FINAL ENGINEERING REPORT

FOR

HYPERBOLIC COORDINATE CONVERTER

This report covers the period 15 September 1959 to 15 January 1960.

MOTOROLA INC.
WESTERN MILITARY ELECTRONICS CENTER
SCOTTSDALE, ARIZONA

FEDERAL AVIATION AGENCY
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ABSTRACT

A four-month study program has been conducted by Motorola to establish the optimum design for a practical, airborne computer that would accept the hyperbolic information available from present low-frequency navigational aids and convert it to a form better suited for air traffic control and general navigation. The results of this investigation are contained in this Final Engineering Report to FAA.

The outputs chosen for display are distance-to-go and cross-track-distance, since these coordinates correspond to displacement from, and distance along, a pre-assigned corridor. The design philosophy employed resolves the conflicting requirements of computer simplicity, long-range operation, and high accuracy report of present-position. The associated computation time is shown to be compatible with the dynamic constraints imposed by atmospheric noise and aircraft velocity considerations. Furthermore, the recommended coordinate converter can be readily combined with other navigational aids to form a complete and integrated control system. Full derivations of the appropriate mathematical results are included along with substantial documentation of the major conclusions reached.

PART I

1. PURPOSE

The intent of this study is to determine definitively the most feasible design of a simple, small, and lightweight coordinate converter (C/C) and to determine the most useful coordinates in which to display the C/C output. The information used is to be available in the hyperbolic receivers The design is to be readily extendable to a complete and integrated flight control system.

1.1 BREAKDOWN OF PROBLEMS

The following is a summary of the more significant problem areas that existed at the inception of this study or that arose as the study proceeded. The list is not necessarily in order of importance, but rather is in the logical order of occurence.

1. Establish operational requirements for a system utilizing hyperbolic information for use in establishing aircraft present-position. The information can be used for other purposes, but the principal objective is to provide a means whereby aircraft can fly a preassigned transoceanic flight path On this mission long-range operation is, of necessity, a major problem

2. Determine a method for changing hyperbolic data into a form more useful for traffic control and navigation. The method developed must accomplish this with relatively simple, light-weight equipment.

3. Determine the applicable hyperbolic systems and devise a method to affect a coordinate converter (C/C) tie-in with existing equipment with a minimum of modification.

4 Devise methods of coordinate conversion for accurate long-range course control. Since long range is an objective, the various approximations normally made for short range navigation must be examined to ascertain their significance. The principal factors which limit the range are

   a. Assumption of a spherical earth.

   b. Extent of correction factor in performing planar computations.

   c. Propagation error at long ranges

   While an interim system may not automatically apply corrections for any of these items, it is important that the C/C be arranged so that correction can be made.

5 For long-range operation input data noise cannot be ignored and consideration must be given to performance characteristics of the equipment in the presence of this noise. This noise is a limiting factor, and while the subject cannot be studied within the frameword of this study program, general methods of solution are discussed with ensuing system benefits

6 The accuracy of the C/C in a hyperbolic field is of concern and analysis must be made to show that the operational requirements can be met with the proposed methods

2 GENERAL FACTUAL DATA

2 1 IDENTIFICATION OF PERSONNEL

Technical personnel who expended effort on this project are listed below in Table I.

TABLE I. TECHNICAL PERSONNEL EFFORT

Name	Title	Hours
M. F Peterson	Group Manager	7
E. J Groth	Assistant Group Manager	39
E. H Lange	Project Leader	524
J P Barto	Electrical Project Leader	381
R. L. Pepin	Development Engineer	454
R. W. Sanneman	Project Electrical Engineer	245
F. T. Gabrielson	Senior Engineer	422
S W. Attwood	Senior Engineer	32
C. R. Williams	Senior Engineer	145
W. A Burns	Electrical Engineer	72

2.2 PATENT DECLARATIONS

2.2.1 Computing System

This invention comprises a novel method for controlling relative drift between ground and airborne oscillators by means of an error signal derived from redundant input range information. The error signal is fed back to control the phase of a local oscillator, thereby allowing a simple computation of present-position, utilizing absolute, drift-free values of ranges from transmitters, rather than range differences.

This invention was submitted under Motorola Patent Application No. 822341, disclosed 2 September 1958 under Contract No. DA-36-039-SC-78020, for action by Patent Office on 23 June 1959.

2.2.2 Hyperbolic Coordinate Converter

This invention achieves an economy of design by the mechanization of simple, exact formulae. Principal features are: (1) formulation of hyperbolic coordinates into an input matrix; (2) multiplication of this matrix by another matrix which depends solely upon the configuration of the stations that produced the hyperbolic information; and (3) use of the elements in the resulting matrix to evaluate the coefficients in a quadratic equation whose solution represents the range of the point in question from one of the stations.

The hyperbolic coordinate converter was disclosed 6 January 1959 under Contract No. DA-36-039-SC-78020. The patent application is presently being prepared for submission to the Patent Office.

2.3 REFERENCES

The following is a list of references cited in portions of this report. The references are given in order of their appearance in the text.

1. Doppler Radar Mark I Computer, ARINC Characteristic No. 543, February 6, 1959.

2. High-Speed Computing Devices, Staff of Engineering Research Associates, New York, McGraw-Hill, 1950.

3. Phase of the Low Frequency Ground Wave, NBS Circular 573.

4. Sampled-Data Control Systems, Ragazzini, J. R. and G. F. Franklin, (Section 10.4), New York, McGraw-Hill, 1954

5. Approximations for Digital Computers, Hastings, Cecil, Princeton University Press, 1955.

6. Journal of the British I. R. E., Vol. 18, No. C, pp 277-292, May 1958.

7. Conformal Projections in Geodesy and Cartography, Thomas, P. D., U. S. Department of Commerce, Coast and Geodetic Survey, Special Bulletin No. 251.

8. Loran, Pierce, J. A , et al, M. I. T. Radiation Laboratory Series, 1947.

9. Simple Computation of Distance on the Earth's Surface, Sitterly, B. W. and J A. Pierce, M. I. T. Radiation Laboratory Report No. 582, July 8, 1944.

2.4 SYSTEM REQUIREMENTS

The general scope of this study as outlined in the specification is to "determine the most feasible design of a simple, small, and lightweight coordinate converter which is readily extendable to a complete and integrated flight control system." The system discussed can be considered a general method for utilizing hyperbolic information to establish aircraft present-position.

The operational aspects of the system are directed toward aircraft flying transoceanic flights, since traffic control problems exist during these flights that presently cannot be solved by other navigational means At the same time the consideration of a specific problem rather than a general navigation method makes the study more definitive of the methods and techniques to be used.

The problem associated with aircraft making transoceanic flights is due to the large number of aircraft flying specified routes. Dead-reckoning is essentially the existing means of navigation for this type of mission. The inaccuracies of present methods of navigation require that aircraft maintain lateral separations of 120 nautical miles and longitudinal separations of 0.5 hour in time. Means of maintaining adequate separations are limited by the desirability or need of aircraft flying long-range missions to take advantage of optimum altitude for efficient engine operation. These restrictions have filled the available airspace to the point where a major problem exists.

The system objective, to satisfy this operational need, is to establish accurately the aircraft present-position relative to an assigned air corridor. The relative position information must be supplied to flight personnel in a direct, convenient form that will enable them to recognize immediately what corrective action is necessary. In the limiting case the data displayed should be in a form suitable for insertion into the flight control system.

The method used to measure basic position information must, of necessity, be usable over long ranges. The hyperbolic system of accurate, long-range navigation offers the greatest potential. The principal disadvantage to the hyperbolic system is that its coordinate system is not amenable to traffic control problems and is not compatible with the coordinate systems of other navigation equipment. Moreover, any hyperbolic system having the required accuracy capabilities cannot be expected to yield instantaneous readings, because of atmospheric noise. To realize full accuracy potential, the system must average-out the inherent noise perturbation. Averaging produces time delays that create errors intolerable for operation of high-performance aircraft. However, if the source of accurate - but noisy - information is suitably combined with the outputs of self-contained navigational aides, a system can be produced that is superior to either of the individual sources of data. More generally, this system can be viewed as one utilizing the radio information to correct the self-contained aides. All of the coordinate conversion methods discussed in this study are compatible with this philosophy, since to neglect this concept would result in a system of limited purpose.

At the beginning of this study, a maximum range of 500 nautical miles from the farthest transmitter was set as the objective for accurate navigation. However, if the equipment is to be useful for transoceanic flights, it must operate at greater ranges. As an example, three existing LORAN-C transmitters located on the east coast of the United States are so spaced that a C/C with maximum range of 500 nautical miles would have essentially zero coverage, since each triad leg would exceed 500 nautical miles. Therefore, a new maximum-range objective of 2000 nautical miles was established. This figure, though somewhat arbitrary, would provide a reasonable requirement in the number of transmitters necessary for congested areas. It is also the upper limit for any existing known hyperbolic system.

To establish realistic accuracy requirements, several geometrical factors must be considered. The accuracy of determining aircraft position in a hyperbolic field is a function of the position in the field. This is due to two factors:

1. A spreading of the lines of constant range difference with range (commonly called lane dilution).

2. Unfavorable angle of intersection of the hyperbolic lines (ideal angle would be 90 degrees).

Both of these effects are readily observable from the two intersecting fields in Figure 1. The effect of these factors on the accuracy of establishing present-position are calculated and discussed in Appendix F, however, it is apparent that no single accuracy figure is applicable to all situations.

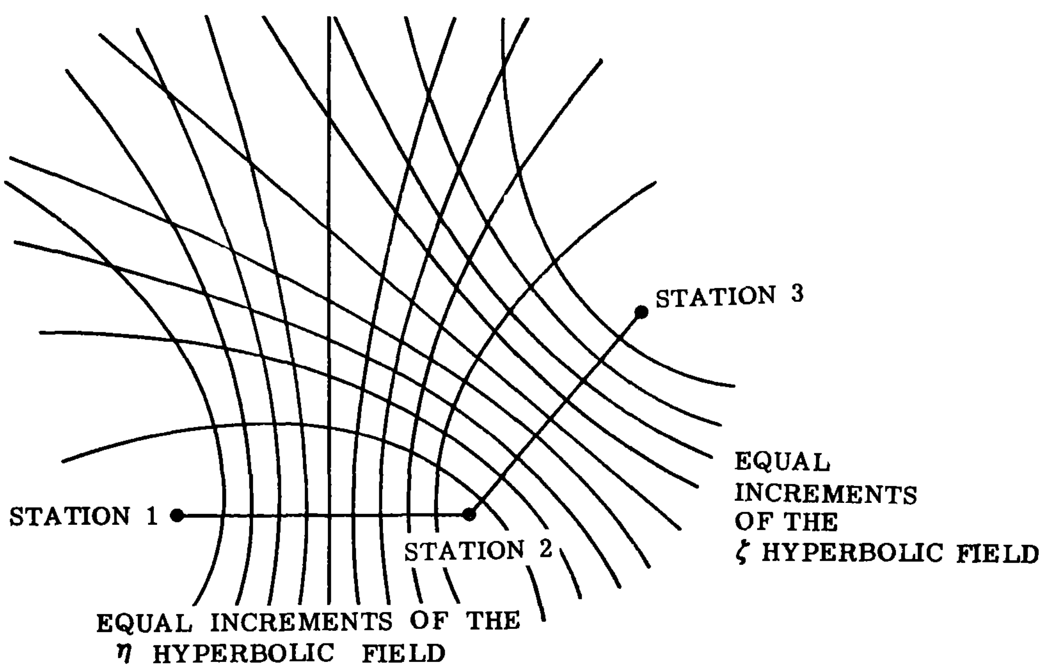


Figure 1. Lane Dilution and Unfavorable Angles of Field Intersection at Extended Range

In the study, it is assumed that operation in the major portion of the hyperbolic field is desired. Boundary conditions of permissible operation and attendent inaccuracies are derived, but, in general the errors will contribute less than 10 nautical miles for a major portion of the field.

In addition to the performance requirements, flexibility is needed, since the specific manner in which the equipment will operate can only be envisioned at this time. Changing flight conditions may result in a variety of techniques; therefore, provisions are incorporated in the design for inserting initial conditions into the C/C either prior to the flight or while enroute.

2.5 LIMITATION OF STUDY

The study report is organized to show the conceptual and detailed methods of mechanizing a C/C without considering other navigation aids The limitations of this system are discussed and means are shown for extending its capabilities with any attendant increase in the equipment discussed

The time available for this study did not permit a complete, thorough treatment of all areas. The immediate problems were emphasized while the long-range aspects were investigated to a lesser extent. The limitations of the hyperbolic system are not treated with rigor, but rather an attempt is made to point out the limitations in general. A quantitative measure of these limitations was not made, since the specific system characteristics that will establish the limitations were not available. If a quantitative measure of the limitations is desired, a more comprehensive study will be necessary. The noise characteristics of the receiver outputs were based upon logical assumptions. While the assumptions seem reasonable, they cannot be substantiated at this time.

Operation of the presently conceived C/C with all known navigation aids was not attempted, but rather a specific navigation aid (and one which is defined by ARINC specifications) is treated in detail This should not imply that the C/C must operate with this specific aid but, rather, demonstrates the techniques used to integrate the two computer outputs into one optimum data source.

It was necessary to make several assumptions regarding the geometric location of the hyperbolic transmitting stations It was assumed that the transmitters will be arranged in a triad configuration with base lines of 500 to 1000 nautical miles and included baseline angles of 90° to 165°. Other configurations may possibly be employed. The C/C operation was assumed to be in an area where hyperbolic ambiguity does not exist; however, this is not necessarily a system limitation.

2.6 APPLICABLE HYPERBOLIC SYSTEMS

The hyperbolic systems that were considered applicable to transoceanic navigation were those with an operating range of 100 nautical miles or more. The following are those hyperbolic systems that may be capable of meeting the range requirement:

1. LORAN C
2. DECTRA
3. OMEGA
4. RADUX

Although all these systems are discussed with reference to converting the outputs of each to a more practical coordinate system, primary consideration was given to the LORAN C system, because at the present time, it appears to be the most probable choice for use on transoceanic flights

2.7 CHOICE OF NAVIGATIONAL COORDINATES

The fundamental objective of this study is to determine the most feasible design of a converter for changing hyperbolic coordinates to ones more useful for both air-traffic control and general navigation The information displayed after coordinate conversion should be in a form most useful to flight personnel and at the same time compatible with the output of other navigational equipment in commercial aircraft. This is of considerable importance, since one expected application is the use of hyperbolic information to correct navigational data of short-term accuracy.

Several types of navigational coordinates were considered, but the obvious choice was a system utilizing some form of latitude and longitude. The problem of establishing aircraft present-position with respect to the earth concerns spherical computations. In addition, all commonly used maps have latitude and longitude grids and are by far the most extensively used aids to long-range navigation.

The limitations of the practical use of such information by flight personnel are quite apparent, since knowing present latitude and longitude and the desired destination in similar coordinates does not immediately convey the desired information. Under such circumstances flight personnel would probably resort to a map or chart to establish the aircraft position relative to some reference. The use of maps and charts to establish and maintain an assigned position in an air corridor requires constant attention. Therefore, it is believed that latitude and longitude coordinates are acceptable for destination coordinates but are not suitable for the display of aircraft position relative to an assigned corridor. For applications requiring traffic control, it is desired that a predetermined flight path can be inserted into the coordinate converter (C/C). With this information, the C/C would be able to display directly any deviation that the aircraft may make from the set flight course. The several methods by which the desired track can be specified are:

1. Range and bearing
2. Initial and final destination coordinates of a leg
3. Coordinates of destination and bearing angle to be flown.

Ideally, any of the above methods will define a given flight path; but due to the probability that several intermediate legs will be required for transoceanic flights, the range and bearing method can accumulate inaccuracies. In this methods, each leg is referenced to the preceding leg; therefore, inaccuracies accumulate that are associated with the initial conditions of the individual legs. In the latter two methods, the initial conditions of the flight legs are referenced to the central coordinate system (latitude and longitude). The accuracy of the position data is, therefore, not affected by inaccuracies of previous initial conditions. The latter two are equally convenient, but method 3 is chosen for discussion, since it is similar to the technique used in ARINC Characteristic 543. Thus, for each leg, flight personnel will insert the latitude and longitude of each destination and the angle at which it is desired to fly through the destination.

With these initial conditions established and with the basic hyperbolic information, the C/C will compute the cross-track-distance and distance-to-go. These quantities are shown in Figure 2 for a planar case.

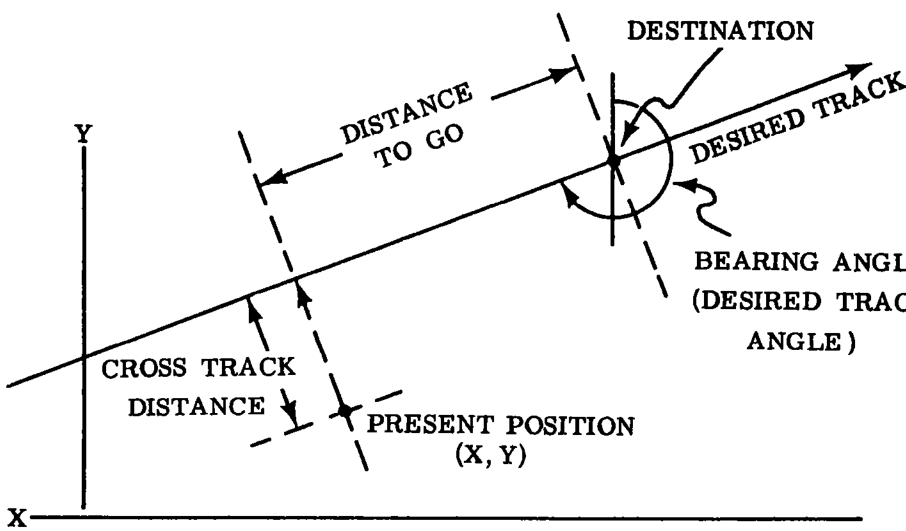


Figure 2. Planar Representation of Cross-Track-Distance and Distance-To-Go.

The latitude and longitude coordinates to be used as destination coordinates appear on most maps and are referenced to the equator and the prime meridian. However, converting actual position to latitude and longitude presents a more difficult problem than converting actual position to a local coordinate system, using the location of a hyperbolic transmitter as the reference.

Actual aircraft position in a local coordinate system is defined by the coordinates of that system Obviously, if one aircraft is utilizing a local coordinate system and a second aircraft is using another local coordinate system, it would not be apparent to anyone, without further mathematical computation, when each aircraft approached the same physical location.

Each local coordinate system could define the geographical aircraft position mathematically, assuming that the location and orientation of the local coordinates are known with respect to general latitude and longitude coordinates. The additional computation required would be simple enough for planar cases but becomes involved for the actual spherical case. Because this computation involves a second coordinate conversion, this method was not considered further and the general latitude and longitude coordinate method is used.

Before distance-to-go and cross-track-distance can be computed, a precise definition must be given to these quantities There are two basic methods commonly used; these are dependent upon whether distances are defined over the actual curved surface of the earth or are represented in a plane corresponding to some mapping projection.

To date, no one mapping projection is universally employed for navigation, and for this reason it was decided to choose converter outputs which are independent of any particular projection. Two types of ground tracks were considered: great circle, and rhumb line, with preference given to the great circle method, since it permits operation at any latitude.

Hence, the navigational coordinates - cross-track-distance and distance-to-go, provided by the proposed coordinate converter - represent great circle distances. In particular, the cross-track-distance reading indicates the deviation of present-position from the desired great circle track, computed along the great circle through present-position and perpendicular to that track The distance-to-go reading corresponds to the distance along the desired great circle track, measured from the point of intersection of that track with the great circle along which cross-track-distance was computed.

On long-range flights, the flight path is usually not a continuous great circle route but is a path that takes advantage of pressure patterns and prevailing wind conditions. To establish this flight trajectory the flight plan is broken into a series of intermediate destinations, as shown in Figure 3.

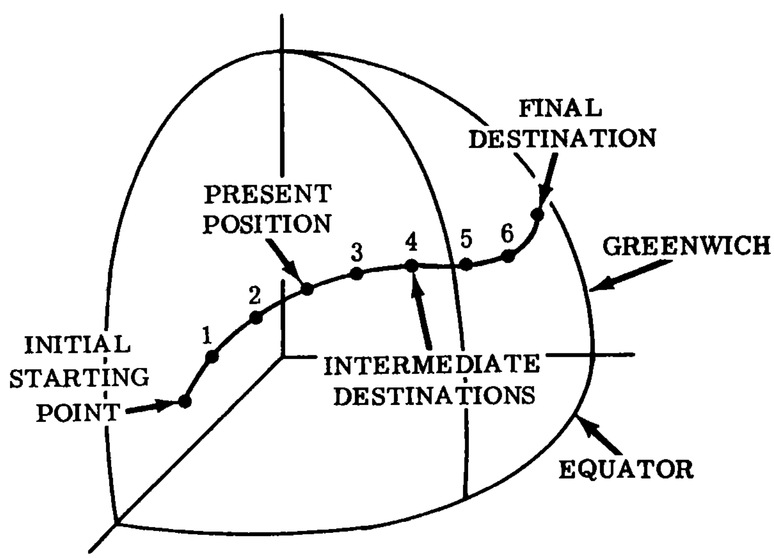


Figure 3. Typical Great Circle Flight Path

The initial condition defining each leg can be inserted into the C/C prior to a mission or while enroute Transfer to a new leg can be made either automatically or manually. In either case, the display registers the projected distance-to-go and cross-track distance from the selected course Each quantity will be a great circle computation registered in nautical miles

The outputs of the C/C are similar to those specified in Reference 1, concerning the Mark I Computer. The Mark I Computer requires the following initial information:

1. Desired track angle
2. Initial distance-to-go
3. Initial cross-track-distance.

The Mark I and the coordinate converter (C/C) differ in two respects: (1) The Mark I track angle is essentially that of a rhumb line while C/C track angle is a great-circle, and (2) the initial distance-to-go and the initial cross-track-distance are not referenced to a specific coordinate system, whereas C/C is referenced to latitude and longitude The differences arise primarily from their application. The Mark I system supplies data of short-term accuracy, while the C/C serves to correct the Mark I. Although the output of the two units differs slightly, they are compatible, since the doppler-heading reference coordinate system does not produce large errors from the true distances indicated by the C/C. When corrections are made to the Mark I (either by manual or automatic means), these small errors will be removed in the same manner as instrumental errors To correct the Mark I manually, distance-to-go and cross-track-distance will be physically adjusted to agree with the C/C outputs. Any errors due to differences in the coordinate systems will be removed in the process with no need for further calculation.

3. DETAILED FACTUAL DATA

3 1 SELECTION OF CONVERTER-DESIGN PHILOSOPHY

3 1 1 Introduction and Formulation of Mathematical Model

Several decades have passed since the advent of low-frequency techniques as a navigational aid. However, the problem of coping with the hyperbolic outputs of such systems has not passed

In the present contract, Motorola was assigned the responsibility for investigating means to alleviate the inherent coordinate conversion problem. In accordance with this contract, Motorola has extended and refined the theory of the feedback approach it suggested originally Moreover, in the interim between its proposal to FAA and the start of the present contract, Motorola continued to investigate the problem of hyperbolic transformations. This investigation established a set of simple, exact hyperbolic transformation formulae whose existence, heretofore, had gone unsuspected. This significant development allows a simplicity in hyperbolic converter design previously considered unobtainable.

Hence, at the present time Motorola has evolved two methods for coping with the hyperbolic coordinate conversion problem: (1) a modification of present Loran-type receivers so that oscillator drift may be controlled through feedback, and (2) a direct transformation of the hyperbolic outputs of such receivers It is the purpose of Section 3. 1 to describe these two approaches, to compare them, and to choose between them.

Before a description of either method is presented, a mathematical model will first be formulated to recapitulate the problem in precise terms. In addition, it will be well to settle the question of analog versus digital techniques at the outset, as this choice has a bearing on the choice of equations. This latter point is treated in the following section.

When properly scaled, the outputs of a Loran-type receiver represent a pair of range differences. More specifically, let the receiver be located on the surface of an assumed spherical earth. Furthermore, let the following nomenclature be introduced.

(ϕ, λ) = latitude and longitude of receiver, located at point P,

(ϕ_n, λ_n) = latitude and longitude of station n; n = 1, 2, 3,

r_n = great circle range of P from station n,

R_o = radius of assumed spherical earth.

The spherical geometry involved is depicted in Figure B-1 of Appendix B. In terms of the above notation, the appropriate equations implicitly defining the receiver's position are:

$$\sin\phi_1 \sin\phi + \cos\phi_1 \cos\phi \cos(\lambda - \lambda_1) = \cos \frac{r_1}{R_o} \quad (1)$$

$$\sin\phi_2 \sin\phi + \cos\phi_2 \cos\phi \cos(\lambda - \lambda_2) = \cos \frac{r_2}{R_o} \quad (2)$$

$$\sin\phi_3 \sin\phi + \cos\phi_3 \cos\phi \cos(\lambda - \lambda_3) = \cos \frac{r_3}{R_o} \quad (3)$$

$$r_{21} = r_2 - r_1 \quad (4)$$

$$r_{23} = r_2 - r_3 . \quad (5)$$

Equations 1 through 3 result from the well known spherical trigonometric formulae relating the great circle distance between two points to their respective latitudes and longitudes. Equations 4 and 5 represent the information normally detected in a hyperbolic receiver. This pair of range differences, by common usage, is termed the hyperbolic coordinates of P, since they locate P by virtue of the intersection of two hyperbolae.

The station locations are known and the range differences are detected Hence, Equations 1 through 5 constitute a set of five simultaneous, nonlinear equations in the five unknowns: ϕ, λ, r_1, r_2 and r_3. An accurate and feasible mechanization of a solution to these equations constitutes the core of the computer design problem.

Actually, as discussed in Section 2 7, latitude and longitude are not the quantities ultimately desired for display However, once they are known, it is a straightforward matter to convert them to coordinates better suited for navigation, such as cross-track and distance-to-go. This additional computation is discussed in Section 3 1. 8.

Equations 1 through 5, as they stand, fail to take into account two important effects, neither of which is negligible: earth's eccentricity, and phase variation due to the secondary factor for radio propagation These topics are discussed individually and in detail in subsequent Sections 3. 1. 6 and 3 1 7. In particular, they will be incorporated into the over-all converter design by a simple iterative process whereby small corrections are computed and added to the detected range differences.

It is believed that the above formulation represents an accurate and realistic description of the original coordinate converter design problem In Section 3. 1. 4 the simplification of Equations 1 through 5 made possible by receiver modification will be discussed.

3. 1 2 Analog Versus Digital Techniques

The question of whether analog or digital techniques should be used to implement equations 1 through 5 can be resolved on the basis of accuracy and storage considerations. The accuracy requirement will be discussed first.

In general, solutions to equations 1 through 5 -- either implicit or explicit -- will involve the loss of two significant figures, owing to successive subtractions. This fact was established by the calculation of points at the ranges of interest for several typical station geometries. Now, as previously discussed in Section 2. 4, it is desired that the maximum error in determining present-position should not exceed 10 nautical miles. For the moment suppose that all computational error can be absorbed by the total allowable system error This implies, for example, that latitude should be known to within at least 10 minutes of angle (since 10 minutes of angle corresponds to a linear error of 10 nautical miles), or three significant figures. As mentioned above, a loss of two significant figures can be expected to be encountered. This means that the inputs should be known to five significant figures and all computations carried out at that level of accuracy. Thus, the maximum error associated with the computation of any range would be 0. 1 nautical mile, which, for ranges less than 2, 000 nautical miles, corresponds to a relative computational accuracy of 1 part in 20, 000. An accuracy of 1 part in 20, 000 is considered to be at least one order of magnitude better than that obtainable by analog computers with the present state of the art.

By restricting the scope of the problem (e. g. , reducing the maximum operating range and perhaps employing a purely local coordinate system), it is possible that an analog design could be used The usefulness of such a device is, however, extremely doubtful. Even with this restricting condition, the accuracy problem is still acute from an analog point of view. Therefore, it is felt that potentially a minimal digital computer would be more reliable and economical than an optimized analog computer.

Moreover, a final converter design, if it is to allow genuine global navigation to be conducted, should be capable of switching from one station triad to another. It must, of necessity, store sets of station coordinates and switch them into the solution, depending upon the vicinity of the nearest triad. The specification of one station triad requires a minimum of 6 coordinates. It is planned to incorporate in the proposed coordinate converter the operational capability of conducting navigation based upon any one of 10 fixed triads. Hence, a minimum of 60 constants would have to be stored in the computer. In addition, there would be constants associated with destination points and other factors. There is no simple, economic way of storing such a large number of constants in an analog device

Both accuracy and storage considerations indicate that the proposed coordinate converter should be based upon a digital design, where neither of these factors presents a problem. In particular, in the converter design recommended, the total error associated with the determination of present-position due to computational sources is not expected to exceed 2 nautical miles

3.1.3 Hyperbolic Mechanizations

As concluded in the previous section, the mechanization of equations 1 through 5 will be based upon a digital design. A minimal digital computer must be a serial-type machine in which computations are performed in a sequential fashion with storage of intermediate results. For the intended application the computer must operate in real time while located in aircraft having 600 knot velocities Furthermore, the receiver outputs contain noise perturbations which will require due regard for the sampling frequency. The capability of the proposed coordinate converter to meet the sampling rates imposed by these constraints is shown in detail in Section 3.2.3.

The important point here is that the solution time required by a serial digital computer is determined effectively by the total number of multiplications and/or divisions it is called upon to perform Therefore, the major design problem consists of constructing a solution to equations 1 through 5 in terms of the elementary operations of addition and multiplication in such a manner that the total number of multiplications and/or divisions is a minimum. This latter requirement provides one basic criterion for comparing alternative design procedures and choosing between them.

It is possible, through the use of an iterative process, to implement equations 1 through 5 as they stand A description of the design required by this kind of approach can be found starting on page 253 of Reference 2. The problem treated there is formulated in terms of a three-dimensional geometry involving three range differences (thereby requiring four stations). However, the method used (an extension of Newton's) could be applied to the case of three stations involving spherical geometry. The philosophy of the procedure is akin to the arrangement indicated in Figure 4.

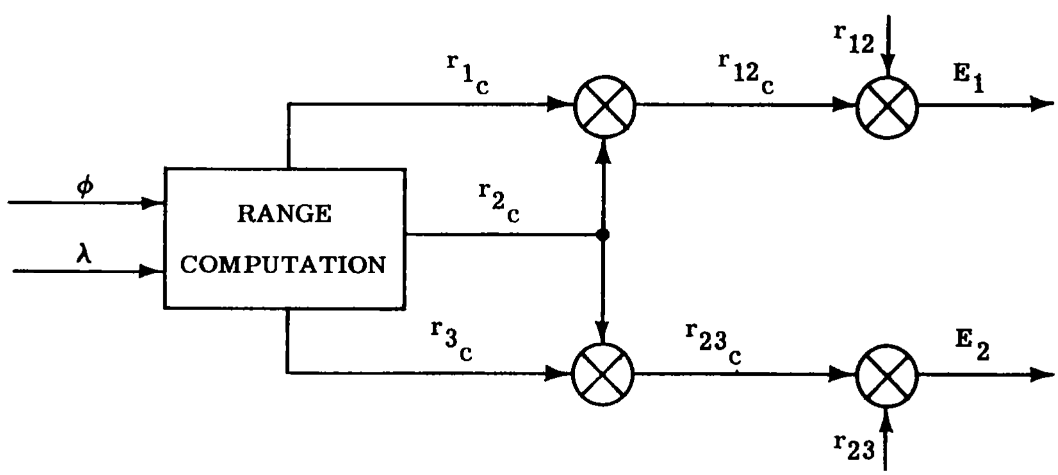


Figure 4. Iterative Scheme for Solution of Hyperbolic Equations

From initial estimates of ϕ and λ the three ranges could be computed from equations 1 through 3 and differenced to form estimated values of the hyperbolic coordinates. These, in turn, would be differenced with the detected hyperbolic coordinates to generate a pair of errors. The errors could then be used to alter the original estimates of ϕ and λ Under the assumptions of restricted aircraft accelerations and sufficiently short solution times, the entire process may be linearized through the introduction of partial derivatives. But even the most cursory inspection of this approach shows that an exorbitant number of arithmetical operations are required.

One is next led to consider explicit formulae whereby ϕ and λ may be calculated, avoiding the cumbersome procedures inherent in iterative processes. For example, if the ranges are eliminated from equations 4 and 5 by virtue of equations 1 through 3, a pair of simultaneous equations in ϕ and λ result. Each represents the locus of a spherical hyperbola, and the intersection of the two determines the point P (hence, ϕ and λ). The elementary character of this observation encourages the specious conclusion that it constitutes the fundamental definition of the problem. However, inasmuch as the solution of this pair of simultaneous equations is formidable, one is led to cast about for a new approach.

Fortunately, a third method of attack exists. In Appendix B a simple, exact set of formulae is derived, giving the solution of equations 1 through 5. The artifice employed there involves solving for one of the ranges before any attempt is made to determine latitude and longitude. Since the derivation in the Appendix is given in some detail, the method used will not be elaborated upon here. The results, collected from Appendix B, are summarized below and labeled as Equation Set I.

Equation Set I

$$\eta_{21} = \cos \frac{r_{21}}{R_o} \quad (6)$$

$$\eta_{23} = \cos \frac{r_{23}}{R_o} \quad (7)$$

$$\xi_{21} = \sin \frac{r_{21}}{R_o} \quad (8)$$

$$\xi_{23} = \sin \frac{r_{23}}{R_o} \quad (9)$$

STATION-CONFIGURATION MATRIX

$$\begin{pmatrix} A & B \\ C & D \\ E & F \end{pmatrix} = \begin{pmatrix} \Lambda_{11} & \Lambda_{12} & \Lambda_{13} \\ \Lambda_{21} & \Lambda_{22} & \Lambda_{23} \\ \Lambda_{31} & \Lambda_{32} & \Lambda_{33} \end{pmatrix} \begin{pmatrix} \eta_{21} & \xi_{21} \\ \eta_{23} & \xi_{23} \\ 1 & 0 \end{pmatrix} \quad (10)$$

$$J = B^2 + D^2 + F^2 - 1 \quad (11)$$

$$K = AB + CD + EF \quad (12)$$

$$L = A^2 + C^2 + E^2 - 1 \quad (13)$$

$$T_{\sigma_2} = \frac{-K \pm \sqrt{K^2 - JL}}{J} \quad (14)$$

$$\phi = \sin^{-1}\left[\frac{A + B\ T_{\sigma_2}}{\sqrt{1 + T_{\sigma_2}^{2}}}\right] \tag{15}$$

$$\lambda = \tan^{-1}\left[\frac{E + F\ T_{\sigma_2}}{C + D\ T_{\sigma_2}}\right] \tag{16}$$

$$r_2 = R_o \tan^{-1} T_{\sigma_2} \tag{17}$$

$$r_1 = r_2 - r_{21} \tag{18}$$

$$r_3 = r_2 - r_{23} \tag{19}$$

The trigonometric functions would be approximated by the first few terms of a series expansion. The square root operation could be handled by a simple iterative process. These matters are discussed further in Section 3.2.4.

The lambdas appearing in the station-configuration matrix are constants whose values depend solely upon the geometry of the station configuration. Their values in terms of the geographical coordinates of the stations may be found at the end of Appendix B. In practice, they would be precomputed and stored in the coordinate converter as fundamental constants. Thus, for a change in operation from one station triad to another, the airborne computer would simply switch to a new station-configuration matrix.

The occurrence of the ± sign before the first radical reflects the ambiguity inherent in a pair of hyperbolic coordinates. Its significance is shown in Figure 5. The clear areas correspond to the regions where the minus sign must be chosen, while the cross-hatched areas correspond to the plus sign.

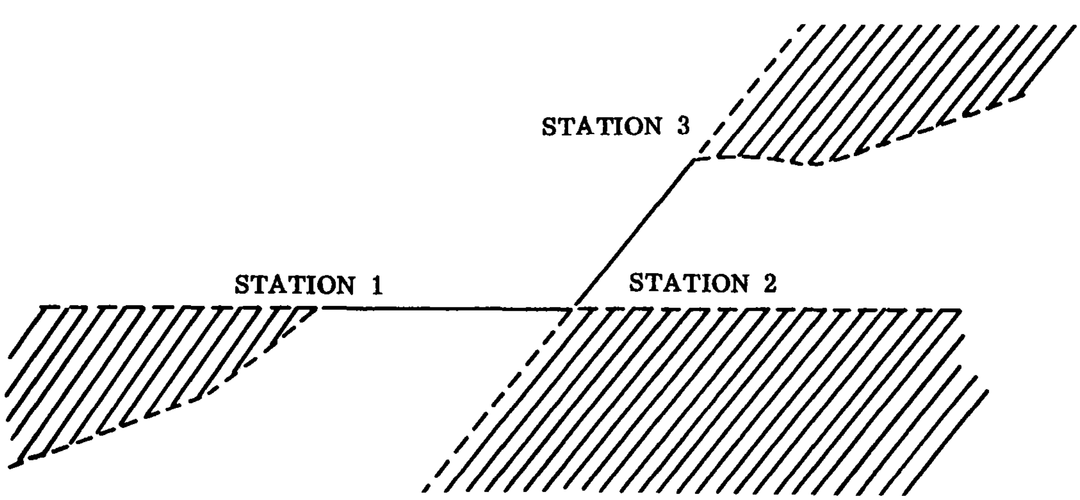


Figure 5. Regions of Hyperbolic Ambiguity

The solution for latitude and longitude is sign sensitive, enabling them to be located with reference to the proper quadrant of the earth Furthermore, the three ranges are readily available for calculating the corrections required by earth's eccentricity and the secondary factor of radio propagation.

It can be asserted that Equation Set I exhibits the minimum amount of digital computation required to solve equations 1 through 5, as they stand, in an exact fashion.

Actually, as discussed previously in Section 2.7, latitude and longitude are not the quantities ultimately desired for navigation and display. Moreover, as shown in Section 3.1 8, the desired coordinates -- distance-to-go and cross-track -- are essentially linear combinations of T, U, and V, where $T = \sin\phi$, $U = \cos\phi\cos\lambda$, and $V = \cos\phi\sin\lambda$ These parameters have the following relations to the variables appearing in Equation Set I:

If

$$C_{\sigma_2} = \frac{1}{\sqrt{1 + T_{\sigma_2}^2}} \tag{20}$$

$$S_{\sigma_2} = \frac{T_{\sigma_2}}{\sqrt{1 + T_{\sigma_2}^2}} \tag{21}$$

then

$$\begin{pmatrix} T \\ U \\ V \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \\ E & F \end{pmatrix} \begin{pmatrix} C_{\sigma_2} \\ S_{\sigma_2} \end{pmatrix} \tag{22}$$

In the recommended design mechanization, the explicit solutions for ϕ and λ would be omitted and replaced by equations 20 through 22. However, for present purposes, it will be convenient to continue presenting results in terms of latitude and longitude.

For reasons that appear in Section 3.1.5, the hyperbolic transformation based upon plane geometry is of interest The planar statement of the problem, corresponding to equations 1 through 5 for the spherical case, assumes the form:

$$r_1 = \sqrt{(x - x_1)^2 + (y - y_1)^2} \tag{23}$$

$$r_2 = \sqrt{(x - x_2)^2 + (y - y_2)^2} \tag{24}$$

$$r_3 = \sqrt{(x - x_3)^2 + (y - y_3)^2} \tag{25}$$

$$\eta_{12} = \frac{1}{2} (r_1 - r_2) \tag{26}$$

$$\eta_{23} = \frac{1}{2} (r_2 - r_3) \tag{27}$$

where

(x, y) = Cartesian coordinates of point P at which the receiver is located,

(x_n, y_n) = Cartesian coordinates of station n; n = 1, 2, 3,

r_n = range of receiver from station n,

(η_{12}, η_{23}) = detected hyperbolic coordinates of P

As before, equations 23 through 27 constitute a set of five simultaneous nonlinear equations in five unknowns. Here again, the most obvious procedures for obtaining a solution are not the best. In Appendix A a solution that would be optimum for serial digital design is derived in detail. The results, collected from that Appendix, are summarized below and labeled as Equation Set II.

Equation Set II

$$\eta_{12} = \frac{1}{2} (r_1 - r_2) \tag{28}$$

$$\eta_{23} = \frac{1}{2} (r_2 - r_3) \tag{29}$$

STATION-CONFIGURATION MATRIX

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} \Lambda_{11} & \Lambda_{12} & \Lambda_{13} \\ \Lambda_{21} & \Lambda_{22} & \Lambda_{23} \end{pmatrix} \begin{pmatrix} \eta_{12} & \eta_{12}^2 \\ \eta_{23} & \eta_{23}^2 \\ 0 & 1 \end{pmatrix} \tag{30}$$

$$J = A^2 + C^2 - 1 \tag{31}$$

$$K = A(B - x_2) + C(D - y_2) \tag{32}$$

$$L = (B - x_2)^2 + (D - y_2)^2 \tag{33}$$

$$r_2 = \frac{-K \pm \sqrt{K^2 - JL}}{J} \tag{34}$$

$$x = Ar_2 + B \tag{35}$$

$$y = Cr_2 + D \tag{36}$$

$$r_1 = r_2 + 2\eta_{12} \tag{37}$$

$$r_3 = r_2 - 2\eta_{23}. \tag{38}$$

The structural similarity of these equations to those appearing in Equation Set I is apparent. The lambdas again are station-configuration constants, dependent entirely upon the geometry of the station triad. Their values in terms of the original station coordinates can be found at the end of Appendix A. As before, they would be precomputed and stored in the coordinate converter as basic geometry constants

Further discussion of the use of these equations in the coordinate converter design problem will be deferred until Section 3.1 5

3.1.4 Control of Oscillator Drift Through Feedback

With a view towards simplifying the computations involved to produce shorter solution times, Motorola has evolved a novel approach, characterized by feedback control. Several statements of this method are possible and a variety of procedures for mechanizing it have been found; however, only one procedure will be described here. This one is considered typical of the group and exhibits the virtues of the feedback approach.

As a starting point for the explanation of this method, it is convenient to review the reason why the somewhat circuitous method of locating a point by curvilinear hyperbolic coordinates is employed with Loran-type receivers Thus, heuristically, if the absolute phases (or propagation times) corresponding to actual ranges could be detected - rather than range differences - the computation of position would become a simple matter and the third station could be discarded (resulting in a considerable cost saving). While there is no problem in bringing out the individual ranges, a difficulty arises from the nature of heterodyne detection processes. In this type of detection a local oscillator is employed for reference. Since an unavoidable drift occurs between it and the ground oscillator controlling the phase of the transmitted signals, a common error is introduced in each range channel. However, the detection of three ranges and their subsequent subtraction from one another cancels the relative drift between ground and airborne oscillators, thereby generating a pair of hyperbolic coordinates.

But the possibility arises that if each of the three ranges were detected individually, the redundant information afforded by the third station could be used to drive the relative oscillator drift to zero through feedback control of the phase of the local oscillator. If the relative oscillator drift were always zero, approximations to the computation could then be made that could not be tolerated in a hyperbolic transformation. Hence, the total computation could be expected to be simpler than that required by a proper hyperbolic transformation, which is the case.

For the case where the individual ranges are brought out, equations 1 through 5 would be modified to appear as:

$$\sin\phi_1 \sin\phi + \cos\phi_1 \cos\phi \cos(\lambda - \lambda_1) = \cos\frac{r_1^*}{R_o} \qquad (39)$$

$$\sin\phi_2 \sin\phi + \cos\phi_2 \cos\phi \cos(\lambda - \lambda_2) = \cos\frac{r_2^*}{R_o} \qquad (40)$$

$$\sin\phi_3 \sin\phi + \cos\phi_3 \cos\phi \cos(\lambda - \lambda_3) = \cos\frac{r_3^*}{R_o} \qquad (41)$$

$$r_1 = r_1^* + \delta \qquad (42)$$

$$r_2 = r_2^* + \delta \qquad (43)$$

$$r_3 = r_3^* + \delta \qquad (44)$$

where:

r_1, r_2 and r_3 = the detected ranges,

r_1^*, r_2^* and r_3^* = the correct ranges,

δ = the relative drift between ground and airborne oscillators in linear units.

Therefore, equations 39 through 44 constitute a set of six simultaneous equations in the six unknown: ϕ, λ, δ, r_1^*, r_2^* and r_3^* Moreover, in the solution, it is permissible to employ the approximation that δ is approximately zero. A detailed derivation of the solution is given in Appendix C. The results, collected from that Appendix, are summarized below and labeled as Equation Set III.

Equation Set III

$$\eta_1 = \cos \frac{r_1}{R_o} \tag{45}$$

$$\eta_2 = \cos \frac{r_2}{R_o} \tag{46}$$

$$\eta_3 = \cos \frac{r_3}{R_o} \tag{47}$$

$$\zeta_1 = \sin \frac{r_1}{R_o} \tag{48}$$

$$\zeta_2 = \sin \frac{r_2}{R_o} \tag{49}$$

$$\zeta_3 = \sin \frac{r_3}{R_o} \tag{50}$$

STATION-CONFIGURATION MATRIX

$$\begin{pmatrix} A & B \\ C & D \\ E & F \end{pmatrix} = \begin{pmatrix} \Lambda_{11} & \Lambda_{12} & \Lambda_{13} \\ \Lambda_{21} & \Lambda_{22} & \Lambda_{23} \\ \Lambda_{31} & \Lambda_{32} & \Lambda_{33} \end{pmatrix} \begin{pmatrix} \eta_1 & \zeta_1 \\ \eta_3 & \zeta_3 \\ \eta_2 & \zeta_2 \end{pmatrix} \tag{51}$$

$$L = A^2 + C^2 + E^2 - 1 \tag{52}$$

$$K = AB + CD + EF \tag{53}$$

$$\delta = - \frac{LR_o}{2K} \tag{54}$$

$$\phi = \sin^{-1} A \tag{55}$$

$$\lambda = \tan^{-1} \frac{E}{C} \tag{56}$$

As predicted, these equations are simpler than those listed under Equation Set I. For example, these do not require a square-root operation. The definitions of the lambdas appearing in the station-configuration matrix are identical to those of Equation Set I. They would be treated in the same fashion as before -- precomputed and stored. It can be noted that the whole process is surprisingly linear.

A simplified block diagram of the mechanization of this scheme is shown in Figure 6. Since this approach was not ultimately selected for the application at hand, no discussion of the dynamics involved in the loop will be included.

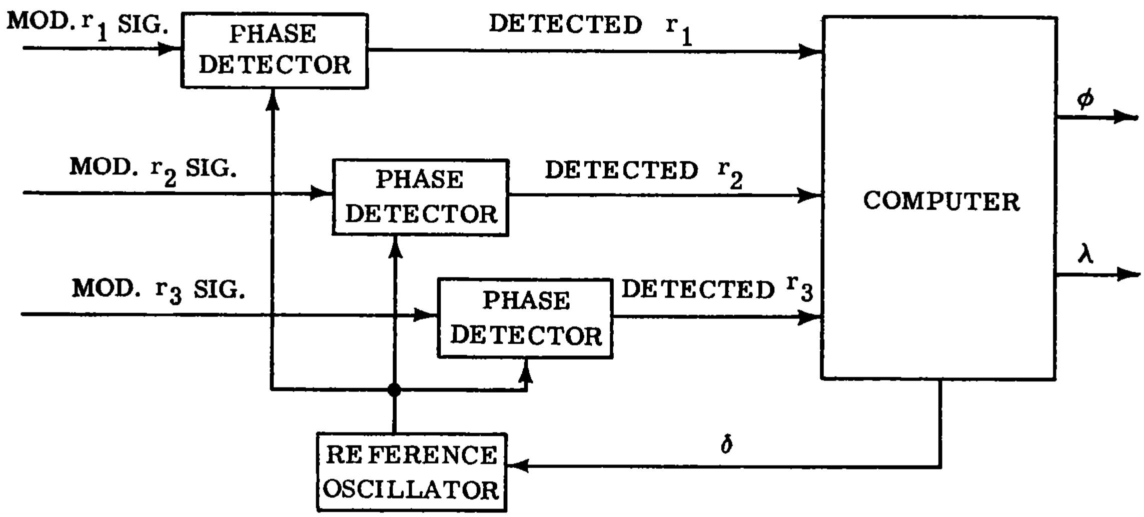


Figure 6. Simplified Block Diagram of Feedback Oscillator Control System

3.1.5 Comparison and Choice of Methods

Any one of the three Equation Sets listed in the preceding sections could be used as the basis for a feasible design of an airborne digital coordinate converter. The problem at hand is to select the one best suited for the application intended. A decision between Equation Sets I and II will first be made.

The possibility of using Equation Set II arose from the consideration that the over-all computation might be simpler if the majority of it were performed in a plane. To test this hypothesis, formulae for converting great circle ranges to ranges in a plane corresponding to a stereographic projection were derived. This projection, which was chosen because of its simplicity, is discussed in Appendix D. However, once x and y are obtained, it is necessary to convert them to latitude and longitude. When these additional computations are added to those already contained in Equation Set II, it was concluded that the artifice of introducing a plane does not simplify the solution.

The other -- and more basic -- decision regarded the choice between oscillator control and a proper hyperbolic transformation, as posed by Equation Sets I and III. Other things being equal, one would be disposed towards an oscillator control approach, since it requires about 20 percent less solution time. However, the coordinate converter under consideration is intended for use with Loran-type receivers, and a modification of them would be required to bring out the individual ranges. The extent of the modification necessary is examined in Section 3.4 and is found to be more involved than one might have supposed. Hence, to avoid a modification of existing equipment, it is concluded that a proper hyperbolic transformation (as manifested by Equation Set I) should be mechanized -- especially since the penalty paid in solution time does not exceed the required sampling rates of the problem.

3.1.6 Correction for Earth Eccentricity

The equations formulated in the previous sections for converting hyperbolic coordinates to geodetic longitude and latitude are based on an assumed spherical earth, that is, an earth with zero eccentricity. This assumption was made because of the extreme difficulty, if not impossibility, of solving the analogous set of equations based on an ellipse of revolution, which, of course, is a better model of the earth's surface. The position expressed in longitude and latitude calculated by means of the spherical equations is therefore in error. The fundamental questions, as far as the FAA problem is concerned, are (1) How large is this error over the field of operation? and (2) If necessary, how may it most suitably be corrected? A study was made to answer these questions and is documented in Appendix F.

3.1.6.1 Magnitude of the Error

To answer the first question, namely, how large is the error, it is first necessary to assume specific baseline lengths, baseline intersection angles, station latitudes, and the orientation of the configuration with respect to north. Two configurations have been selected for study: one with 500 nautical mile baselines intersecting at an included angle of 150 degrees, another with 1000 nautical mile baselines also intersecting at an included angle of 150 degrees. The configurations were studied with the center station at latitude 45 degrees north, and at latitude 90 degrees north, with the station configuration oriented north, south, and east. It is felt that these cases are sufficient to define the error for the Northern Hemisphere transoceanic problem. Ranges up to 2000 nautical miles from each station were considered.

Determination of the error in computed position was divided into two phases. First, for present-position and station locations all specified in terms of geodetic latitudes and longitudes, geodetic ranges between present-positions, and the stations were calculated for the eccentric earth using the Lambert-Andoyer formula. The great circle ranges between the same points specified by the same longitudes and latitudes were calculated for the spherical earth assumed for the coordinate conversion. If the latter ranges are introduced into the coordinate converter, the computed position is correct. However, the former ranges are actually used and the differences between these ranges can be taken as errors in the range inputs. The second phase of the computation consisted of deriving linear error expressions which relate the error in computed outputs to errors in the range inputs. The range errors are then introduced into the linear expressions to arrive at the error in computed position.

Considering the operating field to extend to a range of 2000 nautical miles from each station and to no closer than 25 degrees from the baselines, the error in computed position ranged up to 15 nautical miles for the 1000 nautical-mile baselines and up to 25 nautical miles for the 500 nautical-mile baselines. It is desired to keep the error resulting from eccentricity to within 2 nautical miles in order to restrict the total error in computed position to 10 nautical miles. Clearly, a correction for eccentricity is required. It should be mentioned here that these errors apply to all of the coordinate conversion schemes discussed in previous paragraphs, whether they contain oscillator feedback or not. This fact has been established previously by Motorola.

3.1.6.2 Correction Factors

The next question is, How may the corrections be applied? The most direct scheme that comes to mind is to express the corrections as functions of the ranges or range differences and to add these corrections to the range or range difference inputs of the coordinate converter. The derivation of such correction factors is an arduous task in view of the fact that the corrections are functions, not just of individual ranges but of the orientation of the ranges with respect to north and of the latitude of the stations. For this reason, derivations of this type are not pursued at this time although future work in this direction might prove fruitful. Rather, the approach is taken of using the Lambert-Andoyer formula itself, or approximations thereof, to compute the correction factors. The Lambert-Andoyer formula expresses the correction factor as a function of range and longitude and latitude of the end points of the range, namely, the station and present-position. The approach entails computing the correction factors from the ranges and longitude and latitude of present-position as computed in the coordinate converter. Both digital mechanization of the exact formula and of an approximation to it have been investigated. It was found that the exact formula could be mechanized with the least computing time since many functions appear which are common to other parts of the coordinate conversion. It is estimated that use of this correction factor will reduce the error in computed position as a result of eccentricity to less than 0.2 nautical mile over the usable field of operation. The correction factors to be subtracted from the range difference inputs are summarized below

$$\left.\begin{aligned} \Delta r_{12} &= \Delta r_1 - \Delta r_2 \\ \Delta r_{23} &= \Delta r_2 - \Delta r_3 \end{aligned}\right\} \text{correction factors} \tag{57}$$

$$\Delta r_n = (R_e - R_m)\sigma_n + \frac{R_e^f}{4}\left[(3\sin\sigma_n - \sigma_n)\frac{(T+a_{n_1})^2}{1+\cos\sigma_n} - (3\sin\sigma_n + \sigma_n)\frac{(T-a_{n_1})^2}{1-\cos\sigma_n}\right] \tag{58}$$

where:

$\cos\sigma_n = a_{n_1}T + a_{n_2}\dot{U} + a_{n_3}V$

$T = \sin\phi$

$U = \cos\phi \cos\lambda$

$V = \cos\phi \sin\lambda$

$a_{n_1} = \sin\phi_n$

$a_{n_2} = \cos\phi_n \cos\lambda_n$

$a_{n_3} = \cos\phi_n \sin\lambda_n$

R_e = equatorial radius of the earth

R_m = mean radius of the earth at station 2

f = polar flattening of the earth

$\sigma_n = \dfrac{r_n}{R_m}$, and

r_n = geodetic range from station n, n = 1, 2, 3

3.1.7 Propagation Corrections

The equations that are used to relate the hyperbolic coordinates to latitude and longitude coordinates assume that the hyperbolic coordinates can be scaled to obtain range differences. This assumption would be valid if the ground-wave velocity of propagation were equal to the velocity of propagation in free space. Since this is not true, a correction factor must be inserted into the coordinate converter (C/C) to correct the data. The correction factor (usually referred to as secondary phase factor) can be inserted into the C/C as a function of range. Reference 3 provides data on the manner in which the secondary factor varies with range for 100 kc signals propagated over sea water. The correction is given by:

$$\Delta\phi = cd^k \tag{59}$$

where:

$\Delta\ \phi$ = microseconds of phase shift due to secondary factor, and

d = distance from transmitter in miles.

Data is also provided in Reference 3 for the required secondary phase correction as a function of altitude As previously discussed the C/C has the capability of performing a correction for this type of error. However, work was not carried any further than indicated because the original work statement directed that propagation errors due to altitude be neglected

3.1.8 Computation of Distance-to-Go and Cross-Track Distance

In the previous sections equations have been derived which will be used to convert the hyperbolic coordinates, as measured by the radio system, into geodetic longitude and latitude. These earth coordinates are sufficient to locate the present-position of the aircraft, but the information is not in a form convenient for steering and for traffic control. Also, it is not compatible with the output of simple dead-reckoning computers such as the Mark I Doppler Radar Computer. It is necessary, therefore, that the longitude and latitude of present-position be converted into more suitable coordinates.

As previously discussed in Section 2.7, coordinates which have all of the desired characteristics are cross-track or normal distance from a desired great circle course and distance-to-go along the course to a destination. The cross-track-distance gives flight personnel a direction-sensitive measure of error off the desired course that can be used to steer the aircraft Distance-to-go pinpoints position along the desired course. Furthermore, these coordinates are compatible with the outputs of simple dead-reckoning computers and, as such, can be used directly in an automatic combination of radio and dead-reckoning information. Once these coordinates have been selected for computation and subsequent display, it is necessary to specify the parameters determining the desired track before the required mathematical expressions can be derived The most convenient way to specify the desired great circle course is to set into the coordinate converter the longitude and latitude of the destination of the course and the track angle at the destination - although, longitude and latitude of a starting point may, in some instances, be preferable to track angle at the destination.

To meet the preceding requirements, equations have been derived expressing cross-track-distance and distance-to-go as functions of the longitude and latitude of present-position, the longitude and latitude of a destination, and the desired track angle at the destination. This derivation is given in Appendix E Exact expressions are derived, but an approximation is selected for use in the FAA coordinate converter that is much simpler to mechanize and results in a negligible error in cases of interest Since in some instances it may be desirable to replace the track angle at the destination by the longitude and latitude of a starting point, the expressions necessary for this conversion are also derived in Appendix E. The recommended set of computations is summarized below:

$$\frac{r_c}{R_M} = D_{11}\ T + D_{12}\ U + D_{13}\ V \qquad (60$$

$$\sin \frac{r_R}{R_M} = D_{21} T + D_{22} U + D_{23}\ V \qquad (61)$$

where:

T = $\sin \phi$

U = $\cos\phi \cos \lambda$

V = $\cos\phi \sin \lambda$

D_{11} = $-\cos\phi_D \ \sin\beta_R$

D_{12} = $\sin\phi_D \ \cos\lambda_D \ \sin\beta_R - \sin\lambda_D \ \cos\beta_R$

D_{13} = $\sin\phi_D \ \sin\lambda_D \ \sin\beta_R + \cos\lambda_D \ \cos\beta_R$

D_{21} = $-\cos\phi_D \ \cos\beta_R$

D_{22} = $\sin\phi_D \ \cos\lambda_D \ \cos\beta_R + \sin\lambda_D \ \sin\beta_R$

D_{23} = $\sin\phi_D \ \sin\lambda_D \ \cos\beta_R - \cos\lambda_D \ \sin\beta_R$

r_c = cross-track distance

r_R = distance-to-go

ϕ = latitude of present position

λ = longitude of present position

ϕ_D = latitude of destination

λ_D = longitude of destination, and

β_R = desired track angle at destination

3 1.9 Conclusions and Summary of Equations Chosen for Mechanization

Accuracy and storage considerations dictate that the airborne coordinate converter should be based upon a digital design. To be minimal, it should employ serial computation with the storage of intermediate results To avoid modification of existing hyperbolic receivers, the converter should mechanize hyperbolic transformation formulae. This can be done in a straightforward manner with tolerable solution times, within the present state of the art. Since earth eccentricity and phase variation due to the secondary factor for radio propagation cannot be neglected, they should be incorporated in the over-all design by a simple iterative process whereby small corrections are computed and added to the detected range differences Distance-to-go and cross-track-distance should appear as the display to flight personnel To allow global navigation to be conducted, the computer should store sufficient information to permit operation from any one of 10 fixed station triads.

Figure 7 presents a flow diagram of the set of equations that the airborne coordinate converter should mechanize.

3.2 COMPUTER REQUIREMENTS

3.2.1 Choice of Computer Type

It was shown in Section 3 1 2 that accuracy requirements make digital computation the logical choice for the coordinate converter Three digital-system approaches have the potential to fulfill the requirements

The first approach considered used operational digital techniques to solve the equations. Equipment requirements to implement an operational digital computer were estimated. The estimate showed that, since all operational elements function effectively in parallel, the amount of equipment required for implementation is prohibitive. The computer is also slow, since it uses counting techniques. These disadvantages discouraged effort on this type of computer

The second approach used a digital-differential-analyzer-incremental computer. The possible simplifications of the equations for use in an incremental computer are attractive. However, extensive analysis of possible implementation was discouraged by the incorrectable errors that accumulate in this type of computer, in this application, when the

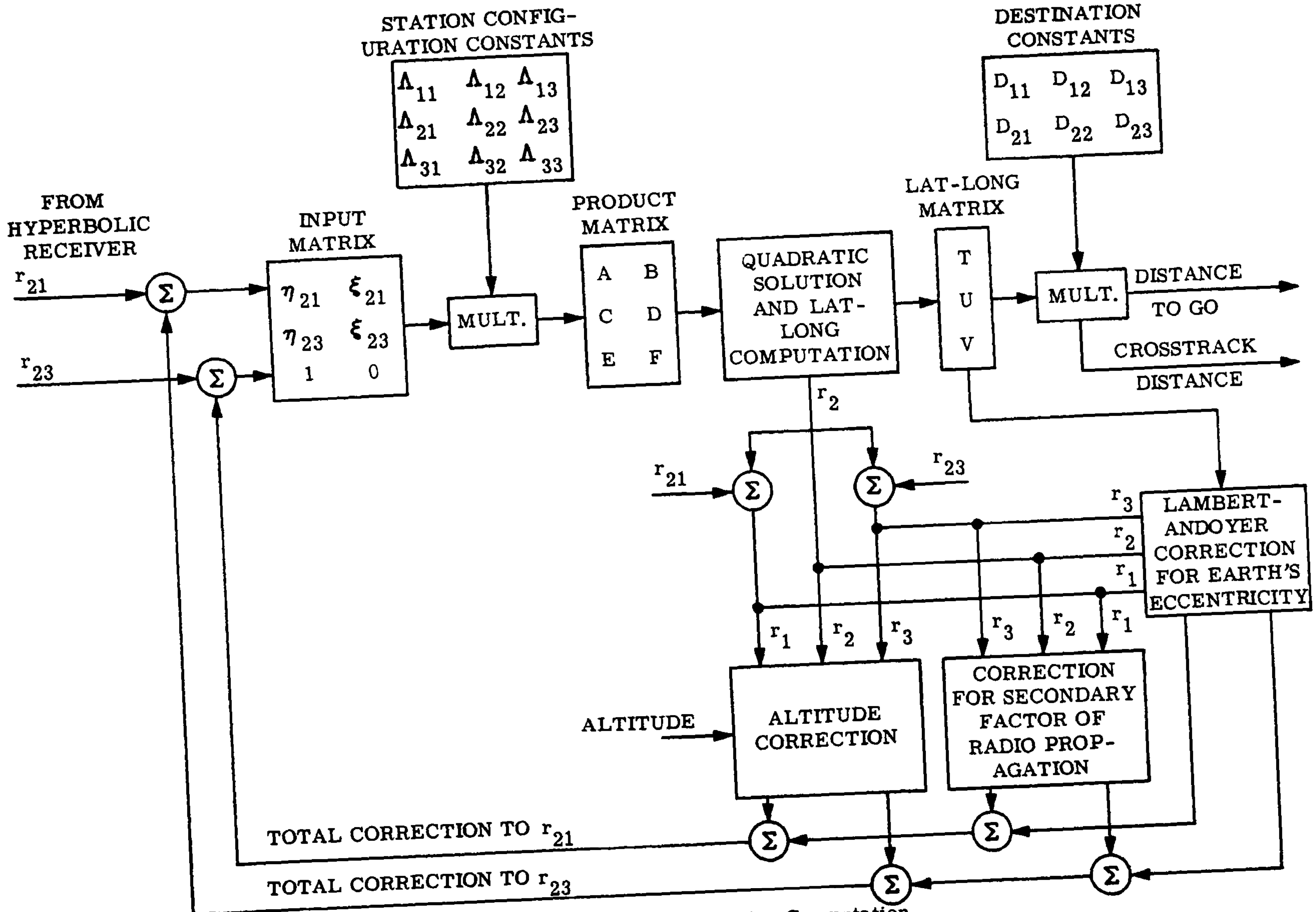


Figure 7. Flow Diagram of Coordinate Conversion Computation

computer power is off for an extended period, or when the computer is started at some point in the field other than the initial point

The third approach, which uses a stored program computation, was the one selected for computer implementation This computer can be visualized by considering a general-purpose, stored-program computer, the complexity of which has been reduced to the specific requirements of this problem The main advantage of this computer is that each solution of an equation is independent of the previous solution Even though several of the routines depend on previous solutions, two or three new solutions are sufficient to up-date this information However, this is of no consequence since several solutions are made each second

Flexibility is another advantage to the stored-program computer. This flexibility is achieved because the program is not wires, allowing it to be easily modified

3.2 2 Sampling Frequency

The requirements for the sampling frequency are derived from consideration of the total spectrum of frequencies present at the outputs of the receiver. This includes not only the signal frequencies of interest but also the spectrum of any noise signals.

The two principal factors to be considered are the signals due to aircraft motion with respect to each radio transmitter site and the output noise. As stated in the original work statement, the primary mission is to supply long-range position information where aircraft maneuvers are not expected to be consistently maintained Accordingly, the only significant dynamic conditions are caused by direct changes of range due to aircraft velocity The sampling frequency required to satisfy this condition is established by the permissible transport delay. The magnitude of error in the coordinate converter output is proportional to the product of the velocity of the aircraft and the time it takes the computer to accept hyperbolic inputs and produce output information in the desired coordinates. The original work statement specified an error of ±0.5 -nautical mile at short ranges for aircraft velocities of 600 knots. This condition will be satisfied with a sampling rate corresponding to one sample every 2.5 seconds.

The noise contents on the receiver outputs can be a serious problem when long ranges are considered To obtain finite sampling frequencies in a sampled data system, the sampling process must be preceded by bandwidth limiting filters. Output filters, called tracking loops, are present in all known long-range hyperbolic receivers. The filters are required whether the receivers are used in conjunction with a C/C or with an operator, because without these filters the operator would be unable to properly interpret the dial readings.

In the case of a C/C the sampling rates must be sufficiently high so as not to produce any magnification of the low-frequency components of this noise. This is considered to be an important factor due to the large amount of noise to be expected and the consequences of inadequate sampling rates.

Reference 4 discusses the effects of sampling band-limited random noise. By applying this theory to the problem, it is concluded that the sampling time must satisfy equation 62

$$\omega_1 T_1 = 1/2 \qquad (62)$$

where:

ω_1 = low pass filter cutoff in radians/second

T_1 = time between samples in seconds

The effects of the tracking-filter time constant are discussed in Section 4.1, where it is concluded that for airborne LORAN-C equipment the low-pass filter cutoff will be about 1/6 radians/second. Application of the above criterion yields a sampling rate of one sample every 3 seconds. It is thus seen that the sampling rate is dictated by the dynamic consideration rather than the noise and is therefore set to be one sample every 2.5 seconds.

However, it is clear that the noise considerations dictate a sampling rate close to this value, and where the errors due to the dynamic properties can be corrected by relatively simple means (to be discussed) the sampling rate must be kept at least one every 3 seconds.

3.2.3 Minimum Access Programming of Computer

In the application of "real time computation," the problem of access time to a word stored in a magnetic drum memory is serious. However, the problem can be solved by proper programming and computer design. If the program is written, using a single-address computer, with complete disregard for the location of words in memory, the solution time is quite long. But, if a word is placed in a memory location so that it arrives at a reading head at the exact time it is desired to read from the memory, the access, and therefore computation, time is minimum.

The minimum access program was written for the airborne computer in order to establish the memory and computation time requirements. The minimum access program is one in which data and instruction are placed on the drum of the computer in such a way as to minimize nonproductive searching time. This is accomplished in programming the airborne computer by using only the sector location and not the track location to determine if a data or instruction order is optimum or nonoptimum.

Table II shows the 12 instructions that the airborne computer is capable of performing. The number of word-times are shown after the instruction. The address of the operand must be in order for the instruction to be of minimum access time.

TABLE II. COMPUTER CAPABILITY

Order	Number of Word-Times
Add, Subtract, Shift Left, Shift Right, Bring, Output, Input	1
Multiply	25
Divide	25
Test, A Delay and B Delay	Any Odd Number

The equations in this report were programmed for the airborne computer and the results are shown in Table III. The results compiled in this table are based upon the characteristics given in Appendix G.

TABLE III. TIME AND MEMORY REQUIREMENTS FOR VARIOUS COMPUTATIONS

Type of Computation	Time Required Per Solution (Seconds)	Approximate Memory Cells Required
Hyperbolic Coarse and Fine Radio Output and Cross-Track-Distance and Distance-To-Go	0.5	760
New Leg Destination and Test Program	0.2	440 (Provides Storage of 32 Legs)
Change of Stations	0.04	200 (Provides Storage for 10 Station Triads)
	Total	1400

The hyperbolic coarse and fine radio outputs and the distance-to-go and cross-track-distance computations require approximately one-half second of computer time, which is well within the sampling rates derived in the previous section Correction terms are included to compensate for errors such as those due to eccentricity and propagation. Approximately 760 memory locations are required for this program, which includes a short section for a limited test program to insure that the control elements of the computer are operating correctly. If a malfunction is detected the DATA GOOD light on the computer front panel extinguishes to alert the operator

When the distance-to-go becomes zero, the computer has the capability of automatically switching to the next programmed leg. Approximately 0.2 second and 440 memory cells are required to switch the new leg coordinates into the distance-to-go and cross-track-distance program. Storage for 32 course legs is provided in the computer. The new-destination program also includes a test program that fully exercises all functions of the computer and assures the operator that the computer is operating correctly. If a malfunction exists, the DATA GOOD indicator will extinguish, as in the short test program, alerting the operator. In the event the radio information is erratic and not usable, a light on the front panel will illuminate, indicating an unsatisfactory condition.

When it becomes necessary to change to a new set of stations, the operator selects a station triad from the ten station triads stored in the computer Approximately 0.04 second and 200 memory cells are required to insert these constants into the distance-to-go and cross-track-distance program.

The operator can run through the test program at any time to assure proper operation of the computer. Approximately 0 05 second is required for this test program. Again, a malfunction is indicated by the extinguishing of the DATA GOOD light on the front panel.

3.2.4 Computer Simulation

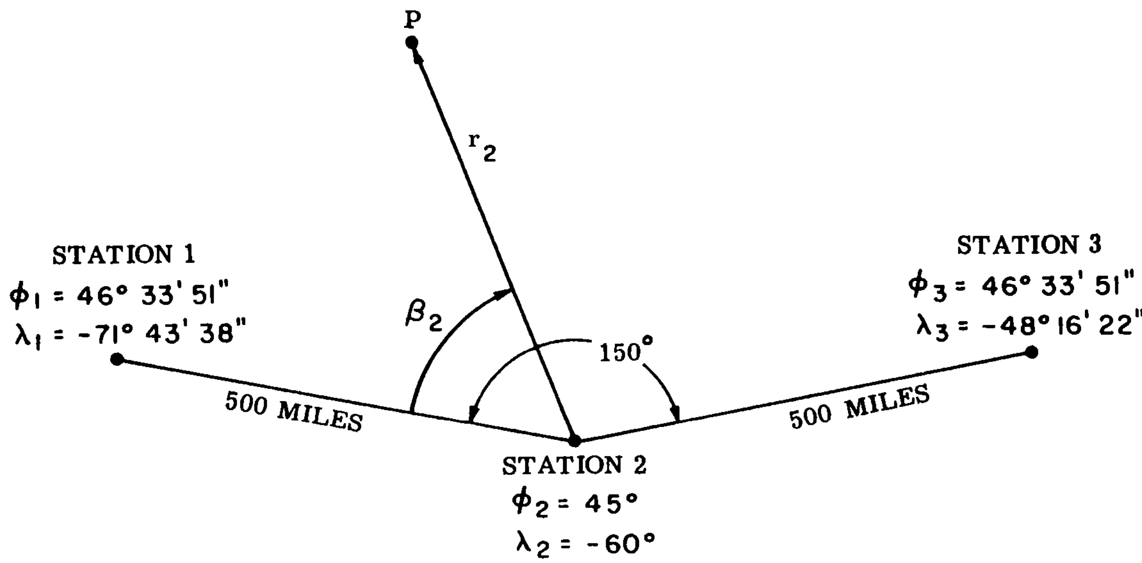
A portion of the minimum access program of Section 3.2.3 was simulated on a Model LGP-30 general-purpose digital computer to verify the validity of the equations for various station configurations and locations. The number of bits required in the computer word also was established at this time.

The equations verified with the LGP-30 digital computer are those giving latitude and longitude from a pair of range differences (refer to page 28). It was felt unnecessary to simulate the remaining equations since they are straightforward and time was limited.

Basically the simulation problem is one of computing latitude and longitude from the radio range differences and comparing these computations with the theoretical latitude and longitude. The theoretical latitude and longitude and the radio range differences were computed using r_2 and β_2 The number of bits in the word of the simulated airborne computer was varied to observe the effect on over-all accuracy.

3.2.4.1 Method of Simulation

Computations are made for the entire field for several station configurations. The pair of range differences are readily computed by assuming a point P in the field given by the coordinates r_2 and β_2, as shown in Figure 8.



r_2 = the great circle distance joining P and radio station n (in this case r_2 is the great circle distance or range joining P and radio station 2)

β_2 = the angle between the great circle distance joining radio stations No. 1 and 2 and the great circle distance joining point P and radio station No. 2.

Figure 8. Station Configuration

These equations are given by the law of cosines and are presented in Appendix B, arranged as follows (equation 63 does not appear in Appendix B, but is the simple cosine law relating range and bearing to latitude):

$$\sin\phi = \sin\phi_2 \cos\frac{r_2}{R_o} + \cos\phi_2 \sin\frac{r_2}{R_o} \cos(\beta_a + \beta_2) \quad (63)$$

$$\cos(\lambda - \lambda_2) = \frac{\cos\frac{r_2}{R_o} - \sin\phi_2 \sin\phi}{\cos\phi_2 \cos\phi} \quad (64)$$

$$\cos\frac{r_1}{R_o} = \sin\phi_1 \sin\phi + \cos\phi_1 \cos\phi \cos(\lambda - \lambda_1) \quad (65)$$

$$\cos\frac{r_3}{R_o} = \sin\phi_3 \sin\phi + \cos\phi_3 \cos\phi \cos(\lambda - \lambda_3) \quad (66)$$

$$r_{21} = r_2 - r_1 \quad (67)$$

$$r_{23} = r_2 - r_3 \quad (68)$$

where:

ϕ_n, λ_n = latitude and longitude of radio station

R_o = radius of the earth

β_a = the angle between the great circle distance joining radio stations 1 and 2 and the great circle distance joining the north pole and radio station 2.

For each point P in the field given by r_2, β_2, the corresponding values of latitude and longitude, ϕ and λ, respectively, are computed from equations 63 and 64. The pair of radio range differences are then computed by equations 65, 66, 67 and 68. These equations were programmed into the LGP-30 digital computer in the fixed decimal point system using the full accuracy of 32 bits per word.

The binary readouts representing the pair of radio range differences are then rounded off and limited to 18 bits and sign to simulate the actual output of the LORAN-C receivers. Since the pair of radio range differences is limited to 18 bits, or approximately six significant figures, the simulation of the airborne program was programmed to preserve this input.

Having simulated the pair of radio range differences, it remained to simulate the actual computer and its program for the computation of latitude and longitude. The equations used are located in Appendix B and summarized below

$$\eta_{21} = \cos \frac{r_{21}}{R_o} \tag{69}$$

$$\eta_{23} = \cos \frac{r_{23}}{R_o} \tag{70}$$

$$\xi_{21} = \sin \frac{r_{21}}{R_o} \tag{71}$$

$$\xi_{23} = \sin \frac{r_{23}}{R_o} \tag{72}$$

$$\begin{pmatrix} A & B \\ C & D \\ E & F \end{pmatrix} = \begin{pmatrix} \Lambda_{11} & \Lambda_{12} & \Lambda_{13} \\ \Lambda_{21} & \Lambda_{22} & \Lambda_{23} \\ \Lambda_{31} & \Lambda_{32} & \Lambda_{33} \end{pmatrix} \begin{pmatrix} \eta_{21} & \xi_{21} \\ \eta_{23} & \xi_{23} \\ 1 & 0 \end{pmatrix} \tag{73}$$

$$J = B^2 + D^2 + F^2 - 1 \tag{74}$$

$$K = AB + CD + EF \tag{75}$$

$$L = A^2 + C^2 + E^2 - 1 \tag{76}$$

$$T_{\sigma_2} = \frac{-K - \sqrt{K^2 - JL}}{J} \tag{77}$$

$$\phi = \sin^{-1}\left[\frac{A + BT\sigma_2}{\sqrt{1 + T^2\sigma_2}}\right] \tag{78}$$

$$\phi = \tan^{-1}\left[\frac{E + FT\sigma_2}{C + DT\sigma_2}\right] \tag{79}$$

The transcendental function cosine and sine expressions 69 through 72 are computed from the pair of radio range differences Since in the argument

$$\left|\frac{r_{21}}{R_o}\right| \text{ and } \left|\frac{r_{23}}{R_o}\right| < .3 \text{ radians}$$

the Maclaurin series is used with a limited number of terms to compute the cosine and sine functions.

To preserve six significant figures the first three terms of the sine function are used,

$$\sin\left(\frac{r_{21}}{R_o}\right) = \left(\frac{r_{21}}{R_o}\right) - \frac{1}{3!}\left(\frac{r_{21}}{R_o}\right)^3 + \frac{1}{5!}\left(\frac{r_{21}}{R_o}\right)^5 \tag{80}$$

This produces a maximum error of:

$$\frac{1}{7!}\left(\frac{r_{21}}{R_o}\right)^7 < .1 \times 10^{-6}$$

The factored sine series can then be evaluated "from the inside out," which is called nesting, in order to reduce the computing time. The number of multiplications and the time required for computation are thus reduced. The series becomes:

$$\sin\left(\frac{r_{21}}{R_o}\right) = \left(\frac{r_{21}}{R_o}\right)\left\{1 - \left(\frac{r_{21}}{R_o}\right)^2\left[\frac{1}{3!} - \frac{1}{5!}\left(\frac{r_{21}}{R_o}\right)^2\right]\right\} \tag{81}$$

In order to preserve the same accuracy in the cosine series, the first three terms are used giving:

$$\cos\left(\frac{r_{12}}{R_o}\right) = 1 - \left(\frac{r_{21}}{R_o}\right)^2\left[\frac{1}{2!} - \frac{1}{4!}\left(\frac{r_{21}}{R_o}\right)^2\right] \tag{82}$$

and producing a maximum error of

$$\left|\frac{1}{6!}(.3)^6\right| < 1 \times 10^{-6}$$

The most economical form for these series, from the standpoint of attaining maximum accuracy with the minimum number of terms, is obtained by a power series arranged in terms of the Tchebysheff polynomials. The greatest possible absolute error at any point inside the range is reduced to a minimum by the Tchebysheff polynomials. Again, because of the limited time available, this was not pursued. Some of these functions can be found in Reference 5.

The various coefficients are computed from equations 73 through 76. The lambdas in the three-by-three matrix are functions of the station configuration. Using these coefficients, T_σ, ϕ, and λ are computed from equations 77, 78, and 79, respectively

The square root is obtained by the Newton-Raphson iteration. This procedure consists of using the previous value of the square root as first estimate for the next square root. A corrected second approximation is applied and this procedure is repeated. By using the value of square root obtained in the prior solution, three iterations are sufficient to compute a value to the desired accuracy.

The terms ϕ and λ have a range of -90° to $+90^\circ$ and -180° to $+180^\circ$, respectively, and are computed by a power series arranged in terms of the Tchebysheff polynomials.

The rms error is computed by the following expression:

$$\sigma_{rms} = R_o \sqrt{(\Delta\phi)^2 + (\Delta\lambda \cos\phi)^2} \tag{83}$$

where:

$$\Delta\phi = \phi \text{ calculated} - \phi \text{ theoretical}$$

$$\Delta\lambda = \lambda \text{ calculated} - \lambda \text{ theoretical}$$

3.2.4.2 Results of Computer Simulation

Computational errors were established with the LGP-30 digital computer for several station configurations. With the pair of radio range differences rounded off to 18 bits and using the full 32 bit precision of the LGP-30 for computations, the error in the field computed from equation 83 is found to be less than 0.5 mile. This compares favorably with the theoretical computations, where error in the field is less than 0.2 mile when the pair of radio range differences are limited to 18 bits.

The resulting error computed from equation 83 for one station configuration is shown in Figures 9 and 10. The stations are located 500 miles apart and have an included baseline angle of 150 degrees.

The maximum error is less than 2 miles in the main part of the field between β_2 (equal to 25° to 125°) and r_2 (from 300 to 2100 miles), as shown in Figure 9. Around the baselines from approximately 0° to 25° and 125° to 150° the error is greater, but is due simply to the limited number of bits used in the simulated computer. This error can be corrected readily by rescaling the program. However, due to the limited time, rescaling of the program was not attempted, since it is time-consuming and not a limitation to the equation being solved.

The reason for the large baseline error is readily understood when equations 74, 75, 76, and 77 are examined. When $\beta_2 = 0$ to 25° and 125° to 150°, J, K, and L have magnitudes much less than 1. When K is squared and J is multiplied by K, even smaller numbers result. Since the airborne computer and the simulated computer operate at a fixed binary point with a limited number of bits per word, the error becomes large when this operation is performed. These numbers are, therefore, in large error and the same square root of equation 77 is in error by a still larger amount.

The field also is examined at closer ranges as shown in Figure 10. Again the errors are small, i.e., less than 2 miles except around the baseline and station 2. The cause of the errors around the baseline was discussed previously and can be corrected. Errors around station 2 are also caused by the scaling and inaccuracies of the square root in equation 76. However, in this case, J is large while K and L are very small. Here again, K^2 and JL are both small numbers and lost in the computer accumulator due to the limited number of bits. Several other configurations were examined with similar results.

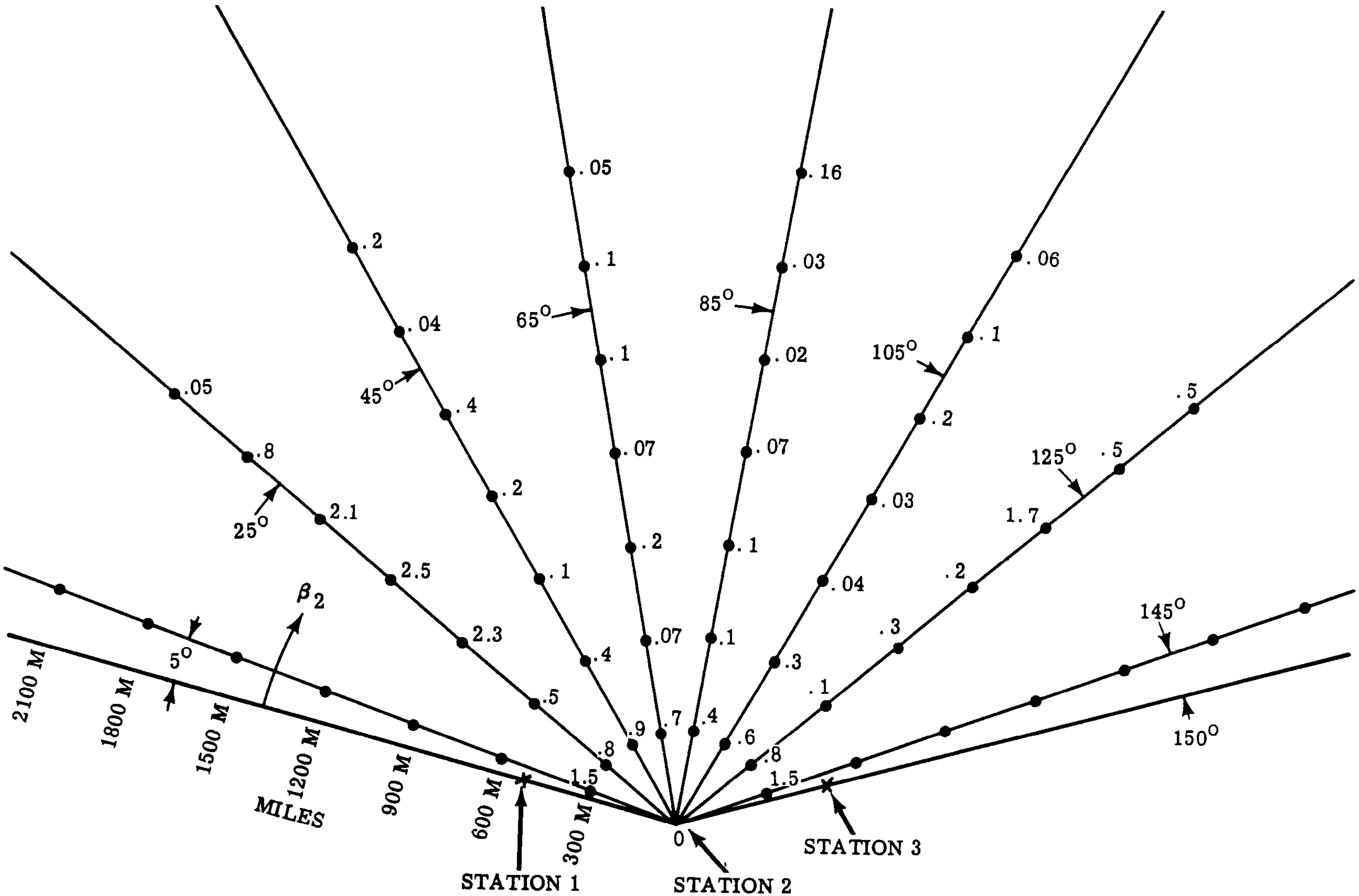


Figure 9. Plot of Error vs Position ($r_2 \beta_2$) in Field (For 500 Mile Baseline at 150°)

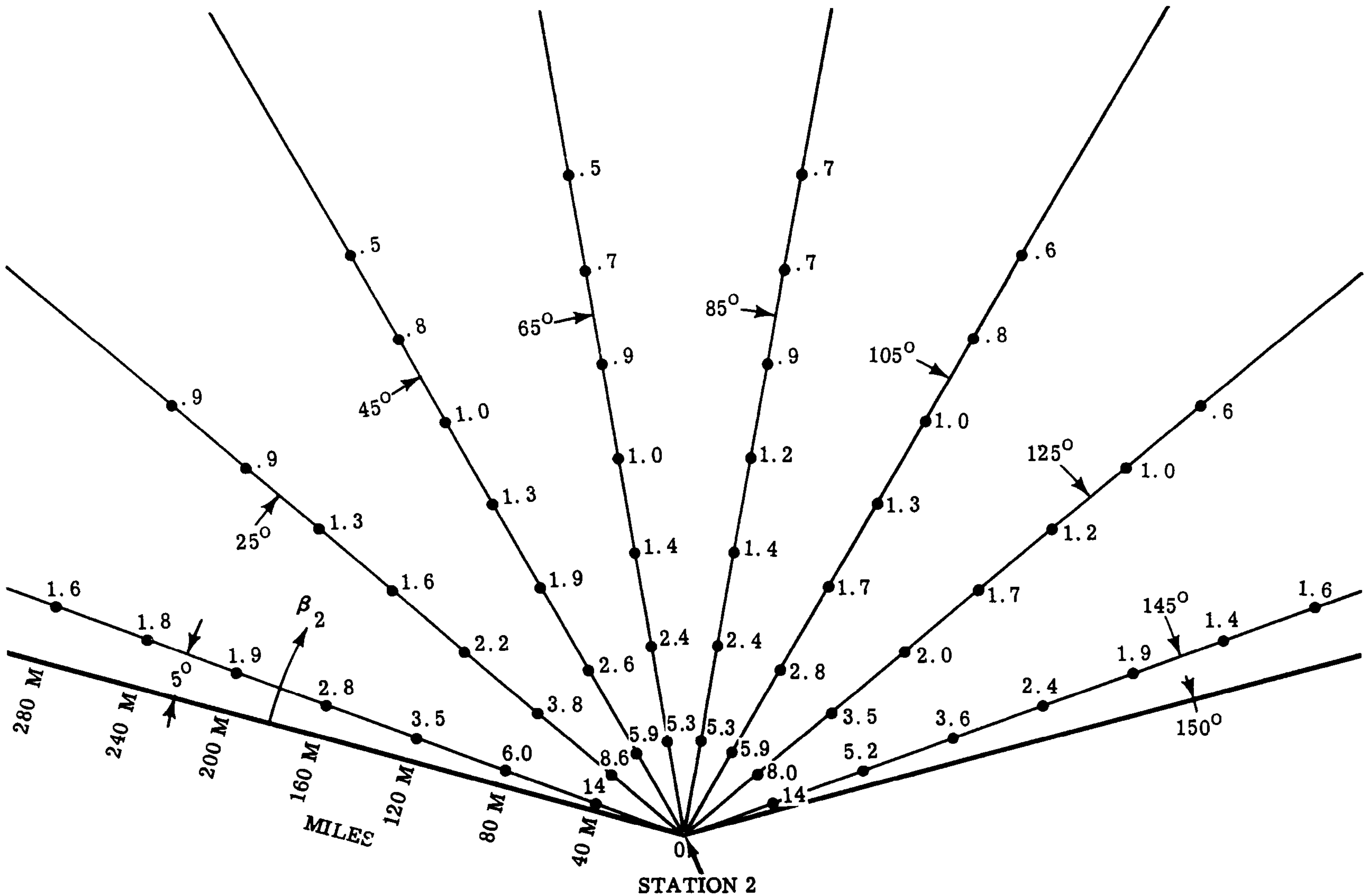


Figure 10. Plot of Error vs Position (r_2 β_2) in Near Field (For 500 Mile Baseline at 150°)

Since the radio station can be located anywhere on the earth it will be necessary to examine all types of station configurations to obtain the range of all constants and variables. At this time a universal program with the desired scaling can be written. Accuracies will be on the order of that found for the typical configuration examined.

The LGP-30 digital computer was programmed so that the bit length of each word could be varied. It was found that for one configuration an optimum scaled program that produced excellent results could be written using a 20-bit word However, to cover all configurations it was necessary to increase the word length to 24 bits. The increase does not enhance accuracy but simply increases the dynamic range of the variable, thereby permitting the program to be applicable to many station configurations.

3.2.5 Memory Requirements

The airborne computer memory, which consists of a magnetic drum, is used to store data, the program, and partial results of computation

As a result of minimum-access programming and computer simulation, it was found that the magnetic drum should have the following memory requirements:

1. Word length of 24 bits (23 bits plus sign)

2 Total storage capacity of 2048 words. (A total of approximately 1400 memory cells or words are required as indicated in Section 3.2.3. A drum having 2048 words was chosen, since it is a convenient number and is readily available. Additional memory cells are also provided.)

3.3 EQUIPMENT MECHANIZATION

3.3.1 Computer Design

3.3.1.1 Logic Design

The results of the minimum-access programming demonstrated that a serial computer can be used. The advantages of a serial computer over a parallel computer in size, weight, and complexity are apparent.

A magnetic drum was chosen for storage because it is ideally suited for the serial type computer in which most of the arithmetic unit and some of the control unit consists of short loops on the drum itself. The approximate storage capacity required is estimated at 1400 words or memory cells, with a 24-bit word length. The magnetic drum memory requires the computer word to have a spacer bit which increases the word length to 25 bits. The estimate of words required is minimum and does not allow a safety margin. Therefore, a drum of 2048 words was chosen.

The coordinate converter is a whole-word computer; that is, for each cycle of computation, the whole word is introduced into the computer. Transmission links can be established between the main storage unit, the arithmetic unit, and input-output devices under control of an internally stored program Minimum access programming is utilized, and no alterations on the instruction words stored on the drum are allowed under program control. Because of the latter restriction, all tracks in general storage are not provided with writing amplifiers, and erasures or unwanted insertions in the program cannot occur.

A block diagram of the coordinate converter computer is shown in Figure 11. The core of the computer is the magnetic drum. In addition to providing a general storage of 2048 words, the drum contains two control tracks that supply synchronizing and clock signals to the entire system, two tracks for the temporary storage of data, and one track that contains three circulating registers A bit counter operating at the clock rate provides a variety of control signals that are gated by the control logic Other control signals originate in the sector counter. The sector counter also doubles as a word counter when multiplication or division are performed. As each command is followed, it is stored in the command register and decoded by the command decoder. If an address is associated with

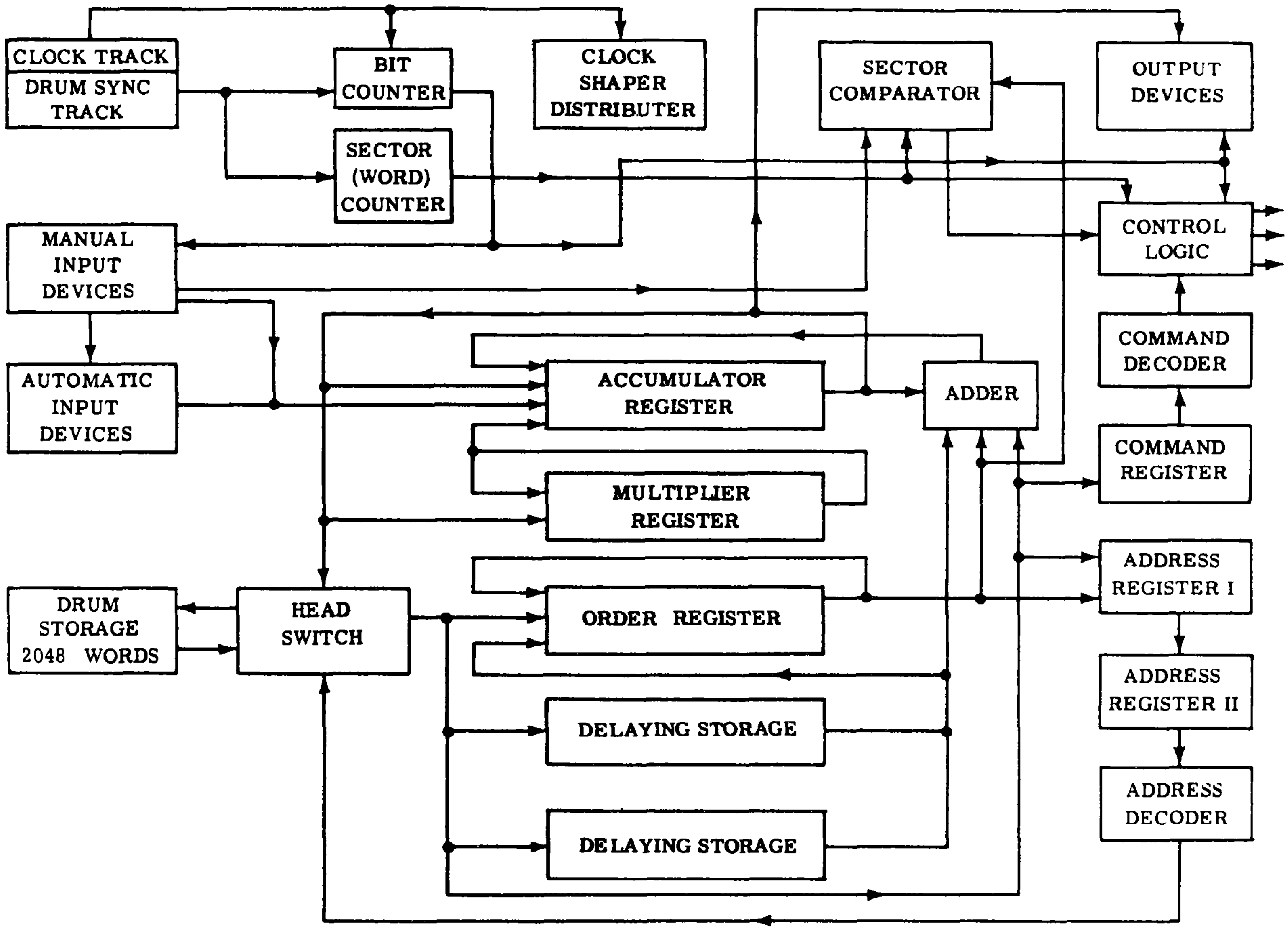


Figure 11. Coordinate Converter Computer, Block Diagram

a particular command, it is held in address register II, having reached its position through address register I. Data enters and leaves the computer via the accumulator register.

Twelve instructions are provided. They are:

Add
Subtract
Multiply
Divide
Shift Right
Shift Left
Test
A Delay
B Delay
Input
Output (Store)
Special Transfer

The first six are arithmetic instructions and the last six are control instructions. The organization of the command into a computer word and a more detailed description of the computer are given in Appendix G.

Arithmetic is carried out serially with operations performed on one digit at a time, beginning with the least significant digit. All transfers of words from one register to another take place serially, one digit at a time and at a rate governed by the system clock. Arithmetic is performed using the two's-complement number system. Each storage track contains 64 computer words, with the 24 digits of each word arranged serially and each word separated from the next by a space bit. Thus there are 1600 bit areas on each track, or 25 bits per sector (word).

3.3.1.2 Circuitry Considerations

The coordinate converter contains relatively few different circuits. The computer is an assembly of these elementary circuits, each of which performs according to predetermined logic. The following basic circuits serve as the building blocks for the C/C.

1. J-K-T flip-flop
2. Delay flip-flop
3. Inverter
4. Inverter-amplifier
5. Diode logic
6. Read amplifier
7. Write amplifier
8. Clock pulse shaper

Solid-state circuitry is used throughout and the number of different components used is held to a minimum.

The semiconductor components are used because of the advantages they have over vacuum tubes. Some of the major advantages are: small size that facilitates the miniaturization of equipment, low power dissipation, long service-free life, and high reliability. Germanium products are selected over silicon because of greater reliability and lower cost, silicon products still being somewhat in the handmade state of development.

Standard transistors and diodes are used throughout the computer. The circuits are designed to make selection of components unnecessary. Components that meet the vendors' published specification are suitable for use in any portion of the computer where they are designed to be used.

The computer is composed of basic printed-wiring modules interchangeable within their own type. Because of this interchangeability, troubleshooting is simplified to substitution of modules, following a brief preliminary analysis. Modules can be repaired on the bench and returned to service later. Thus only a minimum number of modules need be maintained as spares.

The C/C will contain an integral power supply designed to operate from a 115 volt, 400 cps aircraft supply. The supply will utilize semiconductors and will have sufficient capability to supply the modest voltage - regulation requirements of the digital circuits.

3.3.1.3 Estimate of Equipment

Table IV is a summary of the component elements contained within the coordinate converter and is based upon the logic design presented in Appendix G. In addition, the C/C will contain a magnetic drum approximately 4 inches in diameter and 3 inches in axial length, and a power supply to convert 400-cps input into the required voltages.

TABLE IV. SUMMARY OF PARTS USED IN COORDINATE CONVERTER COMPUTER

Arithmetic and Control	
Delay Flip-flops	4
J-K-T Flip-flops	36
Inverters	25
Inverter-Amplifiers	5
Logic Function Diodes	513
Memory	
Diodes	182
Transistors	62
Diodes in Active Devices	686
Totals	
Diodes	1375
Transistors	226
Resistors	918

By utilizing standard printed-wiring modules, the C/C will occupy a volume of approximately 0.5 cubic foot and weighs less than 25 pounds. Further reduction in size is possible, but this is believed to be a reasonable compromise between ease of maintenance and total unit size.

3.3.1.4 Cooling Considerations

A preliminary study was made of the probable maximum thermal environments associated with the coordinate converter. Since the unit is to utilize germanium transistors, the study was directed toward assuring that the transistors would always be able to dissipate heat adequately, without an excessive operating-temperature condition. Analysis indicated that by circulating ambient air through the unit with a small blower, a minimum size could be obtained without compromising performance or reliability.

It was initially assumed that the anticipated maximum external ambient would be 55°C at an altitude of 20,000 feet. Considering an expected 200-watt dissipation in the computer, calculations showed that a prohibitive size would be required if free convection and radiation alone were depended upon to transfer this heat to the external ambient. A much more efficient and desirable situation results from using a small blower to bring ambient air into the unit, circulate the air and discharge it. If such a blower is used, it becomes entirely feasible to hold the unit's internal temperature rise over ambient to 5 to 10°C. The resulting 60 to 65°C internal ambient, under the severest operating conditions, will still allow satisfactory reliable operation of the contemplated germanium transistors. The fan size can be kept below 15 cubic inches and its power requirement below 20 watts.

The use of internal forced circulation provides not only a lower internal ambient, but also makes it possible to direct air to specific areas and to increase the heat transfer rates within the unit. The printed-wiring construction particularly lends itself to this type of cooling. A typical configuration would place the blower at the rear of the unit discharging out, with the air inlet suitably placed on or near the front or aisle panel of the unit.

It is felt that, within the framework of cooling considerations given above, if detailed consideration is given to specific problems, no difficulty should be incurred in operating the germanium transistorized C/C unit under the severest expected aircraft ambients.

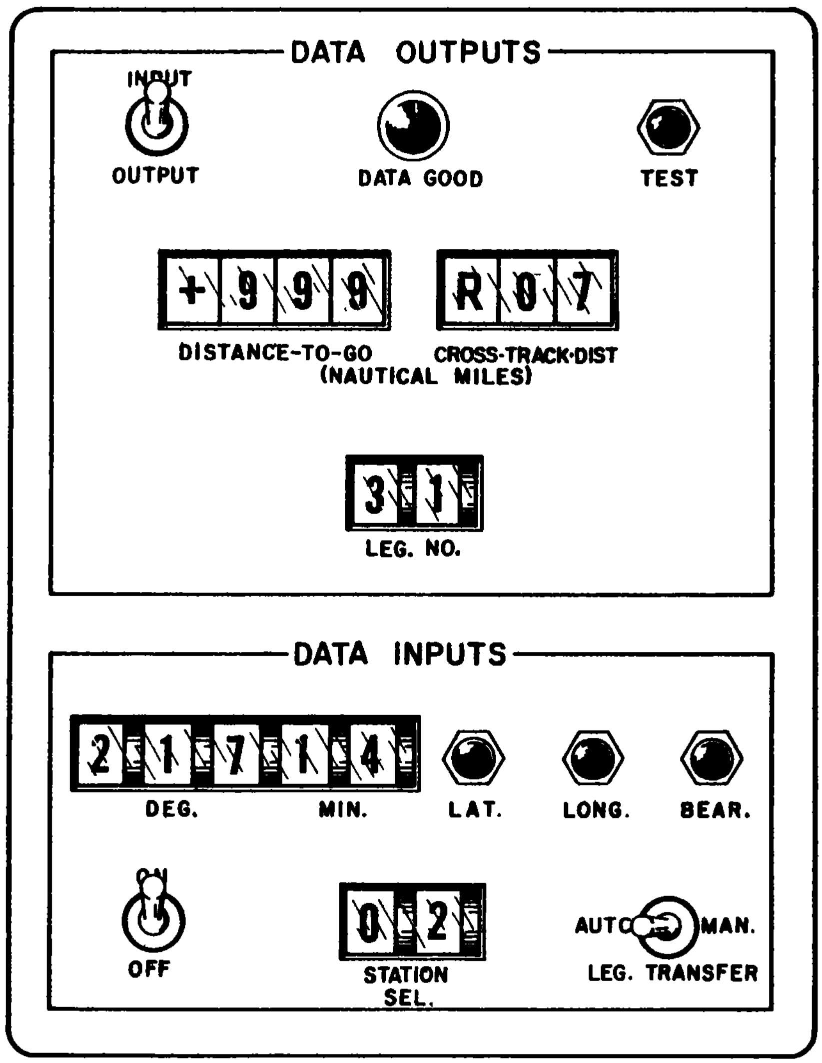


Figure 12. Data Display Unit Control Panel

3.3.2 Display

The minimum requirement of the data display unit is a visual presentation of distance-to-go and the cross-track -distance (lateral error) with respect to the desired flight path. These quantities will appear as illuminated digits on the data display unit panel. The maximum range of the indicators will be ± 999 nautical miles on the DISTANCE-TO-GO indicator and ± 99 nautical miles on the CROSS-TRACK-DISTANCE indicator. Each channel will have a resolution of ± 1 nautical mile.

The display unit is divided into two separate areas, the "input" area and the "output-data" area The "input" area will contain the necessary controls for inserting the initial conditions (latitude, longitude, and destination bearing), for energizing the unit, and for selecting the station-triad to be used. The "output-data" area will contain the readouts of distance-to-go and cross-track-distance, the data good light, and the self-test control.

Figure 12 is a sketch of a basic display configuration illustrating the significant features of the basic display. The display unit is provided with a means for manually inserting the initial conditions into the computer and setting the controls.

3.3.2.1 Information Output

The information displayed by the data display unit is in a concise form and should immediately indicate the aircraft position relative to a preselected flight path. It is felt that this manner of numerically presenting the aircraft position will be adequate.

Two widely known techniques for displaying the data were considered. One is an all-electronic design in which the digital representation of the quantities to be displayed is converted to a binary coded decimal and stored in registers. The output of the registers is then used to control optically illuminated decimal digits, such as nixie tubes or inline incandescent readouts. This method provides a clear visual data representation and is easily read at large incident angles while in the presence of high ambient lighting. It can also be used to read out any desired quantity with the airborne computer. Although longitude and latitude are not the desired display data under normal conditions, they may be desired under emergency conditions and could be displayed by means of a selector switch.

Even though the all-electronic method has several advantages, a number of basic disadvantages restrict its applicability. First, the memory of the unit will be lost, along with all display data, if power is removed. Second, the electronic design is uneconomical because it can only be used to display information concerning aircraft location.

A second and more suitable method is a combination analog-digital display with a mechanical counter. A block diagram of the analog-digital layout is shown in Figure 13.

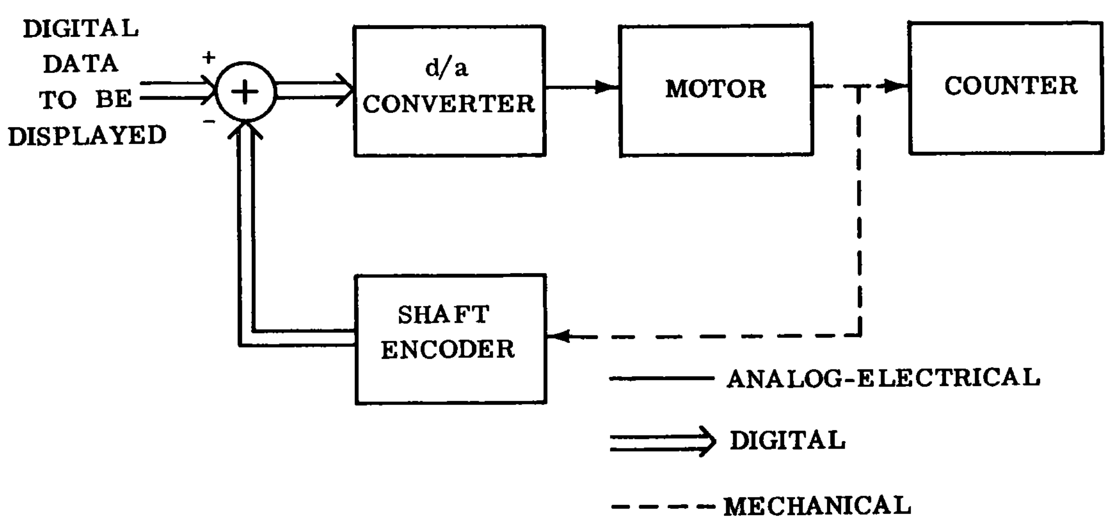


Figure 13. Typical Display Servo, Block Diagram

The analog-digital method functions in the following manner. The data to be displayed is in digital form. In order to compare it with the output analog data, the output data is changed to digital form by a shaft encoder. The output and input data are then compared digitally at the summing point and the difference is converted to an analog error voltage by the digital-to-analog converter. Any difference between the two values will result in an analog voltage being applied to the motor in such a manner as to reduce the error to zero and cause the counter to display the input digital data in analog form. The shaft encoder assures that the motor shaft feedback will be converted to an accurate digital value.

The analog-digital method retains the output reading when equipment power is removed, providing permanent information storage and preservation of output data. Another factor influencing the choice of the analog-digital method is its ability to combine doppler and radio information without converting doppler data to digital form. The methods and techniques used for doppler and radio information combinations are discussed in Section 4.2.1.

3.3.2.2 Information Input

At the beginning of a mission the scheduled flight plan information is programmed into the airborne computer. After the flight plan has been inserted, the correct leg number is located on the LEG ON indicator. In order to fly within a preselected corridor, the computer must contain the longitude, latitude, and the desired track angle for each leg. Even though a means could be provided for automatically inserting the information into the computer, it is felt that a serious loss of flexibility would occur if flight personnel could not manually change the aircraft flight plan. Also, the many variables that are encountered in establishing a flight plan make it desirable to be able to change it readily.

To insert the initial conditions see Figure 12 and proceed as follows:

1. Break the flight plan information into a sequence of legs with the three initial conditions (longitude, latitude, and track angle) known for each leg. (The procedural steps are based on the assumption that the desired track angle is inserted. A similar procedure would be followed for the alternate method of establishing the fixed line by means of initial and end coordinates).

2. Insert the first leg number by turning the LEG NO. dials until the desired digits appear.

3. Rotate the degree (DEG.) and minute (MIN.) dials until the value corresponding to the end coordinates of the latitude appears.

4. Depress the latitude button (LAT.) until the button is illuminated. The illumnation indicates that the computer has stored the inserted information. The longitude and bearing may now be inserted using a similar procedure. When all information has been inserted for the first leg, advance LEG NO. indicator to read 2.

By following the above procedure, initial conditions for 32 flight legs may be inserted and stored. The transfer control switch (LEG TRANSFER) enables the operator to advance from one leg to another, either manually or automatically, in the course of the flight. To change the flight plan, set the LEG NO. indicator to the number of the leg to be changed, set the LEG TRANSFER switch to OFF, and program the computer with the new flight path information.

The remaining input control is the station selector (STATION SEL.). The information associated with each triad of stations is permanently stored within the unit. The STATION SEL. control allows the operator to select the group of transmitting stations which are most applicable under the given conditions.

For reasons of economy, the longitude, latitude, and bearing at any time during the flight are not indicated by the data display unit. However, this information is available within the converter and could be displayed by the addition of display units similar to those discussed.

3.4 TIE-IN WITH HYPERBOLIC SYSTEMS

3.4.1 Description of LORAN C

LORAN C is a hyperbolic navigation system that measures the difference in time of arrival of 100 pulses which are transmitted in synchronism from three or more separate transmitters. Measurements are made on the pulse envelopes to obtain coarse difference measurements to within ±5 microseconds. Measurements of difference in phase between the 100-kc carriers of each pulse supply the fine range difference information. Fine information is measured to a presicison of better than ±0 1 microsecond. Measurements of envelope arrival time are made with respect to a point 30 microseconds after the start of the pulse. Minimum skywave delay at 100 kc is greater than 30 microseconds; therefore, LORAN-C readings are very accurate, since skywave contamination is eliminated.

The system outputs consist of a coarse and a fine readout for each pair of stations. The coarse and fine data are normally displayed in line to facilitate interpretation of the readings. The operator must determine the final readings by combining the initial readings in such a manner as to assume the difference between coarse and fine to be less than 5 microseconds.

It is possible to combine the coarse and fine information into one unambiguous reading and eliminate the need for operator judgement in making readings. A method of combining the information is shown in Appendix H.

The present receivers do not have one unambiguous output, but have two outputs that must be combined by the operator. The coordinate converter is therefore required to combine automatically the two readings into one unambiguous reading. The basic assumption is that the difference between the two readings is less than 5 microseconds. In order to combine the fine (cycle) and coarse (envelope) data in the coordinate converter, the coarse data is assumed to agree with the fine data so that their difference is always less than 5 microseconds. An analog-to-digital (a/d) converter is connected to the coarse and the fine output shafts. The coarse converter will input to 0.1 microsecond out of a maximum time difference of ±12,500 microseconds (18 bits). The fine converter will input to 0.05 microsecond out of a maximum time difference of 10 microseconds (8 bits). The coarse and fine data are compared and the fine data replaces the coarse data in such a manner that it does not change the original coarse data by more than 5 microseconds. This computation is performed every time the C/C samples the outputs of the a/d converters on the fine and coarse shafts. Under extremely noisy conditions the two readings will have a difference greater than 5 microseconds, causing a certain percentage of the total reading to have a large error. These erroneous readings will be distributed randomly within ±10 microseconds of the correct reading and will appear as high noise levels at the computer outputs. This is especially true if operation is considered at unfavorable portions of the field where this lane error is magnified. The magnitude of the rms value of the noise is a function of how often the lane errors appear. In order to keep the noise variations on the tow LORAN-C outputs to a minimum, the LORAN-C measuring loop bandwidths must be kept to a minimum

3.4.2 Adapting Oscillator Feedback Technique to LORAN C

The LORAN-C system measures difference in phase between the received 100-kc carriers of each of the slave signals and the master signal. A reference oscillator is phase locked to the received master signal. By using the phase locked oscillator as a reference for measuring the phase of each received slave signal, the phase difference between each slave and the master is measured directly.

In order to use the oscillator feedback technique, it is necessary to control the oscillator with a digitally computed error signal computer output. At the same time, a measurement of master range (R_m) is necessary. A measurement of range to master requires the addition of an electromechanical loop to replace the electronic loop used previously to phase lock the oscillator. Because of existing space limitations, modification of existing equipment to incorporate the electromechanical loop is not a simple task.

Figure 14 shows a block diagram of the basic LORAN C modified for oscillator feedback. The addition of range-rate and information are shown by dotted lines.

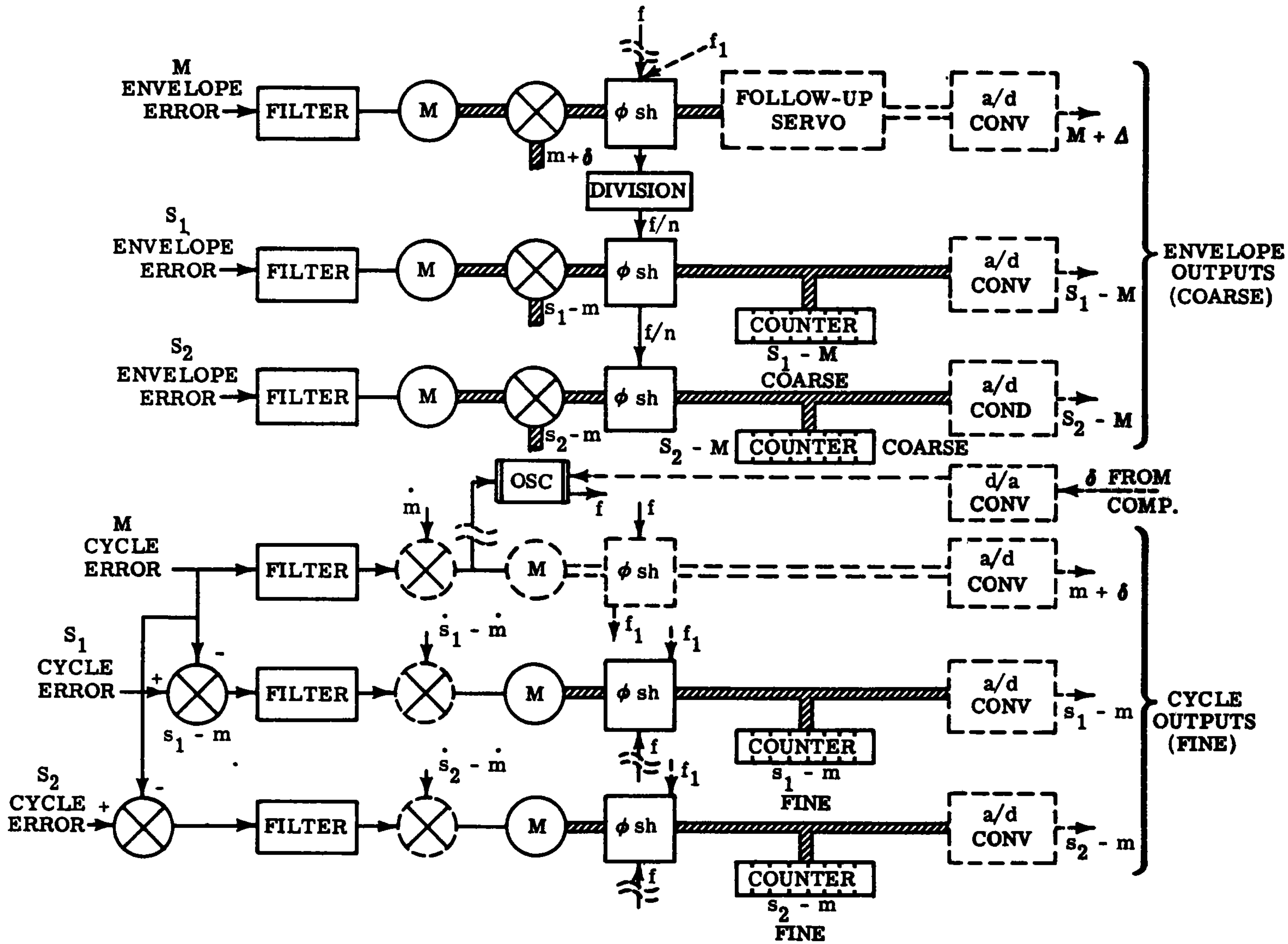


Figure 14. LORAN C Mechanization

The coarse system contains the basic components which are required with the use of the oscillator feedback technique. The master envelope (coarse) servo output is not normally available as an output and a follow-up servo would be required in order to make it available for use with the coordinate converter. The three outputs, S_1-M, S_2-M, and M ± δ will have a/d converters connected to them and the computation of $S_1 + \Delta$ and $S_2 + \Delta$ are made by the converter. The computer calculates a correction signal which drives Δ to zero providing in the steady state the three correct derived ranges (S_1, S_2, M). After is reduced to a defined minimum, the computer calculates δ for the fine system.

The fine system would contain its normal outputs (S_1 - M) and (S_2 - M) plus the additional output of M + δ . Analog-to-digital converters will be connected to the three outputs and the computation of ($S_1 + \delta$) and ($S_2 + \delta$) will be made by the converter. Using the three signals $S_1 + \delta$, $S_2 + \delta$, and M + δ , δ is computed and is fed as a correction signal to the 100 kc reference oscillator, thereby correcting the derived ranges.

3.4.3 Direct Hyperbolic Conversion Adapted to LORAN C

In order to perform the coordinate conversion with the direct hyperbolic conversion technique, it is not necessary to modify the basic measuring loops in the receivers. All that is required is to connect a/d converters to the readout shafts. The C/C can then combine the course and fine information into unambiguous range differences and compute the final coordinates.

In summary, LORAN C can be modified for use with a coordinate converter of the feedback oscillator type or the direct hyperbolic conversion type. The modifications required in order to use the oscillator feedback are relatively difficult* compared to the simple addition of four a/d converters as required by the direct hyperbolic conversion method.

3.4.4 Description of DECTRA

The DECTRA System, Figure 15 is a lane counting system in which the variations in reading due to propagational characteristics are not considered by its advocates to be too derogatory for transoceanic flights.

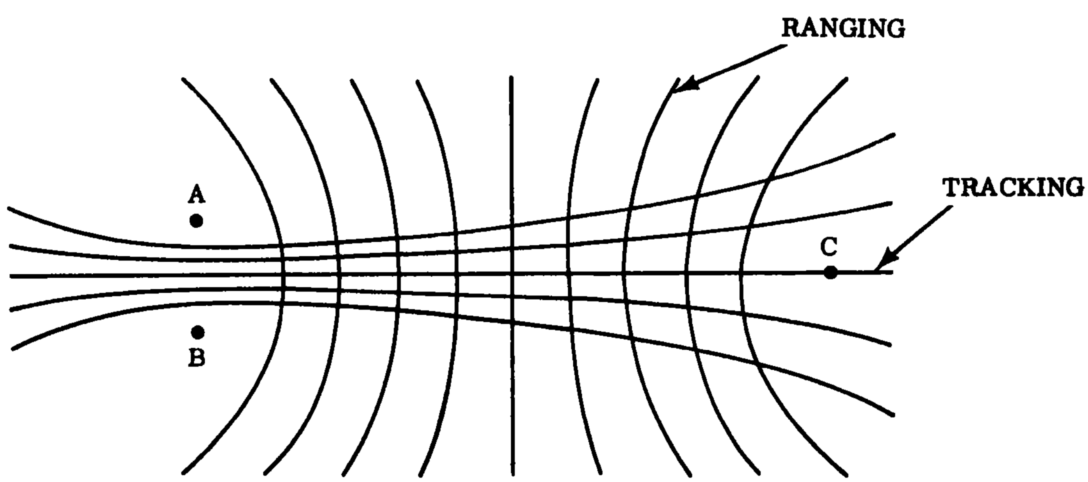


Figure 15. DECTRA System Plot

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* The feedback oscillator type requires the following modifications: One follow-up servo, one electromechanical loop for obtaining master cycle reading, wiring changes to AFC oscillator from C/C, a/d converters for each shaft (6), and a d/a converter to use in the oscillator feedback loop.

Stations A and B (Figure 15) are used for track information. The difference in their readings due to skywave contamination is not expected to exceed 1/4 cycle of phase difference (Reference 6) This 1/4 cycle represents about 1/4 mile on the baseline between stations A and B, but represents a much larger number (25 miles) at a range of 1000 miles, with a station A to B baseline length of 100 miles

Stations A and C are used for range information. Several lanes of phase will probably be lost because of the distance separating stations A and C. Ten lanes is a reasonable number to lose and represents a range error of approximately 10 miles.

Each output has two d-c voltages that when combined in quadrature produce an angle whose tangent is the ratio of the two voltages

3.4 5 DECTRA with Oscillator Feedback

To use oscillator feedback with the DECTRA System, consideration must be given to the DECTRA tracking loop and ranging loop.

It is necessary to add an oscillator (F_{IT}) to the tracking loop to utilize oscillator feedback. This oscillator will serve as the reference for phase measurements of signals on F_1 from stations A and B. The oscillator, originally phase locked to the F_1 transmission from station A, is kept in its original form so that it can perform its function in the measurements made in the ranging loop A phase discriminator circuit will be added so that the two separate outputs are available from the tracking loop. The two outputs ($A + \delta$) and ($B + \delta$), replace the single output of A-B which was previously used. The term δ is the error in each reading due to the oscillator phase error The added oscillator (F_{IT}) will be controlled from the coordinate converter (C/C) and the error δ will be reduced to approximately zero The end outputs with the oscillator feedback loop closed are range A and range B Figure 16 shows a functional block diagram, with the original system in solid lines and the necessary modifications and additions in dotted lines.

The ranging loop measures the difference in phase between the F_1 transmissions from station A and the F_2 transmission from station C. This measurement is made in such a way as to make the information equivalent to a phase difference of the frequency F_2. In order to use the oscillator feedback method, range $C + \delta$ must be derived. Since the outputs of the tracking loop are ($A + \delta$) and ($B + \delta$) and the output of the ranging loop is $A - C$, the $C + \delta$ information is present and can be obtained by subtracting ($A-C$) from ($A + \delta$). This subtraction will be made in the C/C after the phase discriminator outputs have been transformed into a more useful form of computing The ranging loop information will become a part of the oscillator feedback loop. The internal ranging loop will be unmodified and unaffected and will continue to operate in its normal fashion.

3.4.6 Direct Hyperbolic Conversion Adapted to DECTRA

With the direct hyperbolic conversion method of transforming coordinates it is unnecessary to make extensive modifications to the equipment. The only requirement is that the measured difference in range data be available in some form that can be used readily for computing. In this case, each phase discriminator has two outputs that are quadrature and are used to compute the phase difference angle. The only modification required of DECTRA is to bring out four wires from the equipment (two from each phase discriminator). Figure 17 shows a method of deriving the phase information from the phase discriminator outputs

To summarize, DECTRA can be modified so that a C/C operating with the principle of either oscillator feedback or direct computation can be adapted successfully to the receivers. The modifications required to use the oscillator feedback principle are extensive (addition of one oscillator, one phase discriminator circuit, and one a/d converter, and the bringing of four leads out of the equipment), as compared to those required for use with the direct hyperbolic conversion techniques (wiring change only). It will be necessary for the C/C to keep track of the lanes regardless of which principle is used.

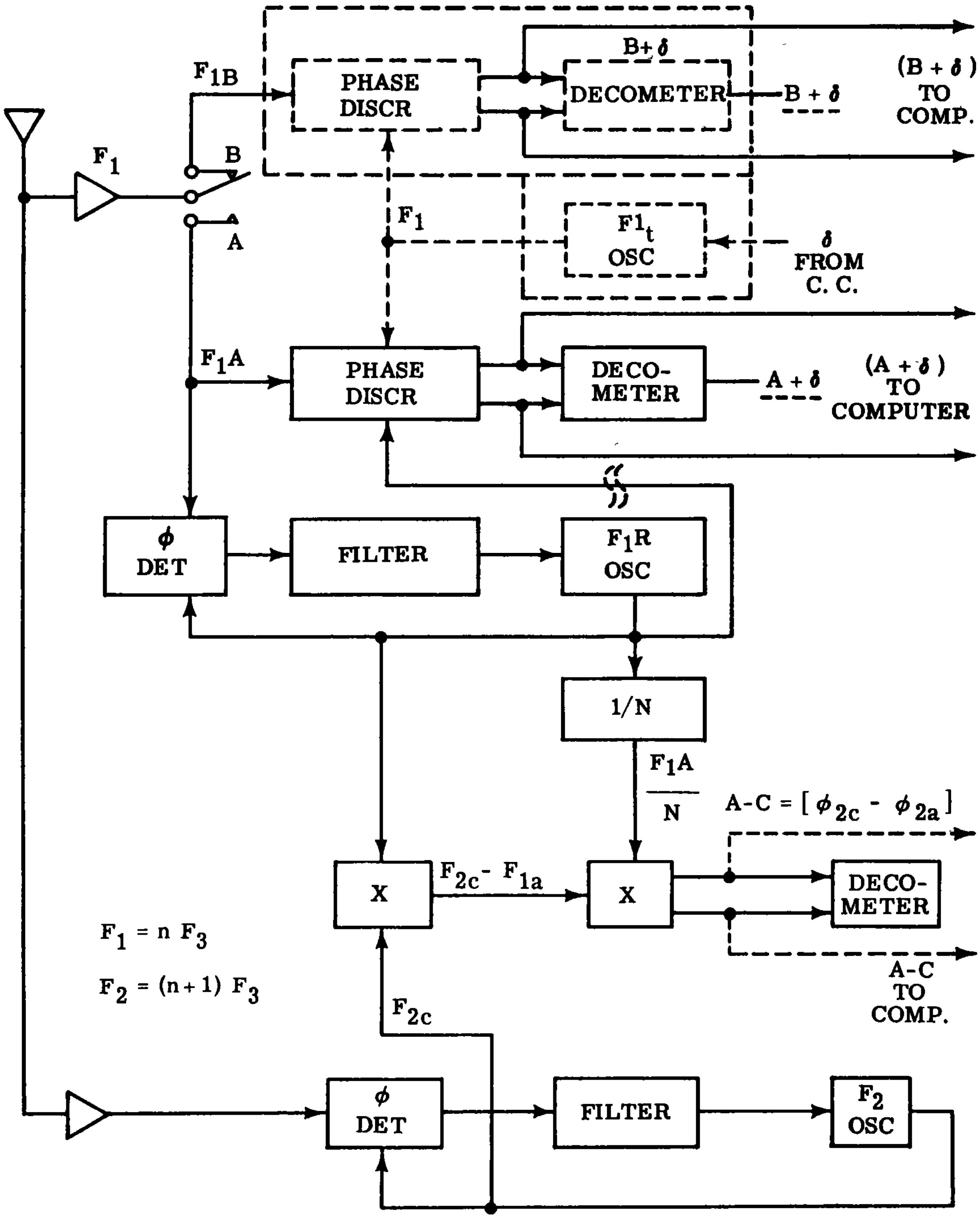


Figure 16. Modification of DECTRA to use Oscillator Feedback

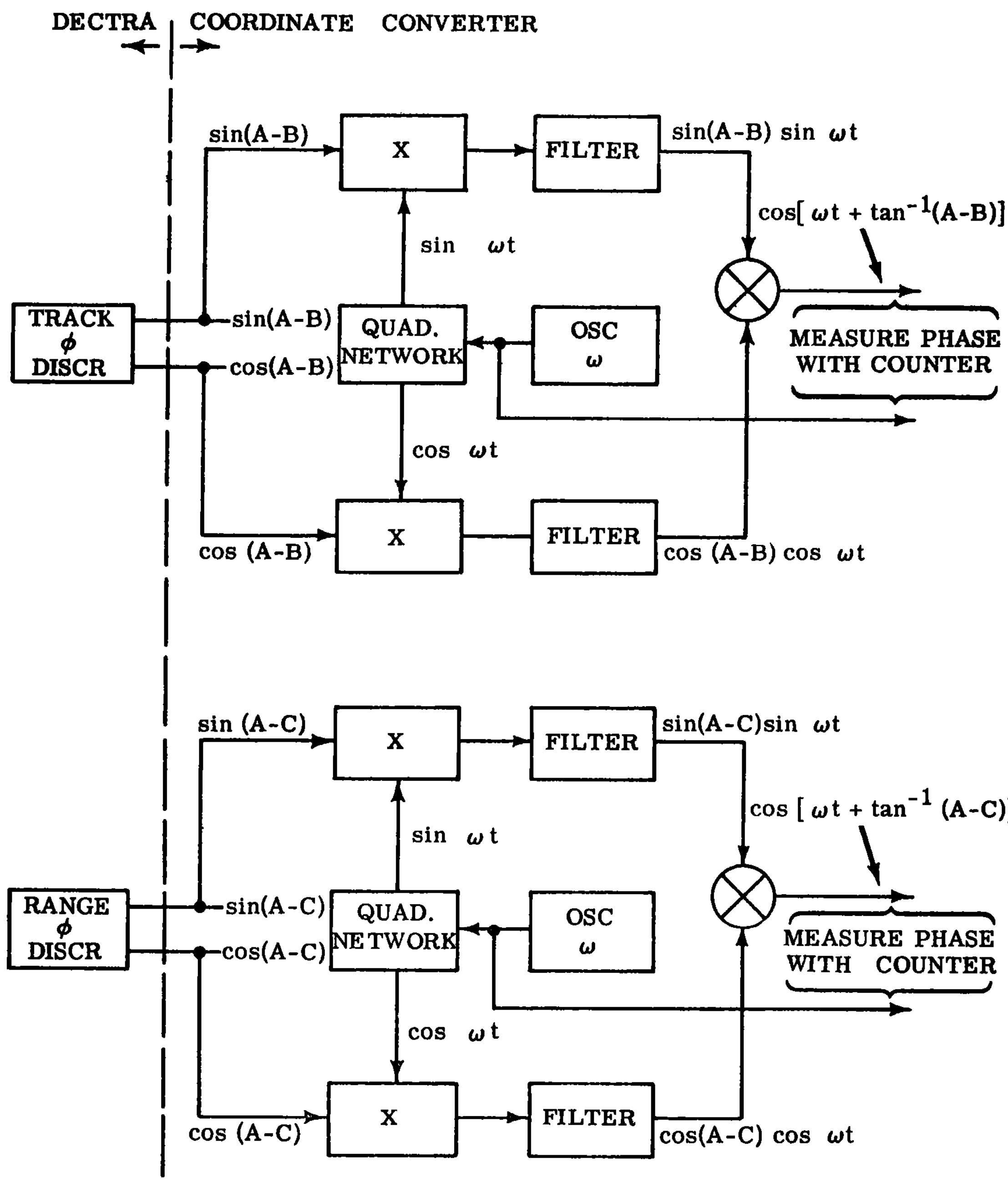


Figure 17. Method of Measuring Phase Difference from DECTRA Phase Discriminator Outputs

3.4.7 Description of RADUX and OMEGA

RADUX and OMEGA are similarly mechanized hyperbolic navigation systems. A fix is derived by the intersection of two hyperbolic lines of position. Each system essentially measures range plus reference oscillator error to each of the three stations. The reference oscillator is phase locked to any one of the three stations so that it can be used with the strongest received signal. Synchronization of the equipment is necessary before readings can be taken, because the transmissions are time shared on the same frequency. The reference oscillator can be locked to an error signal from the C/C, instead of one of the input signals, by adding a varicap to the oscillator.

RADUX uses 200-cps, phase-modulated, 40-kc carrier. Measurements are made directly on the 200-cps phase modulation. The equipment has a 0.003-cps information bandwidth and is capable of tracking a 40-knot rate down a baseline. The standard deviation is approximately 10 microseconds for a one-half voltage signal-to-noise ratio in a 600-cps noise bandwidth at the receiver input. The amgibuities of RADUX occur as a function of the 200-cps modulation at approximately every 400 miles on the baselines.

OMEGA is a low frequency system (10 kc to 15 kc) in which the phase of the carrier frequency is measured directly. At 10 kc the ambiguities are about every 8 miles on the baseline. A coarse system to resolve ambiguities is deemed unnecessary because dead-reckoning techniques are considered adequate to resolve lane identification. The bandwidth of the present monitor system is approximately 0.001 cps and is intended for a receiver that is relatively stationary.

Because three ranges are measured in each of these systems (and then combined for range differences) range-rate information can be readily inserted into each measuring loop at the proper place. The equipment is therefore capable of tracking in a 600-knot airplane with an information bandwidth of approximately 0.001 cps, without any significant information lag.

It is necessary to count lanes with either system; however, if a lane ambiguity arises, the proper lane can be identified with dead-reckoning techniques or with integrated range-rate information.

The RADUX and OMEGA systems are easily modified to use a coordinate converter and are readily usable with range-rate information.

3.4.8 Adapting Oscillator Feedback to RADUX and OMEGA

To use oscillator feedback with the RADUX and OMEGA systems, it is necessary to change the system outputs from range differences to ranges. This is accomplished by locking one side of each differential used in the equipment (Figure 18) and adding a C/C controlled varicap to the reference oscillator.

3.4.9 Direct Hyperbolic Conversion Adapted to RADUX and OMEGA

Modification of the RADUX and OMEGA systems to use the direct hyperbolic conversion method requires the addition of an a/d converter to any two of the range difference outputs. The two outputs can then be used to compute directly (without feedback) the coordinates in the compatible system.

3.4.10 Summary of Coordinate Converter Tie-In with Hyperbolic Systems

Two alternate methods have been shown for accomplishing a tie-in between the coordinate converter and various hyperbolic receivers. It has been concluded that the direct hyperbolic conversion is the preferred method. This method is especially appealing if the C/C is to operate with existing receivers.

The LORAN-C receivers must be modified by the addition of an output shaft encoder to be compatible with C/C. However, when these receivers are designed for airborne use, it may be that digital outputs will be used. The use of electronic techniques in contrast to

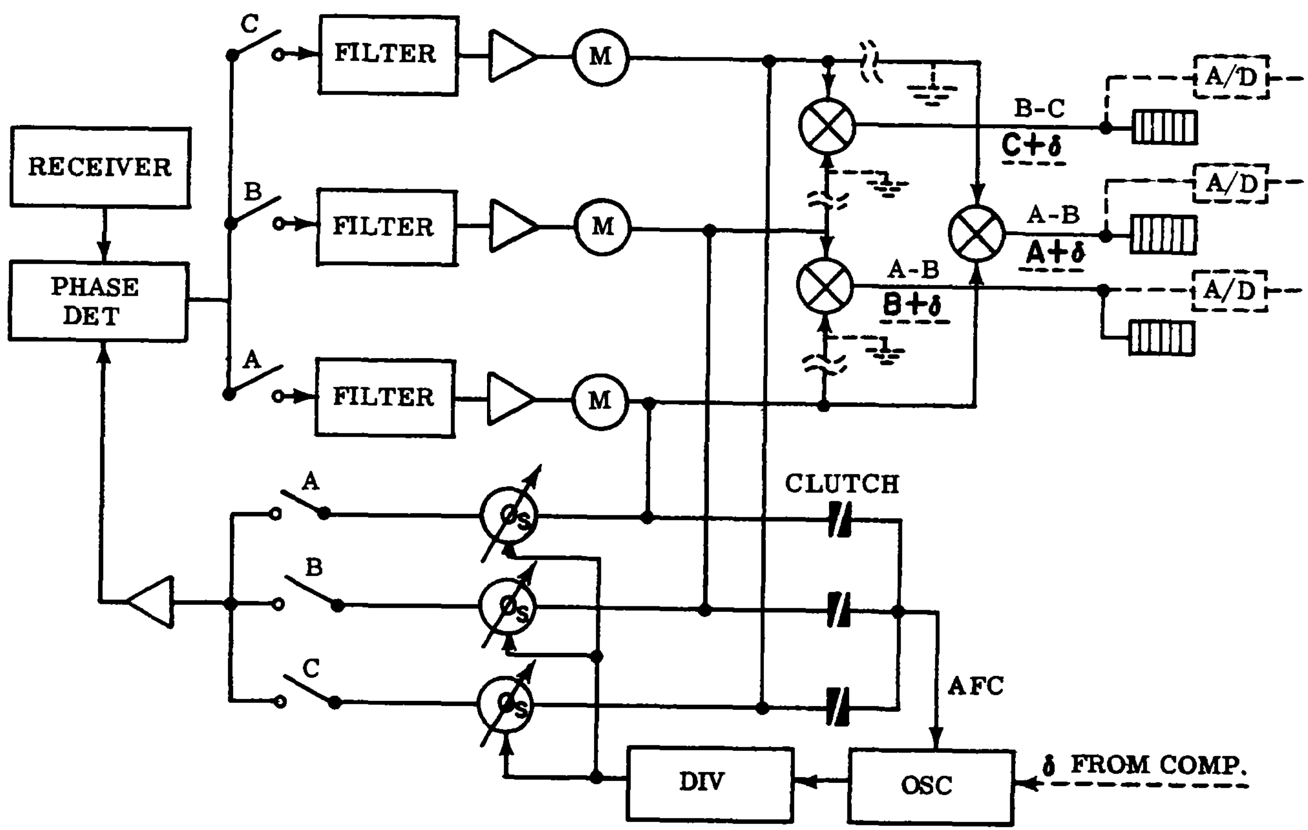


Figure 18. RADUX-OMEGA, Simplified Functional Diagram

the electromechanical approach now used is a distinct possibility. In this case a receiver and C/C designed to operate as a single piece of equipment could considerably simplify the total amount of equipment required. Such a combined unit would be economical since the capability of the two equipments will be combined and the redundant equipment characteristics eliminated.

4. INTEGRATED NAVIGATION SYSTEM

4.1 LIMITATIONS OF HYPERBOLIC SYSTEMS

All of the applicable hyperbolic systems discussed possess varying degrees of limitations in performing transoceanic flights. The precise limitation and efficiency of these systems would only be conjectural within the framework of this study. However one problem -- that of output noise -- is considered to be a fundamental limitation. Output noise is the principal factor limiting the range of the system and is, therefore, of concern in any system that attempts to cover transoceanic applications. This is true, not only because of the desire to have a minimum number of stations, but also because of the unavailability of land sites to provide adequate coverage on many of the possible transoceanic air routes. The desirability of having maximum possible range from any one group of transmitting sites is apparent.

A crude estimate can be made of the ability of the LORAN-C system to operate at long range by the following considerations:

1. For a 100-kw peak power, eight-pulse LORAN C, with a pulse group rate of 10 to 30 groups per second, the average power output will be about 100 watts if the pulse width is 50 to 60 microseconds. At one mile this yields an average field strength of about 60-millivolts per meter. For a 1000-mile path over sea water, the received average signal strength will be about 6-microvolts per meter.

2. In the equatorial Atlantic, it is indicated by the Coast Guard report that the atmospheric noise level may be 55 db above 1 microvolt per meter in a 1000 cps bandwidth during certain times of the year and day. This corresponds well with the predictions of the International Radio Consultative Committee (CCIR). This yields a noise level of about 68 db above 1 microvolt per meter in the 20 kc input bandwidth of a LORAN-C receiver. The input noise-to-average signal level is then approximately 53 db.

3. Receiver performance specifications require an output rms error of about 0.15 microsecond in a carrier-cycle phase-difference measurement. This represents a phase error standard deviation of about 1/10 radian, which requires an output signal-to-noise ratio of 20 db. The bandwidth of the output must be narrow enough to reduce the -53 db input signal-to-noise ratio to a +20 db output signal-to-noise ratio, or a bandwidth improvement ratio of 73 db. The output bandwidth must be about 1.0×10^{-3} cps. Allowing for some pessimism in the above assumptions, the output bandwidth must be of the order of 0.001 cps.

4. Any low frequency system is subject to the same limitations caused by atmospheric noise. The performance at a given range is measured almost entirely by the ratio of average received power to the average power per cycle of noise. For equal ratios, one system performs better than another against atmospheric noise, only insofar as one system has a narrower output bandwidth.

5. The ability of systems to operate in high performance aircraft with this narrow bandwidth is questionable. The loops would normally be of the two-servos variety in which there would be no error or dynamic lag when tracking a constant velocity. This removes the large source of error, but dynamic errors still remain due to the most modest of maneuvers.

6. The receiver bandwidth is thus a compromise between the reduction of noise, by decreasing the bandwidth, and, on the other hand, increasing the bandwidth to provide adequate dynamics-tracking capabilities.

For civilian aircraft, large and violent maneuvers will be the exception rather than the rule; therefore, it seems unnecessary to design the equipment to conditions that are rarely experienced. However, the equipment must possess sufficient dynamic capabilities to cope with modest maneuvers, turbulent conditions, and any conditions imposed by geometrical considerations.

For example, consider Figure 19, where the vector v represents an aircraft flying a constant-velocity, non-maneuvering path with respect to the earth. Distance r is the range of the aircraft from a radio transmitting site. It is apparent from inspection that the quantity r will change with respect to time and with respect to the radio coordinate system. There will be apparent dynamic conditions imposed on any loop that is to track the quantity v. It is thus seen that, even though the aircraft is flying straight and level, the tracking loops must possess a finite bandwidth.

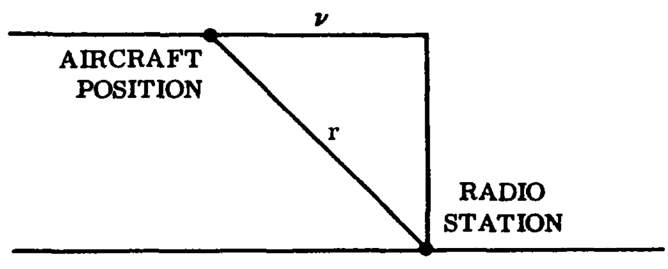


Figure 19. Dynamics Caused by Geometrical Considerations

Another facet to the problem is the adverse effects that short-transient errors could have on the system under poor signal-to-noise conditions. If the equipment is in synchronism, it can track under these poor conditions; however, if a transient error should cause the equipment to lose synchronism, it is a far more difficult problem to require synchronization.

4.2 USE OF DOPPLER INFORMATION

Two methods are available for relieving the dynamic capability. Both use an additional source of positional rate information. The requirement for this additional source of information is that it does not need to be precise in its positional determination but rather that it contain a relatively small amount of noise. In the ensuing discussions it will be assumed that the source is a doppler radar, because current plans for commercial aircraft indicate the use of such equipment. However, the techniques are valid for both inertial and dead reckoning devices with varying degrees of performance. The principal factor involved with these methods is to combine two sources of information in a distortionless manner.

Consider a general case, referring to Figure 20.

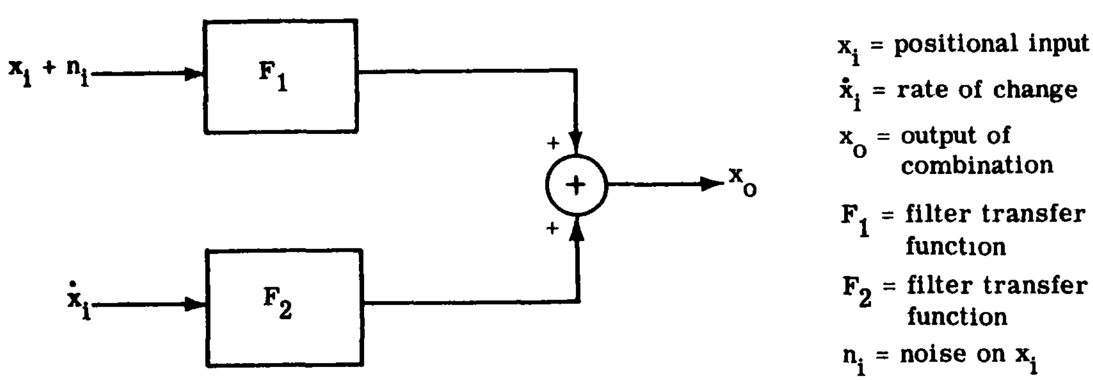


Figure 20. Combining Data in General Case

If x_i is a source of accurate positional data but contains a noise input, n_i, and it is desired to remove a portion of the noise by a simple low pass filter, F_1 will be equal to $1/\tau_1 S + 1$, where τ_1 is the time constant of the filter. This filter has attenuated a portion of the noise content but the information x_i is now also delayed in time. If F_2 is made equal to $\tau_1/\tau_1 S + 1$ the output will become distortionless and contain no position lag, because:

$$x_o = x_i F_1 + S x_i F_2$$

$$= S_1(1/\tau_1 S + 1) + SX_i(\tau 1/\tau_1 S + 1) \qquad (84)$$

$$= x_i \frac{\tau_1 S + 1}{\tau_1 S + 1}$$

$$x_o = x_i$$

Physically this can better be seen by referring to Figure 21.

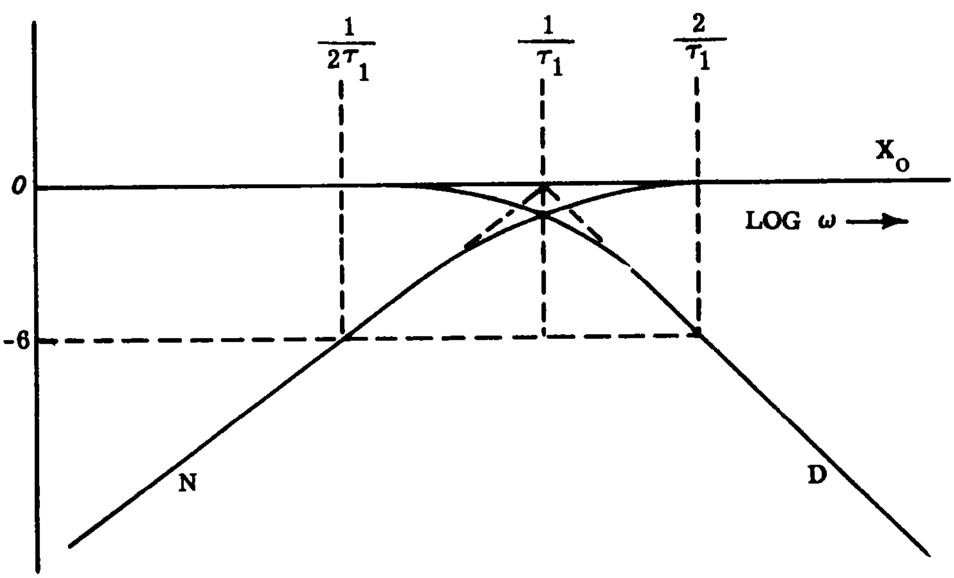


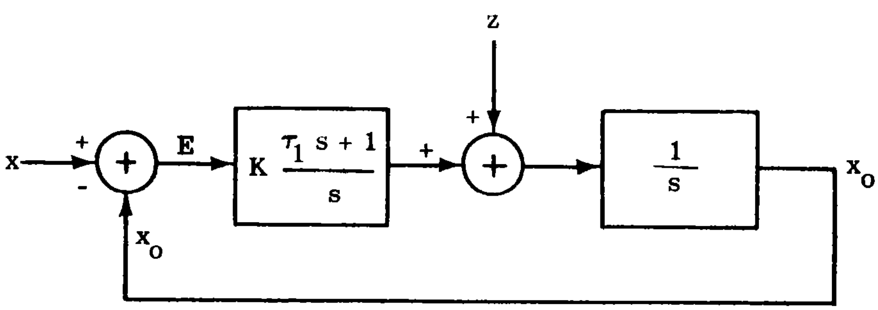
Figure 21. Bode Plot of Data Combination

This is simply a case where the higher frequency information on x_i is attenuated while the low frequency information on x_i is attenuated. If the two filtered outputs are added, the output becomes wide band. This is precisely what is desired, because if x_i is treated as the radio information and $\dot{x}_i$ as doppler information, each contributes where the data is most useful. The radio is an accurate source of low frequency information but contains noise, whereas the d-c component of the doppler is attenuated and need only supply the higher frequency component of information.

This technique can be used in two different ways. Each will be discussed in the following section.

4.2.1 Input Data Combination

The doppler range rate (z) and radio low frequency (x) position information inputs can be combined in the navigation receiver in an optimum manner as shown in Figure 22. This combination offers several advantages.



$$E = \frac{\left[z/s - x_1\right] s^2}{s^2 + K\tau_1 s + K}$$

if $z = s\, x_i$

$E \rightarrow 0$ or the filter is distortionless

Figure 22. Data Combination Loop with Two Integrators

First, it is desirable to narrow the measuring bandwidths of the radio system so that the perturbations on the output data do not exceed a maximum tolerable for ambiguity resolution. The output data combination (Section 4.2.2) will not aid in increasing the useful range of the navigation receiver, because it does not reduce the output noise of the receiver for ambiguity resolution purposes and does not aid in keeping the receiver synchronized under poor signal-to-noise ratio conditions. It follows that the usable range of the radio system is limited by the permissible amount of narrow banding. The combination of range-rate information from the doppler with the radio information in the measuring loops relieves the radio system of the necessity of following aircraft maneuvers. Without this requirement, the measuring loops of the radio system can be made as narrow as possible and limited only by characteristics of the reference oscillator.

Second, since the loops can be made much narrower, the reliability with which synchronization can be held is increased and the range at which the system can be utilized is likewise increased.

The importance of maintaining synchronization is paramount because it is doubtful whether manual assistance in obtaining synchronization will be possible due to the poor signal-to-noise ratios expected. Because of the increase in reliability of synchronization and the extending of usable operating range, the combination of data at the inputs is extremely attractive.

4.2.2 Output Data Combination

A combination of radio and doppler output data will provide for improvement in the system by virtue of the effective filtering that can be provided. As discussed in Section 4.2 the effect of the time constant can be removed. A more optimum type filter than the simple low-pass filter previously discussed is of the form shown in Figure 23.

The radio information filtering characteristics of this loop is a low-pass filter since the closed loop response determines the filtering characteristics.

The principal advantage of this combination over the simple low-pass filter discussed previously is that it provides an automatic trim of the rate information. If the quantity z is made equal to hsx the deviation of K from unity is equivalent to a calibration error of the doppler radar. If z = hsx is then introduced into the equation and the final limit theorem applied, the error in the steady state becomes zero. Physically this can be seen to be true, since the input z has a constant error. A constant output will be obtained at the first integration without any steady error E.

Therefore, by a relatively simple combination of data the steady-state calibration error can be effectively removed. The limitation on removing these sources of error is determined by the manner in which calibration errors change with time. This follows since the data combination loop will be a long time constant loop to provide filtering of the radio information. For civilian aircraft flying transoceanic missions, it is reasonable to assume the error will change slowly with time. For example, one major source of error is in the alignment of the doppler antenna to the aircraft. The error is then a function of the direction the aircraft is flying.

This loop also provides one other feature that is used to remove computer transport delay. The entire loop is a sampled data loop in which the output position of the final integration will be indicated by the shaft position encoder. The shaft position is compared to the new value of position (X_i) and the subtraction performed digitally; it is then converted to an analog value as shown in Figure 23.

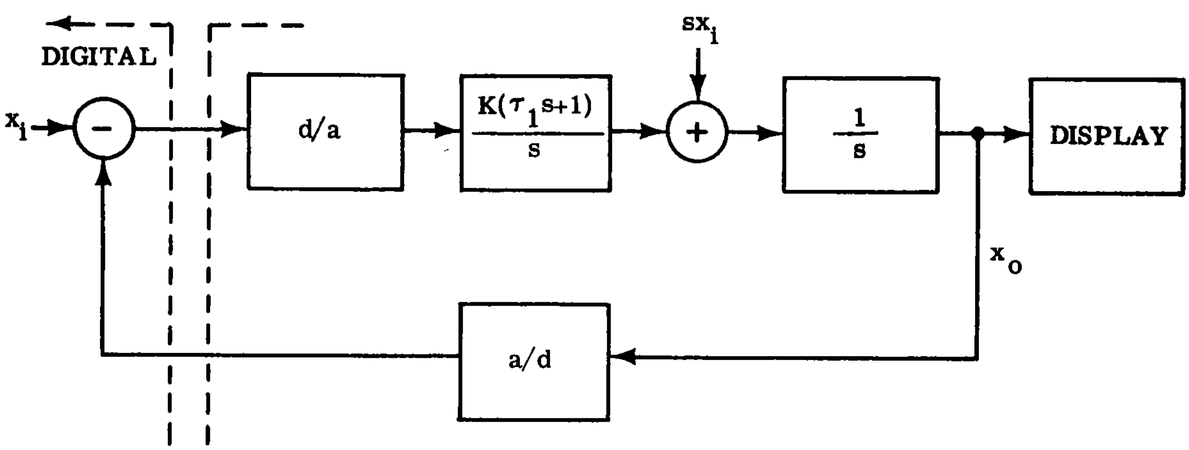


Figure 23. Output Data Combination Loop

Simultaneous with the introduction of radio information into the computer, the shaft position Xo will be sampled. The computer error signal will be delayed by the computation time, but the displayed information will be up to date, since it is integrated doppler information. The effects of transport delay will appear only within the loop.

The filtering action of this loop is predicated on transferring the radio noise of the output of the coordinate converter, where it is combined with the doppler information in such a manner that the superior characteristics of each system are brought out. To achieve this combination successfully, it is necessary to take a sufficient number of samples of the information that passes this noise, without amplifying the lower frequency portion of the noise spectrum. As previously discussed, an adequate sampling rate was selected that would achieve this. The sampling rate can actually be decreased by a factor of two with this data combination, since the previous limiting factor was the transport delay, which can be overcome with the data combination.

Selection of the parameters of the loop is a compromise between the radio noise filtering characteristics and the errors due to change of doppler-calibration errors and doppler noise. Note that on the output of the C/C, the noise has been magnified due to geometrical considerations of the hyperbolic field. Sufficient information is not known on the magnitude of the compromising factors to establish these parameters. However, from previous experience it is estimated that the time constant will be in the order of magnitude of several minutes.

This method of approach is a relatively simple means of exploiting the advantages of a combination of the two navigation systems. In principal, it is one of correcting a navigation system that possesses excellent dynamic capability but limited accuracy. It is also one that requires a minimum of interaction between the two systems that can be made to have excellent fail-safe characteristics. It will afford significant advantages, but is limited to the amount of radio noise that can be tolerated. Whe the radio noise becomes equal to $\pm 1/2$ lane it is doubtful whether sufficient smoothing is possible to average out the errors adequately, without sufficiently increasing the accuracy requirements of the doppler-heading reference. Utilizing this data combination as such will not produce the increase in allowable range that utilizing range rate may produce.

The more useful and efficient a navigation system becomes the greater are the implications that its loss can have. Accordingly, this loop will have fail-safe characteristics of the following nature.

1. In the event that a signal is received from the radio equipment that is not within specification limits, a switch on the output of the first integrator will remove any further corrections from the radio information and the doppler will be simply integrated to obtain present-position. However, there is a significant difference in utilizing integrating doppler independently, since all the errors that would have accumulated prior to loss of the radio will have been removed. In other words, the integrator has the proper initial condition at the time of loss of the radio information and will begin to build up as of this time.

   It is to be noted that the interruption could take place at the point ϵ (Figure 23) where the correction for doppler calibration would remain in the loop. The advantage to be gained by an interruption at this point is outweighed by the consideration that, over long periods, the required calibration correction may change by a large amount; in an extreme case, where the aircraft makes a 180 degree turn, the correction may even become opposite in sign.

2. The coordinate converter will have sample self-test to verify its own operation. An ALERT light will indicate a malfunction and the data combination loop will open.

3. The C/C can be programmed to measure the difference between previous samples and the latest value of computed error signal, ϵ . If significantly different, it will reject the reading. This precludes the possibility of injecting any large sources of errors into the loop.

4. The equipment will be mechanized by analog techniques so that any malfunction of the central elements of the computer will not cause loss of all present-position information. With analog methods the present-position will be stored in shaft position form.

4.2.2.1 Mechanization

A simplified functional block diagram of the output data combination is shown in Figure 24-a. A block diagram showing the basic mechanization of the output data combination is shown in Figure 24-b. The loop is closed inside the computer and only the mechanization external to the computer is shown.

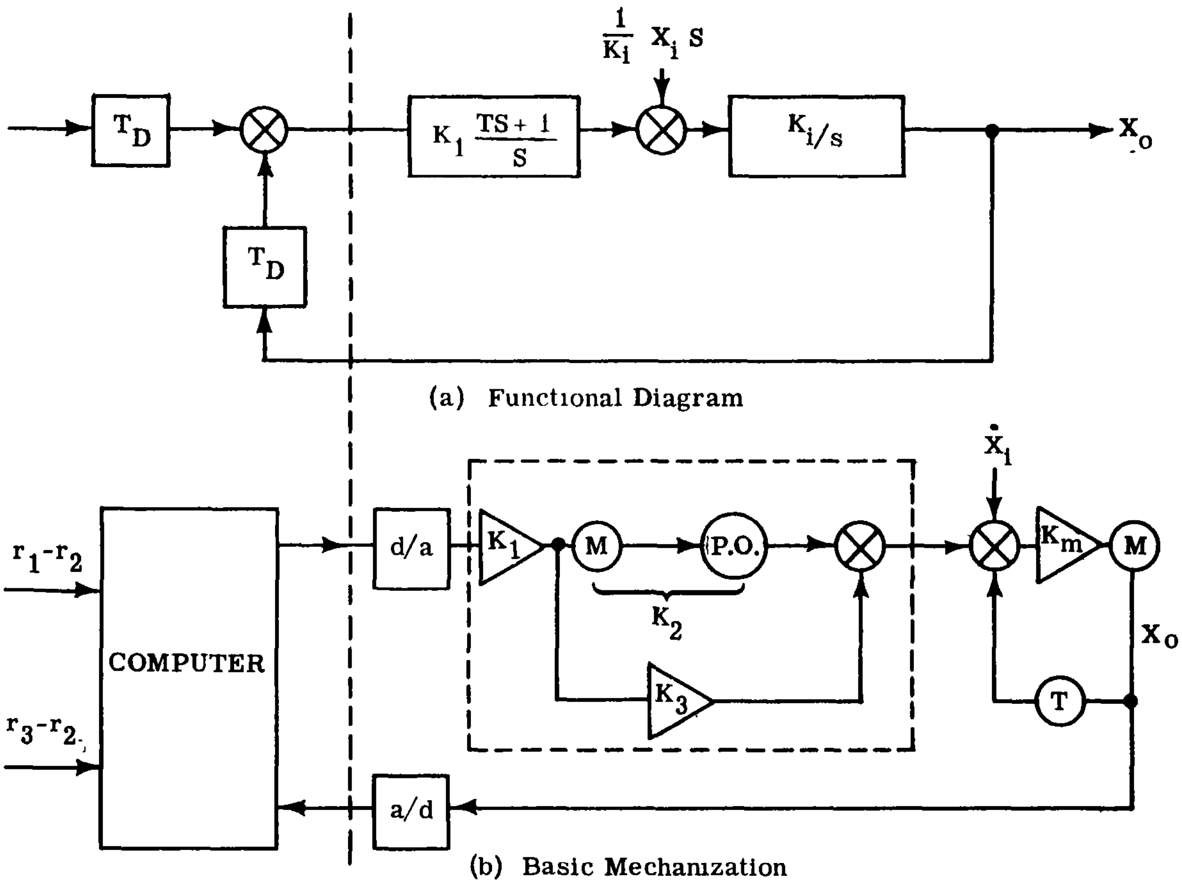


Figure 24. Output Data Combination

The input to the data combination loop is in digital form as it comes from the computer, and must be converted to analog information by a d/a converter. Conversely, the loop output is a shaft position which must be converted to digital form in order to be used by the computer.

Filter $T_s + 1/s$ is mechanized, as shown in the dotted line box of Figure 24-b. The filter has a transfer function of $e_o/e_i(s) = k_1 k_2 \frac{1 + k_3/k_2 s}{s}$. This mechanization requires one amplifier (k_3 will be an attenuator) and a mechanical filter, which is made up of a motor and an a-c pickoff. Estimated size and weight of the mechanical filter are the same as a size eleven servo motor. Amplifier k_1 is a four-transistor design. The output motor, with its tachometer and a four-transistor amplifier, completes the mechanization. The counters connected to the output shaft will read out to the closest mile. Maximum reading on distance-to-go will be ±999 nautical miles and cross-track-error will be ±99 nautical miles. The a/d converter used to feed back the output to the computer will have a resolution of 0.1 nautical mile in ±999, or 11 bits, on distance-to-go and 0.1 nautical mile in ±99 or 8 bits on cross-track. The d/a converter at the loop input has an input resolution of 1 nautical mile in ±50, or 7 bits.

It should be noted at this point that the mechanization of the output would be very similar to the above method, even though range rate were not combined with the radio output.

The major difference is that the mechanical filter would probably not be used and the tachometer would not be required with the motor.

Major parts used in the mechanization are as follows:

1 - a/d converter (11 bits for distance-to-go and 8 bits for cross-track).

1 - d/a converter (7 bits).

2 - 4-transistor amplifiers.

1 - motor-tachometer with gearhead.

1 - mechanical filter.

1 - counter indicator.

4.2.3 Range Rate Computation

The values of range-rate can be computed using the following equations:

From the law of cosines

$$\cos \frac{r_m}{R_m} = \sin\phi \sin \phi_m + \cos \phi \cos \phi_m \cos (\lambda - \lambda_m); \; m = 1, 2, 3, \qquad (86)$$

Differentiating and collecting terms

$$-\sin \frac{r_1}{R_M} \left(\frac{\dot{r}_1}{R_M}\right) = \cos\phi \sin \phi_1 \phi - \cos\phi \cos \phi_1 \sin (\lambda - \lambda_1) \lambda$$

$$- \sin\phi \cos \phi_1 \cos (\lambda - \lambda_1) \phi \qquad (87)$$

$$\dot{r}_m = \frac{\cos \phi_m \cos (\lambda - \lambda_m) \sin \phi_m \cos \phi}{\sin \frac{r_m}{R_M}} V_N$$

$$+ \frac{\cos \phi_m \sin (\lambda - \lambda_m)}{\sin \frac{r_m}{R_M}} V_E; \; m = 1, 2, 3. \qquad (88)$$

where:

λ_n, ϕ_n = the station latitude and longitude coordinates.

λ, ϕ = the latitude and longitude coordinates of present position.

r_m = range of present-position to station n.

R_M = mean radius of the earth.

V_E = velocity in respect to east.

V_N = velocity in respect to north.

From the equations for range-rate, it is obvious that to obtain a solution the following inputs are required:

1. Coordinates of each transmitter site (6).
2. Aircraft heading.
3. Forward and lateral velocities.
4. Aircraft present-position coordinates.

An analog computer can be used to solve for the range-rates, because the accuracy and dynamics requirements are readily attainable with analog techniques. However, the straight analog mechanization would have to be fairly complex to handle the large number of variables required to solve the problem.

A combination analog-digital method that has been successfully used in a previous problem provides a more efficient way of solving the problem, because the C/C already has stored the coordinates of the transmitting stations and the aircraft's present-position. Assuming that the C/C has adequate memory available, it can solve this problem in an efficient manner. The additional equipment required to introduce the inputs into the computer are a time-shared a/d converter and heading follow-up servo. Three d/a converters would be required to supply the range-rate information to each loop.

The combination analog-digital mechanization is more economical than the all-analog mechanization because it utilizes existing equipment in the C/C and takes advantage of the ability of the C/C to store some of the input data for other purposes.

It is again emphasized that the discussion of range-rate is not a complete treatment of the problem; it is only included to show the problem areas and generalized methods of solution.

4.3 GYRO ERROR CORRECTION TECHNIQUE

Once the basic hyperbolic data is converted into a form compatible with the outputs of other navigational equipment, it becomes a source of accurate information that can be usefully applied to many phases of navigation problems. As an example of this, a method for the correction of gyroscopic drift will be discussed.

For purposes of simplicity consider a planar representation of an aircraft flight path (ξ_A) (Figure 25). The coordinates (x_1, y_1) and (x_2, y_2) are two separate positions of the aircraft. A fix on these positions is obtained by means of radio information. The error

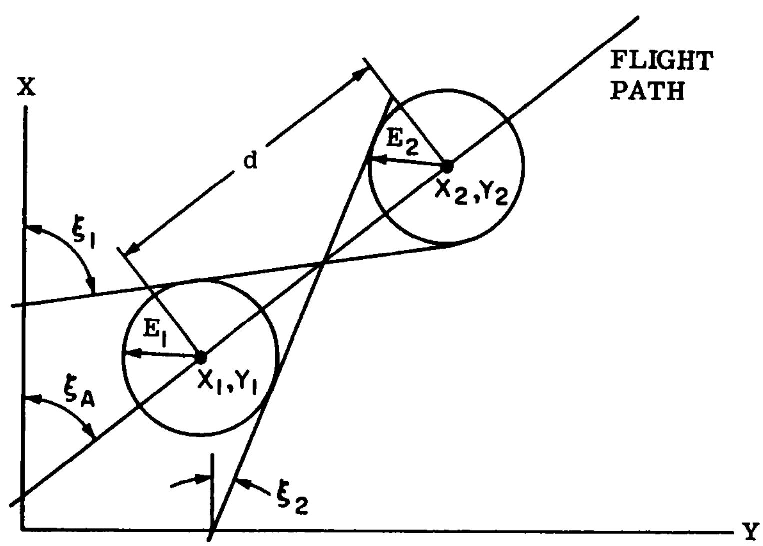


Figure 25. Flight Path Terminations

in each fix is ξ_1 and ξ_2 and is due principally to the radio noise. If the flight path is established by ordinary geometrical means it is apparent that:

$$\xi_1 < \xi_0 < \xi_2 \tag{89}$$

The difference between the determined angle and the actual flight path angle becomes smaller as the distance d is increased, assuming that the aircraft is flying a straight line.

The doppler radar can measure the same quantity (aircraft flight path), which is the sum of the aircraft heading (ψ) and drift angle (β). If this angle (β) is compared to the flight path angle established by radio data, a measure of gyro drift can be obtained as shown in Figure 26. This difference is used as the gyro reference by torquing the gyro as a function of this error signal. Under these conditions the heading reference is independent of a magnetic reference and is slaved to the radio coordinate system.

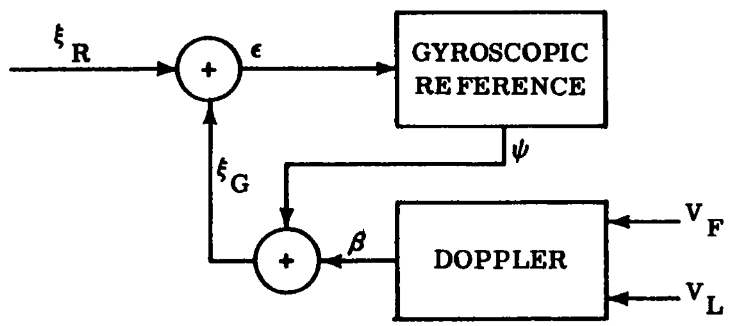


Figure 26. Measure of Gyroscope Drift

This method not only allows for gyro drift rate correction, but has an advantage in that the heading reference is independent of magnetic vagaries. Operation at high latitudes would therefore be more practical.

Previous work done in this area has shown that in areas of reasonably good radio reception it is possible to slave a directional gyro to within ±0.1 degree accuracy.

4.4 APPLICABILITY OF OTHER NAVIGATIONAL DATA

Utilization of rho-theta type radio derived navigational data, such as VORTAC, has limited usefulness when used in conjunction with the discussed coordinate converter (C/C). The C/C is intended for aircraft performing transoceanic missions where long range operation is essential, whereas, VORTAC is limited to line-of-sight ranges and is applicable only when approaching terminal areas. The inclusion of VORTAC data into the C/C would not serve any vital need. Because the traffic control problem in terminal areas is far more complex than the one discussed in this study, it is doubtful that the C/C used with or without the rho-theta would have a vital role.

It should be noted that C/C outputs are in a form very similar to the rho-theta if the destination is the location of the VORTAC transmitter. Each provides range to destination. The track angle to destination supplied by VORTAC can be compared to the cross-track-distance and desired bearing of the C/C without need for additional computations.

One possible application for VORTAC in this operational picture occurs when flying along coastal areas prior to making the oceanic crossing. Such a condition occurs when flying great circle flights from New York to England. Here it is improbable that the VORTAC stations will be intermediate check points and the outputs of C/C would not be compatible with the data displayed by VORTAC. To satisfy this specific case it would be desirable to store the coordinates of each VORTAC station (and the magnetic declination of a limited number of VORTAC stations) and provide a display of the data in a manner similar to that previously discussed. This data can be used in place of the hyperbolic, if hyperbolic coverage is not present, or to cross check the hyperbolic data, if hyperbolic coverage is present.

Neither the means for introducing the rho-theta data into the computer, nor the computations, present any difficult problems; however, it is doubtful whether the added costs of the equipment are justified. It is therefore concluded that for the present, this data should not be included.

5. CONCLUSIONS

It is concluded that a simple, lightweight, practical airborne coordinate converter (C/C) can be designed and built to convert the hyperbolic outputs of existing receivers to a more desirable set of coordinates. In particular:

1. The cross-track-distance from a selected great circle course and the distance-to-go along the course to a destination were chosen as the output coordinates, because they are readily interpreted by flight personnel and are compatible with the outputs of simple dead-reckoning computers

2. The coordinate conversion must utilize digital computer techniques to achieve the desired accuracy at maximum range.

3. Several methods of coordinate conversion that are sufficiently simple to permit a real-time solution with a minimum digital computer were developed, but the method chosen minimizes the modification to existing receivers. In this method, conversion is made from hyperbolic coordinates to distance-to-go and cross-track-distance, by means of a simple set of explicit mathematical relationships.

4. The required accuracy can only be obtained if the earth eccentricity and propagation effects are taken into consideration. The effects of these errors can be removed from the output by applying correction factors to the input hyperbolic coordinates.

5. The C/C can be packaged into a volume of approximately 0.5 cubic feet, and weighing less than 25 pounds.

6. The equipment discussed is practical and has many operational features that would be valuable in manned aircraft application. Some of these are as follows:

   a The design and fabrication of the C/C requires only the use of readily available components utilizing established design techniques.

   b Equipment has fail-safe characteristics and extensive provisions for self-test.

   c. Simplified but yet flexible means are provided for manual insertion into the machine of all initial flight conditions.

7. The C/C has the potential for integration into a complete flight control system without extensive additional equipment.

PART II

1. RECOMMENDATIONS

1. It is recommended that an evaluation program be conducted to verify all the assumptions made and positively demonstrate the validity of the method. To undertake such a program will require that a specific type of hyperbolic system be selected for evaluation and the modifications made to a receiver in accordance with the method discussed in this report. The evaluation should be conducted under dynamic conditions to obtain conclusive results and should include tests to show the advantages of the output data combination under expected service conditions. To accomplish the evaluation, two coordinate converters would be required. These units would be equivalent to service test models. They would have all the operational capabilities discussed, but need not be packaged in their ultimate form.

2. It is believed that a problem area exists with LORAN-C receivers when applied to high-performance aircraft. Specifically: When the output noise level is reduced to a point where proper lane identification can be made and equipment synchronism maintained at long ranges, the output tracking filters become narrow banded to a point where they do not have sufficient dynamic capability to cope with the dynamics of the aircraft motion. This is not based on sustained large maneuvers but on the apparent dynamics of the hyperbolic field and the most modest of aircraft perturbations. This report briefly discusses methods of solving this problem without penalizing the maximum permissible range; however, further analysis of this problem is needed.

3. This latter effort could be conducted in parallel with the first recommendation, since it is of importance when long ranges are to be considered. As such it is an important factor.

PART III

APPENDICES

APPENDIX A

THE PLANAR HYPERBOLIC TRANSFORMATION

1. INTRODUCTION

A planar hyperbolic transformation is defined by the set of equations:

$$r_1^2 = (x-x_1)^2 + (y-y_1)^2 \tag{A-1}$$

$$r_2^2 = (x-x_2)^2 + (y-y_2)^2 , \tag{A-2}$$

$$r_3^2 = (x-x_3)^2 + (y-y_3)^2 , \tag{A-3}$$

$$\eta_{12} = \frac{1}{2}(r_1 - r_2) \tag{A-4}$$

$$\eta_{23} = \frac{1}{2}(r_2 - r_3) . \tag{A-5}$$

where the following nomenclature has been employed:

x_n, y_n = Cartesian coordinates of appropriate station; $n = 1, 2, 3$

x, y = Cartesian coordinates of point P at which signals are received

r_n = Range of point P from appropriate station

(η_{12}, η_{23}) = Hyperbolic coordinates of P.

The geometry is depicted in Figure A-1. The known quantities are the station locations and the η's, which are the hyperbolic coordinates of the point P. Hence, equations A-1 through A-5 constitute a set of five simultaneous equations in the five unknowns: x, y, r_1, r_2, and r_3.

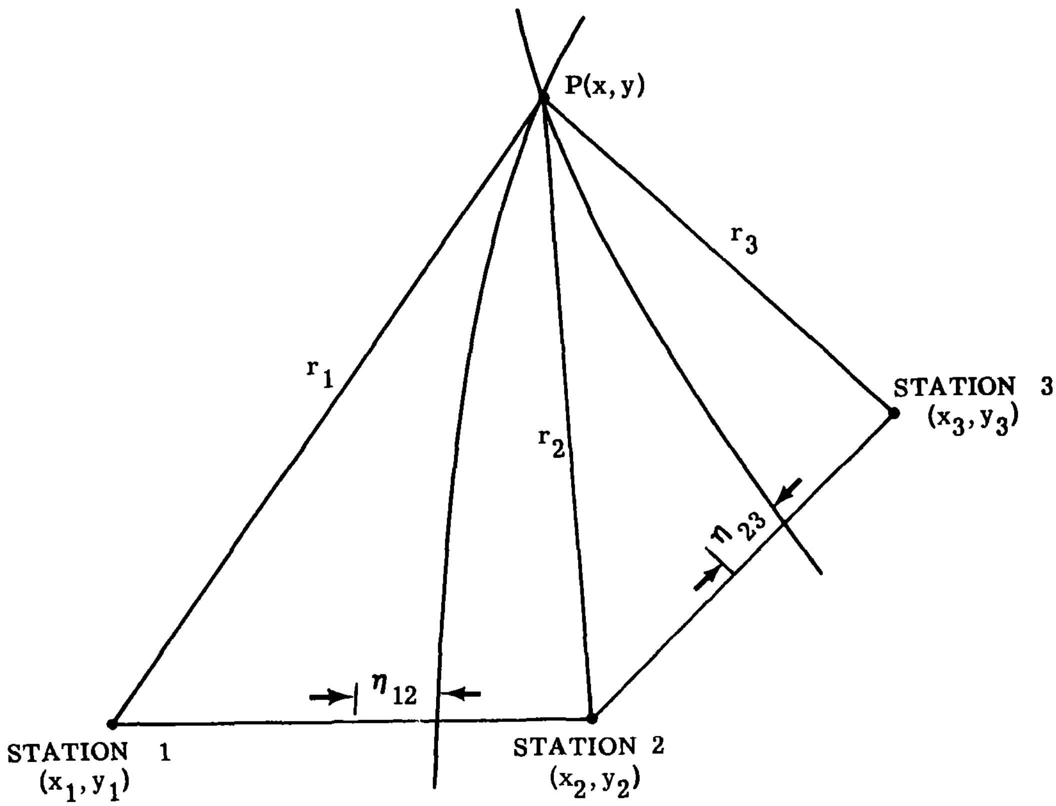


Figure A-1 Plane Hyperbolic Geometry

2. DERIVATION

To effect a solution, we begin with equation A-4 and A-5. Thus

$$r_1 = r_2 + 2\eta_{12} \ , \tag{A-6}$$

$$r_3 = r_2 - 2\eta_{23} \ . \tag{A-7}$$

Equation A-2 is now subtracted from A-1 and equation A-3 is subtracted from A-2, while eliminating r_1 and r_3 by means of equation A-6 and A-7.

$$(r_2 + 2\eta_{12})^2 - r_2^2$$

$$= (x-x_1)^2 + (y-y_1)^2 - (x-x_2)^2 - (y-y_2)^2 \ , \tag{A-8}$$

$$r_2^2 - (r_2 - 2\eta_{23})^2$$

$$= (x-x_2)^2 + (y-y_2)^2 - (x-x_3)^2 - (y-y_3)^2 \qquad \text{(A-9)}$$

Eliminating two ranges while preserving the third may be viewed as the most significant step of the solution.

Expanding the preceding pair of equations, collecting like terms, and dividing through by a factor of four gives:

$$\frac{x_2 - x_1}{2} x + \frac{y_2 - y_1}{2} y =$$

$$\eta_{12} r_2 + \eta_{12}^2 + \frac{1}{4}\left[(x_2^2 + y_2^2) - (x_1^2 + y_1^2)\right] \qquad \text{(A-10)}$$

$$\frac{x_3 - x_2}{2} x + \frac{y_3 - y_2}{2} y =$$

$$\eta_{23} r_2 - \eta_{23}^2 + \frac{1}{4}\left[(x_3^2 + y_3^2) - (x_2^2 + y_2^2)\right] . \qquad \text{(A-11)}$$

Put

$$a_{11} = \frac{x_2 - x_1}{2} \qquad a_{12} = \frac{y_2 - y_1}{2}$$

$$a_{21} = \frac{x_3 - x_2}{2} \qquad a_{22} = \frac{y_3 - y_2}{2}$$

and

$$P = \frac{1}{4}\left[(x_2^2 + y_2^2) - (x_1^2 + y_1^2)\right]$$

$$Q = \frac{1}{4}\left[(x_3^2 + y_3^2) - (x_2^2 + y_2^2)\right] .$$

Then equations A-10 and A-11 become

$$a_{11} x + a_{12} y = \eta_{12} r_2 + \eta_{12}^2 + P \qquad \text{(A-12)}$$

and

$$a_{21}x + a_{22}y = \eta_{23}r_2 - \eta_{23}^2 + Q \; . \qquad \text{(A-13)}$$

The procedure will be to solve equations A-12 and A-13 for x and y as linear functions of r_2, and then use equation A-2 to obtain a quadratic in r_2. Define:

$$\Lambda = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \; . \qquad \text{(A-14)}$$

It is readily verified that Λ will not be zero unless the stations are collinear. Hence, under this restriction, we may write:

$$x = \frac{1}{\Lambda} \begin{vmatrix} \eta_{12}r_2 + \eta_{12}^2 + P & a_{12} \\ \eta_{23}r_2 - \eta_{23}^2 + Q & a_{22} \end{vmatrix} \qquad \text{(A-15)}$$

$$y = \frac{1}{\Lambda} \begin{vmatrix} a_{11} & \eta_{12}r_2 + \eta_{12}^2 + P \\ a_{21} & \eta_{23}r_2 - \eta_{23}^2 + Q \end{vmatrix} \; . \qquad \text{(A-16)}$$

Which on expansion give

$$x = \left[+ \frac{a_{22}}{\Lambda}\eta_{12} - \frac{a_{12}}{\Lambda}\eta_{23} \right] r_2$$

$$+ \frac{a_{22}}{\Lambda}\eta_{12}^2 + \frac{a_{12}}{\Lambda}\eta_{23}^2 + \frac{1}{\Lambda}\left[a_{22}P - a_{12}Q \right] \qquad \text{(A-17)}$$

and

$$y = \left[- \frac{a_{21}}{\Lambda}\eta_{12} + \frac{a_{11}}{\Lambda}\eta_{23} \right] r_2$$

$$- \frac{a_{21}}{\Lambda}\eta_{12}^2 - \frac{a_{11}}{\Lambda}\eta_{23}^2 + \frac{1}{\Lambda}\left[-a_{21}P + a_{11}Q \right] \cdot \qquad \text{(A-18)}$$

Define

$$\Lambda_{11} = \frac{a_{22}}{\Lambda}, \quad \Lambda_{12} = \frac{a_{12}}{\Lambda}, \quad \Lambda_{13} = \frac{1}{\Lambda}\left[a_{22}P - a_{12}Q\right]$$

and (A-19)

$$\Lambda_{21} = -\frac{a_{21}}{\Lambda}, \quad \Lambda_{22} = -\frac{a_{11}}{\Lambda}, \quad \Lambda_{23} = \frac{1}{\Lambda}\left[-a_{21}P + a_{11}Q\right]$$

Then equations A-17 and A-18 become:

$$x = (\Lambda_{11}\eta_{12} - \Lambda_{12}\eta_{23})r_2 + \Lambda_{11}\eta_{12}^2 + \Lambda_{12}\eta_{23}^2 + \Lambda_{13} \qquad \text{(A-20)}$$

$$y = (\Lambda_{21}\eta_{12} - \Lambda_{22}\eta_{23})r_2 + \Lambda_{21}\eta_{12}^2 + \Lambda_{22}\eta_{23}^2 + \Lambda_{23} \ . \qquad \text{(A-21)}$$

Next, make

$$A = \Lambda_{11}\eta_{12} - \Lambda_{12}\eta_{23} \qquad \text{(A-22)}$$

$$B = \Lambda_{11}\eta_{12}^2 + \Lambda_{12}\eta_{23}^2 + \Lambda_{13} \qquad \text{(A-23)}$$

$$C = \Lambda_{21}\eta_{12} - \Lambda_{22}\eta_{23} \qquad \text{(A-24)}$$

$$D = \Lambda_{21}\eta_{12}^2 + \Lambda_{22}\eta_{23}^2 + \Lambda_{23} \ . \qquad \text{(A-25)}$$

Note that the set of equations A-22 through A-25 can be compressed into the single matrix equation:

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} \Lambda_{11} & \Lambda_{12} & \Lambda_{13} \\ \Lambda_{21} & \Lambda_{22} & \Lambda_{23} \end{pmatrix} \begin{pmatrix} \eta_{12} & \eta_{12}^2 \\ -\eta_{23} & \eta_{23}^2 \\ 0 & 1 \end{pmatrix} . \qquad \text{(A-26)}$$

In the new notation equations A-20 and A-21 become

$$x = Ar_2 + B \ , \quad \text{and} \quad y = Cr_2 + D . \qquad \text{(A-27), (A-28)}$$

Substituting equation A-27 and A-28 into A-2, gives

$$r_2^2 = \left[Ar_2 + B - x_2\right]^2 + \left[Cr_2 + D - y_2\right]^2 \ . \qquad \text{(A-29)}$$

Expanding and collecting like terms:

$$\left[A^2+C^2-1\right] r_2^2 + 2\left[A(B-x_2)+C(D-y_2)\right] r_2 + \left[(B-x_2)^2+(D-y_2)^2\right] = 0. \qquad \text{(A-30)}$$

If

$$J = A^2 + C^2 - 1$$
$$K = A(B-x_2) + C(D-y_2) \qquad \text{(A-31)}$$
$$L = (B-x_2)^2 + (D-y_2)^2$$

then equation A-30 may be written

$$Jr_2^2 + 2Kr_2 + L = 0 \qquad \text{(A-32)}$$

the solution of which is

$$r_2 = \frac{-K \pm\sqrt{K^2 - JL}}{J}\ . \qquad \text{(A-33)}$$

The occurrence of the ± sign before the radical reflects the ambiguity inherent in the pair of hyperbolic coordinates. Since r_2 is now known, equations A-27 and A-28 may be used to get x and y, which completes the transformation. If all ranges are desired, equations A-6 and A-7 may be used to get r_1 and r_3.

The lambdas defined in equation A-19, when evaluated in terms of the station coordinates, assume the form:

$$\Lambda = \begin{vmatrix} \frac{x_2-x_1}{2} & \frac{y_2-y_1}{2} \\ \frac{x_3-x_2}{2} & \frac{y_3-y_2}{2} \end{vmatrix} \qquad \text{(A-34)}$$

$$A_{11} = \frac{y_3 - y_2}{2\Lambda}, \qquad A_{12} = \frac{y_2 - y_1}{2\Lambda} \tag{A-35}$$

$$A_{13} = \frac{1}{2\Lambda}\left[\frac{(y_3 - y_2)}{4}\left\{(x_2^2 + y_2^2) - (x_1^2 + y_1^2)\right\} - \frac{(y_2 - y_1)}{4}\left\{(x_3^2 + y_3^2) - (x_2^2 + y_2^2)\right\}\right] \tag{A-36}$$

$$A_{21} = -\frac{(x_3 - x_2)}{2\Lambda} \qquad A_{22} = -\frac{x_2 - x_1}{2\Lambda} \tag{A-37}$$

$$A_{23} = \frac{1}{2\Lambda}\left[\frac{-(x_3 - x_2)}{4}\left\{(x_2^2 + y_2^2) - (x_1^2 + y_1^2)\right\} \frac{+(x_2 - x_1)}{4}\left\{(x_3^2 + y_3^2) - (x_2^2 + y_2^2)\right\}\right]. \tag{A-38}$$

APPENDIX B

THE SPHERICAL HYPERBOLIC TRANSFORMATION

1. INTRODUCTION

The spherical hyperbolic transformation is defined by the following set of equations:

$$\sin\phi_1 \sin\phi + \cos\phi_1 \cos\phi \cos(\lambda - \lambda_1) = \cos \frac{r_1}{R_0} \tag{B-1}$$

$$\sin\phi_2 \sin\phi + \cos\phi_2 \cos\phi \cos(\lambda - \lambda_2) = \cos \frac{r_2}{R_0} \tag{B-2}$$

$$\sin\phi_3 \sin\phi + \cos\phi_3 \cos\phi \cos(\lambda - \lambda_3) = \cos \frac{r_3}{R_0} \tag{B-3}$$

$$r_{21} = r_2 - r_1 \tag{B-4}$$

$$r_{23} = r_2 - r_3 . \tag{B-5}$$

Where the following nomenclature has been employed:

$\phi n, \lambda n$ = latitude, longitude of the appropriate station

ϕ, λ = latitude, longtitude of a general point P.

r_n = the great circle distance joining P and the appropriate station

R_0 = assumed radius of the earth

$\left.\begin{array}{l} r_{21} = r_2 - r_1 \\ r_{23} = r_2 - r_3 \end{array}\right\}$ essentially the hyperbolic coordinates of P.

The geometry is depicted in Figure B-1. The known quantities are the station locations and the pair of range differences, which are essentially the hyperbolic coordinates of the point P. Hence, equations B-1 through B-5 constitute a set of five simultaneous equations in the five unknowns: ϕ, λ, r_1, r_2, and r_3.

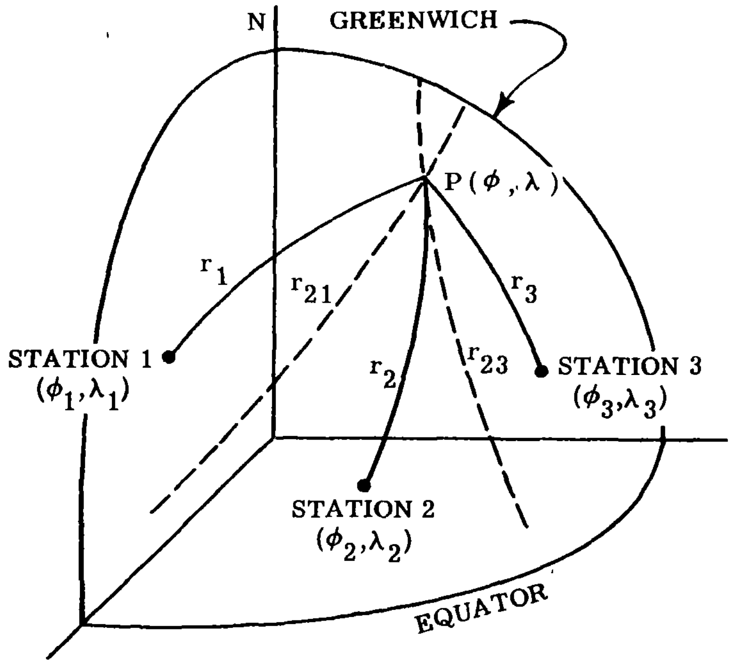


Figure B-1. Geometry

2. DERIVATION

To effect a solution, we begin with equations B-4 and B-5. Thus

$$r_1 = r_2 - r_{21} \tag{B-6}$$

$$r_3 = r_2 - r_{23} \tag{B-7}$$

and

$$\cos\frac{r_1}{R_0} = \cos\left[\frac{r_2}{R_0} - \frac{r_{21}}{R_0}\right] = \cos\frac{r_{21}}{R_0}\cos\frac{r_2}{R_0} + \sin\frac{r_{21}}{R_0}\sin\frac{r_2}{R_0}\,, \tag{B-8}$$

$$\cos\frac{r_3}{R_0} = \cos\left[\frac{r_2}{R_0} - \frac{r_{23}}{R_0}\right] = \cos\frac{r_{23}}{R_0}\cos\frac{r_2}{R_0} + \sin\frac{r_{23}}{R_0}\sin\frac{r_2}{R_0}\,. \tag{B-9}$$

Expanding the left hand members of equations B-1 through B-3, while substituting equations B-8 and B-9 on the right:

$$\sin\phi_1\ \sin\phi + \cos\phi_1\ \cos\lambda_1\ \cos\phi\ \cos\lambda + \cos\phi_1\ \sin\lambda_1\ \cos\phi\ \sin\lambda =$$

$$\cos\frac{r_{21}}{R_0}\cos\frac{r_2}{R_0} + \sin\frac{r_{21}}{R_0}\sin\frac{r_2}{R_0} \tag{B-10}$$

$$\sin\phi_2 \sin\phi + \cos\phi_2 \cos\lambda_2 \cos\phi \cos\lambda + \cos\phi_2 \sin\lambda_2 \cos\phi \sin\lambda =$$

$$\cos\frac{r_2}{R_0} \qquad \text{(B-11)}$$

$$\sin\phi_3 \sin\phi + \cos\phi_3 \cos\lambda_3 \cos\phi \cos\lambda + \cos\phi_3 \sin\lambda_3 \cos\phi \sin\lambda =$$

$$\cos\frac{r_{23}}{R_0}\cos\frac{r_2}{R_0} + \sin\frac{r_{23}}{R_0}\sin\frac{r_2}{R_0} \; . \qquad \text{(B-12)}$$

Eliminating two ranges while preserving the third is perhaps the most significant step in the solution. Set

$$T = \sin\phi \qquad U = \cos\phi\cos\lambda \qquad V = \cos\phi\sin\lambda$$

$$a_{11} = \sin\phi_1 \qquad a_{12} = \cos\phi_1 \cos\lambda_1 \qquad a_{13} = \cos\phi_1 \sin\lambda_1$$

$$a_{21} = \sin\phi_2 \qquad a_{22} = \cos\phi_2 \cos\lambda_2 \qquad a_{23} = \cos\phi_2 \sin\lambda_2$$

$$a_{31} = \sin\phi_3 \qquad a_{32} = \cos\phi_3 \cos\lambda_3 \qquad a_{33} = \cos\phi_3 \sin\lambda_3$$

$$\eta_{21} = \cos\frac{r_{21}}{R_0} \qquad \xi_{21} = \sin\frac{r_{21}}{R_0} \qquad \text{(B-13)}$$

$$\eta_{23} = \cos\frac{r_{23}}{R_0} \qquad \xi_{23} = \sin\frac{r_{23}}{R_0}$$

$$\sigma_2 = \frac{r_2}{R_0} \qquad C_{\sigma_2} = \cos\sigma_2$$

$$S_{\sigma_2} = \sin\sigma_2$$

then equations B-10 through B-12 assume the form

$$T^2 + U^2 + V^2 = 1 \qquad \text{(B-14)}$$

$$a_{11}T + a_{12}U + a_{13}V = \eta_{21}C_{\sigma_2} + \xi_{21}S_{\sigma_2} \qquad \text{(B-15)}$$

$$a_{21}T + a_{22}U + a_{23}V = C_{\sigma_2} \qquad \text{(B-16)}$$

$$a_{31} T + a_{32} U + a_{33} V = \eta_{23} C_{\sigma_2} + \xi_{23} S_{\sigma_2} . \tag{B-17}$$

The procedure will be to solve for T, U, and V in terms of r_2, then use equation B-14 to obtain a quadratic in tan $\frac{r_2}{R_0}$.

Let

$$\Lambda = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \tag{B-18}$$

Provided $\Lambda \neq 0$,

$$T = \frac{1}{\Lambda} \begin{vmatrix} \eta_{21} C_{\sigma_2} + \xi_{21} S_{\sigma_2} & a_{12} & a_{13} \\ C_{\sigma_2} & a_{22} & a_{23} \\ \eta_{23} C_{\sigma_2} + \xi_{23} S_{\sigma_2} & a_{32} & a_{33} \end{vmatrix} \tag{B-19}$$

$$U = \frac{1}{\Lambda} \begin{vmatrix} a_{11} & \eta_{21} C_{\sigma_2} + \xi_{21} S_{\sigma_2} & a_{13} \\ a_{21} & C_{\sigma_2} & a_{23} \\ a_{31} & \eta_{23} C_{\sigma_2} + \xi_{23} S_{\sigma_2} & a_{33} \end{vmatrix} \tag{B-20}$$

$$V = \frac{1}{\Lambda} \begin{vmatrix} a_{11} & a_{12} & \eta_{21} C_{\sigma_2} + \xi_{21} S_{\sigma_2} \\ a_{21} & a_{22} & C_{\sigma_2} \\ a_{31} & a_{32} & \eta_{23} C_{\sigma_2} + \xi_{23} S_{\sigma_2} \end{vmatrix} \tag{B-21}$$

Further,

$$T = \frac{1}{\Lambda}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} \left[\eta_{21} \quad C_{\sigma_2} + \xi_{21} \quad S_{\sigma_2}\right] - \frac{1}{\Lambda}\begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} C_{\sigma_2}$$

$$+ \frac{1}{\Lambda}\begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} \left[\eta_{23} \quad C_{\sigma_2} + \xi_{23} \quad S_{\sigma_2}\right] \qquad \text{(B-22)}$$

$$U = -\frac{1}{\Lambda}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} \left[\eta_{21} \quad C_{\sigma_2} + \xi_{21} \quad S_{\sigma_2}\right] + \frac{1}{\Lambda}\begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} C_{\sigma_2}$$

$$- \frac{1}{\Lambda}\begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix} \left[\eta_{23} \quad C_{\sigma_2} + \xi_{23} \quad S_{\sigma_2}\right] \qquad \text{(B-23)}$$

$$V = \frac{1}{\Lambda}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} \left[\eta_{21} \quad C_{\sigma_2} + \xi_{21} \quad S_{\sigma_2}\right] - \frac{1}{\Lambda}\begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix} C_{\sigma_2}$$

$$+ \frac{1}{\Lambda}\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \left[\eta_{23} \quad C_{\sigma_2} + \xi_{23} \quad S_{\sigma_2}\right] \qquad \text{(B-24)}$$

Define

$$\Lambda_{11} = \frac{1}{\Lambda}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} \qquad \Lambda_{23} = \frac{1}{\Lambda}\begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} \qquad \text{(B-25)}$$

$$\Lambda_{12} = \frac{1}{\Lambda}\begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} \qquad \Lambda_{31} = \frac{1}{\Lambda}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$\Lambda_{13} = -\frac{1}{\Lambda}\begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} \qquad \Lambda_{32} = \frac{1}{\Lambda}\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

$$\Lambda_{21} = -\frac{1}{\Lambda}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} \qquad \Lambda_{33} = -\frac{1}{\Lambda}\begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix}$$

$$\Lambda_{22} = -\frac{1}{\Lambda}\begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix}$$

Then equations B-22 through equation B-24 may be written as

$$T = \Lambda_{11}(\eta_{21} C_{\sigma_2} + \xi_{21} S_{\sigma_2}) + \Lambda_{13} C_{\sigma_2} + \Lambda_{12}(\eta_{23} C_{\sigma_2} + \xi_{23} S_{\sigma_2}) \qquad \text{(B-26)}$$

$$U = \Lambda_{21}(\eta_{21} C_{\sigma_2} + \xi_{21} S_{\sigma_2}) + \Lambda_{23} C_{\sigma_2} + \Lambda_{22}(\eta_{23} C_{\sigma_2} + \xi_{23} S_{\sigma_2}) \qquad \text{(B-27)}$$

$$V = \Lambda_{31}(\eta_{21} C_{\sigma_2} + \xi_{21} S_{\sigma_2}) + \Lambda_{33} C_{\sigma_2} + \Lambda_{32}(\eta_{23} C_{\sigma_2} + \xi_{23} S_{\sigma_2}) \qquad \text{(B-28)}$$

or

$$T=(\Lambda_{11}\eta_{21}+\Lambda_{12}\eta_{23}+\Lambda_{13})\,C_{\sigma_2}+(\Lambda_{11}\xi_{21}+\Lambda_{12}\xi_{23})S_{\sigma_2} \qquad \text{(B-29)}$$

$$U=(\Lambda_{21}\eta_{21}+\Lambda_{22}\eta_{23}+\Lambda_{23})\,C_{\sigma_2}+(\Lambda_{21}\xi_{21}+\Lambda_{22}\xi_{23})S_{\sigma_2} \qquad \text{(B-30)}$$

$$V=(\Lambda_{31}\eta_{21}+\Lambda_{32}\eta_{23}+\Lambda_{33})\,C_{\sigma_2}+(\Lambda_{31}\xi_{21}+\Lambda_{32}\xi_{23})S_{\sigma_2}\,. \qquad \text{(B-31)}$$

Next define

$$A=\Lambda_{11}\eta_{21}+\Lambda_{12}\eta_{23}+\Lambda_{13} \qquad \text{(B-32)}$$

$$B=\Lambda_{11}\xi_{21}+\Lambda_{12}\xi_{23}$$

$$C=\Lambda_{21}\eta_{21}+\Lambda_{22}\eta_{23}+\Lambda_{23}$$

$$D=\Lambda_{21}\xi_{21}+\Lambda_{22}\xi_{23}$$

$$E=\Lambda_{31}\eta_{21}+\Lambda_{32}\eta_{23}+\Lambda_{33}$$

$$F=\Lambda_{31}\xi_{21}+\Lambda_{32}\xi_{23}\,.$$

In view of the fortuitous labeling of the lambdas, the six equations numbered in B-32 may be compressed into a single matrix equation. Thus,

$$\begin{pmatrix} A & B \\ C & D \\ E & F \end{pmatrix} = \begin{pmatrix} \Lambda_{11} & \Lambda_{12} & \Lambda_{13} \\ \Lambda_{21} & \Lambda_{22} & \Lambda_{23} \\ \Lambda_{31} & \Lambda_{32} & \Lambda_{33} \end{pmatrix} \begin{pmatrix} \eta_{21} & \xi_{21} \\ \eta_{23} & \xi_{23} \\ 1 & 0 \end{pmatrix}. \qquad \text{(B-33)}$$

In the new notation equations B-29 through B-31 become

$$T = AC_{\sigma_2} + BS_{\sigma_2} \qquad \text{(B-34)}$$

$$U = CC_{\sigma_2} + DS_{\sigma_2} \qquad \text{(B-35)}$$

$$V = EC_{\sigma_2} + FS_{\sigma_2} . \qquad \text{(B-36)}$$

Replacing the 1 by $C_{\sigma_2}^2 + S_{\sigma_2}^2$ and substituting equation B-34 through equation B-36 into equation B-14,

$$\left[AC_{\sigma_2} + BS_{\sigma_2}\right]^2 + \left[CC_{\sigma_2} + DS_{\sigma_2}\right]^2 + \left[EC_{\sigma_2} + FS_{\sigma_2}\right]^2 =$$

$$C_{\sigma_2}^2 + S_{\sigma_2}^2 . \qquad \text{(B-37)}$$

Squaring and collecting like terms

$$\left[B^2 + D^2 + F^2 - 1\right] S_{\sigma_2}^2 + 2\left[AB + CD + EF\right] C_{\sigma_2} S_{\sigma_2} +$$

$$\left[A^2 + C^2 + E^2 - 1\right] C_{\sigma_2}^2 = 0 . \qquad \text{(B-38)}$$

Define

$$J = B^2 + D^2 + F^2 - 1 \qquad \text{(B-39)}$$

$$K = AB + CD + EF \qquad \text{(B-40)}$$

$$L = A^2 + C^2 + E^2 - 1 . \qquad \text{(B-41)}$$

then

$$JS_{\sigma_2}^2 + 2\,KC_{\sigma_2}S_{\sigma_2} + LC_{\sigma_2}^2 = 0 \tag{B-42}$$

Dividing by $C_{\sigma_2}^2$ (which will not be zero unless $r_2 = \frac{\pi}{2} R_a$, etc.) and substituting T_{σ_2} for $\tan \sigma_2$

$$JT_{\sigma_2}^2 + 2KT_{\sigma_2} + L = 0 \tag{B-43}$$

the solution of which is

$$T_{\sigma_2} = \frac{-K \pm \sqrt{K^2 - JL}}{J}. \tag{B-44}$$

The occurrence of the ± sign in the equation B-44 reflects the original hyperbolic ambiguity of the pair of range differences.

Since T_{σ_2} is now known,

$$C_{\sigma_2} = \frac{1}{\sqrt{1 + T_{\sigma_2}^2}} \tag{B-45}$$

$$S_{\sigma_2} = \frac{T_{\sigma_2}}{\sqrt{1 + T_{\sigma_2}^2}}. \tag{B-46}$$

Returning to the definitions of T, U, and V, we have

$$\sin\phi = T = AC_{\sigma_2} + BS_{\sigma_2} = \frac{A}{\sqrt{1 + T_{\sigma_2}^2}} + \frac{BT_{\sigma_2}}{\sqrt{1 + T_{\sigma_2}^2}} = \frac{A + BT_{\sigma_2}}{\sqrt{1 + T_{\sigma_2}^2}} \tag{B-47}$$

$$\tan\lambda = \frac{V}{U} = \frac{EC_{\sigma_2} + FS_{\sigma_2}}{CC_{\sigma_2} + DS_{\sigma_2}} = \frac{E + FT_{\sigma_2}}{C + DT_{\sigma_2}}. \tag{B-48}$$

The conditions under which the solution holds will now be established.

Expanding equation B-18,

$$\begin{aligned}\Lambda = \; & a_{11}\, a_{22}\, a_{33} - a_{11}\, a_{32}\, a_{23} \\ & - a_{21}\, a_{12}\, a_{33} + a_{21}\, a_{32}\, a_{13} \\ & + a_{31}\, a_{12}\, a_{23} - a_{31}\, a_{22}\, a_{13}\end{aligned} \qquad \text{(B-49)}$$

which, in view of the definitions set forth in B-13, becomes

$$\begin{aligned}\Lambda = \; & \sin\phi_1 \cos\phi_2 \cos\lambda_2 \cos\phi_3 \sin\lambda_3 \\ & - \sin\phi_1 \cos\phi_3 \cos\lambda_3 \cos\phi_2 \sin\lambda_2 \\ & - \sin\phi_2 \cos\phi_1 \cos\lambda_1 \cos\phi_3 \sin\lambda_3 \\ & + \sin\phi_2 \cos\phi_3 \cos\lambda_3 \cos\phi_1 \sin\lambda_1 \\ & + \sin\phi_3 \cos\phi_1 \cos\lambda_1 \cos\phi_2 \sin\lambda_2 \\ & - \sin\phi_3 \cos\phi_2 \cos\lambda_2 \cos\phi_1 \sin\lambda_1 .\end{aligned} \qquad \text{(B-50)}$$

This expression can be rearranged so that

$$\begin{aligned}\Lambda = \; & \sin\phi_1 \cos\phi_2 \cos\phi_3 \sin(\lambda_3 - \lambda_2) \\ & - \sin\phi_2 \cos\phi_1 \cos\phi_3 \sin(\lambda_3 - \lambda_1) \\ & + \sin\phi_3 \cos\phi_1 \cos\phi_2 \sin(\lambda_2 - \lambda_1) .\end{aligned} \qquad \text{(B-51)}$$

For present purposes, let an equator be chosen through Stations 1 and 2. Then

$$\phi_1 = \phi_2 = 0$$

and the expression for Λ reduces to

$$\Lambda = \sin\phi_3 \sin(\lambda_2 - \lambda_1) \qquad \text{(B-52)}$$

from which it is seen that, providing station 3 does not lie on the great circle joining stations 1 and 2 (and that 1 and 2 are not coincident), the solution is valid. The condition is indicated in Figure B-2.

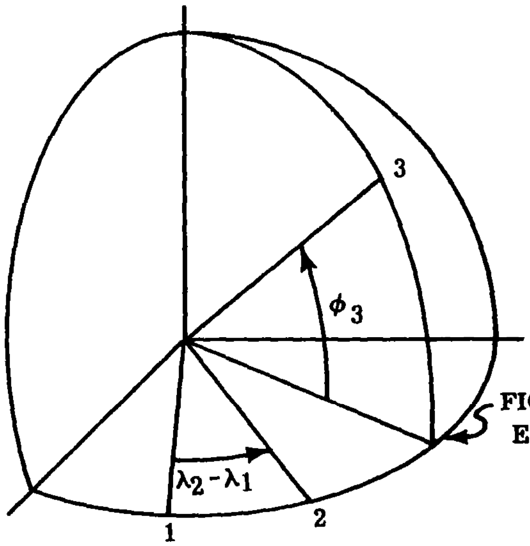


Figure B-2. Geometry

The remaining task is to evaluate, in terms of the geographical coordinates of the stations, the lambdas defined in B-25 Thus,

$$\Lambda_{11} = \frac{1}{\Lambda}(a_{22}a_{33} - a_{32}a_{23})$$

$$= \frac{1}{\Lambda}\left[\cos\phi_2 \cos\lambda_2 \cos\phi_3 \sin\lambda_3 - \cos\phi_3 \cos\lambda_3 \cos\phi_2 \sin\lambda_2\right]$$

$$= \frac{1}{\Lambda}\cos\phi_2 \cos\phi_3 \sin(\lambda_3 - \lambda_2)$$

$$\Lambda_{12} = \frac{1}{\Lambda}(a_{12}\ a_{23} - a_{22}a_{13})$$

$$= \frac{1}{\Lambda}\left[\cos\phi_1 \cos\lambda_1 \cos\phi_2 \sin\lambda_2 - \cos\phi_2 \cos\lambda_2 \cos\phi_1 \sin\lambda_1\right]$$

$$= \frac{1}{\Lambda}\cos\phi_1 \cos\phi_2 \sin(\lambda_2 - \lambda_1)$$

$$\Lambda_{13} = -\frac{1}{\Lambda}(a_{12}\ a_{33} - a_{32}\ a_{13})$$

$$= -\frac{1}{\Lambda}\left[\cos\phi_1\ \cos\lambda_1 \cos\phi_3\ \sin\lambda_3 - \cos\phi_3 \cos\lambda_3\ \cos\phi_1\ \sin\lambda_1\right]$$

$$= -\frac{1}{\Lambda}\cos\phi_1\ \cos\phi_3 \sin(\lambda_3 - \lambda_1)$$

$$\Lambda_{21} = -\frac{1}{\Lambda}(a_{21}\ a_{33} - a_{31}\ a_{23})$$

$$= -\frac{1}{\Lambda}\left[\sin\phi_2\ \cos\phi_3 \sin\lambda_3 - \sin\phi_3\ \cos\phi_2 \sin\lambda_2\right]$$

$$\Lambda_{22} = -\frac{1}{\Lambda}(a_{11}\ a_{23} - a_{21}\ a_{13})$$

$$= -\frac{1}{\Lambda}\left[\sin\phi_1\ \cos\phi_2\ \sin\lambda_2 - \sin\phi_2\ \cos\phi_1\ \sin\lambda_1\right]$$

$$\Lambda_{23} = \frac{1}{\Lambda}(a_{11}\ a_{33} - a_{31}\ a_{13})$$

$$= \frac{1}{\Lambda}\left[\sin\phi_1\ \cos\phi_3\ \sin\lambda_3 - \sin\phi_3 \cos\phi_1\ \sin\lambda_1\right]$$

$$\Lambda_{31} = \frac{1}{\Lambda}(a_{21}\, a_{32} - a_{31}\ a_{22})$$

$$= \frac{1}{\Lambda}\left[\sin\phi_2 \cos\phi_3 \cos\lambda_3 - \sin\phi_3 \cos\phi_2 \cos\lambda_2\right]$$

$$\Lambda_{32} = \frac{1}{\Lambda}(a_{11}\ a_{22} - a_{21}\, a_{12})$$

$$= \frac{1}{\Lambda}\left[\sin\phi_1\ \cos\phi_2 \cos\lambda_2 - \sin\phi_2 \cos\phi_1\ \cos\lambda_1\right]$$

APPENDIX C

SOLUTION FOR OSCILLATOR DRIFT

1. INTRODUCTION

In heterodyne detection where the individual ranges are brought out, the equations relating the detected signals to the receiver's position are:

$$\sin \phi_1 \sin \phi + \cos \phi_1 \cos \phi \cos (\lambda - \lambda_1) = \cos \frac{r_1^*}{R_o} \qquad (C-1)$$

$$\sin \phi_2 \sin \phi + \cos \phi_2 \cos \phi \cos (\lambda - \lambda_2) = \cos \frac{r_2^*}{R_o} \qquad (C-2)$$

$$\sin \phi_3 \sin \phi + \cos \phi_3 \cos \phi \cos (\lambda - \lambda_3) = \cos \frac{r_3^*}{R_o} \qquad (C-3)$$

$$r_1 = r_1^* + \delta \qquad (C-4)$$

$$r_2 = r_2^* + \delta \qquad (C-5)$$

$$r_3 = r_3^* + \delta \qquad (C-6)$$

where the following nomenclature has been employed:

ϕ_n, λ_n = latitude, longitude of the appropriate station; n = 1, 2, 3

ϕ, λ = latitude, longitude of point P at which signals are received

r_n = the detected great circle distance joining P and the appropriate station

r_n^* = correct great circle distance joining P and the appropriate station

δ = relative oscillator drift in linear units

R_o = radius of the earth.

The outputs from the individual receivers are r_1, r_2, and r_3, each being incorrect by the same amount, δ, which represents the relative drift between the ground and airborne oscillators. Since the station locations are known, equations C-1 through C-6 constitute six simultaneous equations in the six unknowns: ϕ, λ, δ, r_1^*, r_2^*, and r_3^*.

2. DERIVATION

To effect a solution, the starred ranges may first be eliminated. Thus:

$$r_1^* = r_1 - \delta \qquad \text{(C-7)}$$

$$r_2^* = r_2 - \delta \qquad \text{(C-8)}$$

$$r_3^* = r_3 - \delta \qquad \text{(C-9)}$$

and

$$\cos\frac{r_1^*}{R_o} = \cos\left[\frac{r_1}{R_o} - \frac{\delta}{R_o}\right] = \cos\frac{r_1}{R_o}\cos\frac{\delta}{R_o} + \sin\frac{r_1}{R_o}\sin\frac{\delta}{R_o}, \qquad \text{(C-10)}$$

$$\cos\frac{r_2^*}{R_o} = \cos\left[\frac{r_2}{R_o} - \frac{\delta}{R_o}\right] = \cos\frac{r_2}{R_o}\cos\frac{\delta}{R_o} + \sin\frac{r_2}{R_o}\sin\frac{\delta}{R_o} \qquad \text{(C-11)}$$

$$\cos\frac{r_3^*}{R_o} = \cos\left[\frac{r_3}{R_o} - \frac{\delta}{R_o}\right] = \cos\frac{r_3}{R_o}\cos\frac{\delta}{R_o} + \sin\frac{r_3}{R_o}\sin\frac{\delta}{R_o}. \qquad \text{(C-12)}$$

Expanding the left members of equations C-1 through C-3, while substituting C-10 through C-12 on the right:

$$\sin\phi_1\sin\phi + \cos\phi_1\cos\lambda_1\cos\phi\cos\lambda + \cos\phi_1\sin\lambda_1\cos\phi\sin\lambda$$
$$= \cos\frac{r_1}{R_o}\cos\frac{\delta}{R_o} + \sin\frac{r_1}{R_o}\sin\frac{\delta}{R_o} \qquad \text{(C-13)}$$

$$\sin\phi_2\sin\phi + \cos\phi_2\cos\lambda_2\cos\phi\cos\lambda + \cos\phi_2\sin\lambda_2\cos\phi\sin\lambda$$
$$= \cos\frac{r_2}{R_o}\cos\frac{\delta}{R_o} + \sin\frac{r_2}{R_o}\sin\frac{\delta}{R_o} \qquad \text{(C-14)}$$

$$\sin\phi_3\sin\phi + \cos\phi_3\cos\lambda_3\cos\phi\cos\lambda + \cos\phi_3\sin\lambda_3\cos\phi\sin\lambda$$
$$= \cos\frac{r_3}{R_o}\cos\frac{\delta}{R_o} + \sin\frac{r_3}{R_o}\sin\frac{\delta}{R_o}. \qquad \text{(C-15)}$$

Put

$$a_{11} = \sin\phi_1 \qquad a_{12} = \cos\phi_1 \cos\lambda_1 \qquad a_{13} = \cos\phi_1 \sin\lambda_1$$

$$a_{21} = \sin\phi_2 \qquad a_{22} = \cos\phi_2 \cos\lambda_2 \qquad a_{23} = \cos\phi_2 \sin\lambda_2$$

$$a_{31} = \sin\phi_3 \qquad a_{32} = \cos\phi_3 \cos\lambda_3 \qquad a_{33} = \cos\phi_3 \sin\lambda_3$$

$$\eta_1 = \cos\frac{r_1}{R_0} \qquad \xi_1 = \sin\frac{r_1}{R_0}$$

$$\eta_2 = \cos\frac{r_2}{R_0} \qquad \xi_2 = \sin\frac{r_2}{R_0}$$

$$\eta_3 = \cos\frac{r_3}{R_0} \qquad \xi_3 = \sin\frac{r_3}{R_0}$$

$$\eta_\delta = \cos\frac{\delta}{R_0} \qquad \xi_\delta = \sin\frac{\delta}{R_0}.$$

Make the following change of variable:

$$T = \sin\phi, \qquad U = \cos\phi \cos\lambda, \qquad V = \cos\phi \sin\lambda.$$

Then equations C-13 through C-15 assume the form:

$$a_{11}T + a_{12}U + a_{13}V = \eta_1\eta_\delta + \xi_1\xi_\delta \tag{C-16}$$

$$a_{21}T + a_{22}U + a_{23}V = \eta_2\eta_\delta + \xi_2\xi_\delta \tag{C-17}$$

$$a_{31}T + a_{32}U + a_{33}V = \eta_3\eta_\delta + \xi_3\xi_\delta \tag{C-18}$$

and

$$T^2 + U^2 + V^2 = 1. \tag{C-19}$$

The procedure will be to solve for T, U, and V in terms of δ, and then use equation C-19 to obtain a quadratic in δ. Let

$$\Lambda = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

The condition for solution here is the same as that given in Appendix B, namely, that the stations do not all lie on the same great circle, so that $\Lambda \neq 0$.

Hence, under this restriction,

$$T = \frac{1}{\Lambda} \begin{vmatrix} \eta_1 \eta_8 + \xi_1 \xi_8 & a_{12} & a_{13} \\ \eta_2 \eta_8 + \xi_2 \xi_8 & a_{22} & a_{23} \\ \eta_3 \eta_8 + \xi_3 \xi_8 & a_{32} & a_{33} \end{vmatrix} \qquad \text{(C-20)}$$

$$U = \frac{1}{\Lambda} \begin{vmatrix} a_{11} & \eta_1 \eta_8 + \xi_1 \xi_8 & a_{13} \\ a_{21} & \eta_2 \eta_8 + \xi_2 \xi_8 & a_{23} \\ a_{31} & \eta_3 \eta_8 + \xi_3 \xi_8 & a_{33} \end{vmatrix} \qquad \text{(C-21)}$$

$$V = \frac{1}{\Lambda} \begin{vmatrix} a_{11} & a_{12} & \eta_1 \eta_8 + \xi_1 \xi_8 \\ a_{21} & a_{22} & \eta_2 \eta_8 + \xi_2 \xi_8 \\ a_{31} & a_{32} & \eta_3 \eta_8 + \xi_3 \xi_8 \end{vmatrix} \qquad \text{(C-22)}$$

$$T = \frac{1}{\Lambda} \left[\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} (\eta_1 \eta_8 + \xi_1 \xi_8) - \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} (\eta_2 \eta_8 + \xi_2 \xi_8) + \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} (\eta_3 \eta_8 + \xi_3 \xi_8) \right] \qquad \text{(C-23)}$$

$$U = \frac{1}{\Lambda}\left[-\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}(\eta_1\eta_8 + \xi_1\xi_8) + \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix}(\eta_2\eta_8 + \xi_2\xi_8) - \begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix}(\eta_3\eta_8 + \xi_3\xi_8)\right] \quad \text{(C-24)}$$

and

$$V = \frac{1}{\Lambda}\left[\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}(\eta_1\eta_8 + \xi_1\xi_8) - \begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix}(\eta_2\eta_8 + \xi_2\xi_8) + \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}(\eta_3\eta_8 + \xi_3\xi_8)\right]. \quad \text{(C-25)}$$

Put

$$\Lambda_{11} = \frac{1}{\Lambda}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} \qquad \Lambda_{12} = \frac{1}{\Lambda}\begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}$$

$$\Lambda_{13} = -\frac{1}{\Lambda}\begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} \qquad \Lambda_{21} = -\frac{1}{\Lambda}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$$

$$\Lambda_{22} = -\frac{1}{\Lambda}\begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix} \qquad \Lambda_{23} = \frac{1}{\Lambda}\begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix}$$

$$\Lambda_{31} = \frac{1}{\Lambda} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} \qquad \Lambda_{32} = \frac{1}{\Lambda} \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

and

$$\Lambda_{33} = -\frac{1}{\Lambda} \begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix} .$$

Then

$$T = \Lambda_{11} (\eta_1 \eta_8 + \xi_1 \xi_8) + \Lambda_{12} (\eta_3 \eta_8 + \xi_3 \xi_8) + \Lambda_{13} (\eta_2 \eta_8 + \xi_2 \xi_8) \tag{C-26}$$

$$U = \Lambda_{21} (\eta_1 \eta_8 + \xi_1 \xi_8) + \Lambda_{22} (\eta_3 \eta_8 + \xi_3 \xi_8) + \Lambda_{23} (\eta_2 \eta_8 + \xi_2 \xi_8) \tag{C-27}$$

and

$$V = \Lambda_{31} (\eta_1 \eta_8 + \xi_1 \xi_8) + \Lambda_{32} (\eta_3 \eta_8 + \xi_3 \xi_8) + \Lambda_{33} (\eta_2 \eta_8 + \xi_2 \xi_8) ; \tag{C-28}$$

or

$$T = (\Lambda_{11} \eta_1 + \Lambda_{12} \eta_3 + \Lambda_{13} \eta_2) \eta_8 + (\Lambda_{11} \xi_1 + \Lambda_{12} \xi_3 + \Lambda_{13} \xi_2) \xi_8 \tag{C-29}$$

$$U = (\Lambda_{21} \eta_1 + \Lambda_{22} \eta_3 + \Lambda_{23} \eta_2) \eta_8 + (\Lambda_{21} \xi_1 + \Lambda_{22} \xi_3 + \Lambda_{23} \xi_2) \xi_8 \tag{C-30}$$

and

$$V = (\Lambda_{31} \eta_1 + \Lambda_{32} \eta_3 + \Lambda_{33} \eta_2) \eta_8 + (\Lambda_{31} \xi_1 + \Lambda_{32} \xi_3 + \Lambda_{33} \xi_2) \xi_8 . \tag{C-31}$$

Define

$$A = \Lambda_{11}\eta_1 + \Lambda_{12}\eta_3 + \Lambda_{13}\eta_2$$

$$B = \Lambda_{11}\xi_1 + \Lambda_{12}\xi_3 + \Lambda_{13}\xi_2$$

$$C = \Lambda_{21}\eta_1 + \Lambda_{22}\eta_3 + \Lambda_{23}\eta_2$$

$$D = \Lambda_{21}\xi_1 + \Lambda_{22}\xi_3 + \Lambda_{23}\xi_2$$

$$E = \Lambda_{31}\eta_1 + \Lambda_{32}\eta_3 + \Lambda_{33}\eta_2$$

and

$$F = \Lambda_{31}\xi_1 + \Lambda_{32}\xi_3 + \Lambda_{33}\xi_2 .$$

The above six equations may be compressed into the single matrix equation:

$$\begin{pmatrix} A & B \\ C & D \\ E & F \end{pmatrix} = \begin{pmatrix} \Lambda_{11} & \Lambda_{12} & \Lambda_{13} \\ \Lambda_{21} & \Lambda_{22} & \Lambda_{23} \\ \Lambda_{31} & \Lambda_{32} & \Lambda_{33} \end{pmatrix} \begin{pmatrix} \eta_1 & \xi_1 \\ \eta_3 & \xi_3 \\ \eta_2 & \xi_2 \end{pmatrix} . \tag{C-32}$$

In the new notation, equations C-29 through C-31 become:

$$T = A\eta_\delta + B\xi_\delta \tag{C-33}$$

$$U = C\eta_\delta + D\xi_\delta \tag{C-34}$$

$$V = E\eta_\delta + F\xi_\delta . \tag{C-35}$$

Observing that $\eta_\delta^2 + \xi_\delta^2 = 1$ and substituting C-33 through C-35 into C-19

$$\left[A\eta_\delta + B\xi_\delta\right]^2 + \left[C\eta_\delta + D\xi_\delta\right]^2 + \left[E\eta_\delta + F\xi_\delta\right]^2 = \eta_\delta^2 + \xi_\delta^2 . \tag{C-36}$$

Squaring and collecting like terms

$$\left[B^2+D^2+F^2-1\right]\xi_\delta^2 + 2\left[AB+CD+EF\right]\eta_\delta\xi_\delta$$

$$+\left[A^2+C^2+E^2-1\right]\eta_\delta^2 = 0. \qquad \text{(C-37)}$$

Define:

$$J = B^2 + D^2 + F^2 - 1$$

$$K = AB + CD + EF$$

and

$$L = A^2 + C^2 + E^2 - 1$$

then

$$J\xi_\delta^2 + 2K\eta_\delta\xi_\delta + L\eta_\delta^2 = 0 \ . \qquad \text{(C-38)}$$

Dividing by η_δ^2 and substituting T_δ for $\tan\frac{\delta}{R_0}$

$$JT_\delta^2 + 2KT_\delta + L = 0 \ . \qquad \text{(C-39)}$$

When feedback is employed, δ is continually driven to zero, so that the following two approximations may be made:

$$T_\delta^2 \ll T_\delta$$

$$T_\delta \rightarrow \frac{\delta}{R_0} \ .$$

Hence, under these conditions, equation C-39 reduces to

$$2K\left(\frac{\delta}{R_0}\right) + L = 0 \ , \qquad \text{(C-40)}$$

which gives:

$$\delta = - \frac{R_0 L}{2K} \quad . \qquad \text{(C-41)}$$

When $\delta \rightarrow O$, $\eta_\delta \longrightarrow 1$ and $\xi_\delta \longrightarrow O$, so that equations C-33 through C-35 reduce to

T = A, U = C, and V = E .

But

$$T = \sin\phi \text{ and } \tan\lambda = \frac{V}{U} \quad .$$

Hence,

$$\phi = \sin^{-1} A \text{ and } \lambda = \tan^{-1} \frac{E}{C} \quad . \qquad \text{(C-42)}$$

The only problem remaining is the evaluation of the lambdas appearing in the matrix of equation C-32. This work has been done in Appendix B; however, the results are repeated below:

$$\Lambda = \sin\phi_1 \cos\phi_2 \cos\phi_3 \sin(\lambda_3 - \lambda_2)$$
$$- \sin\phi_2 \cos\phi_1 \cos\phi_3 \sin(\lambda_3 - \lambda_1)$$
$$+ \sin\phi_3 \cos\phi_1 \cos\phi_2 \sin(\lambda_2 - \lambda_1)$$

$$\Lambda_{11} = \frac{1}{\Lambda} \cos\phi_2 \cos\phi_3 \sin(\lambda_3 - \lambda_2)$$

$$\Lambda_{12} = \frac{1}{\Lambda} \cos\phi_1 \cos\phi_2 \sin(\lambda_2 - \lambda_1)$$

$$\Lambda_{13} = -\frac{1}{\Lambda} \cos\phi_1 \cos\phi_3 \sin(\lambda_3 - \lambda_1)$$

$$\Lambda_{21} = -\frac{1}{\Lambda} \left[\sin\phi_2 \cos\phi_3 \sin\lambda_3 - \sin\phi_3 \cos\phi_2 \sin\lambda_2 \right]$$

$$A_{22} = -\frac{1}{\Lambda}\left[\sin\phi_1 \cos\phi_2 \sin\lambda_2 - \sin\phi_2 \cos\phi_1 \sin\lambda_1\right]$$

$$A_{23} = \frac{1}{\Lambda}\left[\sin\phi_1 \cos\phi_3 \sin\lambda_3 - \sin\phi_3 \cos\phi_1 \sin\lambda_1\right]$$

$$A_{31} = \frac{1}{\Lambda}\left[\sin\phi_2 \cos\phi_3 \cos\lambda_3 - \sin\phi_3 \cos\phi_2 \cos\lambda_2\right]$$

$$A_{32} = \frac{1}{\Lambda}\left[\sin\phi_1 \cos\phi_2 \cos\lambda_2 - \sin\phi_2 \cos\phi_1 \cos\lambda_1\right]$$

and

$$A_{33} = -\frac{1}{\Lambda}\left[\sin\phi_1 \cos\phi_3 \cos\lambda_3 - \sin\phi_3 \cos\phi_1 \cos\lambda_1\right].$$

APPENDIX D

CORRECTION FACTORS TO TRANSFORM GREAT CIRCLE DISTANCES ON THE EARTH TO A STEREOGRAPHIC PROJECTION

1. INTRODUCTION

One philosophy used in coordinate conversion is to convert the measured aircraft-to-station great circle ranges into corresponding straight line ranges on a plane of projection and then to compute in this plane. The stereographic projection is advantageous for this purpose, because:

1. It is a local projection in which the error between the great circle distances on the plane are small for a local region.

2. It is conformal.

3. It is well established in the literature.

This Appendix contains derivations of the necessary conversion expressions to correct the aircraft-to-station great circle ranges into corresponding straight line distances as a stereographic projection line. The projected ranges are simple functions of the great circle ranges and baseline lengths. The projection is described on page 127 of Reference 7.

2. DERIVATION OF CORRECTION FACTORS

Figure D-1 is a pictorial representation of the stereographic projection. Stations 1, 2, and 3 are located on the earth at points P_1, P_2, and P_3 respectively and the aircraft is located above point P_D. Station 2 is chosen as the point of tangency for the plane of projection.

Point O, which is on the surface of the earth diametrically opposing point P_2, is the center of projection. Points P_1' , P_2' , P_3' , and P_D' are the projections of points P_1, P_2, P_3, and P_D, resepectively, on the stereographic projection plane. The distances are defined as follows:

$r_n = P_n P_d$ = earth great circle distance between station n and aircraft.

$r_{np} = P_n' \; P_d'$ = straight line distance between the projections of station n and the aircraft.

$2a = P_1 P_2$ = earth great circle distance between stations 1 and 2.

$2b = P_2 P_3$ = earth great circle distance between stations 2 and 3.

$2a_p = P_1' \; P_2'$ = straight-line distance between the projections of stations 1 and 2.

$2b_p = P_2' \; P_3'$ = straight-line distance between the projections of stations 2 and 3.

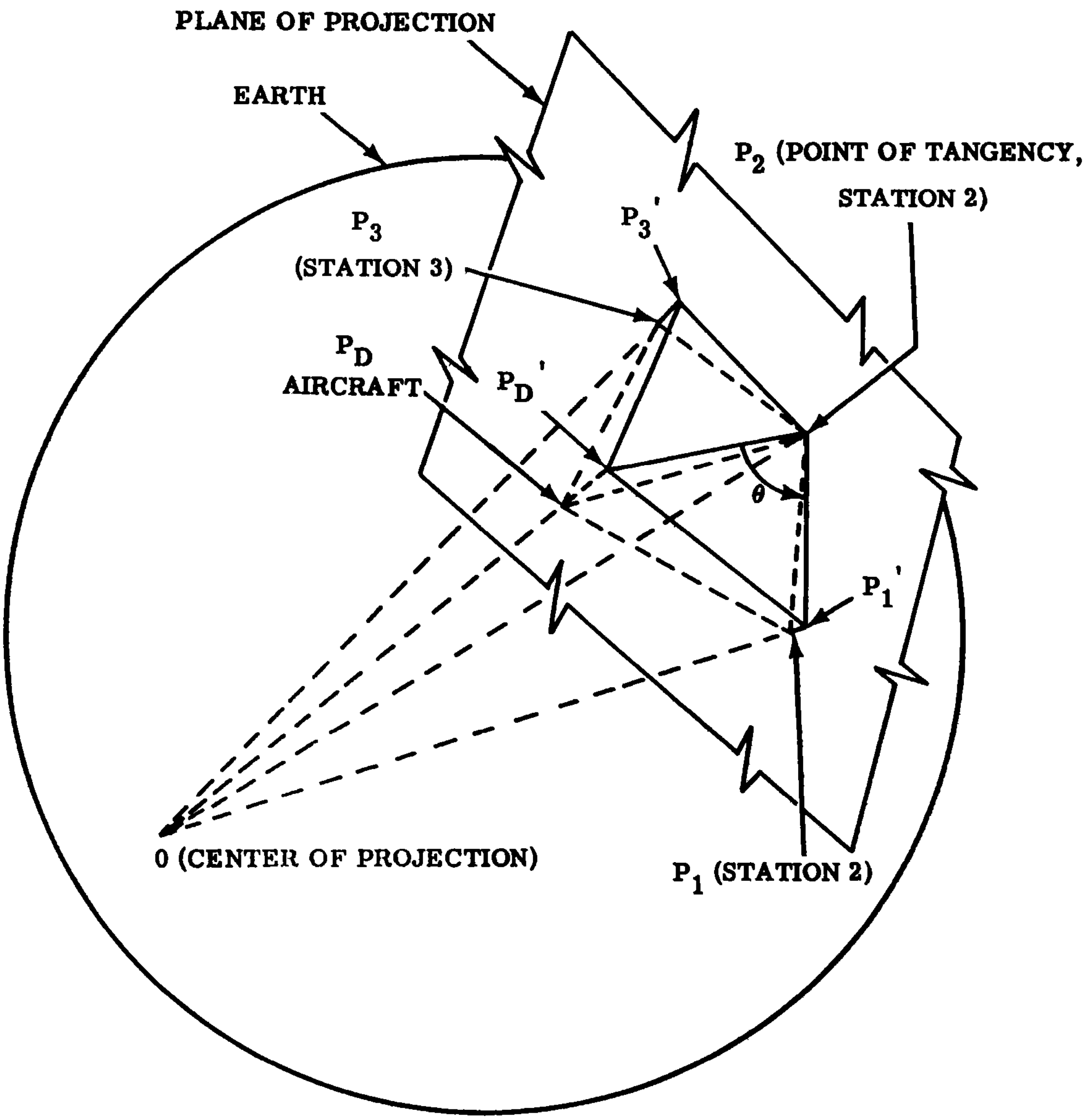


Figure D-1. Stereographic Projection

Let:

R_M = radius of the earth and

θ = angle between P_2P_D and P_2P_1, and $P_2'\ P_D'$ and $P_2'\ P_1'$.

On the surface of the earth

$$\cos\frac{r_1}{R_o} = \cos\frac{2a}{R_o}\cos\frac{r_2}{R_o} + \sin\frac{2a}{R_o}\cos\frac{r_2}{R_o}\cos\theta \cdot \qquad \text{(D-1)}$$

On the plane of projection

$$r_{1p}^2 = (2a_p)^2 + r_{2p}^2 - 2(2a_p)(r_{2p}) \cos\theta \quad . \qquad \text{(D-2)}$$

The equations of projection are.

$$2a_p = 2R_M \tan\frac{2a}{2R_M} \qquad \text{(D-3)}$$

$$r_{2p} = 2R_M \tan\frac{r_2}{2R_M} \quad . \qquad \text{(D-4)}$$

The problem, then, is to solve equations D-1 through D-4 for r_{1p} as a function of r_1, r_2, a and R_m.

Substituting equations D-3 and D-4 into D-2,

$$r_{1p}^2 = 4R_M^2 \tan^2\frac{a}{R_M} + 4R_M^2 \tan^2\frac{r^2}{2R_M}$$

$$- 8R_o^2 \tan\frac{a}{R_M} \tan\frac{r_2}{2R_M} \cos\theta \quad . \qquad \text{(D-5)}$$

By suitable trigonometric manipulation,

$$r_{1p}^2 = 8R_M^2 \frac{1 - \cos\frac{2a}{R_M}\cos\frac{r_2}{R_M} - \sin\frac{2a}{R_M}\sin\frac{r_2}{R_M}\cos\theta}{\left(1 + \cos\frac{r_2}{R_M}\right)\left(1 + \cos\frac{2a}{R_M}\right)} \cdot \qquad \text{(D-6)}$$

Substituting equation D-1 into D-5,

$$r_{1p}^2 = 8R_M^2 \frac{1 - \cos\frac{r_1}{R_M}}{\left(1 + \cos\frac{r_2}{R_M}\right)\left(1 + \cos\frac{2a}{R_M}\right)} \qquad \text{(D-7)}$$

and

$$r_{1p} = \frac{2R_M}{\cos\frac{a}{R_M}} \; \frac{\sin\frac{r_1}{2R_M}}{\cos\frac{r_2}{2R_M}} \; . \tag{D-8}$$

A similar expression can be obtained for r_{3p},

$$r_{3p} = \frac{2R_M}{\cos\frac{b}{R_M}} \; \frac{\sin\frac{r_3}{2R_M}}{\cos\frac{r_2}{2R_M}} \; . \tag{D-9}$$

r_{2p} can be obtained simply from the projection formula,

$$r_{2p} = 2R_M \tan\frac{r_2}{2R_M} \; . \tag{D-10}$$

Equations D-8 through D-10 are then the set of equations necessary for conversion of great circle ranges on the earth to straight line ranges on a stereographic projection.

APPENDIX E

CONVERSION FROM EARTH COORDINATES TO CROSS-TRACK-DISTANCE AND DISTANCE-TO-GO

1. INTRODUCTION

In the FAA problem, it is necessary to compute cross-track-distance and distance-to-go with respect to a selected great circle track. The two distances must be calculated from the longitude and latitude of present-position and the constants defining the track. Section 2 of this Appendix derives cross-track-distance and distance-to-go as functions of present longitude and latitude, destination longitude and latitude, and desired track angle at the destination. In Section 3, approximations of the equations are discussed and in Section 4, a derivation is made of the desired track angle at the destination as a function of the longitude and latitude of the end points of the desired track. Conclusions drawn from the preceding sections are presented in Section 5.

2. DERIVATION

The geometry of the computation is shown in Figure E-1. The symbols are defined as follows:

λ = longitude of present position

ϕ = latitude of present position

λ_D = longitude of destination

ϕ_D = latitude of destination

r_C = cross-course displacement (great circle distance from present position to desired track normal to desired track)

r_R = range-to-go (great circle distance along desired track from intersection with normal from present position to destination)

r_D = great circle distance from present position to destination

β_R = bearing of desired track at destination

β_D = bearing of present position to destination

R_m = radius of the earth.

The following spherical trigonometric relationships can be written from the geometry shown in Figure E-1.

$$\cos \frac{r_D}{r_M} = \sin \phi \sin \phi_D + \cos \phi \cos \phi_D \cos (\lambda_D - \lambda) \qquad \text{(E-1)}$$

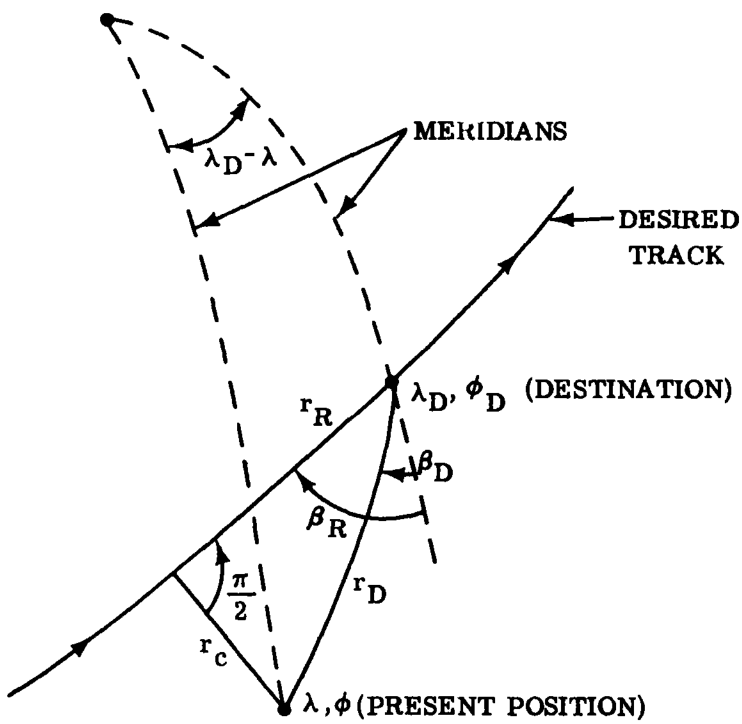


Figure E-1. Geometry of Cross-Track-Distance And Distance-To-Go

$$\frac{\sin(\pi - \beta_D)}{\sin\left(\frac{\pi}{2} - \phi\right)} = \frac{\sin(\lambda_D - \lambda)}{\sin \frac{r_D}{R_M}} \qquad \text{(E-2)}$$

$$\sin \frac{r_c}{R_M} = \sin(\beta_R - \beta_D) \sin \frac{r_D}{R_M} \qquad \text{(E-3)}$$

$$\tan \frac{r_R}{R_M} = \cos(\beta_R - \beta_D) \tan \frac{r_D}{R_M} \quad . \qquad \text{(E-4)}$$

It is desired to obtain $\sin \frac{r_c}{R_M}$ and $\tan \frac{r_R}{R_M}$ as explicit functions of ϕ, ϕ_D, λ, λ_D, and β_D.

Expanding the sin term in equation E-3

$$\sin\frac{r_c}{R_M} = (\sin\beta_R \cos\beta_D - \cos\beta_R \sin\beta_D) \sin\frac{r_D}{R_M}$$

$$= (\sin\beta_R \sqrt{1-\sin^2\beta_D} - \cos\beta_R \sin\beta_D) \sin\frac{r_D}{R_M} \qquad \text{(E-5)}$$

$$= (\sin\beta_R \sin\beta_D \sqrt{\frac{1}{\sin^2\beta_D} - 1} - \cos\beta_R \sin\beta_D) \sin\frac{r_D}{R_M}$$

$$\sin\frac{r_c}{R_M} = (\sin\beta_R \sqrt{\frac{1}{\sin^2\beta_D} - 1} - \cos\beta_R) \sin\beta_D \sin\frac{r_D}{R_M}. \qquad \text{(E-6)}$$

Equation E-2 may be written as

$$\sin\beta_D \sin\frac{r_D}{R_M} = \cos\phi \sin(\lambda_D - \lambda). \qquad \text{(E-7)}$$

Substituting equation E-2 into E-6

$$\sin\frac{r_c}{R_M} = \left(\sin\beta_R \sqrt{\frac{\sin^2\frac{r_D}{R_M}}{\cos^2\phi \sin^2(\lambda_D - \lambda)} - 1} - \cos\beta_R\right) \sin\beta_D \sin\frac{r_D}{R_M} \qquad \text{(E-8)}$$

$$= \sin\beta_R \sqrt{1 - \cos^2\frac{r_D}{R_M} - \cos^2\phi \sin^2(\lambda_D - \lambda)} - \cos\beta_R \cos\phi \sin(\lambda_D - \lambda).$$

Substituting equation E-1 into E-8

$$\sin\frac{r_c}{R_M} = \sin\beta_R \sqrt{\begin{aligned}&1 - \sin^2\phi \sin^2\phi_D - 2\sin\phi \sin\phi_D \cos\phi \cos\phi_D \cos(\lambda_D - \lambda)\\ &- \cos^2\phi \cos^2\phi_D \cos^2(\lambda_D - \lambda) - \cos^2\phi \sin^2(\lambda_D - \lambda)\end{aligned}}$$

$$- \cos\beta_R \cos\phi \sin(\lambda_D - \lambda). \qquad \text{(E-9)}$$

After some computations

$$\sin\frac{r_c}{R_M} = \sin\beta_R\left[\sin\phi_D\cos\phi\cos(\lambda_D-\lambda) - \sin\phi\cos\phi_D\right]$$
$$-\cos\beta_R\cos\phi\sin(\lambda_D-\lambda) \qquad \text{(E-10)}$$

which is the desired expression for $\sin\frac{r_c}{R_M}$.

Expanding the cos term of equation E-4,

$$\tan\frac{r_R}{R_M} = \cos(\beta_R-\beta_D)\tan\frac{r_D}{R_M}$$
$$=\left(\cos\beta_R\sqrt{\frac{1}{\sin^2\beta_D}-1}+\sin\beta_R\right)\frac{\sin\frac{r_D}{R_M}\sin\beta_D}{\cos\frac{r_D}{R_M}} \qquad \text{(E-11)}$$

Substituting equation E-2 into E-11 and performing computations similar to those above,

$$\tan\frac{r_R}{R_M} = \sec\frac{r_D}{R_M}\cos\beta_R\left[\sin\phi_D\cos\phi\cos(\lambda_D-\lambda)\right.$$
$$\left.-\sin\phi\cos\phi_D\right] + \sec\frac{r_D}{R_M}\sin\beta_R\cos\phi\sin(\lambda_D-\lambda) \qquad \text{(E-12)}$$

which is the desired expression for $\tan\frac{r_R}{R_M}$.

Letting $D\phi = \sin\phi_D\cos\phi\cos(\lambda_D-\lambda) - \cos\phi_D\sin\phi$ and (E-13)

$$D\lambda = \cos\phi\sin(\lambda_D-\lambda). \qquad \text{(E-14)}$$

Equations E-10 and E-12 may be written:

$$\sin\frac{r_c}{R_M} = \sin\beta_R D\phi - \cos\beta_R D\lambda \qquad \text{(E-15)}$$

$$\tan\frac{r_R}{R_M} = \cos\beta_R\sec\frac{r_D}{R_M}D\phi + \sin\beta_R\sec\frac{r_D}{R_M}D\lambda. \qquad \text{(E-16)}$$

3. APPROXIMATIONS

Approximations of varying degrees can be made to equations E-15 and E-16, all of which become exact as the cross-track-distance and distance-to-go become small.

3.1 A FIRST APPROXIMATION

One approximation is to let $r_D \rightarrow r_R$. This is a very good approximation since r_c is generally quite small compared to r_R or r_D. Equation E-16 then becomes

$$\tan \frac{r_R}{R_M} = \cos \beta_R \sec \frac{r_R}{R_M} D_\phi + \sin \beta_R \sec \frac{r_R}{R_M} D_\lambda$$

and

$$\sin \frac{r_R}{R_M} = \cos \beta_R D_\phi + \sin \beta_R D_\lambda. \qquad \text{(E-17)}$$

The desired pair of equations then become

$$\sin \frac{r_c}{R_M} = \sin \beta_R D_\phi - \cos \beta_R D_\lambda \qquad \text{(E-18)}$$

and

$$\sin \frac{r_R}{R_M} = \cos \beta_R D_\phi + \sin \beta_R D_\lambda. \qquad \text{(E-19)}$$

The error incurred in using the approximation, equation E-17, rather than the exact, equation E-16, was examined for numerous conditions. It was found that the error was small compared to other system errors. As an example of a pessimistic condition, the error is 47 meters at a distance-to-go of 1000 nautical miles and a cross-course-distance of 25 nautical miles.

Equations E-15 and E-17 can be written compactly and in terms of previously used variables by means of the following identities:

$$D_{11} = -\cos \phi_D \sin \beta_R \qquad \text{(E-20)}$$

$$D_{12} = \sin \phi_D \cos \lambda_D \sin \beta_R - \sin \lambda_D \cos \beta_R \qquad \text{(E-21)}$$

$$D_{13} = \sin \phi_D \sin \lambda_D \sin \beta_R + \cos \lambda_D \cos \beta_R \qquad \text{(E-22)}$$

$$D_{21} = -\cos \phi_D \cos \beta_R \qquad \text{(E-23)}$$

$$D_{22} = \sin \phi_D \cos \lambda_D \cos \beta_R + \sin \lambda_D \sin \beta_R \qquad \text{(E-24)}$$

$$D_{23} = \sin \phi_D \sin \lambda_D \cos \beta_R - \cos \lambda_D \sin \beta_R \qquad \text{(E-25)}$$

$$T = \sin\phi \tag{E-26}$$

$$U = \cos\phi \cos\lambda \tag{E-27}$$

and

$$V = \cos\phi \sin\lambda . \tag{E-28}$$

$\frac{r_c}{R_M}$ can be set equal to $\sin\frac{r_c}{R_M}$ with negligible error and equations E-12 and E-17 then become:

$$\frac{r_c}{R_M} = D_{11}\ T + D_{12}\ U + D_{13}\ V \tag{E-29}$$

$$\sin\frac{r_R}{R_M} = D_{21}\ T + D_{22}\ U + D_{23}\ V. \tag{E-30}$$

3.2 A SECOND APPROXIMATION

If the distances r_c, r_R, and r_D are small compared to the radius of the earth, a second approximation can be obtained by letting

$$\sin(\lambda_D-\lambda) \rightarrow \lambda_D - \lambda \qquad \sin\frac{r_c}{R_M} \rightarrow \frac{r_c}{R_M}$$

$$\cos(\lambda_D-\lambda) \rightarrow 1$$

$$\sin(\phi_D-\phi) \rightarrow \phi_D-\phi \qquad \sin\frac{r_R}{R_M} \rightarrow \frac{r_R}{R_M} .$$

D_ϕ becomes

$$D_\phi = \sin\phi_D \cos\phi \cos(\lambda_D-\lambda) - \cos\phi_D \sin\phi$$

$$= \sin\phi_D \cos\phi - \cos\phi_D \sin\phi = \sin(\phi_D-\phi).$$

$$D_\phi = \phi_D - \phi \tag{E-31}$$

and D_λ becomes

$$D_\lambda = \cos\phi \sin(\lambda_D-\lambda) \tag{E-32}$$

$$D_\lambda = \cos\phi (\lambda_D-\lambda) \tag{E-33}$$

Substituting equations E-32 and E-33 into equations E-15 and E-17, the desired pair become

$$\frac{r_c}{R_M} \doteq \sin\frac{r_R}{R_M} = \sin\beta_R(\phi_D-\phi) - \cos\beta_R \cos\phi (\lambda_D-\lambda) \tag{E-34}$$

and

$$\frac{r_R}{R_M} \doteq \sin\frac{r_R}{R_M} = \cos\beta_R(\phi_D-\phi) + \sin\beta_R \cos\phi (\lambda_D-\lambda) \tag{E-35}$$

4. COMPUTATION OF TRACK ANGLE AT THE DESTINATION

In some instances, it may be desirable to set into the computer, the longitudes and latitudes of the destination and a starting point rather than the longitude and latitude of the destination and the track angle at the destination. In this section expressions are derived for the sine and cosine of the desired track angle in terms of the starting point and destination longitudes and latitudes. These functions may then be used in equations E-29 and E-30 in place of the sine and cosine of the desired track angle. The geometry is depicted in Figure E-2.

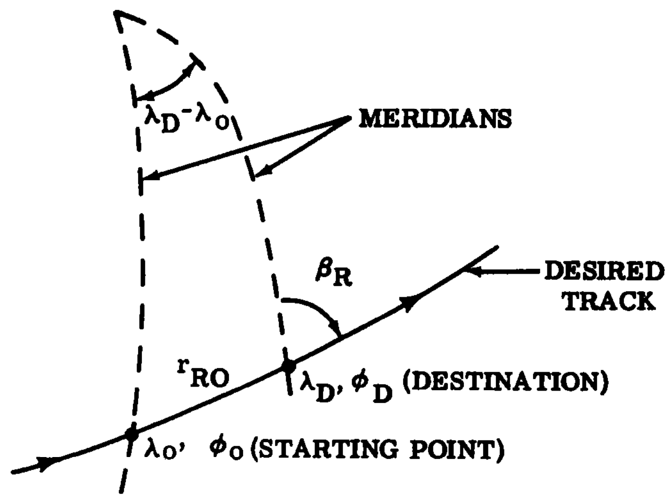


Figure E-2. Geometry of Track Angle Computation.

The symbols used are defined as follows:

λ_O = longitude of starting point

λ_D = longitude of destination

ϕ_O = latitude of starting point

ϕ_D = latitude of destination

β_R = track angle at destination

r_{R_O} = great circle distance between starting point and destination

and

R_m = radius of the earth.

The following relationships follow from the geometry.

$$\frac{\sin(\pi - \beta_R)}{\sin\left(\frac{\pi}{2} - \phi_o\right)} = \frac{\sin(\lambda_D - \lambda_o)}{\sin \frac{r_{R_o}}{R_M}} \tag{E-36}$$

$$\sin \beta_R = \frac{\cos \phi_o \sin(\lambda_D - \lambda_o)}{\sin \frac{r_{R_o}}{R_M}}$$

and

$$\sin \beta_R = \frac{\cos \phi_o \cos \lambda_o \sin \lambda_D - \cos \phi_o \sin \lambda_o \cos \lambda_D}{\sin \frac{r_{R_o}}{R_M}} \tag{E-37}$$

which is one of the desired expressions.

$$\sin \phi_o = \cos \frac{r_{R_o}}{R_M} \sin \phi_D - \sin \frac{r_{R_o}}{R_M} \cos \phi_D \cos \beta_R \tag{E-38}$$

and

$$\cos \beta_R = \frac{\cos \frac{r_{R_o}}{R_M} \sin \phi_D - \sin \phi_o}{\sin \frac{r_{R_o}}{R_M} \cos \phi_D} . \tag{E-39}$$

But

$$\cos \frac{r_{r_o}}{R_M} = \sin \phi_o \sin \phi_D + \cos \phi_o \cos \phi_D \cos(\lambda_D - \lambda_o) \tag{E-40}$$

and equation E-34 becomes

$$\cos \beta_R = \frac{\sin \phi_D \cos \phi_o \cos(\lambda_D - \lambda_o) - \cos \phi_D \sin \phi_o}{\sin \frac{r_{R_o}}{R_M}} \tag{E-41}$$

$$= \frac{\cos \phi_o \cos\lambda_o \sin\phi_D \cos \lambda_D + \cos \phi_o \sin\lambda_o \sin\phi_D \sin \lambda_D - \sin\phi_o \cos\phi_D}{\sin \frac{r_{R_o}}{R_M}} .$$

The required computations are summarized below:

$$\sin \beta_R = \frac{A_o \sin \lambda_D - B_o \cos \lambda_D}{S} \tag{E-42}$$

$$\cos \beta_R = \frac{A_o \sin \phi_D \cos \lambda_D + B_o \sin \phi_D \sin \lambda_D - \sin \phi_o \cos \phi_D}{S} \tag{E-43}$$

$$S = \sqrt{1 - C^2} \tag{E-44}$$

$$C = \sin \phi_o \sin \phi_D + A_o A_D + B_o B_D \tag{E-45}$$

$$A_o = \cos \phi_o \cos \lambda_o \tag{E-46}$$

$$A_D = \cos \phi_D \cos \lambda_D \tag{E-47}$$

$$B_o = \cos \phi_o \sin \lambda_o \tag{E-48}$$

and

$$B_D = \cos \phi_D \sin \lambda_D . \tag{E-49}$$

5. CONCLUSIONS

Equations were derived for finding cross-track-distance and distance-to-go as functions of the latitude and longitude of present-position and destination, and the desired angle at the destination. Approximations of these equations can be made if the distances involved are short. Actually the equations represent a transformation from one sperical coordinate system to another if the desired track is regarded as a local equator. Distance-to-go then becomes a measure of local longitude and cross-course displacement is a measure of local latitude. Equations were also derived which allow the cross-track-distance and distance-to-go to be computed using the latitude and longitude of an initial position rather than track angle at the destination.

Equations E-20 through E-30 involve an approximation but the approximation contributes negligible error to the overall computation and allows the desired quantities to be expressed as linear functions of previously used variables. For this reason equations E-20 through E-30 are recommened for use in the FAA coordinate converter.

APPENDIX F

THE EFFECTS OF RANGE ERRORS AND ECCENTRICITY ON THE ACCURACY OF HYPERBOLIC TO EARTH COORDINATE CONVERSION

1. INTRODUCTION

Two methods are available for converting range information, as measured by a receiver in a three-station radio system, into the latitude and longitude coordinates. In the first, two pairs of ranges are subtracted to obtain two hyperbolic coordinates, thereby cancelling drift common to the three ranges. The ranges are assumed to be geodesic. Longitude and latitude are formulated as explicit functions of the hyperbolic coordinates. In the second, latitude and longitude are found from two ranges and the third range is used to compute a measure of drift common to the three ranges, which is then integrated and fed back to eliminate the drift. In either system, the equations involved are based upon a spherical earth and do not include the effects of eccentricity. Errors due to this omission are investigated in this appendix and are found to be larger than tolerable for the FAA problem. Two correction factors are suggested that can be added to each range. The error in computed position with either correction applied is found to be sufficiently small. To arrive at the above errors, it is necessary to first find the coefficients relating error in computed position to the errors in the radio range inputs. These coefficients are general and can be used in any error analysis of the system. They are not limited only to investigation of eccentricity effects. It should be emphasized that the coefficients and errors so derived apply to both methods of coordinate conversion.

Section 2 contains a derivation of the error coefficients. In section 3 the coefficients are plotted relating total rms position error to rms range error. The error in computed position as a result of eccentricity effects is found in Section 4. The suggested correction factors and their associated errors are given in Section 5.

2. DERIVATION OF ERROR COEFFICIENTS

The system geometry is depicted in Figure 1. (The earth is assumed spherical.)

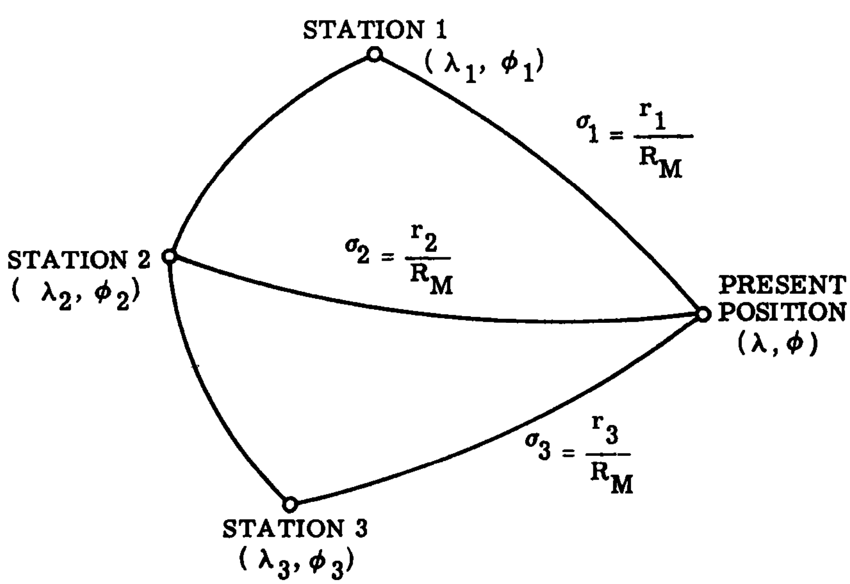


Figure F-1. System Geometry

The symbols are defined as follows:

λ = longitude of present position

ϕ = latitude of present position

λ_n = longitude of station "n"

ϕ_n = latitude of station "n"

r_n = great circle distance from present position to station n

σ_n = spherical angle subtended by $r_n = \frac{r_n}{R_M}$

$r_{12} = r_1 - r_2$ = hyperbolic coordinate

$r_{23} = r_2 - r_3$ = hyperbolic coordinate

$\sigma_{12} = r_{12}/R_M = \sigma_1 - \sigma_2$

$\sigma_{23} = r_{23}/R_M = \sigma_2 - \sigma_3$

R_M = mean radius of the earth at station 2.

The following three equations relate the latitude and longitude to the ranges:

$$\cos \sigma_1 = \sin\phi_1 \sin\phi + \cos\phi_1 \cos\phi \cos(\lambda - \lambda_1) \tag{F-1}$$

$$\cos \sigma_2 = \sin\phi_2 \sin\phi + \cos\phi_2 \cos\phi \cos(\lambda - \lambda_2) \tag{F-2}$$

$$\cos \sigma_3 = \sin\phi_3 \sin\phi + \cos\phi_3 \cos\phi \cos(\lambda - \lambda_3) \cdot \tag{F-3}$$

Differentiating equations F-1, F-2, and F-3 with respect to σ_{12} and solving for the appropriate derivatives,

$$\frac{\partial\sigma_1}{\sigma_{12}} = \kappa_1 \frac{\partial\phi}{\partial\sigma_{12}} + \ell_1 \frac{\partial\lambda}{\partial\sigma_{12}} \tag{F-4}$$

$$\frac{\partial\sigma_2}{\sigma_{12}} = \kappa_2 \frac{\partial\phi}{\partial\sigma_{12}} + \ell_2 \frac{\partial\lambda}{\partial\sigma_{12}} \tag{F-5}$$

$$\frac{\partial\sigma_3}{\sigma_{12}} = \kappa_3 \frac{\partial\phi}{\partial\sigma_{12}} + \ell_3 \frac{\partial\lambda}{\partial\sigma_{12}} \tag{F-6}$$

where:

$$\kappa_1 = \frac{\cos\phi_1 \sin\phi \cos(\lambda-\lambda_1) - \sin\phi_1 \cos\phi}{\sin\sigma_1} \tag{F-7}$$

$$\kappa_2 = \frac{\cos\phi_2 \sin\phi \cos(\lambda-\lambda_2) - \sin\phi_2 \cos\phi}{\sin\sigma_2} \tag{F-8}$$

$$\kappa_3 = \frac{\cos\phi_3 \sin\phi \cos(\lambda-\lambda_3) - \sin\phi_3 \cos\phi}{\sin\sigma_3} \tag{F-9}$$

$$\ell_1 = \frac{\cos\phi_1 \cos\phi \sin(\lambda-\lambda_1)}{\sin\sigma_1} \tag{F-10}$$

$$\ell_2 = \frac{\cos\phi_2 \cos\phi \sin(\lambda-\lambda_2)}{\sin\sigma_2} \tag{F-11}$$

$$\ell_3 = \frac{\cos\phi_3 \cos\phi \sin(\lambda-\lambda_3)}{\sin\sigma_3} \tag{F-12}$$

By definition

$$\sigma_{12} = \sigma_1 - \sigma_2 \tag{F-13}$$

and

$$\sigma_{23} = \sigma_2 - \sigma_3 \quad . \tag{F-14}$$

Differentiating equations F-13 and F-14 with respect to σ_{12}

$$1 = \frac{\partial\sigma_1}{\partial\sigma_{12}} - \frac{\partial\sigma_2}{\partial\sigma_{12}} \tag{F-15}$$

$$0 = \frac{\partial\sigma_2}{\partial\sigma_{12}} - \frac{\partial\sigma_3}{\partial\sigma_{12}} \quad . \tag{F-16}$$

Substituting equations F-4, F-5, and F-6 into equations F-15 and F-16

$$1 = (\kappa_1 - \kappa_2) \frac{\partial \phi}{\partial \sigma_{12}} + (\ell_1 - \ell_2) \frac{\partial \lambda}{\partial \sigma_{12}} \qquad \text{(F-17)}$$

$$0 = (\kappa_2 - \kappa_3) \frac{\partial \phi}{\partial \sigma_{12}} + (\ell_2 - \ell_3) \frac{\partial \lambda}{\partial \sigma_{12}} \qquad \text{(F-18)}$$

Solving equations F-17 and F-18

$$\frac{\partial \phi}{\partial \sigma_{12}} = \frac{\ell_2 - \ell_3}{D} \qquad \text{(F-19)}$$

$$\frac{\partial \lambda}{\partial \sigma_{12}} = -\frac{\kappa_2 + \kappa_3}{D} \qquad \text{(F-20)}$$

where:

$$D = (\kappa_1 - \kappa_2)(\ell_2 - \ell_3) - (\kappa_2 - \kappa_3)(\ell_1 - \ell_2) \; . \qquad \text{(F-21)}$$

By differentiating equations F-1, F-2, and F-3 with respect to σ_{23} and proceeding in a similar manner, it can be shows that

$$\frac{\partial \phi}{\partial \sigma_{23}} = \frac{-\ell_1 + \ell_2}{D} \qquad \text{(F-23)}$$

$$\frac{\partial \lambda}{\partial \sigma_{23}} = \frac{\kappa_1 - \kappa_2}{D} \qquad \text{(F-24)}$$

$$d\phi = \frac{\partial \phi}{\partial \sigma_{12}} d\sigma_{12} + \frac{\partial \phi}{\partial \sigma_{23}} d\sigma_{23} \qquad \text{(F-25)}$$

$$d\lambda = \frac{\partial \lambda}{\partial \sigma_{23}} d\sigma_{23} + \frac{\partial \lambda}{\partial \sigma_{23}} d\sigma_{23} \qquad \text{(F-26)}$$

$$d\sigma_{12} = d\sigma_1 - d\sigma_2 \qquad \text{(F-27)}$$

$$d\sigma_{23} = d\sigma_2 - d\sigma_3 \; . \qquad \text{(F-28)}$$

Substituting equations F-23 and F-24 into equations F-20 and F-21

$$d\phi = a_{\phi 1}\, d\sigma_1 + a_{\phi 2}\, d\sigma_2 + a_{\phi 3}\, d\sigma_3 \qquad \text{(F-29)}$$

$$d\lambda = a_{\lambda 1}\, d\sigma_1 + a_{\lambda 2}\, d\sigma_2 + a_{\lambda 3}\, d\sigma_3 \qquad \text{(F-30)}$$

where:

$$a_{\phi 1} = \frac{\partial \phi}{\partial \sigma_{12}} = \frac{\ell_2 - \ell_3}{D} \qquad \text{(F-31)}$$

$$a_{\phi 2} = \frac{-\partial \phi}{\partial \sigma_{12}} + \frac{\partial \phi}{\partial \sigma_{23}} = \frac{\ell_3 - \ell_1}{D} \qquad \text{(F-32)}$$

$$a_{\phi 3} = -\frac{\partial \phi}{\partial \sigma_{23}} = \frac{\ell_1 - \ell_2}{D} \qquad \text{(F-33)}$$

$$a_{\lambda 1} = \frac{\partial \lambda}{\partial \sigma_{12}} = -\frac{\kappa_2 + \kappa_3}{D} \qquad \text{(F-34)}$$

$$a_{\lambda 2} = -\frac{\partial \lambda}{\partial \sigma_{12}} + \frac{\partial \lambda}{\partial \sigma_{23}} = -\frac{\kappa_3 + \kappa_1}{D} \qquad \text{(F-35)}$$

$$a_{\lambda 3} = -\frac{\partial \lambda}{\partial \sigma_{23}} = -\frac{\kappa_1 + \kappa_2}{D} \qquad \text{(F-36)}$$

and where

$a\phi_n$ = ratio of error in latitude ϕ to error in range angle σ_n

$a\lambda_n$ = ratio of error in longitude λ to error in range angle σ_n.

In terms of distances, the coefficients become

$a_{xn} = a_{\lambda n} \cos\phi$ = ratio of component of error in position along a parallel to error in range r_n and (F-37)

$a_{yn} = a_{\phi n}$ = ratio of component of error in position along a meridian to error in range r_n. (F-38)

Summarizing the coefficients

$$a_{x1} = \frac{-\kappa_2 + \kappa_3}{D} \cos\phi \quad \text{(F-39)}$$

$$a_{x2} = \frac{-\kappa_3 + \kappa_1}{D} \cos\phi \quad \text{(F-40)}$$

$$a_{x3} = \frac{-\kappa_1 + \kappa_2}{D} \cos\phi \quad \text{(F-41)}$$

$$a_{y1} = \frac{\ell_2 - \ell_3}{D} \quad \text{(F-42)}$$

$$a_{y2} = \frac{\ell_3 - \ell_1}{D} \quad \text{(F-43)}$$

$$a_{y3} = \frac{\ell_1 - \ell_2}{D} \quad \text{(F-44)}$$

$$k_n = \frac{\cos\phi_n \sin\phi \cos(\lambda - \lambda_n) - \sin\phi_n \cos\phi}{\sin\sigma_n} \quad \text{(F-45)}$$

$$\ell_n = \frac{\cos\phi_n \cos\phi \sin(\lambda - \lambda_n)}{\sin\sigma_n} \quad \text{(F-46)}$$

where D equals the value given in equation F-21.

The coefficients in equations F-35 through F-40 define the error in computed position in terms of errors in each range and are used in the paragraphs that follow.

3. RATIO OF RMS POSITION ERROR TO RMS RANGE ERROR

It is desirable to find one factor which is a meaningful measure of the ratio of error in computed position to error in the range inputs. Probably the factor which best satisfies this condition is the ratio of rms error in computed position to rms error in each range input, assuming that the errors in the ranges are statistical in nature, independent, but equal in rms value. This factor is derived as follows:

$$\Delta x = a_{x1} \Delta r_1 + a_{x2} \Delta r_2 + a_{x3} \Delta r_3 \quad \text{(F-47)}$$

$$\Delta y = a_{y1}\ \Delta r_1 + a_{y2}\ \Delta r_2 + a_{y3}\ \Delta r_3\ . \qquad \text{(F-48)}$$

where

Δ denotes error in the prefixed variable

x = distance along a parallel

y = distance along a meridian.

$$(\Delta p)^2 = (\Delta x)^2 + (\Delta y)^2 \qquad \text{(F-49)}$$

where

$$\overline{(\Delta p)^2} = \overline{(\Delta x)^2} + \overline{(\Delta y)^2} \qquad \text{(F-50)}$$

where the bar denotes ensemble average.

Since Δr_1, Δr_2, and Δr_3 are assumed independent

$$\overline{(\Delta p)^2} = (a_{x1}^{\ 2} + a_{y1}^{\ 2})\overline{(\Delta r_1)^2} + (a_{x2}^{\ 2} + a_{y2}^{\ 2})\overline{(\Delta r_1)^2} + (a_{x3}^{\ 2} + a_{y3}^{\ 2})\overline{(\Delta r_3)^2}\ . \qquad \text{(F-51)}$$

letting

$$\overline{(\Delta p)^2} = \sigma p^2$$

$$\overline{(\Delta r_1)^2} = \overline{(\Delta r_2)^2} = \overline{(\Delta r_3)^2} = \sigma r^2$$

then

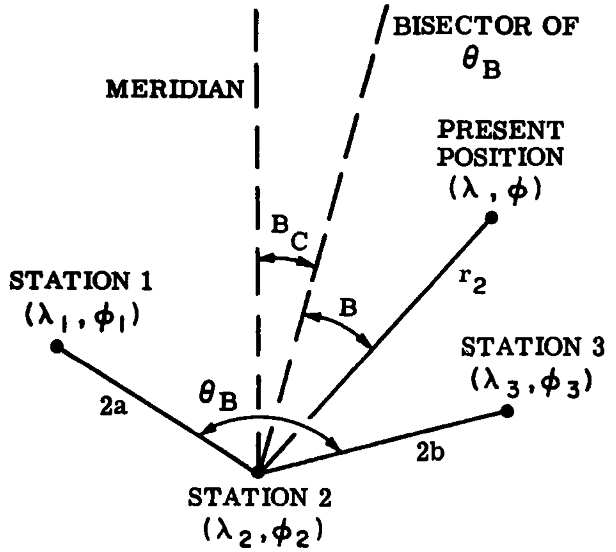
$$\sigma_p = \sigma_r \sqrt{a_{x1}^{\ 2} + a_{x2}^{\ 2} + a_{x3}^{\ 2} + a_{y1}^{\ 2} + a_{y2}^{\ 2} + a_{y3}^{\ 2}} \qquad \text{(F-52)}$$

and the desired ratio of rms error in computed position to rms error in the ranges is

$$\frac{\sigma_p}{\sigma_r} = \sqrt{a_{x1}^{\ 2} + a_{x2}^{\ 2} + a_{x3}^{\ 2} + a_{y1}^{\ 2} + a_{y2}^{\ 2} + a_{y3}^{\ 2}} \qquad \text{(F-53)}$$

This factor was calculated for various positions in two different station configurations using equations F-53 and F-39 through F-46. A general station configuration is shown in Figure F-2.

Both configurations selected contain a baseline angle θ_B of 150 degrees. Configuration 1 has baseline lengths, 2a and 2b, of 500 nautical miles while configuration 2 has baseline lengths of 1000 nautical miles. These configurations are considered typical of those to be encountered in the FAA problem. In fact, configuration 1 closely resembles the Cytac test configuration on the Atlantic coast. It should be noted that the factor $\frac{\sigma P}{\sigma r}$ is independent of the direction in which the station configuration is oriented (β_C) and the latitude and longitude of the stations, but depends upon the baseline angle and length



θ_B = ANGLE BETWEEN BASE-LINES,

B_C = BEARING OF BISECTOR OF BASELINE ANGLE WITH RESPECT TO A MERIDIAN,

B = BEARING OF RANGE LINE γ_2 WITH RESPECT TO BISECTION OF BASELINE ANGLE, AND

2a AND 2b = BASELINE LENGTHS.

Figure F-2. General Station Configuration

and the present position relative to the stations. This occurs even though the coefficients α_{x1}, α_{x2}, etc., do change with the orientation and location of the station configuration relative to the earth coordinates.

In the following figures, $\frac{\sigma_p}{\sigma_r}$ is plotted versus r_2 for values of β from $+70^o$ to -70^o (within 5^o of the baselines). Figure F-3 is for configuration 1 (500 nautical mile baselines) and Figure F-4 is for configuration 2 (1000 nautical mile baselines). It is assumed that the maximum range from any station is 2000 nautical miles. The curves are plotted only to the point where the range to each of the three stations is less than 2000 nautical miles.

It is seen that the error factor is less than 14 for all positions in the field of the 1000 nautical mile baseline configuration and less than 9 for all positions to within 15^o of the baselines. However, in the 500 nautical mile baseline configuration, the factor is 34 in the middle of the field at maximum range and increases to 115 at 15^o from the baselines and 375 at 5^o from the baselines. Hence, the 500 nautical mile configuration is very unfavorable for a range coverage of 2000 nautical miles, except possibly in the center of the field.

4. ERROR IN COMPUTED POSITION DUE TO ECCENTRICITY

In the computation of present-position, assuming the earth to be spherical, the measured radio range r_n is assumed to subtend a spherical angle σ_n, such that

$$\sigma_n = \frac{r_n}{R_M} \qquad \text{(F-54)}$$

where R_M is the mean radius of earth curvature at station 2. The calculation of earth coordinates, latitude and longitude, is then carried out in terms of the σ_n using spherical trigonometry. The r_n are geodetics, and because of eccentricity, do not yield precisely the correct geodetic latitude and longitude. If r_n' represents the ranges necessary to compute the correct longitude and latitude then

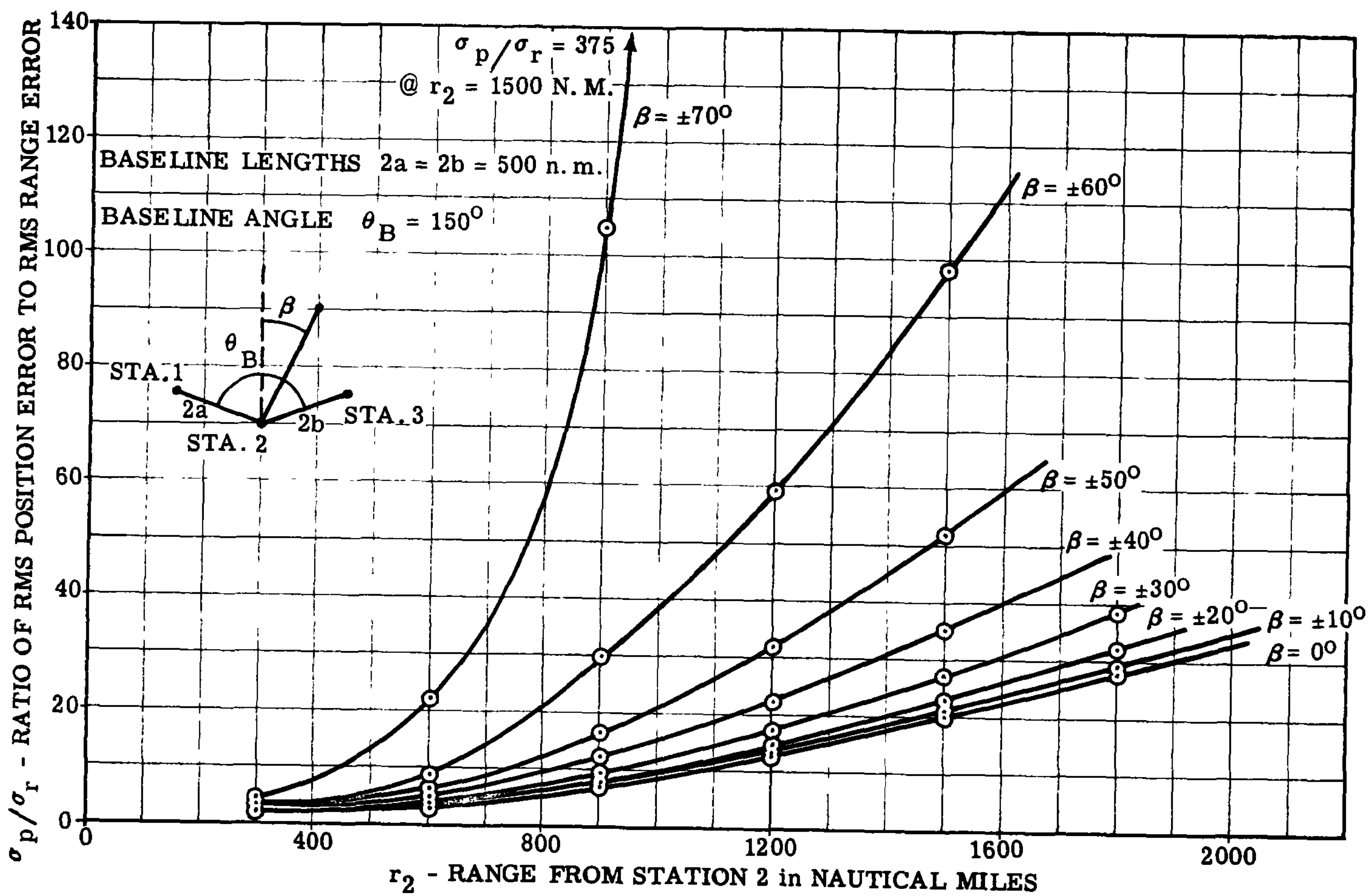


Figure F-3. Ratio of RMS Error in Computed Position to RMS Error in Each Range

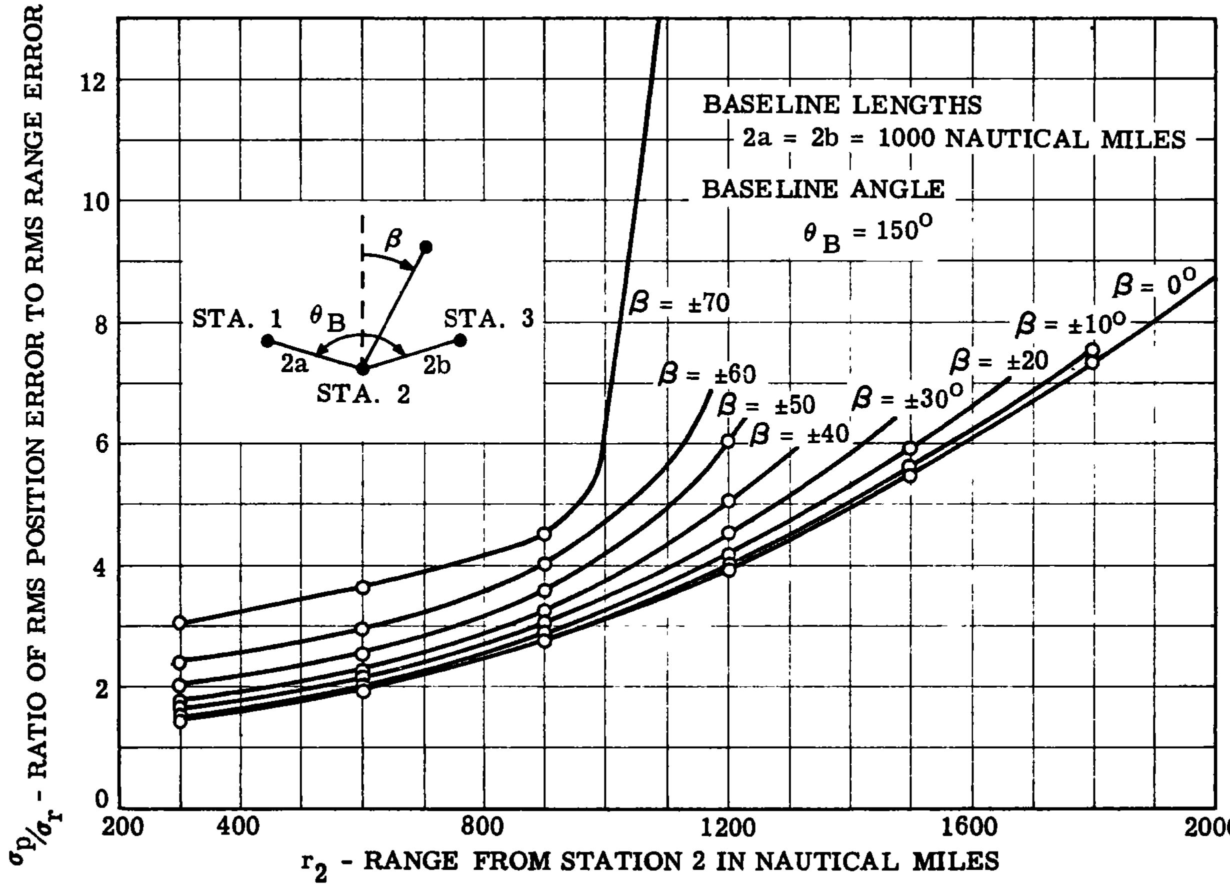


Figure F-4. Ratio of RMS Error in Computed Position to RMS Error in Each Range

$$\Delta r_n = r_n - r_n' \qquad \text{(F-55)}$$

where

Δr_n = error in input range due to eccentricity.

A formula for the geodetic distance between two points on the surface of the earth due to Lambert and Andoyer, is given in References 8 and 9. The formula, which is accurate to better than one part in 200, 000, is applied to r_n, giving

$$r_n = R_e \sigma_n' + \frac{R_e f}{2}\left[(3 \sin \sigma_n' - \sigma_n') \frac{\cos^2\left(\frac{\phi-\phi_n}{2}\right)}{\cos^2\left(\frac{\sigma_n'}{2}\right)} \sin^2\left(\frac{\phi+\phi_n}{2}\right) - (3 \sin \sigma_n' + \sigma_n') \frac{\sin^2\left(\frac{\phi-\phi_n}{2}\right)}{\sin^2\left(\frac{\sigma_n'}{2}\right)} \cos^2\left(\frac{\phi+\phi_n}{2}\right)\right] \qquad \text{(F-56)}$$

where

R_e = equatorial radius of the earth = 3443.956 nautical miles

f = polar flattening of the earth = .00339008.

σ_n' is found from

$$\cos \sigma_n' = \sin\phi \sin\phi_n + \cos\phi \cos\phi_n \cos(\lambda - \lambda_n) \qquad \text{(F-57)}$$

$$r_n' = R_M \sigma_n' . \qquad \text{(F-58)}$$

Substituting equations F-56 and F-58 into F-55

$$\Delta r_n = (R_e - R_M)\sigma_n' + \frac{R_e f}{2}\left[(3 \sin \sigma_n' - \sigma_n') \frac{\cos^2\left(\frac{\phi-\phi_n}{2}\right)}{\cos^2\left(\frac{\sigma_n'}{2}\right)} \sin^2\left(\frac{\phi+\phi_n}{2}\right) - (3 \sin \sigma_n' + \sigma_n') \frac{\sin^2\left(\frac{\phi-\phi_n}{2}\right)}{\sin^2\left(\frac{\sigma_n'}{2}\right)} \cos^2\left(\frac{\phi+\phi_n}{2}\right)\right] . \qquad \text{(F-59)}$$

The total error in computed position due to eccentricity, where the computation is based on a spherical earth, can now be determined as follows:

1. Given the longitude and latitude of present position and the three stations, compute σ_1', σ_2', and σ_3' from equation F-57.

2. Compute Δr_1, Δr_2, and Δr_3 from equation F-59.

3. Compute α_{x1}, α_{x2}, α_{x3}, α_{y1}, α_{y2}, and α_{y3} from equations F-39 through F-44.

4. Compute Δ_x and Δ_y from equations F-47 and F-48.

5. Compute total position error Δ_p from equation F-49.

These steps were programmed on the LGP-30 digital computer. Seven runs were made in which Δ_p was computed for various positions in the field of operation. The results are plotted in Figures F-5 through F-11. All of the station configurations contained a baseline angle (θ_B) of 150°. In Figures F-5, F-6, and F-7, the baseline lengths (2a and 2b) were 500 nautical miles, whereas in Figures F-8 through F-11 they were 1000 nautical miles. Station 2 was located at longitude 60°E and latitude 45°N in Figures F-5 through F-10 and at the North Pole in Figure F-11. The baseline angle bisector was oriented north ($\beta_c = 0°$) in Figures F-5 and F-8, East ($\beta_c = 90°$) in Figures F-6 and F-9, and south ($\beta_c = 180°$) in Figures F-7 and F-10. It is felt that these configurations adequately define the error for Northern Hemisphere coverage.

In the figures, Δ_p is plotted against range (r_2) from the second station, for various angles (β) from the baseline angle bisector. The curves are plotted only for positions within 2000 nautical miles of each station. In Figure F-5, F-6, and F-7, which represent the 500 nautical mile baseline configuration, the error ranges from 2 to 25 nautical miles, except for $\beta = \pm 70°$, where the error is as high as 200 nautical miles. In Figures F-8, F-9, and F-10, which represent the 1000 nautical mile baseline configuration at 45° latitude, the error ranges from 1 to 16 nautical miles. In Figure F-11, which represents the 1000 nautical mile baseline configuration at the North Pole, the error is less than 4 nautical miles. In the FAA problem it is desired to keep the error in computed position to less than 2 nautical miles. Thus, it is necessary to include an eccentricity correction in the computations.

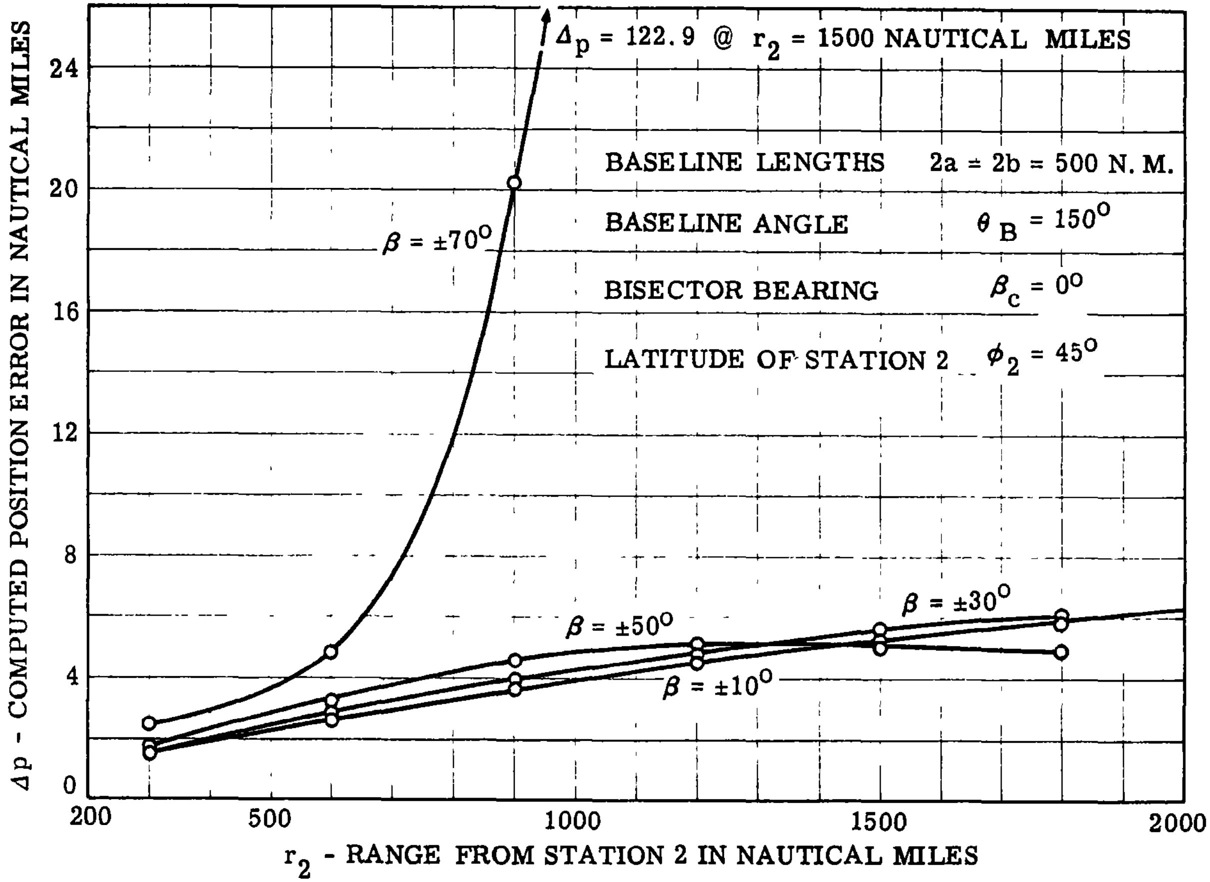


Figure F-5. Error in Computed Position Due to Eccentricity

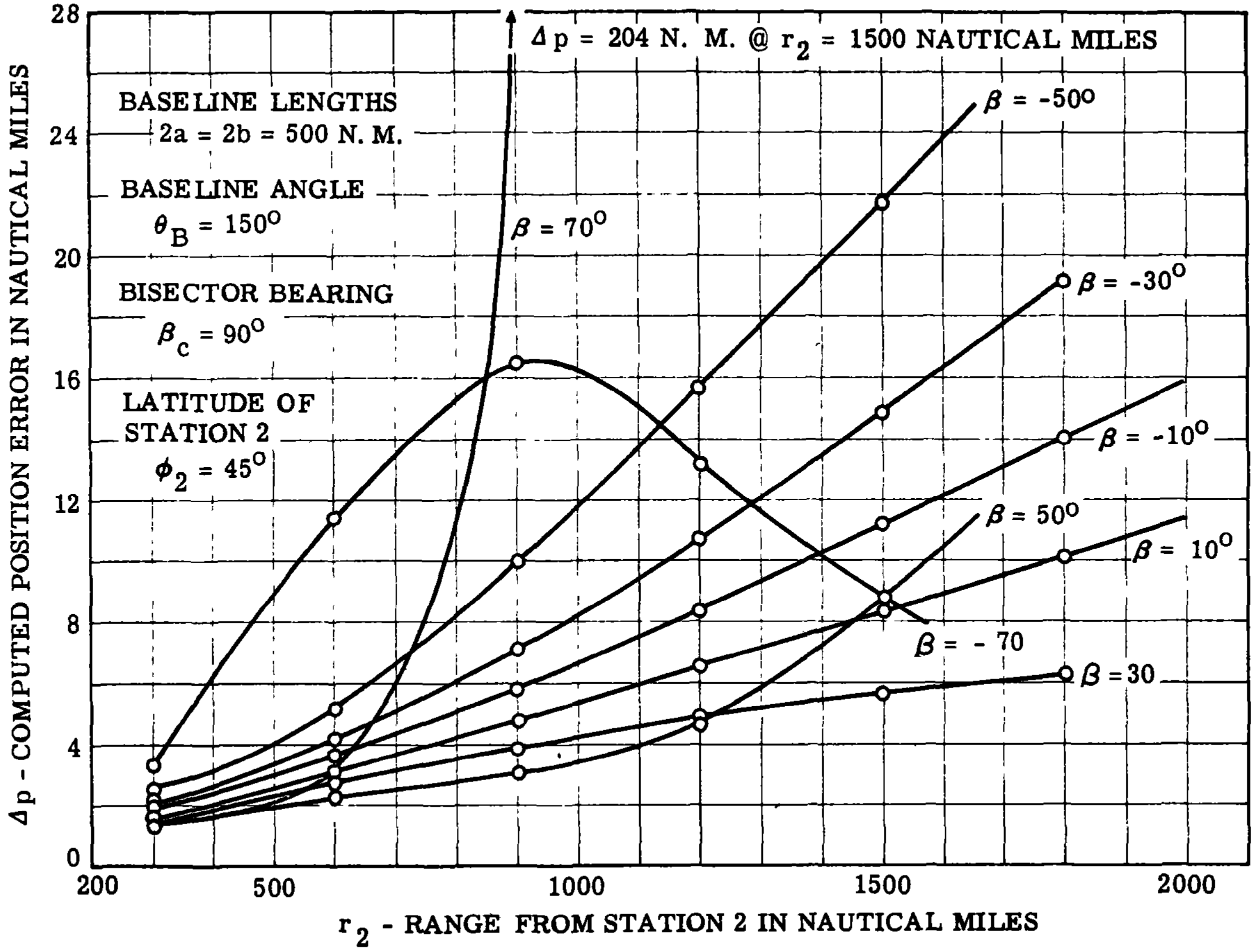


Figure F-6. Error in Computed Position Due to Eccentricity

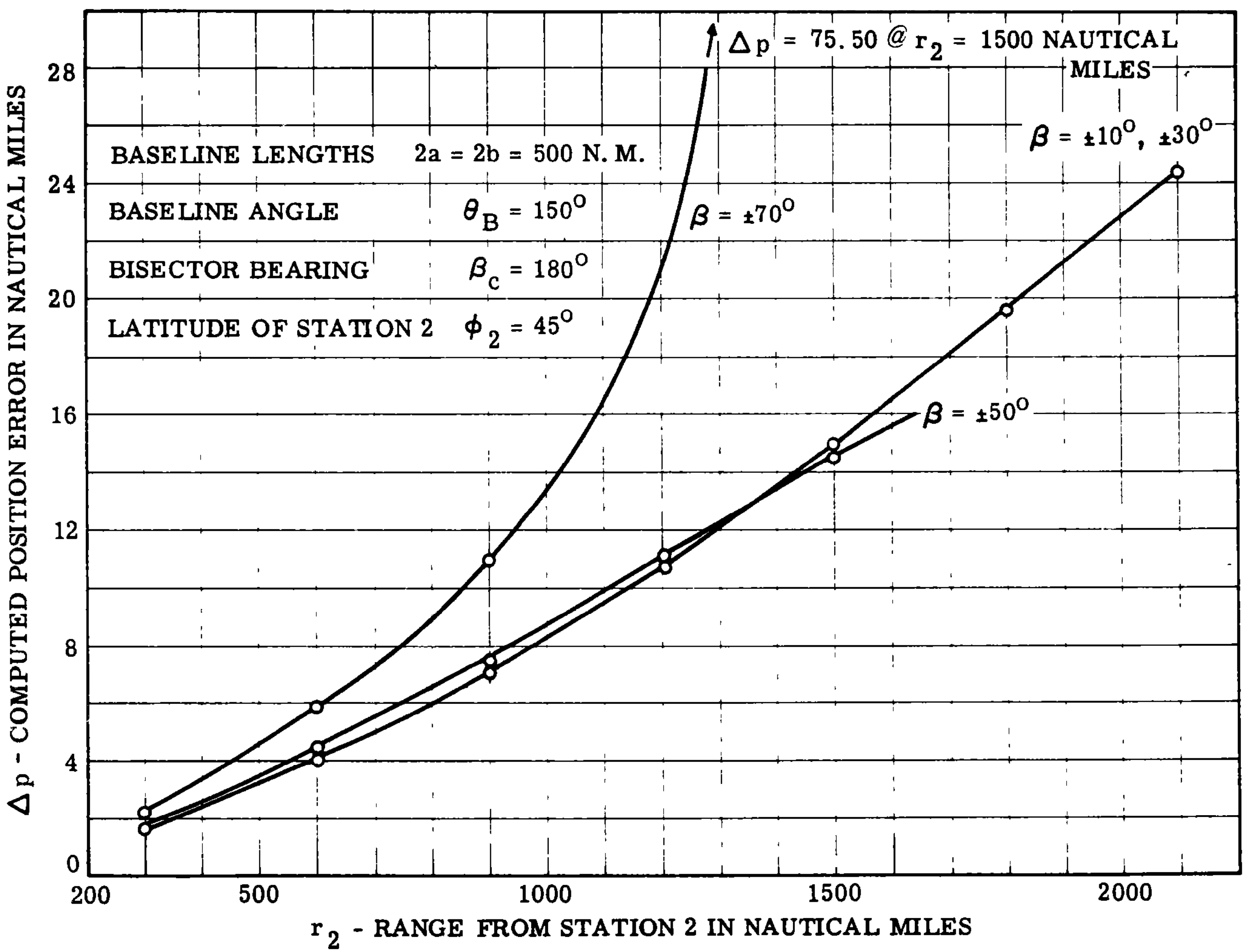


Figure F-7. Error in Computed Position Due to Eccentricity

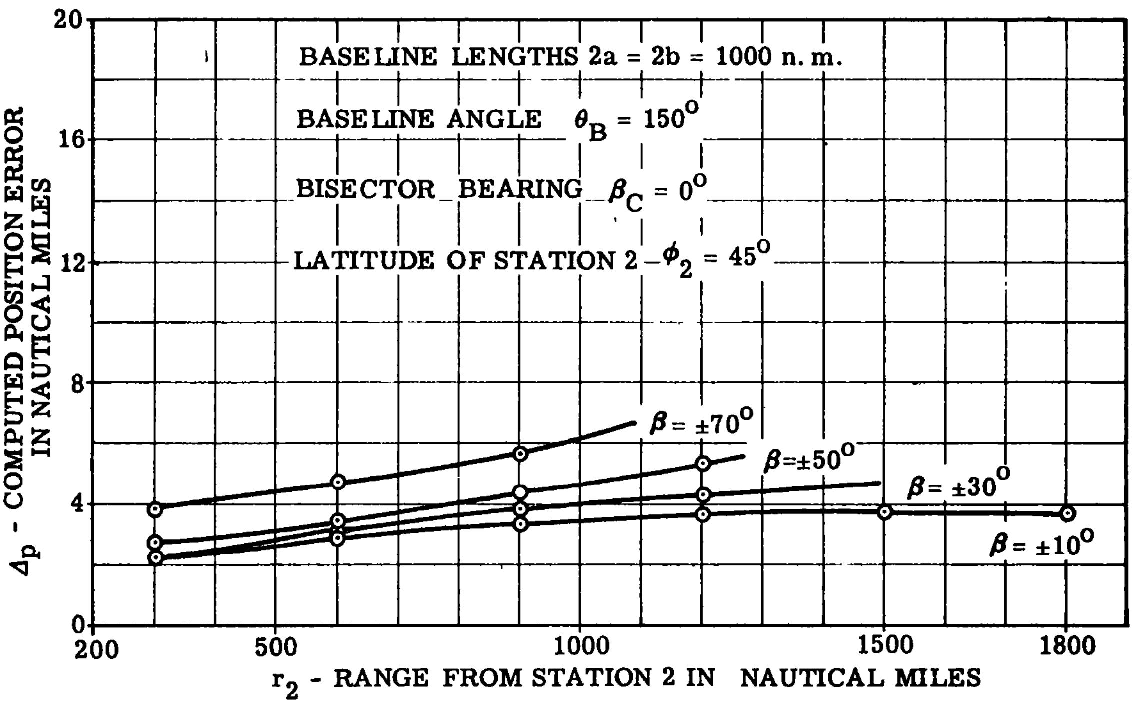


Figure F-8. Error in Computed Position Due to Eccentricity

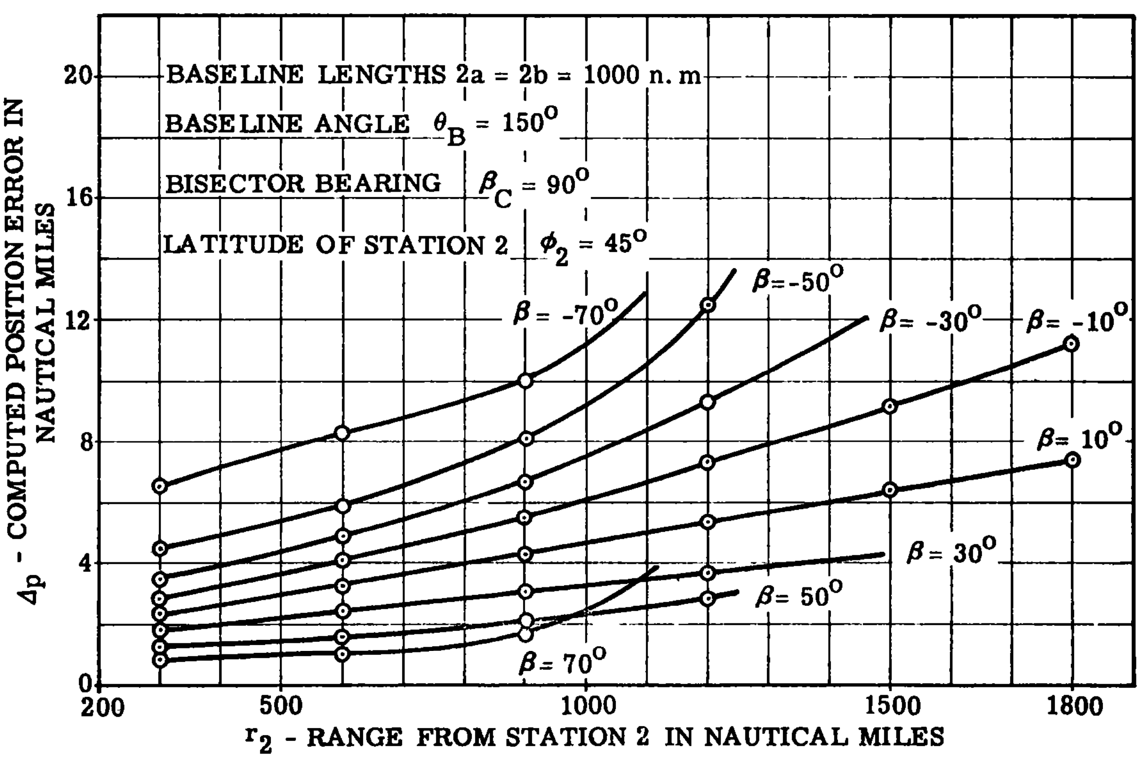


Figure F-9. Error in Computed Position Due to Eccentricity

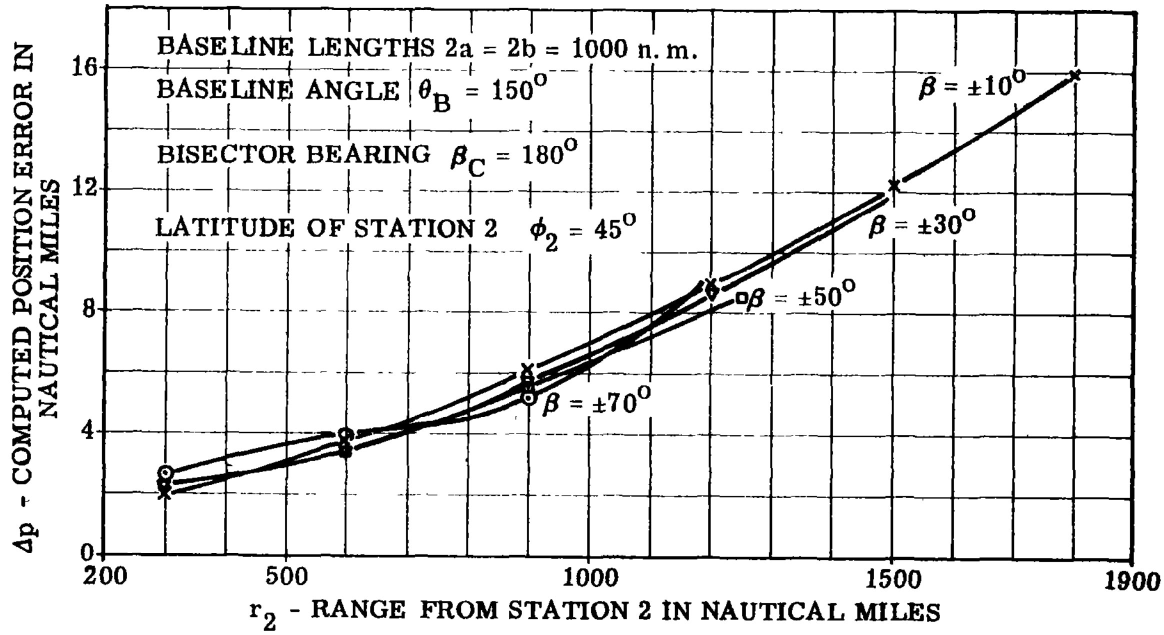


Figure F-10. Error in Computed Position Due to Eccentricity

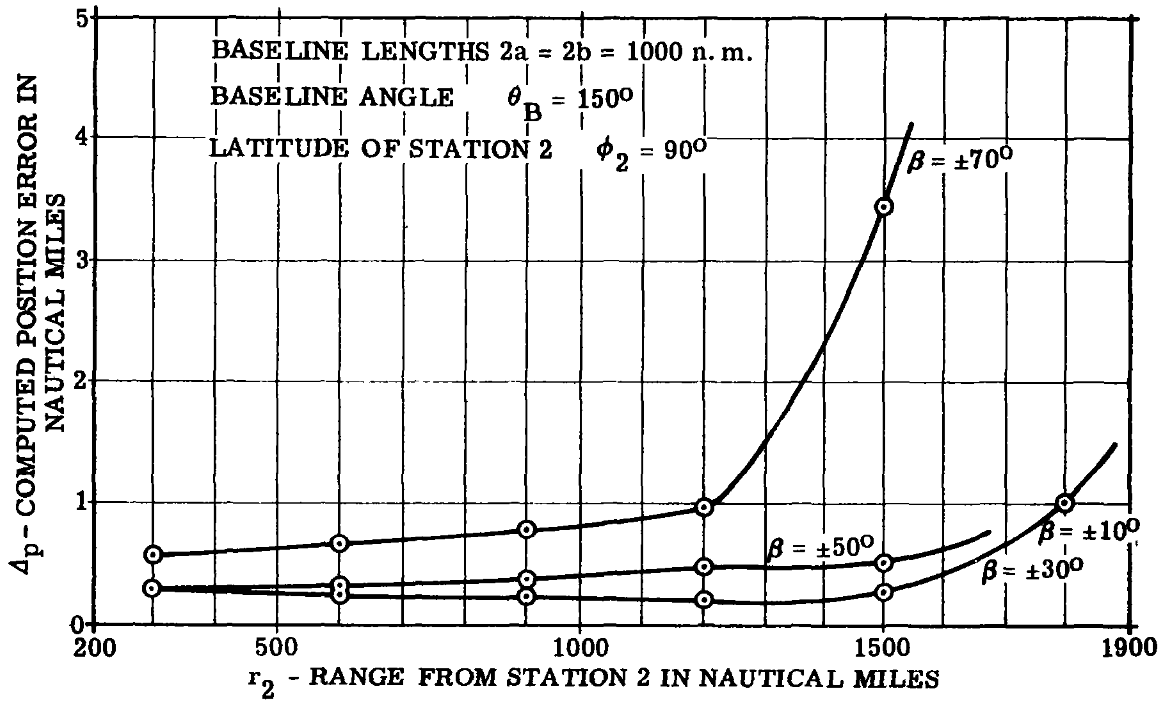


Figure F-11. Error in Computed Position Due to Eccentricity

5. CORRECTION FOR ECCENTRICITY

5.1 LAMBERT-ANDOYER FORMULA

The most obvious way to correct for eccentricity is to correct each range or range difference input to the coordinate converter to the value necessary to give the correct latitude and longitude. This can be done by computing corrections Δr_1, Δr_2, and Δr_3 from equation F-59 and subtracting these quantities from each range r_1, r_2 and r_3, respectively. Where range differences $r_{12} = r_1 - r_2$ and $r_{23} = r_2 - r_3$ are the inputs, $\Delta r_1 - \Delta r_2$ is subtracted from r_{12} and $\Delta r_2 - \Delta r_3$ is subtracted from r_{23}. Equation F-59 can be written in terms of variables common to other parts of the coordinate conversion by using the following relationships

$$\sin \phi = T \tag{F-60}$$

$$\cos \phi \cos \lambda = U \tag{F-61}$$

$$\cos \phi \sin \lambda = V \tag{F-62}$$

$$\sin \phi_n = a_{n1} \tag{F-63}$$

$$\cos \phi_n \cos \lambda_n = a_{n2} \tag{F-64}$$

$$\cos \phi_n \sin \lambda_n = a_{n3} \; . \tag{F-65}$$

σ_n may be substituted for σ_n' without introducing any more error than is inherent in equation F-59. Making appropriate substitutions, equation F-59 becomes

$$\Delta r_n = (R_e - R_n)\sigma_n + \frac{Ref}{4}\left[(3 \sin \sigma_n - \sigma_n)\frac{(T + a_{n1})^2}{(1 + \cos \sigma_n)} - (3 \sin \sigma_n + \sigma_n)\frac{(T - a_{n1})^2}{(1 - \cos \sigma_n)}\right] \tag{F-66}$$

and

$$\cos \sigma_n = a_{n1} T + a_{n2} U + a_{n3} V \; . \tag{F-67}$$

It is estimated that the correction factors Δr_n are in error by less than .01 nautical mile and that the resulting error in computed position is less than 0 2 nautical mile for positions that are more than 25° from the baselines.

5.2 APPROXIMATION

Equation F-66 is rather difficult to mechanize in the airborne computer and furthermore, the precision that it yields is much better than is necessary. For these reasons an approximation was sought. In Reference 9 (Section 4) a power series expansion is made of the second part of equation F-59 and from it an approximation is made. Using this approximation, equation F-59 becomes

$$\Delta r_n = \sigma_n' \left[A + B(\sin^2 p - 2r^2 \cos^2 p) \right] \quad \text{(F-68)}$$

$$A = R_e - R_M \quad \text{(F-69)}$$

$$B = fR_e = 11.675 \text{ nautical miles} \quad \text{(F-70)}$$

$$p = \frac{\phi + \phi_n}{2} \quad \text{(F-71)}$$

$$r = \frac{\phi - \phi_n}{\sigma_n'} \quad \text{(F-72)}$$

$$\sigma_n' = \frac{r_n'}{R_M} \doteq \frac{r_n}{R_M} \; . \quad \text{(F-73)}$$

The error in computed position using the correction defined by equation F-68 was calculated for the configurations used in Figures F-5 through F-10. This was accomplished by differencing the range errors calculated from equation F-68 with those calculated from equation F-59 and substituting the differences as range errors into equations 47 and 48. The errors in computed position at maximum range are plotted in Figures F-12 and F-13. Figure F-12 is plotted for the 500 nautical mile baseline configuration and Figure F-13 for the 1000 nautical mile baseline configuration. The error never exceeds 0.6 nautical mile for the 1000 nautical mile baseline configuration, or 2.1 nautical mile for the 500 nautical mile baseline configuration except at 5° from the baselines. This correction factor is satisfactory, since an error of 2 nautical miles due to eccentricity is permissible in the FAA problem. Although equation F-68 is simpler than equation F-59, it takes more steps to mechanize in the computer, since some of the functions used in other parts of the coordinate conversion can be used in equation F-59.

6. CONCLUSIONS

A set of error coefficients that can be used in any error analysis of a system converting hyperbolic to spherical coordinates was derived. These coefficients were used to derive a factor expressing rms error in computed position as a function of rms error in each range. The coefficients were also used in finding the error in computed position as as a result of assuming the earth to be a perfect sphere with zero eccentricity. This

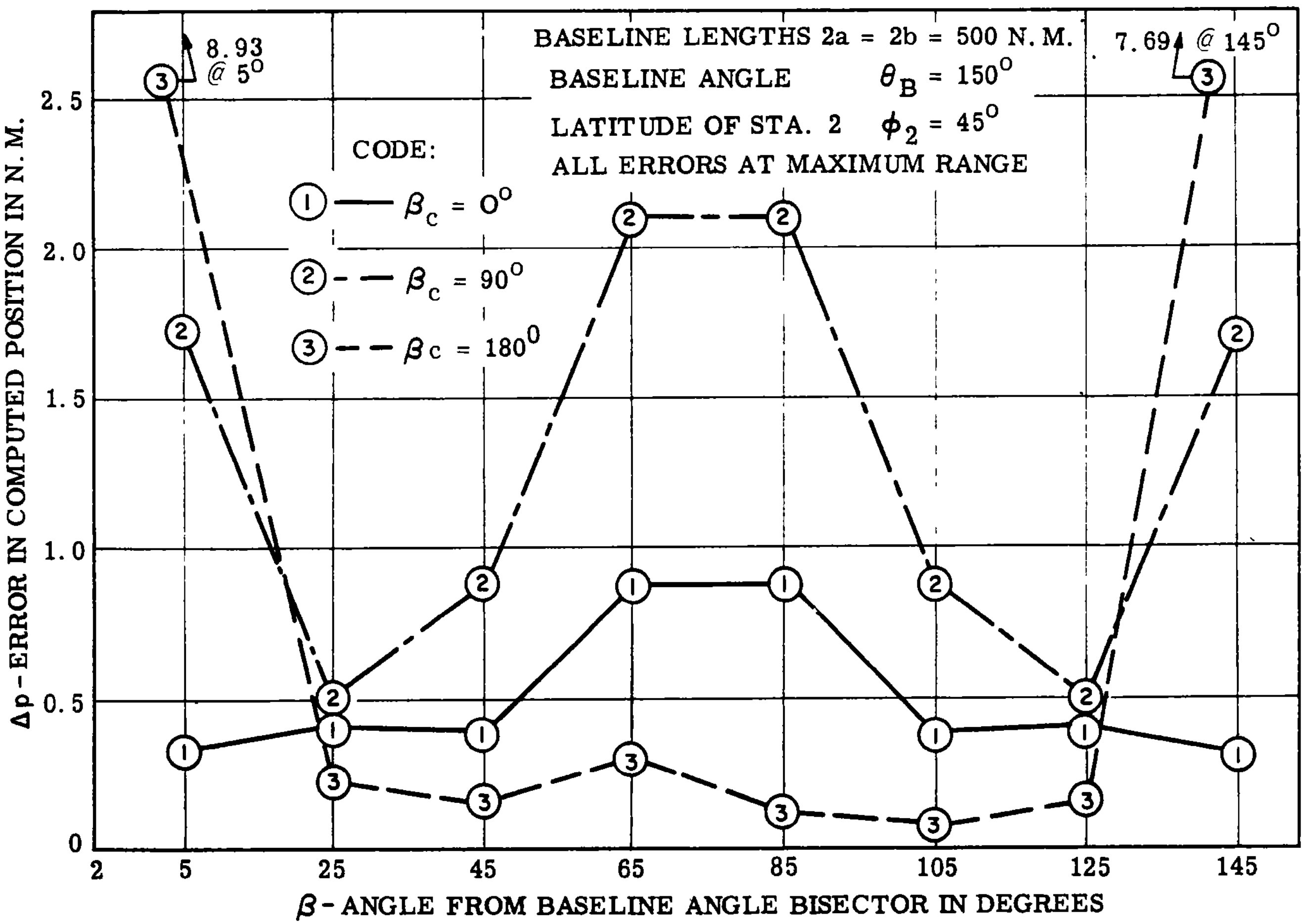


Figure F-12. Error in Computed Position After Correction is Made for Eccentricity

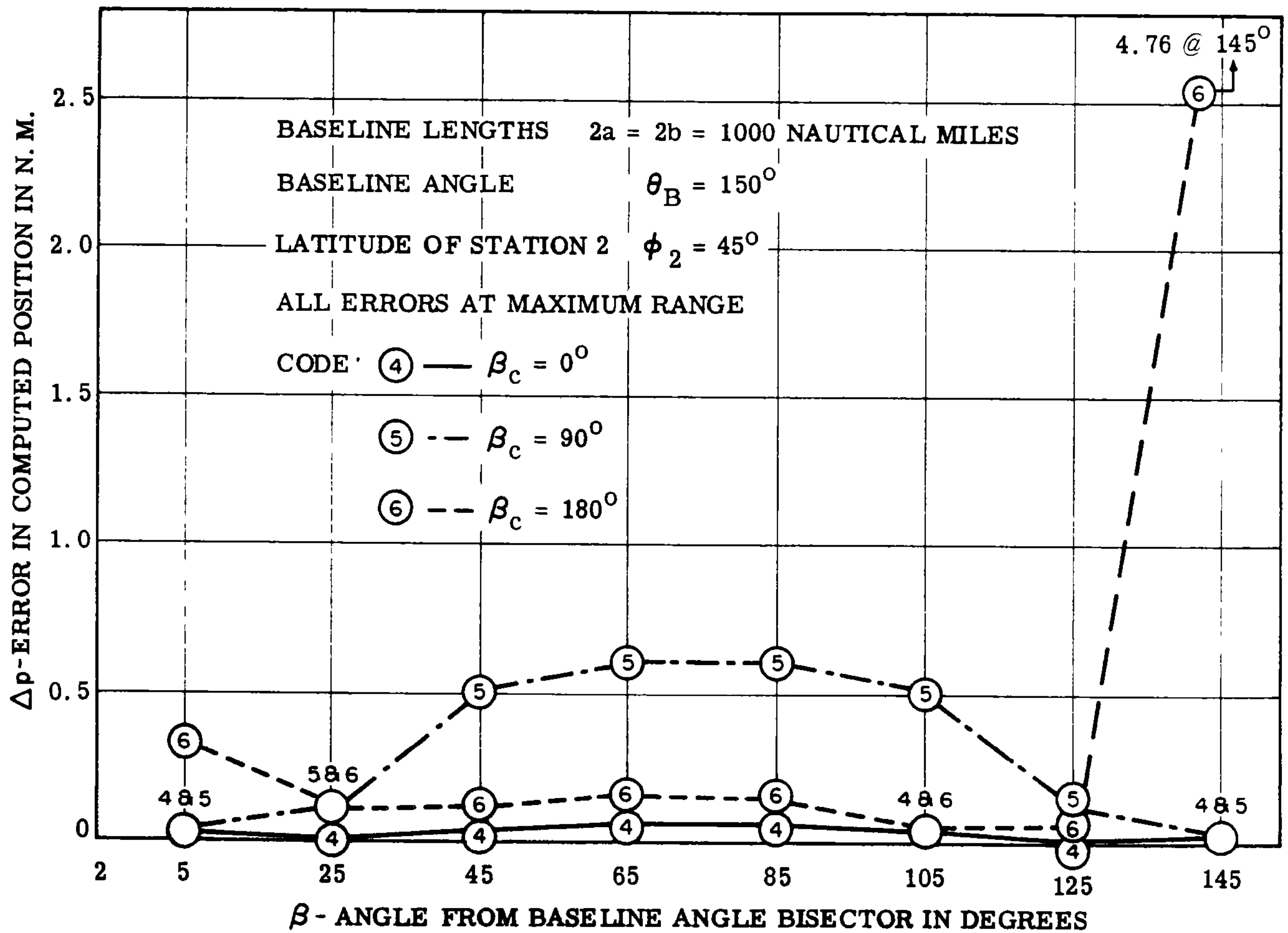


Figure F-13. Error in Computed Position After Correction is Made for Eccentricity

error is too large to be acceptable in the FAA problem and a correction factor is required. A correction factor is suggested which when applied to each range, reduces the error in computed position due to eccentricity to a negligible value. The computations required for the correction factors are summarized below.

$$\Delta r_{12} = \Delta r_1 - \Delta r_2 \quad \text{(F-74)}$$

$$\Delta r_{23} = \Delta r_2 - \Delta r_3 \quad \text{(F-75)}$$

$$\Delta r_n = (R_e - R_M)\sigma_n + \frac{Ref}{4}\left[(3 \sin \sigma_n - \sigma_n)\frac{(T + a_{n1})^2}{1 + \cos \sigma_n} - (3 \sin \sigma_n + \sigma_n)\frac{(T - a_{n1})^2}{1 - \cos \sigma_n}\right], \quad \text{(F-76)}$$

$$\cos \sigma_n = a_{n1} T + a_{n2} U + a_{n3} V \quad \text{(F-77)}$$

where

$T = \sin \phi$ $\qquad a_{n1} = \sin \phi_n$

$U = \cos \phi \cos \lambda$ $\qquad a_{n2} = \cos \phi_n \cos \lambda_n$

$V = \cos \phi \sin \lambda$ $\qquad a_{n3} = \cos \phi_n \sin \lambda_n$.

APPENDIX G

COORDINATE CONVERTER COMPUTER

1. INTRODUCTION

This appendix describes the coordinate converter computer in detail. Included also is a description of the unit characteristics and a discussion of the function of the C/C computer.

2. COUNTERS

2.1 SECTOR COUNTER

A sector counter referenced to a given point on the surface of the drum is advanced as each sector passes beneath its reading head and thus provides a method of recognizing a particular sector on a track. The least significant stage of the sector counter alternates with each succeeding sector, and the states of this stage are used to divide the memory into two portions (Figure G-1). One state is called "control". It indicates that the sector passing under the reading head at that time is a control sector and contains an instruction word. The other state is called "operate" and indicates that the sector may contain an operand. It is only during control sectors that new instructions can be read from the drum and stored in the control registers. Commands are decoded and carried out during operate sectors, with the exception of certain commands that require more than one word time to complete.

2.2 BIT COUNTER

The individual bit time intervals are generated by a bit counter. The counter is synchronized with the drum by the drum sync pulse, and is advanced by each clock pulse received thereafter. The clock pulses are not shown but are assumed to be present at each flip-flop. When 25 bit intervals have been counted (T_{24}, Figure G-1), the counter returns to its initial state (T_0), and the sector counter is advanced.

Synchronization of the sector counter with the drum also is accomplished by the drum sync pulse. The logic for the counters is also shown in figure G-1. Combinational logic circuits in the output develop the timing and gating signals for the computing processes.

3. DRUM UNIT CHARACTERISTICS

The approximate dimensions for the magnetic drum are 4.0 inches in diameter and 3.0 inches in axial length. Sixty-four words, each word occupying 25 bit areas, including spacer are stored on every track of general storage. The resulting packing density is 127 bits per inch, a value well within the limits for excellent reliability. The estimated speed is 8000 rpm, making the clock rate 213 kc and the word interval 117 microseconds. Reading and recording heads are mounted around the surface of the drum so as to provide 37 active tracks having center-to-center spacings of approximately 0.075 inch.

3.1 GENERAL STORAGE

Thirty two tracks are reserved for general storage, each track containing 64 words. Of the 2048 words thus provided, one half (1024 words) is reserved exclusively for storage of instructions, and one half is reserved for operands. One head is mounted on each track.

3.2 CONTROL TRACKS

Two tracks provide control signals to the system. The control information is permanently recorded around the circumference of the drum by suitably engraved marks, yielding control tracks that are immune to complete destruction. The heads mounted on the control tracks are connected to reading amplifiers only. The clock track is engraved with 1600 marks for synchronizing the system. The logical elements of the computer are

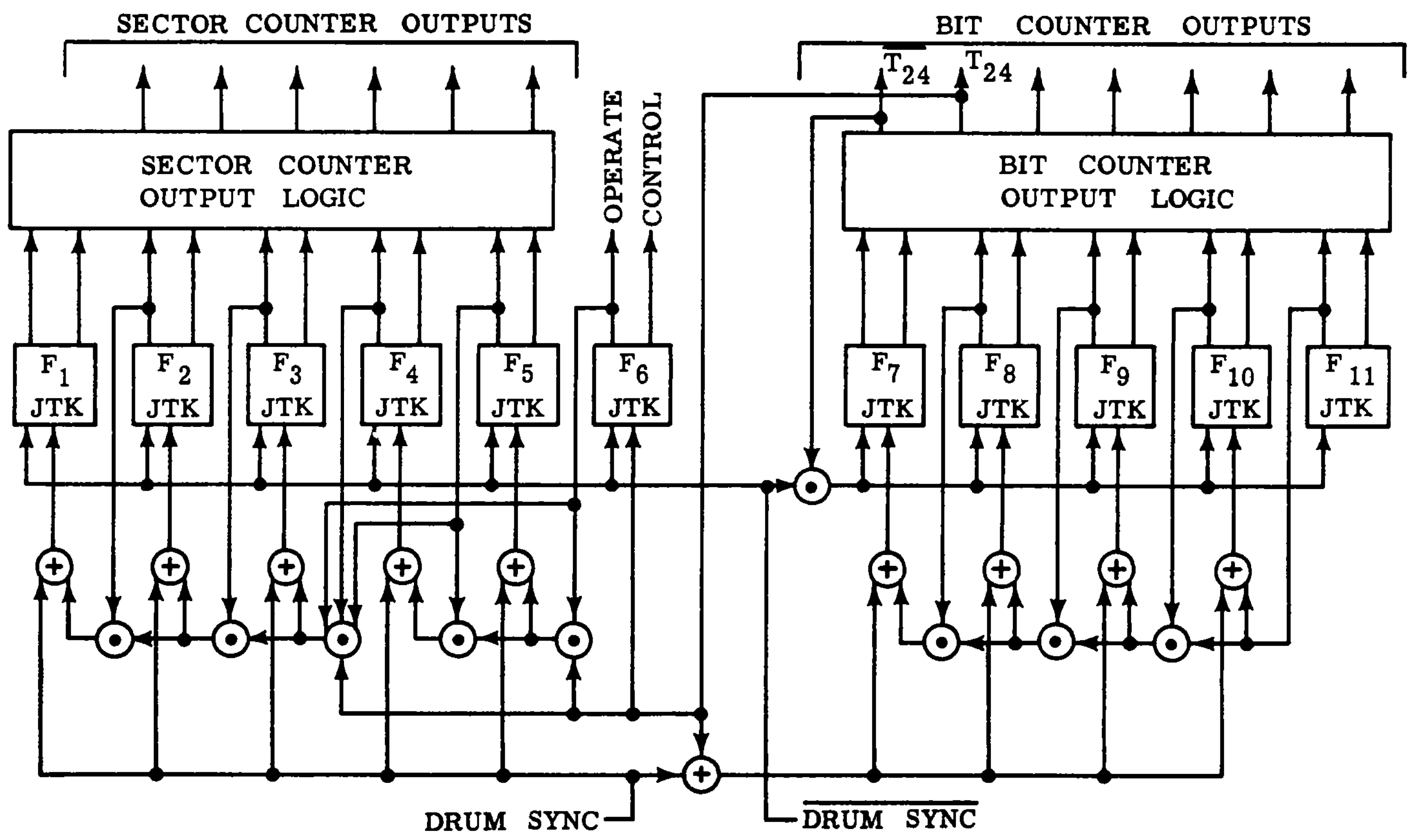


Figure G-1. Logic of the Bit Counter and Sector Counter asynchronous in operation, and signals which ripple through the various paths formed by these elements may arrive at their destinations at different times. These elements, when combined into the units of the computer, are synchronized by the timing signal, which marks the beginning of each time interval. One time interval in this computer is approximately 4 7 microseconds; however, deviations between the intervals may exist. A small non-uniformity arises from the inability to hold stringent tolerance on the spacings between the timing marks on the drum and from fluctuations in the speed of rotation. Variations as great as ±15% can be permitted without introducing errors into the computer. The second control track is engraved with a single mark that produces a signal coincident with one of the clock marks. Its use is to provide a reference for time intervals and storage addresses, and it is called the DRUM SYNC.

3. 3 REGISTERS

The accumulator register, multiplier register, and order register are all circulating loops mounted on a single track. Of the 25 bits that form the complete accumulator loop, 23 are on the drum at any given bit time interval while the remaining bits are held in flip-flops. In this way it is possible to make selected bits of the word available to be sensed as required in the performance of the computer operations. For the same reasons, the other loops include flip-flops. Circulating loops require exact spacings between reading and recording heads in order for a given number of bits to be stored between them. The following table lists the characteristics of the three registers

TABLE G-I. HEAD-TO-HEAD SPACING FOR ARITHMETIC CIRCULATING LOOPS

Register	Head-to-Head Spacing	
	Bits	Inches
Accumulator	23	.181
Multiplier	22	.173
Order	24	.189

A more detailed description of the function of the registers is given in the description of the instructions.

4. OPERAND DELAY

With the minimum-access or relative programming system in use in the computer, the situation often arises in which one of the operands that is to be used in a particular step of the computation has, just previous to the need for it, passed under its reading head. In this case it would be necessary to await the major part of a complete revolution of the drum for the operand to be in position to be read Inclusion of suitable delaying orders in the program would be necessary with a resulting loss in computing time. Two data delay channels are provided to alleviate this problem. By making proper use of these channels, operands known to be required at a specific time in the future can be temporarily stored in one of the channels and used later when the proper time arrives. Each channel consists of a single writing head and four reading heads placed at various word lengths from the reference. On channel A, the reading heads are placed so that a word may be read 2 + 64n, 8 + 64n, 10 + 64n, and 32 + 64n word times after it has been written, where n equals any desired integral number of drum revolutions 0, 1, 2, etc. A word written on channel B may be read 4 + 64n, 6 + 64n, 20 + 64n, and 44 + 64n word times later. A writing amplifier and a reading amplifier serves both channels, the selection of the appropriate reading or recording head being made by a suitable switching matrix.

As an example of the use of the delaying channels, consider a situation in which an operand is in position to be used immediately; however, a few steps in the calculation remain before the program calls for its use. It is determined that by interrupting the program flow and inserting an order that will read the operand from the main storage and record it on a delaying channel, it can be made available at a later time. It is then determined that three additional orders must be carried out before that point in the program is reached where the operand is to be used and that each intervening order requires two word times (including order access). Hence a delay of 8 word times is required and it can then be stated that channel A was the proper channel on which to record the operand; and that it can be read, when needed, by the head that produces a delay of 8 + 64n word times (n = 0).

Each delay channel can store up to 32 data words simultaneously for later reference, and each location may be used again once a desired delaying function has been fulfilled. When delays greater than ten word times are required, it may be necessary to rewrite a delayed operand into a delaying channel for further delay.

5. CONTROL REGISTERS

In addition to the registers which are a part of the drum three shorter registers are provided (Figure G-2). The command register, address register I and address register II are composed of 4, 6, and 6 flip-flops, respectively Their functions are to hold the command digits and address digits for decoding purposes.

6. THE INSTRUCTION WORD

The organization of an instruction word is shown in Table G-II. The first four digits of the serial word are the command digits. A command is decoded by storing the digits in the command register. Digits 5 through 10 in general represent the storage address from which the operand is obtained. Six digits are required since the number of locations from which the operand may be obtained is greater than 32 but less than 64. Digits 11 through 15 specify the drum track from which the next order is to be obtained. In this case five digits are sufficient since an instruction can be obtained only from one of 32 storage tracks. The next five digits of the instruction are, with the exception of the test order and Z order, not used. The final four digits are not used in any order.

The computer is a two-cycle machine. The control cycle requires the reading-out or "fetching" of an instruction word from storage. The operate cycle occurs when the command is carried out. The cycles alternate for as long as the computer is in operation.

7. ORDER "FETCH" CYCLE

An order word is obtained from storage only during a control sector. It must be assumed in the following description that at some time previous to a fetch cycle the track number from which the order word is obtained had been placed in the address register II. The desired reading head was then activated through the address decoder.

As an order word is read from its stored location on the drum, it is gated into the order register. In addition, the four command digits are shifted into the command register where, beginning with bit interval T_4, they are decoded by the command decoder. As bits 5 through 10 are transferred to the order register, they are shifted into address register I where they are held until the last digit of the instruction word has been read from the drum. They are then gated in parallel to address register II for decoding and selection of the forthcoming operand. At this time the operate cycle begins, although all control functions have not been completed. However, the control cycle is completed during the next word time, and in most cases the operate cycle is completed at the same time.

Assume that the command which has been decoded calls for an operand. While the operand is being read from the address held in address register II, the instruction word continues to shift through the order register. When digits 11 through 15 become available they are shifted into address register I. Digits 16 through 20 are compared serially with the corresponding digits of the sector counter. After the final digit of the operand has been read, address register I is again gated into address register II, and the track holding the next order word is selected. At this point the next control cycle is ready to commence.

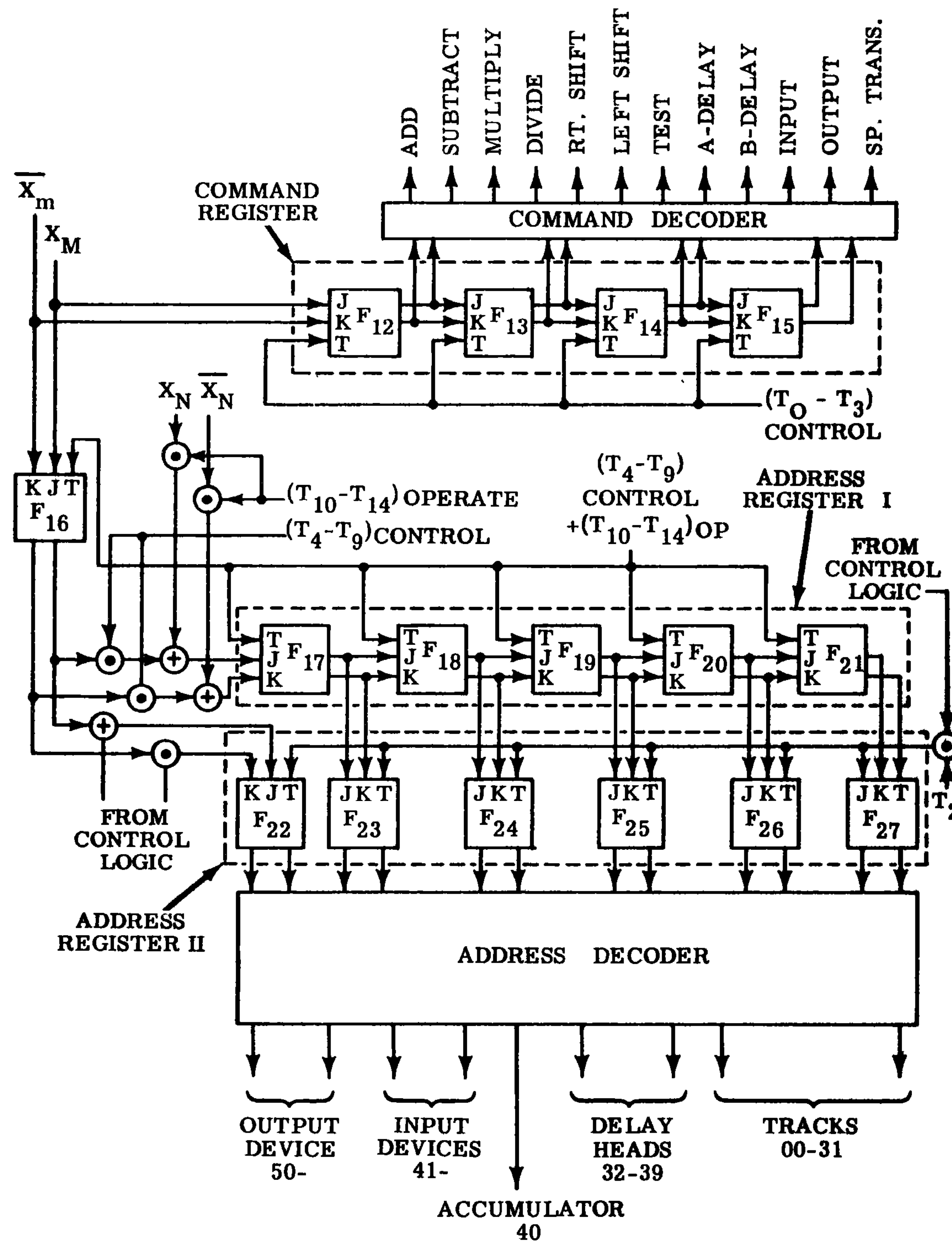


Figure G-2. Organization of the Registers for Holding Control Information

TABLE G-II. COMPUTER INSTRUCTION CODE

	SECTOR Z	N. O. W. TRACK Y	OPERAND X	COMMAND
24 23 22 21	20 19 18 17 16	15 14 13 12 11	10 9 8 7 6 5	4 3 2 1
00	00	Track 00-27	Track 00-31 Delay 32-39 Input Unit 41-	Add (A)
00	00	Track 00-27	Track 00-31 Delay 32-39 Input Unit 41-	Subt. (S)
00	00	Track 00-27	Track 00-31 Delay 32-39 Input Unit 41-	Mult.(M)
00	00	Track 00-27	Track 00-31 Delay 32-39 Input Unit 41-	Div. (D)
00	00	Track 00-27	00	Shift Right (R)
00	00	Track 00-27	00	Shift Left (Y)
00	Sector 00-31	Track 00-27	Track 00-31	Test (T)
00	00	Track 00-27	Track 00-31 Delay 32-39 Accumulator 40	A-Delay
00	00	Track 00-27	Track 00-31 Delay 32-39 Accumulator 40	B-Delay
00	00	Track 00-27	Track 00-31 Delay 32-39 Input Unit 41-	Input (I)
00	00	Track 00-27	Track 00-31 Output Unit 50-	Output (P)
00	Sector 00-31 (N. O. W.)	Track 00-27	Track 00-31	Special Trans. (Z)

8. DESCRIPTION OF THE INSTRUCTIONS (Refer to Section 3.3.1.1).

1. Add: The contents of a storage register, or input unit x, are added to the contents of the accumulator, and the sum is placed in the accumulator register. Time to complete: two word times including order fetch.

2. Subtract: The contents of a storage register, or input unit x, are subtracted from the contents of the accumulator, and the difference is placed in the accumulator register. Time to complete: two word times including order fetch.

3. Multiply: The contents of a storage register, or input unit x, are multiplied by the contents of the accumulator, and the 24 most significant digits of the product are placed in the accumulator register. Time to complete: twenty five word times after passing a pre-set reference.

4. Divide: The contents of the accumulator are divided by the contents of a storage register, or input unit x, and the quotient is placed in the accumulator register. Time to complete: twenty five word times after passing a pre-set reference.

5. Shift Right: The contents of the accumulator register are divided by two with the result remaining in the accumulator register. Time to complete: two word times including order fetch.

6. Shift Left: The contents of the accumulator register are multiplied by two with the result remaining in the accumulator register. Time to complete: two word times including order fetch.

7. Test: If the contents of the accumulator register are positive, obtain the next order from track x, section z; if the contents of the accumulator register are negative, obtain the next order from track y, sector z. Time to complete: variable from two to 64 word times depending upon the relative positions of the old and new orders.

8. A-Delay: Transfer the contents of the storage register or accumulator x to the A-delay track. Time to complete: two word times including order fetch.

9. B-Delay: Transfer the contents of storage register or accumulator x to the B-delay track. Time to complete: two word times including order fetch.

10. Input: Transfer the contents of address x to the accumulator register. Time to complete: two word times including order fetch.

11. Output: Transfer the contents of the accumulator register to the storage track or output unit specified by X. When the output device is the Flight Leg Indicator, the action is to cause it to advance one position. Time to complete: two word times including order fetch.

12. Special Transfer: Transfer the contents of the storage location on track x and on a sector specified by the setting of the flight leg indicator to the accumulator register. Obtain the next order from track y, sector z; Time to complete: variable from four to 64 word times depending upon the relative positions of the operand, old order and new order.

9. ADDER

The adder used in the computer is shown in Figure G-3. The carry formed in each step of an addition is stored in F_c and returned to the adder on the next step. For operations of subtraction, a carry is forced into the adder, on the first step only, through the external line provided. The addend and augend are selected according to the command.

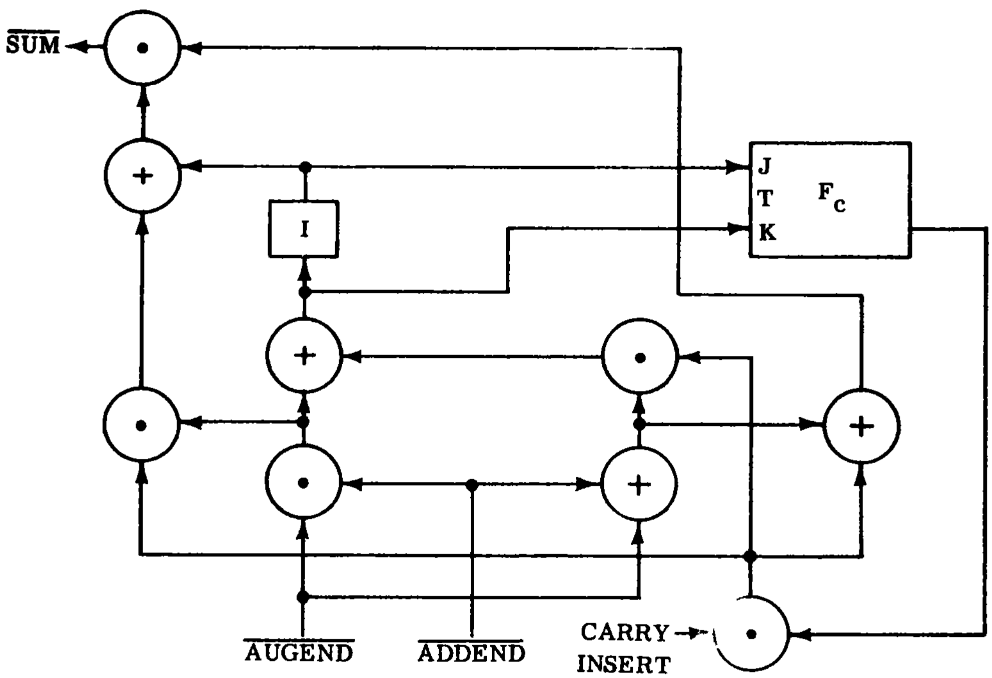


Figure G-3 Full Adder Used in Serial Arithmetic Operations

10. MULTIPLICATION AND DIVISION

Multiplication and division are performed by repetitive additions and subtractions. In order to avoid the necessity of a separate counter, the steps of multiplication and division are counted by the sector counter. For this reason, multiplications are started only at sectors 0 or 32 and divisions only at sectors 16 or 48.

In multiplication, the order register serves a double purpose by receiving and circulating the multiplicand (operand). At the same word time in which the operand is obtained, the contents of the accumulator register are transferred to the multiplier register and the accumulator is cleared to zero. During each of the following 24 word times, two bits of the multiplier are sensed and the accumulator register is modified by adding or subtracting the multiplicand or by simply shifting the accumulator as specified by the bits sensed The addend and augend are shown as boxed-in columns in the order register and accumulator register respectively.

The accumulator register is modified in each division step by additions or subtractions according to the sign of the divisor and the sign of the partial remainder as it is formed in the accumulator. The quotient is developed in the multiplier register and transferred to the accumulator on the last step.

11. SECTOR COMPARATOR

When it is necessary to identify a specific sector in the performance of a command, a serial comparison is made between assigned digits of the instruction word and corresponding digits of the sector counter The comparator circuit is shown in Figure G-4. When there is disagreement between any digit of the order register (flip-flop X_N) and the

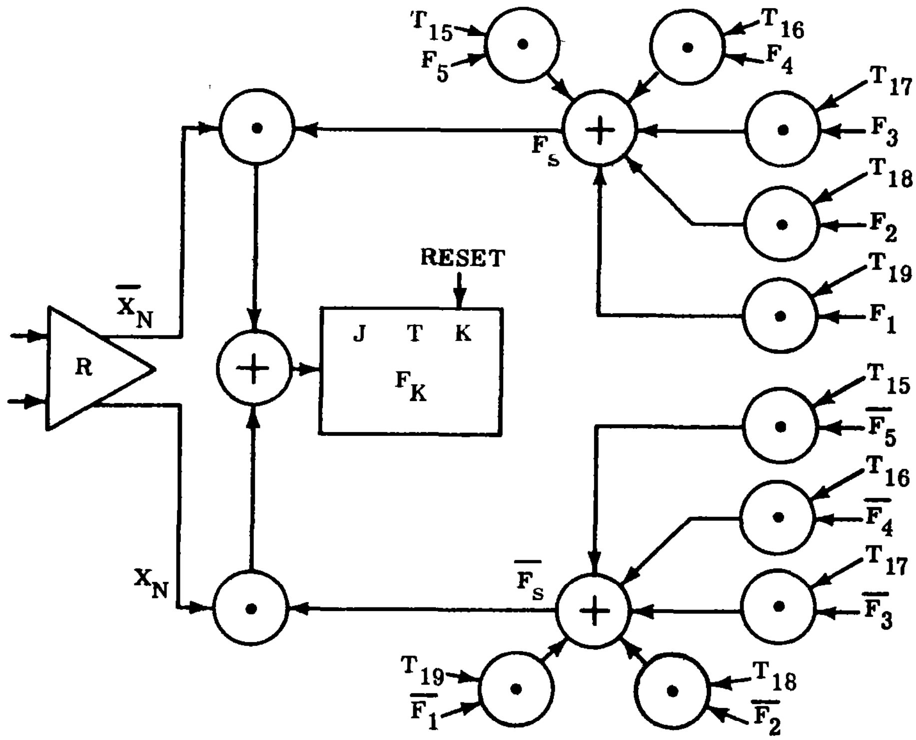


Figure G-4. Comparator Circuit Used to Seek the Location of a Next Order Word

sector counter, flip-flop F_K is set to a "one". Upon full agreement between all digits, F_K remains in its re-set state and enables the word stored in the succeeding sector to be read.

In performing a test command, the instruction word continues to circulate in the order register until a successful sector comparison has been made. At this time a new instruction word is transferred into the order register.

12. READING AND RECORDING

The reading and recording elements are shown in Figure G-5. Head selection for the delay tracks is controlled from the command decoder. The digit input to D_D is obtained from sources specified in the operand address portion of the order. Outputs of the address decoder select the delay track reading heads and general-storage, dual-purpose heads through the appropriate selection matrix. No head selection is required in the three circulating registers at the bottom of Figure G-5. The writing amplifiers for the delay tracks and general storage are turned on by signals generated in the control logic only when it is desired to record a word.

13. INPUT-OUTPUT

Provisions for both manual and automatic input methods are included. Under normal continued operation, the computer obtains its input data from analog-to-digital conversion devices From this data, position information is computed by use of a stored program. Manual input is supplied to enable the operator to change constants while in flight. All input devices, manual and automatic, feed data into the accumulator register. The data

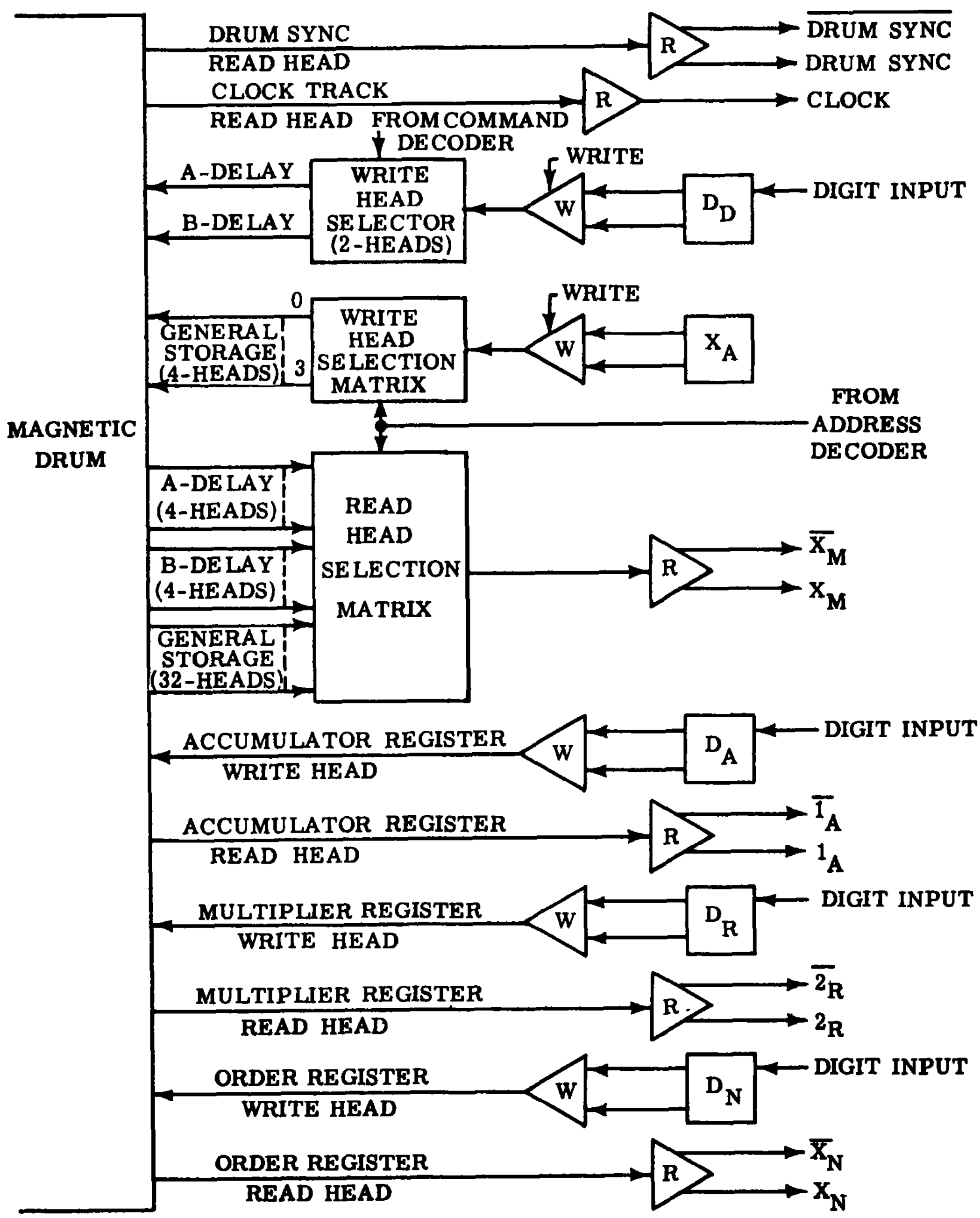


Figure G-5. Magnetic Drum Memory with Reading and Writing Circuits is operated upon and stored under program control. Positional information generated by the computer is transferred from the accumulator register to the output device under program control.

14. J-K-T FLIP-FLOP

The J-K-T flip-flop is a one-bit storage device used in the computer registers and counters and for defining internal control state of the machine. The rules which the flip-flop obeys are listed in Table G-III.

TABLE G-III. TRUTH TABLE FOR A J-K-T FLIP-FLOP

J^n	T^n	K^n	S^{n+1}
0	0	0	S^n
0	0	1	S^n
0	1	0	S^n
0	1	1	0
1	0	0	S^n
1	0	1	S^n
1	1	0	1
1	1	1	$\overline{S^n}$

It is seen that the state of the flip-flop does not change as long as a false signal exists at the T terminal, but when a true signal is present, the flip-flop obeys all the rules of a J-K flip-flop. State changes occur only upon the incidence of a one-microsecond clock pulse.

The schematic diagram of a typical J-K-T flip-flop is shown in Figure G-6 The six diodes located at the top of the figure are the input gates, which with the resistor R_1, perform logic functions on the input signals. When input conditions are satisfied, a clock pulse charges C1 through diode CR8 (CR11) and the ON transistor At the conclusion of the clock pulse, C1 discharges through CR9 (CR10) to the base of the ON transistor, turning it off This in turn causes the other transistor to turn ON. Should the input arrive at a transistor that is already off, the corresponding C1 cannot charge and no change will occur as a result of that input The value of C1 is calculated from the amount of stored base-charge in the base of the conducting transistor

The circuit is triggered by a clock pulse with minimum width of one microsecond and rise time of less than 0.1 microsecond. The pulse swings from -1 volt to -6 volts and supplies 1.3 ma maximum at -1 volt. The J-K-T flip-flop can supply AND current of 15 ma and OR current of 5 ma simultaneously. All circuits used in the computer are capable of operating in a temperature range of -20°C to +55°C at a maximum repetition of 350kpps.

15. DELAY FLIP-FLOPS

With the exception of the delay flip-flops in the input circuits, and an additional transistor used in the input for signal inversion, the delay flip-flop is basically the same as the J-K-T flip-flop. The flip-flop is controlled by a single input line. The state of the flip-flop during any given bit interval is identical to the information carried on the input line during the preceding interval A delay of one bit results as tabulated below.

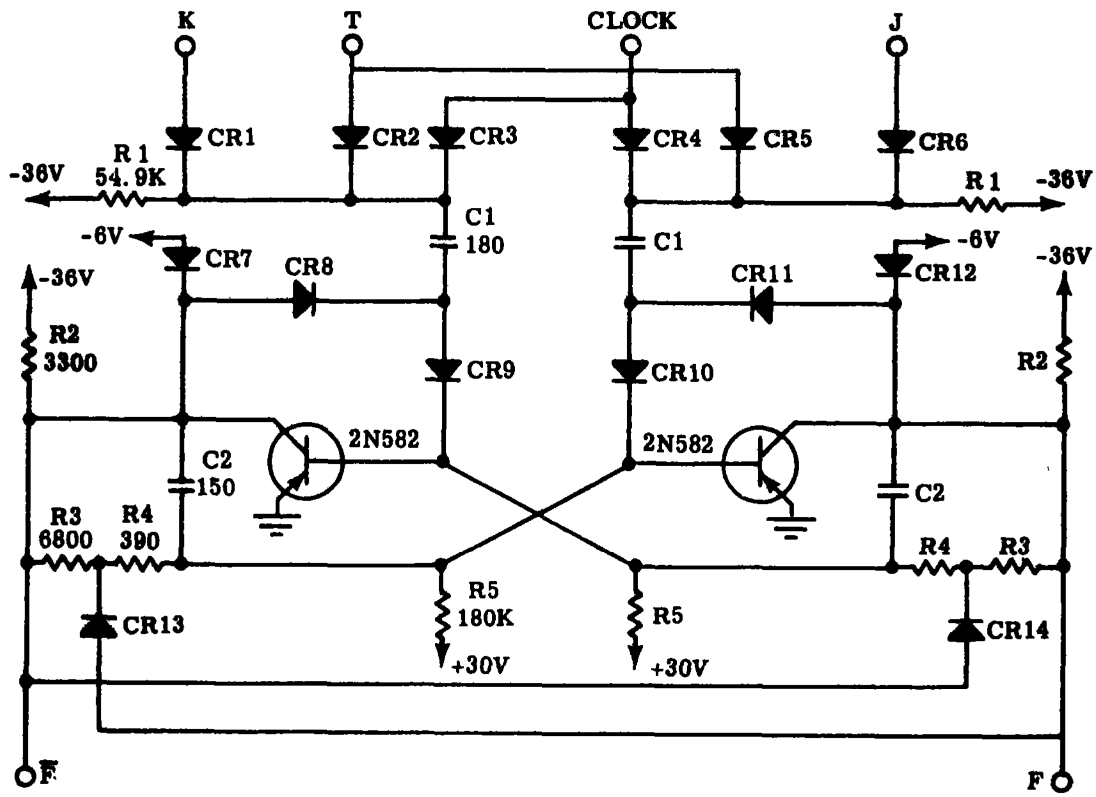


Figure G-6. Circuit Diagram of a Typical J-K-T Flip-Flop

TABLE G-IV. TRUTH TABLE FOR A "D" FLIP-FLOP

D^n	S^{n+1}
0	0
1	1

The major uses of the delay flip-flop in the coordinate converter computer are as a driver for the recording amplifiers and as a method of re-establishing a time reference for signals converging from different-length logic chains. Since a "D" flip-flop has a single input terminal, it has another advantage of eliminating the need for duplicate logic on the J and K input of the J-K-T flip-flop.

16. INVERTER

When many switching elements built of passive components are connected in cascade between storage elements, deterioration of the binary-valued voltages may be great enough to prevent distinction between the binary values. Signal inverters are placed periodically into the logic chains to avoid this situation. These inverters function to restore the d-c levels and to provide the current gain for driving large amounts of logic. In addition, their complementing action is used to advantage in deriving desirable switching functions.

The schematic diagram of a typical inverter is shown in Figure G-7. It utilizes an NPN and a PNP transistor to obtain a 0.3 microsecond maximum delay when capacitively loaded with 150 $\mu\mu f$. It can supply 5 ma of OR current and 15 ma of AND current simultaneously to the logic it drives. A two-stage inverter is used because circuit capacitance needed at the input to a single-stage inverter inherently destroys the input waveform and this situation cannot be allowed when the logic branches at the input terminal.

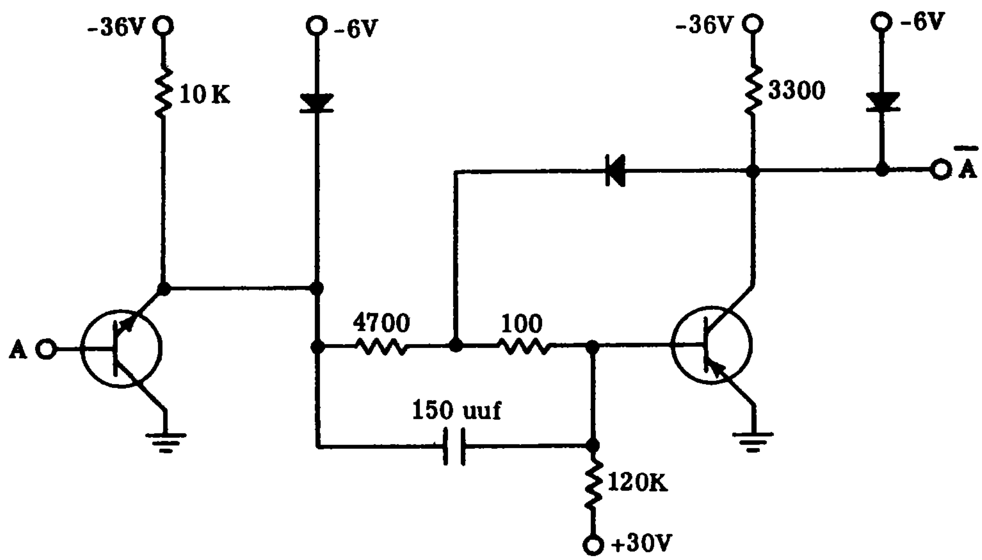


Figure G-7. A Typical Logic Inverter For Driving AND Loads

17. INVERTER-AMPLIFIER

At several points in the computer it is desirable to regenerate a function and at the same time make it available in both its regenerated and complemented forms for further use in the switch logic. A three-transistor element is designed for this purpose. Like the inverter described above, it provides current gain to drive additional logic chains while restoring the d-c voltage levels to their binary values. Both the true and complemented outputs are capable of supplying 5 ma of OR current and 15 ma of AND current simultaneously.

18. DIODE LOGIC

All switching functions are accomplished through the use of diodes combined in such a way as to produce the functions AND and OR. Figure G-8a and G-8b respectively are typical two-input AND and OR circuits. Additional inputs are added to the functions by including one diode for each additional input.

Due to the build up of currents and deterioration of the voltages carrying intelligence, a limit of four cascade-connected switching elements is allowed before an inverter, inverter-amplifier, or flip-flop is included in the chain. Relatively long switching paths are permitted in order to reduce the number of diodes required to perform a function. This is practical with the bit interval selected.

The diode logic acts as a channel between storage elements. Through it "ripples" a signal that governs the state of the flip-flop at the terminal end according to the states of flip-flops at the source. The flip-flop at each end of a logic chain can be, in some

TABLE G-V (Continued)

Bit Counter	No. Units	Transistors	Diodes
J-K-T Flip-Flops	5	10	70
Switch Logic	-	0	90
Inverter	8	16	24
Inverter/Amplifier	2	6	10
Sector Counter			
J-K-T Flip-Flops	5	10	70
Switch Logic	-	0	40
Inverter/Amplifier	1	3	5
Sector Comparator			
J-K-T Flip-Flop	1	2	14
Switch Logic	-	0	55
Adder			
J-K-T Flip-Flop	1	2	14
Inverter	4	8	12
Switch Logic	-	0	16
Command Register			
J-K-T Flip-Flop	4	8	56
Address Register I			
J-K-T Flip-Flop	6	12	84
Switch Logic	-	0	12
Address Register II			
J-K-T Flip-Flop	6	12	84
Switch Logic	-	0	6
Command Decoder			
Inverters	3	6	9
Inverter/Amplifier	1	3	5
Switch Logic	-	0	50
Address Decoder			
Inverter	5	10	25
Switch Logic	-	0	80
Control Logic			
J-K-T Flip-Flop	3	6	42
Inverter	4	8	12
Switch Logic	-	0	110
Inverter/Amplifier	1	3	5

TABLE G-V (Continued)

Delay Registers	No. Units	Transistors	Diodes
D Flip-Flop	1	3	11
Clock System			
Clock Pulse Shapers	2	10	12
Clock Amplifiers	5	5	10
Clock-Sync Inverter	1	2	6
TOTAL	92	226	1375

APPENDIX H

COMBINATION OF COARSE AND FINE DIFFERENCE IN TIME MEASUREMENTS

1. INTRODUCTION

This appendix illustrates a method of combining the envelope (coarse) and cycle (fine) measurements into one unambiguous measurement.

2 PROCEDURE

To combine the envelope and cycle measurements (Figure H-1) the counter is driven initially by the coarse servo. When the error in the envelope loop indicates that the coarse measurement is within range of the measurement made by the cycle measuring loop, the clutch energizes. When the clutch closes, the difference between the coarse and fine readings are manually inserted and the counter readings are corrected by the difference between the coarse and fine measurement. All perturbations on the envelope measuring shaft are cancelled and will not appear on the data combination output. A refinement of this procedure would be an automatic addition of the difference between coarse and fine readings at the time the combination is made. It is also possible to automatically close the clutch at the proper time.

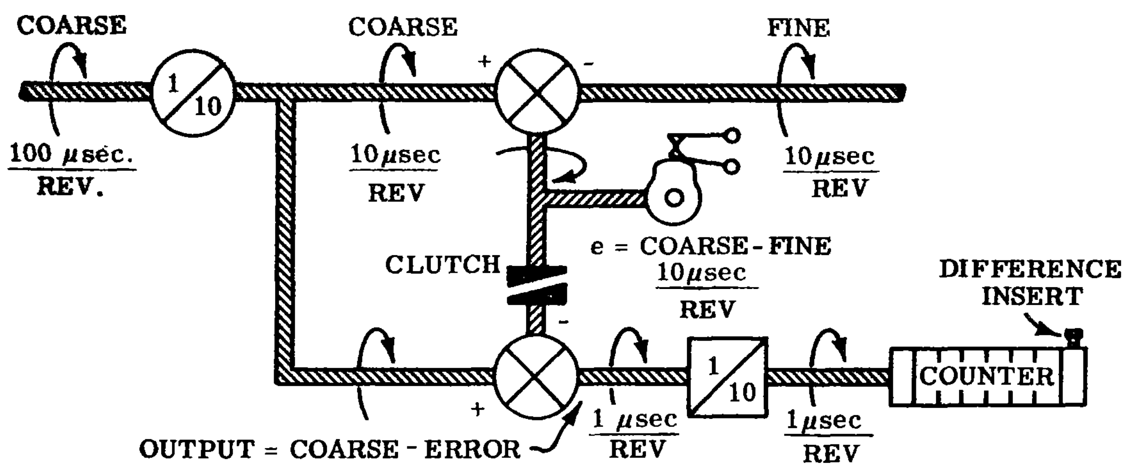


Figure H-1. Combining Envelope and Cycle Measurements