THE CELESTIAL ALTITUDE DIFFERENTIAL COMPUTER

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INTRODUCTION

The idea for the Celestial Altitude Differential computer was conceived within the Aeronautical Charts Section of Technical Development Service and the design was accomplished by that Section. The experimental model of the instrument was constructed by the Civil Aeronautics Administration Experimental Station at Indianapolis, Indiana. The purpose of this instrument is to compute the change in altitude of a celestial body for a short interval of time, such change being due both to the rotation of the earth and to movement of an observer over the surface of the earth.

In celestial navigation, altitude observations upon two celestial bodies determine a fix, or known position. In almost any method of computing this position it is an advantage to have the observations simultaneous and in several methods it is imperative. However, in air navigation it is impractical to make the observations simultaneously, since this would necessitate two observers and two sextants. The Celestial Altitude Differential Computer enables the observer to compute a correction to the first altitude observed, thereby reducing it to a condition of simultaneity with the second observation and thus accomplishing the same result as would obtain if both observations had been taken at the same time.

HISTORY OF THE COMPUTER

The design of a computer to accomplish the above-mentioned result was originally suggested by the proposal of a system of navigation known as Spherographical Navigation This system proposed use of an accurately ground sphere with plastic coating, upon which the prominent navigational stars were plotted. By the use of special instruments it was proposed to solve celestial problems graphically upon the surface of this sphere In using this system, with the sphere representing the celestial sphere, it was necessary in most cases to reduce two celestial observations to a condition of simultaneity, in order to insure the proper degree of accuracy in the result. There are several ways of accomplishing this. An additional observation may be taken on the first star after the second star has been observed, and the altitude of the first star adjusted for time of second star sight by means of the two observations. H O No. 218, a set of astronomical navigation tables published by the Hydrographic Office of the Navy, includes several tables which may be used for this purpose. The Celestial Altitude Differential Computer furnishes another method of solving the same problem, and its original conception was the result of an effort to solve the problem for Spherographical Navigation

BASIC THEORY OF THE COMPUTER

The change in altitude of a star to an observer in a rapidly moving airplane is due to two causes, the rotation of the earth and the movement of the airplane with reference to the surface of the earth If $\frac{dh}{dt}$ equals the rate of change of altitude in minutes of arc per minute of time due to the earth's rotation, then

$$\frac{dh}{dt} = 15 \sin Z \cos L,$$

where Z = azimuth of the body from the observer's meridian
and L = latitude of the observer.
If $\frac{dh'}{dt}$ equals the rate of change of altitude in minutes of arc per minute of time due to motion of the airplane over the surface of the earth, then

$$\frac{dh'}{dt} = \frac{S \cos \phi}{60},$$

where S = ground speed of airplane in knots
and ϕ = angle between the track of the plane and the bearing to the star.

While these rates are theoretically correct for only an instant of time, they may be assumed to be constant for such short periods as usually elapse between sextant observations - usually less than ten minutes - without appreciable error Therefore, for short intervals, we may say that

$$\Delta H = 15 \sin Z \cos L \qquad (1)$$

$$\text{and } \Delta H' = \frac{S \cos \phi}{60}, \qquad (2)$$

where Δ H = change in altitude in one minute of time due to the rotation of the earth and ΔH' = change in altitude in one minute of time due to motion of airplane over the earth's surface

The solution of these two equations for ΔH and $\Delta H'$, the algebraic addition of the results, and the multiplication by the number of minutes in the time interval will evidently give the total altitude change in the star between the first and second observations The problem, then, was to design an instrument to perform these operations mechanically

Since the two equations to be solved contain operations of multiplication and division only, it was evident that the logarithmic principle used in the conventional slide rule would apply to the case The values 15 in equation (1) and 1/60 or 0 0167 in equation (2) could be engraved on a slide rule, since they remain constant The value S, being known for each problem, could be set on the slide rule without further reference However, the values sin Z, cos L, and cos ϕ, since they represent functions of angles, could not be obtained without reference to tables The idea was then conceived of using a base plate with logarithmic sine and cosine curves engraved thereon and a movable slide rule, sliding across the base plate, in order that the necessary functional values of each angle involved might be set into the slide rule from the curves

CONSTRUCTION

The experimental model computer, as finally constructed, (see Fig 1), consists of a 7-1/2 inch square brass plate, known as the base plate, with a sliding logarithmic rule, and a sliding addition scale. The base plate has a line, known as the base line, engraved near its right hand edge This line is subdivided into equal spaces representing the degrees from 0 to 90 Near the left edge of the base plate is a parallel line, similarly subdivided, and with angles noted every 10 degrees, including all angles from 0 to 360 with equal numerical values for their sines and cosines Above are noted the functional signs of the star azimuths and track-to-star angles Assigning a numerical value of one to the base line, the logarithmic sine and cosine curves are engraved upon the base plate, with the logarithmic value of the function as the abscissa and the angle as the ordinate The sine curve is labeled STAR AZIMUTH, since that curve is used to obtain the sine of the star azimuth, while the cosine curve is labeled both LATITUDE and TRACK-TO-STAR ANGLE, since that curve is used to obtain the cosines of these two angles Along the left and right edges of the base plate are two projections with stops at the upper ends, which act as tracks upon which the sliding logarithmic rule moves.

The sliding logarithmic rule is composed of three parts, (1) the fixed scale, which slides across the base plate and has a logarithmic scale engraved upon it with the right hand 1 in line with the base line, (2) the sliding scale, which likewise has a logarithmic scale engraved upon it, and which slides along the fixed scale with the two logarithmic scales in juxtaposition, and (3) the indicator, which slides in a groove in the sliding scale and has an index mark both at the fiducial edge of the rule and at the logarithmic scales. The fixed scale is graduated logarithmically from 0 04 through 1. The sliding scale is graduated from 0.4 through 15. The constants 15 in equation (1) and 1/60 or 0.0167 in equation (2) are represented on the sliding scale by arrows at their proper values and by the letters A and B respectively.

A simple addition scale is provided near the upper edge of the base plate, for the convenience of anyone desiring to perform mechanically the algebraic addition of the change in altitude per minute due to the earth's rotation and the change in altitude per minute due to movement over the earth's surface. On the base plate is engraved a scale running both directions from zero to plus or minus 10 minutes, with graduations each two-tenths of a minute. A sliding scale is provided with a scale running both directions from zero to plus or minus twenty-five minutes and with graduations likewise each two-tenths of a minute A groove in this sliding scale provides for a sliding indicator with an index mark

OPERATION OF THE COMPUTER

In the use of the logarithmic rule the standard procedure is to always read the result of any operation on the sliding scale The fixed scale and the sine and cosine curves are used to set off the various increments or decrements on the sliding scale by moving the previous or carried value on the sliding scale to the base line, and then moving the indicator to the multiplier on the fixed scale or on the functional curves, at which time the

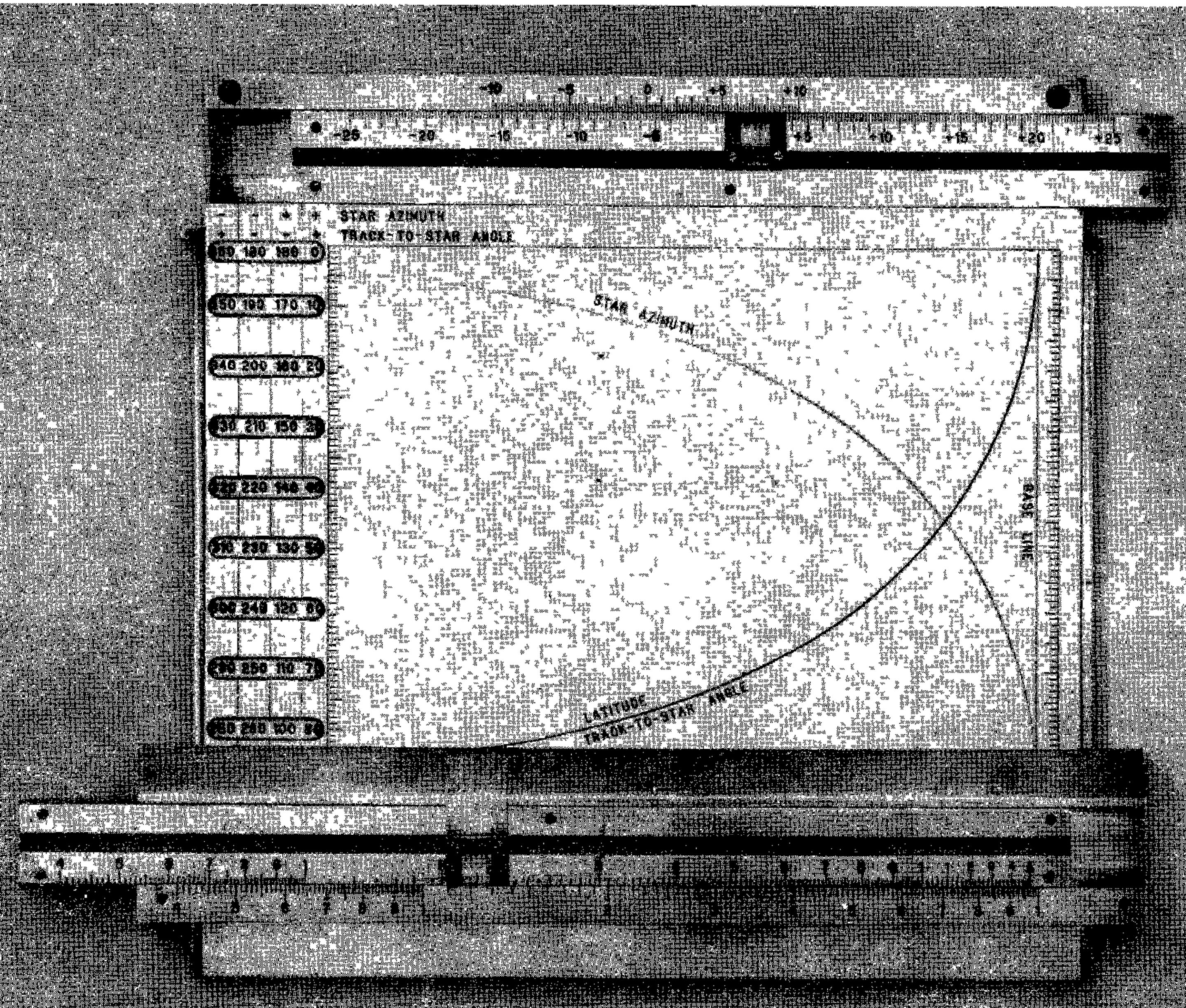

Fig 1 The Celestial Altitude Differential Computer

indicator will read the result of the operation on the sliding scale

The following rules indicate the method of solving a problem with the instrument

1 Set A on sliding scale to base line.

2 Move fiducial edge of rule to latitude angle

3 Set index pointer to latitude curve.

4 Move sliding scale until index pointer is at base line

5 Move fiducial edge of rule to azimuth angle

6. Set index pointer to azimuth curve.

7 Read minutes at index pointer on sliding scale, record and note sign from azimuth angle

8. Set B on sliding scale to left hand 1 on fixed scale.

9 Set index pointer to ground speed in nautical miles on fixed scale.

10 Move sliding scale until index pointer is at base line.

11. Move fiducial edge of rule to track-star angle.

12 Set index pointer to track-star curve.

13 Read minutes at index pointer on sliding scale, record and note sign from track-star angle

14. Add algebraically (7) and (13)

15. Multiply (14) by time interval in minutes

The result of operation 15 gives the number of minutes of altitude change, with sign of change, of the celestial body during the elapsed time interval. Operation 14 may be performed either with pencil and paper or with the addition scale on the instrument.

The following sample problem is given as an illustration

Two star shots are taken from an airplane, with a time interval of 6 minutes and 36 seconds between them. It is desired to compute the altitude of the first star observed as it would have appeared at the time of the second observation. The following data prevail

Latitude	39° N
Star azimuth	220°
Ground speed of plane	
(nautical miles per hour)	165
Track-star angle	30°
Elapsed time	6 min 36 sec.

Set A on sliding scale to the base line and move the fiducial edge of the rule to 39 degrees Set index pointer to latitude curve and read 11 60 on sliding scale. Move sliding scale until index pointer is at base line Move fiducial edge of rule to 220 degrees, set index pointer to azimuth curve, and read 7 45 on sliding scale with a minus sign taken from the azimuth angle This indicates that the star will decrease in altitude 7.45′ for every minute of elapsed time, due to rotation of the earth Record this value and proceed with the next operation Set B on sliding scale to left hand 1 on fixed scale and set index pointer at 165 on fixed scale, reading 2.75 on sliding scale Move sliding scale until index pointer is at base line and set fiducial edge of rule to 30 degrees. Set index pointer to track-star curve and read 2.38 on sliding scale with a plus sign taken from the track-star angle. This indicates that the star will increase in altitude 2.38′ for every minute of elapsed time, due to the motion of the airplane. The algebraic sum of -7 45 and +2 38 gives the total change in altitude per minute due to both motions, namely -5.07′. Multiplying this on the slide rule by the elapsed time 6.6 minutes gives the value -33.4′, which is the total change in altitude of the star during the time interval between observations This must, of course, be applied to the observed altitude, whereupon we have the altitude of both stars simultaneously.

CONCLUSIONS

Use of the Celestial Altitude Differential Computer is by no means the only method of obtaining a condition of simultaneity of two celestial observations for determining position. Tables exist for determining the change in altitude, due both to the earth's rotation and to movement over the earth's surface. An additional observation may be taken on the first star after the second has been observed to attain the same result In the tabular method of precomputed astronomical triangles and with the astrograph the element of time difference is taken care of in other ways.

Below are listed some of the methods used in obtaining a fix from two celestial altitudes and the possible use of the computer in each case.

1. The conventional method of using

precomputed tables or computers to obtain the altitude and azimuth from an assumed position and using the difference between the observed and computed altitudes to establish a position line for each observation. In this case the element of time enters into the computation for altitude and it is only necessary to adjust for the run of the plane between observations. This may be done in the conventional way by advancing the first position line parallel to itself the distance and direction of the plane's run between observations It may also be done by using the computer to obtain the difference in star altitude due to the plane's run between observations and applying this difference to the first observed altitude, whereupon no advance of the first position line is necessary Use of the computer in this case merely substitutes a computation for a drafting operation and is a matter of personal choice.

2. The Astrograph - The same conditions prevail in this case as in case (1) and the computer can be put to the same use, if desired.

3 Star Altitude Curves - When two stars other than Polaris are used, it is necessary to make graphically, adjustments on the curve sheet for change in altitude due to both the earth's rotation and the run of the plane between observations. These two graphical adjustments may be eliminated by using the computer to obtain the total altitude change in the first star observed and then applying this change to the first observation Again, it is a matter of personal choice between the two methods.

4. Spherographical Navigation - Use of the Spherograph, mentioned in the paragraph on History of the Computer, has not, to our knowledge, come into general use Using the Spherographical two star fix solution with the sphere as a celestial sphere, it would be necessary to make both adjustments for the time interval between observations. We know of no practical way of accomplishing this graphically upon the sphere, but use of either the computer or tables of altitude change will accomplish the necessary result. Here it is a matter of choice between the computer, tables, or an additional observation.

5. Navigator Sphere - This is a new instrument now under construction for the Air Navigation Devices Development Division of Technical Development Service It is designed to give a direct reading of position latitude and longitude when the altitudes of two stars and the sidereal time are set into the instrument. It is necessary with the Navigator Sphere to have a condition of simultaneity of observations, which can be obtained with either the computer or tables, as desired.

The foregoing illustrations indicate some of the ways in which the computer may be used. Again, it should be emphasized that by means of an additional observation or the use of tables the same result may be achieved in all cases. The computer is probably somewhat faster than an additional observation and probably somewhat more accurate than tables such as appear in H O 218, although great accuracy is not essential in this computation, since the basic data is usually not extremely accurate. Its use in preference to other methods would probably depend to a great extent upon the personal preference of the navigator.