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DETERMINATION OF SOME GEOMETRIC RELATIONSHIPS PERTAINING TO COLLISION FLIGHT PATHS

by
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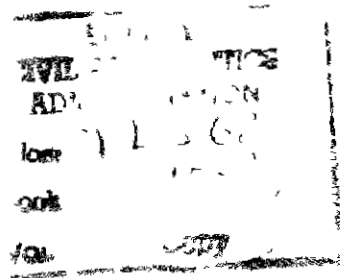
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TABLE OF CONTENTS

	Page
SUMMARY	1
INTRODUCTION	1
GENERAL PRESENTATION AND LIMITATIONS OF BASIC DATA	2
COLLISION CONDITION A STRAIGHT-AND-LEVEL FLIGHT	3
COLLISION CONDITION B STRAIGHT CLIMB AND DESCENT	9
COLLISION CONDITION C ONE AIRPLANE TURNING AND ONE FLYING STRAIGHT AND LEVEL	16
COLLISION CONDITION D TWO AIRPLANES TURNING IN THE SAME DIRECTION	39
CONCLUSIONS	105
ACKNOWLEDGEMENTS	105
APPENDIX I, DERIVATION OF FORMULAS	106
APPENDIX II, FLIGHT-PATH ANALYSIS	120



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DETERMINATION OF SOME GEOMETRIC RELATIONSHIPS PERTAINING TO COLLISION FLIGHT PATHS*

SUMMARY

A study has been made to develop a basic means, in the form of graphs and charts, for permitting a better understanding of cockpit visual problems and aircraft conspicuity problems as they are related to mid-air collisions

Formulas are derived and visual-angle and distance charts are presented for certain combinations of straight-and-level, climbing, descending, and turning flight conditions. Using only aircraft headings and speeds, the visual angles and distances between aircraft at assumed instances of time prior to collision may be determined from these charts. It is also possible to obtain critical collision-course headings by entering the charts with the effective cockpit cut-off vision angles and speeds.

Tables are presented showing actual airplane attitudes during flight.

Actual flight paths recorded in the vicinity of airports are presented. Some performance parameters used in the design of the visual-angle charts contained in this report were established from these flight-path data.

From the graphs contained in this report, it is evident that

1. No apparent motion from pilots' viewpoints exists between two aircraft which are flying straight and level on collision courses at constant speeds.

2. The most severe mid-air collision hazard from a visual-angle standpoint exists between aircraft that are flying at small differences of heading and speed. During this condition, even when the closing rates are small and much time is available for evasive maneuver, the visual-angle restrictions of present-day transport aircraft do not permit the pilots to observe each other.

3. Mid-air collisions cannot be avoided by use of the windshield area alone, since extreme visual angles may be required during many collision conditions. However, certain minimum visual angles in aircraft-cockpit design will tend to decrease the probability of mid-air collisions.

INTRODUCTION

The danger of aircraft colliding in mid-air is becoming increasingly serious as speeds and traffic densities increase. Cockpit visibility angles, or angles of vision required by the pilot of one aircraft to see another aircraft, and pilot alertness are two major factors in mid-air collisions. Another major factor is the conspicuity of aircraft.

During 1948, the Technical Development and Evaluation Center of the Civil Aeronautics Administration commenced an investigation of the cockpit-visibility problem. Since that time, much work has been accomplished and reported^{1,2,3} in an effort to arrive at a basis for establishing minimum cockpit-visibility standards for aircraft and thus to improve the safety of aircraft.

*Manuscript submitted for publication December 1954

¹George L. Pigman and Thomas M. Edwards, "Airline Pilot Questionnaire Study on Cockpit Visibility Problems," CAA Technical Development Report No. 123, September 1950.

²Thomas M. Edwards, "Development of an Instrument for Measuring Aircraft Cockpit Visibility Limits," CAA Technical Development Report No. 153, January 1952.

³Thomas M. Edwards and Wayne D. Howell, "A Study of Pilots' Eye Movements During Visual Flight Conditions," CAA Technical Development Report No. 179, June 1952.

Since 1938, development activities designed for the improvement of aircraft conspicuity have been in progress. To date, these have chiefly involved external aircraft lighting. Two technical reports covering this work have been published ^{4,5}

Continued work in these fields of cockpit visibility and aircraft conspicuity requires the solution of equations showing the relationship that exists among the fundamental parameters of aircraft in flight on collision courses.

The ultimate objective of this report is to present a simple means of calculating cockpit visual angles and distance-time separation as they are related to collision courses. In order to accomplish this task, a series of formulas was developed. From these formulas, charts and nomographs were designed to simplify and facilitate their solutions.

The information and data of this report are presented as follows:

- Collision Condition A - Straight-and-Level Flight
- Collision Condition B - Straight Climb and Descent
- Collision Condition C - One Aircraft Turning and One Flying Straight and Level
- Collision Condition D - Two Aircraft Turning in Same Direction

Graphical presentations covering two aircraft turning in opposite directions were found to vary over such wide limits that it was preferable to achieve solutions by the use of the equations rather than by graphical methods. These equations are presented in the Appendix.

Each collision condition is discussed separately and is presented to conform to the following outline:

- 1 Description
- 2 Procedure and Formulas
- 3 Sample Solution
- 4 Additional Considerations
- 5 Discussion of Significant Trends

The derivation of the formulas and the actual measured flight paths in the vicinity of airports are presented in the Appendix.

This report is limited to a study of collision courses. It is primarily intended to serve as a basic tool to aid in the solution of cockpit-vision and aircraft-conspicuity problems from the standpoint of collision avoidance. Its intended utility also includes investigations of mid-air collision accidents.

GENERAL PRESENTATION AND LIMITATIONS OF BASIC DATA

Charts were drawn to facilitate the calculation of the visual angles required for the pilots of each of two aircraft on a collision course to see the other aircraft. The charts are designed to encompass a wide range of aircraft speeds and collision approach angles, and it is assumed that the aircraft are point sources. It is also possible to determine the distance separating two airplanes at any particular instant of time prior to collision.

Charts for determining separation distances and visual angles were drawn for the most common collision courses. Simple charts shown in Figs 1, 2, and 3 cover straight-and-level flight conditions. Charts for climbing and descending conditions are shown in Figs 1, 2, 5, and 6. A more complicated system of charts is presented for computation of distance of separation and visual angles from aircraft, one of which is flying straight and level and one of which is turning, and also from aircraft which are both turning. These families of charts are shown in Figs 10 through 21 and 28 through 82, respectively. In all cases, sample calculations showing the manner in which the charts are used are presented.

⁴Cecil B. Phillips and Alan L. Morse, "A Review of Aircraft External Lighting Activities," CAA Technical Development Report No. 215, September 1953.

⁵Marvin J. Anderson and Cecil B. Phillips, "Development of Blade Tip Lighting," CAA Technical Development Report No. 248, October 1954.

It was possible to limit the coverage of the charts for straight-and-level, climbing, and descending flight by the assumption of logical limits for the data. However, it was much more difficult to establish practical parameters for the turning conditions of flight. The limits for turning flight were therefore established from recordings of 250 flight paths of aircraft in the vicinity of airports. These flight-path data are shown in the Appendix. The collision cases presented in this report are designed around the assumption that the airplanes are flying at constant speeds, constant rates of climb or descent, and constant angles of bank during level turns, as these conditions are applicable. By necessity, the collision conditions covered by this report constitute only a small part of the total number of possible collision cases.

In using this report as a tool for determining the visual angles required for the pilot of one airplane to see the other airplane and determining the distances between airplanes, the accuracy of the results obtained will sometimes be limited by the authenticity of the input parameters rather than by the design of the charts themselves. If it is found that none of the collision cases represented in this report apply exactly to the one being investigated, the selection of the collision case that is most similar will provide approximate visual-angle requirements.

COLLISION CONDITION A STRAIGHT-AND-LEVEL FLIGHT

Description

Calculation of visual angles to the right or left required for the pilots of each airplane to see the other airplane is facilitated by the use of the charts designed for the straight-and-level flight condition. These charts are designed around the hypothesis that two airplanes are flying along straight lines and on a collision course in the same horizontal plane and that they are approaching each other from any direction at any relative speed.

Procedure and Formulas

The level-cruise charts are shown in Figs. 1, 2, and 3. The procedure for using these charts is as follows:

1. To determine speed relationship A , enter Fig. 1 with the true air speeds of the two aircraft. A is simply the ratio of the speed of the slower airplane to the speed of the faster airplane.
2. To determine visual angles θ (for the faster airplane) and ϕ (for the slower airplane) at which it is possible for the pilot of each aircraft to see the other aircraft, enter Fig. 2 with the absolute difference in the true headings ψ , and the value of A .
3. To determine the distance Z between the airplanes at any instant of time prior to collision, enter Fig. 3 with the velocity V_s of the slower airplane, time t before collision, the absolute difference in true heading ψ , and the visual angle θ required from the faster airplane.
4. To determine closing velocity between the two aircraft, enter Fig. 3 with the distance Z between the aircraft and the time t before collision.
5. To determine the blind areas caused by specific cockpit cutoff angles of any two aircraft, enter the chart shown in Fig. 2 with the visibility cutoff angle of the slower airplane, proceed to the intersection of the relative speed A in question, and follow up the diagonal line to ψ absolute difference in true heading. This value of ψ will be the minimum absolute difference in heading of the two aircraft that will permit them to be visible to each pilot.

The speed-relationship chart shown in Fig. 1 is presented for the convenience of the reader and is constructed to permit a speed relationship value to be obtained for any speeds as long as the same scale is used on both axes. This chart was constructed from the equation

$$A = \frac{V_s}{V_f} \tag{1}$$

Where

A = Speed relationship

V_s = Velocity of slower airplane, true air speed, in mph

V_f = Velocity of faster airplane, true air speed, in mph

The visual-angle chart shown in Fig 2 for straight-and-level flight was constructed by substituting example values in Equation (2)*.

$$\theta = \arcsin \left(\frac{A \sin \psi}{\sqrt{A^2 + 1 - 2A \cos \psi}} \right) \quad (2)^*$$

and $\phi = 180^\circ - (\psi + \theta)$

Where

θ = Visual angle from longitudinal axis required from fast airplane, in degrees

ϕ = Visual angle from longitudinal axis required from slow airplane, in degrees

A = Speed relationship (Fig 1)

ψ = Absolute difference in true heading of two aircraft, in degrees (always 180° or less)

The distance-of-separation nomograph shown in Fig 3 was constructed by using Equation (3)

$$Z = \frac{V_s t \sin \psi}{60 \sin \theta} \quad (3)$$

Where

V_s = Velocity of slower airplane, true air speed, in mph

t = Time of investigation before collision, in minutes

ψ = Absolute difference in true heading of two airplanes, in degrees

θ = Visual angle required from faster airplane, in degrees

Z = Distance between airplanes at time t of investigation, in miles

Sample Solution

It is desired to know the visual angles that are required for the pilots of each of two aircraft to see the other aircraft while on a collision course and to determine the distance between them at a given time t . Refer to Fig 4. In this case, both airplanes are flying straight and level.

Given

$V_s = 180$ mph, velocity of slower airplane, true air speed

$V_f = 200$ mph, velocity of faster airplane, true air speed

*Equations (2) through (6), (8) through (14), (16), and (20) through (29) have their derivations in the Appendix.

$\psi = 45^\circ$, absolute difference in true heading of two aircraft

In this case, assume that the slower aircraft is on the left of the faster aircraft

= 1 minute, time of investigation before collision

Solution

1 Enter Fig 1 with $V_s = 180$ mph and $V_f = 200$ mph and determine the speed relationship $A = 0.90$

2 Enter the chart shown in Fig 2 with the difference in true heading $\psi = 45^\circ$ and follow down this diagonal line until an intersection is made with the speed-relationship curve $A = 0.9$. Follow the horizontal line from this point and read the angle $\theta = 60^\circ$. This is the angle required for the pilot of the faster airplane to see the slower airplane. Follow the vertical line from this point and read the angle $\phi = 75^\circ$. This is the angle required for the pilot of the lower airplane to see the faster airplane.

3 The distance between the two aircraft at $t = 1$ minute, or one minute before collision, is found by using the nomograph shown in Fig 3

a Enter nomograph with $V_s = 180$ mph

b Connect 180 to $t = 1$ minute, find T_1

c. Connect T_1 intersection with $\psi = 45^\circ$ (absolute difference in true heading) and find T_2

d Connect T_2 intersection with $\theta = 60^\circ$ (visual angle required from the faster airplane)

e Read $Z = 2.5$ miles apart

4. The closing velocity at $t = 1$, one minute before collision, is found by using the nomograph shown in Fig 3

a Enter nomograph with $Z = 2.5$ miles

b Connect 2.5 to $t = 1$ minute

c Read $V_c = 220$ feet per second, or 150 mph

Additional Considerations

The formulas and charts are designed for using true air speed and true heading of the aircraft. In the event that the ground track and ground speeds are used, it is necessary to correct the visual angles by adding or subtracting the crab angle or wind-drift angle.

Discussion of Significant Trends

By inspecting the curves shown in Fig 2, it is evident that the mid-air collision problem is greatest at small differences in heading and at small differences in speed. These are the conditions that make the chances of either pilot seeing the other airplane very small.

Since time is not involved in obtaining visual angles from Fig 2, it is evident that as the time from collision varies the visual angles obtained from this chart will remain constant throughout the specific collision course being investigated. If the required visual angle remains constant, there is no relative motion between the airplanes flying on a straight-and-level collision course.

For any possible combination of speed relationship and absolute difference in heading that would result in a collision, the visual angle required from the faster airplane will always be less than 90° while the visual angle required from the slower airplane could be as great as 180° . These maximum visual angles do not occur at the same time.

The locus line drawn on Fig 2 indicates that the maximum visual angle required from the faster airplane occurs when a 90° visual angle is required from the slower airplane.

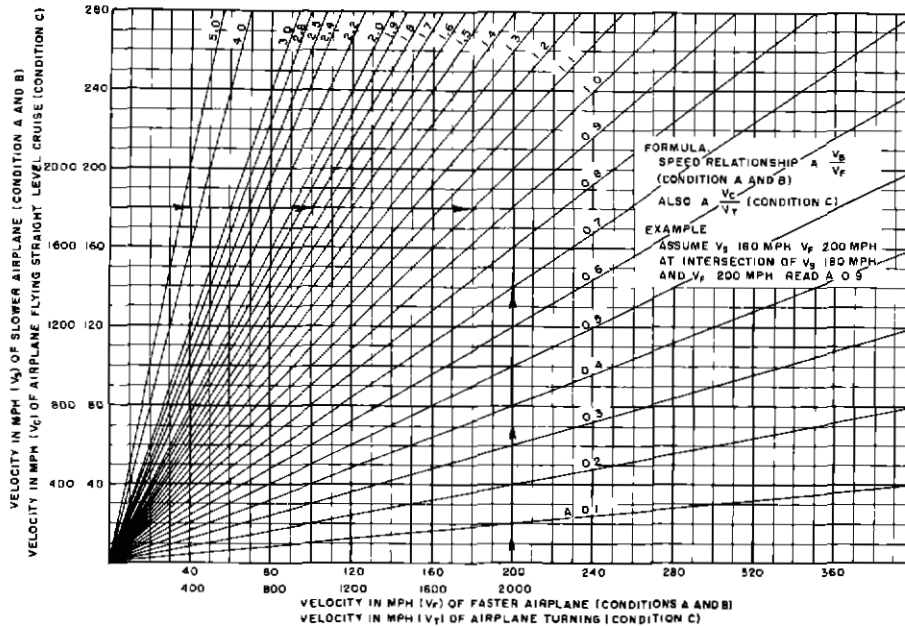


Fig 1 Determination of Speed Relationship

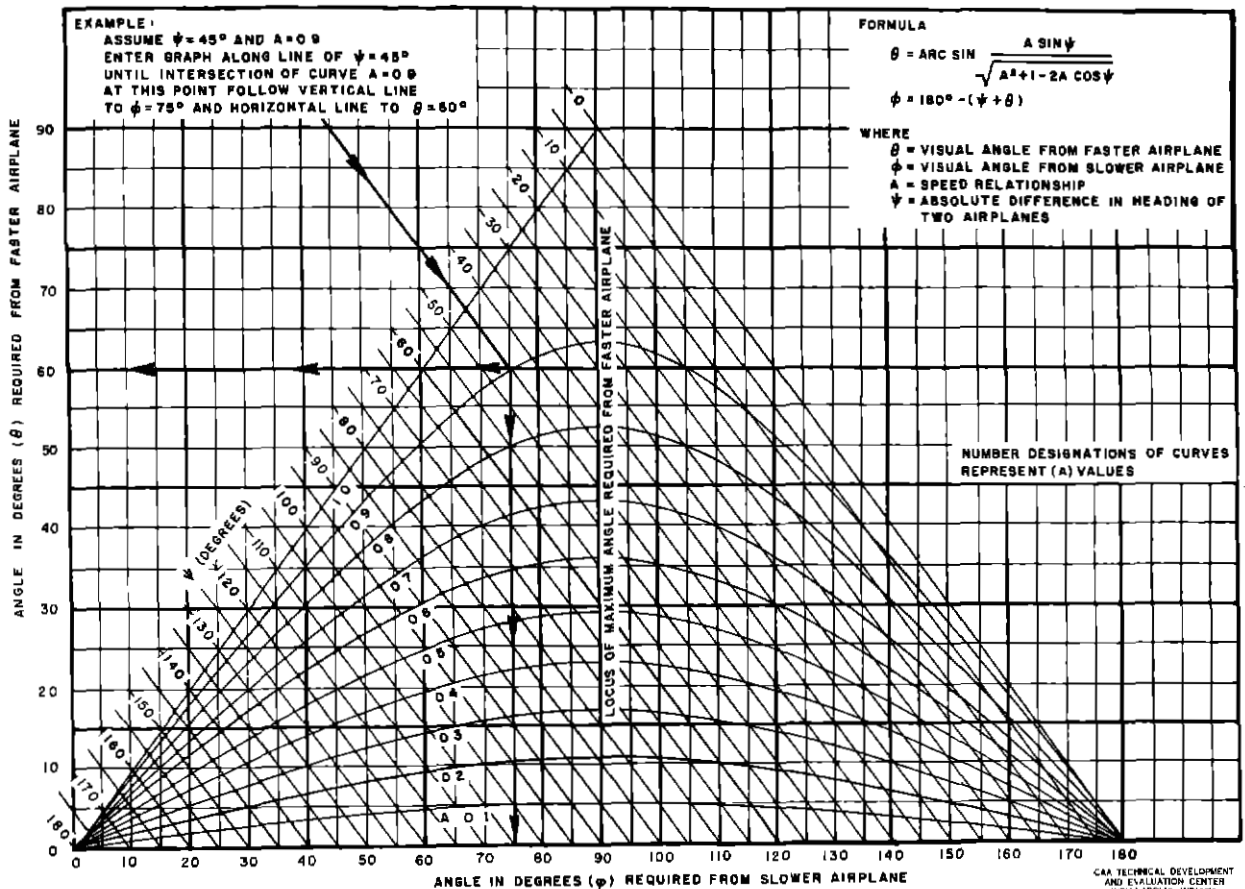
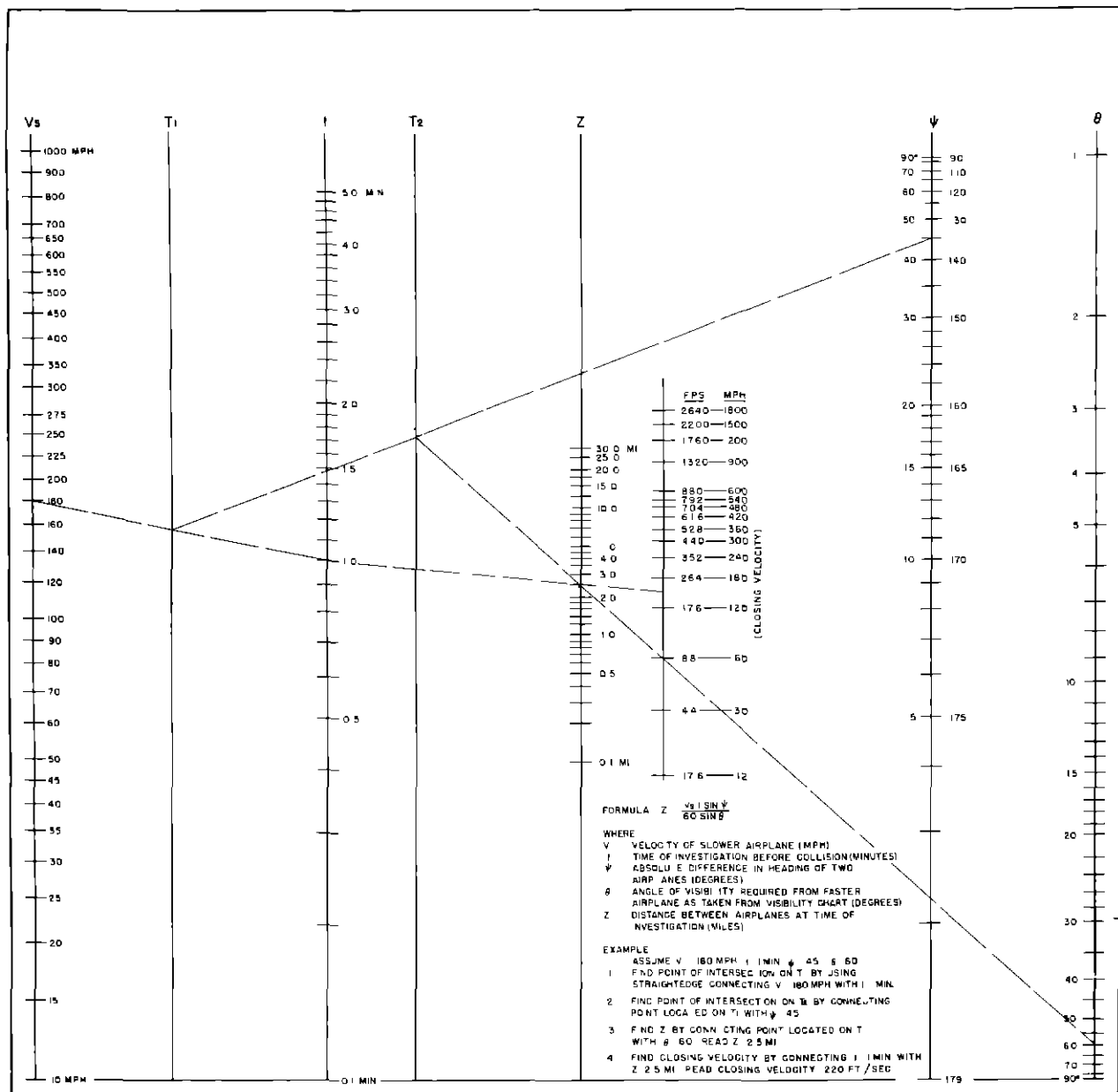


Fig 2 Horizontal Visual Angles Required From Two Airplanes for Level-Flight, Climb, and Descent Collision Conditions



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Fig 3 Determination of Distance of Separation for Level-Flight Condition

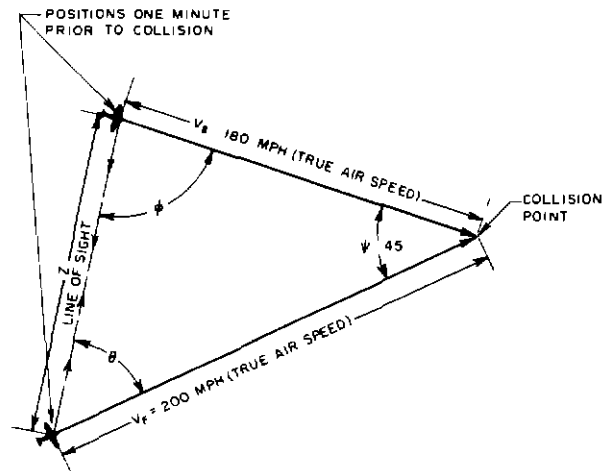


Fig 4 Vector Diagram of Sample Solution for Level-Flight Collision Condition

COLLISION CONDITION B STRAIGHT CLIMB AND DESCENT

Description

Calculation of visual angles to right or left and up or down for the pilots of each aircraft to see the other aircraft is facilitated by the use of the charts designed for the climb-and-descent condition. These charts are designed around the hypothesis that two airplanes are climbing or descending relative to one another and that they are approaching each other on straight courses from any direction at any relative speed.

Procedure and Formulas

The climb and descent charts are shown in Figs 1, 2, 5, and 6. The procedure for using these charts is as follows:

1 To determine speed relationship A, enter Fig 1 with the true airspeeds of the two aircraft.

2 To determine horizontal visual angles θ and ϕ required for the pilots of each aircraft to see the other aircraft, enter Fig 2 with the absolute difference in true heading ψ and the value of A.

3 To determine V, the projection upon the horizontal plane of the closing velocity (Fig 8), enter Fig 5 with horizontal visual angle θ required from faster airplane, velocity V_s of the slower airplane, and the absolute difference in true heading ψ .

4 To determine the vertical angle ρ (which is measured from the horizon and is equal and opposite from each airplane), enter Fig 6 with the algebraic sum of the rate of change of altitudes between the two aircraft $\Delta_1 \pm \Delta_2$ and V, the projection upon the horizontal plane of the closing velocity.

5 To determine the true vertical visual angle required, the fuselage attitude with respect to the horizon must be considered and the correction must be applied to the vertical angle ρ . The fuselage attitude with respect to the horizon during climb or descent varies with different airplanes. A listing of aircraft attitudes during climb or descent, as compiled from studies made at Indianapolis and Chicago airports, may be found on Table I. The data for this table were computed from photographs of the flight paths as illustrated in Fig 7. These data are presented in order to obtain a sample of fuselage attitudes with respect to the horizon for different types of aircraft.

6 To determine the distance Z between the airplanes at any instant of time prior to collision, enter Fig 5 with time t before collision, the vertical angle ρ , and V, the projection upon the horizontal plane of the closing velocity.

7 To determine the closing velocity between the two aircraft, enter Fig 5 with the distance Z between aircraft and time t before collision.

In determining the velocity V, the quantity needed for entering the vertical-angle chart (Fig 6), the nomograph⁶ shown in Fig 5 was constructed by using Equation (4)

$$V = \frac{V_s \sin \psi}{\sin \theta} \quad (4)$$

Where

V = Vertical projection of the closing velocity upon the horizontal plane, in mph

V_s = Velocity of the slower airplane, in mph

ψ = Absolute difference in true heading of the two aircraft, in degrees

θ = Horizontal visual angle required from the faster airplane, in degrees

⁶Lee H. Johnson, "Nomography and Empirical Equations," John Wiley & Sons, Inc., New York, pp 33-43, 1952.

The vertical-angle chart, shown in Fig 6, was constructed by substituting example values in the following equation

$$\rho = \arctan \frac{\Delta_1 \pm \Delta_2}{88V} \quad (5)$$

Where

Δ_1 or Δ_2 = Rate of climb or rate of descent of the respective airplanes, in feet per minute

$\Delta_1 \bullet \Delta_2$ = The algebraic sum of the rate of change of altitude between the two airplanes, in feet per minute (If both Δ_1 and Δ_2 are rate of climb or if both are rate of descent, then a minus sign is used, for any other combination, a plus sign is used)

ρ = The vertical angle measured from the horizon, in degrees

V = Vertical projection of the closing velocity upon the horizontal plane, in mph

For determination of the distance between the aircraft at time t , Fig 5 was constructed by using Equation (6)

$$Z = \frac{Vt}{60 \cos \rho} \quad (6)$$

Where

V = Vertical projection of the closing velocity upon the horizontal plane, in mph

t = Time of investigation before collision, in minutes

ρ = Vertical angle, in degrees

Z = Distance between aircraft at time of investigation, in miles

Sample Solution

It is desired to know the visual angles that are required for the pilots of each aircraft to see the other aircraft while flying on a collision course and to determine the distance they are apart at a given time t . Refer to Fig 8. In this case, one airplane is climbing and one is descending.

$V_s = 200$ mph, the velocity (true air speed) of the slower airplane, which is climbing

$\Delta_1 = 1220$ fpm, the rate of climb

$V_f = 220$ mph, the velocity (true air speed) of the faster airplane, which is descending

$\Delta_2 = 2000$ fpm, the rate of descent

$\psi = 45^\circ$, the absolute difference in true heading of the two aircraft

In this case, the climbing airplane is approaching the descending airplane from the right

$t = 0.5$ minute, the time of investigation before collision

Solution

- 1 Enter Fig 1 with $V_s = 200$ mph and $V_f = 220$ mph, and read $A = 0.9$
- 2 Enter horizontal-visual-angle chart, Fig 2, with $\psi = 45^\circ$ and $A = 0.9$ Read $\theta = 60^\circ$ right, the horizontal visual angle (measured from straight ahead of the pilot-eye position) required for the slower airplane to be seen from the faster airplane Read $\phi = 75^\circ$ left, the horizontal visual angle (measured from straight ahead of the pilot-eye position) required for the faster airplane to be seen from the slower airplane
- 3 Enter nomograph, Fig 5, with $\theta = 60^\circ$, $V_s = 200$ mph, and $\psi = 45^\circ$, and read $V = 160$ mph, the velocity needed for entering the vertical-angle chart
- 4 Enter Fig 6, the vertical-angle chart, with $\Delta_1 + \Delta_2 = 3220$ feet per minute, which is the rate of change of altitude added algebraically between the two aircraft, and $V = 160$ mph Read $\rho = 13^\circ$, the vertical angle (measured from the horizon) required for either airplane to be seen from the other The airplane descending would require 13° down from the horizon, and the airplane climbing would require 13° up from the horizon
- 5 To determine the true vertical visual angle required from each airplane, the fuselage attitude with respect to the horizon must be considered
For example, if it is assumed that the fuselage of the airplane climbing is in a 2° nose-up attitude, then the true vertical visual angle required from this airplane with respect to the cockpit would be $13^\circ - 2^\circ$, or 11° up Also, if it is assumed that the fuselage of the airplane descending is in a 5° nose-down attitude, then the true vertical visual angle required from airplane with respect to the cockpit would be $13^\circ - 5^\circ$, or 8° down Table I, which shows a variety of attitudes compiled from photographic studies made of actual take-off and landing-approach flight paths at Indianapolis and Chicago airports, is presented for the convenience of the reader
- 6 Enter nomograph, Fig 5, with $V = 160$ mph, previously determined in Step 3 In this case, $t = 0.5$ minute and $\rho = 13^\circ$ Read $Z = 1.40$ miles, the distance between the airplanes at time = 0.5 minute Note There can be similar solutions for other conditions such as both aircraft climbing, both aircraft descending, one airplane flying level and one climbing, or one airplane flying level and one descending
If the airplanes are climbing or descending on a difference in heading equal to 0° or 180° , the visual angles required from either airplane will be in the vertical plane only This condition, then, becomes the same as Condition A, except that in this case the relative motion occurs in the vertical plane while in Condition A the relative motion occurs in the horizontal plane If the vertical plane is rotated 90° , it becomes coincident with the horizontal plane, therefore, Condition B becomes hypothetically the same as Condition A and the visual-angle chart designed for Condition A can be used In using this chart, the difference in heading in this condition is determined by the angle of climb or descent of each airplane

Additional Considerations

The same additional considerations that were discussed in Condition A apply similarly to this collision condition

Discussion of Significant Trends

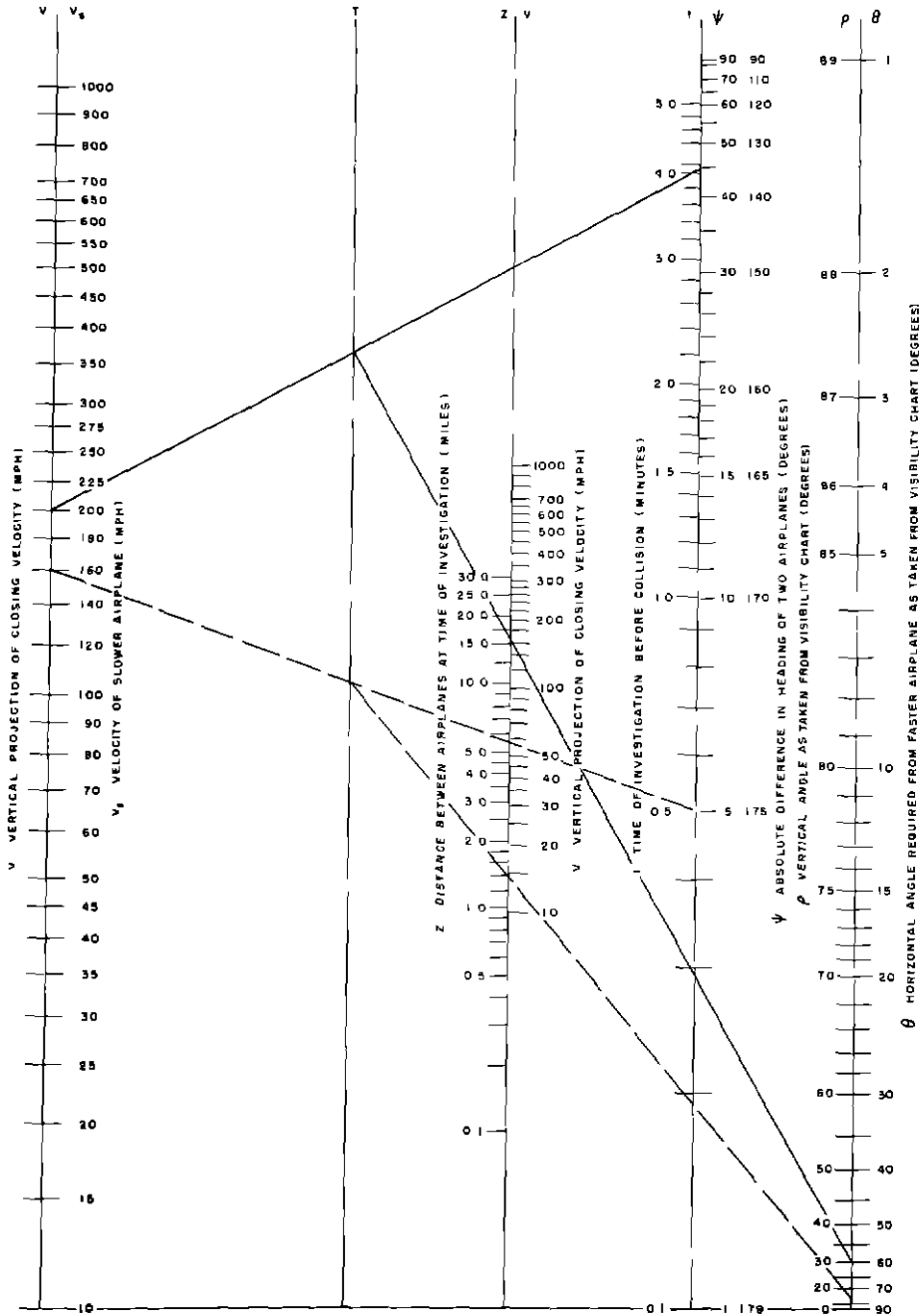
The trends discussed in Condition A apply similarly to this collision condition The vertical-visual-angle requirements from both airplanes are greatest at small differences in heading and small differences in speed

TABLE I

TYPICAL FLIGHT-PATH ANGLES AND FUSELAGE-
LONGITUDINAL-AXIS ATTITUDES

Aircraft Type	Flight-Path Angle Relative to Horizon		Fuselage Attitude Relative to Flight- Path Angle		Fuselage Attitude Relative to Horizon	
	Takeoff (degrees)	Landing (degrees)	Takeoff (degrees)	Landing (degrees)	Takeoff (degrees)	Landing (degrees)
Martin 404	+ 7	- 5 25	-6	+0 75	+ 1	- 4 50
	6	- 8 0	-3	-3 75	+ 3	-11 75
	4 5	- 5 0	-4 5	-4 25	0	- 9 25
	5 25	- 4 0	-3 25	-3 0	+ 2	- 7 0
	4	- 5 0	-2 5	-4 0	+ 1 5	- 9 0
	4 75	- 7 5	-4 5	-8 0	+ 0 25	-15 5
	5	- 8 25	-4	-3 5	+ 1	-11 75
	7		0		+ 7	
	6		-4 5		+ 1 5	
	5		+2 5		+ 7 5	
	5		+2 5		+ 7 5	
	Average	5 41	-2 48	-3 68	2 93	- 9 82
Convair 240	11	-10	+1	+1	12	- 9
	6 25	- 4	+1	-1	7 25	- 5
	1 50	- 5	0	+1	1 5	- 4
	5 0	- 5 75	+0 25	0	5 25	- 5 75
	Average	5 94	±0 56	+0 25	6 50	- 5 94
Douglas DC-3	4 5	- 4 0	0	+1 5	4 5	- 2 5
	5 25	- 6 0	0	+2 0	5 25	- 4
	1 5	- 5 5	-0 5	2 75	1 0	- 2 75
	4 0	- 6 0	0	0	4 0	- 6
	4 0	- 5 5	-2 0	0	2 0	- 5 5
	4 5	- 5	-3	0	1 5	- 5
	4 0	- 5	+3	+2	7 0	- 3
	2 0	- 5	±3 5	1 5	5 5	- 3 5
	4 0		2 5		6 5	
	Average	3 75	0 39	+1 22	+4 14	- 4 03
Lockheed Constellations	+ 3	- 5	-2 5	-1 0	+0 5	- 6 0
	+ 4	- 5 5	+1	-6 0	+5	-11 5
	+ 5		0		+5	
	+ 6		+2 5		+8 5	
	Average	+ 4 5	+0 25	-3 5	4 75	- 8 75
Douglas DC-4	5 5		+1 0		4 5	
Douglas DC-6	3 5		0		3 5	
	4		3 5		7 5	
	Average	+ 3 7	+1 7		+5 5	
Beech D-18S	+ 5 0	- 4	-5 0	-2	0	- 6 0
	3		0		3	
	Average	+ 4	-2 5	-2	+1 5	- 6

VELOCITY AND DISTANCE NOMOGRAPH FOR CLIMB AND DESCENT CONDITION



$$\text{FORMULA } V = \frac{V_s \sin \psi}{\sin \theta}$$

EXAMPLE ILLUSTRATED BY SOLID LINE
ASSUME $V_s = 200$ MPH $\psi = 45^\circ$ $\theta = 60^\circ$

1 FIND POINT OF INTERSECTION ON T
BY USING STRAIGHTEDGE CONNECT-
ING $V_s = 200$ MPH WITH $\psi = 45^\circ$

2 FIND V VELOCITY NEEDED FOR
ENTERING VERTICAL VISIBILITY
ANGLE CHART BY CONNECTING
POINT LOCATED ON T WITH
 $\theta = 60^\circ$ READ V 160 MPH

$$\text{FORMULA } Z = \frac{V T}{60 \cos \rho}$$

EXAMPLE ILLUSTRATED BY DASHED LINE

ASSUME $V = 160$ MPH $\rho = 0.5$ MIN
 $\theta = 13^\circ$

1 FIND POINT OF INTERSECTION ON T
BY USING STRAIGHTEDGE CONNECT-
ING $V = 160$ MPH WITH $\rho = 0.5$ MIN

2 FIND Z BY CONNECTING POINT
LOCATED ON T WITH $\theta = 13^\circ$
READ Z 1.40 MI

Fig 5 Determination of Velocity and Distance of Separation for Climb-and-Descent Conditions

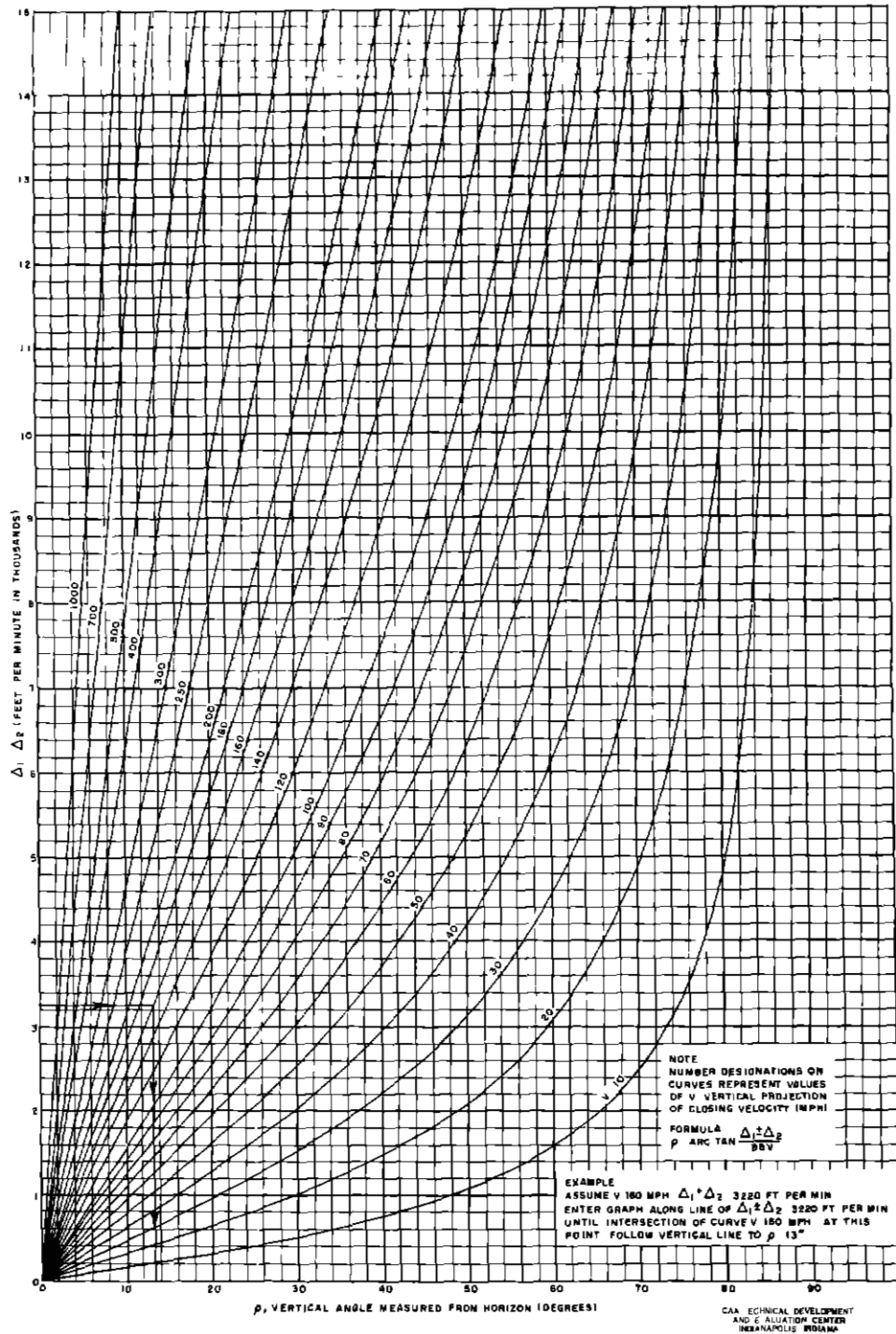


Fig 6 Vertical Angle, Measured From Horizon, Required From Two Airplanes for Climb-and-Descent Collision Condition



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Fig 7 Example of Flight-Path Attitude Analysis

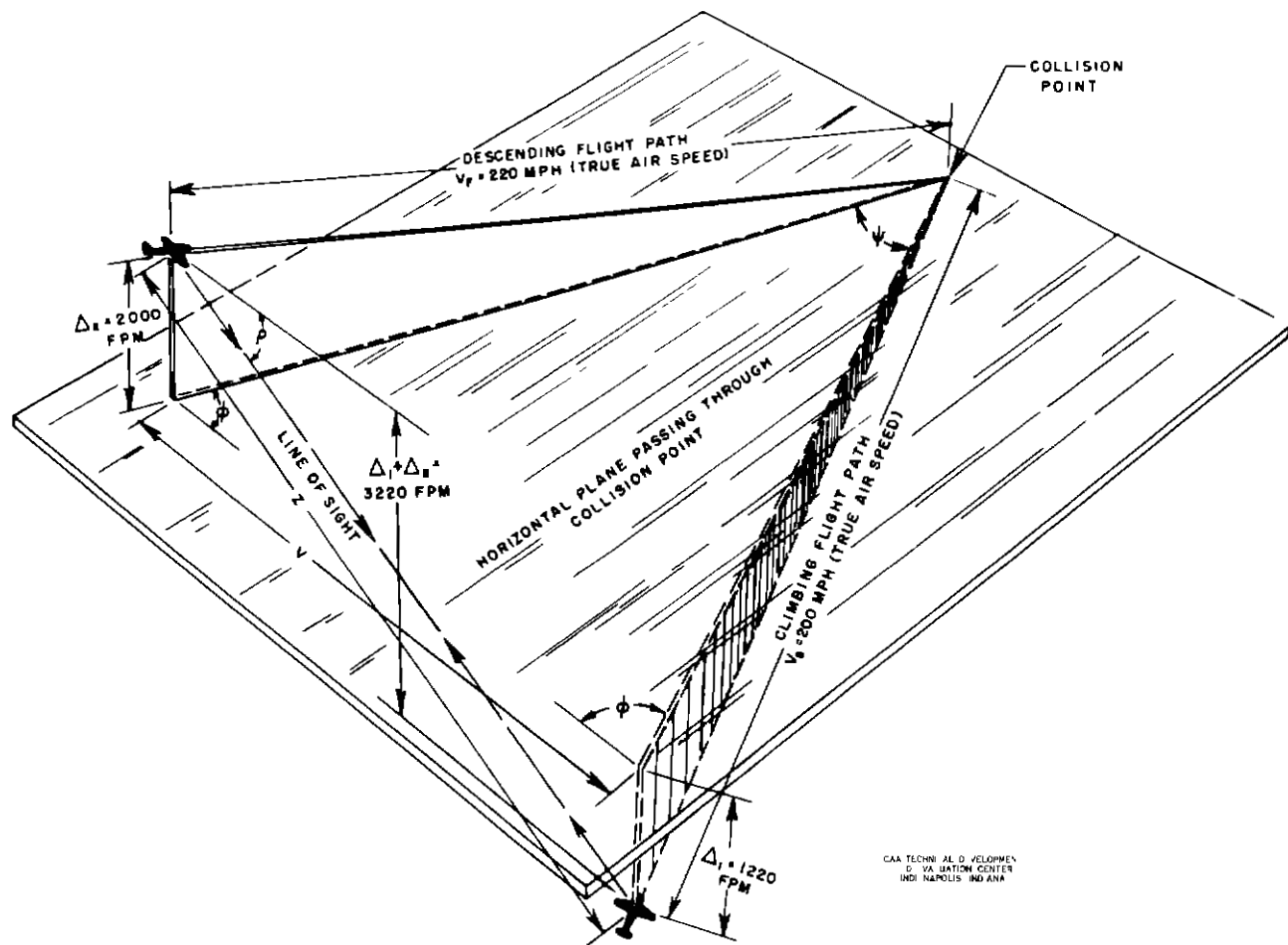


Fig 8 Vector Diagram of Sample Solution for Climb-and-Descent Collision Conditions

COLLISION COURSE C

ONE AIRPLANE TURNING AND ONE FLYING STRAIGHT AND LEVEL

Description

Calculation of the visual angles to the right or left which are required for the pilots of each airplane to see the other airplane is facilitated by the use of the charts designed for one airplane turning left and one airplane flying straight and level. For the solution of a condition involving a right turn, see page 22.

The turning-and-level-flight visual-angle charts are designed around the hypothesis that one airplane is flying straight and level while another airplane is making a left turn and both are on a collision course, flying in the same horizontal plane, and approaching each other at any relative speed from any direction.

Procedure and Formulas

The turning and straight-and-level charts are shown in Figs. 1 and 9 through 25. The procedure for using these charts is as follows:

1. To determine speed relationship A , enter Fig. 1 with the true air speeds of the two aircraft.

2. To determine the time value t_0 required for entering the horizontal-visual-angle charts, enter Fig. 25 with the angle of bank λ of the turning airplane, the velocity V_T of the turning airplane, and the time t of investigation before collision.

3. To determine horizontal visual angles α and β (where α = the visual angle for the airplane flying straight and level and β = the visual angle for the turning airplane) required for the pilots of each aircraft to see the other aircraft, select from Figs. 10 through 21 the horizontal-visual-angle chart that applies to the absolute difference in heading between the two aircraft at collision points. Enter this chart with the time value t_0 and the speed relationship A .

4. To determine the distance factor Z_0 , enter with speed relationship A and time value t_0 the Z_0 table located on the same horizontal-visual-angle chart that was selected in Step C-3.

5. To determine actual distance separation Z between aircraft at any time prior to collision, enter Fig. 24 with distance factor Z_0 and radius of turn R_T . If the radius of turn is not known, enter Fig. 23 with the angle of bank λ and the velocity in turn V_T .

6. To determine the vertical visual angle μ as affected by the angle of bank λ , enter Fig. 22 with the angle of bank λ and the horizontal visual angle β .

The speed-relationship chart, designed for Conditions A and B and shown in Fig. 1, is used to obtain the speed relationship A . In this case, however,

$$A = \frac{V_c}{V_T} \quad (7)$$

Where

V_c = Velocity of the airplane flying straight and level, in mph

V_T = Velocity of the airplane turning, in mph

The t_0 nomograph shown in Fig. 25 was constructed by using the equation

$$t_0 = \frac{209.2 \tan \lambda \cdot t}{V_T} \quad (8)$$

Where

V_T = Velocity of the turning airplane, in mph

λ = Angle of bank of turning airplane, in degrees

t = Actual time of investigation before collision, in minutes

t_0 = Required time value for entering visual-angle charts, in minutes

The quantity t_0 is a relative time value determined by proportional analysis as described in the Appendix

The visual-angle charts shown in Figs 10 through 21 were constructed by substituting example values in the following equations

$$\alpha = \arctan \frac{Y}{X} \quad (9)$$

$$\beta = \alpha + \omega - 90^\circ \quad (10)$$

$$Z = \sqrt{X^2 + Y^2} \quad (11)$$

To solve Equations (9), (10), and (11), the following has to be known

$$X = V_c t + R_T (\cos \psi - \cos \omega) \quad (12)$$

$$Y = R_T (\sin \psi + \sin \omega) \quad (13)$$

$$\omega = Kt - \psi \quad (14)$$

Where

α = the horizontal visual angle that is needed from straight ahead for the turning airplane to be seen from the airplane flying straight and level, in degrees

β = the horizontal visual angle that is needed from straight ahead for the airplane flying straight and level to be seen from the turning airplane, in degrees

Z = Distance between the aircraft at time t , in feet

t = Time before collision, in minutes

V_c = Velocity (true air speed) of airplane flying straight and level, in feet per minute

R_T = Radius of flight path of turning aircraft, in feet

K = Rate of turn of the turning aircraft, in degrees per minute

ψ , ω , X , and Y are illustrated in Fig 86 of the Appendix

Note In case it is necessary to calculate visual angles by using the formulas, use of Table II will be necessary to determine that value of ψ which corresponds to the absolute difference in heading at collision point

The vertical-visual-angle chart shown in Fig 22 was constructed by passing a horizontal plane, which simulates the plane of the horizon, through a right cylinder at different bank angles and then plotting the points of intersection. The plane of the horizon is the horizontal plane in which both aircraft are flying, therefore, the pilot flying either aircraft must look to the horizon to see the other aircraft

TABLE II
RELATIONSHIP BETWEEN ψ AND ACTUAL HEADING DIFFERENCE

ψ (degrees)	Difference in Heading at Collision Point (degrees)	Condition
0	90	With turning airplane on the left of cruising airplane at collision point
30	60	With turning airplane on the left of cruising airplane at collision point
60	30	With turning airplane on the left of cruising airplane at collision point
90	0	Both airplanes heading in the same direction
120	30	With turning airplane on the right of cruising airplane at collision point
150	60	With turning airplane on the right of cruising airplane at collision point
180	90	With turning airplane on the right of cruising airplane at collision point
210	120	With turning airplane on the right of cruising airplane at collision point
240	150	With turning airplane on the right of cruising airplane at collision point
270	180	Head on
300	150	With turning airplane on the left of cruising airplane at collision point
330	120	With turning airplane on the left of cruising airplane at collision point

The radius-of-turn chart shown in Fig 23 was constructed by substituting example values in the equation⁷

$$\tan \lambda = \frac{0.067 V_T^2}{R_T} \quad (15)$$

Where

λ = Angle of bank, in degrees

V_T = Velocity of turning airplane, in mph

R_T = Radius of turn, in feet

⁷Bradley Jones, "Elements of Practical Aerodynamics," Eqn (22 2 1) p 365, John Wiley & Sons, Inc , New York, 1950

The distance chart shown in Fig 24 was constructed by substituting values in the equation

$$Z = \frac{Z_0 R_T}{2801} \quad (16)$$

Where

Z = Actual distance between aircraft at time t before collision, in feet

Z_0 = Proportional distance between aircraft as obtained from visual-angle chart, in feet

R_T = Radius of turn of turning airplane, in feet

Sample Solution

It is desired to know what visual angles are required for the pilots of each aircraft to see the other aircraft while on a collision course and to determine the distance they are apart at a given time. Refer to Fig 9. In this case, one airplane is flying straight and level and one airplane is turning left.

V_C , the velocity of the airplane cruising straight and level, = 250 mph, true air speed

V_T , the velocity of the turning airplane, = 100 mph, true air speed

λ , the angle of bank of the turning airplane, = 20°

t , the time before collision to be investigated, = 0.5 minute

absolute difference in heading at collision point = 90°

Solution

- 1 Enter Fig 1 with $V_C = 250$ mph and $V_T = 100$ mph, and read $A = 2.5$
- 2 Enter Fig 25 with $\lambda = 20^\circ$, $t = 0.5$ minute, and $V_T = 100$ mph, and read $t_0 = 0.38$
- 3 Since the absolute difference in heading at collision is 90° with the turning airplane on the left at collision point, Fig 13 of the horizontal-visual-angle charts is selected. Enter this chart with $t_0 = 0.38$ and $A = 2.5$, and read α , which is the horizontal visual angle required from the airplane flying straight and level, equal to 4° left, also read β , which is the horizontal visual angle required from the turning airplane, equal to 52° left.

To find the distance between the aircraft at 0.5-minute separation, enter the Z_0 table on Fig 13 with A equal to 2.5 and t_0 equal to 0.38. No value of t_0 equal to 0.38 appears, therefore, the following interpolation is made:

$A = 2.5$	$t_0 = 0.40$	$Z_0 = 22,727$
$A = 2.5$	$t_0 = 0.30$	$Z_0 = 17,076$
		5,651 = Difference

Since 0.38 is 0.8 of the interval between 0.30 and 0.40, $0.8 \times 5651 = 4521$ for $t_0 = 0.38$ and $A = 2.5$, $Z_0 = 17,076 + 4521 = 21,597$ feet, or approximately 21,600 feet. This is not a true distance but a proportional distance which will be used in Step 5 to determine Z , the true distance.

If the absolute difference in heading at collision does not exactly correspond to one of the charts, then a similar interpolation between charts is necessary.

- 4 If R_T , the radius of turn of the turning aircraft is not known, Fig 23 is entered with λ equal to 20° and V_T equal to 100 mph, and read R_T equal to 1840 feet.

5 Enter Fig 24 with Z_0 equal to 21,600 feet, previously determined, and R_T equal to 1840 feet Read $Z = 14,000$ feet

6 To find the vertical visual angle μ , required at 0.5 minute from the turning aircraft while it is in a 20° bank, enter Fig 22 with λ equal to 20° and β equal to 52° left and read vertical visual angle required equal to 16° up

Additional Considerations

The formulas and charts are designed for using true air speed and true heading of the aircraft. In the event that ground track and ground speeds are used, it is necessary to correct the visual angles by adding or subtracting the crab angle or the wind-drift angle.

Visual angles may be determined from one airplane flying straight and level and one airplane making a right turn instead of a left turn by using the following procedure:

Given

The absolute difference in heading at collision point is equal to 150° with the turning airplane on the right of the cruising airplane at the collision point and making a right turn.

1 Enter the horizontal-visual-angle chart designed for a difference in heading at collision point equal to 150° and with the airplane making a turn on the left of the airplane flying straight and level at the collision point. This chart is entered with the parameters determined by the given condition, the only difference being that left is substituted for right in using the charts as related to heading at collision point.

2 In reading α and β , the magnitudes of the angles are correct but the directions of the angles need to be reversed to apply correctly to the given condition.

3 The distance between the airplanes at a specified time before collision and the vertical visual angles are determined in the same manner as described for one airplane flying straight and level and one airplane making a left turn.

Discussion of Significant Trends

The rate of change of the horizontal visual angle required from the airplane flying at level cruise is greatest when the velocity of the turning airplane is the greater of the two.

The rate of change of the horizontal visual angle required from the turning airplane is greatest and varies over the widest range when the velocity of the airplane flying at level cruise is the greater of the two.

When both airplanes are traveling at the same speed, the rate of change of the horizontal visual angle required from the turning airplane is generally greater than that of the airplane flying at level cruise.

When the rate of change of the horizontal visual angle required is the greatest from one airplane, it is a minimum for the other airplane.

In analyzing all horizontal-visual-angle charts for Condition C, it will be noted that when all curves cross at one common intersection point then the turning airplane is directly ahead or directly behind the airplane flying straight and level.

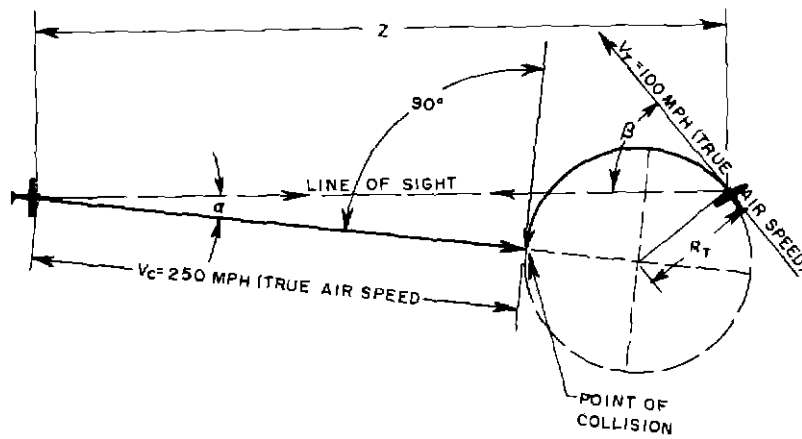


Fig 9 Vector Diagram of Sample Solution for One Airplane Turning Left and One Airplane Flying Straight and Level

The following charts are designed for the determination of horizontal-visual-angle requirements and distance factors for one airplane turning and one airplane flying straight and level

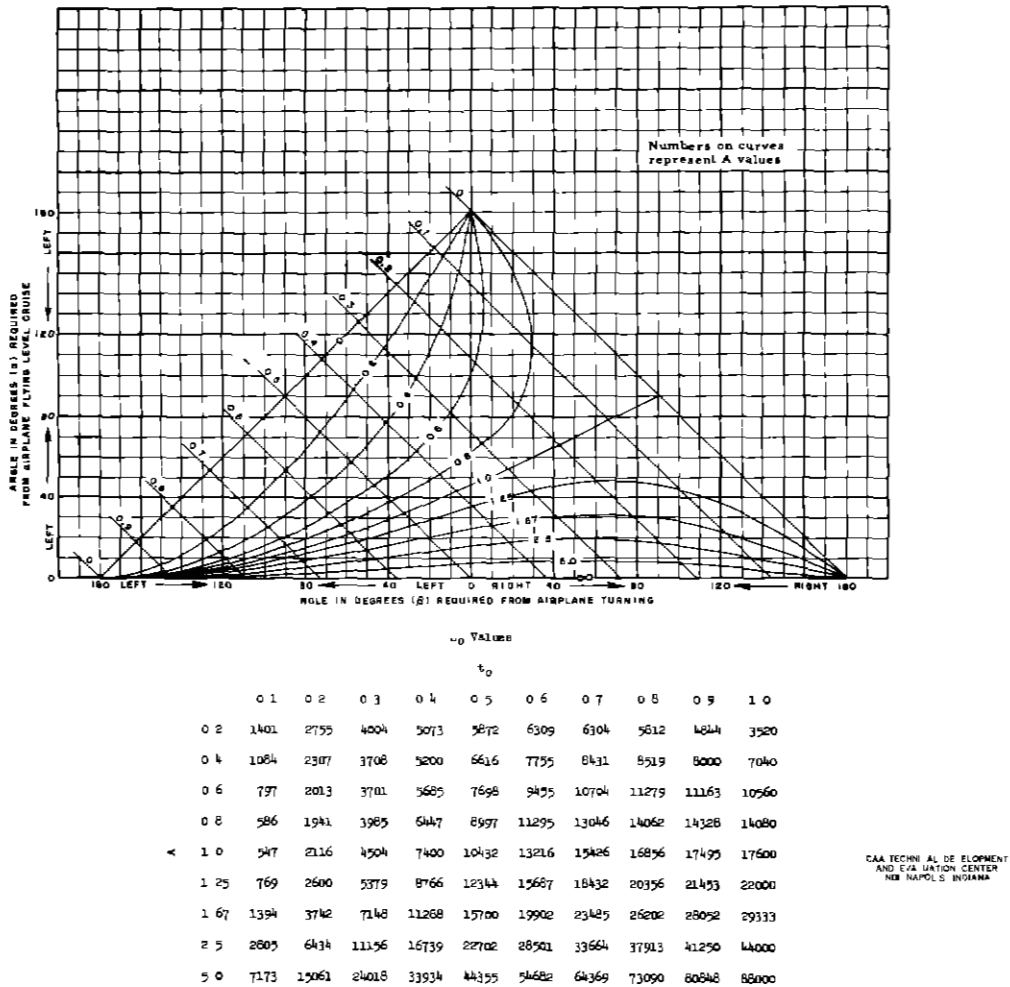
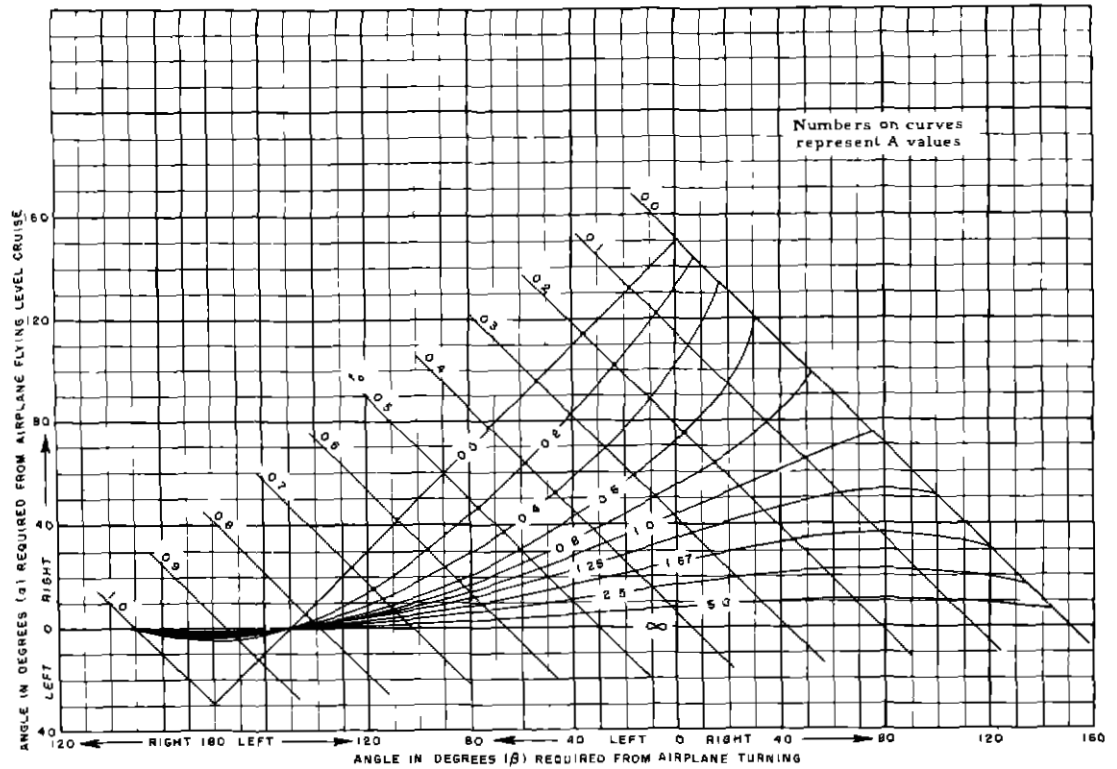


Fig 10 At Collision Point, the Difference in Heading = 0° and the Turning Airplane is on the Left of the Airplane Flying Straight and Level


 Z_0 Values

 t_0

	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.2	1512	3075	4545	5787	6659	7041	6857	6101	4675	3520
0.4	1364	3009	4796	6524	7968	8926	9254	8914	8037	7040
0.6	1290	3106	5251	7456	9426	10895	11679	11727	11203	10560
0.8	1310	3351	5864	8520	10972	12910	14117	14543	14370	14080
1.0	1420	3716	6580	9672	12575	14951	16563	17358	17537	17600
1.25	1655	4292	7606	11195	14629	17526	19627	20878	21496	22000
1.67	2192	5436	9468	13852	18129	21852	24743	26744	28056	29333
2.5	3488	8044	13501	19420	25271	30568	34989	38476	41295	44000
5.0	7749	16537	26315	36680	47052	56871	65766	73676	80894	88000

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Fig 11 At Collision Point, the Difference in Heading = 30° and the Turning Airplane is on the Left of the Airplane Flying Straight and Level

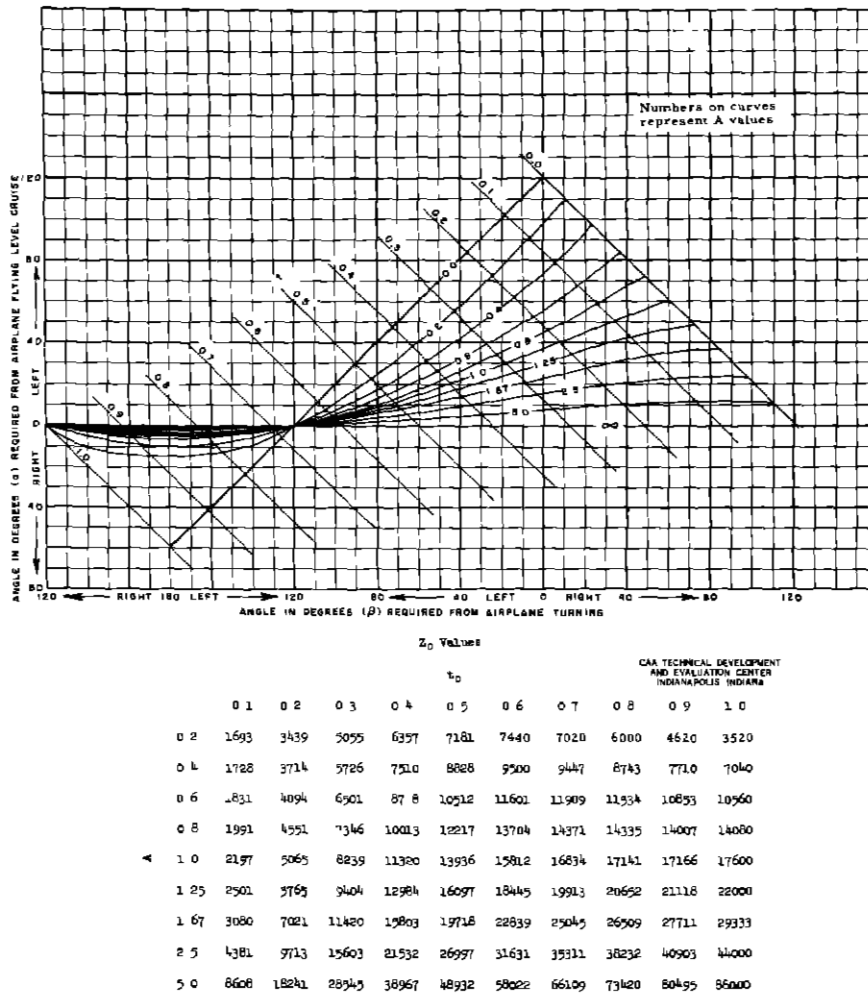
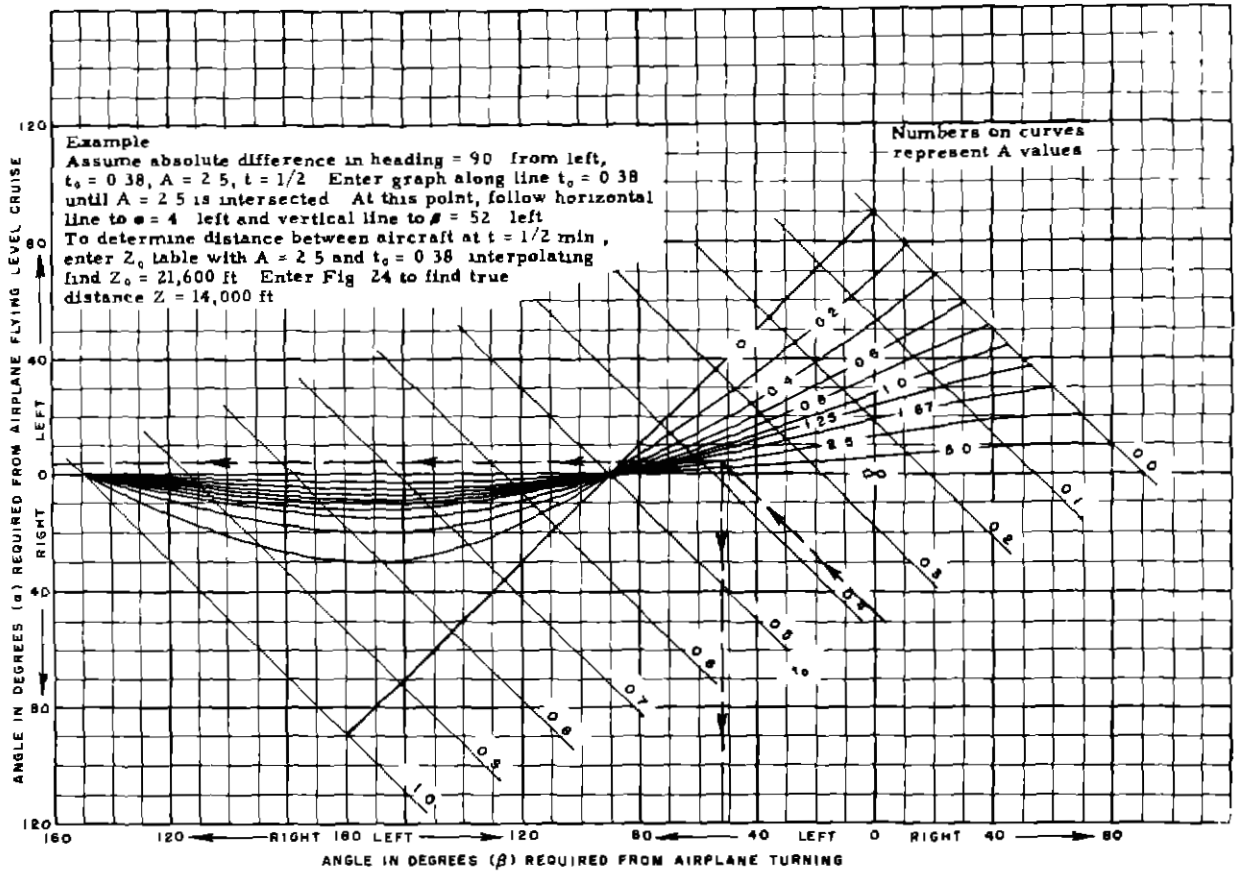


Fig 12 At Collision Point, the Difference in Heading $\approx 60^\circ$ and the Turning Airplane is on the Left of the Airplane Flying Straight and Level



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Z_0 Values

	t_0									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.2	1870	3750	5422	6681	7362	7366	6685	5447	4053	3520
0.4	2061	4275	6363	8053	9122	9436	8998	8023	7065	7040
0.6	2290	4846	7336	9436	10882	11522	11375	10720	10173	10560
0.8	2547	5447	8328	10825	12642	13615	13783	13466	13309	14080
< 1.0	2825	6071	9335	12219	14402	15714	16207	16236	16458	17600
1.25	3192	6873	10607	13965	16602	18341	19252	19716	20402	22000
1.67	3839	8244	12748	16881	20269	22727	24346	25542	26985	29333
2.5	5202	11061	17076	22727	27602	31510	34570	37231	40169	44000
5.0	9479	19716	30185	40301	49602	57891	65321	72385	79752	88000

Fig. 13 At Collision Point, the Difference in Heading = 90° and the Turning Airplane is on the Left of the Airplane Flying Straight and Level

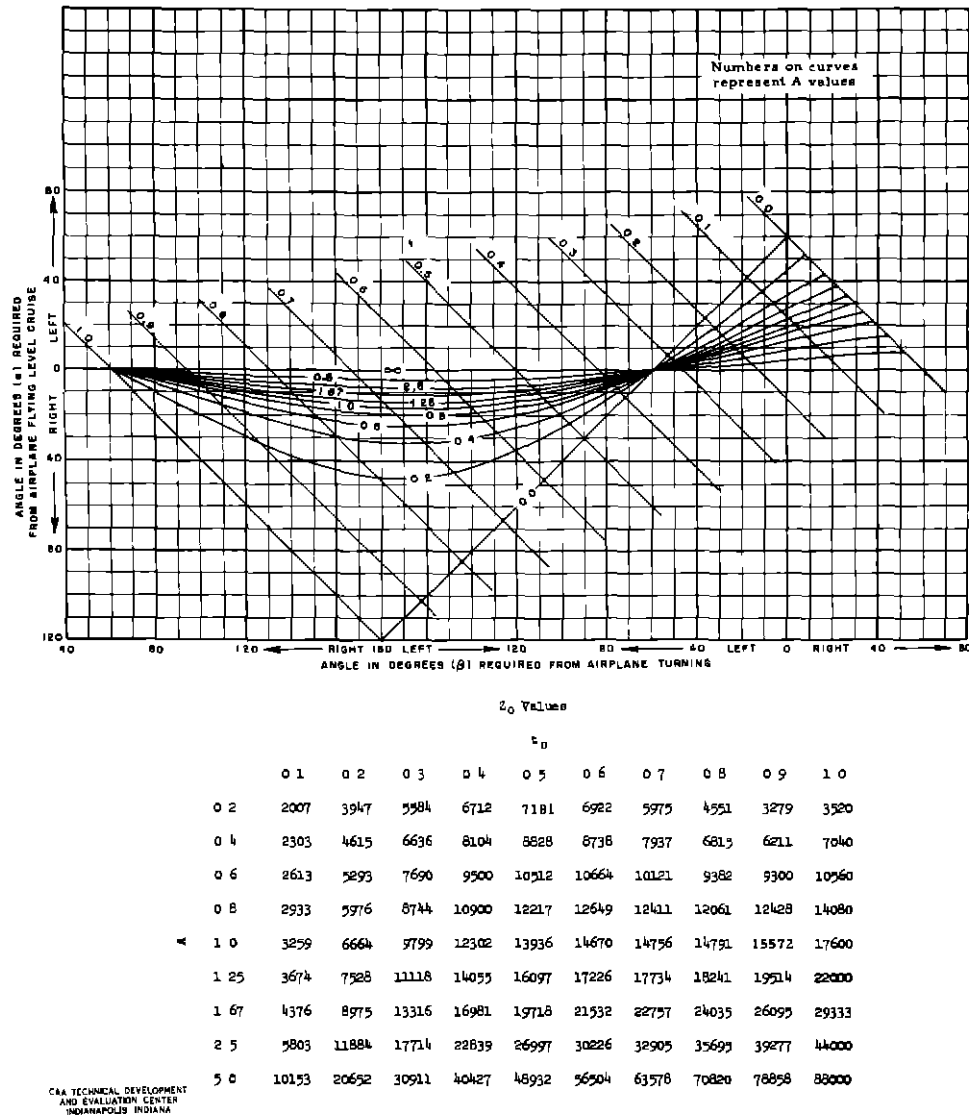


Fig 14 At Collision Point, the Difference in Heading = 120° and the Turning Airplane is on the Left of the Airplane Flying Straight and Level

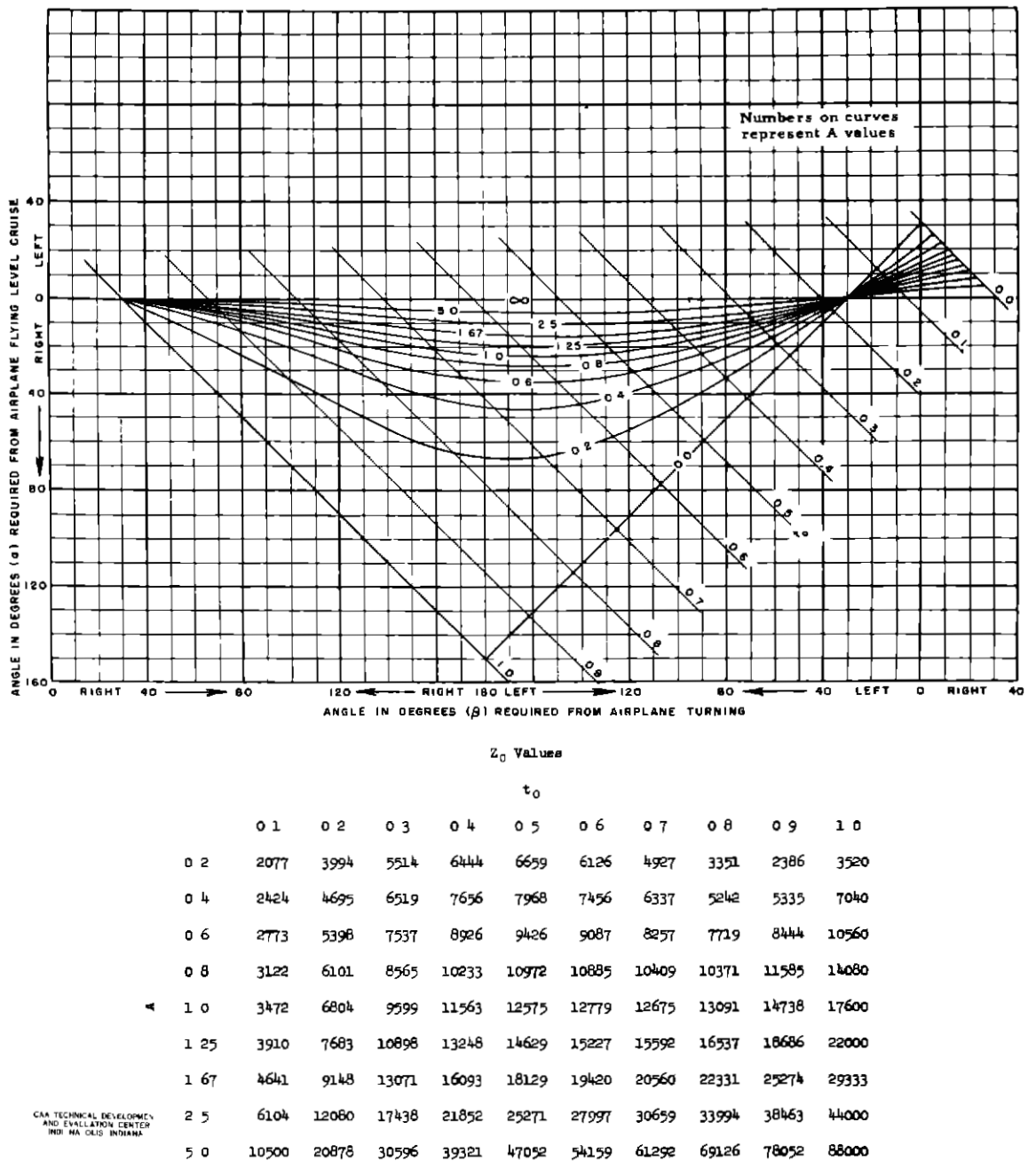
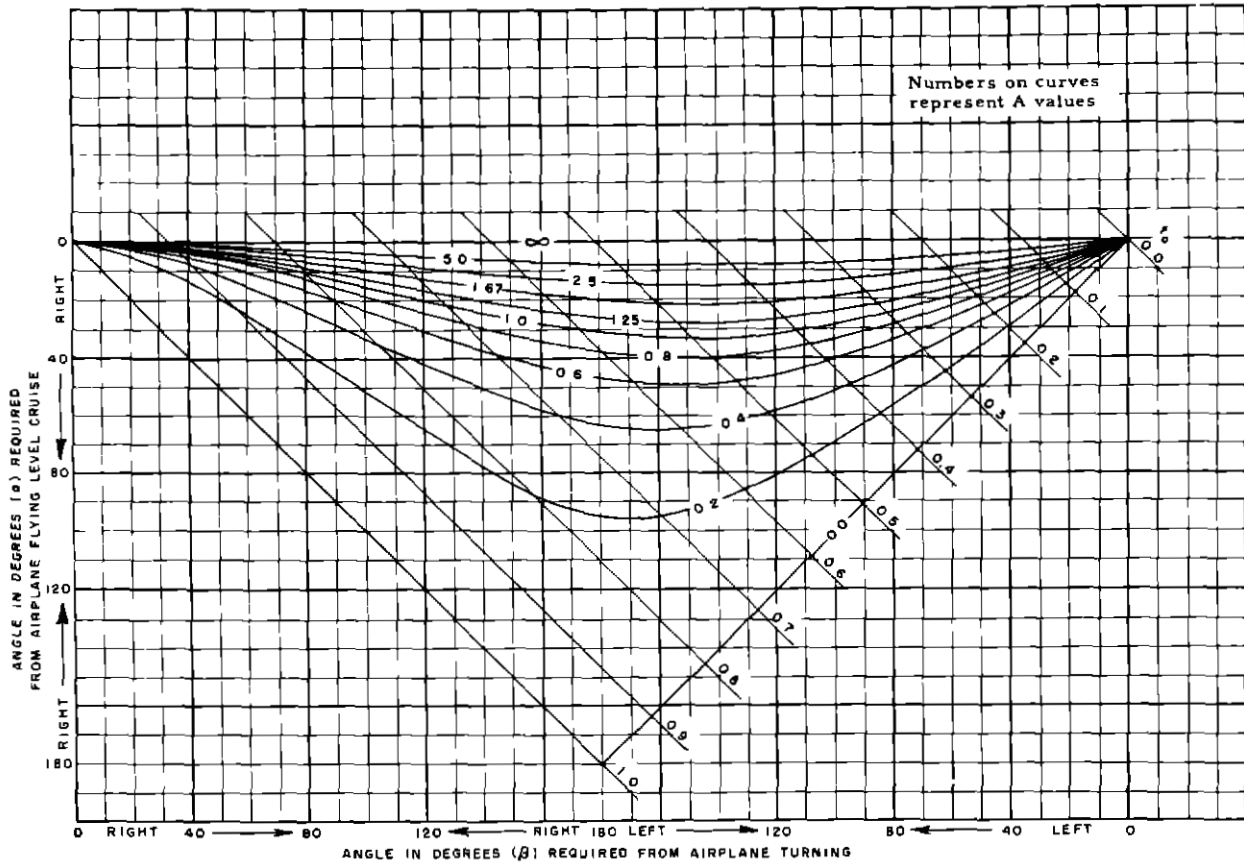


Fig 15 At Collision Point, the Difference in Heading = 150° and the Turning Airplane is on the Left of the Airplane Flying Straight and Level

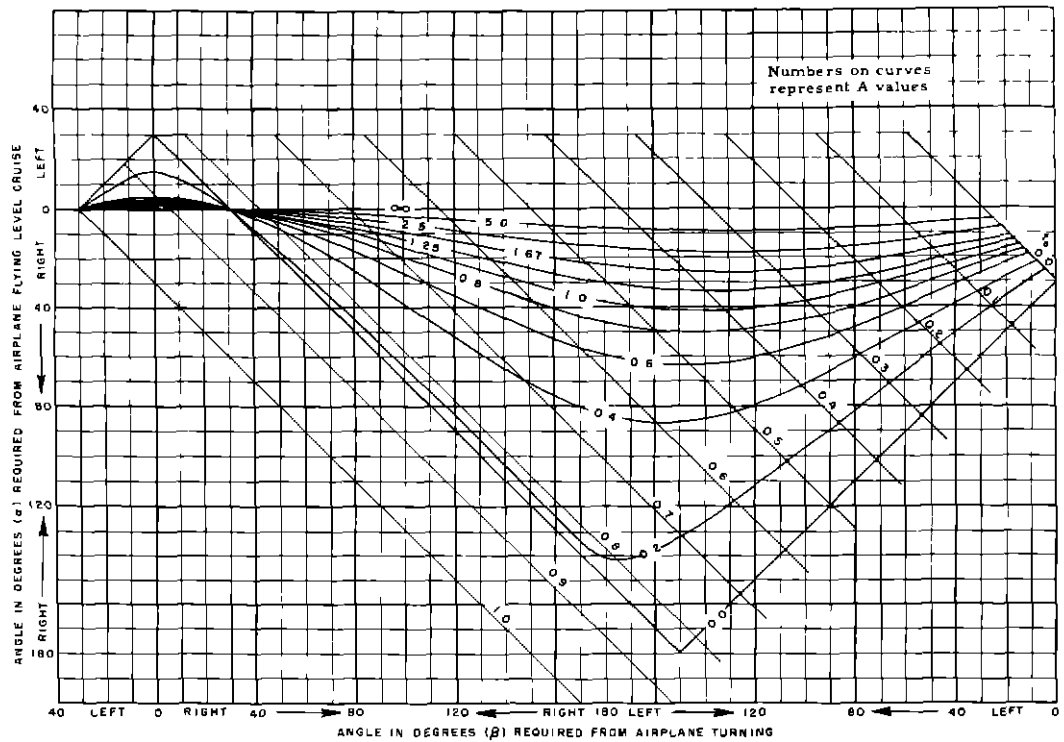


Z_0 Values

	t_0									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.2	2069	3885	5223	5917	5872	5089	3672	1941	1613	3520
0.4	2411	4509	6021	6752	6616	5685	4309	3543	4720	7040
0.6	2755	5153	6889	7755	7698	6904	5983	6099	7876	10560
0.8	3101	5812	7803	8869	8997	8482	8073	8815	11039	14080
1.0	3448	6480	8749	10056	10432	10253	10329	11579	14204	17600
1.25	3883	7324	9963	11611	12344	12616	13253	15061	18161	22000
1.67	4611	8748	12036	14307	15700	16739	18242	20892	24759	29333
2.5	6070	11626	16282	19902	22702	25267	28374	32593	37957	44000
5.0	10460	20356	29294	37193	44355	51404	59050	67764	77555	88000

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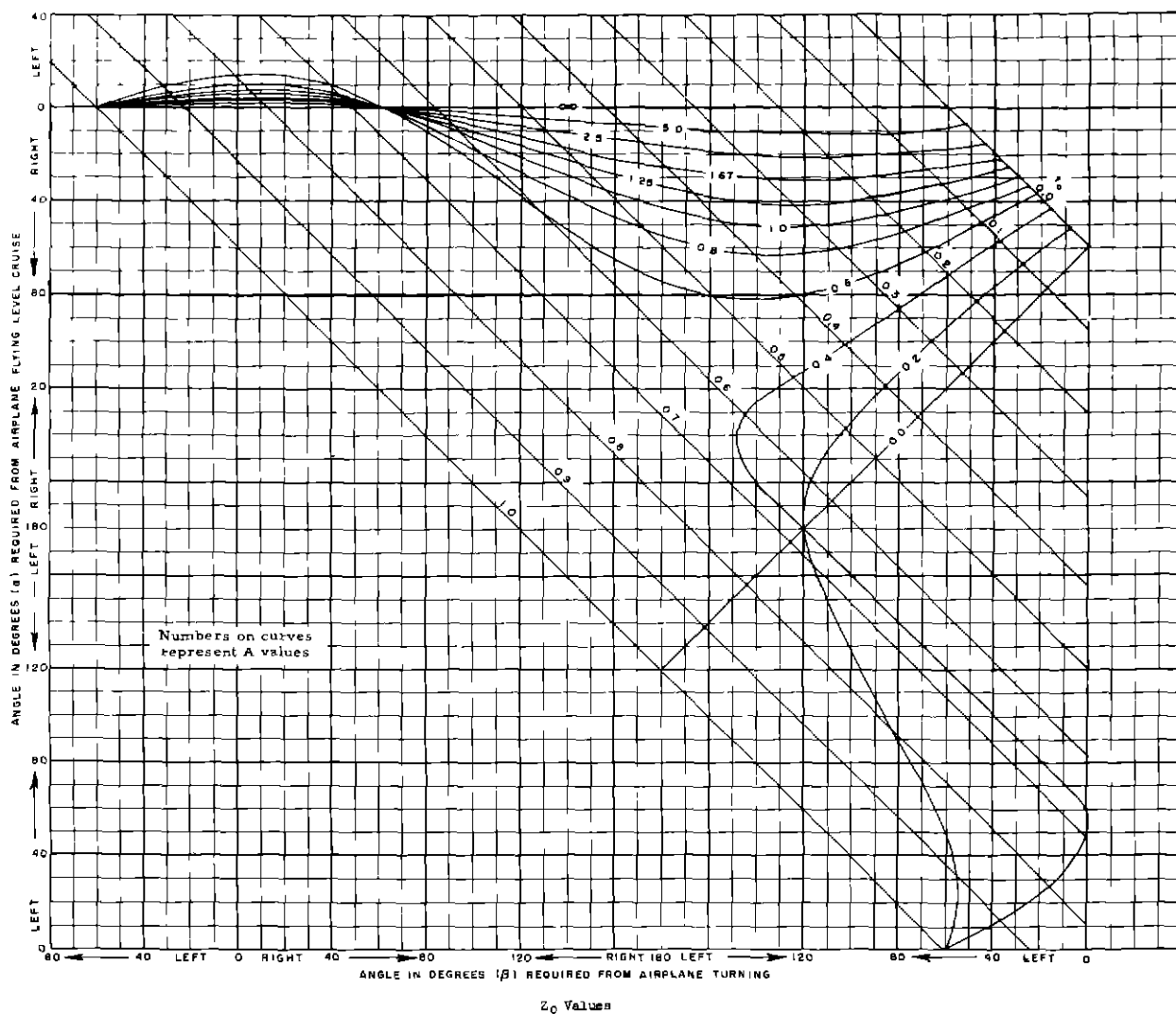
Fig 16 At Collision Point, the Difference in Heading = 180° and the Turning Airplane is on the Right of the Airplane Flying Straight and Level



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Z_0 Values		t_0									
		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
α	0.2	1984	3637	4760	5220	4962	4015	2492	574	1518	3520
	0.4	2263	4074	5197	5484	4905	3575	2004	2382	4657	7040
	0.6	2561	4578	5795	6072	5448	4285	3738	5185	7819	10560
	0.8	2871	5130	6511	6901	6443	5732	6005	7997	10985	14080
	1.0	3189	5715	7309	7896	7715	7502	8385	10811	14151	17600
	1.25	3596	6480	8388	9291	9527	9904	11409	14329	18110	22000
β	1.67	4289	7809	10311	11835	12819	14099	16496	20195	24709	29333
	2.5	5705	10576	14398	17296	19802	22722	26723	31927	37908	44000
	5.0	10041	19177	27249	34488	41484	48970	57489	67126	77507	88000

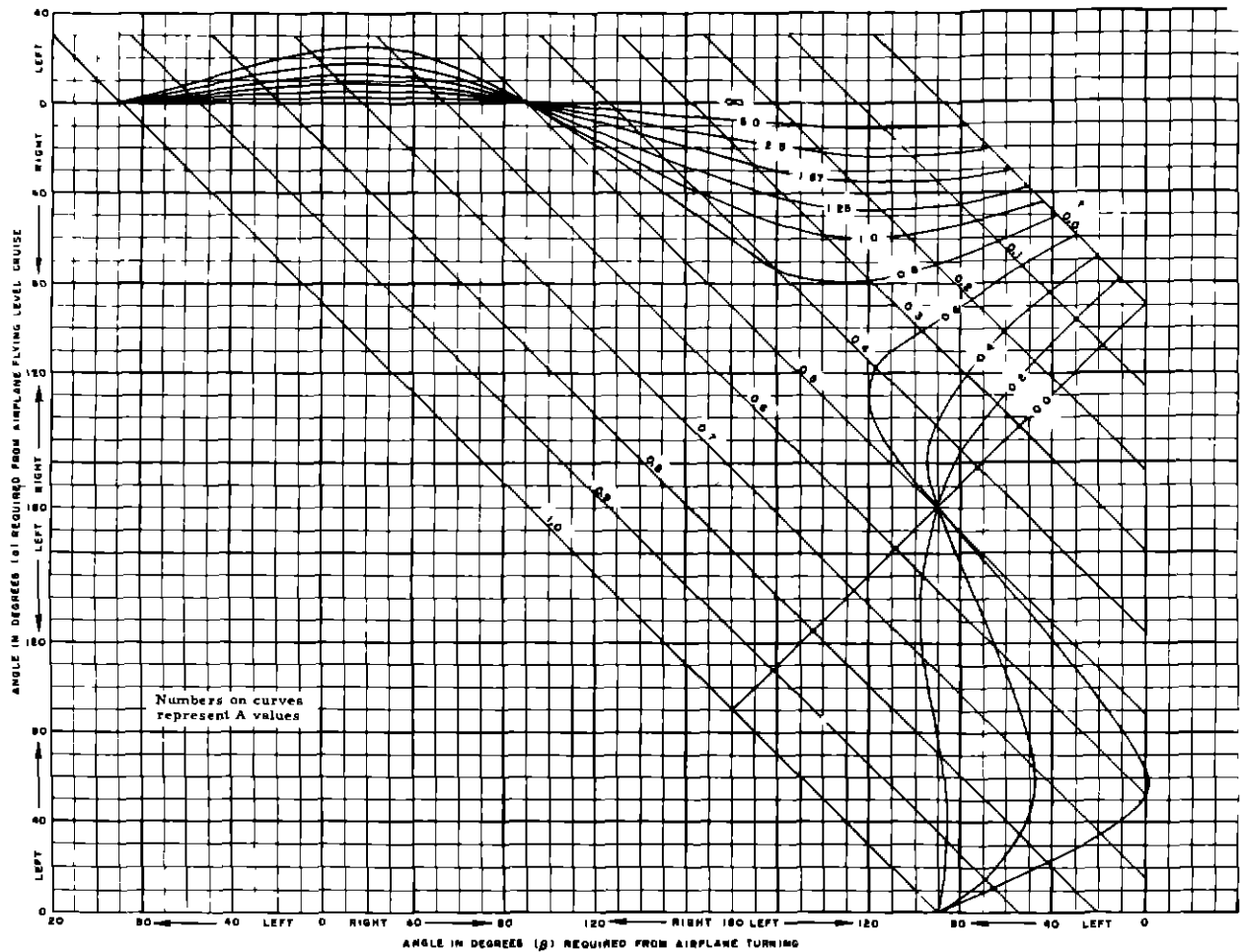
Fig 17 At Collision Point, the Difference in Heading = 150° and the Turning Airplane is on the Right of the Airplane Flying Straight and Level



		Z ₀ Values									
		t ₀									
		0 1	0 2	0 3	0 4	0 5	0 6	0 7	0 8	0 9	1 0
4	0 2	1837	3295	4215	4509	4172	3292	2096	1353	2210	3520
	0 4	2000	3443	4149	4030	3101	1484	633	2946	5181	7040
	0 6	2207	3722	4347	4014	2834	1578	2923	5602	8299	10560
	0 8	2448	4103	4776	4467	3555	3421	5369	8364	11444	14080
	1 0	2713	4562	5381	5268	4241	5462	7827	11152	14599	17600
	1 25	3069	5213	6306	6564	6756	8065	10903	14653	18550	22000
	1 67	3703	6421	8095	9077	10207	12438	16033	20502	25140	29333
	2 5	5052	9068	12088	14583	17376	21217	26297	32220	38331	44000
	5 0	9315	17564	24903	31882	39248	47601	57094	67405	77922	88000

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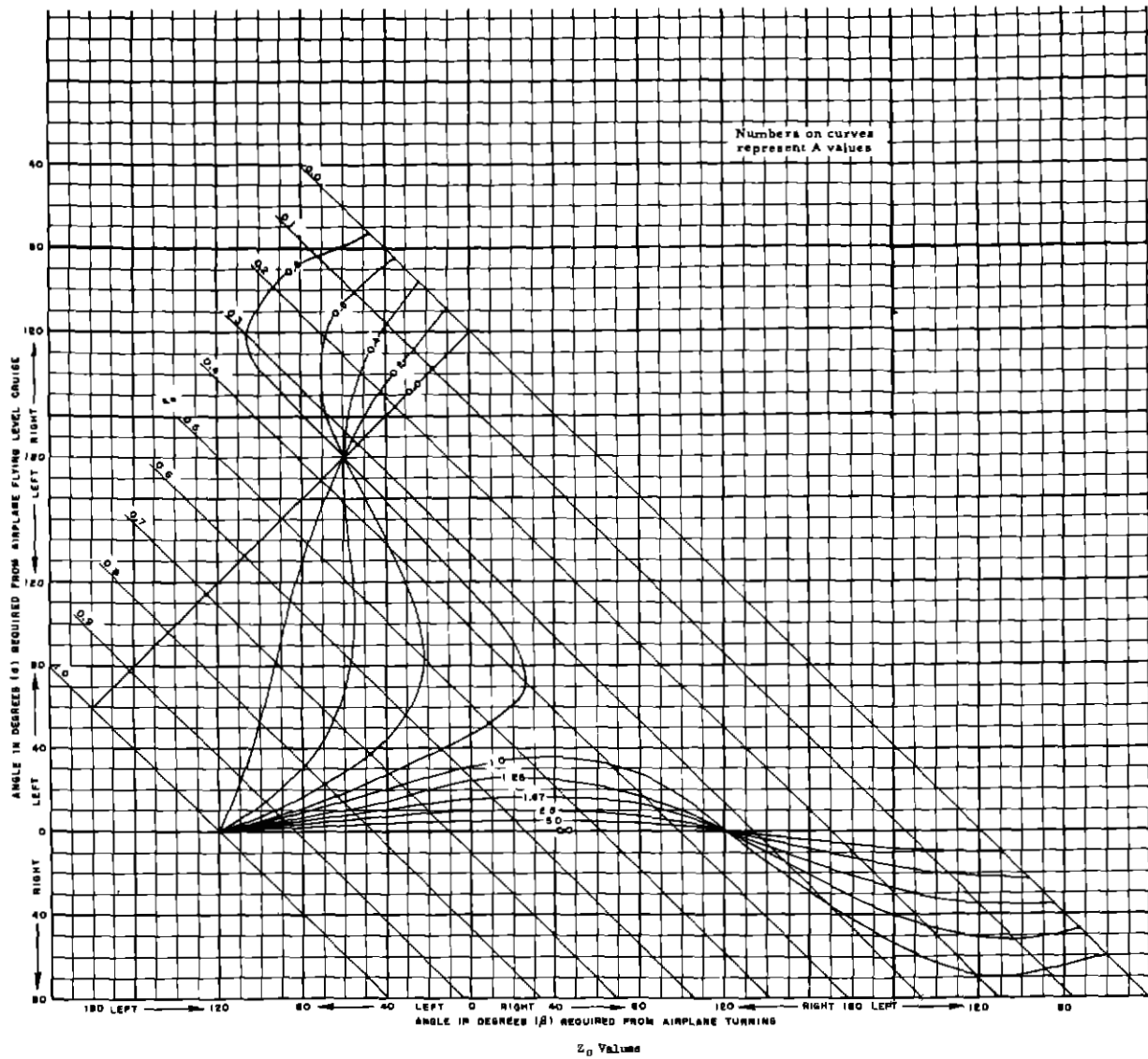
Fig 18 At Collision Point, the Difference in Heading = 120° and the Turning Airplane is on the Right of the Airplane Flying Straight and Level



		Z_0 Value									
		t_0									
		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.2	1657	2935	3730	4013	3842	3383	2923	2806	3105	3520	
0.4	1655	2716	3084	2789	2082	1850	2948	4556	6030	7040	
0.6	1727	2670	2710	1850	322	2079	4580	7016	9119	10560	
0.8	1864	2806	2722	1741	1438	3760	6738	9701	12248	14080	
< 1.0	2052	3100	3114	2570	3198	5734	9054	12433	15393	17600	
1.25	2342	3629	3962	4080	5398	8298	12032	15689	19335	22000	
1.67	2909	4749	5783	6866	9064	12640	17076	21695	25917	29333	
2.5	4201	7363	9998	12640	16398	21396	27264	33371	39100	44000	
5.0	3427	15889	22889	30178	38398	47761	57994	68516	78682	88000	

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Fig 19 At Collision Point, the Difference in Heading = 90° and the Turning Airplane is on the Right of the Airplane Flying Straight and Level

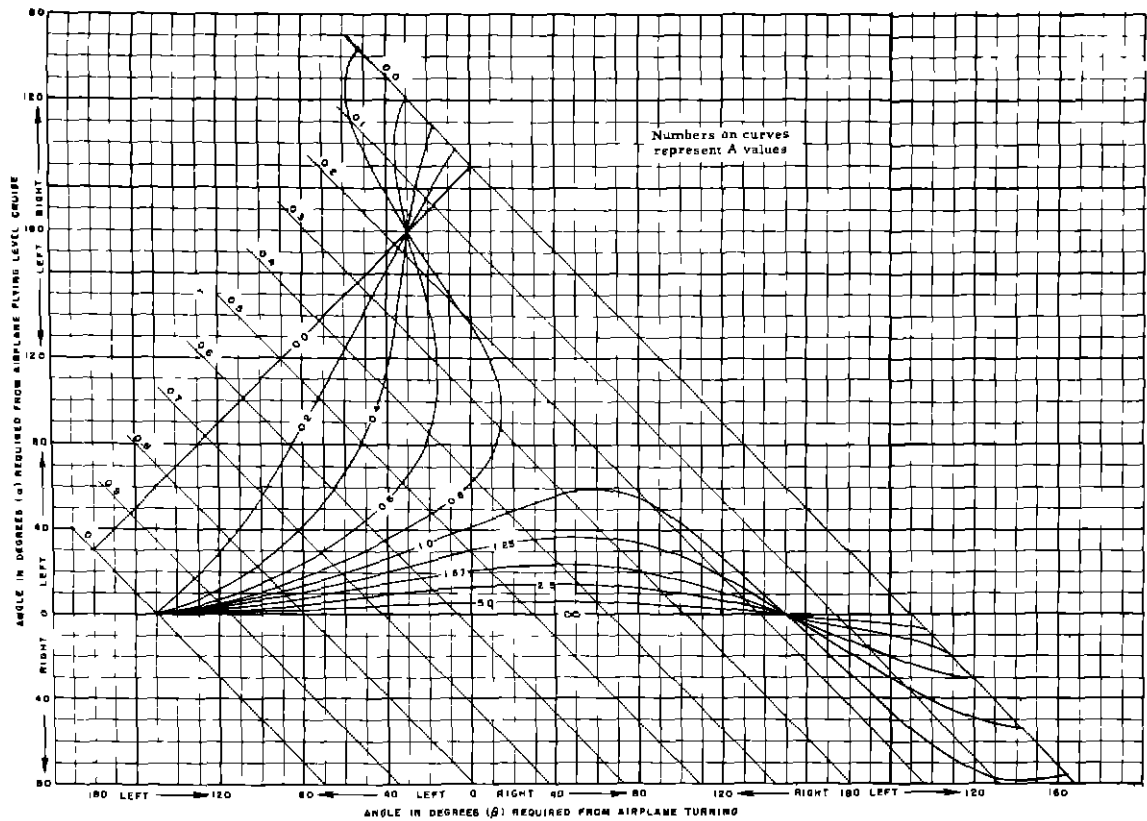


Z_0 Values

	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.2	1488	2665	3484	3962	4172	4218	4187	4103	3913	3520
0.4	1297	2087	2442	2639	3101	4014	5163	6220	6907	7040
0.6	1181	1612	1421	1484	2834	4833	6923	8741	10008	10560
0.8	1165	1353	951	1185	3555	6286	9019	11400	13141	14080
1.0	1251	1434	906	2138	4841	8038	11265	14121	16288	17600
1.25	1475	1932	2145	3755	6756	10417	14175	17564	20231	22000
1.67	2013	3157	4319	6615	10207	14583	19143	23353	26813	29333
2.5	3322	5945	8705	12438	17376	23176	29231	35009	39996	44000
5.0	7602	14653	21898	30009	39248	49394	59900	70132	79978	88000

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Fig. 20 At Collision Point, the Difference in Heading = 60° and the Turning Airplane is on the Right of the Airplane Flying Straight and Level

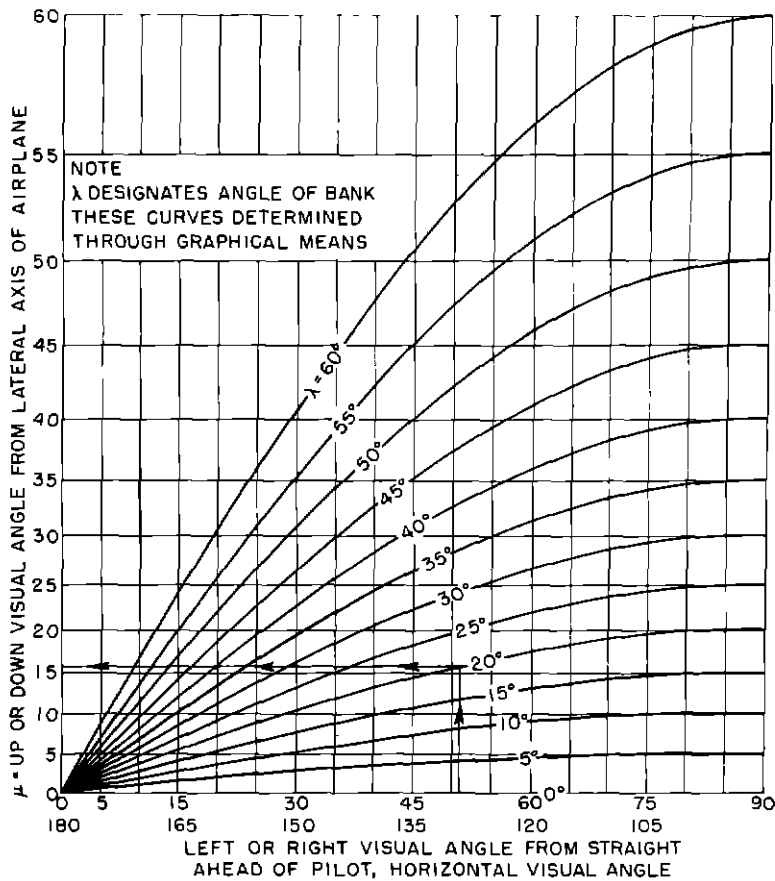


Z_0 Values

	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.2	1389	2594	3593	4384	4962	5397	5780	6130	6454	6750
0.4	1053	1898	2741	3744	4905	6072	7035	7593	7604	7040
0.6	732	1212	2084	3575	5448	7382	9066	10239	10740	10560
0.8	459	574	1845	3938	6443	9002	11270	12957	13890	14090
< 1.0	366	422	2167	4712	7715	10794	13565	15710	17047	17600
1.25	622	1176	3074	6012	9527	13168	16501	19177	20958	22000
1.67	1252	2614	5011	8552	12815	17256	21485	24988	27588	29333
2.5	2731	5536	9245	14099	19803	25824	31597	36663	40779	44000
5.0	7116	14329	22736	31443	41404	51954	62237	71802	80369	88000

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Fig 21 At Collision Point, the Difference in Heading = 30° and the Turning Airplane is on the Right of the Airplane Flying Straight and Level



EXAMPLE
 ASSUME $\lambda = 20^\circ$ LEFT AND VISIBILITY ANGLE = 52° LEFT
 ENTER GRAPH ALONG LINE OF LEFT VISIBILITY ANGLE = 52°
 UNTIL $\lambda = 20^\circ$ IS INTERSECTED AT THIS POINT, FOLLOW
 HORIZONTAL LINE OVER TO UP VISIBILITY ANGLE = 16°
 FROM LATERAL AXIS OF AIRPLANE

Fig 22 Determination of Vertical Visual Angle During Turning Collision Conditions

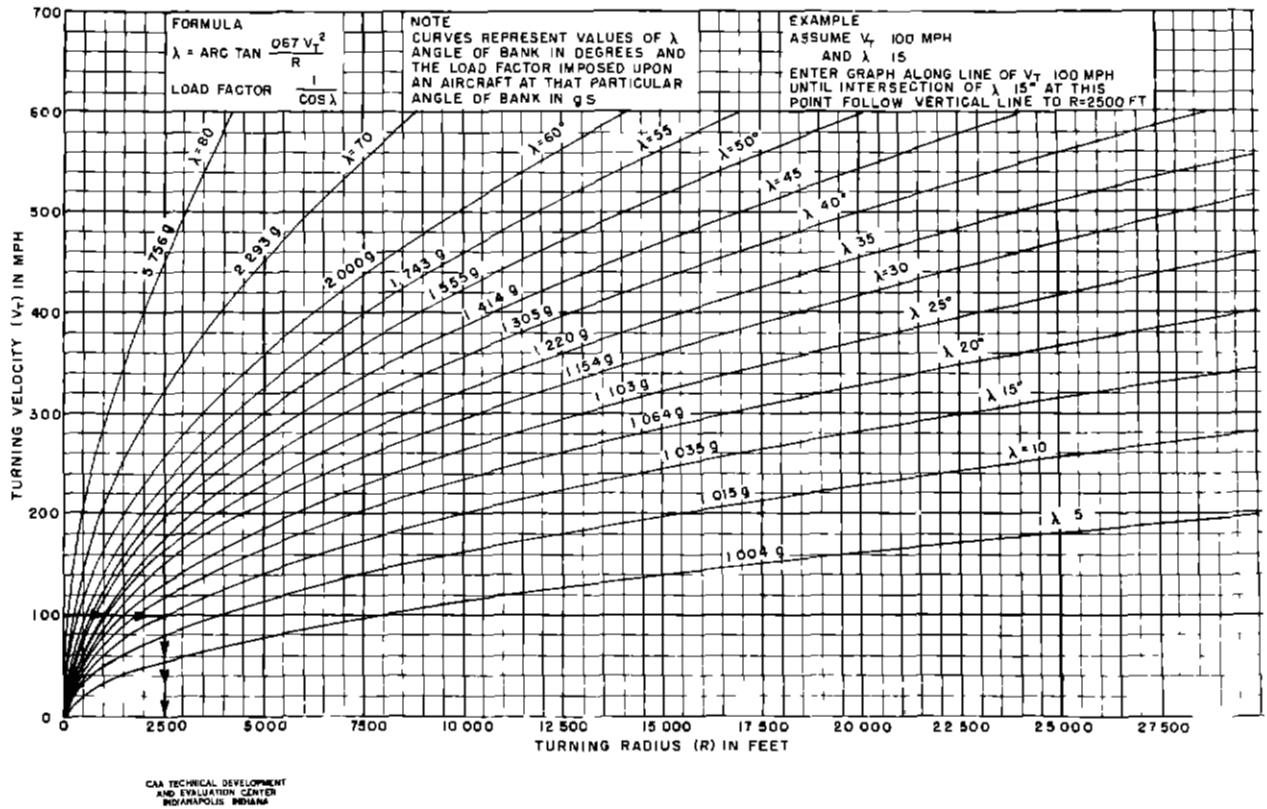
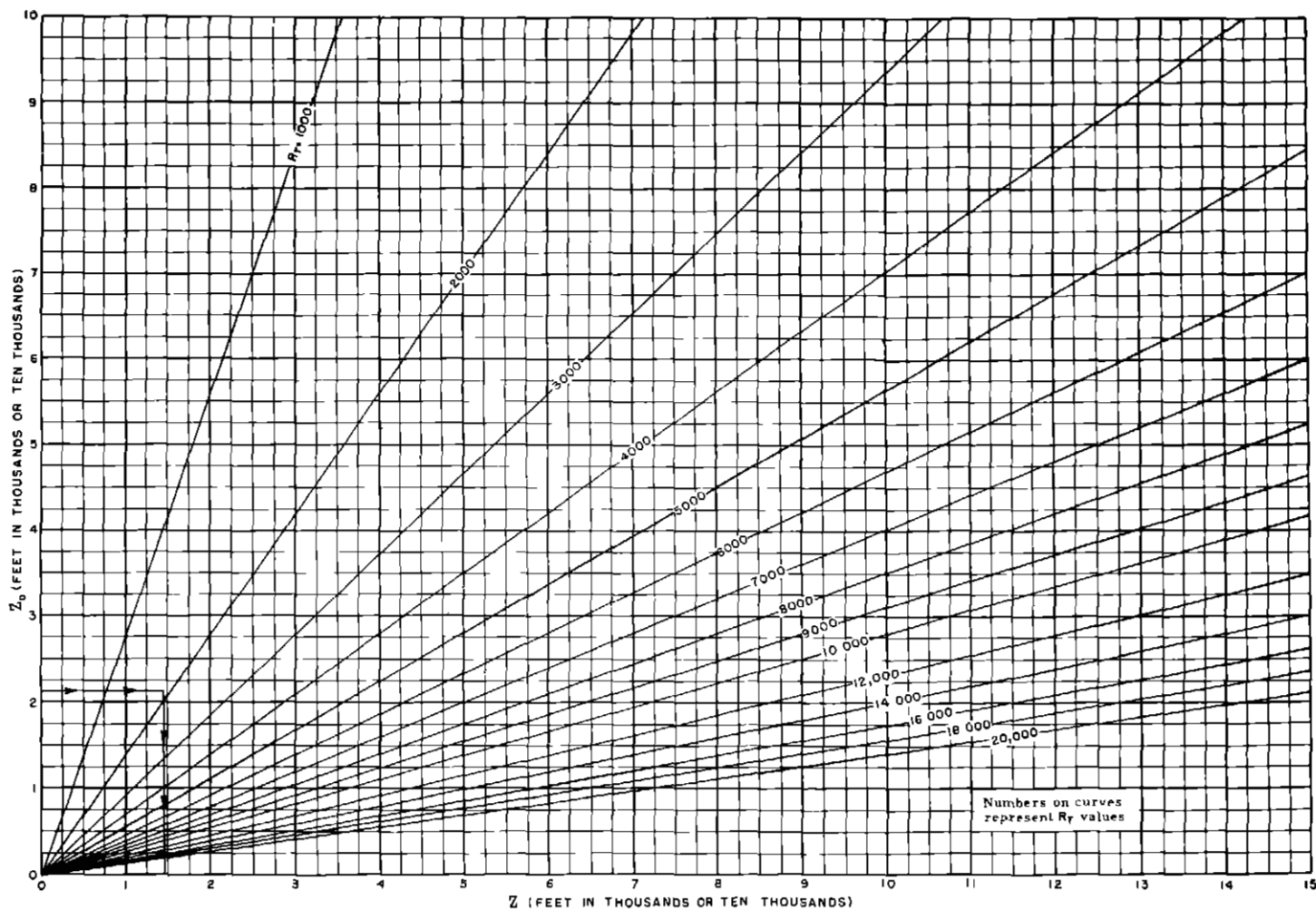


Fig 23 Determination of Turning Radius



NOTE

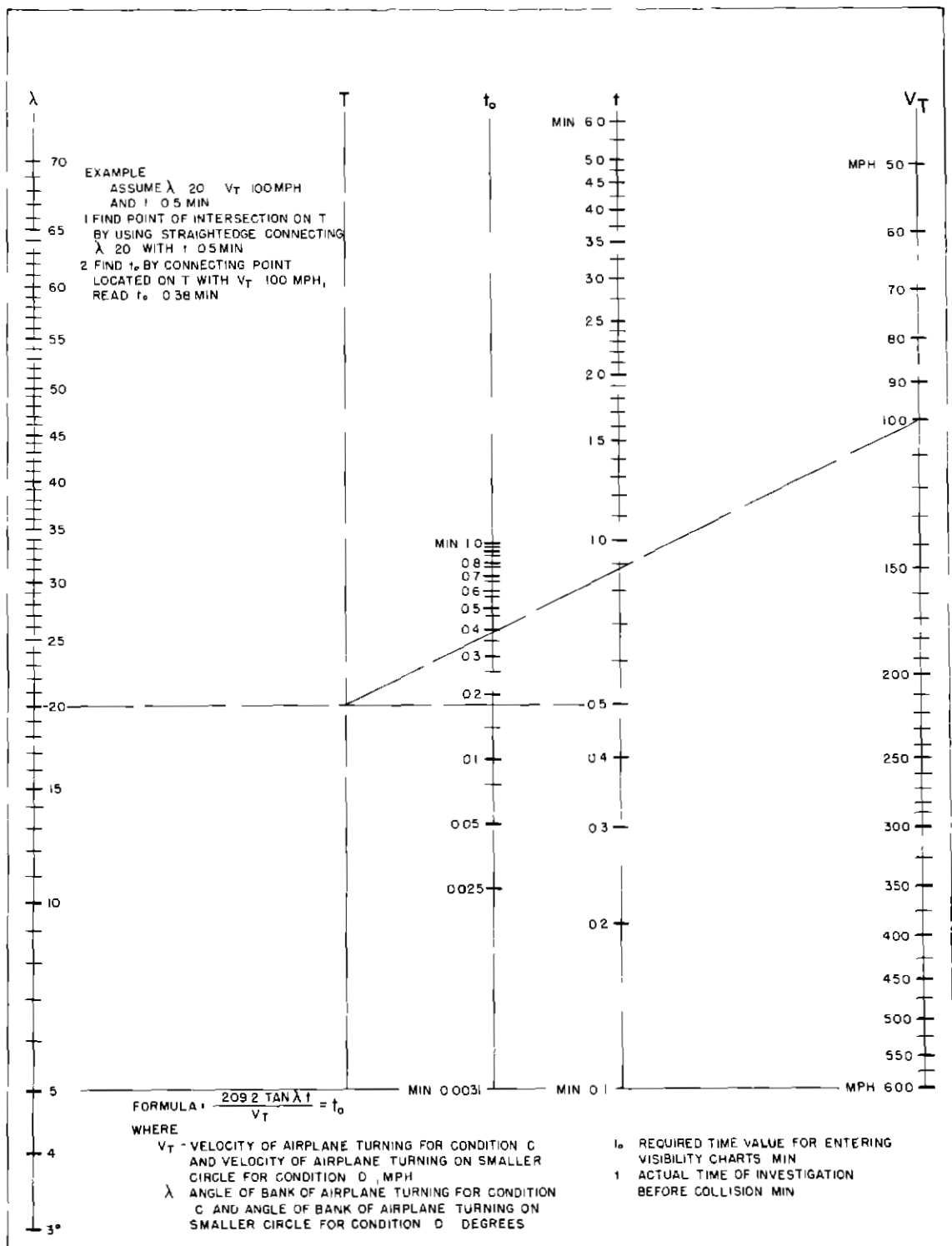
$$Z = \frac{Z_0 R_T}{280115} \text{ (FORMULA)}$$

NUMBER DESIGNATIONS OF LINES REPRESENT R_T , RADIUS OF AIRPLANE TURNING IN FEET

EXAMPLE

ASSUME Z_0 21,600 FEET AND R_T 1840 FEET ENTER GRAPH ALONG LINE $Z_0 = 2.16$ WHICH IS 21 600 FEET, UNTIL LINE $R_T = 1840$ IS INTERSECTED AT THIS POINT FOLLOW VERTICAL LINE TO 1.4, OR $Z = 14,000$ FEET

Fig 24 Determination of Distance of Separation for One Airplane Turning and One Airplane Flying Straight and Level

Fig. 25 Determination of t_0 Value for Turning Collision Conditions

COLLISION CONDITION D TWO AIRPLANES TURNING IN THE SAME DIRECTION

Description

The calculation of visual angles required to the right or left for the pilot of each airplane to see the other airplane is facilitated by the use of the charts designed for two airplanes turning in the same direction. The turning-visual-angle charts are designed around the hypothesis that both airplanes are making a turn to the left on a collision course, flying in the same horizontal plane, and approaching each other at any relative speed and direction.

Procedure and Formulas

The turning charts are shown in Figs 23, 25, 26, and 28 through 83. The procedure for using these charts is as follows:

Solution 1

$$R/A \geq 1.0$$

Where

R = Ratio of turning radii

A = Ratio of speeds (speed relationship)

1. To determine the turning radii R_2 and R_3 , enter Fig 23 with the turning velocities V_{T2} and V_{T3} and with the angles of bank λ_2 and λ_3 .

2. To determine speed relationship A, ratio of radii R, and ratio of radii to ratio of velocities R/A, enter Fig 26 with the turning velocities V_{T2} and V_{T3} and with the turning radii R_2 and R_3 . See notes on Fig 26. Note: If R/A is less than one, follow the procedure for Solution 2.

3. To determine the time value t_0 required for entering the horizontal-visual-angle charts, enter Fig 25 with the angle of bank λ_2 of the airplane turning on the smaller circle, the velocity V_{T2} of the airplane turning on the smaller circle, and the actual time t of investigation before collision.

4. To determine the horizontal visual angles α and β required for the pilot of each aircraft to see the other aircraft, select that horizontal-visual-angle chart from Fig 28 through 82 which applies to the absolute difference in heading between the two aircraft at collision point and the R/A value as determined in Step 2. Enter this chart with time value t_0 and with the speed relationship A.

5. To determine the distance factor Z_0 , enter, with speed relationship A and with the time value t_0 , the Z_0 table located on the same horizontal-visual-angle chart that was selected in Step 4.

6. To determine actual distance of separation Z between aircraft, enter Fig 83 with distance factor Z_0 and with the smaller turning radius R_2 .

7. To determine the vertical visual angle μ , as affected by the angles of bank of the turning airplanes, enter Fig 22 with the angles of bank λ_2 and λ_3 and with their respective visual angles α and β .

Solution 2

$$R/A < 1.0$$

The procedure for Solution 2 is the same as that for Solution 1 except that the values which apply to the airplane turning on the smaller circle are substituted for the values of the airplane turning on the larger circle when all charts designed for this condition are entered.

The chart shown in Fig 26 is constructed in the same manner as the speed-relationship chart used in Conditions A, B, and C, but with the change in nomenclature shown in Equation (17)

$$A = \frac{V_{T_3}}{V_{T_2}} \quad (17)$$

Where

V_{T_3} = Velocity of airplane turning on larger circle, in mph

V_{T_2} = Velocity of airplane turning on smaller circle, in mph

$$R = \frac{R_3}{R_2} \quad (18)$$

Where

R_3 = Radius of the larger circle, in feet

R_2 = Radius of the smaller circle, in feet

Therefore,

$$R/A = \frac{R_3/R_2}{V_{T_3}/V_{T_2}} \quad (19)$$

For determination of t_0 , the same four-parallel-scale nomograph constructed for Condition C, Fig 25, will be used. The formula remains the same

$$t_0 = \frac{209.2 \tan \lambda}{V_T} \quad (8)$$

Where

$V_{T_2} = V_T$ = Velocity of the airplane turning on the smaller circle, in mph

$\lambda_2 = \lambda$ = Angle of bank of the airplane turning on the smaller circle, in degrees

t = Actual time of investigation before collision, in minutes

t_0 = Required time value for entering visual-angle charts, in minutes

The visual-angle charts shown in Figs 28 through 82 were constructed for this condition by substituting example values in the following equations. See Fig 89 in the Appendix

$$\alpha = \omega_0 - \theta - 90^\circ \quad (20)$$

$$\beta = \omega_1 - \theta + 90^\circ \quad (21)$$

$$Z = \sqrt{(X_0 - X_1)^2 + (Y_1 - Y_0)^2} \quad (22)$$

To solve these equations, the following must be known

$$X_1 = R_1(\cos \psi - \cos \omega_1) \quad (23)$$

$$Y_1 = R_1(\sin \psi - \sin \omega_1) \quad (24)$$

$$\omega_1 = K_1 t - \psi \quad (25)$$

$$X_0 = R_0(1 - \cos \omega_0) \quad (26)$$

$$Y_0 = -R_0 \sin \omega_0 \quad (27)$$

$$\omega_0 = K_0 t \quad (28)$$

$$\theta = \arctan \frac{Y_1 - Y_0}{X_0 - X_1} \quad (29)$$

Where

α = The horizontal visual angle (from straight ahead) needed for the airplane turning on the larger circle to be seen from the airplane turning on the smaller circle, in degrees

β = The horizontal visual angle (from straight ahead) needed for the airplane turning on the smaller circle to be seen from the airplane turning on the larger circle, in degrees

Z = The distance between the aircraft at time t , in feet

R_1 = The turning radius of the airplane flying on the larger circle, in feet

R_0 = The turning radius of the airplane flying on the smaller circle, in feet

t = The time before collision, in minutes

K_1 = The rate of turn of the airplane flying on the larger circle, in degrees per minute

K_0 = The rate of turn of the airplane flying on the smaller circle, in degrees per minute

In case it is necessary to calculate visual angles by means of the formulas, use of Table III will be necessary to determine the relationship between ψ and the absolute difference in heading at collision point $\omega_0, \omega_1, X_0, Y_0, Y_1$, and θ are illustrated in Fig 89 of Appendix I

The chart shown in Fig 83 was constructed by substituting values in the equation

$$Z = \frac{Z_0 R_2}{1400.56} \quad (30)$$

Where

Z = The actual distance between aircraft at time t before collision, in feet

Z_0 = The proportional distance between aircraft as obtained from the visual-angle chart, in feet

R_2 = The radius of turn of the airplane flying on the smaller circle, in feet

TABLE III
RELATIONSHIP BETWEEN ψ AND ACTUAL HEADING DIFFERENCE

ψ (degrees)	Difference in Heading at Collision Point (degrees)	Condition
0	0	Heading in same direction
45	45	With airplane turning on the larger circle on the right of airplane turning on smaller circle at collision point
90	90	With airplane turning on the larger circle on the right of airplane turning on smaller circle at collision point
135	135	With airplane turning on the larger circle on the right of airplane turning on smaller circle at collision point
180	180	Head on
225	135	With airplane turning on the larger circle on the left of airplane turning on smaller circle at collision point
270	90	With airplane turning on the larger circle on the left of airplane turning on smaller circle at collision point
315	45	With airplane turning on the larger circle on the left of airplane turning on smaller circle at collision point

Sample Solution

It is desired to know what visual angles are required for the pilots of each aircraft to see the other aircraft while on a collision course and to determine the separation of the aircraft at a given time. Refer to Fig. 27. In this case, both airplanes are turning in the same direction.

Solution 1

Let

V_{T_2} , the velocity of the airplane flying on the smaller circle, = 100 mph, true air speed,

λ_2 , the angle of bank of the airplane turning on the smaller circle, = 15° ,

V_{T_3} , the velocity of the airplane flying on the larger circle, = 100 mph, true air speed,

λ_3 , the angle of bank of the airplane turning on the larger circle, = 7.5° ,

t , the time before the collision to be investigated, = 1 minute

The absolute difference in heading at collision point = 45° with the airplane that is turning on the larger circle on the right of the airplane turning on the smaller circle.

1 Enter Fig. 23 with $V_{T_2} = 100$ mph and with $\lambda_2 = 15^\circ$, and read $R_2 = 2500$ feet. Also enter with $V_{T_3} = 100$ mph and with $\lambda_3 = 7.5^\circ$, and read $R_3 = 5000$ feet.

2 Enter Fig. 26 with $V_{T_2} = 100$ mph and with $V_{T_3} = 100$ mph, and read $A = 1.0$, enter with

$R_2 = 2500$ feet and with $R_3 = 5000$ feet, and read $R = 2.0$, then enter with $R = 2.0$ and with $A = 1.0$, and read $R/A = 2.0$. Note: If R/A becomes less than 1.0, use Solution 2.

3 Enter Fig 25 with $\lambda_2 = 15^\circ$, $V_{T2} = 100$ mph, and $t = 1$ minute, and read $t_0 = 0.56$ minute.

4 Since the absolute difference in heading at collision point $= 45^\circ$ and $R/A = 2.0$ and since the airplane turning on larger circle is on the right of the airplane turning on the smaller circle, Fig 36 of the visual-angle charts is selected and is entered with $A = 1.0$ and $t_0 = 0.56$. Read α , which is the horizontal visual angle required from the airplane turning on the smaller circle, equal to 46° left, also read β , which is the horizontal visual angle required from the airplane turning on the larger circle, equal to 80° left. If the distance between the aircraft at 1 minute is also desired, then enter the Z_0 chart on the same figure with $A = 1.0$ and $t_0 = 0.56$. No value of t_0 equal to 0.56 appears, therefore, the following interpolation is made:

$A = 1.0$	$t_0 = 0.60$	$Z_0 = 5620$
$A = 1.0$	$t_0 = 0.50$	$Z_0 = 4872$
		748 = Difference

Since 0.56 is 0.6 of the interval between 0.50 and 0.60, then $0.6 \times 748 = 448.8$ or approximately 449.

For

$t_0 = 0.56$ and $A = 1.0$, $Z_0 = 4872 + 449 = 5321$ feet, or approximately 5300 feet.

This is not a true distance but is a proportional distance and is used in the following step.

5 Enter Fig 83 with $Z_0 = 5300$ feet and $R_2 = 2500$ feet, and read $Z = 9400$ feet.

6 To find the vertical visual angle μ at 1 minute before collision required from the airplane turning on the smaller circle at a 15° bank angle, enter Fig 22 with $\lambda_2 = 15^\circ$ and $\alpha = 46^\circ$ left and read required vertical visual angle $\mu = 11^\circ$ up. Also, to find the vertical visual angle at 1 minute prior to collision required from the airplane turning on the larger circle, enter Fig 22 with $\lambda_3 = 7.5^\circ$ and $\beta = 80^\circ$ left and read required vertical visual angle $\mu = 7.5^\circ$ up.

Solution 2

If the R/A value is computed to be less than 1.0 by using procedures outlined in sample Solution 1, then the following process is to be used in computing the visual angles required from two airplanes making a left turn.

Given (Reference Fig 27)

V_{T2} , the velocity of the airplane flying on the smaller circle, = 100 mph, true air speed

λ_2 , the angle of bank of the airplane turning on the smaller circle, = 5°

V_{T3} , the velocity of an airplane flying on the larger circle, = 250 mph, true air speed

λ_3 , the angle of bank of the airplane turning on the larger circle, = 22° .

t , the time before collision to be investigated, = 1 minute

The absolute difference in heading at collision point = 45° , with the airplane turning on the larger circle on the right of the airplane turning on the smaller circle.

1 Enter Fig 23 with $V_{T2} = 100$ mph and $\lambda_2 = 5^\circ$, and read $R_2 = 8000$ feet. Also enter $V_{T3} = 250$ mph and $\lambda_3 = 22^\circ$, and read $R_3 = 10,000$ feet.

2 Enter Fig 26 with $V_{T2} = 100$ mph and $V_{T3} = 250$ mph, and read $A = 2.5$, then also enter with $R_2 = 8000$ feet and $R_3 = 10,000$ feet, and read $R = 1.25$, then enter with $R = 1.25$ and $A = 2.5$, and read $R/A = 0.5$.

Since there are no visual-angle charts designed for an R/A value less than 1.0, the following procedure is to be followed. The ratios R , A , and R/A are inverted, that is, enter Fig 26 with $V_{T2} = 250$ mph instead of 100 mph and $V_{T3} = 100$ mph instead of 250 mph and read $A = 0.40$.

Also enter with $R_2 = 10,000$ feet instead of 8000 feet and $R_3 = 8000$ feet instead of 10,000 feet, and read $R = 0.8$. Then entering Fig 26 with $R = 0.8$ and $A = 0.4$, read $R/A = 2.0$.

3 Enter Fig 25 with $\lambda_3 = 22^\circ$, $V_{T3} = 250$ mph, and $t = 1$ minute, and read $t_0 = 0.34$.

4 The absolute difference in heading at collision point = 45° , with the airplane turning on the larger circle at the right of the airplane turning on the smaller circle at collision point. For this situation, however, is selected the horizontal-visual-angle chart that was designed for an absolute difference in heading at collision point = 45° but with the airplane turning on the larger circle at the left of the airplane turning on the smaller circle at collision point. Since $R/A = 2.0$, Fig 77 is selected and is entered with $A = 0.4$ and $t_0 = 0.34$. Read α , which is the horizontal visual angle required from the airplane turning on the larger circle, equal to 74° left. Also read β , which is the horizontal visual angle required from the airplane turning on the smaller circle, equal to 123° right.

Normally, α is the visual angle required from the airplane turning on the smaller circle, and β is the visual angle required from the airplane turning on the larger circle, but, as the note on Fig 77 indicates, when entering with an A value less than 0.5, α becomes the visual angle required from the airplane turning on the larger circle and β becomes the visual angle required from the airplane turning on the smaller circle.

5 In determining the distance between the aircraft at 1 minute, the Z_0 table on Fig 77 is entered with $t_0 = 0.34$ and $A = 0.4$. No value of t_0 equal to 0.34 appears, therefore, the following interpolation is made:

$A = 0.4$	$t_0 = 0.40$	$Z_0 = 1383$
$A = 0.4$	$t_0 = 0.30$	$Z_0 = 1346$
		37 = Difference

Since 0.34 is 0.4 of the interval between 0.30 and 0.40, $0.4 \times 37 = 14.8$, or approximately 15. Then, because $t_0 = 0.34$ and $A = 0.4$, $Z_0 = 1346 + 15 = 1361$ feet.

This is not a true distance but is a proportional distance and is used in the following step.

6 Enter Fig 83 with $Z_0 = 1361$ feet and $R_3 = 10,000$ feet, and read $Z = 9800$ feet. Normally, R_2 , the radius of the smaller circle, is used in entering Fig 83, but for this solution R_3 , the radius of the larger circle, is used.

7 The vertical visual angles are determined in the same manner as described in Solution 1, Step 6.

Additional Considerations

Visual angles may be determined from two airplanes making a right turn instead of from both making a left turn by using the following procedure.

Given

The absolute difference in heading at collision point is equal to 45° , with the airplane turning on the larger circle on the right of the airplane turning on the smaller circle at the point of collision and with both airplanes making right turns.

1 Enter the horizontal-visual-angle chart designed for a difference in heading at collision point equal to 45° with the airplane that is turning on the larger circle on the left of the airplane turning on the smaller circle at the collision point. This chart is entered with the parameters determined by the given condition, the only difference being that in using the charts left is substituted for right as related to the collision point.

2 In reading α and β , the magnitudes of the angles are correct but the directions of the angles need to be reversed to apply correctly to the given condition.

3 The distance between the airplanes at a specified time before collision and the vertical visual angles are determined in the same manner as described for two airplanes making left turns.

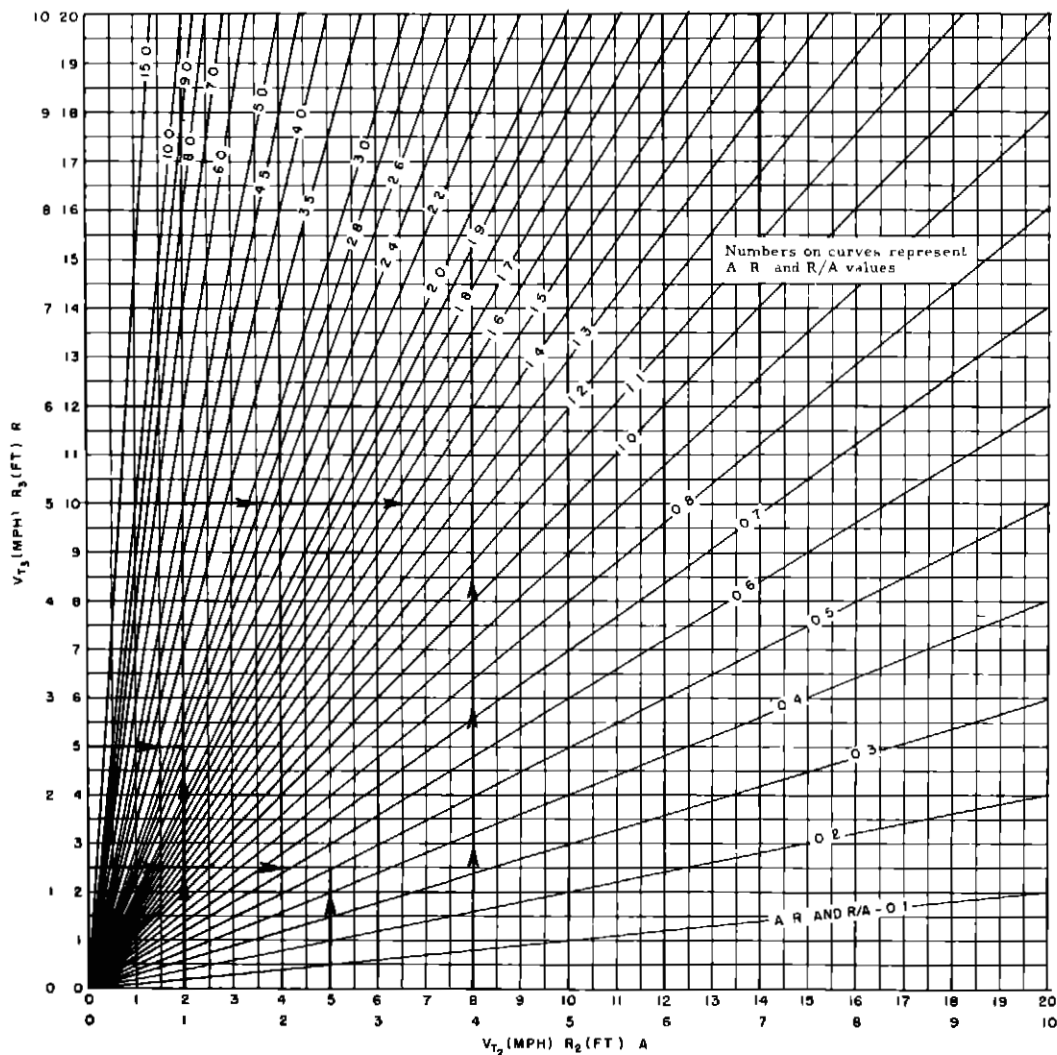
4 Additional considerations mentioned in Collision Condition C apply similarly to this condition.

Visual angles for two airplanes turning in opposite directions may be determined as indicated in the Appendix.

Discussion of Significant Trends

The rate of change of the horizontal visual angle is generally greater from the airplane turning on the smaller circle

In a condition where the curves cross, it indicates that a straight line can be drawn through the two turning airplanes and the collision point at that instant of time before collision



EXAMPLES

To determine speed relationship $A = \frac{V_{T3}}{V_{T2}}$
 Where V_{T3} velocity of airplane turning on larger circle
 V_{T2} velocity of airplane turning on smaller circle

Example
 Assume $V_{T3} = 250$ mph and $V_{T2} = 100$ mph
 Using larger scales read $A = 2.5$

To determine radius relationship $R = \frac{R_3}{R_2}$
 Where R_3 radius of larger circle
 R_2 radius of smaller circle

Example
 Assume $R_3 = 10,000$ ft and $R_2 = 8000$ ft
 Using smaller scales read $R = 1.25$

To determine the ratio of radius relationship to speed relationship

$$R/A = \frac{R_3/R_2}{V_{T3}/V_{T2}}$$

Example
 Since $R = 1.25$ and $A = 2.5$ using larger scales read $R/A = 0.5$

Note

If $R/A \geq 1.0$ the actual value of R/A is used in connection with the charts (Figs 28-82). See solution 1 of collision condition D.

If $R/A < 1.0$ the reciprocal value of R/A is used in connection with the charts (Figs 28-82). See solution 2 of collision condition "D".

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Fig 26 Determination of A, R, and R/A for Two Turning Airplanes

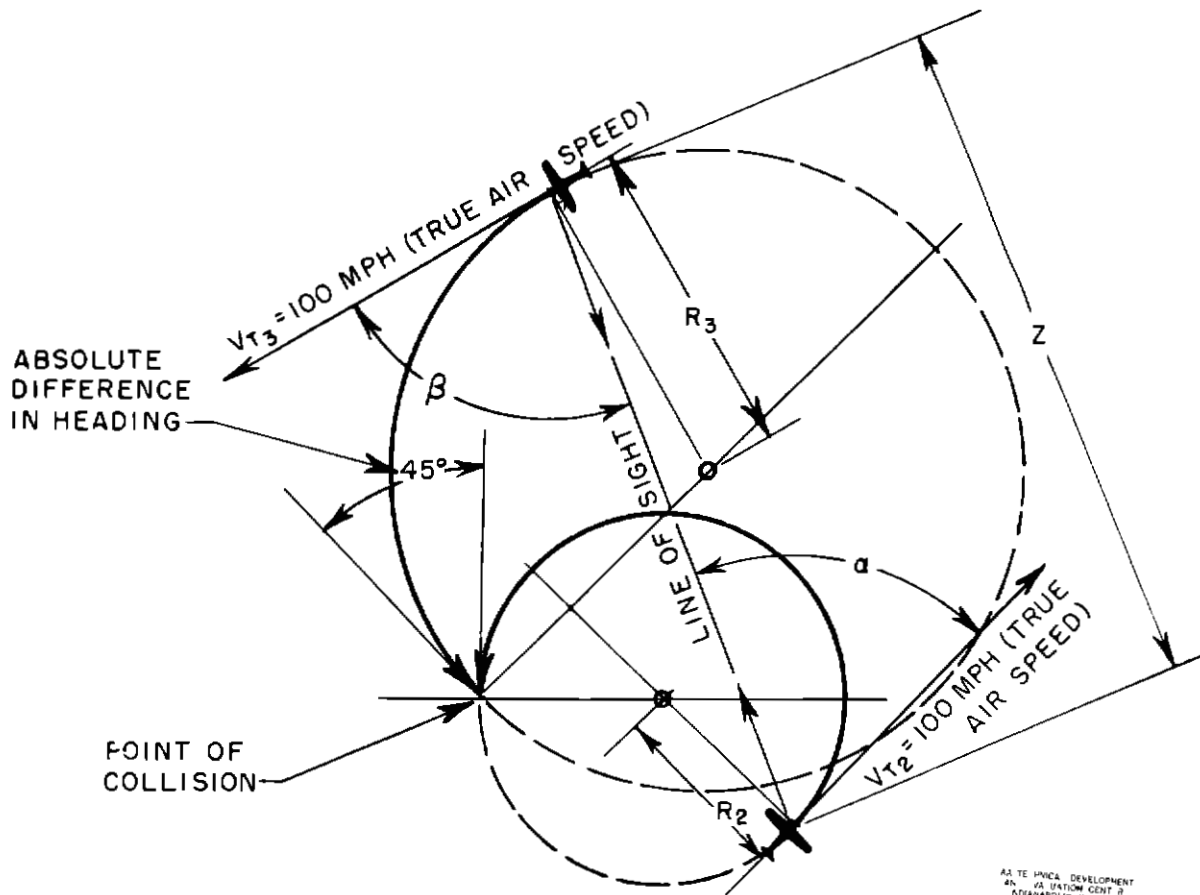


Fig 27 Vector Diagram of Sample Solution for Two Airplanes Turning Left

The following charts are designed for the determination of horizontal-visual-angle requirements and distance factor for two airplanes turning in same direction

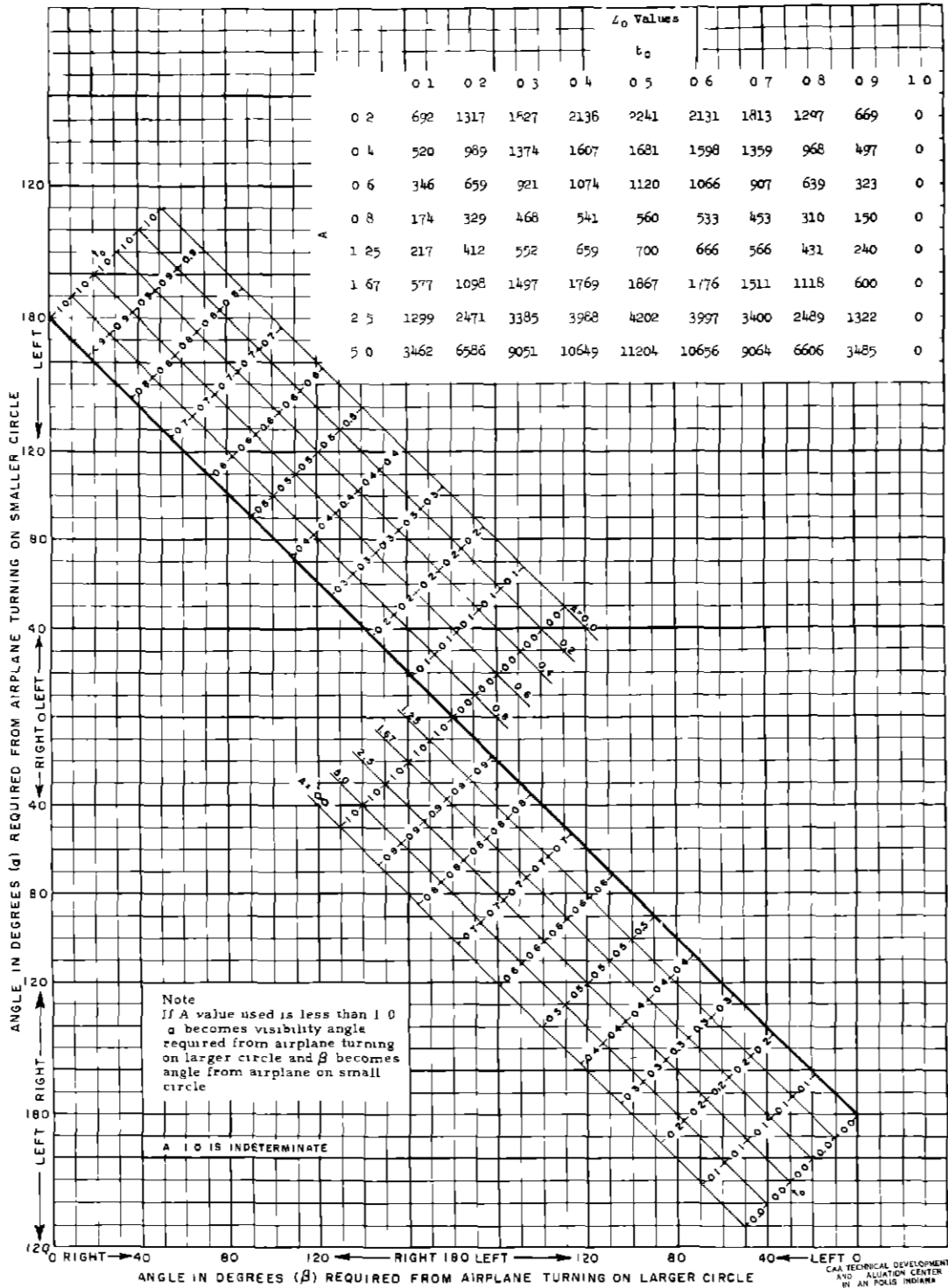
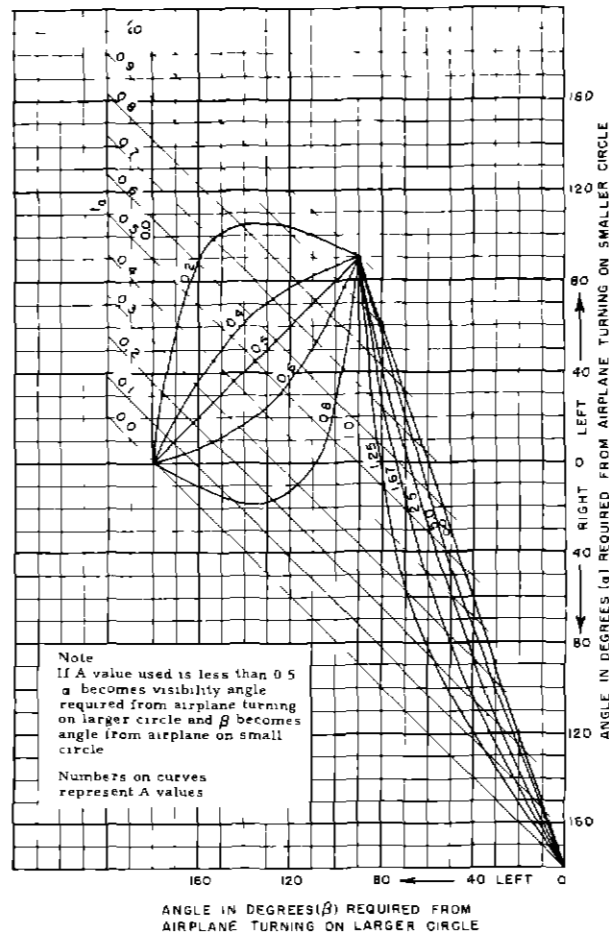


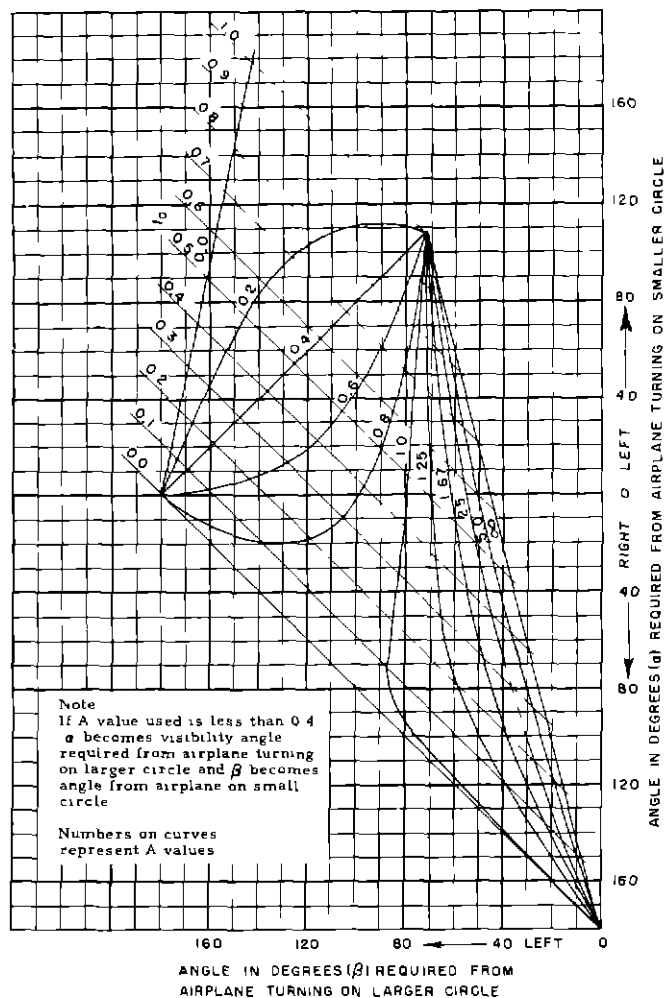
Fig 28 At Collision Point, the Difference in Heading = 0° , $R/A \approx 1.0$, and the Airplane Turning on the Larger Circle is on the Right of the Airplane Turning on the Smaller Circle



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Z ₀ Values										
t ₀										
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.2	693	1322	1845	2178	2310	2253	2020	1655	1288	1120
0.4	523	1011	1456	1792	2020	2170	2239	2259	2249	2241
0.6	356	733	1164	1599	2020	2444	2818	3120	3302	3361
0.8	205	539	1053	1662	2310	2962	3568	4080	4382	4482
< 1.0	128	520	1152	1920	2801	3660	4440	5072	5468	5602
1.25	277	785	1564	2530	3571	4631	5589	6363	6843	7003
1.67	620	1414	2453	3670	5028	6364	7566	8534	9136	9338
2.5	1342	2809	4462	6289	8166	10000	11627	12925	13733	14006
5.0	3529	7109	10746	14405	17936	21205	24014	26197	27555	28011

Fig 29 At Collision Point, the Difference in Heading = 0°, R/A = 2, and the Airplane Turning on the Larger Circle is on the Right of the Airplane Turning on the Smaller Circle



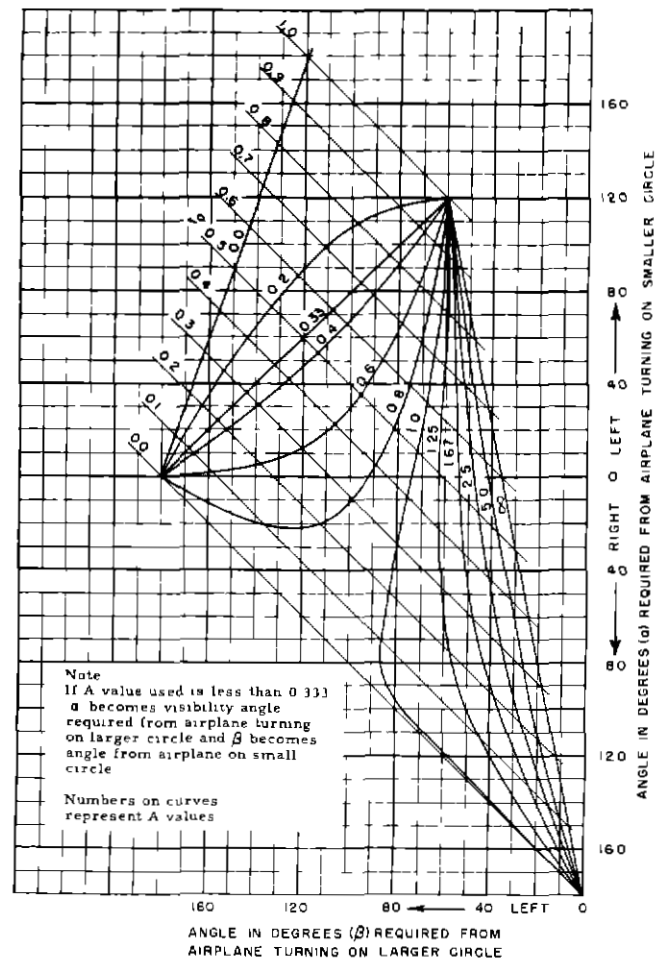
Z_0 Values

t_0

	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.2	695	1329	1868	2233	2412	2417	2254	1959	1613	1333
0.4	525	1031	1522	1935	2266	2535	2713	2796	2776	2664
0.6	362	778	1292	1848	2412	2974	3460	3822	4002	3997
0.8	219	622	1248	1999	2801	3620	4348	4918	5249	5328
1.0	165	642	1405	2342	3352	4382	5308	6047	6503	6661
1.25	295	886	1802	2938	4168	5424	6560	7481	8077	8325
1.67	633	1499	2685	4128	5685	7266	8711	9900	10706	11101
2.5	1394	2887	4696	6759	8936	11116	13110	14784	15975	16650
5.0	3542	7202	11044	15051	19069	22961	26506	29520	31805	33301

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Fig 30 At Collision Point, the Difference in Heading = 0° , $R/A = 2.5$, and the Airplane Turning on the Larger Circle is on the Right of the Airplane Turning on the Smaller Circle



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Z_0 Values

	t_0									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.2	695	1335	1889	2277	2490	2538	2423	2160	1813	1456
0.4	527	1047	1571	2041	2442	2784	3022	3131	3092	2911
0.6	368	810	1384	2021	2672	3317	3862	4251	4419	4367
0.8	231	679	1379	2221	3120	4023	4821	5426	5762	5822
< 1.0	182	713	1557	2592	3705	4833	5839	6629	7111	7277
1.25	309	954	1999	3203	4552	5920	7157	8152	8802	9097
1.67	643	1560	2839	4407	6105	7830	9409	10715	11625	12129
2.5	1362	2940	4852	7064	9421	11801	14003	15879	17283	18194
5.0	3550	7263	11233	15450	19757	24008	27969	31450	34268	36387

Fig 31 At Collision Point, the Difference in Heading = 0° , $R/A = 3.0$, and the Airplane Turning on the Larger Circle is on the Right of the Airplane Turning on the Smaller Circle

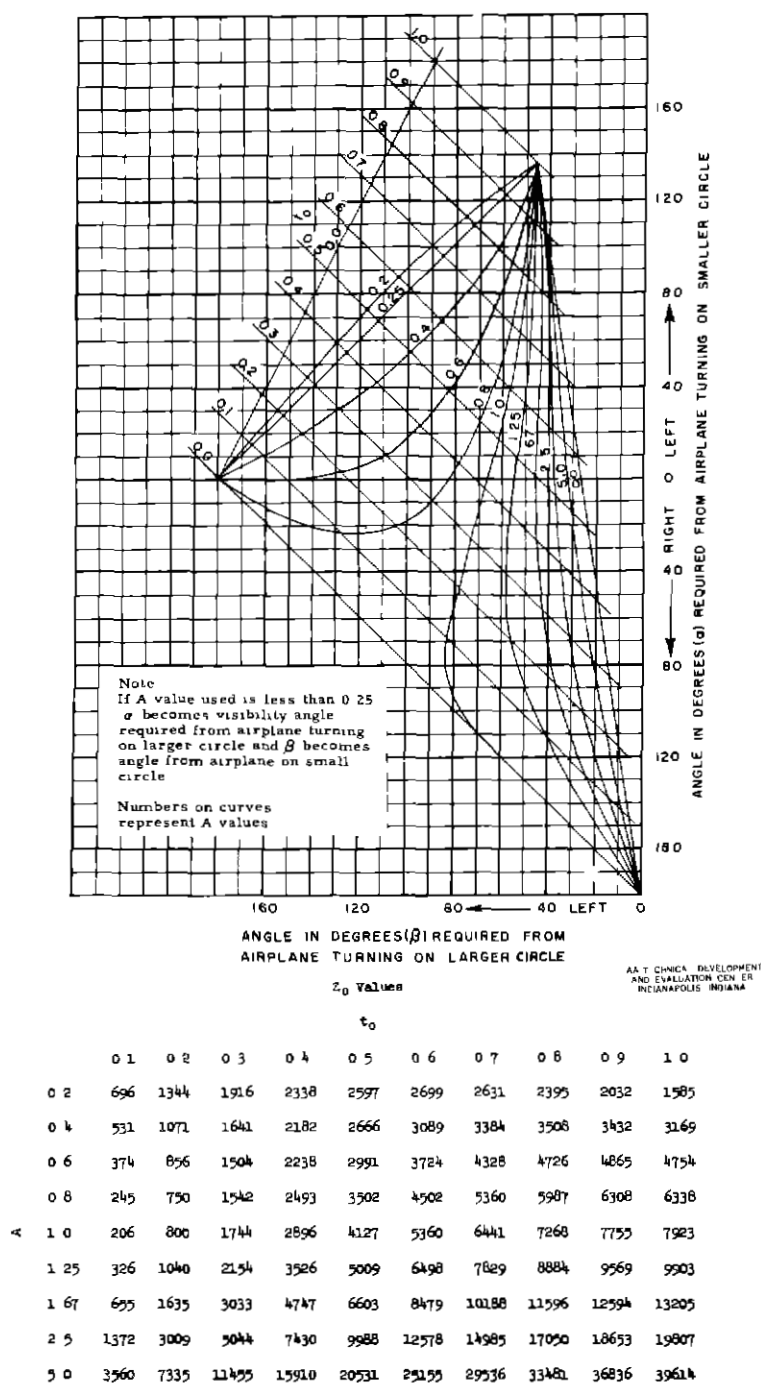
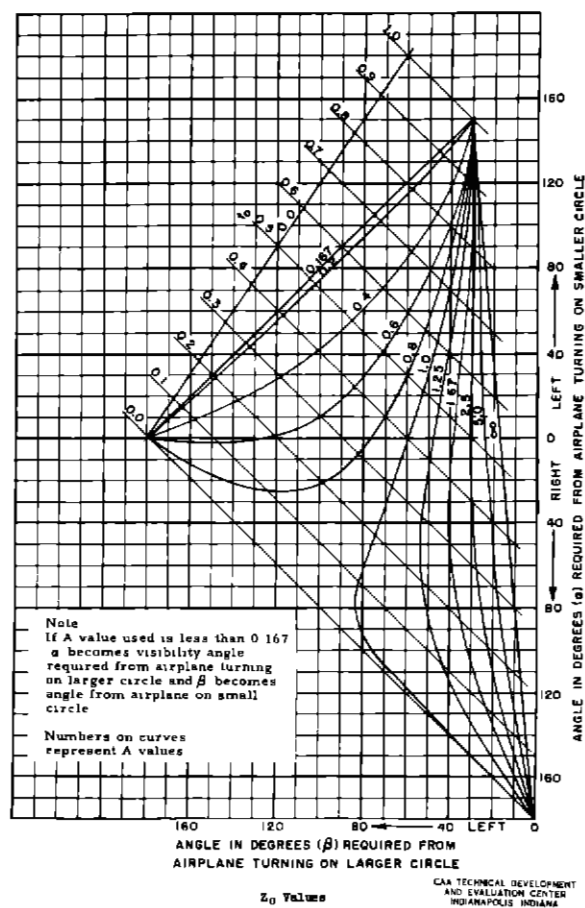
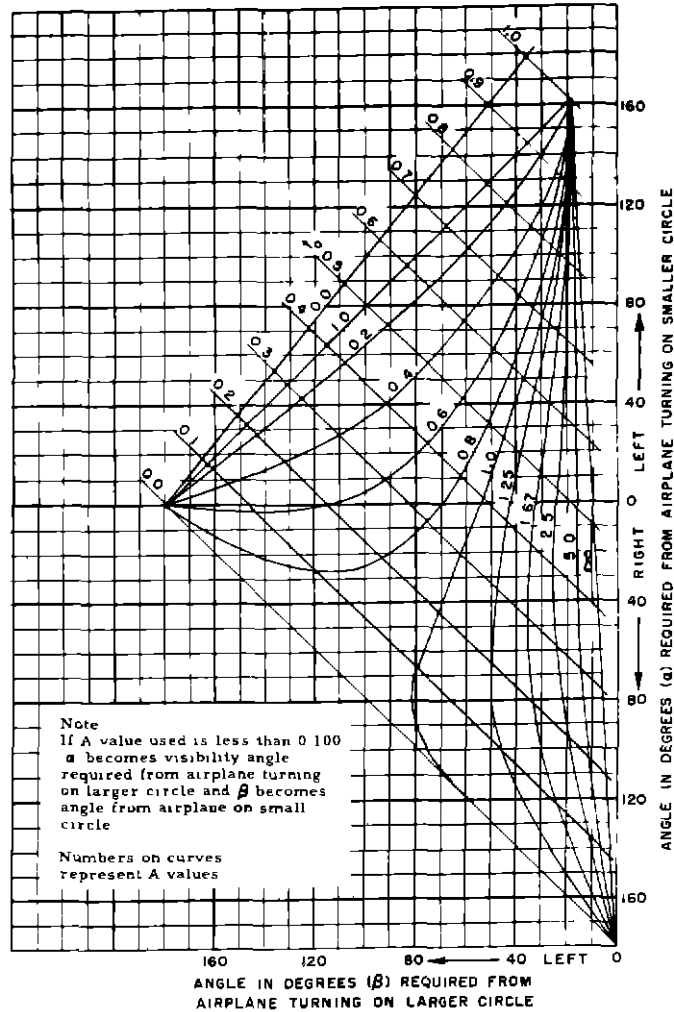


Fig 32 At Collision Point, the Difference in Heading = 0° , $R/A = 4.0$, and the Airplane Turning on the Larger Circle is on the Right of the Airplane Turning on the Smaller Circle



A	Z ₀ Values									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.2	696	1354	1949	2406	2709	2859	2828	2602	2207	1680
0.4	534	1096	1716	2327	2830	3379	3710	3826	3700	3342
0.6	381	903	1426	2053	2698	3400	4137	5123	5212	5042
0.8	261	823	1304	2056	2961	4035	5032	6449	6731	6723
1.0	228	887	1326	2187	3223	4538	6061	7788	8252	8403
1.25	346	1127	2344	3837	5434	7017	8406	9475	10155	10504
1.67	669	1713	3224	5073	7064	9057	10848	12259	13331	14005
2.5	1381	3079	5234	7775	10504	13255	15796	17971	19685	21008
5.0	3569	7404	11661	16323	21198	26104	30780	35034	38756	42017

Fig 33 At Collision Point, the Difference in Heading = 0° , $R/A = 6.0$, and the Airplane Turning on the Larger Circle is on the Right of the Airplane Turning on the Smaller Circle



	Z ₀ Values									
	t ₀									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.2	698	1362	1976	2462	2801	2984	2972	2740	2311	1732
0.4	537	1119	1779	2445	3063	3590	3938	4032	3855	3462
0.6	388	944	1726	2622	3529	4373	5018	5373	5411	5194
0.8	273	683	1831	2959	4131	5247	6150	6737	6968	6924
< 1.0	246	956	2069	3409	4817	6177	7313	8110	8529	8056
1.25	361	1197	2494	4074	5749	7384	8787	9834	10479	10819
1.67	680	1776	3374	5321	7401	9458	11277	12719	13734	14426
2.5	1390	3133	5381	8036	10874	13713	16310	18504	20243	21641
3.0	3577	7457	11614	16616	21649	26709	31521	35899	39778	43279

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Fig 34 At Collision Point, the Difference in Heading = 0°, R/A = 10.0, and the Airplane Turning on the Larger Circle is on the Right of the Airplane Turning on the Smaller Circle

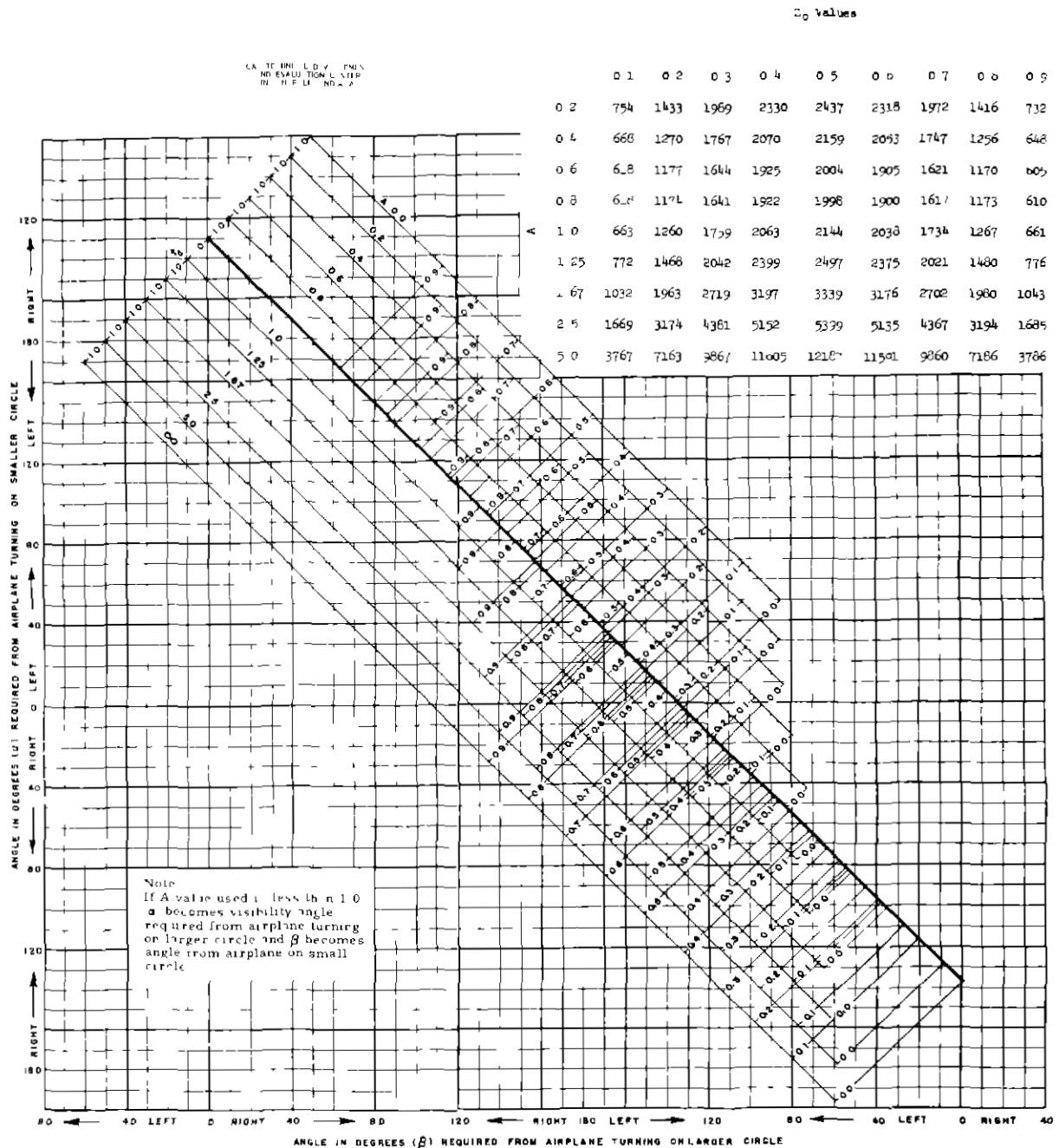
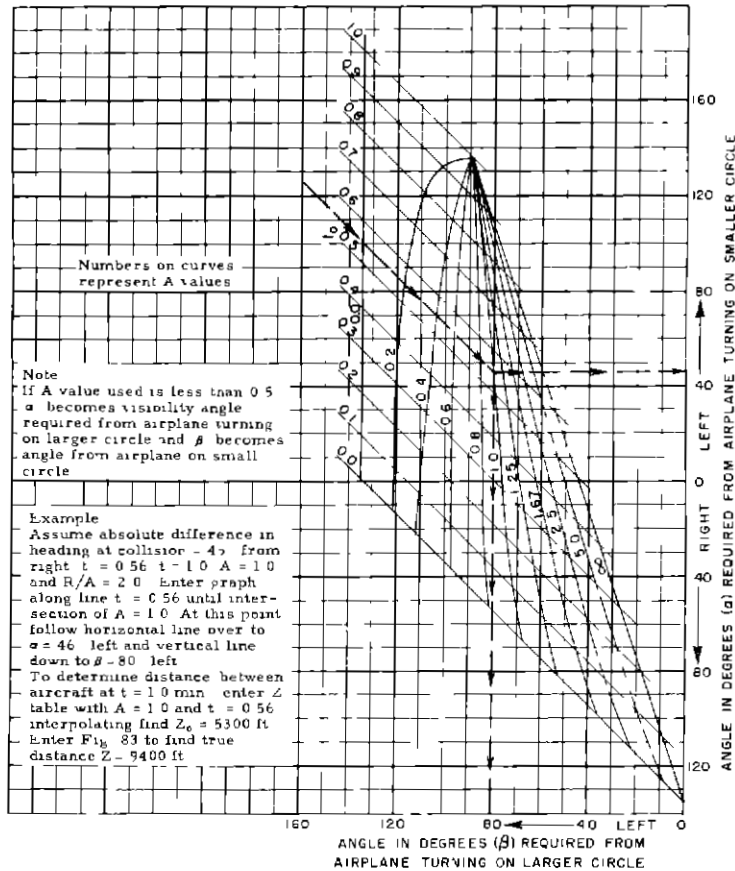


Fig 35 At Collision Point, the Difference in Heading = 45° , $R/A = 1.0$, and the Airplane Turning on the Larger Circle is on the Right of the Airplane Turning on the Smaller Circle



Z_0 Values

t_0

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	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.2	771	1521	2182	2656	2911	2945	2744	2319	1745	1121
0.4	718	1469	2000	2708	3203	3449	3453	3223	2797	2240
0.6	701	1496	2131	3079	3674	4094	4278	4203	3880	3361
0.8	726	1500	2560	3463	4230	4824	5165	5217	4975	4482
1.0	800	1750	2560	3920	4872	5620	6096	6240	6002	5600
1.25	914	2040	3306	4564	5690	6627	7267	7551	7449	7004
1.67	1182	2593	4166	5738	7172	8394	9273	9739	9747	9337
2.5	1823	3869	6076	8271	10292	12038	13354	14146	14350	14005
5.0	3916	8044	12226	16285	20004	23225	25749	27430	28176	28011

Fig 36 At Collision Point, the Difference in Heading = 45° , $R/A = 2.0$, and the Airplane Turning on the Larger Circle is on the Right of the Airplane Turning on the Smaller Circle

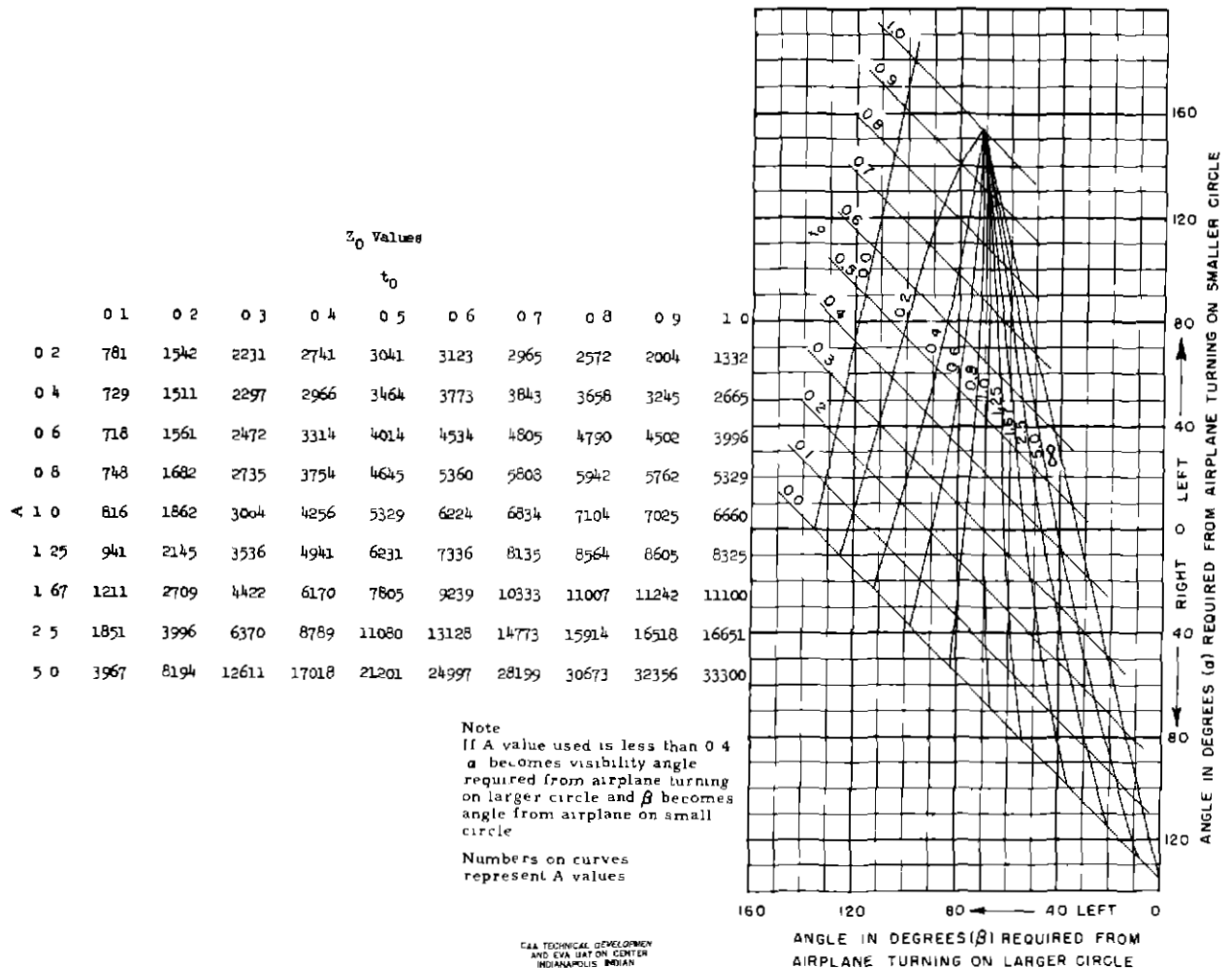


Fig 37 At Collision Point, the Difference in Heading = 45° , $R/A = 2.5$, and the Airplane Turning on the Larger Circle is on the Right of the Airplane Turning on the Smaller Circle

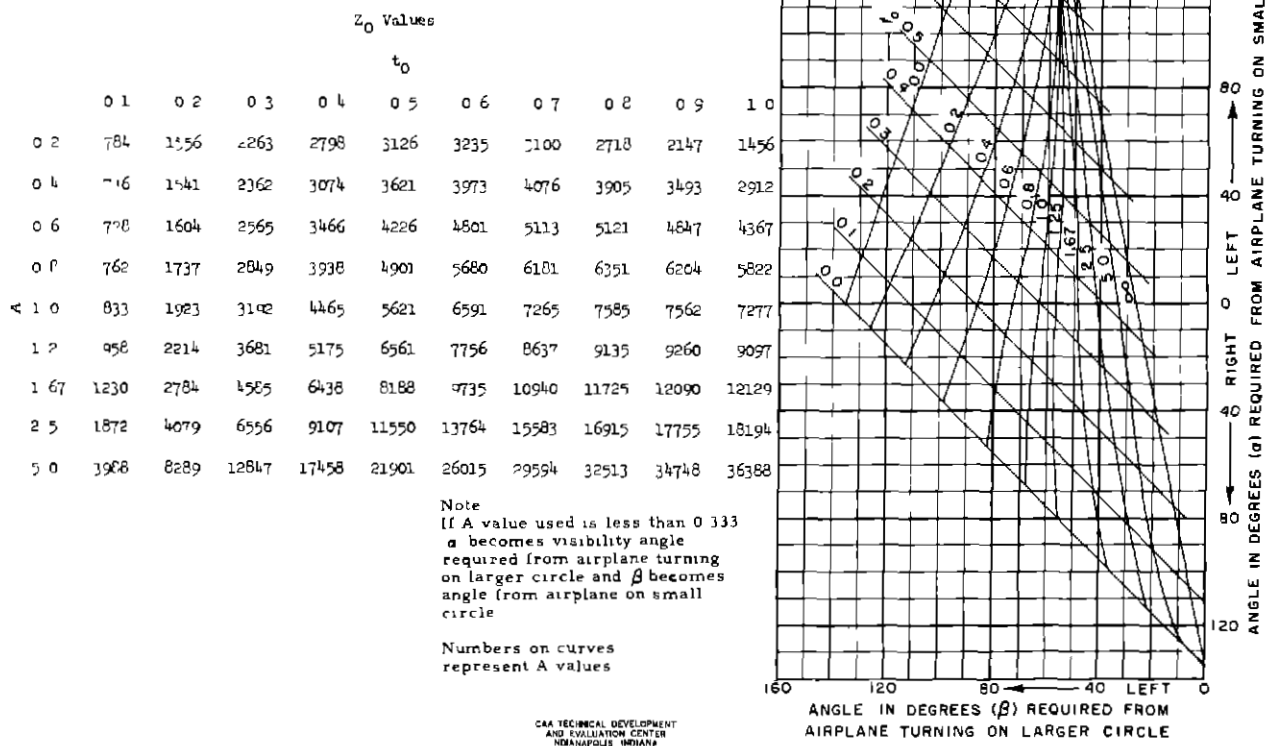


Fig 38 At Collision Point, the Difference in Heading = 45° , $R/A = 3.0$, and the Airplane Turning on the Larger Circle is on the Right of the Airplane Turning on the Smaller Circle

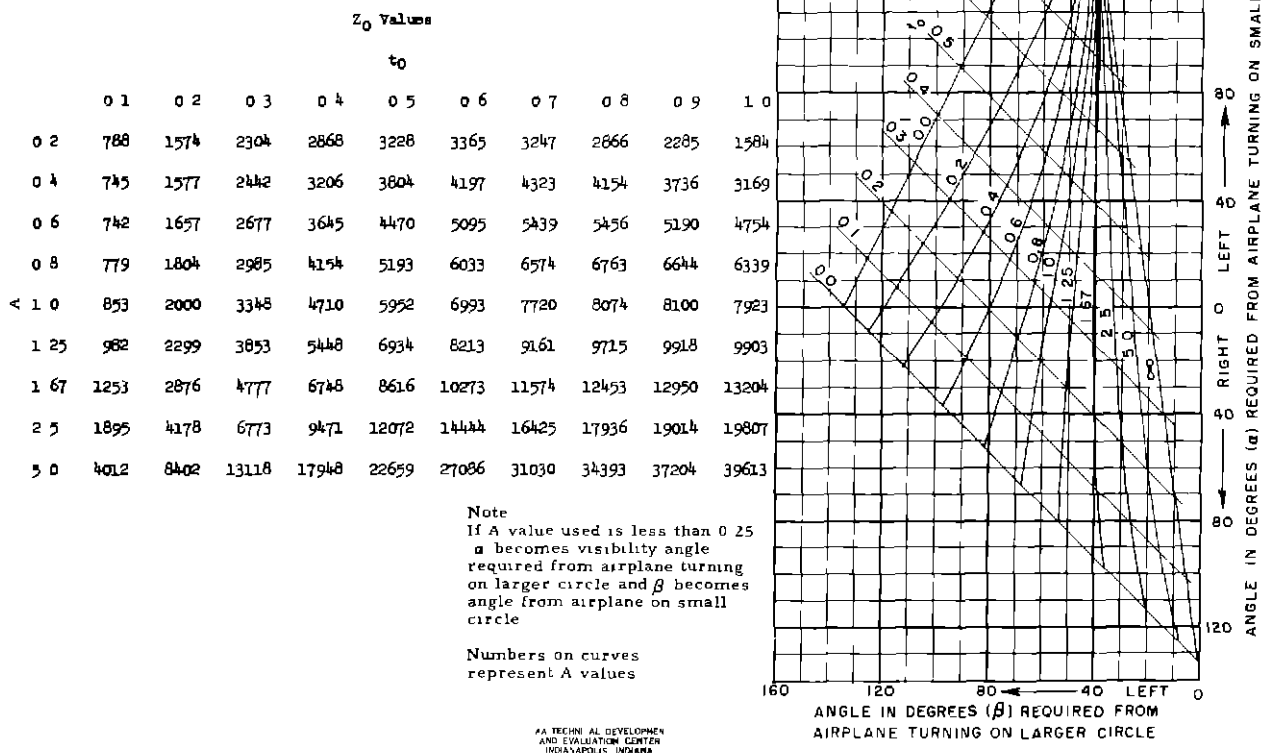


Fig 39 At Collision Point, the Difference in Heading = 45°, R/A = 4.0, and the Airplane Turning on the Larger Circle is on the Right of the Airplane Turning on the Smaller Circle

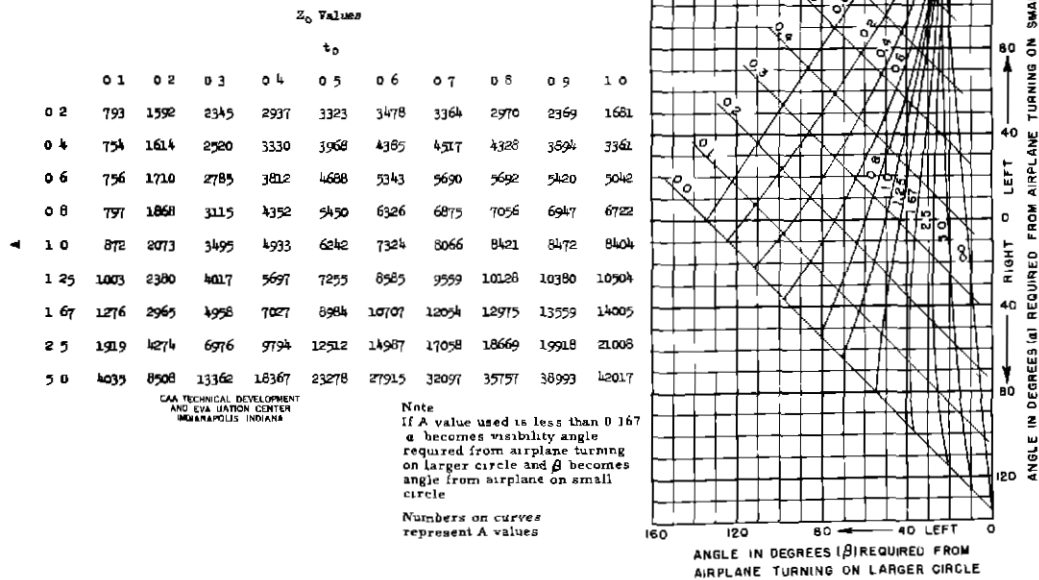
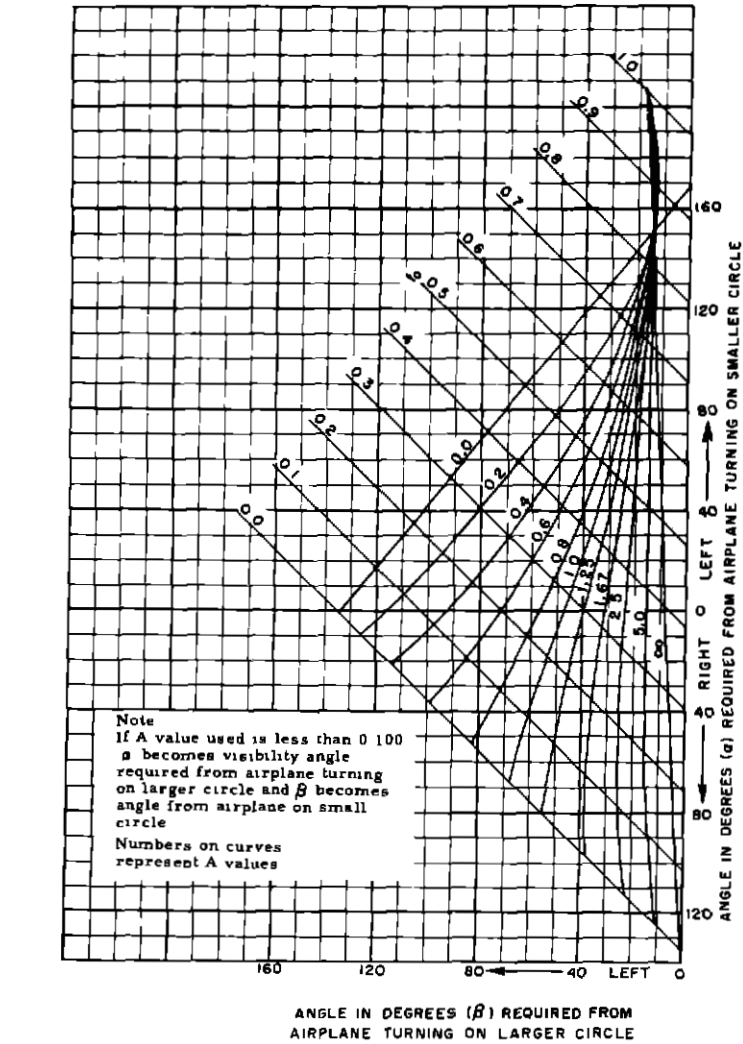


Fig 40 At Collision Point, the Difference in Heading = 45° , $R/A = 6.0$, and the Airplane Turning on the Larger Circle is on the Right of the Airplane Turning on the Smaller Circle



Z ₀ Values										
	t ₀									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.2	796	1607	2378	2989	3391	3552	3433	3018	2396	1731
0.4	762	1643	2581	3423	4086	4511	4627	4409	3996	3463
0.6	787	1751	2866	3934	4839	5503	5833	5802	5517	5194
0.8	811	1918	3214	4498	5628	6512	7045	7194	7079	6925
< 1.0	889	2132	3606	5097	6440	7533	8260	8588	8642	8696
1.25	1021	2442	4139	5877	7476	8818	9781	10329	10596	10819
1.67	1295	3033	5093	7227	9232	10976	12320	13232	13852	14426
2.5	1937	4346	7126	10022	12804	15314	17403	19036	20364	21640
5.0	4054	8586	13536	18650	23665	28396	32671	36451	39901	43280

CAA TECHNICAL DEVELOPMENT
AND EVALUATION CENTER
INDIANAPOLIS, INDIANA

Fig 41 At Collision Point, the Difference in Heading = 45°, R/A = 10.0, and the Airplane Turning on the Larger Circle is on the Right of the Airplane Turning on the Smaller Circle

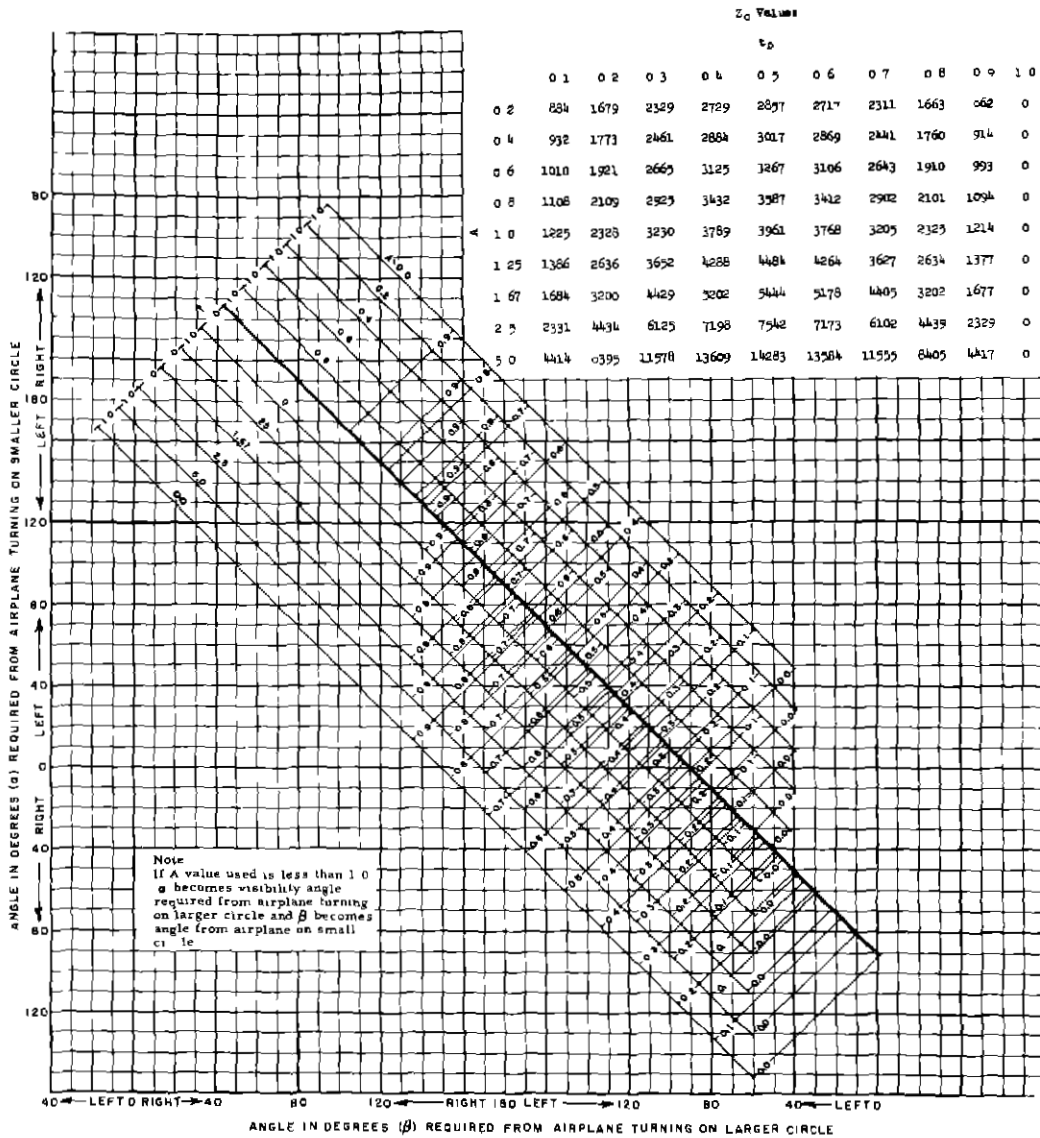


Fig 42 At Collision Point, the Difference in Heading = 90° , $R/A = 1.0$, and the Airplane Turning on the Larger Circle is on the Right of the Airplane Turning on the Smaller Circle

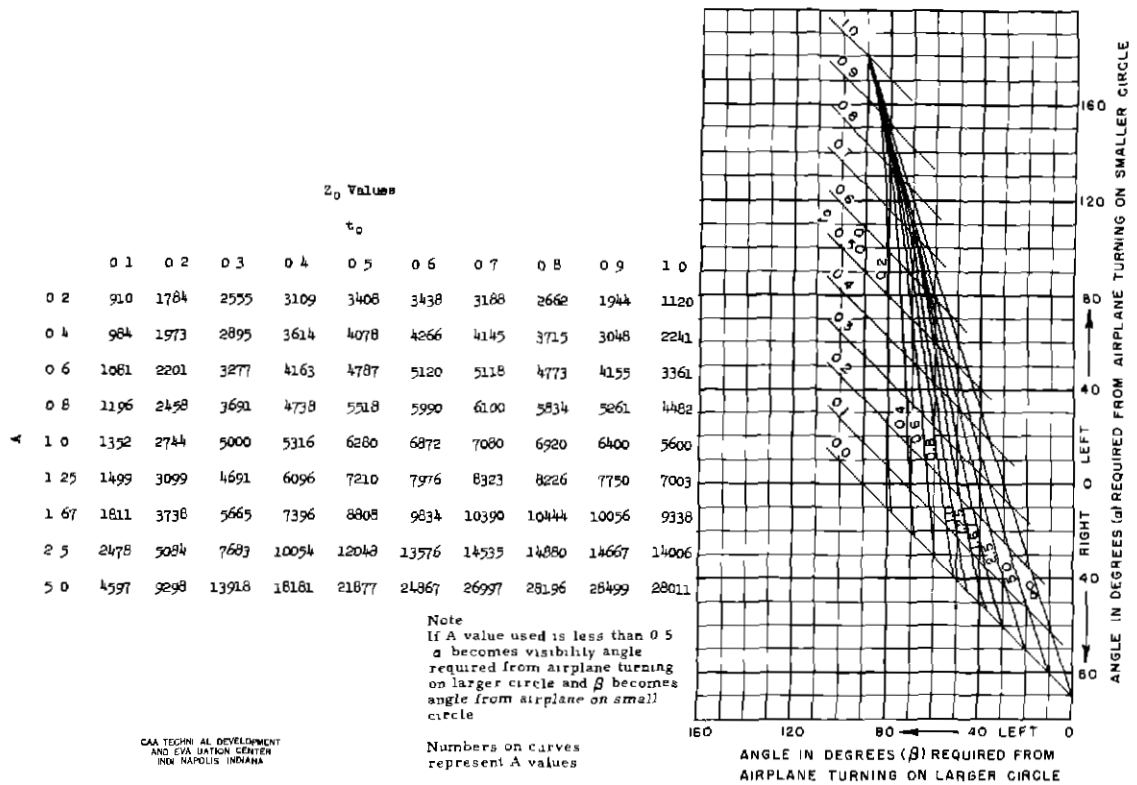


Fig 43 At Collision Point, the Difference in Heading = 90° , $R/A = 2.0$, and the Airplane Turning on the Larger Circle is on the Right of the Airplane Turning on the Smaller Circle

Z_0 Values t_0

	0 1	0 2	0 3	0 4	0 5	0 6	0 7	0 8	0 9	1 0
0 2	915	1604	2597	3175	3501	3555	3322	2806	2105	1333
0 4	993	2010	2970	3734	4245	4474	4390	3990	3371	2664
0 6	1095	2252	3380	4326	5015	5407	5461	5173	4638	3997
0 8	1212	2519	3816	4958	5798	6347	6536	6355	5904	5328
1 0	1343	2802	4270	5565	6592	7293	7612	7538	7172	6661
1 25	1520	3177	4855	6363	7592	8479	8958	9015	8757	8325
1 67	1834	3828	5857	7714	9274	10462	11203	11479	11395	11101
2 5	2505	5191	7928	10460	12664	14440	15694	16407	16677	16650
5 0	4630	9439	14257	19813	22906	26403	29179	31168	32516	33301

Note
If A value used is less than 0 4
 α becomes visibility angle
required from airplane turning
on larger circle and β becomes
angle from airplane on small
circle

Numbers on curves
represent A values

CAA TECHNICAL DEVELOPMENT
AND EVALUATION CENTER
INDIANAPOLIS INDIANA

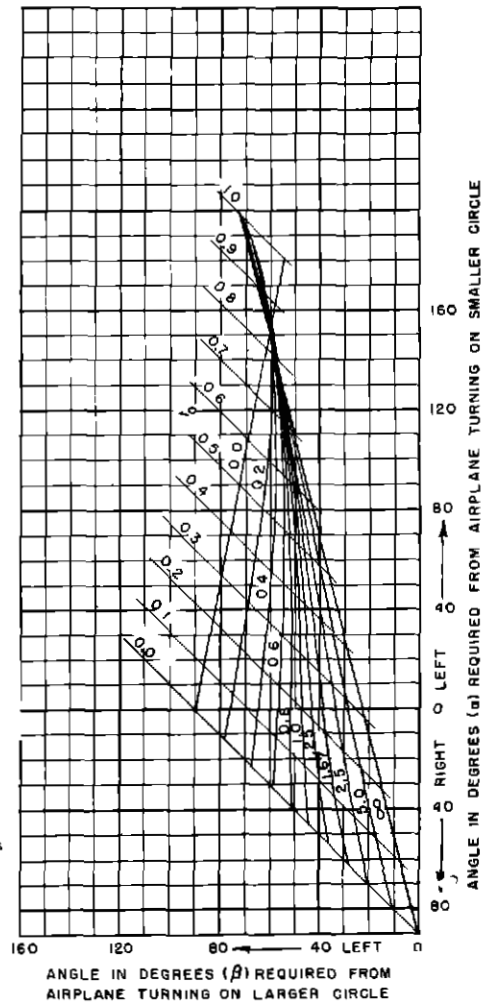


Fig 44 At Collision Point, the Difference in Heading = 90°, R/A = 2 5, and the Airplane Turning on the Larger Circle is on the Right of the Airplane Turning on the Smaller Circle

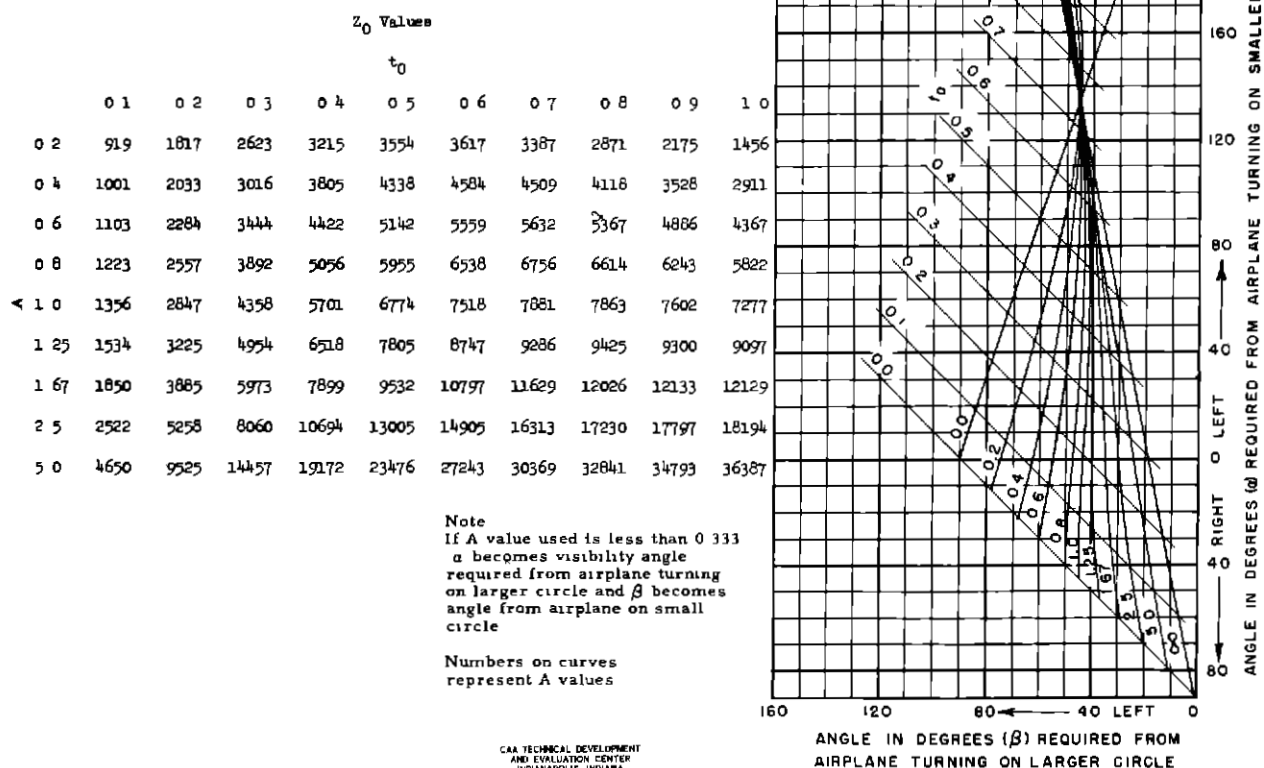


Fig 45 At Collision Point, the Difference in Heading = 90° , $R/A = 3.0$, and the Airplane Turning on the Larger Circle is on the Right of the Airplane Turning on the Smaller Circle

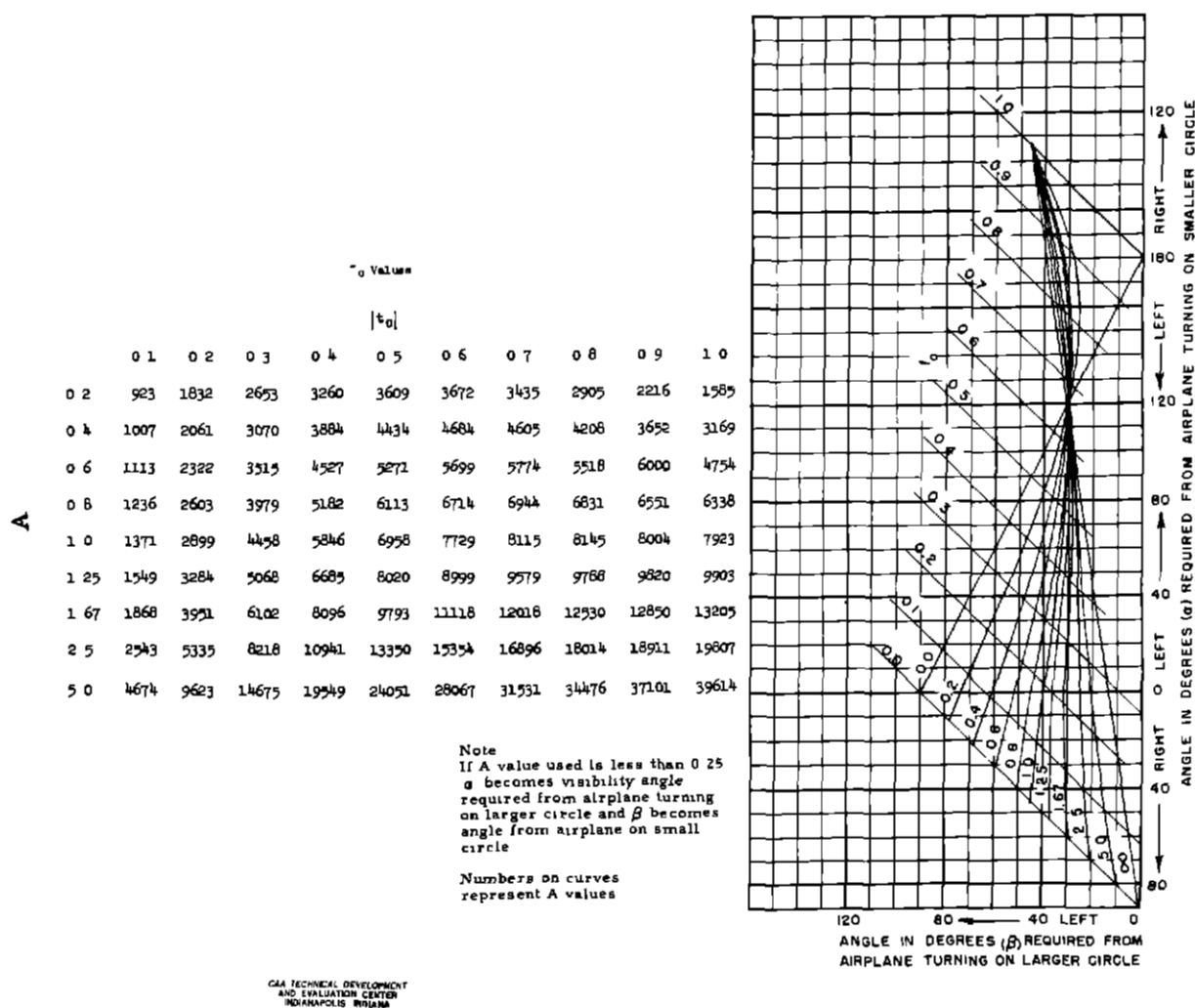


Fig 46 At Collision Point, the Difference in Heading = 90°, R/A = 4 0, and the Airplane Turning on the Larger Circle is on the Right of the Airplane Turning on the Smaller Circle

		Z ₀ Values									
		t ₀									
		0 1	0 2	0 3	0 4	0 5	0 6	0 7	0 8	0 9	1 0
4	2 0	926	1848	2681	3297	3648	3702	3444	2889	2200	1680
	4 0	1016	2089	3119	3948	4504	4742	4636	4218	3692	3362
	6 0	1124	2358	3580	4614	5365	5780	5833	5565	5203	5042
	8 0	1250	2646	4053	5288	6228	6819	7032	6920	6721	6723
	1 0	1384	2949	4546	5967	7093	7898	8233	8279	8243	8403
	1 25	1566	3338	5166	6822	8175	9155	9736	9981	10146	10504
1 67	1885	4013	6218	8295	9982	11320	12241	12821	13321	14005	
2 5	2563	5407	8354	11139	13600	15643	17256	18511	19675	21008	
5 0	4697	9712	14858	19840	24466	28632	32309	35591	38745	42017	

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Note
If A value used is less than 0.167
 α becomes visibility angle
required from airplane turning
on larger circle and β becomes
angle from airplane on small
circle

Numbers on curves
represent A values

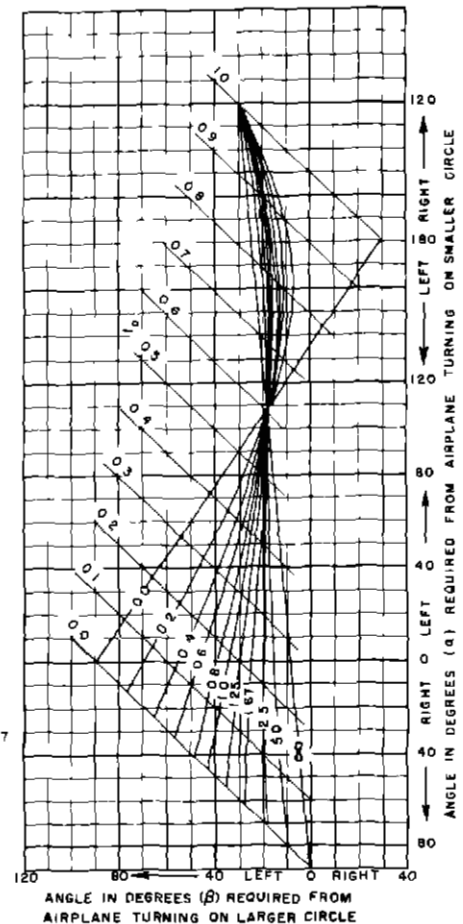
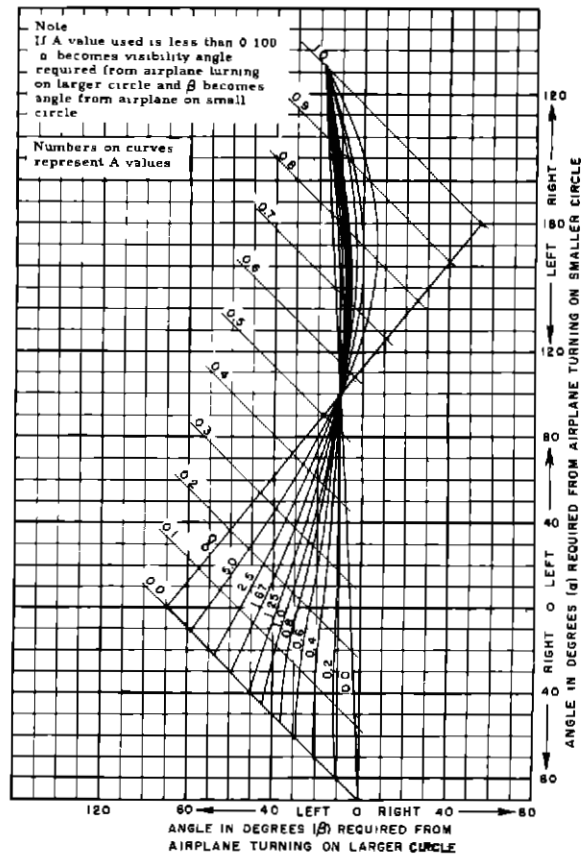
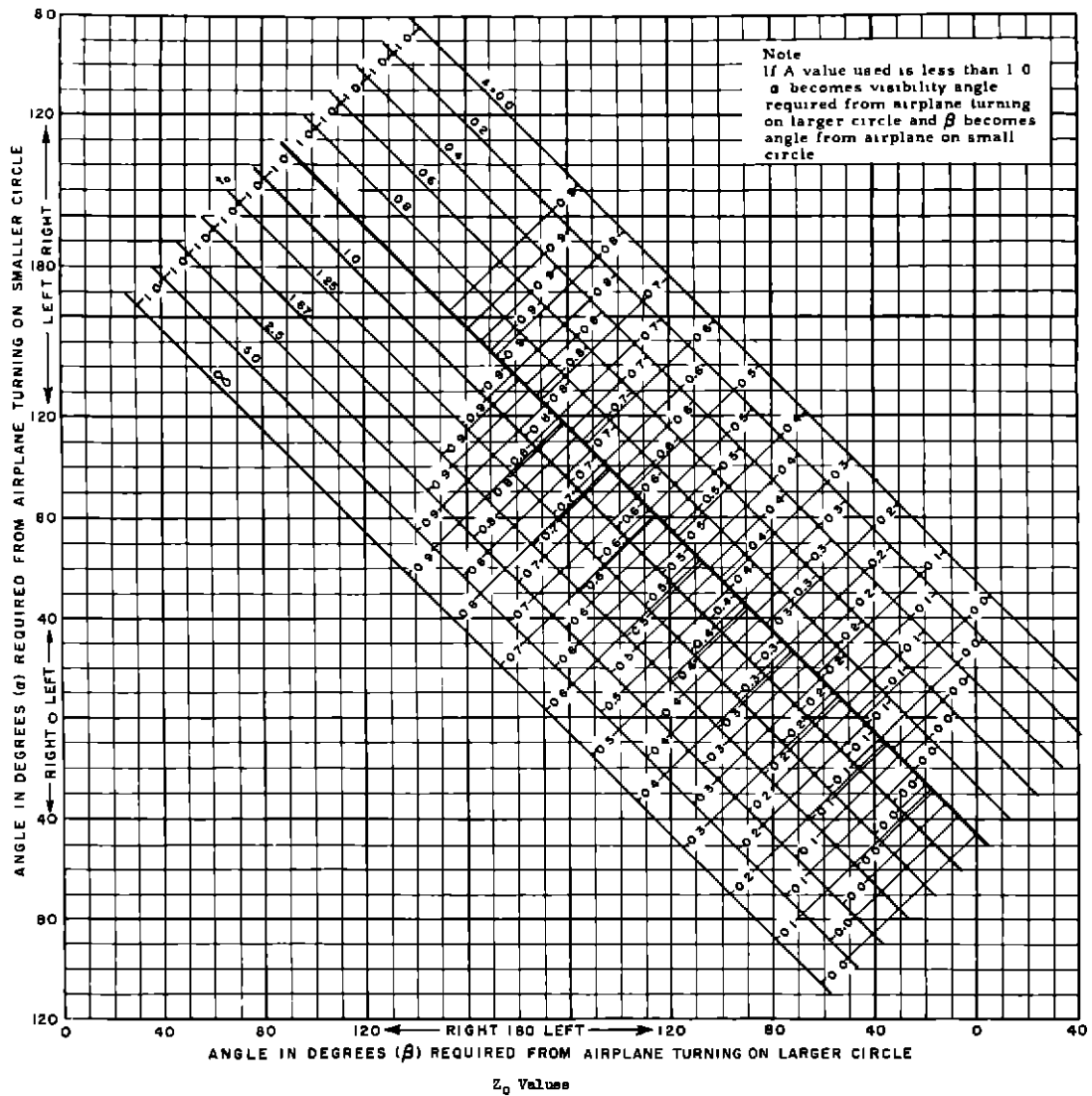


Fig 47 At Collision Point, the Difference in Heading = 90° , $R/A = 6/0$, and the Airplane Turning on the Larger Circle is on the Right of the Airplane Turning on the Smaller Circle



A	Z ₀ Values										CAA TECHNICAL DEVELOPMENT AND EVALUATION CENTER INDY NAPO IS INITIATIVE
	t ₀										
	0 1	0 2	0 3	0 4	0 5	0 6	0 7	0 8	0 9	1 0	
0 2	931	1859	2701	3323	3669	3707	3423	2838	2149	1732	
0 4	1021	2109	3154	3951	4541	4755	4613	4164	3661	3162	
0 6	1133	2385	3627	4669	5413	5802	5816	5524	5004	4594	
0 8	1259	2679	4114	5354	6287	6850	7026	6897	6756	6364	
1 0	1396	2984	4602	6042	7162	7898	8240	8277	8312	8656	
1 25	1578	3381	5236	6907	8256	9211	9755	10007	10261	10819	
1 67	1900	4059	6297	8353	10079	11395	12296	12897	13151	14426	
2 5	2579	5459	8447	11259	13728	15768	17380	18689	20017	21641	
5 0	4715	9776	14979	20012	24680	28889	32645	36090	39548	43271	

Fig 48 At Collision Point, the Difference in Heading = 90° , R/A = 10 0, and the Airplane Turning on the Larger Circle is on the Right of the Airplane Turning on the Smaller Circle



Z_0 Values

	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.2	995	1894	2623	3074	3222	3065	2607	1876	974	0
0.4	1137	2163	2995	3512	3680	3499	2976	2147	1116	0
0.6	1286	2447	3388	3973	4164	3959	3368	2432	1266	0
0.8	1441	2741	3793	4450	4662	4435	3773	2726	1422	0
1.0	1600	3042	4208	4938	5175	4923	4187	3030	1580	0
1.25	1801	3426	4737	5561	5829	5543	4716	3415	1783	0
1.67	2144	4078	5635	6616	6938	6599	5614	4068	2127	0
2.5	2843	5408	7465	8767	9200	8749	7442	5398	2827	0
5.0	4978	9469	13055	15340	16108	15321	13033	9461	4964	0

CAA TECHNICAL DEVELOPMENT
AND EVALUATION CENTER
INDIANAPOLIS, INDIANA

Fig 49 At Collision Point, the Difference in Heading = 135° , $R/A = 1.0$, and the Airplane Turning on the Larger Circle is on the Right of the Airplane Turning on the Smaller Circle

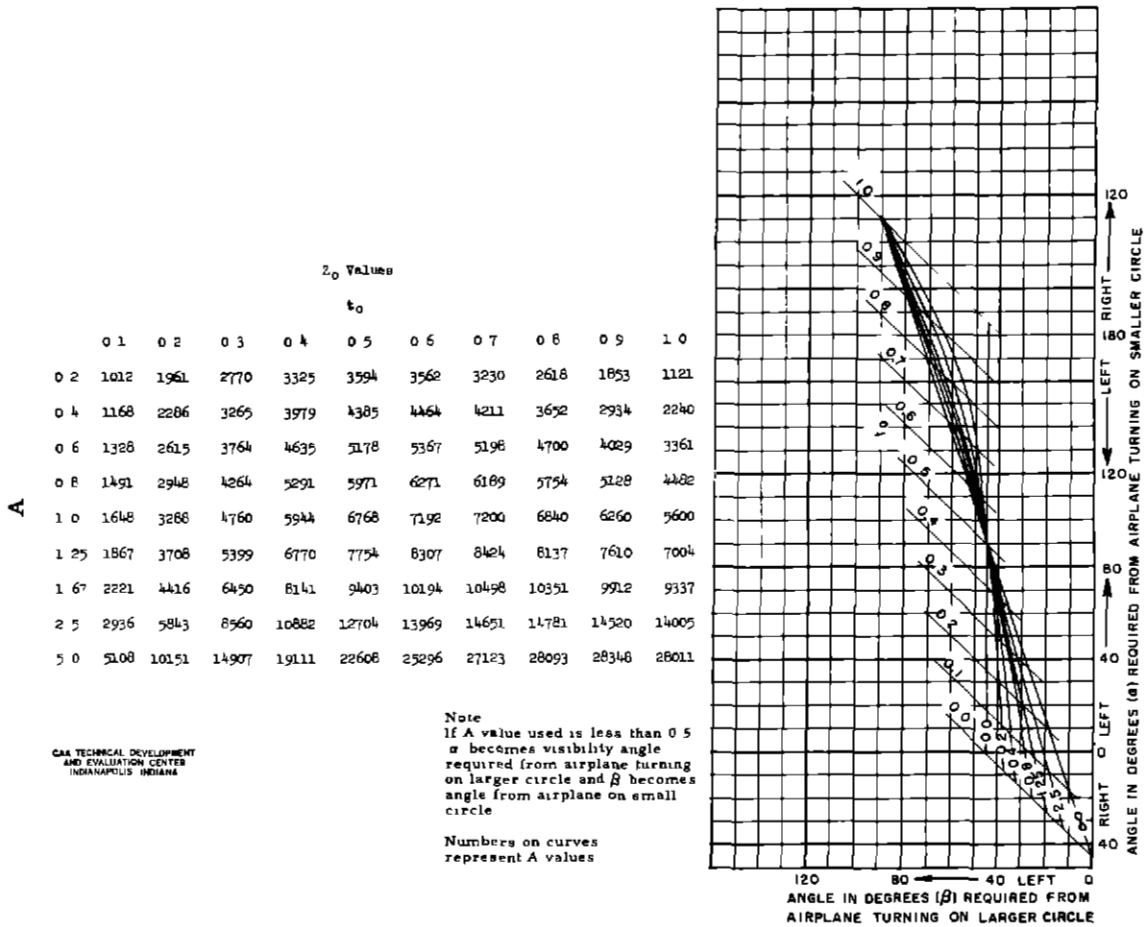


Fig 50 At Collision Point, the Difference in Heading = 135° , $R/A = 2.0$, and the Airplane Turning on the Larger Circle is on the Right of the Airplane Turning on the Smaller Circle

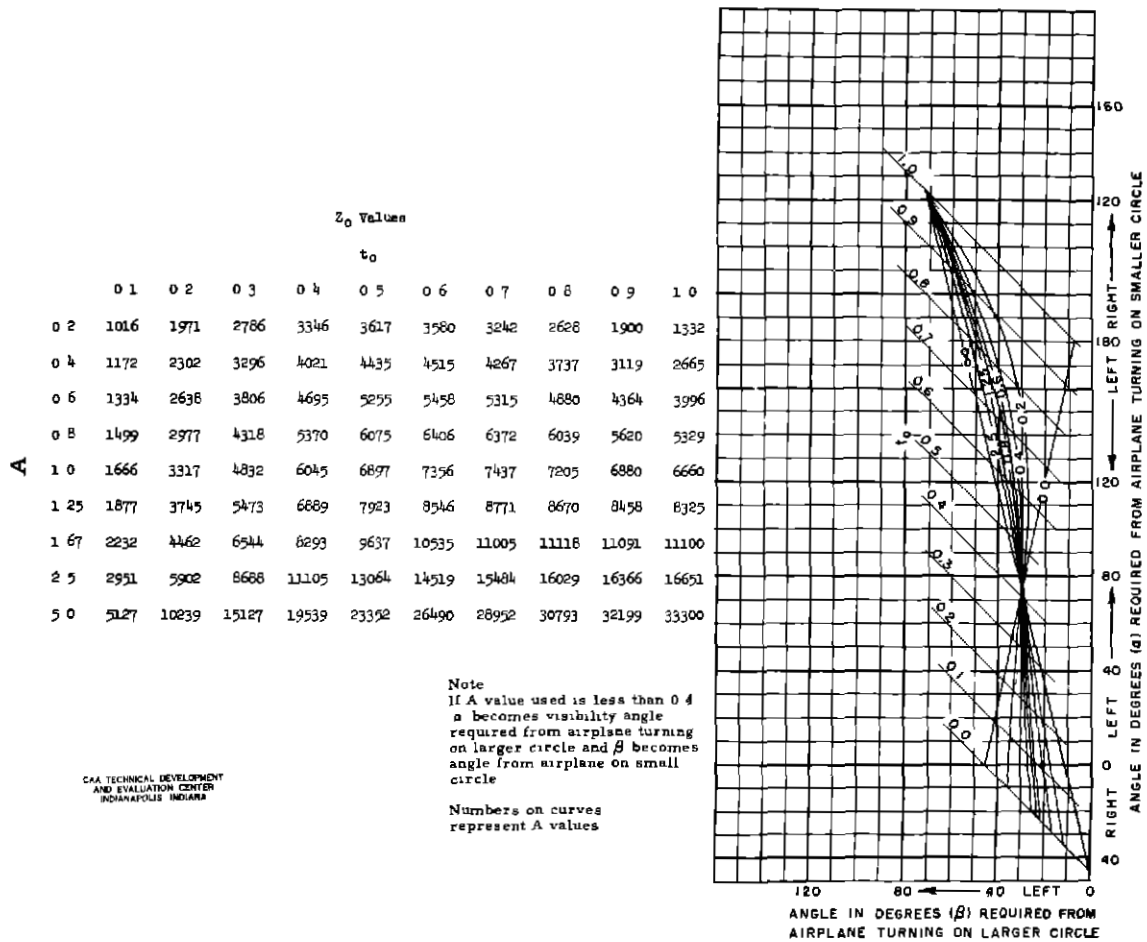


Fig 51 At Collision Point, the Difference in Heading = 135° , $R/A = 2.5$, and the Airplane Turning on the Larger Circle is on the Right of the Airplane Turning on the Smaller Circle

	z_0 Values									
	t_0									
A	0 1	0 2	0 3	0 4	0 5	0 6	0 7	0 8	0 9	1 0
0 2	1018	1977	2795	3355	3620	3573	3219	2595	1893	1456
0 4	1176	2313	3311	4037	4446	4514	4257	3735	3186	2912
0 6	1339	2652	3829	4720	5277	5473	5330	4927	4519	4367
0 8	1505	2994	4346	5403	6110	6440	6419	6142	5863	5822
1 0	1672	3337	4864	6087	6946	7412	7520	7368	7214	7277
1 25	1884	3767	5512	6941	7991	8633	8905	8910	8906	9097
1 67	2240	4489	6592	8364	9736	10675	11223	11492	11732	12129
2 5	2961	5935	8754	11213	13229	14771	15882	16673	17389	18194
5 0	5139	10290	15243	19757	23725	27098	29909	32262	34377	36388

Note
If A value used is less than 0 333
 α becomes visibility angle
required from airplane turning
on larger circle and β becomes
angle from airplane on small
circle

Numbers on curves
represent A values

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AND EVALUATION CENTER
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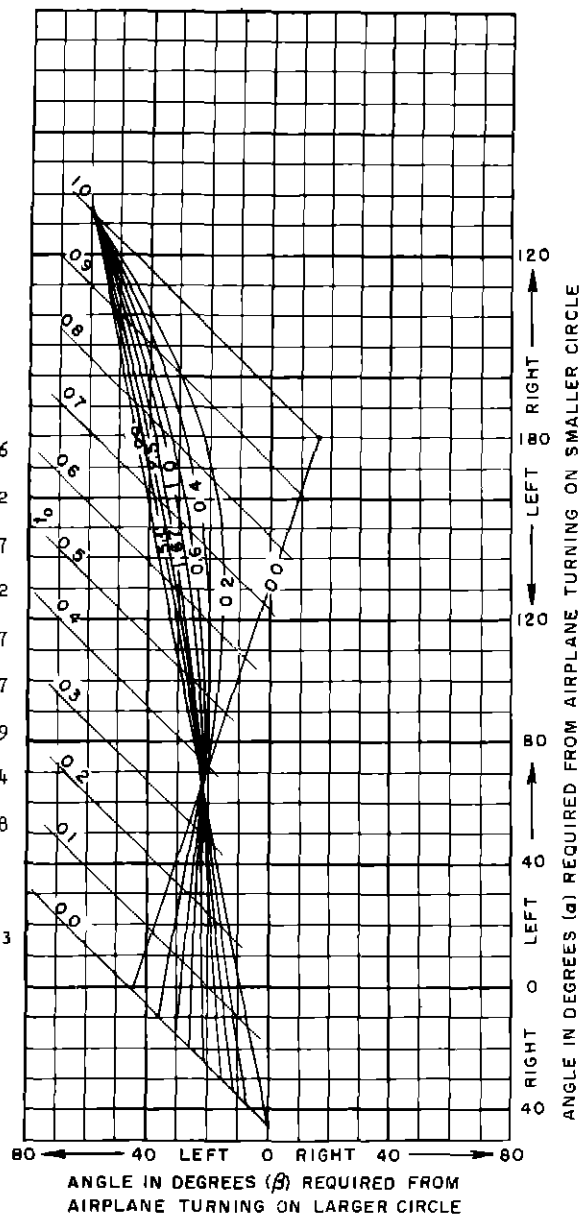


Fig 52 At Collision Point, the Difference in Heading = 135° , $R/A = 3.0$, and the Airplane Turning on the Larger Circle is on the Right of the Airplane Turning on the Smaller Circle

	Z ₀ Values									
	t ₀									
A	0 1	0 2	0 3	0 4	0 5	0 6	0 7	0 8	0 9	1 0
0 2	1020	1983	2803	3357	3609	3538	3155	2508	1839	1584
0 4	1180	2323	3326	4044	4434	4473	4185	3662	3208	3169
0 6	1343	2668	3848	4733	5271	5438	5274	4898	4629	4754
0 8	1511	3012	4311	5423	6113	6418	6392	6169	6067	6339
1 0	1678	3358	4893	6113	6958	7409	7527	7455	7512	7923
1 25	1892	3792	5546	6978	8020	8657	8958	9076	9321	9903
1 67	2249	4516	6637	8417	9793	10748	11360	11792	12345	13204
2 5	2970	5969	8816	11299	13350	14954	16199	17251	18400	19807
5 0	5153	10342	15354	19952	24051	27634	30792	33686	36583	39613

Note
If A value used is less than 0 25
a becomes visibility angle
required from airplane turning
on larger circle and β becomes
angle from airplane on small
circle

Numbers on curves
represent A values

CAA TECHNICAL DEVELOPMENT
AND EVALUATION CENTER
INDIANAPOLIS, INDIANA

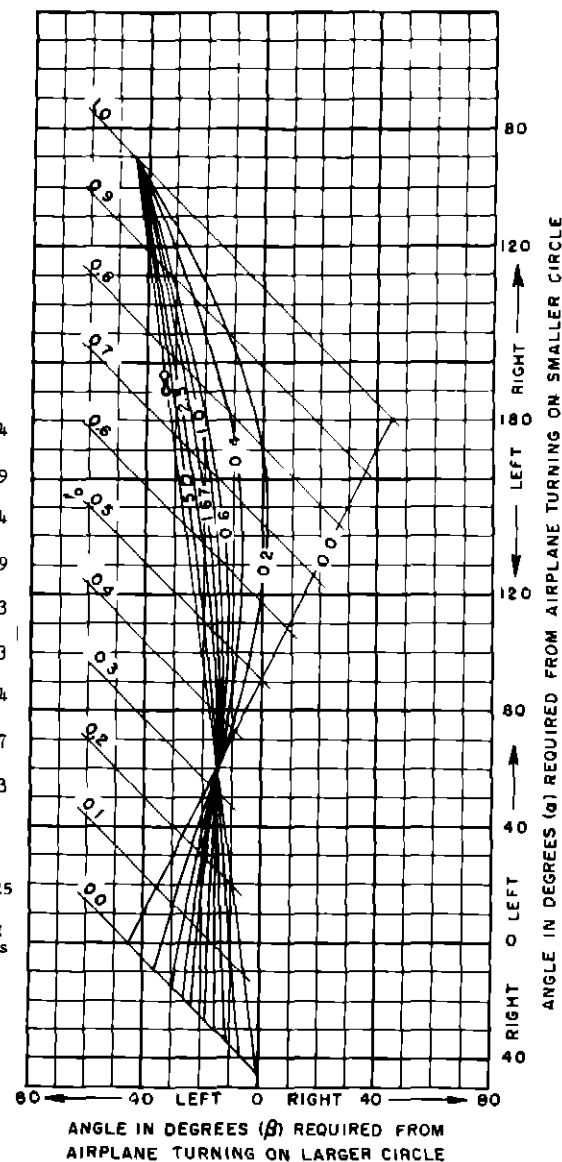


Fig 53 At Collision Point, the Difference in Heading = 135° , $R/A = 4.0$, and the Airplane Turning on the Larger Circle is on the Right of the Airplane Turning on the Smaller Circle

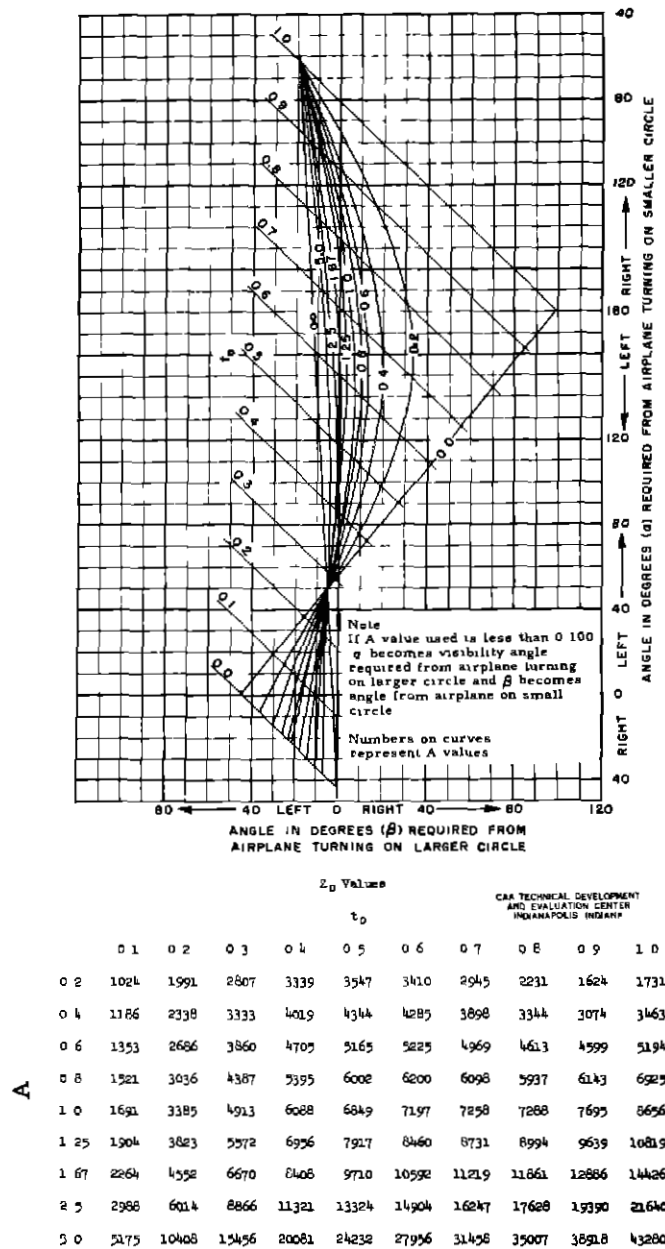


Fig 54 At Collision Point, the Difference in Heading = 135° , $R/A = 10.0$, and the Airplane Turning on the Larger Circle is on the Right of the Airplane Turning on the Smaller Circle

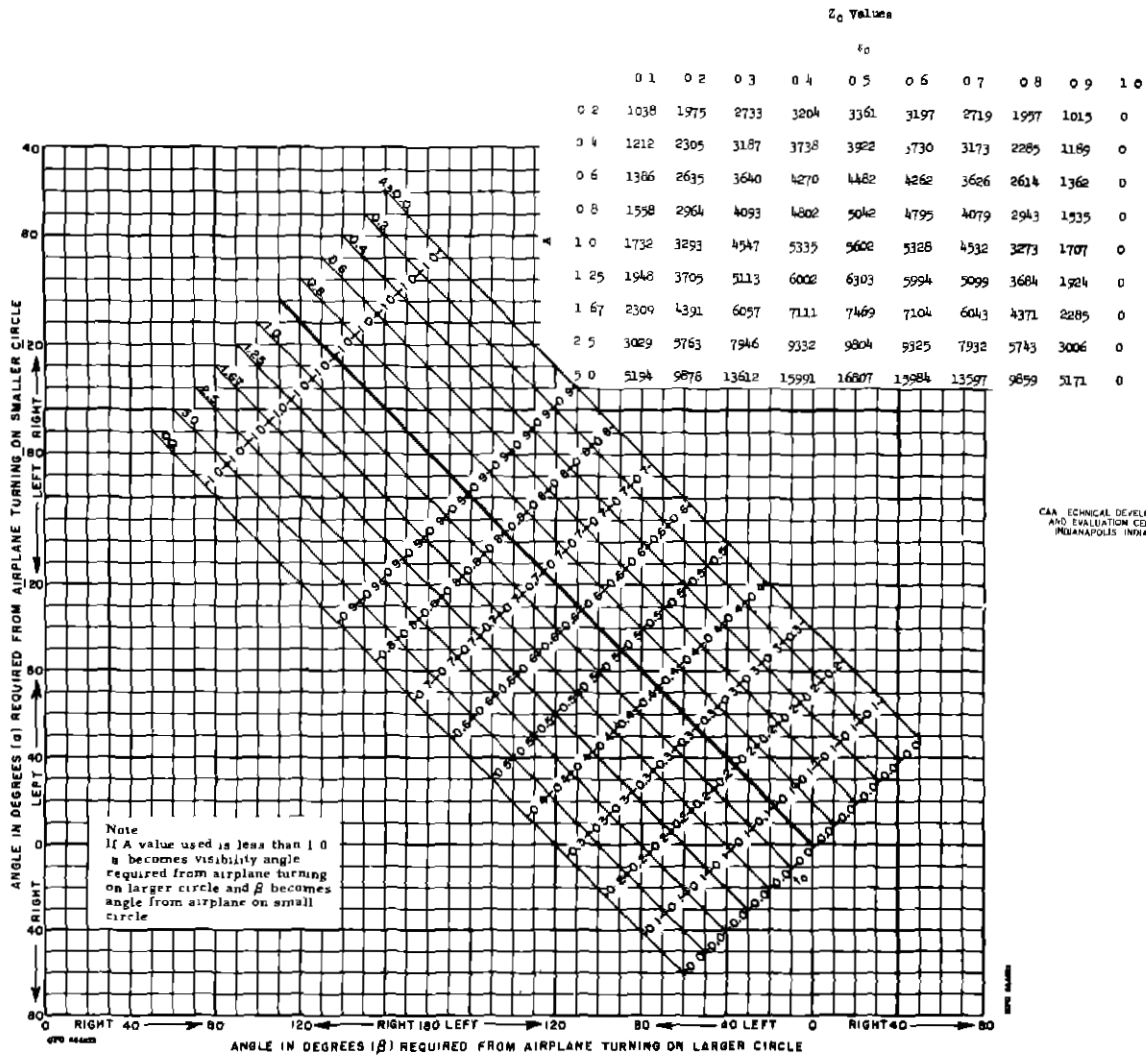


Fig 55 At Collision Point, the Difference in Heading = 180° , $R/A = 1.0$, and the Airplane Turning on the Larger Circle is on the Left of the Airplane Turning on the Smaller Circle

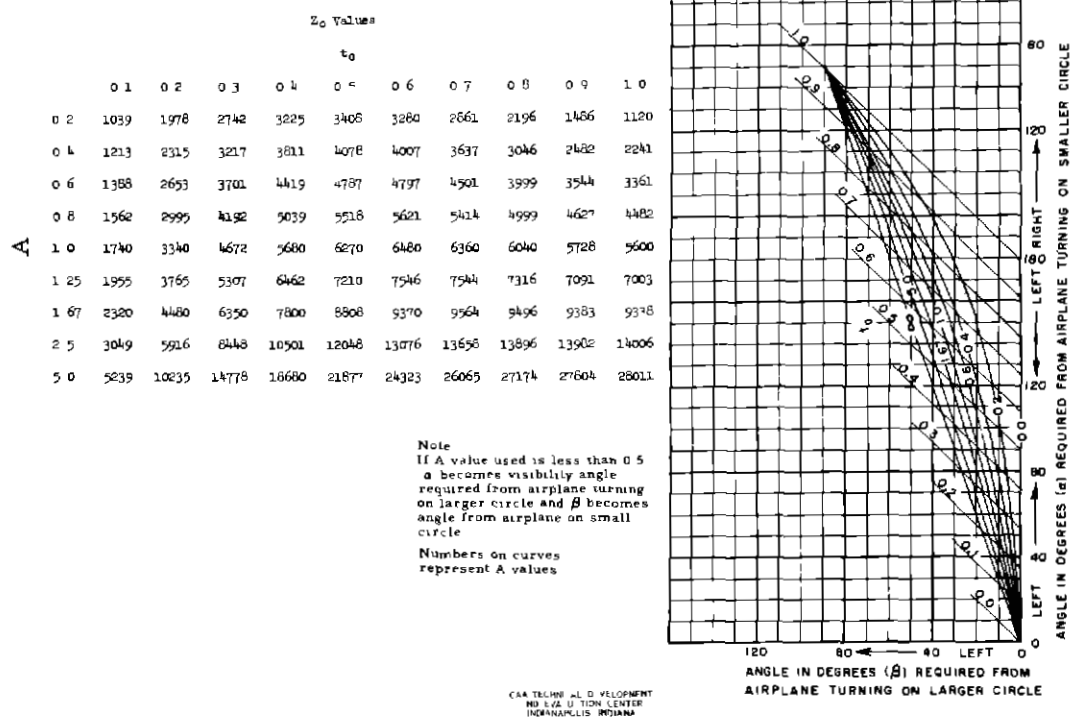


Fig 56 At Collision Point, the Difference in Heading = 180° , $R/A = 2.0$, and the Airplane Turning on the Larger Circle is on the Left of the Airplane Turning on the Smaller Circle

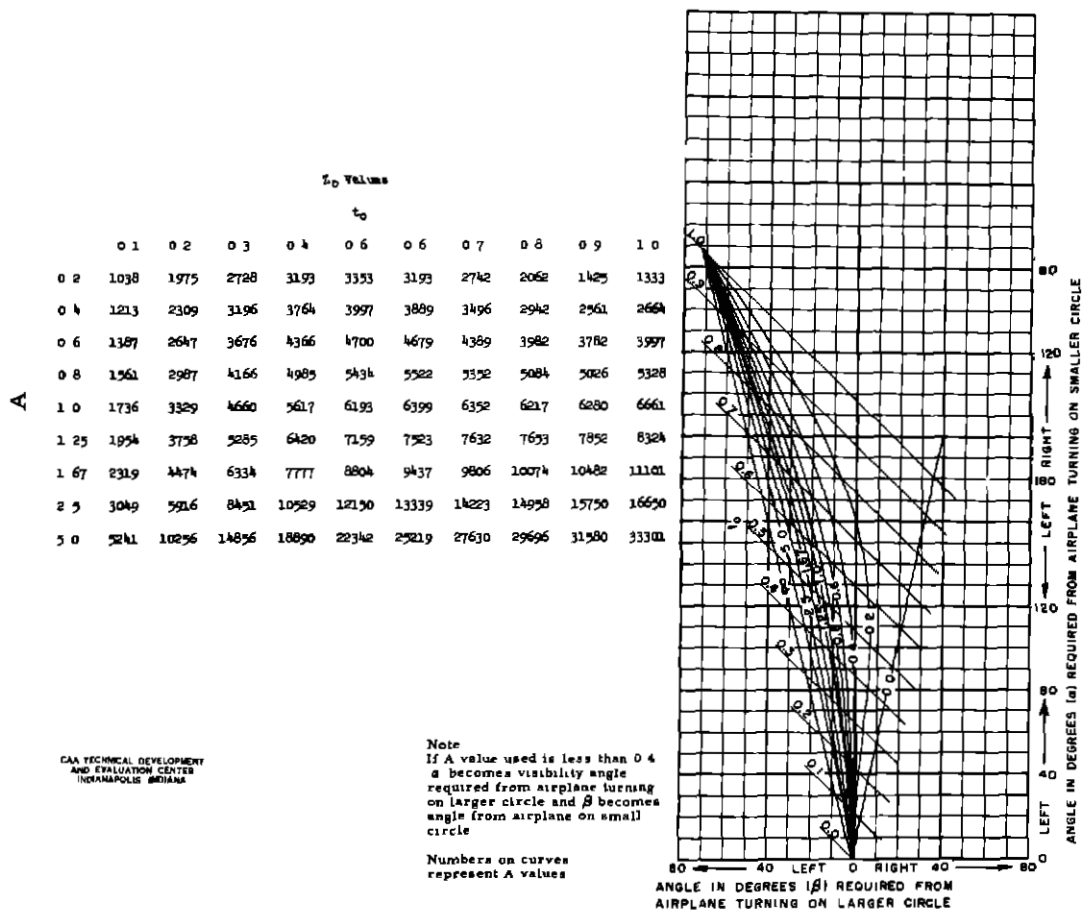


Fig 57 At Collision Point, the Difference in Heading = 180° , $R/A = 2.5$, and the Airplane Turning on the Larger Circle is on the Left of the Airplane Turning on the Smaller Circle

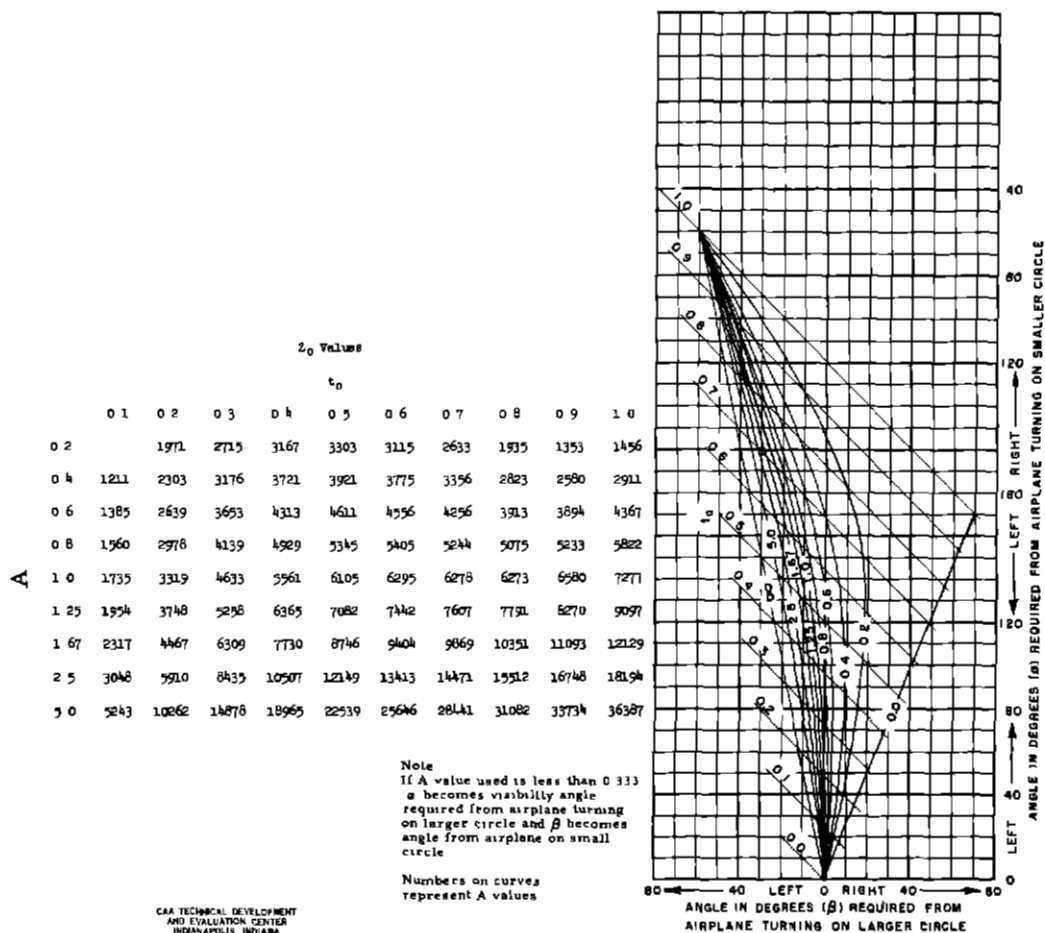
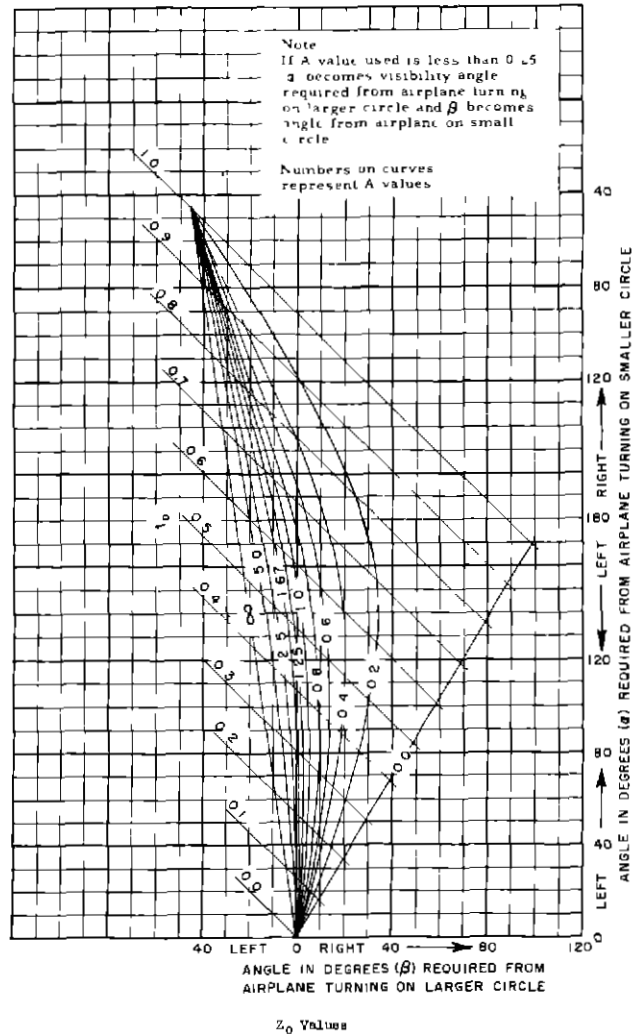


Fig 58 At Collision Point, the Difference in Heading = 180° , $R/A = 3.0$, and the Airplane Turning on the Larger Circle is on the Left of the Airplane Turning on the Smaller Circle



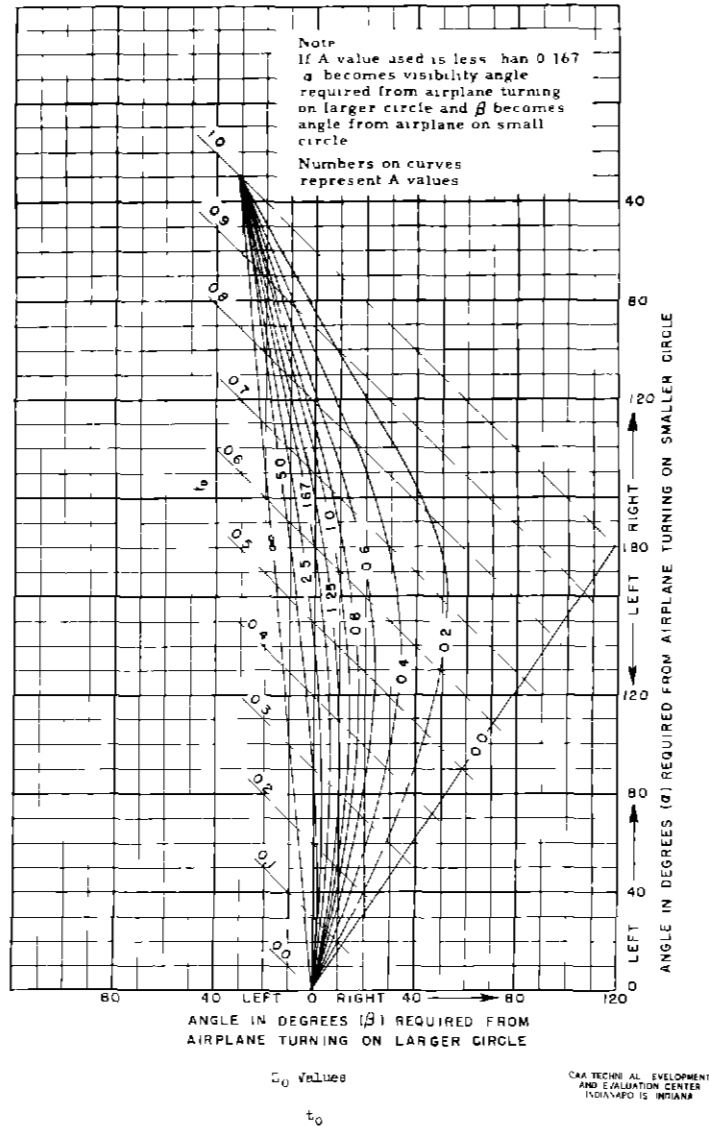
α_0 Values

t_0

	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.2	1038	1965	2698	3125	3228	2997	2468	1738	1237	1585
0.4	1210	2294	3146	3654	3804	3597	3129	2620	2587	3160
0.6	1384	2628	3614	4231	4470	4354	4027	3766	3985	4754
0.8	1557	2965	4096	4838	5193	5201	5040	4997	5422	6338
1.0	1733	3305	4588	5465	5952	6100	6110	6263	6867	7923
1.25	1951	3734	5211	6269	6934	7268	7489	7869	8679	9903
1.67	2316	4451	6262	7638	8616	9276	9842	10574	11704	13205
2.5	3046	5896	8394	10437	12072	13394	14633	16023	17760	19607
5.0	5242	10258	14874	18988	22659	25984	29181	32450	35944	39614

AA TECHNICAL DEVELOPMENT
AND EVALUATION CENTER
IN AN FOLS, INDIANA

Fig 59 At Collision Point, the Difference in Heading = 180° , $R/A = 4.0$, and the Airplane Turning on the Larger Circle is on the Left of the Airplane Turning on the Smaller Circle



	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.2	1036	1959	2675	3077	3141	2859	2274	1502	1098	1620
0.4	1209	2282	3109	3574	3661	3378	2849	2369	2521	3362
0.6	1382	2614	3568	4129	4294	4100	3736	3564	4021	5042
0.8	1557	2949	4044	4723	5000	4936	4765	4851	5536	6723
1.0	1730	3268	4530	5342	5751	5837	5661	6171	7055	8403
1.25	1949	3714	5149	6141	6730	7017	7276	7845	8957	10504
1.67	2313	4431	6198	7509	8422	9057	9702	10660	12131	14005
2.5	3044	5876	8332	10817	11911	13255	14636	16325	18484	21008
5.0	5239	10242	14834	18931	22637	26105	29610	33384	37554	42017

Fig 60 At Collision Point, the Difference in Heading = 180° , $R/A = 6.0$, and the Airplane Turning on the Larger Circle is on the Left of the Airplane Turning on the Smaller Circle

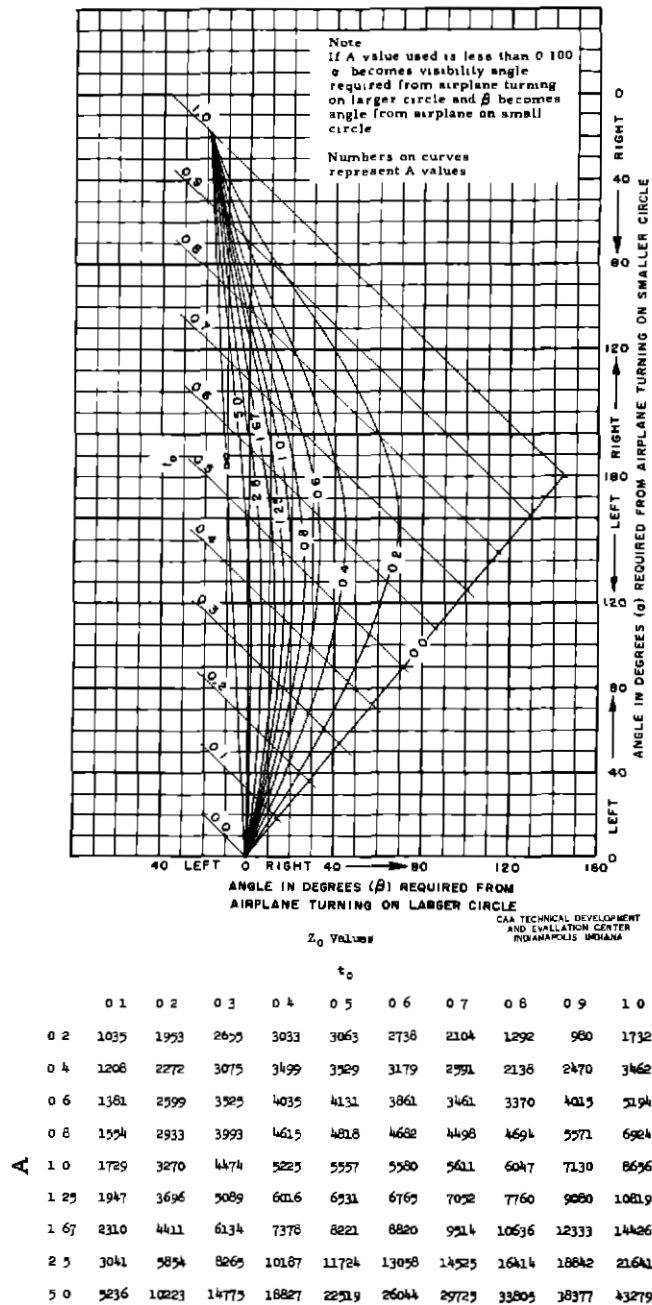
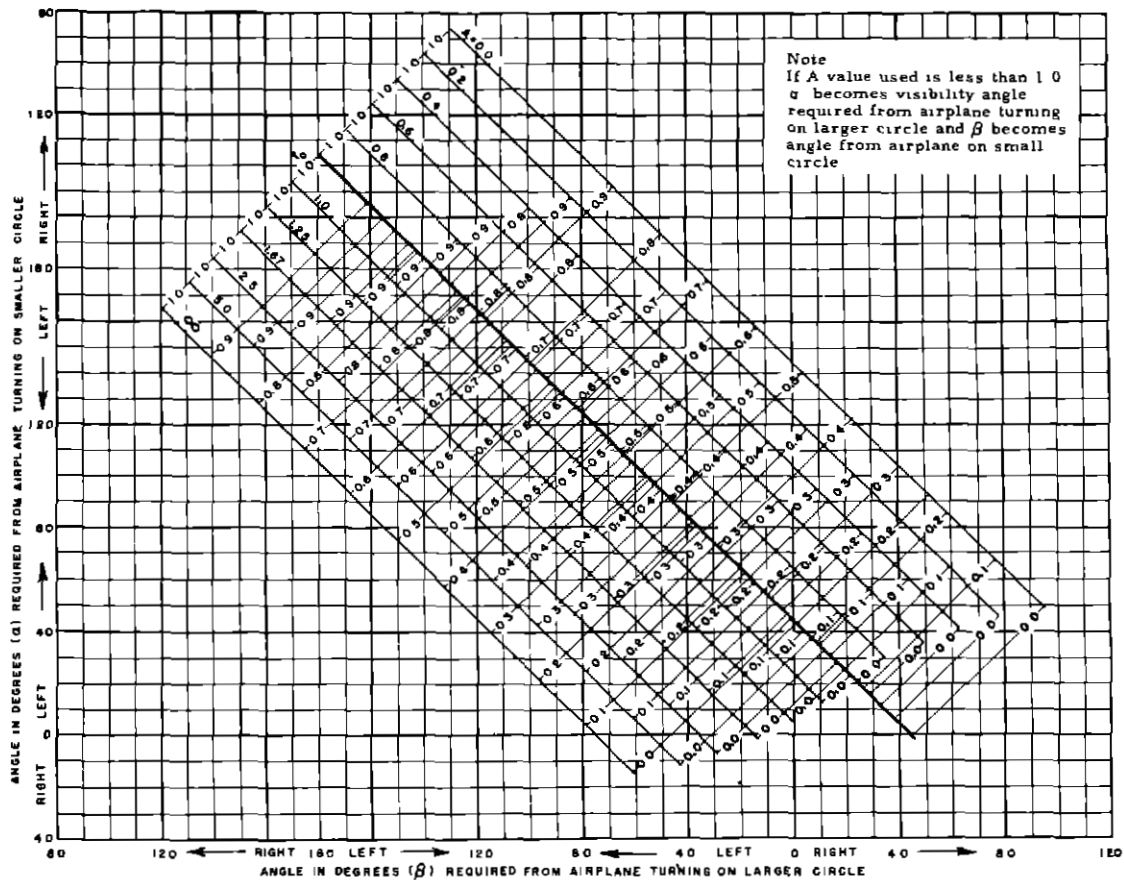


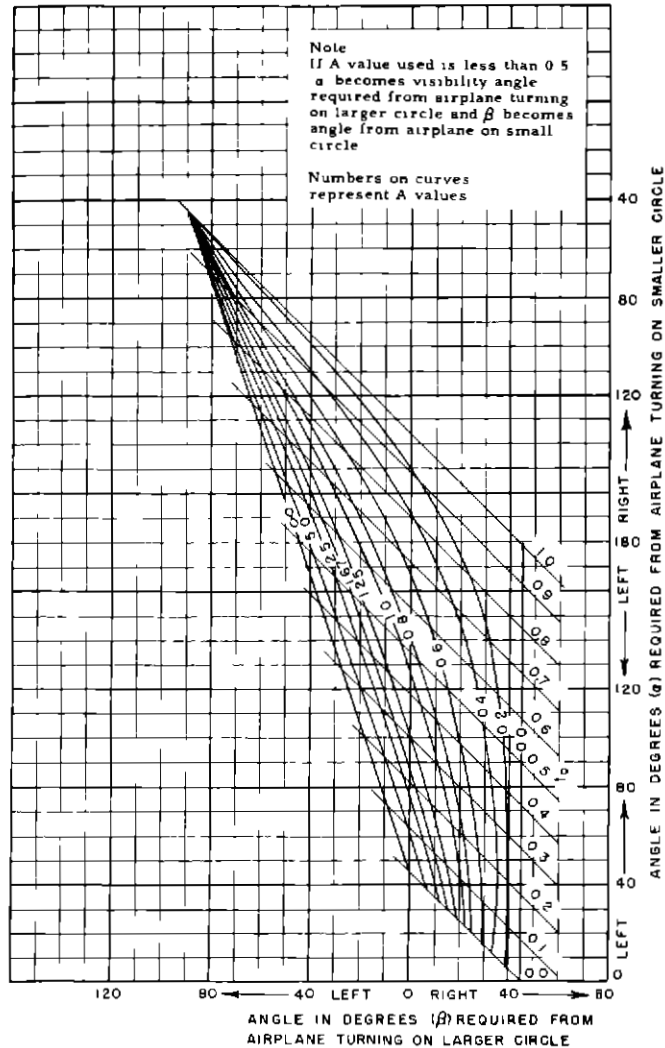
Fig 61 At Collision Point, the Difference in Heading = 180° , $R/A = 10.0$, and the Airplane Turning on the Larger Circle is on the Left of the Airplane Turning on the Smaller Circle

Z₀ Values

		t ₀									
		0 1	0 2	0 3	0 4	0 5	0 6	0 7	0 8	0 9	1 0
x	0 2	996	1893	2618	3069	3222	3064	2607	1872	972	0
	0 4	1137	2163	2987	3503	3680	3500	2977	2140	1113	0
	0 6	1286	2447	3376	3960	4164	3959	3368	2424	1262	0
	0 8	1441	2741	3779	4434	4662	4435	3773	2717	1416	0
	1 0	1600	3042	4194	4920	5175	4923	4187	3019	1575	0
	1 25	1801	3427	4720	5541	5829	5544	4716	3402	1777	0
	1 67	2145	4078	5616	6594	6938	6599	5614	4054	2120	0
	2 5	2844	5407	7444	8743	9200	8749	7442	5382	2819	0
	5 0	4978	9469	13031	15312	16108	15320	13033	9444	4955	0

CAA TECHNICAL DEVELOPMENT
AND EVALUATION CENTER
INDIANAPOLIS, INDIANA

Fig 62 At Collision Point, the Difference in Heading = 135°, R/A = 1.0, and the Airplane Turning on the Larger Circle is on the Left of the Airplane Turning on the Smaller Circle

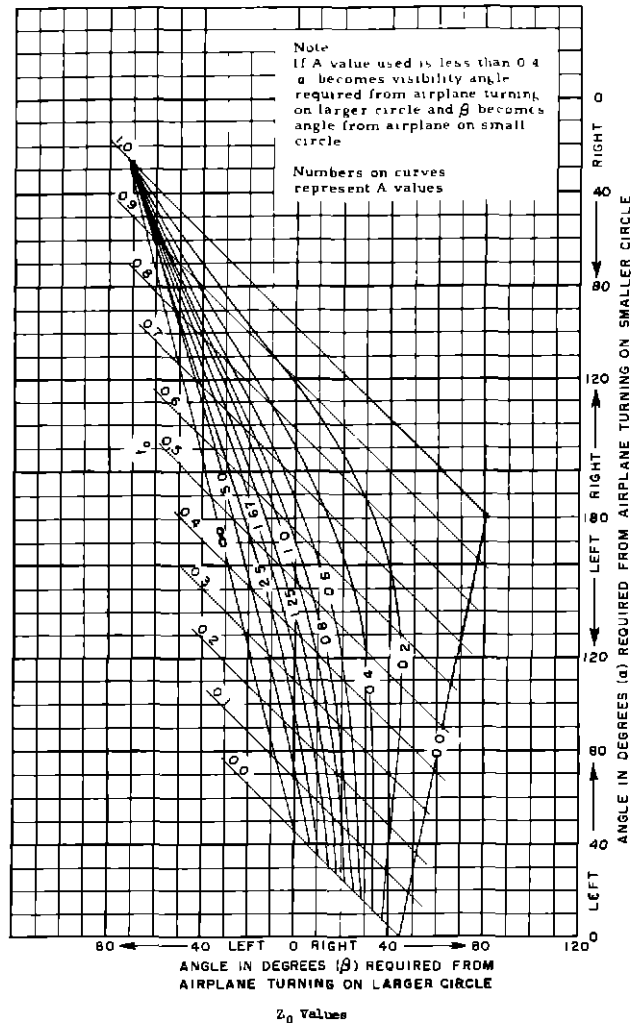


Z_0 Values

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AND EVALUATION CENTER
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	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.2	979	1830	2481	2845	2921	2676	2175	1477	908	1121
0.4	1108	2056	2762	3149	3218	2979	2516	1996	1842	2240
0.6	1250	2312	3102	3550	3674	3497	3149	2840	2899	3361
0.8	1398	2588	3483	4019	4230	4150	3936	3796	3984	4482
1.0	1560	2880	3892	4552	4800	4880	4800	4820	5000	5600
1.25	1751	3261	4434	5228	5690	5872	5944	6063	6452	7004
1.67	2091	3918	5384	6449	7172	7607	7918	8256	8747	9337
2.5	2789	5284	7372	9023	10292	11227	11974	12652	13349	14005
5.0	4941	9518	13578	17066	20004	22399	24354	25930	27274	28011

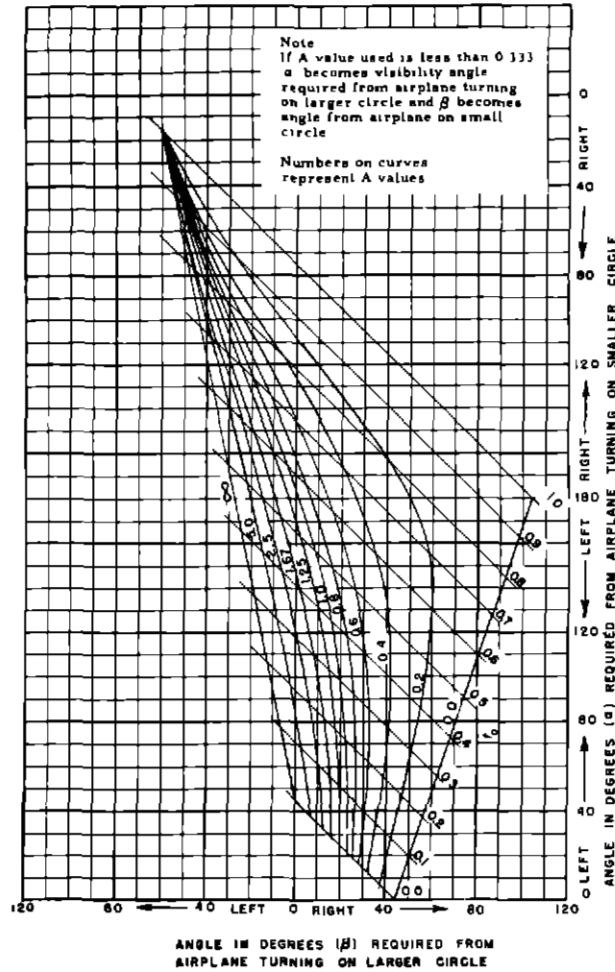
Fig 63 At Collision Point, the Difference in Heading = 135° , $R/A = 2.0$, and the Airplane Turning on the Larger Circle is on the Left of the Airplane Turning on the Smaller Circle



	t_0									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.2	975	1813	2440	2770	2793	2508	1953	1215	785	1332
0.4	1102	2026	2692	3020	3019	2704	2194	1762	1933	2665
0.6	1241	2273	3012	3389	3433	3191	2855	2744	3172	3996
0.8	1388	2544	3380	3841	3975	3856	3716	3840	4429	5329
1.0	1541	2832	3783	4351	4603	4629	4669	4976	5689	6660
1.25	1738	3209	4320	5043	5459	5674	5921	6421	7268	8325
1.67	2077	3864	5270	6280	6993	7517	8078	8851	9904	11100
2.5	2775	5231	7274	8904	10234	11362	12489	13748	15179	16651
5.0	4927	9483	13552	17138	20332	23201	25896	28502	31016	33300

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Fig 64 At Collision Point, the Difference in Heading = 135° , $R/A = 2.5$, and the Airplane Turning on the Larger Circle is on the Left of the Airplane Turning on the Smaller Circle

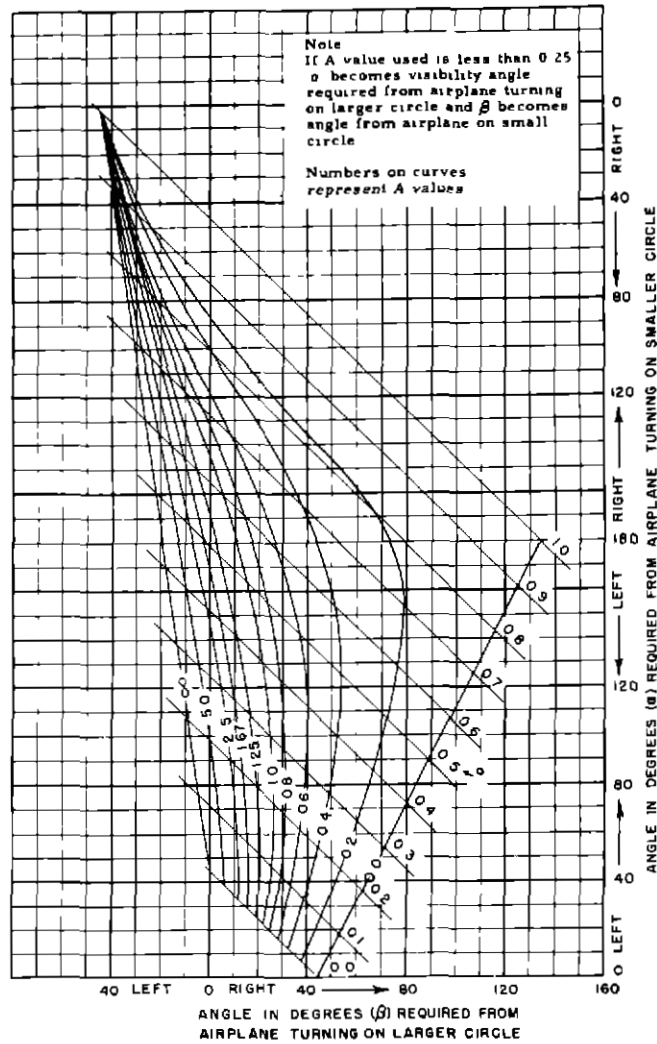


Z_0 Values

	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.2	972	1801	2412	2716	2709	2384	1787	1013	713	1496
0.4	1098	2007	2642	2926	2869	2494	1944	1290	2001	2912
0.6	1234	2247	2947	3268	3248	2951	2626	2073	3347	4367
0.8	1381	2513	3304	3704	3778	3627	3541	3899	4700	5822
< 1.0	1532	2797	3701	4208	4406	4422	4551	5074	6057	7277
1.25	1729	3172	4234	4899	5274	5502	5873	6611	7753	9097
1.67	2067	3825	5182	6143	6837	7414	8138	9191	10584	12129
2.5	2764	5191	7192	8795	10146	11388	12753	14374	16246	18194
5.0	4918	9491	13511	17135	20463	23608	26746	29968	33241	36388

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Fig 65 At Collision Point, the Difference in Heading = 135° , $R/A = 3.0$, and the Airplane Turning on the Larger Circle is on the Left of the Airplane Turning on the Smaller Circle

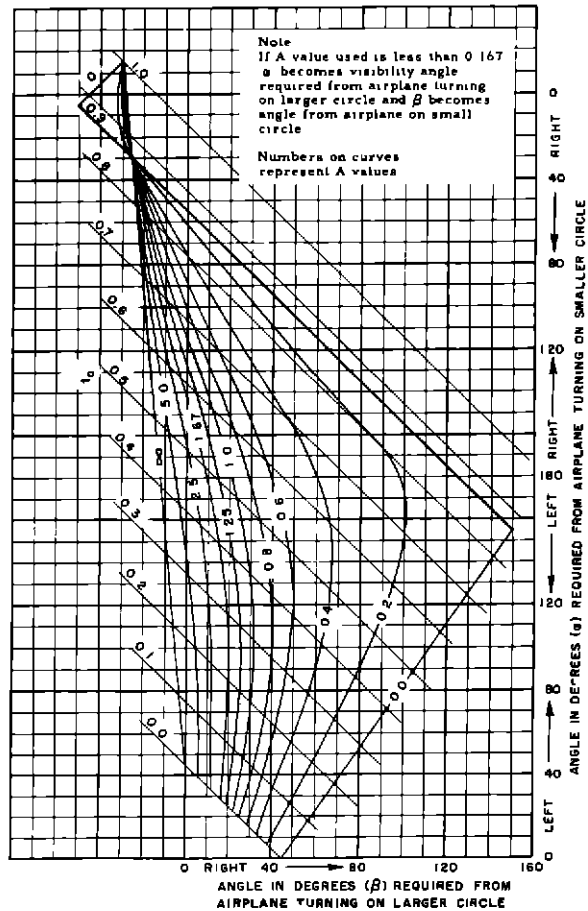


20 Values

	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.2	969	1787	2375	2647	2797	2825	1572	738	662	1584
0.4	1092	1979	2574	2799	2666	2206	1597	1364	2098	3169
0.6	1226	2211	2857	3102	2991	2619	2318	2599	3550	4754
0.8	1372	2471	3200	3517	3502	3304	3306	3881	5004	6338
1.0	1522	2751	3588	4009	4127	4128	4385	5181	6459	7923
1.25	1717	3122	4115	4696	5009	5255	5786	6815	8278	9903
1.67	2054	3771	5058	5946	6603	7240	8168	9548	11309	13204
2.5	2751	5135	7073	8627	9986	11358	12996	15027	17372	19807
5.0	4903	9403	13431	17075	20531	23965	27587	31483	35563	39613

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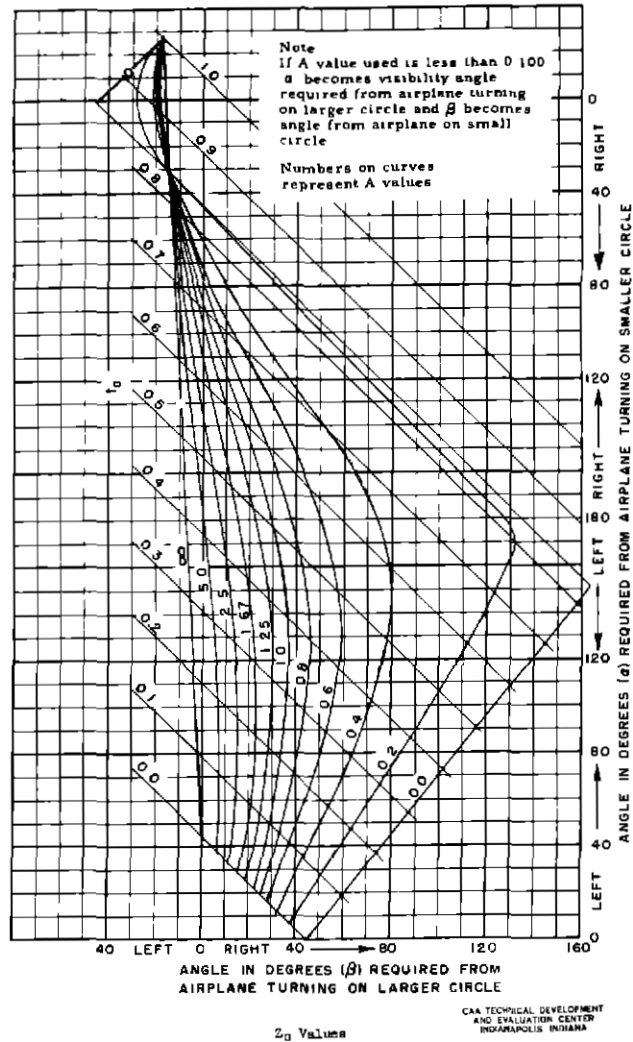
Fig 66 At Collision Point, the Difference in Heading = 135° , $R/A = 4.0$, and the Airplane Turning on the Larger Circle is on the Left of the Airplane Turning on the Smaller Circle



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A	Z ₀ Values									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.2	965	1770	2336	2774	2483	2064	1363	458	684	1681
0.4	1086	1990	2503	2664	2450	1894	1217	1229	221.0	3361
0.6	1219	2173	2762	2922	2711	2252	2003	2560	3735	5042
0.8	1361	2426	3090	3311	3196	2450	3073	3915	5261	6722
1.0	1512	2701	3465	3789	3819	3808	4216	5277	6788	8404
1.25	1705	3067	3984	4470	4709	4978	5681	6982	8696	10904
1.67	2042	3713	4922	5722	6332	7029	8135	9827	11875	14005
2.5	2737	5074	6938	8425	9779	11259	13145	15520	18234	21008
5.0	4888	9346	13322	16995	20494	24157	28175	32607	37309	42017

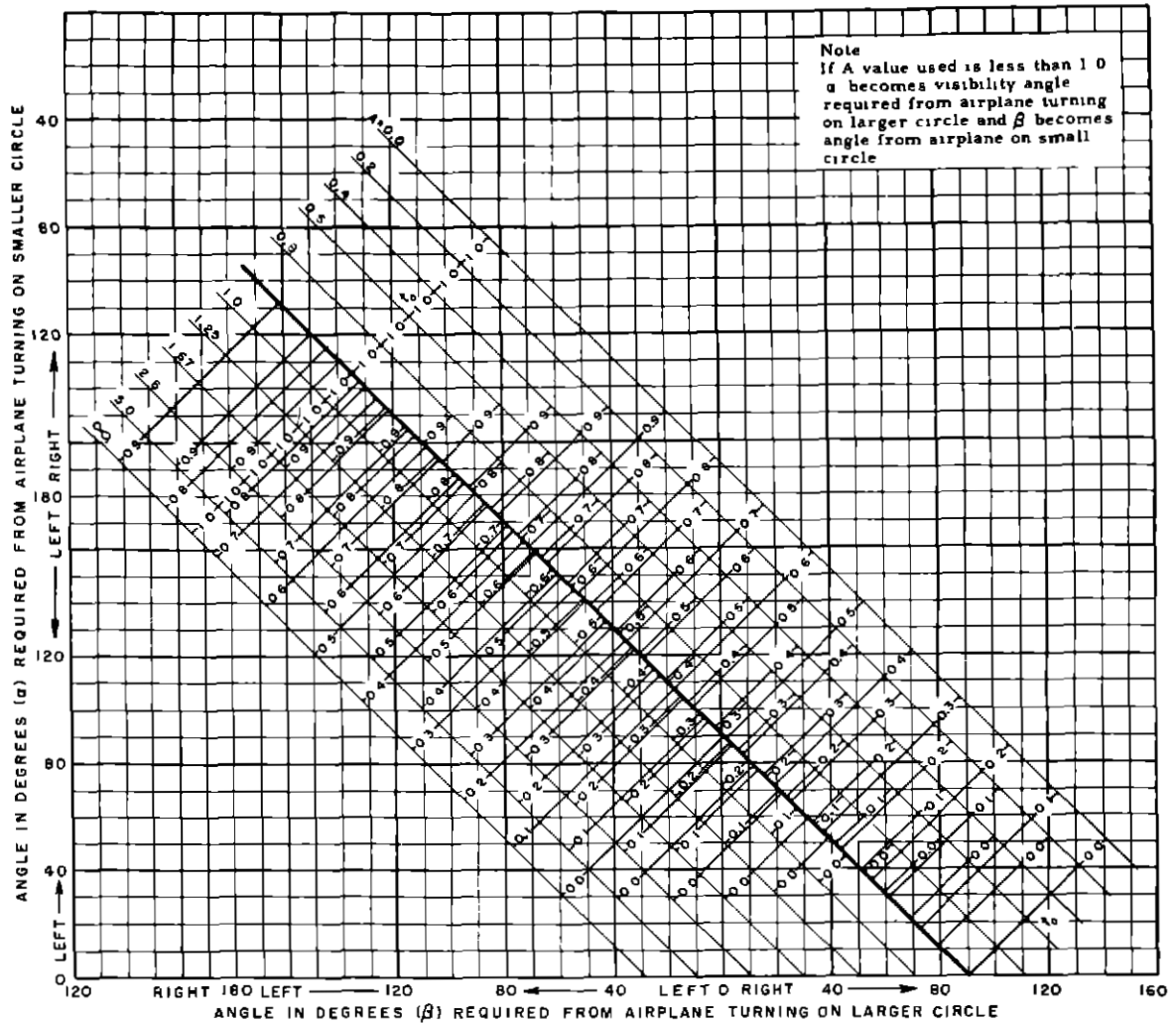
Fig 67 At Collision Point, the Difference in Heading = 135°, R/A = 6.0, and the Airplane Turning on the Larger Circle is on the Left of the Airplane Turning on the Smaller Circle



t_0 Values

	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.2	962	1758	2304	2516	2393	1945	1217	266	755	1731
0.4	1080	1927	2444	2551	2269	1632	899	1174	2305	3463
0.6	1212	2140	2681	2769	2471	1938	1768	2563	3866	5194
0.8	1353	2388	2994	3133	2931	2696	2904	3954	5429	6925
1.0	1503	2659	3360	3599	3552	3543	4090	5348	6991	8656
1.25	1696	3081	3870	4273	4451	4746	5594	7088	8945	10819
1.67	2031	3662	4804	5526	6094	6841	8119	9991	12200	14426
2.5	2725	5020	6818	8240	9581	11137	13195	15795	18712	21640
5.0	4876	9893	13215	16815	20190	24194	28457	33210	38249	43280

Fig 68 At Collision Point, the Difference in Heading = 135° , $R/A = 10.0$, and the Airplane Turning on the Larger Circle is on the Left of the Airplane Turning on the Smaller Circle



Z_0 Values

		t_0									
		0 1	0 2	0 3	0 4	0 5	0 6	0 7	0 8	0 9	1 0
A	0 2	883	1679	2320	2720	2857	2717	2311	1657	858	0
	0 4	932	1774	2446	2867	3017	2869	2440	1750	908	0
	0 6	1009	1921	2645	3102	3267	3107	2643	1896	986	0
	0 8	1109	2109	2901	3403	3587	3411	2902	2084	1086	0
	1 0	1225	2328	3202	3757	3961	3768	3205	2305	1203	0
	1 25	1386	2636	3621	4251	4484	4265	3628	2612	1366	0
	1 67	1683	3200	4395	5162	5444	5178	4405	3178	1665	0
	2 5	2331	4434	6089	7154	7542	7174	6102	4413	2315	0
	5 0	4414	8395	11539	13563	14283	13584	11555	8378	4402	0

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Fig 69 At Collision Point, the Difference in Heading = 90° , $R/A = 1.0$, and the Airplane Turning on the Larger Circle is on the Left of the Airplane Turning on the Smaller Circle

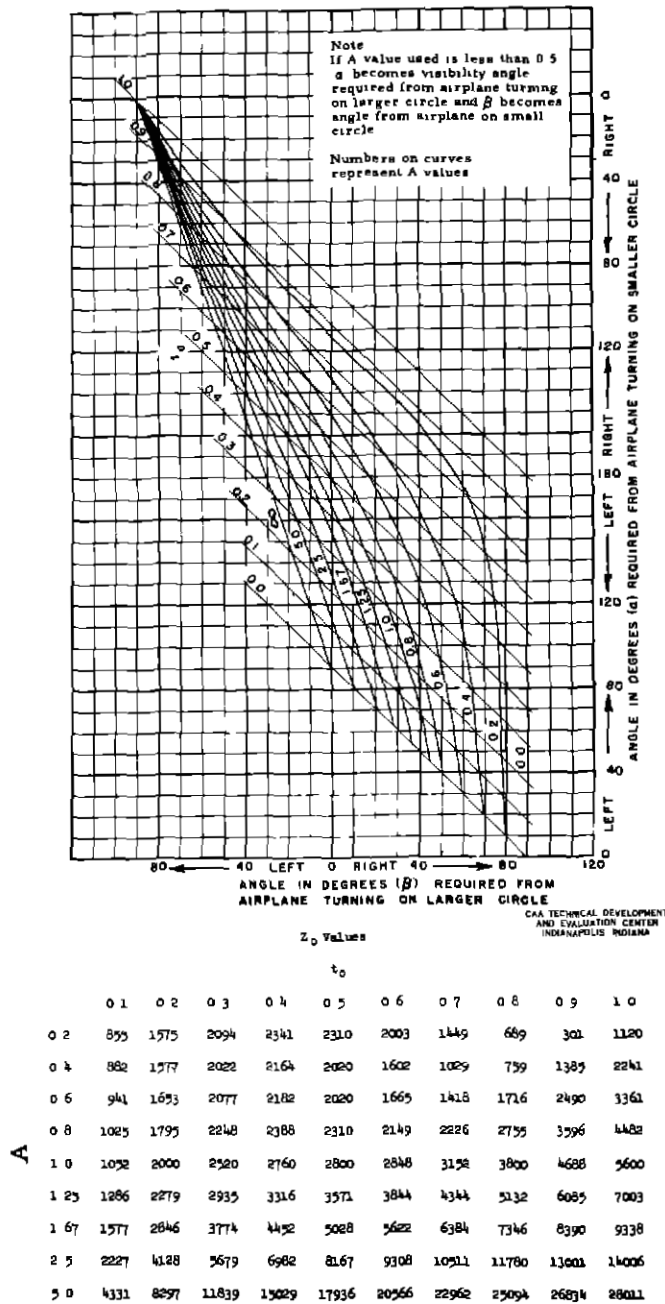


Fig 70 At Collision Point, the Difference in Heading = 90° , $R/A = 2.0$, and the Airplane Turning on the Larger Circle is on the Left of the Airplane Turning on the Smaller Circle

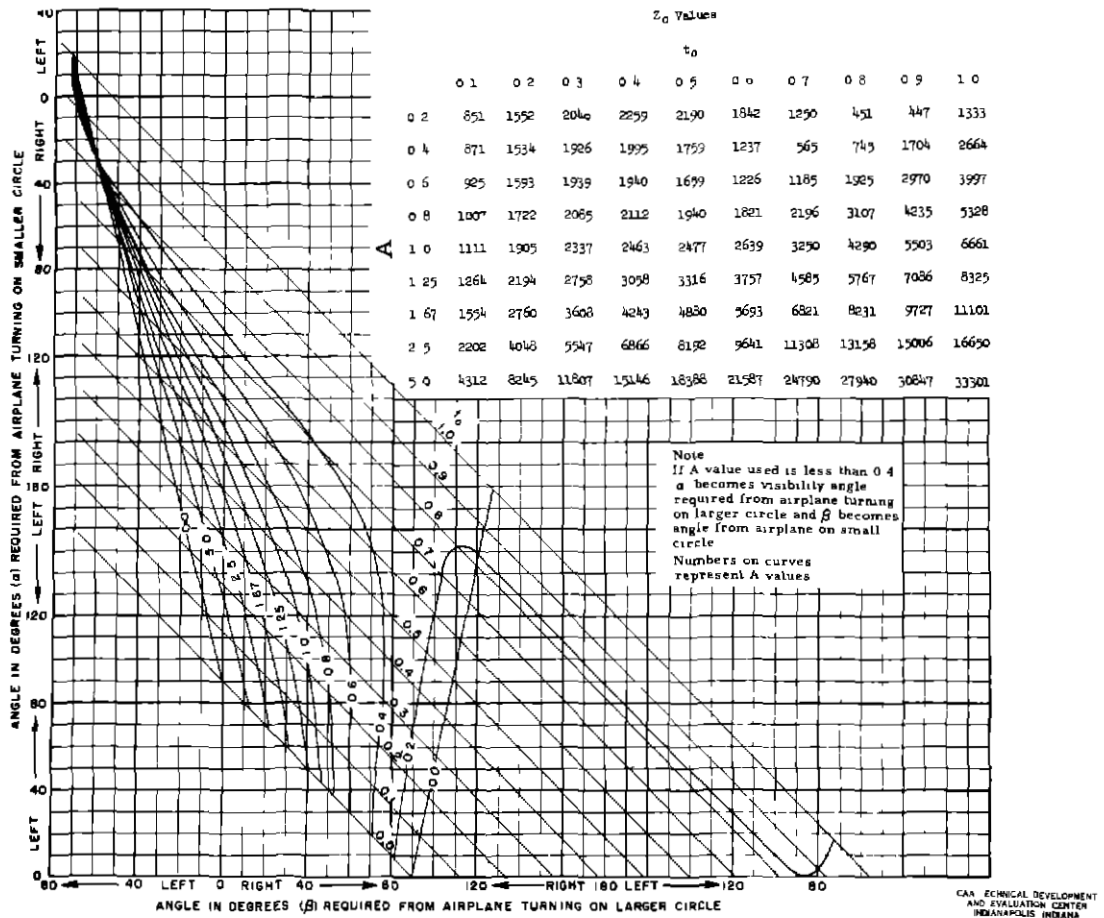


Fig 71 At Collision Point, the Difference in Heading = 90° , $R/A = 2.5$, and the Airplane Turning on the Larger Circle is on the Left of the Airplane Turning on the Smaller Circle

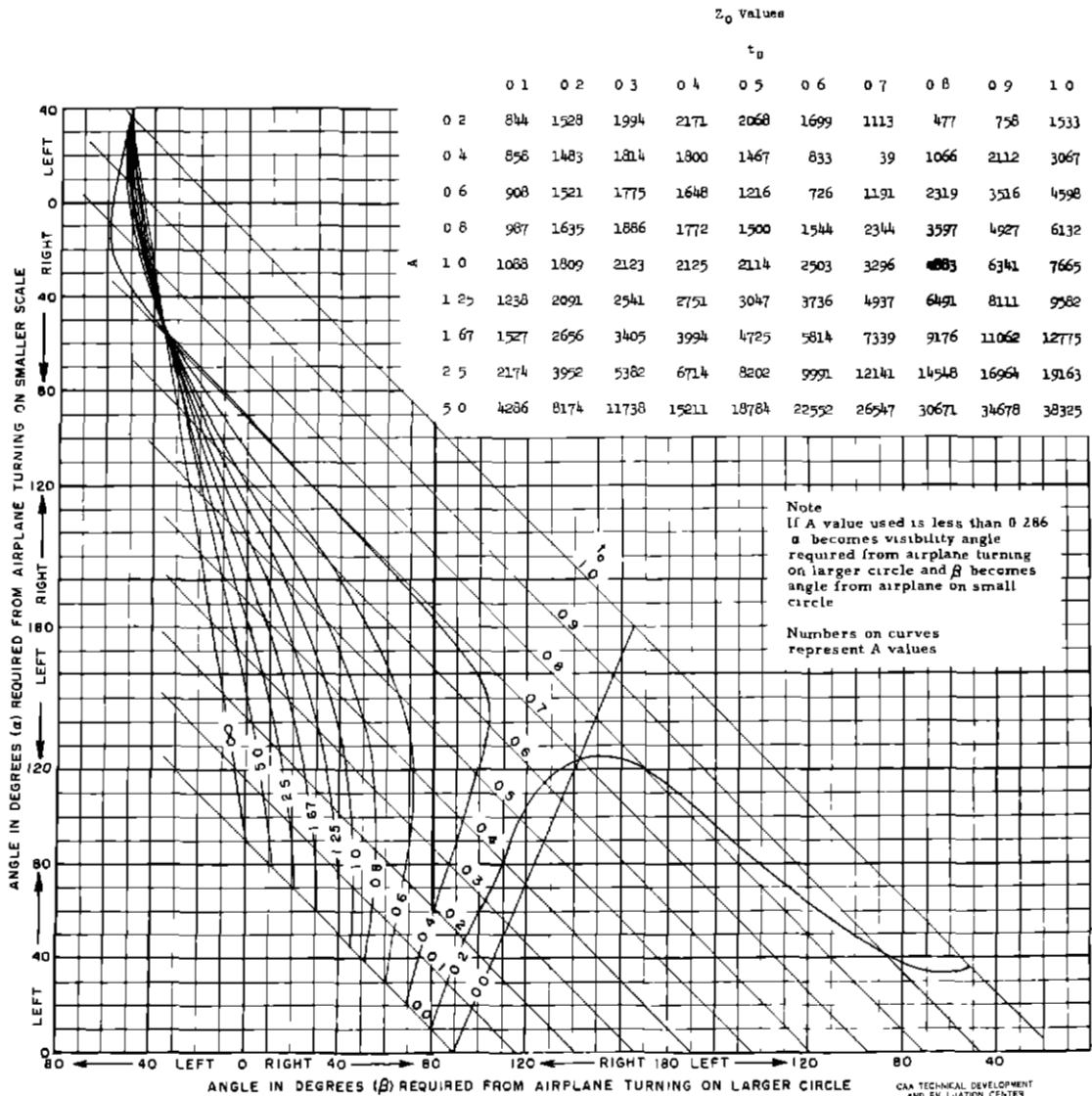


Fig 72 At Collision Point, the Difference in Heading = 90° , $R/A = 3.5$, and the Airplane Turning on the Larger Circle is on the Left of the Airplane Turning on the Smaller Circle

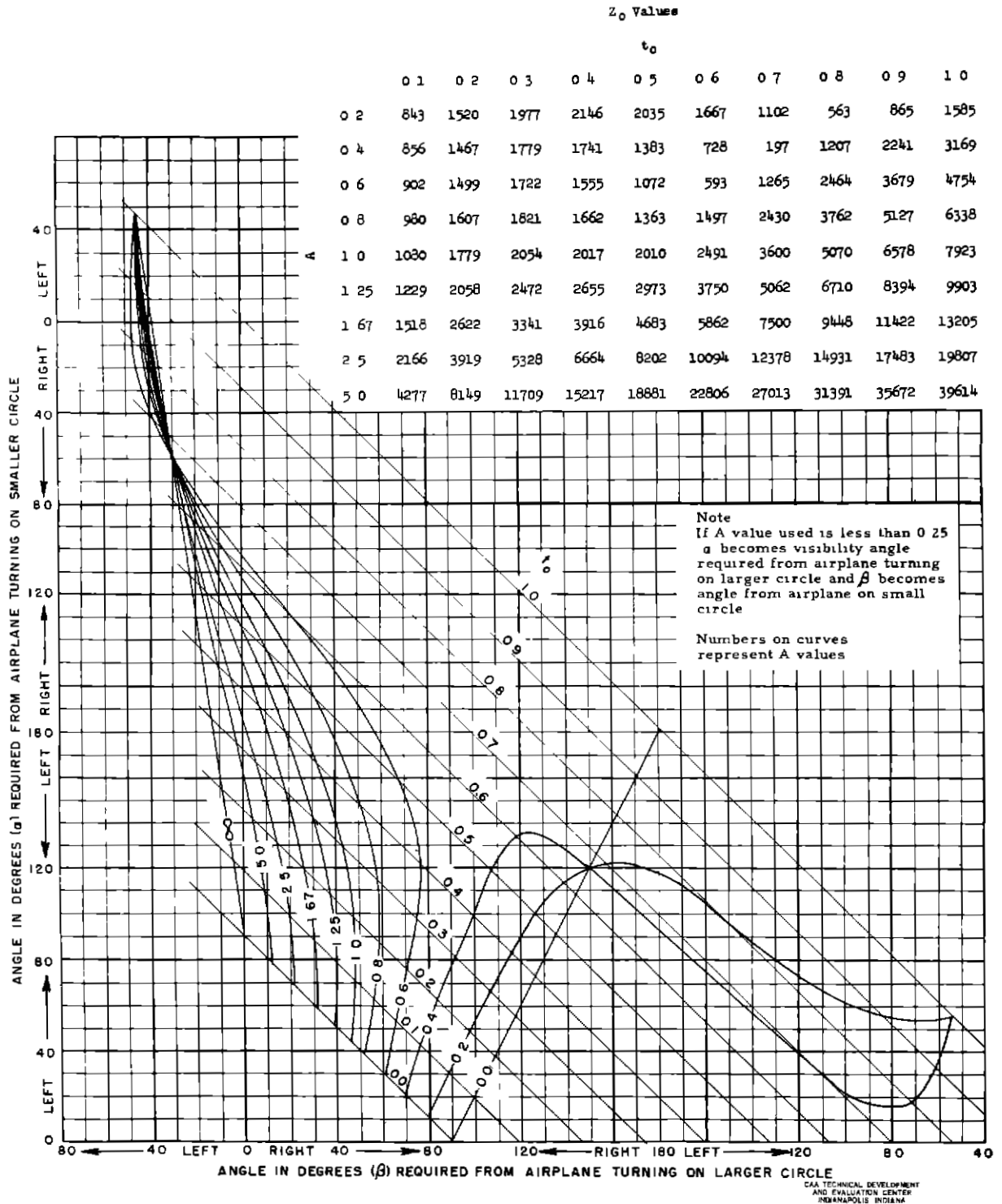


Fig 73 At Collision Point, the Difference in Heading = 90° , $R/A = 4.0$, and the Airplane Turning on the Larger Circle is on the Left of the Airplane Turning on the Smaller Circle

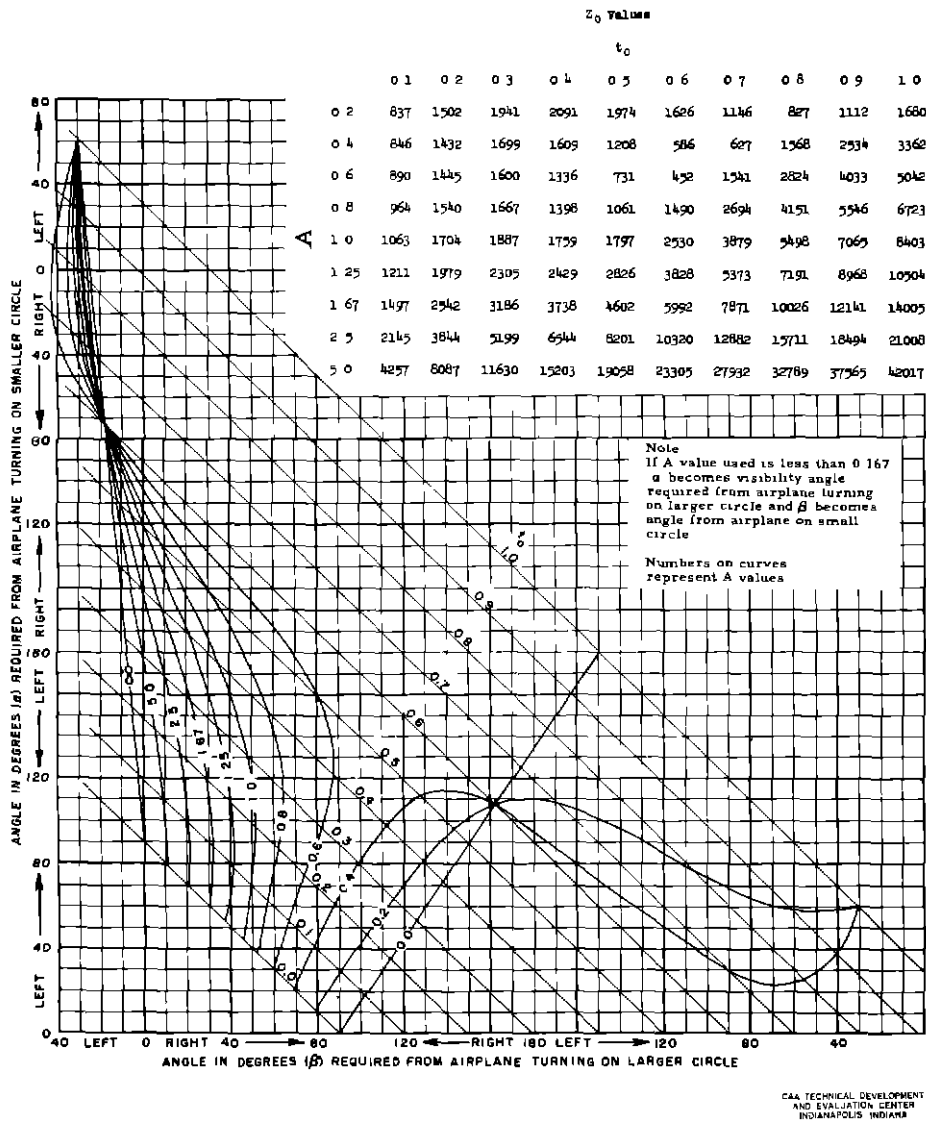


Fig 74 At Collision Point, the Difference in Heading = 90° , $R/A = 6.0$, and the Airplane Turning on the Larger Circle is on the Left of the Airplane Turning on the Smaller Circle

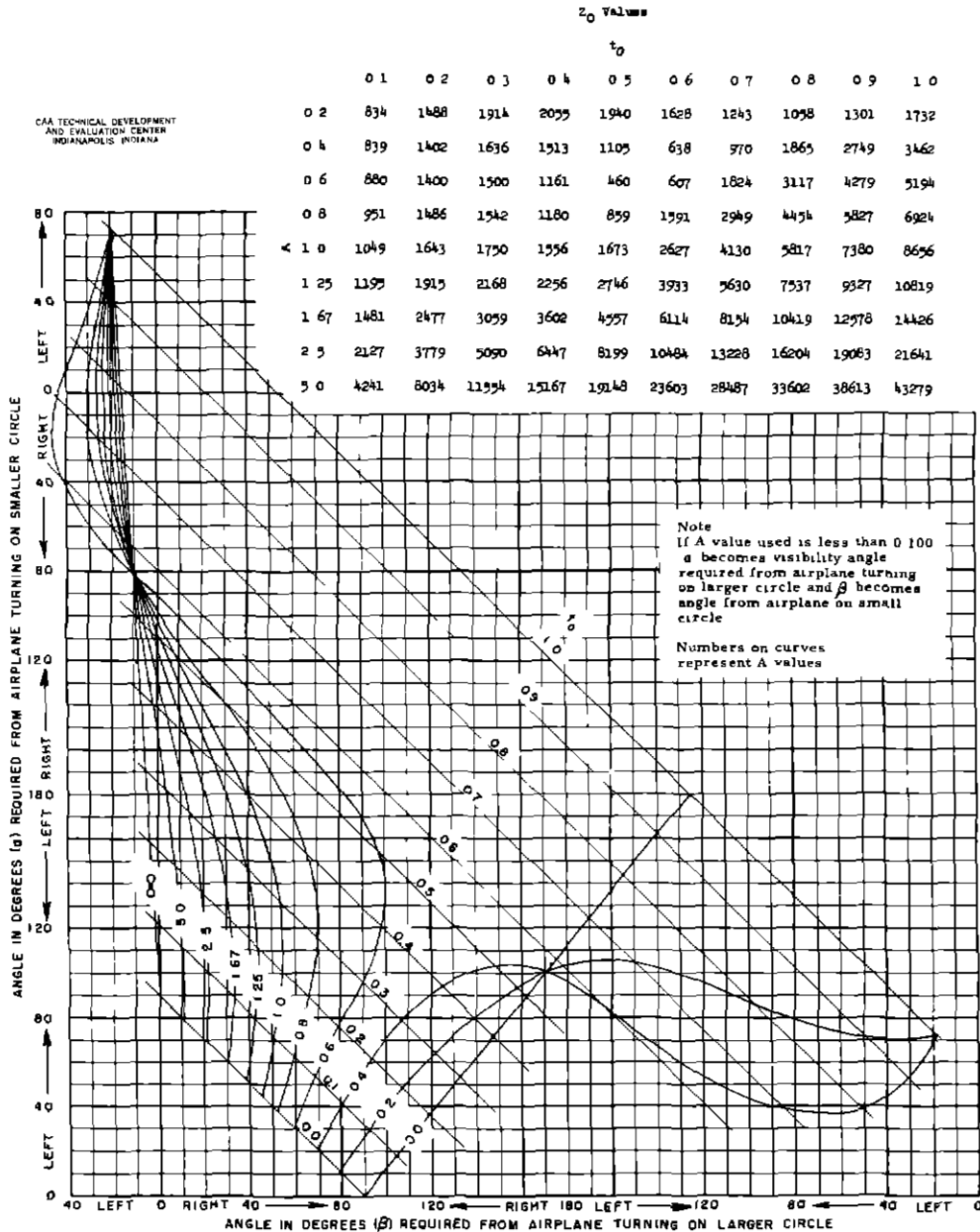


Fig 75 At Collision Point, the Difference in Heading = 90° , $R/A = 10.0$, and the Airplane Turning on the Larger Circle is on the Left of the Airplane Turning on the Smaller Circle

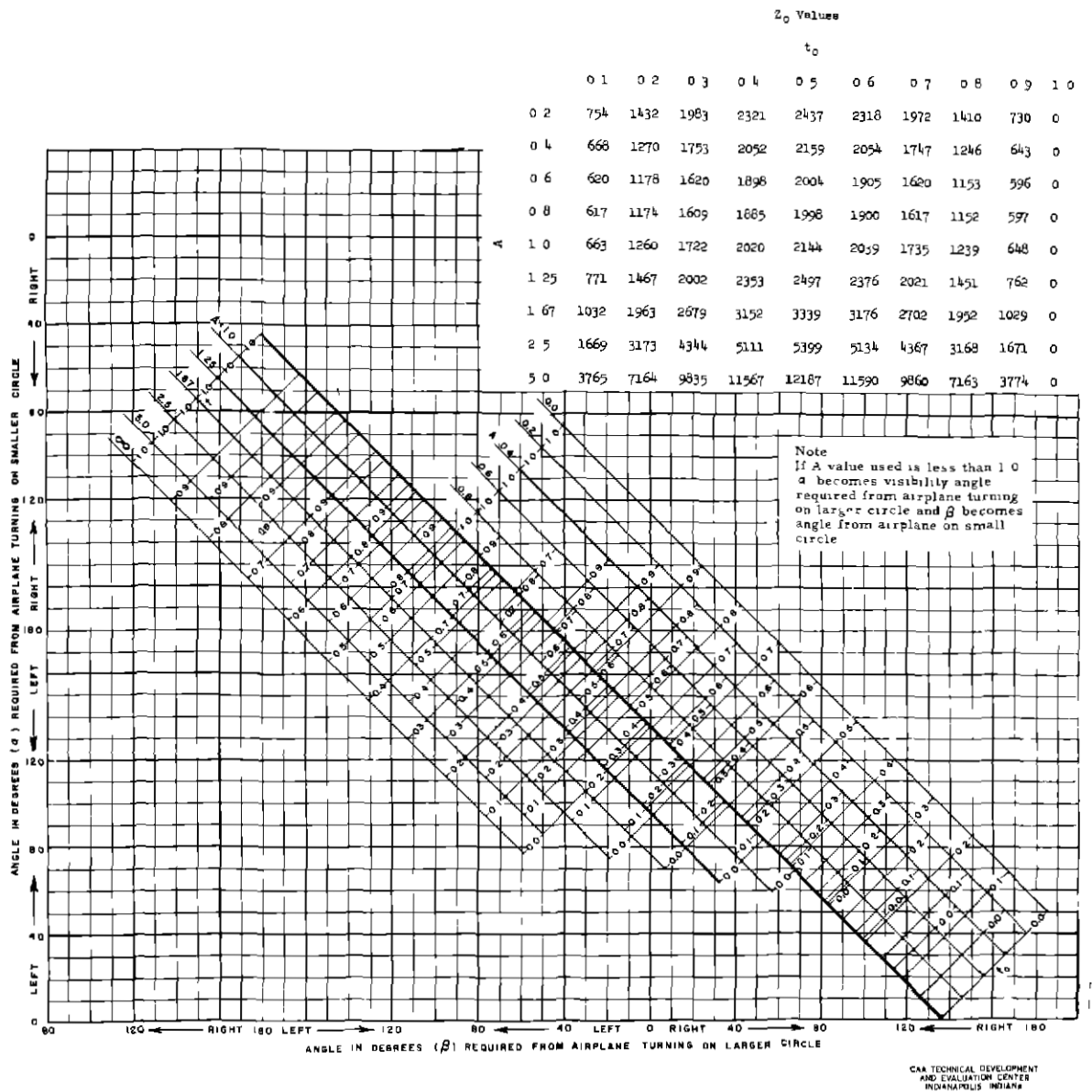


Fig 76 At Collision Point, the Difference in Heading = 45° , $R/A = 1.0$, and the Airplane Turning on the Larger Circle is on the Left of the Airplane Turning on the Smaller Circle

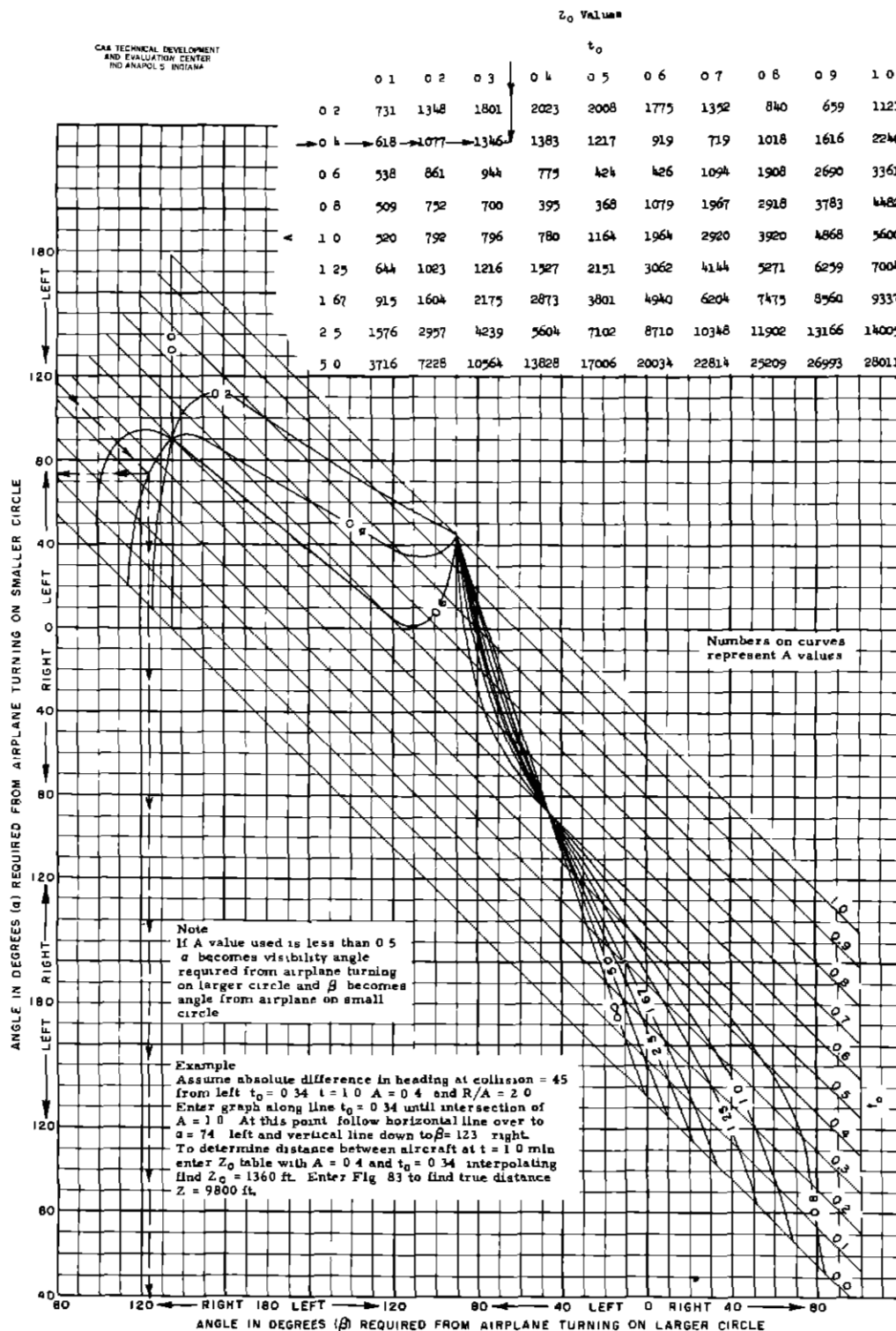


Fig. 77 At Collision Point, the Difference in Heading = 45°, $R/A = 2.0$, and the Airplane Turning on the Larger Circle is on the Left of the Airplane Turning on the Smaller Circle

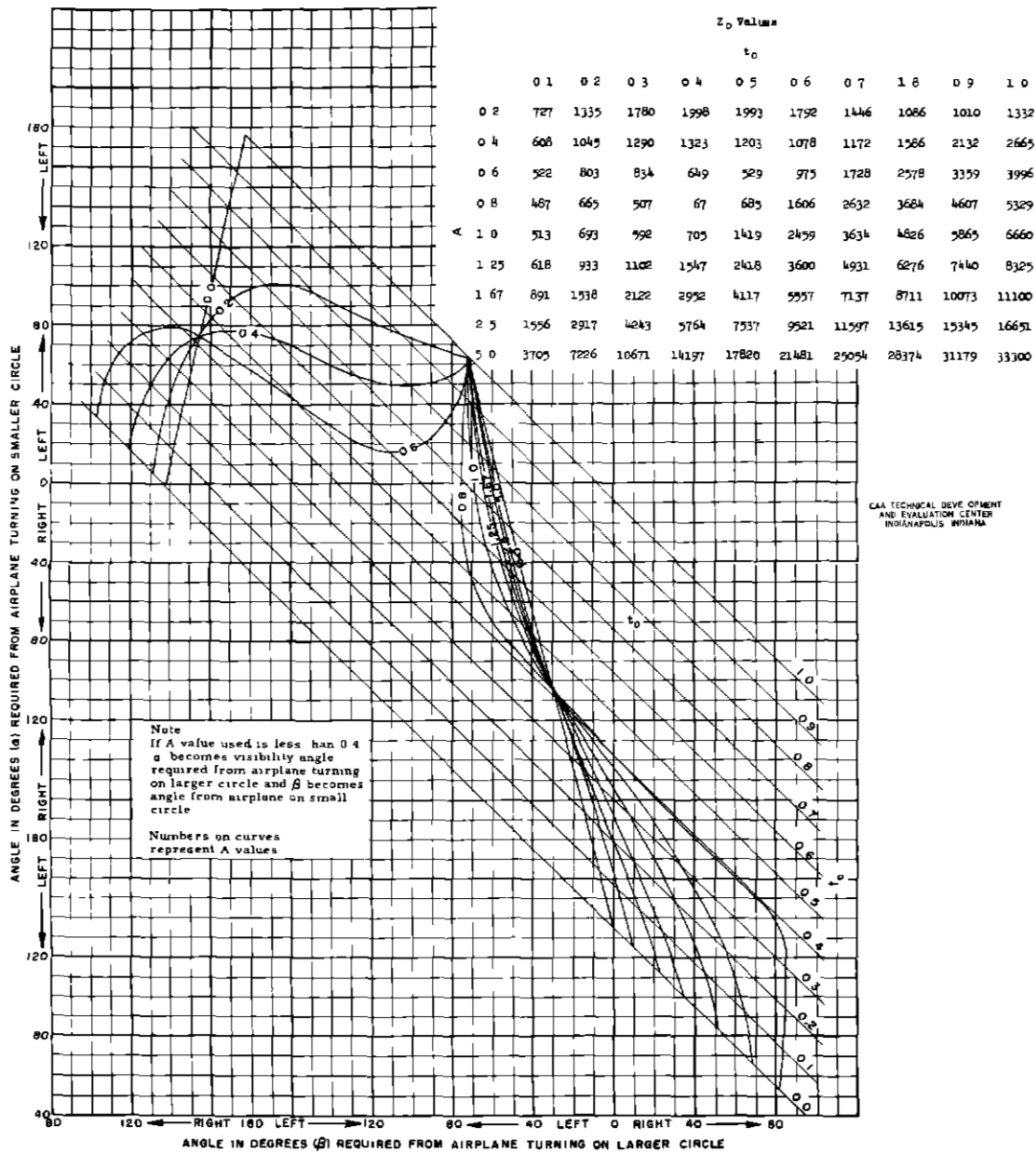


Fig 78 At Collision Point, the Difference in Heading = 45° , $R/A = 2.5$, and the Airplane Turning on the Larger Circle is on the Left of the Airplane Turning on the Smaller Circle

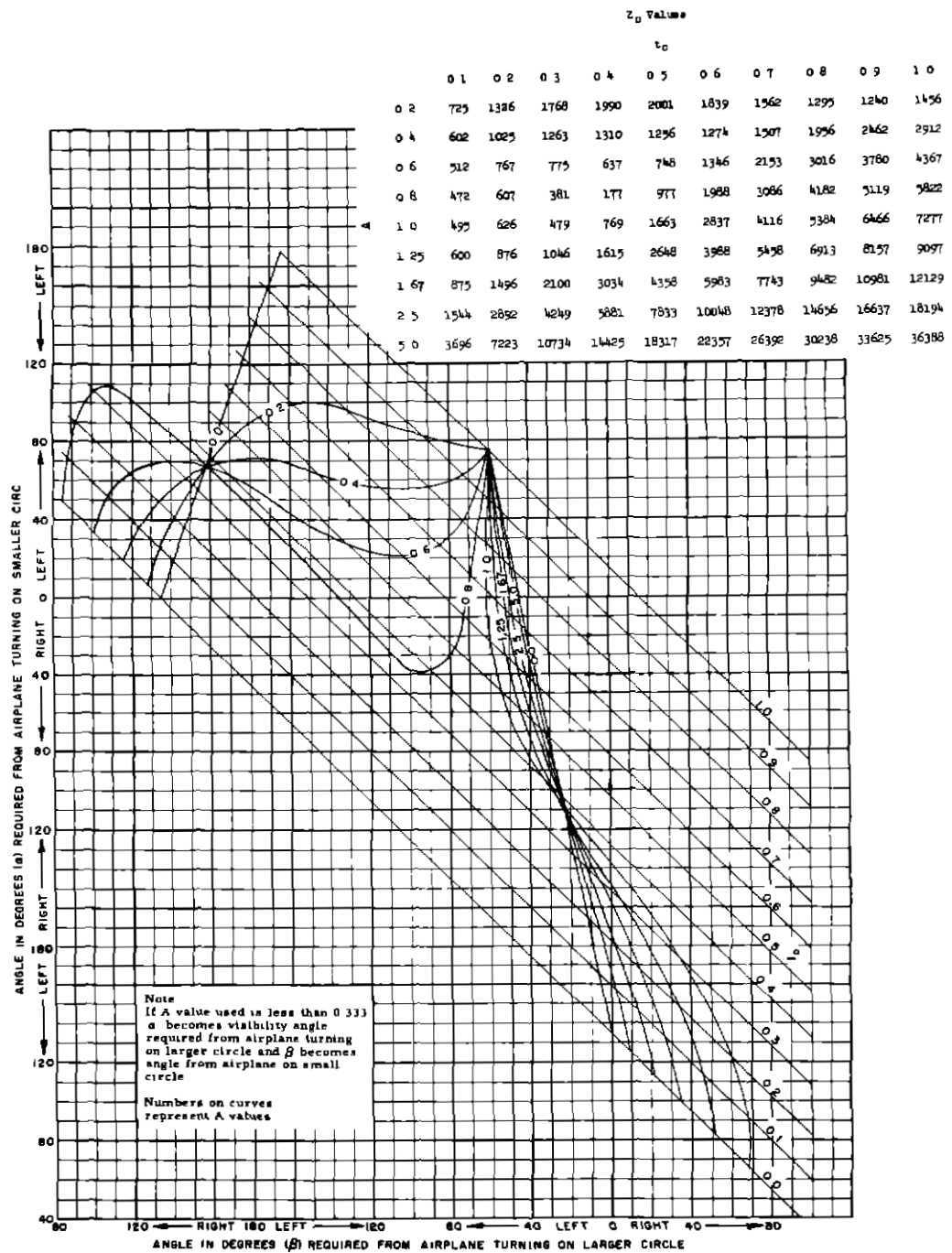


Fig 79 At Collision Point, the Difference in Heading = 45° , $R/A = 3.0$, and the Airplane Turning on the Larger Circle is on the Left of the Airplane Turning on the Smaller Circle

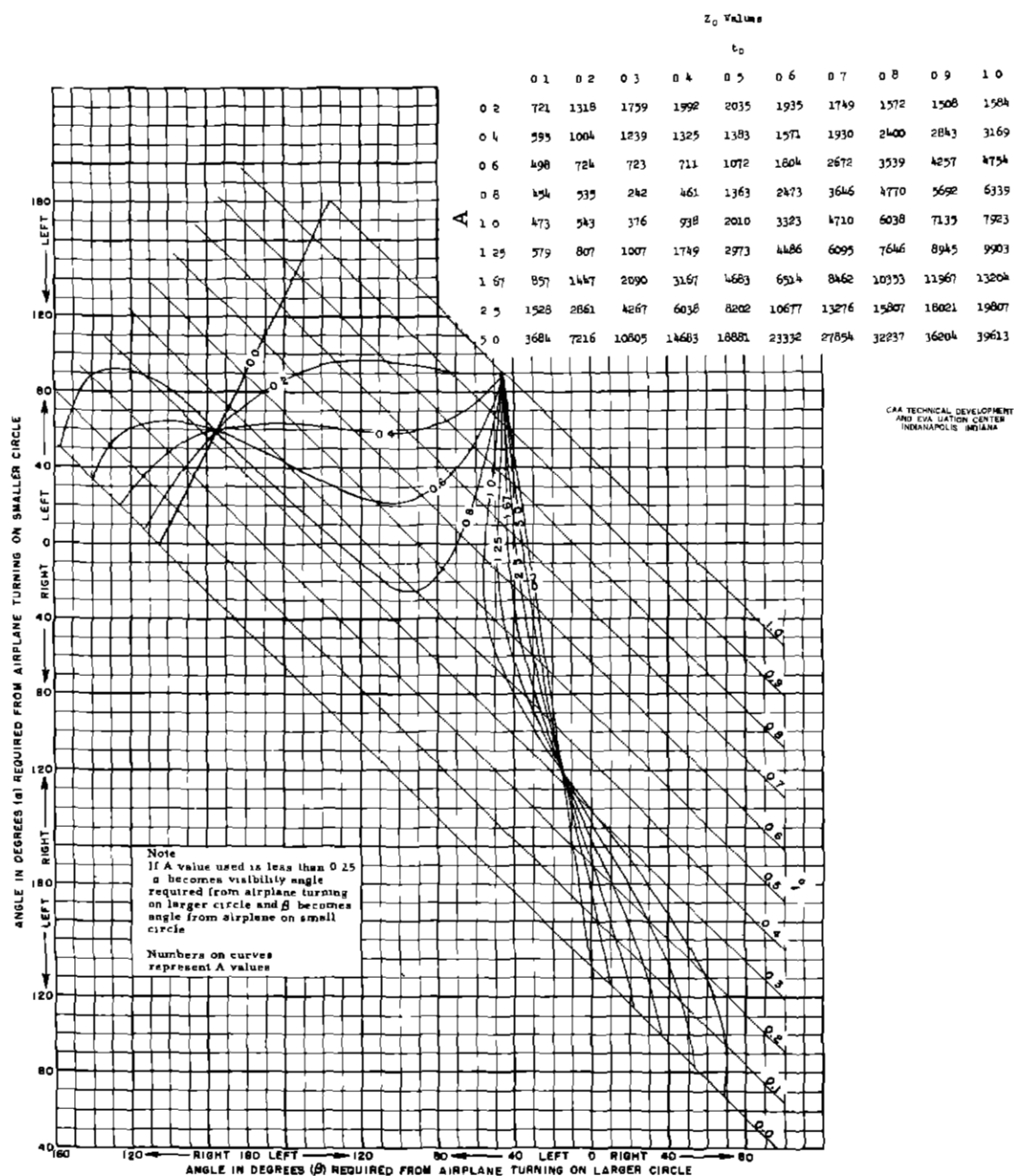


Fig 80 At Collision Point, the Difference in Heading = 45°, R/A = 4.0, and the Airplane Turning on the Larger Circle is on the Left of the Airplane Turning on the Smaller Circle

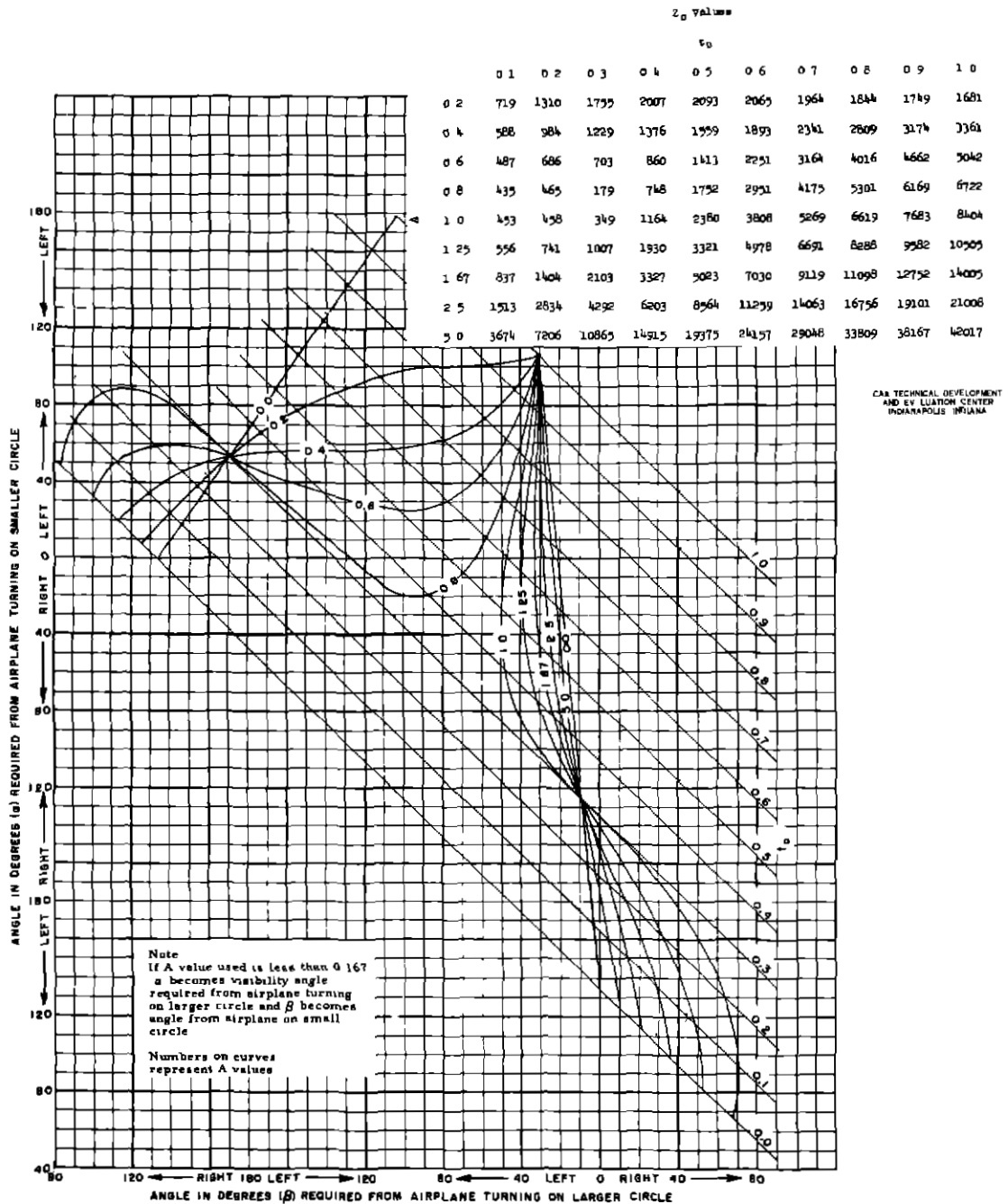


Fig 81 At Collision Point, the Difference in Heading = 45° , $R/A = 6.0$, and the Airplane Turning on the Larger Circle is on the Left of the Airplane Turning on the Smaller Circle

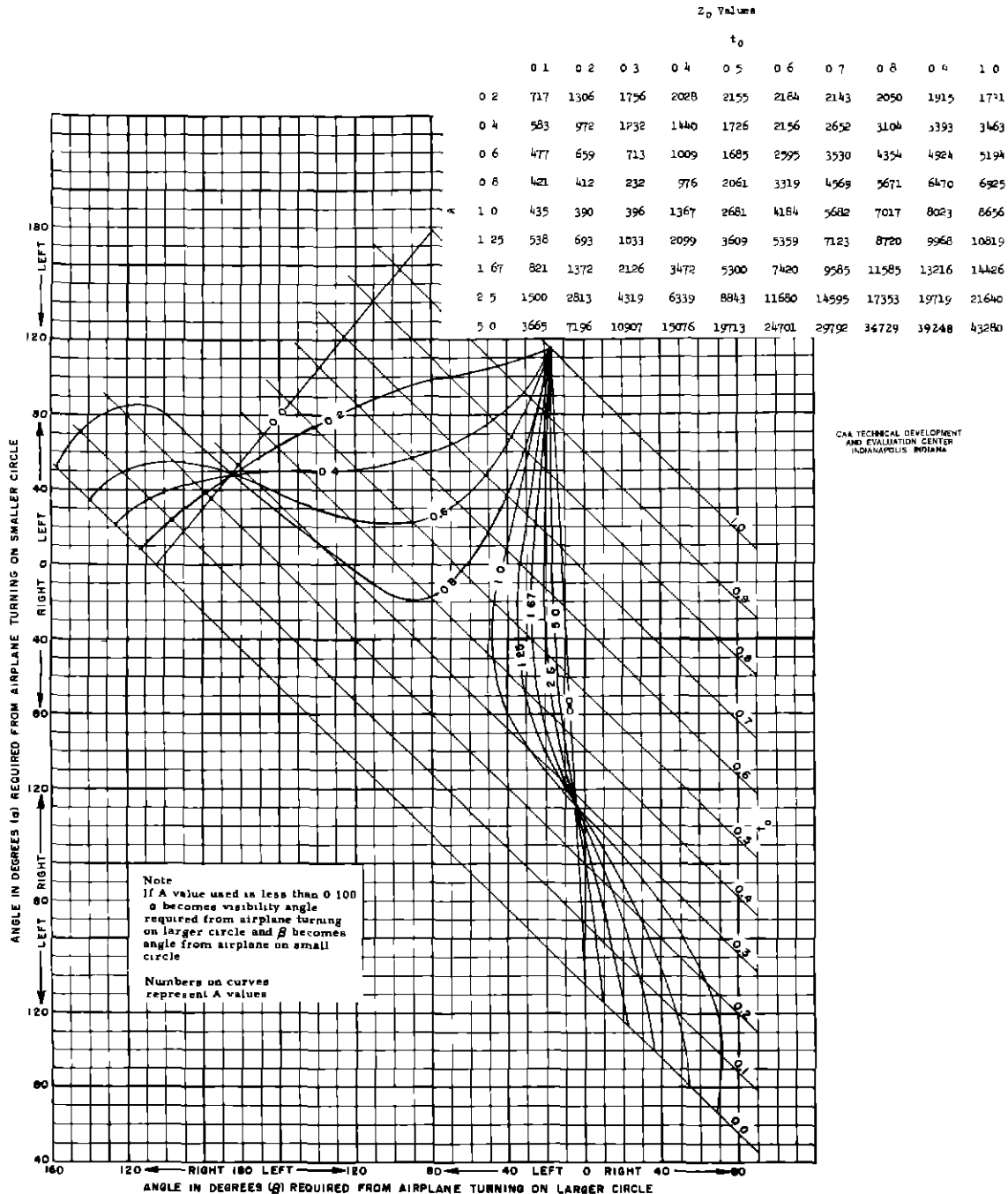
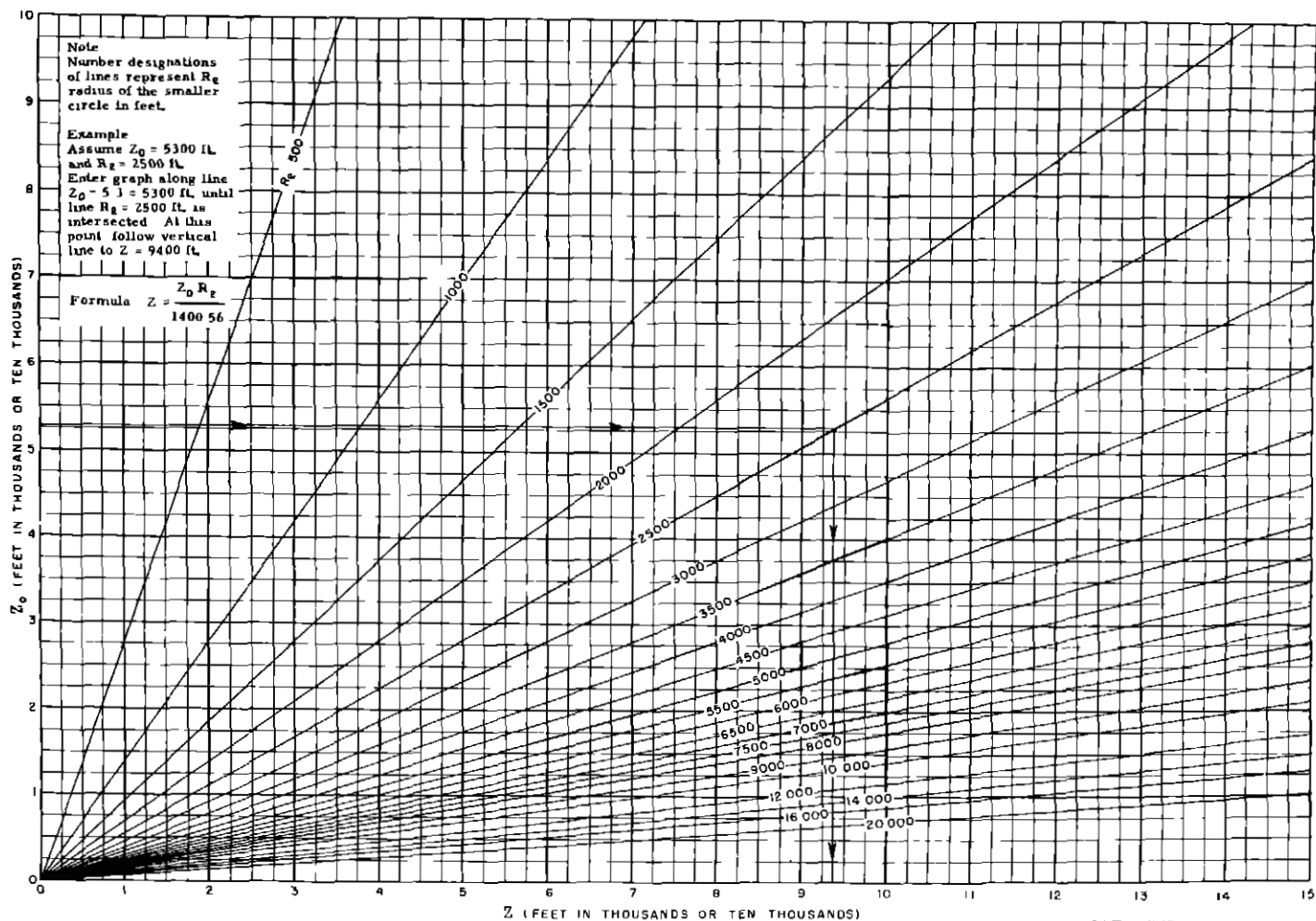


Fig 82 At Collision Point, the Difference in Heading = 45° , $R/A = 10.0$, and the Airplane Turning on the Larger Circle is on the Left of the Airplane Turning on the Smaller Circle



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Fig 83 Determination of Distance of Separation for Two Airplanes Turning

CONCLUSIONS

1 It is apparent from studying the visual-angle curves that, to prevent all mid-air collisions, it is necessary for a pilot to be able to see or detect the approach of aircraft from any direction. Since this cannot be accomplished by adding windshield area alone, it must be done by other means.

2 Since the visual angle does not change during Collision Conditions A and B, there is no apparent motion of the other aircraft to be observed by either pilot. This is true only if both aircraft are flying on straight courses and at constant speeds. The closing speeds also remain constant.

3 This study has pointed out that a severe collision hazard exists where aircraft are flying in the same general direction and at approximately the same speeds. It must also be pointed out, however, that during this critical condition the closing rate is small and, if an auxiliary means of viewing this area were available, the pilots would have sufficient time to execute evasive maneuvers in time to avoid collision.

4 In the turning collision conditions, both the visual angles and the closing rate are constantly varying. Under these circumstances, there is apparent relative motion between the aircraft which will tend to increase the noticeability in the peripheral field of vision. When the airplane is turning, however, it is possible for the cockpit structure to completely block out the horizon and, consequently, to block out another aircraft at the same altitude even though the area was cleared as carefully as possible prior to the turn. It is noteworthy that maximum collision protection can be obtained in present-day airplanes only if both pilots clear the area to their rear prior to a turn in either direction.

5 In the event of a mid-air collision, the scratch angle or the angle of physical marks on both damaged aircraft, when measured from the longitudinal axis, are the same as the visual-angle requirements of each aircraft to see the other immediately prior to the collision. In making this statement, it is assumed that neither aircraft is deflected or bumped from its original heading during the collision impact.

ACKNOWLEDGEMENTS

Acknowledgement is made to the United States Naval Ordnance Plant, Numerical Analysis Branch, Indianapolis, Indiana, who made the visual-angle curves for Collision Condition C possible by computing the values on their digital electronic computers. Also thanks are due the airport managers and the CAA air traffic controllers at Municipal Airport, Birmingham, Alabama, Weir Cook Municipal Airport, Indianapolis, Indiana, and the Midway Airport, Chicago, Illinois, who co-operated wholeheartedly during flight-path measurements recorded at those airports.

APPENDIX I

DERIVATION OF FORMULAS FOR COLLISION CONDITION A

For this condition, two airplanes are assumed to be on a collision course, flying on straight courses in the same horizontal plane, and approaching each other from any direction at any relative speed. Figure 84 shows the physical significance of the horizontal visual angles and distances involved.

Using the law of sines

$$\frac{V_s}{\sin \theta} = \frac{V_c}{\sin \psi} \quad (31)$$

Solving for V_c from the law of cosines

$$V_c = \sqrt{V_s^2 + V_f^2 - 2 V_s V_f \cos \psi} \quad (32)$$

Substituting Equation (32) in Equation (31)

$$\sin \theta = \frac{V_s \sin \psi}{\sqrt{V_s^2 + V_f^2 - 2 V_s V_f \cos \psi}} \quad (33)$$

Since $A = \frac{V_s}{V_f}$, Equation (33) may be written

$$\theta = \arcsin \frac{A \sin \psi}{\sqrt{A^2 + 1 - 2A \cos \psi}} \quad (2)$$

From the geometry of Fig 84,

$$\phi = 180^\circ - (\psi + \theta)$$

Solving for the distance between the airplanes at time t minutes before collision, we have, using the law of sines,

$$Z = \frac{V_s t \sin \psi}{60 \sin \theta} \quad (3)$$

where 60 is the factor for changing mph to mpm

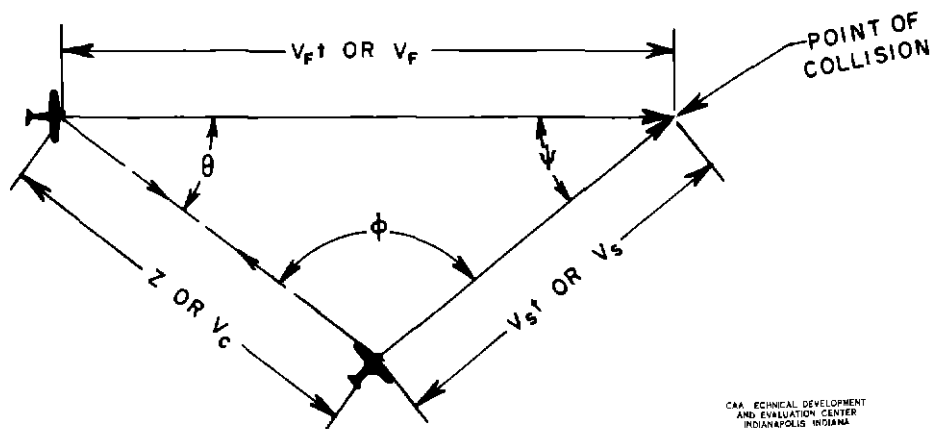


Fig 84 Vector Diagram for Derivation of Formulas for Level-Flight Collision Condition

DERIVATION OF FORMULAS FOR COLLISION CONDITION B

For this condition, two airplanes are assumed to be on a collision course with one airplane climbing and one airplane descending, with both airplanes climbing, or with both airplanes descending, also, one airplane can be flying at level cruise and the other one either climbing or descending. Figure 85 shows the physical significance of the visual angles and of the distances involved in this condition.

In order to use the visual-angle chart designed for Condition A in determining the horizontal visual angles required from the climbing and descending airplanes, V_{CLH} , the vertical projection of the climbing velocity onto the horizontal plane passing through the collision point, and V_{DH} , the vertical projection of the descending velocity onto the horizontal plane passing through the collision point, should be determined, however, for simplicity the values of V_{CL} and V_D are used. In using V_{CL} and V_D instead of V_{CLH} and V_{DH} , very little error is introduced into the visual angles obtained since the angles at which airplanes climb and descend are relatively small, and small climb and descent angles make the difference very small between V_{CL} and V_{CLH} or V_D and V_{DH} . Consequently, V_{CL} and V_D , either of which can be entered as slower or faster, and ψ , which is the absolute difference in heading, can be used in Fig 2 for determining the horizontal visual angles with a relative degree of accuracy.

Referring to Fig 85 and using the law of sines

$$V = \frac{V_s \sin \psi}{\sin \theta} \quad (4)$$

V_s , the velocity of the slower airplane, is V_D in the figure

Solving for ρ , the vertical angle measured from the horizon and required from either airplane,

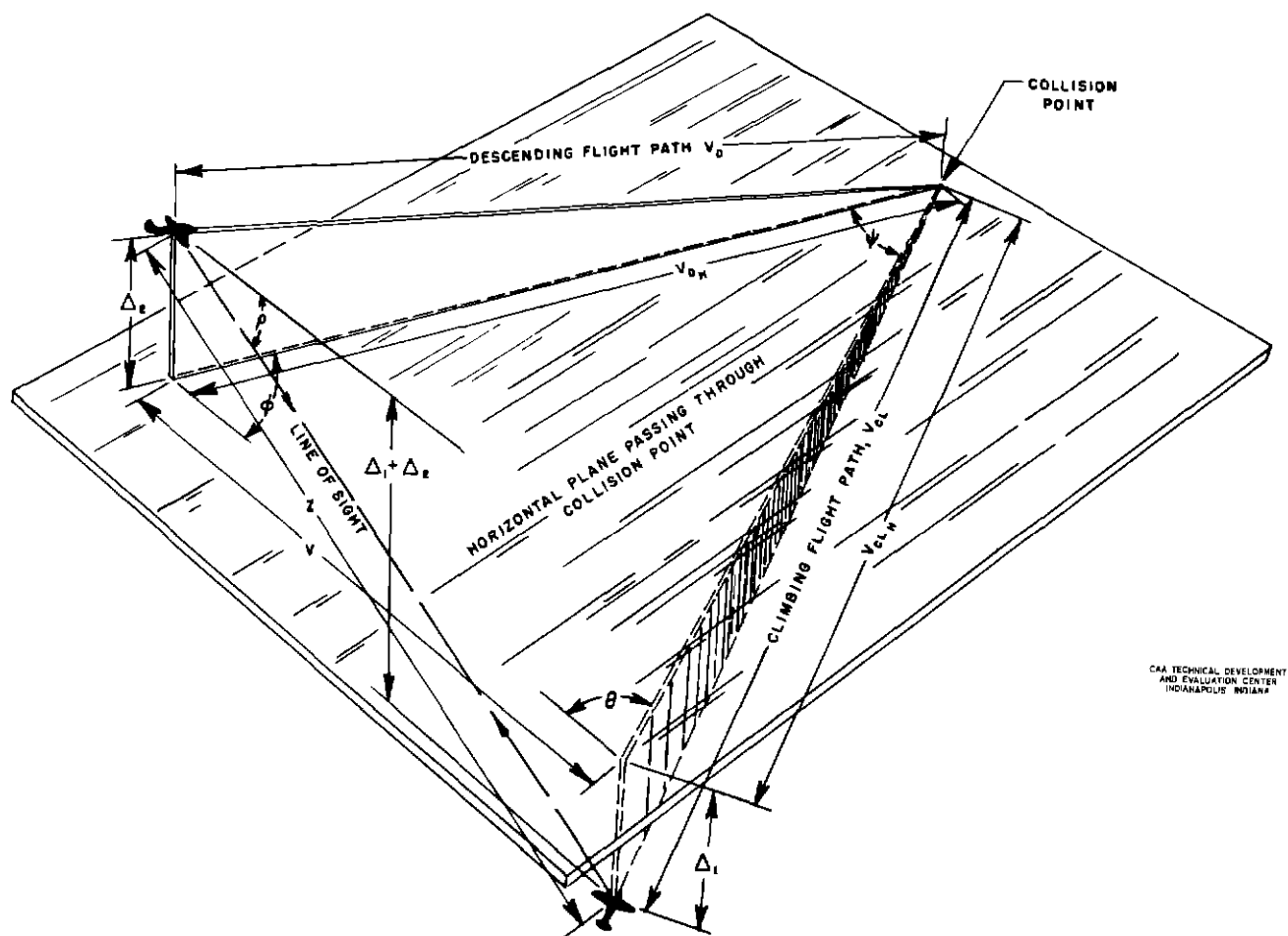
$$\rho = \arctan \frac{\Delta_1 \pm \Delta_2}{88V} \quad (5)$$

If both airplanes are descending or climbing, then Δ_1 and Δ_2 are added, and if one is descending and one is climbing, then the difference between Δ_1 and Δ_2 is used.

Solving for Z , the distance between the airplanes at time t measured along the line of sight,

$$Z = \frac{Vt}{60 \cos \rho} \quad (6)$$

The factor 60 is used to change mph to mpm.



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Fig. 85 Vector Diagram for Derivation of Formulas for Climb-and-Descent Conditions

DERIVATION OF FORMULAS FOR COLLISION CONDITION C

For this condition, two airplanes are assumed to be on a collision course, one flying straight and level (cruising) and the other making a left turn. Figure 86 shows the physical significance of the horizontal visual angles and of the distances involved in this condition. Horizontal visual angles are measured from straight ahead of the pilot at the time of investigation.

Locating the center of the circle of the turning airplane with respect to the cruising airplane (the origin of the X and Y axis)

$$a = R_T \sin \psi \quad (34)$$

Distance traveled, from time t to collision,

$$D = V_c t$$

$$E = R_T \cos \psi$$

since

$$b = D + E$$

Then

$$b = V_c t + R_T \cos \psi \quad (35)$$

If K = rate of turn, in degrees per minute,

Then

$$\theta = Kt$$

Also

$$\theta = \psi + \omega$$

Therefore

$$\omega = Kt - \psi \quad (14)$$

The co-ordinates of the turning airplane with respect to the cruising airplane at time t before collision are then given by,

$$X = b - X_1$$

Also

$$X_1 = R_T \cos \omega$$

Substituting for b , Equation (35), and X_1

$$X = V_c t + R_T(\cos \psi - \cos \omega) \quad (12)$$

$$Y = a + Y_1$$

Also

$$Y_1 = R_T \sin \omega$$

Substituting for a , Equation (34), and Y_1

$$Y = R_T(\sin \psi + \sin \omega) \quad (13)$$

Solving for the horizontal visual angle α , measured from straight ahead of the cruising airplane,

$$\alpha = \arctan \frac{Y}{X} \quad (9)$$

Substituting in Equation (12) and Equation (13)

$$\alpha = \arctan \frac{R_T(\sin \psi + \sin \omega)}{V_c t + R_T(\cos \psi - \cos \omega)} \quad (36)$$

From the geometry of Fig 86 the horizontal visual angle β , measured from straight ahead of the turning airplane, is

$$\beta + (90^\circ - \alpha) + (90^\circ - \omega) = 90^\circ$$

Simplifying

$$\beta = \alpha + \omega - 90^\circ \quad (10)$$

Distance between airplanes at time t is

$$Z = \sqrt{X^2 + Y^2} \quad (11)$$

After developing the formulas, it was noted that several visual-angle charts would be required to represent all possible combinations that could occur between the variables involved in this condition. To construct a minimum number of charts that represent all possible combinations of the variables, the law of geometric similarity was applied. Figures 87 and 88 are geometrically

similar. Therefore, the corresponding angles are equal and the corresponding sides are proportional.

The charts for Condition C were developed with the use of assumed specific values of V_{T0} , V_{C0} , t_0 , R_{T0} , and ψ_0 . These parameters are shown in Fig 87. The actual case to be investigated is represented by Fig 88. It is necessary, then, for the reader to establish the correct proportion between the parameters shown in Fig 87 and the parameters used in the actual case shown in Fig 88 in order to enter the charts.

To establish these proportions, the following formulas are obtained from the geometric similarity between Figs 87 and 88.

$$a_0 = a$$

$$\beta_0 = \beta$$

$$\theta_0 = \theta$$

Also

$$\frac{Z_0}{Z} = \frac{R_0}{R_T}$$

$$\frac{V_{C0}}{V_{T0}} = \frac{V_C}{V_T}, \text{ and}$$

$$\psi_0 = \psi$$

The parameters needed for entering the visual-angle charts are A , the absolute difference in heading at collision as related to ψ , and t_0 , the time before collision.

$$\text{Since } A = \frac{V_{C0}}{V_{T0}}, \frac{V_{C0}}{V_{T0}} = \frac{V_C}{V_T}, \text{ and } \psi_0 = \psi, \text{ then } t_0 \text{ is the only unknown.}$$

From Figs 87 and 88, $\theta_0 = K_0 t_0$ and $\theta = Kt$, where K or K_0 = rate of turn, in degrees per minute. Since $\theta_0 = \theta$, then

$$t_0 = \frac{Kt}{K_0} \tag{37}$$

If V_T is in mph and K (rate of turn) is in degrees per minute,

$$K = \frac{5042.03 V_T}{R_T} \tag{38}$$

and

$$\tan \lambda = \frac{0.067 V_T^2}{R_T} \quad (15)$$

Substituting Equation (15) in Equation (38),

$$K = \frac{75,254.18 \tan \lambda}{V_T} \quad (39)$$

Substituting Equation (39) and Equation (37),

$$t_0 = \frac{75,254.18 \tan \lambda \cdot t}{K_0 V_T} \quad (40)$$

In computing the points necessary for drawing the curves, the following values of the parameters were used

$K_0 = 360$ degrees per minute, $R_0 = 2801.13$ feet, and $V_{T0} = 200$ mph

The following values of the independent variables used

$\psi_0 = 0^\circ$ to 360° in 30° intervals

$\frac{V_{c0}}{V_{T0}}$ is used instead of V_{c0}

$\frac{V_{c0}}{V_{T0}} = 0.2, 0.4, 0.6, 0.8, 1.0, 1.25, 1.67, 2.5, 5.0, \infty$

$t_0 = 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0$

Since the value $K_0 = 360$ degrees per minute was held constant, then Equation (40) becomes

$$t_0 = \frac{209.2 \tan \lambda \cdot t}{V_T} \quad (8)$$

Since the value $R_0 = 2801.1$ feet was held constant, then

$$Z = \frac{Z_0 R_T}{2801.1} \quad (16)$$

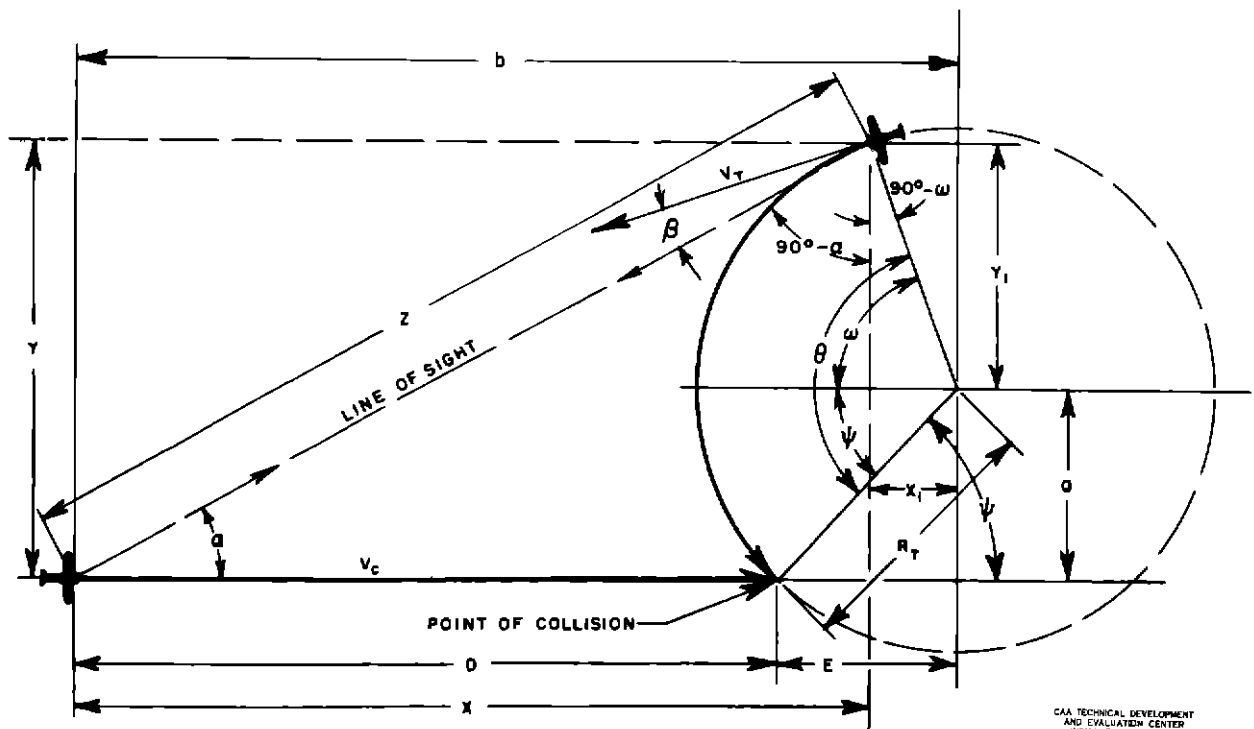


Fig 86 Vector Diagram for Derivation of Formulas for One Airplane Turning Left and One Airplane Flying Straight and Level

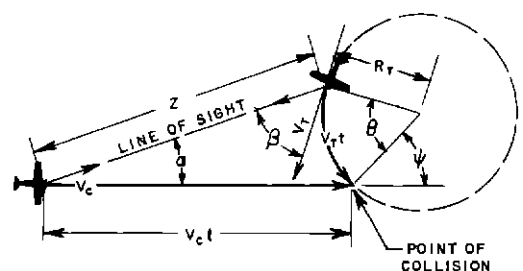
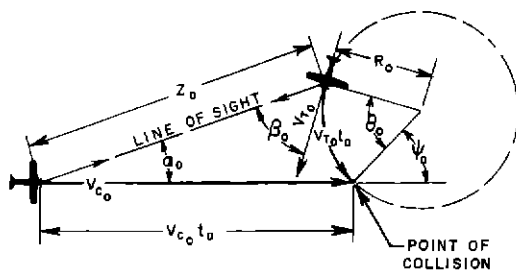


Fig 87 Vector Diagram Showing Similarity to Fig 88 for Derivation of Formulas for One Airplane Turning Left and One Airplane Flying Straight and Level

Fig 88 Vector Diagram Showing Similarity to Fig 87 for Derivation of Formulas for One Airplane Turning Left and One Airplane Flying Straight and Level

DERIVATION OF FORMULAS FOR COLLISION CONDITION D

For this condition, two airplanes are assumed to be on a collision course, both airplanes making a left turn. Figure 89 shows the physical significance of the horizontal visual angles and of the distances involved in this condition. The horizontal visual angles are measured from straight ahead of the pilot at the time of investigation.

Locating the center of the larger circle with respect to the point of collision or the origin of the X and Y axis

$$a = R_1 \sin \psi \quad (41)$$

$$b = R_1 \cos \psi \quad (42)$$

$$\omega_0 = K_0 t \quad (28)$$

Where

K_0 = the rate of turn of the airplane turning on the smaller circle, in degrees per minute

t = time of investigation before collision, in minutes

Also

$$\omega_1 = K_1 t - \psi \quad (25)$$

Where

K_1 = the rate of turn of the airplane turning on the larger circle, in degrees per minute

t = the time of investigation before collision, in minutes

The co-ordinates of the airplane turning on the smaller circle, with respect to the airplane turning on the larger circle at time t , are then given by

$$X_0 = R_0 (1 - \cos \omega_0) \quad (26)$$

$$Y_0 = -R_0 \sin \omega_0 \quad (27)$$

And the co-ordinates of the airplane turning on the larger circle are

$$X_1 = b - R_1 \cos \omega_1$$

When Equation (42) is substituted for b,

$$X_1 = R_1 (\cos \psi - \cos \omega_1) \quad (23)$$

$$Y_1 = a - R_1 \sin \omega_1$$

When Equation (41) is substituted for a,

$$Y_1 = R_1 (\sin \psi - \sin \omega_1) \quad (24)$$

When these co-ordinates are used, the following is derived

$$\tan \theta = \frac{Y_1 - Y_0}{X_0 - X_1} \quad (29)$$

Substituting in Equations (26), (27), (23), and (24)

$$\theta = \arctan \frac{R_1 (\sin \psi - \sin \omega_1) + R_0 \sin \omega_0}{R_0 (1 - \cos \omega_0) - R_1 (\cos \psi - \cos \omega_1)} \quad (43)$$

In solving for α , the horizontal visual angle measured from straight ahead of the airplane turning on the smaller circle,

$$\alpha = \omega_0 - \theta - 90^\circ \quad (20)$$

In solving for β , the horizontal visual angle measured from straight ahead of the airplane turning on the larger circle,

$$\beta = \omega_1 - \theta + 90^\circ \quad (21)$$

The distance between airplanes at time t is as follows:

$$Z = \sqrt{(X_0 - X_1)^2 + (Y_1 - Y_0)^2} \quad (22)$$

To construct a minimum number of visual-angle charts to represent the variables involved in this condition, the same law of geometric similarity is applied as was explained in detail in the derivation of formulas for Collision Condition C. In this case the similarity between Figs 90 and 91 is used

$$t_0 = \frac{209.2 \tan \lambda t}{V_T} \quad (8)$$

Where

$$V_T = V_{T_2}$$

$$\lambda = \lambda_2$$

Also

$$Z = \frac{R_2 Z_0}{1400.56} \quad (30)$$

In computing the points necessary for drawing the curves, the following values of the parameters were used

Held constant $K_0 = 360$ degrees per minute, $R_0 = 1400.56$ feet, $V_{T_0} = 100$ mph

The following values of the independent variables were used

$$\psi_0 = 0^\circ, 45^\circ, 90^\circ, 135^\circ, 180^\circ, 225^\circ, 270^\circ, 315^\circ$$

$$A = \frac{V_{T_1}}{V_{T_0}} = 0, 0.2, 0.4, 0.6, 0.8, 1.0, 1.25, 1.67, 5.0, \infty$$

$$t_0 = 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0$$

$$R/A = 1.0, 2.0, 2.5, 3.0, 3.5, 4.0, 6, 8, 10$$

$$\text{Note } R/A = \frac{R_1/R_0}{V_{T_1}/V_{T_0}}$$

Formulas Needed to Determine Visual Angles and Distances for Two Airplanes on a Collision Course While Flying in the Horizontal Plane and Turning in Opposite Directions (Derivation is Similar to Two Airplanes Turning in Same Direction)

$$\omega_0 = -K_0 t \quad (44)$$

$$\omega_1 = \psi + K_1 t \quad (45)$$

$$\tan \theta = \frac{R_1 (\cos \psi - \cos \theta_1) - R_0 (1 - \cos \omega_0)}{R_1 (\sin \psi - \sin \omega_1) + R_0 \sin \omega_0} \quad (46)$$

$$\alpha = \omega_0 + \theta + 180^\circ \quad (47)$$

$$\beta = \omega_1 + \theta + 180^\circ \quad (48)$$

$$Z = \frac{X}{\sin \theta} \quad (49)$$

$$X = R_1 (\cos \psi - \cos \omega_1) - R_0 (1 - \cos \omega_0) \quad (50)$$

Where

R_1 = Radius of the larger turning circle, in feet

R_0 = Radius of the smaller turning circle, in feet

K_0 = Rate of turn of the airplane on the smaller circle, in degrees per minute

K_1 = Rate of turn of the airplane on the larger circle, in degrees per minute

t = Time of investigation prior to collision, in minutes

α = Visual angle required from the airplane flying on the smaller circle to see the airplane on the larger circle at time t , in degrees

β = Visual angle required from the aircraft flying on the larger circle to see the airplane flying on the smaller circle at time t , in degrees

Z = Distance between the airplanes at time t , in feet

ψ = Difference in heading at collision point as established in Table IV

TABLE IV

RELATIONSHIP BETWEEN ψ AND THE ABSOLUTE DIFFERENCE IN HEADING AT COLLISION POINT FOR TWO AIRPLANES TURNING IN OPPOSITE DIRECTION

Difference in Heading at Collision (degrees)	Condition	Value of ψ in Formulas (degrees)
180		0
90	With the airplane turning on the larger circle on the left of the airplane turning on the smaller circle at collision point	90
0		180
90	With the airplane turning on the larger circle on the right of the airplane turning on the smaller circle at collision point.	270

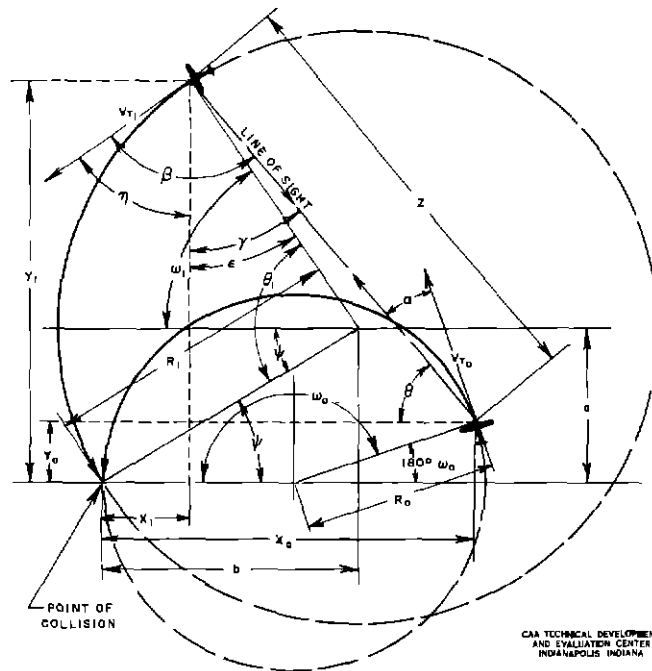


Fig 89 Vector Diagram for Derivation of Formulas for Two Airplanes Turning Left

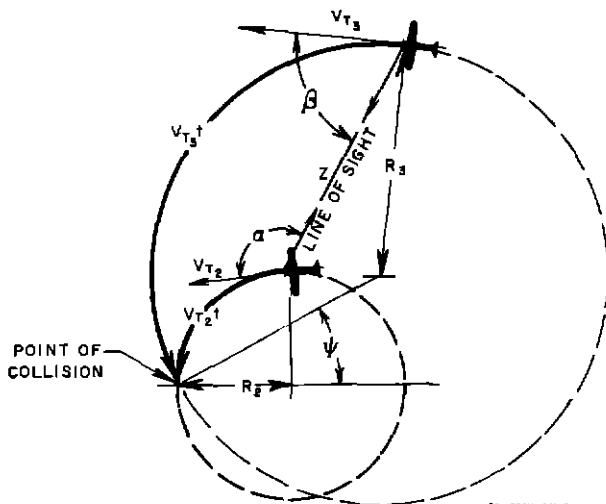


Fig 90 Vector Diagram Showing Similarity to Fig 91 for Derivation of Formulas for Two Airplanes Turning Left

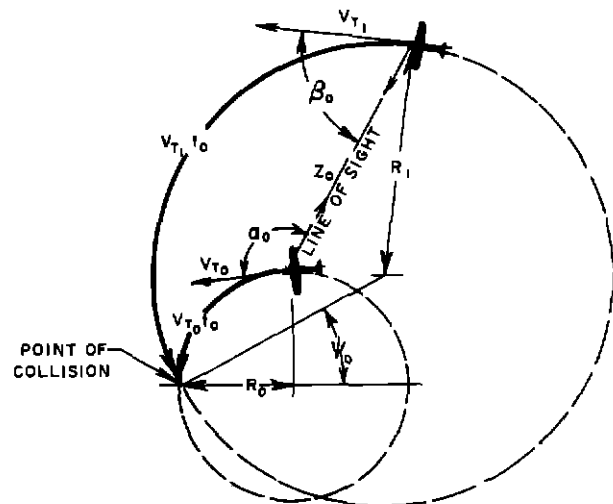


Fig 91 Vector Diagram Showing Similarity to Fig 90 for Derivation of Formulas for Two Airplanes Turning Left

APPENDIX II

FLIGHT-PATH ANALYSIS

The flight paths that are shown in Figs 92 through 108 are actual samples of the typical flight paths of most types of aircraft. They were made so that a better understanding of aircraft flight paths in the vicinity of airports would be possible. Day and night flights were recorded.

Figures 92 through 96 show flight paths that were recorded at Indianapolis, Indiana. The Indianapolis Weir Cook Municipal Airport was considered to be the average airport with respect to traffic and terrain conditions.

Figures 97 through 101 show flight paths that were recorded at the Birmingham Municipal Airport at Birmingham, Alabama. This airport was selected because of its rough terrain immediately surrounding the airport. A 20-foot-increment elevation-contour grid is overlaid on the airport. Figure 110 shows the mountainous terrain at Birmingham Airport.

Figures 102 through 108 show flight paths recorded at the Chicago Midway Airport at Chicago, Illinois. This airport was selected because of the large number of aircraft that use it.

RECORDING PROCEDURE

Three sighting stations were employed to record the aircraft flight paths. Each sighting station was equipped with a David White Company Theodolite Model No. 6061, one two-way radio, one stop watch, and two personnel to man the station. A typical sighting station is shown in Fig. 109.

The sighting stations were located on each airport as far apart as possible (3000-foot minimum) and as near the formation of an equilateral triangle as terrain and obstructions permitted. In all cases, each sighting station was visible from the others to permit azimuth triangulation readings and altitude information. The theodolites were leveled and the azimuth scale was zeroed on a point that was visible from all three stations. This point in most cases was the control tower on the airport. Each station recorded elevation and azimuth locations of another common point in order to obtain further reference for accurately locating the sighting stations.

Upon sighting an approaching aircraft, radio contact with the other two stations was made and stop watches were synchronized. Azimuth and elevation readings of the aircraft were made at 15-second intervals. Later, these points were plotted and the curves were drawn.

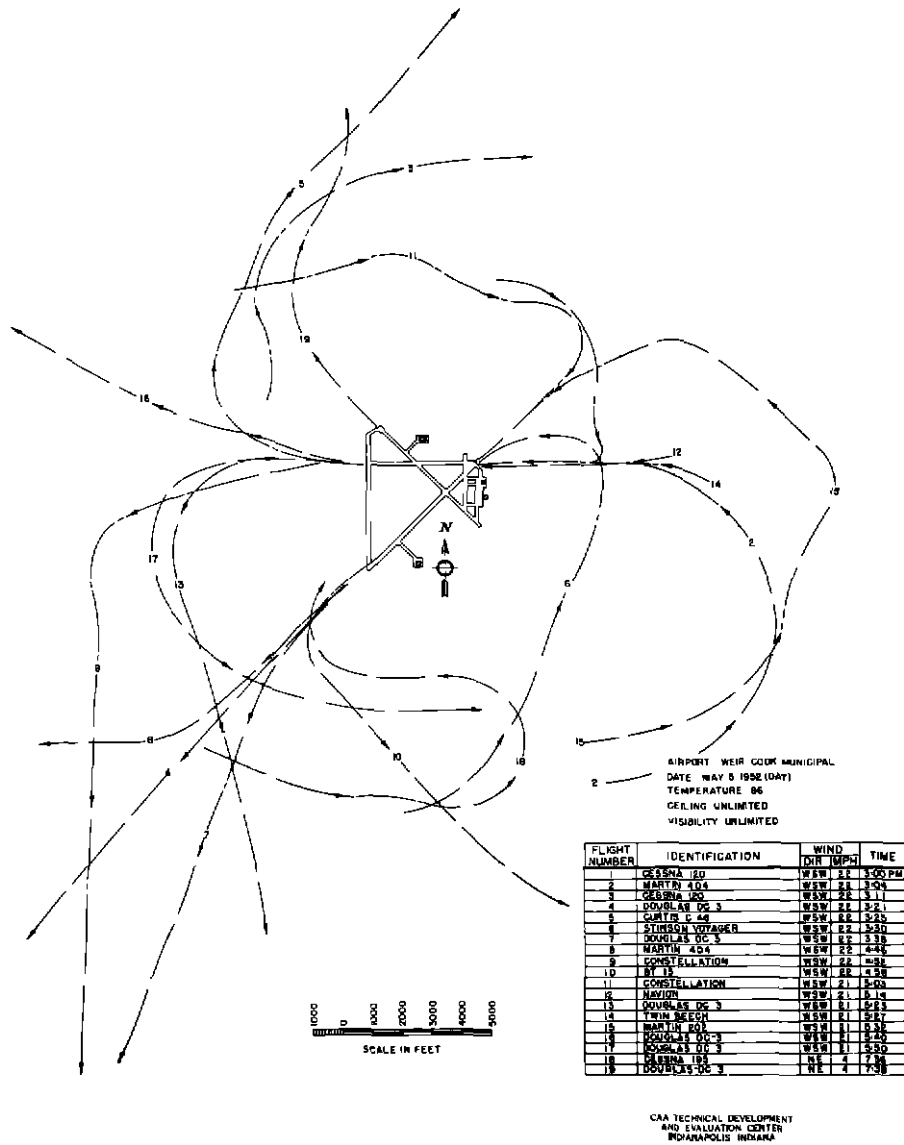


Fig 92 Typical Flight Paths in the Vicinity of Airports

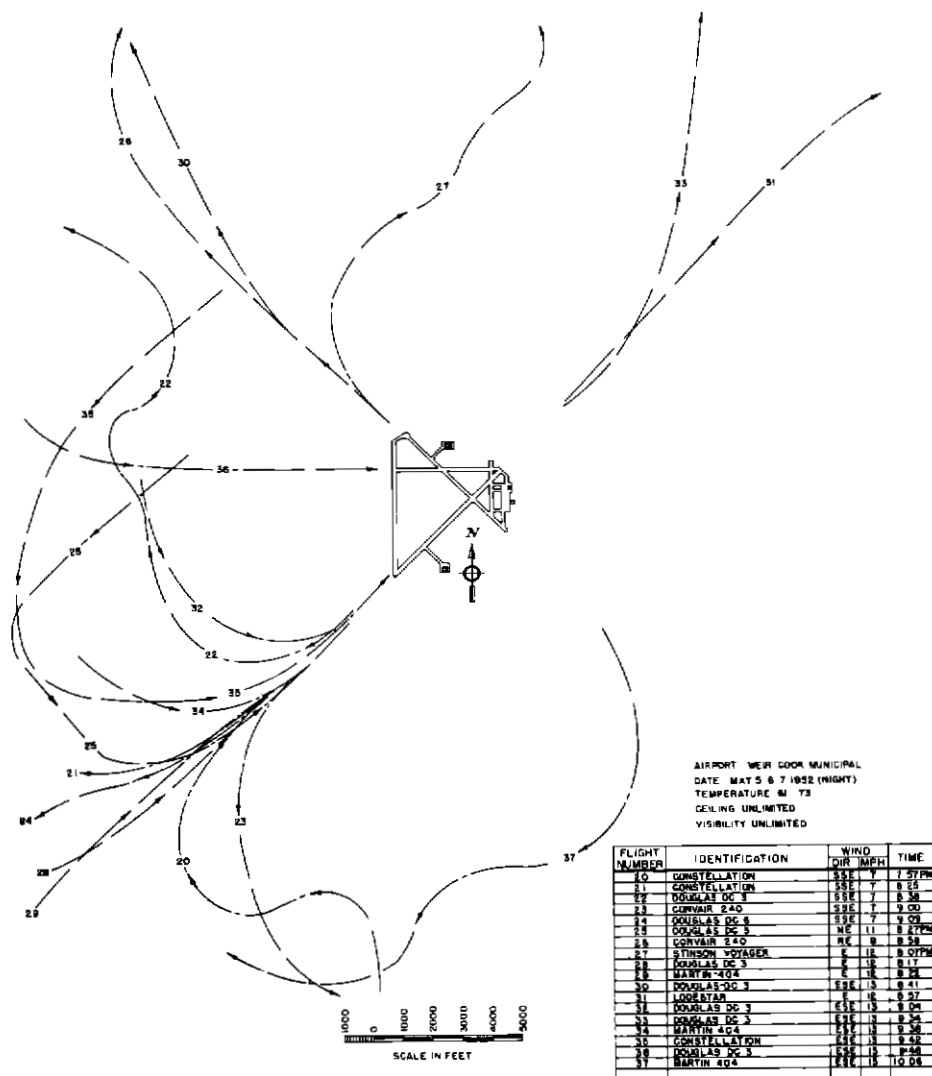


Fig 93 Typical Flight Paths in the Vicinity of Airports

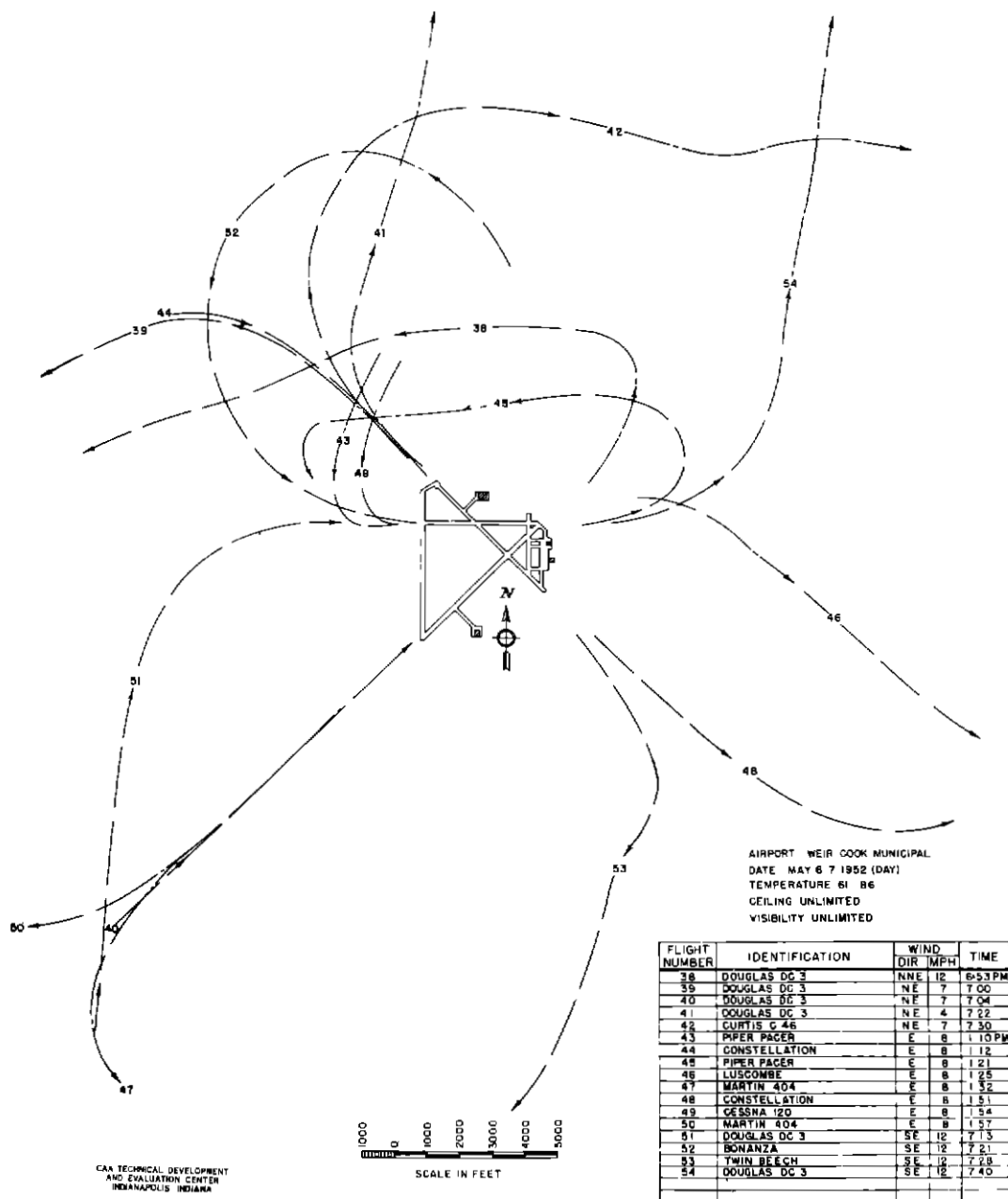
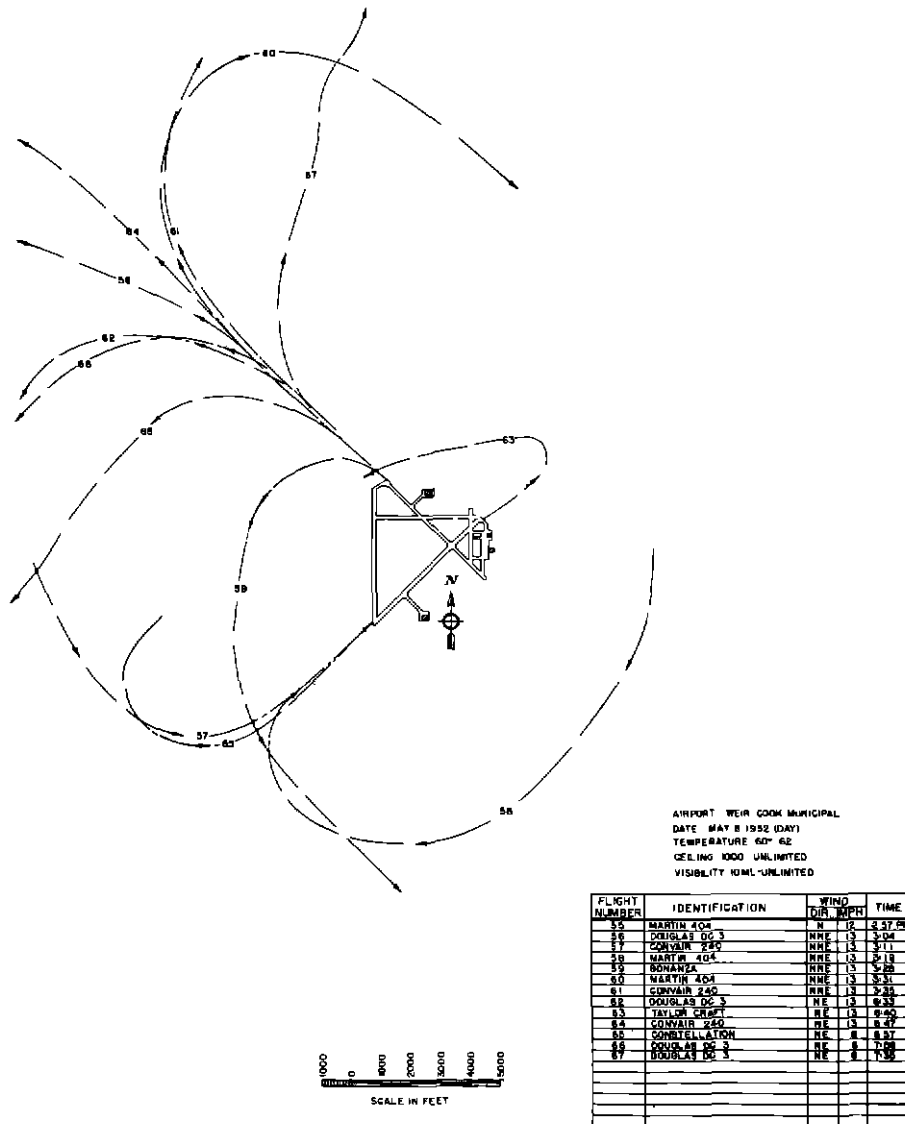


Fig 94 Typical Flight Paths in the Vicinity of Airports



CAA TECHNICAL DEVELOPMENT
AND EVALUATION CENTER
INDIANAPOLIS, INDIANA

Fig 95 Typical Flight Paths in the Vicinity of Airports

Fig 96 Typical Flight Paths in the Vicinity of Airports

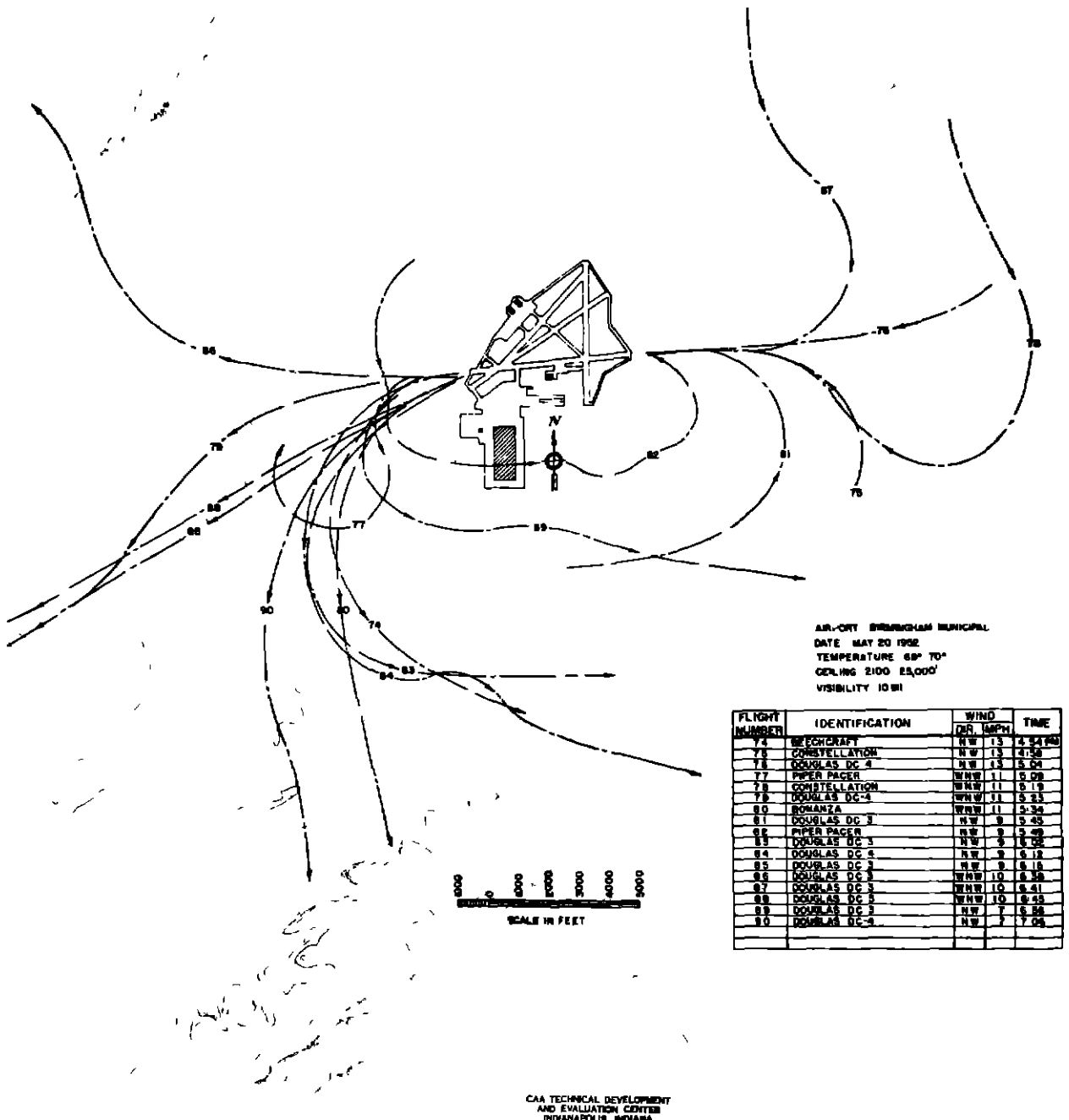


Fig 97 Typical Flight Paths in the Vicinity of Airports

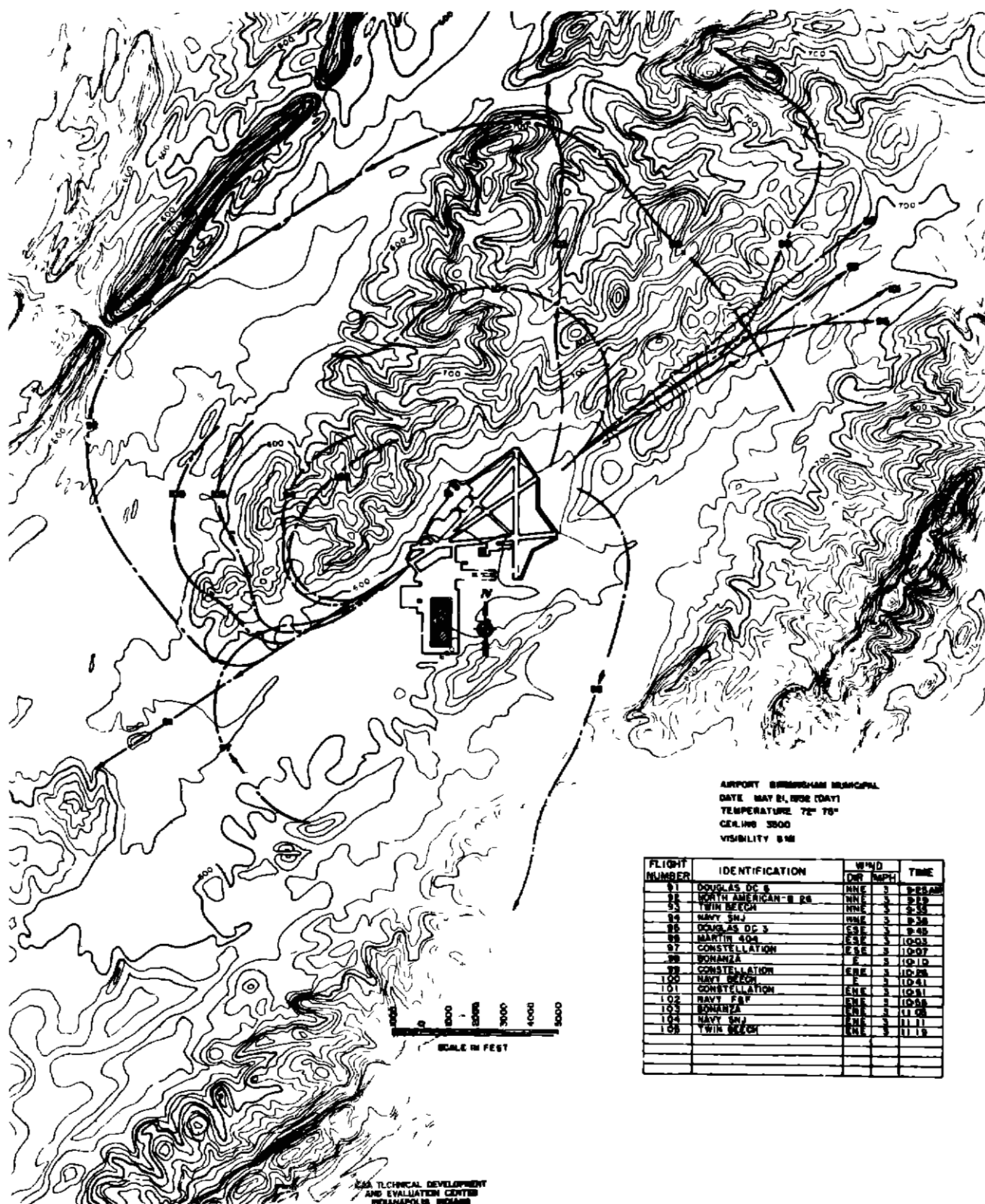


Fig 98 Typical Flight Paths in the Vicinity of Airports

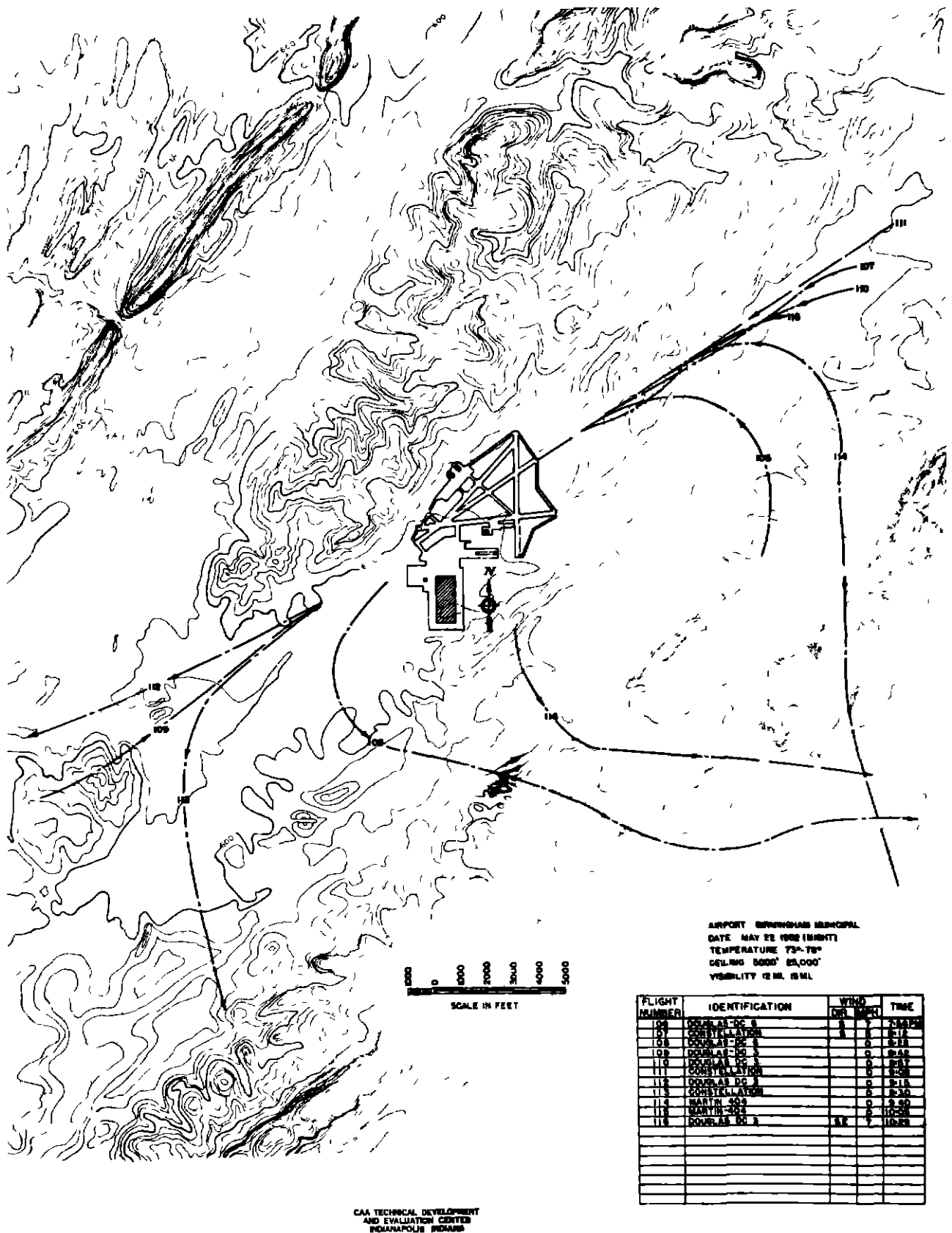


Fig 99 Typical Flight Paths in the Vicinity of Airports

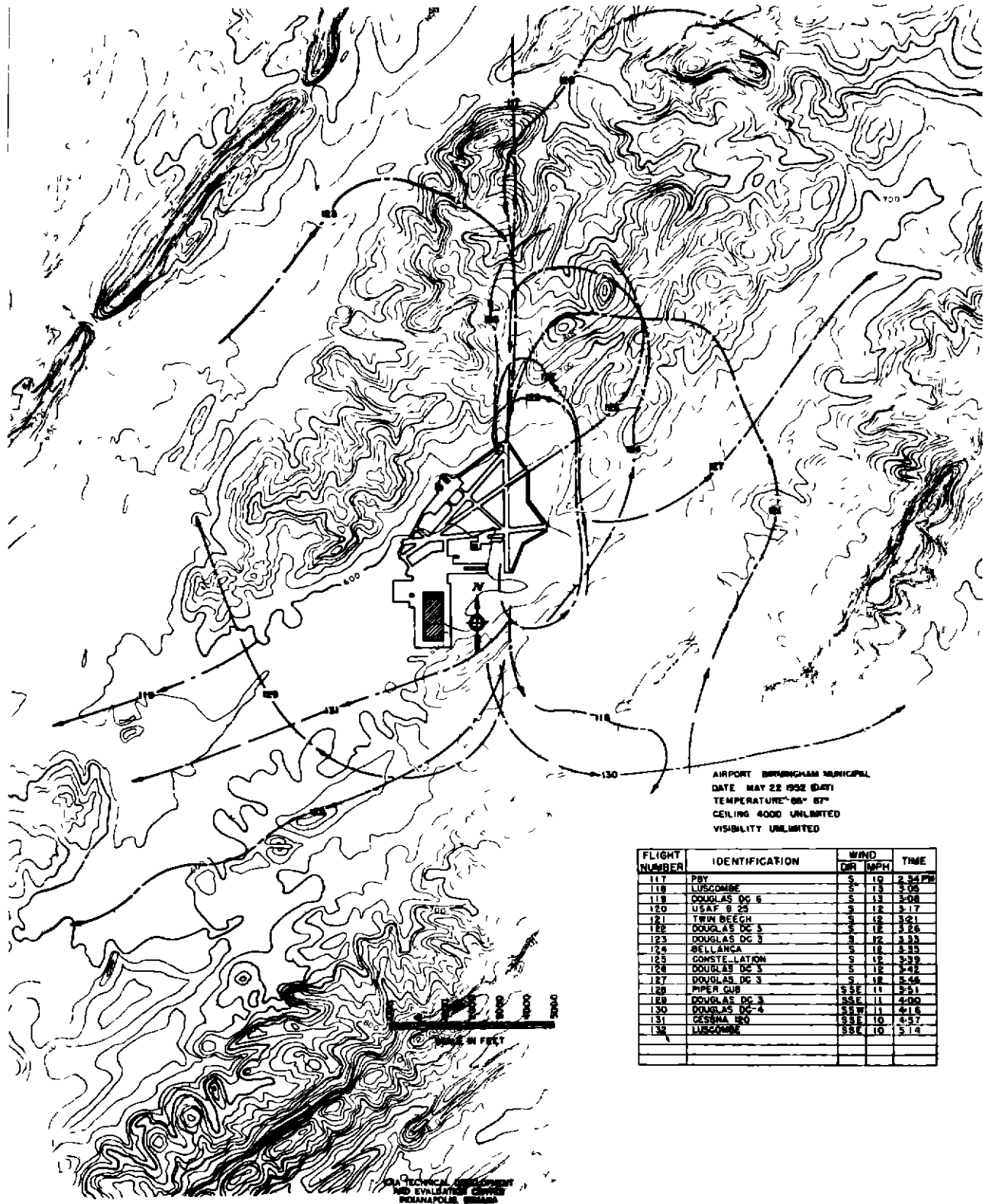


Fig 100 Typical Flight Paths in the Vicinity of Airports

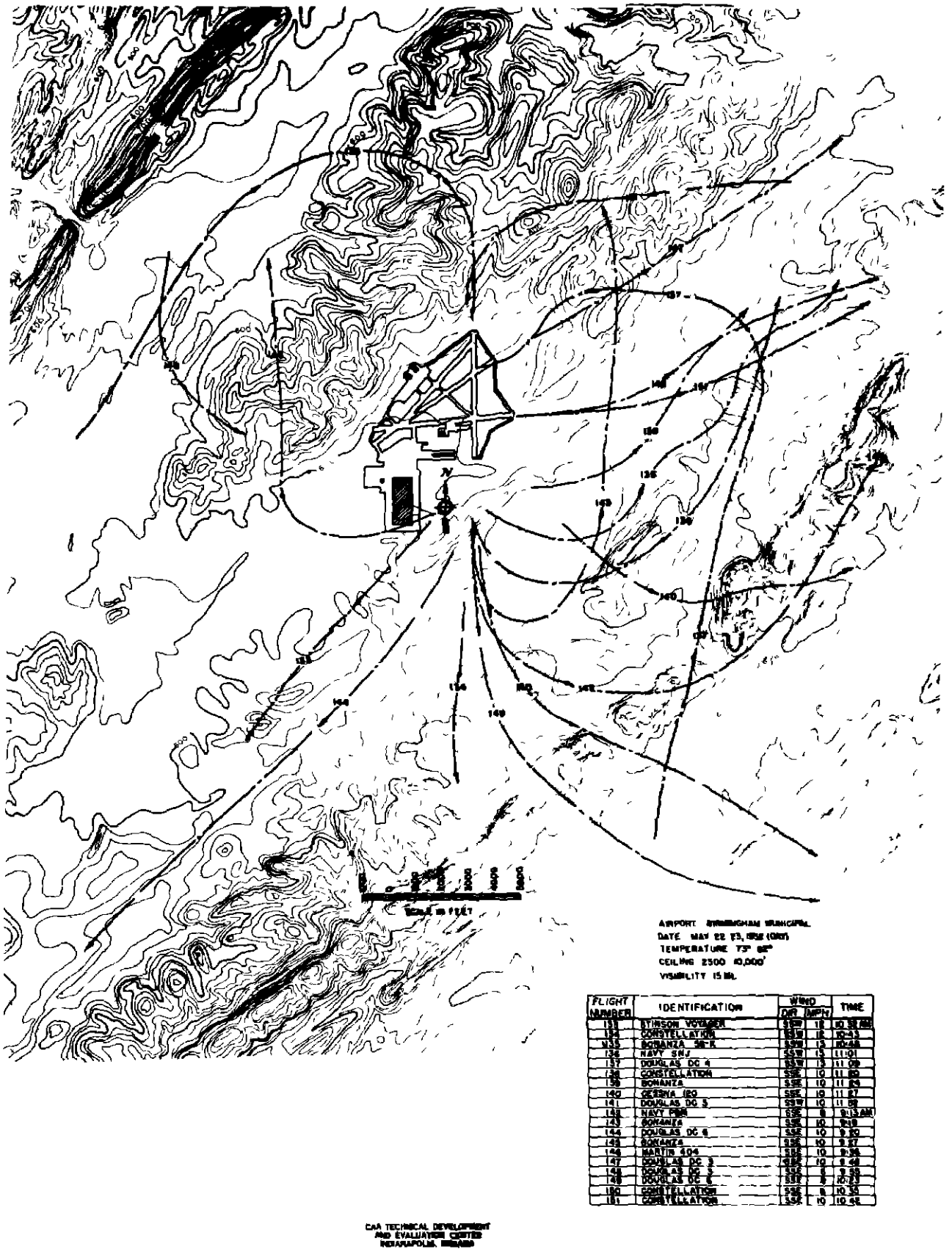


Fig 101 Typical Flight Paths in the Vicinity of Airports

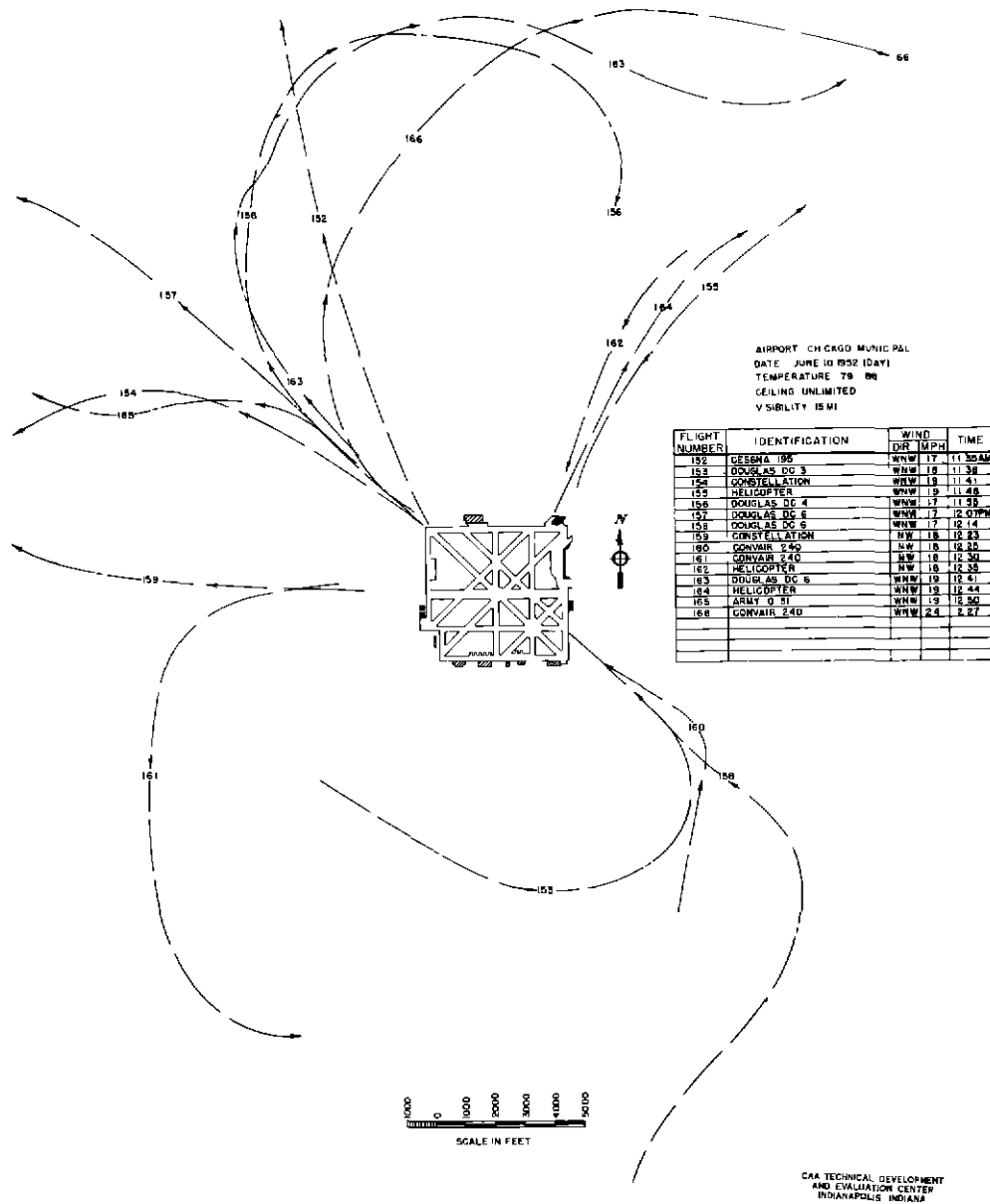


Fig 102 Typical Flight Paths in the Vicinity of Airports

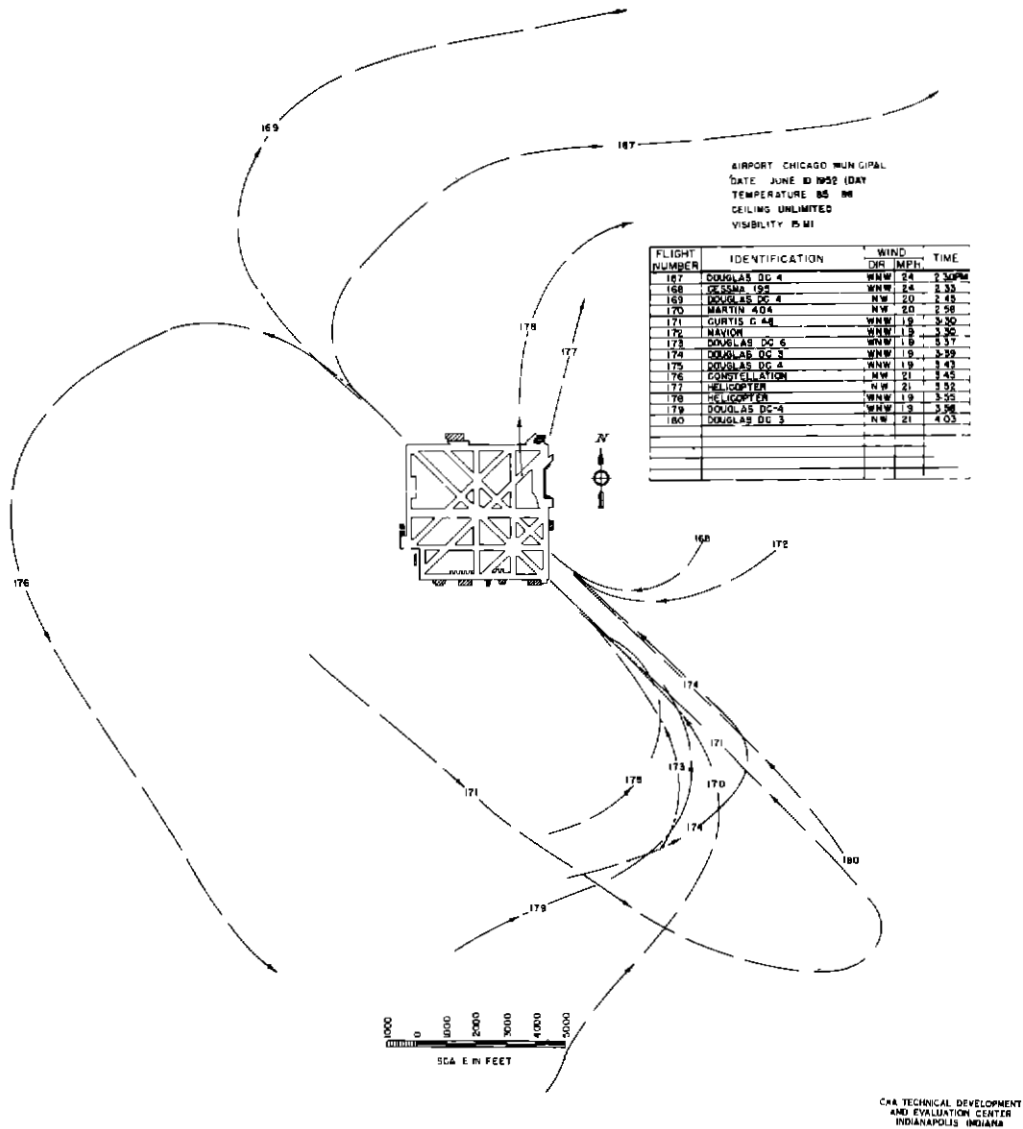


Fig 103 Typical Flight Paths in the Vicinity of Airports

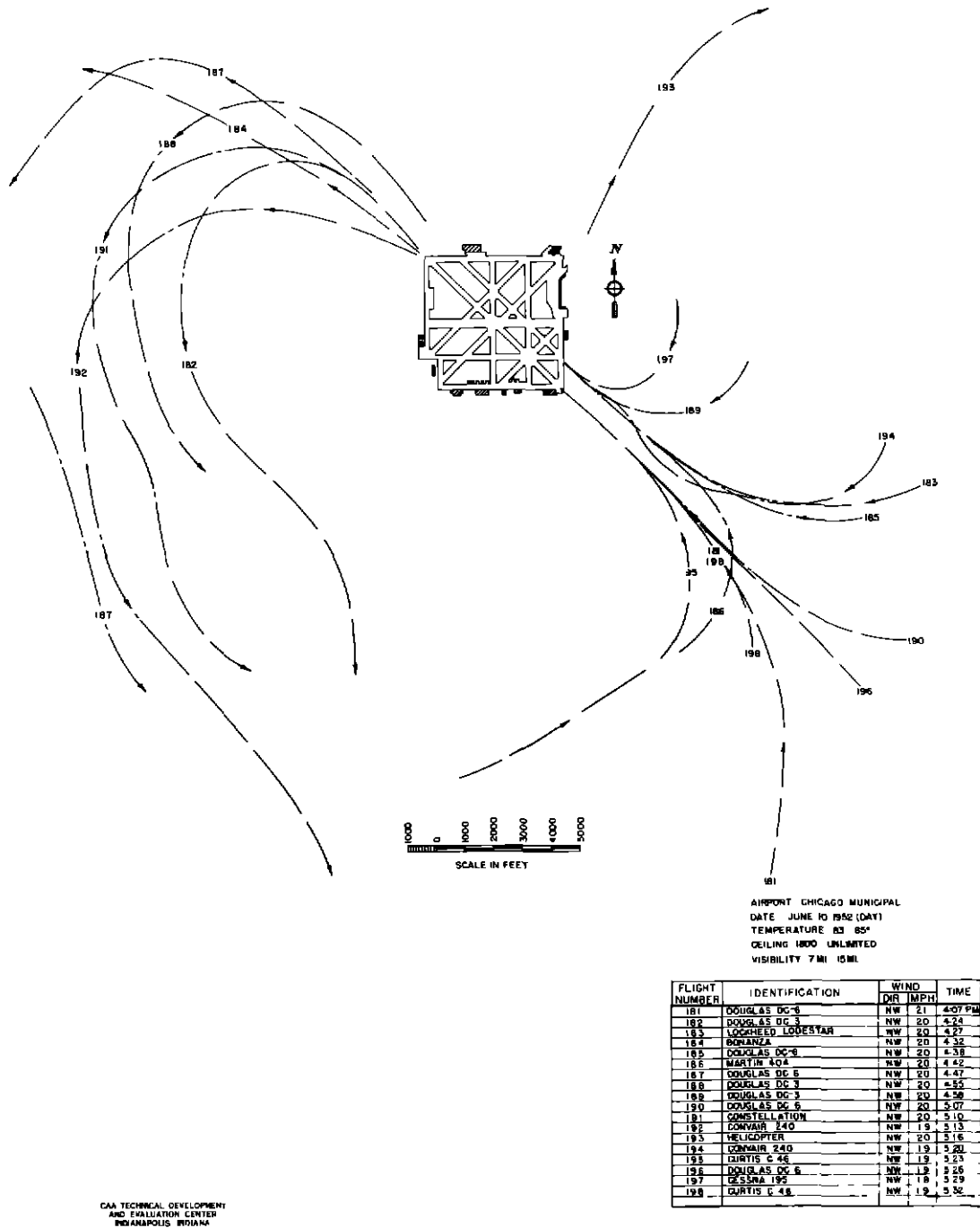


Fig 104 Typical Flight Paths in the Vicinity of Airports

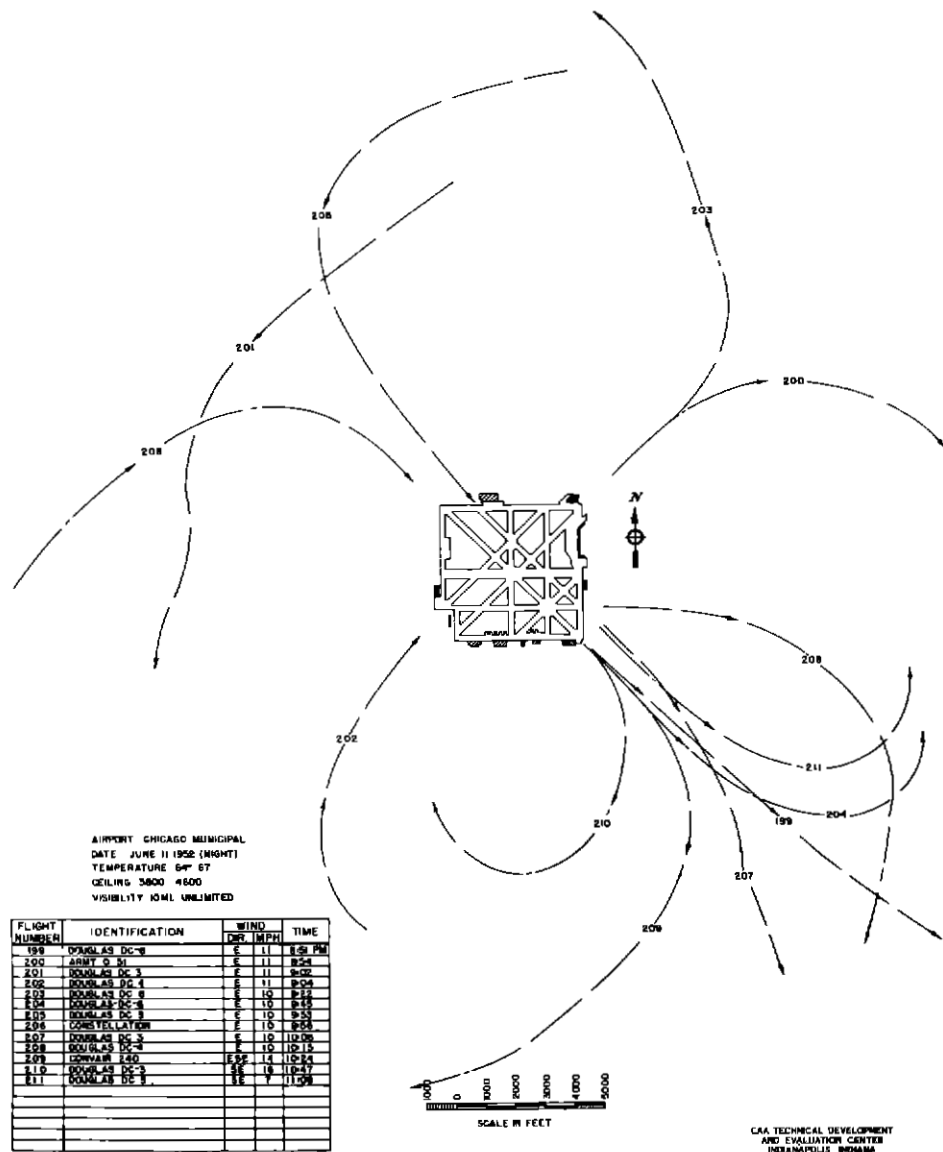


Fig 105 Typical Flight Paths in the Vicinity of Airports

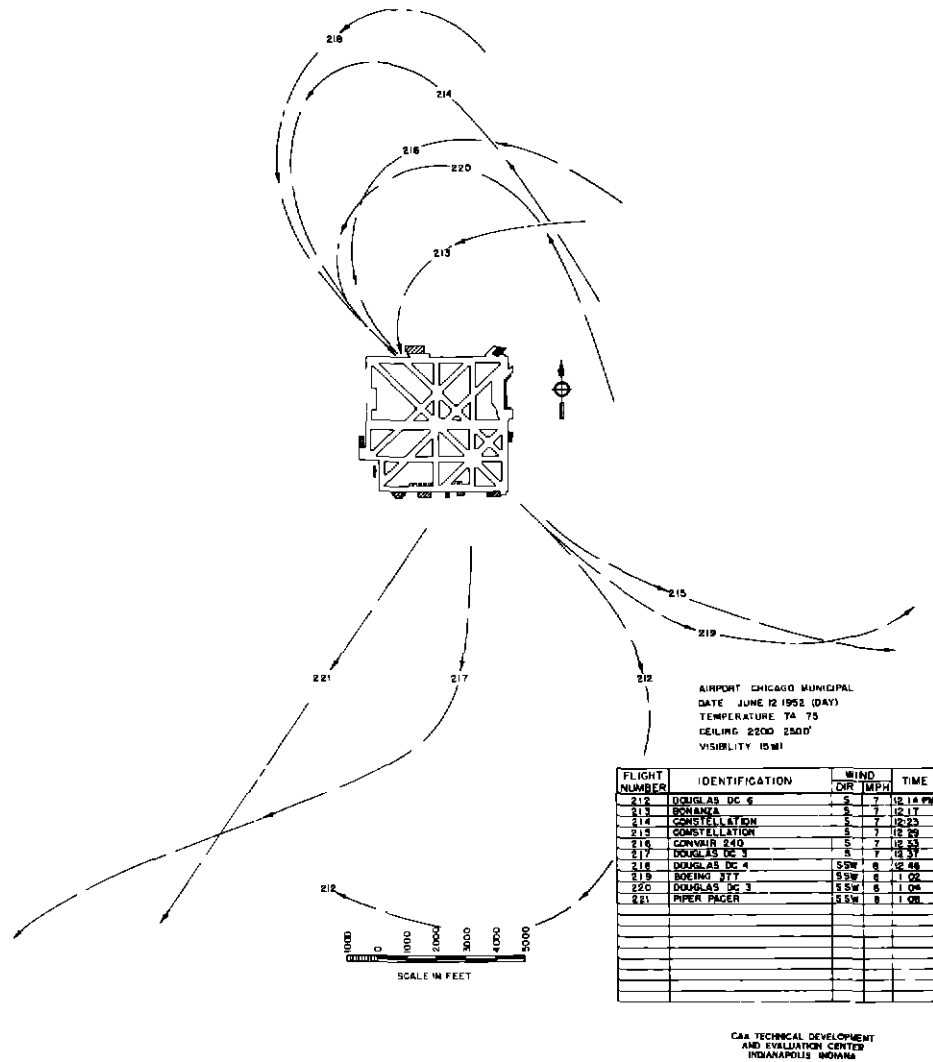


Fig 106 Typical Flight Paths in the Vicinity of Airports

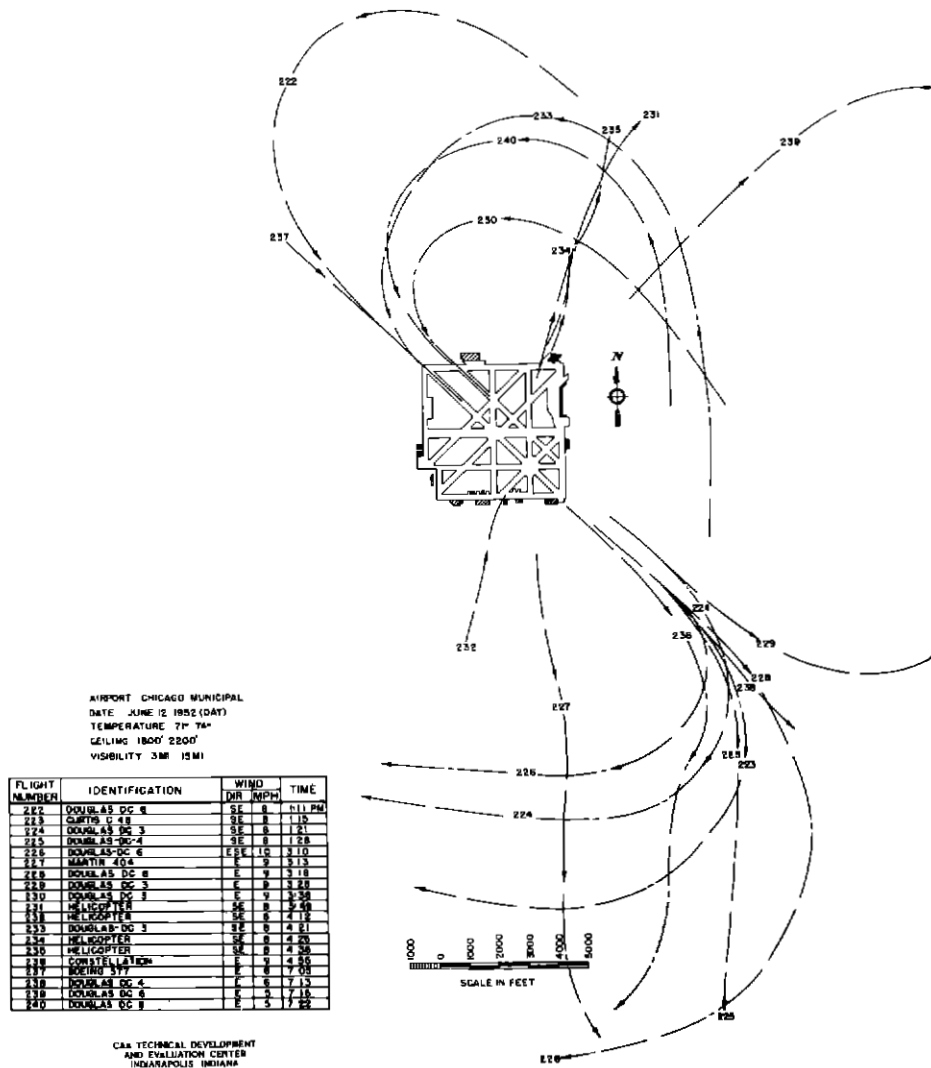


Fig 107 Typical Flight Paths in the Vicinity of Airports

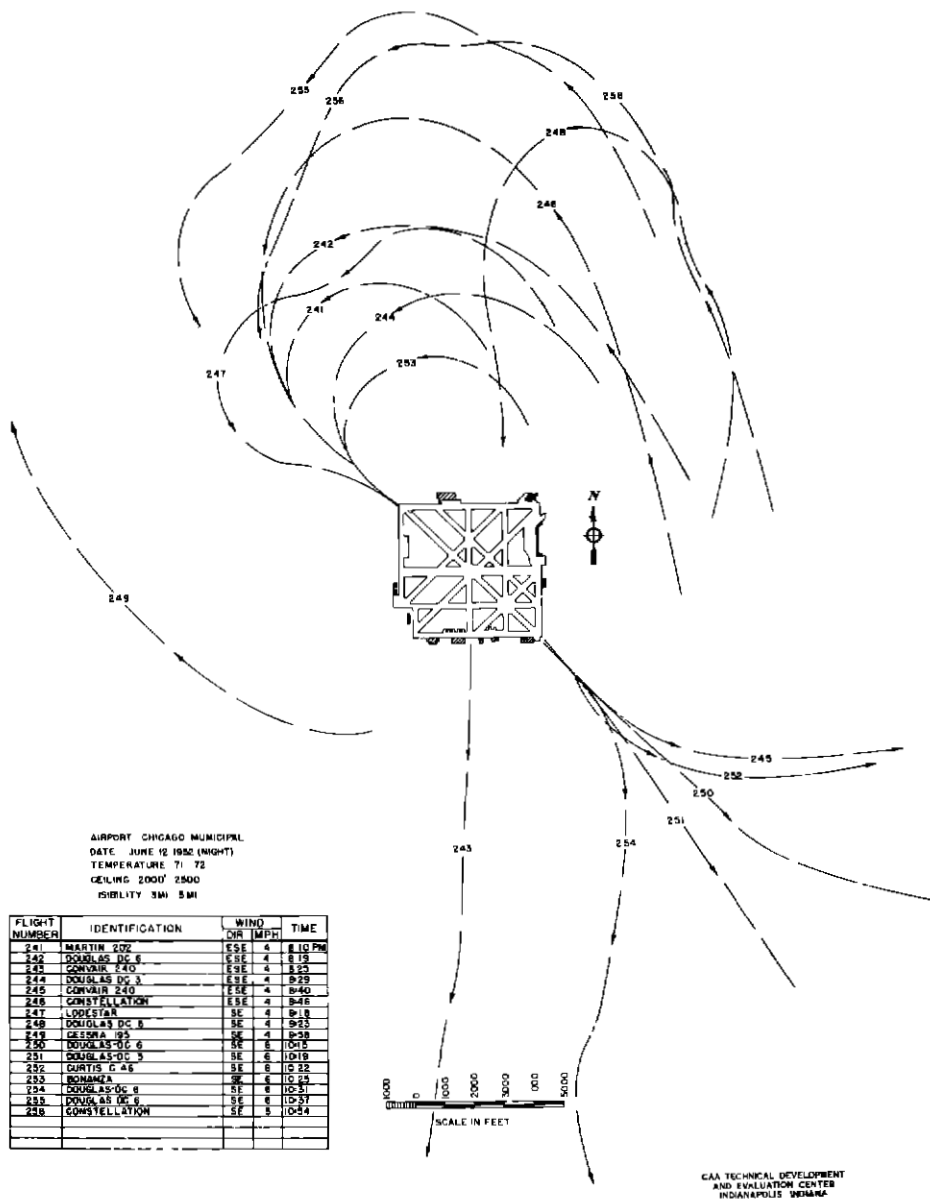


Fig 108 Typical Flight Paths in the Vicinity of Airports



Fig 109 Typical Flight-Path Recording Station

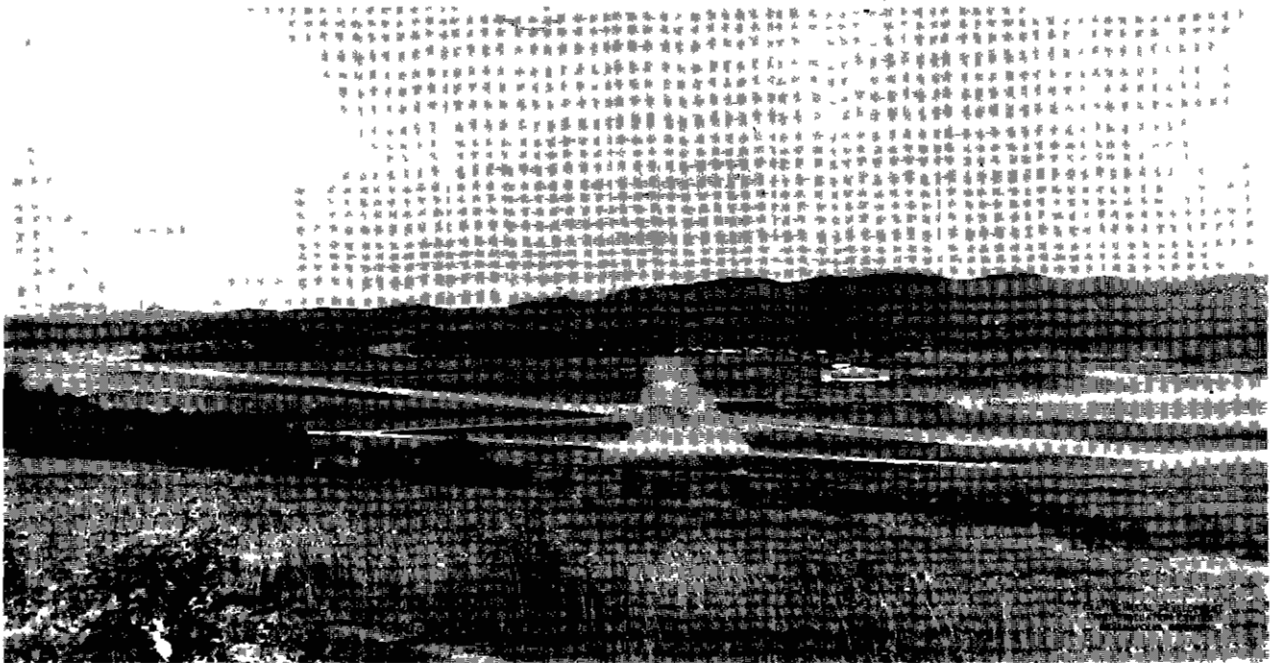


Fig 110 Birmingham Municipal Airport