same runway.

These limitations make it desirable to have one runway direction with a second cross-runway direction to be added only if wind conditions require. If a lesser capacity is provided in the second direction, greater delays may be expected at those times when the controller or pilots choose to use that direction.

The landing operation begins when the aircraft has crossed the runway threshold and continues until the aircraft has taxied off to clear the runway edge by 100 feet. The runway occupancy time is thus affected by the location and configuration of the exit taxiway. Exit taxiways can join the runway at right angles (requiring slow turning movements by aircraft) or at large-radii curves (permitting turning movements at speeds of about 90 feet per second). Exit taxiways should be located at points consistent with the class of the airport and the aircraft population. For operations from both ends of the runway, a symmetrical taxiway layout is indicated. Section IV has shown the effect of the location and type of turn-off on runway operations.

The most efficient route from the exit taxiway to the runway to the terminal ramp and to the assigned gate position is the most direct straight-line distance with a minimum of turns consistent with the surface limitations of the airport and the cost of construction. Unidirectional taxiways with a minimum of active taxiway crossings will provide for taxiing at the highest speed consistent with required maneuvers and the least delay at crossings when holding for other aircraft movements.

Since taxi speed on the active terminal apron, even on marked taxi lanes, is usually very low compared with taxi speeds on taxiways, taxi time can be reduced by providing bypass taxiways that will enable aircraft to reach the assigned gate position on the apron without taxiing along the edge of the apron. In this way, the taxi distance on the apron to the gate is kept to a minimum.

The operation in reverse, from the gate position to the queue, is also dependent upon the taxi route. These considerations apply equally to the route to the queue or to the take-off holding apron. In addition, the configuration at the holding apron on large airports should provide for those aircraft on the apron awaiting approval for take-off (generally, three or four) and a bypass taxiway for those aircraft in the queue that receive clearance prior to the first aircraft in the queue. The delay time while holding in the queue and prior to movement down the runway for take-off is considered to be part of this operation. The take-off operation includes the aircraft take-off ground roll and continues until the aircraft has cleared the boundary or far end of the runway.

B. TYPICAL LAYOUTS

Each airport must be designed to fit the particular conditions of traffic, topography, meteorology, etc. Reasonable standardization of basic elements and layouts is desirable. Optimization for a specific site should begin by determining the general type of layout to be used and modifying it for the specific location.

As a guide to initiating the layout for a particular location, 12 typical layouts are shown.

1. Local airport with low volume,
2. Local airport with high volume,
3 Trunk airport A with high volume,
4. Trunk airport surface restrictions,
5. Trunk airport with low volume,
6. Trunk airport B with high volume,
7. Continental airport with high volume,
8. Continental airport surface restrictions,
9. Continental airport with low volume,
10. Intercontinental airport with low volume,
11. Intercontinental airport surface restrictions,
12. Intercontinental airport with high volume.

These are not optimum layouts unless every condition that affects the various elements of design is met by the layout discussed. Such a situation is unlikely and these layouts must not be considered ideal. Comments on the relative efficiency of runway layouts are based on the analysis in Section IV, where the operating efficiency of various runway layouts is covered in detail.

Efficiency of airport operations is often reduced by design features that cause pilots to request the use of a different runway from that planned by the controller. The outstanding factor causing this is a configuration with different runway lengths. Under the same conditions--similar aircraft and constant wind--different pilots will request different runways. For example, one pilot will request a shorter runway into the wind and another pilot will request a longer runway with a cross wind. This situation exists if three or four runway directions are available. In general, the greatest efficiency is obtained with a minimum of runway directions, all runways of equal length, and all runways usable in both directions.

1 LOCAL AIRPORT WITH LOW VOLUME (FIGURE 6-1)

A single-runway airport is efficient until the volume of traffic or the cross-wind limitation justifies a second runway. Air carrier traffic must be able to land on 0.6 of the runway length and will generally be able to exit near the midpoint of the runway. For the local airport with a high percentage of single-engine general-aviation aircraft, the holding areas at the end of the runway can best be divided into two connecting taxiways, since the predominant movement is the bypass rather than the usual holding of larger aircraft for engine run-up or IFR clearance. A low volume of traffic will not provide enough saving in time to pay for high-speed exits. For a single-runway airport, the most efficient operations are obtained where the terminal area is parallel to the runway and near the center of the runway length if the two directions are used about equally, or toward the downwind end for the prevailing wind, with all general-aviation aircraft located between the air carrier terminal and the downwind end of the runway.

2. LOCAL AIRPORT WITH HIGH VOLUME (FIGURE 6-2)

High-volume operation with aircraft requiring only local runway criteria can use to advantage a configuration that places parallel runways on opposite sides of the terminal area exactly symmetrical to each other with both runways used for landings and take-offs. This layout requires less land area than one with offset parallel runways and has greater flexibility since simultaneous landings or take-offs can be accomplished. It also affords greater flexibility when one

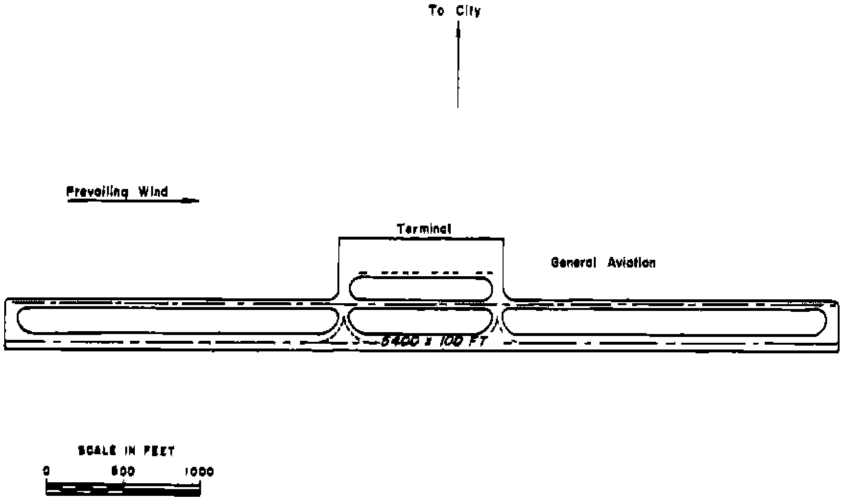


FIGURE 6-1 LOCAL AIRPORT WITH LOW VOLUME (1962 POPULATION)

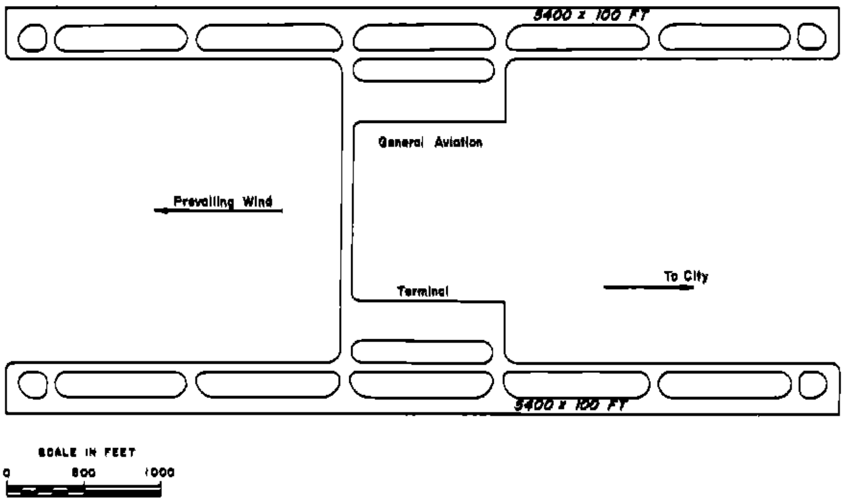


FIGURE 6-2. LOCAL AIRPORT WITH HIGH VOLUME (1975 POPULATION)

of the runways is closed for snow removal, repairs, or construction. One right-hand and one left-hand air traffic pattern make excellent use of available airspace.

3. TRUNK AIRPORT A WITH HIGH VOLUME (FIGURE 6-3)

Where the prevailing wind and strong wind or low visibility approach requires two runways for greater coverage, the open-vee layout can effect a high traffic capacity. Maximum capacity is obtained by segregated operations with one runway for landing and the other for take-off.

4. TRUNK AIRPORT SURFACE RESTRICTIONS (FIGURE 6-4)

Surface restrictions that prohibit the use of optimum configurations require some compromise of runway layout, terminal location, or taxiway routes. With reference to two runway layouts, surface restrictions may require overlapping of runways. Crossing of runways by aircraft landing, taking off, or taxiing will cause some delay. A terminal location determined by surface restrictions may cause a substantial increase in taxi time over the minimum time obtainable at the most efficient location.

5. TRUNK AIRPORT WITH LOW VOLUME (FIGURE 6-5)

As for other low-volume traffic, the single runway is all that is required if proper wind coverage is obtainable. Larger aircraft will require longer runways and usually a larger terminal area. The holding apron at the ends of the runway must be adequate for run-up and holding of larger air carrier aircraft.

6. TRUNK AIRPORT B WITH HIGH VOLUME (FIGURE 6-6)

A high percentage of light general-aviation aircraft requiring increased airport capacity can be served by adding a runway that will accommodate the large population of small aircraft. Where wind conditions permit, advantages are provided by parallel symmetrical runways at a minimum centerline-

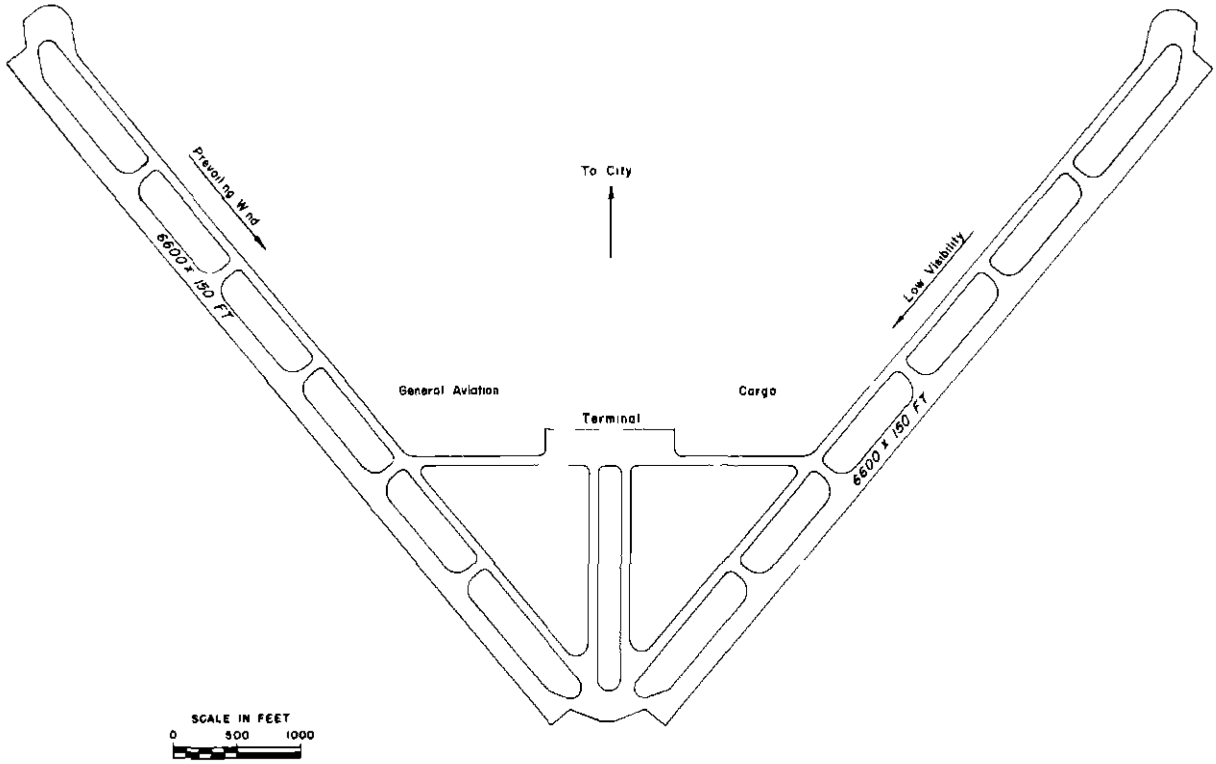


FIGURE 6-3 TRUNK AIRPORT WITH HIGH VOLUME (1962 POPULATION)

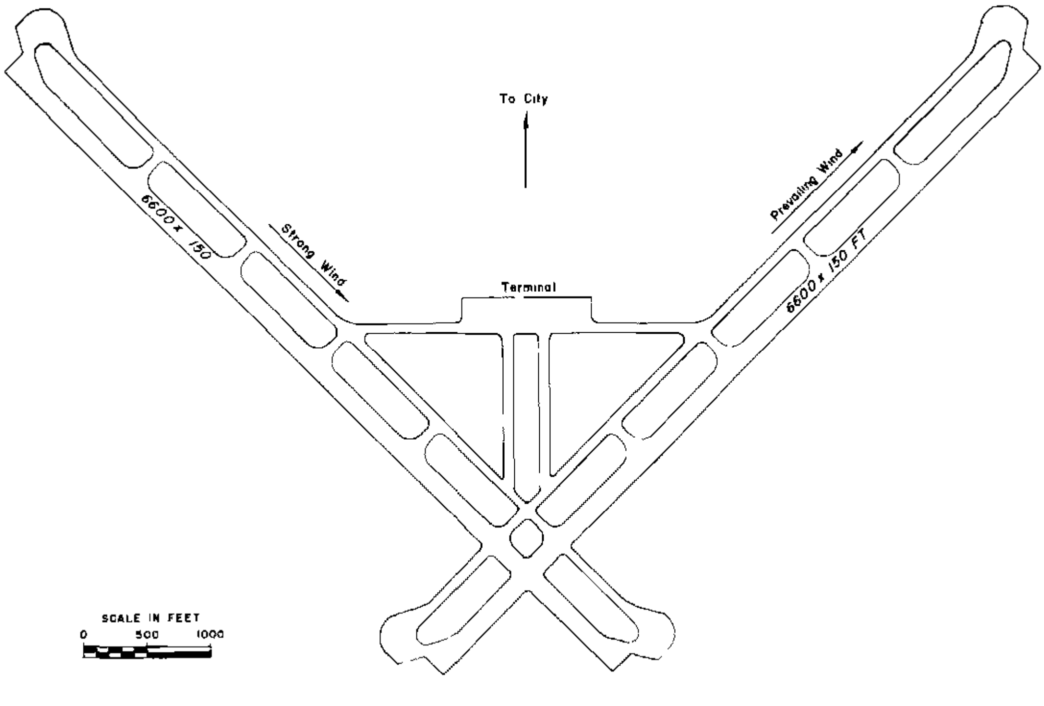


FIGURE 6-4 TRUNK AIRPORT SURFACE RESTRICTIONS (1962 POPULATION)

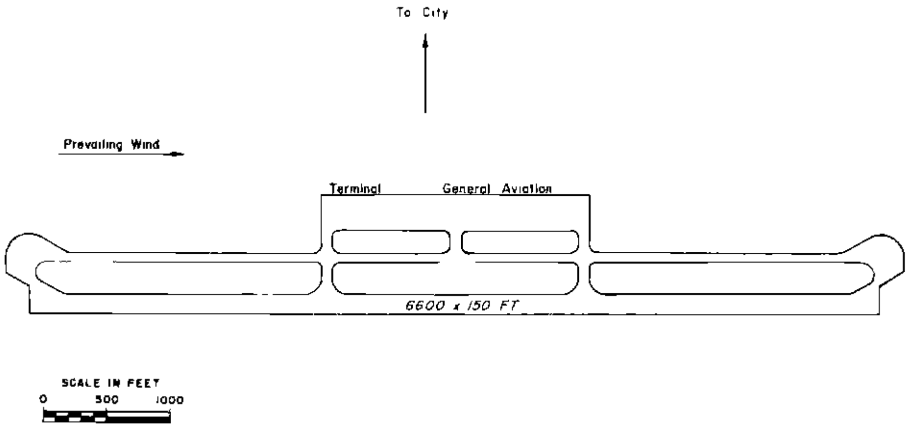


FIGURE 6-5 TRUNK AIRPORT WITH LOW VOLUME (1962 POPULATION)

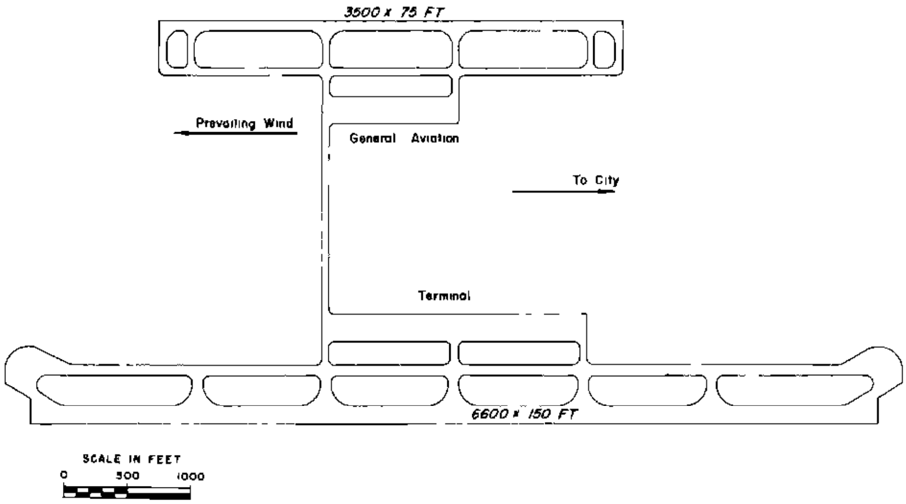


FIGURE 6-6 TRUNK AIRPORT WITH HIGH VOLUME AND HIGH PERCENT OF LIGHT AIRCRAFT (1962 POPULATION)

to-centerline spacing of runways consistent with the requirements of the air carrier terminal and other operations for general-aviation aircraft. Where segregated operations of this type are possible, pavement areas for the general-aviation operation can be kept to a minimum by use of runways 75 feet wide and taxiways 50 feet wide, thus saving considerable development expense.

7. CONTINENTAL AIRPORT WITH HIGH VOLUME (FIGURE 6-7)

Where the prevailing wind and the strong wind are at right angles, an airport can use two parallel prevailing-wind runways, separated by the terminal area, with a single runway aligned into strong wind. The runways can be positioned so that when landings are made into strong wind, one or both of the other runways can be used for concurrent but independent departures.

8. CONTINENTAL AIRPORT SURFACE RESTRICTIONS (FIGURE 6-8)

Where limited land area or other restrictions do not permit long runways to be constructed exactly opposite to each other at spacings of 5000 feet or more, the runways can be offset by varying amounts to fit the area available

9. CONTINENTAL AIRPORT WITH LOW VOLUME (FIGURE 6-9)

With a continental population and low volume, the single runway is all that is needed, provided that adverse winds do not indicate other requirements. Larger aircraft will require larger runways and usually a larger terminal area to increase the number of gate positions. Where a system other than the frontal terminal is advantageous, certain modifications of the taxiway configurations will be required. Economic analysis will show whether high-speed exits and a correspondingly greater separation of parallel taxiways are justified.

10. INTERCONTINENTAL AIRPORT WITH LOW VOLUME (FIGURE 6-10)

With a low traffic volume, a single runway is most efficient if it can be aligned with the strong wind Operations

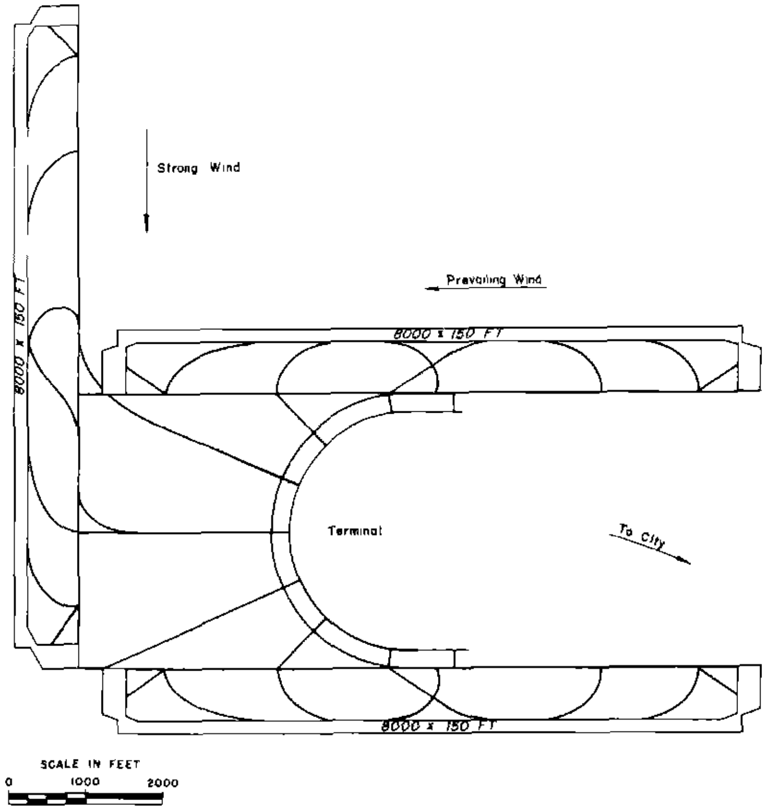


FIGURE 6-7 CONTINENTAL AIRPORT WITH HIGH VOLUME (1962 POPULATION)

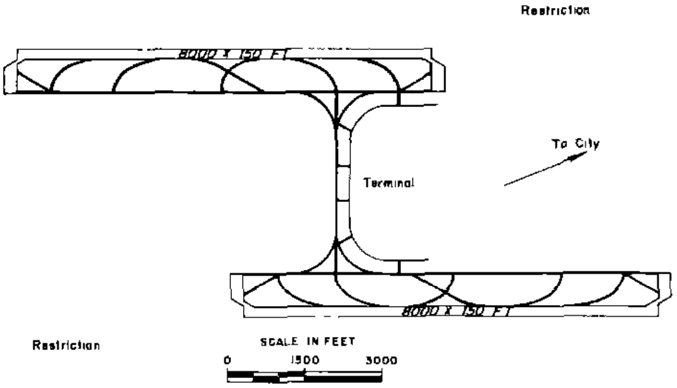


FIGURE 6-8 CONTINENTAL AIRPORT SURFACE RESTRICTIONS (1962 POPULATION)

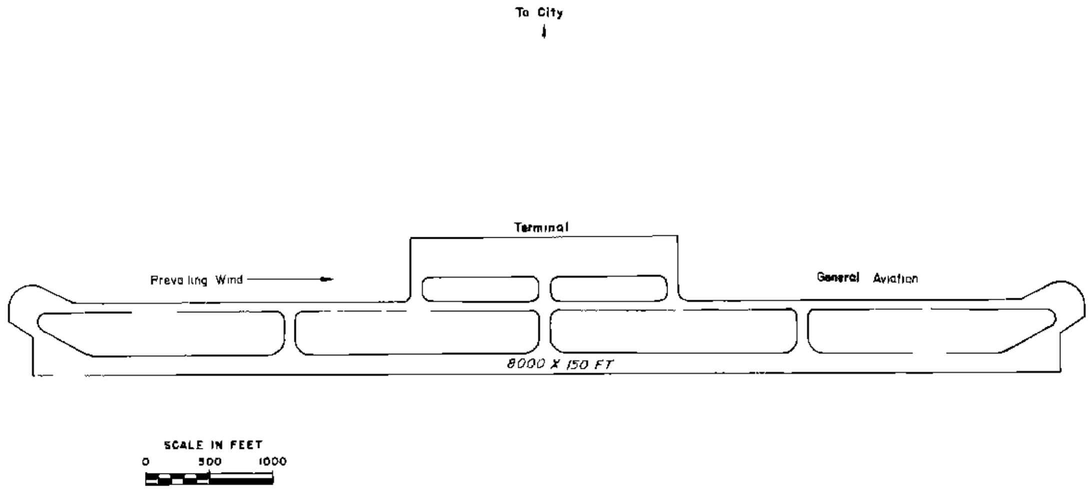


FIGURE 6-9 CONTINENTAL AIRPORT WITH LOW VOLUME (1962 POPULATION)

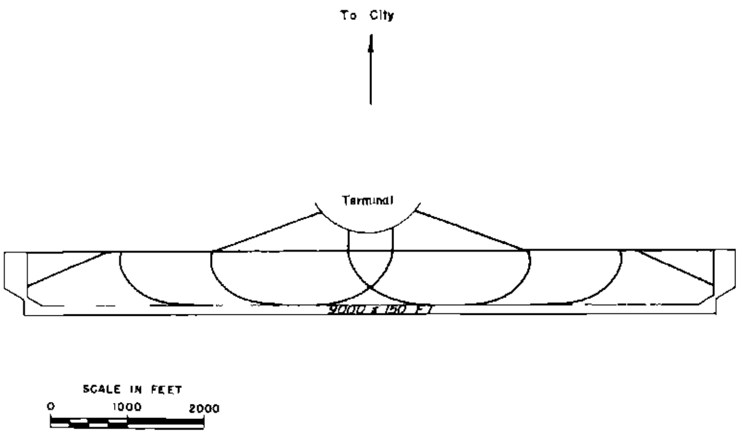


FIGURE 6-10 INTERCONTINENTAL AIRPORT WITH LOW VOLUME (1962 POPULATION)

are thus simplified and protection is required for only two approaches

Air carrier landings will require about half the runway length. Therefore, the terminal area should be opposite the center of the runway. Only one parallel taxiway is required for through movements In addition, extra taxiways may be required to serve maintenance areas.

11. INTERCONTINENTAL AIRPORT SURFACE RESTRICTIONS (FIGURE 6-11)

Surface restrictions that prohibit aligning a single runway into the wind may require two runways placed at an angle of 60 to 90 degrees. If these restrictions do not permit non-intersecting runways, the terminal can be placed so that landing aircraft will go directly from the exit taxiway to the terminal area, and then to the take-off runway without crossing either runway and with a short taxi distance for prevailing wind.

12 INTERCONTINENTAL AIRPORT WITH HIGH VOLUME (FIGURE 6-12)

For maximum traffic-handling capability, an airport should have four parallel runways, two on each side of the terminal. All runways should be capable of accommodating regular landings or take-offs in either direction. A capacity analysis for the projected aircraft population may indicate that it is desirable to assign runways for landing only and take-off only, but past experience has shown that mixed operations are preferable At times, the two pairs of runways may be used for traffic in opposing directions to permit a maximum of straight-in approaches and departures.

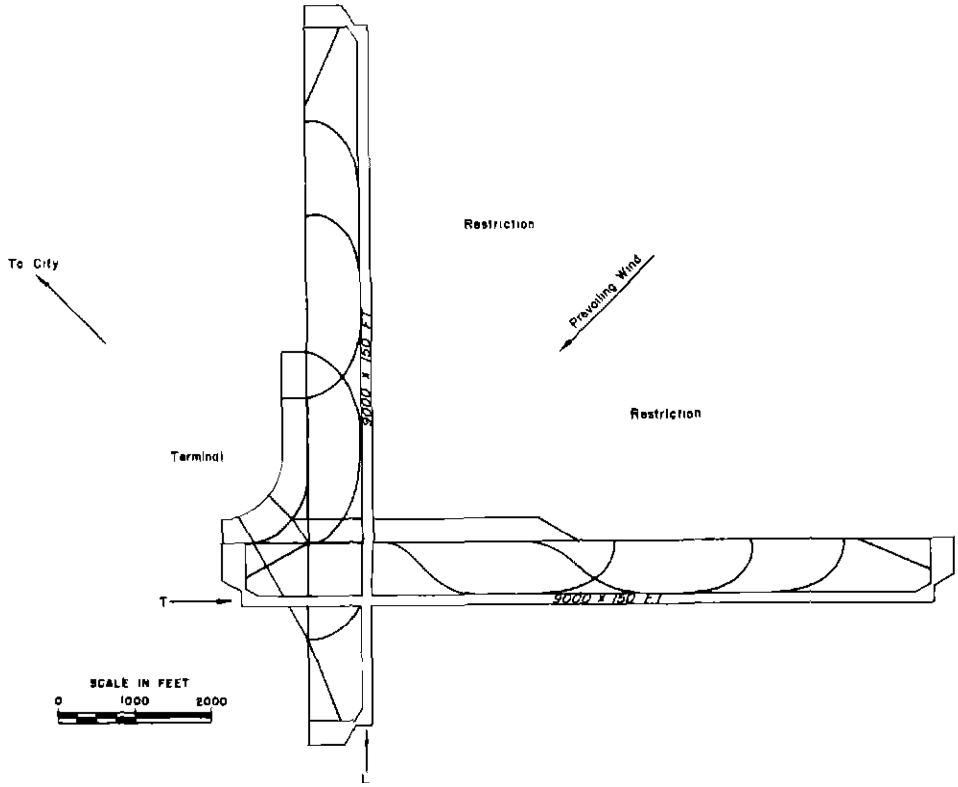


FIGURE 6-11 INTERCONTINENTAL AIRPORT SURFACE RESTRICTIONS (1962 POPULATION)

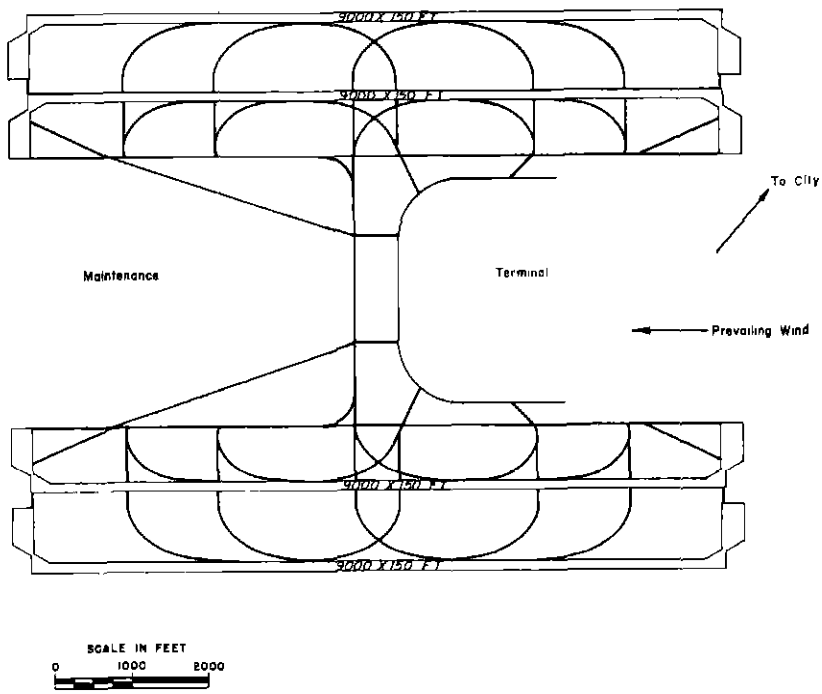


FIGURE 6-12 INTERCONTINENTAL AIRPORT WITH HIGH VOLUME (1975 POPULATION)

VII. OPTIMIZING DESIGN BY ECONOMIC ANALYSIS

An optimum airport configuration is applicable only to a specific site at a specific time. Therefore, this section will provide guidance to assist in the selection of an optimum design for particular situations.

It is obvious that a parallel runway system with a capacity of 400,000 operations per year--an optimum design for some locations--is not a sound investment for a local service airport with only 40,000 annual operations. In addition to volume of traffic, many other factors that influence the services to be provided as well as the cost of providing these services must be considered in developing the airport facilities.

The only common denominator for a comparison of airport services and development is money. The value of the services can be estimated and compared with the costs of furnishing facilities, to determine the economic feasibility of proposed improvements

The optimum design is that which will return the greatest net benefit to the user. It is possible to evaluate airport user benefits in much the same manner as in other comparative economic studies. The relationship of user benefit to cost provides the basis for evaluation. The maximum ratio rather than the maximum net benefit is often considered optimum, but any analysis should consider both. As in other fields of transportation, the comparative economic evaluation can be used for programming construction from the initial requirement to the ultimate master plan. In master planning, the optimum layout for the estimated ultimate use should be developed and stages of construction defined for optimum return at various steps in traffic growth. Projects with a benefit-cost ratio of less than one should not be undertaken and those projects with the highest ratio should be given priority for construction.

In all cases, design standards for runway length, width, lighting equipment, and approaches must be followed to ensure proper safety. User benefits are measured by comparative operating costs. Any change that reduces aircraft delay or operating time will result in a definite saving to the operator.

These direct benefits to the aircraft operators can be paid to the airport owner in fees to cover the costs of improvements. This is now done at many airports. The surplus, and any indirect benefits, are distributed by natural economic processes to the owner of the airport, aircraft operators, passengers, airport customers, and the general public.

If the airport site has not been selected, preliminary studies for a particular location will include economic analysis of the major factors that affect site selection. Economic analysis of user benefits for the ground transportation access to the airport is one important study that can be included. User benefits in this study, as in highway benefit-cost analyses, can include direct transportation costs plus time savings for the individual passengers. Indirect benefits may include greater utility of the airport and its related facilities.

Airport development and operations can be restricted at specific locations by the topography of the site, topography at approaches or turning areas, and the limitation of usable airspace by other airports. Noise abatement is a current but, hopefully, temporary airspace problem that should be corrected by means other than limitations on airspace usage.

Ideally, a site with restrictions should not be selected since this lowers the efficiency of operations. However, this will not be true when the optimum airport for a particular location can accept certain inefficiencies so as not to incur excessive development costs or an unreasonable ground travel distance The effect of restrictions and the cost of reducing the extent of limitations must be fully explored by economic analysis

Site limitation may dictate a less efficient layout, such as intersecting runways, closely spaced parallel runways, or less efficient placement of terminal area. Restricted approaches are usually more important in instrument operations where certain approaches are unusable or where turns are restricted. Many sites will have combinations of topography and airspace restrictions. In addition, national planning and control of airspace assignments has been too long delayed.

The standards of design, the consideration of any surface restrictions, and the determination of a preliminary runway layout will provide the initial typical configuration for which an economic evaluation can be made. The optimum design then is determined by the economic analysis of the factors that affect airport operations. In this analysis, direct benefits are limited to savings in aircraft operating costs.

The mathematical models developed and the procedures set forth in Section IV can be used to determine anticipated delays caused by runway operations. In other operations (excluding terminal servicing), taxiing, taxi routes, and taxi times are important and will partly determine the extent of delay.

Taxiing times can be computed simply by using average speeds for different sections and average delays for points of conflict. Preliminary studies such as those used in this report can be based on weighted average figures. A more exact analysis can be made of a specific configuration if the respective portions that make up the aircraft population are handled separately in all calculations. To obtain a more accurate analysis we can make use of exit taxiways selected by aircraft type, taxi routes at probable taxi speeds of designated aircraft types, probable assigned gate positions, etc.

Much can be done in developing basic principles for optimum configurations. The typical configurations shown for a particular set of conditions, and the discussion in Sections V and VI, point out many of the factors to be considered in the selection of a layout. Similarly, many design features require detailed analysis to obtain an optimum configuration. This section will point out examples of the methods by which specific items can be studied

A. FACTORS TO BE USED IN ECONOMIC ANALYSIS

Aircraft population is one of the basic factors that influence airport design and must be forecast through the anticipated period of airport use. Ordinarily, this population cannot be controlled except through the provision of other airports, but is a result of customer demands for various airport services. Forecasting is very important, but obviously it cannot be highly accurate because of unexpected technological developments and changing economic conditions. It is, of course, not feasible to break the population down into a great number of aircraft types. As a practical standpoint, three to six categories should be sufficient to estimate airport performance and permit reasonable forecasting by categories. The classifications given in Section IV will suffice for the studies of optimum design. The tabulation used in this report follows (The actual or projected population should be used to study any specific location.)

Airport	Suggested 1962 Population by Class (in percent)				
	A	B	C	D	E
Intercontinental	40	30	10	10	10
Continental	20	30	20	10	20
Trunk	0	30	30	20	20
Local	0	0	30	20	50

Scheduled traffic is quite uniform with a typical hourly pattern repeating daily. These patterns vary in number and in time of peaks depending upon the type of traffic and the flight times to principal intermediate points or to final destinations. Generally, the periods of high activity for light aircraft and air carrier transports are the same, but the peak hours do not coincide. It has been found that when the hourly rates are plotted in order of magnitude, the patterns are very uniform (Figures 7-1, 7-2, and 7-3). For reasonably high volumes there is a straight-line distribution of air carrier traffic with a maximum of 8 to 10 percent of daily operation in the busy hour.

At busier airports (more than 200,000 annual movements), the hours of operation are usually extended, at airports with less than 100,000 movements, the hours are concentrated over shorter periods. The distribution has a great effect on the delays from landings and take-offs, but has much less effect on time used for long taxi routes and for taxiway intersections.

To simplify computations, the "design day" for scheduled traffic is taken as having a straight-line distribution over 13, 19, or 24 hours, with a maximum of 16, 10, or 8 percent in the highest hour and handling 1/360 of the annual traffic. Since we are most interested in locations where traffic records are available, modifications can be

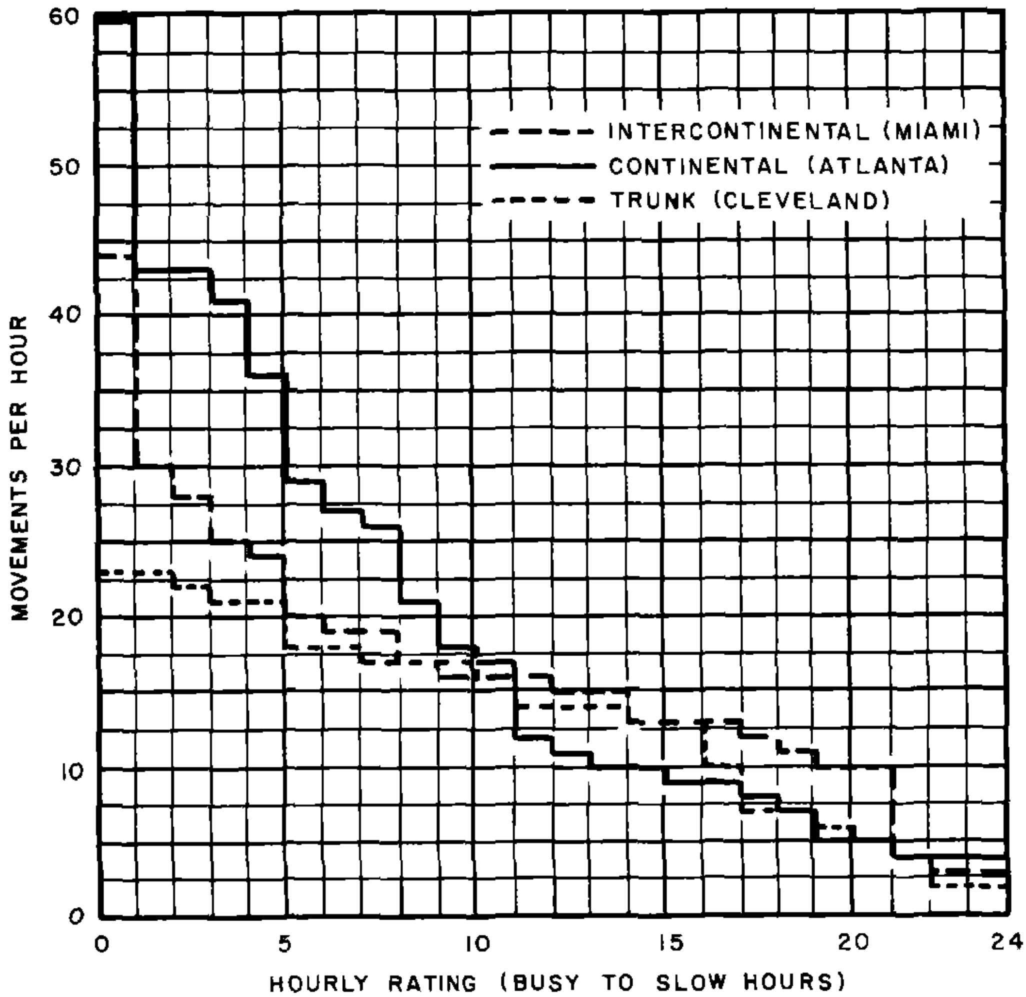


FIGURE 7-1 HOURLY DISTRIBUTION OF MOVEMENTS FOR AIR CARRIER OPERATIONS

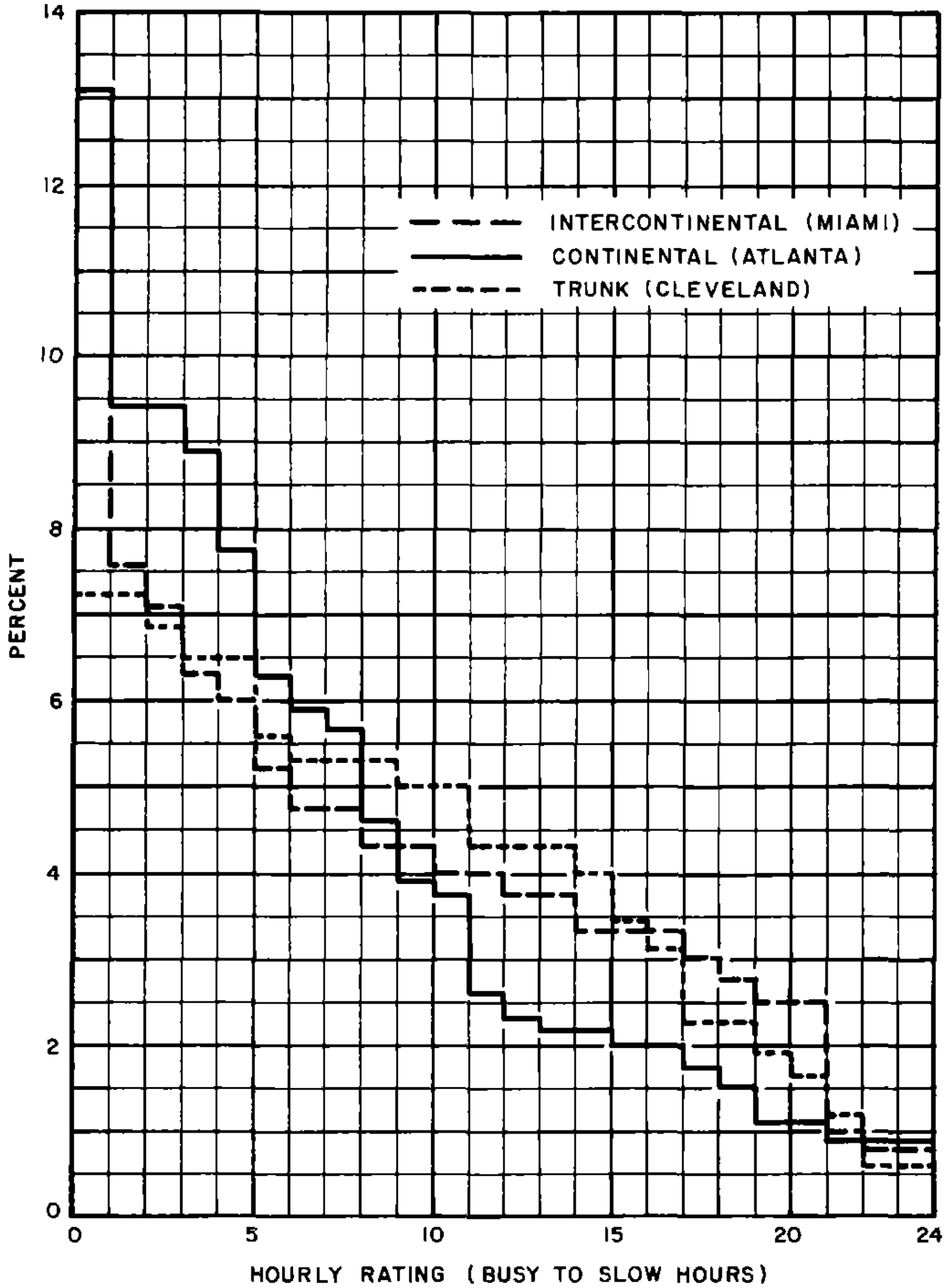


FIGURE 7-2 HOURLY DISTRIBUTION OF AIR CARRIER MOVEMENTS BY PERCENTAGE

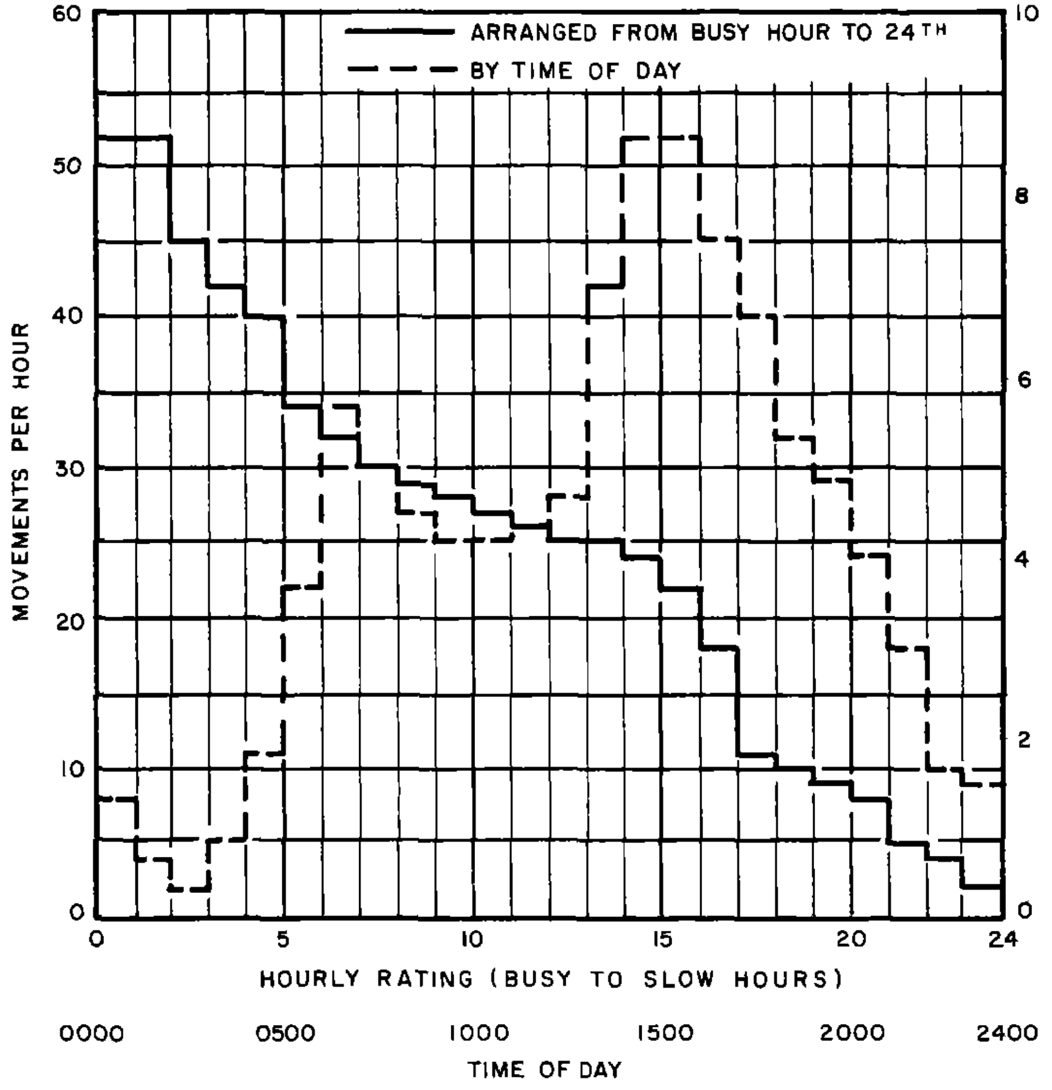


FIGURE 7-3 TYPICAL HOURLY DISTRIBUTION FOR TOTAL OPERATIONS

made where the daily distribution is atypical. The design day is defined differently than the "average peak day" discussed in Section IV. Similarly, the definition of a design day differs from the "peak day" commonly used in reports of traffic and in forecasts. Therefore, the design day, rather than either of these two, will be used in design analyses

The distribution is much more variable for general aviation than for scheduled aircraft. A logical assumption for the design day is a 10-hour straight-line distribution with a 20-percent daily peak and is made on the basis of 1/300 of the annual movements. General aviation aircraft are largely operated under VFR and can be considered only for a VFR design day.

Typical distributions of traffic (hours) in order of magnitude are given in Table 7-I by percentage of design-day volume.

TABLE 7-I

TYPICAL TRAFFIC DISTRIBUTIONS BY HOURS

Hour	Low	Air Carrier Medium	High	General Aviation All Airports
1	16	10	8	20
2,3	12	9	8	16
4,5	10	8	7	11
6,7	8	7	6	7
8,9	6	6	5	4
10,11	4	5	4	2
12,13	2	4	3	0
14,15	0	3	3	0
16,17	0	2	3	0
18,19	0	1	2	0
20,21	0	0	2	0
22,23,24	0	0	2	0

The percentage of aircraft exiting at individual taxiways is another factor that can be considered in optimized design. This data establishes the taxi routes taken by the various aircraft.

The percentages in Table 7-II have been determined from analyses of actual performance, and are the basis for the runway performance analyses in Section IV.

TABLE 7-II

EXIT PERFORMANCE ON 9000-FOOT RUNWAY

Right-Angle Exits Every 1000 Feet

Aircraft Class	Percent Exiting* $\lambda_1 = 10$	$\lambda_1 = 30$	Exit Distance from Threshold
A	-	9.5	4000
	7.0	50.0	5000
	39.0	36.5	6000
	41.0	4.0	7000
	13.0	-	8000
B	-	13.0	3000
	11.5	53.0	4000
	38.5	34.0	5000
	39 5	-	6000
	10.5	-	7000
C	6.5	40.0	3000
	47.5	56.0	4000
	41.5	4.0	5000
	4.5	-	6000
D	5.0	17.5	2000
	32.5	59.0	3000
	46.5	23.5	4000
	16.0	-	5000
E	-	5.0	1000
	31.0	66.5	2000
	58.5	28.5	3000
	10 5	-	4000

* λ_1 is the arrival rate per hour.

Right-Angle Exits Every 1500 Feet

Aircraft Class	Percent Exiting* $\lambda_1 = 10$	Percent Exiting* $\lambda_1 = 30$	Exit Distance from Threshold
A	-	29.5	4500
	39.0	66.5	6000
	55.0	4.0	7500
	6.0	-	9000
B	-	13.0	3000
	28.5	76.0	4500
	61.5	11.0	6000
	10.0	-	7500
C	6.5	40.5	3000
	75.0	59.5	4500
	18.5	-	6000
D	-	4.5	1500
	38.5	72.5	3000
	57.0	23.0	4500
	4.5	-	6000
E	38.5	31.0	1500
	57.0	69.9	3000
	10.0	-	4500

Right-Angle Exits Every 3000 Feet

Aircraft Class	Percent Exiting* $\lambda_1 = 10$	Percent Exiting* $\lambda_1 = 30$	Exit Distance from Threshold
A	39.0	96.0	6000
	61.0	4.0	9000
B	-	13.0	3000
	90.0	87.0	6000
	10.0	-	9000
C	6.5	40.5	3000
	93.5	59.5	6000
D	38.5	77.0	3000
	61.5	23.0	6000
E	90 0	100.0	3000
	10.0	-	6000

* λ_1 is the arrival rate per hour.

Aircraft Class	Percent Exiting** $\lambda_1 = 10$	$\lambda_1 = 30$	Exit Distance from Threshold
High-Speed Exits at 2550, 3950, and 5950 Feet*			
A		41.0	3950
		59.0	5950
B		22.0	2550
		69.5	3950
		8.5	5950
C		69.5	2550
		30.5	3950
D		90.0	2550
		10 0	3950
E		100.0	2550
High-Speed Exits at 1700, 2550, 3950, and 5950 Feet*			
A		41.0	3950
		59.0	5950
B		22 0	2550
		69.5	3950
		8.5	5950
C		69.5	2550
		30.5	3950
D		25.0	1700
		66.0	2550
		9.0	3950
E		82.0	1700
		18.0	2550

* λ_1 is assumed to have no effect on runway performance.

** λ_1 is the arrival rate per hour.

Taxi speeds are assumed that represent normal maximum speeds. These speeds were confirmed by field observations. It was found that the major factor in taxi speed is the length of the section of taxiway between turns or intersections. Typical observed speeds are shown in Appendix A. In the economic analysis that follows (paragraph C), the following figures were used.

Assumed taxi speeds for various straight distances:

Distance (feet)	Speed (fps)
0 to 1000	25
1000 to 1500	30
1500 to 2000	40
2000 to 2500	50
Over 2500	60

Maximum taxi speeds for various aircraft:

Aircraft Class	Speed (fps)
A	60
B	60
C	60
D	50
E	40

User benefits in these analyses are measured by estimated savings in aircraft operating costs. The values used were developed from data recently reported to the Civil Aeronautics Board (CAB) by the various airlines. These data can be properly weighted by types of aircraft for the airport population at a specific time to determine aircraft operating cost per minute for an airport (Tables 7-III and 7-IV).

TABLE 7-III

DETERMINATION OF AIRCRAFT OPERATING COSTS PER UNIT TIME

Aircraft Type	Class	Projected	Cost per Minute
Boeing 707, Douglas DC-8	A	1975	$15.00
Medium Jet	B	1975	10 00
Electra	B	1962	7.00
Four-Engine Piston	B	1962	6.00
Viscount	B	1962	4.00
Average	B	1962	6.00
Average	B	1975	7.00
Fairchild F-27	C	1962	3.00
Twin-Engine Piston, Executive	D	1962	1.00
Single Engine	E	1962	0.25

TABLE 7-IV

DETERMINATION OF OPERATING COSTS FOR PROJECTED POPULATION

Projected	Airport	Aircraft Class	Cost	Per-cent	Total per Minute
1975	Intercontinental	X*	Assume $20.00	10	$2.00
		A	15 00	40	6.00
		B	7.00	20	1 40
		C	3.00	10	0 30
		D	1.00	10	0.10
		E	0.25	10	0.03
			Total	100	$9.83
1962	Intercontinental	A	$15.00	40	$6.00
		B	6.00	30	1.80
		C	3.00	10	0.30
		D	1.00	10	0.10
		E	0.25	10	0.03
			Total	100	$8.23
1962	Continental	A	$15.00	20	$3.00
		B	6.00	30	1 80
		C	3 00	20	0.60
		D	1.00	10	0 10
		E	0.25	20	0.05
			Total	100	$5.55
1962	Trunk	B	$ 6.00	20	$1 20
		C	3.00	30	0.90
		D	1.00	20	0.20
		E	0.25	30	0.08
			Total	100	$2.38
1962	Local	C	$ 3.00	30	$0.90
		D	1.00	20	0.20
		E	0.25	50	0.13
			Total	100	$1 23
1975	Local	C	$ 3.00	10	$0.30
		D	1 00	30	0 30
		E	0.25	60	0.15
			Total	100	$0 75

* Encompasses all supersonic aircraft.

Construction, amortization, and maintenance costs can be estimated to a degree of accuracy consistent with the purpose of the study. In this report, however, only the basic cost factors have been considered. Construction costs are estimated at cost per square yard for pavement, lighting, etc., and grading, drainage, etc. The figures used are:

	Cost in Dollars per Square Yard
Pavement, lighting, etc.	
Intercontinental	8.00
Continental	7.00
Trunk	6.00
Local	5.00
Grading, drainage, etc.	
All airports	2.00

Annual cost of construction, including amortization of capital expenditure, is assumed to be 1/15 of the total cost of construction.

Annual cost of maintenance, including repairs and replacement, is assumed to be another 1/15 of the total cost (or equal to the annual cost of construction).

Total annual cost is, therefore, 2/15 of the total cost of construction.

B EXAMPLES OF ECONOMIC ANALYSIS

The method of evaluating various portions of an airport design by economic analysis is shown in the examples to follow. The procedure can be extended to an entire airport wherever the input data are available.

Comparative evaluations can be made of various layouts for airport taxiway routes. This method of analysis is equally applicable to studies of proposed improvements of existing airports. These airports have been grouped together into several composite layouts.

Taxiing times are largely determined by the distance traveled. The benefits derived from improved taxiway routes are almost directly proportional to the volume of traffic. Therefore, the "break-even volume" for benefit-cost ratio = 1 can be determined for any layouts involving new taxiways. Typical examples of such layouts follow

1. <u>TAXIWAY CONSTRUCTION AT COMPOSITE CONTINENTAL AIRPORT</u>

With a triangular intersecting runway system (Figure 7-4), new taxiways to the take-off holding apron are studied to determine the break-even volume, with savings in taxi time for take-off only. Average taxi speeds, taxiway and runway crossing delays, composite population, wind rose runway utilization, and construction costs are assumed.

A summary of results gave the following.

Average saving for each take-off aircraft·	45 seconds
User benefit or estimated savings per operation for 1962 continental population	$\$5.55 \times \frac{45}{60}$ = $4.16 per take-off aircraft (or $2.08 per operation)
Cost of construction	$2,156,000
Annual cost of construction	144,000
Annual cost of maintenance	144,000
Total annual cost:	288,000

Benefit cost ratio per 100,000 operations = 2.08 × 100,000 = 0.72

Break-even volume for benefit-cost ratio = 1 = $\frac{288,000}{2.08}$ = 139,000 operations per year

Additional taxiways to improve landing operations can be evaluated separately or combined with this study.

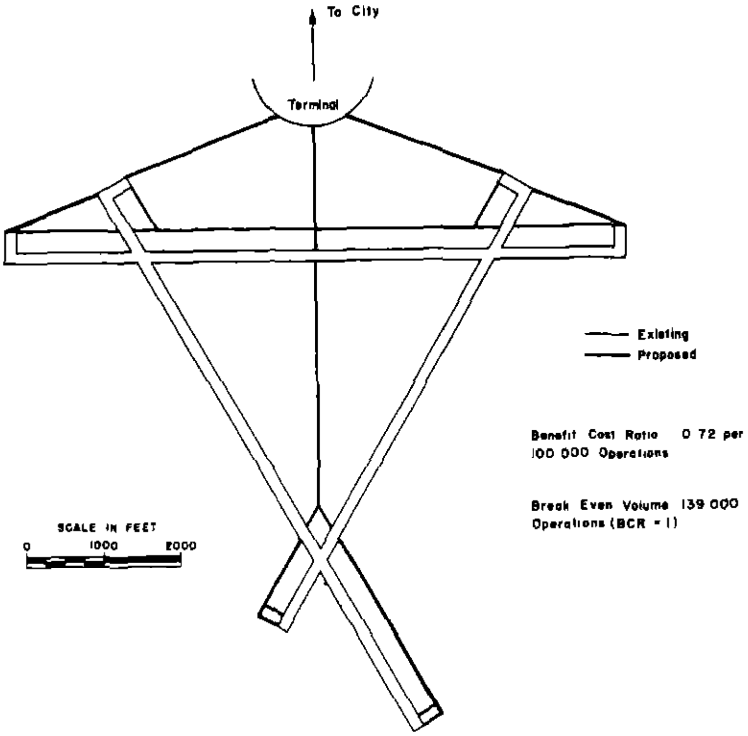


FIGURE 7-4 COMPOSITE CONTINENTAL AIRPORT WITH ADDITIONAL TAXIWAYS

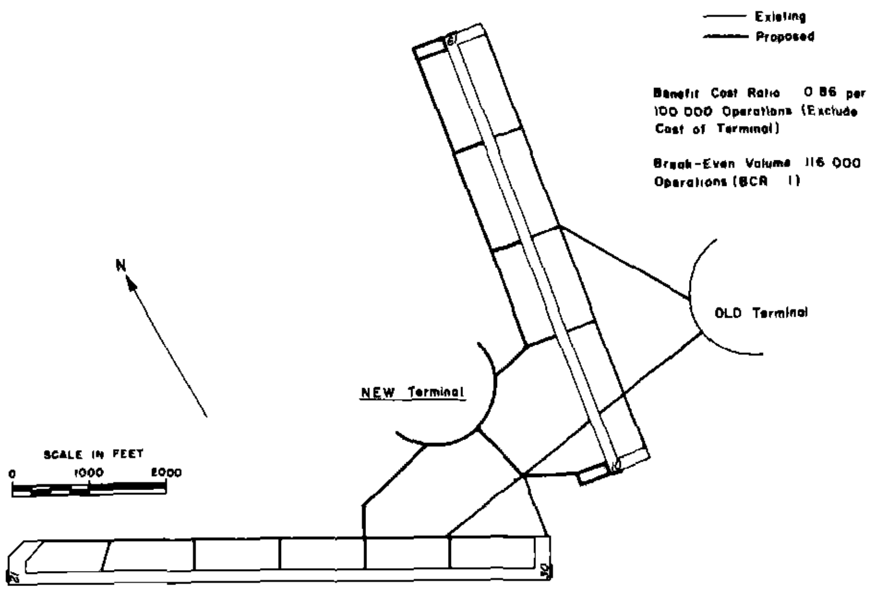


FIGURE 7-5 TERMINAL RELOCATION OF COMPOSITE AIRPORT TRUNK

2. TERMINAL RELOCATION AT COMPOSITE TRUNK AIRPORT

A further example demonstrates how the introduction of additional factors affects the economic evaluation of terminal relocation at a composite trunk airport (Figure 7-5). The object is to determine if it is economical to relocate the airport terminal and to consider the break-even volume required with taxi time savings only. The development of the new terminal area is excluded. Thus, only the taxiway development and operational savings in aircraft taxiing are considered.

Average taxi speeds, taxiway and runway crossing delays, composite population, and construction costs are assumed. The following percent utilization of runways is also assumed.

Runway Preference	Landing	Take-off	Utilization (percent)
Combination A	12	10	35
Combination B	19	30	20
Combination C	30	10	15
Combination D	10	30	5
Single Runway	12	12	10
Single Runway	30	30	5
Single Runway	10	10	5
Single Runway	19	19	5

The aircraft populating exiting from the landing runway at various exits from the threshold is assumed.

Landing Runway	Exit Number (percent) 1	2	3	4	5	6
12-30, 30-12	0	10	30	30	20	10
19-01	0	20	30	40	10	-
01-19	0	10	40	40	10	-

Table 7-V is a sample calculation of taxi time and taxi time saved for landing runway 12-30.

TABLE 7-V

TAXI TIME SAVED FOR LANDING RUNWAY 12-30

Exit	Old Taxi Time (feet)	Old Taxi Time (fps)	Old Taxi Time (seconds)	New Taxi Time (feet)	New Taxi Time (fps)	New Taxi Time (seconds)
2	500	25	20	-	-	20
	3000	60	50	2000	40	50
	300 (turn)	30	10	300	25	12
	2000	40	50	300 (turn)	30	10
	Runway Delay		25	-	-	-
	400	25	16	-	-	-
	Taxiway Delay		5	-	-	-
	1300	40	32	1300	40	32
			208			124
	Saving 84 seconds (10 percent)					
3	-3000	60	-50	-2000	40	-50
	+2000	40	+50	+1000	25	+40
						-10
			208			114
	Saving 94 seconds (30 percent)					
4	-2000	40	-50	-1000	25	-40
	+1000	25	+40	Taxiway Delay		+ 5
			-10			-35
			198			69
	Saving 129 seconds (30 percent)					
5	Recalculate		20	-	-	20
	2000	40	50	1200	25	48
	Runway		25	-	-	-
	400	25	16	-	-	-
	Taxiway		5	-	-	5
	1300	40	32	800	25	32
			148			105
	Saving 43 seconds (20 percent)					
6	End		20	-	-	20
	800	25	32	-	-	32
	Taxiway		5	-	-	5
	1000	25	40	-	-	-
	Runway		25	-	-	-
	400	25	16	-	-	-
	Taxiway		5	-	-	-
	1300	40	32	-	-	32
			175			89
	Saving 86 seconds (10 percent)					

On the basis of those assumptions, the total saving in taxi time for aircraft landing on runway 12-30 can be determined.

Exit	Time (seconds)	Time (percent)	Saving (seconds)
2	84	10	8 4
3	94	30	28 2
4	129	30	38.7
5	43	20	8.6
6 (end)	86	10	8 6
			92.5
			(use 92)

Table 7-VI gives the total savings in taxi time for all aircraft operations at the airport for all runways.

TABLE 7-VI

TOTAL SAVINGS IN TAXI TIME*

Runway Preference	Utilization (percent)	Savings in Seconds: Landing Aircraft	Total	Aircraft Take-off	Total
Combination A	35	92	32.2	-16	-5.6
Combination B	20	10	2 0	65	13.0
Combination C	15	83	12 5	-16	-2.4
Combination D	5	10	0.5	65	3.3
Single Runway 12-30	10	92	9 2	82	8.2
Single Runway 30-12	5	83	4 1	65	3.3
Single Runway 10-19	5	10	0.5	-16	-0.8
Single Runway 19-10	5	10	0.5	1	-

* Landing aircraft 61.5 seconds. Take-off aircraft 19.0 seconds.

The user benefit or estimated operating saving can be determined on the basis of the criteria mentioned. The average saving per operation is.

$$\frac{61.5 + 19.0}{2} \times \frac{\$2.38}{60} = \$1.60$$

A summary of the costs of the new taxiways, holding aprons, etc., exclusive of the terminal area development, is as follows.

Cost of construction of parallel taxiway for runway 10-19	$685,000
Cost of construction of connecting taxiways for runway 10-19 to new terminal area	$492,000
Cost of construction of connecting taxiway for runway 12-30 to new terminal area	$210,000
Total cost of construction, excluding new terminal area (building, gate positions, ramp)	$1,387,000
Annual cost of construction	$93,000
Annual cost of maintenance	$93,000
Total annual cost	$186,000

On the basis of this and the taxi time savings, the break-even volume is

$$\frac{186,000}{1.60} = 116,000 \text{ total, operations per year.}$$

3. STAGING OF TAXIWAY CONSTRUCTION AT LOCAL AIRPORT

The benefit-cost analysis can be used to determine at what stage in airport development additional construction is warranted by increased traffic. As an example, a local airport with a single-runway configuration (Figure 7-6) might

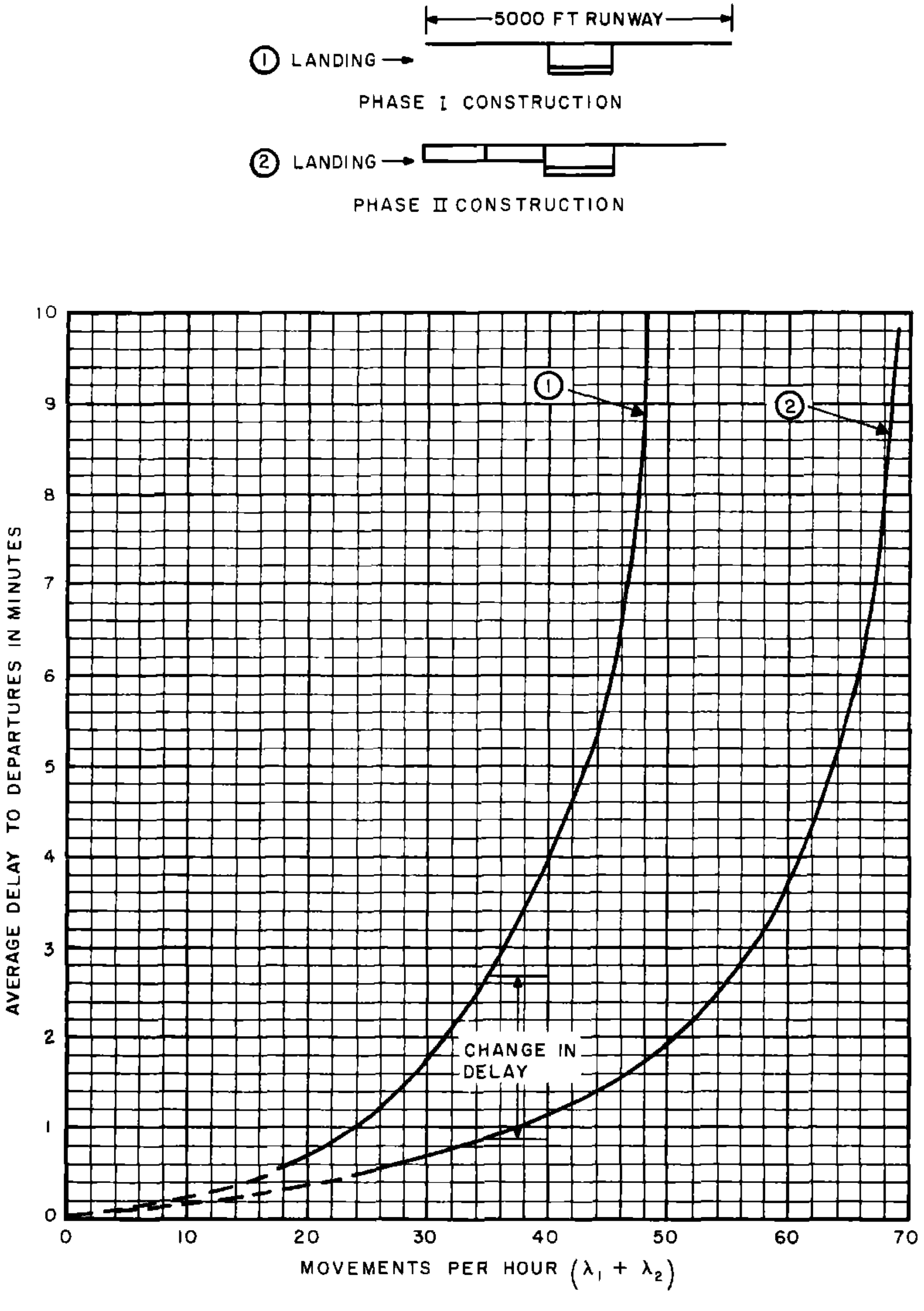


FIGURE 7-6 CAPACITY OF SINGLE RUNWAY WITH LIMITED TAXIWAY

be constructed with a 5000 by 100 foot runway, and apron area with connecting taxiways direct to the runway. The initial phase of construction would cost about $1,006,000 (or a total annual cost of $134,000). Phase I would be adequate for a practical operating rate of 45 movements per hour.

It would be most economical to begin the second phase of airport construction at some point below the rate of 45 movements per hour and 6-minute average delay. Phase II might include the addition of a parallel taxiway from the downwind end of the runway to the apron.

Phase II construction costs would be.

Cost of construction	$131,000
Annual cost of construction	$9,000
Annual cost of maintenance	$9,000
Total annual cost	$18,000

To determine the break-even volume for Phase II construction, consider the completed runway-taxiway system. It can be seen that taxi time, excluding delays, will not be appreciably different after construction The change in delay versus the movements per hour is determined from the curves in Figure 7-6.

The previously discussed factors of time distribution of traffic, typical population, percentage of aircraft exiting at various turn-offs, and cost data are all considered. The following is a tabulation of the difference in delay for the two layouts and the time saved after completion of Phase II construction.

Design day assumed to be 100 movements per day.

Hour	Per-cent	Movements	Saving in Delay per Departure (sec)	Saving per Day (sec)
Busy	20	20	16	160
2-3	16	16	8	128
4-5	11	11	2	22
6-7	7	7	N.C.	
			Total	310

$$\frac{310 \times 300}{60} = 1550 \text{ minutes per year}$$

Design day assumed to be 150 movements per day.

Hour	Per-cent	Movements	Saving in Delay per Departure (sec)	Saving per Day (sec)
Busy	20	30	66	990
2-3	16	24	30	720
4-5	11	16.5	10	165
6-7	7	10 5	2	21
8-9	4	6	N C	
10-11	2	3	--	
			Total	1896

$$\frac{1896 \times 300}{60} = 9480 \text{ minutes per year}$$

Design day assumed to be 175 movements per day.

Hour	Per-cent	Movements	Saving in Delay per Departure (sec)	Saving per Day (sec)
Busy	20	35	107	1890
2-3	16	28	52	1456
4-5	11	19.2	14	279
6-7	7	12.2	4	49
8-9	4	7	N.C	
10-11	2	3.5	--	
			Total	3674

$$\frac{3674 \times 300}{60} = 18{,}370 \text{ minutes per year.}$$

Savings in dollars to break even would equal \$18,000--the total annual cost of the improvement. The cost per minute for this population is \$1.23. Minutes saved per year to break even equal $\frac{18,000}{\$1.23} = 14,625$ minutes. Figure 7-7 indicates the intersection of the total annual cost with the curve drawn through various points determined by the annual benefit representing the time saving to the aircraft operators.

Break-even traffic in Figure 7-7 is thus 166 movements per design day, which represents the total annual operations of 49,800 movements for the one direction of operation and a savings per year of 14,625 minutes. The total airport operations can be found by including the effect of the other operating directions. If the prevailing-wind direction represents 70 percent of total wind, then the break-even volume for this construction would be 71,000 operations per year for the airport. The break-even volume for the same improvement at the other end of the runway, which accommodates only 30 percent of the operation, would be 166,000 operations per year for the airport.

4. PARALLEL RUNWAY AT CONTINENTAL AIRPORT

Another example of the benefit-cost analysis is the determination of the stage in airport development at which additional runways are warranted by the increasing traffic. Consider, for example, a continental airport with a single 9000 by 150 foot runway, parallel taxiway, exit taxiways every 1000 feet, and terminal apron area. The initial stage of construction would be adequate for a practical operating rate of 46 movements per hour (curve 4 in Figure 4-6).

To increase the capacity of the airport beyond 46 movements per hour at the anticipated 6-minute average delay to all departing aircraft, it would be necessary to begin the second phase of construction. Phase II construction might include the addition of a parallel runway system

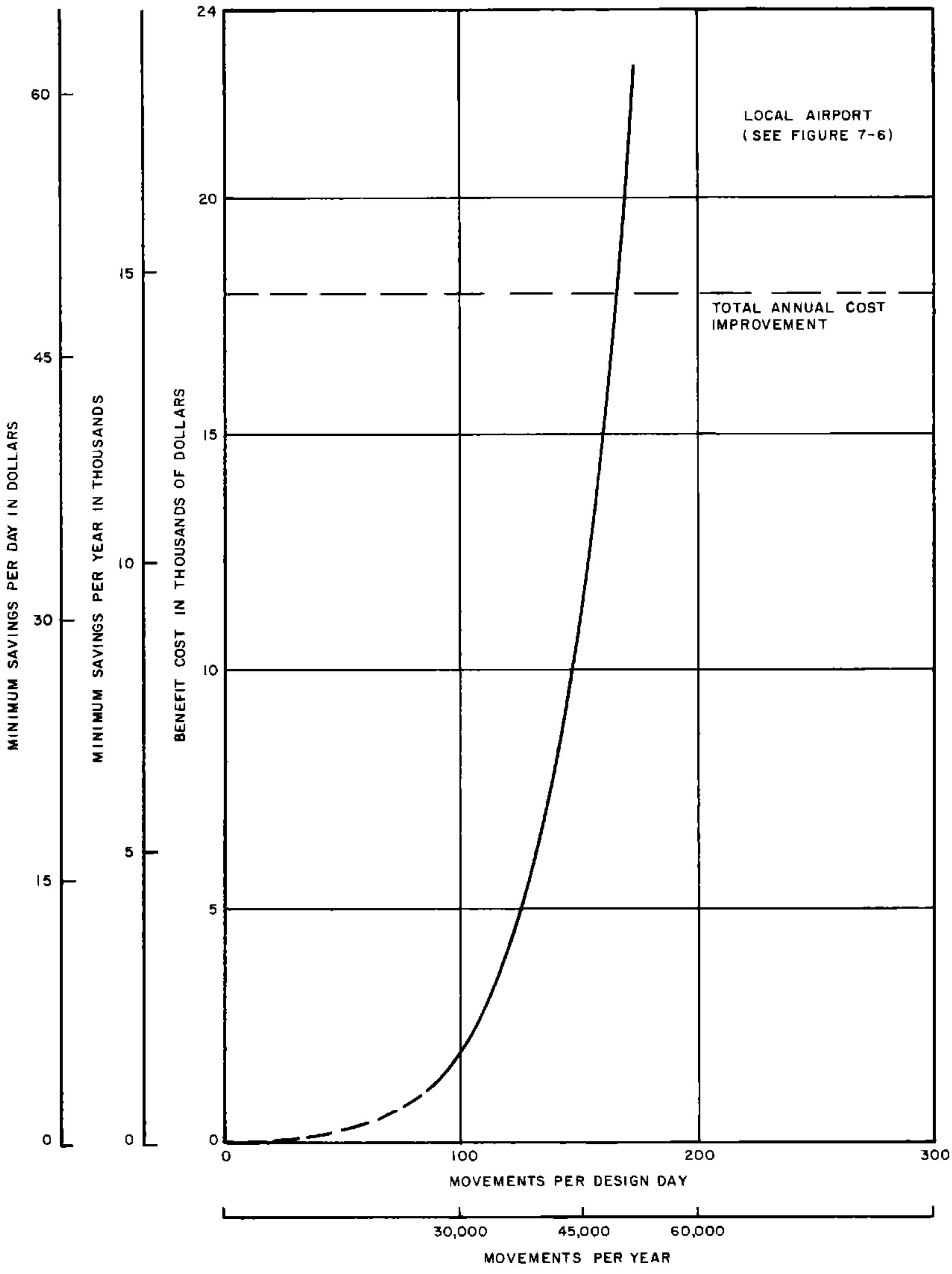


FIGURE 7-7 BENEFIT-COST ANALYSIS OF PARALLEL TAXIWAY

complete with parallel taxiways, exit turn-offs every 1000 feet, and connecting taxiways to the terminal.

Phase II construction costs would be

Cost of construction $4,145,000
Annual cost of construction $276,000
Annual cost of maintenance $276,000
Total annual cost $552,000

In determining the break-even volume for Phase II construction, consider the completed runway-taxiway system. The change in delay for each number of movements per hour can be determined as the difference between curves 1 and 4 in Figure 4-6.

The previously discussed factors of time distribution of traffic, typical population, percent of aircraft exiting at various turn-offs, and cost data are all considered The following is a tabulation of the difference in delay for the two layouts and the time saved after completion of Phase II construction.

Design day assumed to be 200 movements per day.

Hour	Movements Air Carrier	Movements General Aviation	Total	Saving in Departure Delay (sec)	Saving per Day (sec)
	70% = 140	30% = 60			
Busy	14	12	26	60	780
2	12.6	9.6	22.2	44	985
4	11 2	6.6	17.8	32	570
6	9.8	4.2	14.0	24	336
8	8.4	2.4	10 8	18	194
10	7	1.2	8.2	12	98
12	5.6	0	5.6	8	45
14	4.2	-	-	6	25
16	2.8	-	-	4	14
18	1.4	-	-	2	3
20	0	-	-	-	-
				Total	3050

$$\frac{3050 \times 300}{60} = 15{,}300 \text{ minutes per year.}$$

By similar computation, other design days will show savings in time as follows

Movements per Day (assumed)	Minutes per Year
300	48,600
350	90,800
375	134,600

Savings in dollars to break even would equal $552,000--the total annual cost of the improvement. The cost per minute for this population is $5.55. Minutes saved per year to break even would equal 552,000 ÷ 5.55 = 99,600 minutes. This relation is shown in Figure 7-8, which indicates the intersection of the curve for the total annual cost with the curve drawn through various points determined by the annual benefit representing the time saving to the aircraft operators. The break-even volume in Figure 7-8 is thus 352 movements per design day. This represents a total annual operation of 105,600 movements.

This method of analysis is equally applicable to intercontinental and other airports. With the appropriate change in values and/or the addition of other factors under consideration, the analysis can be extended over a wide range.

C. CONCLUSION

Comparative evaluations of various configurations become the basis for trial configurations to be evaluated at specific sites. In general, the runway with entrances and exits can be optimized for specific traffic conditions. The time spent by the aircraft between the runway and the point of origin or destination on the airport should be held to a minimum. This can be achieved by designing a runway pattern and a system of taxiways that conforms with the flow patterns necessitated by these points of origin and destination. This

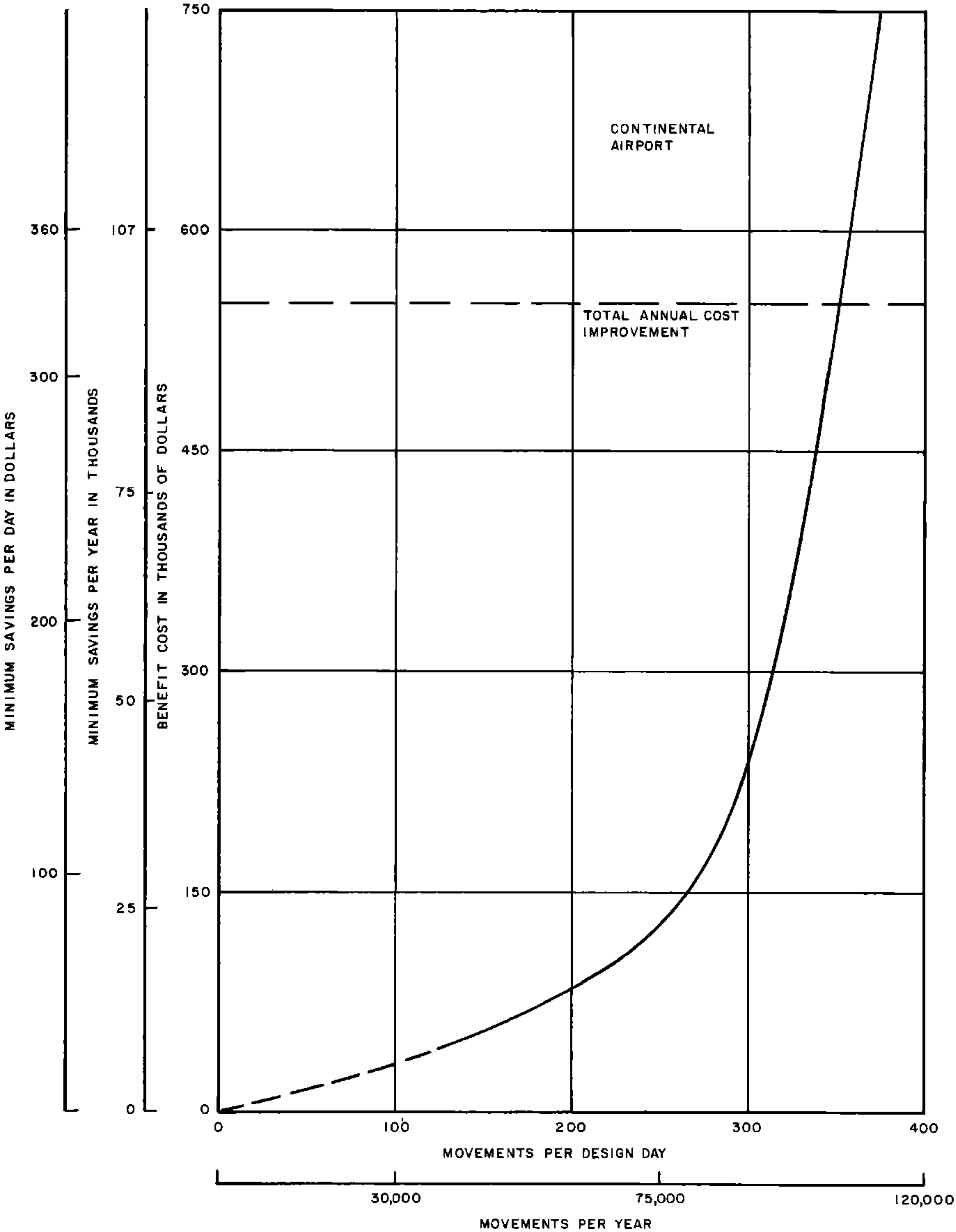


FIGURE 7-8 BENEFIT-COST ANALYSIS OF PARALLEL RUNWAY

goal is difficult to achieve because the number of possible destinations desired increases in proportion to the volume of traffic handled. To some extent, this can be influenced by the choice of terminal building design--that is, whether frontal, fingers, unit terminals, or mobile lounges.

For a free-flowing taxiway system, the travel should be direct without sharp turns, with a minimum of intersections with other taxiways, and without crossing an active runway. In evaluating an airport layout, the time spent in taxiing can be the most important element in unproductive aircraft time.

Except for those airports where the cross-wind limitation must be considered, single or parallel runways offer the greatest advantages and highest efficiency.

Multiple runways increase capacity and reduce delays only when two or more are used concurrently. If a number of runways are used separately that only permit operations into the wind, then the greater the number of runways, the more delays that will be occasioned by changing traffic from one runway to another This can also become a serious matter if the runways are unequal in length or differ in approach conditions since pilots may request a runway other than the one currently in use. This will tend to cause delays. Situations may arise at an airport limited to a single runway when certain traffic, particularly light aircraft, are restricted from operating. This is not as insubstantial as it might appear for high wind conditions will frequently ground such operations though a number of runway directions are available.

A second runway direction can show a direct benefit in proportion to the additional traffic that could not use the primary runway. A secondary benefit may result from improved reliability of service Since this additional traffic will normally be only 1 to 5 percent of the total, and since the airport is often closed for a much greater time for other weather conditions, this secondary benefit should not exceed the direct benefit.

To compute the benefit of a cross runway, the percentage of additional traffic by types should be determined and doubled to include indirect benefits. The benefit will be a percentage of the income of the airport. It may also be estimated that if such a runway increases utilization by 5 percent, this will justify a capital expenditure of 10 percent of the total airport development cost. If such a cross runway can make savings in operational times, these savings should also be included as benefits. It must be remembered that the simultaneous use of two runways is undesirable without a traffic control tower.

VIII. DETERMINING PRACTICAL OPERATING RATES FOR SPECIFIC AIRPORT DESIGNS

The methods developed to predict practical operating rates for various runway configurations use mathematical models (formulas) that predict the delays that will result for various operating rates. The practical operating rate is then selected, based upon the amount of delay acceptable for the operation. This study has verified that these models can be applied to analyze numerous problems of the airport surface.

The mathematical theory and the derivation of the mathematical models is developed in Section XII of this report. Here we will describe briefly the operational basis for each model, show examples of how the models can be used, and give guidance for their application.

Some care must be used in selecting the proper formula to be applied to a particular type of conflict since the formulas make very specific assumptions about

1 The time pattern with which requests for use of the facility occur

2 The lengths of time consumed by the users of the facility.

3 The order or priority assigned to the user

Moreover, a great deal of care must be used in determining the identity and the numerical values of the elements to be substituted into the formulas.

A. SELECTION OF MODEL TO FIT AIRPORT PROBLEM

Three models are suggested for the analysis of airport problems known by the names of

1 First-Come, First-Served Model (FIM)

2. Pre-emptive Spaced Arrivals Model (SAM)
3. Pre-emptive Poisson Arrivals Model (PAM)

Figures 8-1 and 8-2 illustrate specific problems for which traffic flow ratings can be found with a corresponding probable delay using the mathematical models.

B. OPERATIONAL BASIS FOR AND COMPUTATIONS WITH FIRST-COME, FIRST-SERVED QUEUING MODEL

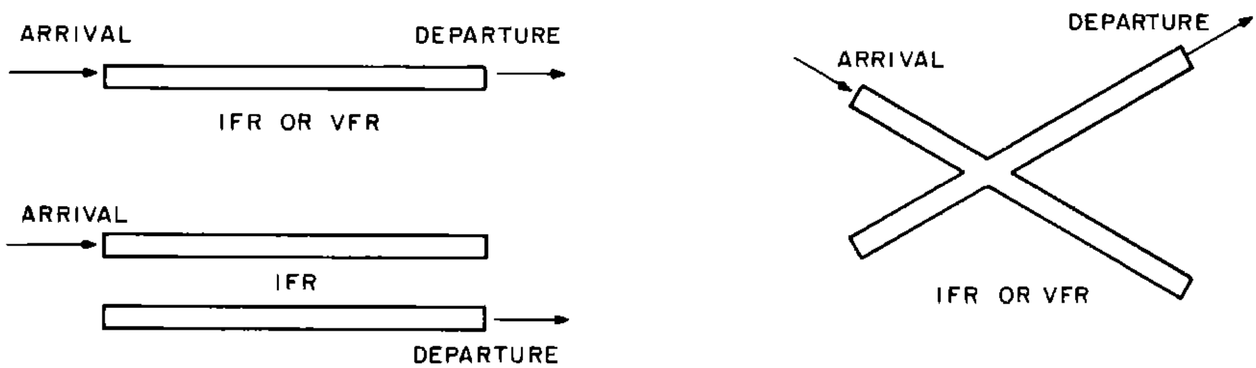
The first-come, first-served queuing model is one of the simpler (yet frequently applicable) queuing models. Its simplicity, however, does not in any way reduce the effectiveness of this tool It is useful and entirely suitable in several airport problems Such problems include the delay to arrivals or departures on parallel runways where the runways are operated independently, one for arrivals and the other for departures. It also applies to two classes of customers (taxiing aircraft passing through a common intersection), each with an independent movement rate and service characteristic using a common facility

This model is based upon a simple concept. The customers (arriving or departing aircraft or both) that require the use of a facility (runway or taxiway) are given the use of the facility in the order in which they request such use All customers (whether arriving or departing aircraft) are treated with equal priority. In normal operation, runways handling landings only or departures only will use this concept of operations.

If the case under study is arrivals only, the formula and elements are

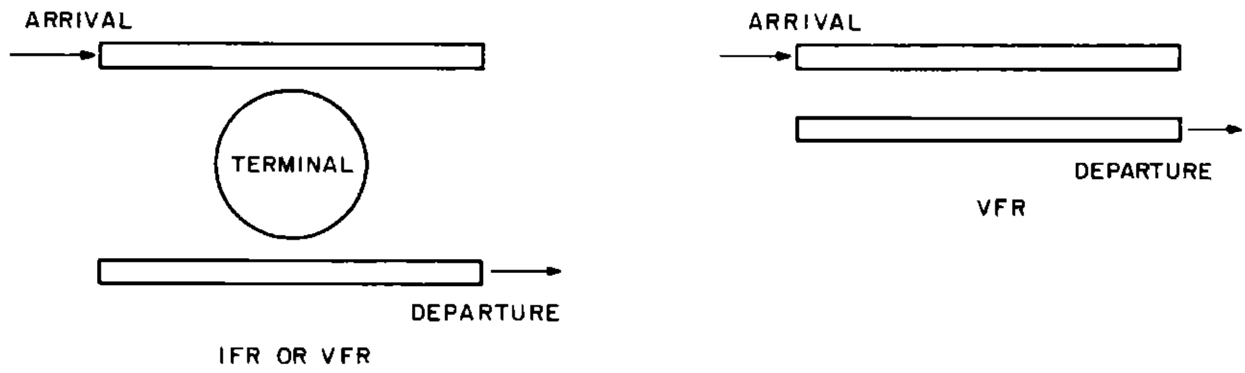
$$w_1 = \frac{\lambda_1 S_{12}}{2(1 - \lambda_1 S_{11})}$$

RUNWAY OPERATIONS WHERE ARRIVALS AND DEPARTURES ARE INTERDEPENDENT



A PRE-EMPTIVE SPACED ARRIVALS MODEL (SAM)

RUNWAY OPERATIONS WHERE ARRIVALS AND DEPARTURES ARE INDEPENDENT



B FIRST-COME, FIRST-SERVED MODEL (FIM)

FIGURE 8-1 APPLICATION OF MATHEMATICAL MODELS TO ARRIVALS AND DEPARTURES

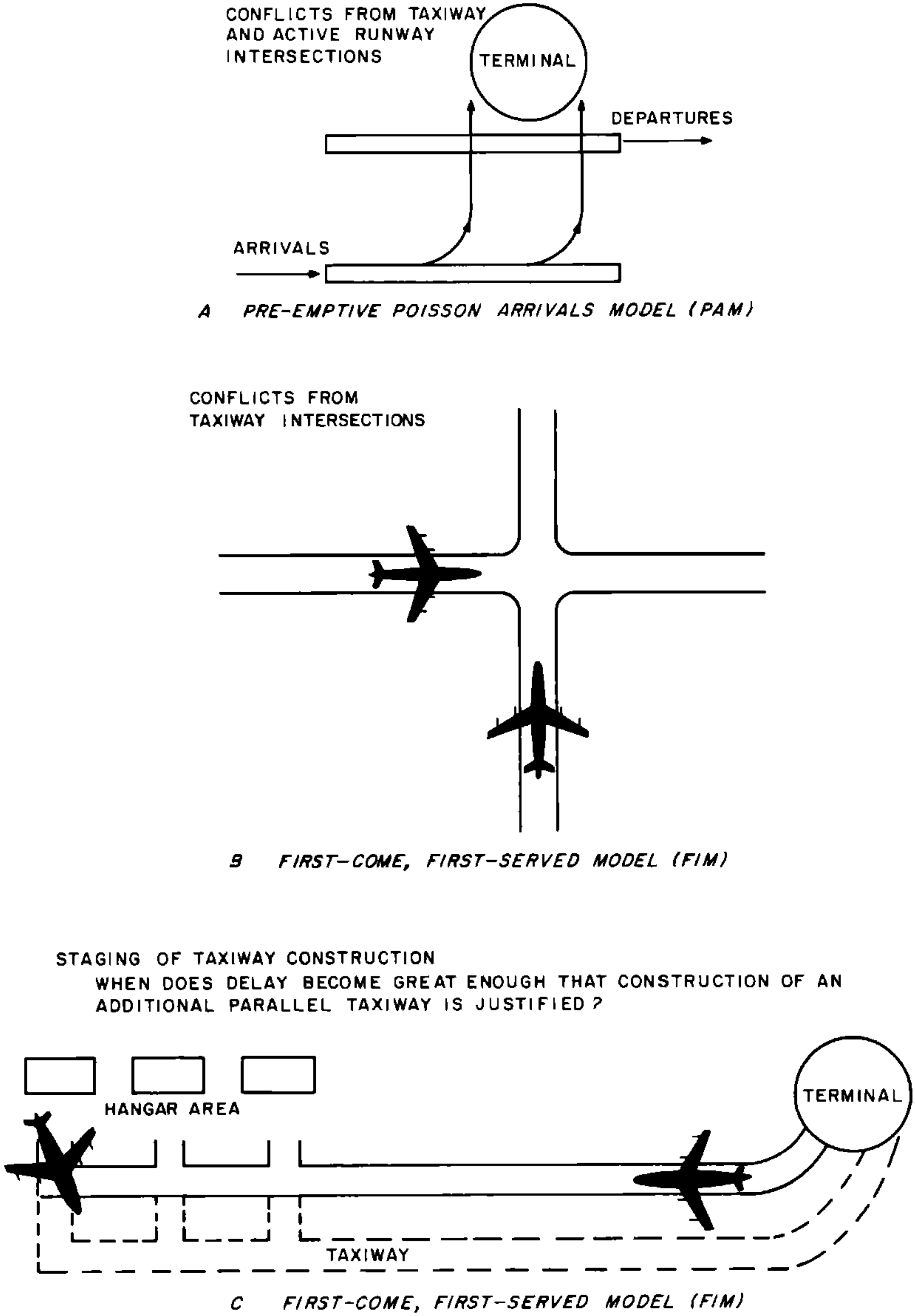


FIGURE 8-2 APPLICATION OF MATHEMATICAL MODELS TO TAXIWAY PROBLEMS

where

w_1 = steady-state average delay for arrivals,

λ_1 = arrival rate,

S_{11} = first moment (mean) of arrival service time,

S_{12} = second moment (mean square) of arrival service time

$$S_{12} = \frac{S_1^2 + S_2^2 + S_3^2 \qquad S_n^2}{n}$$

where S is the individual service time

Appendix A contains data for selection of values of these elements.

When a runway is used for departures only, the corresponding formula is

$$w_2 = \frac{\lambda_2 S_{22}}{2(1 - \lambda_2 S_{21})}$$

where

w_2 = steady-state average delay for departures,

λ_2 = departure rate,

S_{21} = first moment (mean) of departure service time,

S_{22} = second moment (mean square) of departure service time

Sometimes, arrivals and departures share a common path--such as a taxiway or taxiway crossing--with equal priority

In these cases, the first-come, first-served model is quite applicable with the following formula

$$w = \frac{\lambda_1 S_{12} + \lambda_2 S_{22}}{2(1 - \lambda_1 S_{11} - \lambda_2 S_{21})}$$

The symbols would be defined as above, except that w is the steady-state average delay for all users (departures or arrivals).

C OPERATIONAL BASIS FOR AND COMPUTATIONS WITH PRE-EMPTIVE SPACED ARRIVALS MODEL

This model is a complex analytical expression that can be used for forecasting operating rates and delays for mixed runway operations. In mixed runway operations, the normal procedure is that landings are given priority and departures await a gap in the landing sequence before they can be cleared for take-off. In the most serious cases of congestion, the controllers will vary this discipline and ask pilots to delay to permit them to handle departures, however, this is the exceptional case Thus, the pre-emptive spaced arrivals model is based on the general case where landing aircraft will be given priority over departing aircraft.

As in the simpler first-come, first-served queuing model already discussed, the choice of the elements used in applying the pre-emptive renewal model are very important. These elements are selected to include the various spacing factors that exist in day-to-day operations and which can be adjusted for VFR or IFR conditions The factors are shown schematically in Figure 8-3. Those elements of the model that must be determined and supplied as inputs are then described as.

λ_1 = arrival rate of landings (high-priority units) per hour,

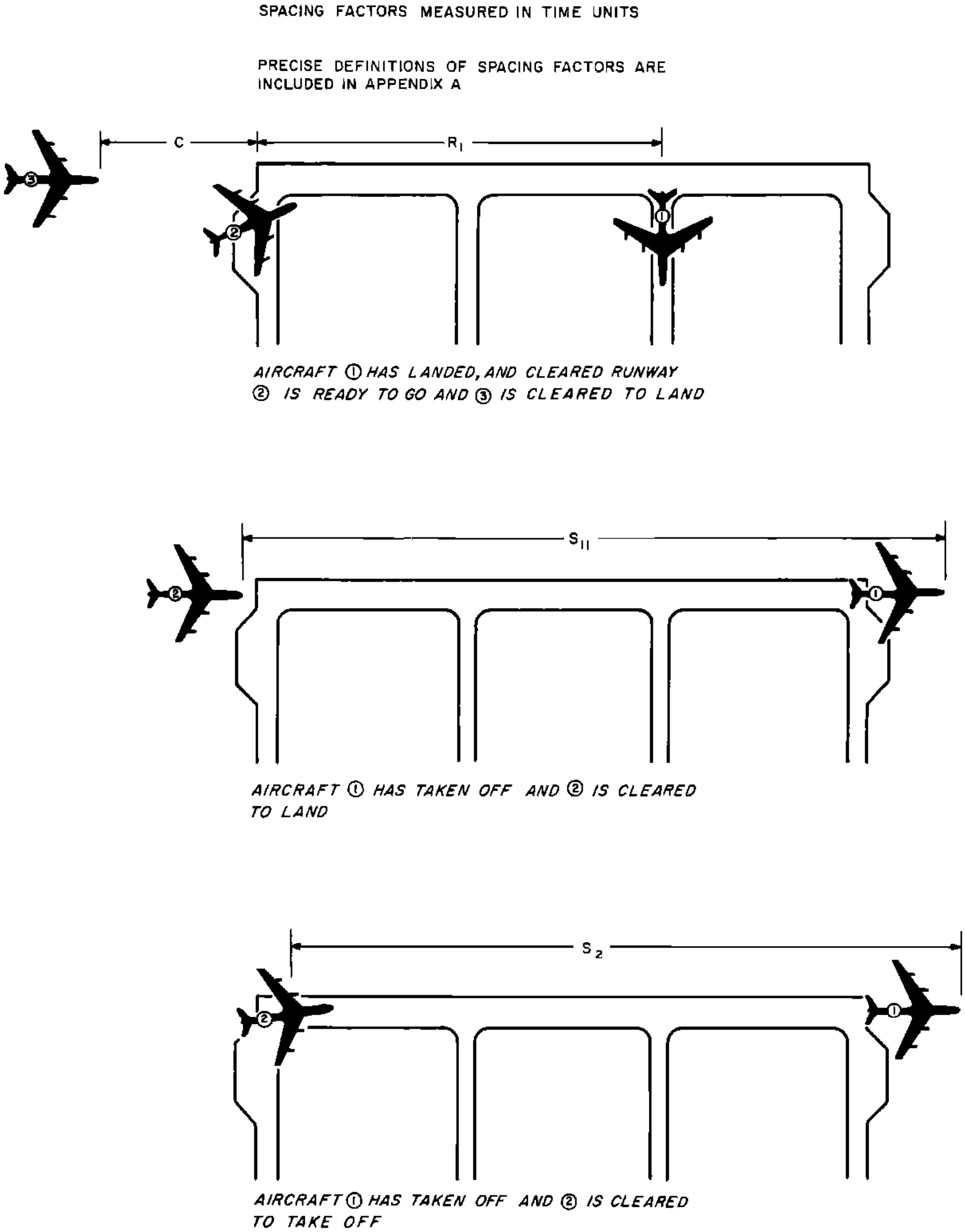


FIGURE 8-3 SPACING FACTORS FOR PRE-EMPTIVE SPACED ARRIVALS MODEL (SAM)

r_1 = mean runway time of landings (high-priority units) in seconds,

c_1 = mean spacing factor for landings from wave-off to runway threshold in seconds,

λ_2 = arrival rate of take-offs (low-priority units) per hour,

S_{11} = mean service time for take-off when next user is landing (in seconds),

S_2 = mean service time for take-off when next user is take-off (in seconds),

$\sigma(R + C)$ = standard deviation of $R + C = \sigma(OT - OT)$ minimum.

Appendix A contains data for selection of values for these elements

The output of the spaced arrivals model is the steady-state average delay for the departures (low-priority users).

It is practical to solve airport problems by use of the pre-emptive spaced arrivals model with manual computation. However, when many answers are needed for developing curves (see Section IV), a computer is most efficient and a relatively simple computer program is required. However, where manual computation can be used, there are definite steps involved in the computation. The total delay W can be computed from these steps as follows

1. $a_1 = \frac{3600}{\lambda_1}$

2. $g_1 = a_1 - r_1 - c_1$

3. $a_2 = g_1^2 + a_1^2 + \sigma^2 (R + C)$

4 $g = \frac{1}{g_1}$

5. $S_1 = S_{11} - c_1$

6 $T_1 = e^{gS_1}$

7. $T_2 = e^{-gS_2}$

8 $T_3 = 1 - T_2$

We can now compute, from the previous eight steps, the following

1. $W_{21} = a_1 \left\{T_1 - 1\right\} - S_1$

2 $J_1 = a_1 T_1 T_3$

3 $\frac{J_2}{2} = J_1 W_{21} + T_1 \left\{\frac{a_2 T_3}{2} - a_1 S_2 T_2\right\}$

4 $T_4 = \frac{\lambda_2 J_1}{3600}$ = utilization for departures

5. $W_o = \frac{J_2}{2} \frac{\lambda_2}{3600\ (1 - T_4)}$

6 $W_1 = \frac{a_2}{2a_1} - g_1$

7. $W = W_{21} + W_1 + W_o$

D. OPERATIONAL BASIS FOR AND COMPUTATIONS WITH PRE-EMPTIVE POISSON ARRIVALS MODEL

The pre-emptive Poisson arrivals model, like the spaced arrivals model, assumes that one group of aircraft being handled has a priority. However, the basis for the mathematical treatment is somewhat different than the spaced arrivals model since it assumes arrivals have a Poisson input distribution This pre-emptive model applies, for example, to taxiing aircraft waiting to cross an active runway In those cases where an aircraft lands on a runway away from the terminal and then must cross a runway used for departures to get to the terminal, the pre-emptive queuing model can be used to forecast the delay encountered by the taxiing aircraft since, of course, the aircraft on the runway will have priority. The choice of elements for use in this model is also critical, and they are

λ_1 = arrival rate per hour for high-priority user,

λ_2 = arrival rate per hour for low-priority user,

S_{11} = mean service time (seconds) of high-priority user,

S_{12} = mean square service time of high-priority user

$$S_{12} = \frac{s_1^2 + s_2^2 \; . \; . \; . \; s_n^2}{n}$$

where

s = individual service times,

S_{21} = mean service time (seconds) of low-priority user

Appendix A contains data useful in selecting values for the elements of this model.

The step-by-step computation is as follows

1. $$T_1 = \frac{\lambda_1}{3600}$$

2. $$s_2(-\lambda_1) = e^{T_1 S_{21}}$$

$$s_2(-2\lambda_1) = e^{2T_1 S_{21}}$$

$$s_2'(-\lambda_1) = -S_{21}\, e^{T_1 S_{21}}$$

3. $$T_2 = \frac{\lambda_2}{3600}$$

4. $$\rho_1 = T_1 S_{11}$$

5. $$T_3 = 1 - \rho_1$$

6. $a_1 = T_1 T_3$

7. $T_4 = s_2(-\lambda_1) - 1$

8. $J_1 = \frac{T_4}{a_1}$

9. $J_2 = \frac{2}{a_1^2}\left[s_2(-2\lambda_1) - 1\right] + T_4\left[\frac{S_{12}}{T_3^3} - \frac{2(1+\rho_1)}{a_1^2}\right] + \frac{2}{a_1} s_2'(-\lambda_1)$

10. $W_1 = \text{delay in seconds} = J_1 - S_{21} + \frac{T_2 J_2}{2(1 - T_2 J_1)} + \frac{T_1 S_{12}}{2T_3^2}$

To obtain a more precise answer in step 2, the following should be used

$$s_2(-\lambda_1) = \frac{1}{n} \sum_{i=1}^{n} e^{(\lambda_1/3600)\, t_i}$$

$$s_2(-2\lambda_1) = \frac{1}{n} \sum_{i=1}^{n} e^{(2\lambda_1/3600)\, t_i}$$

$$s_2'(-\lambda_1) = -\frac{1}{n} \sum t_i\, e^{(\lambda_1/3600)\, t_i}$$

t_i are the sample low-priority service times (seconds) for $1 \leq i \leq n$

A brief discussion of average delay follows to assist in making practical interpretations of the results obtained with the models A more thorough treatment of delay and its variations can be found in Section X

E. INTERPRETATION OF DELAY AND OPERATIONAL RATES AT AIRPORTS

The mathematical formulas discussed, when used to evaluate airport configurations, produce an average delay for the selected operating rates It is important to realize the meaning of this average delay in order to make proper use of it

1. BUILD-UP OF DELAY WITH MOVEMENT RATE

In a typical analysis of runway operations, the average delay to an aircraft will increase if the number of landings and take-offs per hour are increased. This is illustrated in Figure 8-4 for a sample configuration. This shows the increase in steady-state average delay that occurs as operating rates are increased for a single runway having mixed operations. Because of this build-up in average delay when operational rates are increased, and other considerations of delay in the following text, the practical operating rate for this runway configuration was considered to be about 59 movements per hour, which is indicated with a resulting steady-state average delay of 6 minutes.

2. VARIATION OF AVERAGE DELAY

The average delay produced by the mathematical formula is what is known as the steady-state average delay and is to be interpreted as the average delay that would be expected after a lengthy period of operation at the selected operating rate. If you were to keep observing the cumulative average delay value for an operation at a given hourly rate as it progressed, for example, after every 25 operations, you would find that the average delay varies considerably though as the operation continues and the sample size becomes increasingly larger, the cumulative average delay will gradually settle down to the steady-state average. On another day's operation at the same rate, the variation of delay with increasing sample size may be considerably different from the previous day's record, but if the sample size were again large enough, the cumulative average delay would settle down to the same steady-state average as was obtained for the previous day.

This subject is treated more thoroughly in Section X and quantitative examples are presented.

ARRIVAL RATE (λ_1) = DEPARTURE RATE (λ_2)
VFR SPACING
TRUNK AIRPORT POPULATION

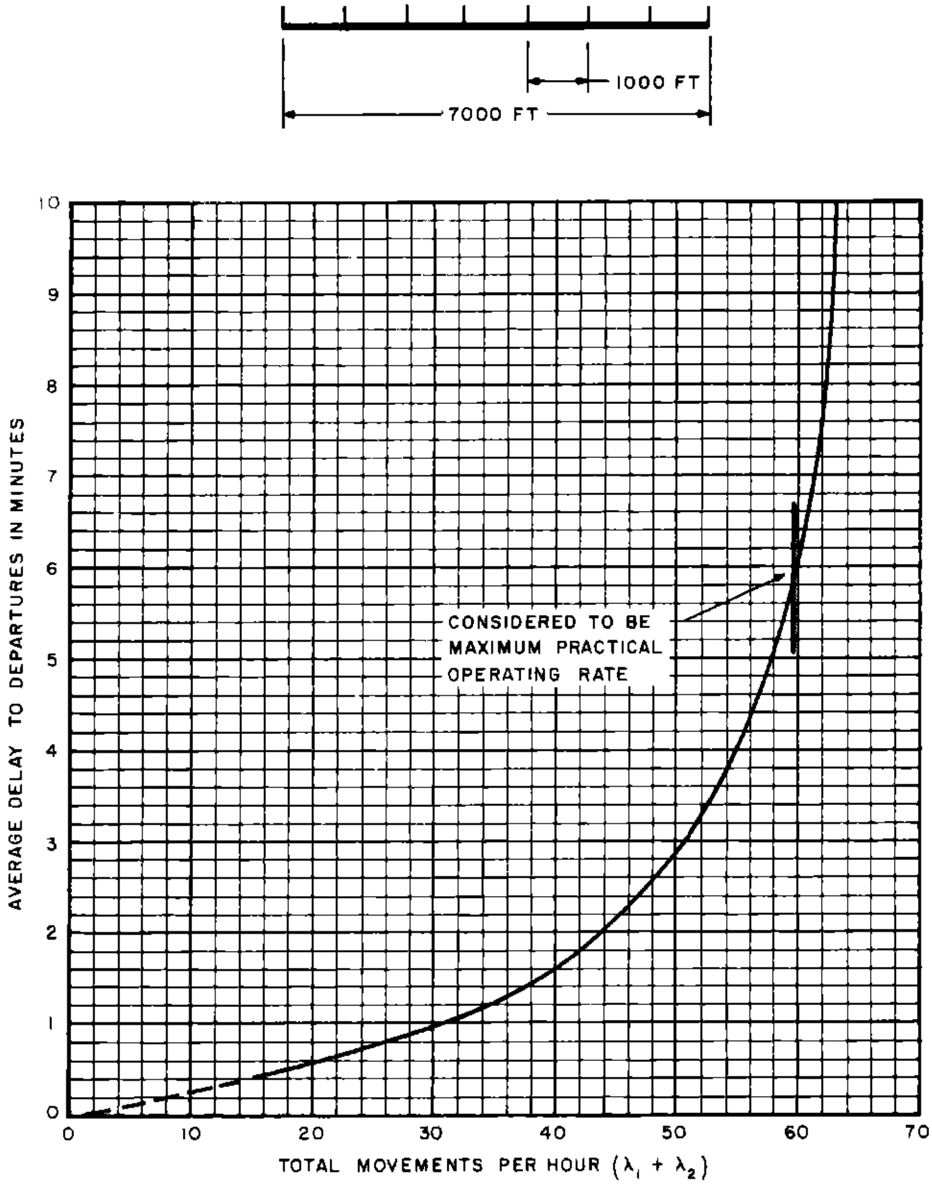


FIGURE 8-4 PRACTICAL CAPACITY OF SINGLE RUNWAY

Thus, in observing actual operations and determining the average delay, it is important to realize that one can observe considerable variation and still not exceed the practical operating rate conforming to the steady-state average delay This also indicates that if one selects too high a value for the steady-state average delay, undesirable operating conditions would frequently result

3. VARIATION OF INDIVIDUAL DELAY

The individual delay that each movement encounters, and that is totaled to develop the sample average delay, varies considerably in daily operations--even for the same operating rate. To understand this variation, several cases have been analyzed and three of these are shown in Figures 8-5, 8-6, and 8-7. Figures 8-5 and 8-6 apply to departure delay during VFR weather, Figure 8-7 shows arrival delay during IFR weather These three cases indicate that a tremendous variation is possible for operations at different times with the same aircraft populations and operating rates.

4. SELECTION OF STEADY-STATE AVERAGE DELAY VALUE

The value of steady-state delay to be selected for evaluation of an airport will depend upon realization of the information already indicated, as well as the following

The delay distribution acceptable at one airport, say a short-haul airport, may be different from that acceptable at another airport, such as a long-haul airport. The delay at a short-haul airport should generally be minimized to retain the time advantage that air travel develops A longer delay may be acceptable for longer trips. Similarly, the type of aircraft involved will influence the tolerable delay--for example, the new turbojet aircraft generally require an efficient operation because of fuel consumption This will become even more critical when supersonic transports come into operation.

AVERAGE DELAY = 2 8 MINUTES

MAXIMUM DELAY = 14 4 MINUTES

VFR PERIOD FROM 1420 TO 1730 HOURS
4 DECEMBER 1959 WITH 79 ARRIVALS
AND 77 DEPARTURES

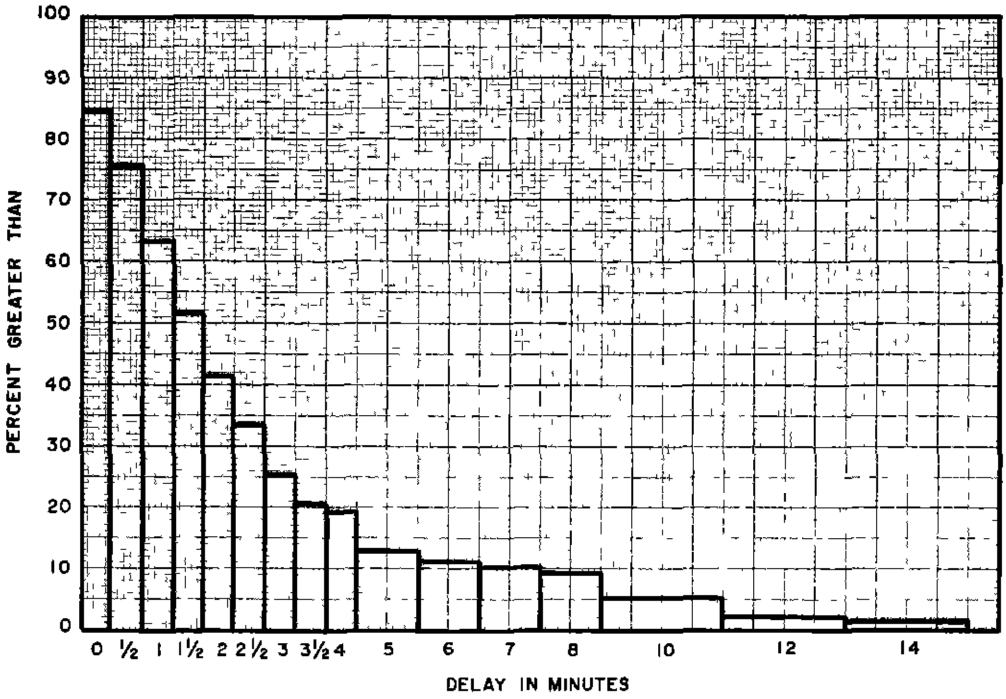


FIGURE 8-5 DISTRIBUTION OF DEPARTURE DELAY AT MIAMI AIRPORT

AVERAGE DELAY = 2 2 MINUTES

MAXIMUM DELAY = 10 MINUTES

VFR PERIOD FROM 1210 to 1610 HOURS
13 NOVEMBER 1959 WITH 81 ARRIVALS
AND 92 DEPARTURES

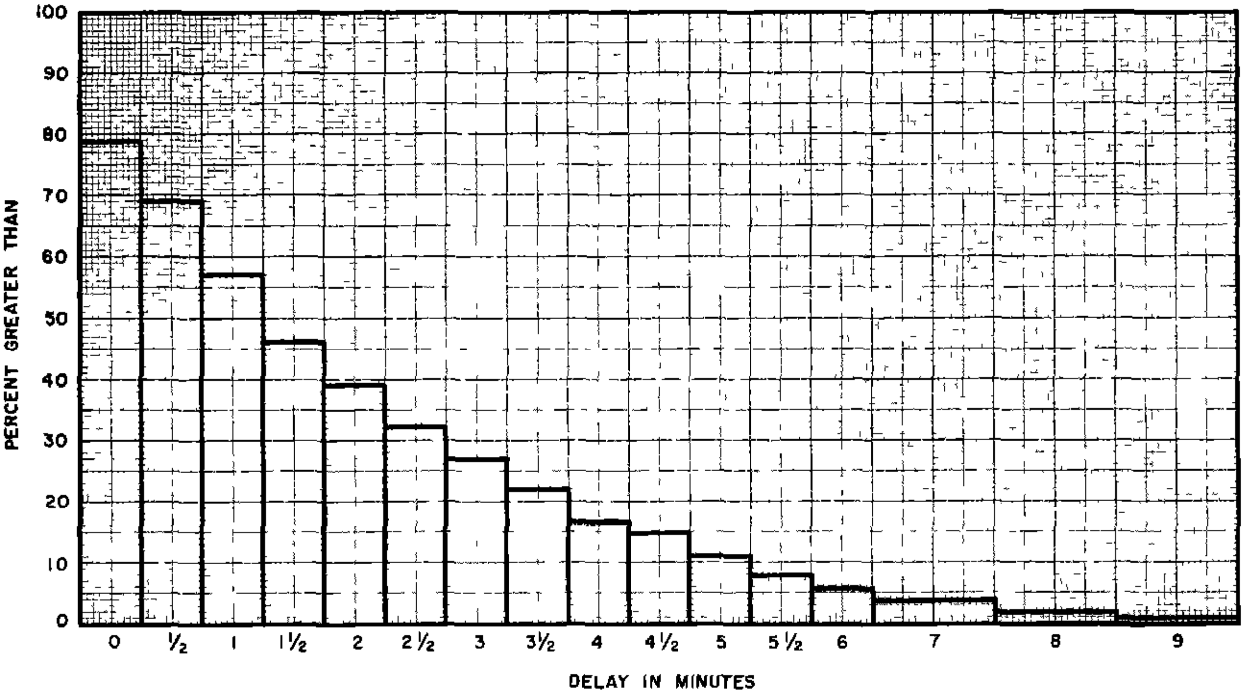


FIGURE 8-6 DISTRIBUTION OF DEPARTURE DELAY AT LA GUARDIA AIRPORT

AVERAGE DELAY = 15 5 MINUTES

MAXIMUM DELAY = 80 to 89 MINUTES

IFR WEATHER CONDITIONS WITH
279 ARRIVALS FOR 27 APRIL 1959

SOURCE
"CHICAGO AREA AIR TRAFFIC FLOW AND DELAY ANALYSIS," VOL II, COOK RESEARCH LABORATORIES, SEPTEMBER 1959

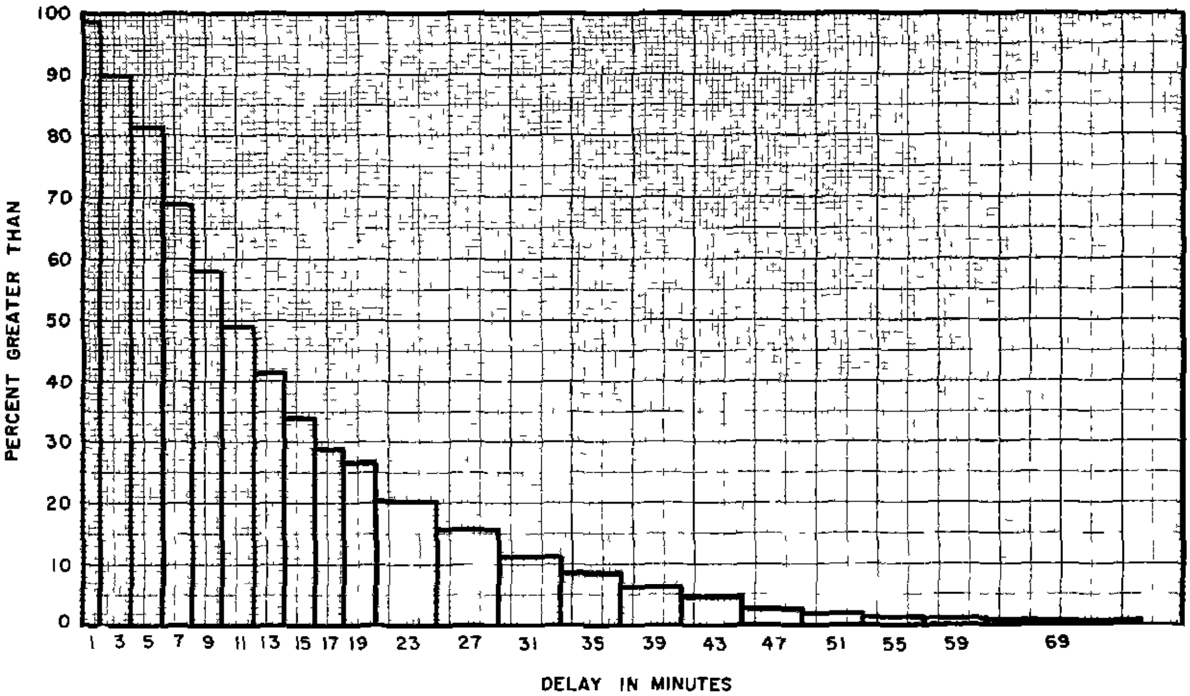


FIGURE 8-7 DISTRIBUTION OF ARRIVAL DELAY AT MIDWAY AIRPORT

A complete delay analysis must include analysis of the delay build-up as the operating rate increases to correspond to a typical distribution of daily operations by hours (see Figure 7-3) The build-up in delay may be such that an airport can be temporarily overloaded without exceeding acceptable delay criteria, particularly if the increase in movement rate to the overloaded hour is abrupt. On the other hand, when the practical operating rates are exceeded for any length of time, the delay builds up very rapidly and an intolerable delay situation can very easily develop This, of course, can be made still worse by equipment failures that temporarily reduce the airport operating capacity. Thus, the higher the airport utilization, the more dependable should be the equipment used for control procedures.

Delay that occurs either to landings or departures is very expensive to the aircraft operators. With an average delay of 6 minutes as shown in Figure 4-2 and using the continental population and operating at the rate of 46 movements per hour, the cost to the aircraft operators would be approximately $750 per hour. This cost does not include any consideration of the delay to any passenger that might be aboard the aircraft However, this operating cost does indicate the important economical considerations involved. These costs are discussed further in Section VII.

IX. RESULTS OBTAINED FROM OBSERVATIONS OF AIRPORT OPERATIONS

During the project, airports were visited to observe operations, discuss operating problems with local personnel, and gather data as needed. The airports visited were New York International, LaGuardia, Newark, Pittsburgh, Wichita, Midway, O'Hare, Cleveland, San Francisco, Oakland, Atlanta, Miami, Bristol, and Utica. Their layouts are shown in Figures 9-1, 9-2, 9-3, and 9-4.

A. TECHNIQUES USED TO GATHER DATA

The only usable data available from previous efforts was that contained in the FAA reports of terminal area studies.* Not all of the airport data were useful because of problems in airport procedures, or lack of operating situations with delay. We did obtain information on runway operations at Atlanta, Midway, Newark, and New York International airports, with corresponding delay information. Since more data were desired on operating rates with delay, particularly at higher capacities, it was necessary to obtain data while on the field trips to the various airports.

An efficient data-taking and recording system was therefore developed that may be of interest in similar studies.

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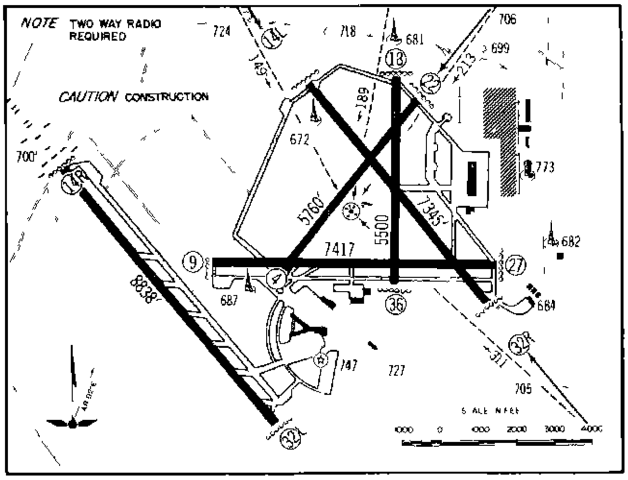
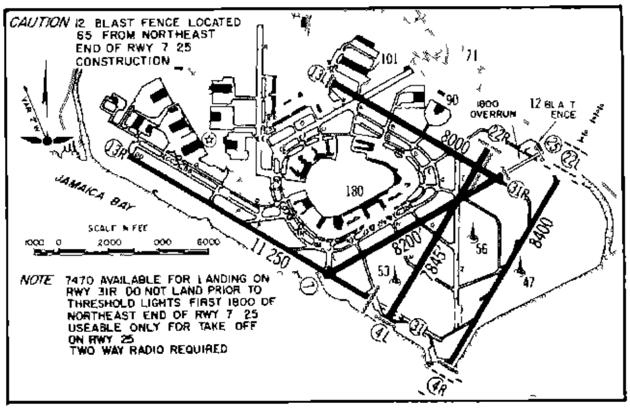
* "Terminal Area and Airport Surface Traffic, New York, Winter 1957-1958," Report No. 4851-1, Airborne Instruments Laboratory, November 1958, "Chicago Area Air Traffic Flow and Delay Analysis," Vol II, Cook Research Laboratories, September 1959, "Atlanta Area Air Traffic Analysis, Spring 1959," Franklin Institute and Philco Corp., October 1959.

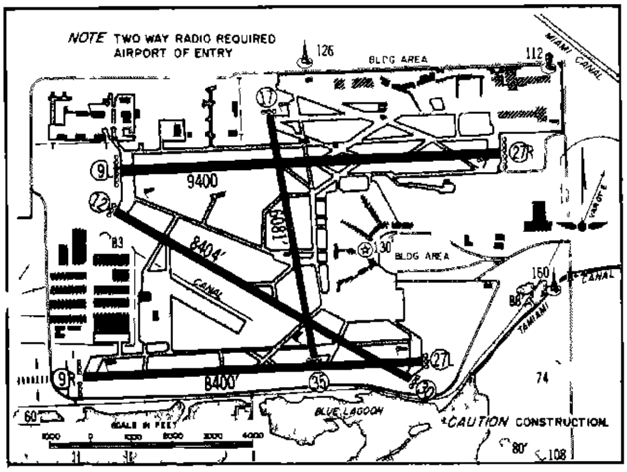
CHICAGO, ILLINOIS
CHICAGO OHARE INT L APT ORD
TERMINAL OMNI Rwys ML 18 22 32R



NEW YORK (IDLEWILD), N Y
NEW YORK INT L IDL
ILS/ADF Rwy 4R/L



MIAMI, FLORIDA
MIAMI INTERNATIONAL APT MIA (AFA)
GCA (ASR) approach



SAN FRANCISCO, CALIF
SAN FRANCISCO INT L APT SFO
GCA (ASR) app ch

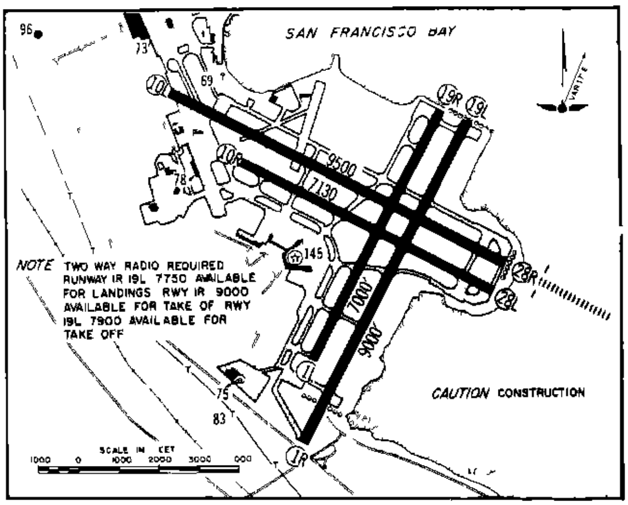
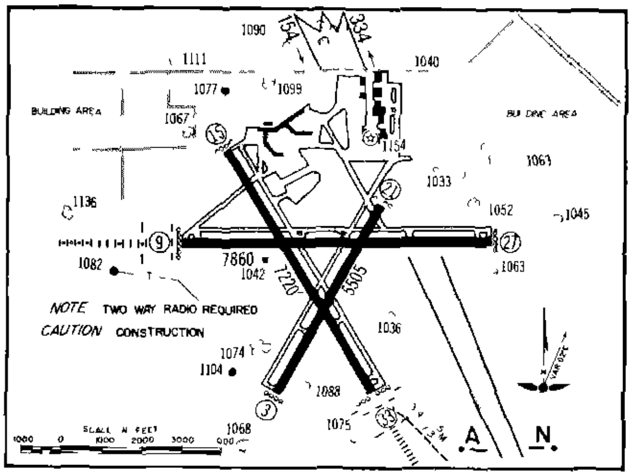
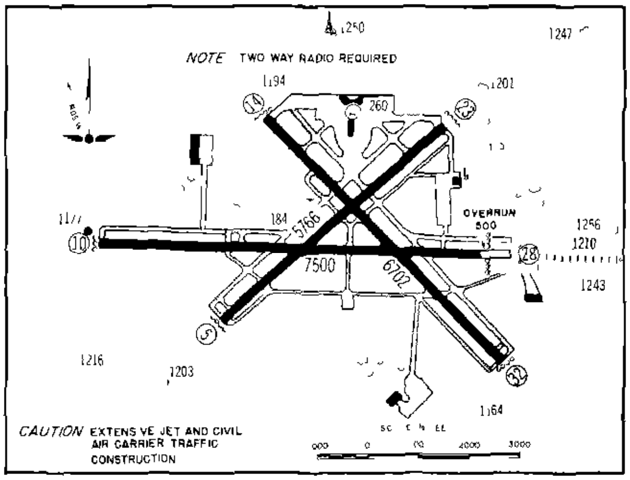


FIGURE 9-1 INTERCONTINENTAL AIRPORTS

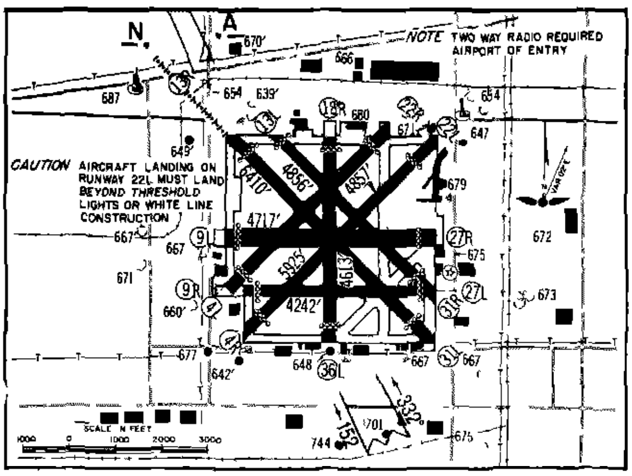
ATLANTA, GA
ATLANTA APT ATL
LF RANGE approach



PITTSBURGH, PA
GREATER PITTSBURGH APT PT
GCA (ASR) approach



CHICAGO, ILLINOIS
CHICAGO-MIDWAY APT MDW
LF RANGE approach



NEWARK, NEW JERSEY
NEWARK EWR
ILS Rwy 22

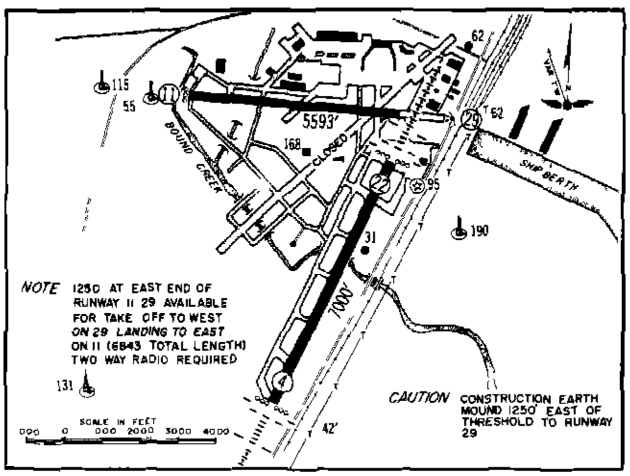
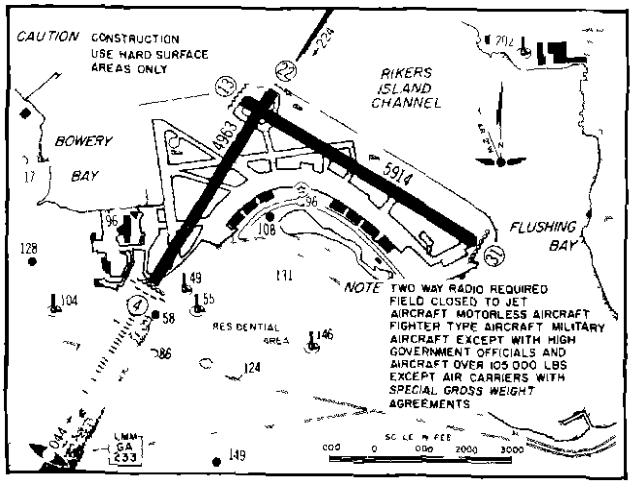
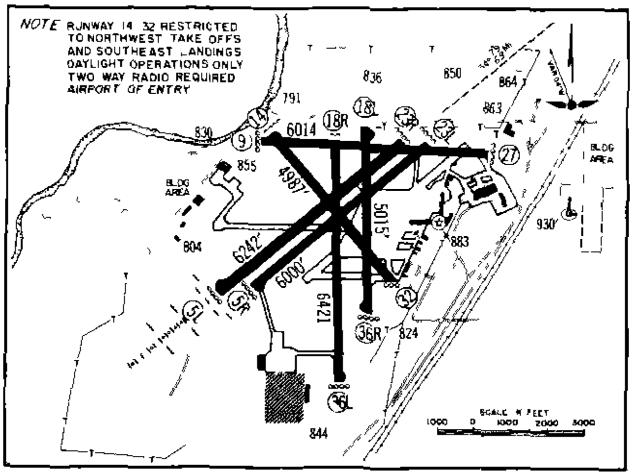


FIGURE 9-2 CONTINENTAL AIRPORTS

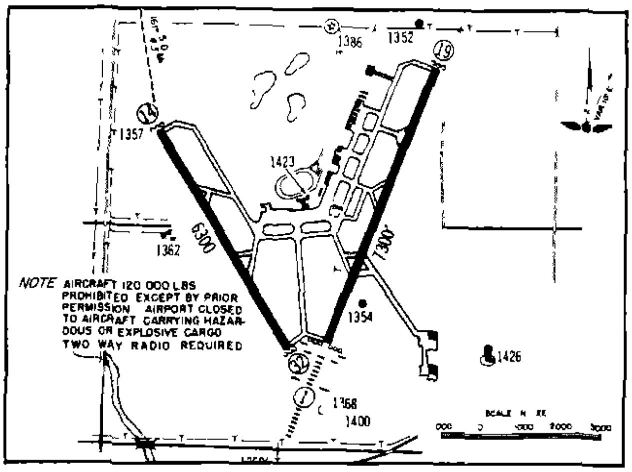
NEW YORK N Y
LA GUARDIA LGA
ILS/ADF Rwy 4



CLEVELAND, OHIO
CLEVELAND-HOPKINS APT CLE
ADF No 2



WICHITA, KANSAS
WICHITA APT ICT
OMNI RANGE approach



OAKLAND, CALIF
METRO OAKLAND INT L APT (NAS) OAK
LF RANGE pp ach

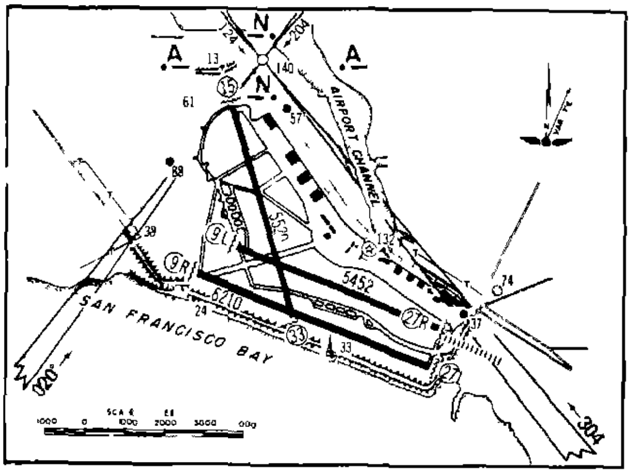
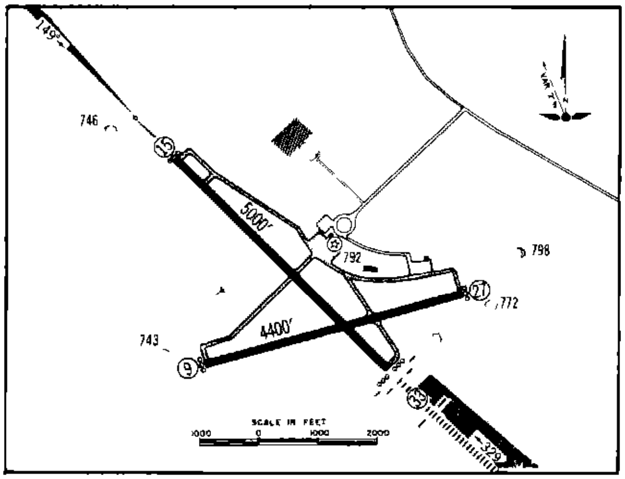


FIGURE 9-3 TRUNK AIRPORTS

UTICA, NEW YORK
ONEIDA CO UCA
ILS/ADF Rwy 33



BRISTOL (TRI CITY), TENN
TRI-CITY TRI
ILS Rwy 27

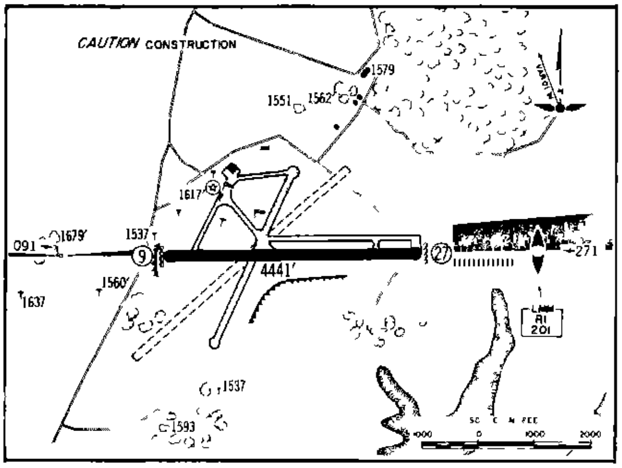


FIGURE 9-4 LOCAL AIRPORTS

1. RECORDING DATA IN CONTROL TOWERS

It was found that a most efficient and practical technique for gathering data in the control towers was to use two observers trained in control tower and airport operations. The observers are both equipped with microphones connected to a tape recorder. One observer also had earphones to hear the local control frequency. The two observers were located in the control tower or at a proper vantage point to observe airport operations, and a clock with a sweep second hand was placed in view of both observers. Observer 1, equipped with earphones to monitor the local control frequency, tape recorded the times of significant calls for ready to go, clear to take-off, and aircraft identification. He would likewise verbally record other significant movement times--for example, enter active runway. The second observer would record the precise times needed--such as over threshold, off runway, and clear boundary. Occasionally, the precise time is recorded as a reference that can be used to correlate the local control tape in later analyses. A second recorder was used to record the local control frequency for later analyses of the ready to go and clear to take-off times.

This technique can be expanded to gather more data by recording the ASDE and ASR radars at the same time. However, deriving all the data available in the recording is a rather difficult data reduction program.

2. PLOTTING OF OBSERVED DATA

Reducing the field data into useful inputs for the models is a rather lengthy task and requires techniques that adequately display the data to properly interpret it. The technique shown in Figure 9-5 gave an excellent display. Color was used to differentiate operations on the original plot. With these plots, the personnel reducing the data could

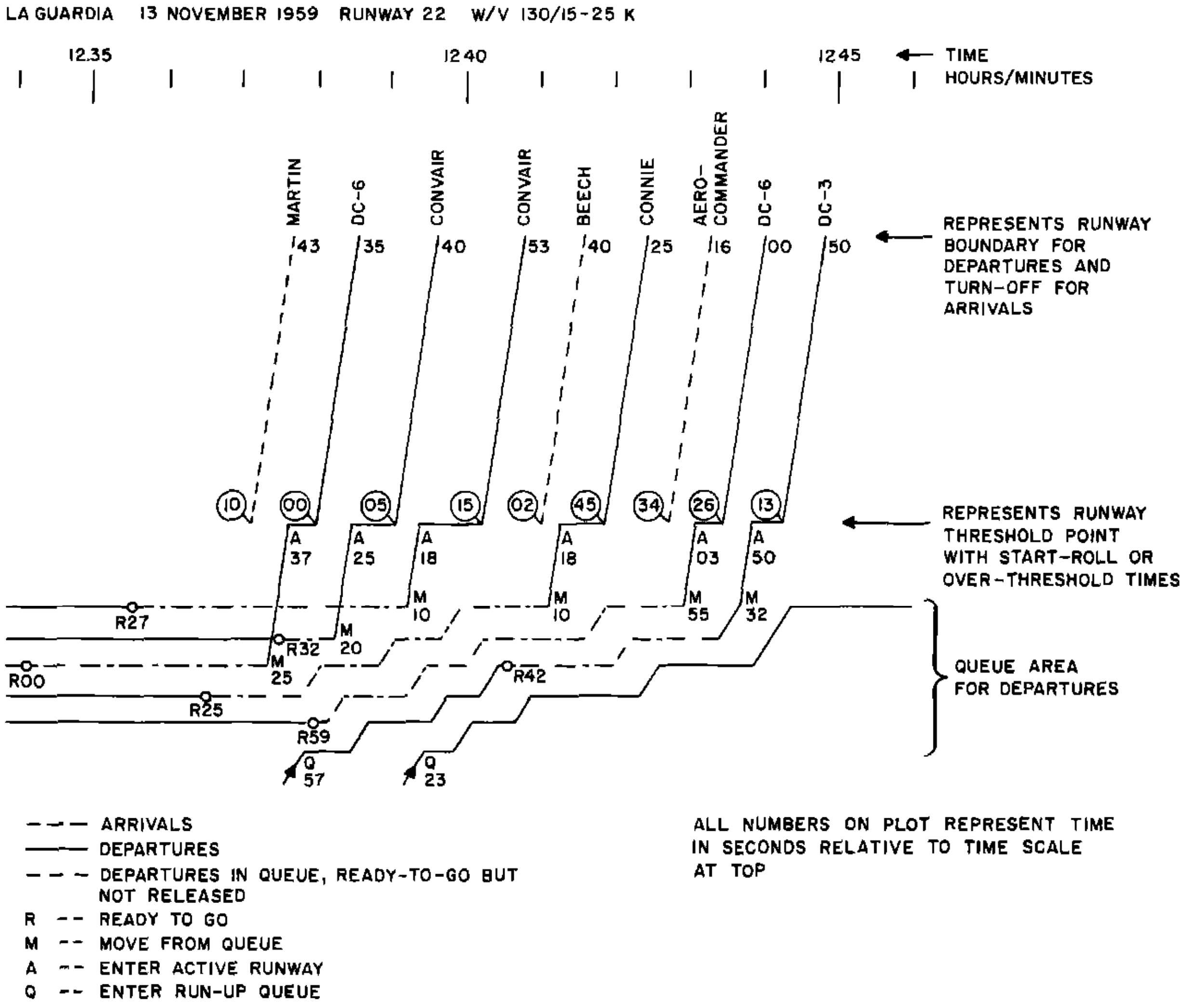


FIGURE 9-5 EXAMPLE OF RUNWAY OPERATION PLOT

graphically display all operations and thereby determine whether they had correctly interpreted the data on each aircraft. Those using the data could easily read off the values desired and could quickly see areas of major delay. In addition, it provided a permanent record that was found invaluable as work progressed, for we were constantly seeking new interpretations of the data not thought necessary upon earlier investigations.

B. DATA ANALYSIS

In reducing the data for use as inputs to the models, it was helpful to group the data for analysis as follows

1 An arrival followed by an arrival when an aircraft was ready to go and being delayed by the arrival

2. An arrival followed by a departure when the departure was ready to go before the arrival crossed the runway threshold.

3. A departure followed by a departure when the second departure was ready to go before the first aircraft started its take-off roll.

4. A departure followed by an arrival when a second departure was ready to go before the first departure started its take-off roll.

From this analysis it was possible to document the pressure factor that is evident at airports when the operational rate increases Our field observations had indicated to us, in a qualitative sense, that both pilots and controllers operated more efficiently at higher operating rates, since they seemed to sense the urgency and reacted to it. In fact, operations at the same airport--observed on different days and at different operating rates--showed a difference in runway use time An indication of this effect is shown in Figures 9-6 through 9-9 where each figure corresponds to one of the operational groupings tabulated previously.

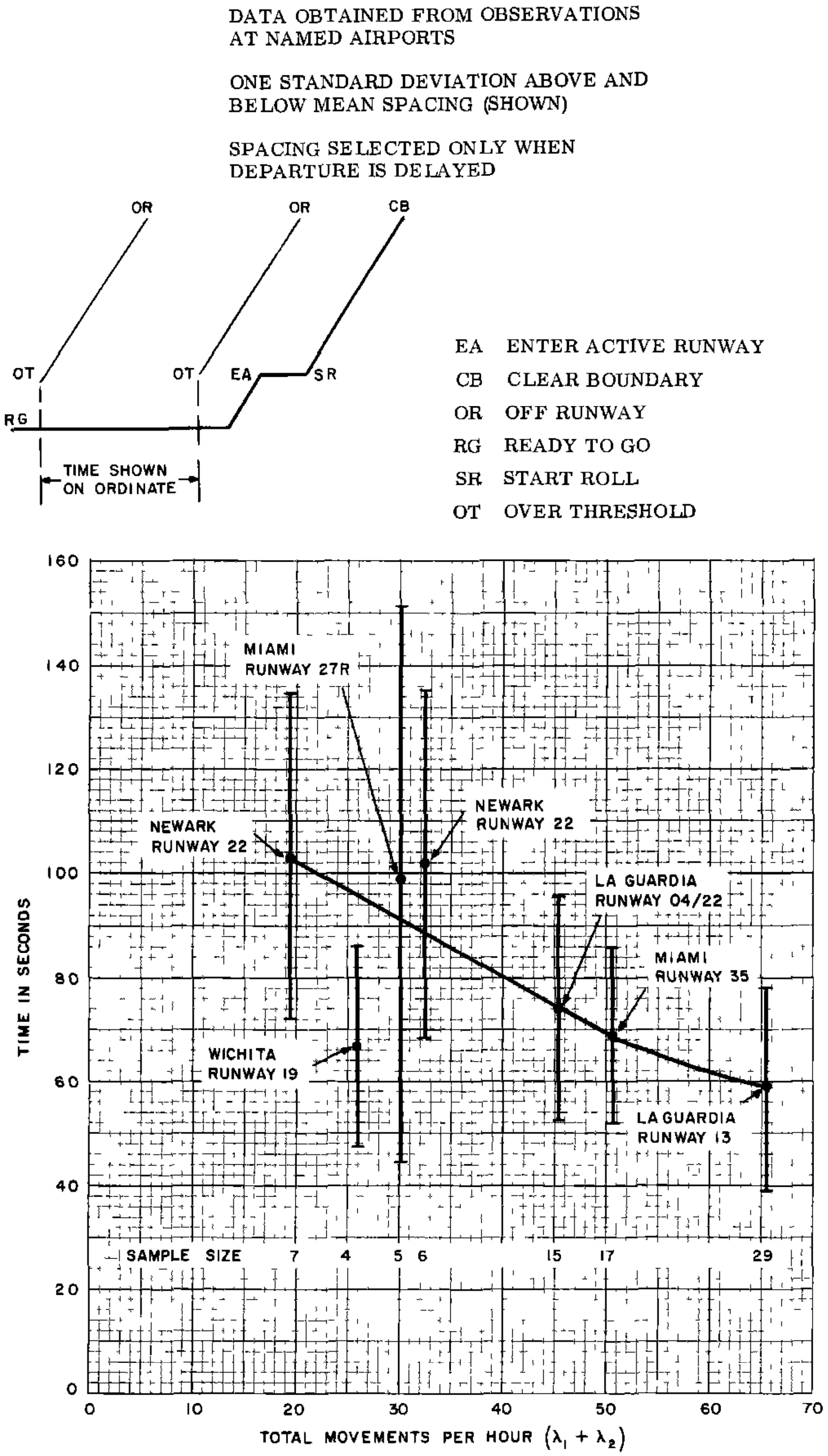


FIGURE 9-6 VFR SPACING FOR ARRIVAL FOLLOWED BY ARRIVAL

DATA OBTAINED FROM OBSERVATIONS AT NAMED AIRPORTS

ONE STANDARD DEVIATION ABOVE AND BELOW MEAN SPACING (SHOWN)

SPACING SELECTED ONLY WHEN DEPARTURE IS DELAYED

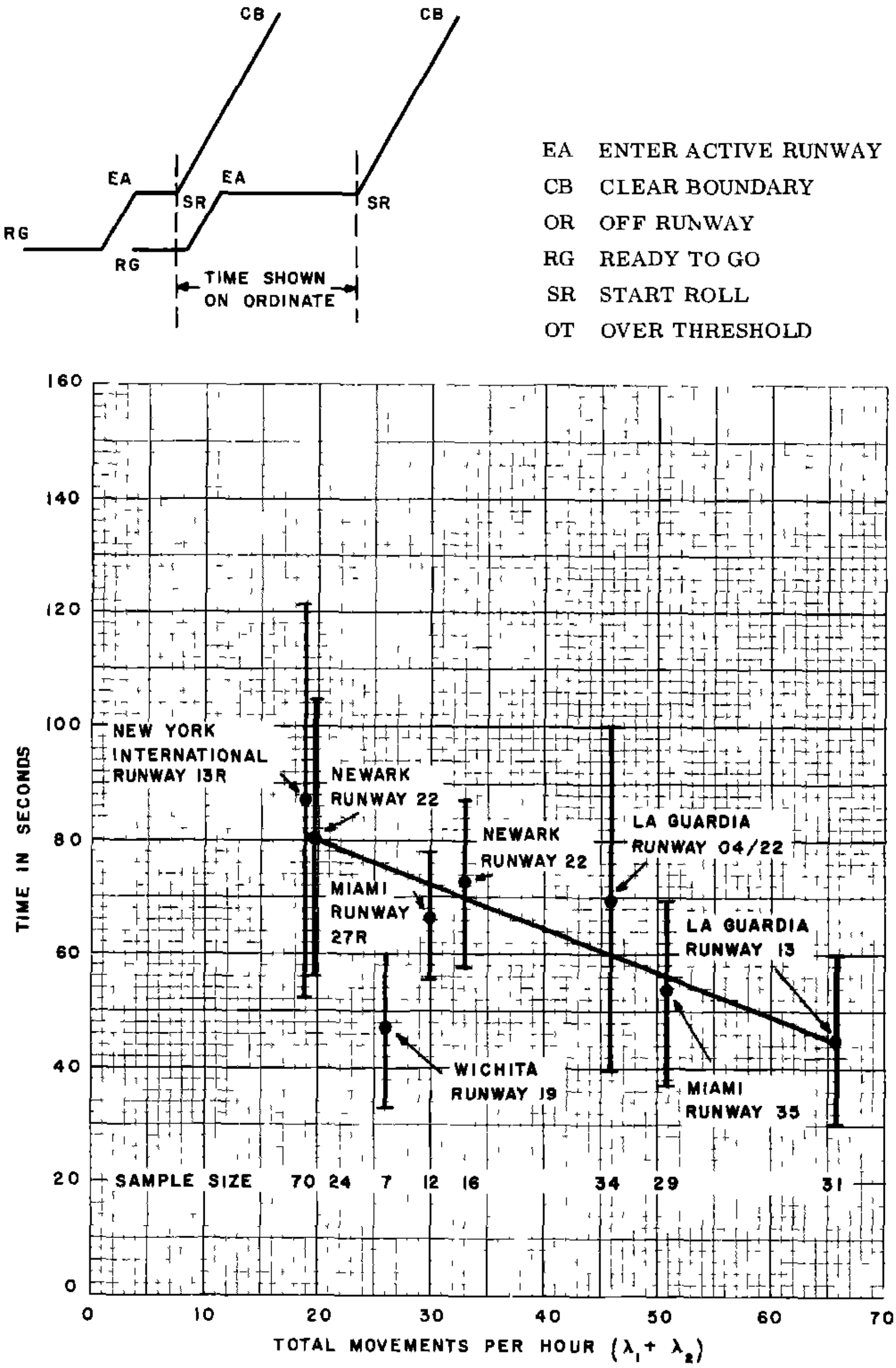


FIGURE 9-7 VFR SPACING FOR DEPARTURE FOLLOWED BY DEPARTURE

DATA OBTAINED FROM OBSERVATIONS
AT NAMED AIRPORTS

ONE STANDARD DEVIATION ABOVE AND
BELOW MEAN SPACING (SHOWN)

SPACING SELECTED ONLY WHEN
DEPARTURE IS DELAYED

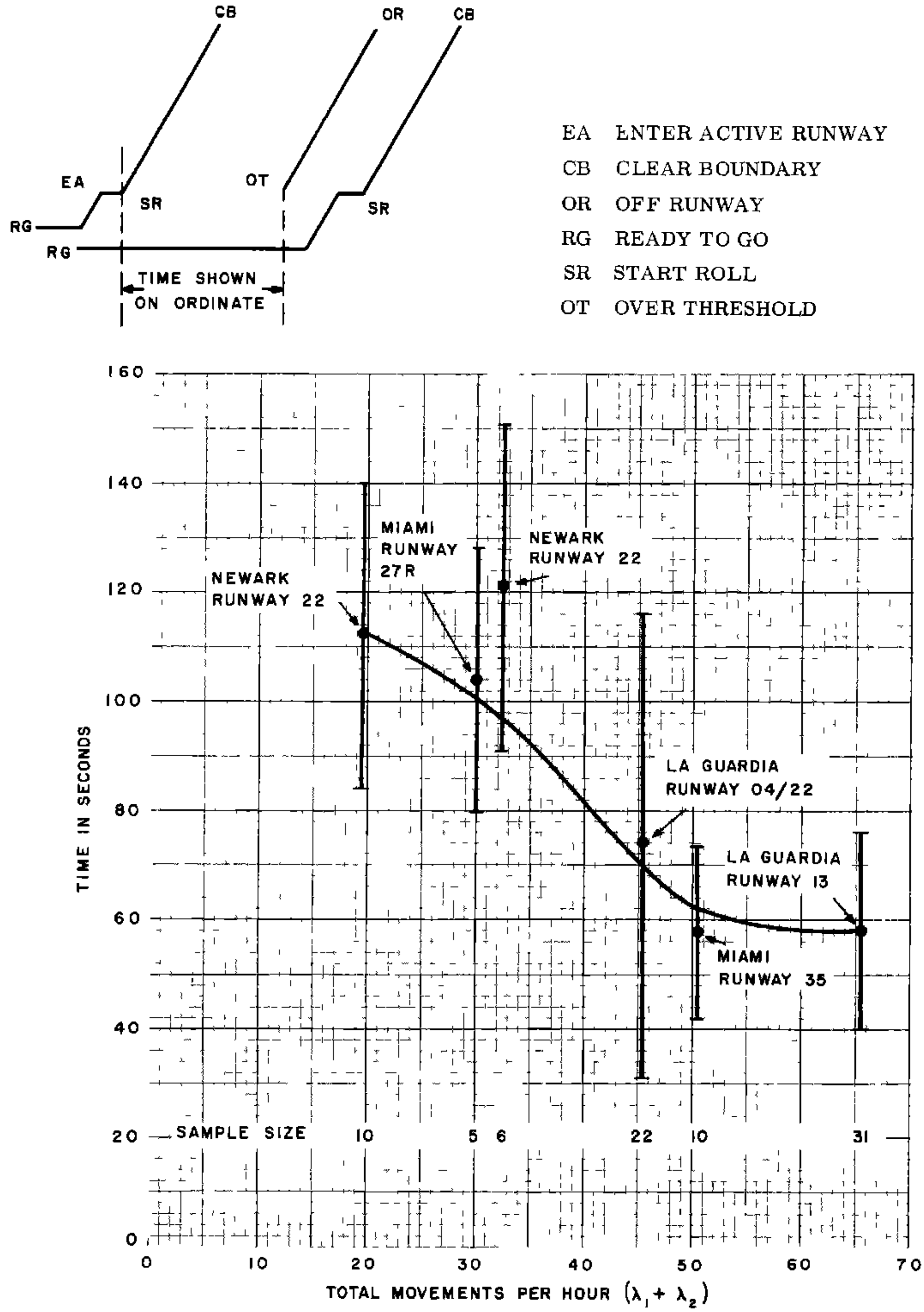
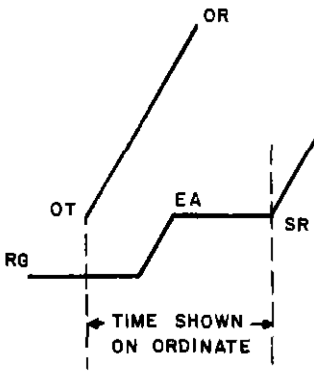


FIGURE 9-8 VFR SPACING FOR DEPARTURE FOLLOWED BY ARRIVAL

DATA OBTAINED FROM OBSERVATIONS
AT NAMED AIRPORTS

ONE STANDARD DEVIATION ABOVE AND
BELOW MEAN SPACING (SHOWN)

SPACING SELECTED ONLY WHEN
DEPARTURE IS DELAYED



EA ENTER ACTIVE RUNWAY
CB CLEAR BOUNDARY
OR OFF RUNWAY
RG READY TO GO
SR START ROLL
OT OVER THRESHOLD

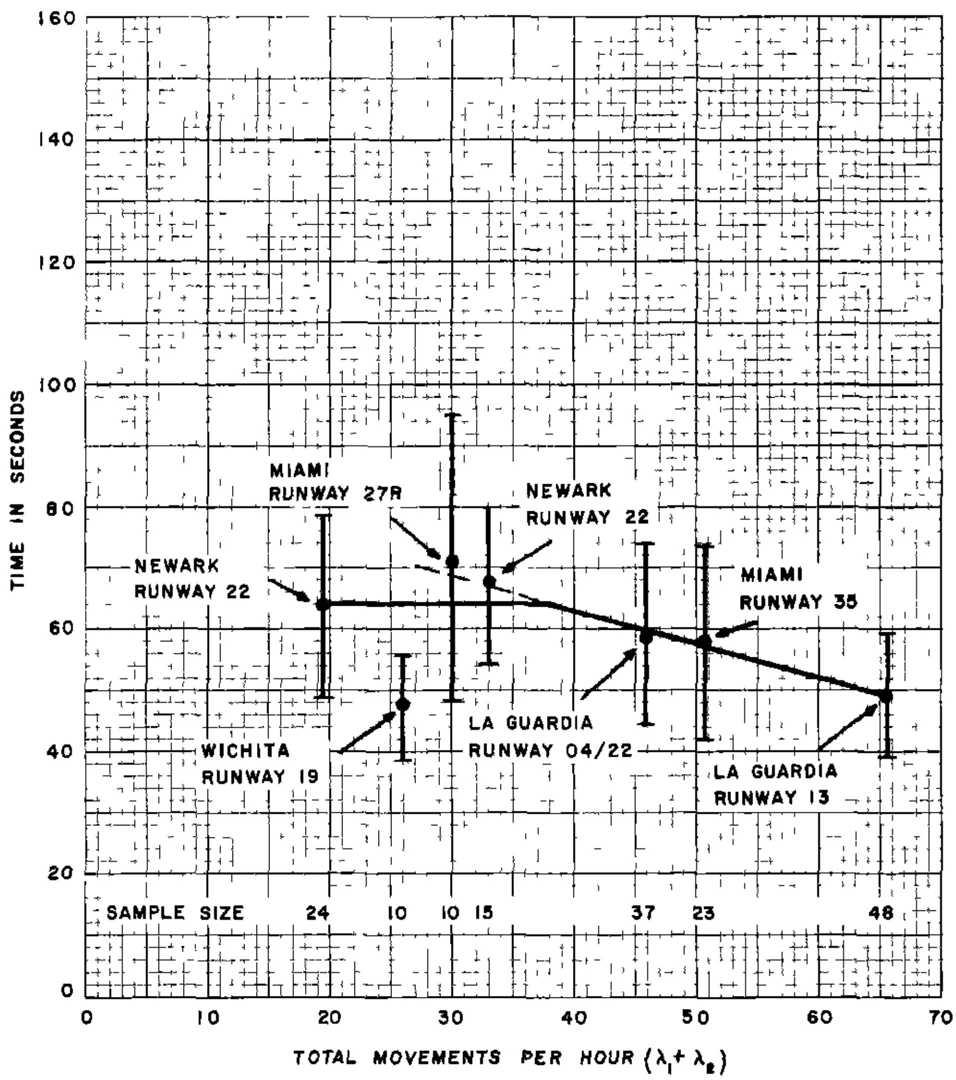


FIGURE 9-9 VFR SPACING FOR ARRIVAL FOLLOWED BY DEPARTURE

These figures indicate that the time between operations gradually diminishes as the movement rate increases. These data need further amplification and verification to determine the precise slope of the curve for each operational combination However, the trend certainly is evident, even when one considers the difference in the runway layouts and aircraft types that are included in the observation data, for it must be realized that the slopes can be much influenced by runway length and aircraft population.

C. TESTING OBSERVED DATA FOR MATHEMATICAL-MODEL APPLICATION

The various mathematical models make certain assumptions about the arrival distributions of the aircraft. All take-offs are assumed to have Poisson distributions with a constant parameter λ. With some models, this is also the assumption required for landings. In all cases of interest, the inter-arrival time is a random variable.

When these models are compared with observed data, it is necessary to test the data to determine if they statistically satisfy the assumptions of the particular model.

Appendix C gives details of the tests that were used to determine if a distribution is random, steady state, or Poisson with a constant parameter λ.

The ready-to-go times for departures or departure-input times were tested and were found to be in statistical agreement with a random Poisson input. These test results are shown in Figures 9-10 and 9-11. The input times for arrivals, however, are not directly observable. Therefore, they cannot be tested directly.

If the arrivals can be imitated by a queuing process, then they necessarily must have an input distribution and a service-time distribution. Furthermore, since the

TEST FOR POISSON INPUT

INTERVALS BETWEEN SUCCESSIVE DEPARTURES MEASURED WHEN PILOT IS READY TO GO

RUNWAY 22 WITH MIXED OPERATIONS

SAMPLE SIZE 91

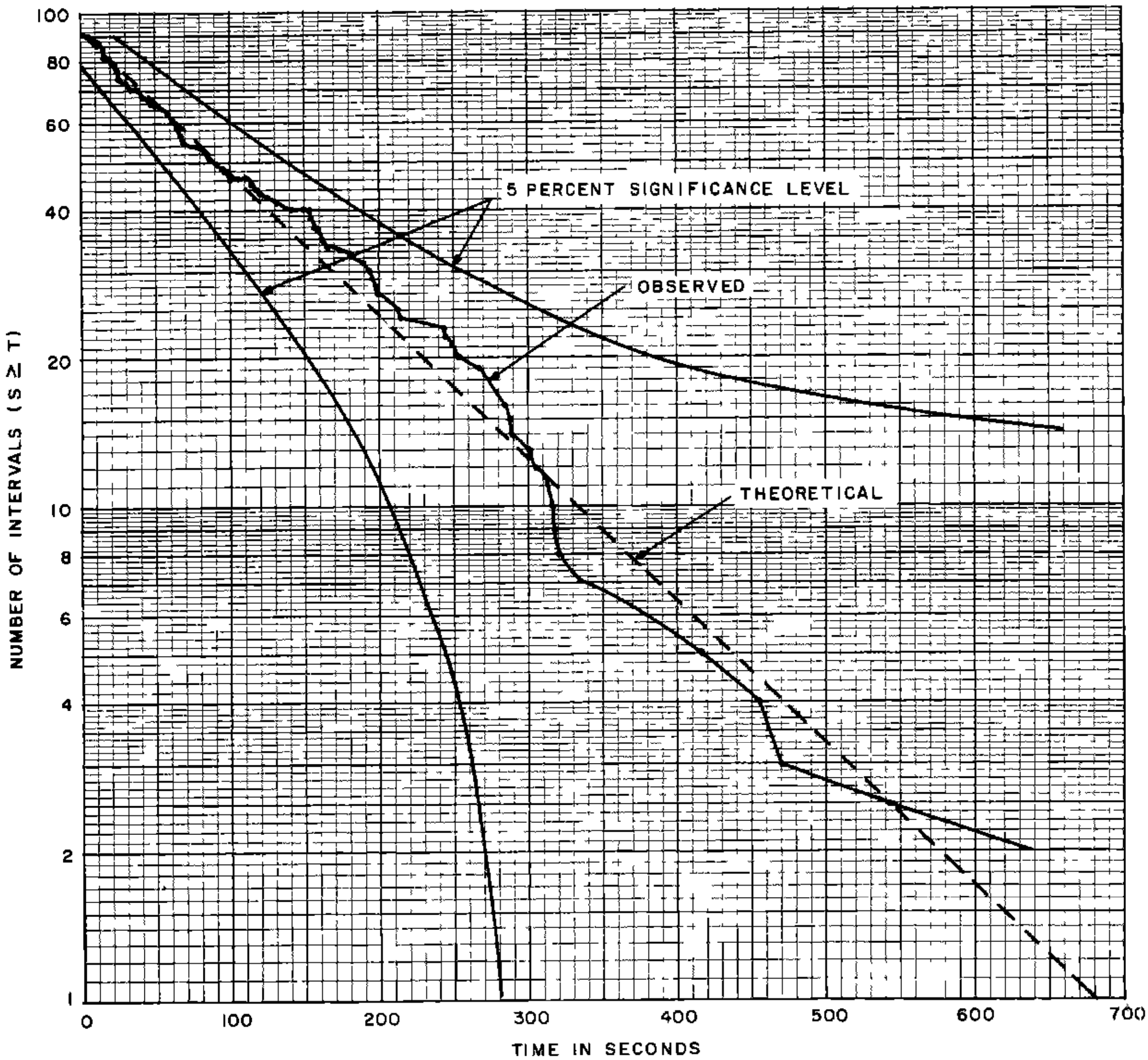


FIGURE 9-10 CUMULATIVE INPUT DISTRIBUTION FOR LA GUARDIA DEPARTURES (13 NOVEMBER 1959)

TEST FOR POISSON INPUT

INTERVALS BETWEEN SUCCESSIVE DEPARTURES
MEASURED WHEN PILOT IS READY TO GO

RUNWAY 13R WITH DEPARTURES ONLY

SAMPLE SIZE 158

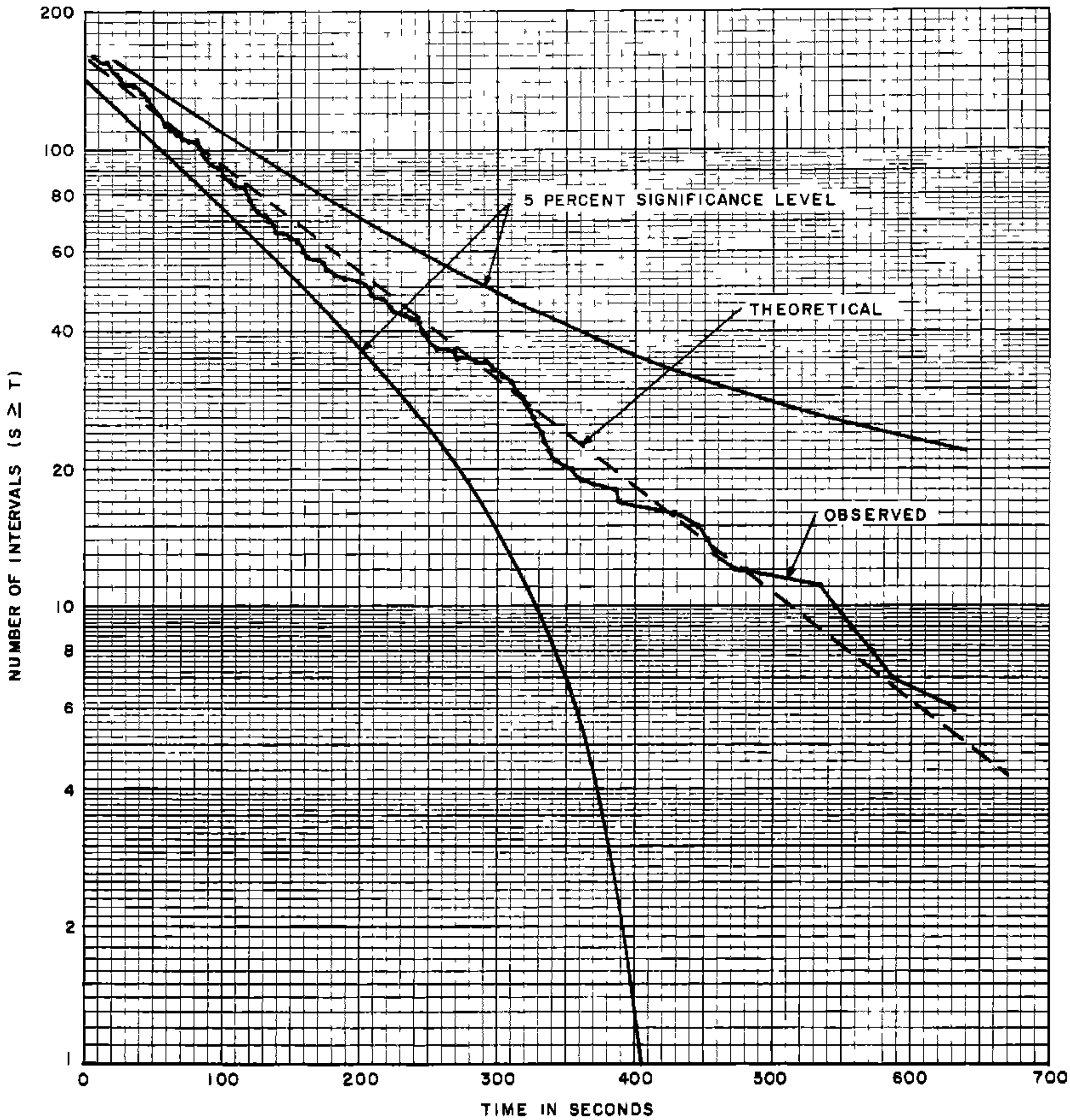


FIGURE 9-11. CUMULATIVE INPUT DISTRIBUTION FOR IDLEWILD DEPARTURES (4 APRIL 1958)

arrivals must obviously queue up while in the air, the time that they reach the runway threshold would represent the time the arrivals are discharged from the queue. Thus, the difference between successive threshold times can be called the inter-output times, and these times are directly observable and measurable.

To test a measured inter-output distribution against a theoretical one, the service-time distribution must be specified, since the distribution of service times affects the inter output distribution. The first and second moments (mean and mean square) of arrival service were estimated from the measured inter-output times in the following way

First, consider the list of measured inter-output times to be sorted and ranked from lowest to highest If the arrivals form a queue in the air, the inter-output times under queue conditions represent true service times, since successive discharge intervals from a queue are actually a definition of service time. Such minimum-separation times represent the true service times we are seeking and they are found near the top of our ranked list. The question is how to select the gapless inter-output times and obtain samples of true service times

The selection was made for VFR mixed runway operations by arbitrarily selecting only those inter-output times that occurred when a departure was waiting to go during the entire inter-output interval. Since the controller did not release the departure, such inter-output intervals are assumed to be of the minimum variety and thus represent a sample of arrival service times.

Since the distribution of the samples of arrival service times obtained did not appear particularly constant, a simple distribution function was needed to describe it. Perhaps the simplest type of distribution between a constant and a pure random (exponential) distribution is the Erlang. The Erlang distribution has an associated parameter k that is defined as the square of the ratio of the mean to the standard deviation. For constant service time, k becomes infinite, whereas for exponential service time, k becomes 1. The arrival service times obtained from airport data had values of k ranging from about 6 to 16.

1. <u>TEST FOR POISSON ARRIVALS</u>

The theoretical, cumulative inter-output distribution for Poisson input with Erlang service, is given as

$$p > t = \left(1 - \frac{m_s}{m_i}\right)\left(1 - \frac{m_s}{km_i}\right)^{-k} e^{-\frac{t}{m_i}} \sum_{n=k}^{\infty} \frac{\left[\left(\frac{k}{m_s} - \frac{1}{m_i}\right)t\right]^n}{n!} e^{-\left(\frac{k}{m_s} - \frac{1}{m_i}\right)t} + \sum_{n=0}^{k-1} \frac{\left(\frac{kt}{m_s}\right)^n e^{-\frac{kt}{m_s}}}{n!}$$

where

$p > t$ = probability of obtaining an inter-output time greater than t,

m_s = mean service time,

m_i = mean input separation = $1/\lambda$,

k = Erlang factor = $(\text{mean/standard deviation})^2$

Theoretical inter-output curves were computed using values of k, m_s, and m_i as obtained from the data. The Kolmogorov-Smirnov test (Appendix C) was applied and the results are shown in Figures 9-12 through 9-16. The cumulative plots all fall well within the 5-percent significance level except the plot of Figure 9-13, which is a plot for LaGuardia on a day of unusually heavy traffic. Figure 9-17 shows three simulation runs using the appropriate Erlang k and mean value obtained from observed data to demonstrate that a good fit results. Other cases were also tested.

TEST FOR POISSON INPUT AND ERLANG SERVICE

INTERVALS BETWEEN SUCCESSIVE ARRIVALS MEASURED FROM OVER-THRESHOLD TO OVER-THRESHOLD

RUNWAY 22 WITH MIXED OPERATIONS

SAMPLE SIZE 81

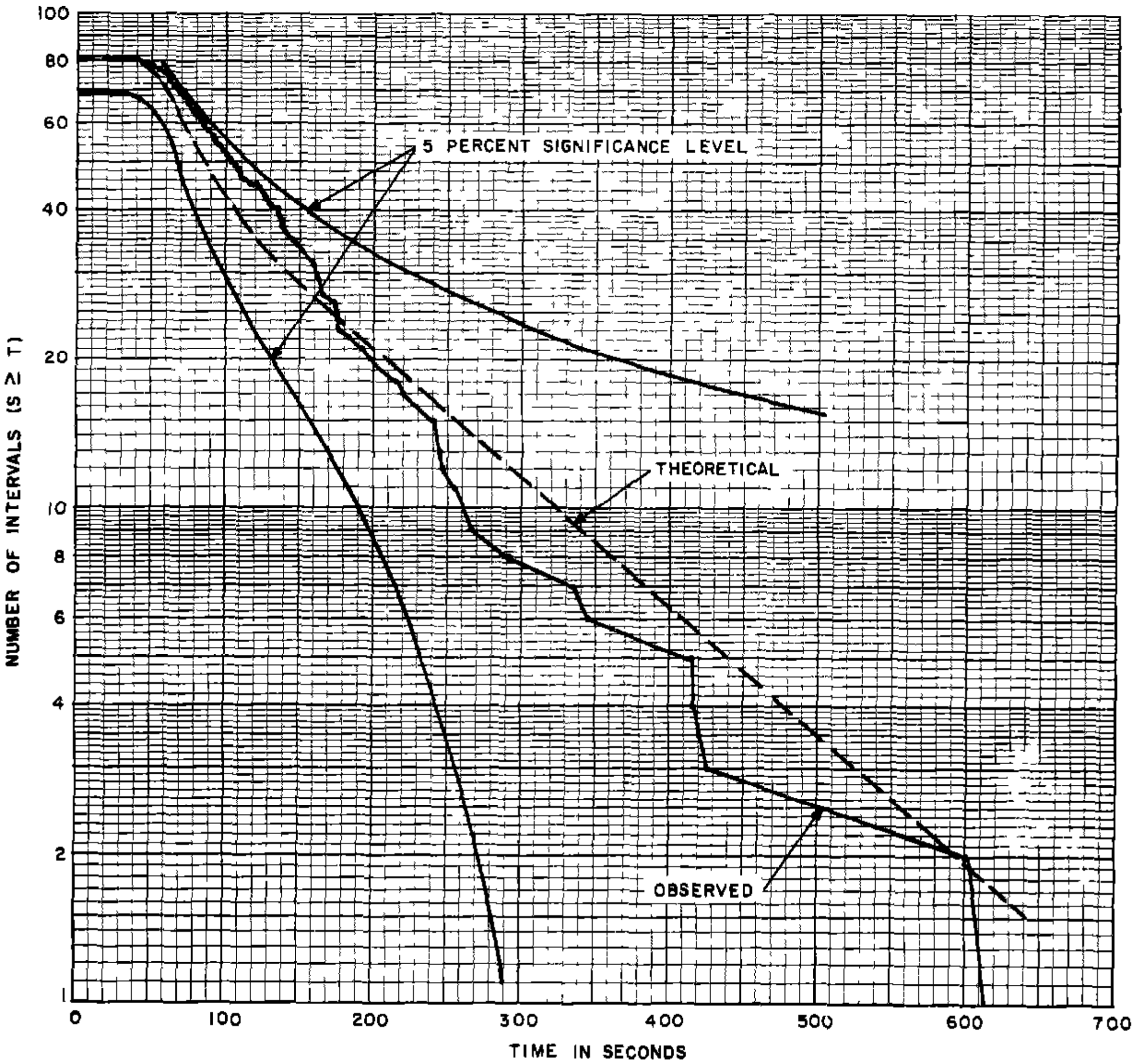


FIGURE 9-12 CUMULATIVE INTER-OUTPUT DISTRIBUTION FOR LA GUARDIA ARRIVALS (13 NOVEMBER 1959)

TEST FOR POISSON INPUT AND ERLANG SERVICE

INTERVALS BETWEEN SUCCESSIVE ARRIVALS MEASURED FROM OVER-THRESHOLD TO OVER-THRESHOLD

RUNWAY 13 WITH MIXED OPERATIONS

SAMPLE SIZE 156

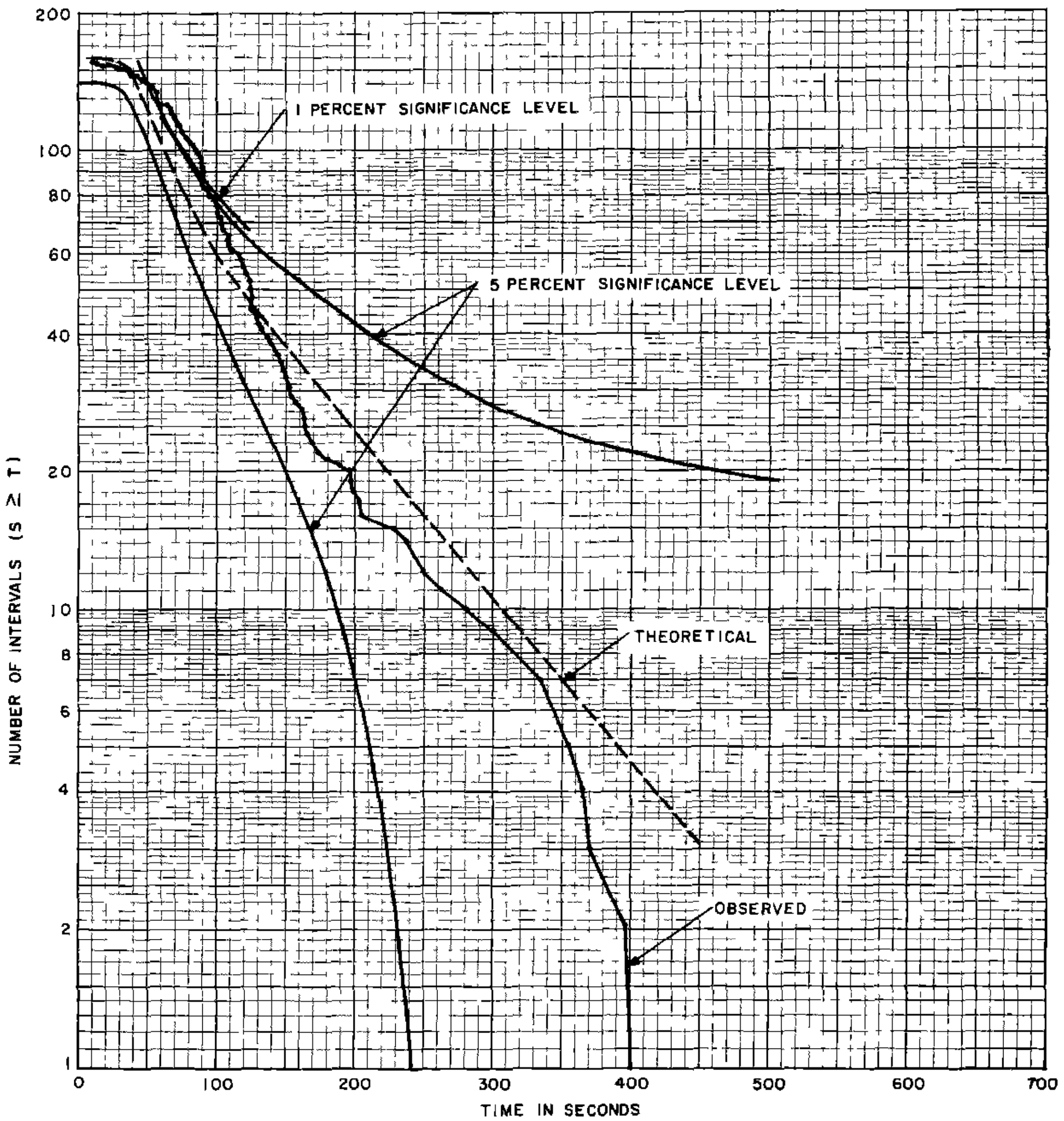


FIGURE 9-13 CUMULATIVE INTER-OUTPUT DISTRIBUTION FOR LA GUARDIA ARRIVALS (4 SEPTEMBER 1959)

TEST FOR POISSON INPUT AND ERLANG SERVICE

INTERVALS BETWEEN SUCCESSIVE ARRIVALS MEASURED FROM OVER-THRESHOLD TO OVER-THRESHOLD

RUNWAY 35 WITH MIXED OPERATIONS

SAMPLE SIZE 76

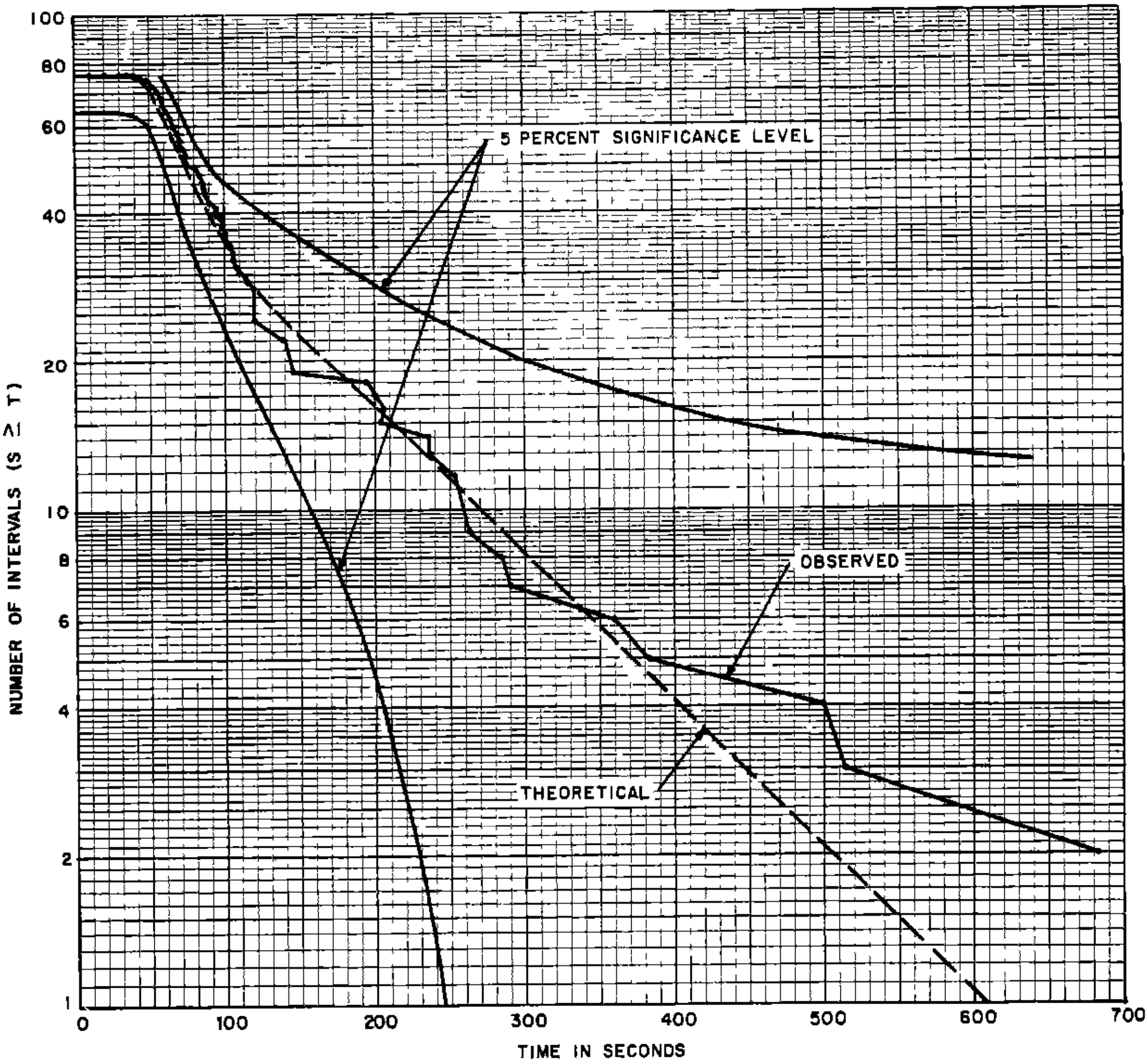


FIGURE 9-14 CUMULATIVE INTER-OUTPUT DISTRIBUTION FOR MIAMI ARRIVALS (4 DECEMBER 1959)

TEST FOR POISSON INPUT AND ERLANG SERVICE

INTERVALS BETWEEN SUCCESSIVE DEPARTURES
MEASURED FROM START-ROLL TO START-ROLL

RUNWAY 22 WITH MIXED OPERATIONS

SAMPLE SIZE 91

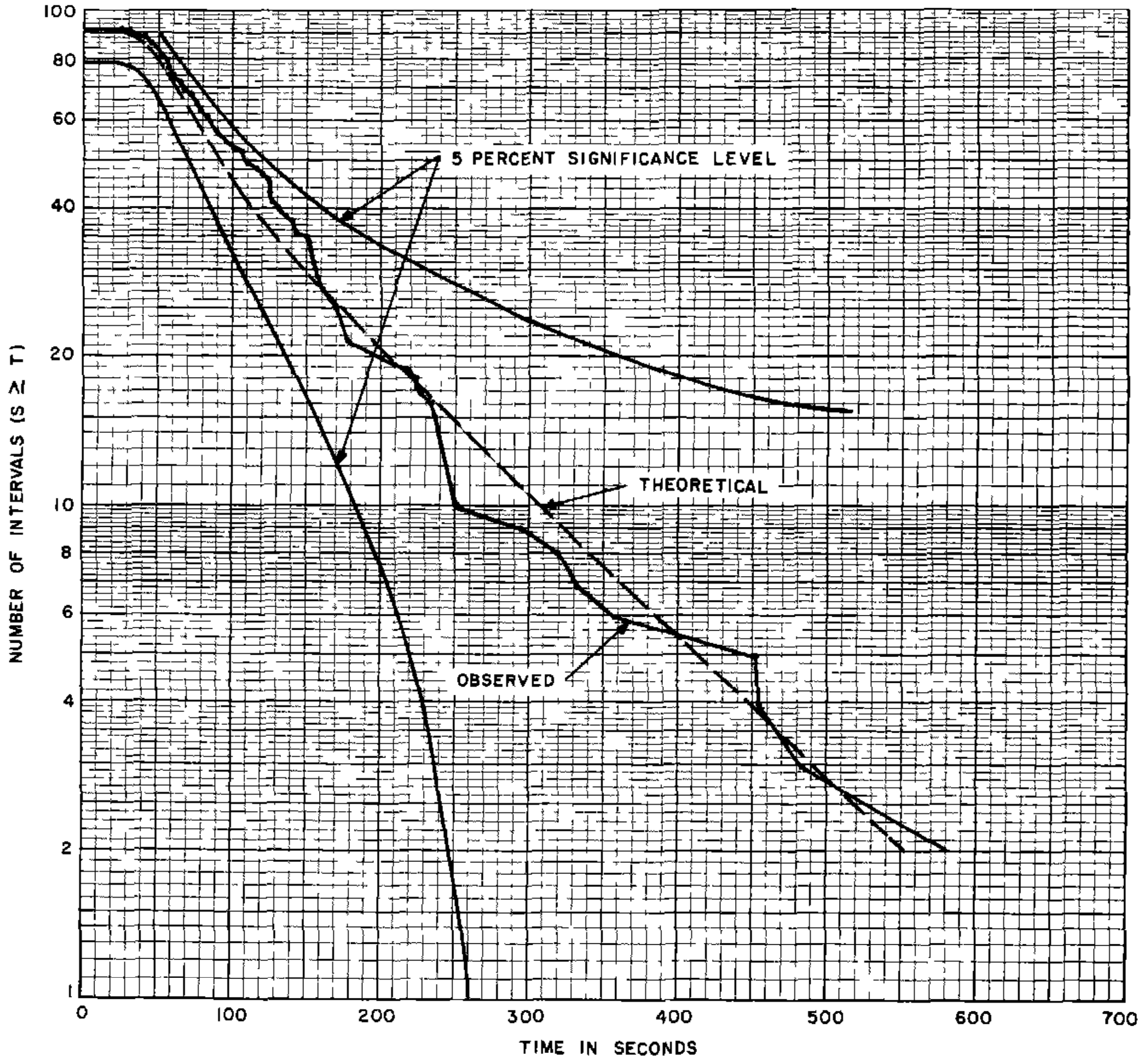


FIGURE 9-15 CUMULATIVE INTER-OUTPUT DISTRIBUTION FOR LA GUARDIA DEPARTURES (13 NOVEMBER 1959)

TEST FOR POISSON INPUT AND ERLANG SERVICE

INTERVALS BETWEEN SUCCESSIVE DEPARTURES
MEASURED FROM START-ROLL TO START-ROLL

RUNWAY 13R WITH DEPARTURES ONLY

SAMPLE SIZE 158

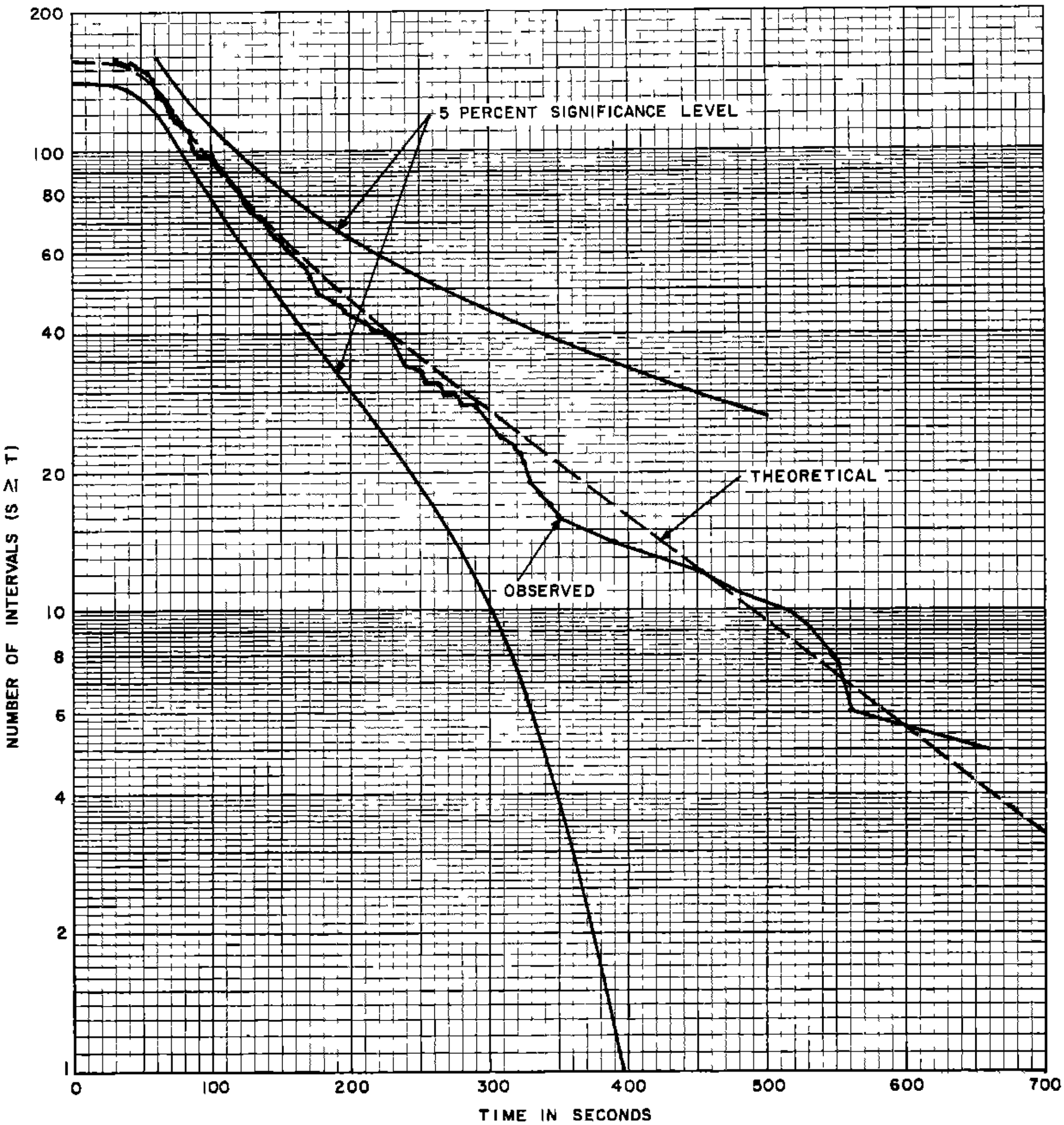


FIGURE 9-16 CUMULATIVE INTER-OUTPUT DISTRIBUTION FOR IDLEWILD DEPARTURES (4 APRIL 1958)

TEST FOR POISSON INPUT AND ERLANG SERVICE

INTERVALS BETWEEN SUCCESSIVE ARRIVALS MEASURED FROM OVER-THRESHOLD TO OVER-THRESHOLD

RUNWAY 22 WITH MIXED OPERATIONS

SAMPLE SIZE 81

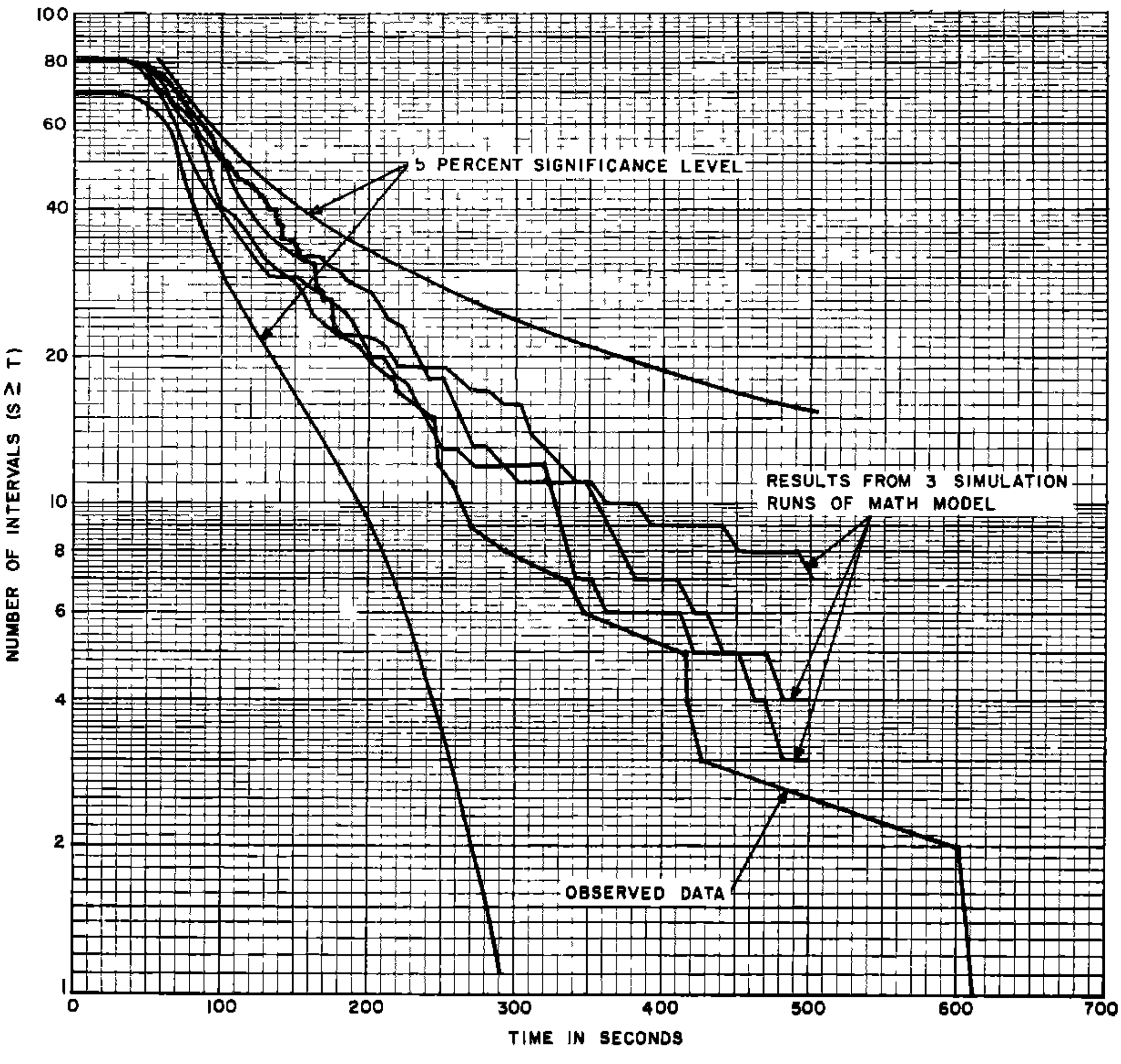


FIGURE 9-17 CUMULATIVE INTER-OUTPUT DISTRIBUTION FOR SIMULATED LA GUARDIA ARRIVALS (13 NOVEMBER 1959)

2 TEST FOR SPACED ARRIVALS

The arrival process in mixed operations may not be a queuing process at all. During VFR operations, arrival queues are typically not present or observable even with high movement rates An alternative form of input for arrivals is a pre-emptive spaced arrivals input, where the arrival service times are considered independent and the gap between arrivals is exponentially distributed Details of this model are developed in Section XII.

The theoretical inter-output distribution for the spaced arrivals model is given as

$$P > t = \left(\frac{\mu}{\mu - g}\right)^k e^{-gt} \sum_{n=k}^{\infty} \frac{c^n e^{-c}}{n!} + \sum_{n=o}^{k-1} \frac{a^n e^{-a}}{n!}$$

where

$$g = \frac{1}{\text{mean gap}} = \frac{1}{m_1 - m_s},$$

$$\mu = \frac{k}{m_s},$$

$$c = (\mu - g)t \text{ provided that } \mu > g,$$

$$a = \mu t.$$

Theoretical inter-output curves were computed, again using values of k, m_s, m_1, and the results are shown in Figures 9-18, 9-19, and 9-20 Notice that in Figure 9-19, where the observed data fell outside the significance level in the Poisson case, it now falls nicely within the significance level for spaced arrivals. On the other hand, Figure 9-20 satisfies the Poisson criterion nicely, but fails the spaced arrivals significance test. Thus, both types of input may be inferred from these inter-output tests.

TEST FOR SPACED ARRIVALS INPUT

INTERVALS BETWEEN SUCCESSIVE ARRIVALS MEASURED FROM OVER-THRESHOLD TO OVER-THRESHOLD

RUNWAY 22 WITH MIXED OPERATIONS

SAMPLE SIZE 81

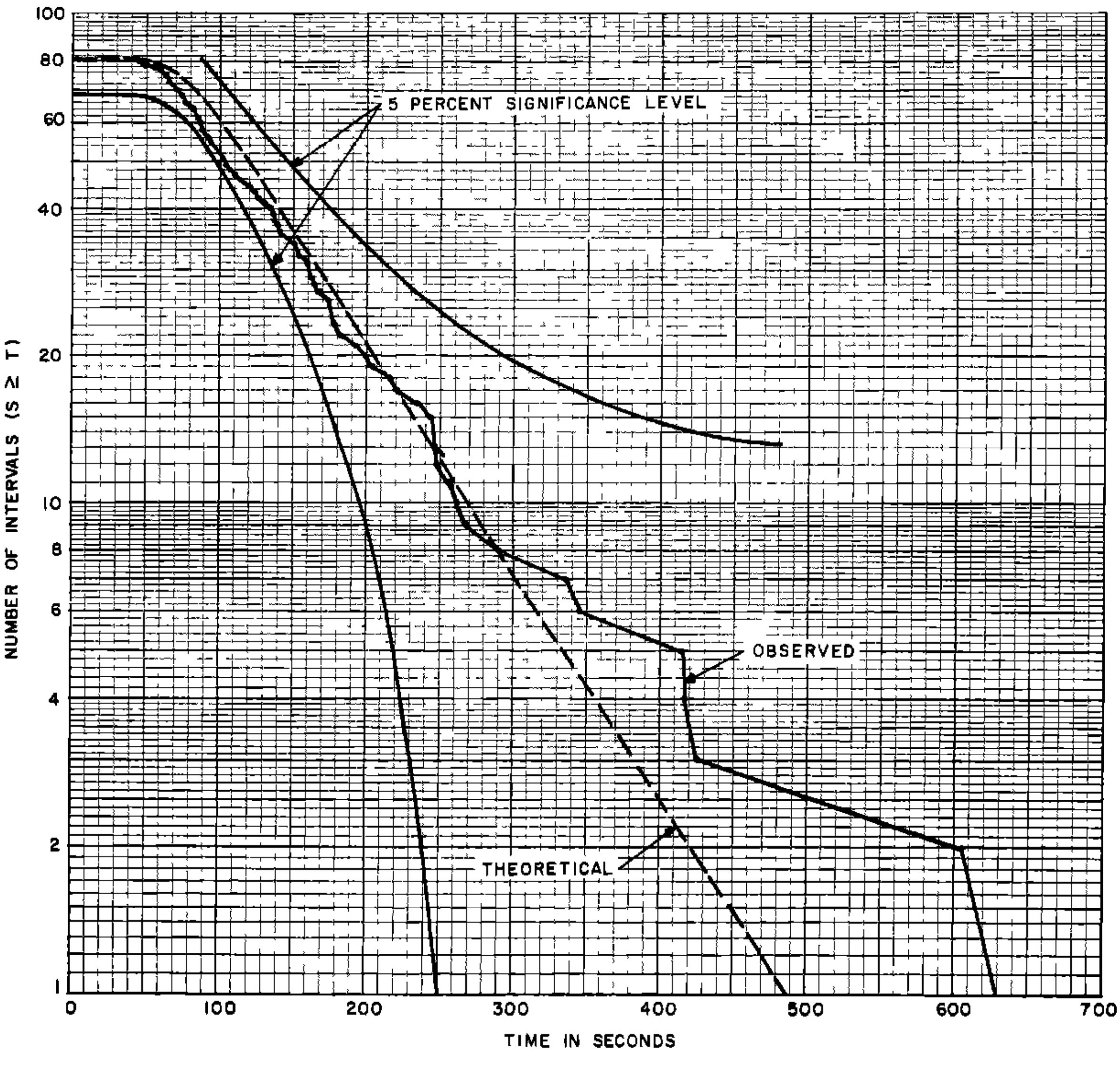


FIGURE 9-18 CUMULATIVE INTER-OUTPUT DISTRIBUTIONS FOR SPACED LA GUARDIA ARRIVALS (13 NOVEMBER 1959)

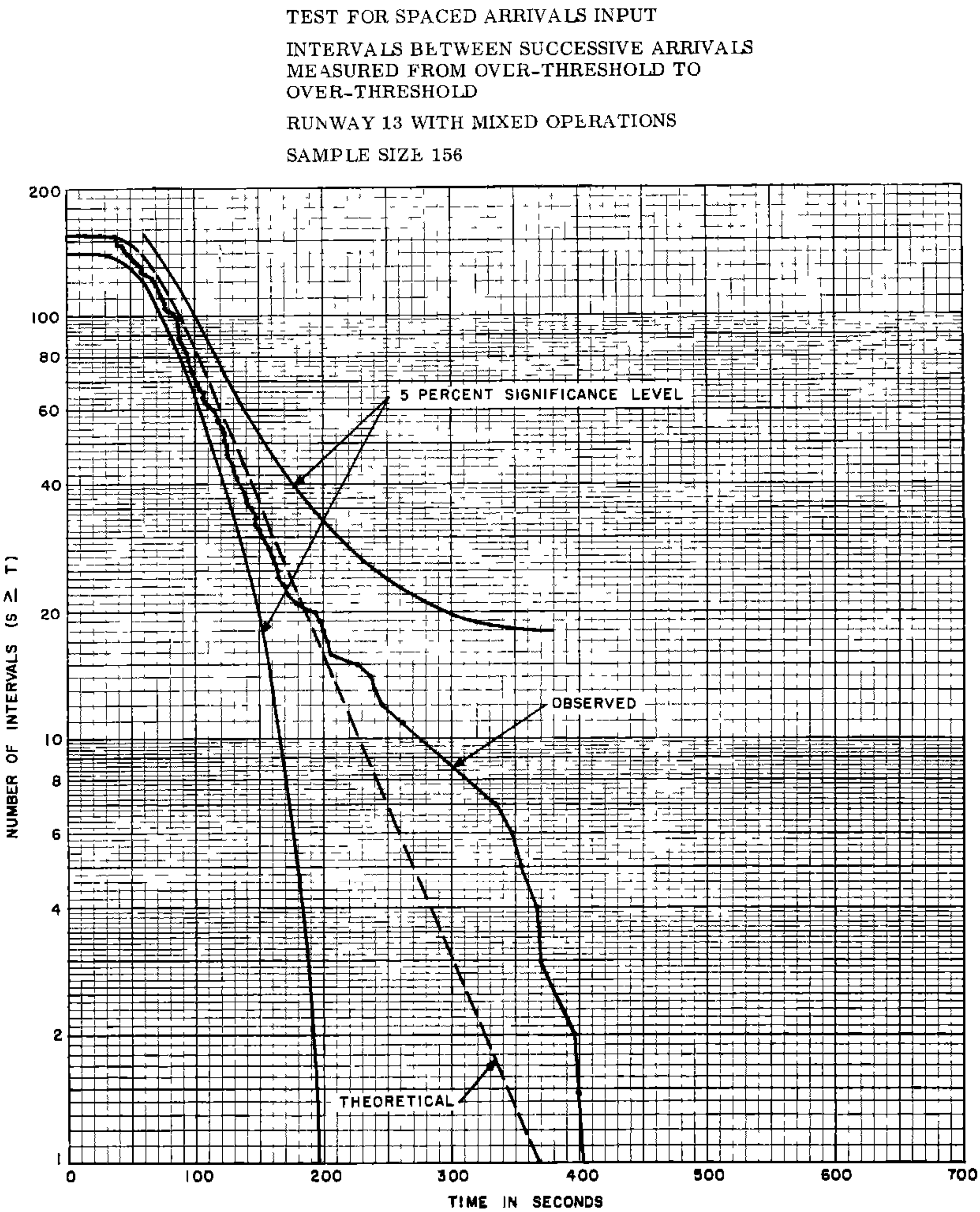


FIGURE 9-19 CUMULATIVE INTER-OUTPUT DISTRIBUTIONS FOR SPACED LA GUARDIA ARRIVALS (4 SEPTEMBER 1959)

TEST FOR SPACED ARRIVALS INPUT

INTERVALS BETWEEN SUCCESSIVE ARRIVALS MEASURED FROM OVER-THRESHOLD TO OVER-THRESHOLD

RUNWAY 35 WITH MIXED OPERATIONS

SAMPLE SIZE 76

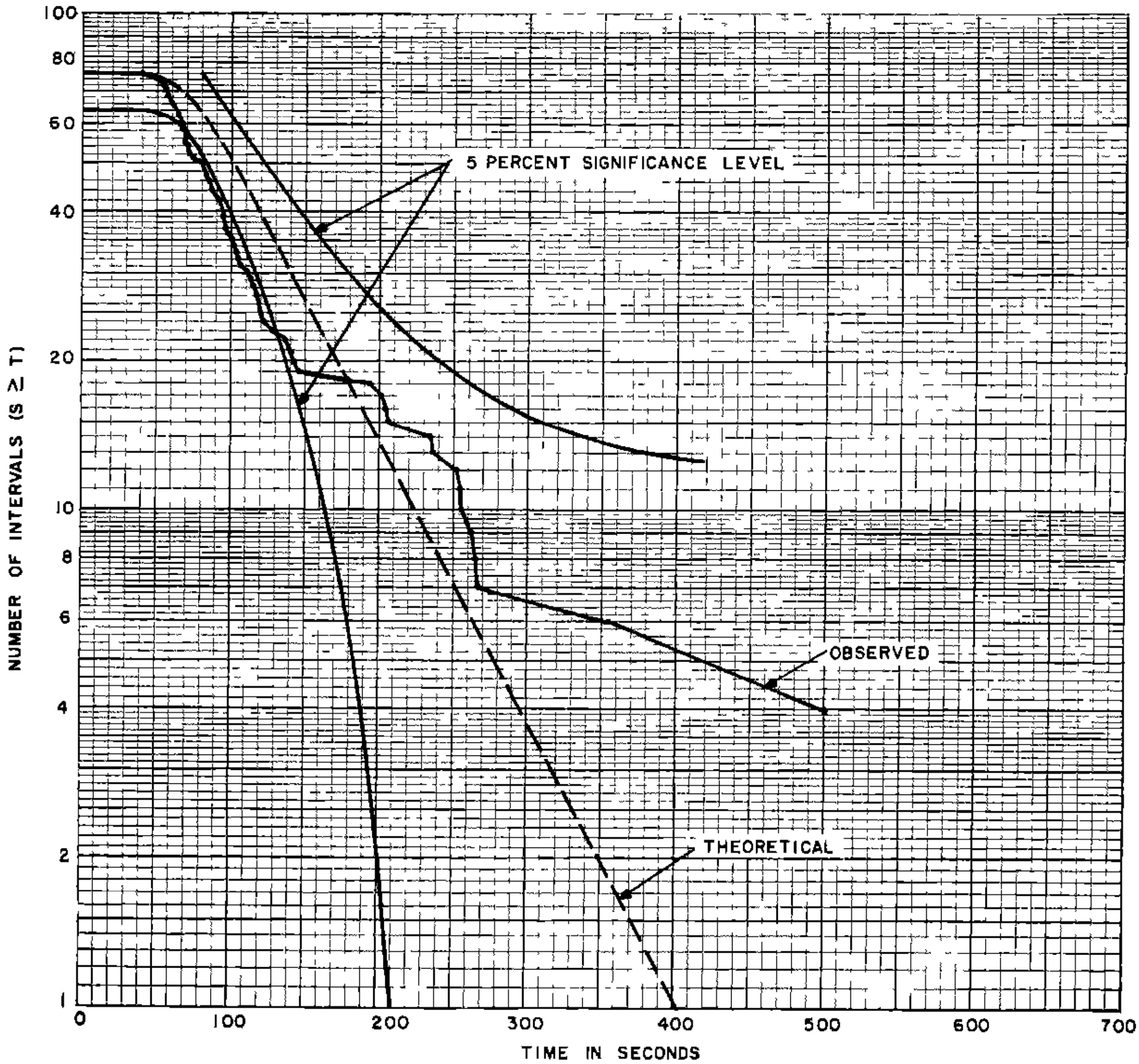


FIGURE 9-20 CUMULATIVE INTER-OUTPUT DISTRIBUTIONS FOR SPACED MIAMI ARRIVALS (4 DECEMBER 1959)

D. RESULTS OF MODEL TESTING

Shortly after the start of our project, testing of the mathematical models against observed data was begun and continued throughout the project. The testing program proved invaluable in helping us to define elements of the delay models and then observe the variation between actual and computed delay obtained by using such elements. The definitions of model elements and the method of defining the delay changed several times during this project.

The key results of this testing program are tabulated and discussed in this section as a guide to understanding the derivation of the models, and for future reference in similar programs

The delay observed and computed is normally a departure delay Thus, it would include, as a general definition, the time a departure is delayed after it reports ready to go until take-off is started Other delays, such as delay while awaiting IFR clearance, would not be predicted by these models.

The definitions of delay for departures, and the definitions of the inputs to the first-come, first-served model and to the pre-emptive spaced arrivals model, are shown in Figures 9-21, 9-22, and 9-23. These diagrams use the same basic plot (position versus time) that is shown in Figure 9-5.

It was necessary to be precise in these definitions and to assume that only one aircraft can be in service at any one time, even if two or three aircraft are physically occupying the runway.

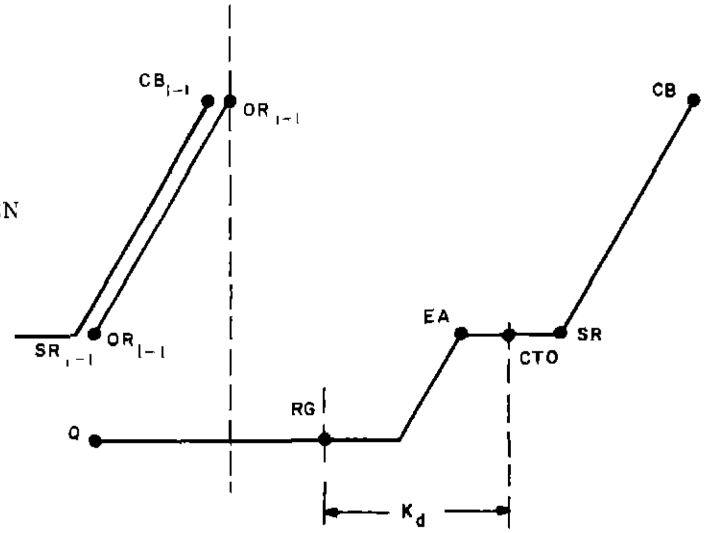
Examples of undelayed and delayed departures are shown in Figure 9-21. Note that the interval K_d is determined from undelayed departures but is used in determining the delay for delayed departures

UNDELAYED DEPARTURE

$RG_i \geq CB_{i+1}$ or

$RG_i \geq OR_{i-1}$

IF CTO OCCURS BEFORE EA, THEN
$K_d = (EA_i - RG_i)$

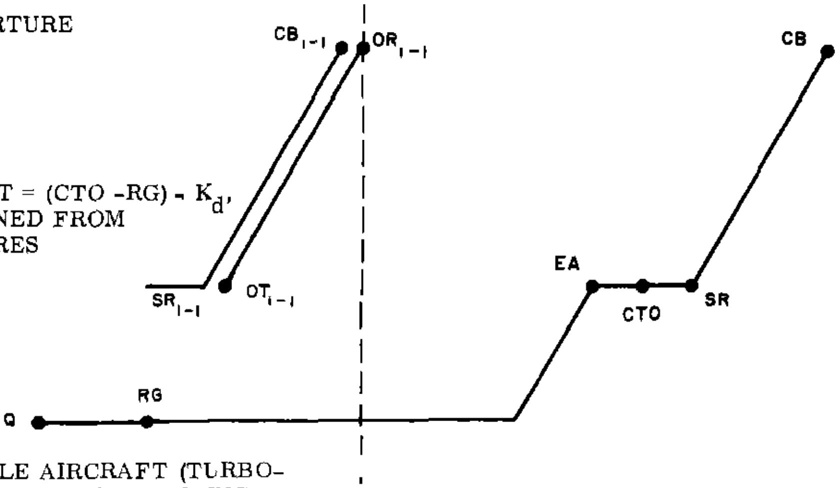


DELAYED DEPARTURE

$RG_i < CB_{i-1}$ or

$RG_i < OR_{i-1}$

DELAY OF i^{th} AIRCRAFT = (CTO -RG) - K_d,
WHERE K_d IS DETERMINED FROM
UNDELAYED DEPARTURES



IF RG OCCURS WHILE AIRCRAFT (TURBO-
JET OR TURBOPROP) IS STILL TAXIING,
DELAY IS COMPUTED FROM Q TIME
ALSO, IF CTO OCCURS BEFORE AIRCRAFT
ENTERS ACTIVE RUNWAY (EA), DELAY IS
COMPUTED BY (EA - RG) - K_d

EA	ENTER ACTIVE RUNWAY
CB	CLEAR BOUNDARY
CTO	CLEAR TO TAKE-OFF
OR	OFF RUNWAY
RG	READY TO GO
SR	START ROLL
OT	OVER THRESHOLD
M	MOVE FROM QUEUE
Q	ENTER RUN-UP QUEUE

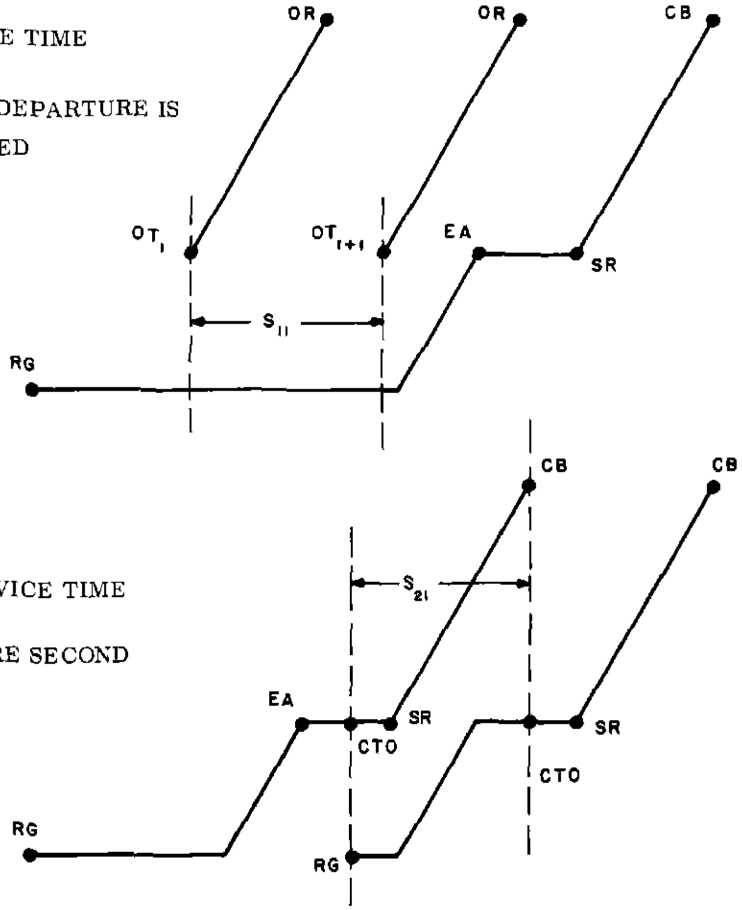
FIGURE 9-21 DEFINITION OF DEPARTURE DELAY

S_{11} ARRIVAL SERVICE TIME

$S_{11} = OT_{i+1} - OT_i$ WHERE DEPARTURE IS $RG < OT_i$ BUT NOT RELEASED

S_{21} DEPARTURE SERVICE TIME

$S_{21} = CTO_{i+1} - CTO_i$ WHERE SECOND DEPARTURE $RG < SR_i$

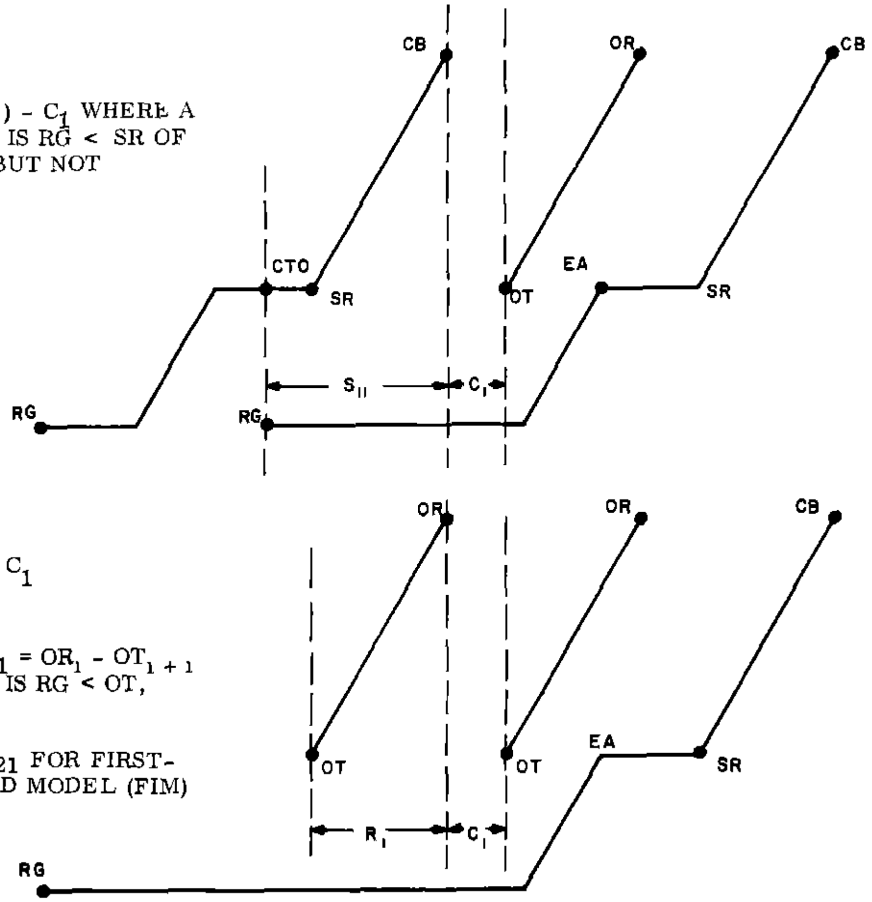


EA ENTER ACTIVE RUNWAY
CB CLEAR BOUNDARY
CTO CLEAR TO TAKE-OFF
OR OFF RUNWAY
RG READY TO GO
SR START ROLL
OT OVER THRESHOLD
M MOVE FROM QUEUE
Q ENTER RUN-UP QUEUE

FIGURE 9-22 DEFINITION OF INPUTS FOR FIRST-COME, FIRST-SERVED MODEL (FIM)

S_{11}

$S_{11} = (CTO_1 - OT_{1+1}) - C_1$ WHERE A SECOND DEPARTURE IS $RG < SR$ OF FIRST DEPARTURE, BUT NOT RELEASED



R_1 AND C_1

$R_1 = OT - OR_1$ AND $C_1 = OR_1 - OT_{1+1}$ WHERE DEPARTURE IS $RG < OT$, BUT NOT RELEASED

S_2 IS THE SAME AS S_{21} FOR FIRST-COME, FIRST-SERVED MODEL (FIM)

EA	ENTER ACTIVE RUNWAY
CB	CLEAR BOUNDARY
CTO	CLEAR TO TAKE-OFF
OR	OFF RUNWAY
RG	READY TO GO
SR	START ROLL
OT	OVER THRESHOLD
M	MOVE FROM QUEUE
Q	ENTER RUN-UP QUEUE

FIGURE 9-23 DEFINITION OF INPUTS FOR PRE-EMPTIVE SPACED ARRIVALS MODEL (SAM)

Table 9-I shows the results of some of the final tests with the values of the parameters used to obtain them. The results of testing the spaced arrivals model are considered quite satisfactory Similarly, the testing of Idlewild data for departures only, using the first-come, first-served model, indicates reasonable agreement with the actual test data. These models have been selected for use in analyzing airport runway operations Their use in many runway analyses have produced results that appear very practical.

Our field observations, and the simulation and analytical work done as part of the project, make it apparent that we cannot expect precise agreement between observed data and computed data The reason for this is the tremendous variation that can occur for a given operating rate. The average delay of a sample number of operations has a distribution spread that can be quite sizable for even a short period of operation, and this distribution spread gradually diminishes as the number of operations increases and the average delay approaches a steady-state delay This is discussed more thoroughly in Section XI

1 RUNWAY CROSSING PROBLEMS

Although the principal item of concern in airport design is the ability to analyze runway operations, an allied problem that can affect runway capacity is the problem of taxiing aircraft crossing an active runway, particularly with parallel runways both on the same side of the terminal building In this case, all aircraft using the outer runway must cross the inner runway to come to the terminal. If the delay for this activity becomes too serious, the controller will slow down use of the inner runway to clear up the crossing situation Often, the local controller who handles all landings and take-offs will also control the runway crossings, thus the workload itself can be a limiting condition.

TABLE 9-I

MODEL TESTS

Airport*	λ_1	λ_2	Actual Average Delay (min)	Pre-emptive Spaced Arrivals: Computed Average Delay (min)	S_{11} (sec)	S_2 (sec)	R_1 (sec)	C_1 (sec)	C_2 (sec)	K	First-Come, First-Served: Computed Average Delay (min)	S_{21} (sec)	S_{22} (sec)
Idlewild (Departures only) 4 April 1958	--	19	1.26	--	--	--	--	--	--	--	1.54	108.0	14510
Newark A	9	10	1.21	1 00	80 3	76 7	62 6	34 4	1316	16 9			
Newark B	13	20	1 92	2 17	64 4	70 9	58 2	30 5	1126	19 7			
Wichita 22 September 1959	14	12	0 24	0 55	49 8	50 9	39 6	24.1	909	6.9			
LaGuardia 4 September 1959	31	35	4 5	4 39	35 6	47 6	36.8	21 3	617	12 6			
LaGuardia 13 November 1959	22	24	2.20	1.77	44.3	58 3	41 0	29 4	1086	17 8			
Miami 4 December 1959	25	25	2 79	4 57	46.5	60.4	44 9	20 8	491	16.4			

* Newark A includes 0800 to 1815 hours, 27 March 1958, Newark B includes 1530 to 1815 hours, 27 March 1958

The pre-emptive Poisson arrivals model that is developed and discussed in this report provides a tool for analyzing this problem. The computations made and actual data analyzed are summarized in Table 9-II The model will also predict the queue length for the crossing aircraft.

TABLE 9-II

RUNWAY CROSSING DELAYS

Number of Aircraft per Hour Crossing Runway 7	16	14	5
Number of Aircraft per Hour Operating on Runway 7	31	24	24
Observed Delay in Minutes	0.6*	0.5*	0 3**
Theoretical Delay in Minutes	1.07	0.61	0.75

* Taxiway F.

** Taxiway 31R

These delays apply to New York International Airport with landings on runway 4R and departures on runway 7

In this comparison all computed wait times are higher than observed, but there is an indication of agreement With other model trials, it becomes evident that the number of take-offs and the time required by each are the controlling elements in predicting delay In general, the model may predict delays somewhat on the high side because the delay with the model is predicted on the basis of a strict discipline of no crossings after an aircraft is cleared for take-off, and the crossing cannot be started until the take-off aircraft passes the taxiway where the crossing aircraft is waiting. In actual practice, however, it is likely that the discipline will be more relaxed and the controller and pilot will work together for higher efficiency and reduce somewhat the observed delay over that which would be computed.

The data used for computing delay has been based on runway crossing observations made at Midway Airport in Chicago.

These data are included in Appendix A for general application. Additional data should be gathered as time permits. It appears, however, from limited observation, that the runway crossing times of larger aircraft that are not in operation at Midway are about the same as those observed at Midway, and the use of the Midway data will give reasonable answers when using the delay model.

2. TAXIWAY PROBLEMS

Delay occurring at busy taxiway intersections has not been documented with field data since time has not permitted this. However, based on knowledge of ground traffic flow, use of the first-come, first-served model to analyze such cases is reasonable To use this model would require field observations or examination of aircraft operating data to develop the service times needed for crossing typical intersections.

3. STAGING OF TAXIWAY CONSTRUCTION

The first-come, first-served model is recommended for determining the staging of construction--for example, parallel taxiways Here, again, time has not permitted actual field observations or testing Such testing would be difficult because one could not easily observe the situation with and then without parallel taxiways However, experience with the first-come, first-served queuing model indicates it is an obvious application and its usefulness would depend principally on the proper selection of the service times for the taxiing aircraft

Section VII includes an analysis of the need for a taxiway paralleling a runway The pre-emptive spaced arrivals model was used to predict operating rates and delay with and without the taxiway, thus providing a means of deciding when the parallel taxiway should be built.

X TIME-DEPENDENT DELAY

BUILDUP OF DELAY WITH TIME FOR CONSTANT MOVEMENT RATE

The principal mathematical models used in this study are steady-state or stationary models because the steady-state concepts generally meet the requirements of airport design and analysis. However, certain associated problems cannot be handled adequately with steady-state concepts. Among these is the problem of estimating delay during periods of high utilization or during overloaded periods.

It is well-known that steady-state formulas fail completely for overloaded conditions because a steady state is never attained (that is, the average delay continues to rise with time indefinitely). At high utilizations (slightly less than 1), the steady-state average delay obtained may not apply to airport operations since the steady-state condition is likely to take many more operations to achieve than are encountered in daily airport traffic. This is one reason why, when very high average delays are predicted for airports by steady-state models, such delays are not generally found The number of operations in a day is insufficient to build up to the steady-state condition

Thus it is important to determine exactly what is meant by high utilization, how fast will delays build up under these conditions and under overloaded conditions, and how to estimate delay when the steady-state methods do not apply

To examine the quantitative nature of delays as a function of time, a time-dependent analytical model with Poisson input was developed for a first-come, first-served discipline with constant service time. Details of the mathematical development of this model are found in Appendix D. (The mathematical approach discussed in Appendix E was reviewed before development of the model in Appendix D, and though not used, it is included as a matter of information.) This model was used because the delay can be normalized and reduced to dependence upon a single parameter (utilization). Other models, such as the pre-emptive spaced arrival or the pre-emptive Poisson, require several parameters to specify the models. Thus, for the first-come, first-served model, a general family of curves can be developed that apply to many specific situations.

The service time used in the analytical model was taken as constant. One reason for the selection of constant service was that the measured service times for departures revealed that the measured distributions have an Erlang k factor ranging from 6 to 33 (Figure 9-7). Since the steady-state average delay for the Erlang service with Poisson input is $\frac{k+1}{k}$ times the average delay for constant service, the steady-state average delay, for the most variable service-time distribution k = 6, is but 7/6 of the constant service case (only 17 percent more). Thus, for departure operations, the selection of constant service for an analysis of delay buildup appears reasonable.

Figure 10-1 shows how the average delay builds up as a function of time for a variety of different utilizations.*

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* Curves of this type are presented in S. M. Berkowitz and E. L. Fritz, "Analytical and Simulation Studies of Terminal-Area Air Traffic Control," Franklin Institute Laboratories, Technical Development Report No 251, May 1955. Their curves were prepared by extrapolation and did not cover the range of interest of the present analysis or furnish any information on the distribution of delays.

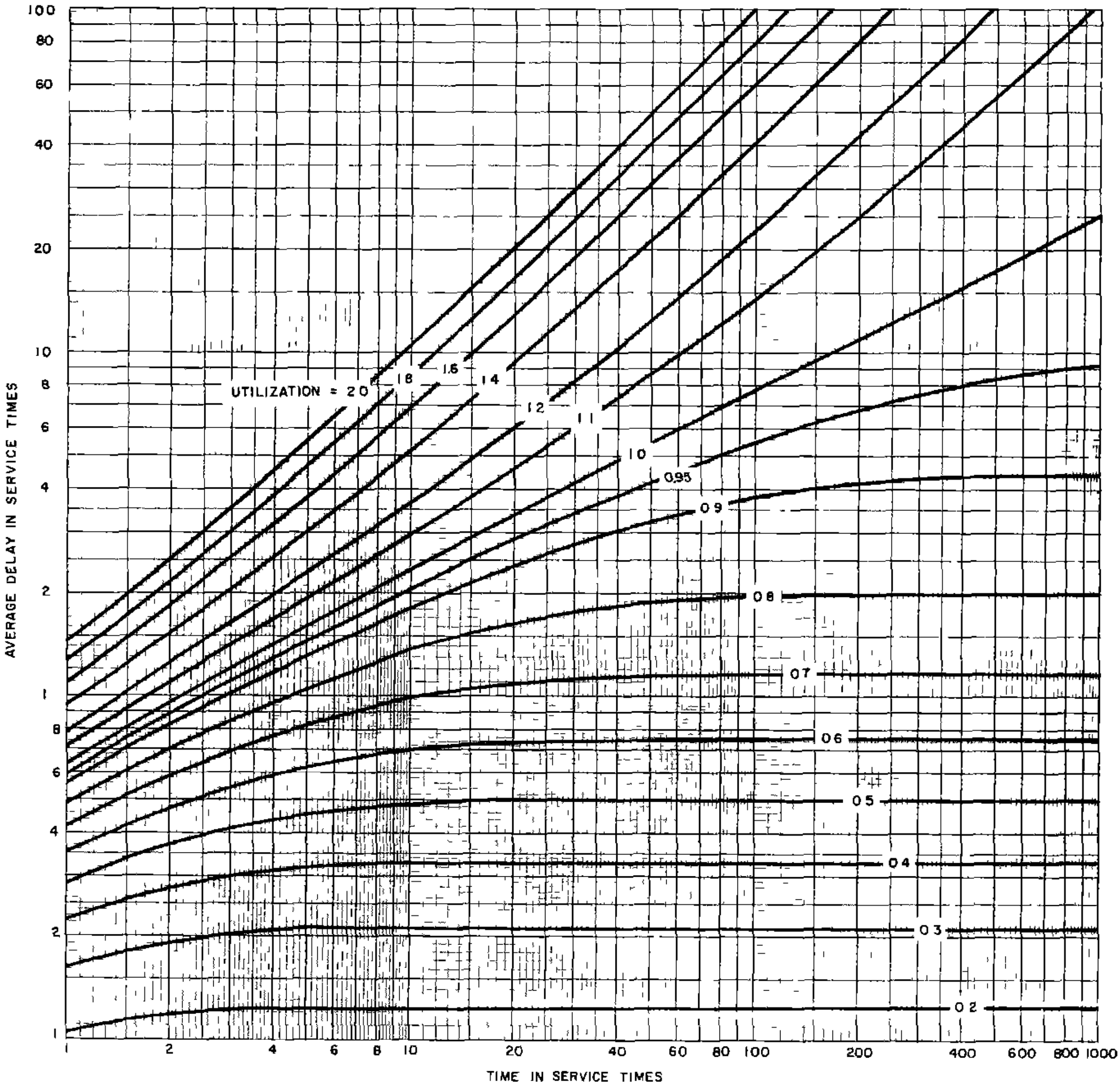

FIGURE 10-1 TIME DEPENDENT AVERAGE DELAY FOR FIRST-COME, FIRST-SERVED CONSTANT SERVICE QUEUE

The horizontal time axis is a logarithmic scale in units ranging from 1 to 1000 service times. The vertical scale, also logarithmic, gives the average delay in service-time units for an aircraft arriving at time t. Individual curves in Figure 10-1 are for constant utilization.

The assumption underlying this family of curves is that the system is initially empty. Thus, at time t = 10 service times, the delay given is interpreted as the average delay (or expected delay) to an aircraft desiring to use the runway 10 service times after the runway has been opened for traffic

Notice that for utilizations considerably less than 1, the average delay curves quickly level out to their steady-state values. However, for the higher utilizations such as 0.9, many service times are required to achieve the steady-state value In fact, for utilizations of 0.95, the steady state is not reached in 1000 service times. Figure 10-2 shows just how long it takes to achieve a given fraction of the steady-state value for various utilizations. This is one reason why steady-state models are always in error on the high side when used for relatively short periods of time to approximate delays in airport queues.

With utilizations of 1 and greater, the curves in Figure 10-1 do not level out at any steady-state value but keep rising indefinitely. However, for relatively short periods of time, accumulated average delay may not become excessive.

Figure 10-3 shows how the average delay varies as a function of utilization at given times after the runway has opened for traffic. This family of curves is also useful for interpolating the values of utilization plotted in Figure 10-1.

The time-dependent distributions of delays are shown in Figure 10-4. In some figures where the higher percentiles

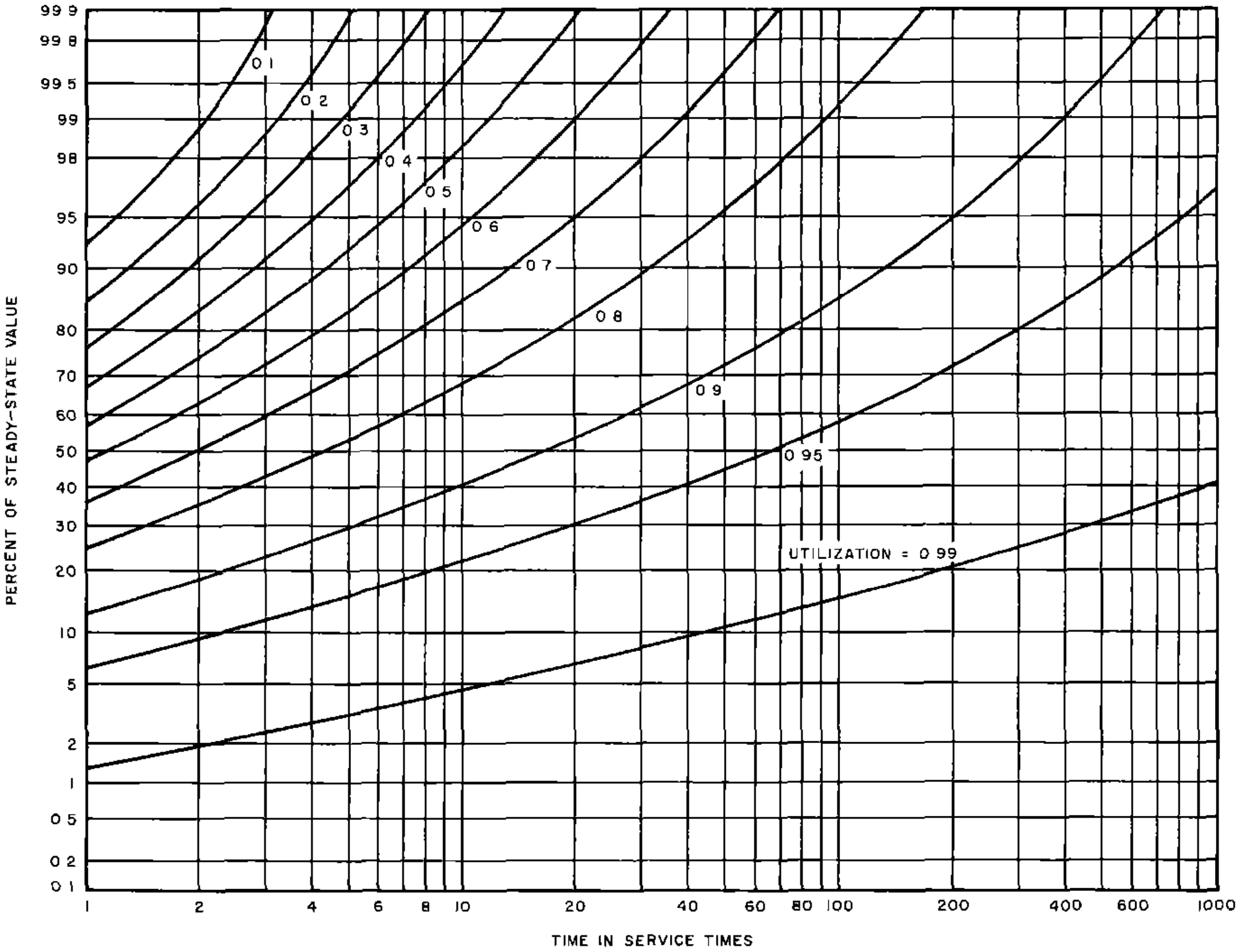

FIGURE 10-2 TIME DEPENDENT AVERAGE DELAY SHOWN AS FRACTION OF STEADY-STATE VALUE

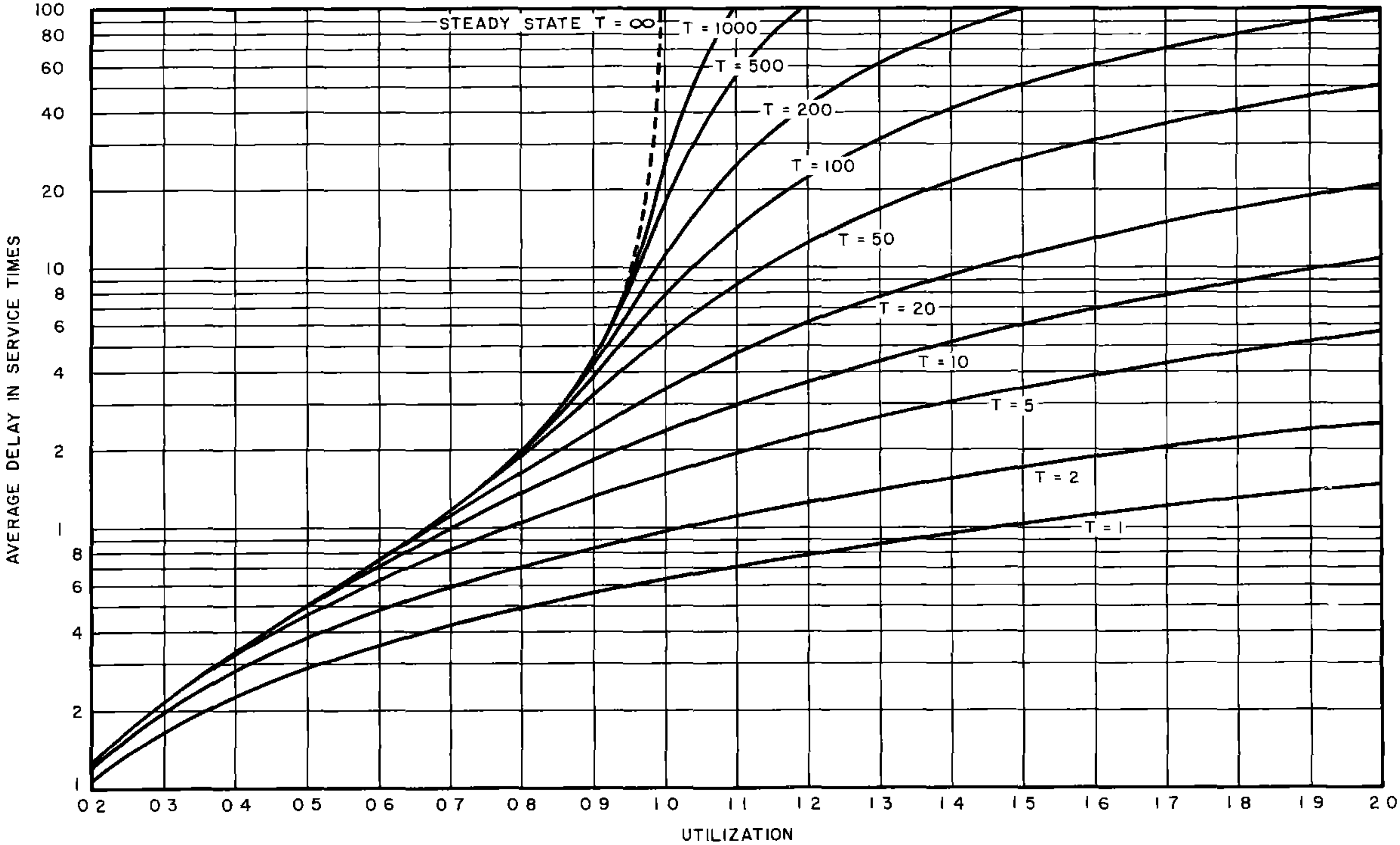

FIGURE 10-3 AVERAGE DELAY AS FUNCTION OF UTILIZATION

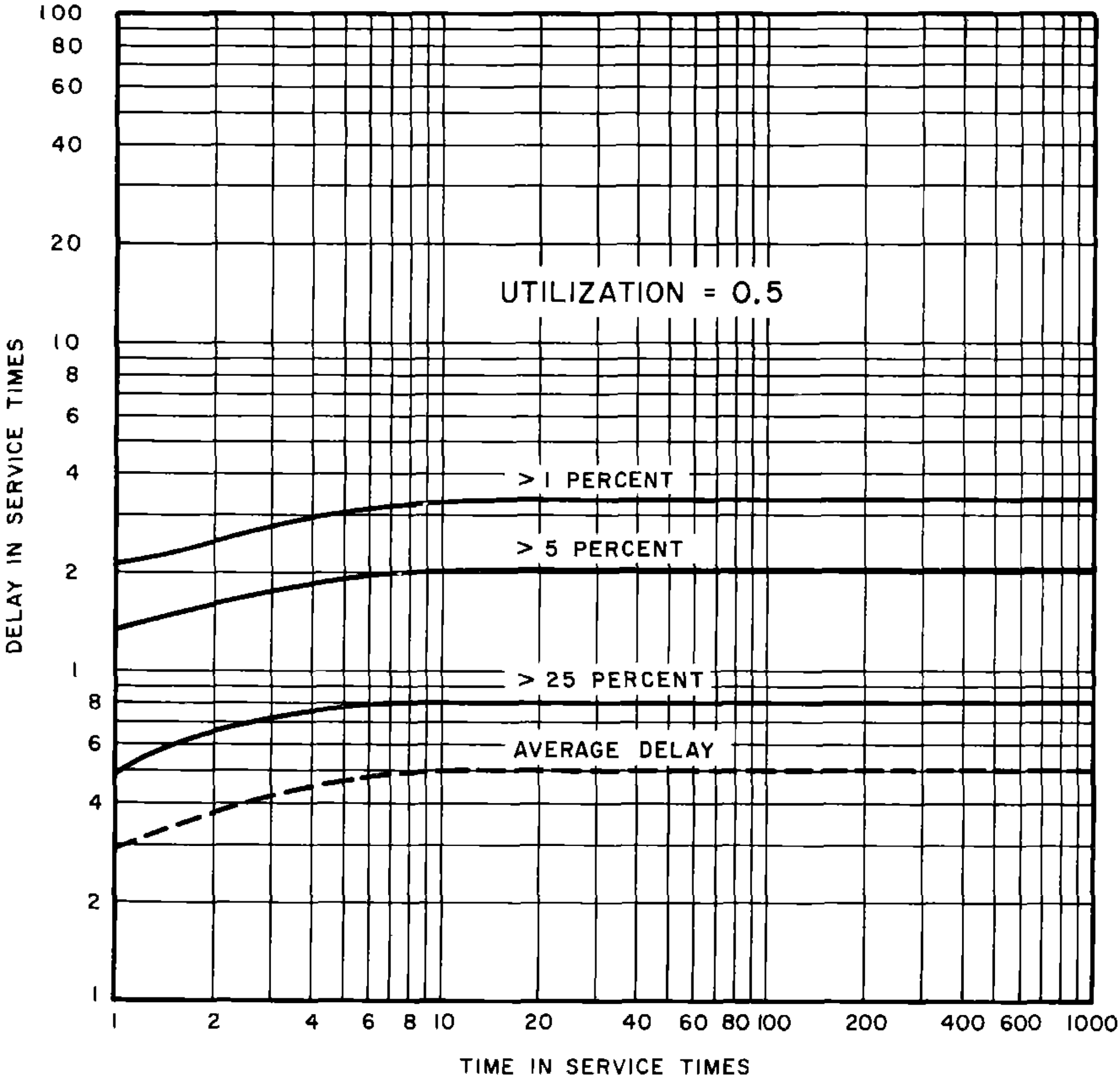

FIGURE 10-4 TIME DEPENDENT DISTRIBUTION OF DELAYS FOR FIRST-COME, FIRST-SERVED QUEUE

(SHEET 1 OF 5)

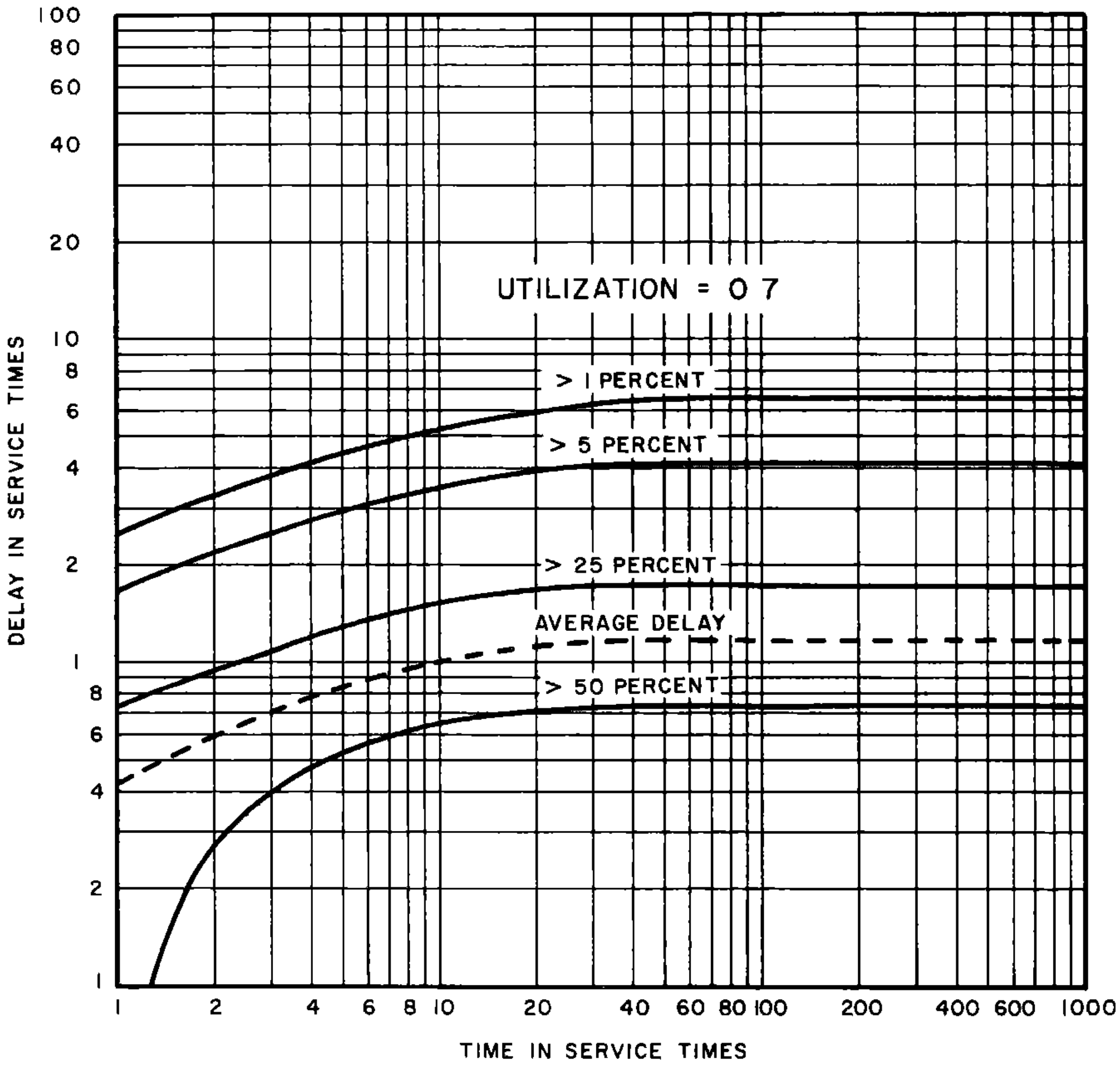

FIGURE 10-4

(SHEET 2 OF 5)

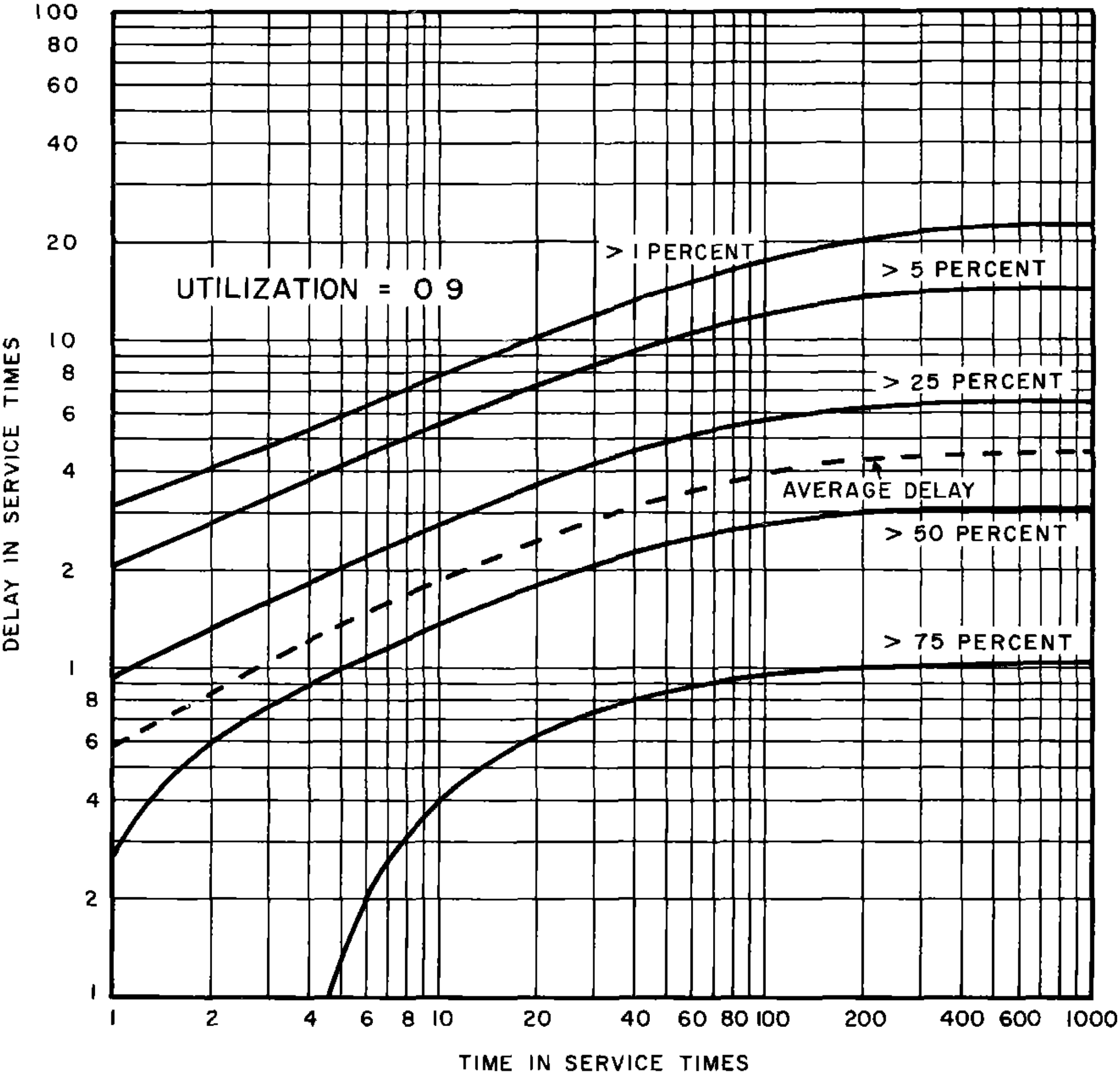

FIGURE 10-4

(SHEET 3 OF 5)

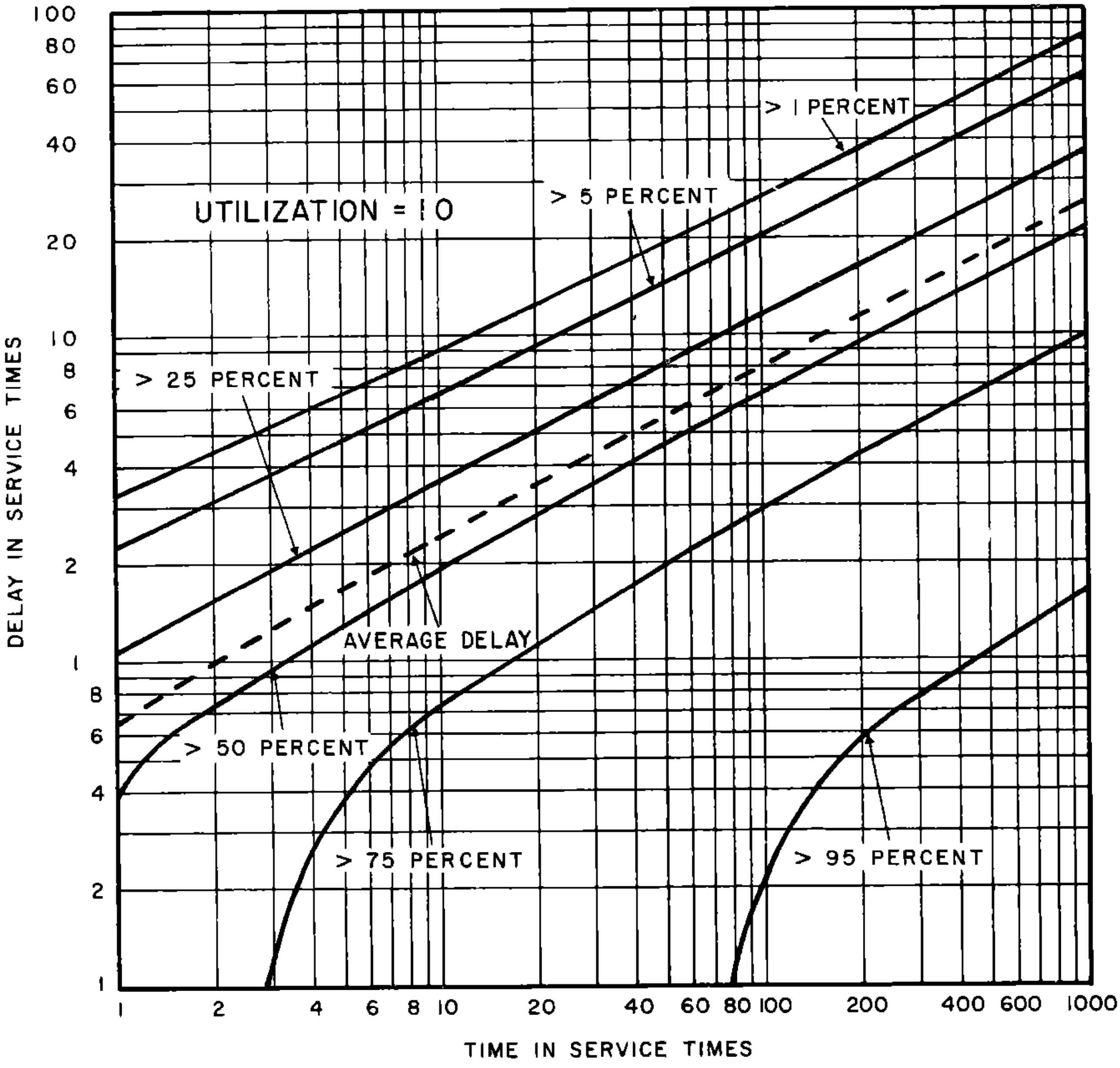

FIGURE 10-4
(SHEET 4 OF 5)

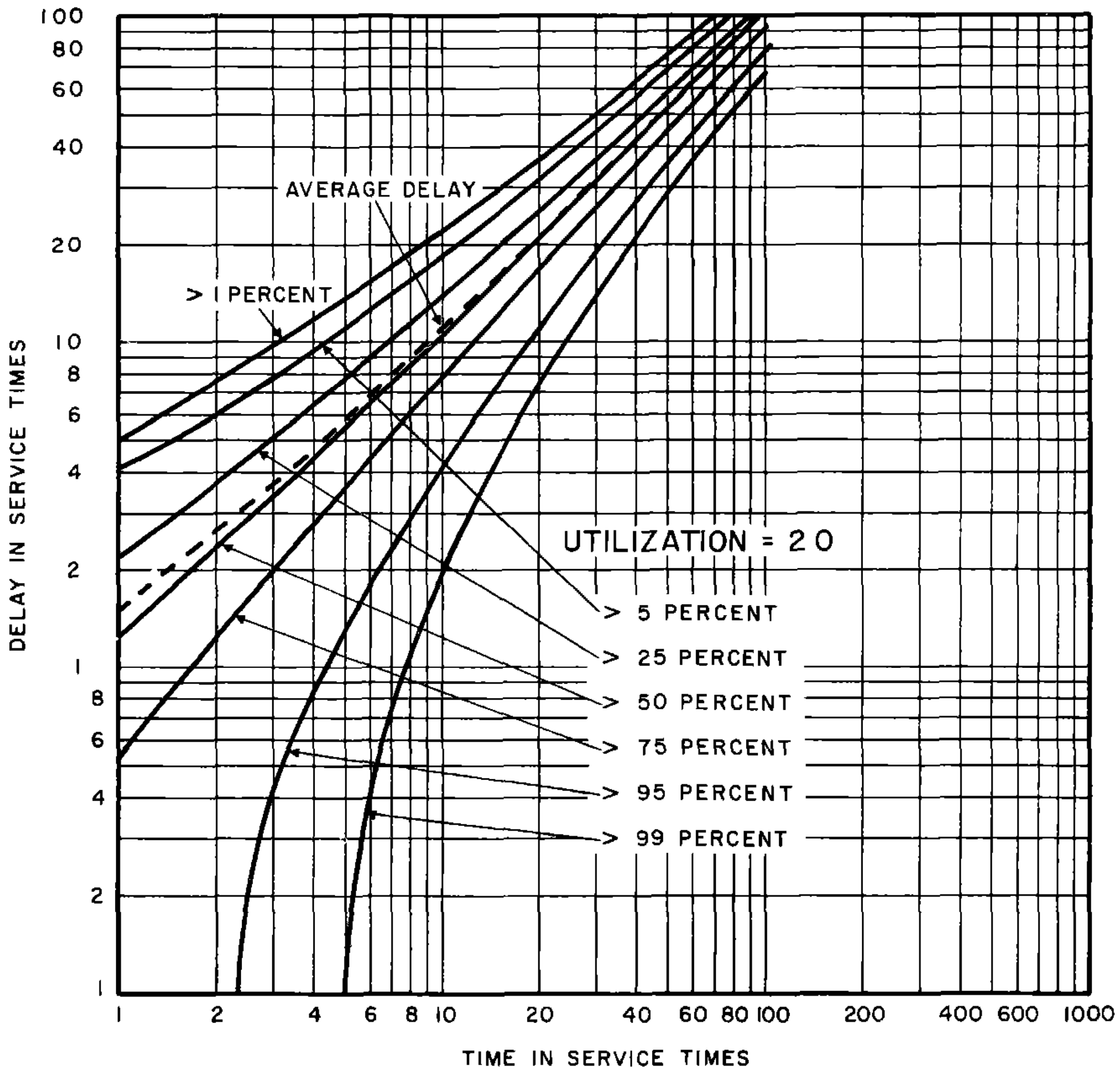

FIGURE 10-4

(SHEET 5 OF 5)

(75, 90, and 99 percent) appear to be missing, it is because they fall below 0.1 service time. It will be noticed from this family of curves that the percentiles appear to build up at about the same rate as the average delay.

Thus, we have found the distribution and average values of delay for Poisson input, constant movement rate, and constant service as a function of time, measured from when the airport runway opens for traffic.

One application of the family of curves in Figure 10-1 is estimation of delay at the end of a given hour. To do this, one must know the average delay at the beginning of the given hour, and the utilization during that hour If the average delay at the beginning of the given hour is 0, then the curves in Figure 10-1 apply directly. The utilization may be computed from the movement rate during the given hour and the runway capacity (both assumed to be known) by dividing the first quantity by the second. Since the runway capacity is the number of service times per hour, Figure 10-1 is entered using this number of service times, and the average delay is obtained from the corresponding utilization curve.

The average delay in service times thus obtained can be used as a starting value to estimate the average delay at the end of the next hour provided that the utilization during the next hour is at least as high as that of the given hour. To do this, one simply finds (using the curve for the next hour's utilization) the time corresponding to the average delay in service times obtained for the given hour. Adding the number of service times per hour to this time, we obtain the corresponding average delay for the end of the next hour.

In this way, hour-by-hour steps may be taken as long as the utilization does not decrease. This approximate graphical technique furnishes estimates of delay at the end of peak hours, whether overloaded or not. For example, suppose the service time were assumed to be 1 minute, then, 1 hour would represent 60 service times. Suppose the airport opens up and the traffic during the first hour has a utilization of 0.5, the second hour 0.8, and is overloaded during the third hour with a utilization of 1.2. At the end of the first hour, at 60 service times, the average delay for 0 5 utilization is 0.5 service time. This delay intersects the 0.8 utilization curve at about t = 1 service time. Adding 60 to this, and using the 0.8 curve, the average delay corresponding to 61 service times is about 1.9 service times. This delay intersects the 1 2 curve at about 4 service times, so the average delay at the end of the peak hour (using t = 64) with utilization 1.2, is about 15 service times (or 15 minutes).

This type of step-by-step analysis has been tested and agrees quite closely with the actual computation. In a previous paper by Galliher and Wheeler,* a nonstationary analysis was applied using first-come, first-served constant service queuing. Applying these approximate step-by-step relations to the exact programming used for that effort showed that the agreement in average delay was within 2 percent of the peak-hour value.

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* H. P. Galliher and R. C. Wheeler, Jr., "Nonstationary Queuing Probabilities for Landing Congestion of Aircraft," Operations Research, Vol 6, No. 2, p 264, March-April 1958.

XI. DISTRIBUTION OF SAMPLE AVERAGES

A person who wanted to measure the delay experienced by departing aircraft might go to the airport and, starting at a convenient idle period, observe and record the times when departures informed the controller they were ready for take-off and the times when the controller cleared the departures for take-off. The observer might continue to do this for several hours of operation during which time the number of departures may have accumulated to a sample size of 50 or 100. The observer could then total the individual delays obtained for each aircraft of the sample, divide the total by the number of departures in the sample, and thus obtain a sample average delay. The average movement rate can be estimated from the elapsed time, and other parameters such as runway time or start-roll to start-roll time, can be recorded. Using this information, the parameters for the steady-state models can be determined, and the steady-state average delay can be calculated.

The measured sample average may agree or disagree with the answer obtained from the model. In either case, it is natural to ask whether the difference between the model answer and the measured sample average is significant. In other words, how much variation can be expected in a sample average if the sampling experiment is repeated on successive days?

To obtain this variability of a given sample average, an investigation was made to see if the distribution of sample averages could be determined analytically. The analytical approach proved to be rather formidable, so the desired distributions were obtained by simulation techniques.

Accordingly, a simulation program was developed for the spaced arrivals model with the first-come, first-served queuing model as a special case. Details of this program are given in Appendix F. For the first-come, first-served queuing model, constant service time was used and three utilizations were selected 0.5, 0.7, and 0.9. In a single run, cumulative sample averages were recorded for 29 different sample sizes, ranging from 1 to 1000. The number of runs per case was generally 500 For each sample size, the 500 sample means were sorted and ranked and the percentiles were obtained. The mean and standard deviation (of the 500 sample means) were also calculated for each sample size. The percentiles, the mean, and the standard deviation thus obtained are shown in Figures 11-1, 11-2, and 11-3. These curves are all normalized to the steady-state value for each utilization, so that one unit on the vertical scale represents the steady-state value.

Several interesting points will be noted about these families of curves First, when plotted in the normalized units, the families for the different utilizations appear quite similar, with the principal difference being the location of the peaks. The irregularities in the curves are due to the finite number of simulation runs (generally 500) and would smooth out if the number of runs were increased.

With a sample size of 1, of course, all the percentiles, the average delay, and the standard deviation are identically 0, since it was assumed that the sampling process starts with an empty system. Therefore, the first aircraft always has 0 delay. As the sample size increases, the standard deviation is seen to rise to a peak and then drop again. The location of the standard-deviation peak occurs in different places--for a utilization of 0.5, the peak occurs at a sample size of about 13, for 0.9 utilization, the sample size is about 200. The reason for a peak, of course, is that

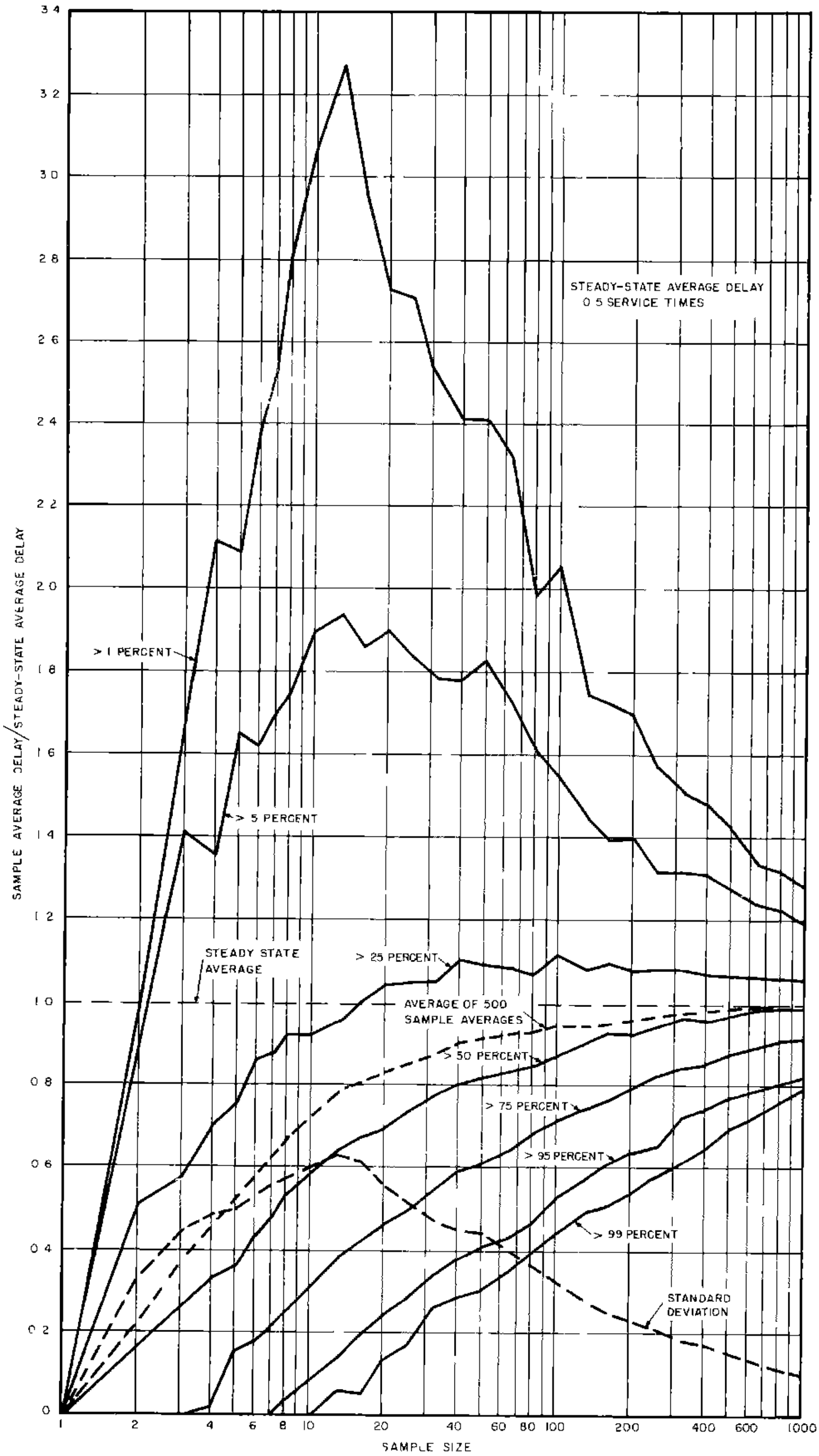

FIGURE 11-1 DISTRIBUTION OF 500 SAMPLE AVERAGES FOR FIRST-COME, FIRST-SERVED CONSTANT SERVICE QUEUE (UTILIZATION 0 5)

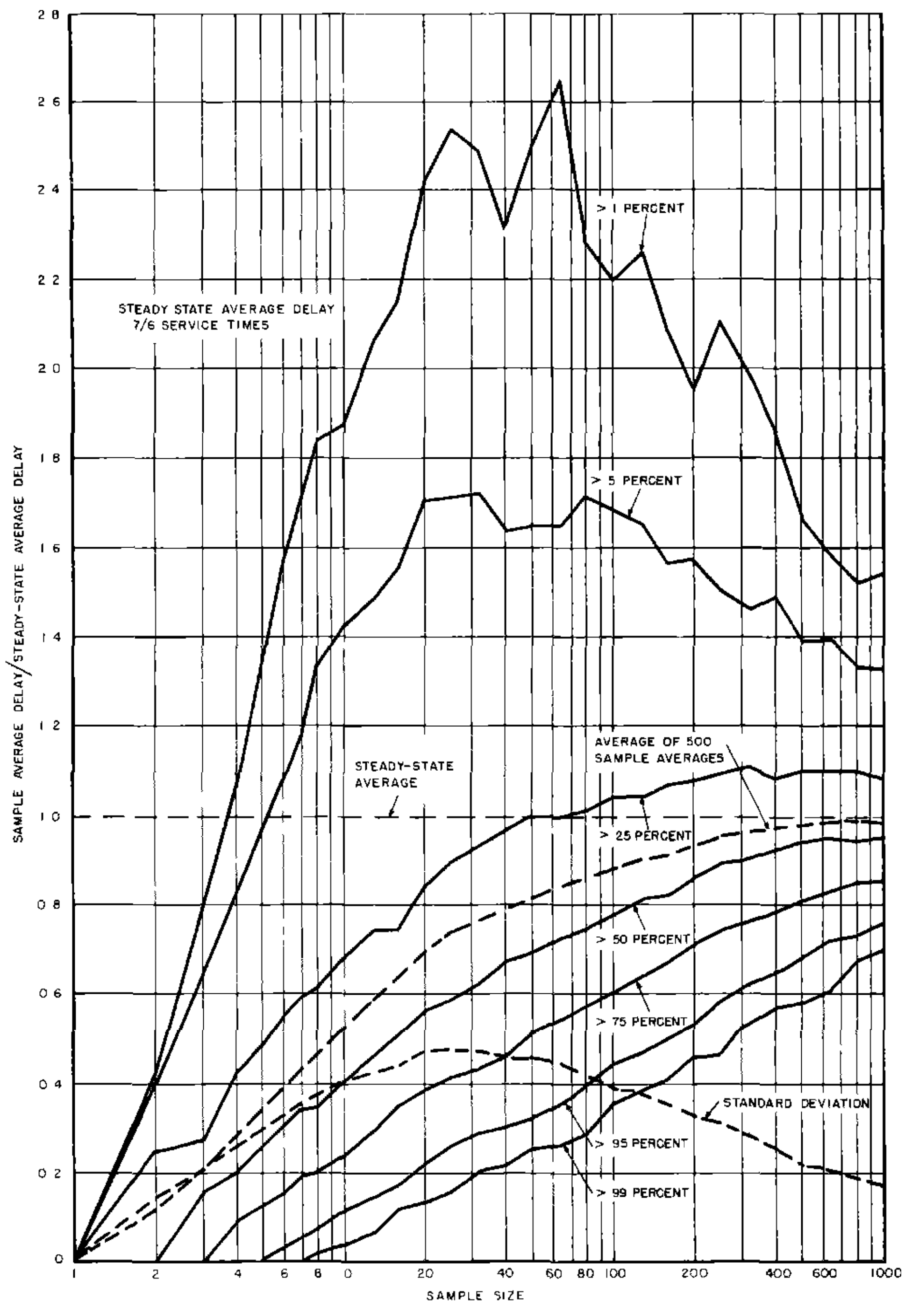

FIGURE 11-2 DISTRIBUTION OF 500 SAMPLE AVERAGES FOR FIRST-COME, FIRST-SERVED CONSTANT SERVICE QUEUE (UTILIZATION 0 7)

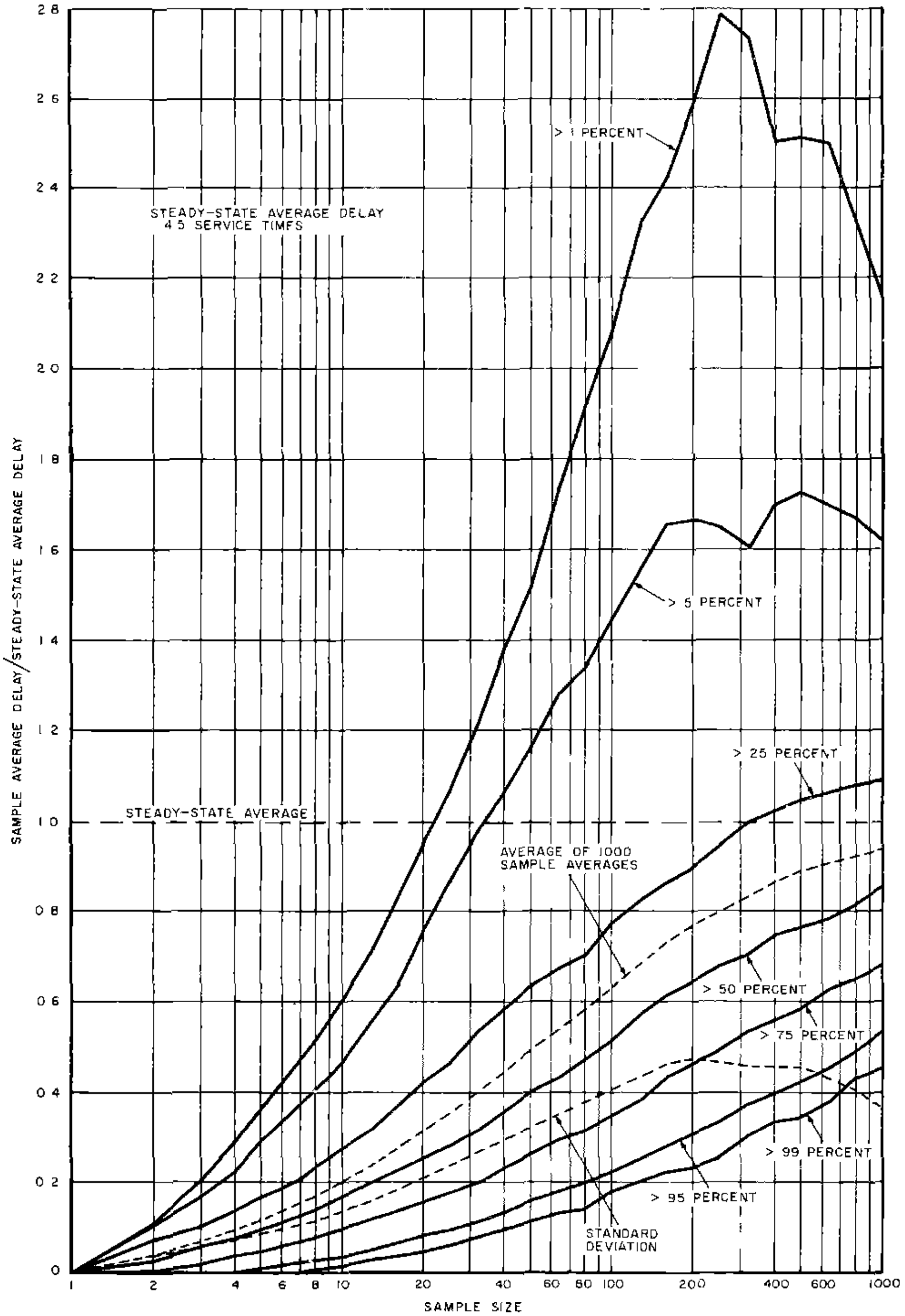

FIGURE 11-3 DISTRIBUTION OF 1000 SAMPLE AVERAGES FOR FIRST-COME, FIRST-SERVED CONSTANT SERVICE QUFUE (UTILIZATION 0 9)

for a sample size of 1, the standard deviation is necessarily 0, and as the sample size increases without bound, the standard deviation again tends to become 0 since the difference between sample means as the sample size increases indefinitely, asymptotically approaches 0 for all distributions.

The peak of the 5 percentile is seen to vary from about 1.7 to 1.9 times the steady-state average. This means that for 5 percent of the time (or 1 chance in 20), one can obtain a single sample average greater than 1.7 times the steady-state average. The sample size would be only 13 for a utilization of 0.5, but 500 for a utilization of 0.9 Using the 95-percent curves to obtain the lower limits, we see that the 90-percent middle range contains a 4 to 1 range of sample average for 0.9 utilization, and a 14 to 1 range for 0.5 utilization. This is, of course, a rather remarkable spread, and, as we shall soon see, ties in directly with the problem of correlating data with model.

Two cases were selected for the spaced arrival simulation models. One was for the 9000-foot runway continental airport with 1000-foot turn-offs (Figure 11-4). The other used parameters obtained from measured data taken at Miami Airport on 4 December 1959 because data from this particular airport had what seemed like the poorest agreement in average delay between the spaced arrivals model theory and the observed data (Figure 11-5). The steady-state spaced arrivals model in Figure 11-5 indicated a steady-state delay for departures of 274 seconds. The measured delay was 167 seconds (for a sample size of 82), which is only about 60 percent of the steady-state value. This seemingly large discrepancy can be explained directly by means of these curves. For example, in Figure 11-5 for sample size 82, sample averages can be expected to fall within the interval from 156 to 210 seconds or 25 percent of the time. Moreover, 50 percent

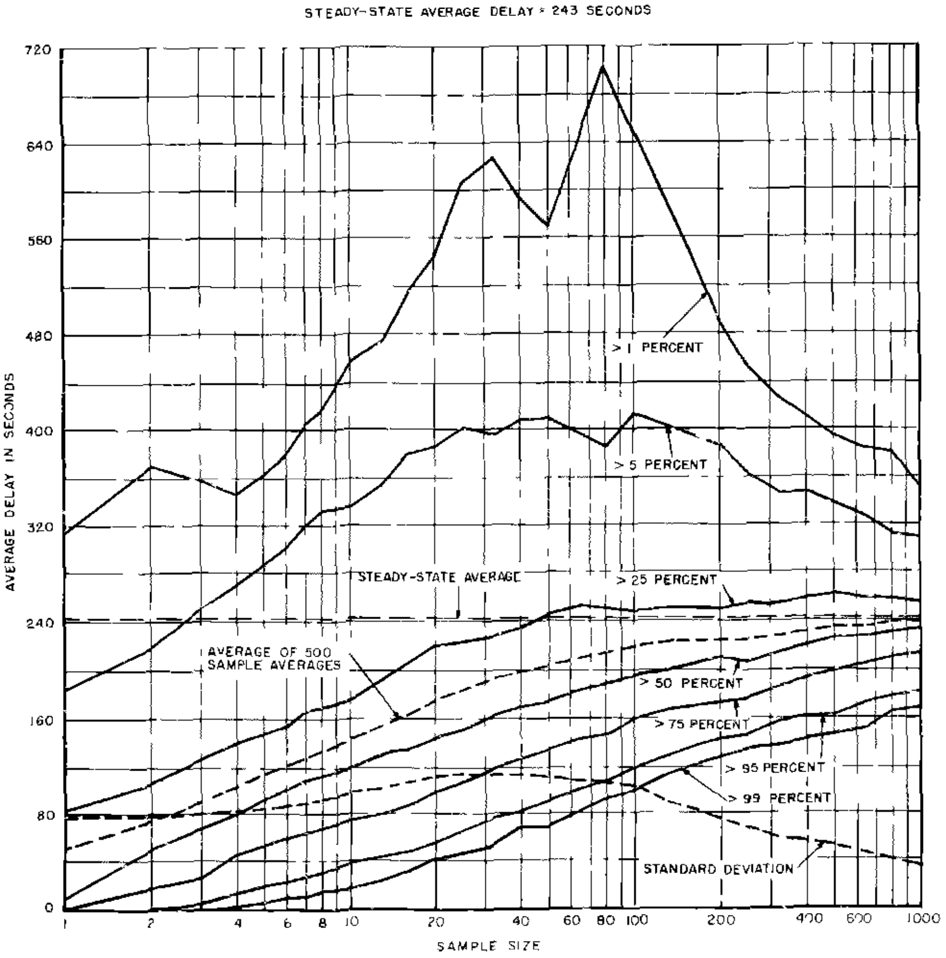

FIGURE 11-4 DISTRIBUTION OF 500 SAMPLE AVERAGES FOR SPACED ARRIVALS ON TYPICAL 9000-FOOT RUNWAY

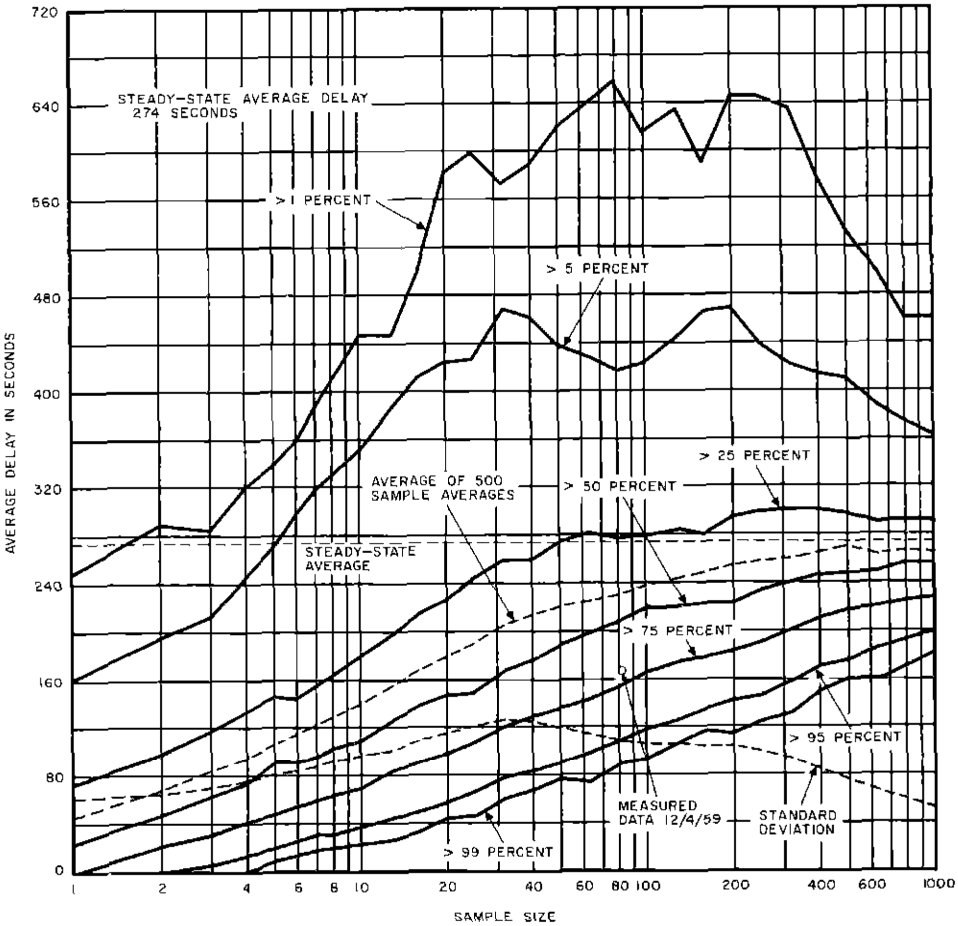

FIGURE 11-5 DISTRIBUTION OF 500 SAMPLE AVERAGES FOR SPACED ARRIVALS FOR MIAMI AIRPORT

of the sample averages can be expected to fall between 156 and 277 seconds, and 90 percent between 110 and 416 seconds. Thus, the measured delay agrees statistically with the spaced arrivals model. To disagree (at the 5-percent significance level), the measured sample mean would have to be below 110 seconds or above 416 seconds.

This analysis of the distribution of sample averages may not have been made before. A very important conclusion to be drawn from this analysis is that queuing operations (either simple queues like first-come, first-served with constant service, or involved models like spaced arrivals) may have sample averages with such a broad distribution that tests of a queuing model with sample averages of delay from observed data must be treated with extreme caution before any conclusions can be made regarding the validity of the model.

What does all this mean as far as airport operations are concerned? First, it indicates that if the airport operation follows perfectly the rules of the spaced arrivals model for mixed operations on a single runway, sample average delays should fall within the large ranges described. However, such wide variations in sample averages are not likely to be found at airports. There are three principal reasons for this.

1 Local pressure factors are used.
2. Model is not rigidly followed.
3. Steady-state conditions have not been reached

Any particular sample of airport operations consists, as we have seen, of busy periods and slack periods. During the busy periods, the airport controllers can be expected to react to the pressure factor that reduces delay, during the slack periods this pressure factor is relaxed. The simulation model, however, used constant factors for all periods--busy and slack--that were determined from busy-period data. Thus, the variation in pressure factor by the controllers could be expected to reduce the spread of sample average delay.

The high sample averages that the curves indicate as occurring quite frequently must involve large departure queues so that the sample mean achieves a high value However, it is a fact that whenever a local busy period for departures occurs, the controllers may occasionally delay arrivals (for example, by path stretching) to get departures off, thus reducing their delay. This changing of the rules does not happen in the rigid simulation model. Thus, in practice, the occasional delay to arrivals tends to reduce the long delays that would be expected to happen to departures Such action by controllers has been observed at several airports.

As was shown in this section, perhaps hundreds of operations are required to achieve a steady state at the higher utilizations The simulation model for Miami shows that for a sample size of 82, the average delay to be expected is 230 seconds, not the steady-state answer of 274 seconds.

XII. DERIVING MATHEMATICAL FORMULAS

This section presents the mathematical derivations of the formulas Their application has been discussed in previous sections

The discussion is presented in three parts

A Departure Delays in Mixed Operation on Single Runway

B. Departure Delays and Capacities on Departures-Only Runway

C. Arrival Patterns, Delays, and Capacities

A. DEPARTURE DELAYS IN MIXED OPERATION ON SINGLE RUNWAY

This section presents two sets of formulas for delays encountered by departures when landings are using the same runway and are being given complete priority over departures. One set of formulas is for when arriving aircrafts are spaced in a certain fashion, the other set is for when arriving aircraft are tightly sequenced in queues. Although the formulas describe the distribution of delay, only the average delay is explicitly derived--and this only for sustained operation at an average rate below operational capacity.

1. GENERAL CONSIDERATIONS

In mixed operations (landings and take-offs) on a single runway, landings are normally given complete priority for use of the runway Departures must thus be released in the intervals between landings, where the pattern of such intervals is determined by the sequences of landing aircraft as if no departures were present. Although such priority is occasionally withdrawn to permit the release of an excessive departure queue, we shall in the following analysis ignore this occasional procedure. Indeed, with the increase in use of radar approach control, the natural tendency is to use approach control to achieve a wider spacing of approaching landings and intervals of sufficient size to release waiting departures. This way of spacing landings is very important in making the most efficient use of the runway, that is, in minimizing the average delay to both landing and departing aircraft.

a. TIME INTERVALS BETWEEN LANDING AIRCRAFT

We must thus focus attention on the time intervals between landing aircraft. A convenient reference point is the approach threshold of the runway. When a landing passes over this threshold (OT designates "over threshold"), there begins an interval R (runway occupancy time of the aircraft) that terminates when the aircraft turns off the runway (OR). This is followed by a time interval G (possibly 0 in length), commencing at that instant and terminating when the next aircraft approaching to land is committed to land. This time of commitment is evidently not a completely precise moment, even for a single aircraft, since at a potential exchange of safety, an aircraft may be viewed as committed sooner or later. Plots of time intervals between landing aircraft that were actually observed exhibit such commitment intervals C. This interval begins sometime prior to the time that the aircraft crosses the threshold and terminates at OT It is typically short for light or highly maneuverable aircraft, longer for less maneuverable aircraft (older piston-engine commercial carriers), and quite long for large jets

If we denote by A the total time interval between runway threshold crossing by two successive landing aircraft, then A is the sum of the intervals R, G, and C,

$$A = R + G + C.$$

Although the threshold is the natural reference point for describing these intervals, for purposes of analysis, it is more natural to shift the origin to the time that the aircraft turns off the runway

$$A = G + C + R.$$

The primary reason for this is that neither during the interval C nor during the interval R can a departure be safely released. Generally, we need not even consider C and R as separate intervals as far as departure release is concerned Accordingly, we shall denote the sum of C and R by B,

$$B = C + R$$

so that

$$A = G + B.$$

Plots of the intervals A are shown in Figures 9-12, 9-13, and 9-14. Further discussion of the nature of the intervals A will be found in paragraph C of this section As we know, the interval G is the most difficult to account for adequately, for it contains the effects of several factors

1. The minimal separation, in addition to C, required between two arriving aircraft when the following aircraft is on final approach.
2. The effect of deliberate spacing of aircraft by approach control (particularly under IFR conditions) and by pilots.
3. The basic time pattern of demand for landing clearance is acted on by approach control and final separation requirements to produce the intervals A

Departure delays are also sensitive to the nature of the intervals G, since departures cannot be released safely in intervals G that are too short

b TWO TYPES OF INTERVAL G

The formulas for departure delays developed in this section apply when the intervals G are of either but not both of two types

(1) POISSON DEMAND FOR LANDING CLEARANCE WITH NO DEPARTURES RELEASED IN FRONT OF DELAYED ARRIVALS

In this case, G = 0 when a landing aircraft has been delayed by a landing aircraft ahead. Otherwise G has an exponential distribution with a mean equal to the average time interval between arrivals The interval C may be enlarged to include the total separation between two landing aircraft when the second is delayed by the first.

(2) SPACED ARRIVAL AT THE THRESHOLD G OF EXPONENTIAL DISTRIBUTION

This is a form of separation in addition to C where the intervals A are assumed statistically independent. The general effect is to produce a more regular pattern of arrivals at the threshold than in case 1, and lower departure delays. Data from observations made of current operations at airports suggest that case 1 characterizes actual landing patterns, but that case 2 gives better agreement between observed delays and those predicted by the formula developed below. The reason for this anomaly may be that the intervals G are of neither type, or that, though case 1 is correct, the steady-state conditions assumed by the formulas were not achieved during the actual operations observed

Mathematically, case 1 is a special case of 2 Accordingly, the discussion will now proceed for case 2. At the end of the discussion the solution for case 1 is easily derived as a special case

c REQUIREMENTS FOR DEPARTURE RELEASE

In addition to the obvious requirement that the runway be clear of aircraft, there are definite requirements that must be fulfilled before a waiting departure can be released One is that sufficient time clearance exists before the next oncoming landing is committed to land.

This clearance, which we call "departure to landing clearance," is required to achieve communication of release between the controller and the departing aircraft, but mostly to allow the departing aircraft time either to clear the far end of the runway or to be at least sufficiently airborne that it is no longer committed to the runway. Obviously, safety is an important factor in determining the value of this clearance as is controller skill. We denote this required time clearance by S_1. Since no departure is released in the interval B, the release requirement for departure to landing clearance is $G \geq S_1$.

When, as often happens, more than one departure is released in a single interval G, the second departure is released in some remainder of the original full interval G. We shall for simplicity refer to any such remaining residual of a full interval G as also an interval G.

The interval S_1 can vary depending upon the type of departing aircraft and is quite long for large jets and aircraft requiring long runway occupancy time prior to the beginning of their roll In the following analysis, provision is made for such variation among aircraft of the departure population Numerical determination of S_1 is described in Appendix A.*

d. DEPARTURE TO DEPARTURE CLEARANCE

Before departure clearance there must exist a sufficient time separation S_2 since release of the last departure. The value of S_2 may depend upon the identity of both the last departing aircraft and the one to be released, particularly where the chief purpose is to prevent post take-off route con-

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* In Appendix A the quantity S_{11} is calculated to satisfy the requirement $G + c \geq S_{11}$ or $G \geq S_{11} - c$. In the above text $G \geq S_1$. S_1 can be approximated from Appendix A by the relationship $S_1 = S_{11} - c$.

flicts between aircraft of very different velocities However, the value of S_2 usually may be made to depend primarily upon the identity of the aircraft to be released We assume this in the following analysis

With respect to both of the clearance requirements discussed, note that in practice they will not necessarily be met as precisely as the statement of them might imply The departure to landing clearance in particular is often estimated by controllers in terms of distances or positions of approaching landings rather than times However, the experienced controller probably learns what time intervals go with what positions, especially since times are of essential importance from the standpoint of real safety The clearance requirements stated here are posited as norms and are consistent with safety and other operating requirements.

e. DEPARTURE INPUT

By "departure input" we mean the set of times at which departing aircraft report themselves ready to go. This means that if the aircraft are immediately cleared for departure, they will take off immediately These times show a considerable fluctuation despite any regularizing effects of scheduling departures because of such factors as lateness of commercial flights, variation in time used in getting to the warm-up pad check-out time, and nonscheduled aircraft in the departure population Measurements of actual ready-to-go times display a perfectly random set of the familiar Poisson type.

f. MEANING OF DEPARTURE DELAY

The basic notion of runway delay for a departure is the time spent waiting once the departure has requested runway clearance for take-off. Sometimes this delay is expended entirely while the departure is waiting off the runway. In

Finally, the method of analysis may also be used to calculate delays even under sustained overloads though no equations or numerical values are given

2. QUANTITATIVE ANALYSIS OF DEPARTURE DELAY

Let us trace what happens to a departure that becomes ready to go (RG) at some time t We can simplify this by noting that when any departure is released, no further departure can be cleared until a special interval $K(S_1, S_2)$ has elapsed·

a If S_2 terminates in an interval G, then $K(S_1, S_2) = S_2$,

b. If S_2 terminates in an interval B, then $K(S_1, S_2) = S_2$ + the remaining time to the end of this interval B

We can now define the time spent by a departure that becomes ready to go at some time t as follows.

a. A wait W_0 occurs until the end of the interval $K(S_1, S_2)$ of the last of all preceding departures W_0 may be 0. We shall use $w_0(x, t)$ as the density distribution of values of $W_0 > 0$.

b If $W_0 = 0$, a wait W_1 until a first interval G occurs W_1 and W_0 may both be 0. We denote this probability by p(t). We shall use $w_1(x, t)$ for the density distribution of values of $W_1 > 0$. The existence of any such value automatically implies that $W_0 = 0$

c. When both waits W_0 and W_1 have been endured, the controller may entertain release of our departure There now begins a further wait $W_2(S_1)$ until an interval $G \geq S_1$ first occurs, at which time $W_2\ (S_1)$ terminates. $W_2(S_1)$ may of course be 0. We shall use $W_2(x|S_1)$ for the distribution function of $W_2\ (S_1)$ and $W_2(x|S_1)$ for the density distribution of positive values.

d. Once release occurs, the interval $K(S_1, S_2)$ is determined for this departure We shall use $K(x|S_1, S_2)$ for its distribution function and $k(x|S_1, S_2)$ for the density distribution of positive values

Clearly the total wait of departure $W(t|S_1)$ is the sum of the first three waits; that is,

$$W(t|S_1) = W_0(t) + W_1(t) + W_2(S_1).$$

We shall find separately each of the terms in this sum. It is most convenient to begin with $W_2(S_1)$, take up the interval $K(S_1, S_2)$, and then compute $W_0(t)$ and $W_1(t)$. The interval $J(S_1, S_2) = W_2(S_1) + K(S_1, S_2)$ is of special significance since it is the nearest thing to a service time for a departure.

The remainder of this paragraph is written for the mathematical reader and may be omitted by those not familiar with the terminology.

a. NOTATION FOR POPULATION CHARACTERISTICS

We use the following notations throughout

$A(t)$ = probability $(A \leq t)$,

e^{-gt} = probability $(G > t)$,

$B(t)$ = probability $(B \leq t)$,

$\frac{1}{a_1}$ = average arrival rate of landing aircraft,

λ_2 = average departure rate.

b. INTERVAL $W_2\ (S_1)$

This interval always begins in an interval G and ends in the first interval G (possibly the same one), which is $\geq S_1$. Hence,

$$W_2(0|S_1) = e^{-gS_1}$$

and for $t > 0$.

$$W_2(t|S_1) = e^{-gS_1} \sum_{n=1}^{\infty} \left[\int_0^{S_1} ge^{-gx}\ b(t - x) \right]^n$$

These relationships are conveniently summarized in Laplace transforms. If we let

$$w_2(\bar{\theta}|S_1) = \int_0^{\infty} e^{-\theta x}\, dW_2(x),$$

then

$$w_2(\bar{\theta}|S_1) = e^{-gS_1}\left[1 - (1 - e^{-(g+\theta)S_1})\, a(\bar{\theta})\right]^{-1}$$

We use the following notational conventions throughout:

For any function $F(. \quad x \quad . .)$, $f(. . . \bar{\theta} \quad . \quad)$ denotes $\int_0^{\infty} e^{-\theta x}\, dF(\quad . \quad x \quad .)$

For any function $F_1(\quad x . . .)$, $f_{1n}(. . \quad)$ denotes $\int_0^{\infty} x^n\, dF_1(\quad . . x . . .)$

Thus, $W_{21}(S_1)$, the average value of $W_2(S_1)$, is:

$$W_{21}(S_1) = a_1\left[e^{gS_1} - 1\right] - S_1$$

When S_1 varies among the departure population, we denote by W_2 the expected interval $W_2(S_1)$, the expectation being taken over the distribution of S_1. In that case, we also have an average numerical value W_{21} of the interval W_2.

c. INTERVAL $K(S_1, S_2)$

Actual observations as well as general considerations indicate that ordinarily in single runway operation

$$B + S_1 > S_2.$$

As a consequence,

$$K(x; S_1, S_2) = 0 \text{ for } x < S_2$$

$$K(S_2; S_1, S_2) = e^{-g(S_2 - S_1)}$$

and for $t > S_2$:

$$k(t | S_1, S_2) = \int_{S_1}^{\min\left[t - S_2 + S_1, S_2\right]} ge^{-gx} \, b(t - x) \, dx \, e^{gS_1}$$

If we let

$$k(\overline{\theta} | S_1, S_2) = \int_0^{\infty} e^{-\theta x} \, dK(x | S_1, S_2),$$

then

$$k(\overline{\theta} | S_1, S_2) = e^{-\theta S_1} \left[a(\theta) + e^{-(g + \theta)(S_2 - S_1)} \left[1 - a(\overline{\theta})\right]\right]$$

d. INTERVAL $J(S_1, S_2)$

If we denote the density and the distribution functions of

$$J(S_1, S_2) = W_2(S_1) + K(S_1, S_2)$$

by $j(x|S_1, S_2)$ and $J(x|S_1, S_2)$, respectively, and if we let

$$j(\overline{\theta}|S_1, S_2) = \int_0^{\infty} e^{-\theta x}\, dJ(x|S_1, S_2),$$

then

$$J(\overline{\theta}|S_1, S_2) = W_2(\overline{\theta}|S_1)\; k(\overline{\theta}|S_1, S_2).$$

For average total delay, the following are needed·

$$j_1\,(S_1, S_2) = a_1 e^{gS_1}\left[1 - e^{-gS_2}\right]$$

$$\frac{j_2(S_1, S_2)}{2} = j_1(S_1, S_2)\, W_{21}(S_1) + \frac{a_2}{2} e^{gS_1}\left[1 - e^{-gS_2}\right] - a_1 S_2 e^{-g(S_2 - S_1)}$$

e INTERVAL J

When S_1 and S_2 vary considerably among the aircraft of the departure population, it can become important to average the interval $J(S_1, S_2)$ over the population of values of S_1 and S_2. If $U(S_1, S_2)$ denotes the joint distribution function of S_1 and S_2 among departures, we then write formally:

$$j(x) = \int_{S_1, S_2} J(x|S_1, S_2)\; dU(S_1, S_2).$$

In practice, aircraft that require an abnormally large value of S_2 will probably require a larger than average value of S_1

f. INTERVAL $W_1(t)$

The density $W_1(x;\ t)$ will be found to satisfy the following differential equation:

$$w_1(x,\ t + dt) = w_1(x + dt,\ t)(1 - \lambda_2 dt) + p(t)\ g\ dB(x)$$

For large t this approaches

$$0 = \frac{d}{dx} w_1(x) - \lambda_2 w_1(x) + g\ p\ b(x)$$

which is

$$0 = (\theta - \lambda_2)\ w_1(\overline{\theta}) + \lambda_2\ W_{10} - g\ p\left[1 - b(\overline{\theta})\right]$$

From this we find two results that will be useful

$$\lambda_2 W_{11} = g b_1 p - W_{10}$$

$$\lambda_2 W_{12} = g b_2 p - 2W_{11}$$

g. MAXIMUM UTILIZATION

As a necessary condition for $W_1(t)$ not to become large without bound as t becomes large, we find by direct analysis that

$$\lambda_2 j_1 < 1$$

Thus, $\lambda_2 j_1$ can be termed the departure traffic load (or utilization).

h. TOTAL WAIT W OF DEPARTURE

As the length t of operation becomes large, the wait $W(t|S_1)$ has a limiting value W when $\lambda_2 j_1 < 1$. Denoting by $W(x)$ the distribution function of $W(t|S_1)$ averaged over values of S_1, we have

$$W(\theta) = \left[p + W_1(\theta) + W_0(\theta)\right] W_2(\theta).$$

Consequently, the average total wait W_1 of a departure is given by

$$W_1 = W_{21} + W_{11} + W_{01}$$

$$= W_{21} + \frac{\lambda_2 j_2}{2\left[1 - \lambda_2 j_1\right]} + \frac{b_2}{2a_1} .$$

3 POISSON ARRIVALS

Earlier we stated that from this solution we can obtain, as a special case, the delay when the demand for landing clearance is Poisson and no departures are released in front of delayed arrivals. We now consider this case.

Ignoring departures, the arrival operation becomes a simple single-server queuing operation The interval C + R can be taken as the total time separation at the threshold between two successive landings when the second landing is delayed by the preceding aircraft For reasons that will soon be clear, we prefer to denote the interval C + R by H, and its n^{th} moment by h_n.

The landing arrival rate will be denoted by λ_1, since the interval a_1 of case 2 (discussed earlier) will receive a different interpretation

The arrival operation now divides itself into an alternation between idle and busy periods. An idle period I begins when an arriving aircraft turns off the runway if the next arrival is not delayed. The idle period ends when the next arrival reaches a time H prior to turning off the runway (that is, enters service). A busy period begins at the end of an idle period I and terminates at the beginning of the next idle period I. Thus, a busy period consists of a sequence of one or more landings each of which (except the first) is delayed by the preceding landing.

We may now choose the interval B of case 2 as such a busy period--that is, as a sequence of intervals H--since no departure is released between two landing aircraft, the second of which is delayed. Consequently, the interval A of case 2 must be taken as

$$A = I + B.$$

An idle period I has an exponential probability distribution with an average length a_1. Busy periods B have, however, distinctive distributions that can be derived from the distribution of H. It may be shown from queuing theory that

$$b(\overline{\theta}) = \int_0^{\infty} e^{-\left(\theta + \left[1 - b(\overline{\theta})\right]\right) x} \, dH(x)$$

where H(x) is the probability $(H \leq x)$.

As a consequence it may be verified that

$$b_1 = \frac{h_1}{1 - \rho_1}$$

$$b_2 = \frac{h_2}{(1 - \rho_1)^3}$$

where $\rho_1 = \lambda_1 h_1$ is the landing utilization.

Accordingly, we may still use the formulas derived in paragraph B for case 2 if we also substitute λ_1 for g in addition to the above changes for b_1 and b_2.

This implies that now

$$a_1 = \frac{1}{\lambda_1} + b_1 \frac{1}{\lambda_1(1 - \rho_1)}$$

$$a_2 = b_2 + \frac{2a_1}{g} = \frac{h_2}{(1 - \rho_1)^3} + \frac{2}{\lambda_1{}^2(1 - \rho_1)}$$

The expressions for W_{21} (S_1), j_1 (S_1, S_2), $\frac{j_2}{2}$ (S_1, S_2) and W_1 become

$$W_{21}(S_1) = \frac{\left[e^{\lambda_1 S_1} - 1\right]}{\lambda_1(1 - \rho_1)} - S_1$$

$$j_1(S_1, S_2) = \frac{e^{\lambda_1 S_1}\left[1 - e^{-\lambda_1 S_2}\right]}{\lambda_1(1 - \rho_1)}$$

$$\frac{J_2(S_1, S_2)}{2} = J_1(S_1, S_2)W_{21}(S_1) + \frac{1}{2}\left[\frac{h_2}{(1 - \rho_1)^3} + \frac{2}{\lambda_1^2(1 - \rho_1)}\right] e^{\lambda_1 S_1}\left[1 - e^{-\lambda_1 S_2}\right]$$

$$- \frac{S_2\, e^{-\lambda_1(S_2 - S_1)}}{\lambda_1 (1 - \rho_1)}$$

$$W_1 = W_{21} + \frac{\lambda_2 J_2(S_1, S_2)}{2\ 1 - \lambda_2 J_1 (S_1, S_2)} + \frac{\lambda_1 h_2}{2(1 - \rho_1)^2}$$

To apply these formulas to taxiing aircraft crossing an active runway, we use the pre-emptive Poisson arrivals model by letting $S_1 = S_2$, and identifying the low-priority service time with S_2 (and moments S_{21}, S_{22}) and the high-priority service time H with S_1 (and moments S_{11}, S_{12}), we obtain, after averaging over $S_2(t)$,

$$W_{21}(S_2) = \frac{s_2(-\lambda_1) - 1}{\lambda_1(1 - \rho_1)} - S_{21}$$

$$J_1(S_2) = \frac{s_2(-\lambda_1) - 1}{\lambda_1(1 - \rho_1)}$$

$$\frac{J_2(S_2)}{2} = J_1(S_2)\, W_{21}\, (S_2) + \frac{1}{2}\left[\frac{S_{12}}{(1 - \rho_1)^3} + \frac{2}{\lambda_1^2(1 - \rho_1)}\right]\left[s_2(-\lambda_1) - 1\right] - \frac{S_{21}}{\lambda_1(1 - \rho_1)}$$

$$= \frac{s_2(-2\lambda_1) - 1}{\lambda_1^2(1 - \rho_1)^2} + \frac{\left[s_2(-\lambda_1) - 1\right]}{2}\left[\frac{S_{12}}{(1 - \rho_1)^3} - \frac{2(1 + \rho_1)}{\lambda_1^2(1 - \rho_1)^2}\right] + \frac{S_2'(-\lambda_1)}{\lambda_1(1 - \rho_1)}$$

$$W_1 = W_{21}(S_2) + \frac{\lambda_2 j_2(S_2)}{2\left[1 - \lambda_2 j_1(S_2)\right]} + \frac{\lambda_1 S_{12}}{2(1 - \rho_1)^2}$$

Here, we define

$$s_2(\theta) = \int_0^\infty e^{-\theta t}\, d\, S_2(t).$$

B. DEPARTURE DELAYS AND CAPACITIES ON DEPARTURES-ONLY RUNWAY

1. AVERAGE DELAY UNDER CONSTANT TRAFFIC LOAD

When a single runway is being used for departures only, a fairly simple formula results for average delay when operating conditions remain at a constant average traffic load below the capacity of the runway. Under such conditions the average delay W_D per aircraft departing, expressed in units of the average minimal departure separation time, may be estimated by.

$$W_d = \text{average departure delay} = \frac{\rho}{2(1-\rho)}\left[1 + C^2\right] \text{ separation times} \tag{1a}$$

where ρ = utilization = (rate at which aircraft depart) x average minimal time separation between two departing aircraft = λS_{21}, C = coefficient of variation of the minimal time separations S_{21} allowable between releases of departing aircraft, that is, $C = \sigma(S)/S_1$.

Another way of expressing the variability of the service time S_{21} is in terms of the Erlang K factor where

$$K = \left(\frac{\text{mean service time}}{\text{standard deviation}}\right)^2$$

$$= \left[\frac{S_{11}}{\sigma(S)}\right]^2 = \frac{1}{C^2}$$

The expression for average departure delay in this case is

$$\frac{\rho}{2(1 - \rho)} \left(1 + \frac{1}{k}\right) \text{ service times.} \qquad (1b)$$

When the minimal time separations S are constant for all departing aircraft, $\sigma(S) = 0$ and, therefore, $C = 0$ and $K = \infty$. Thus the delay is less than when differences in characteristic climb-path velocities, runway occupancy times, or other reasons vary the values of S considerably among pairs of departures The measured value of C^2 is seldom greater than 0.17 for typical mixtures present at airports.

An alternative form of equation 1a gives the delay directly in time units

$$W_d = \frac{\lambda S_{22}}{2(1 - \lambda S_{21})} \qquad (1c)$$

Here S_{22} is the average value of the square of the minimum departure to departure separation times

In formulas 1a and 1b, the utilization ρ is expressed as a fraction of capacity. That is,

$$\rho = \text{utilization} < 1$$
$$\text{capacity} = \frac{1}{S_{21}} \text{ aircraft} \qquad (2)$$

2. MODERATE SURGES IN TRAFFIC LOAD

Most airports encounter periods when the traffic load ρ is typically greater than at other periods in the day.

Although formulas 1a, 1b, and 1c may not then correctly predict delays to aircraft, they nevertheless afford a sound basis for determining requirements for runway capacity. This is particularly true for long-range planning, since there is economic pressure on commercial air traffic to avoid schedules producing severe peaks in the traffic load. An alteration of only a few minutes in departure schedules reduces these peaks.

The formula itself (and other average delay formulas in this report) allow for considerable fluctuation in short periods of the actual load because the ready-to-go-times of departures have a Poisson distribution. Table 12-I illustrates magnitudes of fluctuations from average load tolerated by the above formula.

TABLE 12-I

AVERAGE TOLERATED LOADS

λ_2 = Average Number of Departure Requests per Hour	Actual Number of Departure Requests per Hour: Middle 50 Percent Range	Actual Number of Departure Requests per Hour: Middle 90 Percent Range
10	8 to 12	6 to 16
20	17 to 23	14 to 28
30	26 to 34	21 to 39
40	36 to 44	30 to 50
50	45 to 55	39 to 61
60	55 to 65	48 to 72

A 10-hour period, during which a total of 10 λ_2 aircraft depart, might easily consist of the following hourly number of departures occurring in any single hour

$10\ \lambda_2$	Departures by Hour									
	1	2	3	4	5	6	7	8	9	10
100	17	11	6	7	10	5	9	13	13	9
300	34	24	19	32	27	29	36	42	28	29

Such patterns of variation are incorporated in the derivation of the above delay formula as long as the <u>average</u> load ρ is less than 1.

3. DETAILED ANALYSIS OF VARYING LOAD

When the traffic load varies greatly with the time of day, rising to a high peak (several times the average load) at certain typical hours, and perhaps exceeding the capacity of the runway, then the above formula for delay may not be sufficiently informative. In that case it may be necessary to formulate in complete detail equations for the probabilities of delay of all magnitudes. These equations are presented now, beginning with a mathematical account The disinterested reader may omit this and skip from here to Primary Effect of Overloading.

4. DEFINITIONS AND ASSUMPTIONS

We divide the population of departing aircraft into those classes that generate very different departure-to-departure minimal tolerable separation times (for example, large jets versus light aircraft, classes determined by length of common climb path, or post take-off route differences). Denoting these classes by i, j, etc., let S_{ij} denote the minimal separation time allowable when an aircraft of type j follows an aircraft of type i These time separations S_{ij}

are measured from start-roll of the i^{th} aircraft to start-roll of the j^{th} aircraft

It is mathematically convenient and not unrealistic to assume that S_{ij} varies from one individual pair of aircraft to another in the same i and j classes. Moreover, we must recognize the possibility that S_{ij} depends upon the time t at which the j^{th} aircraft becomes ready to start-roll so as to cover the pressure factor change in S_{ij} with change in the departure movement rate. Therefore, we replace S_{ij} by $S_{ij}(t)$ to denote this time dependence, and for fixed i and j we treat $S_{ij}(t)$ as a random variable Accordingly, we let

$$S_{ij}(x,t) = \text{probability} \left\{S_{ij}(t) \leq x\right\}. \tag{3a}$$

All values of $S_{ij}(t)$ are assumed to be statistically independent. For mathematical convenience we shall, however, work with the "imputed" density function

$$s_{ij}(x,t) = \text{probable density} \left\{S_{ij}(t) = x\right\} \tag{3b}$$

which may contain integrable impulses at jumps of $S_{ij}(x,t)$

The rate at which departures of type i become ready to start-roll at time t will be denoted by

$$\begin{aligned} \lambda_i(t) &= \text{average departure demand rate for aircraft of type } i, \\ \lambda(t) &= \text{average departure demand rate of all aircraft.} \end{aligned} \tag{3c}$$

This is assumed to be a Poisson process for aircraft of each type and for aircraft of all types (not necessarily stationary) thus, for example, the number $N(t_1,t_2)$ of all types of departures becoming ready to start-roll between t_1 and t_2 has the probability

If we let

$$w(\overline{\theta},t) = W(o,t) + \int_0^\infty e^{-\theta x}\, w(x,t)\, dx$$

and

$$S(\overline{\theta};t) = \int_0^\infty e^{-\theta x}\, S(x,t),$$

these equations are summarized as

$$\frac{\partial}{\partial t} w(\theta,t) = \left[\theta - \lambda(t) + \lambda(t)\, S(\overline{\theta};t)\right] w(\overline{\theta},t) - \theta\, W(o,t) \qquad (8)$$

Equation 7 constitutes a scheme that can be solved for specified value of W(x,o) recursively in t by numerical computation A detailed procedure for doing this on a high-speed computer is included at the end of this section The solution permits tabulation of waiting-time probabilities at any time t*

6 PRIMARY EFFECT OF OVERLOADING

By direct inspection of equation 7 we may find the primary effect of overloading

Let $w_1(t)$ be the average delay of an aircraft arriving at time t. Then

$$\frac{\partial}{\partial t} w_1(t) = p(t) - 1 + W(o,t) \qquad (9)$$

---

* The special case when all S(t) have the same value was computed for typical overloads, and is reported by H. P Galliher and R. C Wheeler, "Nonstationary Queuing Probabilities for Landing Congestion of Aircraft," Operations Research, March-April 1958.

where

$$p(t) = \lambda(t) \times (\text{average minimal separation time at } t)$$
$$= \text{traffic load at time } t$$

When the typical overload occurs sharply and is sustained at a constant rate above capacity, the average delay tends to increase in time at a rate which, though initially larger than the excess load by $W(0;t)$, finally equals the excess load. Since $W(0,t)$ decays rapidly with high overload, its effect may be ignored for such approximate arguments.

For example, if the average delay was 2 minutes just prior to a peak hour, the average delay at a peak-hour utilization of 1.2 will increase about $0.2 \times 60 = 12$ minutes per hour, reaching an average of 14 minutes delay at the end of an hour. For time-dependent delay, this approximation is quite close to the exact increase in delay presented.

Denoting the variance of W_t by $\sigma^2(W_t)$, we find that

$$\frac{\partial}{\partial t} \sigma^2(W_t) = \lambda(t) \left\{\sigma^2\left[S(t)\right] + S_1{}^2(t)\right\} - 2W(0,t). \qquad (10)$$

Thus, as $W(0,t) \longrightarrow 0$, the variance of the wait increases without bound.

The effectiveness of compressing separation times can also be seen in equation 9 If an increase in $\lambda(t)$ to $\lambda^*(t)$ is matched by a change in $S(x;t)$ to $S^*(x,t)$ then

$$\lambda^*(t) \quad 1 - S^*(\overline{\theta},t) \quad = \lambda(t) \quad 1 - S(\theta,t) \ . \qquad (11a)$$

The delay will not be affected by the increase in $\lambda(t)$. Another requirement implied for $S^*(x,t)$ is that

$$\begin{aligned} &\lambda^*(t)\ S^*(t) = \lambda(t)\ S(t) \\ &\text{i.e., } p^*(t) = p(t) \end{aligned} \qquad (11b)$$

This requirement cannot, however, usually be met for high overloads.

7 SUSTAINED OPERATION

When $\lambda(t)$ can be approximated by a constant λ, and $S_{ij}(t)$ by random variable S_{ij} that does not vary in time, the asymptotic solution (for large t) of equation 8 becomes

$$w(\overline{\theta}) = \frac{1 - \rho}{\left[\theta - \lambda \quad 1 - S(\theta)\right]} \qquad (12)$$

By algebraic analysis we find that

$$W_d = \text{average delay} = \frac{\lambda S_2}{2(1 - \rho)}$$

and

$$\sigma_d^2 = \text{variance of delay} = W_d^2 + \frac{\lambda S_3}{3(1 - \rho)}$$

C. ARRIVAL PATTERNS, DELAYS, AND CAPACITIES

Two theories are presented and discussed Poisson arrivals and spaced arrivals

1. CONCEPT OF ARRIVAL DELAY

To give meaning to the term "delay," it is necessary to establish what would be the arrival times of aircraft if delay did not occur. Delay is then the difference between such potential arrival times and the actual arrival time. For departures, the potential departure times can be quite accurately determined from the times at which requests for take-off clearance occur. For arrivals, however, no such determination can be made. Furthermore, delay itself cannot be readily computed, since it is usually consumed in maneuvers, and sometimes at a considerable distance from the airport.

We are thus forced to establish potential arrival times by theoretical argument which limits us to deducing the amount of delay. These deductions can be checked against common experience grossly, and the theory can be checked for time patterns in which arriving aircraft pass a fixed point such as the runway threshold, etc

2. POTENTIAL LANDING TIMES UNDER AMPLE EN ROUTE CAPACITY

When the volume of air traffic between airports does not seriously strain the capacity of the en route airspace or approach zone, it would appear quite safe to assume that potential landing times are Poisson (with possibly a time-varying average rate). As long as sufficient altitude assignments are available to accommodate aircraft traveling on identical lanes, the many altitudes, the many remote origins, and the variable departure times and en route times make the potential arrival times independent and produce a Poisson pattern.

3. ACTUAL LANDING TIMES

Delay occurs only because the potential landing time of one aircraft is too close to that of another. The trailing aircraft must establish a separation in time from the aircraft it is following. This separation time extends in front of the trailing aircraft and is analogous to service time in queuing operations. It also limits runway access to any third trailing aircraft A third aircraft must establish, in turn, a separation time behind the second.

Thus, actual sequences of over-threshold times by successive aircraft will consist of a set of times for an aircraft queue, all but the first of which has been delayed by the aircraft ahead, and a passage time of an aircraft that has not been delayed--that is, whose potential landing time was later than the over-threshold time of the last previous aircraft by more than minimal separation time

If the inter-threshold time separations are governed by requirements of separation beyond the threshold (for example, on the runway), then passing the threshold is analogous to start of service in a queue. However, if requirements of separation on final approach dominate, then the over-threshold times are analogous to the service times for completion of a queue

This report shows plots of the time intervals of successive aircraft in passing the threshold, these data were observed at a number of airports The plots indicate that the time patterns observed are very similar to those for completion of service The following are formulas for such service times in the ideal case of a Poisson-fed queue with independent service times

4. INTER-OUTPUT AND INTER-START SERVICE TIMES IN POISSON-FED QUEUE

a. INTER-OUTPUT INTERVALS

Suppose a single-server queuing operation is fed by Poisson arrivals with mean rate λ, and that service-time B is statistically independent with probability-density $b(t)$ and n^{th} moment b_n. The time-interval I begins when one unit finishes service and ends when the next unit finishes service. This interval is the inter-output interval. As interval I begins, either of two situations arises

1. Another unit is waiting to be served. The probability of this we denote by p (in this case the interval I is a service-time B).
2. No unit is waiting to be served. The probability of this is 1 - p and the interval I exceeds a service time by an interval G (the next unit arriving at the end of G).

If we denote by $f(t)$ the probability-density function of interval I, then

$$f(t) = p\ b(t) + (1-p) \int_0^t \lambda\ e^{-\lambda(t-x)}\ b(x)dx$$

Imposing the requirement that the average value of interval I be equal to the average interval $1/\lambda$ between arrivals, we find that

$$p = \lambda b_1 = \rho = \text{utilization.}$$

A more useful formula is

$$G(t) = \text{prob } (I < t)$$

and this will be found to be

$$G(t) = \int_t^{\infty} b(x)dx + (1 - p)\, e^{-\lambda t} \int_0^t e^{\lambda x}\, b(x)dx.$$

Ordinarily, B has some maximum value B_{max} (particularly in this application to aircraft). Consequently, for $t > B_{max}$,

$$G(t) = c\, e^{-\lambda t}$$

where

$$c = (1 - p) \int_0^{\infty} e^{\lambda x}\, b(x)dx.$$

If we were to plot observed values of G(t) on semi-logarithm paper, we would find that for values of $t > B_{max}$, G(t) is ideally a straight line, with a slope equal to the arrival rate (This linearity also occurs when the intervals B are of exponential distribution.)

b. ERLANG TIMES B

When the service-times B can be approximated as Erlang intervals, G(t) takes on a form that can be tabulated numerically. When

$$b(t) = \mu^k\, t^{k-1}\, e^{-\mu t}/(k - 1)!$$

$$b_1 = \frac{k}{\mu}$$

G(t) takes the form

$$G(t) = \sum_{j=o}^{k-1} a_1^j\, e^{-a_1}/j! + (1 - p)\frac{e^{-\lambda t}}{\left[1 - p/k\right]^k} \sum_{j=k}^{\infty} a_2^j\, e^{-a_2}/j!$$

where

$$a_1 = \mu t \text{ and } a_2 = (\mu - \lambda)t$$

For Erlang intervals B note that $c < 1$. This is displayed in the graphs, for

$$\int_0^\infty e^{\lambda x}\, b(x)dx = (\mu/\mu - \lambda)^k = (1 - p/k)^{-k} < (1 - p)^{-1}$$

We shall return to this point later.

c. INTER-START SERVICE INTERVALS

Consider the interval I^1 that begins when one unit begins service and ends when the next unit begins service. As I^1 begins, let q denote the probability that in addition to the unit beginning service, still another unit is waiting. If we let e(t) denote the probability-density function of I^1, we then find that

$$e(t) = q\, b(t) + (1 - q)\left[(1 - e^{-\lambda t})\, b(t) + \int_0^t b(x)\, e^{-\lambda x}\, \lambda e^{-\lambda(t-x)} dx\right] =$$

$$= b(t)\left[1 - (1 - q)\, e^{-\lambda t} + (1 - q)\, \lambda e^{-\lambda t} \int_0^t b(x)dx\right]$$

When we require I^1 to have an average value of $1/\lambda$,

$$(1 - q) = (1 - p)\int_0^\infty e^{-\lambda x}\, b(x)dx < 1$$

If we denote by H(t) the probability that $I^1 > t$, we find that

$$H(t) = \int_t^{\infty} b(x)dx + (1 - q)\, e^{-\lambda t} \int_0^t b(x)dx$$

For $t > B_{max}$,

$$H(t) = (1 - q)\, e^{-\lambda t}$$

that is, linearity on a semilogarithm plot.

d. ARRIVAL DELAYS

When arrivals at a single runway are Poisson, the delay may be computed exactly as it is for departures. Since the inter-output curves for arrivals (Section IX) agree with the hypothesis of Poisson input, this can be an effective method for estimating arrival delay. Therefore, the reader is referred to paragraph B of this section.

5. SPACED ARRIVALS (RENEWAL INPUT WITH EXPONENTIAL GAPS)

The theory of Poisson queuing of arrivals assumes that the runway is the first point at which arriving aircraft conflict with each other. That is, it assumes that the flow of aircraft toward the point where final approach begins is essentially uncongested flow

Projected increases in traffic volume will make this assumption less valid in the future. Moreover, it is not so true today of IFR operation, in which traffic backs up beyond the approach zone Finally, increased use of approach control will tend to discourage sharp queuing before landing.

Consequently, it was thought desirable to develop an alternative theory of arrivals with a somewhat greater regularity of arrival flow to the threshold. During high movement rates, such a theory is particularly necessary to avoid the excessively high delays encountered by departures when arrivals occur in Poisson queues. Even in operations at present-day rates, when an excessive number of departures are waiting, there is a tendency to space arrivals to release departure congestion.

The theory that we now present has already been used in this report to compute projected capacities for various airport configurations and aircraft populations. It is not a theory of arrival delays, though it does provide a basis (not developed in this report) for evaluating the net capacities of the airway-approach-airport complex to move aircraft. It also provides a means of evaluating the airport's contribution to such delay--that is, the landing runway, when the runway is studied in conjunction with approach and en route control. Such a study was beyond the scope of this project.

Finally, the theory to be presented affords a basis for predicting departure delays and capacities that have proved quite satisfactory.

a INTER-THRESHOLD TIMES

We assume that the interval A between the passage of the threshold by one aircraft and the next such passage is the sum of two intervals B and G.

$$A = B + G$$

where B exceeds the time R from threshold to turn-off by an interval C, and where G has an exponential distribution.

The interval C may ordinarily correspond to the normal minimum commitment time of the aircraft that is to land next. The interval G is essentially an error-type interval, which includes the effects of all other variables, such as potential landing times, lagging of delayed aircraft by pilots, effects of approach control, etc.

Values of R, C, and G are assumed to be statistically independent, and for periods of time they are assumed to have a common distribution. During such periods of time

$$\frac{1}{a_1} = \frac{1}{\text{average value of A}} = \text{average arrival rate}$$

is assumed to be constant. Such an assumption will not tolerate as wide a variation from the average of the actual number of arrivals during sub-intervals of the period as is tolerable under the assumption of Poisson arrivals. The reason for this is that interval B produces a regularizing effect.

b. CAPACITIES

The response of B to pressure has been noted. As a_1 decreases, the average value b_1 of B decreases. Except for this response, all of the decrease in a_1 would have to be accomplished as a decrease in $1/g$ (the average length of the interval G).

However, it is likely that there are limits as to how much decrease in G can be achieved. Consequently, $1/b_1$ should not be considered the ultimate capacity.

For example, in automobile traffic this renewal model fits the observed traffic flow, but the same warnings about capacity are substantiated. In a single-expressway lane, b_1 is about 1 second when a_1 is 3 seconds. Theoretically, this implies a lane capacity of 3600 vehicles per hour, whereas flows of 1800 vehicles per hour are exceptionally high in practice. This suggests that $1/g$ has a minimum achievable value of about 1 second--about equal to b_1. As higher vehicular flows are attempted, there is a tendency for the flow to break down from a smooth flow to shockwaves.

c. PROBABILITY DISTRIBUTION OF INTER-THRESHOLD TIMES

Denoting the probability densities of A, B, and G, respectively, by a(t), b(t), and ge^{-gt},

$$a(t) = \int_0^t ge^{-g(t-x)}\ b(x)dx$$

The more convenient equation is that used for Q(t) = the probability that A > t. This is

$$Q(t) = \int_t^\infty b(x)dx + e^{-gt} \int_0^t e^{gx}\ b(x)dx.$$

Again we note that, for $t > B_{max}$,

$$Q(t) \rightarrow r\ e^{-gt} \text{ (linear or semilogarithmic)}$$

where

$$r = \int_0^\infty e^{gx}\ b(x)dx > 1.$$

In contrast to the Poisson arrivals, the asymptote of G(t) plotted on semilogarithm paper intercepts the top axis $[G(t) = 1]$ to the right of the origin t = 0; whereas in the Poisson case for Erlang intervals B, the asymptote intercepts the axis to the left of t = 0.

d. ERLANG INTERVALS B

When the intervals B have an Erlang probability distribution of a type described earlier--that is,

$$b(t) = \mu^k\ t^{k-1}\ e^{-\mu t}/(k-1)!$$

we may develop Q(t) in a form for computation, provided that $\mu > g$,

$$Q(t) = \sum_{j=o}^{k-1} a_1^j e^{-a_1}/j! + \frac{e^{-gt}}{(1-g/\mu)^k} \sum_{j=k}^{\infty} a_2^j e^{-a_2}/j!$$

where

$$a_1 = \mu t,$$

$$a_2 = (\mu-g)t.$$

XIII. RECOMMENDATIONS

The analytical models devised in this report should be further tested in additional operational situations to determine how they conform to actual situations. This should be done on a broader basis than has thus far been possible. As this work progressed, the analytical models would be refined when necessary to make their application more universally useful for all airport configurations and operating situations.

Comprehensive IFR operating data should be obtained to properly apply the analytical techniques in IFR. Before undertaking such a program, the elements to be observed should be carefully designed to provide the proper inputs for the models.

Future data taken on airport operations (such as the FAA terminal area studies) should include data collection that would apply directly to the analytical techniques developed in this report.

Similar analytical techniques should be devised for the analysis of operations and space layout involved in the loading, unloading, and servicing of aircraft.

APPENDIX A

BASIC DATA USEFUL IN SELECTING VALUES FOR ELEMENTS OF ANALYTIC DELAY MODELS

Three mathematical models were evolved during this contract, and they are described completely in Sections VII and X. Each is suitable for specified uses in computing average delay to aircraft at an airport

In this section the inputs and the methods used in their derivation are described

Pre-emptive Spaced Arrivals Model (SAM) --This model is used for mixed operations on single, cross, open vee, or interdependent parallel runways Inputs are λ_1, λ_2, S_{11}, S_2, R_1, K, C_1, C_2, and, in addition--for other than single runways, R_2 Output is average delay to all departures. Note that K, R_1, C_1, and C_2 are all interrelated and were used here in a computer program. For manual computation, K and C_2 are replaced by J(R + C) (Section VIII).

First-Come, First-Served Model (FIM).--This model is used for computing the average delay to departures on a departures-only runway, the average delay to arrivals on an arrivals-only runway, the average delay to aircraft at a taxiway intersection, and other taxiway problems. For departures, the inputs are λ_2, S_{21}, and S_{22}. For arrivals, the inputs are λ_1, S_{11}, S_{12} For taxiways, the inputs are λ_1, λ_2, S_{11}, S_{12}, S_{21}, and S_{22}--the latter having more inputs than the first because two separate streams of traffic are considered.

Pre-emptive Poisson Arrivals (PAM).--This model is suggested for use in the computation of the average delay to all aircraft on a taxiway crossing a live runway. The inputs are λ_1, λ_2, S_{11}, S_{12}, and S_{21}

It must be emphasized that for each specific use of each model the inputs are not necessarily directly related--for example, S_{11} in the pre-emptive spaced arrivals model is entirely different from the S_{11} used in the first-come, first-served model as applied to a taxiway intersection. Because of this the inputs to each model will be described separately under their respective headings.

Throughout certain abbreviations will be used to simplify the text

Arrivals

- λ_1 arrival rate - aircraft per hour
- OT over threshold of runway
- OR off runway, or tail cleared

Departures

- λ_2 departure rate - aircraft per hour
- RG ready to go, normally, on piston and turbo-prop aircraft, RG occurs off the runway at end of engine run-up (if any)
- CTO clear to take-off (controller's call)
- SR start roll of take-off
- X cross time for either arrivals or departures at intersections

The spacing intervals and times required as inputs to the mathematical models are fairly straightforward However, it should be stressed that great care was required in their compilation

The person making up the inputs for such work should be familiar with total airport operations, because it is generally true that the inputs are, or should be the actual results of pilots and controllers actions at an airport

To use the various mathematical models for projected airports and mixtures of aircraft, it is necessary to extract certain spacing intervals and times from the observed data or, where observed data are not available or unsuitable, generate new data that should be checked as much as practical against observed data.

As described in Section IX, with very few exceptions, all the model time inputs vary inversely with λ_1 and/or λ_2. In addition, the runway time for arrivals (OT to OR) depends upon the design of the runway and turn-offs, as well as the length of the runway Also, many of the inputs are interdependent, so that any errors in compilation will have a cumulative effect

A PRE-EMPTIVE SPACED ARRIVALS MODEL (SAM)

The pre-emptive spaced arrivals model can be used for the evaluation of either VFR or IFR operating conditions However, since only VFR data are available, only VFR operations are discussed We can visualize the modifications to the several model elements for IFR by comparing VFR and IFR operating rules. The following inputs are required

1. RUNWAY TIME, ARRIVALS (R_1)

This is the mean OT-to-OR time for any given mixture of aircraft on a predetermined runway design

It is a direct input and the standard deviation is required, though the latter in itself is not a direct input

R_1 depends upon

a. Arrival rate λ_1,

b Total length of the runway,

c Number, position, and type of turn-offs,

d Aircraft types

In addition to these factors, R_1 may be affected by the departure rate λ_2, the use of ILS or GCA (affecting point of touchdown), and the position of the terminal-gate area relative to the runway At low movement rates it has been observed that if the gate area is at the far end of the runway from touchdown, pilots prefer to keep taxiing straight ahead on the runway. In compiling R_1 these factors have been ignored as it was considered that λ_2 and GCA/ILS have a very small effect. At the higher movement rates, the effect of gate position is also reduced to the point where it is negligible

To the compilation of R_1, we must know the runway performance of aircraft types involved This basic performance consists of the mean time and distance to 60 mph (or 46 mph for very light aircraft) and the respective standard deviations In addition, the deceleration rates below 60 mph must be known, as well as the time and distance required for the actual turn-off

With the exception of the turn-off data, which was obtained in part from a Franklin Institute Report,* the runway performance was available in AIL Report No. 5791-15 † Not all the types required were covered by these reports, thus, where necessary, estimates were made on the basis of our airport observations (This information is presented in Table A-I.)

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* A A JeSchonek, "Radar Measurements of Approach and Landing Characteristics of Transport Aircraft," Report No. I-2185-3, Franklin Institute Laboratories, 31 December 1952

† "Runway Characteristics and Performance of Jet Transports in Routine Operation," Report No 5791-15, Vol 1 and 2, Airborne Instruments Laboratory, March 1960, "Runway Characteristics and Performance of Selected Propeller-Driven Aircraft in Routine Operation," Airborne Instruments Laboratory, April 1960

TABLE A-I

BASIC RUNWAY PERFORMANCE FOR 8000-FOOT RUNWAY

	Time to 60/mph λ_1 = 10 (seconds)		Distance to 60/mph λ_1 = 10 (feet)		Deceleration Below 60 mph (fps)		Right-Angle Turn-Off Starting at 27 mph	
	mean	σ	mean	σ	λ_1 10	λ_1 = 30	Time (seconds)	Distance (feet)
Boeing 707	33 6	4 3	5610	805	3 4	5 6	9 0	200
DC-8	34 4	2 7	5180	640	3 4	5 6	9 0	200
Convair 880*	34 4	2 7	5180	640	3 4	5 6	9 0	200
Convair 600*	33 5	4 7	4960	620	3 4	5 6	9 0	200
Caravelle*	31 3	3 7	4000	630	3 4	5 6	9 0	200
DC-6, DC-7, Connie	30 0	6 5	4170	1000	2 6	5 5	7 0	150
Electra	26 2	3 2	3750	560	3 0	6 5	7 0	150
Viscount	29 8	2 7	4060	380	2 5	4 5	7 0	150
Convair/Martin	24 0	4 6	3290	580	[illegible] 2	6 0	7 0	150
DC-3	22 8	3 5	2750	400	2 9	4 5	7 0	150
Beech 18	20 0	2 0	2360	300	3 8	4 5	7 0	150
F-27*	27 6	4 0	3470	500	3 4	4 5	7 0	150
Lodestar*	31 5	4 0	4140	500	3 4	4 5	7 0	150
Gulfstream*	23 7	3 0	2930	350	3 4	4 5	7 0	150
Apache	14 4	4 5	1460	500	2 9	4 5	6 0	125
Travelair*	16 8	4 0	1790	500	3 2	4 5	6 0	125
A Commander*	24 0	3 0	1810	450	3 8	4 5	6 0	125
Twin Cessna*	19 1	3 5	2200	400	3 6	4 5	6 0	125
Twin Bonanza*	22 6	3 5	2770	450	3 8	4 5	6 0	125
Queen Air*	20 8	2 0	2460	300	3 8	4 5	6 0	125
Aztec*	18 6	3 0	2080	350	3 6	4 5	6 0	125
Bonanza*	16 6	5 0	1670	500	2 9	4 5	6 0	125

* Estimated

TABLE A-1 (cont)

	Time to 46 mph $\lambda_1 = 10$ (seconds)		Distance to 46 mph $\lambda_1 = 10$ (feet)		Deceleration Below 46 mph (ips)		Turn-Off Starting at 20 mph	
	mean	σ	mean	σ	$\lambda_1 = 10$	$\lambda_1 = 30$	Time (seconds)	Distance (feet)
Light Aircraft Class E* Including Cessna 150-210, Piper Series etc	17 4	6 1	1550	560	2 9	4 5	6 0	75

* Estimated.

The data available from AIL Report No. 5791-15 was collected at Idlewild. At this airport all runways have a length in excess of 8000 feet In addition, the data were taken generally at periods of fairly low movement rates (about λ_1 = 10) Since both λ_1 and runway length affect R_1, it is necessary to compensate for these two factors. After a considerable amount of checking and testing against observed runway times at various airports, suitable reduction factors were obtained These reduction factors (Figure A-1) are applied to both the means and the standard deviations of time and distance to 60 mph as required.

In obtaining R_1 for the configurations described in this report, the various types of aircraft were grouped into classes (Section IV) These classes had to be further subdivided to make use of Table A-I. For example, at a continental airport, Class A was made up of Boeing 707 (25 percent), DC-8 (25 percent), Convair 880 (20 percent), Convair 600 (20 percent), and Caravelle (10 percent).

Thus, Class A has a time and distance to 60 mph of 33 7 seconds (σ3 8) and 5125 feet (σ810) at λ_1 = 10, while at λ_1 = 30 these figures are reduced to 26.9 seconds (σ3 0) and 4100 feet (σ650)

To obtain the complete runway time, it is best to first plot this data on a scale drawing of the runway (complete with turn-offs) This will include two standard deviations on either side of the mean, thus encompassing 96 percent of the population For all practical purposes, it was assumed that this was the equivalent of 100 percent. The calculation of time from 60 mph to off runway is somewhat complex, but it can be described briefly.

The deceleration rate for each class is known at λ_1 = 10 and λ_1 = 30 The optimum performance for each turn-off

IT IS ASSUMED THAT RUNWAY HAS ADEQUATE
RIGHT-ANGLE TURN-OFFS (AT LEAST 3)

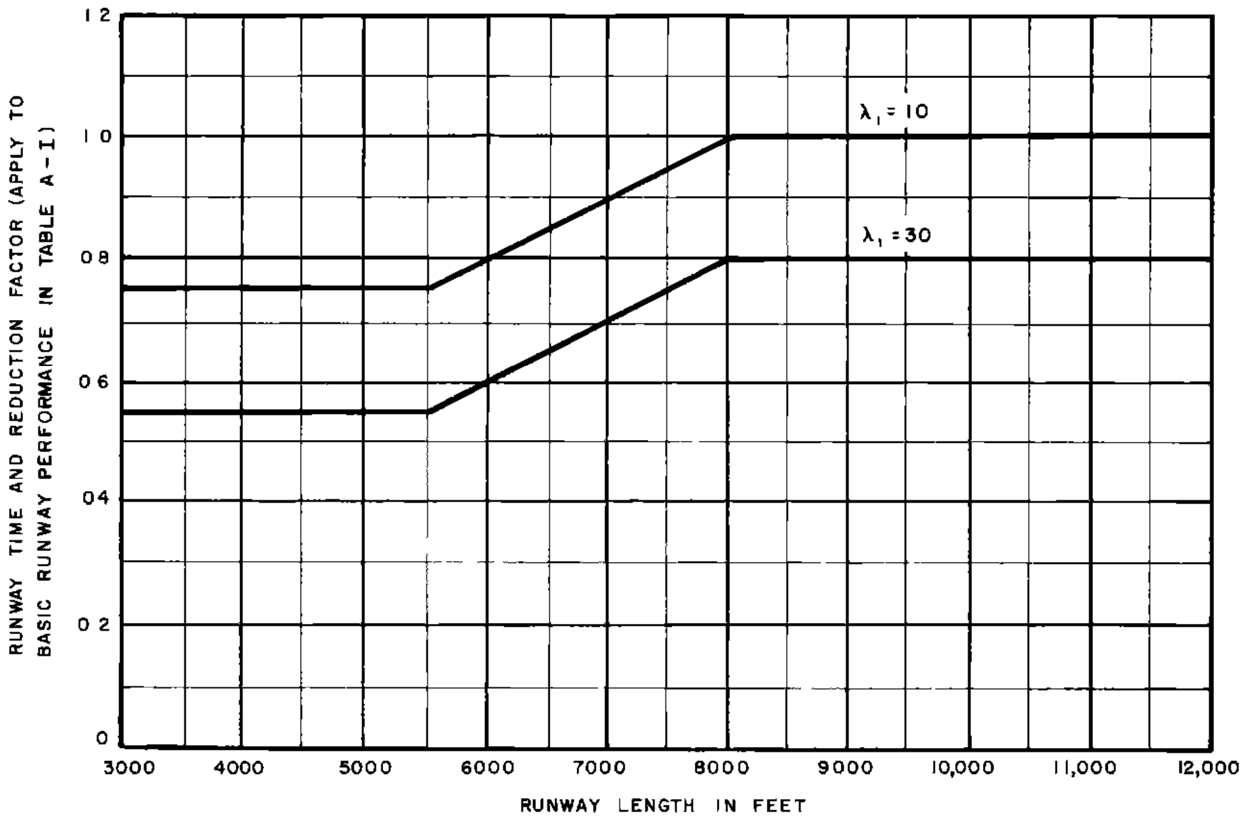

FIGURE A-1 EFFECT OF RUNWAY LENGTH AND MOVEMENT RATE ON TIME AND DISTANCE TO 60 MPH is used to calculate distance traveled and time elapsed from 60 mph This information is then used to subdivide the runway 60-mph plot into percentages of aircraft using the respective turn-offs The mean time is calculated from the average of the "best" and the "worst" performance of each block of aircraft, the spread being the standard deviation This procedure, applied to Class A on a 9000-foot runway at $\lambda_1 = 30$, is shown in Figure A-2

By taking each class it is possible to arrive at a complete picture of runway performance (Figure A-3). Using the weighted means of all classes, a final mean and standard deviation of the time OT-OR at $\lambda_1 = 10$ and $\lambda_1 = 30$ can be determined At most airports, the runway time reaches a minimum at an arrival rate slightly in excess of $\lambda_1 = 30$. Plotted graphically (Figure A-4), a shallow curve joining $\lambda_1 = 10$ to $\lambda_1 = 30$ and flattening out beyond $\lambda_1 = 30$, gave R_1 at intermediate movement rates.

Although this is apparently an approximate method of determining R_1, it does work well and bears a very close relationship to the performance of aircraft on known runways with right-angle turn-offs

To obtain R_1 for runways having high-speed turn-offs, a different procedure was required It was assumed that, in general, pilots would match their runway performance to the turn-offs This is the intention of the designers of the turn-offs. Therefore, the known runway performance between $\lambda_1 = 10$ and $\lambda_1 = 30$ was examined, and the best fit to the appropriate turn-off was taken. Data taken at Idlewild (Appendix C) indicated that if aircraft missed their optimum turn-off, they reduced speed down the runway to the next turn. This is taken into account in our computations of R_1, and the deceleration figures at $\lambda_1 = 10$ are used, since it was observed that generally the decelerations were fairly low

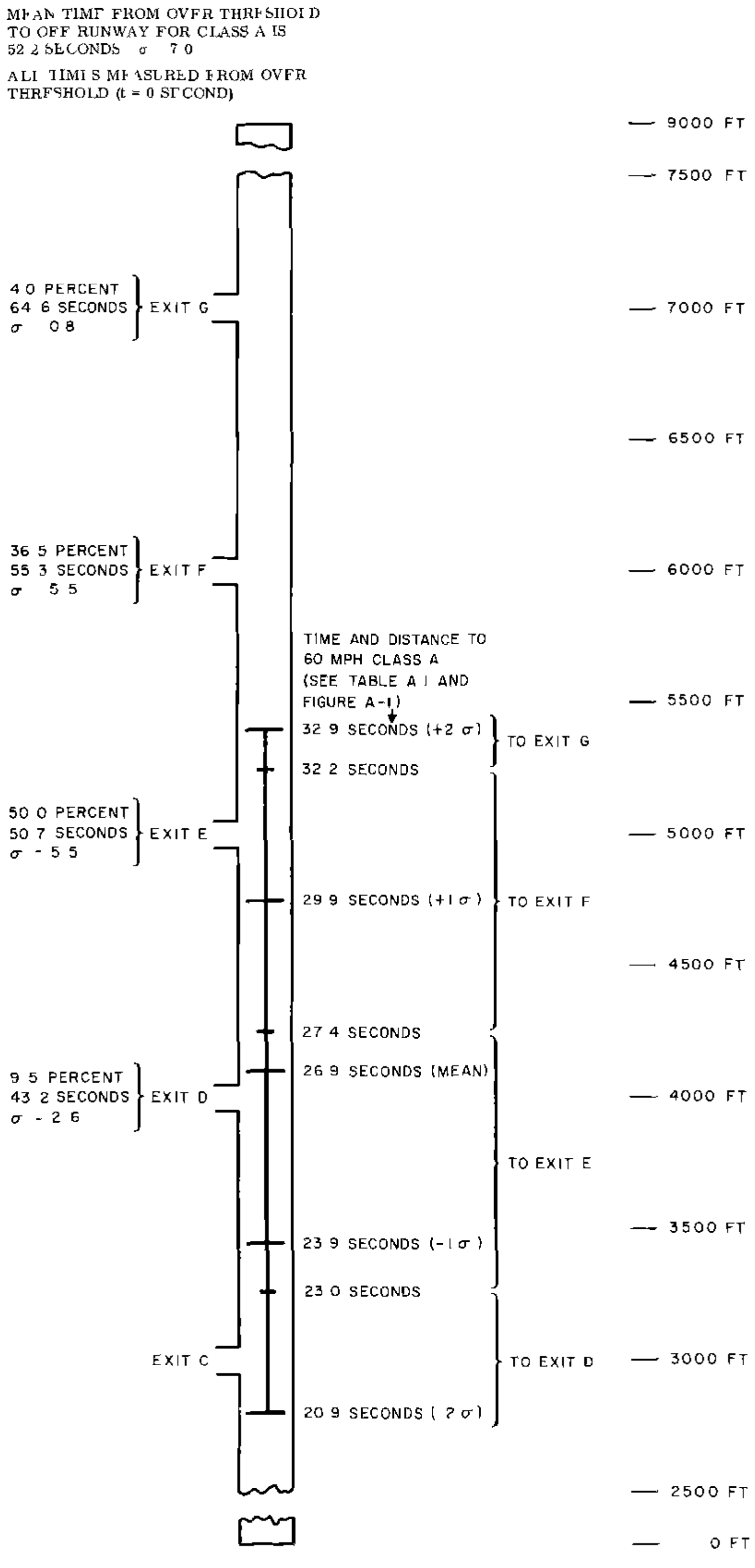

FIGURE A-2 PLOT OF 9000-FOOT RUNWAY PERFORMANCE FOR CLASS A AT $\lambda_1 = 30$

CONTINENTAL POPULATION AND MEAN
TIME OVER THRESHOLD TO OFF RUNWAY

CLASS	SECONDS	σ	PERCENT
A	52 2	7 0	20
B	44 4	6 5	30
C	44 4	6 0	20
D	39 9	6 5	10
E	39 2	8 3	20

44 47 SECONDS
(σ = 8 2)

ALL TIMES MEASURED FROM OVER
THRESHOLD (t = 0 SECOND)

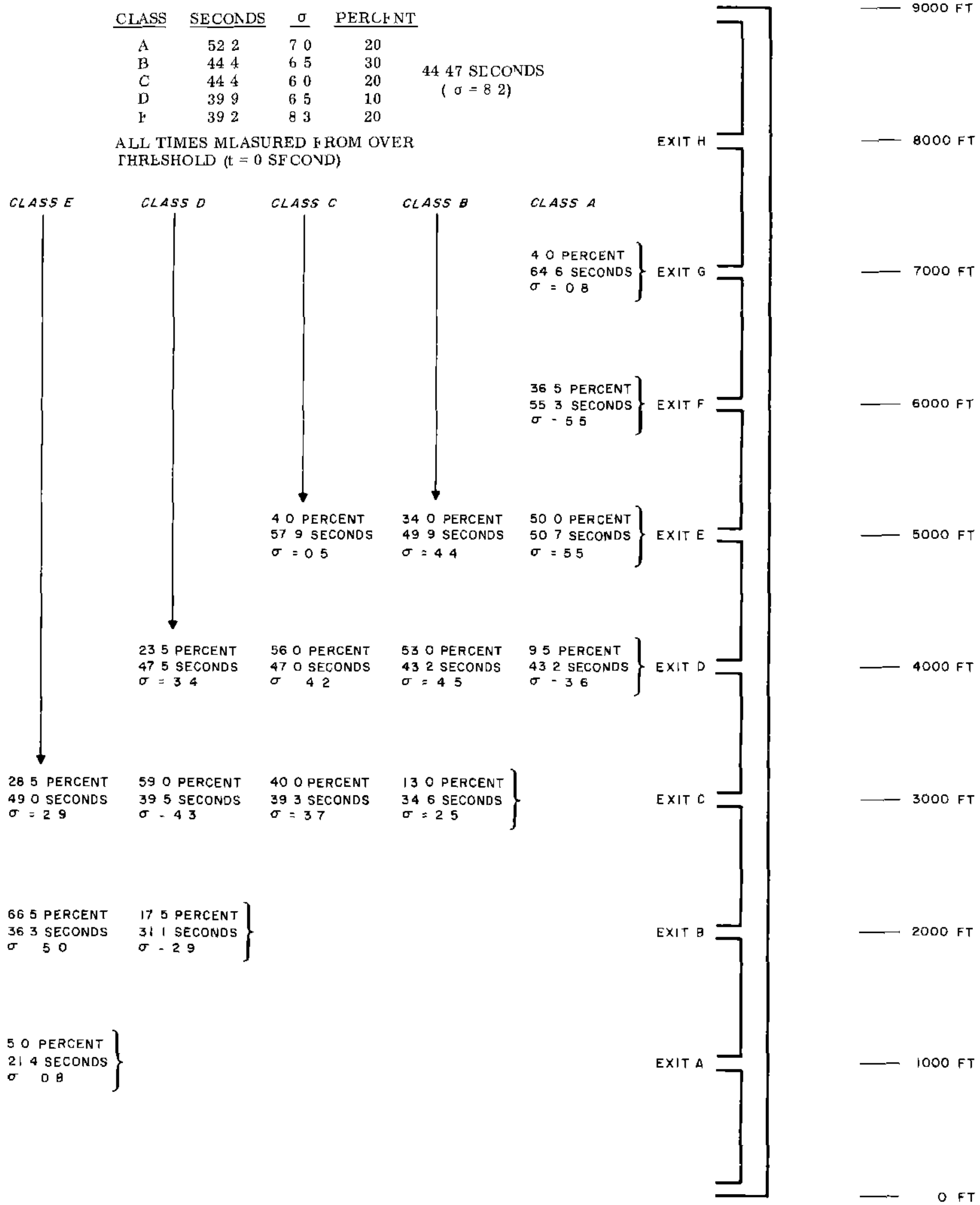

FIGURE A-3 FINAL COMPUTATION OF RUNWAY TIME OF ALL CLASSES ON 9000-FOOT RUNWAY WITH 1000-FOOT TURN-OFFS AT $\lambda_1 = 30$

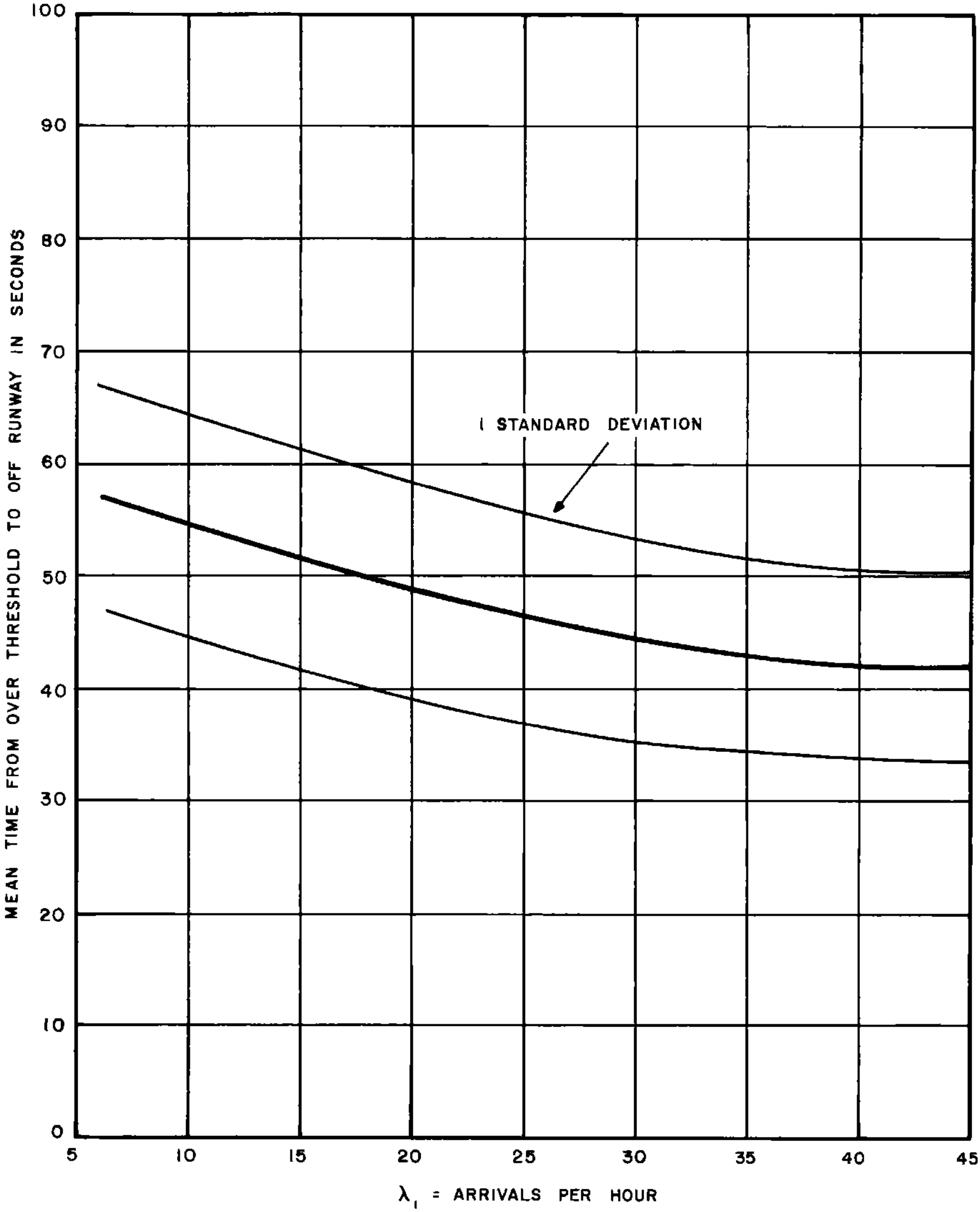

FIGURE A-4 RUNWAY TIME FOR ALL CLASSES ON 9000-FOOT RUNWAY WITH 1000-FOOT TURN-OFFS AND CONTINENTAL AIRPORT POPULATION AT VARYING ARRIVAL RATE

In the absence of actual airport observations on the high-speed turn-offs (positioned according to recommendations* and used in computing R_1), the computations used are realistic though not perfect It will be noticed that λ_1 will not affect R_1, since the position of the high-speed turn-offs and the aircraft types are the governing factors

2 RUNWAY TIME, ARRIVALS (R_2)

This is the mean over-threshold to cross time for intersecting runways. It is also used in other applications, such as the open vee layout (with landings and take-offs toward the apex) and IFR operation of close interdependent parallel runway operations where a take-off cannot be released until a previous arrival has covered a certain distance or the other runway--that is, definitely committed to the landing roll

To determine R_2, it is necessary to know the runway performance of each class of aircraft, the percentage using each exit, and their respective times. This data is already available as the result of the computation of R_1 However, to compute R_2 we must replot the data graphically A typical example is shown in Figure A-5, which is a replot of Figure A-2 using the means and standard deviation of time to 60 mph, start of turn-off, and off runway as the points on the graph. A weighted mean of the respective cross times at any desired point will give R_2 for that class of aircraft

Similar graphs must be made up for each class at $\lambda_1 = 10$ and $\lambda_1 = 30$, and the cross times weighted to give the

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* R. Horonjeff, G Ahlborn, and R Read, "Exit Taxiway Locations," Institute of Transportation and Traffic Engineering, University of California

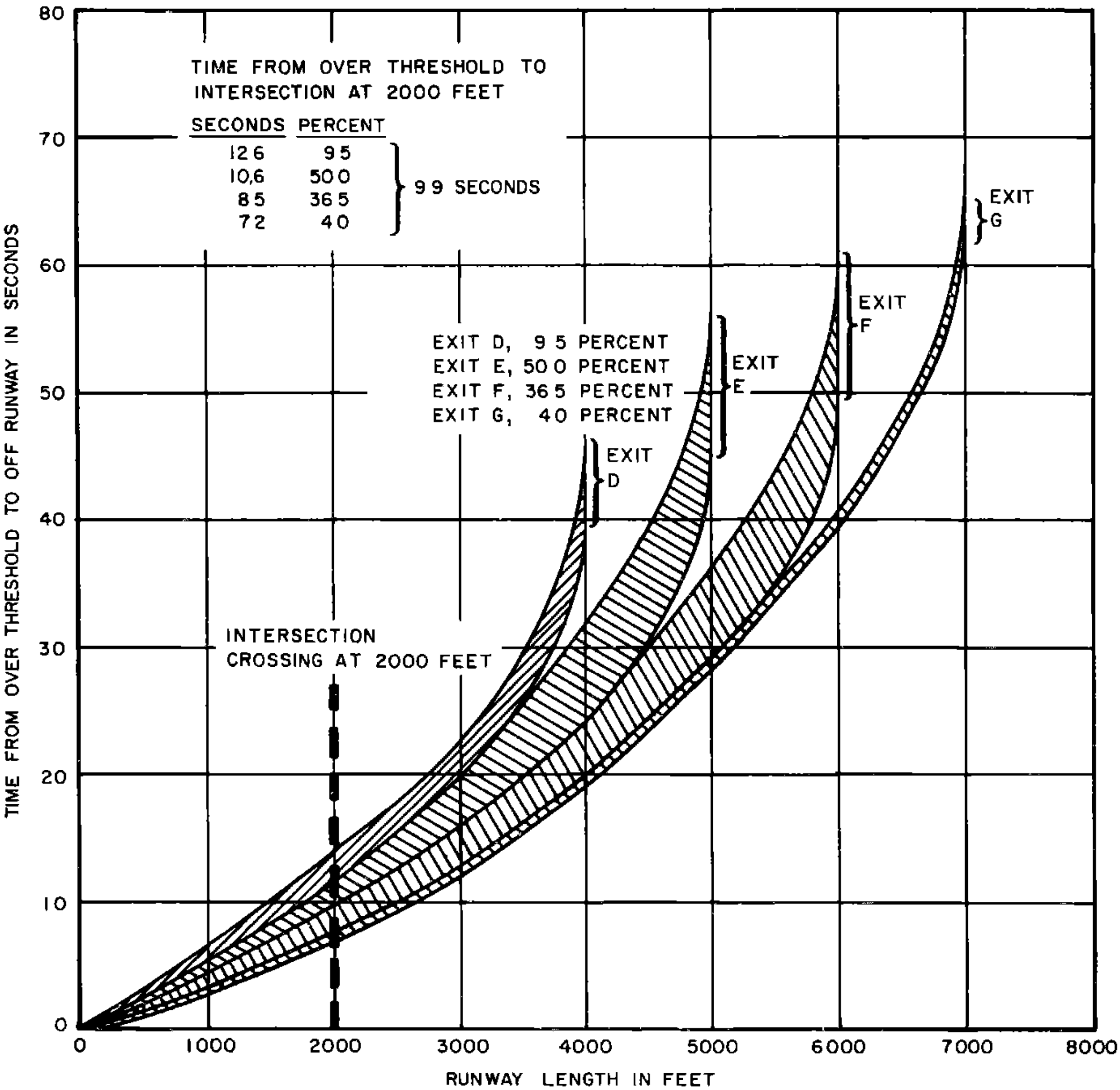

FIGURE A-5 REPLOT OF 9000-FOOT RUNWAY (2) PERFORMANCE WITH 1000-FOOT TURN-OFFS FOR CLASS A AT $\lambda_1 = 30$

final average R_2. In certain configurations of cross runways it is possible that a certain percentage of arrivals do not in fact go across the intersection but turn off before it At these airports the controller waited until the arrival had clearly slowed down and was about to start his turn-off before releasing a departure on the other runway

To obtain R_2 for this percentage of aircraft, the turn-off time (Table A-I) is subtracted from OT-OR time of the noncrossing aircraft. Neither the second moment nor the standard deviation is required

3 MINIMAL ARRIVAL SPACING (OT-OT)

This is the average minimum spacing between two arrivals on the same runway It is not a direct input, but together with the standard deviation is required for the computation of C_1, C_2, and K (If, however, a computer program is not used and manual computation is made, K and C_2 may be omitted and the standard deviation used as a direct input--Section VIII.) It is measured from OT of the first aircraft to the OT of the second aircraft, when the spacing is at a minimum

OT-OT spacing is affected by

a Mixture of aircraft types,

b Arrival rate λ_1

It may also be affected by the circuit pattern and control procedures in use at any given airport, and also by very poor runway turn-off facilities The latter is discussed in Section IV, and its application under C_1 of this appendix. The effect of the circuit pattern is very difficult to put into figures Therefore, some averaging must be made The control procedures vary considerably between IFR and VFR, but only VFR is considered here

A detailed analysis of the observed data was made to extract the minimum OT-to-OT times. Generally, this time was measured when two arrivals occurred and a departure was RG before OT of the first arrival, and the departure was not released between the two arrivals Also some data were collected at LaGuardia when the average λ_1 over a 5-hour period averaged $\lambda_1 = 30$ When these data were taken, a complete record of the number of aircraft on final approach had been taken, and it was quite easy to obtain the minimum spacings These spacings corresponded very closely with those taken on the same day when a departure was RG (but not released) before OT of the first arrival.

As a result of this and other data, it was possible to make up tables showing the variance of OT-OT time with λ_1 and type of aircraft. Four basic types of aircraft were used

Jet .	Class A
Heavy	Class B
Medium . .	Class C
Light	Classes D and E

Breaking down these types into their combinations gives a total of 16--that is, light followed by light, light followed by medium, etc.

In some cases there was a scarcity of data at the lower movement rates, but the figures presented in Table A-II are considered to be very realistic indications of pilots' performance, particularly at the higher movement rates.

To use the tables, it is necessary to know the percentages of aircraft types at a given airport From this the probability of each sequence occurring can be determined. The sum of the individual probabilities multiplied by their individual times gives the mean OT-OT time for a given λ_1 Obtaining the standard deviation is also comparatively simple

(continued on page A-22)

KEY TO TERMINOLOGY USED IN TABLE A-II

λ_1 = arrival rate (aircraft per hour),

$\lambda_1 + \lambda_2$ = total runway movement rate of arrivals and departures (aircraft per hour),

SM = second moment ($\text{mean}^2 + \sigma^2$).

All times are given in seconds

The terms "Light," "Heavy," etc , refer to the following classes of aircraft.

Jet . . . . Class A

Heavy . . . . Class B

Medium . . . . . . Class C

Light . . . . . . Classes D and E

TABLE A-II

OT-OT SPACING*

	Light Followed by Light			Light Followed by Medium		
λ_1	Mean	σ	SM	Mean	σ	SM
10	57 0	21 0	3690	66.0	23 5	4897
15	51 5	19.0	3013	58.5	20 0	3822
20	46 0	17.0	2405	52 5	16 5	3029
25	41.0	15.0	1906	47 0	13.5	2385
30	35 5	13 5	1436	43.5	11 0	2013
35	31.5	11 5	1119	41 0	9 5	1767
40	29 0	9.5	912	39.5	8.5	1629
45+	28.0	8 5	856	39 5	8 5	1629

	Light Followed by Heavy			Light Followed by Jet		
λ_1	Mean	σ	SM	Mean	σ	SM
10	88 5	24.5	8433	88.5	24 5	8433
15	77 0	20 0	6329	77 0	20.0	6329
20	66 5	16 0	4678	66 5	16.0	4678
25	57 5	12 5	3463	58.5	12 5	3422
30	51.5	10 5	2658	53.0	10 5	2925
35	49 0	9 5	2491	50 5	9.5	2641
40	48 5	9.5	2443	50.0	9.5	2590
45+	48.5	9 5	2443	50 0	9.5	2590

* Average minimal spacing between successive arrivals at increasing arrival rate.

TABLE A-II (cont)

	Medium Followed by Light			Medium Followed by Medium		
λ_1	Mean	σ	SM	Mean	σ	SM
10	98.5	27.5	10507	75.0	18.5	5977
15	81 5	24 0	7259	70.0	16 5	5172
20	69 0	22 5	5256	65 0	14 0	4414
25	58.5	20 5	3843	61 0	11 5	3853
30	54.5	19 0	3331	59.0	10 5	3481
35	53 5	18 0	3213	58.0	10 0	3464
40	53 0	17 5	3115	57 0	9 5	3249
45+	53 0	17 5	3115	57 0	9 5	3249

	Medium Followed by Heavy			Medium Followed by Jet		
λ_1	Mean	σ	SM	Mean	σ	SM
10	78 0	18.5	6084	78 0	18 5	6426
15	70 5	16 0	5226	71 0	16 0	5297
20	64.5	14.0	4356	66 5	14 0	4618
25	59 0	12.5	3637	63 0	12 5	4125
30	54 5	10.5	3080	60 5	10.5	3770
35	52 0	9 5	2704	60 0	9 5	3690
40	50 0	9 0	2581	60.0	9 0	3681
45+	49 5	9 0	2531	60 0	9 0	3681

TABLE A-II (cont)

	Heavy Followed by Light			Heavy Followed by Medium		
λ_1	Mean	σ	SM	Mean	σ	SM
10	90.0	28.0	8903	98.5	22.5	10209
15	80.0	24.0	6976	88.0	17.0	8033
20	71.5	19.5	5493	79.5	13.5	6496
25	64.5	16.0	4416	74.0	11.0	5597
30	60.0	14.5	3810	70.5	10.5	5081
35	57.5	14.0	3502	69.0	10.5	4871
40	56.5	13.5	3375	68.5	10.0	4792
45+	56.0	13.5	3318	68.5	10.0	4792

	Heavy Followed by Heavy			Heavy Followed by Jet		
λ_1	Mean	σ	SM	Mean	σ	SM
10	98.0	21.0	10045	98.0	21.0	10045
15	87.5	17.0	7937	87.5	17.0	7937
20	79.5	13.5	6496	79.5	13.5	6496
25	74.5	11.5	5683	74.5	11.5	5683
30	71.5	11.0	5233	71.5	11.0	5233
35	70.0	10.5	5010	70.0	10.5	5010
40	69.0	10.0	4861	69.0	10.0	4861
45+	69.0	10.0	4861	69.0	10.0	4861

TABLE A-II (con't)

	Jet Followed by Light			Jet Followed by Medium		
λ_1	Mean	σ	SM	Mean	σ	SM
10	102 0	28 0	11202	106 0	22.5	11742
15	92 0	24 0	9040	95 5	17.0	9409
20	84 0	19 5	7436	87.0	13 5	7745
25	76 5	16 0	6108	81 0	11 0	6682
30	72.0	14 5	5394	78.0	10 5	6194
35	69 5	14 0	5026	76.5	10.5	5963
40	68.5	13.5	4875	76.0	10 0	5876
45+	68.5	13 5	4875	76 0	10 0	5876

	Jet Followed by Heavy			Jet Followed by Jet		
λ_1	Mean	σ	SM	Mean	σ	SM
10	103 0	21 0	11050	103 0	21.0	11050
15	92.5	17 0	8837	92.5	17.0	8837
20	84 5	13.5	7316	84.5	13 5	7316
25	80 0	11.5	6532	80.0	11.5	6532
30	77 0	11 0	6050	77.0	11.0	6050
35	75.0	10 5	5735	75.0	10.5	5735
40	74 0	10.0	5576	74.0	10 0	5576
45+	74.0	10.0	5576	74 0	10 0	5576

by multiplying the individual probability by the second moment at the λ_1 required The square root of the sum of these minus the mean^2 is the standard deviation.

For the pre-emptive spaced arrivals model the mean and second moment are used to compute K, C_1, and C_2

4 K (ERLANG) FACTOR

Input to the pre-emptive spaced arrivals model only when a computer program is used $\left(\frac{OT-OT}{\sigma}\right)^2$ calculated for the appropriate λ_1

5. COMMITMENT INTERVAL C_1

This time, referred to in Section VIII as the "Commitment Interval," is the result of OT to OT minus R_1 That is, OT to OT, or the term B, is equal to $R_1 + C_1$, both R_1 and C_1 being direct inputs

Because it has been found that there is no dependence of C_1 on R_1 (B, or OT to OT, being the governing factor), C_2 for the computer program is calculated from the equation

$$C_2 = C_1^{\ 2} + \left[\sigma^2(B) - \sigma^2(R_1)\right]$$

However, it must be noted that under certain conditions C_1 could go to 0 or even negative--that is, if R_1 became excessive due to lack of turn-offs

Although this has not been observed (on an average basis) at any of the airports visited, it is felt that such a condition would not be tolerated over a period of time To compensate for this, a simple rule of thumb is used. For Class E, the average minimum C_1 is considered to be 0 second, Class D, 5 seconds, Class C, 7 5 seconds, Class B, 10 seconds, and Class A, 20 seconds For a continental population (see Section IV), this would give a minimum acceptable C_1 of 9 seconds.

If, for example, in the process of computation, any value of C_1 less than 9 seconds occurs (this would normally occur at a relatively high movement rate), C_1 must be increased and maintained at 9 seconds. At the same time, B (OT to OT) must be increased to maintain $B = R_1 + C_1$ The standard deviation of C_1 (C_2) is obtained by using the standard deviations of B and R_1 as before, assuming that the alteration to B has no effect on the standard deviation

Having placed a restriction on the minimum value of C_1, it is necessary to place a maximum value on this factor.

Generally, the need for restricting C_1 between the maximum and the minimum does not arise. Most runway configurations are such that C_1 (OT to OT minus R_1) does not lie outside the limits. Some runway designs, however, result in such a short R_1 (for example, very short runways with excellent turn-off facilities at high movement rates) that C_1 becomes excessive

Therefore, for each test of runway capacity the maximum and minimum value must be computed The maximum value varies with λ_1 and the OT-to-OT spacing, thus we can extract the following from observed data

λ_1	Reduction Factor Applied to OT-to-OT Spacing to Obtain C_1
10	0.53
15	0 50
20	0 47
25	0 43
30	0 40
35+	0.39

In an actual application this would be used as follows

λ_1 = 20 aircraft per hour (reduction factor = 0 47),

R_1 = 34 seconds,

OT-OT = 67 seconds,

Therefore, $C_1 max = 67 \times 0.47 = 31.5$ seconds

In this example, with no limitation, C_1 would have been 33 seconds, but it has now been reduced to 31.5 seconds

It is still necessary, however, to ensure that $R_1 + C_1$ = OT-OT With the new lower value for C_1, this becomes $34 + 31.5 = 65.5$ seconds

The new OT-to-OT time is now lower than the previously used figure This does not imply that the aircraft will actually reduce their spacing, but that their <u>effective</u> spacing has been reduced by 1 5 seconds--the 1.5 seconds becoming an unused time interval or gap. (An explanation of this gap is given in Section X.)

Note that if the OT-to-OT spacing is reduced, a new value for K must be derived.

6. DEPARTURE SPACING S_2

This is the average minimum spacing between CTO of the first departure and CTO of the second On an average basis this is the same as SR to SR and was extracted from the data as a minimum where the second departure was RG before SR of the first.

S_2 depends upon

a. $\lambda_1 + \lambda_2$ (total movement rate),

b. aircraft types

As in OT to OT, the control procedures can affect S_2 Again only VFR is considered. All the data used for the compilation of S_2 were taken under VFR conditions at airports having entrance taxiways at the runway threshold. Under these conditions the second departure normally has time to enter and line up on the runway before the first aircraft has completed the take-off maneuver At airports where it is necessary to backtrack down the runway before take off, S_2 can be altered considerably. Such cases must be treated individually and minimum spacing data taken for the individual configuration at various movement rates

As in OT to OT, 16 combinations of aircraft types were considered.

To obtain S_2 for any given $\lambda_1 + \lambda_2$, Table A-III is used. By using the probabilities of the 16 combinations with the means of S_2 and their respective standard deviations, a final average value of S_2 is computed Note that for the pre-emptive spaced arrivals model only S_2 is required, with no standard deviation.

7. DEPARTURE RELEASE TIME S_{11}

This is the average minimum time required to release and service a departure between the sequence of an arrival followed by an arrival or a departure followed by an arrival.

Of all the inputs to the spaced arrival model, it is the most difficult to define and therefore requires the most careful attention in its formation

Since most runways possess good entrance taxiways, a departure can normally be on the runway and ready to go before the previous arrival or departure has completed its maneuvers Therefore, S_{11} clearly starts at CTO of the departure. The time at which S_{11} ends obviously cannot be in excess of the time OT of the subsequent arrival.

In a sense, S_{11} is the time interval that a controller must allow for a departure in the gap that may exist between an arrival or departure and a subsequent arrival. In the observed data, the time from CTO of the departure to OT of the arrival is not, in itself, a perfect measure of S_{11}.

(continued on page A-31)

KEY TO TERMINOLOGY USED IN TABLE A-III

λ_1 = arrival rate (aircraft per hour),

$\lambda_1 + \lambda_2$ = total runway movement rate of arrivals and departures (aircraft per hour),

SM = second moment ($\text{mean}^2 + \sigma^2$).

All times are given in seconds.

The terms "Light," "Heavy," etc., refer to the following classes of aircraft

Jet . . . Class A

Heavy . . . Class B

Medium . . . . . Class C

Light . . . . Classes D and E

TABLE A-III
SR-TO-SR SPACING*

	Light Followed by Light			Light Followed by Medium		
$\lambda_1 + \lambda_2$	Mean	σ	SM	Mean	σ	SM
20	46.0	15 5	2356	61 5	13.0	3951
30	42.5	14.5	2017	59 0	12 0	3625
40	39 0	13.5	1703	56.0	10 5	3246
50	35 5	12 5	1417	53 5	10.0	2962
60	32 0	11.5	1156	50.5	8.0	2614
70	29.0	11 0	962	48.0	7.0	2353
80	25.5	11 0	771	46 0	7 0	2165
90+	25 0	10 5	735	46.0	7.0	2165

	Light Followed by Heavy			Light Followed by Jet		
$\lambda_1 + \lambda_2$	Mean	σ	SM	Mean	σ	SM
20	61.5	12.0	3926	87 0	15.0	7794
30	59.5	11.0	3661	85 0	14 0	7421
40	57.0	10.0	3349	84 0	13.5	7238
50	55 0	9 0	3106	82.0	13.0	6893
60	52 5	9 0	2837	81 0	12.5	6717
70	51 0	7 0	2650	80.0	11 5	6532
80	50.5	7 0	2599	80 0	11 0	6521
90+	50 0	6 5	2542	80 0	11 0	6521

* Average minimal spacing between two successive departures at increasing movement rate.

TABLE A-III (cont)

	Medium Followed by Light			Medium Followed by Medium		
$\lambda_1 + \lambda_2$	Mean	σ	SM	Mean	σ	SM
20	44.5	9.0	2061	68.0	18 5	4966
30	43 0	9 0	1930	64.0	17 5	4402
40	41 5	8.5	1795	59 0	15.0	3706
50	40.0	8.0	1664	53.5	12 5	3019
60	39.0	8.0	1585	48.0	9.0	2385
70	37 5	7 5	1463	43 0	7 0	1898
80	36.0	7.5	1352	40 5	7.0	1689
90+	34.5	7.5	1247	40 0	7.0	1649

	Medium Followed by Heavy			Medium Followed by Jet		
$\lambda_1 + \lambda_2$	Mean	σ	SM	Mean	σ	SM
20	70.5	16.5	5243	93.0	18.5	8991
30	69 0	14.5	4971	92.0	17 5	8770
40	65.0	12.0	4369	89.0	15.5	8161
50	58 0	10 0	3464	83 0	14.0	7085
60	48.0	10.0	2404	74.0	14.0	5672
70	42.5	7.0	1855	69.5	12.0	4974
80	42.0	7 0	1813	69.5	11.5	4963
90+	42 0	7 0	1813	69.5	11 5	4963

TABLE A-III (con)

$\lambda_1 + \lambda_2$	Heavy Followed by Light Mean	σ	SM	Heavy Followed by Medium Mean	σ	SM
20	68 0	16 5	4896	72.0	18 5	5526
30	63 0	15.0	4194	67.0	17 5	4795
40	58 0	13 0	3533	62 5	16.5	4179
50	52.5	11 5	2889	58 0	15.0	3589
60	48 0	10.0	2404	53.5	14.0	3058
70	43.5	8 5	1965	49.5	13.0	2619
80	42.0	8 0	1828	47 0	12 0	2353
90+	41 5	8 0	1786	46 5	12.0	2306

$\lambda_1 + \lambda_2$	Heavy Followed by Heavy Mean	σ	SM	Heavy Followed by Jet Mean	σ	SM
20	83 5	24.0	7548	103 5	25.5	11363
30	76 5	21.0	6293	97 0	23.5	9961
40	69.5	18.0	5154	91.0	21.0	8722
50	62 5	15 0	4131	85.0	19.0	7586
60	59 0	12.5	3637	82 0	15.5	6964
70	57 0	10.5	3359	81.5	13.5	6825
80	56 5	10.5	3303	81 5	13 5	6825
90+	56 0	10.0	3236	81 5	13.5	6825

TABLE A-III (cont)

	Jet Followed by Light			Jet Followed by Medium		
$\lambda_1 + \lambda_2$	Mean	σ	SM	Mean	σ	SM
20	80.5	16.5	6753	79.5	18 5	6663
30	75 0	15.0	5850	75 0	17 5	5931
40	70 5	13 0	5139	71 0	16.5	5313
50	65.5	11.5	4423	66.5	15 0	4647
60	64 0	10.0	4196	62.5	14.0	4102
70	63 5	8.5	4105	60 0	13 0	3769
80	63 5	8 0	4096	59 0	12.0	3625
90+	63 5	8.0	4096	59.0	12 0	3625

	Jet Followed by Heavy			Jet Followed by Jet		
$\lambda_1 + \lambda_2$	Mean	σ	SM	Mean	σ	SM
20	96.0	24.0	9792	98 5	25 5	10353
30	87 5	21.0	8097	92 0	23.5	9016
40	79 5	18 0	6644	86 0	21.0	7837
50	72 5	15 0	5481	80.0	19 0	6761
60	68 0	12 5	4780	77.5	15.5	6247
70	65.0	10 5	4335	76 5	13.5	6035
80	64 5	10 5	4271	76.5	13 5	6035
90+	64.0	10 0	4196	76.5	13 0	6021

This time will be in excess of the actual time required to service the departure for a variety of reasons

One reason why OT is not the end of S_{11} is illustrated by a light aircraft that calls for departure when another aircraft of the same class is in the process of arrival At LaGuardia, when $\lambda_1 + \lambda_2 = 65$, the average minimum SR to OT (CTO to SR being very small in this case) for the light aircraft was measured at 58 seconds At Zahn's Airport, at a slightly higher movement rate, the average minimum was only 28.5 seconds The reason for the discrepancy lies in the type of circuit pattern in use at the two airports At LaGuardia, the heavy aircraft (DC-6, Constellation, etc.) dictate the circuit pattern, so that even though the SR to OT of the light aircraft is measured at a minimum, it is a comparatively long interval since the arrival cannot close the gap because the circuit pattern does not permit this. At the same time it is apparent that the light aircraft does not on departure require the full 58 seconds for the maneuver. At Zahn's, the circuit pattern is very much tighter because the majority of the aircraft is light aircraft The average minimum SR to OT measured at this airport is therefore a much more realistic <u>direct</u> value of S_{11}

After considerable checking and testing against measured data, it was possible to obtain a very close approximation of the S_{11} required

This input is a combination of two easily measured factors--the interval CTO to SR plus the interval SR to OT (measured at the average minimum value), minus C_1 measured at the same λ_1. (However, the resulting S_{11} must <u>always</u> be greater than C_1. This is discussed more fully later)

S_{11} varies with $\lambda_1 + \lambda_2$ and with aircraft types. For a light aircraft followed by a light aircraft it will depend upon the circuit pattern The circuit pattern may possibly affect some of the other combinations of SR to OT, but it is extremely difficult to detect and may be ignored here.

To determine S_{11}, it is necessary to build up the respective intervals step by step.

First, we must consider the interval CTO to SR. For aircraft other than jet transports, this interval is inversely proportional to $\lambda_1 + \lambda_2$. However, for jet transports, the interval CTO to SR constitutes the run-up time. A large proportion of the jet transports in service require a period of time after CTO to bring engines up to full power to check and synchronize them. On some aircraft, water injection is applied at this time The average run-up time has been measured at 25 seconds, but there is considerable variance of this value--some aircraft taking as long as 45 seconds. This variance causes some difficulty to tower controllers in estimating S_{11} of jet aircraft. Thus, the controllers allow for the possibility of the long run-up time rather than the average. In computing the CTO-to-SR segment of S_{11}, 40 seconds have been used for the jet aircraft, as an average maximum.

By knowing the percentage of jet and other aircraft types (Classes B to E) at an airport, a weighted mean of CTO to SR can be obtained Table A-IV shows the CTO-to-SR times for the two types as a function of $\lambda_1 + \lambda_2$.

For the portion SR to OT, a procedure similar to that used for computing S_2 and B (OT to OT) is used. The 16 type combinations and their probabilities are used in conjunction with Table A-V Note that if the model is being used to determine capacity of a small airport, where no heavy aircraft (Classes A and B) exist, the appropriate table must be used

(continued on page A-40)

TABLE A-IV

C-TO-S SPACING*

Jet Aircraft (Class A)

Movement rate has no effect on this time interval
Average maximum value: 40 seconds.

Piston and turboprop aircraft (Class B to E)

Movement Rate $\lambda_1 + \lambda_2$	Average Time
0 - 20	10 0
30	9.2
40	8.3
50	7.5
60	6 7
70	5 8
80+	5 0

* The average time interval between take-off clearance (tower controller) and start roll for any one departure.

KEY TO TERMINOLOGY USED IN TABLE A-V

λ_1 = arrival rate (aircraft per hour),

$\lambda_1 + \lambda_2$ = total runway movement rate of arrivals and departures (aircraft per hour),

SM = second moment $(\text{mean}^2 + \sigma^2)$.

All times are given in seconds.

The terms "Light," "Heavy," etc., refer to the following classes of aircraft

Jet . . . . . Class A

Heavy . . . . Class B

Medium . . . . Class C

Light . . . . Classes D and E

TABLE A-V

SR-OT SPACING*

$\lambda_1 + \lambda_2$	Light Followed by Light (Small Airport-- No Heavy Traffic) Mean	Light Followed by Light (Large Airport with Heavy Traffic) Mean
20	63.5	65 0
30	55 5	63.5
40	48 0	62 0
50	41 5	60 5
60	36 0	59 0
70	32.0	57.5
80	29.5	56 0
90+	28.5	54 5

$\lambda_1 + \lambda_2$	Light Followed by Medium Mean	Light Followed by Heavy Mean
20	68 5	81 0
30	61.0	67 0
40	55.0	57 0
50	50 0	49 0
60	47.0	44 5
70	45.5	43 0
80	45.0	42 5
90+	45.0	42 5

* Average minimal spacing between a departure followed by an arrival at increasing movement rate.

TABLE A-V (cont)

$\lambda_1 + \lambda_2$	Light Followed by Jet Mean	Medium Followed by Light Mean
20	88.0	55 0
30	74 0	47 5
40	64.0	42 5
50	56 0	40.0
60	51 5	38.5
70	50.0	38 0
80	49.5	38 0
90+	49 5	38 0

$\lambda_1 + \lambda_2$	Medium Followed by Medium Mean	Medium Followed by Heavy Mean
20	82 0	109.0
30	75.5	94.5
40	70 0	80.0
50	65 0	65 0
60	61.0	51.0
70	58.0	43.5
80	55 5	40.5
90+	54.0	39.0

TABLE A-V (cont)

$\lambda_1 + \lambda_2$	Medium Followed by Jet Mean	Heavy Followed by Light Mean
20	109.0	72.5
30	96 5	66.0
40	84.0	61.5
50	71 0	58.5
60	59.0	57 0
70	50 0	55.5
80	47.0	55.0
90+	46 5	55.0

$\lambda_1 + \lambda_2$	Heavy Followed by Medium Mean	Heavy Followed by Heavy Mean
20	92.0	108.0
30	74 5	94.0
40	65 5	81.0
50	57.5	69.0
60	51 5	59.0
70	46.0	51.0
80	43.0	46.0
90+	42.0	44.0

TABLE A-V (cont)

$\lambda_1 + \lambda_2$	Heavy Followed by Jet Mean	Jet Followed by Light Mean
20	108.0	82.5
30	94 0	76.0
40	81.0	71.5
50	69.0	69.0
60	59.0	67.0
70	51 0	65.5
80	46 0	65.0
90+	44 0	65 0

$\lambda_1 + \lambda_2$	Jet Followed by Medium Mean	Jet Followed by Heavy Mean
20	92.0	108 0
30	76 0	94.0
40	68.0	81.0
50	62 5	70.0
60	58.0	61.5
70	55 0	56 0
80	52.5	53 0
90+	51 5	51.5

TABLE A-V (cont)

Jet Followed by Jet

$\lambda_1 + \lambda_2$	Mean
20	108.0
30	94.0
40	81.0
50	70.0
60	61.5
70	56.0
80	53.0
90+	51.5

When the final mean of SR to OT has been obtained, it is added to the CTO-to-SR interval After this, we subtract C_1, this value having already been obtained.

At an airport serving a mixture of jet, heavy, medium, and light aircraft, C_1 is normally in excess of about 9 seconds and affords quite a reduction in the CTO-to-OT interval to give S_{11}. This, therefore, compensates for the larger interval for a light aircraft followed by a light aircraft at such an airport

For a runway handling light aircraft only, C_1 will normally come very close to 0 at the higher movement rates, therefore, reducing CTO to SR by a very small amount--which is consistent with the comments already expressed.

The subtraction of C_1 from the CTO-to-OT interval is presently the only method available for determining S_{11}. When testing certain runway configurations, it may happen that C_1 is at its maximum value. This can result in a too small value for S_{11}. To prevent this possibility, a minimum value for S_{11} must be determined. After some detailed analysis of the observed data these minimum values were obtained and are presented in Table A-VI.

To determine the average minimum S_{11} for any given runway, the probabilities of each sequence of classes must be determined. Multiplying the individual times by their probabilities gives the minimum S_{11}

If, upon comparison with the S_{11} previously obtained, the minimum S_{11} is higher, then this higher value must be used. Because S_{11} decreases with an increase of $\lambda_1 + \lambda_2$, it will normally only be necessary to apply the cutoff at the higher movement rates.

For cross-runway and open "V" configurations, the method of obtaining S_{11} differs from the foregoing. For

(continued on page A-43)

KEY TO TERMINOLOGY USED IN TABLE A-VI

λ_1 = arrival rate (aircraft per hour),

$\lambda_1 + \lambda_2$ = total runway movement rate of arrivals and departures (aircraft per hour),

SM = second moment ($\text{mean}^2 + \sigma^2$).

All times are given in seconds.

The terms "Light," "Heavy," etc., refer to the following classes of aircraft

Jet . . . . Class A

Heavy . . . . Class B

Medium . . . . Class C

Light . . . . Classes D and E

TABLE A-VI

MINIMUM S_{11}*

Light Followed by Light	Light Followed by Medium	Light Followed by Heavy
30	35	40
Light Followed by Jet	Medium Followed by Light	Medium Followed by Medium
45	40	35
Medium Followed by Heavy	Medium Followed by Jet	Heavy Followed by Light
38	45	44
Heavy Followed by Medium	Heavy Followed by Heavy	Heavy Followed by Jet
36	36	40
Jet Followed by Light	Jet Followed by Medium	Jet Followed by Heavy
74	69	66
	Jet Followed by Jet	
	66	

* This is the minimum time interval in which a departure can be released for take-off in front of an incoming arrival. This interval is compared with the computed value of S_{11} (after the subtraction of C_1 from that value).

cross runways with departures on one runway and arrivals on the other, S_{11} is now the average interval CTO to cross time (X) of the departure. The interval CTO to SR is computed as before and the time from SR to X can be derived (by class) from Figure A-6. Figure A-6 was derived from our airport observations and not from Appendix B, which was completed only toward the end of this project. However, for the jet aircraft, AIL Report No. 5791-15 was used Class A in this example consists of Boeing 707 (25 percent), DC-8 (25 percent), Convair 880 (20 percent), Convair 600 (20 percent), and Caravelle (10 percent)

For a cross-runway continental airport configuration, where the take-off runway crosses the landing runway between 4500 feet and the far end, the times from Figure A-6 can be used directly. If, however, on this type of airport, the intersection occurs at less than 4500 feet on both the landing and take-off runway, the cross time at 4500 feet appears to represent the minimum S_{11} that can be accepted at that airport. This is because pilots and controllers desire adequate separation between crossing aircraft.

For open "V" configurations with take-offs and landings toward the apex, a value of S_{11} must be chosen to allow sufficient clearance between a take-off and an arrival which has waved off. As no operations of this kind were observed during this project, no general rule is stated for the determination of S_{11} under these conditions. It will be governed by the types of aircraft using the airport, the position of the intersection beyond the airport, and the control procedures.

Thus, for special cases such as those already mentioned, each airport must be treated individually, and operating data at various movement rates obtained from observations.

(continued on page A-49)

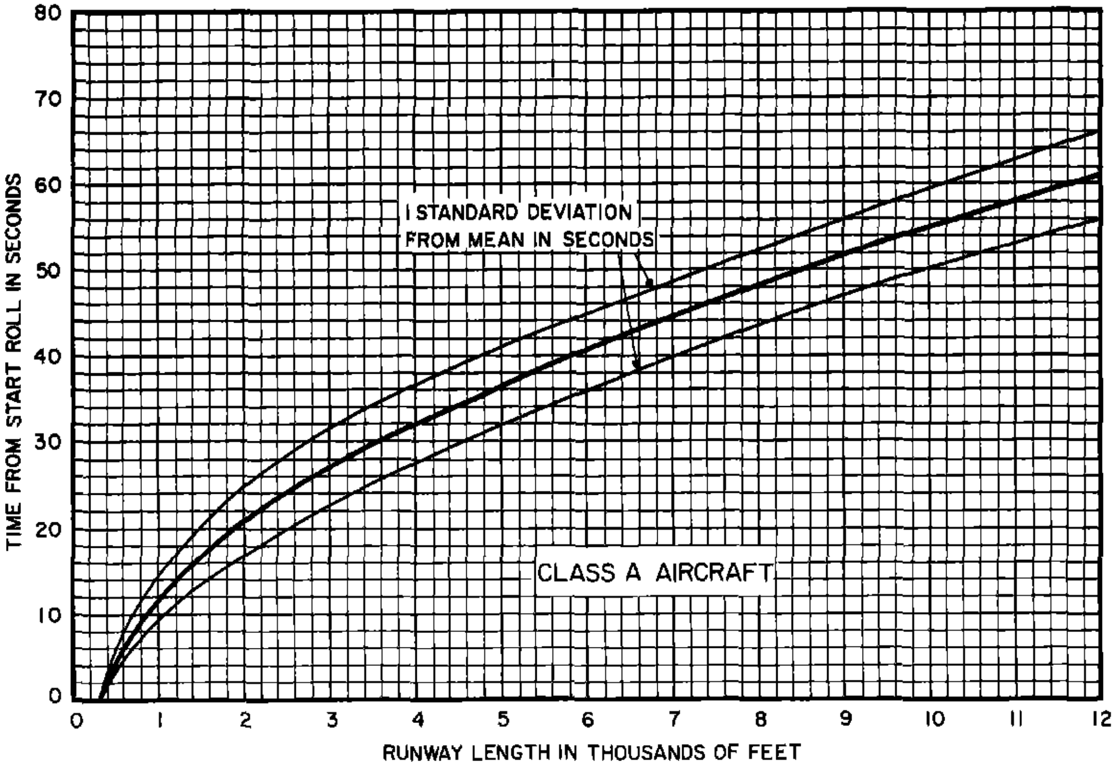

FIGURE A-6 TAKE-OFF PERFORMANCE
(SHEET 1 OF 5)

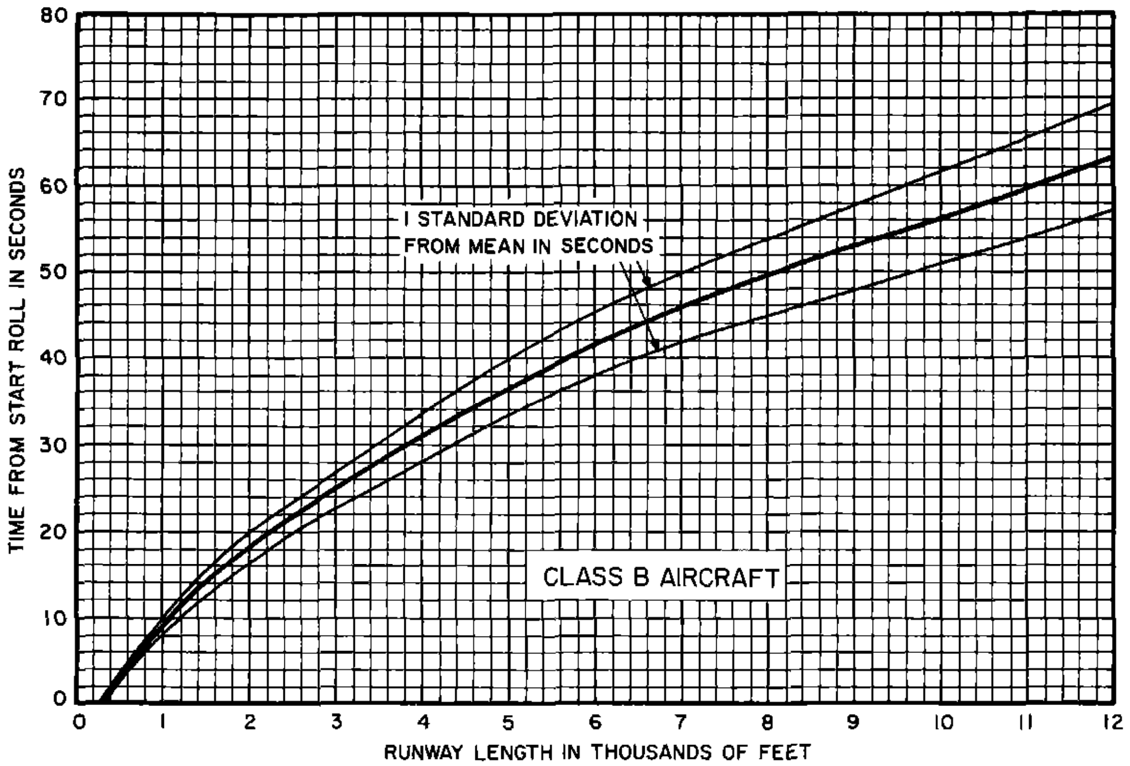

FIGURE A-6

(SHEET 2 OF 5)

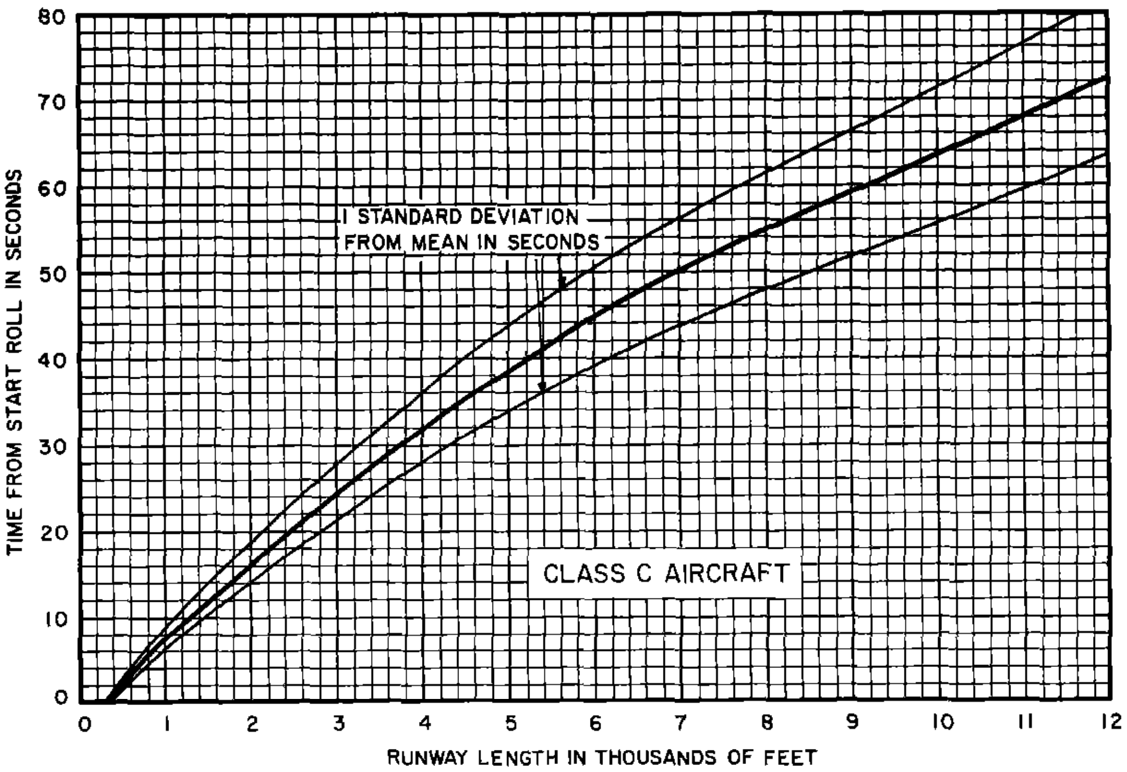

FIGURE A-6

(SHEET 3 OF 5)

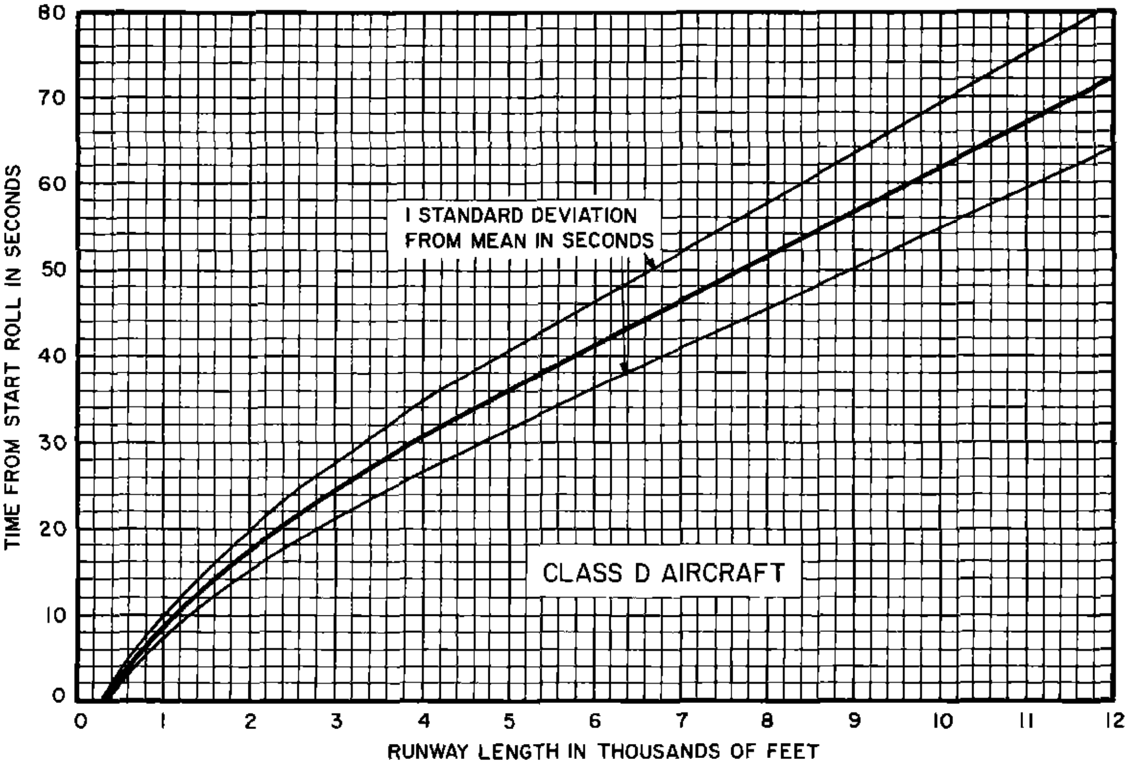

FIGURE A-6

(SHEET 4 OF 5)

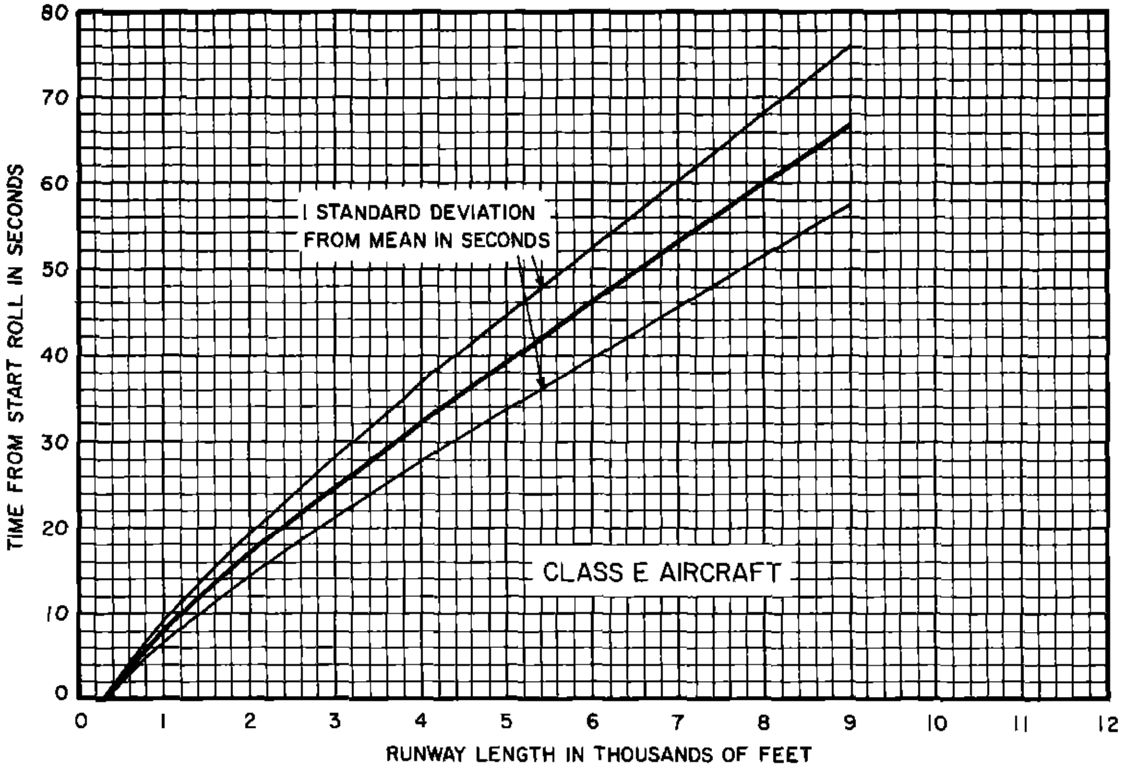

FIGURE A-6

(SHEET 5 OF 5)

When finally applying S_{11} to the mathematical model, it must be noted that S_{11} must always be greater than C_1. If this condition is not fulfilled, the final figure of average delay will be in error It has been found in practice that this occurs in a very few cases.

8. PROCEDURE FOR OBTAINING INPUTS FOR SPACED ARRIVALS MODEL FOR SINGLE RUNWAY (MIXED OPERATION)

a. SPECIFY RUNWAY DESIGN

1 Length

2 Number, type, and position of turn-offs.

b SPECIFY MIXTURE OF AIRCRAFT

1 Percentages of Classes A, B, C, D, and E

2. Compute probabilities of class sequences.

c. CALCULATE RUNWAY TIME OT TO OR (R_1)

1 Calculate average R_1 and standard deviation for each class at $\lambda_1 = 10$ and $\lambda_1 = 30$.

2. Using percentages of each class (from step b above) calculate mean R_1 and σ at $\lambda_1 = 10$ and $\lambda_1 = 30$.

3 Draw up graph of R_1 against λ_1.

d. CALCULATE OT-TO-OT TIME

1 Using Table A-II, calculate mean and standard deviation at increasing λ_1. Commence at $\lambda_1 = 10$ and increase in increments of 5.

2. Draw up graph of OT to OT against λ_1

e CALCULATE MINIMUM C_1 AND MAXIMUM C_1

1. Maximum C_1 will vary with λ_1

2. Plot both minimum C_1 and maximum C_1 against λ_1

f. CALCULATE C_1

1. OT to OT minus R_1.
2. Plot on same graph as step e above.
3 Check that C_1 does not go above or below the limits.
4 If this is the case, adjust OT to OT accordingly

g. CALCULATE C_2

$$C_1^2 + \left[\sigma^2(OT-OT) - \sigma^2(R_1)\right]$$

h. CALCULATE K

$$\left(\frac{OT-OT}{\sigma}\right)^2$$

i. CALCULATE MINIMUM S_{11}

1. Use Table A-VI (does not vary with $\lambda_1 + \lambda_2$).
2 Plot on graph against $\lambda_1 + \lambda_2$.

j. CALCULATE S_{11}

1. Compute run-up time, CTO to SR, from Table A-IV.
2. Compute SR-to-OT time from Table A-V.
3. Add steps 1 and 2 above.
4. Subtract C_1 appropriate to each increment of $\lambda_1 + \lambda_2$.
5 Plot on same graph as step i above.
6. Check minimum limit

k CALCULATE S_2

1 Use Table A-III.
2 Plot on graph against $\lambda_1 + \lambda_2$.

l. INSERT MODEL INPUTS IN COMPUTER PROGRAM

Using the graphs obtained in steps c, d, f, j, and k above, the final inputs may be derived against any movement rate desired.

Table A-VII shows the final inputs necessary for the pre-emptive spaced arrivals model actually calculated for the 9000-foot single runway (turn-offs every 1000 feet) with a continental airport population (Section IV).

TABLE A-VII
INPUTS

λ_1	λ_2	S_{11} (sec)	S_2 (sec)	C_1 (sec)	C_2 (sec)	R_1 (sec)	K
9	9	71 0	74.5	36.0	1927	55.0	11.4
12	12	67.5	72.0	31.0	1490	53.5	11.4
15	15	65 0	69.5	27 0	1189	51.5	11.2
18	18	62.5	67 0	23.5	924	50 0	11.7
21	21	60.0	64.5	21 0	735	48 5	12.7
24	24	57 5	61.5	18.5	581	47.0	13.2
27	27	55 5	59 0	17.0	514	45 5	12.7

B. FIRST-COME, FIRST-SERVED MODEL

1. DEPARTURES-ONLY RUNWAY

λ_2 is simply the departure rate in aircraft per hour, measured at their respective R times

S_{21} is the mean of the departure service times, measured at the average minimum from CTO of the first departure to CTO of the second. In this respect, S_{21} is derived in exactly the same manner as S_2 in the pre-emptive spaced arrivals model, and the remarks concerning S_2 apply to S_{21} here.

S_{22} is the second moment of S_{21} and can be derived using Table A-III. The aircraft population of the runway under study must be divided into the various classes (jet, heavy, medium, and light) and used to compute the probabilities of each sequence occurring--that is, light aircraft followed by a light aircraft, etc. By multiplying the various probabilities by the second moments of each spacing at each movement rate, a final figure of S_{22} is derived.

2. ARRIVALS-ONLY RUNWAY

λ_1 is the arrival rate in aircraft per hour demanding service.

S_{11} is the mean of the arrival service times, measured at the average minimum, from OT of the first arrival to OT of the second. It is therefore identical to the minimal OT to OT (or B) of the pre-emptive spaced arrivals model and can therefore be derived in exactly the same manner.

S_{12} is the second moment of S_{11} Because the second moment is mean^2 + σ^2, S_{12} can be derived from Table A-II in a similar manner to that used for deriving the standard deviation of OT to OT in the spaced arrivals model

3. TAXIWAY INTERSECTION

In the simple case it may be assumed that the rate, population, and time to cross are identical for each stream of traffic In this case, $\lambda_1 = \lambda_2$, $S_{11} = S_{21}$, and $S_{12} = S_{22}$, where λ_1 is the traffic rate per hour, S_{11} is the mean of the cross time, and S_{12} is the second moment of the cross time.

In a more complex situation--for example, heavy aircraft crossing a light aircraft taxiway in the terminal area--the respective rates and cross times might well be different.

To use this model to determine the need for a parallel taxiway (Figure 8-2), the service times will also involve the time to taxi the distances involved

4. AIRCRAFT TAXI SPEEDS

Some typical taxi speeds were obtained from the extensive film library compiled as part of the project described in AIL Report No 5791-15 By use of this film and successive plots of aircraft position, taxi speeds were obtained in an area on the airport where moderate speeds might be expected--since the taxiway was relatively clear and unobstructed, but not too distant from the terminal apron area. The measured speeds are plotted in Figure A-7.

C. PRE-EMPTIVE POISSON ARRIVALS MODEL

1. RUNWAY-TAXIWAY INTERSECTION

λ_1 is the movement rate of the runway aircraft per hour.

λ_2 is the movement rate of the taxiway aircraft per hour.

S_{11} is the mean service time of the runway aircraft. If, for example, the runway aircraft were all departures, the combined service time (CTO to intersection) can be obtained by the use of Figure A-6 and Table A-IV.

S_{12} is the second moment of S_{11}.

S_{21} is the mean service time of the crossing time of the taxiway aircraft from Clear to Taxi to off runway.

2 RUNWAY CROSSING TIME OBSERVATIONS

To develop service times for the above application, some useful data was taken at Midway on 13 October 1959 during daylight with a 2300-foot ceiling, 6-mile visibility, and on runways 175 feet wide. From spot checks at other airports, it appears to be quite representative.

The data in Table A-VIII were taken when either runways 31R and L were being used, or when runways 4R and L

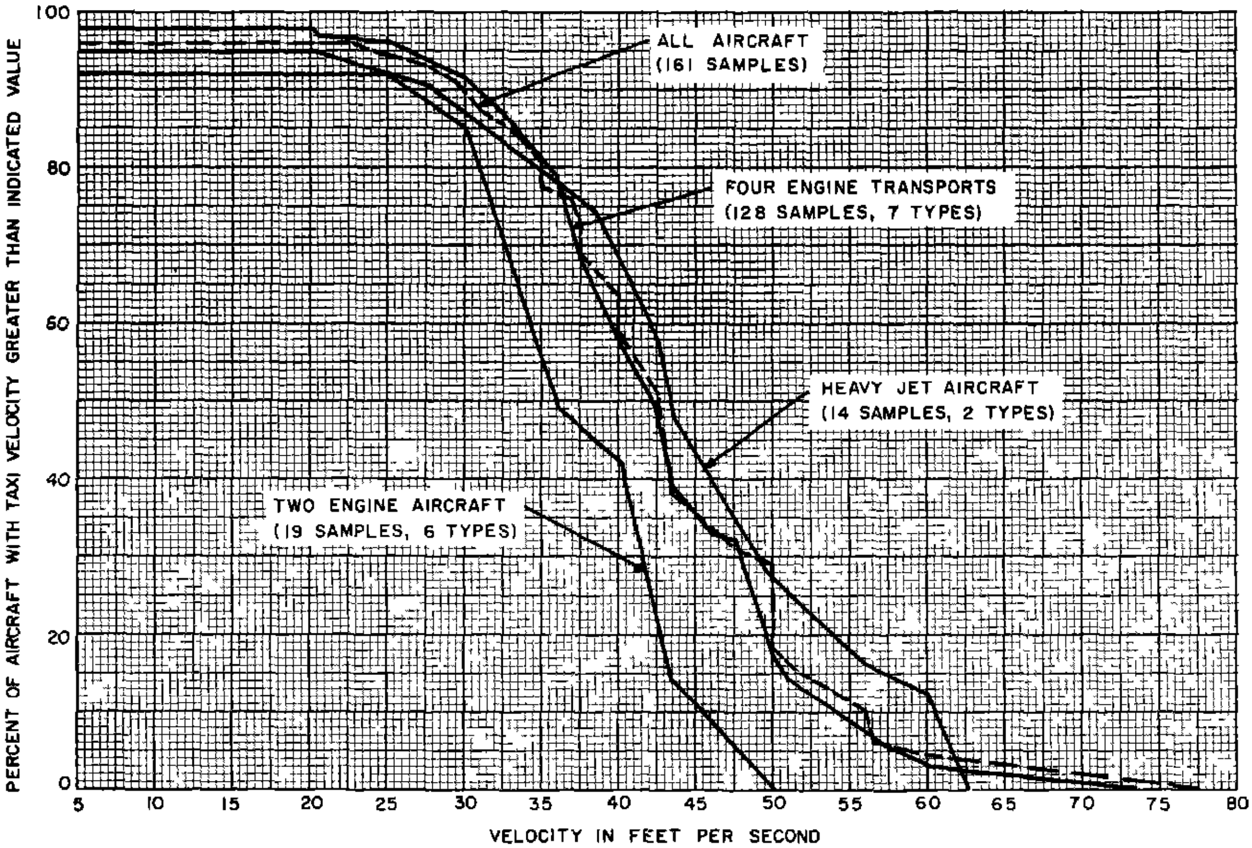

FIGURE A-7 TAXI SPEEDS MEASURED ON UNOBSTRUCTED TAXIWAY AT NEW YORK INTERNATIONAL AIRPORT

TABLE A-VIII

RUNWAY CROSSING OBSERVATIONS

	Viscount	DC-3	Convair	Convair 240	DC-6	DC-7	202	Electra	Light Aircraft
	19	27	23	18	27	31	24	29	15
	29	23	22	25	20	30	27	18	16
	25	22	25	25			22		32
	30	20	29	22					20
	20	16	31	29					23
	19	22	30						30
	29	9	19						
		18	19						
		23							
		20							
		20							
		22							
		19							
		23							
Means	24.4	20.3	24.7	23.8	23.5	30.5	24.3	23.5	22.6

Mean for 49 samples is 23.2 seconds.

Standard deviation is 4 899.

were being used. The time is tabulated in the seconds required for an aircraft to cross a runway when starting from standstill. It is measured from the time a local controller cleared the aircraft for the runway crossing until the aircraft tail was clear of the runway.

APPENDIX B

ANALYSIS OF ACCIDENT PROBABILITY AND RUNWAY AND TAXIWAY SEPARATION

If an accident occurs during landing or take-off, it is possible that the damaged aircraft will leave the runway. It was considered useful to find the probability that damaged aircraft would affect operations on a parallel taxiway or runway.

The accident data used in the analysis was obtained in nine years (1950-1958) of commercial aviation operations. In this period there were 50,000,000 operations (landing or take-off). The number of accidents occurring during landing or take-off that terminated adjacent to the runway was 63. During this period there were about 46,200,000 general aviation operations Since the accident data available did not give adequate information on aircraft position, this latter class of aviation is not included in the analysis.

Figure B-1 indicates the termination point of the accidents with respect to a runway centerline. The distribution of the lateral separation of the terminal position of the accident from the centerline of the runway is given in Table B-I. The separations are measured from the midpoint of the aircraft. Table B-I does not distinguish among accidents terminating on either side of the runway.

The following quantities will be of interest in the analysis.

p = Probability of an accident during landing or take-off = $1\ 26 \times 10^{-6}$.

d = Distance runway centerline to midpoint injured aircraft.

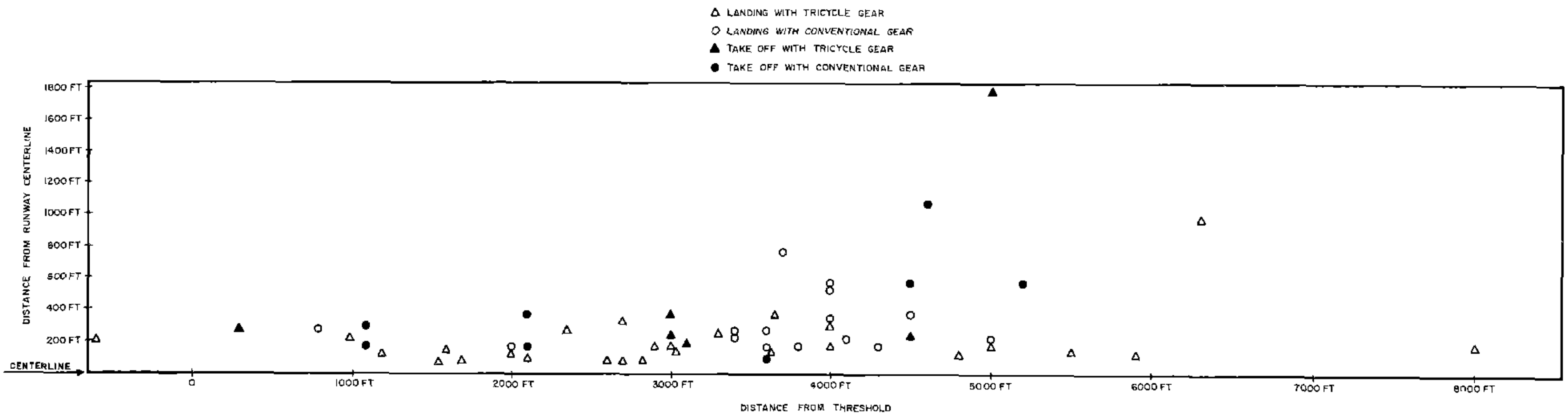

FIGURE B-1 AIRPORT ACCIDENT TERMINATION POINTS

TABLE B-I

ACCIDENT TERMINATION POINTS

Lateral Separation d	Number of Aircraft	Cumulative Number of Aircraft
75 to 100	8	55
101 to 125	3	52
126 to 150	5	47
151 to 175	12	35
176 to 200	2	33
201 to 225	3	30
226 to 250	2	28
251 to 275	5	23
276 to 300	2	21
301 to 325	2	19
326 to 350	1	18
351 to 375	4	14
525	1	13
575	3	10
775	1	9
975	1	8
1075	2	6
1375	1	5
1775	1	4
Unaccounted	4	-
Total	63	

p(d) = Probability that the terminal position of an accident is greater than d feet from the centerline of the runway.

q = Probability that a given rate of aircraft moving at uniform velocity will be passing over a particular position on a taxiway or runway.

p(d)q = Probability that an accident due to a landing or a take-off will cause an accident to an aircraft passing over a particular position on a parallel taxiway or runway.

N = Number of years necessary to achieve one collision of an aircraft, as a result of either a landing or a take-off with an aircraft on a parallel taxiway or runway. The rate of aircraft and its velocity will be the same as given in the definition of q

r = Yearly movement rate of aircraft on either a taxiway or a runway.

l+s = Length plus wingspread of aircraft involved.

v = Uniform (average) velocity of aircraft on a parallel runway or taxiway.

One of the considerations in airport design is the separation distance between an active runway and a parallel taxiway or a parallel active runway. This separation should be large enough to minimize the number of possible collisions of an aircraft that has become disabled on an active runway with an aircraft that is using an adjacent taxiway or runway.

One possibility is to make all separation distances so large that a chance collision is impossible or improbable. However, a collision involves two aircraft, and if the number of operations on the adjacent taxiway or runway is few, then the probability of a collision will still be small even with smaller separations. This situation is given by p(d)q. Tables B-II and B-III give this probability for various separations d for taxiway and runway, respectively. Such a collision will occur when either the length or wingspread of one aircraft (l+s) can occupy the same space as the other. This collision also depends on the uniform velocity v and the average rate r of the aircraft on the adjacent taxiway or runway.

Given l+s, r, and v, the following quantities may be computed·

p(d) = cumulative probability as given in Table B-I

$$q = \frac{r}{365} \times \frac{(l+s)}{v} \times \frac{1}{24 \times 3600} =$$

$$\frac{\text{time required for aircraft to move } (l+s)}{\text{average separation time between aircraft}} =$$

number aircraft ×

$$\frac{\text{time (seconds) required for aircraft to move through } (l+s) \text{ feet}}{\text{time (seconds) in day}}$$

$$N = \frac{1}{p(d)q \times r}$$

Because d is the distance from the centerline of the runway to the midpoint of the injured aircraft, the centerline-to-centerline distance between the runway and taxiway (or runway and runway) has been assumed as $\frac{l+s}{2}$.

TABLE B-II

PROBABILITY OF TAXIWAY COLLISION

Runway-Taxiway Separation	Lateral Separation d	Number Aircraft >d	p(d)	p(d)q	N
200	75	63	$12\ 6 \times 10^{-7}$	$5\ 70 \times 10^{-8}$	209
225	100	55	$11\ 0 \times 10^{-7}$	4.98×10^{-8}	239
250	125	52	$10\ 4 \times 10^{-7}$	$4\ 71 \times 10^{-8}$	254
275	150	47	$9\ 4 \times 10^{-7}$	4.26×10^{-8}	280
300	175	35	$7\ 0 \times 10^{-7}$	$3\ 17 \times 10^{-8}$	377
325	200	33	$6\ 6 \times 10^{-7}$	$2\ 99 \times 10^{-8}$	399
350	225	30	$6\ 0 \times 10^{-7}$	$2\ 72 \times 10^{-8}$	439
375	250	28	$5\ 6 \times 10^{-7}$	$2\ 53 \times 10^{-8}$	472
400	275	23	$4\ 6 \times 10^{-7}$	$2\ 08 \times 10^{-8}$	574
425	300	21	$4\ 2 \times 10^{-7}$	1.90×10^{-8}	628
450	325	19	3.8×10^{-7}	$1\ 72 \times 10^{-8}$	694
475	350	18	$3\ 6 \times 10^{-7}$	$1\ 62 \times 10^{-8}$	737
500	375	14	$2\ 8 \times 10^{-7}$	$1\ 27 \times 10^{-8}$	940
750	625	10	$2\ 0 \times 10^{-7}$	$0\ 91 \times 10^{-8}$	1312
1000	875	9	$1\ 8 \times 10^{-7}$	$0\ 81 \times 10^{-8}$	1474

r = 83,750 (comparable to LaGuardia)

$l+s$ = 250 feet

v = 10 feet

$q = 4.52674 \times 10^{-2}$

TABLE B-III

PROBABILITY OF RUNWAY COLLISION

Runway-Runway Separation	Lateral Separation d	Number >d	p(d)	p(d)q	N	p(d)q	N
200	75	63	12.6 × 10^{-7}	1 42 × 10^{-8}	282	2.84 × 10^{-8}	71
225	100	55	11 0 × 10^{-7}	1.24 × 10^{-8}	323	2.48 × 10^{-8}	81
250	125	52	10.4 × 10^{-7}	1.17 × 10^{-8}	342	2.34 × 10^{-8}	86
275	150	47	9.4 × 10^{-7}	1.06 × 10^{-8}	377	2.12 × 10^{-8}	95
300	175	35	7.0 × 10^{-7}	7 88 × 10^{-9}	508	1.57 × 10^{-8}	127
325	200	33	6.6 × 10^{-7}	7.43 × 10^{-9}	539	1.49 × 10^{-8}	135
350	225	30	6.0 × 10^{-7}	6 76 × 10^{-9}	592	1 35 × 10^{-8}	148
375	250	28	5.6 × 10^{-7}	6.31 × 10^{-9}	634	1 26 × 10^{-8}	159
400	275	23	4 6 × 10^{-7}	5 18 × 10^{-9}	772	1.04 × 10^{-8}	193
425	300	21	4.2 × 10^{-7}	4.73 × 10^{-9}	846	9.46 × 10^{-9}	212
450	325	19	3 8 × 10^{-7}	4.28 × 10^{-9}	935	8.56 × 10^{-9}	234
475	350	18	3.6 × 10^{-7}	4.05 × 10^{-9}	988	8.10 × 10^{-9}	247
500	375	14	2 8 × 10^{-7}	3.15 × 10^{-9}	1270	6.30 × 10^{-9}	318
750	625	10	2 0 × 10^{-7}	2.52 × 10^{-9}	1588	5.04 × 10^{-9}	397
1000	875	9	1.8 × 10^{-7}	2 03 × 10^{-9}	1970	4.06 × 10^{-9}	493

$l+s$ = 250 feet

v = 60 feet

r = 125,000 (LaGuardia), 250,000 (Midway)

q = 1 126 × 10^{-2}, 2.252 × 10^{-2}

The computation of N includes the interaction of both runways

In Table B-III the value of N takes into consideration the fact that either runway could affect the other.

The rates r that were used in Tables B-II and B-III are roughly comparable to rates at LaGuardia and Midway, where the rates are 250,000 and 400,000 operations per year, respectively. The lower rate of 250,000 would correspond to the current activity level at LaGuardia. The higher rate of 500,000 would be about the maximum number of annual movements that can be anticipated in the near future.

Note that the expressions for N are conservative since not all accidents will terminate on the same side of the runway.

The analysis indicates that the probability of collision between landing and taxiing aircraft (or landings on adjacent runways) is very low for all separations analyzed--some of which are less than the actual separation criteria in use today.

APPENDIX C

TEST FOR RANDOM DISTRIBUTIONS

A. TESTS FOR RANDOMNESS

A sequence of numbers (t_i), $i = 1, N$

$$t_1, t_2, t_3, \ldots, t_i, \ldots, t_N \tag{1}$$

is random if knowledge of t_i does not imply precise knowledge of any t_ℓ where $\ell > i$. The serial correlation test can be applied to sequence 1 to determine if the sequence is random. In the application of these tests, Anderson* assumes that the distribution of sequence 1 is normal. However, Wald** makes no assumption about this distribution. Both Anderson and Wald use the "circular" definition of the serial correlation-coefficient, and since this is easier to apply than the definition given by Kendall,*** a description of Kendall's test is omitted.

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* R. L. Anderson, "Distribution of the Serial Correlation Coefficient," Vol 13, p 1, Annals of Mathematical Statistics, 1942.

** A. Wald and F. Wolfowitz, "An Exact Test for Randomness in the Non-Parametric Case Based on Serial Correlation," Vol 14, p 378, Annals of Mathematical Statistics, 1943.

*** M. G. Kendall, "The Advanced Theory of Statistics," Vol 2, Hafner Publishing Co., 1951.

From 1, the following quantities are computed

$$S_r = \sum_{i=1}^{N} t_i^r \quad (2)$$

where r = 1, 2, 3, 4

$$E(R) = \frac{S_1^2 - S_R}{N - 1} \quad (3)$$

$$\sigma_A^2(R) = \left(S_2 - \frac{S_1{}^2}{N}\right) \frac{(N - 2)}{(N - 1)2} \quad (4)$$

$$\sigma_W^2(R) = \frac{S_2^2 - S_4}{N - 1} + \frac{S_1^4 - 4S_1^2S_2 + 4S_1S_3 + S_2^2 - 2S_4}{(N - 1)(N - 2)} - \left[\frac{S_1^2 - S_2}{N - 1}\right]^2 \quad (5)$$

$$R_h = \sum_{i=1}^{N} t_i t_{i+h} \quad (6)$$

when $i+h > N$, replace $i+h$ by $i+h - N$.

If 1 is a random sequence, the expected value of R_h is E(R). The variance of R_h is given by Anderson in 4 and Wald in 5. For this test, h and N must have no common factors If the distribution of 1 is normal, the variance in 4 may be used. However, when the distribution of 1 is unknown, the variance in 5 should be used.

To apply the test, form a normalized variable α

$$\alpha = \frac{R_h - E(R)}{\sigma} \quad (7)$$

where σ is obtained from either 4 or 5. Depending upon the significance level desired, the acceptance or rejection of the assumption that 1 is a random sequence is determined by α. For 95-percent limits, the assumption is accepted if $|\alpha| \leq 1.96$, and for 99-percent limits, if $|\alpha| \leq 1.99$.

A summary of the sequences tested is given in Table C-I.

It is possible for a sequence of "random" numbers to fail the test. Four sets of sequences were obtained from the exponential distribution by solving

$$e^{-\lambda t_i} = r_i \quad \begin{cases} r_i \text{ a random number} \\ 0 \leq r_i \leq 1 \end{cases} \tag{8}$$

for λ = 10, 25, 50, 100. For each λ, the sequence t_i passed the serial correlation test. However, when the four were grouped to form one sequence, the new sequence failed the test because it exhibited a trend effect.

B TESTS FOR POISSON DISTRIBUTION

Epstein* describes many tests that can be used if the underlying distribution of life is exponential His test 3 can be applied to sequence 1 to determine if its distribution is Poisson with a constant parameter λ. In this

* B. Epstein, "Tests for the Validity that the Underlying Distribution of Life is Exponential, Part I," Vol 2, p 83, Technometrics, 1960.

TABLE C-I

RANDOM DISTRIBUTIONS

Airport	Sample Size	Function	E(R)	σ_W	σ_A	R_I	α_W	α_A
LaGuardia--November	91	R	2,038,556	185,991	510,805	867,006	-0.922	-0.469
Newark--All Day	106	R	12,452,603	-	1,250,951	15,942,425	-	2.789
Newark--1st	49	R	1,462,017	-	2,827,807	13,804,956	-	4.365
Newark--2nd	46	R	1,542,965	152,612	164,021	1,442,990	-0.655	-0 610
Wichita--1st	28	R	3,271,830	595,437	664,005	3,743,568	0.792	0 710
Wichita--2nd	53	R	3,829,019	404,338	440,730	4,050,424	0 547	0 502
Miami--December 3	47	R	2,454,312	580,259	621,172	1,891,544	-0.970	-0 910
Miami--December 4	73	T	1,281,843	93,916	102,074	1,288,724	0.073	0.067
LaGuardia--November	79	T	2,222,971	129,914	135,741	2,211,997	-0 085	-0.007
Newark--1st	53	T	11,100,100	2,702,724	-	10,609,730	-0.181	-
Newark--2nd	31	T	2,368,621	184,840	-	2,675,695	1.661	-
Miami--December 3	43	T	3,176,672	326,792	-	3,556,918	1.164	-
Wichita--1st	43	T	1,745,886	278,717	-	1,579,993	-0 595	-
Wichita--2nd	41	T	4,247,775	651,800	-	4,766,810	0 796	-
Miami--December 4	76	T	1,600,103	155,963	-	1,808,727	1.338	-
Newark--All Day	91	T	14,239,520	2,477,779	-	14,564,230	0 131	-

R is the sequence formed by the time differences of the ready-to-go times of successive departing aircraft.

T is the sequence formed by the time differences of the over-threshold times of successive landing aircraft.

case, t_i can be interpreted as the elapsed time between two events (for example, the ready-to-go time of a departure). If we form

$$\begin{aligned} T_1 &= t_1 \\ T_2 &= T_1 + t_2 \\ T_{i+1} &= T_i + t_{i+1} \\ T_N &= T_{N-1} + t_N \end{aligned} \tag{9}$$

where the probability $(t > t_o) = e^{-\lambda t_o}$ λ constant, then the distribution T_i, when the T_i are considered as unordered, is rectangular over $(0,T_N)$. If we form

$$T = \sum_{i=1}^{n-1} T_i \tag{10}$$

the expected value and variance of T is

$$E(T) = \frac{(N - 1)}{2} T_N \tag{11}$$

and

$$\sigma^2(T) = \frac{(N - 1)}{12} T_N^2 \tag{12}$$

To use this test, we again form a normalized variable

$$\alpha = \frac{T - E(T)}{\sigma(T)} \tag{13}$$

and the assumption that sequence in 1 is Poisson with constant parameter λ, at some level of significance is accepted or rejected if (α) is less than or exceeds that level, as described before

The results of this test are given in Table C-II.

TABLE C-II
POISSON DISTRIBUTIONS

Airport	Sample Size	Function	T	E(T)	σ(T)	α
Wichita--2nd	53	R	106.193	103.711	8.3	0.298
Wichita--1st	28	R	34.940	36.5	4 06	-0.39
Newark--2nd	49	R	52.975	56.613	4.718	-0 771
Miami-- December 3	47	R	80 715	69.900	5.950	1.8175
LaGuardia-- November 13	91	R	183.46	171.099	10.413	1.187
Miami-- December 4	76	R	90.203	97.19	6.613	-1.057
Newark-- All Day	106	R	608.3861	532.437	29.999	2 5317
Newark--1st	49	R	172.001	36.76	3.063	44.148

R is the sequence formed by the time differences of the ready-to-go times of successive departing aircraft.

A further test using the cumulative frequency distribution is the Kolmogorov-Smirnov test.* If $F(x)$ is the

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* F. J Massey, Jr., "The Kolmogorov-Smirnov Test for Goodness of Fit," Vol 46, p 68, Journal of the American Statistical Association, 1951

assumed cumulative frequency distribution and $S_N(x)$ is the observed cumulative frequency distribution of sample size N, the following function can be formed.

$$d = \max_x \left| F(x) - S_N(x) \right| \qquad (14)$$

The distribution of d is independent of F(x) if F(x) is continuous, which is the case for an assumed exponential distribution. If d exceeds some critical value $d_\alpha(N)$ where α is the level of significance, then F(x) is rejected as the assumed distribution.

Draw the function F(x) and then the two functions $F(x) \pm d_\alpha(N)$. If at any point x, $S_N(x)$ extends beyond $F(x) \pm d_\alpha(N)$, then F(x) is rejected at the level α as the assumed curve. Applications of this test are given in Figures 9-10, 9-11, and 9-12.

APPENDIX D

MATHEMATICAL FORMULATION OF TIME-DEPENDENT, FIRST-COME, FIRST-SERVED MODEL WITH CONSTANT SERVICE TIME

This section gives the equations for the time-dependent, first-come, first-served model with constant service time. These equations were used to develop the expected delay for several system loadings (Section IX) by writing a Fortran program and using an IBM 709 Computer

1 NOTATION AND DEFINITIONS

Service time = constant = 1 unit

$\Delta t = \frac{1}{m}$ = increments of unit time at which delay is computed

$w(n,i)$ = probability of delay of $n\Delta t$ units at time $i\Delta t$. $n = 0, 1, 2, 3, \ldots,$ $i = 1, 2, \ldots$

$G(n,i) = \sum_{k=n}^{\infty} w(k,i)$ = cumulative probability of delay of $n\Delta t$ units or more at time $i\Delta t$. $n = 0, 1, 2, 3, \ldots,$ $i = 1, 2, 3, \ldots$

$a(n)$ = probability of n aircraft arriving in time Δt

$A(n) = \sum_{k=n}^{\infty} a(k)$ = cumulative probability of n or more aircraft arriving in time Δt

λ = rate at which aircraft arrive

$A_n = \sum_{k=o}^{\infty} k^n a(k)$ = nth moment of $a(n)$

$W_n(i) = \sum_{k=o}^{\infty} (k)^n w(k,i)$ = nth moment of $w(n,i)$

$w(n) = \lim_{i \to \infty} w(n,i)$

$$G(n) = \lim_{i \to \infty} G(n,i)$$

$$W_n = \lim_{i \to \infty} W_n(i)$$

2. MODEL AND DISCIPLINE

The model has a constant service time with a first-come, first-served discipline. The aircraft arrive, begin, and end service only at times $n\Delta t$ The technique permits a general stationary independent arrival distribution $a(n)$. This will later be specialized into a Poisson distribution with a parameter λ. Since this model is discrete, the delay probabilities $w(n,i)$ may be in error when compared with a continuous model. The error decreases for $\Delta t \to 0$.

3. DERIVATION OF EQUATIONS FOR GENERAL ARRIVAL DISTRIBUTION

The delay at time $(i+1)\Delta t$ depends upon the delay at time $i\Delta t$ and the arrival distribution $a(n)$.

The equations of delay are

$$w(0, i+1) = a(0)\left[w(0, i) + w(1, i)\right]$$

$$w(1, i+1) = a(0)\, w(2, i)$$

$$w(2, i+1) = a(0)\, w(3, i)$$

$$w(m-1, i+1) = a(0)\, w(m, i)$$

$$w(m, i+1) = a(0)\, w(m+1, i) + a(1)\left[w(0, i) + w(1, i)\right]$$

$$w(m+1, i+1) = a(0)\, w(m+2, i) + a(1)\, w(2, i)$$

$$w(2m - 1, i + 1) = a(0)\ w(2m, i) + a(1)\ w(m, i)$$

$$w(2m, i + 1) = a(0)\ w(2m + 1, i) + a(1)\ w(m + 1, i) + a(2)\left[w(0, i) + w(1, i)\right]$$

$$w(2m + 1, i + 1) = a(0)\ w(2m + 2, i) + a(1)\ w(m + 2, i) + a(2)\ w(2, i)$$

----

(1)

The equations of cumulative delay are.

$$G(0, i + 1) = 1$$

$$G(1, i + 1) = a(0)\ G(2, i) + A(1)$$

$$G(2, i + 1) = a(0)\ G(3, i) + A(1)$$

$$G(m - 1, i + 1) = a(0)\ G(m, i) + A(1)$$

$$G(m, i + 1) = a(0)\ G(m + 1, i) + A(1)$$

$$G(m + 1, i + 1) = a(0)\ G(m + 2, i) + a(1)\ G(2, i) + A(2)$$

$$G(2m - 1, i + 1) = a(0)\ G(2m, i) + a(1)\ G(m, i) + A(2)$$

$$G(2m, i + 1) = a(0)\ G(2m+1, i) + a(1)\ G(m + 1, i) + A$$

$$G(2m + 1, i + 1) = a(0)\ G(2m + 2, i) + a(1)\ G(m + 2, i) + a(2)\ G(2, i) + A(3)$$

----

(2)

In general, let n = km + p $\qquad m > p \geq 0$

$$G(n, i + 1) = a(0)\, G(n + 1, i) + a(1)\, G(n + 1 - m, i) + a(2)\, G(n + 1 - 2m, i) + \quad . + a(k - 1)\, G\left[n + 1 - (k - 1)\, m, i\right] + F \qquad (3)$$

where

$$F = a(k)\, G(n + 1 - km, i) + A(k + 1) \qquad p > 0$$
$$= A(k) \qquad p = 0$$

If we let $\vec{G}(i)$ be the column vector of cumulative delay at time (i), equation 3 may be represented by the matrix equation of the form.

$$\vec{G}(i + 1) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 -- 0 & & & \\ 0 & 0 & a(0) & 0 & 0 -- 0 & & & \\ 0 & 0 & 0 & a(0) & 0 -- 0 & & & \\ 0 & 0 & 0 & 0 & a(0)\ 0 & & & \\ \hline 0 & 0 & a(1) & 0 & 0 & a(0) & 0 & 0 \\ 0 & 0 & 0 & a(1) & 0 -- 0 & & a(1) & 0 \\ \hline \end{bmatrix} \vec{G}(i) + \begin{bmatrix} A(0) \\ A(1) \\ A(1) \\ A(1) \\ - \\ A(2) \\ A(2) \\ - \end{bmatrix}$$

(4)

Since G(0, i) = 1, the first row and column may be deleted from the scheme.

By multiplying the n^{th} equation 1 by $k^n (k = 0, 1, 2, 3, \quad . .)$ and summing, the expressions for the moments $W_k(i)$ can be obtained. By summing the right side in terms of the columns, we have (for n = 1)

$$W_1(i+1) = a(0) \sum_{k=1}^{\infty} (k-1)\, w(k, i) + a(1) \sum_{k=1}^{\infty} (k+m-1)\, w(k, i)$$

$$+ a(2) \sum_{k=1}^{\infty} (k+2m-1)\, w(k, i) + \ldots$$

$$+ w(0, i) \sum_{k=0}^{\infty} kma(k) = a(0) \left[W_1(i) - G(1, i) \right]$$

$$+ a(1) \left[W_1(i) + (m-1)\, G(1, i) \right]$$

$$+ a(2) \left[W_1(i) + (2m-1)\, G(1, i) \right] + \ldots + mw(0, i)\, A_1$$

$$= W_1(i) + G(1, i) \left[\sum_{k=0}^{\infty} (km-1)\, a(k) \right] + mw(0, i)\, A_1$$

$$= W_1(i) + mA_1\, G(1, i) - G(1, i) + mw(0, i)\, A_1$$

$$= W_1(i) + mA_1 - G(1, i)$$

By a similar technique, the second and third moments can be obtained We summarize

$$W_1(i+1) = W_1(i) + mA_1 - G(1, i) \tag{5}$$

$$W_2(i+1) = W_2(i) + m^2A_2 + 2W_1(i) \left[mA_1 - 1 \right] + G(1, i) \left[1 - 2mA_1 \right] \tag{6}$$

$$W_3(i+1) = W_3(i) + m^3A_3 + 3W_2(i) \left[mA_1 - 1 \right] + 3W_1(i) \left[m^2A_2 - 2mA_1 + 1 \right] - G(1, i) \left[1 - 3mA_1 + 3m^2A_2 \right] \tag{7}$$

If there is a steady-state solution, we can obtain (for $i \longrightarrow \infty$) the following from equations 5, 6, and 7

$$G(1) = mA_1 = \text{probability of delay} \tag{8}$$

$$W_1 = \frac{m^2 A_2 + mA_1 - 2m^2 A_1{}^2}{2(1 - mA_1)} \tag{9}$$

$$W_2 = \frac{m^3 A_3 + 3W_1 \left[m^2 A_2 - 2mA_1 + 1\right] - mA_1 \left[1 - 3mA_1 + 3m^2 A_2\right]}{3(1 - mA_1)} \tag{10}$$

The limit distribution of $G(n, i)$ from equation 4 can be obtained in a similar manner.

These results have been obtained with a general arrival distribution $a(n)$, and with an arbitrary interval size $\Delta t = \frac{1}{m}$. Note that equations 5, 6, and 7 are expressions in terms of Δt, and can be written in terms of unit service time (unity) by dividing them respectively by m, m^2, m^3. Note further that equations 8, 9, and 10 are now in this form.

4. SPECIALIZATION OF POISSON ARRIVAL DISTRIBUTION

Let $$a(n) = \frac{(\lambda/m)^n e^{-\lambda/m}}{n!},$$

then

$$A_1 = \frac{\lambda}{m}$$

$$A_2 = \left(\frac{\lambda}{m}\right)^2 + \left(\frac{\lambda}{m}\right)$$

$$A_3 = \left(\frac{\lambda}{m}\right)^3 + 3\left(\frac{\lambda}{m}\right)^2 + \left(\frac{\lambda}{m}\right)$$

Equations 8, 9, and 10 can now be rewritten as

$$G(1) = \lambda = \text{probability of delay} \tag{11}$$

$$W_1 = \frac{\lambda}{2(1 - \lambda)} + \frac{\lambda}{2m} \tag{12}$$

$$W_2 = \frac{\lambda}{3(1 - \lambda)} + \frac{\lambda^2}{2(1 - \lambda)^2} + \frac{\lambda + \lambda^2}{6m^2} + \frac{\lambda}{2(1 - \lambda)m} \tag{13}$$

The continuous, steady-state values (as $m \rightarrow \infty$) are

$$G(1) = \lambda = \text{probability of delay} \tag{14}$$

$$W_1 = \frac{\lambda}{2(1 - \lambda)} \tag{15}$$

$$W_2 = \frac{\lambda}{3(1 - \lambda)} + \frac{\lambda^2}{2(1 - \lambda)^2} \tag{16}$$

The probability of delay in equation 11 does not depend on m, and equations 12 and 13 exceed the steady-state values by expressions of the order $\frac{1}{m}$ If we consider

$$W_{nq} = \sum_{k=1}^{\infty} (k - 1)^n \, w(k, i) \tag{17}$$

we can obtain, for W_{nq}, expressions analogous to equations 12 and 13, that is,

$$W_{1q}(i) = W_1(i) - G(1, i) \tag{18}$$

$$W_{2q}(i) = W_2(i) - 2W_1(i) + G(1, i) \tag{19}$$

and in the limit for $i \rightarrow \infty$

$$W_{1q} = W_1 - \frac{\lambda}{m} \tag{20}$$

$$W_{2q} = W_2 - 2\frac{W_1}{m} + \frac{\lambda}{m^2} \tag{21}$$

If we form a linear combination of equations 12, 13, and 20. Then expressions can be derived that agree with equation 15 and differ from equation 16 by the order of $\left(\frac{1}{m^2}\right)$

The correct linear combination is one-half of each, that is,

$$\frac{W_1 + W_{1q}}{2} = \frac{\lambda}{2(1 - \lambda)} \tag{22}$$

$$\frac{W_2 + W_{2q}}{2} = \frac{\lambda}{3(1 - \lambda)} + \frac{\lambda^2}{2(1 - \lambda)^2} + \frac{\lambda + \lambda^2}{6m^2} \tag{23}$$

Expressions 5 and 6 should be modified to

$$\overline{W}_1(i + 1) = W_1(i) - \frac{G(1, i + 1)}{2} \tag{24}$$

$$\overline{W}_2(i + 1) = W_2(i) - \overline{W}_1(i + 1) \tag{25}$$

If the values 24 and 25 are used, we obtain expressions for the first and second moments of delay that depend least upon the interval Δt.

5. COMPARISON WITH WORK OF GALLIHER AND WHEELER*

A study similar to the present one has been made by Galliher and Wheeler. Both models assume that the arrival distribution is Poisson and the service time is constant. The final output of each model is the average delay or wait.

Although the format of the equations is the same, the equations in this model are interpreted as equations of delay, whereas in the Galliher and Wheeler model, the equations are equations of state (or the number of units present in the system). They show how the average delay may be derived from the state equations, whereas, in the present work, the moments of delay are computed directly

By treating the equations as equations of delay, the delay at time intervals less than the service-time interval can be computed in a direct fashion In this way, the effect of interval size on the distribution and moments of delay can be studied.

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* H P Galliher and R C Wheeler, Jr , "Nonstationary Queuing Probabilities for Landing Congestion of Aircraft," Vol 6, p 264, Operations Research, March-April 1958.

APPENDIX E

NUMERICAL COMPUTATION OF TIME-DEPENDENT DELAY FOR VARIABLE SERVICE TIME USING DISCRETE TRIALS

The following technique can be used to compute the probability of delay and the average delay for a first-come, first-served model when both the service time and the arrival distribution vary as functions of time.

A. NONSTATIONARY BERNOULLI ARRIVALS

Arrivals are a Bernoulli process with a nonstationary success probability. That is, a sequence of trials numbered $i = 1,2,$. with the i^{th} trial occurring at time $t(i)$, and with the trials separated in time. For arrivals, there are only two possible outcomes of a trial:

Outcome	Probability of Being the Outcome of the i^{th} Trial
1 arrival	$p(i)$
0 arrivals	$1 - p(i)$

Nonstationarity of arrivals thus means that the value of $p(i)$ is not a constant, independent of the value of i, but depends upon the value of i.

B NONSTATIONARY DISCRETE SERVICE TIMES

Service times begin and end only as outcomes of the same discrete set of the above trials $i = 1,2,$ . . That is, a service time that begins as an outcome of a trial i will end as an outcome of some later trial $j > i$ ($j = i$ will not be allowed). Such a service time will have a duration of $j - i$ trials, and a length in time equal to $t(j) - t(i) > 0$.

For each service time that begins as an outcome of some trial i, assume a probability $s(n,i)$ that the servicing has a duration of n trials. For any given n, $s(n,i)$ may vary with i. Therefore, the service-time distribution is nonstationary. Of course, for each i,

$$\sum_{n=1}^{\infty} s(n,i) = 1.$$

We also need the complementary distribution function $K(n,i)$, defined by

$$K(n,i) = \sum_{j=n+1}^{\infty} s(j,i).$$

C MULTIPLE OUTCOMES OF TRIAL

Both an arrival and a completion of service can be simultaneous outcomes of a trial. Obviously, we want no delay in start of service. Thus, if a unit arriving as the outcome of a trial finds the server idle as the outcome of that trial, its service time begins as an outcome of that same trial. The same applies to a unit that has been forced to wait for service, its service begins as the outcome of the same trial as an outcome of which the server has completed the service of all units served before the waiting unit.

D WAITING TIME IF SERVICE IS ARRIVAL

Suppose that a unit arrives as an outcome of some trial i. If its service time begins as an outcome of trial $j \geq i$, then its wait has by definition a duration of $j - i$ trials, and a time-length equal to $t(j) - t(i)$.

Let $w(n,i)$ be the probability that a unit arriving as an outcome of trial i has a duration of wait equal to n trials. If we consider the actual outcome of the i^{th} trial, we can write the following recursive relation.

$$w(0,i+1) = p(i)w(0,i)s(1,i) + [1 - p(i)]\,[w(0,i) + w(1,i)]$$

$$w(n,i+1) = p(i)\,[w(0,i)s(n+1,i) + \sum_{k=1}^{n} w(k,i)s(n+1-k,\, i+k)]$$

$$+ [1 - p(i)]\, w(n+1,i)$$

We can verify that the sum of w(n,i) equals the sum of w(n,0). Thus, we need only make the latter sum equal to 1 as an initial constraint. The recursion would presumably start by setting i = 0, so that the specific values of w(n,0) would be assumed

E COMPUTATION

To compute the mean duration of wait

$$\overline{n}(i) = \sum_{n=1}^{\infty} nw(n,i)$$

and also the probabilities

$$G(n,i) = \sum_{j=n+1}^{\infty} w(j,i)$$

that the wait has a duration greater than n intervals, it is more convenient to work from equations especially derived for this purpose. Moreover, if the arrival probabilities and service-time distributions are piecewise-stationary (that is, do not change with every value of i, and preferably only from time to time), then further advantages are obtained by using equations especially adapted for that purpose.

F. COMPUTATION IN PIECEWISE-STATIONARY CASE

We make the following assumptions

1. The intervals i are equally spaced in time. This assumption is not always necessary, but eliminates consideration of the function t(i), the duration of wait provides for measurement

2. The value of p(i) changes on a set i_h of values of i, h = 1, 2, . . .,H. A table a(h) of the values i_h is assumed available, with a(1) = 0 A table p(h) of values then provides values of p(i) The value of h corresponding to i at any stage in the computations is understood to be the largest value of h, so that $a(h) \le i$.

3. The values s(n,i) change for some n on a set i_k of values, with k = 1,2, . .,K. A table b(k) of the values i_k is assumed available, with b(1) = 0. A table s(n,k) provides values of s(n,i). The value of k corresponding to a value of i is understood to be the largest value of k, so that $b(k) \le i$.

4. We assume further that the following tables are also provided

u(k) = maximum value of n for which s(n,k) > 0.

v(k) = minimum value of n (it must be ≥ 1) for which s(n,k) > 0.

$$\bar{s}(k) = \sum_{v(k)}^{u(k)} ns(n,k) = \text{mean service-times}$$

$$K(n,k) = \sum_{j=n+1}^{u(k)} s(j,k), \text{ for } 0 \le n \le u(k) - 1$$

$$\left.\begin{aligned} \bar{d}(m,k) &= \bar{s}(m) - \bar{s}(k) \\ f(m,k) &= \min\,[v(k), v(m)] \\ g(m,k) &= \max\,[u(k), u(m)] \\ d(n,m,k) &= K(n,m) - K(n,k),\ f(m,k) \le n \le g(m,k) - 1 \end{aligned}\right\} m = k+1,\ k+2,\ \ldots,\ K$$

With the above functions available as a result of preliminary tabulation, we can then arrange the computation. We can validate the equations that follow.

G. MEAN WAIT

$$\overline{w}(i + 1) = \overline{w}(i) - G(0,i) + p(h) \left\{ \sum_{m=k+1}^{K-1} \overline{d}(m,k) \left[G[b(m) - 1,i] - G[b(m + 1) - 1,i] \right] - 1 + \overline{s}(k) + G(0,i) \right\}$$

It is assumed tacitly that $G[b(K) - 1,i] = 0$. The value of $b(K)$ must be large enough to ensure this. In practice this is done by the use of the quantity $e(i)$ (defined below).

H PROBABILITY THAT WAIT EXCEEDS n INTERVALS

$$G(n,i + 1) = p(h) \left\{ K(n + 1,k) + \sum_{j=A(n,k,i)}^{B(n,k)} s(j,k)\, G(n + 1 - j,i) + \sum_{m=k+1}^{K-1} \sum_{j+C(m,n,k)}^{D(m,n,k,i)} [G(j - 1,i) - G(j,i)]\, d(n + 1 - j,m,k) \right\}$$

$$+ [1 - p(h)]\, G(n + 1,i)$$

where

$$A(n,k,i) = \max\,[v(k),\ n + 2 - e(i)]$$
$$B(n,k) = \min\,[n + 1,\ u(k)]$$
$$C(m,n,k) = \max\,[b(m) - i,\ n + 2 - g(m,k)]$$
$$D(m,n,k,i) = \min\,[n, b(m + 1) - i - 1,\ n + 1 - f(m,k)\ e(i)]$$
$$e(i) = \text{the least value of } n \text{ for which } G(n,i) = 0.$$

APPENDIX F

SIMULATION MODEL TO OBTAIN DISTRIBUTION OF SAMPLE AVERAGES

A computer program has been developed to simulate a simplified version of either the pre-emptive spaced arrivals model or the first-come, first-served queuing model. Both models are incorporated into the same program, the first-come, first-served model can be obtained from the renewal model by the deletion of a few steps Because of this the spaced arrivals model will be discussed first. In both cases program output is a distribution of average delay as a function of sample size.

A. SIMULATION SPACED ARRIVALS MODEL

The random variables that explicitly enter the spaced arrivals model are R, C, S_1, and S_2 (Section X). However, for simplicity in the model, the above quantities will be constant--as determined by their means r_1, c_1, S_{11}, S_{21} After the parameters λ_1 and λ_2 (Section X) have been specified, the spaced arrivals model will be completely determined.

The random elements of the model are the inter-arrival times for departures and the random variable G for landings. Now (Section X),

$$g_1 = \frac{3600}{\lambda_1} - r_1 - c_1$$

Parameters	Units
λ_1, λ_2,	aircraft/hour
r_1, c_1, g_1,	seconds
t, S_{11}, S_{21}	seconds

then, prob $(G > t) = e^{-t/g_1}$. The cumulative "arrival" distribution for departures is $e^{-\lambda_2 t/3600}$

B. TECHNIQUE OF SIMULATION

To describe the logic used in the simulation program, the following notations are made.

- α = random number drawn from pseudo-random number generator, where the set (α) has a rectangular distribution. All α are positive and less than 1
- RTGL = ready-to-go time (cumulative) of the last departure
- RTGN = ready-to-go time (cumulative) of the next departure
- OTN = over-threshold time (cumulative) of the next landing
- TC = latest time (cumulative) that a departure has either cleared the boundary or a landing has turned off the runway
- TEMP = temporary value used for intermediate computation
- DT = delay to this departure

Assuming that a departure has just cleared the boundary, the delay (if any) to the next departure can be obtained from the following steps.

1. $RTGN = RTGL - \left(\frac{3600}{\lambda_2}\right) \log \alpha$
2. RTGL = RTGN
3. TEMP = Max (RTGN, TC)
4. If TEMP + S_{11} < OTN proceed to step 5, otherwise a landing will take place next (step 8)
5. DT = TEMP - RTGN
6. TC = TEMP + S_{21}
7. Exit from this routine
8. TC = OTN + r_1
9. OTN = TC + c_1 + $(-g_1) \log \alpha$
10. Return to step 3

C. LOGIC USED IN SIMULATION

The spaced arrivals model assumes that landing aircraft are never impeded by departing aircraft. Moreover, in the interval between over threshold of two successive landings, there are three elements--R, G, and C (Section XII). If a departure is released between two successive landings, it may be released only in an interval G. Moreover, the departure must finish S_{11} in time G + C This requirement is given in steps 3 and 4. If a departure cannot complete S_{11} before the expiration of G + C, then he must wait until the off-runway time of the next landing (step 8), and the departure must now be considered again if the interval G + C is sufficient for his release (steps 9, 3, 4). After a departure has finally been released, its delay is computed. The delay is defined as the difference in times between the time the departure was able to initiate service and the time service was desired (RTGN).

D. PROGRAM OUTPUT

As a result of the n^{th} departure, the output is the average total delay for the first n departures--that is, if d_i is the delay to the i^{th} departure, the output for the n^{th} departure is

$$\frac{1}{n} \sum_{i=1}^{n} d_i$$

If many replications are taken, the final output of the program is the distribution of these averages for certain values of n.

E. FIRST-COME, FIRST-SERVED MODEL

This model is obtained by deleting step 4 in the logic used for the renewal model The final output is also a distribution of the average delay.