ANALYTICAL AND SIMULATION STUDIES OF TERMINAL-AREA AIR TRAFFIC CONTROL

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FOREWORD

The Air Navigation Development Board (ANDB) was established by the Departments of Defense and Commerce in 1948 to carry out a unified development program aimed at meeting the stated operational requirements of the common military/civil air navigation and traffic control system This project, sponsored and financed by the ANDB, is a part of that program The ANDB is located within the administrative framework of the Civil Aeronautics Administration for housekeeping purposes only Persons desiring to communicate with ANDB should address the Director, Air Navigation Development Board, Civil Aeronautics Administration, W-9, Washington 25, D C

SUMMARY

This report summarizes the program of terminal-area simulation work completed jointly at the Franklin Institute Laboratories for Research and Development and at the Technical Development and Evaluation Center of the Civil Aeronautics Administration during a nine-month period The specific tasks involved the setup, description, and analysis of the mathematical and physical system analogs and a determination of the interaction of the many variables involved Advanced radar-vectoring techniques previously developed to a high degree by TDEC were employed As a matter of convenience the Washington National Airport terminal area, which is 40 miles in diameter, was used in establishing certain generalized problems with many specific applications and solutions

A configuration described as WNA Phase 2 was used as the main reference system In this and in previous investigations, this configuration was found to be a most effective and practical setup for a manual radar-vectoring system and yet was relatively inexpensive It used two independent approach-control sectors with symmetrically located, close-in, twin, inner stacks One approach controller was employed for each sector Aircraft were vectored from these inner stacks to the approach gate of a single airport, which was assumed to have a single instrument runway with properly designed high-speed runway turn-offs, entrances, and taxiways

A multitrack configuration known as WNA Phase 2A was also investigated When a track-selection computer known as EMTAC (electronic multitrack approach computer) was used and when it was assumed that aircraft were equipped with airborne pictorial computers, it was found that traffic-control procedures were very simple and that the controller's job was simplified to the lowest level ever attained in simulation tests of high-capacity approach systems Furthermore, the interspersing of jet traffic with conventional aircraft was a smooth and efficient operation which did not necessitate any significant preferential treatment or additional workload

Representative high-density traffic samples of either arrivals only or of arrivals and departures, with peak durations of one to four hours, were employed The experiments included theoretical acceptance rates ranging from 30 to 41 landings per hour and used random arrival rates of 35 aircraft per hour and peak departure rates of 31 aircraft per hour Three methods of analyzing the problems were used, an FIL ideal technique, an FIL graphical space-time technique, and the TDEC dynamic simulator Correlation among the three was found to be excellent for the wide variety of traffic samples and of conditions tested

Tests showed that controller judgment is a variable factor which can have a marked influence on the effectiveness of a system It was found that various features of the WNA Phase 2 configuration provided the controllers with a relaxed system of terminal-area traffic control These features included (1) clear-cut, nonconflicting paths, (2) simple, "party-line," direct

*Manuscript submitted for publication June 1954 (simplex) communications, (3) reliable radar, (4) a limited degree of velocity control and of path-stretching, and (5) close-in, twin, inner stacks With these features, present systems could probably double their present instrument-flight-rule (IFR) capacities and, moreover, could profit from the standpoint of safety

Tests showed that, contrary to popular belief, stacks can be useful reservoirs and coarse filters for de-randomizing the inbound traffic and for maintaining a steady supply of aircraft to the final approach without alternately starving the system and then causing cumulative delays Furthermore, the proper use of stacks keeps the workload at a minimum Because adequate outer feeding stacks are available and because a practical stack-and-fixed-block system was followed between the outer and inner twin feeder stacks, random traffic could be handled at theoretical or effective densities greater than 100 per cent for periods of more than one hour without saturation or breakdown

Limiting the flow of arrivals to 35 per hour every hour instead of allowing aircraft to enter the terminal area randomly at an average rate of 35 per hour for 3 hours resulted in significant decreases in over-all delay The differences in delays were slight for short periods of about 1/2 hour However, when random entries were used, it was more difficult to keep the flow constant This factor increased the workload

It was also found that when inbound traffic only was handled the capacity of the system, including the human element in the manual controlling of random arrival traffic over several relaxed hours, varied from about 26 per hour in a 20-mph average headwind to 32 per hour in still air when present 3-mile separation standards were used Although loadings exceeded 100 per cent, the delays remained within tolerable limits For short periods of 15 minutes to 1/2 hour, a capacity of about 36 arrivals per hour or an average interval of 1 minute and 40 seconds could be achieved Compared with the theoretical capacities of 32 to 39 arrivals per hour, on the average approximately 80 per cent of the ideal optimum was approached with the dynamic simulator In cases of mixed arrival and departure traffic on a single properly designed runway, about 50 operations per hour were found feasible It is believed that these landings or mixed landing and take-off rates could also be achieved in actual operation with adequate facilities plus some additional training and a little co-operation from pilots Furthermore, a graphical study showed that 65 to 70 operations could be handled on intersecting runways and that a much smaller amount of controller co-ordination would be required

On the basis of available operational data and of established probability concepts, minimum outer-marker separations were established for various outer-marker distances and wave-off criteria The effect of force-feeding a runway was also considered A theoretical capacity of 39 landings per hour for a single runway was found to be about the maximum if present glide-slope and runway separation restrictions, aircraft-performance estimates, and a high-speed runway turn-off at 3400 feet from the approach end of the runway were used

The steady-state traffic-delay theory now in use was found to be inadequate for the analysis of air traffic control Contrary to steady-state concepts, delays do not approach infinity when loadings approach or exceed 100 per cent Instead, they build up almost linearly with time, and their treatment must be considered on a transient, rather than on a steady-state basis The number of landing intervals or so-called "holding" or "service" times required to reach equilibrium conditions was found to be a function of loading At low loadings, the number of intervals are relatively small Since low loadings present no problem, we are concerned only with peak loadings It was found that though the delays became intolerable after several hours they did not approach infinity This fact is considered quite important in grasping a better perspective and a better knowledge of the fundamentals of properly designing the entire traffic-control system, including the en route, origin, and destination-airport elements of the system

Another significant result of these investigations concerned the amount and type of communications involved Even with high-density traffic such as 35 random arrivals per hour, it was found that the communications or simplex densities, the busy times, and the number of messages delayed were relatively low, contrary to random-communication theory The origin of calls was ordered to a large degree by the direct simplex (party-line) communications and by the many stereotyped situations This reduced the degree of randomness a great deal It became evident that the WNA Phase 2 or Phase 2A systems or any other similar set-ups with their simple procedures are efficient and relaxed systems of terminal-area radar control, regardless of the duration or density of traffic

INTRODUCTION

The work undertaken by Franklin Institute Laboratories under this contract was the supplying of engineering services to the Technical Development and Evaluation Center (TDEC) in the technical supervision of ANDB Project 6.7, the Simulation of Air Traffic Control. This project specified a simulation program for determining (1) the factors which influence the acceptance rate of a terminal area, and (2) the quantitative effect of each of these factors and of different combinations of these factors on the acceptance rate. "Factors" means both the physical environment such as aircraft characteristics, airport characteristics, and traffic demand rates and the operating environment.

The studies undertaken under a prior contract[1] were extended as a continuing simulation activity. In this activity were undertaken the formulation and execution of a co-ordinated two-stage program in which the experience and capabilities of the Franklin Institute Laboratories (FIL) analytical-, the FIL graphical-, and the TDEC dynamic-simulation techniques were integrated into a logical, concerted effort. It is intended that the results of this program will lead to a more conclusive and realistic determination of those fundamentals which are necessary for the logical design of a safe and efficient common air-traffic-control system.

The design of the associated experiments and the preliminary evaluations were first undertaken with the use of analytical and graphical techniques developed and described previously. The graphical techniques emphasize the use of S-t (space-time) curves. The S-t curve is a graphical plot in which the ordinate S is the projected distance, or space, of the flight flight path and the abscissa t is time. A separate curve is drawn for each aircraft in the traffic situation, and altitude data are noted at appropriate points. A typical S-t plot of a hypothetical situation is shown in Fig. 1. This type of simulation is inexpensive, fast, and versatile. It is especially useful in establishing and studying the individual and interrelated effects, on traffic operations, of various operating restrictions, separation standards, navigational-facilities layouts, traffic flow, and traffic distribution. The results can then be used as an index of perfection. After this screening, certain problems were then programmed for investigation on the TDEC dynamic simulator as the next, more conclusive step – a closer approach to real life. The differences between the paper-and-pencil and the dynamic-simulation studies are a good measure of the effectiveness of a system comprised of human, equipment, and facility elements. Thus calibration factors can be determined.

The dynamic simulator[2] furnishes the means of simulating, to a high degree, actual flights of aircraft through any specific area. Although the dynamic simulator is a more expensive and complex device to maintain and operate than the graphical simulator, it has the distinct and important advantage of introducing the human element such as controllers, communicators, and, to a degree, pilots quite realistically into both the air and ground portions of the system.

Critics of this approach may claim that much time and money are expended to determine solutions which have already been accepted on the basis of past experience, usually opinion and inconclusive data, or on the basis of actual flight tests. However, flight tests are expensive and involve much risk. They cannot feasibly represent the many situations, procedures, traffic composition, rates, and other variables to be explored. It is essential that there be used a rational and conclusive approach in which the results not only are meaningful for those situations and rules under which the experiments were performed but also are capable of extension to many future situations with reasonable conclusiveness. Simulation helps to make this possible without the usual trial-and-error construction, operational tests, and their accompanying long lead times and expense.

An idea fo the scale and of the value of the simulation tests described herein can be gleaned from the fact that in a few months of working time over 10,000 simulated flights were flown through the various tests under a wide number of conditions. The results to date show that this FIL-TDEC two-stage simulation effort serves as a logical and valuable aid in understanding the basic laws affecting traffic flow and in solving present and future air-traffic problems.

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[1] Samuel M. Berkowitz and Ruth R. Doering, "Analytical and Simulation Studies of Several Radar-Vectoring Procedures in the Washington, D. C., Terminal Area," CAA Technical Development Report No. 222, April 1954.

[2] Richard E. Baker, Arthur L. Grant, and Tirey K. Vickers, "Development of a Dynamic Air Traffic Control Simulator," CAA Technical Development Report No. 191, October 1953.

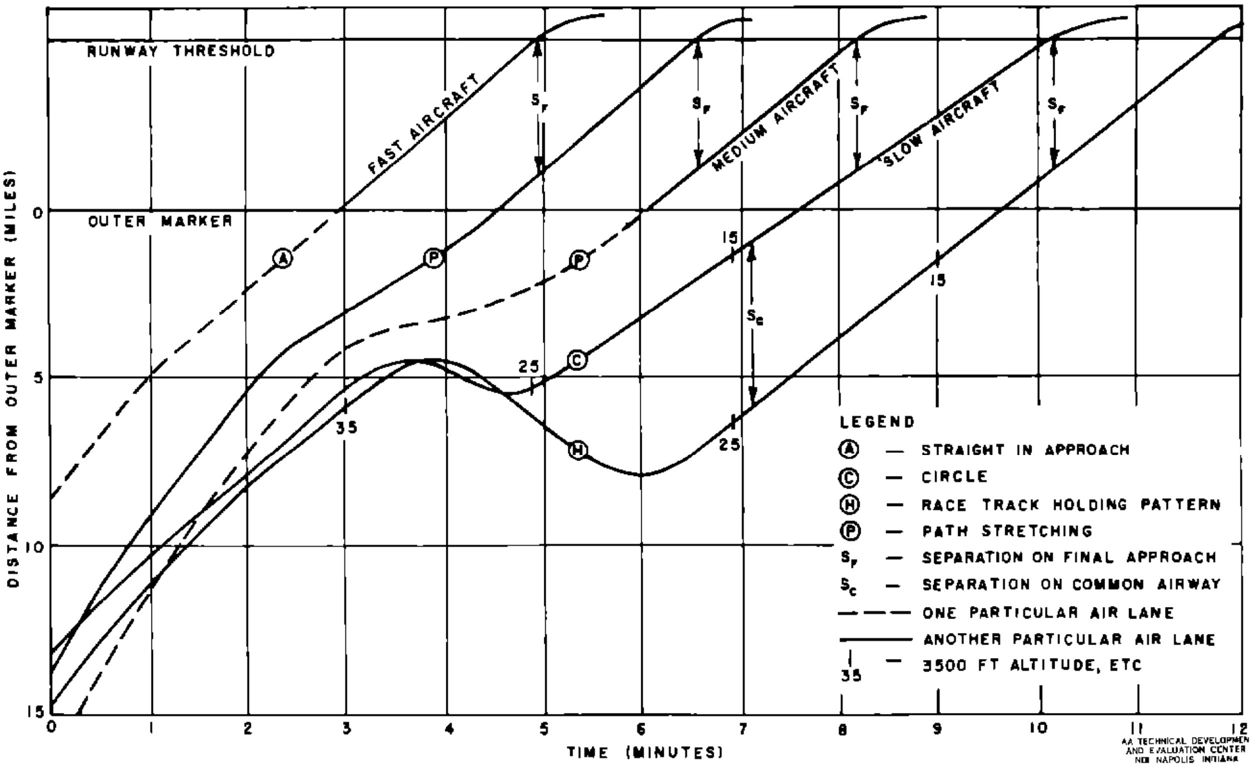

Fig 1 Section of Typical S-t Representation

OBJECTIVES

The scope of these terminal-area investigations was limited by contract time, budget, and the availability of associated simulation equipment and personnel It became necessary to consider certain fundamental questions and criteria and then to list them in an order of relative importance Consequently, the consideration of certain immediate and semiacademic problems indicated the needs (1) to be sure that the experiments are designed to produce the very maximum of information from each run, and (2) to set up mathematical analogs and descriptions of the many variables involved, including the unpredictable human judgment Although completely rational formulas might not be forthcoming immediately, the development of semi-empirical relationships would be valuable and would permit short extrapolations beyond the quantitative values actually determined

It was evident that the criterion relating to loading (demand versus capacity, or acceptanc rate) was basic and needed further study, because the complexity had outrun generally accepted mathematical techniques Studies to date have shown that loading, or density, per se is almost meaningless as far as delays are concerned unless (1) equilibrium conditions have been reach (2) the duration of the loading is considered, and (3) the actual or effective over-all system capacity is known Since air-traffic capacity and demand are fluctuating variables of relatively short periods, equilibrium can be expected to occur rarely and then only at low loadings, therefore the study of equilibrium conditions is of academic interest only In this work, attention was devoted to the more practical and meaningful transient conditions slightly below and above so-called theoretical saturation, a loading $\left(\frac{\text{arrival rate}}{\text{acceptance rate}}\right)$ of 1 0 In developing an over-all terminal-area system criterion, the results of these analytical and simulative studies were closely correlated with the theory of the build-up of queues as treated by Kendall, Palm, Pollaczek, and others [3]

[3] This has been treated partly by Carl Hammer, "Transients and Steady State for Tower-Controlled Stacks of Aircraft," FIL Working Paper No 3, Project 2343, (Limited Distribution) March 26, 1953

It is recognized that the design of experiments in air-traffic-control simulation involving the numerous random variables is quite difficult However, significant statistical results may be obtained by limiting the number of random parameters to those which have overriding influence on the traffic flow

The FIL ideal analysis shows the best a system can do with only the one rule of maintaining the specified separations at the outer marker This method corresponds somewhat to applicable transient-delay theory, except that use is made of particular representative random samples of traffic instead of theoretically infinite samples

The FIL graphical analysis shows what can best be done for the particular system or configuration being studied and includes the requirement (and consequent delays) for following specific airways, speeds, descent rates, aircraft turning rates, holding patterns, separation rules, and other such limitations that might occur in reality if the decisions, judgment, and transfer of information by the human elements such as controllers and pilots were perfect

The dynamic-simulation experiments, as the next step, are a closer approach to realism but are more expensive, and the differences between the graphical and dynamic results then reflect the effect of the human element on the system

The FIL-TDEC simulation program under discussion was directed toward investigation of the following specific factors:

1 The effect of the sequence of arrival intervals on aircraft delays

2 The correlation between ideal-, graphical-, and dynamic-simulator results to determine the degree of conclusiveness with which the dynamic results, and eventually results in actual practice, can be predicted on the basis of ideal or graphical results The possible extrapolation of these results for other situations and conditions was also investigated

3 The effect of limiting the random flow of arrivals to 35 for each hour, as against the effect of allowing the peak flow of aircraft into the terminal area to be random (at an average rate of 35 per hour for 3 hours)

4 The effect of interspersing departures on single or intersecting runways and the effect of intermixing landings and departures as against segregating them by groups

5 The effects of each particular configuration and the system, including facility layout, traffic rules, and the human factor, on system efficiency and the amount of delay chargeable to each factor

6 The correlation between communications densities, waiting times, number of messages, and lengths of communications and aircraft delays

7 The range of variation in the efficiency of experienced controllers and the effect of time on duty, traffic rates, and traffic loading on individual efficiency

8 The performance or relative efficiency of controllers in manually vectoring aircraft to the outer marker while maintaining safe but not too conservative separations

9. The determination of the length of traffic samples required to approach equilibrium and the effect of demand and capacity on this factor

10 The significance of other parameters such as the number of operations moved per hour, per half hour, and per 15 minutes and the correlation between various parameters such as the number of aircraft in the system versus the number of aircraft being delayed

11 The determination of the ideal, graphical, and dynamic acceptance rates or capacities which are effective for the various conditions of wind, types of traffic samples, lengths of traffic samples, and separation rules

12 The determination, for system efficiency, of a valid comparative index such as an effective acceptance rate, the aircraft delays, or the controller workload

13 The advantages and disadvantages of providing the approach controllers with a type of computer, multipath-selection device for assigning aircraft tracks between the inner feeding stacks and the outer marker

14 The determination of the most effective methods of integrating jet aircraft into the 40-mile-diameter approach system of a terminal area while making due allowance for their high cruising speeds, high cruising altitudes, and large rate of fuel consumption and the effect of such integration on controller workload, safety, communications, and delay to other aircraft

15 The establishment of mathematical analogs to describe the interactions of the variables involved and the determination of the degree of conclusiveness of these analogs in describing and predicting actual situations of the present and future

16 The determination of outer-marker separations which can be used most safely and effectively for present-day distributions of random-aircraft performance

17 The effect of the location of the outer marker on these separations

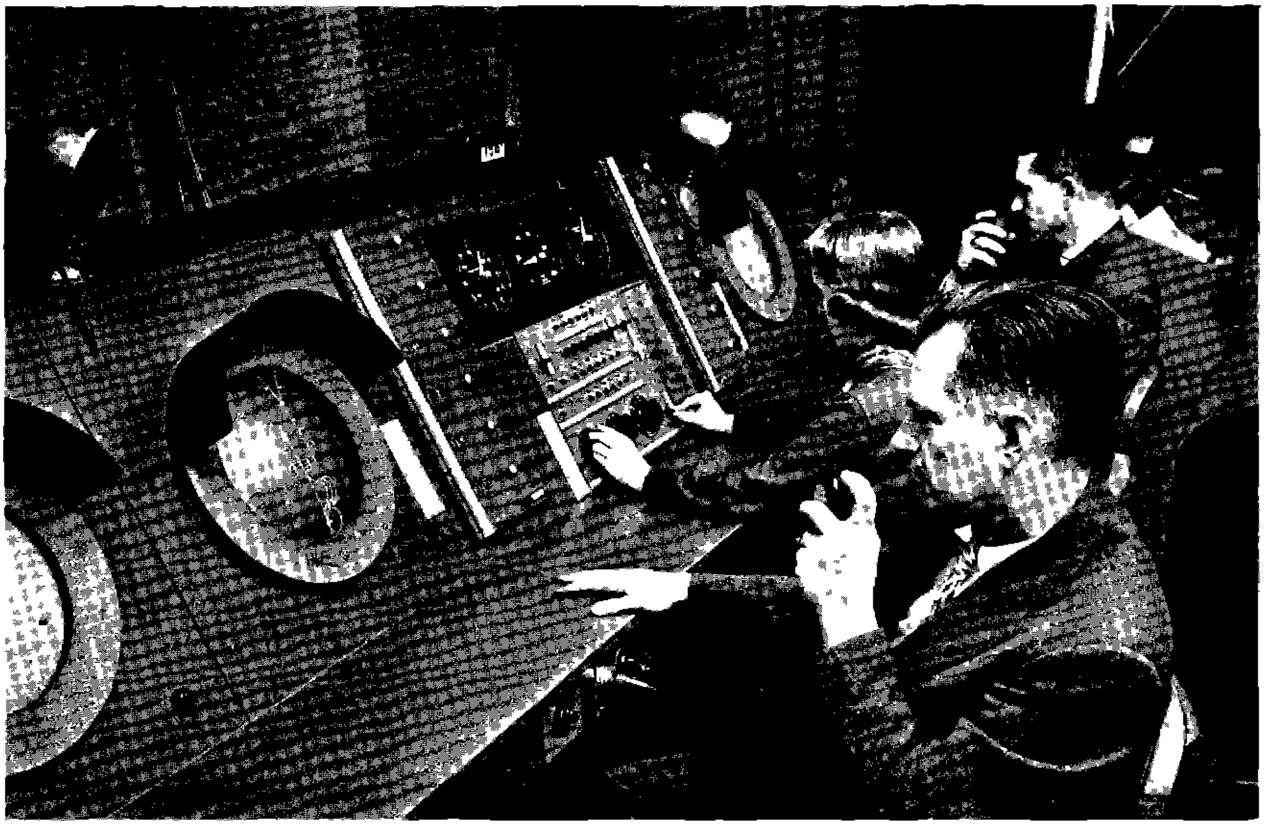

Fig 2 Simulator Approach-Control Arrangement Including EMTAC

18 The determination of the benefits to be obtained from the use of a limited degree of speed control on the glide path

19 The effects of using nonuniform separations for various sequences of slow, medium, and fast aircraft as compared with using constant but conservative separations regardless of sequence

20 The determination of the resulting effects of feeding the runway at increased rates

ESTABLISHMENT OF CONFIGURATIONS, RULES, PROCEDURES, AND TRAFFIC SAMPLES

Terminal-Area Configuration

In all terminal-area configurations employed in these simulation tests, the arrival altitudes and holding patterns were arranged to permit use of standard missed-approach procedures and proper nonconflicting departure routes and altitudes

WNA Phase 2 Twin-Stack Configuration

The terminal-area twin-stack configurations analyzed in prior FIL-TDEC simulation work were identified as WNA Phases 2, 6, and 7 and were described in a previous report [4]

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[4] Berkowitz and Doering, op cit

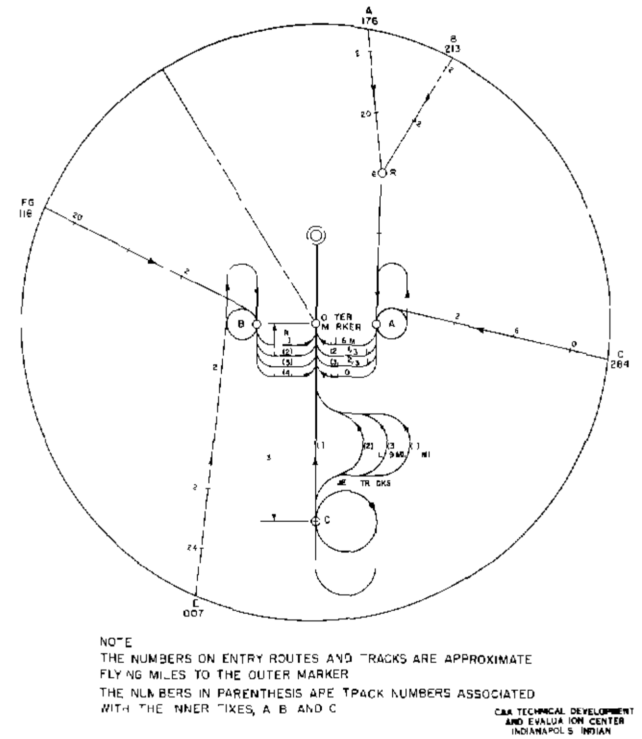

Fig 3 WNA Phase 2A Terminal-Area Multitrack Configuration

Two approach controllers were employed in the dynamic tests, one for the east sector and one for the west sector Although the delays in Phase 2 were slightly less than those in the other two phases for the particular conditions (0 wind and theoretical arrival loading of 97 per cent for one-hour peak) tested, they were of little significance statistically insofar as delays and capacity were concerned However, the close-in symmetrical layout of the Phase 2 inner stacks facilitated the job of establishing precise separation between aircraft Furthermore, with present separation standards, identification and co-ordination problems were held to a minimum since no more than an optimum number of three aircraft need ever be en route simultaneously between the inner fixes and the approach gate This fact also helped in allowing for sandwiching of departures between arrivals at high rates

Another advantage of Phase 2 over the others was that the A-B inner fixes lay closest to the minimum paths between the major entry airways and the approach gate (outer marker) This made for less "dog-legging" and thus for smaller over-all delays during short periods of heavy loading or longer periods of lighter loadings Therefore, except for Groups 1 and 2 of the simulation program, the Phase 2 configuration was featured in the comparative investigations of this contract period A group is defined as a set of conditions involving the type and composition of the traffic sample, the set of separations used, and the wind conditions

WNA Phase 2A Multitrack System

The basic Phase 2 configuration was modified to evaluate a system which used a special approach computer, cockpit pictorial computers, and the associated less complicated communications Conceived by TDEC personnel, this system was known as the Phase 2A multitrack system It is described in another report[5] and is discussed further in Technical Development Report No 222 [6] Phase 2A was essentially the same as Phase 2, except that, instead of point-to-point vectoring instructions, four standardized approach tracks 6, 7 1/3, 8 2/3, and 10 miles in length were provided between the inner A-B twin holding fixes and the outer marker These

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[5] C M Anderson and T K Vickers, "Application of Simulation Techniques in the Study of Terminal-Area Air Traffic Control Problems," CAA Technical Development Report No 192, November 1953

[6] Berkowitz and Doering, op cit

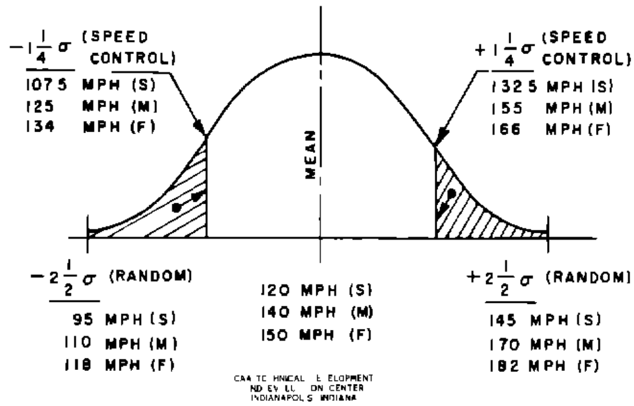


Fig 4 Random Distribution of Approach Speeds

tracks were used as a sort of moving-block system for conventional piston-engined aircraft An approach-control computer especially developed by TDEC for this purpose was used by the approach controllers Figure 2 is a photo of the simulated arrangement of the computer panel, associated radar scopes, east- and west-sector layout, limited data-transfer device, and other components

For jets, a simple jet-letdown path was provided between the outer fixes and the outer marker for the cases where jets cannot make a straight-in approach, and an inner jet fix was established 13 miles south of the outer marker at Fix C See Fig 3 The rules which were assumed to be in effect for jets were as follows

1 They would cruise at an airspeed of 400 mph, with an intermediate speed of 300 mph and an approach speed of 180 mph
2 The average en route descent rate would be 3000 fpm
3 Approaches would be started from the outer approach fixes at 20,000 feet, and the aircraft would descend to 8000 feet by the time they reached the inner C fix
4 Descent would be made at an indicated air speed of 300 mph and would slow to 180 mph for the last stage of the approach
5 Turns would be made at half standard rate, 1 1/2 degrees per second
6 Any holding necessary would be done at 20,000 feet at approach speed
7 The straight-in final-approach path to the outer marker would be at least 13 miles long
8 The approach-control system would establish an irrevocable landing-sequence number before the jet descended below 20,000 feet altitude As seen in Fig 3, jet tracks between the inner fix and the outer marker were 13, 16, 19, and 22 miles long

Rules for Departure Separations

Rules for interspersing departures on a single runway are given in Technical Development Report No 222

Optimum Outer-Marker Arrival Separations and Wave-Off Probabilities for 5 3-Mile Glide Slope With No Speed Control

Prior probability analysis showed that, because of present random variability in aircraft performance during IFR conditions, the attempted use of a uniform radar separation of 3 mile would result in an abnormally high number of wave-offs At Washington National Airport, this condition applied to the 5 3-mile glide slope and to Runway 36

For this reason, nonuniform (called optimum) separations were derived, and minimum outer-marker separations corresponding to each sequence of speed classes were assigned These optimum separations, which averaged out to about 4 miles, reduced the probability of a glide-slope wave-off to about 1 per cent under saturated traffic conditions A total probability of 1 per cent wave-offs on the glide slope was established as the criterion A probable separation of less than 2 5 miles on the glide slope was presumed to result in a wave-off Table I shows the maximum theoretical acceptance or runway feeding rates possible with the use of th optimum separations to feed projected Washington National Airport peak-hour traffic samples

TABLE I

MAXIMUM THEORETICAL ACCEPTANCE RATES WITH OPTIMUM SEPARATIONS AND NO SPEED CONTROL

Condition	Zero Wind	20-mph Headwind
All arrivals land at Washington National Airport	33 0	26 6
Military arrivals land at Bolling Air Force Base or Anacostia Naval Air Station, all other arrivals land at Washington National Airport	36 1	29 8

optimum separations to feed projected Washington National Airport peak-hour traffic samples which consist of the following distribution of aircraft categories

Slow 29 per cent to Washington National Airport, 14 per cent to Bolling Field
Total 43 per cent

Medium. 30 per cent to Washington National Airport, 3 per cent to Bolling Field.
Total 33 per cent

Fast 22 per cent to Washington National Airport, 2 per cent to Bolling Field
Total 24 per cent

Slow aircraft included DC-3, Lodestar, Twin Beech, and C-46 aircraft, the medium category represented Martin, Convair, and DC-4 types, fast aircraft included DC-6, Constellation, and Stratocruiser types No jet aircraft were included

Even with these optimum outer-marker separations, it was found that the present layout of WNA instrument Runway 36 caused the number of probable runway wave-offs due to the runway-occupancy rule still to be too high at approximately 16 per cent Analysis showed that an additional high-speed exit located at 3400 feet would be needed This exit could serve both directions of the 6800-foot instrument runway and would reduce the theoretical number of probable runway wave-offs to about 2 per cent The existence of this additional runway exit was assumed in all subsequent analyses therefore, and the 2 per cent runway wave-off probability was considered acceptable in view of the assumptions and the theoretical nature of the analysis

The implications of the preceding probabilities are evident Large random spreads of aircraft performance in IFR conditions contribute significantly to the conservatively large separations used in a safe traffic-control system As shown later in this report, increasing the acceptance rate just a little can appreciably reduce delays during peak traffic periods Accordingly, other optimum outer-marker minimum separations were derived in which a very limited degree of speed-control on the glide slope was assumed. See Table II For lack of a better term, these separations are called speed-control optimum separations

In the previous derivation of the optimum separations when no speed control was used, the distributions of the performance variables were found from actual measurements to be normal, Gaussian, cut off at the 2 1/2 σ points The symbol σ is the standard deviation, which is equal to the square root of the average of the squares of the performance deviations about the mean This means that about two-thirds of all of the aircraft have a dispersion in performance which falls within $\pm$ 1 standard deviation about the mean, 95 per cent lie within $\pm$ 2 standard deviations, and about 99 per cent lie within $\pm$ 2 1/2 standard deviations, $\pm$ 2 1/2σ

In deriving the speed-control optimum separations and in computing the resulting probable glide-slope wave-offs, it was assumed that the aircraft performance distributions with speed control would be normal truncated types and would be truncated at the $\pm$ 1 1/4 σ points instead of at $\pm$ 2 1/2 σ as in the random case The standard deviations used were based on available IFR operational data on the spread in glide-slope air speeds, 10 mph for slow, 12 mph for medium, and 12 8 mph for fast aircraft These random normal and truncated speed-control distributions are shown in Fig 4

Analytically, control was exercised by the assumption of the speed-up of any random aircraft speeds on the left-hand side (Fig. 4) between the - 2 1/2 σ and the - 1 1/4 σ points (shaded) to the - 1 1/4 σ point, likewise, aircraft on the right-hand side between the + 2 1/2 σ

TABLE II

SUMMARY OF OPTIMUM OUTER-MARKER MINIMUM SEPARATIONS AND PROBABLE GLIDE-SLOPE WAVE OFFS

Conditions Speed control; zero wind; and 5 3-mile glide slope (2 1/2-mile glide-slope wave-off minimum)

Sequence Aircraft No 1	Sequence Aircraft No 2	Outer-Marker Minimum Separation Distance (miles)	Outer-Marker Minimum Separation Time* (seconds)	Probability of Sequence	Probability of Glide-Slope Wave-Offs	Total Probability of Glide-Slope Wave-Offs
S	S	3 6	108	0 18	0 018	0 0032
S	M	4 6	118	0 14	0 017	0 0023
S	F	5 1	122	0 10	0 000	0 0000
M	S	2 8	84	0 14	0 018	0 0025
M	M	3 6	92	0 11	0 014	0 0015
M	F	4 1	97	0 08	0 014	0 0011
F	S	2 5	75	0 10	0 000	0 0000
F	M	3 2	83	0 08	0 017	0 0014
F	F	3 6	86	0 07	0 017	0 0012
Total, all sequences						0 0132 (1 3%)

*Based on mean approach speeds of 150 140 and 120 mph for F, M, and S, respectively

and + 1 1/4 σ points (shaded) are assumed to have been slowed down to the + 1 1/4 σ point Thus we have a normal truncated distribution of approach speeds with a maximum over-all spread between - 1 1/4 σ and + 1 1/4 σ in average glide-slope airspeed of 25 mph for slow, 30 mph for medium, and 32 mph for fast aircraft

Physically interpreted, this means that if a DC-3 were found to be flying down the glide-slope at such a speed as 140 mph the pilot would be asked to slow down but no airspeed figure would be stated It seems reasonable that he might slow down to perhaps 132 mph indicated airspeed Likewise it seems reasonable to assume that if a Constellation were approaching at 120 mph there would be no objection to advising that pilot to speed up, for analytical purposes, it might be assumed that he speeds up to 134 mph, which is still below the Constellation's mean approach speed of approximately 150 mph Although these cases are relatively rare, they must be taken into account in the over-all picture because the controllers must consider them in establishing safe, yet not too conservative, separations It may be argued that in the case of the Constellation the increased speed of 134 mph is still too slow for this type of aircraft In such a case, if the pilot speeds up even more it is so much the better for the analysis

Speed-control optimum outer-marker minimum separations, shown in Table II, were established on the assumption that all aircraft in the infinite traffic samples land at Washington National Airport It is seen that, with these separations and with the small degree of speed control assumed, the total probability of glide-slope wave-offs is 1 3 per cent in zero wind

Speed-control optimum outer-marker separations under 20-mph-headwind conditions were established for the sequences involving military arrivals which are fed through the Washington National Airport outer marker, descend to circling altitude, and go to Bolling or Anacostia fields These separations are shown along with the zero-wind separations in Table III, which also shows the previously determined no-speed-control optimum outer-marker separations

Optimum Separations for 3 0- and 7 0-Mile Outer Marker-to-Runway Distances

In the probability analysis described, optimum nonuniform separations for both zero and 20-mph headwind with and without the small degree of glide-slope speed control were established for only the 5 3-mile glide slope similar to that of Washington National Airport, and any separation of less than 2 1/2 miles on the glide slope was presumed to result in a wave-off

In order to explore the effects of shortening or lengthening the glide slope, similar probability analyses were also performed for glide-slope lengths of 3 and 7 miles and optimum outer-marker separations were established for both the speed-control and the no-speed-control conditions Inasmuch as future practices, radar dependability, pilot education, and other variables may permit smaller glide-slope separations as low as 1 1/2 miles, the analysis was extended to determine the corresponding optimum outer-marker separations This latter analysis included the 5 3-mile distance, since in the previous study only the 2 1/2-mile minimum separation was considered as the wave-off criterion

TABLE III

SUMMARY OF OPTIMUM MINIMUM OUTER-MARKER SEPARATIONS

Conditions: 5 3-mile glide slope, zero and 20-mph wind, both with and without speed control; minimum allowable separation of glide slope, 2 1/2 miles

Sequence		Zero Wind Outer-Marker Minimum Separations				20-mph Wind Outer-Marker Minimum Separations			
		Distance		Time*		Distance		Time*	
Aircraft No 1	Aircraft No 2	No Speed Control (miles)	Speed Control Assumed (miles)	No Speed Control (seconds)	Speed Control Assumed (seconds)	No Speed Control (miles)	Speed Control Assumed (miles)	No Speed Control (seconds)	Speed Control Assumed (seconds)
S	S	4 0	3 5	122	105	4 25	3 75	153	135
S	M	5 0	4 5	129	116	5 75	5 0	172	150
S	F	5 75	5 0	138	120	6 5	5 5	180	153
M	S	3 0	3 0	90	90	3 0	3 0	108	108
M	M	4 0	3 5	103	90	4 25	3 75	127	112
M	F	4 5	4 0	108	96	4 75	4 0	131	111
F	S	3 0	3 0	90	90	3 0	3 0	108	108
F	M	3 5	3 25	90	84	3 75	3 5	112	105
F	F	4 0	3 5	97	84	4 25	3 75	118	104
Military Mixtures									
Any Type	S or S_m	3 0	3 0	90	90	3 0	3 0	108	108
	M or M_m	3,0	3 0	77	77	3 0	3 0	90	90
	F or M_m	3 0	3 0	72	72	3 0	3 0	83	83

Notes: A military mixture is any sequence involving one military and one nonmilitary aircraft The amount of separation is determined from the type of Aircraft No 2 without regard to the type of Aircraft No 1

S_m is a military slow, M_m is a military medium, etc

*Time separations are based on mean approach speeds of 150, 140, and 120 mph for F, M, and S, respectively

As in the previous analyses, the criterion used for determining the outer-marker separations was a total of no more than about 1 per cent of probable glide-slope wave-offs for the nine sequences of slow, medium, and fast aircraft A total of 18 conditions were thus analyzed For each of the three outer-marker distances (3 0, 5 3, and 7 0 miles), two conditions (speed control or no speed control) of glide-slope performance were assumed, and for each of these six conditions, three minimum glide-slope separations (2 1/2, 2, or 1 1/2 miles) were considered

It should be pointed out that the probable glide-slope wave-offs considered in these studies are chargeable to the traffic-control system itself with all its ramifications and anticipated random aircraft performance These probable wave-offs do not include wave-offs caused by pilot error or pilots' choice to go around again. The applicability of the available data and assumptions employed are necessarily limited In order to keep the studies as realistic as possible, these data are used and extended to other situations only where they are considered reasonably valid Consequently, the results are indicative of relative effects but have quantitative implications

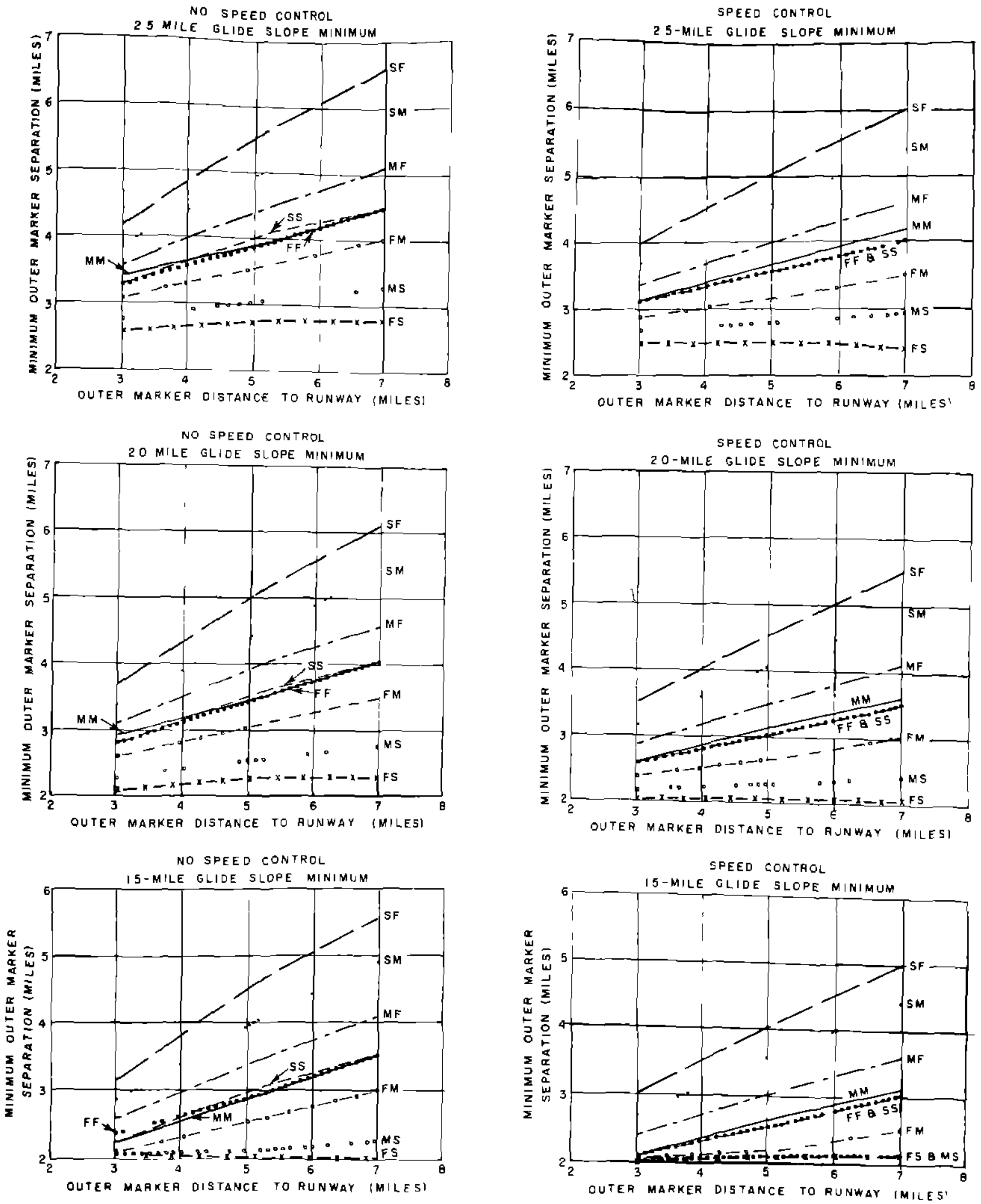
The theoretical optimum outer-marker separations for the 18 conditions of outer-marker distances, performance distributions, and glide-slope wave-off criteria are summarized in

TABLE IV

SUMMARY OF THEORETICAL MAXIMUM RUNWAY FEEDING RATES

Conditions Optimum outer-marker separations, without and with speed control 5 3 mile glide slope zero and 20-mph wind

Condition	Zero Wind		20-mph Headwind	
	No Speed Control	Speed Control	No Speed Control	Speed Control
All arrivals land at Washington National Airport	33 0	36 3	26 6	29 7
Military arrivals land at Bolling Air Force Base or Anacostia Naval Air Station, all others land at Washington National Airport	36 1	38 5	29 8	31 6



NOTE
ZERO WIND

LEGEND
F - FAST AIRCRAFT
M MEDIUM AIRCRAFT
S - SLOW AIRCRAFT
SF SLOW FOLLOWED BY FAST ETC

Fig 5 Optimum Outer-Marker Minimum Separations as a Function of Outer-Marker Distance

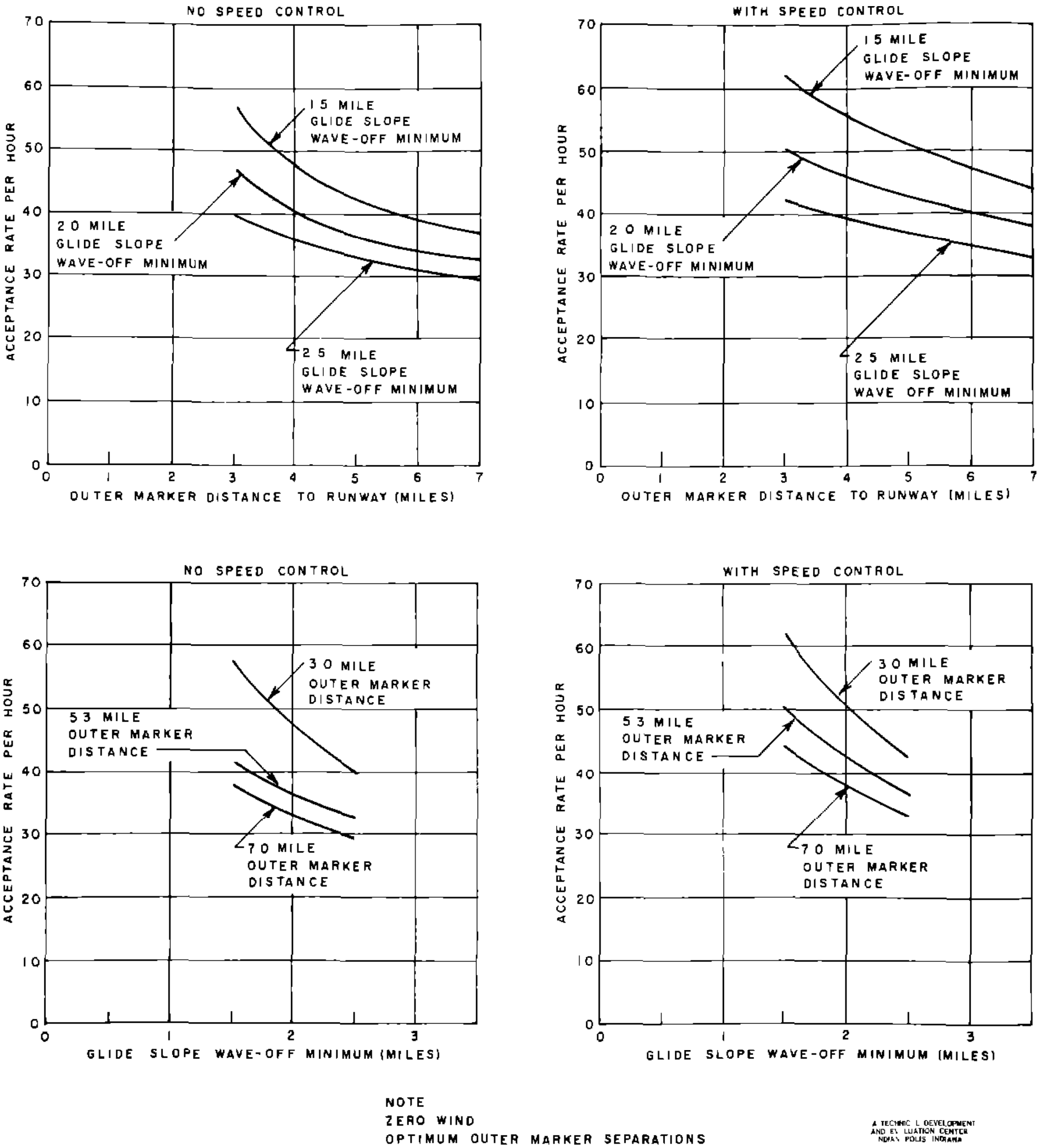


Fig 6 Theoretical Maximum Acceptance Rate as a Function of Glide-Slope Minimum Separation and Outer-Marker Separations

Table V Also shown in this table are the resulting theoretical acceptance or runway feeding rates per hour from 29 5 for the outer-marker distance of 7 miles to 62 2 for the 3-mile outer-marker distance It is assumed for analytical purposes that all aircraft, including the military, land at Washington National Airport Figure 5 is a plot of outer-marker optimum separations for the nine sequences of slow, medium, and fast aircraft as a function of outer-marker

TABLE V

OPTIMUM OUTER-MARKER MINIMUM SEPARATIONS AND RUNWAY FEEDING RATES
FOR VARIOUS OUTER-MARKER DISTANCES AND GLIDE-SLOPE WAVE-OFF CRITERIA

Conditions 0 wind, all aircraft land at Washington National Airport

Sequence		Glide-Slope Minimum (miles)	Separations Without Speed Control			Separations With Speed Control		
Speed Class Aircraft No 1	Speed Class Aircraft No 2		3 0-Mile Glide Slope (miles)	5 3-Mile Glide Slope (miles)	7 0-Mile Glide Slope (miles)	3 0-Mile Glide Slope (miles)	5 3-Mile Glide Slope (miles)	7 0-Mile Glide Slope (miles)
S	S	2 5	3 3	4 1	4 5	3 1	3 6	4 0
S	M	2 5	3 9	5 1	5 9	3 7	4 6	5 4
S	F	2 5	4 2	5 7	6 6	4 0	5 1	6 0
M	S	2 5	2 8	3 1	3 3	2 7	2 8	2 9
M	M	2 5	3 4	4 0	4 5	3 1	3 6	4 1
M	F	2 5	3 6	4 5	5 1	3 4	4 1	4 6
F	S	2 5	2 6	2 8	2 8	2 5	2 5	2 4
F	M	2 5	3 1	3 6	4 0	2 9	3 2	3 5
F	F	2 5	3 3	4 0	4 5	3 1	3 6	4 0
Theoretical Acceptance Rate		2 5	40 0	33 0	29 5	42 3	36 3	33 0
S	S	2 0	2 8	3 6	4 0	2 6	3 1	3 5
S	M	2 0	3 4	4 6	5 4	3 2	4 1	4 9
S	F	2 0	3 7	5 2	6 1	3 5	4 6	5 5
M	S	2 0	2 3	2 6	2 8	2 2	2 3	2 4
M	M	2 0	2 9	3 5	4 0	2 6	3 1	3 6
M	F	2 0	3 1	4 0	4 6	2 9	3 6	4 1
F	S	2 0	2 1	2 3	2 3	2 0	2 0	1 9
F	M	2 0	2 6	3 1	3 5	2 4	2 7	3 0
F	F	2 0	2 8	3 5	4 0	2 6	3 1	3 5
Theoretical Acceptance Rate			47 1	35 8	33 2	50 4	42 1	37 7
S	S	1 5	2 3	3 1	3 5	2.1	2 6	3 0
S	M	1 5	2 9	4 1	4 9	2 7	3 6	4 4
S	F	1 5	3 2	4 7	5 6	3 0	4 1	5 0
M	S	1 5	1 8	2 1	2 3	1 7	1 8	1 9
M	M	1 5	2 4	3 0	3 5	2 1	2 6	3 1
M	F	1 5	2 6	3 5	4 1	2 4	3 1	3 6
F	S	1 5	1 6	1 8	1 8	1 5	1 5	1 4
F	M	1 5	2 1	2 6	3 0	1 9	2 2	2 5
F	F	1 5	2 3	3 0	3 5	2 1	2 6	3 0
Theoretical Acceptance Rate			57 3	41 4	38 0	62 2	50 1	43 9

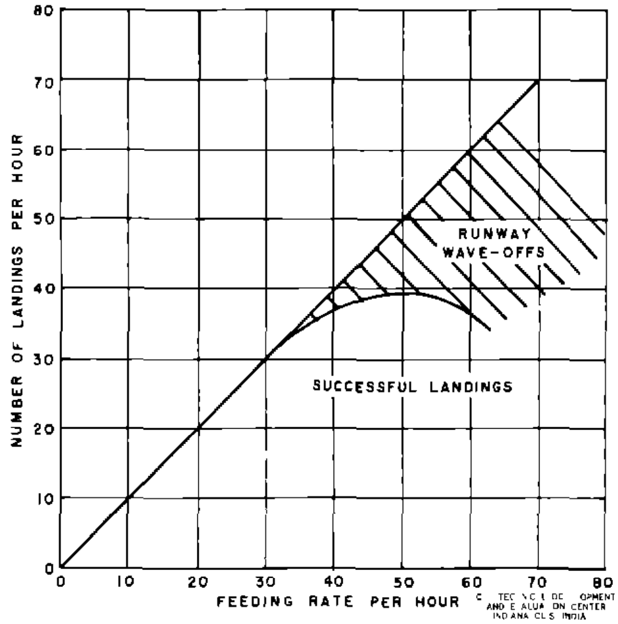


Fig 7 Number of Landings as a Function of Feeding Rates to the Runway With Zero Wind

distance to the runway Figure 6 cross-plots the data to show the theoretical acceptance rates as a function of outer-marker distance with the glide-slope wave-off separation minimum as the independent parameter, and as a function of glide-slope wave-off separation minimum with the outer-marker distance as the independent parameter

The data are sufficiently comprehensive on a comparable basis to permit interpolation for other outer-marker-to-runway distances, glide-slope minima, and aircraft-performance distributions Shorter outer-marker distances, smaller acceptable glide-slope minima, and less spread in aircraft performance permit smaller separations and thus permit increased acceptance rates These trends are clearly shown in Fig 6 Even relatively small changes yield significant gains

In this preliminary study, it will not be argued whether the shorter glide-slope lengths are sufficient for pilots to line up on the localizer and glide slope, whether pilots will submit to the very small degree of speed control described previously, or whether the 2- or even 1 1/2-mile minimum separation on the glide slope is acceptable It suffices that the gains are significant and therefore deserve serious consideration in view of the complexities and cost involved in increasing the capacity of present airports and airways via additional airports, runways, and radio facilities Certainly a little pilot education and appreciation of the cited implications, plus the use of zero readers and the use of automatic or semiautomatic approach couplers,[7] offer attractive possibilities for achieving these gains

Effects of Force-Feeding the Runway at the Higher Acceptance Rates

In view of the described increase in acceptance or runway feeding rates and in view of previous studies showing the possible pyramiding of probable runway wave-offs at higher feeding rates, these force-feeding effects were further analyzed The runway analyzed was Washington National Airport Runway 36 with an assumed high-speed exit at 3400 feet, the rules and random runway-performance distributions were given in Technical Development Report No 222 [8] A probable runway wave-off was presumed to occur if a succeeding landing aircraft appeared beyond the wave-off point (1000 feet from the runway threshold) before the preceding landing aircraft cleared the runway

[7] C M Anderson, N R Smith, T K Vickers, and M H Yost, "A Preliminary Investigation of the Application of the Tangential Approach Principle to Air Traffic Control," CAA Technical Development Report No 149, October 1951 (Limited Distribution)

[8] Berkowitz and Doering, op cit

TABLE VI

SCHEDULED ARRIVAL AND DEPARTURE RATES, TYPE I SAMPLES

Period of Operations (half hour)	Total Number of Scheduled Arrivals		Total Number of Scheduled Departures	
	(half hour)	(hour)	(half hour)	(hour)
0 to 1/2	7		5 (including 1 military)	
1/2 to 1	11		8 (including 1 military)	
		18		13
1 to 1 1/2	15		11 (including 2 military)	
1 1/2 to 2	20		14 (including 3 military)	
		35		25
2 to 2 1/2	12		17 (including 4 military)	
2 1/2 to 3	18		13 (including 2 military)	
		30		30
Totals		73		68

The results are shown in Fig 7 in a plot of successful landings per hour as a function of the runway feeding rate The shaded portion of the graph indicates the number of probable runway wave-offs The percentage of theoretical wave-offs ranged almost exponentially from 1 2 per cent for the feeding rate of 29 5 per hour with a 7-mile outer-marker distance, no speed control, and the 2 1/2-mile glide-slope minimum separation to 45 4 per cent for the feeding rate of 62 2 per hour with a 3-mile outer-marker distance with speed control and the 1 1/2-mile glide-slope minimum separation It is shown that with present rules, unless even more plentiful and more judicious spacing of high-speed runway exits are made, force-feeding the runway at rates higher than about 36 arrivals per hour yields very few more landings (39 maximum) Further force-feeding yields even fewer successful landings At any rate, probable runway wave-offs of more than about 5 per cent are considered intolerable This 5 per cent figure, although too high in actual practice, is considered negligible in these analytical studies in view of its theoretical nature and the limitations in the assumptions made

It is obvious that with imminent increases in terminal-area capacity, more attention should be paid first to the runway layout and next to the airport layout including loading ramps, taxi strips, and warm-up pads in order that any potential gains do not result in other, more serious bottlenecks

Formulation and Construction of the Traffic Samples

Type I Samples (One Hour Peak of 35 Arrivals)

In previous work, three futuristic traffic samples of arrival and departure aircraft were employed [9,10] They were based on a traffic survey of IFR and visual-flight-rule (VFR) activity at Washington National Airport These samples, designated as Type I, were originally of three hours duration and included a one-hour loading period, a one-hour peak period, and a one-hour unloading period The half-hourly rates for each of the three samples are shown in Table V

After a number of trial runs, it was decided to eliminate the first and last half-hours because they proved to be of little consequence for comparative purposes These Type I samples had a one-hour peak period of 35 arrivals, a half-hour loading period, and a half-hour unloading period — a total of 2 hours duration, hours 1/2 to 2 1/2 on Table VI All three samples had similar over-all characteristics such as rates, distribution of aircraft types, and distribution of entry points and differed only in the random distribution and sequencing of time intervals, aircraft types, and entry points The arrival samples consisted of 43 per cent slow (29 per cent to Washington National Airport and 14 per cent military), 32 1/2 per cent medium (30 per cent to Washington National Airport and 2 1/2 per cent military), and 24 1/2 per cent fast aircraft (22 per cent to Washington National Airport and 2 1/2 per cent military) No jets were included Twenty-four per cent of all arrivals were presumed to enter on route A, 22 per cent on route B, 12 per cent on route C, 15 per cent on route E, 8 per cent on route F, and 19 per cent on route G

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[9] Berkowitz and Doering, op cit

[10] Ruth R Doering, "Procedure for Constructing Traffic Samples," FIL Working Paper No 1, Project 2343, February 6, 1953 (Limited Distribution)

TABLE VII

CONSTRUCTION OF TYPE II SAMPLES OF ARRIVALS

Sample No	Hour	Construction*		
		Hour	Type	Sample
1	0 to 1	1 to 2	I	1
1	1 to 2	1 to 2	I	2
1	2 to 3	1 to 2	I	3
1	3 to 4	1 to 2	I	1
2	0 to 1	1 to 2	I	2
2	1 to 2	1 to 2	I	1
2	2 to 3	1 to 2	I	3
2	3 to 4	1 to 2	I	2
3	0 to 1	1 to 2	I	3
3	1 to 2	1 to 2	I	2
3	2 to 3	1 to 2	I	1
3	3 to 4	1 to 2	I	3

*Arrivals only were used

Some of the groups (Groups 1 through 4 and Groups 14 and 22) of the simulation work programmed on the present contract employed these Type I, one-hour-peak samples, as outlined later in this report

Type II Flow-Control Samples

Three 2-hour, three 3-hour, and three 4-hour samples with 35 arrivals scheduled for each hour were constructed They were made up from the peak hours 1 to 2 (35 arrivals) of the three Type I samples and are called Type II flow-control samples No jets were included In the present FIL graphical and ideal analysis, 2-hour, 3-hour, and 4-hour Type II samples of arrivals were used The dynamic-simulator experiments used only the 3-hour samples (see section of this report entitled "Program of Simulation Experiments"), a total of 105 arrivals at 35 per each hour The Type II samples were constructed as indicated in Table VII, and no loading or unloading periods were included

Two wind conditions were specified in the simulation experiments, zero wind and 20-mph headwind Six Type II, 3-hour samples of arrivals were prepared for use at TDEC 2 wind conditions x 3 random samples

Type IIJ Flow-Control Samples

The three Type II, 3-hour samples of arrivals were modified to substitute four jet aircraft during each hour and were therefore called Type IIJ Four of the 35 arrivals during each hour, a total of 12 jets for the 3-hour period of 105 arrivals, were changed arbitrarily from their original random speed classes to the jet class These Type IIJ samples were used in connection with the Group 5J runs and utilized the Phase 2A multitrack configuration shown in Fig 4

Type III, No-Flow-Control Samples

Three new 3-hour random samples of arrivals were constructed for use in the ideal-, graphical-, and dynamic-simulation tests They are called Type III no-flow-control samples

TABLE VIII

SCHEDULED ARRIVAL RATES TYPE III RANDOM SAMPLES OF 3 HOURS

Period of Operation	Sample No 1 Arrivals		Sample No 2 Arrivals		Sample No 3 Arrivals	
	(half hour)	(hour)	(half hour)	(hour)	(half hour)	(hour)
0 to 1/2	18		17		15	
1/2 to 1	20		14		19	
		38		31		34
1 to to 1 1/2	16		25		18	
1 1/2 to 2	18		13		18	
		34		38		36
2 to 2 1/2	22		18		21	
2 1/2 to 3	11		18		14	
		33		36		35
3-hour totals		105		105		105

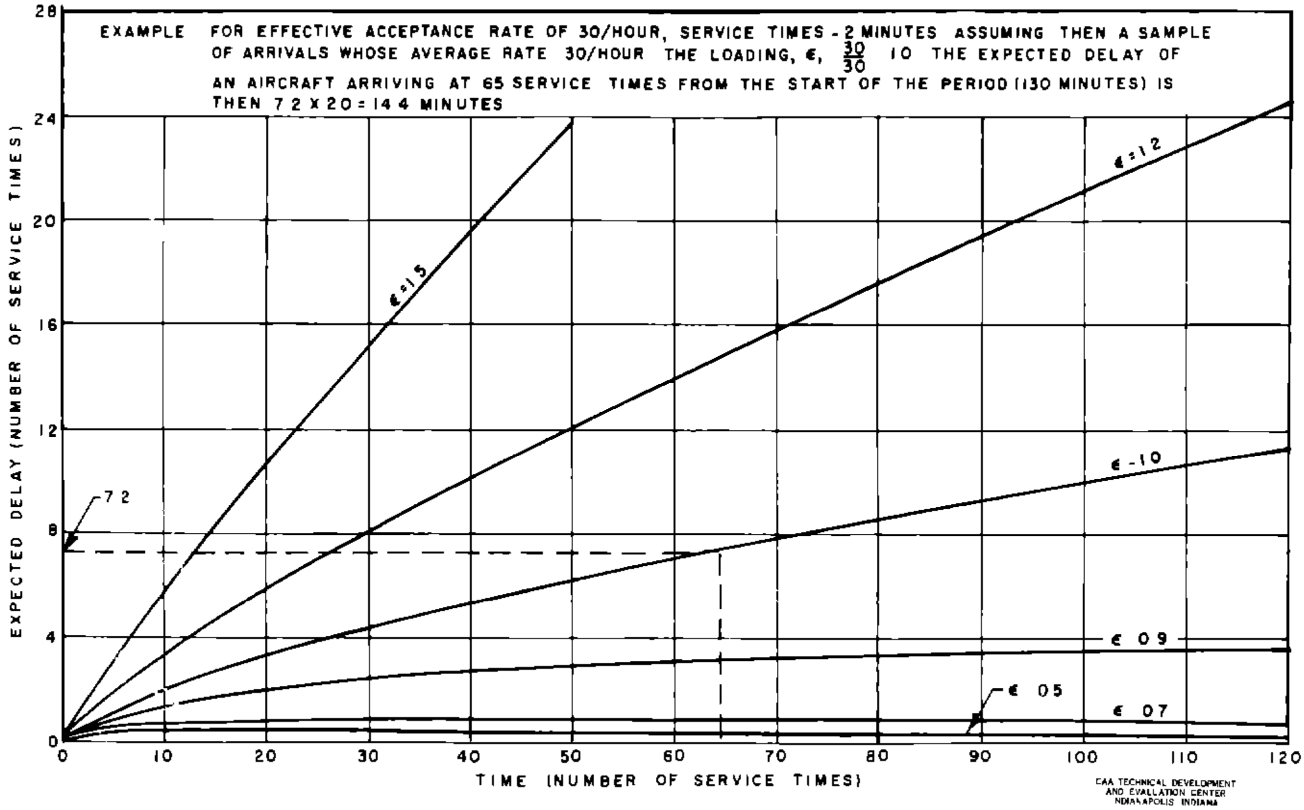


Fig 8 Queue Theory Showing Expected Delay of Aircraft Arriving at Time t for Several System Loadings

because the arrival rates for each period are random according to Poisson's rule, with an average rate of 35 per hour for the total 3 hours a total of 105 aircraft No jets were included The over-all characteristics of these random samples were the same as those used in the Type I samples of arrivals

Again, since two wind conditions (zero wind and 20-mph) were specified, six Type III 3-h samples of arrivals (3 random samples x 2 wind conditions) were constructed for use at TDEC The resulting random half-hourly and hourly arrival rates are as shown in Table VIII

THEORETICAL TRANSIENT AND STEADY-STATE DELAYS

Introduction

In past years, several air-traffic-control studies dealt with the estimation of delays Because of the difficulty of actually measuring delays, the investigators here and abroad have turned to mathematical models for their predictions The model most frequently used is the one developed for telephone traffic by Fry, Molina, and others It has been reasoned that a terminal area with a common glide slope is very similar to a telephone system with a single trunk Therefore, the steady-state telephone delay theory was readily adapted by many investigators to air-traffic delay theory

There are two objections to such an approach First, the utilization or loading of a telephone channel can never exceed 100 per cent because in the telephone system loading is the proportion of time in which the channel is used In air traffic control, loading is measured as a ratio of input (arrival) rate to output (acceptance) rate It is therefore possible, and it frequently happens, that the loading exceeds 100 per cent, especially for relatively short periods of time (one or two hours)

Second, the telephone delay theory assumes that the period being examined is in statistical equilibrium Whereas this assumption may hold for telephone traffic in which

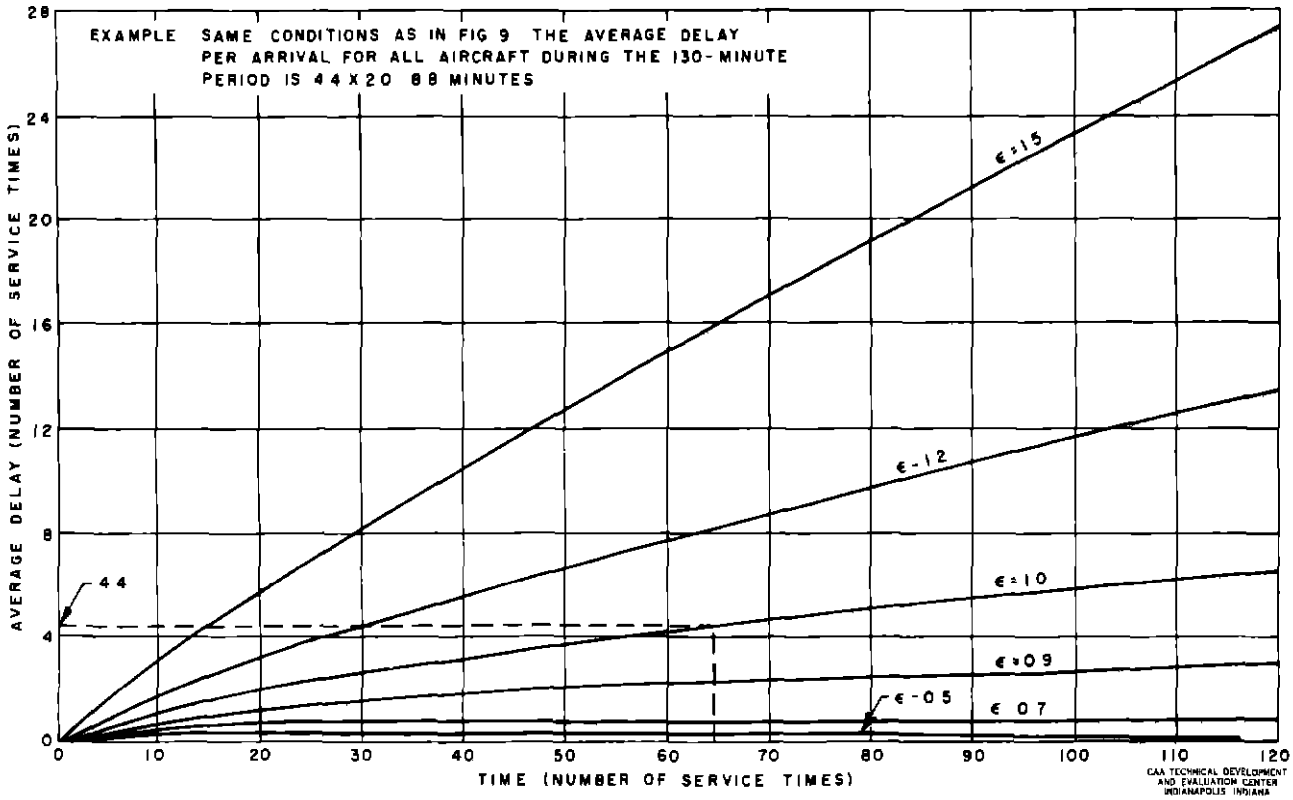


Fig 9 Queue Theory Showing Average Delay of All Aircraft During the Time From 0 to t for Several System Loadings

thousands of calls are processed in a few minutes, it does not necessarily hold true in air traffic control where 35 or 40 IFR arrivals or less per hour may saturate an airport

In view of these objections to the currently popular mathematical delay model, we have adopted here the more recently developed "queue theory " (The dictionary defines queue as a "waiting line ") The build-up of a stack is more analogous to the build-up of a queue than it is to that of telephone delays Furthermore, the queue model incorporates the input-output aspect of terminal-area control and includes the relationship of duration of loading to delays with no assumption about statistical equilibrium

The Model

In order to bring out the fundamentals of the model, the stochastic process [11] of landing aircraft in a terminal area may be described rather simply as follows.

Aircraft arrive essentially at random at the perimeter of the terminal area, from which point they are directed to the outer marker and down the glide slope via some prescribed path However, the restriction is made that once a plane has gone through the outer marker into the glide slope the next aircraft will not be permitted to follow in less than a given period of time This minimum separation time during which each aircraft in a sequence monopolizes the glide slope will here be called the service time

When the interval between arrivals is shorter than this service time, some aircraft are forced to delay, or to form a queue Regardless of the form of delay, be it holding, vectoring, or speed control, it will be assumed in this case that a delayed aircraft enters a stack In the case of a large number of delayed aircraft, they may spill over into outer stacks or may even be delayed en route Because this spillage is not relevant to the analysis, it is assumed that

[11] A stochastic process is one that can be described in general by averages, although the precise outcome or the internal dimensions cannot be predicted with certainty

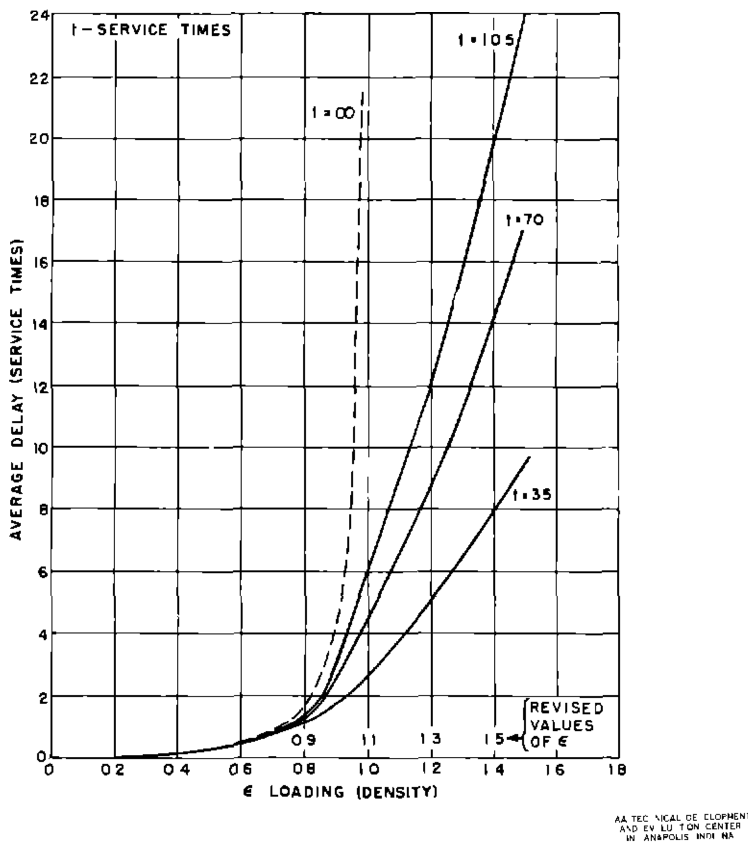


Fig 10 Queue Theory Showing Average Delay as a Function of Loading

all delayed aircraft enter one stack or queue which has an infinite capacity Such a simplification does not detract from any realism, yet it supplies a cleaner model with which to work

One further restriction put into this model is that the service time or separation be a constant for all aircraft using a given airport Although this factor is not precisely true, the effects of the distribution of actual separations are not appreciably different, as shown in Appendix A, and this restriction does not seriously affect the precision of the model

The mechanics of the model are described by the stochastic matrix discussed in Appendix A This matrix approach describes the distribution of stack size at any given time in terms of the conditions at the beginning of the congested period When the original matrix is known, the conditions at any time t are described by raising the matrix to the t^{th} power, time is measured in the number of elapsed service times

Calculation of Delays

In order to obtain a comprehensive picture of delays, seven matrices (0 3, 0 5, 0 7, 0 9, 1 0, 1 2, and 1 5) were formed for the following values of ϵ , when

$$\epsilon = \text{loading} = \frac{\text{arrival rate}}{\text{acceptance rate}} = \text{density}$$

While the matrices were easily obtained, the raising to higher powers presented a problem When the matrices, especially for the higher values of ϵ , were raised as high as the 32nd power, their sizes became formidable and unwieldy It was impractical to carry the expansion any further on desk calculators

For this reason, a different approach was used for obtaining these higher powers The matrices were expanded to the 16th power, and curves were fitted to the expected stack size See Appendix A Subsequent simulation tests indicated that the extrapolations resulting from this projection of curves produced fairly reliable estimates of expected stack size for moderately large values of t such as 100 With an acceptance rate of 30 arrivals per hour, 90 intervals represent three hours, an acceptance rate of 35 per hour gives 105 intervals in

TABLE IX

CONDITIONS ANALYZED AND SIMULATED

Condition	Sample Type	Theoretical Acceptance Rate	Speed Control	Wind (mph)	Theoretical Loading
1	I*	29 8	No	20	35/29 8 x 100 = 117 1/2%
2	I	31 6	Yes	20	35/31 6 x 100 = 111%
3	I	38 5	Yes	0	35/38 5 x 100 = 91%
4	II and III**	29 8	No	20	35/29 8 x 100 = 117 1/2%
5	II and III	29 8	No	20	35/29 8 x 100 = 117 1/2%
6	II and III	31 6	Yes	20	35/31 6 x 100 = 111%
7	II and III	31 6	Yes	20	35/31 6 x 100 = 111%
8	II, IIJ, and III	36 1	No	0	35/36 1 x 100 = 97%
9	II, IIJ, and III	36 1	No	0	35/36 1 x 100 = 97%
10	II, IIJ, and III	36 1	No	0	35/36 1 x 100 = 97%
11	II and III	38 5	Yes	0	35/38 5 x 100 = 91%
12	II and III	38 5	Yes	0	35/38 5 x 100 = 91%

*Type I has a one-hour peak of 35 arrivals with 1/2-hour loading and 1/2-hour unloading period
**Types II IIJ and III have two, three or four continuous hours of 35 arrivals per hour

three hours Therefore, the build-up of air-traffic delays for a period of about three hours can be described by this method

This extrapolation method helped in the determination of the expected delays and the average delays Unfortunately, it provided no measure of the distribution of delays around the expected values Such distributions were left for later study

There is a difference in the delay curves for values of ϵ less than 1 0 and for those values equal to or greater than 1 0 For $\epsilon <$ 1 0, the average delays approach asymptotic values The rate of approach and the asymptotic value attained depend solely on the magnitude

TABLE X

GROUPS AND CONDITIONS OF SIMULATION TESTS PERFORMED

Wind	Outer-Marker Minimum Separation	Acceptance Rate*	3 Type I Samples (2 Hrs of Arrivals Only or of Arrivals and Departures)		3 Type II Samples (3 Hrs of Arrivals)		3 Type III Samples (3 Hrs of Arrivals)		3 Type II Samples (2 Hrs of Arrivals)		3 Type II Samples (4 Hrs of Arrivals)		Sample No 2 of Type II Samples (Arrival Intervals Re equenced)		3 Type IIJ Samples (3 Hrs of Arrivals)	
			Group	Type of Analysis	Group	Type of Analysis	Group	Type of Analysis	Group	Type of Analysis	Group	Type of Analysis	Group	Type of Analysis	Group	Type of Analysis
(mph)																
0	3-mile uniform	41 3	A B***	I** I**	Not Scheduled Not Scheduled		Not Scheduled Not Scheduled		Not Scheduled Not Scheduled		Not Scheduled Not Scheduled		Not Scheduled Not Scheduled		Not Scheduled Not Scheduled	
20	3-mile uniform	35 0	C D***	I** I**	Not Scheduled Not Scheduled		Not Scheduled Not Scheduled		Not Scheduled Not Scheduled		Not Scheduled Not Scheduled		Not Scheduled Not Scheduled		Not Scheduled Not Scheduled	
0	no-speed-control optimum	36 1	E F***	I** G D I** G D	5A 5B 10	D G** D I G D	11	I G D	12	I G	13	I G	15	I	5J	D
20	no speed-control optimum	29 8	1 2***	G D, I** G D I**	16	I G	18	I G	15	I G	17	I G	Not Scheduled		Not Scheduled	
0	speed control optimum	38 5	4 22***	I G D G	6	I G D	8	I G D	Not Scheduled		Not Scheduled		Not Scheduled		Not Scheduled	
20	speed-control optimum	31 6	3 14***	I G D I G D	7	I G, D	9	I G, D	20	I G	21	I G	Not Scheduled		Not Scheduled	
0	constant separation of 100 seconds for all sequence of S M and F		Not Scheduled		Not Scheduled		23	I†	Not Scheduled		Not Scheduled		Not Scheduled		Not Scheduled	

* Acceptance rates shown are theoretical maximum landings per hour
** These tests were run under the previous contract
*** Arrivals and departures were both included in these samples All other samples consisted of arrival only
† Samples 2 and 3 only

NOTE

Each Type I sample of arrivals or of arrivals and departures was run three times in the dynamic simulator a total of nine runs per group per configuration
Each Type II III and IIJ sample of arrivals was run twice in the dynamic simulator; a total of six runs per group per configuration
WBA Phase 2 (see Fig 2) was used in all groups except for Group 5A 5B and 5J which used WBA Phase 2A (see Fig 4) In addition to Phase 2 WBA Phases 6 and 7 were simulated graphically and dynamically in Groups E and F of the previous contract; WBA Phase 7 was simulated graphically in Groups 1 and 2
In Groups 5A 5B and 5J approach controllers used an electronic multitrack-approach computer (METAC); in addition it was assumed in Groups 5B and 5J that aircraft were equipped with pictorial computers

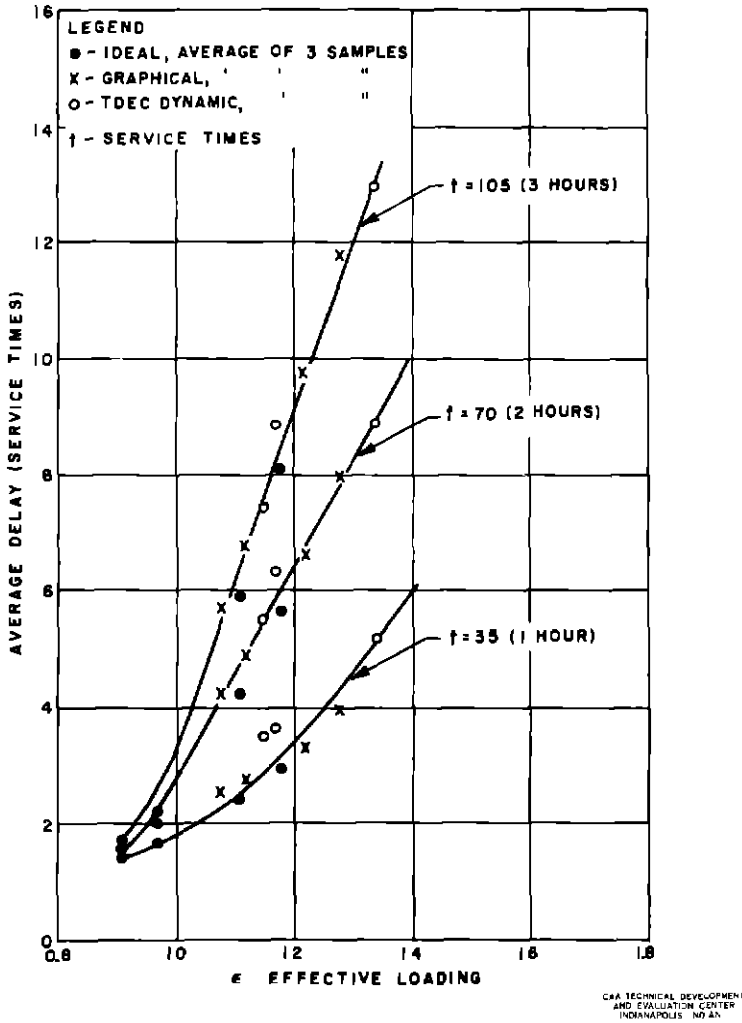


Fig 11 Average Delay as a Function of Effective Loading

of ϵ These asymptotes are precisely the delay figures given by the equilibrium formula for telephone traffic

$$\frac{D}{t_o} = \frac{\epsilon^2}{2(1-\epsilon)},$$

where

D = average delay

ϵ = loading

t_o = minimum separation or service time

Figure 9 shows that equilibrium sets in quickly for small values of ϵ, but for $0.9 \leq \epsilon < 1.0$, the steady-state characteristics are not quite reached even after 100 intervals (3 hours) On the other hand, for values of $\epsilon \geq 1.0$ there are no asymptotic values and no equilibrium conditions It is seen that for these values of ϵ greater than 100 per cent loading the delays become infinite, but only if the saturation continues for an infinite duration However, for relatively short periods of time these curves (Figs 8 - 10) provide a good approximation of the expected finite delays One hundred service times is relatively short, mathematically speaking,

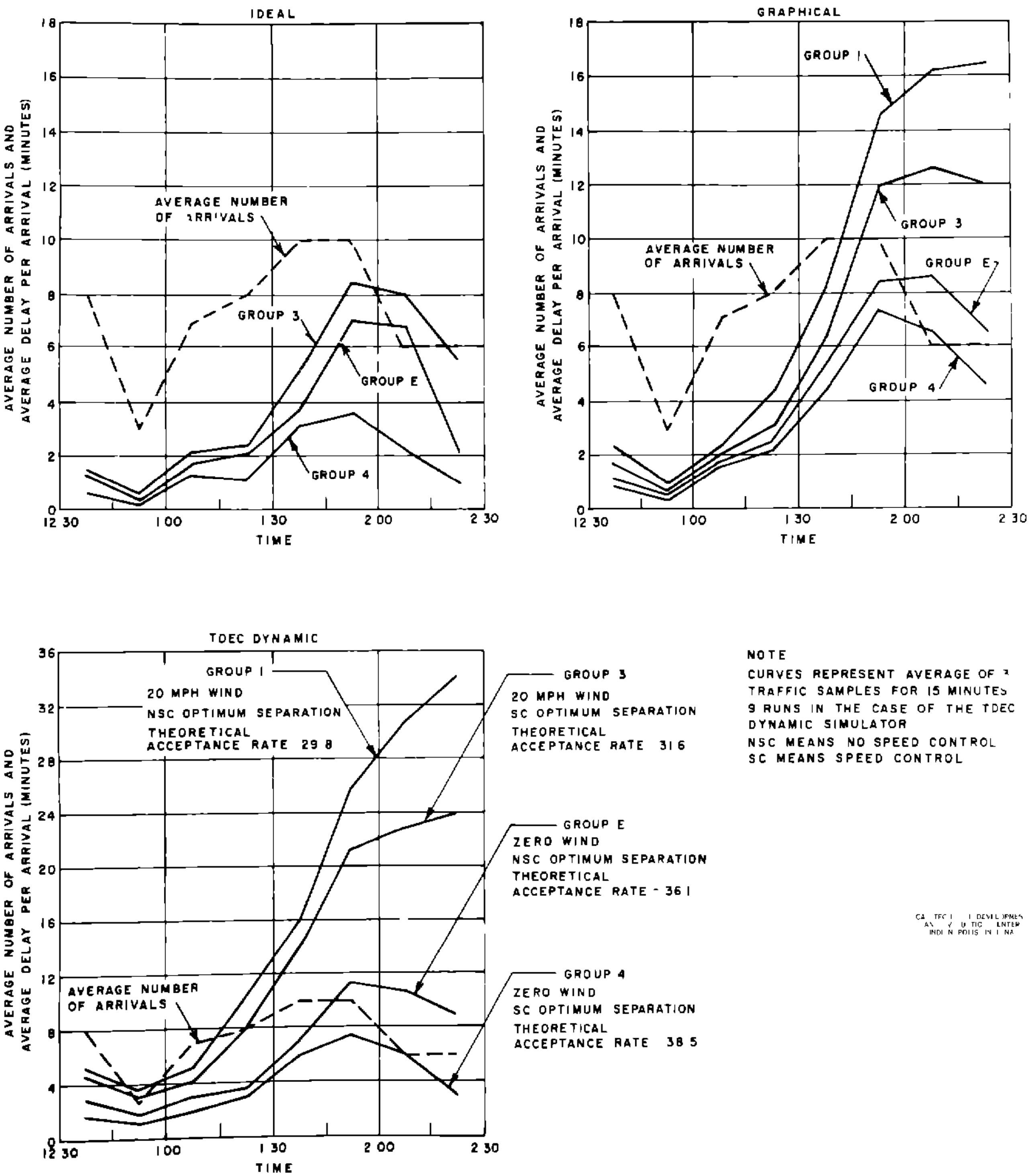


Fig 12 Comparison of Average Delays for Ideal-, Graphical-, and Dynamic-Simulator Analysis, WNA Phase 2, Type I Samples of Arrivals

however, 100 service times (average separations) of 2 minutes is 200 minutes, or 3 hours and 20 minutes From the standpoint of traffic control this is a very long period of time

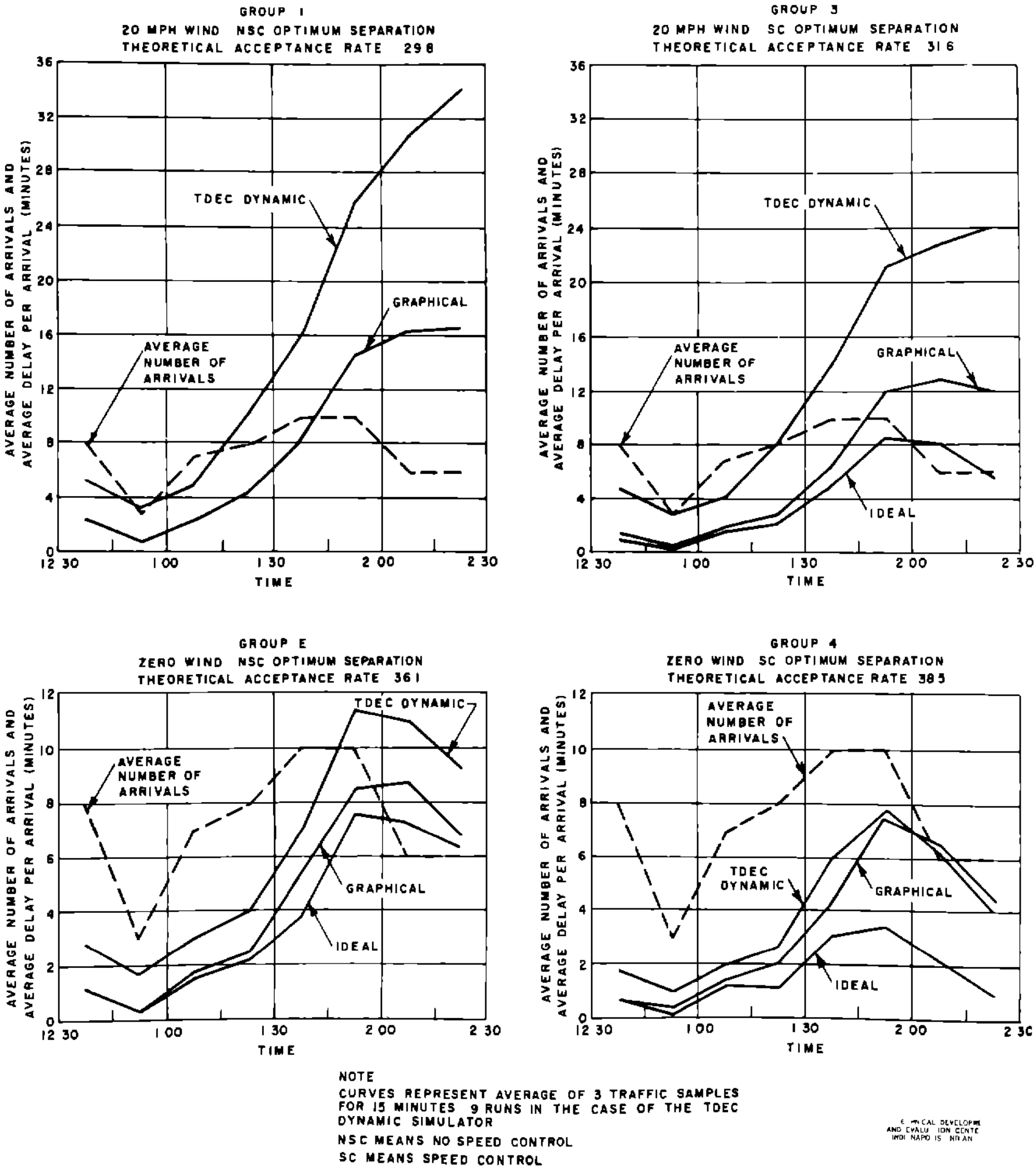
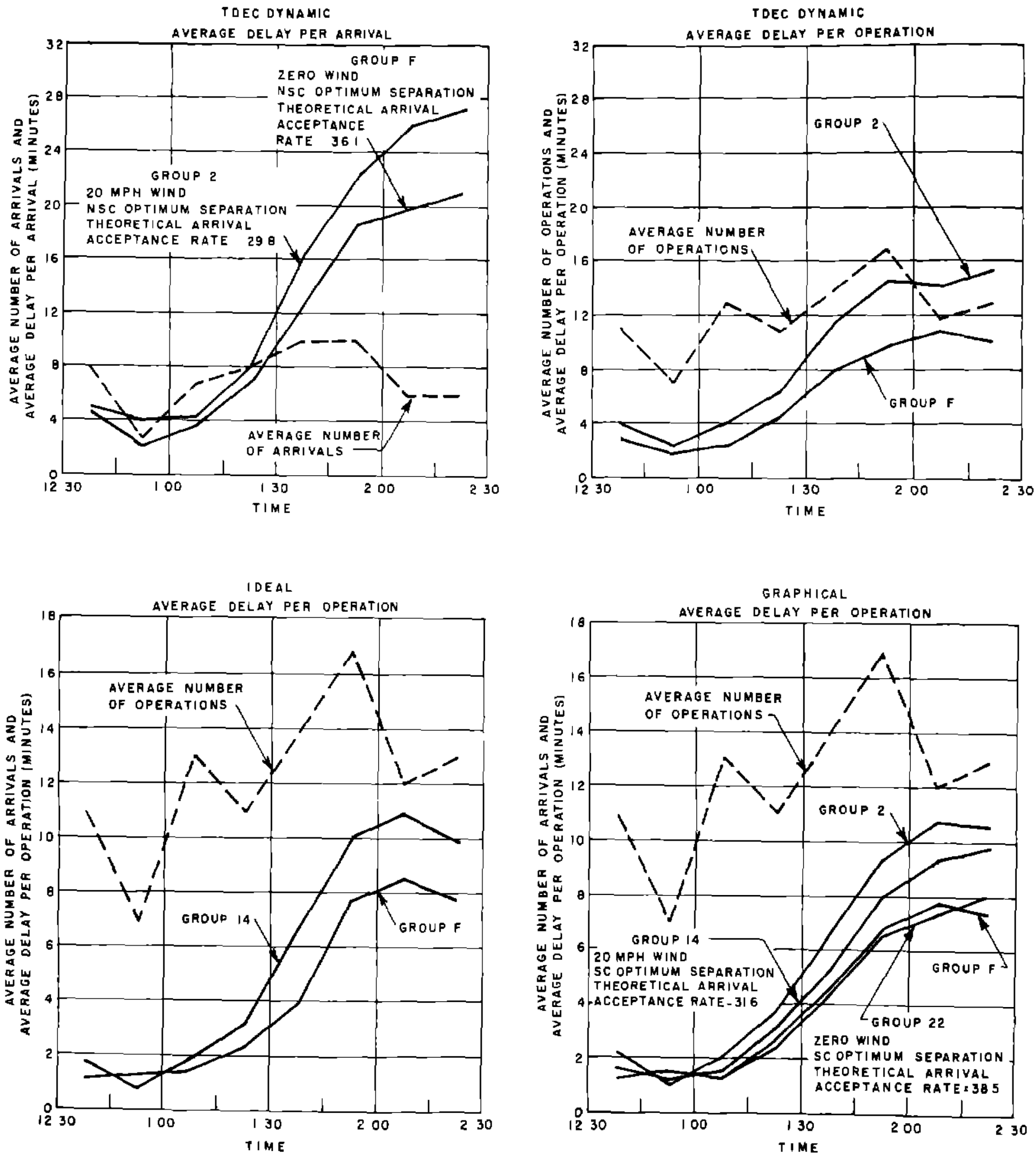


Fig 13 Comparison of Average Delays for Ideal-, Graphical-, and Dynamic-Simulator Analysis, WNA Phase 2, Type I Samples of Arrivals

Validation of Theory

Whenever any new theory is set forth, one immediately asks how well it conforms to reality Since both a theoretical aspect and a simulation or empirical aspect were investigated, it was possible to compare mathematical results with those from simulation



NOTE
CURVES REPRESENT AVERAGE OF 3 TRAFFIC SAMPLES FOR 15 MINUTES 9 RUNS IN THE CASE OF THE TDEC DYNAMIC SIMULATOR
NSC MEANS NO SPEED CONTROL
SC MEANS SPEED CONTROL

Fig 14 Comparison of Average Delays for Ideal-, Graphical-, and Dynamic-Simulator Analysis, WNA Phase 2, Type I Samples of Arrivals Plus Departures

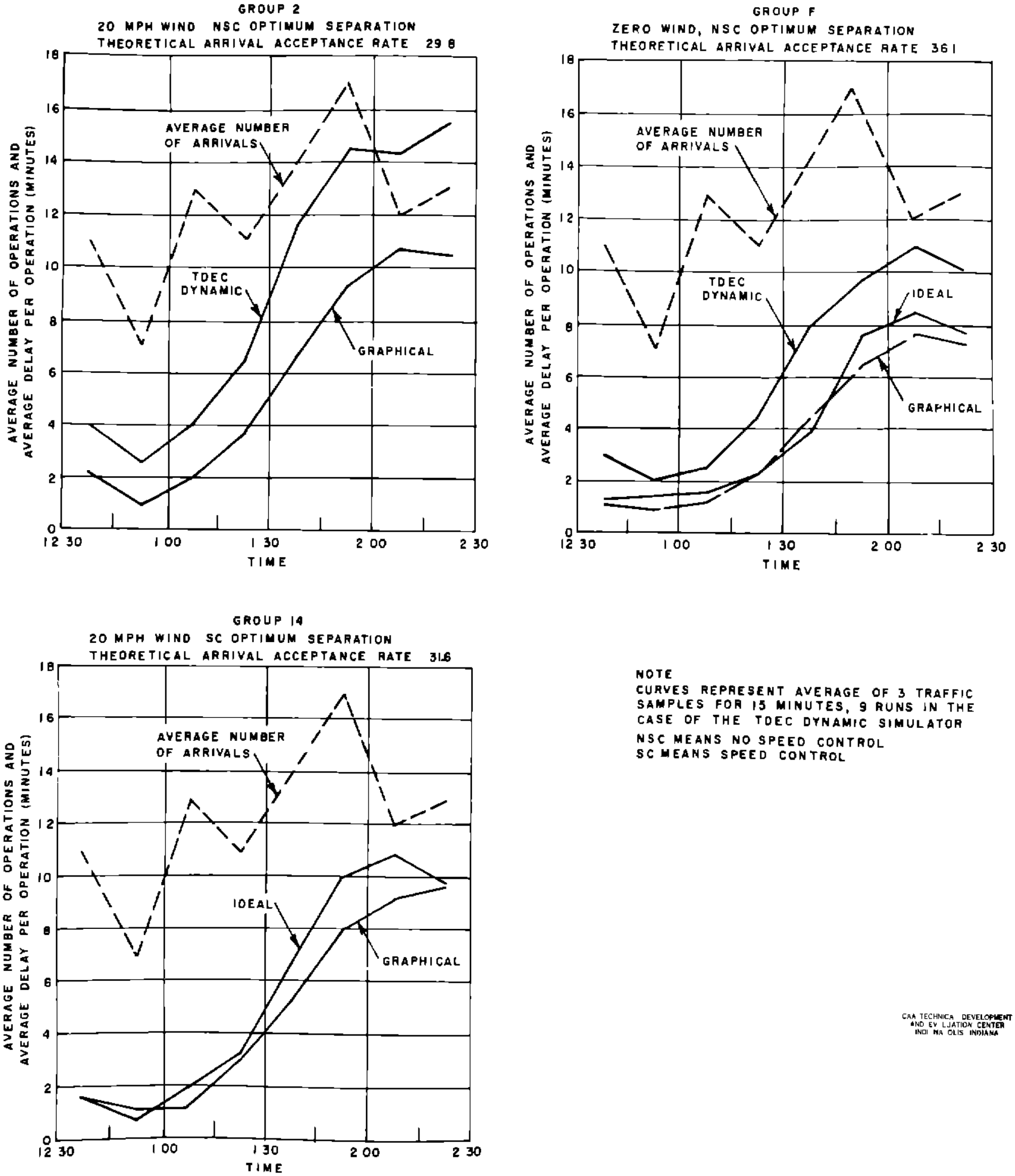


Fig 15 Comparison of Average Delays for Ideal-, Graphical-, and Dynamic-Simulator Analysis, WNA Phase 2, Type I Samples of Arrivals Plus Departures

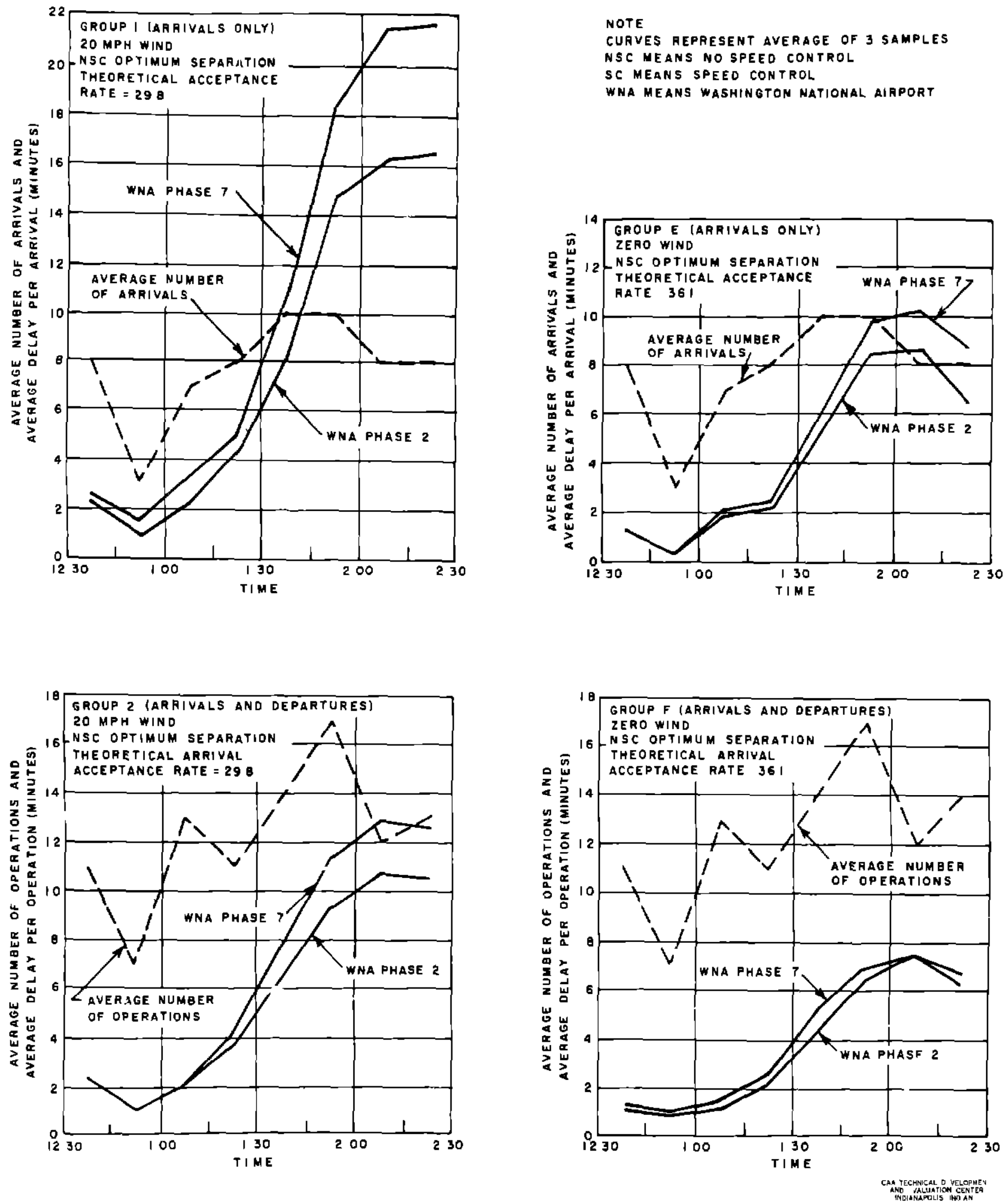


Fig 16 Comparison of Graphical-Analysis Average Delays; WNA Phases 2 and 7, Type I Samples of Arrivals and of Arrivals Plus Departures

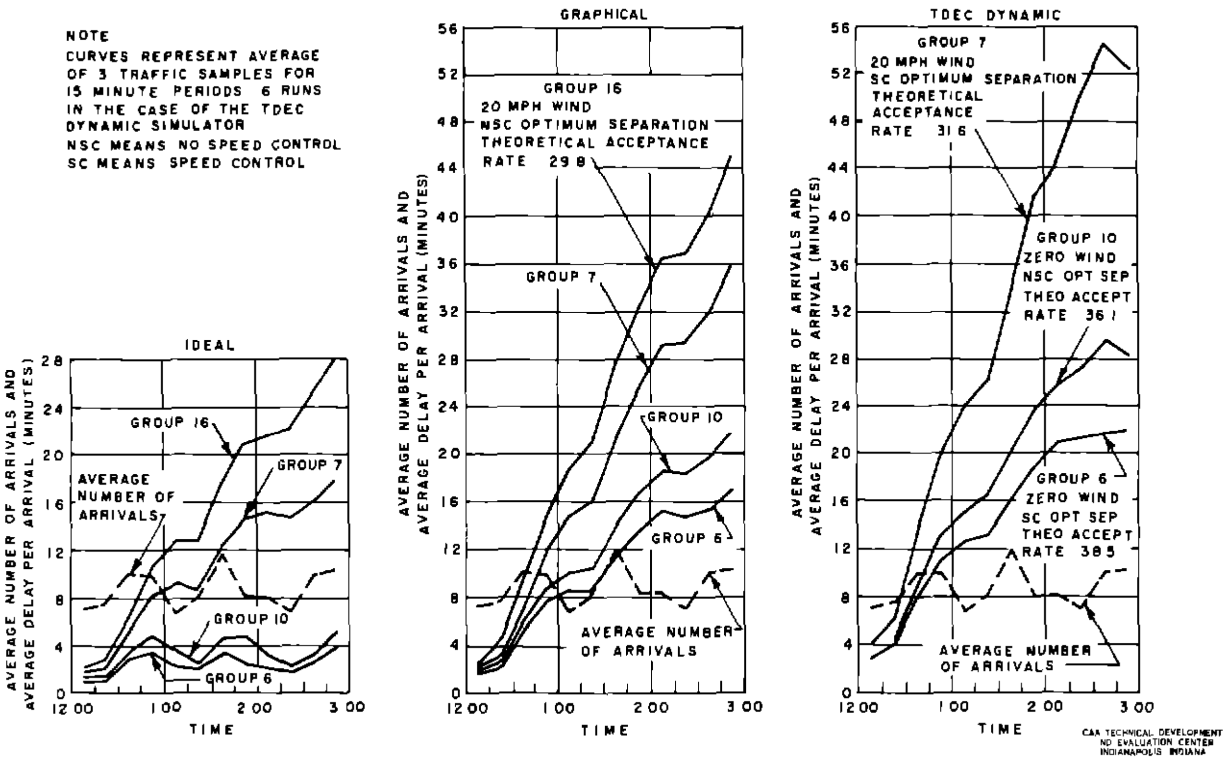


Fig 17 Comparison of Ideal-, Graphical-, and Dynamic-Simulator Average Delays, WNA Phase 2, Type II, 3-Hour Flow-Control Samples of Arrivals

Figures 8 and 9 show that for the conditions of $\epsilon <$ 0 9 the delays are relatively small and the system reaches statistical equilibrium rather quickly The prediction of delays under these conditions is therefore of little importance, and, if necessary, it can be accomplished with little error through the equilibrium formula However, for loadings of greater than 90 per cent, delays increase rapidly with time It is precisely this area of congestion that concerns us most, because most complaints occur under these conditions

Figure 10 is a cross-plot of Fig 9 and shows the relationship of density, or loading, to estimated delays The three different periods of congestion shown are 35, 70, and 105 service times These seemingly arbitrary lengths were chosen because most of the simulation runs consisted of 105 landings and had a constant or average arrival rate of 35 per hour for three hours Thus, these three curves correspond to the simulation delays for the periods of one, two, and three hours of arrivals The simulation delays as a function of effective density are shown in Fig 11 Since the acceptance rate varied for each condition and at each level of simulation, the actual or effective acceptance rate for each case was used whenever it differed from the theoretical ideal rate It is obvious that the slopes of the three curves in this figure agree with those of the theory, Fig 10, but that the delays are less for any given density This discrepancy in the magnitude of delays may at first seem serious, but actually the two sets of curves are virtually identical except for a shift by 10 per cent along the x-axis Thus, if we add 0 10 to each value of ϵ on the delay curves of the mathematical model, Fig 10, or if we subtract 0 10 from the other set, Fig 11, the two sets of curves become identical

This shifting of scale appears to be necessary, but the authors can at present offer no logical reasons for it A number of tests applied both to the model and to the simulation results indicated that both sets of data are accurate More work is necessary to justify this scale-shifting on a sound basis In the meantime, an apparently valid technique has been found to describe delays for relatively short periods at high levels of loading It is again emphasized that in mathematical terms the period is relatively short, in practice, 100 landings will use a relatively long period of time

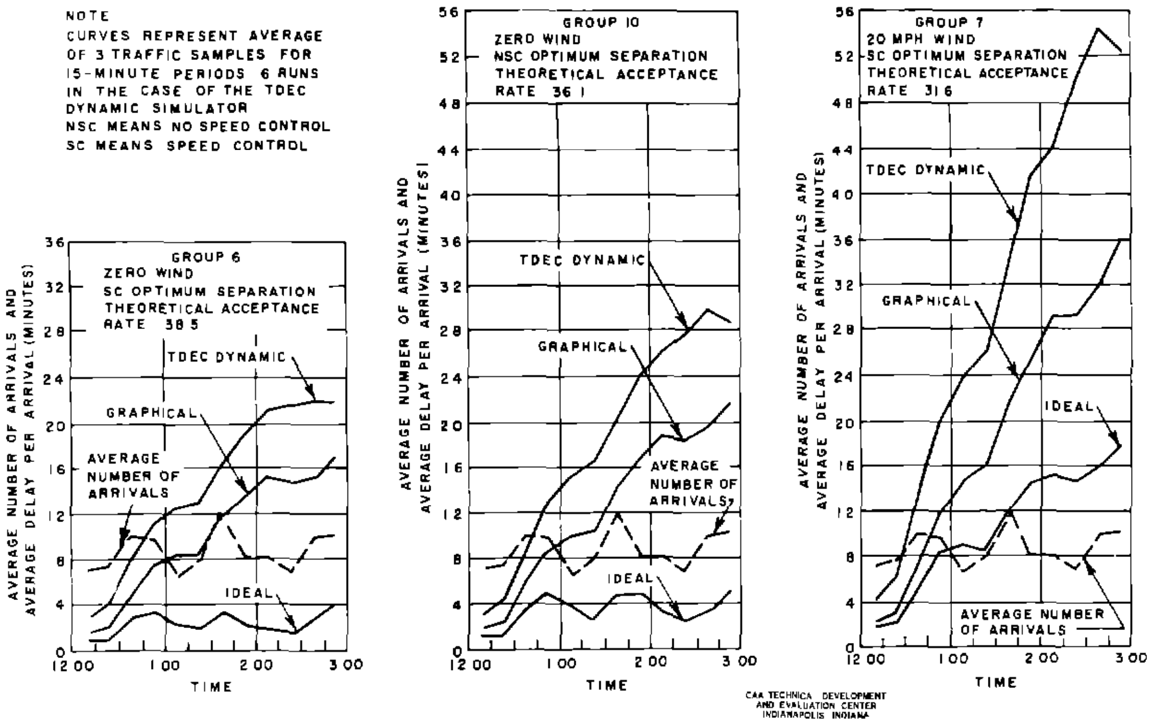


Fig 18 Comparison of Ideal-, Graphical-, and Dynamic-Simulator Delays; WNA Phase 2, Type II, 3-Hour Flow-Control Arrivals

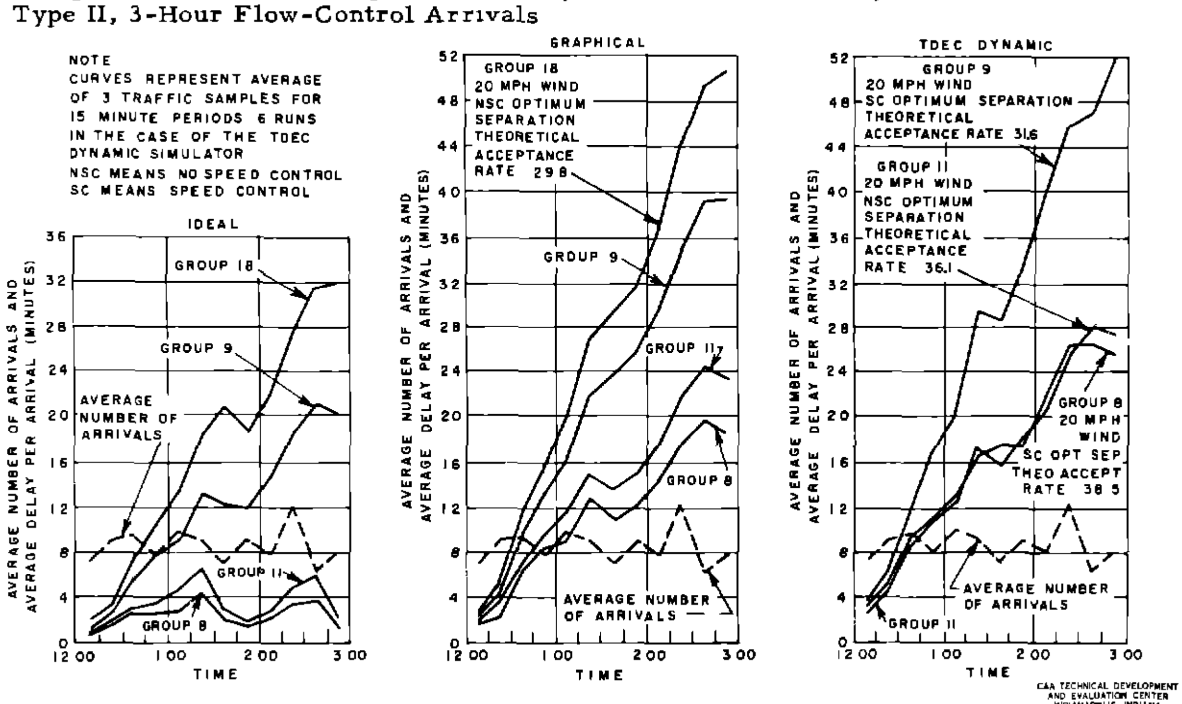


Fig 19 Comparison of Ideal-, Graphical-, and Dynamic-Simulator Delays, WNA Phase 2, Type III, 3-Hour Flow-Control Arrivals

TABLE

DELAY RESULTS OF

Period of Operation	Operations Number	Operations Kind	Type I Samples* Group	Wind (mph)	Acceptance Rate	Optimum Separation	Delay Parameter: Average Delay Per Aircraft (minutes)	Number of Aircraft Delayed (per cent)	Maximum Delay (minutes)	Group	Duration of Sample (hours)	Wind (mph)
0 to 1										6	3	0
										10	3	0
										13	4	0
										7	3	20
										21	4	20
										16	3	20
										17	4	20
1 to 2	35	Arrivals	3	20	31 6	SC	4 8	91 4	14 0	6	3	0
	35	Arrivals	4	0	38 5	SC	2 4	88 6	7 8	10	3	0
	35	Arrivals and	14	20	31 6	SC	8 5	96 2	23 7	13	4	0
	20	WNA Departures	14	20	31 6	SC	1 3	71 7	9 0	7	3	20
	55	Combined	14	20	31 6	SC	5 9	87 3	23 7	21	4	20
										16	3	20
										17	4	20
1 to 2 1/2	47	Arrivals	3	20	31 6	SC	5 3	92 2	17 3			
	47	Arrivals	4	0	38 5	SC	2 2	80 1	7 8			
	47	Arrivals and	14	20	31 6	SC	10 2	97 2	26 6			
	33	WNA Departures	14	20	31 6	SC	3 0	82 8	14 3			
	80	Combined	14	20	31 6	SC	7 2	91 3	26 6			
1/2 to 2 1/2	58	Arrivals	3	20	31 6	SC	4 6	83 9	17 3			
	58	Arrivals	4	0	38 5	SC	1 9	73 0	7 8			
	58	Arrivals and	14	20	31 6	SC	8 6	91 4	26 6			
	40	WNA Departures	14	20	31 6	SC	2 6	80 9	14 3			
	98	Combined	14	20	31 5	SC	6 1	87 1	26 6			
2 to 3										6	3	0
										10	3	0
										13	4	0
										7	3	20
										21	4	20
										16	3	20
										17	4	20
3 to 4										10	3	0
										13	4	0
										7	3	20
										21	4	20
										16	3	20
										17	4	20
Average 0 to 3										6	3	0
										10	3	0
										13	4	0
										7	3	20
										21	4	20
										16	3	20
										17	4	20
Average 0 to 4										10	3	0
										13	4	0
										7	3	20
										21	4	20
										16	3	20
										17	4	20

*Type I utilizes arrivals only for Groups 3 and 4 and utilizes arrivals plus departures for Group 14
**Type II utilizes three- or four-hour samples of 35 arrivals per hour
***Type III utilizes three hour samples averaging 35 arrivals per hour

NOTES
1 Except for maximum delays, all data shown are for the averages of three traffic samples
2 Acceptance rates shown are theoretical maximum landings per hour
3 An operation is either a landing or a take off

II

THE IDEAL ANALYSES

Type II Samples**							Type III Samples***				
Acceptance Rate	Optimum Separation	Delay Parameter			Group	Wind	Acceptance Rate	Optimum Separation	Delay Parameter		
		Average Delay per Aircraft (minutes)	Number of Aircraft Delayed (per cent)	Maximum Delay (minutes)		(mph)			Average Delay per Aircraft (minutes)	Number of Aircraft Delayed (per cent)	Maximum Delay (minutes)
38 5	SC	2 4	86 7	7 8	8	0	38 5	SC	2 1	78 6	7 5
36 1	NSC	3 0	89 5	8 8	11	0	36 1	NSC	2 6	82 6	8 3
36 1	NSC	3 0	89 5	8 8	9	20	31 6	SC	4 5	89 3	11 8
31 6	SC	4 8	90 5	14 0	18	20	29 8	NSC	5 9	89 3	16 0
31 6	SC	4 8	90 5	14 0							
29 8	NSC	6 1	93 3	17 4							
29 8	NSC	6 1	93 3	17 4							
38 5	SC	2 6	91 4	7 8	8	0	38 5	SC	2 8	87 0	7 7
36 1	NSC	4 0	96 2	9 4	11	0	36 1	NSC	4 3	96 3	10 7
36 1	NSC	4 0	96 2	9 4	9	20	31 6	SC	11 7	100 0	19 3
31 6	SC	11 4	100 0	18 7	18	20	29 8	NSC	17 2	100 0	26 6
31 6	SC	11 4	100 0	18 7							
29 8	NSC	16 4	100 0	26 0							
29 8	NSC	16 4	100 0	26 0							
38 5	SC	2 8	90 5	7 8	8	0	38 5	SC	2 9	88 5	8 9
36 1	NSC	3 7	94 3	8 8	11	0	36 1	NSC	4 4	96 2	12 5
36 1	NSC	3 7	94 3	8 8	9	20	31 6	SC	18 5	100 0	28 3
31 6	SC	16 3	100 0	27 8	18	20	29 8	NSC	28 0	100 0	39 3
31 6	SC	16 3	100 0	27 8							
29 8	NSC	24 6	100 0	37 7							
29 8	NSC	24 6	100 0	37 7							
36 1	NSC	5 1	98 1	11 2							
36 1	NSC	5 1	98 1	11 2							
31 6	SC	23 3	100 0	32 4							
31 6	SC	23 3	100 0	32 4							
29 8	NSC	36 0	100 0	46 6							
29 8	NSC	36 0	100 0	46 6							
38 5	SC	2 6	89 5		8	0	38 5	SC	2 6	84 8	
36 1	NSC	3 6	93 3		11	0	36 1	NSC	3 8	91 7	
36 1	NSC	3 6	93 3		9	20	31 6	SC	11 6	96 5	
31 6	SC	10 8	96 8		18	20	29 8	NSC	17 1	96 5	
31 6	SC	10 8	96 8								
29 8	NSC	15 7	97 8								
29 8	NSC	15 7	97 8								
36 1	NSC	4 0	94 8								
36 1	NSC	4 0	94 8								
31 6	SC	14 0	97 6								
31 6	SC	14 0	97 6								
29 8	NSC	20 8	98 3								
29 8	NSC	20 8	98 3								

TABLE III

COMPARISON BY IDEAL ANALYSIS OF AVERAGE DELAYS FOR GROUPS 11 AND 23, TYPE III

Conditions Variable or constant time separations, 3-hour random samples of arrivals 0 wind

Period of Arrivals	Group 11* Sample No 2		Group 11* Sample No 3		Average of 2 Samples		Group 23** Sample No 2		Group 23** Sample No 3		Average of 2 Samples	
(hour)	Average Delay (minutes)	Arrivals	Average Delay (minutes)	Arrivals	Average Delay (minutes)	Arrivals	Average Delay (minutes)	Arrivals	Average Delay (minutes)	Arrivals	Average Delay (minutes)	Arrivals
0 to 1	1 5	31	3 4	34	2 6	32 5	1 7	31	3 5	34	2 6	32 5
1 to 2	4 4	38	4 0	36	4 2	37	5 8	38	4 2	36	5 0	37
2 to 3	2 7	36	7 3	35	5 0	35 5	3 6	36	8 1	35	5 8	35 5
0 to 3	3 0***		4 9***		4 0***		3 7***		5 3***		4 5***	

* No-speed-control optimum separation variable separation for each sequence of S, M, and F Theoretical acceptance rate 36 1

** Constant separation of 100 seconds for any sequence of S, M, and F Theoretical acceptance rate 36 0

*** Average of 35 arrivals

PROGRAM OF SIMULATION EXPERIMENTS

Conditions Analyzed and Simulated

Twenty-four sets of major conditions were initially considered for investigation ideally, graphically, dynamically, or by any combination of two methods, or by all three in this joint FIL-TDEC simulation work. six maximum theoretical acceptance rates (29 8, 31 6, 35 0, 36 1, 38 5, and 41 3) with four types of traffic samples (I, II, IIJ, and III) for each Three of these conditions were investigated under the prior contract Type I samples, arrivals only and arrivals plus departures, with outer-marker separations corresponding to maximum theoretical acceptance rates of 35 0, 36 1, and 41 3 per hour Of the remaining 21 sets of conditions, 12

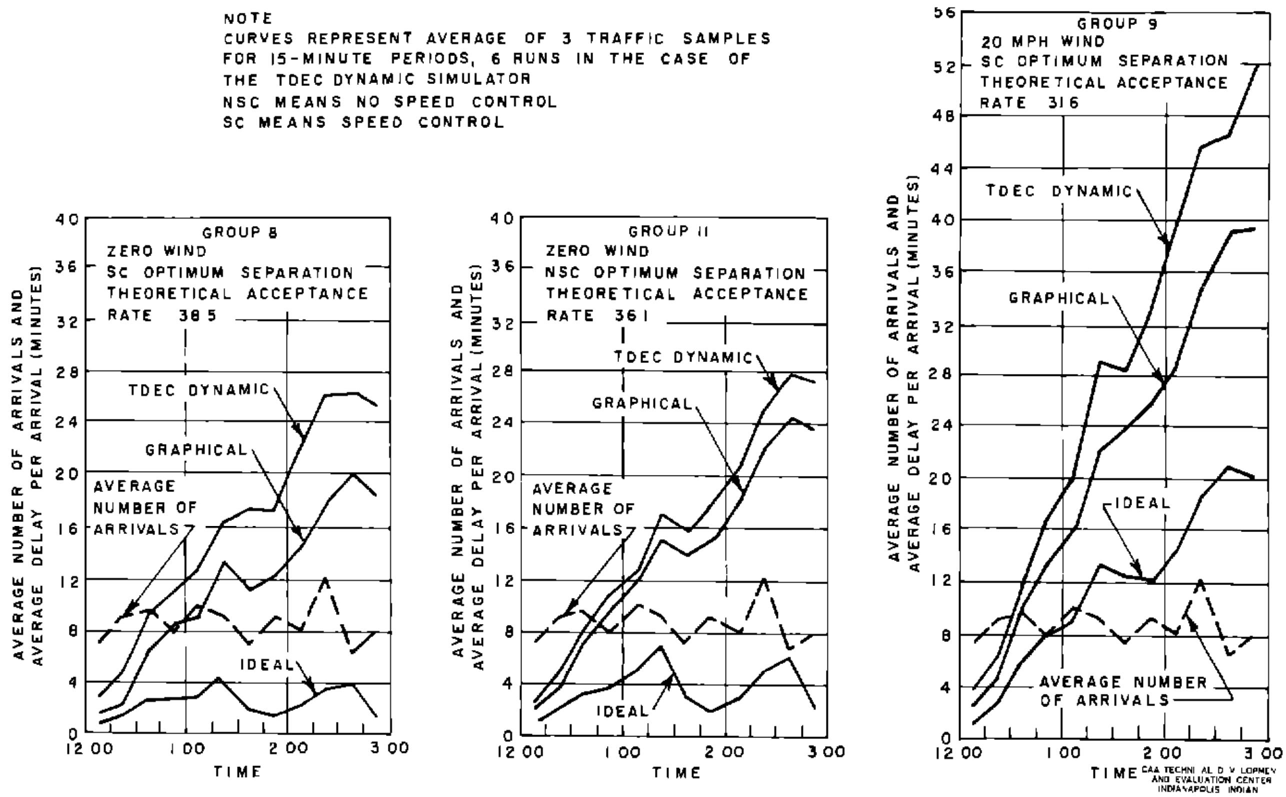


Fig 20 Comparison of Ideal-, Graphical-, and Dynamic-Simulator Delays, WNA Phase 2, Type III, 3-Hour Flow-Control Arrivals

TABLE XIII

COMPARISON BY IDEAL ANALYSIS OF AVERAGE DELAYS RESULTING FROM RANDOM RESEQUENCING OF THE TIME INTERVALS FOR GROUP 19, TYPE III, SAMPLE NO 2

Conditions No-speed-control optimum separation theoretical acceptance rate 36 1 0 wind

Period of Arrivals	Original Sequencing of Time Intervals		Random Resequence No 1 of Time Intervals		Random Resequence No 2 of Time Intervals	
(hours)	Average Delay (minutes)	Arrivals	Average Delay (minutes)	Arrivals	Average Delay (minutes)	Arrivals
0 to 1	1 5	31	2 89	39	5 43	40
1 to 2	4 4	38	7 09	32	8 04	39
2 to 3	2 7	36	2 78	31	2 18	23
0 to 3	3 0*		4 18**		5 86**	

*Average of 35 arrivals
**Average of 34 arrivals

were undertaken during this contract period as indicated in Table IX The acceptance rates are for the optimum separations derived for the 5 3-mile outer-marker distance and for the 2 1/2-mile glide-slope minimum All military operations were presumed to land or to take off at Bolling Air Force Base or at Anacostia Naval Air Station Inbound military aircraft were governed only as far as the outer marker and were then discontinued The corresponding calculations of the acceptance rates were weighted to suit

The final dynamic runs (Groups 5A, 5B, and 5J as listed in Table VIII) involving the Washington National Airport Phase 2A multitrack layout with the electronic approach-control computer, the airborne pictorial computers, and the Type IIJ samples were not completed in time to report in detail

The selection of the groups to be analyzed ideally, graphically, and dynamically was based on a design of experiments planned to yield a maximum of significant information with the fewest number of experiments and conditions

FIL-TDEC Program of Experiments

In order to stabilize the results three samples, random internally but with similar overall characteristics, of each sample type were used In the dynamic-simulator runs, each Type I sample was run 3 times and each Type II, IIJ, or III sample was run twice This repetition was done to explore some of the variabilities expected in practice and to then average them in order to make significant comparisons To eliminate learning, controllers were changed or positions were rotated in repeat runs of any sample In addition, the runs were programmed so that the second or third repeat runs of each sample were not consecutive

The program of ideal, graphical, and dynamic tests for the aforementioned groups of conditions is given in Table X During the previous contract period, the groups listed as A through F were performed ideally, graphically, or dynamically and Groups 1 and 2 were analyzed ideally only These groups are also listed in Table X for convenient reference The acceptance rates shown are the maximum theoretical landing capacities per hour

Excluding Groups A through F, a total of 25 groups are listed Thirteen of these groups were tested on the dynamic simulator, a total of 90 runs 36 runs on Groups 1 through 4 (9 runs per group, 3 Type I samples times 3 runs per sample) and 54 runs on Groups 5A, 5B, 5J, and Groups 6 through 11 (6 runs per group; 3 Type II, Type IIJ, or Type III samples times 2 runs per sample) Twenty-one groups were analyzed graphically, and 21 groups (not the identical 21 analyzed graphically) were analyzed ideally

SIMULATION RESULTS

Presentation of Data

Tables XI, XIV, and XV list the pertinent delay results of the ideal-, graphical-, and dynamic-simulation analyses, respectively Table XVI shows a comparison of typical

TABLE

ABSOLUTE DELAY RESULTS

Period of Operation	Type I Samples: Operations Number	Operations Kind	Group	Phase	Wind (mph)	Acceptance Rate	Optimum Separation	Delay Parameter: Average Delay per Aircraft (minutes)	Number of Aircraft Delayed (per cent)	Maximum Delay (minutes)	Group	Duration of Sample (hours)	Wind (mph)
0 to 1											6	3	0
											10	3	0
											13	4	0
											7	3	20
											21	4	20
											16	3	20
											17	4	20
1 to 2	35	Arrivals	1 & 2	2	20	29 8	NSC	8 4	94 3	23 2	6	3	0
	20	WNA Departures	1 & 2	2	20	29 8	NSC	1 6	81 7	5 8	10	3	0
	55	Combined	1 & 2	2	20	29 8	NSC	5 9	89 7	23 2	13	4	0
	35	Arrivals	1 & 2	7	20	29 8	NSC	10 0	92 4	26 2	7	3	20
	20	WNA Departures	1 & 2	7	20	29 8	NSC	1 5	80 0	6 7	21	4	20
	55	Combined	1 & 2	7	20	29 8	NSC	6 9	87 9	26 2	16	3	20
	35	Arrivals	3 & 14	2	20	31 6	SC	6 5	92 4	17 2	17	4	20
	20	WNA Departures	3 & 14	2	20	31 6	SC	2 3	83 3	10 6			
	55	Combined	3 & 14	2	20	31 6	SC	4 9	89 1	17 2			
	35	Arrivals	4 & 22	2	0	38 5	SC	4 4	90 5	13 3			
	20	WNA Departures	4 & 22	2	0	38 5	SC	3 4	80 0	11 3			
	55	Combined	4 & 22	2	0	38 5	SC	4 0	86 7	13 3			
1 to 2 1/2	47	Arrivals	1 & 2	2	20	29 8	NSC	10 4	95 7	23 2			
	33	WNA Departures	1 & 2	2	20	29 8	NSC	3 0	86 8	9 3			
	80	Combined	1 & 2	2	20	29 8	NSC	7 4	92 1	23 2			
	47	Arrivals	1 & 2	7	20	29 8	NSC	12 9	94 3	29 2			
	33	WNA Departures	1 & 2	7	20	29 8	NSC	2 7	84 8	9 6			
	80	Combined	1 & 2	7	20	29 8	NSC	8 7	90 4	29 2			
	47	Arrivals	3 & 14	2	20	31 6	SC	8 0	94 3	21 5			
	33	WNA Departures	3 & 14	2	20	31 6	SC	4 1	89 9	15 6			
	80	Combined	3 & 14	2	20	31 6	SC	6 4	92 5	21 5			
	47	Arrivals	4 & 22	2	0	38 5	SC	4 6	91 5	14 4			
	33	WNA Departures	4 & 22	2	0	38 5	SC	5 9	87 9	17 6			
	80	Combined	4 & 22	2	0	38 5	SC	5 2	90 0	17 6			
1/2 to 2 1/2	58	Arrivals	1 & 2	2	20	29 8	NSC	8 7	93 4	23 2			
	40	WNA Departures	1 & 2	2	20	29 8	NSC	2 9	85 6	12 3			
	98	Combined	1 & 2	2	20	29 8	NSC	6 2	90 0	23 2			
	58	Arrivals	1 & 2	7	20	29 8	NSC	11 3	92 9	29 2			
	40	WNA Departures	1 & 2	7	20	89 8	NSC	2 4	87 6	9 6			
	98	Combined	1 & 2	7	20	29 8	NSC	7 4	90 6	29 2			
	58	Arrivals	3 & 14	2	20	31 6	SC	6 7	88 5	21 5			
	40	WNA Departures	3 & 14	2	20	31 6	SC	3 6	86 7	15 6			
	98	Combined	3 & 14	2	20	31 6	SC	5 5	87 8	21 5			
	58	Arrivals	4 & 22	2	0	38 5	SC	3 9	86 2	14 4			
	40	WNA Departures	4 & 22	2	0	38 5	SC	5 2	85 0	17 6			
	98	Combined	4 & 22	2	0	38 5	SC	4 4	85 7	17 6			
2 to 3											6	3	0
											10	3	0
											13	4	0
											7	3	20
											21	4	20
											16	3	20
											17	4	20
3 to 4											6	3	0
											10	3	0
											13	4	0
											7	3	20
											21	4	20
											16	3	20
											17	4	20
Average 0 to 3											6	3	0
											10	3	0
											13	4	0
											7	3	20
											21	4	20
											16	3	20
											17	4	20
Average 0 to 4											10	3	0
											13	4	0
											7	3	20
											21	4	20
											16	3	20
											17	4	20
1/2 to 3											6	3	0
											10	3	0
											13	4	0
											7	3	20
											21	4	20
											16	3	20
											17	4	20

* Type I samples are three or four hour samples of 35 arrivals each hour utilizing WNA Phase 2

** Type III samples are three hour samples averaging 35 arrivals per hour and utilizing WNA Phase 2

NOTE

Except for maximum delays all data shown are for the averages of three samples Maximum delays are the maximum for three samples

Acceptance rates shown are theoretical maximum landings per hour

A TIOTAM is a time interval greater than the assigned minima

An operation is either a landing or a take-off

X V

OF THE GRAPHICAL ANALYSES

Type II Samples*						Type III Samples**							
Acceptance Rate	Optimum Separation	Delay Parameter: Average Delay per Aircraft (minutes)	Delay Parameter: Number of Aircraft Delayed (per cent)	Delay Parameter: Maximum Delay (minutes)	Delay Parameter: Average Length of TIGTAMS (seconds)	Group	Wind (mph)	Acceptance Rate	Optimum Separation	Delay Parameter: Average Delay per Aircraft (minutes)	Delay Parameter: Number of Aircraft Delayed (per cent)	Delay Parameter: Maximum Delay (minutes)	Delay Parameter: Average Length of TIGTAMS (seconds)
38 5	SC	4 4	89 5	13 3		8	0	38 5	SC	5 2	87 4	13 1	
36 1	NSC	5 0	90 5	14 5		11	0	36 1	NSC	5 7	87 4	14 0	
36 1	NSC	5 0	90 5	14 5		9	20	31 6	SC	7 7	90 3	18 9	
31 6	SC	6 4	91 4	17 2		18	20	29 8	NSC	9 1	90 3	23 3	
31 6	SC	6 4	91 4	17 2									
29 8	NSC	8 4	93 3	20 0									
29 8	NSC	8 4	93 3	20 0									
38 5	SC	10 6	100 0	18 6		8	0	38 5	SC	11 4	99 1	18 8	
36 1	NSC	13 2	100 0	21 2		11	0	36 1	NSC	13 6	99 1	21 1	
36 1	NSC	13 2	100 0	21 2		9	20	31 6	SC	21 4	100 0	31 1	
31 6	SC	19 9	100 0	29 2		18	20	29 8	NSC	27 1	100 0	40 6	
31 6	SC	19 9	100 0	29 2									
29 8	NSC	25 6	100 0	37 3									
29 8	NSC	25 6	100 0	37 3									
38 5	SC	19 6	100 0	25 3		8	0	38 5	SC	17 5	100 0	26 3	
36 1	NSC	19 8	100 0	30 3		11	0	36 1	NSC	21 8	100 0	31 5	
36 1	NSC	19 8	100 0	30 3		9	20	31 6	SC	35 5	100 0	48 0	
31 6	SC	32 0	100 0	44 3		18	20	29 8	NSC	45 1	100 0	60 1	
31 6	SC	32 0	100 0	44 3									
29 8	NSC	40 1	100 0	52 4									
29 8	NSC	40 1	100 0	52 4									
38 5	SC		-										
36 1	NSC	27 0	100 0	36 5									
36 1	NSC	27 0	100 0	36 5									
31 6	SC	44 7	100 0	56 1									
31 6	SC	44 7	100 0	56 1									
29 8	NSC	56 3	100 0	67 9									
29 8	NSC	56 3	100 0	67 9									
38 5	SC	10 2	96 5			8	0	38 5	SC	11 4	95 6		
36 1	NSC	12 6	96 8			11	0	36 1	NSC	13 7	95 6		
36 1	NSC	12 6	96 8			9	20	31 6	SC	21 6	96 8		
31 6	SC	19 5	97 1			18	20	29 8	NSC	27 1	96 8		
31 6	SC	19 5	97 1										
29 8	NSC	24 7	97 8										
29 8	NSC	24 7	97 8										
36 1	NSC	16 2	97 6										
36 1	NSC	16 2	97 6										
31 6	SC	25 8	97 9										
31 6	SC	25 8	97 9										
29 8	NSC	32 6	98 3										
29 8	NSC	32 6	98 3										
38 5	SC				18 8	8	0	38 5	SC				18 0
36 1	NSC				16 7	11	0	36 1	NSC				15 3
36 1	NSC				16 7	9	20	31 6	SC				10 8
31 6	SC				12 1	18	20	29 8	NSC				10 4
31 6	SC				12 1								
29 8	NSC				11 0								
29 8	NSC				11 0								

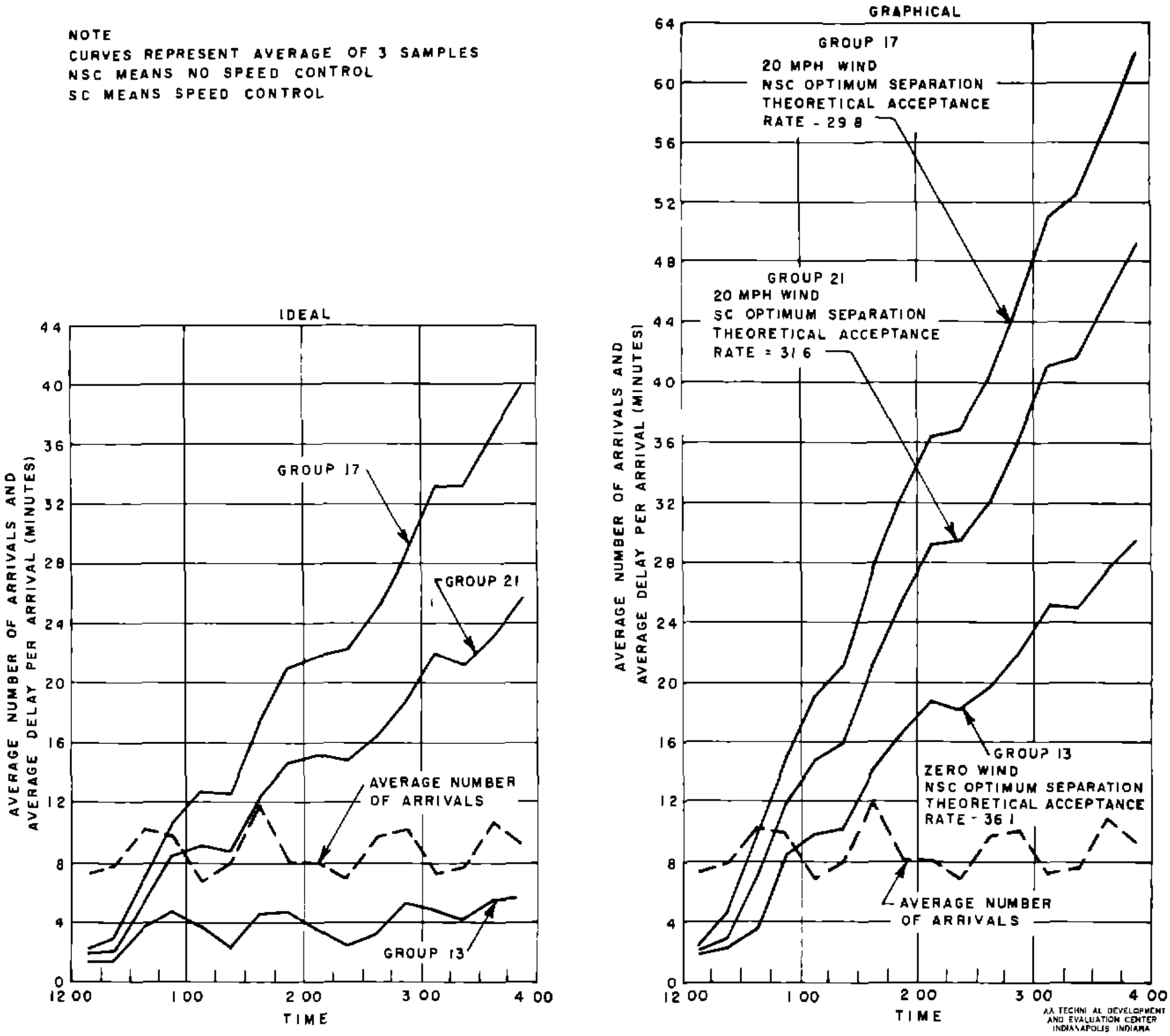


Fig 21 Comparison of Ideal-, Graphical-, and Dynamic-Simulator Delays, WNA Phase 2, Type II, 4-Hour Flow-Control Arrivals

variability between samples and repeat runs for Group 6 when these three simulation techniques are used Table XII shows a comparison, by the ideal technique, of average delays for Group 23 when constant or variable time separations are used Table XIII shows the effect, on average delays, for Group 19, of random resequencing of the time intervals Throughout this report, average delay refers to the average for all aircraft, including those not delayed A delay is defined as that additional time required to meet the specified separations beyond the time normally required when no other traffic is involved Data shown on Table XIV compare both the WNA Phase 2 and WNA Phase 7 delays for Groups 1 and 2 All other data are for WNA Phase 2 only

The delay results obtained by ideal and graphical simulation techniques for the first and second hours of the 2-hour samples used in Groups 12, 15, and 20 (Type II samples) were not affected significantly by the traffic of the third hour in comparable Groups 10, 16, and 7 (Type II 3-hour samples), therefore, they are not listed Also, the delays of the third hour, obtained by ideal and graphical simulation, in all groups using the Type II, 3-hour samples were not affected significantly by the arrivals of the fourth hour

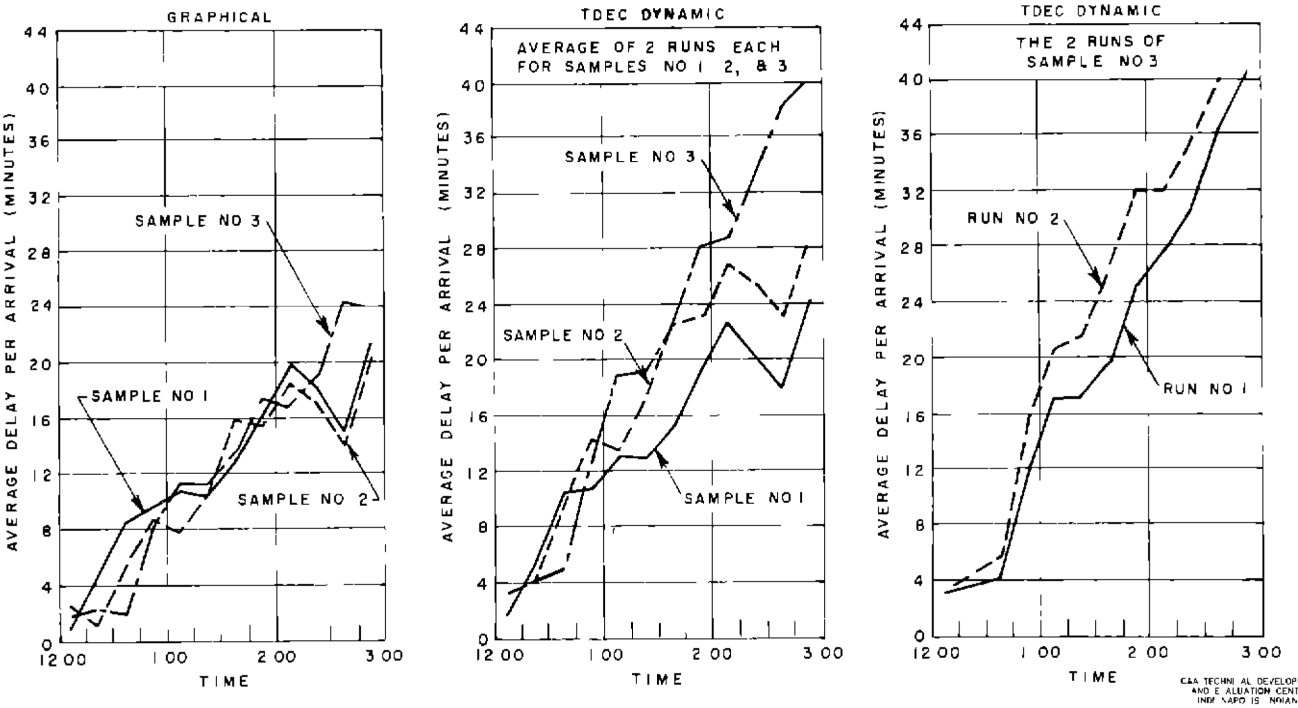


Fig 22 Comparison of Typical Variability in Average Delays Between Traffic Samples and Repeat Runs, Graphical- and Dynamic-Simulator Analysis of Group 10, WNA Phase 2

All figures listed hereafter for average delays are presented in the form of a time series of average delays for those aircraft scheduled to arrive or depart in each successive 15-minute period Figures 12 through 15 show comparisons of the ideal-, graphical-, and dynamic-simulator average delays for Groups 1, 3, 4, and E and Groups 2, 4, 22, and F on WNA Phase 2 Figure 16 compares the WNA Phase 2 and Phase 7 average delays for Groups 1, 2, E, and F The data on Groups E and F (performed on previous work) are shown on some of these figures for convenient comparison and reference

Comparisons of the ideal-, graphical-, and dynamic-simulator average delays (WNA Phase 2) for Groups 6, 7, 10, and 16 (Type II, 3-hour flow-control samples of arrivals) are shown in Figs 17 and 18, Groups 8, 9, 11, and 18 (Type III, 3-hour random samples of arrivals) are shown in Figs 19 and 20 Figure 21 compares the ideal- and graphical-analysis average delays for Groups 13, 17, and 21 (Type II, 4-hour flow-control samples of arrivals)

Figure 22 shows another example of typical variability in both graphical- and dynamic-simulator average delays between traffic samples of characteristics which are similar over-all but which differ in internal randomness and between dynamic-simulator repeat runs on an identical sample, Group 10 was used as the example

Figures 23 and 24 compare the graphical- and dynamic-simulator distributions and timewise averages of the TIGTAMS for Groups 6, 7, and 10 (Type II, 3-hour flow-control samples) and Groups 8, 9, and 11 (Type III, 3-hour random samples), respectively A TIGTAM is defined as a Time Interval Greater Than Assigned Minima In order to load the system, the first half hour is excluded No quantitative data on the Group 5A, 5B, and 5J dynamic runs are given inasmuch as some of the original runs were made under adverse conditions and the necessary reruns were not completed in time to report here However a brief discussion of the qualitative implications is given in the next section of this report

Discussion and Interpretation of the Results

Measures of Effectiveness

Insofar as this study was concerned with operational research in terminal-area traffic control, one of the vital tasks was to define proper measures for discriminating between proposals The measures chosen had to (a) measure the effects of particular rules and

TABLE

ABSOLUTE DELAY RESULTS OF

Period of Operation (hour)	Kind of Operation	Delay Parameter, Type I Samples* Group No	Wind (mph)	Acceptance Rate	Optimum Separation	Average Delay per Aircraft (minutes)	Number of Aircraft Delayed (per cent)	Maximum Delay (minutes)	Number of TILTAMS Less Than 15 Seconds	Number of TILTAMS Less Than 30 Seconds	Delay Parameter, Group No	Wind (mph)	Acceptance Rate	Optimum Separation	Average Delay per Aircraft (minutes)
0 to 1	Arrival										6	0	38 5	SC	7 0
	Arrival										10	0	36 1	NSC	7 7
	Arrival										7	20	31 6	SC	11 7
1 to 2	Arrival	1	20	29 8	NSC	15 8	99 0	47 0			6	0	38 5	SC	15 5
	Arrival	3	20	31 6	SC	13 0	98 0	40 1			10	0	36 1	NSC	19 3
	Arrival	4	0	38 5	SC	5 1	94 9	20 0			7	20	31 6	SC	32 0
	Arrival	2	20	29 8	NSC	13 7	98 0	48 6							
	Departure	2	20	29 8	NSC	2 8	97 0	13 3							
	Operation	2	20	29 8	NSC	9 7	98 0	40 6							
1 to 2 1/2	Arrival	1	20	29 8	NSC	19 9	99 0	49 3							
	Arrival	3	20	31 6	SC	15 8	99 0	42 8							
	Arrival	4	0	38 5	SC	5 0	94 8	20 0							
	Arrival	2	20	29 8	NSC	17 1	99 0	40 6							
	Departure	2	20	29 8	NSC	3 0	98 0	13 3							
1/2 to 2 1/2	Arrival	1	20	29 8	NSC	17 1	99 0	49 3	6 6	2 6					
	Arrival	3	20	31 6	SC	13 7	98 0	42 8	5 4	0 8					
	Arrival	4	0	38 5	SC	4 3	92 3	20 0	3 9	1 0					
	Arrival	2	20	29 8	NSC	14 7	99 0	40 6	7 9	2 3					
	Departure	2	20	29 8	NSC	2 6	98 0	13 3	Not Available						
	Operation	2	20	29 8	NSC	11 2	99 0	40 6	Not Available						
2 to 3	Arrival										6	0	38 5	SC	21 6
	Arrival										10	0	36 1	NSC	28 0
	Arrival										7	20	31 6	SC	50 4
0 to 3 Average	Arrival										6	0	38 5	SC	14 7
	Arrival										10	0	36 1	NSC	18 4
	Arrival										7	20	31 6	SC	31 4
1/2 to 3	Arrival										6	0	38 5	SC	
	Arrival										10	0	36 1	NSC	
	Arrival										7	20	1 6	SC	

*Groups 1 3, and 4 of Type I are of arrivals only, WNA Phase 2, Group 2 uses samples of both arrivals and departures, also WNA phase 2
**Type II utilizes 3 hour samples of 35 arrivals per hour, WNA Phase 2
***Type III utilizes 3-hour samples averaging 35 arrivals per hour, WNA Phase 2

NOTES

1 Except for maximum delays, data shown are for the averages of 3 samples -- 9 runs in the cases of those groups using the Type I samples and 6 runs
2 A TIGTAM is a time interval greater than the assigned minima, in averaging TIGTAMs, all TILTAMs are weighted as zero seconds
3 A TILTAM is a time interval less than the assigned minima, 15 seconds represents about 1/2 mile and 30 seconds represents about 1 mile at average
4 Acceptance rates shown are theoretical maximum landings per hour
5 An operation is either a landing or a take-off

XV

THE DYNAMIC-SIMULATION TESTS

Type II Samples**					Delay Parameter, Type III Samples***									
Number of Aircraft Delayed (per cent)	Maximum Delay (minutes)	Number of TILTAMS Less Than 15 Seconds	Number of TILTAMS Less Than 30 Seconds	Average Length of TIGTAMS (seconds)	Group No	Wind (mph)	Acceptance Rate	Optimum Separation	Average Delay per Aircraft (minutes)	Number of Aircraft Delayed (per cent)	Maximum Delay (minutes)	Number of TILTAMS Less Than 15 Seconds	Number of TILTAMS Less Than 30 Seconds	Average Length of TIGTAMS (seconds)
96 2	24 9				8	0	38 5	SC	7 3	98 6	24 1			
98 1	23 6				11	0	36 1	NSC	6 8	98 1	27 5			
99 1	33 4				9	20	31 6	SC	9 7	99 5	34 9			
99 5	33 2				8	0	38 5	SC	16 0	99 1	39 5			
100 0	37 3				11	0	36 1	NSC	15 8	100 0	38 1			
100 0	57 9				9	20	31 6	SC	27 2	100 0	53 8			
100 0	42 8				8	0	38 5	SC	25 2	100 0	56 3			
100 0	51 2				11	0	36 1	NSC	25 1	100 0	50 8			
100 0	85 7				9	20	31 6	SC	46 1	100 0	85 2			
98 6		4 0	0 2		8	0	38 5	SC	16 2	99 2		2 8	0 2	
99 4		3 2	0 5		11	0	36 1	NSC	15 9	99 4		5 7	0 8	
99 7		6 8	2 3		9	20	31 6	SC	27 7	99 8		10 0	1 5	
				24 6	8	0	38 5	SC						24 6
				22 6	11	0	36 1	NSC						20 1
				26 3	9	20	31 6	SC						21 9

for Type II and Type III samples

approach speeds

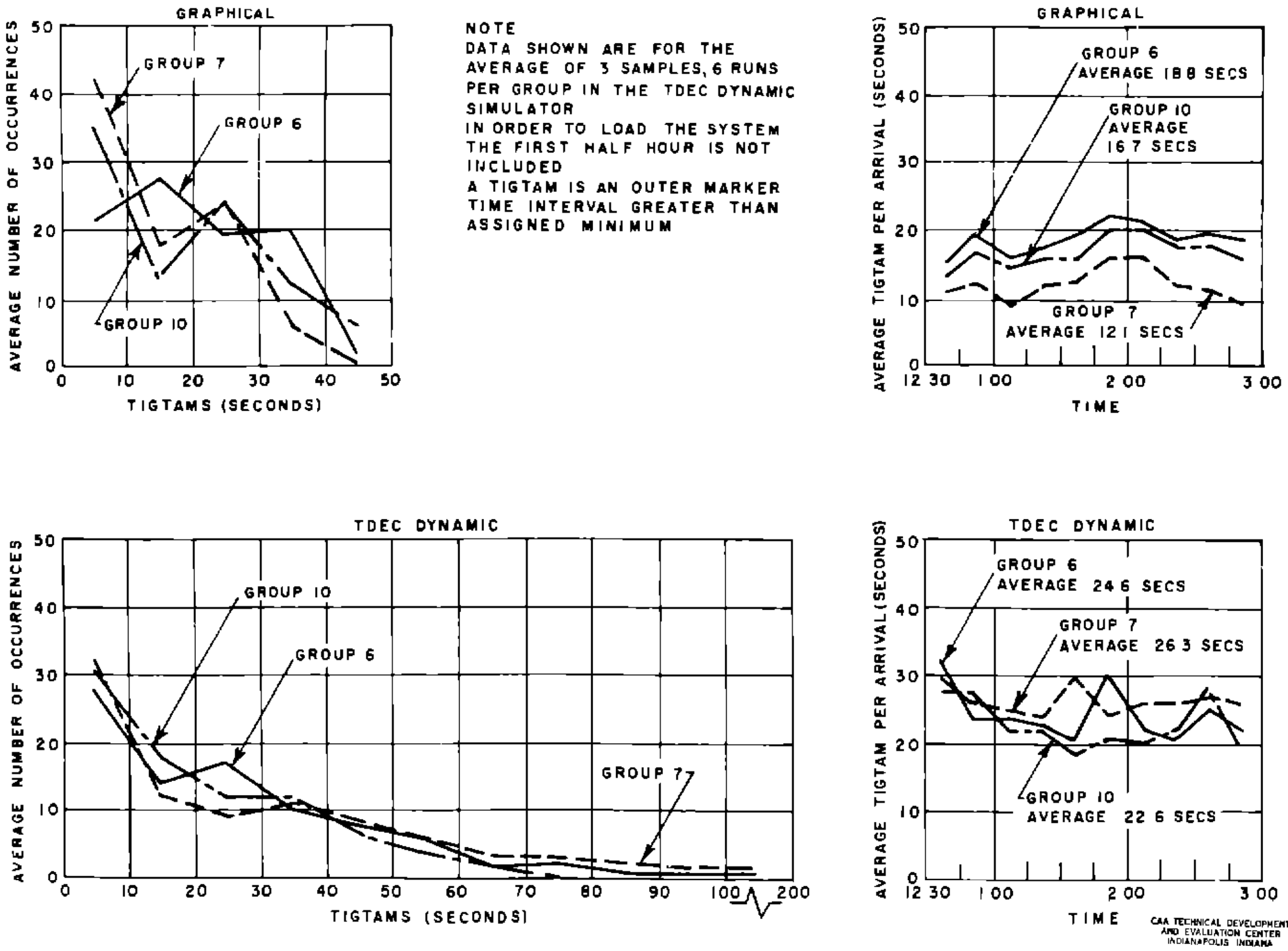


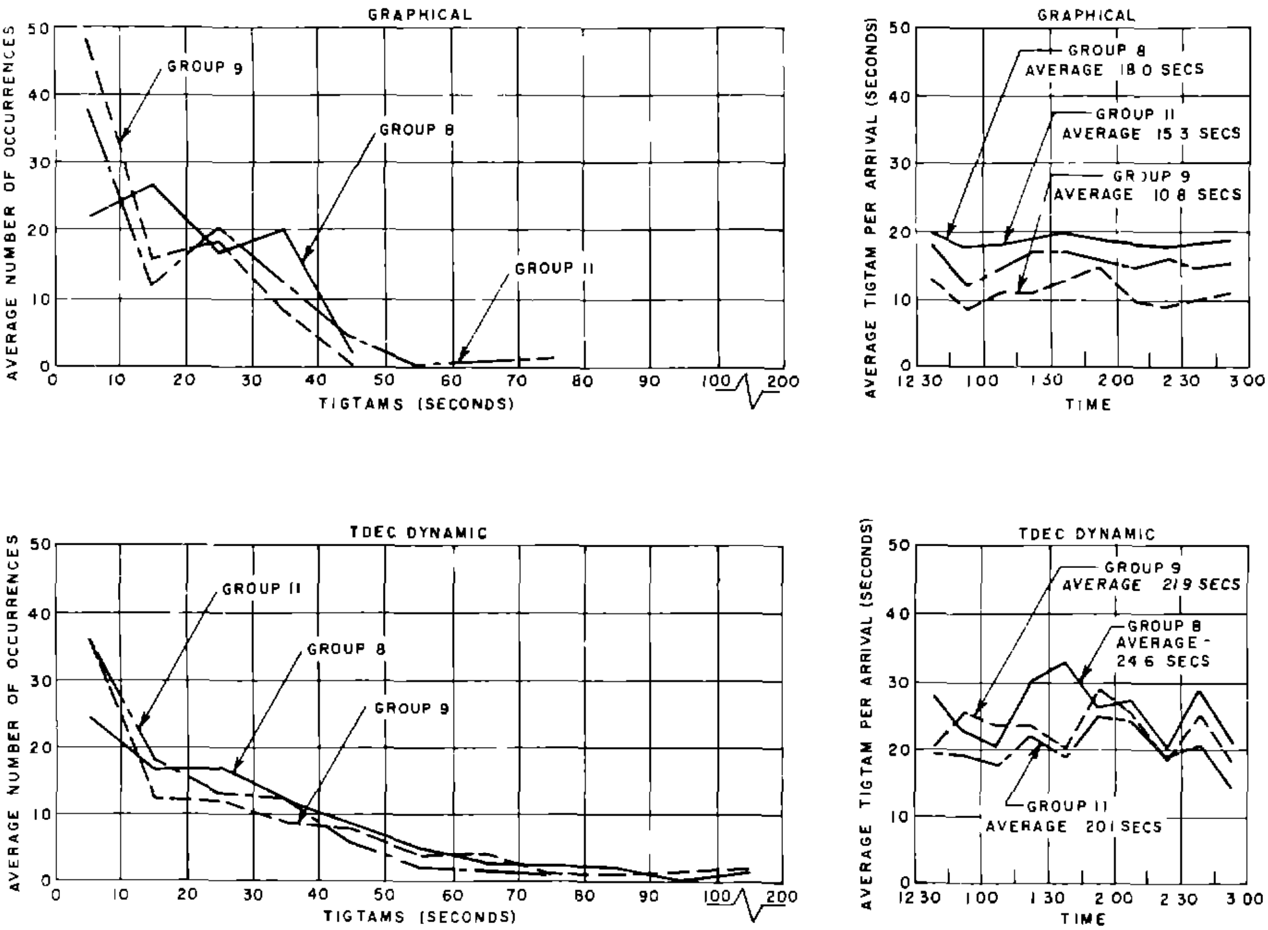
Fig 23 Comparison of Graphical- and Dynamic-Simulator Distributions and Time Averages of TIGTAMS, WNA Phase 2, Type II, 3-Hour Flow-Control Arrival Samples

configurations, (b) measure the human factors involving controllers; (c) determine the effects of changing rules and procedures, and (d) establish the effects of (a), (b), and (c) on the volume and type of communications

Toward these ends the most direct measure is capacity, in number of movements per unit time Since one of the ultimate aims of air-traffic control is to move as many airplanes as possible as quickly and as safely as possible, such a rate measure of system efficiency provides the most direct basis for discrimination between systems Therefore, the acceptance rate or capacity at the outer marker was one of the criteria used in this study

Another valuable efficiency measure is aircraft delay, which takes into account not only capacity but also demand and shows the effects of the interrelationship of these two factors In many respects, delay is a more meaningful measure of effectiveness than is capacity alone First of all, it provides a quantitative basis for assessing cost A capacity that is too low for the demand results in long delays, and the cost to airline operators is high not only in operating costs but also in business lost through inconvenience to passengers On the other hand, a capacity that is more than adequate for the number of arrivals may result in little or no delays, but the cost of providing and maintaining the high airport capacity may be unreasonable on a per-airplane basis Therefore, expected delays provide a sound basis for determining the capacity needed to provide some optimum grade of service

Sensitivity is another feature of the delay measure Whereas a small difference in capacity between two systems may appear negligible, a high congestion may show the differences in delay to be appreciable Although any measure may show up gross differences, delays show up finer, more subtle dissimilarities



NOTE
DATA SHOWN ARE FOR THE AVERAGE OF 3 SAMPLES 6 RUNS PER GROUP IN THE *TDEC DYNAMIC SIMULATOR*
IN ORDER TO LOAD THE SYSTEM THE FIRST HALF HOUR IS NOT INCLUDED
A TIGTAM IS AN OUTER MARKER TIME INTERVAL GREATER THAN ASSIGNED MINIMUM

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Fig 24 Comparison of Graphical- and Dynamic-Simulator Distributions and Time Averages of TIGTAMS, WNA Phase 2, Type III, 3-Hour Flow-Control Arrival Samples

Several other measures were recognized but were not used explicitly For instance, safety and ease of operation are important requisites in any system Although safety was assumed to be constant in all systems and the safety aspects were taken into account in the prescribed separations, the ease-of-operation aspect was discussed in more detail in previous reports[12,13] when the techniques of the relaxed system were first evolved

The results of this investigation are therefore presented from two points of view capacity as measured in the acceptance rate (number of aircraft per hour) and the interaction of demand with capacity as measured by delays (*minutes and number of service times*)

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[12] Berkowitz and Doering, op cit

[13] Anderson and Vickers, op cit

TABLE

COMPARISONS OF TYPICAL VARIATIONS BETWEEN SAMPLES AND REPEAT RUNS

Period of Operation	Run Number	Sample Number	Ideal Simulation				Graphical	
			Average Delay per Aircraft (minutes)	Number of Aircraft Delayed (per cent)	Maximum Delay (minutes)	Average of TIGTAMS (seconds)	Average Delay per Aircraft (minutes)	Number of Aircraft Delayed (per cent)
0 to 1	1 2	1 1	2 6	85 7	7 8		5 4	94 3
	1 2	2 2	2 1	85 7	5 5		3 8	88 6
	1 2	3 3	2 4	88 6	7 2		4 0	85 7
		Average of 3 Samples	2 4	86 7			4 4	89 5
1 to 2	1 2	1 1	2 2	91 4	5 5		10 2	100 0
	1 2	2 2	2 6	88 6	7 8		10 5	100 0
	1 2	3 3	2 8	94 3	7 8		11 2	100 0
		Average of 3 Samples	2 6	91 4			10 6	100 0
2 to 3	1 2	1 1	3 0	94 3	7 2		14 8	100 0
	1 2	2 2	2 8	91 4	7 2		14 8	100 0
	1 2	3 3	2 6	85 7	7 8		17 3	100 0
		Average of 3 Samples	2 8	90 5			15 6	100 0
0 to 3	1 2	1 1	2 6*	90 5*	7 8		10 1*	98 1*
	1 2	2 2	2 5*	88 6*	7 8		9 7*	96 2*
	1 2	3 3	2 6*	89 5*	7 8		10 8*	95 2*
		Average of 3 Samples	2 6	89 5			10 2	96 5
1/2 to 3	1 2	1 1				Not Available		
	1 2	2 2				Not Available		
	1 2	3 3				Not Available		
		Average of 3 Samples				Not Available		

*Average for hours of operation

NOTE

1 The Group 6 conditions included Type II; 3-hour samples of 35 arrivals each hour, speed-control
2 Maximum delays are not averaged Average delay refers to all aircraft
3 A TILTAM is a time interval less than the assigned minimum, 15 seconds represents approximately
4 A TIGTAM is a time interval greater than the assigned minimum in averaging the TIGTAMs, all
5 The configuration used is WEA Phase 2

XVI

OF GROUP 6 FOR IDEAL, GRAPHICAL, AND DYNAMIC SIMULATOR TESTS

Simulation		Dynamic Simulation					
Maximum Delay (minutes)	Average of TIGTAMS (seconds)	Average Delay per Aircraft (minutes)	Number of Aircraft Delayed (per cent)	Maximum Delay (minutes)	Average of TIGTAMS (seconds)	Tiltams Less Than 15	Tiltams Less Than 30
13 3		8 4	94 3	24 9			
		5 9	94 3	13 8			
9 5		8 4	100 0	19 0			
		6 2	97 1	15 0			
12 0		6 5	94 3	19 6			
		6 4	97 1	18 8			
		7 0	96 2				
15 5		15 9	100 0	27 1			
		11 5	97 1	20 6			
18 6		18 8	100 0	33 2			
		10 6	100 0	18 6			
16 4		19 5	100 0	30 5			
		16 6	100 0	25 7			
		15 5	99 5				
20 9		21 5	100 0	34 2			
		17 2	100 0	27 5			
20 9		25 7	100 0	34 2			
		11 0	100 0	21 0			
25 3		30 3	100 0	42 8			
		24 0	100 0	34 8			
		21 6	100 0				
20 9		15 2*	98 1*	34 2		2	0
		11 5*	97 1*	27 5		1	0
20 9		17 6*	100 0*	34 2		5	0
		9 3*	99 0*	21 0		4	0
25 3		18 8*	98 1*	42 8		7	0
		15 6*	99 0*	34 8		5	1
		14 7	98 6			4 0	0 2
	18 4				22 5		
					18 7		
	19 5				29 4		
					20 0		
	18 4				31 3		
					25 7		
	18 6				24 6		

optimum separation 0 wind, and a theoretical maximum acceptance rate of 38 5 per hour

1/2 mile, 30 seconds represents 1 mile at average approach speeds
TILTAMs are weighted to zero seconds

TABLE XVII

EFFECTIVE ACCEPTANCE RATES

Type of Simulation	0 Wind Acceptance Rates		20-mph Wind Acceptance Rates	
	Speed-Control Optimum Separation	No-Speed Control Optimum Separation	Speed-Control Optimum Separation	No-Speed-Control Optimum Separation
Ideal	38 5	36 1	31 6	29 8
Graphical	32 1	31 1	28 6	27 4
Dynamic	30 5	29 8	26 1	Not Done

The Three Stages of Simulation

Within the limits just established, the merits of the three stages of simulation used in this study may be questioned The first stage, or ideal analysis, determines the maximum possible acceptance rate with a given set of prescribed separations The second stage, or graphical analysis, determines what the system can handle when the existing airways, flight-separation rules, types of aircraft, and descent rates are considered The third stage, or dynamic simulation, in addition to the above factors considers the human judgments and estimates, aircraft performance, including rates of turn and variable speeds, and the effects of using standard communications Thus the three stages are three successively more complete approximations to reality

The merit of using this multilevel approach to simulation may also be questioned One feature of this method is that the systemic and the human factors can be isolated and measured The most direct measure of these effects is seen in the effective acceptance rates (EAR) at each level This EAR was calculated by measuring the average time between each successive outer-marker crossing when the system was loaded The average separation (in seconds) was then divided into 3600 This quotient gives the maximum possible arrivals that could be handled in an hour Contrasting with effective acceptance rate is the maximum theoretical rate, which is theoretical in that it presumes that aircraft could make good any path or speed in order to appear at the outer marker exactly within the prescribed separation Table XVII gives the resulting EAR for the three levels of simulation at 0 wind and at 20-mph wind and for the two sets of prescribed separations The two sets of samples, flow control and random, were not differentiated because the average EAR for both groups did not differ by more than 0 1 aircraft per hour It should be noted that the ideal EARs are the maximum theoretical acceptance rates

System Effects

Several interesting facts can be gleaned from Table XVII First, the system effects as measured by the difference between ideal- and graphical-analysis delays increase as the

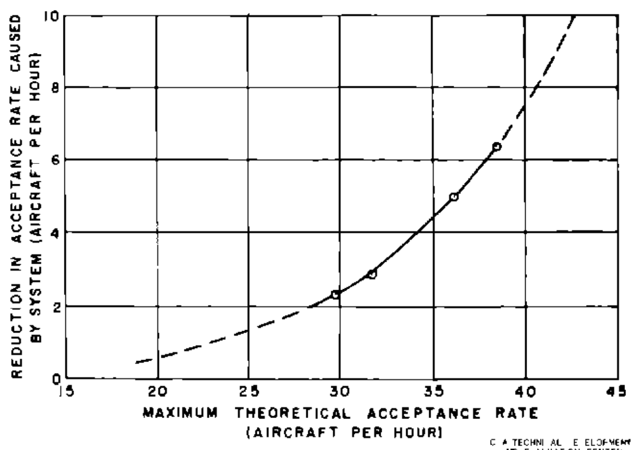

Fig 25 System Effect as a Function of Maximum Theoretical Acceptance Rate

TABLE XVIII

GROUPS COMPARABLE EXCEPT FOR DIFFERENCE IN SEPARATIONS

Optimum Separations	Speed-Control Separations
6	10
7	16
8	11
9	18

traffic-input rate increases This relationship is shown in Fig 25, where it is seen that the system would have a negligible effect at a theoretical maximum acceptance rate of 15 to 20 per hour but that it produces increasingly greater reductions in the EAR at theoretical rates of 30 to 35 or more Therefore, the system seems so flexible that force-feeding is possible with some system resistance The resistance seems to increase exponentially with the degree of forcing From the rate indicated by the graph in Fig 25, this counteraction of the system makes it improbable that when the present separation rules are adhered to a sustained acceptance rate in actual practice could exceed about 35 landings per hour Violations of these separation rules are quite frequent in practice, however, there are many cases where flexibility in judgment would not seem to violate the intent of the safety requirement or of the prescribed separation of 3 miles minimum A more direct measure of system reaction is

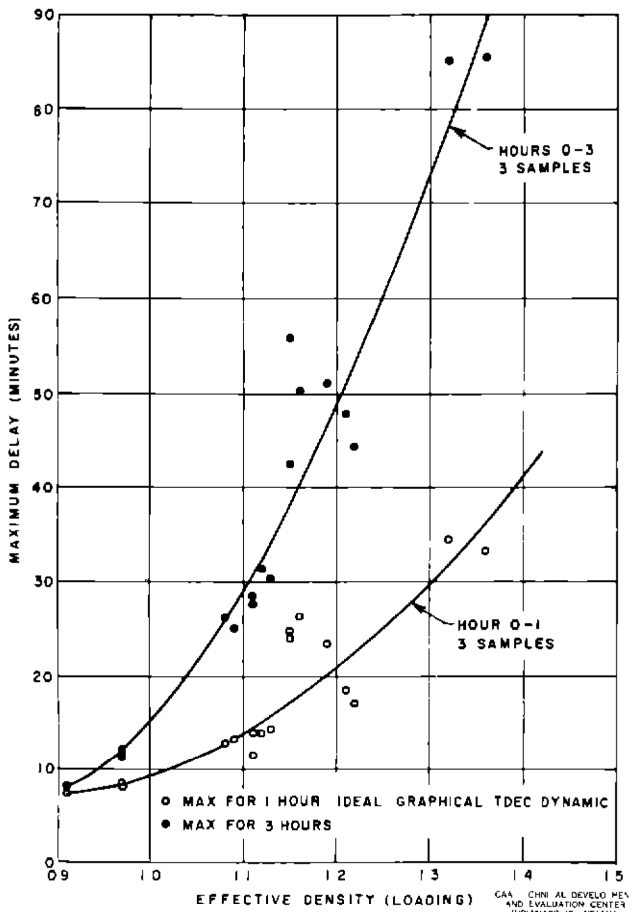

Fig 26 Maximum Delay as a Function of Effective Density

TABLE XIX

COMMUNICATIONS DATA FOR THE AVERAGE OF THE TWO SAMPLE NO 2 DYNAMIC-SIMULATOR RUNS OF GROUP 10

	15-Minute Peak Density (per cent)	Average Density (per cent)	Average Message Length (seconds)	Average Communications Time Per Aircraft (seconds)	Average Number of Messages Per Aircraft	Busy Time (per cent)	Messages Delayed (per cent)
West-Sector Approach Control							
Air-to Ground	22 2	14 7	2 4	39 3	16 4	0 3	2 0
Ground to-Air	25 8	18 0	2 9	47 0	16 2	---	--
Air and-Ground	48 0	32 7	2 6	86 3	32 6	-	---
East-Sector Approach Control							
Air-to-Ground	19 5	15 7	2 3	34 8	15 1	0 4	2 0
Ground to Air	26 5	17 8	3 5	50 0	14 3	---	---
Air-and-Ground	46 0	33 5	2 9	84 8	29 4	---	---
ARTC Sector Transferred to Approach							
Air-to-Ground	20 0	8 9	1 9	11 6	6 1	---	--
Ground-to Air	24 0	15 4	2 8	19 3	6 9	---	- -
Air-and-Ground	44 0	24 3	2 4	30 9	13 0	--	---

shown in Figs 23 and 24 Essentially, TIGTAMS are added separations enforced by the system rules, the randomness of aircraft arrivals, and the ability of aircraft to pull out of the inner fixes and fly the prescribed tracks The graphical-time series clearly shows that the TIGTAMS increase for Groups 7 and 9, 10 and 11, and 6 and 8 with theoretical acceptance rates of 31 6, 36 1, and 38 5 per hour, respectively

The human effects were measured by the difference between the dynamic results and the graphical results One of the conditions (Groups 15, 16, 18, and 19, Type II samples, 20-mph wind, and no speed control) was not run dynamically because it would have resulted in an unrealistically high congestion However, the high density that resulted from the one 20-mph-wind condition (with speed control and with a theoretical acceptance rate of 31 6) apparently caused a deterioration in the controllers' performance In the 0-wind condition, the controllers were able to operate at an effective loading of about 115 per cent for several hours and yet missed the perfection of the system by about one and a half aircraft per hour, a truly creditable feat On the other hand, in the 20-mph-wind condition at an effective density of 134 per cent, the dynamic acceptance rate dropped to 2 7 aircraft below the graphical rate although the drop might have been less than one aircraft per hour for this lower theoretical acceptance rate To illustrate this effect, Figs 26, 27, and 28 are presented to show the average and maximum delays produced in ideal, graphical, and dynamic analysis as a function of effective or theoretical density for the one- and three-hour periods

Traffic Fluctuations

Another advantage of the three-stage simulation program is the predictability of delays with the fluctuation of the incoming traffic rate For realistic problems where traffic builds up to a peak and then declines, it is possible to run simulation studies at the lower, less costly levels and from these studies to predict what the delay picture would be at the higher levels and in reality In a previous report[14] it was pointed out that a high degree of correlation (98 per cent) exists between the time series of delays of the three stages This fact is of great importance because the expense of simulation increases with the introduction of more and more realism Therefore, the most economical simulation program would establish the effective acceptance rates between the three stages and would then run more extensive tests at one of the more elementary levels

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[14]Berkowitz and Doering, op cit

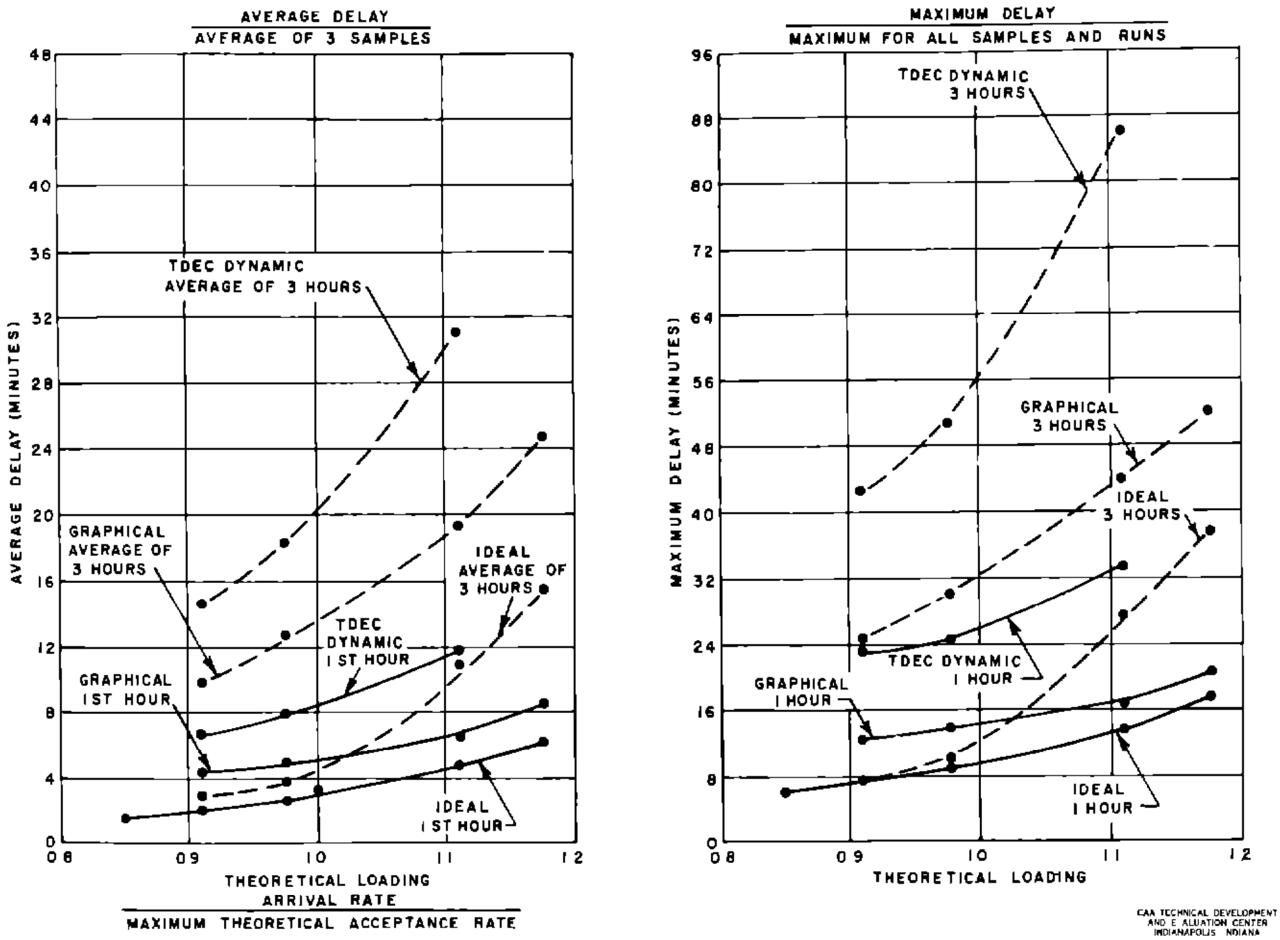

Fig 27 Average of Maximum Delay as a Function of Theoretical Loading, Types I and II Flow-Control Arrival Samples

Before such a program is undertaken, however, it should be remembered that each step toward realism reduces the EAR Therefore a constant input rate increases the effective congestion at each level For prolonged samples at high congestion, as were used in this study, this effective density is of critical importance in making time-oriented estimates of delays For accurate prediction, it is necessary that both stages have effective densities on the same side of 1 0, both should be less or both should be more As an example, Fig 18 shows that the ideal level was of little value in predicting delays of the two higher levels for Groups 6 and 10 but that the ideal level of Group 7 and the graphical levels of all groups provided good predictions of the dynamic, more costly stage The reason for the dissimilarity in the one case and the correspondence in the other lies in the delay curves referred to previously There it can be seen that delays for congestion less than 1 0 reach some equilibrium value, whereas a density of greater than 1 0 results in a constant build-up of delays The two ideal sets, Groups 6 and 10, were both run at densities less than the critical 1 0 and therefore do not reflect the increase in delay at the higher levels reached in simulation tests which were run at densities greater than 100 per cent All the runs at the higher densities show increasing delays as time progresses Because delays build up almost linearly, the only difference between one set and the next is the slope of the build-up Once these slopes have been determined for each level, the simpler level can be used for predicting the more complex one

At low densities, the delay situation very closely follows the arrival situation That is, a surge of arrivals in one period causes an increase in delay for the next period, and a decrease in the number of arrivals is reflected in a decrease in delays This relationship is shown in

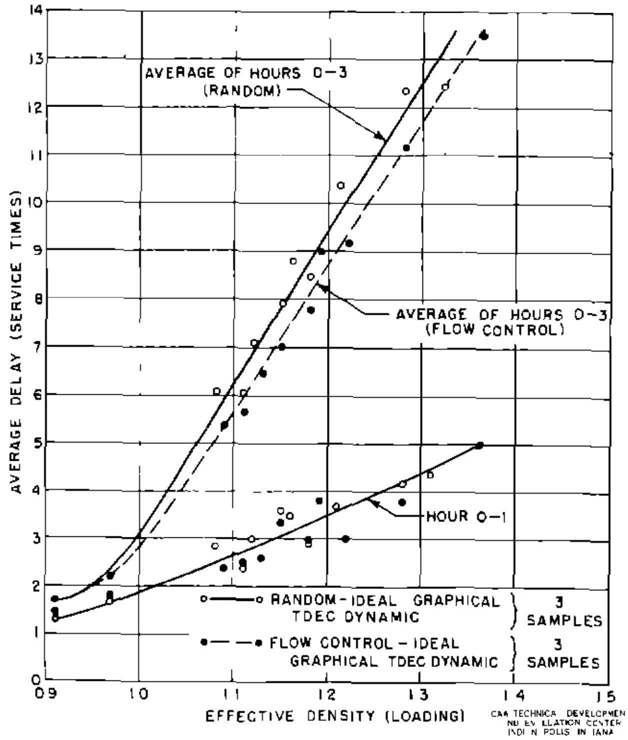

Fig 28 Average Delays for Random and Flow-Control Arrivals as a Function of Effective Density

Figs 12 and 13, especially in the ideal- and graphical-analysis curves However, in the two dynamic-simulation curves for the 20-mph wind condition in which the effective density was more than 120 per cent, the delays continued to accumulate even after the arrival rate decreased

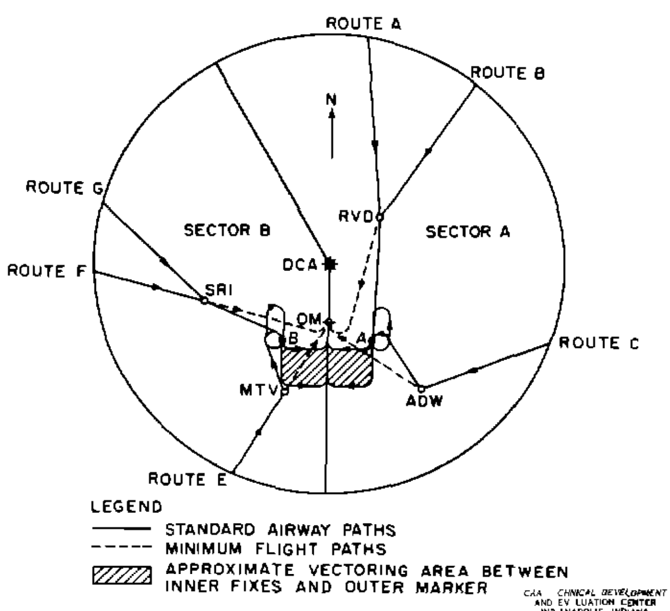

Fig 29 Configuration of WNA Phase 2

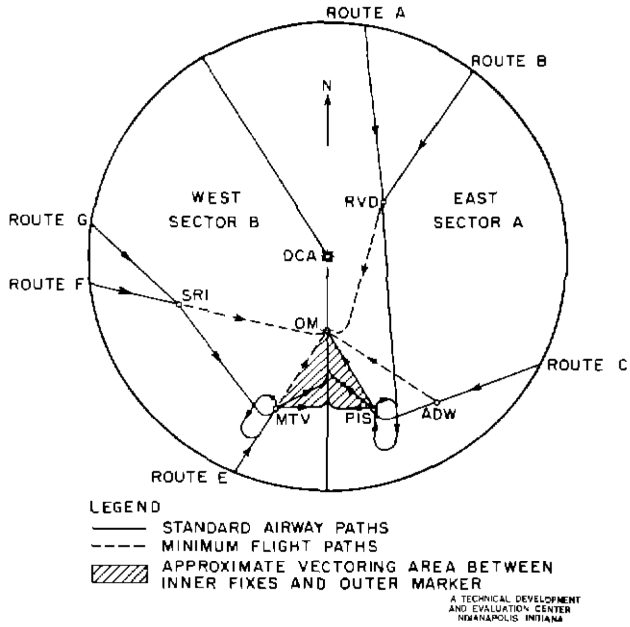

Fig 30 Configuration of WNA Phase 7

Human Considerations

As has already been pointed out, on the average the controllers' performance decreases the acceptance rate of the system to only a small degree However, there is a large amount of variability between the performances of the same controllers on the same problem, and the differences in degrees of experience in this type of control is startling A typical example of the differences between similar samples and repeat runs of identical samples was shown in Fig 22 and Table XVI In fact, several runs in which untrained men did the controlling had to be eliminated from the data because the results, in terms of delays and violations, were so unrepresentative Putting untrained controller personnel in live and heavy IFR traffic situations is not considered representative or at least should not be tolerated in actual operations

The only explanation that can be given for this high variability is that the precision of spacing aircraft properly at the approach gate during high loadings is very critical Although the principle of using twin close-in stacks permits a high degree of precision, increases as small as 10 seconds in the average effective separation can cause large increases in delays under high congestion

This relationship between delays and separations is shown vividly in Tables XV and XVI in the rows headed Delays and TIGTAMS (Time Interval Greater Than Assigned Minimum) Those teams which attained the lower TIGTAMS in effect created the higher acceptance rates and the lowest delay rate A few seconds difference in the TIGTAMS between identical runs resulted in many minutes difference in average and maximum delays Each aircraft in a run was assigned a TIGTAM value if (a) the outer-marker time separation between it and the preceeding aircraft was greater than the prescribed or assigned separation and (b) it was delayed The second restriction assured that a TIGTAM would not be ascribed to an aircraft simply because it did not arrive soon enough These TIGTAMS give a direct measure of the precision with which controllers can meet the prescribed separations In effect, a larger TIGTAM indicates a lower effective acceptance rate, and a small value indicates a higher EAR and, therefore, system efficiency

The rows headed TILTAMS (Time Intervals Less Than Assigned Minimum) indicate the number of violations of the prescribed minimum separations Offhand, one may consider these dangerous from the safety angle of air traffic control However, it must be realized that no human could possibly maintain exactly the required separations There is bound to be some variation about these values If exactly zero TILTAMS were to be attained, the controllers would have to maintain such conservatively large average separations resulting in large TIGTAMS over and above what are already prescribed for safety's sake that the effective acceptance rate would be reduced drastically The most easily reached compromise between the necessity for increasing acceptance rates on the one hand and the necessity for maintaining safety requirements on the other seems to be that some TILTAMS can be tolerated, provided that they are small and infrequent As Tables XV and XVI show, this is precisely what occurred at TDEC As a matter of interest, one-half mile on a 12-inch radar scope covering 40 miles diameter is only about 3/16 of an inch

Because of this lower separation limit needed for safety and because of the need for a small average separation for maximum airport capacity, it is desirable to reduce the amount of variability in meeting the required separations This problem can be solved most directly by such techniques as multitrack layouts and special approach computers These simple devices would serve two purposes first, little training would be required before a controller could attain maximum efficiency and thus small variability with this vectoring system, second, there would be little tendency for trained controllers to get rusty from lack of practice

Randomness Versus Flow Control

There has been much discussion in recent years about the merits of derandomizing traffic to decrease terminal-area delays It was possible to make partial measurements in this study to throw some light on this subject because our two sets of samples were purely random and included a predetermined number of arrivals per half hour and per hour arriving at random within the half hour To be sure, this second group constitutes flow control at a very low level, but the results show some derandomization at work Figure 28 shows plots of average delays as a function of effective loading for each of these two sets of samples after one hour and after three hours As can be seen, for a period of one hour there is only a minor difference between these sets, however, after three hours the minor degree of flow control tends to reduce delays for the loadings greater than 100 per cent Here again is shown the value of using delays to measure effectiveness The smallest differences can be detected by it

The implications of this finding are far-reaching First, even the coarsest level of flow control can have nothing but beneficial results Second, it should be realized that en route flow control will not necessarily reduce over-all delays to aircraft, it primarily transfers the terminal delay further out and, at times, might even make the delay even higher because it could conceivably decrease the degree of precision of feeding the terminal area Third, even delays at a departing airport are delays from the standpoint of inconvenience but seem to be much more economical and safer from the viewpoint of the airline operators Therefore, some technique for predicting the number of arrivals at an airport and for subsequently rescheduling arrivals, as necessary, to avoid extreme congestion would be of significant benefit to the system The best feature of such a program is that the scheduling need not be very precise for the advantages to be felt

Intermixing Landings With Take-Offs

Simulation tests were made with landings and departures intermixed on a single runway See Tables XI, XIV, and XV and Figs 14, 15, and 16 These tests permitted study of the techniques for handling the double-purpose single runway For the conditions of 0 wind and optimum separations, it was found that the average separation time between two successive landings was about 120 seconds When a take-off intervened, the separation between the landings was about 140 seconds The average runway separation between two successive take-offs was 65 seconds On the basis of these figures, it seems obvious that the most efficient method for intermixing landings and take-offs when there is a backlog of both is to alternate them, a take-off requires 65 seconds when following another take-off but adds only 20 seconds when interspersed between two landings This alternating procedure resulted in a single-runway capacity of 50 to 55 operations per hour

For the 20-mph-headwind condition and single-runway operation, arrival separations had to be increased anyway It was therefore found convenient and efficient to intersperse departures between successive arrivals with no additional delay to the arrivals This alternating procedure resulted in 45 to 50 operations per hour on the single runway

A brief graphical analysis on the use of intersecting runways (one for landings, and one for take-offs) showed that about 70 mixed operations per hour were possible Furthermore, it was evident that approach- and departure-controller co-ordination might be reduced very much until the amount is even less than for a dual parallel-runway configuration

Speed Control

One reason for the large outer-marker separations and for the conservatism used today is the random variability in speeds between aircraft This variation makes it difficult to maintain a precise separation between two aircraft even when their types and rated speeds are known. In prescribing present-day separations this factor of ignorance must be taken into account.

To investigate the possibilities of reducing the spread in aircraft performance, a degree of speed control was introduced and new separations were determined. A small sample of the trend of the results was shown in Table XVII. It is seen that at the ideal level these new separations led to an improvement of 2.4 aircraft per hour. When the system and human effects were added, the improvement increased another 0.7 per hour in zero wind. These results may seem insignificant, but the delay curves of Figs. 17 through 20 and Fig. 27 prove that they are not. The additional safety factor resulting from this small amount of speed control should also be borne in mind. For reference, Table XVIII lists the group numbers that are comparable in every respect except for the difference in separations. By comparing the similar groups in Figs. 17 through 20, it can be verified that the seemingly insignificant increase in acceptance rate introduced by speed control had a marked effect in reducing the delays, especially at high loadings.

Effect of Resequencing the Exponential Time Intervals of a Traffic Sample

Table XIII showed the effect of rearranging into new sequence the exponential time intervals of the second sample of the Type III, 3-hour random samples of arrivals. It should be observed how the arrival rates fluctuated for each hourly period and correspondingly how the ideal-analysis delays varied. This is just a sample of the caution that must be exercised in making comparisons between single similar, but internally different, random traffic samples.

Comparison of Delays of WNA Phase 2 and WNA Phase 7 Configurations

Figures 29 and 30 illustrate the differences between these two configurations. It is seen that the locations of the Phase 2 inner fixes lay closest to the minimum flight paths for the various entry airways and also permitted more flexibility for radar guidance, which is commonly known as vectoring. The delay results of the graphical analysis are shown in Table XIV and Fig. 16. The differences between the two are small under the conditions of zero wind and higher theoretical acceptance rates (lower loadings) for either arrivals only or for arrivals and departures. Phase 2 appears to be the more efficient by a small percentage which is statistically insignificant. However, at the lower acceptance rate under conditions of a 20-mph headwind and higher loading, the differences are more pronounced.

Communications Results

The high-density dynamic-simulator runs of three hours placed some limitations on the degree of accuracy of the latest results. The high number of aircraft delays and the length of the runs made it necessary at times to keep the experiments going without usable airplane consoles and their live pilots (eight consoles were available). Instead, when an airplane console became available, aircraft were fed into the experiments at those points in space and time corresponding to where they should have been according to the program script. Consequently, some of the routine communications, the resulting channel busy times, and other variable conditions could not be included or integrated properly with the rest of the live simulation workload. There was little significant difference between groups, samples, or repeat runs. Table XIX is presented to show communications workload in the form of channel densities, number of messages, and communicating time for the two approach sectors and the adjoining Air e Traffic Control (ARTC) sector; the data shown are for the averages of the two Sample No. 2 runs of Group 10 and are typical of the other groups and samples. It is seen that even though the traffic loading exceeded 100 per cent for three hours, average densities of approach-control communications per sector for the entire period were only about 33 per cent and never exceeded 48 per cent for any 15-minute period and that the average communicating time (air plus ground) per aircraft was about 85 seconds.

Contrary to random-communication theory, the various channel densities, numbers of messages delayed, busy times, amount of conversation, and other communications variables were low. Present communication theory is based on the assumption that calls occur at random, but simulation tests indicated that in the WNA Phase 2 type of terminal-area traffic control there are a large number of stereotyped situations such as a fixed block in space between outer and inner fixes, a fixed block in altitude separations, and party-line simplex communications. Such situations cause a high degree of redundancy in the communications nd a low irritability factor. This result is supported by the fact that there were very few y times in the air-to-ground end of the party line, and it was seen that the origination of he calls followed a definite pattern and were not by any means random.

Dynamic-Simulation Tests of Controller's Approach Computer, Airborne Pictorial Computers, and Integration of Jets, Groups 5A, 5B, and 5J

No detailed quantitative results are reported The prototypal electronic multitrack approach computer (EMTAC) was found to be operating improperly, and certain other adverse conditions were also encountered during some of the six runs scheduled for each group Some repeat runs were made, but the analysis of the data is not sufficiently complete at the time of writing The computer malfunctions were subsequently engineered out of the prototypal model, and its operation was satisfactory

The results obtained to date by the use of EMTAC showed that the approach procedures were very simple When it was assumed that pilots followed their computer-assigned tracks by using airborne pictorial computers with radar monitoring and slight but infrequent corrections by the controllers where necessary, the controller's job was simplified to the lowest level ever attained in simulation tests The equipment was simple to operate, and personnel of the clerk grade could operate EMTAC efficiently after only 5 to 10 minutes of dual instruction

It became obvious that with this decrease in workload one controller could handle the usual functions of the two approach controllers (east and west) Some runs recently made employed only the single approach controller, and the results look very encouraging

It also is noteworthy that in the runs involving the integration of jets with conventional aircraft (Group 5J, using either one or two approach controllers), the delay results and communication workload were consistently and significantly less than in those comparable groups involving only conventional aircraft

CONCLUSIONS

1 Generally speaking, the results to date of the joint FIL-TDEC program of simulation activity have yielded a wealth of information In a relatively quick and inexpensive manner and on a rigorous scientific basis, the program has uncovered many requisites for the design of safe and efficient common terminal-area traffic-control systems Some of this information would be impossible to obtain by other means or in some cases would have been inconclusive Many of the principles involved can be extended to other areas of the navigable airspace and to ground facilities and have an immediate application toward increasing system capacity

2 For a wide variety of conditions such as types of traffic, traffic rates, traffic durations, acceptance rates, wind conditions, and constant and variable separations, the correlation between the FIL analytical and graphical analyses and the TDEC dynamic-simulation technique is excellent Thus techniques have been established whereby many of the terminal-area problems relating to delays, capacity, and such factors can be solved by the inexpensive analytical and graphical techniques without resorting to large-scale dynamic simulation or to actual flight tests

3 A preliminary determination of the mathematical formulas was made to describe and to evaluate air traffic flow on a realistic basis The results were validated by the graphical- and dynamic-simulation results Classical steady-state theory was shown to be totally inadequate and misleading for high traffic densities and traffic peaks approaching and exceedi loadings of 100 per cent, infinite delays occur only with infinite traffic congestion, a fact which is of no practical interest in air traffic control

4 As a matter of convenience, a configuration described as WNA Phase 2 was used as the reference terminal-area radar system, and many specific problems which had many generalize applications could be evaluated This configuration was found to be a most practical and effective one With the simple party-line simplex communications, clear-cut, nonconflicting paths, reliable radars, a very limited degree of speed control, and radar vectoring, WNA Phase 2 was found to be a relatively straightforward and relaxed system of terminal-area traffic control An over-all average for arrivals only of about 32 per hour could be handled in zero wind on a single runway for several hours This capacity is based on (a) the use of present IFR radar-separation minima of 3 miles, (b) the assumption that en route control could feed the inner stacks ideally at the top level all of the time, and (c) the use of previously described wave-off considerations In the case of mixed arrival-and-departure traffic, 50 or more IFR operations could be handled on a single instrument runway, by far the most efficient sequence was the alternation of landings with take-offs rather than the moving of a few landin

and then of a few take-offs For short periods of 15 minutes to a half hour, approximately 36 random arrivals per hour could be landed at an average interval between successive approaches of 1 minute and 40 seconds, correspondingly, about 50 to 55 mixed arrival-and-departure operations could be moved on a single runway A total of about 65 to 70 operations per hour was found feasible for intersecting runways and required very little co-ordination

5 Optimum outer-marker separations for outer-marker distances of 3 to 7 miles with or without speed control were established for the condition of a maximum probability of glide-slope wave-offs of 1 per cent In addition, the glide-slope wave-off criterion was assumed to be 2 1/2, 2, or 1 1/2 miles The theoretical acceptance rates ranged from 29 per hour for the 7-mile outer-marker distance when a 2 1/2-mile glide-slope separation was used as the minimum to 62 per hour for the 3-mile outer-marker distance when the 1 1/2-mile glide-slope separation was used as the minimum Probability analyses were undertaken to determine the effects of increasingly force-feeding at these higher rates a single instrument runway such as Washington National Airport 36 with a hypothetical high-speed exit added at 3400 feet The studies showed that the probable runway wave-offs pyramided to the point where feeding the runway at landing rates greater than 36 per hour resulted in very few more successful landings (39 per hour) and subsequently even fewer, the percentage of probable runway wave-offs varied almost exponentially from 1 per cent for a feeding rate of 29 landings per hour to 45 per cent at a feeding rate of 62 per hour Because of the theoretical nature of the work and because of the accuracy of the assumptions, it was concluded that a 5 per cent figure in theory might correspond to a negligible figure in practice This would yield a successful runway landing rate of about 36 per hour in actual operations

6 The implications are evident Significant gains in higher landing rates can be achieved with rather small changes such as shorter glide slopes, smaller allowable glide-slope separations, and a very limited degree of speed control However, the higher feeding rates induce an intolerable percentage of runway wave-offs unless runway layouts are significantly improved Means of achieving shorter glide slopes and smaller acceptable separations, of educating pilots to appreciate the effects of the small degree of speed control, and of judiciously locating even more high-speed runway exits should be given serious consideration The small changes can yield, at little cost, significant gains that could otherwise be attained only by the construction and implementation of new runways, airways, and airports

7 The use of the radar type of terminal-area traffic-control system described as WNA Phase 2 with close-in twin feeder fixes or with any other similar twin-stack system resulted in simple communications procedures and low workloads Tests of a multitrack configuration involving the use of a controller's approach computer and of airborne pictorial computers showed even further simplicity and reduction of workload, furthermore the integration of jet traffic caused no additional delays or increase in workload

RECOMMENDATIONS FOR FUTURE SIMULATION ACTIVITY

Much has been learned about terminal-area control The best way to make the en route and airport elements compatible with the potentially high terminal-area capacity deserves the most serious consideration At the same time, other factors relating to terminal-area control should also be studied if the long-range point of view is properly taken Also, certain items of equipment, new techniques, new aircraft types, and other innovations are being introduced and should be integrated into the over-all scheme in the most efficient manner It is recommended that the present plans, facilities, and techniques for simulation be carefully studied and extended in an orderly manner to suit the solution of many other problems Accordingly, a comprehensive program should be undertaken to

a Investigate the pertinent aspects of safety and efficiency associated with the proper handling of present and future traffic in both en route and terminal areas Consideration should be given to the effects of

(1) Introduction of other aircraft types such as very slow (VS) and helicopter (H) and of a heavier concentration of jet (J) types
(2) Changes in present separation standards
(3) Multiple airway, airport, and runway operations

(4) Various traffic rates and durations
(5) Airport configuration and control of surface traffic
(6) Co-ordination between en route, approach-, and departure-control functions
(7) Centralized versus decentralized control and displays

b Evaluate thoroughly under a wide variety of conditions the systemic contributions of such various equipment and aids to navigation and air traffic control as are under consideration for development and use in the near future These include

(1) Symbolic data-transfer and display devices such as limited data-transfer devices and Type C Boards
(2) ATC signalling system of visual communications
(3) Cockpit pictorial displays and computers
(4) Primary and secondary radar
(5) Approach computers or timing devices for traffic controllers
(6) Pictorial displays including bright-tube displays and new displays such as the type being developed at Camp Evans, New Jersey

c Prepare detailed requirements and specifications for the design and development of new, or expansion of present, dynamic-simulation facilities in order to properly set up and evaluate the listed factor and many other problems, configurations, and systems

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APPENDIX A

SOME FURTHER ASPECTS OF THEORETICAL TRANSIENT DELAYS

Operation of the Model

A verbal description of the stochastic process for landing aircraft in a terminal area has been given It was stated that aircraft arrive at random times, subject to some constant arrival rate, and pass through a gate (the outer marker) at prescribed minimum intervals Such a series of events can be considered stationary and non-Markovian By definition, a system is Markovian if its present state depends only upon its immediately preceding state but not upon earlier states However, if we measure time in units of service times, the end of each service time can be considered a point of regeneration and the whole process can be treated as a Markov chain The following is a description of the air-traffic model from this point of view It should be realized that this is only one of many possible models For a more complete treatment a list of references is given at the end of this appendix The paper by Kendall is particularly interesting and comprehensive

A zero stack exists at the beginning of the first service time If no aircraft arrive at the outer marker during this interval, there will be zero aircraft in the stack at the beginning of the second interval If one aircraft arrives, it proceeds down the glide slope The stack is still zero at the start of next interval Two arrivals during the first period permit only one to start its descent The other will be in the stack at the start of second period The system progresses in a similar fashion

Mathematically, this process can be written as follows

$$_{\circ}Q_2(0) = P_0 + P_1$$

$$_{\circ}Q_2(1) = P_2$$

$$_{\circ}Q_2(2) = P_3$$

$$_{\circ}Q_2(N) = P_{N+1} \qquad (1)$$

Equation (1) is read as follows starting with a 0 stack, the probability that at the beginning of the second interval there will be N aircraft in the stack is equal to the probability that N + 1 aircraft arrive at the outer marker during this first interval

Similar sets of equations may be written for initial stacks of one, two, three, and on for any number Finally, all these equations can be written more succinctly in matrix form

Original Stack Size (i)	Subsequent Stack Size (j)					
	0	1	2	3	N	N + 1
0	$P_{\circ} + P_1$	P_2	P_3	P_4	$P_N + 1$	
1	$P_{\circ}$	P_1	P_2	P_3	P_N	$P_N + 1$
2	0	$P_{\circ}$	P_1	P_2	$P_N - 1$	P
3	0	0	P_0	P_1		

This matrix shows the probability that any original stack size i will change to any subsequent stack size j during one service time Each P_k value in the matrix gives the probability of k arrivals during any one service time

In this model, P_k follows a Poisson distribution The Poisson formula reads

$$P_k = \frac{\epsilon^k e^{\epsilon}}{k!} \qquad (2)$$

with a mean value of ϵ for the loading on the system, where

$$\epsilon = \frac{\text{arrival rate}}{\text{acceptance rate}}$$

The t^{th} power of the stochastic matrix describes the probabilities that any original stack size i has, after t intervals, evolved to a subsequent stack size j Therefore, expansion of the matrix to various powers describes the fluctuations in stack size after any given number of elapsed intervals For values of $\epsilon < 1\ 0$, the transition matrix when expanded to high powers eventually becomes idempotent That is, no matter how many times the final matrix is multiplied by itself, the product matrix remains the same This final matrix describes the equilibrium condition of the telephone delay model However, such a condition never occurs when $\epsilon \geq 1\ 0$ The matrix continues to change as the powers increase and as the sprectral density continually moves to the right As time progresses, it becomes decreasingly probable that the stack size will ever return to 0 or 1 or 2, or to similar small values

Calculation of Delays

As previously mentioned, the calculation of delays by this method is accomplished directly through the "powering up" of the proper stochastic matrices Although in theory these matrices are of infinite dimensions, very little error is introduced if the probabilities are recorded to only 4 decimal places and if any smaller probabilities ($P < 0\ 0005$) are considered as 0 This reduces the matrix to tractable dimensions This is a prodigious task for the higher powers which describe fluctuations in delay after long durations of system loading Therefore, the expansion was carried only to the 16th power Curves were then fitted to these 16 points and extrapolated to higher powers This analytical approach seems valid, especially since graphical extrapolation on log-log paper produced very similar results

As can well be surmised, two types of fitting curves were needed one for $\epsilon < 1\ 0$ and the other for $\epsilon \geq 1\ 0$ The final equations are given in the following

In all equations, t refers to the number of service times during which the system loading has continued It should be noted that for $\epsilon < 1\ 0$ both $\bar{S}$ and $\bar{\bar{S}}$ tend to the equilibrium formula

$$D = \frac{\epsilon^2}{2(1 - \epsilon)} \qquad (3)$$

because $t \rightarrow \infty$ D is the average delay in units of service times

$\underline{\epsilon < 1\ 0}$

Expected stack size at time t

$$\bar{S}_t = \frac{\epsilon^2(e^{bt} - 1)}{2(1 - \epsilon)(e^{bt} - \epsilon)} \qquad (4)$$

where the values of b depend on ϵ as follows

ϵ	b
0 5	0 3000
0 7	0 0800
0 9	0 0047

Average stack size during period 0-t

$$\bar{\bar{S}}_t = \frac{\epsilon}{2t(1-\epsilon)}\left[t - \frac{(1-\epsilon)}{b} \ln\left(\frac{e^{bt}-\epsilon}{1-\epsilon}\right)\right] \qquad (5)$$

$\epsilon \geq 1\ 0$

ϵ	$\bar{S}_t$	$\bar{\bar{S}}_t$
1 0	$0\ 3877\ (t)^{0\ 7049}$	$0\ 5865\ \bar{S}$
1 2	$0\ 5095\ (t)^{0\ 8084}$	$0\ 5530\ \bar{S}$
1 5	$0\ 7226\ (t)^{0\ 8928}$	$0\ 5283\ \bar{S}$

Variable Separations

The techniques used in this study are based on a constant service time However, air traffic control does not use a constant outer-marker separation It tailors the separations to each sequence of two aircraft For instance, a fast aircraft following a slower one certainly needs a larger separation than a slow aircraft following a fast, if the safety requirements of the system are to be met Therefore, the question arises as to how much deviation from theory can be expected because of these variable separations

The pertinent Pollaczek-Khintchine formula is

$$\bar{\bar{S}}_t = \frac{\epsilon^2}{2(1-\epsilon)}\left[1 + \mathrm{Var}\,\frac{h_i}{\bar{h}}\right] \qquad (6)$$

$\bar{\bar{S}}_t$ is the average delay during the period from zero to t Time is again measured in units of service times The h_i are the variable service times with average $\bar{h}$

This formula demonstrates that in theory a constant time separation is most efficient and any variation is bound to increase delays The words "in theory" were used here advisedly In air traffic control, the only constant separation that would meet the safety requirements is the largest of the variable separations (fast following slow) This larger separation reduces the acceptance rate and, in effect, increases delays The question then arises as to which increases delays more a larger constant separation or variable separations with a smaller average

One small example should answer this question The optimum separations for 0 wind and no speed control, as listed in Table II, indicate that the average of the variable separations is about 100 seconds The largest is 138 seconds With an arrival rate of approximately 30 per hour (an average arrival interval of 120 seconds), the resulting densities would be, correspondingly, 83 per cent for the variable separations and 114 per cent for a constant separation of 138 seconds The average delays for one hour for these two densities are about 0 8 and 2 5 service times respectively Therefore, it is seen that the constant service time results in delays about 300 per cent greater than the variable separations On the other hand, the Pollaczek-Khintchine formula indicates that these variable separations increase delays by about 2 4 per cent over what they would be if a constant 100-second separation were used It is therefore obvious that in air traffic control the variable separations result in a higher acceptance rate and in lower delays Furthermore, they do not cause any significant over-all variation from the constant service-time model See Table IX The delay picture is adequately given by the simplified model.

For a more complete description of the distribution of air-traffic separations, the reader is referred to the Franklin Institute Report by Carl Hammer [15] In that paper can be found a theoretical treatment of delays when (a) the service times of the system are constant for each sequence of two aircraft (slow-fast, medium-slow, fast-fast, and so on) and (b) the sequencing of several types of aircraft results in several of these constant separations

[15] Carl Hammer, "Transients and Steady States for Tower Controlled Stacks of Aircraft," FIL Working Paper No 3,(Limited Distribution)

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