THE EFFECT OF VIBRATORY AND SLOW REPETITIONAL FORCES ON THE BEARING PROPERTIES OF SOILS

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PREFACE

The purpose of this investigation, sponsored by the Technical Development Service of the Civil Aeronautics Administration and conducted by the Soil Mechanics Laboratory of the School of Engineering, Princeton University, is to obtain some insight into the relative importance of the various factors affecting the bearing properties of soils in airport sub-grades and base courses when subjected to vibratory and to slow repetitional loading by warming-up and by taxiing aircraft.

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The writers are also indebted to D. M. Stuart, John Easton, F. H. Grieme; David S. Jenkins, and R. C. Mainfort of the Technical Development Service for valuable suggestions.

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I SUMMARY

Facts of practical and theoretical importance are reported as established during a study of the effect of vibrations on airport pavement performance carried out for the Technical Development Service of the Civil Aeronautics Administration by the Soil Mechanics Laboratory of Princeton University A brief summary is given of the limited amount of published work previously performed elsewhere concerning the effect of sustained vibrations on soils Special features presented by the vibrating system plane-tire-pavement are briefly discussed The vibration studies at Princeton are outlined

The findings are explained by which it was demonstrated that controlled strain load tests or CBR tests cannot be applied to the study of vibration effects The results of a series of small-scale controlled stress 2-in and 5-in diameter plunger tests on two types of sand are reported These tests were performed at frequencies below the established resonance range of the vibrating systems Diagrams are given showing the relationship between observed plunger penetrations and the time of vibration, also the relationship between the relative density, the saturation conditions of the sands and the observed plunger penetrations under static and dynamic loading. Comparisons are made between the observed effects of a dynamic vibratory force and an equivalent static force showing the greater relative magnitude of the former Uniformly graded sand was found to be most easily deformed by vibratory loading For equal densities and both static and dynamic plunger penetrations the greatest were recorded for submerged sand and the smallest for dry sand, with sand saturated by capillarity giving intermediate values. Complete submergence more than doubled the plunger penetrations, as compared to dry sand

Vibratory plunger tests on two types of clay were performed and showed that on cohesive soils the deformation producing capacity of a vibratory force is no greater than that of an equivalent static force This finding was confirmed by comparisons of static and vibratory unconfined compressive strength tests and consolidation tests.

Actual measurements of vibratory forces transmitted to pavements by the wheels of an airplane during engine warm-up permitted the adaptation of laboratory tests to field conditions.

The effects of a horizontal force applied to the pavement surface by the blocked wheels of an airplane during engine warm-up are estimated on the strength of a mathematical analysis and are shown to be negligible, except in the upper portion of the pavement proper

Comparisons are made of the results of slow repetitional and of vibratory plunger load tests, bringing out the fact that, outside of the resonance range, only the number of load repetitions and the magnitude of the static and dynamic loads are of importance, irrespective of the frequency.

An outline is given of the manner in which the findings of this investigation can be applied to field problems of airport pavement design

II. INTRODUCTION

The fact has been recognized for some time that foundations of factory engines require special treatment because of the vibrations transmitted to them However, it is only recently that some consideration has been given to possible detrimental effects of the vibrations transmitted to airport pavements by warming or reving up aircraft motors

It is current practice in the design of airport pavements to require an increase in thickness for areas subjected to dynamic forces caused by the warm-up of aircraft engines, these are, aprons, hardstandings and ends of taxiways Design procedures for these areas vary. In one case[1] it is specified that the design load be increased 25 percent over that used for the runways; in another case[2] specifications state that the thickness of pavements shall be 20 percent greater than that found necessary for the runways Some cases of damages to pavements and of strongly increased pavement deflections as a result of the action of aircraft motors have been reported in technical publications[3] Nevertheless, it appeared possible that slow repetitional loading of pavements produced by the repeated passage of taxying airplanes over the same spot might be more severe in its effects than vibratory loading during engine warm-up[4] .

No systematic study appears to have been made so far of the relative importance of the various factors involved in the performance of pavements under the action of vibratory or of slow repetitional forces imposed by aircraft. Some basic research concerning these factors appeared advisable.

Systematic observations on airport pavements during the warming up of airplanes are valuable However, such observations alone are not likely to permit a separation and evaluation of all possibly important variables Further, only isolated and incomplete field observations of this kind appeared to have been made so far[4] They are insufficient to permit any definite conclusions

A basic study of the component parts of this problem appeared advisable. The following considerations influenced the method of approach to this study

The nature of the vibratory forces transmitted by airplanes to airport pavements not only may vary with the type of airplane but may differ for individual planes of the same type. It is even probable that the vibratory forces may change for the same airplane depending upon the adjustment of its motors and the resonance characteristics of its component structural parts. There is also reason to believe that different types of pavements and of soils will react differently to vibrations and to slow repetitional loading It is further possible that the dynamic characteristics of the component parts of a vibrating plane-shock absorber-tire-pavement-soil system will influence each other. Field observations alone are therefore not likely to provide conclusive information concerning the most detrimental possible combinations of relevant factors Hence it appeared advisable to split up the problem and to study separately and in limited groups the effect of the numerous variables involved

The Technical Development Service of the Civil Aeronautics Administration sponsored a research project at the Soil Mechanics Laboratory of Princeton University which undertook the study of airport pavement performance in the presence of vibrations and of slow repetitional loading imposed by aircraft

For purposes of study the above problem can be separated into the following main component parts

(a) The determination of the characteristics of the vibratory forces transmitted to pavements by different types of aircraft. The magnitude and direction of the forces as well as the frequency and the amplitude of the vibration

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[1] "Military Airfields.--A Symposium —Construction and Design Problem," by Colonel James H Stratton Proceedings, American Society of Civil Engineers, January 1944, pp 29.

[2] "Design Manual for Airport Pavements," C A A Publication, Washington, D C , March 1, 1944

[3] "Foundations for Flexible Pavements," by O. J Porter, Proceedings, Highway Research Board, Vol 22, pp. 115-117 (1942).

[4] "Certain Requirements for Flexible Pavement Design for B-29 Planes," U S. Waterways Experiment Station, Vicksburg, Miss., 1 August 1945.

produced have to be considered

(b) The estimation of the changes in magnitude and of the redistribution of the shearing stresses induced in a pavement by the horizontal forces transmitted to it by a warming up airplane when its wheels are blocked, as compared to the corresponding values for vertical loading only

(c) The effect of vibratory and of slow repetitional forces of varying characteristics on the bearing properties of different types of soils of varying degrees of density and saturation. This may be estimated by small-scale laboratory tests of the vibrated plunger and of the vibrating table types under controlled conditions of soil moisture and density

(d) The dynamic characteristics of different plane-shock absorber-tire-pavement-soil systems, special consideration being given to possibilities of resonance

(e) Field control tests on different types of airport pavements by means of forced vibrations of the same characteristics as the most unfavorable ones established by observations of actual aircraft

It may be seen that the problem is a vast one, requiring for its study special equipment and techniques Consideration was given to all of the above parts of the general problem However, for practical and organizational reasons, emphasis had to be placed on the study of the first three items, especially on the laboratory phases of the work A summary of the most important data obtained during this study is given in this Report It is hoped that it may help to clarify somewhat the problem of the bearing properties of soils in the presence of vibratory and of slow repetitional loading, and that it may stimulate field recordings of essential data

First shall be given a brief outline of soil vibration studies previously performed elsewhere

III. PREVIOUS PUBLISHED DATA ON ENGINEERING SOIL-VIBRATION STUDIES

Wave Propagation Studies for Purposes of Soil Exploration

Most early engineering soil vibration studies were performed in Europe for the purpose of exploring soil foundation conditions[1,2]. These studies were based largely on the determination of wave propagation velocities[3]

The investigation of wave propagation velocities through soils has not entered into our studies. It should however be mentioned that the use of forced vibrations for exploration of soil conditions has certain advantages as compared to tests on small samples because vibration recordings provide direct data concerning the average properties of the whole deposit Errors inherent to all extrapolations are thereby avoided. Nevertheless vibration recordings have to be supplemented

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[1] "New Results of Dynamic Investigations of Foundation Soils," (in German) by H. Lorenz Zeitschrift d.V.d I., March 24, 1934

[2] "Determination by Means of Forced Vibrations of Soil Properties of Particular Importance for Construction Work," (in German) by A. Hertwig, G. Fruh and H. Lorenz, Publication No. 1 of the German Research Society for Soil Mechanics ("Degebo"), Berlin, 1933.

[3] "Geophysical Study of Soil Dynamics," by R. K. Bernhard. Technical Publication No. 834, American Institute of Mining and Metallurgical Engineers, January, 1938.

by borings in order to provide the necessary data for foundation design, as the same velocities and other vibratory characteristics may be obtained for a clay and for a sand deposit However, once the nature of the soil has been determined from borings, or where it is known in advance, as is the case with new fills, forced vibrations can provide valuable data concerning the average density of the deposit For instance, in one case the velocity of wave propagation through an unconsolidated sand deposit was 160 meters (523 ft) per sec. After compaction it was increased to 470 m (1540 ft) per sec.

Vibratory Equipment Used

Dynamite cartridges were at first used to create vibration waves through the upper soil layers in a manner similar to the one used by geophysicists for deep explorations Shocks created by a dropping weight were sometimes substituted when only a limited soil area was available. Shock waves of this type could not simulate continuous vibrations caused by operating machinery Special machines were then built to create continuous forced vibrations and transmit them to the soil

The main feature of these vibrators lay in two parallel horizontal shafts placed in the same horizontal plane and geared to revolve at the same speed but in opposite directions Eccentric weights were attached to these shafts in such a manner that the horizontal components of their centrifugal force canceled each other at all times. A continuous vertical harmonic sinusoidal vibration was thus produced

Resonance Studies

Vibrators of the type just described were sometimes used for the study of resonance characteristics of machinery foundations[1]

Every mass system when allowed to vibrate freely has the tendency to do so at a certain definite frequency known as its natural frequency. When the frequency of forced vibrations happens to coincide with the natural frequency of the system the phenomenon known as "resonance" occurs. The amplitude of undamped vibrations is thereby continuously increased as well as the vibratory forces transmitted to the supports of the vibrating system. A basic requirement for the design of all foundations which have to absorb vibratory forces is to so design them that no resonance can occur at operating speeds.

The conventional resonance formula for undamped vibrations reads

$$\omega_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad (I)$$

where

ω_n = natural frequency of the vibrating system

m = mass of the vibrating system

k = spring constant of the support

Damping in soils appears to change only slightly the natural frequency of the vibrator-soil system

It was found that the observed resonance frequencies of the vibrator-soil system could be satisfactorily explained only if it were considered that a certain mass of soil takes part in the vibration.

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[1] "Vibrations of Foundations"-(In Russian with English abstract)-A Symposium-Paper "Experimental Study of Vibrations of Mat Foundations Resting on Cohesive Saturated Soils," by D D. Barkan Transactions, Institute for Engineering Foundations Research ("Vios") P C for Heavy Industry, Moscow, U.S S.R., (1934).

Lorenz[1] transformed Formula I to read as follows

$$\omega_n = \frac{1}{2\pi} \sqrt{\frac{k' \ A g}{W_s + W_v}} \qquad \text{(II)}$$

where

A = contact area between vibrator and soil

$k' = k/A$ = coefficient of subgrade reaction

g = acceleration of gravity

W_v = weight of the vibrator

W_s = weight of the vibrating soil

Lorenz contended that the soil mass absorbing the energy of vibrations--"W_s" in Formula II--could be assumed as having a definite and limited value Its weight was determined experimentally by two independent procedures and it was shown that within the limits of the experiments performed, e g., for $(0\ 14 < W_v/W_s < 0.26)$

(a) The weight of the vibrating soil mass depended only on the energy of vibration it had to absorb, e g , on the dynamic forces expressed by the values of the centrifugal force of the vibrator

(b) The weight of the vibrating soil mass was independent of the weight of the vibrator itself and of the contact area between vibrator and soil

An examination of Formula II shows that, all other factors remaining unchanged, the natural frequency of the system vibrator-soil will increase with

(1) an increase of the modulus of subgrade reaction "k'", e g , with denser soils,

(2) an increase of the contact area between vibrator and soil, e g with a decrease of the unit pressure on the soil surface,

(3) a decrease in the weight of the vibrator;

(4) a decrease in the weight of the vibrating soil, e.g with a decrease in the value of the dynamic (centrifugal) forces of the vibrator.

Field experiments performed by the "Degebo"[2] have confirmed in a general manner the above theoretical considerations It should however be noted that the "vibrating soil mass" as defined above does not necessarily have a definite boundary in a physical sense.

Table 1 shows that, for a large standard type "Degebo" vibrator weighing 5950 lb on a loaded area of 9 3 sq ft., the natural frequency of the system vibrator-soil varied from 750 cycles per min on peat to 2040 cycles per min on sandstone

Table 2 shows that on the same kind of soil (the nature not indicated) the natural frequency of the system vibrator-soil decreased with an increase of the centrifugal force. Further, the natural frequency decreased with an increasing weight of the vibrator.

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[1] "New Results of Dynamic Investigations of Foundation Soils," (in German) by H Lorenz. Zeitschrift d V.d I, March 24, 1934.

[2] German Research Society for Soil Mechanics.

TABLE 1

NATURAL FREQUENCIES OF A "DEGEBO" STANDARD TYPE VIBRATOR ON DIFFERENT KINDS OF SOILS

Nature of Soil	Natural Frequencies	
	Cycles per Second	Cycles per Minute
6 Ft. of Peat, Overlying Sand	12.5	750
6 Ft. Thick Old Fill Consisting of Medium Sand with Remnants of Peat	19.1	1,145
Gravelly Sand with Clay Lenses	19 4	1,165
Old, Well Compacted by Traffic Slag Fill	21 3	1,280
Very Old, Well Compacted Fill of Loamy Sand	21.7	1,300
Tertiary Clay, Moist	21 8	1,310
Lias Clay, Moist	23.8	1,430
Very Uniform Yellow Medium Sand	24 1	1,445
Fine Sand with 30 Per Cent Medium Sand	24.2	1,455
Uniform Coarse Sand	26 2	1,570
Non-Uniform Compacted Sand	26 7	1,600
Quite Dry Tertiary Clay	27 5	1,650
Compact, Medium Size Clay	28.1	1,685
Limestone, Undisturbed Rock	30 0	1,800
Sandstone, Undisturbed	34 0	2,040

TABLE 2

NATURAL FREQUENCIES OF DIFFERENT "DEGEBO" VIBRATORS ON THE SAME KIND OF SOIL

Weight of Vibrator		Pressure on Soil		Natural Frequencies for Eccentricities of				
kg.	lb	kg per cm^2	p.s.i	10°	12°	16°	30°	Lower Limit
2,000	4,540	0 27	3 85	29 0 / 1740	26.9 / 1612	24 6 / 1475	23.2 / 1390	23 / 1380
2,700	5,950	0.27	3 85	26.7 / 1600	25 4 / 1525	23.7 / 1425	22 3 / 1335	22 / 1320

NOTE The centrifugal forces corresponding to the "angular eccentricities" of the 30 4 kg. = 67 lb eccentric weights are not indicated in the paper From data given concerning standard eccentricities, it would appear that for 10° and 29 c.p.s. the centrifugal force equals CF. = 2,300 lbs.

c p s / r.p m.

The determination of the soil constants k and k' used in Formulas I and II presents considerable practical difficulties Russian experiments[1] showed that an approximately correct value of the constant "k" could be obtained from the rebound curve of static load tests only if the unloading was performed very rapidly so that the recorded expansion characteristics could not be affected by time lags.

It may be seen from Table 1 that the natural frequencies of vibrator-soil systems can be used for the overall classification of the compactness of soils However it should be noted that attempts to establish any direct relationship between the damping characteristics of a soil and its engineering properties so far do not appear to have been successful.

Settlements Induced by Vibrations

It is generally known that vibrations of machinery are sometimes liable to produce settlements of adjoining structures Settlements of loaded test plates as the result of forced vibrations transmitted to the soil by an attached vibrator were recorded during the Degebo tests[2] The properties of the underlying soil however were numerically recorded only in a few cases. Settlements were naturally greatly increased at resonance frequencies of the vibrating system.

No systematic studies of the effect of vibrations on the stress-strain characteristics of different soils of varying density and moisture content appear to have been made so far

IV. PRESENT STUDIES AT PRINCETON

The purpose of the first series of tests was to determine the changes in the bearing properties of a sand as a result of vibrations applied under vaying conditions of saturation and density A simple type of test permitting the performance of a large number of experiments had to be selected.

California Bearing Ratio or Other Controlled Strain Load or Plunger Tests Found to Be Unsuitable for the Study of Vibration Effects

It was at first attempted to use the California Bearing Ratio test since, if successful, this method would have provided an easy basis for comparison with known CBR values of other soils. A vibrator of the same type as shown on Figure 1 was inserted between the plunger and the head of a universal testing machine The CBR test was then repeatedly performed with and without the vibrator running. No difference was found between the results. This is not surprising if one stops to consider that a CBR test is of the controlled strain type, that is, the resistance of the soil to the plunger penetration is registered as a function of the induced penetration.

The inertia of the various parts of the testing machines is apparently too great to permit their use in connection with vibration studies. As soon as the downward force of the vibrator causes some additional compression of the soil the contact

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[1] "Vibrations of Foundations"-(In Russian with English abstract)-A Symposium-Paper "Experimental Study of Vibrations of Mat Foundations Resting on Cohesive Saturated Soils," by D. D. Barkan Transactions, Institute for Engineering Foundations Research ("Vios"). P. C. for Heavy Industry, Moscow, U S S R , (1934).

[2] "Determination by Means of Forced Vibrations of Soil Properties of Particular Importance for Construction Work," (in German) by A. Hertwig, G. Fruh and H. Lorenz, Publication No. 1 of the German Research Society for Soil Mechanics ("Degebo"), Berlin, 1933.

pressure between the vibrator and the head of the testing machines decreases The head of the testing machine is then unable to follow the vibrator down in the fraction of a second before the direction of the centrifugal force of the vibrator is reversed The pressure exerted by the machine head on the soil specimen through the vibrator is thereby reduced As a result, the static pressure on the soil specimen can never be exceeded in a controlled strain machine so long as the centrifugal force of the vibrator is smaller than the static pressure applied through the testing machine. This was to be the case in all our tests

It was then concluded that controlled stress tests would be required. These tests would better correspond to actual field conditions when a vibrating load actually rests on the soil surface A preliminary test of this kind showed that vibrations markedly increased the penetration of plungers with a dead weight resting on them.

The foregoing finding should naturally also apply to most field load bearing tests when a hydraulic jack is inserted between the test plate and a loaded truck against which the jack presses. This type of test is essentially a controlled strain test and therefore should not be used for the study of vibrations by means of vibrators inserted above or below the hydraulic jack The insertion of springs below or above the vibrator might change the situation but would complicate it considerably by altering the resonance characteristics of the whole system

Equipment Used for Controlled Stress 2-In Diameter and 5-In Diameter Plunger Tests

Figure 1 shows a small vibrator of a type developed by the General Motor Corp. It is built along the same lines as the vibrators described on Page 4 of this Report The centrifugal force cannot be changed independently of the R P M. of the two horizontal shafts as the eccentric weights form part of these shafts. The centrifugal force will therefore change with the square of the speed. The rotating shafts can however be easily changed before each experiment A set of such shafts with different eccentric weights is available, producing different centrifugal forces at a known R P M

Figures 2 and 3 show a larger vibrator of the Lazan (Baldwin-Southwark) type This vibrator is capable of producing a centrifugal force up to 1600 lb. at speeds up to 3000 r p m The centrifugal force can be changed during a test without stopping the vibrator or changing the r p m.

The amplitude of the vibration produced was recorded by means of a pickup attached to the vibrating frame and connected to a Vibrometer of the Televiso system. On the diagrams of this paper the amplitudes of the vibration are given as recorded from the instrument In order to obtain the peak-to-trough displacements a multiplication factor approximately equal to 3 should be used A later re-calibration of this instrument on a vibration table[1] showed that the multiplication factor remained fairly constant for the same range setting, but could vary by as much as ±100% for different range adjustments. The same range setting was used during each test.

Controlled stress tests using plungers of 2 and of 5 in diameter were performed In the case of the smaller plunger the usual CBR equipment was employed as illustrated by Figure 1 for a test in a completely submerged condition of the sand Figures 2 and 3 show the tests with the larger-sized plunger (5-in. diameter) whereby the dimensions of the cylinder were increased correspondingly (diameter of 15 in. instead of 6 in.) Figure 3 shows the arrangment for the application of the static load The vibrator with enclosing cage, plunger and weights was suspended from a steel rope slung over pulleys with counterweights at its other end The static load was applied in increments by gradual removal of the counterweights. A similar arrangement, not shown on Figure 1, was used for the small plunger.

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[1] "Electromagnetic Vibration Table", by R. K Bernhard and J G. Barry, Bulletin, American Society for Testing Materials, March 1946.

Vibrator Plunger Tests Performed with Relatively High Values of the Centrifugal Force Ranging from ±25% to ±6% of the Original Static Load on the Plunger

Sands Used

Two sands were used for the tests The grain size distribution curves of both sands are given on Figure 4 Microphotographs illustrating the shape of the grains of both sands are given by Figures 5 and 6.

The sand designated in this Report as "Sand A" was a clean (washed) well graded sand from Jamesburg, New Jersey Approximately 5% of the sand grains were composed of limonite (iron oxide), the remainder consisting of quartz

The fine sand designated in this Report as "Sand B" came from the site of the famous racetrack at Daytona Beach, Florida It was composed exclusively of quartz grains of very uniform size (Figure 6)

Compaction and Saturation Procedures Sand "A"

Tests were performed on completely dry sand, on submerged sand (Figure 1) and on sand saturated by capillarity In the latter case the sand was first submerged and then withdrawn from the water container and allowed to drain No tests under conditions of capillary saturation were made on the larger plunger since the depth of the sand in the case of the larger cylinder appreciably exceeded the height of passive capillary rise of the sand. By means of the negative head capillarimeter this height was found to equal 6 in.

To obtain different densities the sand was compacted in a 400,000 lb machine under static pressures of 2000, 1500, 1000, 500, and 100 p s i. applied over the whole area of the sample. The void ratio of the compacted sample was recorded in all cases and it was found that in this way relative densities ranging from about 20 to 100 per cent of A A S.H O optimum could be obtained

A fresh sample of dry sand "A" was used each time because it was found that some crushing of the grains occurred even under 1000 p s i. compaction pressure as can be seen from the sieve curves on Figure 7, the sieving having been performed on the same sample before and after the test. This probably can be explained by the crushing of the weaker luminite grains

Compaction was usually performed with dry sand Several check tests were made with sand "A" which had been compacted in a completely submerged condition. Subsequent plunger tests performed in a submerged condition gave only slightly smaller penetrations for sand compacted after flooding, as compared to sand compacted to the same density in a dry state. Some sand samples were compacted by the standard and the modified A A S H O procedures (tamping) The dynamic penetrations recorded were approximately the same as for samples of equal density compacted by static pressure

Apart from the void ratio "e" the relative density "D_d" of the sand was determined The maximum density was determined in a 4-in diameter cylinder using the standard A A S H O. compaction technique. The least density was obtained by pouring loose dry sand into the cylinder Under high static compaction pressures relative densities over 100 per cent were sometimes obtained, especially in the larger cylinder. In general it was noticed that for the same unit compaction pressure applied over the whole area of the cylinder slightly greater densities were obtained in the 15-in diameter cylinder than in the 6-in diameter cylinder, presumably because of the smaller relative effect of friction on the side walls.

Resonance Characteristics of the Vibrating System

It was necessary to perform the tests without any amplification of the centrifugal force of the vibrator due to resonance. In order to determine the resonance characteristics of the vibrating system a method developed previously by the "Degebo" was used. The vibrator was started at a low speed of about 900 r p.m and was permitted to run for 1 min. The speed was then increased by 100 r.p.m. every minute and the increment of penetration plotted against the r.p m Since the centrifugal force increases with the square of the speed, a continuous increase of the penetration increments could have been expected However, since the dynamic forces are greatly increased at resonance the increment of penetration--r.p.m. curves generally showed a pronounced peak at resonance This method was checked by us and gave satisfactory results in the case of dense sands In addition, the vibrometer readings of the displacement amplitude were also recorded, as shown on Figure 8. Their peak generally coincided very closely with the peak of the increment of penetration-r.p m. curve.

This method however did not produce satisfactory results for loose and medium density sands as shown on Figure 9, since in such cases the increment of penetration-r p.m. curve had two or three peaks, only the last one coinciding with the peak of the amplitude curve.

Table 3 gives a summary of the results of the resonance tests made with the small plunger

It may be seen that the general trends confirm earlier findings of the "Degebo" Denser sands give higher values of the natural frequency "ω_n". A higher value of the centrifugal force at the same density decreased the natural frequency.

It may be seen from Figure 8 that speeds below 1500 r p m. lay well below the natural frequency of the system. It was desired to perform our routine investigations at speeds where no resonance could occur since it appeared likely that no resonance of the whole plane-shock absorber-tire-pavement-soil system would take place. For that reason the first series of 2-in diameter plunger tests were run at 1500 r.p.m. before the larger-sized 5-in diameter plunger and the corresponding cylinder were ready. A later test of the resonance characteristics of the large plunger and cylinder (see Fig. 10) however showed that the speed of 1500 r p.m. lay too close to the resonance range of that system. All further comparative tests on both sizes of plunger were then performed at the speed of 1000 r p m.

Dynamic Plunger Penetrations Recorded Sand "A"

Figures 11, 12 and 13 give some typical time-dynamic plunger penetration curves obtained during the tests In all tests with a surcharge a parabolic shape of the curve of the type shown on Figure 13 was obtained. This was not the case of zero surcharge as shown on Figure 11 when fatigue effects producing effects producing an irregular shape of the curve were often apparent The amplitude of the vibration generally rapidly decreased with time and decreasing rate of dynamic plunger penetration as shown on Figure 12

A surcharge strongly decreased the penetrations.

For later comparison with the results of slow repetitional loading tests, it proved advisable to plot some of the time-dynamic penetration curves on a semi-log scale, as shown on Fig 14, and to substitute the number of load repetitions for the corresponding units of time.

Tests were made with the 2-in. diameter plunger as shown on Figure 1 on dry, moist and submerged sand and on sand saturated by passive capillarity Three types of rotors were used producing at 1500 r.p m 10 3, 23.2, and 40 4 lb centrifugal force. It may be seen from Figure 16 that the amplitude of displacement increases with an increase in the centrifugal force For dry and moist sand and for sand saturated by capillarity the rate of increase is approximately the same. For

TABLE 3

NATURAL FREQUENCY ω_n VALUES OBTAINED AT PRINCETON DURING TESTS WITH 2-IN. DIAMETER PLUNGER

Compaction Pressure p.s.i.	Void Ratio e	Natural Frequency ω_n c.p.m.
I. Centrifugal Force = 40.4 lb. — Dry Sand		
2,000	0 574	2,100
1,500	0.632	2,100
II. Centrifugal Force = 10 4 lb. — Dry Sand		
2,000	0.563	2,700
1,500	0.623	2,550
1,000	0.663	2,500
III Centrifugal Force = 10.4 lb. — Moist Sand		
2,000	0.512	2,750
1,500	0 548	2,700
1,000	0 583	2,600

submerged sand, however, the increase is more rapid Appreciable dynamic plunger penetrations were produced at displacement amplitudes of approximately one thousandth of an inch and less.

Figure 17 shows the increase of the dynamic plunger penetration at 10 min. vibration for different centrifugal force values and varying compaction pressures (and therefore also densities).

Figure 15 shows the total dynamic plunger penetrations after 10 min vibration plotted against the relative density and the void ratio of the sand It should be noted in this connection that penetrations did not entirely stop after 10 min vibration, as can be seen from the time curves on Figures 12 and 13 The greater part of the penetrations had however already occurred after 10 min vibration

It may be seen from Figure 15

(a) that for the same condition of saturation the dynamic penetrations rapidly increased with decreasing density of the sand This is a natural result

(b) for the same density submerged sand showed much larger penetrations than dry sand to an extent greater than could be expected

(c) sand saturated by capillarity gave intermediate values This result is contrary to conventional conceptions concerning increased effective stresses and improved bearing properties due to capillary saturation.

It was at first believed that this latter result could be attributed to an action of water in the pores of the sand which took place only in the presence of vibrations. Studies of static penetrations however produced a similar relationship.

The 5-in diameter plunger showed appreciably greater dynamic penetrations than the 2-in. diameter plunger at the same density The difference was particularly pronounced in the dry state (Fig 26) and to a lesser degree in the submerged state (Fig 25).

Static Plunger Penetrations Recorded   Sand "A"

Figure 18 gives typical static pressure-plunger penetration curves obtained for different densities. Time consuming measures for exactly level initial seating of the plunger were not taken Irregular initial penetrations at the first load increment resulted Because of this, for all later summaries the zero point was taken at an initial pressure of approximately 0 5 T per sq ft. corresponding to a total load of approximately 23 lb

A summary of all static load tests on the 2-in diameter plunger is given on Figure 19. The increment of static penetration from 23 to 93 lb designated by the symbol S_{23}^{93} is plotted against density, different symbols being used for different conditions of saturation It may be seen that for the same density submerged sand produced much greater penetrations than dry sand and that sand saturated by passive capillarity gave intermediate values. In other words, the same relationship as for the dynamic penetrations was obtained.

This result is contrary to conventional conceptions on the matter and therefore requires discussion

Data Obtained from Samples Saturated by Capillarity Shows Limitations of Conventional Effective Stress Theories

It was at first believed that the surprising result obtained on sand samples saturated by capillarity might be attributed to the manner in which the static load was applied, that is, to the removal of the counter-weights in 10-lb increments It was thought that the sudden removal of these weights might change the neutral stresses in the capillary water with a resulting momentary decrease of the effective stresses between the sand particles A large water tank was then substituted for the iron counter-weights and the static load was applied by gradually draining the counter-weight tank over a period of one hour. Three tests were performed in this manner whereby the originally submerged sand was first allowed to drain for 5 min. only, as had been done on all tests It can be seen from Figure 19 that very little decrease of plunger penetrations, if any, was observed for these three tests It was then surmised that the time of drainage might have been insufficient Accordingly, three tests were then performed after 48 hr drainage, but gave, as shown on Figure 19, only a slight further decrease of the static penetrations

A series of tests on the active and passive capillarity of sands, performed at the Princeton Soil Mechanics Laboratory by Mr Valle-Rodas, should be noted in this connection They showed, among other things, that in the case of drainage the water content held by passive capillarity in a uniform sand continued to decrease appreciably during the first five days and probably longer[1] This can be seen from Figure 5 of Mr Valle's paper. On the other hand a test made with the well graded sand "A" used in the present experiments showed, as can be seen from Figure 21, that after 20 min drainage the water content remained practically unchanged along the whole 6-in. depth of the sample, which depth corresponded to the height of passive capillary rise. Decrease of the water content through drainage might therefore continue in this well graded sand for a longer time than in Mr Valle's experiments with uniform sand

It is generally assumed that capillary saturation increases the effective stresses in the sand[2] It is further generally considered that the bearing properties of a soil depend largely on its shearing strength which in turn depends upon the effective stresses between the sand grains. Therefore, the bearing capacity of a sand should be improved by a condition of capillary saturation. Figure 20 illustrates the distribution of effective stresses under the three conditions of saturation determined in accordance with conventional theories on the matter[2]. These conventional ideas appear to be based on the assumption that a sand remains completely saturated by capillarity at least up to the height of passive capillary rise and that up to that level the water is under tension Mr Valle's experiments[1] have however shown that complete saturation by passive capillarity is not maintained The gradual decrease with time in the water content, indicating continued slow downward seepage of the water, and other unevaluated phenomena may be responsible.

Two tests were made with sand which had 6 per cent moisture before compaction As can be seen from Figure 19, the results coincide with those of dry sand, presumably because the slight apparent cohesion produced by the moisture was negligible as compared to the magnitude of the external stresses applied

Deformation Producing Effect of a Vibratory Force on Sand "A" Found to be Relatively Greater Than That of an Equivalent Static Force

Plunger penetrations produced after 10 minutes vibration by an applied centrifugal force were compared to the penetrations produced by an equivalent static

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[1] "Capillarity in Sands," by R. Valle, Proceedings, Highway Research Board, Vol. 24, 1944

[2] "Neutral and Effective Stresses in Soils," by P C Rutledge, Proceedings, Purdue Conference on Soil Mechanics and Its Allications, 1940.

force. In order to make this comparison separate static plunger penetration tests had to be made, the loading being then continued beyond the point reached by the static loading during dynamic tests. Some scattering of the results was inevitable since the same density and gradation could not be exactly reproduced by a second test. Figure 22 shows one set of such comparisons where the dynamic plunger penetrations produced by a centrifugal force of 23 lb after a static load of 93 lb. had been applied, were compared to the increment of penetration produced during a different test at approximately the same density of the sand by a 23-lb increment of the static force during its increase from 93 lb to 116 lb This ratio is indicated on the diagrams by the symbol $D_{10/S}{}^{116}_{93}$ It can be seen from Figure 22 that for that particular set of conditions this effect of the dynamic force is five to ten times greater than that of an equivalent static force Some scattering of the results is noticeable with a few tests on submerged sand giving appreciably higher values of the dynamic to static penetration ratio This, however, was not a general trend and in some other series of tests not reproduced in this Report dry sand gave the highest values of the relative dynamic effect. The ratio $D_{10/S}$ appeared to decrease somewhat with decreasing centrifugal force values

Figure 27 shows the above ratio of equivalent dynamic to static plunger penetrations obtained from the 2-in. diameter and 5-in. diameter plunger tests at 1000 r p m The same scattering and relative magnitude of the ratio was obtained In the series recorded by Figure 27 the centrifugal force for the 2-in diameter plunger was equal to 10 3 lb. and therefore equal to approximately one half of the force during the tests at 1500 r.p m. to which Figure 22 refers Both static and dynamic unit pressures on the larger 5-in diameter plunger were equal to the unit pressures on the 2-in diameter plunger It may be seen from Figure 27 that for submerged sand the ratio of dynamic to static plunger penetrations shows considerable scattering with no definite trend as to effect of plunger size being noticeable For dry sand as shown on Figure 28 the effect of size is noticeable, indicating an increase of the relative effect of the dynamic forces produced by the 5-in. diameter plunger.

The Fine-Grained Uniform Sand "B" from Daytona Beach Racetrack Found to be Most Strongly Affected by Vibratory Loading

It was desired to check the results of the first sand "A" series of vibratory plunger tests by means of identical tests on a different kind of sand Selected was sand from the Daytona Beach racetrack in Florida, which is referred to as sand "B" in this Report. This sand had been credited with having a particularly high bearing value due to the effects of capillary saturation It was of further interest because of its extremely uniform size (see Figures 4 and 6) Tests were performed with oven dried sand "B", with completely submerged sand, with sand saturated by capillarity from the bottom of the sample, and with sand which originally had 6% moisture. Some samples were compacted by static pressure only, others were slightly tapped during their compression

Figure 29 gives a summary of the results of the static loading Static plunger penetration is plotted against the void ratio "e" and the relative density "D_d". Several facts emerge from a study of this diagram

First, it may be seen that the density of the samples is not appreciably affected by the compaction pressure with the sole exception of the samples which originally had 6% moisture These latter samples showed an increase in density with increasing compaction pressure. Presumably this was due to the fact that this very fine sand had highly viscous adsorbed water films which could be deformed by increasingly high compaction pressures. The remaining samples gave erratic pressure - density results

The second observation is that the sand saturated by capillarity from the bottom on the whole did not show any higher bearing power as compared to the dry material. The results were on the average approximately the same for both conditions of saturation. In general, there was a considerable scattering of the results, but submerged sand appeared to show higher penetrations

Figure 30 gives a summary of the results of the dynamic plunger tests. The dynamic plunger penetration after 10 minutes vibration ("D_{10}") is plotted against the soil density The same general remarks made concerning Figure 29 (static penetration) are also valid for the results of the dynamic penetration

Figure 31 illustrates the relative effect of a vibratory force on plunger penetrations as compared to that of an equivalent static force It may be seen that this dynamic effect is quite considerable and is from 9 to 142 times greater than the effect of a corresponding static force It is decreased by the presence of some 6% moisture in the sand, for this condition the D_{10}/S^{118}_{95} ratio varying from 15 to 50 No definite effect of the conditions of saturation on the value of this ratio is discernible

Figure 32 illustrates the shape of the "number of load repetions - plunger penetration" curves when plotted on a semi-log scale This diagram is of importance in connection with similar curves plotted during slow repetitional load tests

High Bearing Power of Daytona Beach Sand ("B") Is Due to Its Improved Resistance, When Moist, to Tangential Forces

A simple test was performed as follows A wheelbarrow with a pneumatic tire was placed on our 15-in diameter cylinder filled with sand "B". During the first test the sand was dry, during the second it was saturated by capillarity from the bottom of the cylinder The height of passive capillary rise was determined in the laboratory by means of the usual procedure as being equal to 15 in A tangential force was applied to the rim of the wheel by means of weights and a cable slung over a pulley.

It was found that the tangential load which produced a slipping of the tire over the surface of the sand was greater by about 50% in the case of sand saturated by capillarity as in the case of dry sand Also, the depth to which the wheel sunk while it spun around was appreciably greater in the case of dry sand than in the case of sand saturated by capillarity.

This finding is only of theoretical interest, but serves to explain the good performance of the Daytona Beach sand, instead of the earlier theories on the matter which were in contradiction with our own findings made during the first stages of this program when it was found that resistance to vertical loading was not improved by capillary saturation.

The Two Types of Clay Used for the Plunger Tests

Two types of clay were used, which, for purposes of brevity, shall be referred to as "Red Clay" and "Blue Clay" Actually, the Red Clay is a material consisting mainly of particles of silt size The grain size accumulation curves of both clays are given on Figure 33. The density - moisture relationship of the Red Clay and its consistency limits are given on Figure 34 The corresponding values of the Blue Clay are given on Figure 35, which also indicates the variations in density of that material after different periods of swelling.

Static and Dynamic Plunger Penetrations Recorded for the Two Clays

Figure 36 gives a summary of the static loading results and Figure 37 summarizes the dynamic loading results for the Red Clay. It may be seen that in both cases a rapid increase of both static and dynamic penetrations is observed when the water content of the clay approaches the plastic limit. The time - dynamic penetration curves have the same general shape as the corresponding curves on sand illustrated by Figure 13, when plotted at a plain coordinate scale However, when plotted at a semi-log scale, they have a somewhat different shape, as illustrated by Figure 42.

Figure 38 gives the ratio of the dynamic penetrations to the penetrations produced by an equivalent static force. It may be seen that for this clay the effect of the dynamic force is approximately equal to the effect of an equivalent static force and is therefore relatively smaller than during corresponding tests on sand.

Plunger tests with the Blue Clay produced the same general trends, as shown on Figures 39, 40 and 41

Vibratory Plunger Tests with Sand-Clay Mixtures

Tests were performed with five different mixtures of Sand "A" and of Red Clay, containing respectively 10%, 20%, 30%, 40% and 50% of the latter. At low water contents even a small addition of clay was found to appreciably decrease the $D_{10/S}$ ratio, e.g , the relative importance of vibratory forces, as compared to the deformation producing capacity of static forces of equivalent magnitude However, at water contents exceeding the optimum moisture, the $D_{10/S}$ ratio increased somewhat, especially for low clay contents

Field Measurements of Vibratory Forces Transmitted to Pavements by Aircraft During the Period of Engine Warm-up

At the time this project was started (1943) no data was available concerning the actual magnitude of vibratory forces transmitted to pavements by warming-up airplanes. The centrifugal force values used during the plunger tests and varying from ±25% to ±6% of the original static load were selected subject to later substantiation by actual field measurements.

Some uncertainty was attached to the accuracy of response of soil pressure cells when measuring vibratory forces. For that reason, Dr R K Bernhard, acting as Special Consultant to the Project, was requested to develop a special device for the purpose of measuring in the field vibratory forces transmitted to pavements by aircraft during the period of engine warm-up This development and the results of some trial measurements are described in Reference[1].

It was shown by these trial measurments that for the type of plane tested with properly adjusted engines the vibratory force transmitted to the pavement had the order of magnitude of ±1% to ±2% of the original static load, but could reach a maximum value of ±4.3% This maximum value did not correspond to the highest speed of the motors, indicating possibilities of resonance of certain component parts of the plane at intermediate speeds. Indications were also obtained of possible much higher values of vibratory forces transmitted to pavements by planes with improperly adjusted motors.

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[1] "Study on Vibrations Transmitted to Pavements during Warm-Up Period of Airplanes," by R K Bernhard, CAA Technical Development Note No. 48, 1947

At about the same time the results of other field measurements became known[1], which had given results of the same order of magnitude.

Some supplementary plunger tests with lower values of the centrifugal force appeared necessary and were performed.

Vibratory Plunger Tests Performed with Relatively Low Values of the Centrifugal Force Ranging from ± 2.0% to ± 0 9% of the Original Static Load on the Plunger

Tests Without Partial Release of Static Load

Figure 43 illustrates the effect of the relative density of Sand "A" on the ratio D_{10}/S^{165}_{25}, as a result of the application of a vibratory force equal to ±0.9% of the original static load on the plunger It may be seen that the value of this ratio increases from app. 5% for dense sand to 30% for loose sand In other words, a vibratory force equal to less than ±1% of the static load will produce deformations equal up to 30% of the deformations caused by that static load

Figure 44 illustrates a similar relationship for Sand "B" In the case of this uniform sand the effect of density was less pronounced, both loose and dense sand being more sensitive to vibrations than Sand "A" The value of the ratio D_{10}/S^{165}_{25} varied from 28% to 58%.

Tests with Partial Release of Static Load

During field tests it was observed that the warming up of motors produced a redistribution of load between the wheels In one case[1] this redistribution was found to produce a decrease of the load on the main wheels by some 15%

A series of plunger tests were then performed to simulate these conditions. Figure 45 illustrates a typical test result on Sand "A" and Figures 46 and 47 on Sand "B".

Table 4 summarizes the test results on Sand "A" and Table 5 on Sand "B".

In the case of Sand "A", the partial removal of the static load appreciably decreases the residual additional dynamic penetration only when the vibratory force is very small and equal to less than 1% of the original static load

Values of the vibratory force equal to 2% of the static load produced additional residual dynamic penetrations vaying from 1% to 6% of the penetration caused by the original static load for dry sand; of app 10% for sand saturated by capillarity, and 15% to 47% for submerged sand.

In all cases the relative density of the sand samples exceeded 60%, so that they could be classified as "dense"

In the case of Sand "B", the partial removal of the static load did not noticeably decrease the residual dynamic penetrations of this poorly graded sand. When the vibratory force was particularly small and equal to only 0.9% of the original static load, the residual additional dynamic penetration varied from 17% to 42% of the penetration produced by the original static load

When the vibratory force equalled 2% of the original static load, the corresponding relative dynamic penetration values were increased to values varying from 33% to 138% for dry sand; app. 73% for sand saturated by capillarity, and from 46% to 143% for submerged sand.

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[1] "Certain Requirements for Flexible Pavement Design for B-29 Planes," U. S Waterways Experiment Station, Vicksburg, Miss., 1 August 1945.

TABLE 4

SAND "A"

Void Ratio "e"	Centrifugal Force		Ratio of Dynamic to Static Penetrations		Relative Density "D_d"	Condition of Saturation
	Lbs.	% of Static Load	D_{10}/S^{165}_{25}	R_{10}/S^{165}_{25}		
.658	3 31	2 0	.0664	.0586	63%	Dry
.603	"	"	.0326	0326	87%	"
.658	"	"	.0407	0116	63%	"
585	"	"	1582	.1582	97%	Submerged
590	"	"	.1522	.1509	91%	"
.656	"	"	.4748	.4754	63%	"
.648	"	"	.1019	.1005	68%	Capillary Saturation
581	1.47	0.9	.0403	0000	96%	Dry
.658	"	"	.0442	0110	63%	"
.661	"	"	.0237	.0047	62%	"

Note R_{10} = Residual static penetration after 15% reduction of the static load

TABLE 5

SAND "B"

Void Ratio "e"	Centrifugal Force		Ratio of Dynamic to Static Penetrations		Relative Density "D_d"	Condition of Saturation
	Lbs	% of Static Load	D_{10}/S^{165}_{25}	R_{10}/S^{165}_{25}		
850	3 31	2 0	.3892	.3323	43%	Dry
847	"	"	1 414	1 387	44%	"
829	"	"	1 370	1 357	51%	"
817	"	"	4552	4672	58%	Submerged
821	"	"	1.427	1.428	55%	"
834	"	"	7277	.7297	50%	Capillary Saturation
836	1 47	0.9	.1597	.1885	?	Dry
804	"	"	2049	.2029	62%	"
831	"	"	.2757	.2676	52%	"
846	"	"	1929	.1786	45%	"
855	"	"	.4032	.4194	40%	"

Note: R_{10} = Residual static penetration after 15% reduction of the static load.

In all cases the relative density varied from 40% to 62%, so that the sand could be classified as being of medium density. The above percentages refer to 10 minutes vibration. Continued vibration appreciably increased the penetrations.

Our above tests have clearly shown the fallacy of the reasoning according to which the effect of vibratory forces transmitted by airplane wheels to pavements cannot be of importance since the vibratory forces are usually equal to only 1% to 3% of the static load, whereas the static load was reduced by some 15%[1]

It has now been demonstrated that, in so far as sands are concerned, even a small vibratory force has a much greater deformation producing capacity than a static force of corresponding magnitude. As a result, any decrease of static load accompanying engine warm-up is not only compensated for by the effect of vibrations, but deformations of the sand may be greatly amplified

Vibratory Consolidation Tests with Three Clay Soils

Vibratory plunger tests were not possible to perform on very plastic and soft clay mixtures. For that reason it was decided to perform a few consolidation tests on a vibration table to determine the effect if any of a vibratory motion of the whole system on the compressibility characteristics of the clay

Comparative vibratory and standard type consolidation tests were performed on three types of clays A vibration table designed for the Project by Dr R K Bernhard and Dr J G. Barry[2] was used for the purpose. The main use of this table consisted, however, in the calibration of instruments[3]

Two parallel tests were performed with the same material, using standard type 2 5-in diameter and 1-in. high consolidometers. One test was performed under static conditions, and the other was performed in a consolidometer which was attached to the vibration table

Three clay soils were tested The first soil was the Princeton Red Clay, identical to the material used during the plunger tests The second soil was some undisturbed clay from a boring at the Flushing Meadow site in New York This clay was used in its natural condition and had a structure which was easily disturbed and weakened by remolding. The third soil was powdered Wyoming Bentonite, of a type stated by its manufacturers to have particularly pronounced thixotropic properties

Figures 48, 49 and 50 give the pressure-compression curves of all three soils It can be seen that the vibrated specimens did not show any appreciable increase of compression, as compared to identical specimens tested under static conditions

It is the opinion of the writers that vibration table tests are not suitable for the investigation of the compressibility of soils in the presence of vibrations The entire soil and loading plunger system, as well as its base, performs a vibratory motion without any relative displacement occurring. Therefore, this type of vibratory motion corresponds more to the type of vibration produced by shock waves, rather than to the motion of the soil particles in the immediate vicinity of a source of vibrations when the particles closer to the source are bound to have a greater amplitude of motion than there is further away from it.

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[1] "Certain Requirements for Flexible Pavement Design for B-29 Planes," U S Waterways Experiment Station, Vicksburg, Miss , 1 August 1945

[2] "Electromagnetic Vibration Table", by R K Bernhard and J G Barry, Bulletin, American Society for Testing Materials, March 1946.

[3] "Study on Vibrations Transmitted to Pavements during Warm-Up Period of Airplanes," by R. K Bernhard, CAA Technical Development Note No. 48, 1947

It is the latter type of motion which is of practical interest in connection with the studies of this Project, and it is believed that only such relative displacements of soil particles are liable to cause permanent deformations of the soil system This type of condition is much better simulated by the plunger tests which have been performed so far

For this reason no further consolidation or other tests were performed with the vibration table.

The above remarks apply naturally only to a cohesive soil In the case of a granular, non-cohesive material, a vibratory motion imposed on the entire system may cause the individual particles to move in respect to each other However, consolidation tests cannot very well be performed on sands, and they were not attempted on the table, since the results could not be directly applied to practical problems of pavement design

Vibratory Unconfined Compressive Strength Tests
With Red Clay

Four trial tests have been performed, the results of which are given by Figure 51 Four cylinders were prepared in an identical manner Two of them were loaded by means of the arrangements shown on Figure 1, whereby a flat plate was substituted instead of the plunger in order to provide a uniform bearing surface During two of these tests the vibrator was not running During the two other tests it was in operation with a centrifugal force of 3 31#, which produced a vibratory stress increment of ± 0 026 T/sq ft It may be seen from Figure 51 which gives the stress-strain curves for all four samples that in spite of the relatively high water content of these samples the vibratory force did not produce any decrease of the shearing strength of the specimen in excess of the strain corresponding to the value of the vibratory stress increment

Triaxial Shear Control Tests with Sands "A" and "B"

A series of triaxial shear tests were performed at the suggestion of the junior author for the purpose of determining any possible relationship which might exist between the so-called "critical void ratio" and observed bearing properties of the soil. The "critical void ratio" is a standard term used in soil mechanics to designate the density of a sand at which shearing deformations can occur without any volume changes. A sand denser than the "critical void ratio" will have to expand in order to shear A looser sand may shear with simultaneous decrease of volume.

Figure 52 summarizes the results of this test. At lateral pressures of 1/4, 1/2, and 1 T/sq ft a certain scattering of the results was observed which was particularly pronounced in the case of Sand "B" Therefore, only general ranges of the variation in the "critical void ratio" are indicated.

It can be seen that in the case of Sand "A" the "critical void ratio" varied from 0.55 to 0 65 This range corresponded to the density range which during our plunger tests was found to be of a critical nature It can be seen from Figure 19 that static plunger penetrations increase rapidly for this sand at densities looser than a void ratio of 0 65.

For Sand "B" the scattering of the triaxial test results was particularly pronounced and the "critical void ratio" varied from 0 54 to 0 90. It can be seen from Figure 29 that a range of considerable instability and a scattering of our results during static plunger penetration tests existed for densities in the vicinity of a void ratio of 0 9

The need for the compaction of sands well above the "critical void ratio" is therefore apparent This compaction should be well above 70 to 80 percent of the maximum possible density

Slow Repetitional Plunger Load Tests

Taxiways, aprons and hardstandings are subjected to vibrations transmitted to pavements during engine warm-up and are also repeatedly stressed by the weight of taxiing airplanes

Laboratory plunger tests were performed to ascertain the effect of slow repetitional loading of varying magnitude as compared to the effects of vibratory loading. The results of preliminary tests with slow repetitional loading and sand "B" are given on Figure 59, where the number of load repetitions is plotted against the plunger penetrations. It may be seen from this diagram that the slow repetitional loading performed at the rate of 1 cycle per minute gave an almost identical curve as did two vibratory tests performed with sands of almost equal density It therefore appeared that the only thing of importance is the number of load repetitions irrespective of the frequency. Of course this was true only so long as no resonance of the soil vibrating mass system was present to magnify the actual forces transmitted to the soil

Photograph Figure 53 illustrates the construction of the 6-bank plunger testing machine designed and built by the School of Engineering, Princeton University. The motor and the reduction gear box "F" drive a horizontal shaft which operates 2 crank shafts "H" at each end of the horizontal shaft. These crank shafts are connected to a piston "J" moving in an up and down direction in the bronze-lined cylinder "I". The pistons "J" are connected to a horizontal beam "D". The whole system is geared for one repetition of the up and down movement of the beam "D" per minute. The number of these movements is registered by a counter "G".

The machine can simultaneously operate with six soil containers of the standard CBR type. These containers are marked "A" on the photograph. The usual 2-in diameter (3 sq in area) plungers are employed and are loaded to the desired amounts by the weights "E" The plungers are connected by means of wire ropes and pulleys to the counter weights "C" These counter weights are raised by the upward motion of the beam "D" and released when the beam is in its downward position

Figure 53 shows the downward position of the beam with the counter weights hanging on the ropes and relieving by the amount of their weight the load "E" on the plunger "B"

Results of Tests with Sand "A"

Figure 58 gives the number of load repetitions versus plunger penetration curves plotted on semi-log paper. Figures 59 and 60 summarize the test results

It may be seen that for the same number of load repetitions and the same magnitude of the repetitional load expressed in percent of the original load, the plunger penetrations increase with the intensity of the original static load Further, that for the same value of the original static load, the plunger penetrations increase with the magnitude of the repetitional load. The same holds true in a general manner for the ratio of the repetitional plunger penetrations as compared to the penetrations under one load application

Under a unit pressure of 55 psi, 10,000 full load repetitions increased the original plunger penetration from the first loading by some 750% (Figure 60) From Figure 55 it may be estimated that 1,000 full load repetitions would have increased the original first plunger penetration by some 330%, 100 full load repetitions by some 150%, and 10 full load repetitions by some 40%.

The Results of Tests with Sand "B"

Figure 56 gives the results of tests with sand "B", whereby the number of load repetitions is plotted against the number of plunger penetrations at a semi-log scale. Figures 61 and 62 summarize the test results.

It may be seen that sand "B" followed the same trends as sand "A", but in a much more pronounced manner. Under a unit pressure of 55 psi, 10,000 full load repetitions increased the original plunger penetration from the first loading by some 4300% (Figure 62) From Figure 56 it may be seen that 1,000 full load repetitions would have increased the original first plunger penetration by some 1660%, 100 full load repetitions by some 380%, and 10 full load repetitions by some 100%

A comparison of this data with the information given by Table 5 shows that 10 full load repetitions increase the plunger penetrations in approximately the same proportion as 10 minutes vibration (e g 1500 repetitions) of a centrifugal force equal to approximately ± 2 0% of the same original static load

Results of Tests with Red Clay

Figure 57 gives the number of load repetitions - plunger penetration curves for tests with red clay. It may be seen from these diagrams and also from Figure 42 showing a similar type of curves for vibratory loading of red clay and Figures 54-56 giving typical similar curves for vibratory loading of sands, that when plotted at a semi-log scale, the number of load repetitions - plunger penetration curves has a different shape for cohesive and for granular non-cohesive soils, as illustrated by the following sketch, Figure 58

The shape of the curve "a" closely corresponds to the one obtained by Mr Kersten[1] for the slow repetitional tests he had performed at the University of Minnesota. It shows even on a semi-log scale a progressively decreasing effect of the number of load repetitions on the penetrations

Mr Kersten had not made any tests with clean sands, and it may be seen from our tests that the sands follow the shape of curve "b" of Figure 58 indicating that they are most susceptible to any type of load repetitions, whether vibratory or slow repetitional This finding is of considerable practical importance

Redistribution of Shearing Stresses in a Pavement Beneath an Airplane Wheel during Warm-up of the Motors

The wheels of an airplane are blocked (by means of brakes or otherwise) during the warming up of its motors for the purpose of keeping it stationary. This means that a horizontal force equal to the thrust of the propellers is transmitted by the wheels of the plane to the surface of the pavement

So far the effect of this horizontal force on the stability of a pavement was not known Opinions have been expressed at times that the detrimental effect of this horizontal force might be even greater than that of vibrations This did not appear likely, but the matter needed investigation

A great number of variables is involved in the study of the effect of warming up operations of a plane on an airport pavement This circumstance imposed a form of study which was based on a separation of variables It was therefore impractical in the present stages of our study to actually introduce a horizontal force during load tests It was then decided to attempt a mathematical analysis of the redistribution of stresses in a pavement as a result of the application of a horizontal force to its surface which might help to gain some insight into the problem Since failures of pavements are to be mainly attributed to excessive shearing stresses, a study of the redistribution of such stresses in a pavement as a result of the action of a horizontal force would be liable to give an approximate idea of the magnitude of the detrimental effect of horizontal forces applied to the pavement surface Professor Frank Baron of Yale University was requested to perform this study

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[1] "Repeated Load Tests on Highway Subgrade Soil and Bases", by Miles S Kersten, University of Minnesota Engineering Experiment Station Technical Paper No. 43, July 1943

It should be mentioned that, as shown by his illustrations, the analysis made by Professor Baron refers to a rectangular loaded area and to a homogeneous semi-infinite solid beneath it.

The above assumptions are naturally not quite exact. However, the assumption of an elliptical loaded area, although corresponding more closely to the imprint of a tire, would have required extremely complicated computations The difficulties of analysis would have then become so great that any slight additional analysis obtained thereby would in no way compensate for the several times greater amount of mathematical work required to achieve it.

The second approximation, that of a homogeneous semi-infinite solid, is used as a point of departure for all studies of that kind. On the strength of similar comparisons made for stresses produced by vertical loads only, it is possible to estimate the changes in the stress distribution to be expected in a layered system as compared to the distribution in a homogeneous body.

In order to adapt the results of Professor Baron's investigation[1] to conditions existing during the warm-up of airplane engines, certain additional computations have to be performed.

In the case of a warming-up airplane a horizontal force is applied to the surface of the pavement simultaneously with a vertical force. The ratio of the two forces changes with the increase of the thrust of the propellers and with a redistribution of the vertical load between the main and the secondary wheels. From published data on the matter[2] it can be estimated that the maximum value of the horizontal force is approximately equal to one third of the vertical force, which is simultaneously reduced by approximately 15% as compared to the original condition when the motors are not running.

The four curves shown on Figure 63 were obtained by applying the above considerations to the results obtained by Professor Baron

If we want to compare the maximum shear stresses produced by a stationary plane when its motors are not running to the maximum shear values developed when its motors are running at high speed the comparison should be made by means of curves "1" and "4" shown on Figure 63. It may be seen that greater shearing stresses are created by the second condition - (motors running) - only at a depth "z" smaller than 0.34a. In other words, an increase of shearing stresses will be noticeable only at a depth smaller than 0.17 of the length of the major diameter of a tire imprint.

In the case of a layered system, or, to be more precise, for the case of a semi-rigid pavement and base overlying softer subgrade below it - (a condition which will normally occur on airports) - the shearing stress concentration in the upper layer will be accentuated still further. It can therefore be safely assumed that an increase of the shearing stresses due to the warming up of airplane motors will occur only within the depth of the pavement proper.

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[1] "A Mathematical Study of Shearing Stresses Produced in a Pavement by the Locked Wheels of an Airplane during the Warm-Up of Its Engines," by Frank Baron, C.A.A. Technical Development Note No. 47, 1947

[2] "Certain Requirements for Flexible Pavement Design for B-29 Planes," U. S. Waterways Experiment Station, Vicksburg, Miss., 1 August 1945

V. CONCLUSIONS

(1) Soil load tests of the controlled strain type, including the California Bearing Ratio test, are unsuited for the study of vibration effects. Load or plunger tests of the controlled stress type have to be used.

(2). A series of controlled stress 2-in diameter and 5-in diameter plunger tests performed at frequencies below the resonance range on a well graded clean sand with a surcharge on its surface around the plunger showed that

(a) At all densities and under all conditions of saturation the effect on plunger penetrations of a vibratory force was several times greater than that of an equivalent static force

(b) At all densities both static and dynamic plunger penetrations on submerged sand were appreciably greater than plunger penetrations on dry sand. The bearing capacity of the sand was decreased by more than 50% as a result of complete submergence.

(c) At all densities both static and dynamic plunger penetrations on sand saturated by passive capillary action were larger than for dry sand.

(d) The penetrations of the 5-in. diameter plunger on dry sand were larger than the 2-in. diameter plunger penetrations in approximately the same ratio both under static and dynamic loading. The same relationship held for submerged sand, but the increase of the penetrations for the larger plunger was relatively less pronounced

(3). Conventional theories concerning the increase of effective stresses in sand due to passive capillary saturation appear to be in need of revision, since evidence of a slight decrease was obtained

(4). Saturation conditions appear to influence the bearing properties of a sand in the same manner both under static and dynamic loading.

(5). Sand of uniform grading is particularly susceptible to the action of vibratory or slow repetitional forces. In the case of a fine sand of uniform size, the continued application for 10 minutes of a vibratory force was found to produce deformations up to 140 times greater than those produced by a static force of equivalent magnitude.

(6) On a sand of uniform grading a very small vibratory force of the same order of magnitude as in the case of warming-up airplanes more than offsets any reduction of static pressure corresponding to the redistribution of load between airplane wheels during the period of engine warm-up.

(7). The deformations produced in a soil by vibratory or by slowly repeated forces do not depend on the frequency of vibration or load repetition so long as the vibratory force itself is not magnified by resonance occurring somewhere within the vibrating system. Identical plunger penetrations have been obtained for the same number of load repetitions irrespective of whether the frequency of load repetition was 1 cycle per minute or 1,000 cycles per minute.

(8) Clay soils do not appear to be appreciably affected by repeated loading either of a vibratory or of a slow repetitional character.

(9). Sand-clay mixtures occupy an intermediate position. Even small admixtures of clay appreciably reduce the deformations produced by repeated loading of a vibratory character, especially at low water contents.

(10). The well-known improved bearing capacity of a fine sand due to saturation by capillarity was shown to be mainly due to improved resistance against rupture under the effect of tangential surface forces, rather than to its improved resistance to vertical loading

(11). Both static and dynamic plunger penetrations were found to appreciably increase at sand densities smaller than those corresponding to the "critical void ratio".

(12). For the same number of load repetitions on sand, plunger penetrations increased with the intensity of the slow repetitional or vibratory load. For the same fractional value of the slow repetitional or vibratory force expressed as a fraction of the original load, the plunger penetrations naturally increased also with the value of the original load However, a slow repetitional force of the same intensity was found to produce somewhat higher penetrations with a decrease of the intensity of the original static load (Figure 64)

(13). It appears possible to make the following conclusions which are of direct practical importance for the design of base courses of flexible airport pavements

(a) Vibratory forces of the order of magnitude transmitted to airport pavements by airplane wheels during the warm-up period of properly balanced engines are liable to cause pavement damage only under certain extreme conditions such as A subgrade consisting of a not too dense sand of uniform grading and/or a rise of the ground water table with a resulting strong reduction of the bearing capacity of a submerged sand base course or subgrade. Special precautions are indicated on airports where such soil conditions prevail.

(b) Improperly adjusted engines warmed up on a pavement underlain by soil of the above character may magnify the vibratory forces transmitted to the soil.

(c) Load repetitions caused by repeated slow passage of taxiing airplanes are liable to cause most trouble on uniform sand subgrades or on sand base courses and are liable to produce more detrimental effects than vibration during engine warm-up if the wheels repeatedly pass over the same place. Distributed traffic is liable to have a beneficial action by producing additional compaction of base courses and subgrades of not too greatly underdesigned flexible pavements

(d) Maximum possible compaction of sandy base courses and subgrades and provision of adequate drainage to prevent their saturation at any time to an extent where the sand grains are buoyed is essential, especially in taxi-way, hard-standing, and warm-up areas

(14). A study of the performance of concrete pavements had not been included in the program of this investigation. Nevertheless, our findings may in some respects also be applied to sand base courses and subgrades beneath concrete paving.

(15). The effects of a horizontal force applied to the pavement surface by the blocked wheels of an airplane during engine warm-up are negligible, except in the upper portion of the pavement proper.

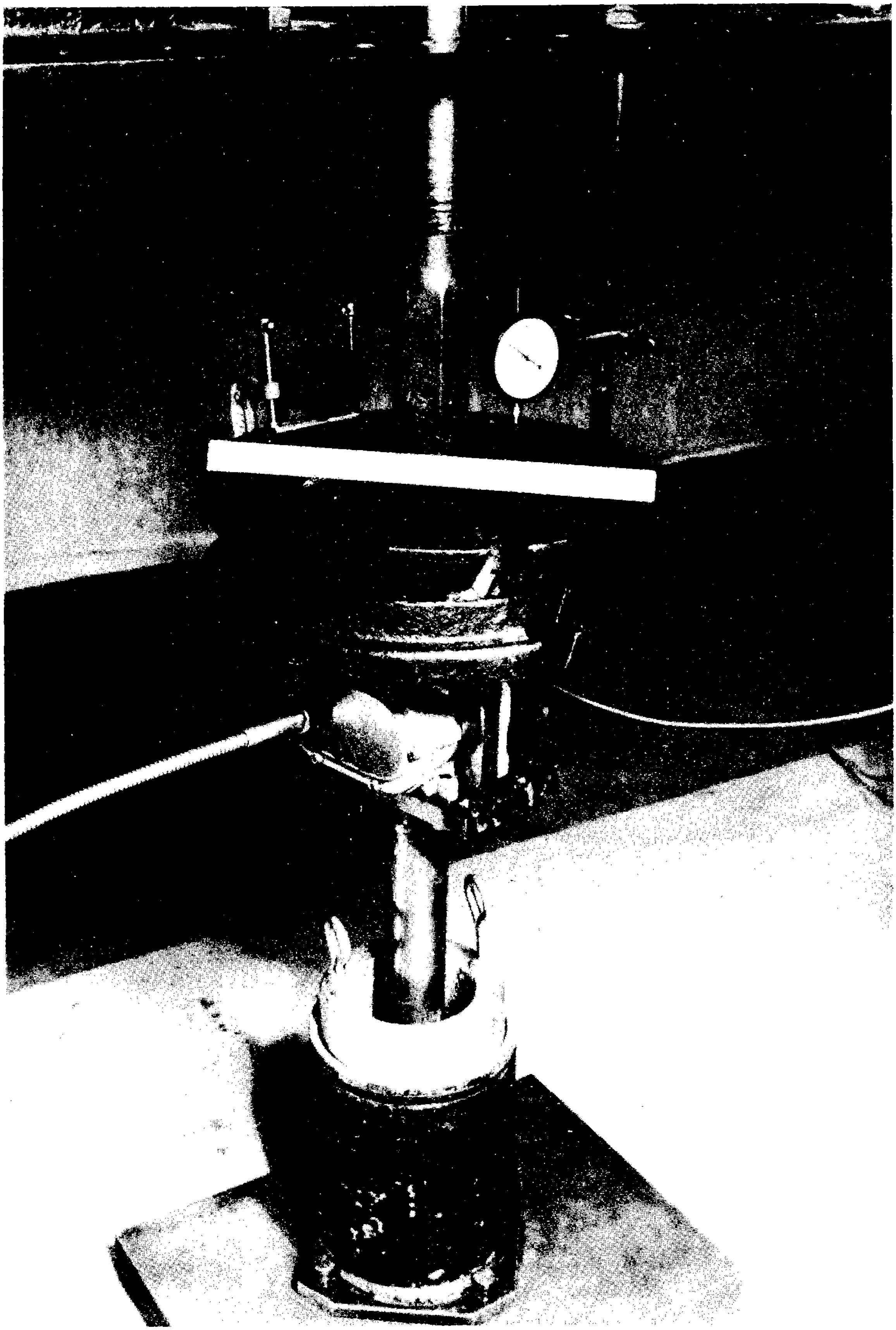


Figure 1. Photo of Set-up for Controlled Stress 2 inch Diameter Vibratory Plunger Test.

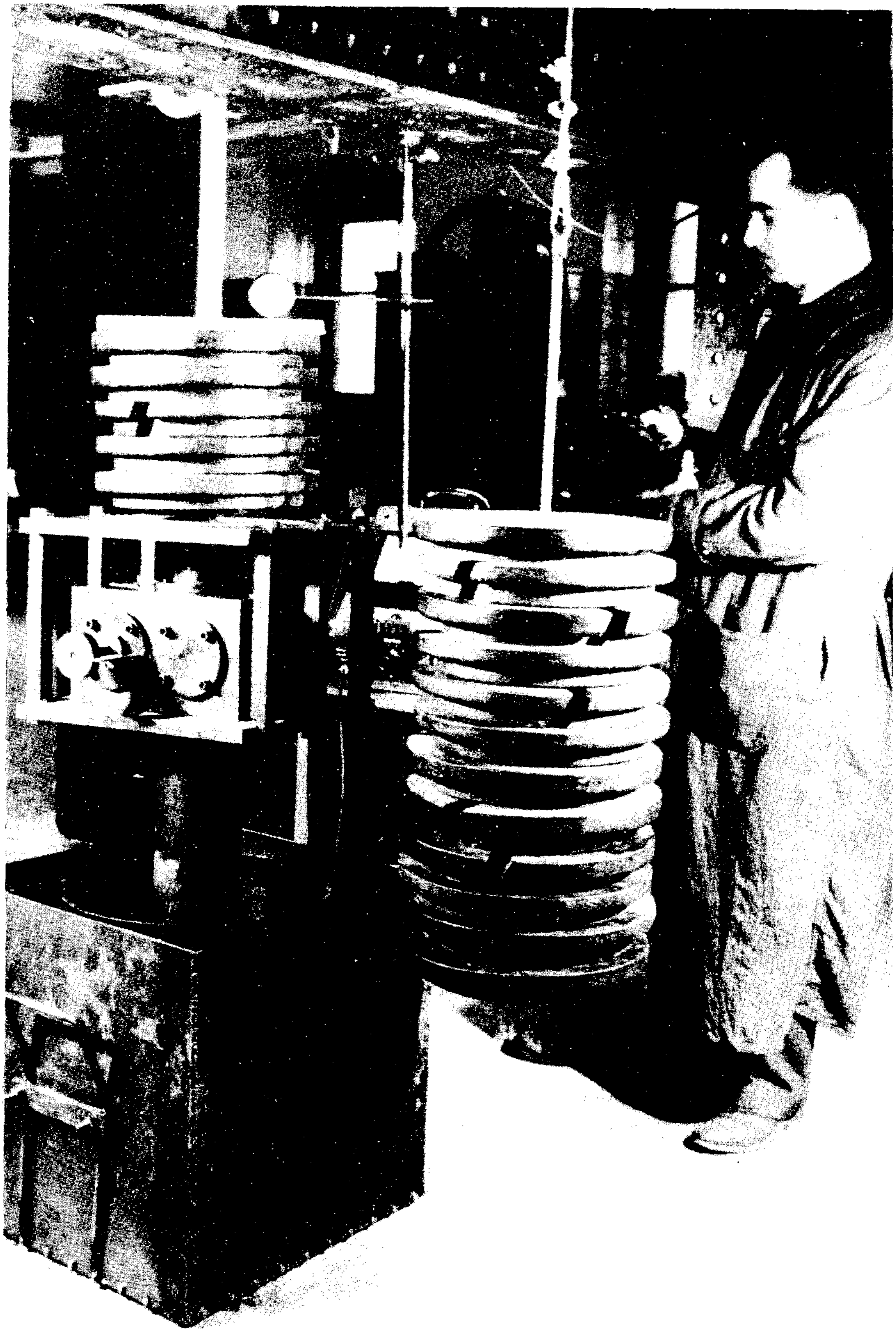


Figure 2. Close-up Photo of Controlled Stress 5 inch. Diameter Vibratory Plunger Test.

Figure 3. General View Photo of Set-up for the Controlled Stress 5 inch. Diameter Vibratory Plunger Test.

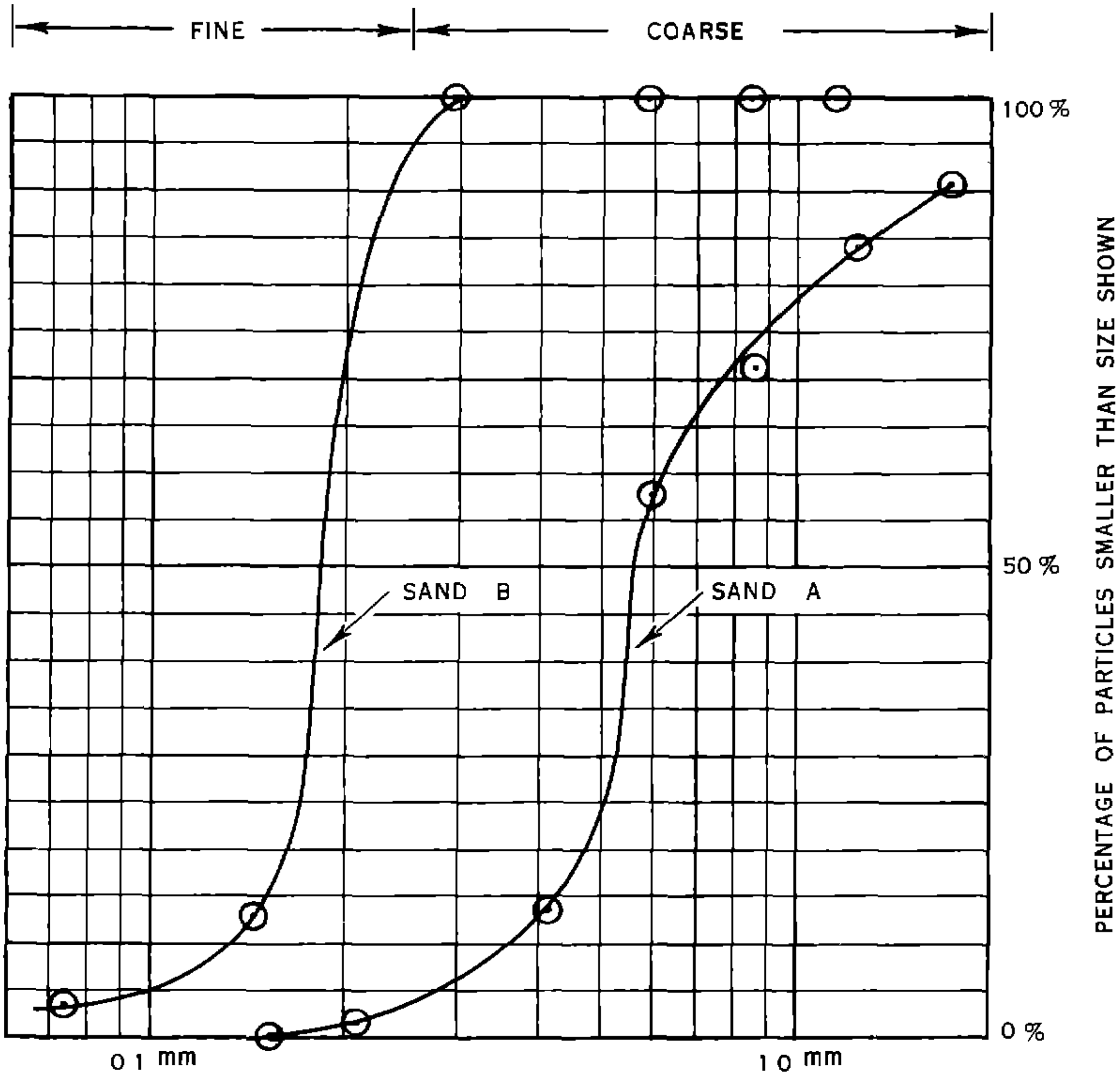


Figure 4 Grain Size Distribution Curves of Sands "A" and "B'

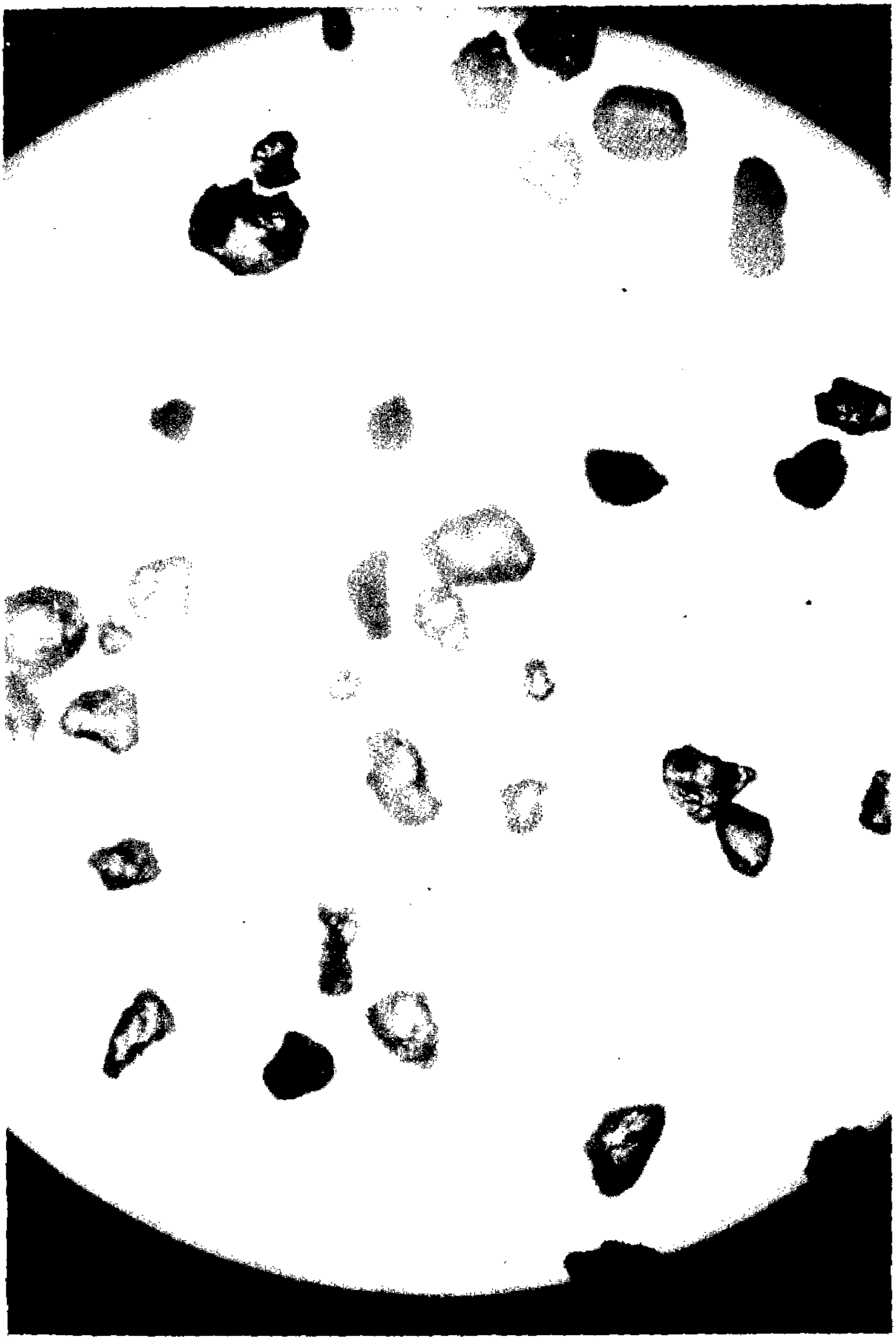


Figure 5. Microphotograph of Sand "A".

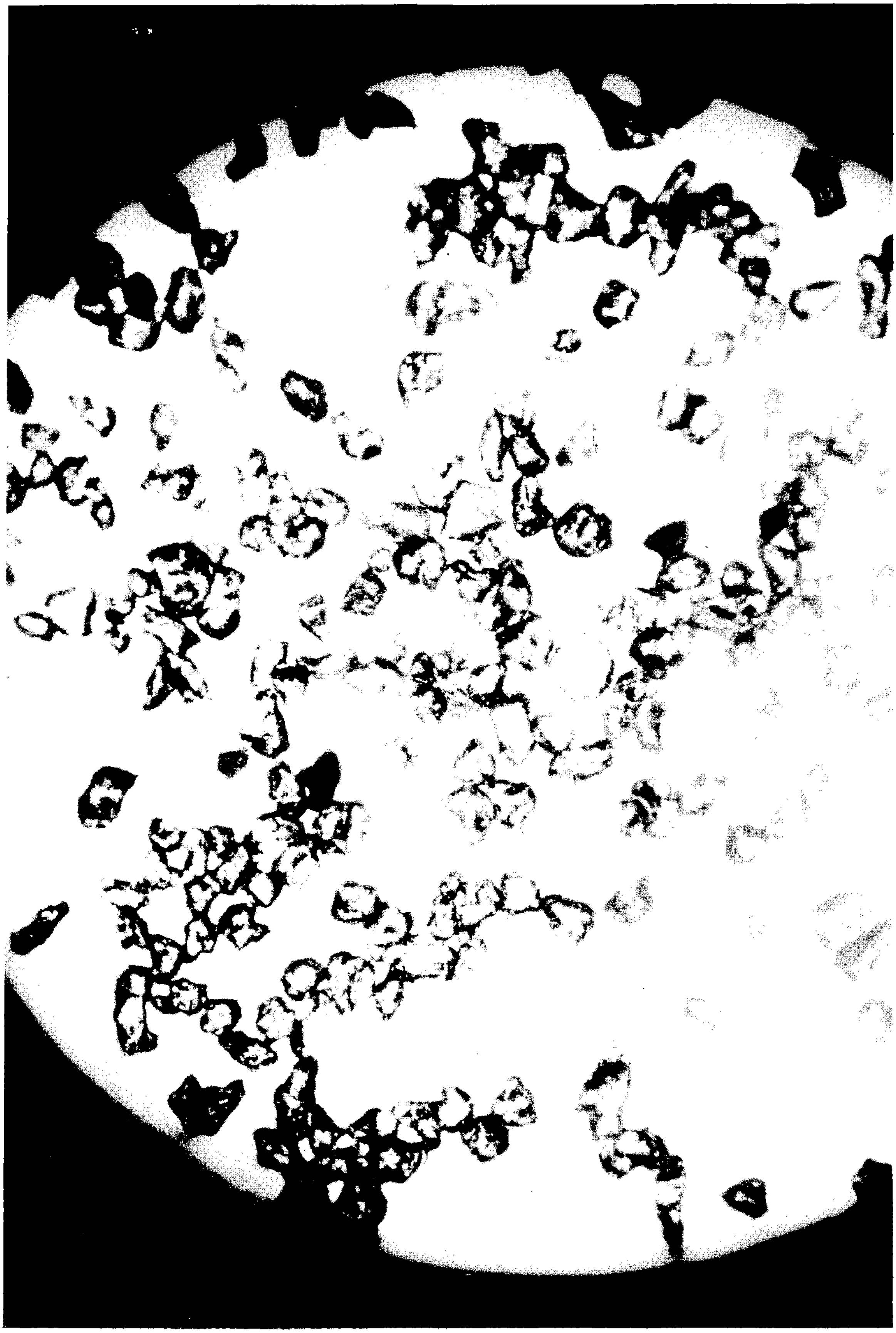


Figure 6. Microphotograph of Sand "B".

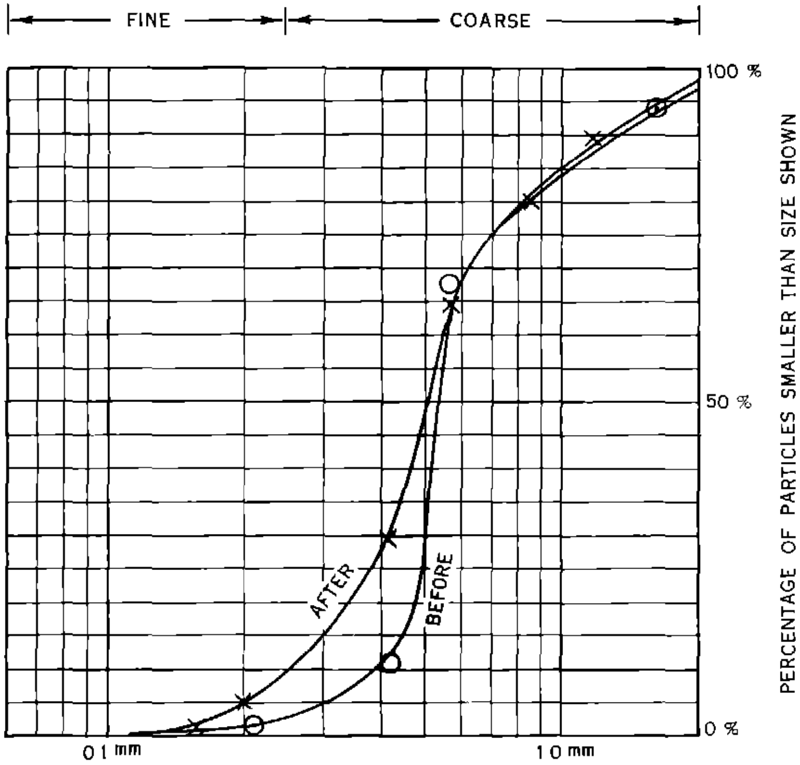
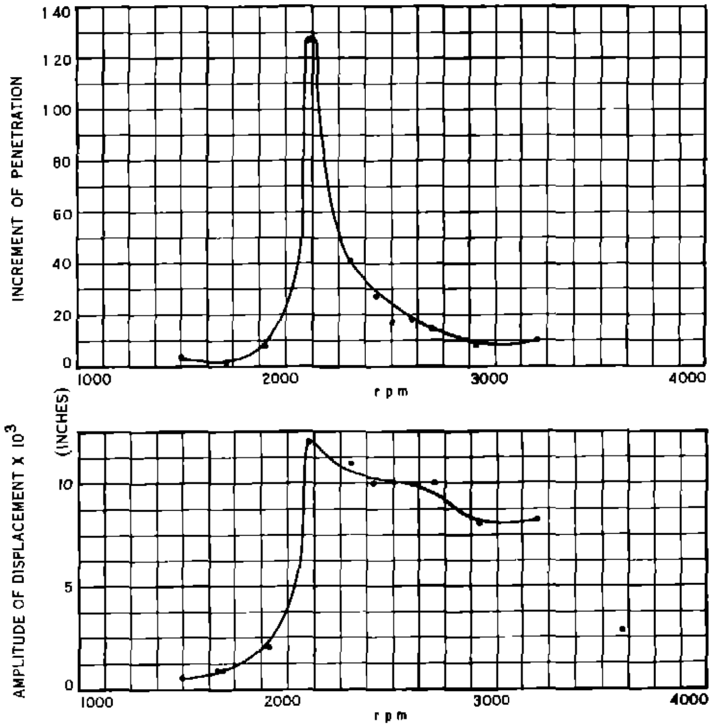
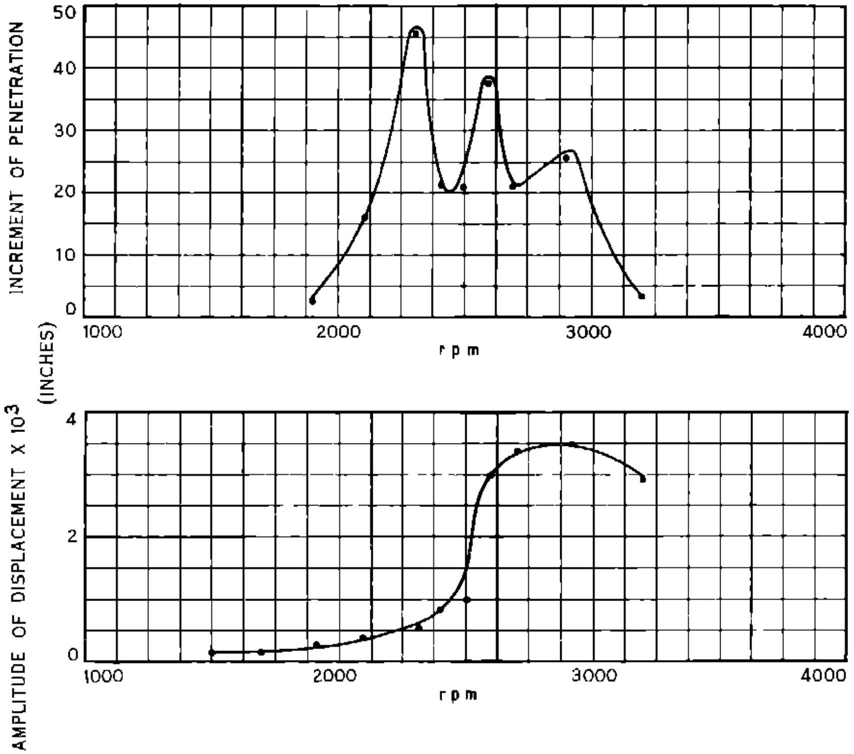


Figure 7 Grain Size Distribution Curves of Sand "A' Before and After 3 Tests Under Compaction Pressure of 1,000 psi



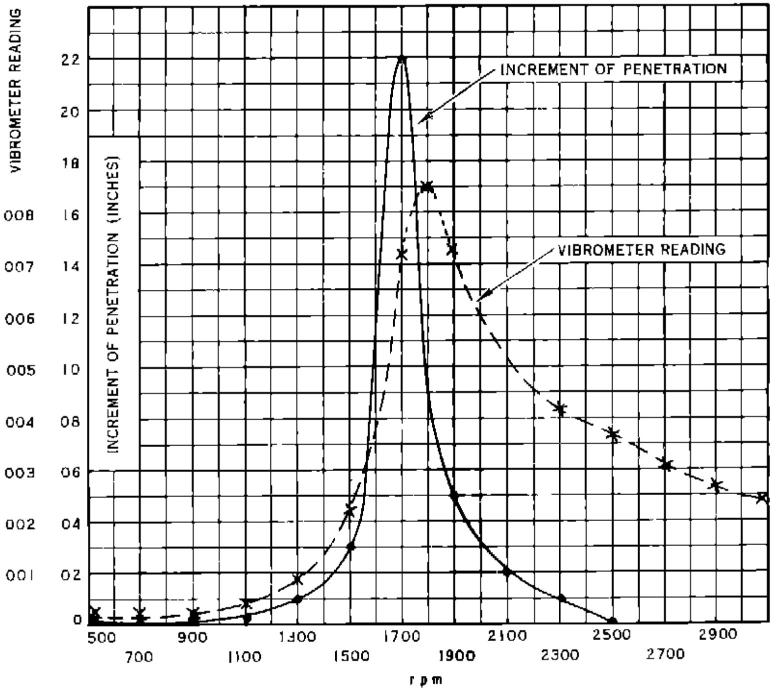
TEST NUMBER	119
VIBRATOR NUMBER	B
r p m	VARYING
CENTRIFUGAL FORCE	VARYING (40 4 AT 1500 r p m)
AREA OF PLUNGER	3 sq in
DIAMETER OF CONTAINER	6 in
SURCHARGE	12 3 lb 0 49 psi
STATIC LOAD	165 lb 55 psi
SAND TYPE	A
CONDITIONS OF SATURATION	DRY (W 0 %)
COMPACTION PRESSURE	1500 psi e 632

Figure 8 Determination of Resonance Range of 2 inch Diameter Plunger on Dense Sand "A ' from Increment of Penetration vs r p m Curve



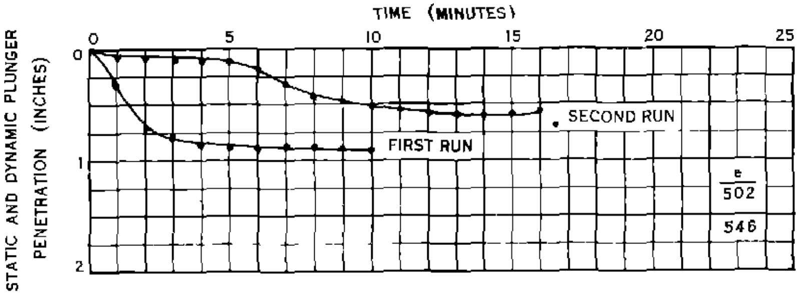
TEST NUMBER	112
VIBRATOR NUMBER	B
r p m	VARYING
CENTRIFUGAL FORCE	VARYING (10 4 AT 1500 r p m)
AREA OF PLUNGER	3 sq in
DIAMETER OF CONTAINER	6 in
SURCHARGE	12 3 lb = 0 49 psi
STATIC LOAD	165 lb 55 psi
SAND TYPE	A
CONDITIONS OF SATURATION	MOIST (W - 6%)
COMPACTION PRESSURE	500 psi e 663

Figure 9 Increment of Penetration vs r p m Curve of 2 inch Diameter Plunger on Loose Sand "A"



TEST NUMBER	287
COMPACTION PRESSURE	2000 psi
VOID RATIO "e"	535
SATURATION	DRY
SAND TYPE	A
AREA OF PLUNGER	19 635 sq in
SURCHARGE	121 5 lb 0 78 psi
DIAMETER OF CONTAINER	15 5 in
STATIC LOAD	610 lb 31 07 psi
CENTRIFUGAL FORCE	VARIABLE
VIBRATOR	A 15 ECCENTRIC 67 lb AT WEIGHT 1000 r p m

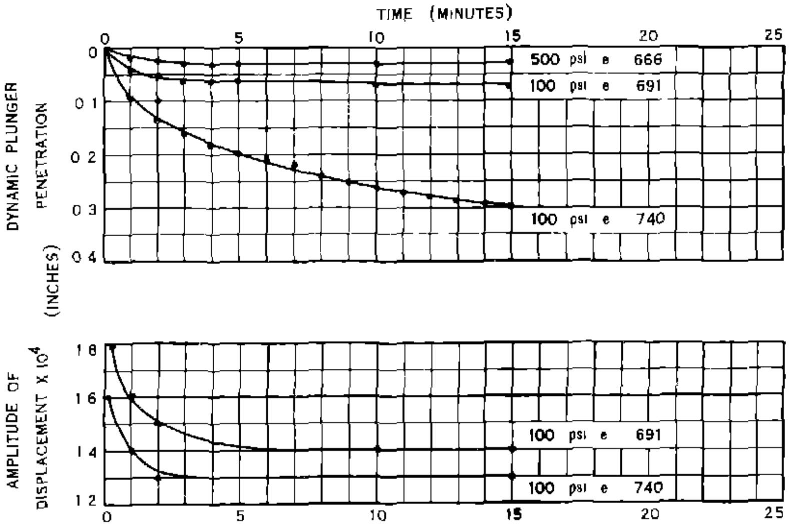
Figure 10 Determination of Resonance Range of 5 inch Diameter Plunger on Dense Sand "A" from Increment of Penetration vs r p m Curve



TEST NUMBERS	39 40
VIBRATOR NUMBER	B
r p m	1500
CENTRIFUGAL FORCE	40 4 lb
AREA OF PLUNGER	3 sq in
DIAMETER OF CONTAINER	6 in
SURCHARGE	NONE
STATIC LOAD	165 lb 55 psi
SAND TYPE	A
CONDITIONS OF SATURATION	MOIST (W 6%)
COMPACTION PRESSURE	1500 psi

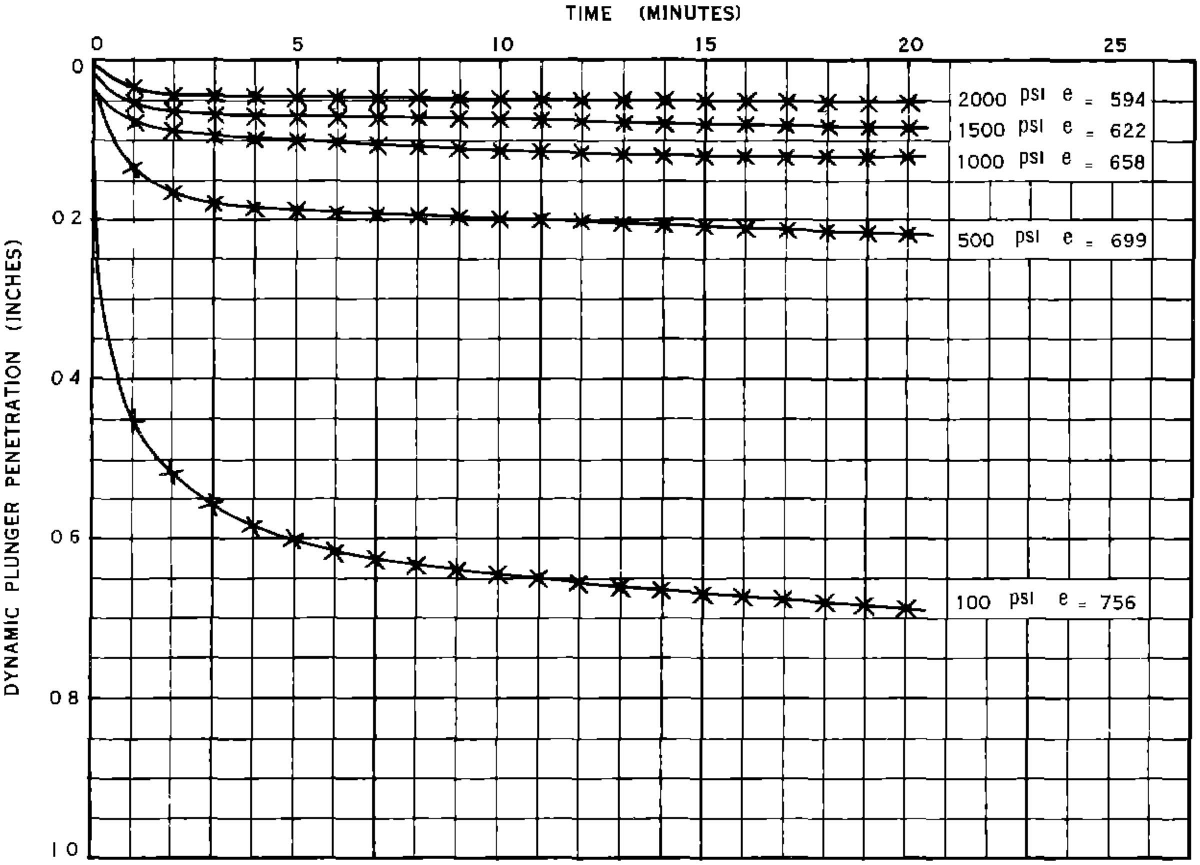
NOTE THE SAME SAMPLE OF SAND WAS USED FOR BOTH RUNS

Figure 11 Time vs Dynamic 2 inch Plunger Penetration Curves on Sand without Surcharge



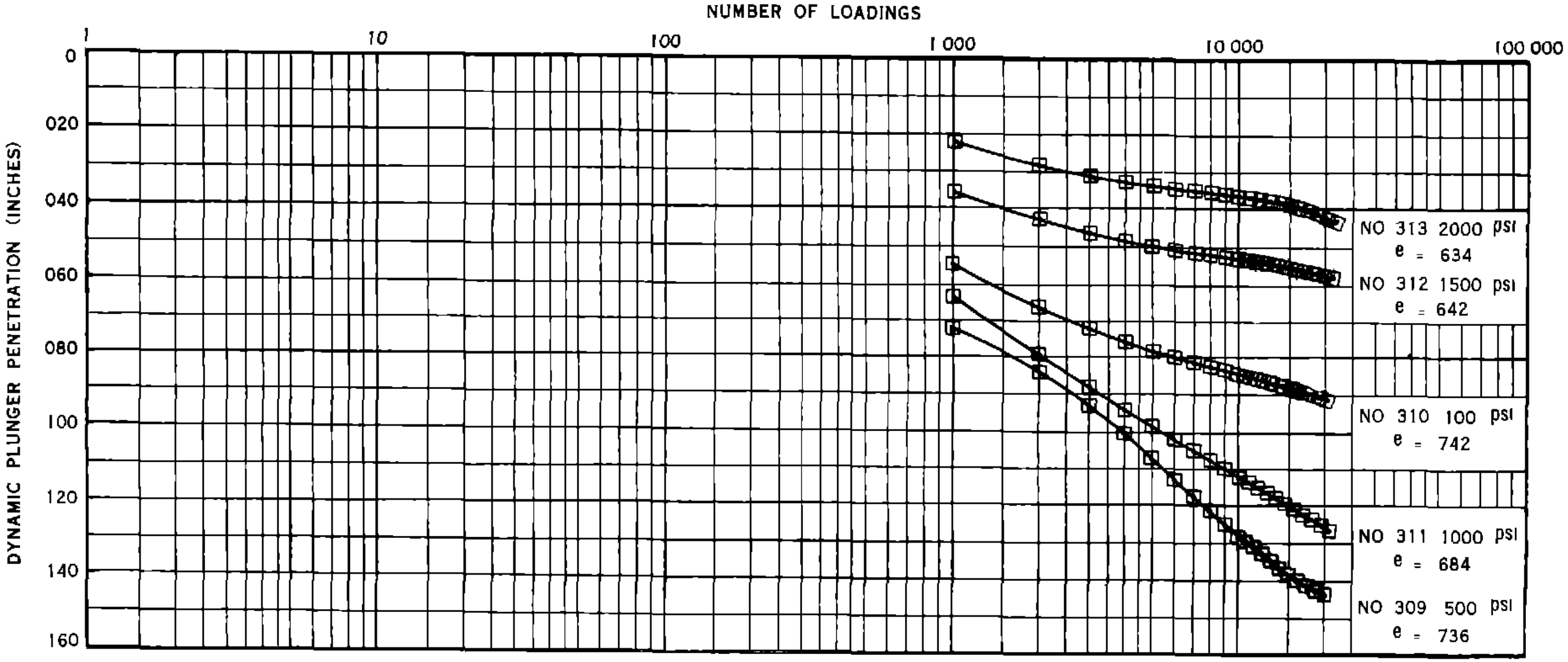
TEST NUMBERS	74 81 97
VIBRATOR NUMBER	B
r p m	1500
CENTRIFUGAL FORCE	10 4 lb
AREA OF PLUNGER	3 sq in
DIAMETER OF CONTAINER	6 in
SURCHARGE	12 3 lb 0 49 psi
STATIC LOAD	165 lb 55 psi
SAND TYPE	A
CONDITIONS OF SATURATION	DRY (W 0%)
COMPACTION PRESSURE	*NOTED ABOVE*

Figure 12 Time vs Dynamic 2 inch Plunger Penetration and Time vs Amplitude of Displacement Curves on Sand "A" with Surcharge



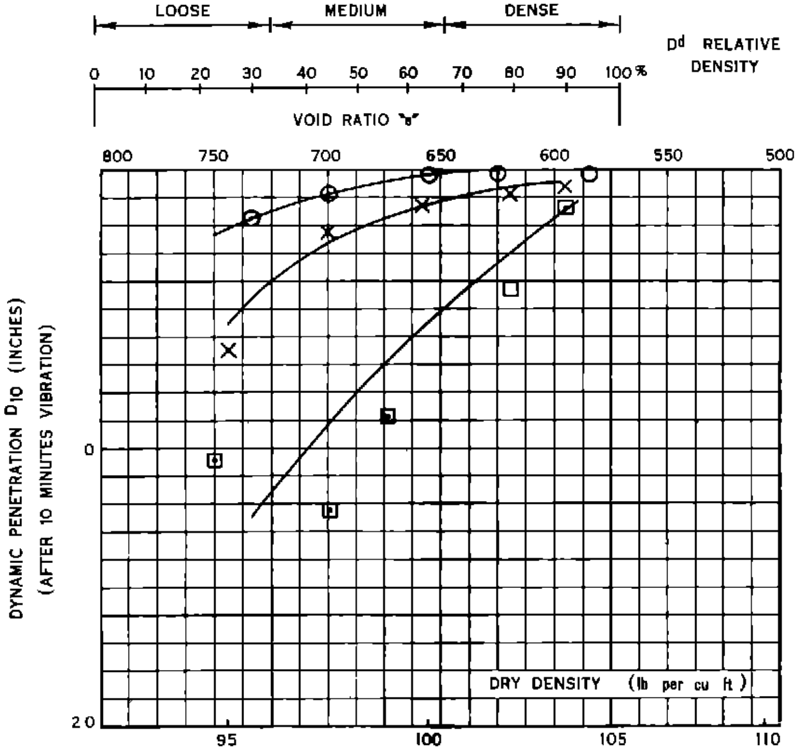
TEST NUMBERS	209 210 211 212 213
VIBRATOR NUMBER	B 1
r p m	1500
CENTRIFUGAL FORCE	23 2 lb
AREA OF PLUNGER	3 sq in
DIAMETER OF CONTAINER	6 in
SURCHARGE	12 3 lb = 0 49 psi
STATIC LOAD	93 lb = 31 psi
SAND TYPE	A
CONDITION OF SATURATION	CAPILLARY SATURATION
COMPACTION PRESSURE	NOTED ABOVE

Figure 13 Time vs Dynamic 2 inch Plunger Penetration Curves on Sand A' with Surcharge



TEST	VIBRATORY
MATERIAL	SAND A
CONDITION OF SATURATION	SUBMERGED
COMPACTION METHOD	NOTED ABOVE STATIC LOAD
VOID RATIO "e"	NOTED ABOVE
TEST NUMBERS	309 313
DIAMETER OF CONTAINER	6 in
AREA OF PLUNGER	3 sq in
SURCHARGE	12 3 lb = 0 49 psi
CENTRIFUGAL FORCE	10 3 lb
FREQUENCY	1000 r p m
STATIC LOAD	95 lb = 31 7 psi

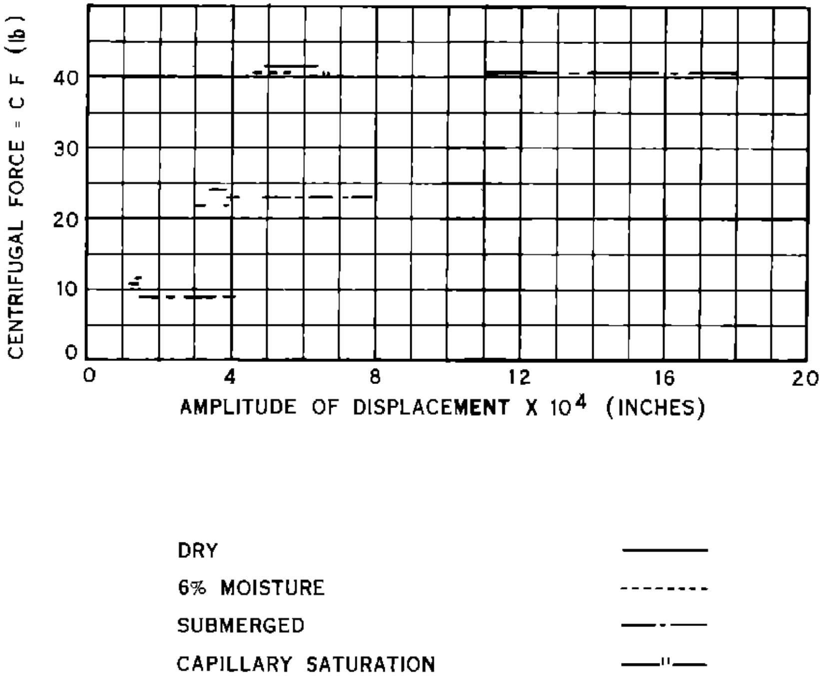
Figure 14 Number of Load Repetitions vs Dynamic 2 inch Plunger Penetrations Curves on Sand "A" Plotted on Semi-Log Scale



AREA OF PLUNGER	3 sq in
DIAMETER OF CONTAINER	6 in
SURCHARGE	12 3 lb 0 49 psi
CENTRIFUGAL FORCE	23 2 lb
STATIC LOAD	93 lb 31 psi
TEST NUMBERS	204 207 209 214 262 266

DRY	○
CAPILLARY SATURATION	×
SUBMERGED	⊡

Figure 15 Density vs Dynamic 2 inch Plunger Penetrations Curves, Sand "A"



NOTE 1 BOTH MAXIMUM AND MINIMUM AMPLITUDES INDICATED

MIN ———— MAX

NOTE 2 FOR TYPICAL CURVES SHOWING VARIATION OF AMPLITUDE WITH TIME SEE FIGURE 12

Figure 16 Variation of Vibration Amplitude with Changing Centrifugal Force, Sand "A"

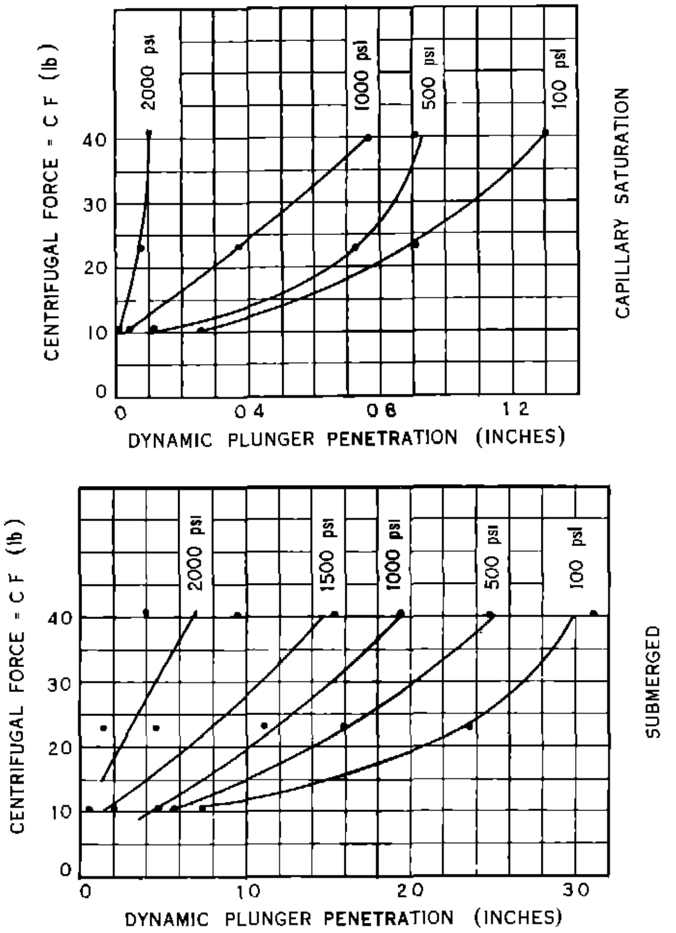
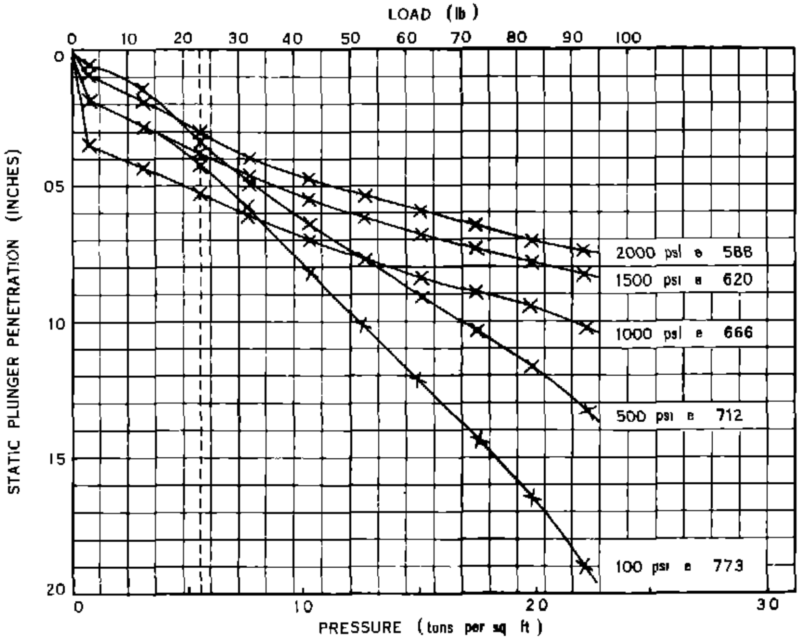
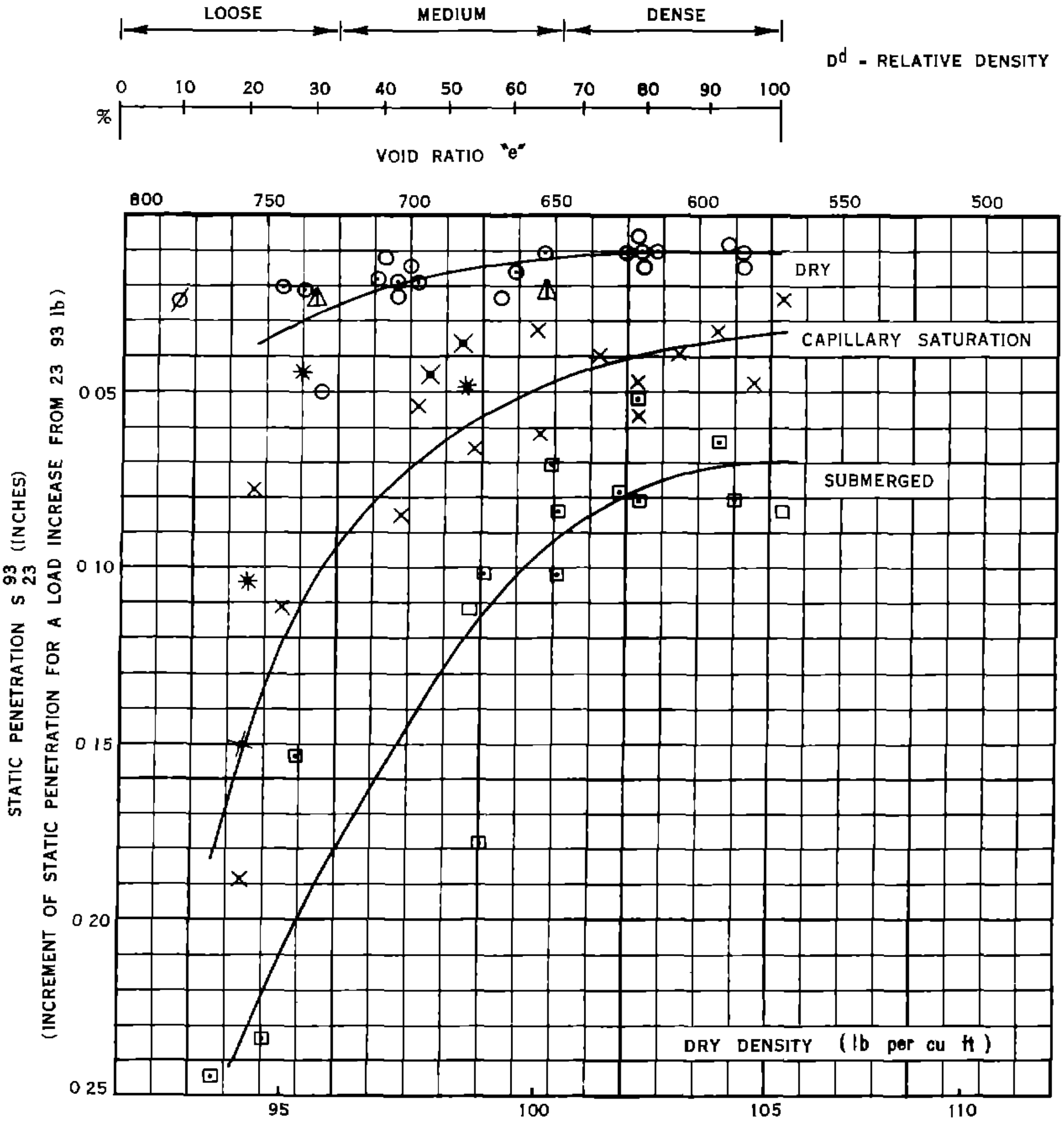


Figure 17 Variation of Dynamic 2 inch Plunger Penetration with Changing Centrifugal Force, Sand "A"



TEST NUMBERS	220 221 222 223 224
AREA OF PLUNGER	3 sq in
DIAMETER OF CONTAINER	6 in
SURCHARGE	12 3 lb 0 49 psi
SAND TYPE	A
CONDITION OF SATURATION	CAPILLARY SATURATION
COMPACTION PRESSURE	NOTED ABOVE

Figure 18 Load vs Static 2 inch Plunger Penetration Curves Sand "A"



AREA OF PLUNGER 3 sq in
DIAMETER OF CONTAINER 6 in
SURCHARGE 12 3 lb
TEST NUMBERS
187 202
204 207
209 214
225 226
229 236
238 240
242 243
258 259
262 266
272 280

LOAD APPLICATION IN INCREMENTS OF 10 lb
DRY ○
CAPILLARY SATURATION ×
SUBMERGED □

GRADUAL LOAD APPLICATION BY FILLING TANK WITH WATER OVER A PERIOD OF 1 HOUR
DRY Ø
CAPILLARY SATURATION
DRAINED FOR 5 MINUTES ✖
DRAINED FOR 48 HOURS ✳
6% MOISTURE ▲

Figure 19 Density vs Static 2 inch Plunger Penetration Curves Sand "A"

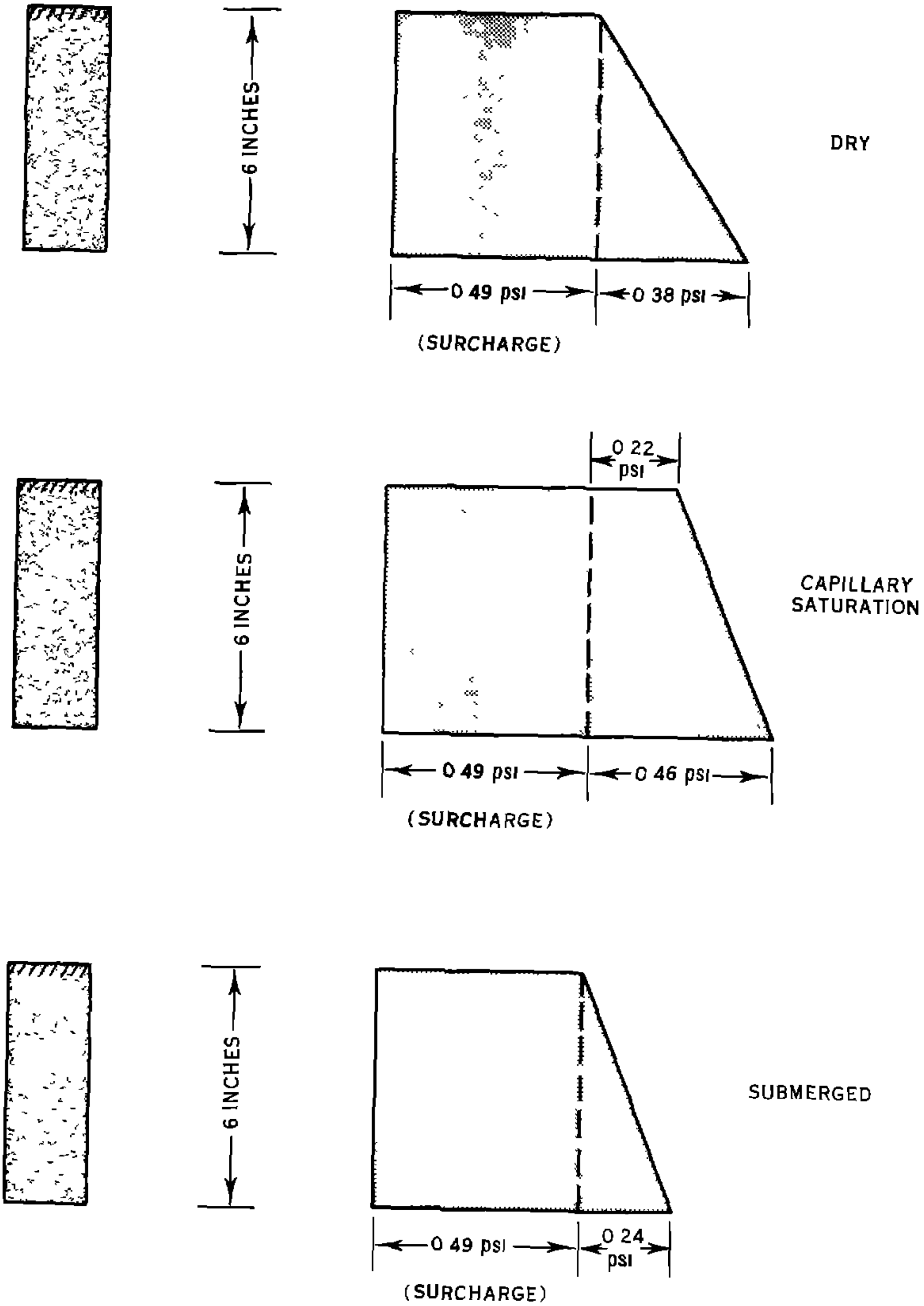


Figure 20 Theoretical Variation of the Effective Stresses Throughout the Depth of the Sand Sample

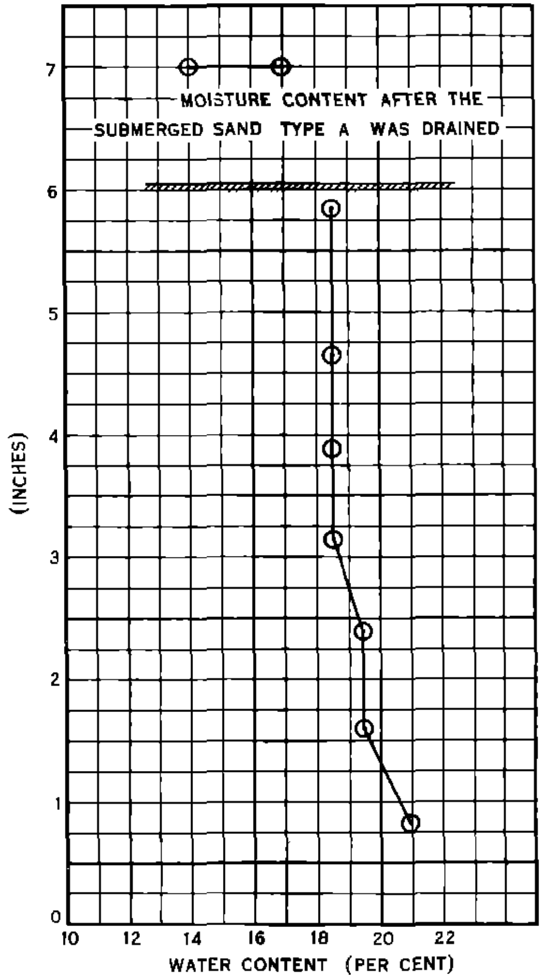
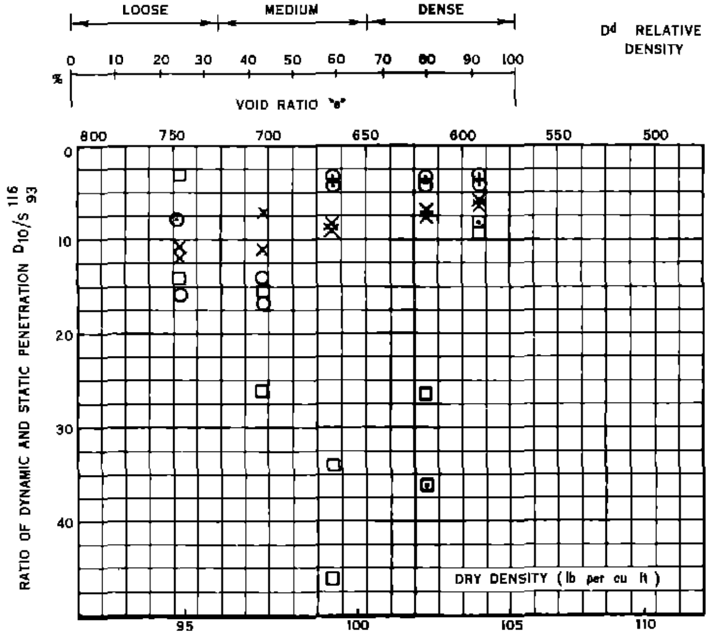


Figure 21 Water Content Distribution Throughout Depth of Sand "A" Sample after Passive Capillary Saturation

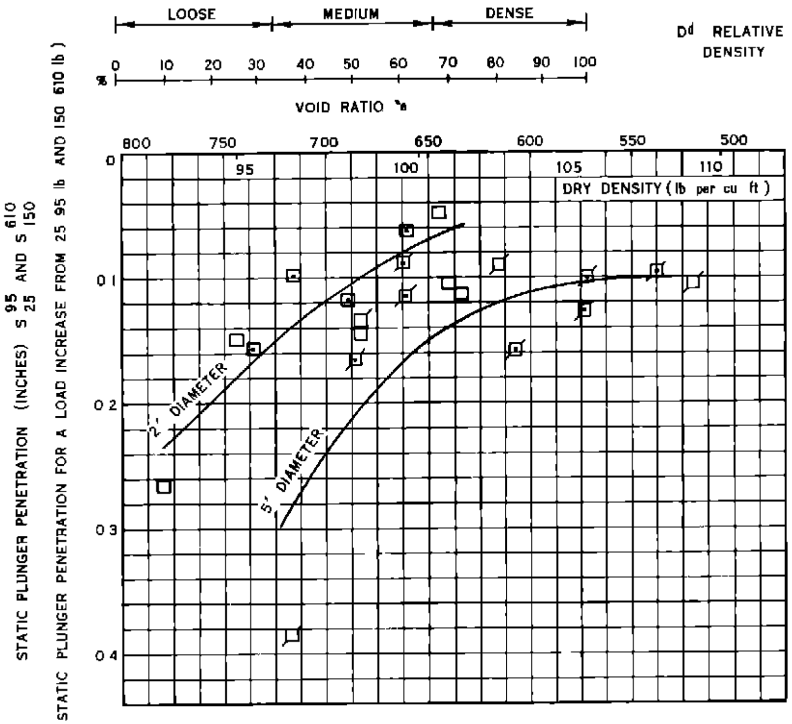


AREA OF PLUNGER	3 sq in	
DIAMETER OF CONTAINER	6 in	
SURCHARGE	12 3 lb	0 49 psi
CENTRIFUGAL FORCE	23 2 lb	
STATIC LOAD	93 lb	31 psi
TEST NUMBERS	187 188	
	190 202	
	204 207	
	209 214	
	225 236	
	238 240	
	242 243	
	262 266	

LOAD APPLICATION IN INCREMENTS OF 10 lb

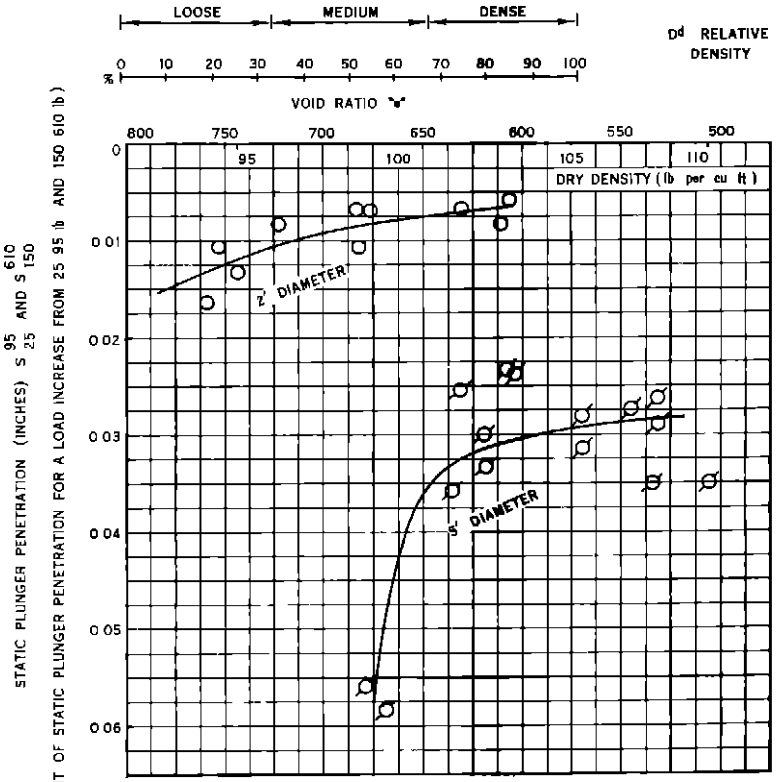
DRY	○
CAPILLARY SATURATION	X
SUBMERGED	□

Figure 22 Diagram Showing Effect of Density on the Ratio of Dynamic and Static 2 inch Plunger Penetrations Sand "A"



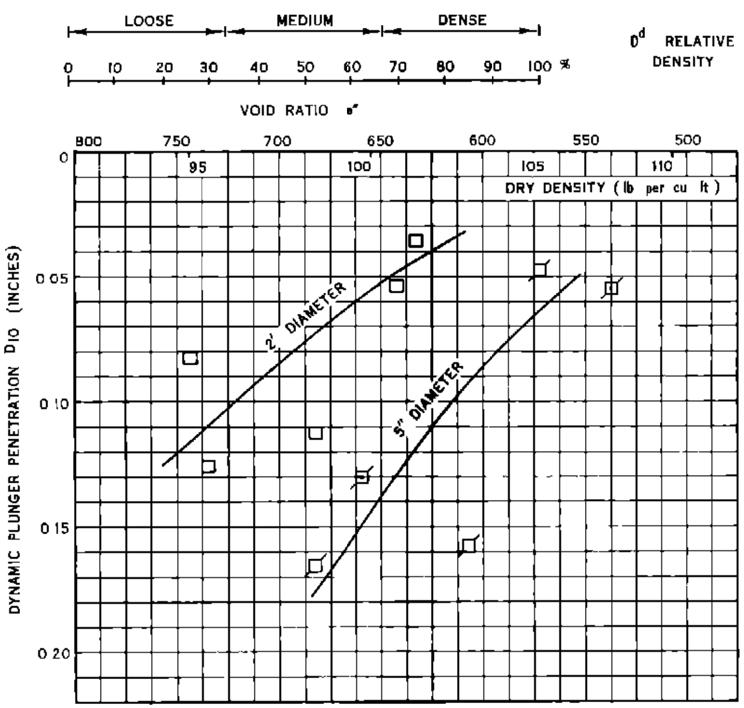
SYMBOL	□	□
TEST NUMBERS	309 313 325 329	281 298 307
CONDITION OF SATURATION	SUBMERGED	SUBMERGED
MATERIAL	SAND A	SAND A
AREA OF PLUNGER	3 sq in	19 64 sq in
SURCHARGE	18 6 lb 0 78 psi	121 5 lb 0 78 psi
DIAMETER OF CONTAINER	6 in	15 5 in
INCREMENT OF PENETRATION PLOTTED FOR	S_{25}^{95}	S_{150}^{610}

Figure 23 Density vs Static 2 inch and 5 inch Plunger Penetrations Submerged Sand "A"



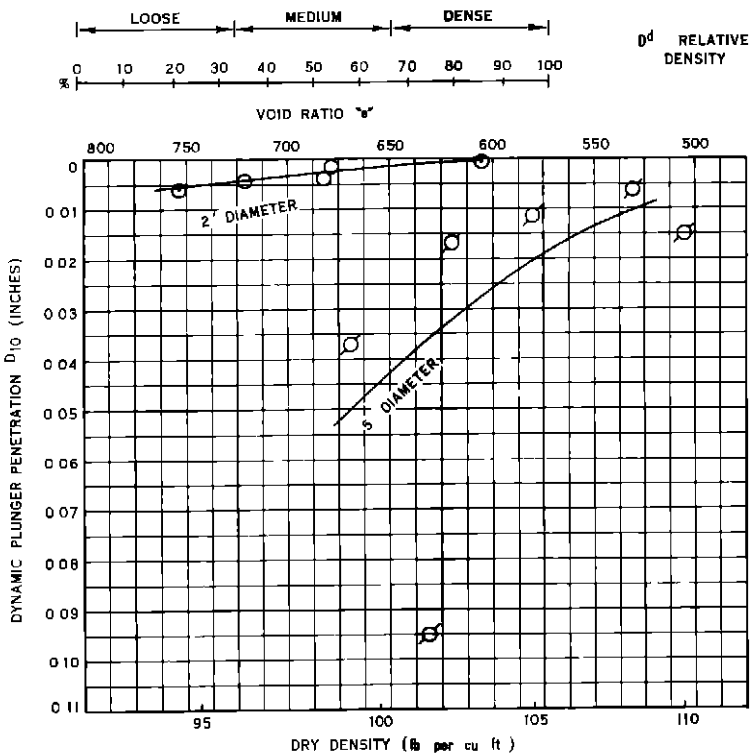
SYMBOL	○	ø
TEST NUMBERS	308 314 316 318 323	282 285 287 297
CONDITION OF SATURATION	DRY	DRY
MATERIAL	SAND A	SAND A
AREA OF PLUNGER	3 sq in	19 64 sq in
SURCHARGE	18 6 lb 0 78 psi	121 5 lb 0 78 psi
DIAMETER OF CONTAINER	6 in	15 5 in
INCREMENT OF PENETRATION PLOTTED FOR	S_{25}^{95}	S_{150}^{610}

Figure 24 Density vs Static 2 inch and 5 inch Plunger Penetrations Dry Sand "A"



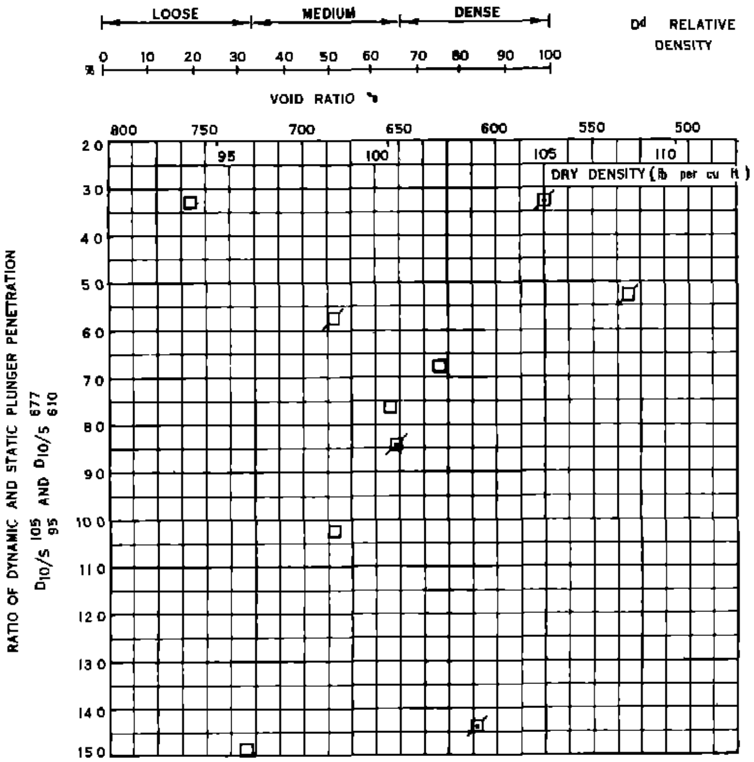
SYMBOL	□	⧄
TEST NUMBERS	309 313	303 307
CONDITION OF SATURATION	SUBMERGED	SUBMERGED
MATERIAL	SAND A	SAND A
AREA OF PLUNGER	3 sq in	19 64 sq in
SURCHARGE	18 6 lb 0 78 psi	1215 lb 0 78 psi
DIAMETER OF CONTAINER	6 in	15 5 in
STATIC LOAD	95 lb 31 7 psi	610 lb 31 07 psi
CENTRIFUGAL FORCE	10 3 lb	67 lb
FREQUENCY	1000 r p m	1000 r p m

Figure 25 Density vs Dynamic 2 inch and 5 inch Plunger Penetrations Submerged Sand "A"



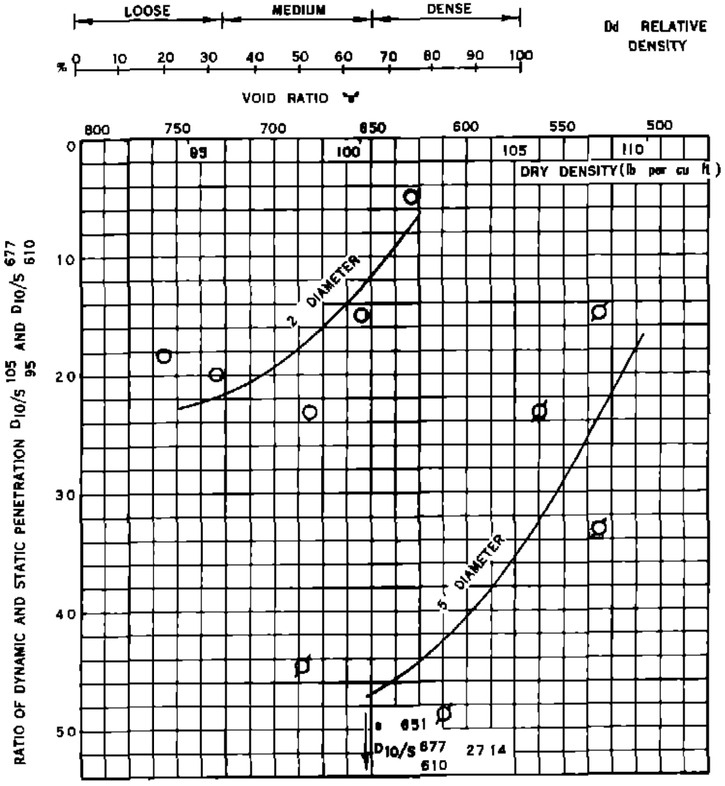
SYMBOL	○	⌀
TEST NUMBERS	308 314 316 318	285 288 292
CONDITION OF SATURATION	DRY	DRY
MATERIAL	SAND A	SAND A
AREA OF PLUNGER	3 sq in	19 64 sq in
SURCHARGE	18 6 lb 0 78 psi	1215 lb 0 78 psi
DIAMETER OF CONTAINER	6 in	15 5 in
STATIC LOAD	95 lb 31 7 psi	610 lb 31 07 psi
CENTRIFUGAL FORCE	10 3 lb	67 lb
FREQUENCY	1000 r p m	1000 r p m

Figure 26 Density vs Dynamic 2 inch and 5 inch Plunger Penetrations Dry Sand "A"



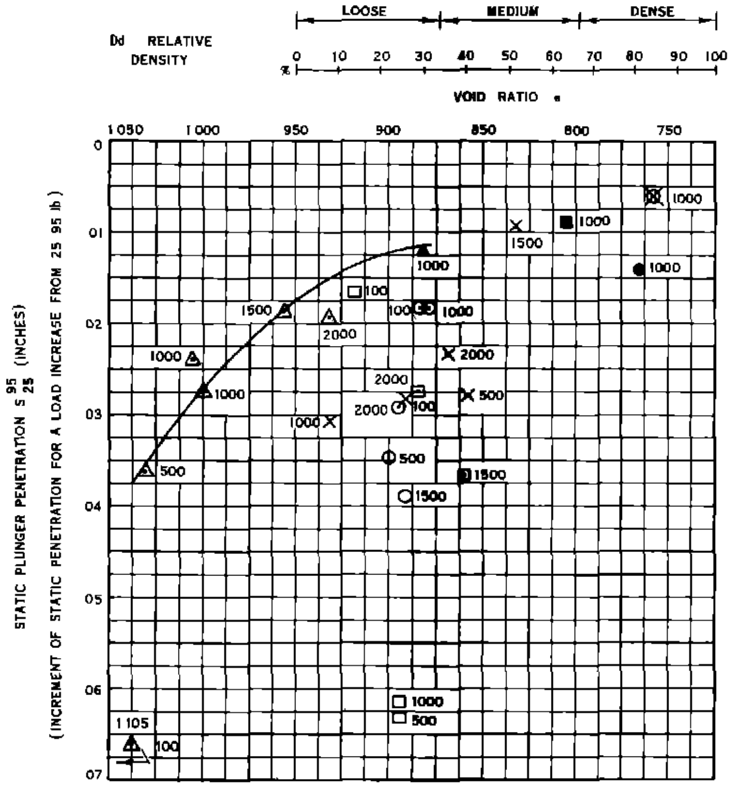
SYMBOL	□	⧄
TEST NUMBERS	309 313 325 329	298 307
CONDITION OF SATURATION	SUBMERGED	SUBMERGED
MATERIAL	SAND A	SAND A
AREA OF PLUNGER	3 sq in	19 64 sq in
SURCHARGE	18 6 lb 0 78 psi	121 5 lb 0 78 psi
DIAMETER OF CONTAINER	6 in	15 5 in
STATIC LOAD	95 lb 31 7 psi	610 lb 31 07 psi
CENTRIFUGAL FORCE	10 3 lb	67 lb
FREQUENCY	1000 r p m	1000 r p m
RATIO PLOTTED	$D_{10}/S \frac{105}{95}$	$D_{10}/S \frac{677}{610}$

Figure 27 Density vs Ratio of Dynamic and Static 2 inch and 5 inch Plunger Penetrations Submerged Sand "A"



SYMBOL	○	Ø
TEST NUMBERS	285, 288 297	308 314 316 318 323
CONDITION OF SATURATION	DRY	DRY
MATERIAL	SAND A	SAND A
AREA OF PLUNGER	3 sq in	19 64 sq in
SURCHARGE	18 6 lb 0 78 psi	121 5 lb 0 78 psi
DIAMETER OF CONTAINER	6 in	15 5 in
STATIC LOAD	95 lb 31 7 psi	610 lb 31 07 psi
CENTRIFUGAL FORCE	10 3 lb	67 lb
FRFOUENCY	1000 r p m	1000 r p m
RATIO PLOTTED	$D_{10}/S \frac{105}{95}$	$D_{10}/S \frac{677}{610}$

Figure 28 Density vs Ratio of Dynamic and Static 2 inch and 5 inch Plunger Penetrations Dry Sand "A"



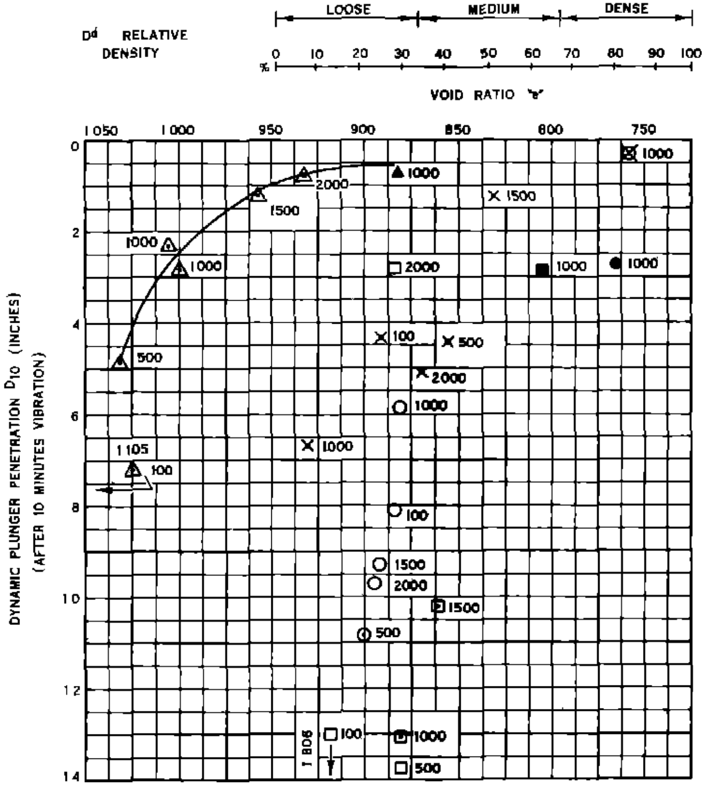
MATERIAL SAND B

CONDITION OF SATURATION

		UNTAPPED	TAPPED WHILE COMPRESSING
	DRY	○	●
	6 % MOISTURE	△	▲
	CAPILLARY SATURATION FROM BOTTOM	×	☒
	SUBMERGED	□	■

COMPACTION METHOD STATIC LOAD NOTED ABOVE
TEST NUMBERS 378 383 86 391 409 410-411
DIAMETER OF CONTAINER 6 in
AREA OF PLUNGER 3 sq in
SURCHARGE 12 3 lb 0 49 psi

Figure 29 Density vs Static 2 inch Plunger Penetration Sand "B"



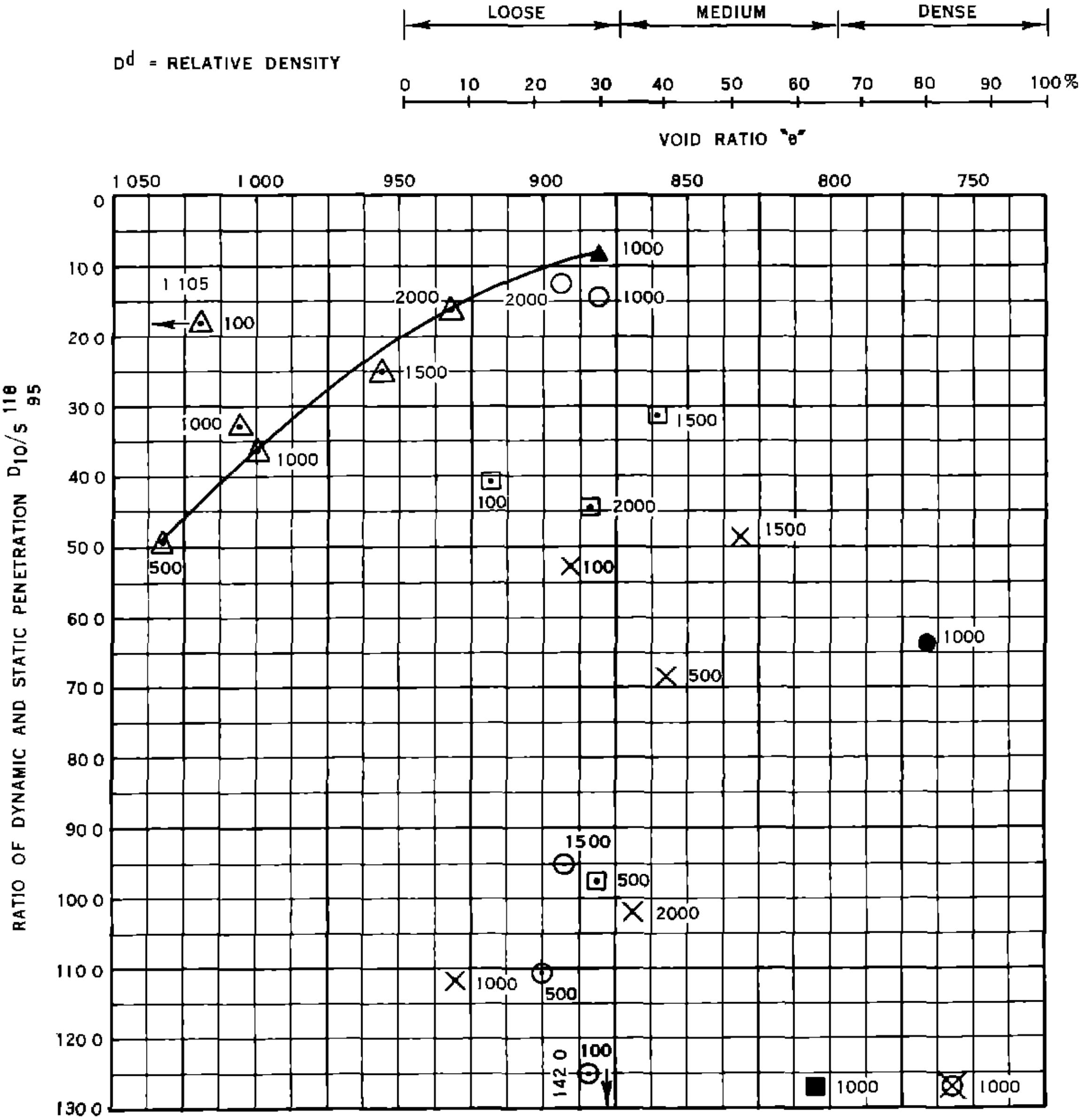
MATERIAL SAND B

CONDITION OF SATURATION

		UNTAPPED	TAPPED WHILE COMPRESSING
	DRY	○	●
	6 % MOISTURE	△	▲
	CAPILLARY SATURATION FROM BOTTOM	×	☒
	SUBMERGED	□	■

COMPACTION METHOD STATIC LOAD NOTED ABOVE
TEST NUMBERS 378 383 386 391 409 410 411
DIAMETER OF CONTAINER 6 in
AREA OF PLUNGER 3 sq in
SURCHARGE 12 3 lb 0 49 psi
CENTRIFUGAL FORCE 23 2 lb
FREQUENCY 1500 r p m
STATIC LOAD 95 lb 31 7 psi

Figure 30 Density vs Dynamic 2 inch Plunger Penetration Sand "B"



MATERIAL	SAND B		
CONDITION OF SATURATION		UNTAPPED	TAPPED WHILE COMPRESSING
DRY		○	●
6% MOISTURE		△	▲
CAPILLARY SATURATION FROM BOTTOM		×	⊠
SUBMERGED		⊡	■

COMPACTION METHOD	STATIC LOAD NOTED ABOVE
TEST NUMBERS	378 383 384 386 391 409 410 411
DIAMETER OF CONTAINER	6 in
AREA OF PLUNGER	3 sq in
SURCHARGE	12 3 lb = 0 49 psi
CENTRIFUGAL FORCE	23 2 lb
FREQUENCY	1500 r p m
STATIC LOAD	95 lb = 31 7 psi

Figure 31 Density vs Ratio of Dynamic and Static 2 inch Plunger Penetrations Sand "B"

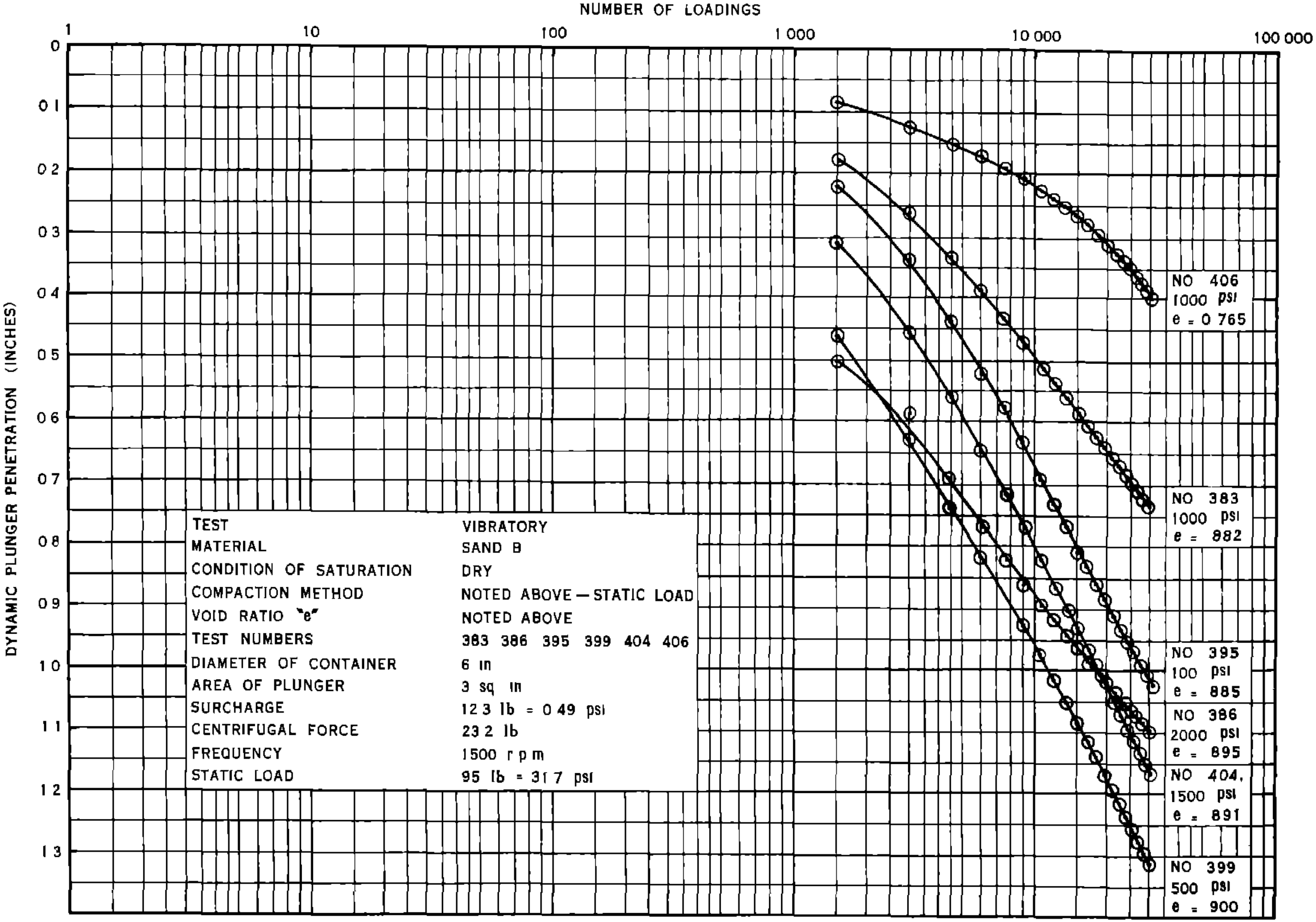


Figure 32 Number of Load Repetitions vs Dynamic 2 inch Plunger Penetrations Semi Log Scale Sand "B"

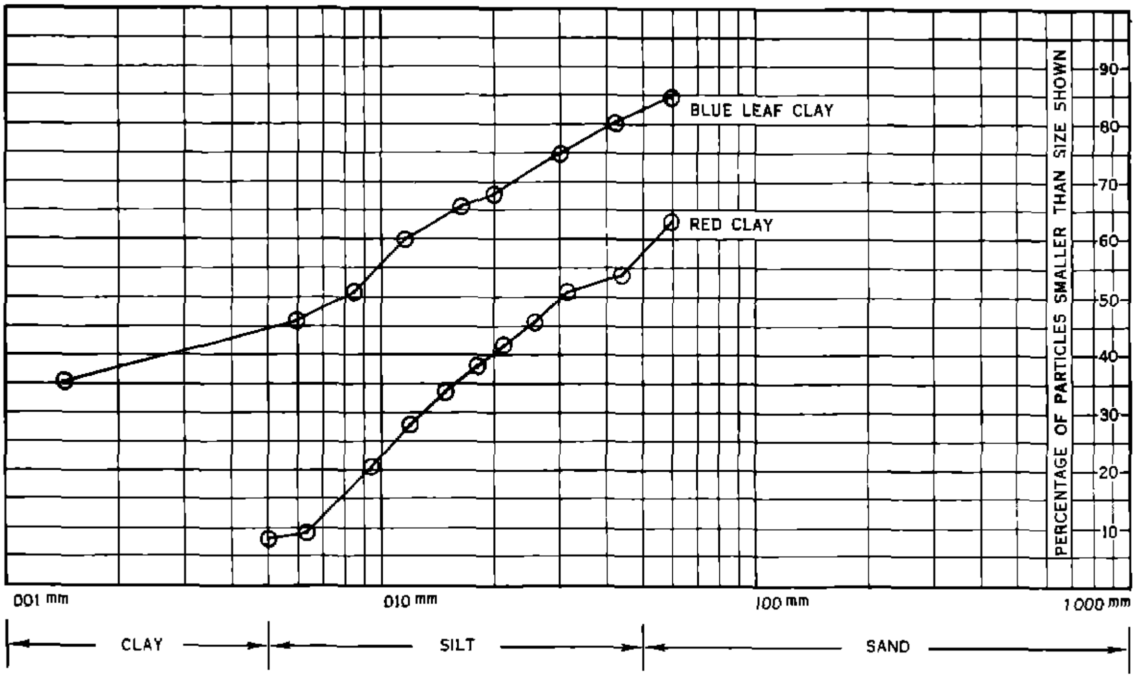
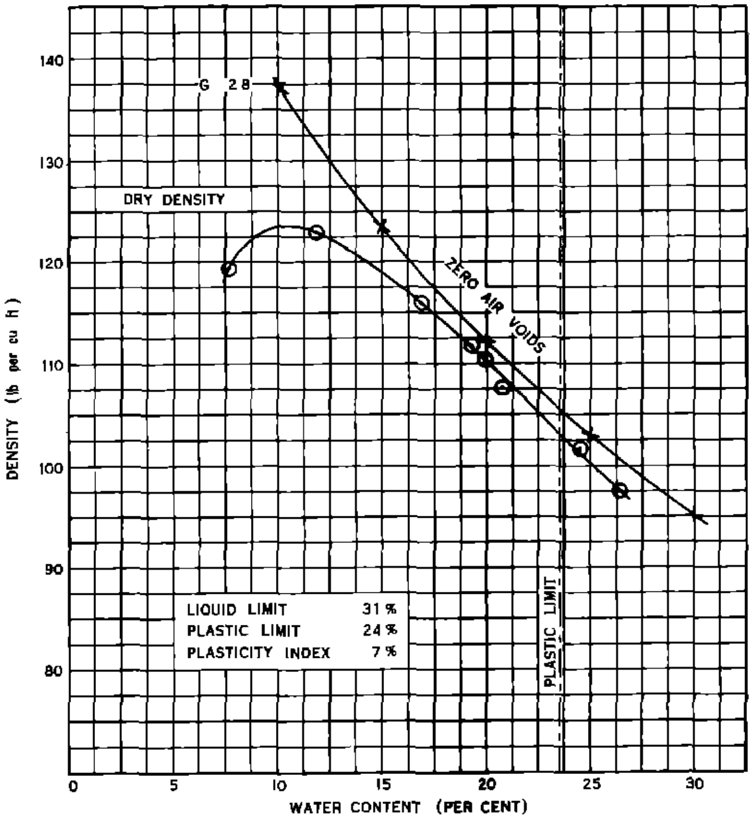
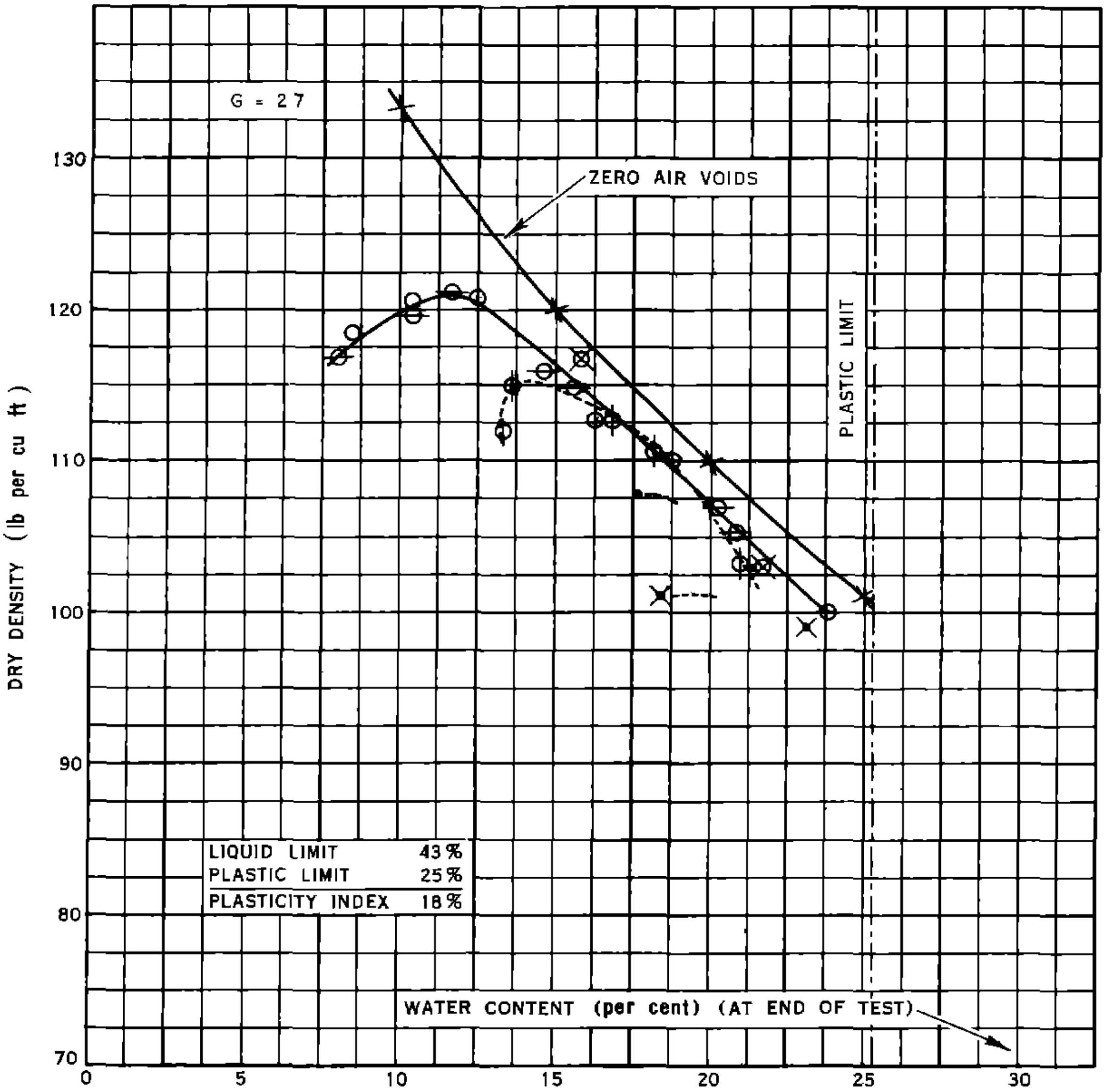


Figure 33 Grain Size Accumulation Curves Red Clay and Blue Leaf Clay



TEST NUMBERS	330 336
MATERIAL	RED CLAY
COMPACTION METHOD	2000 psi STATIC LOAD
WATER CONTENT DETERMINED AFTER TEST	

Figure 34 Density vs Moisture Red Clay



MATERIAL	BLUE LEAF CLAY
COMPACTION METHOD	2000 psi, STATIC LOAD
TEST NUMBERS	338 341 343 344 346, 347 349 357 361 367
DIAMETER OF CONTAINER	6 in
AREA OF PLUNGER	3 sq in
SURCHARGE	18 6 lb = 0 78 psi

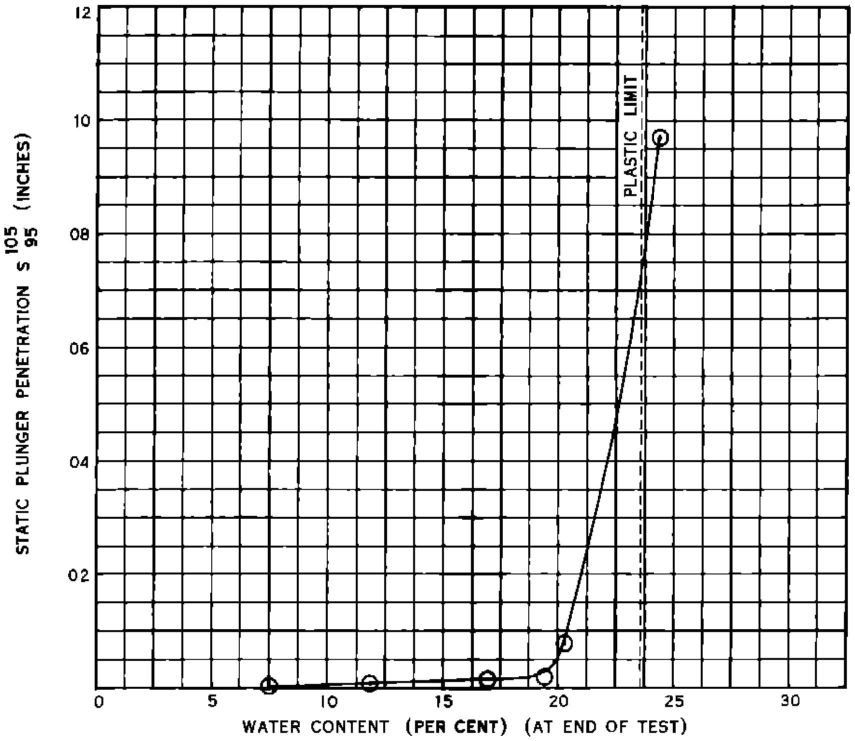
○ LOAD APPLIED BEFORE SWELLING WAS COMPLETED (WITHOUT WATER ABSORPTION)

× LOAD APPLIED AFTER SWELLING WAS COMPLETED 1 DAY AFTER COMPACTION (WITHOUT WATER ABSORPTION)

⊖ LOAD APPLIED AFTER SWELLING (WITH WATER ABSORPTION THRU BOTTOM OF SAMPLE)

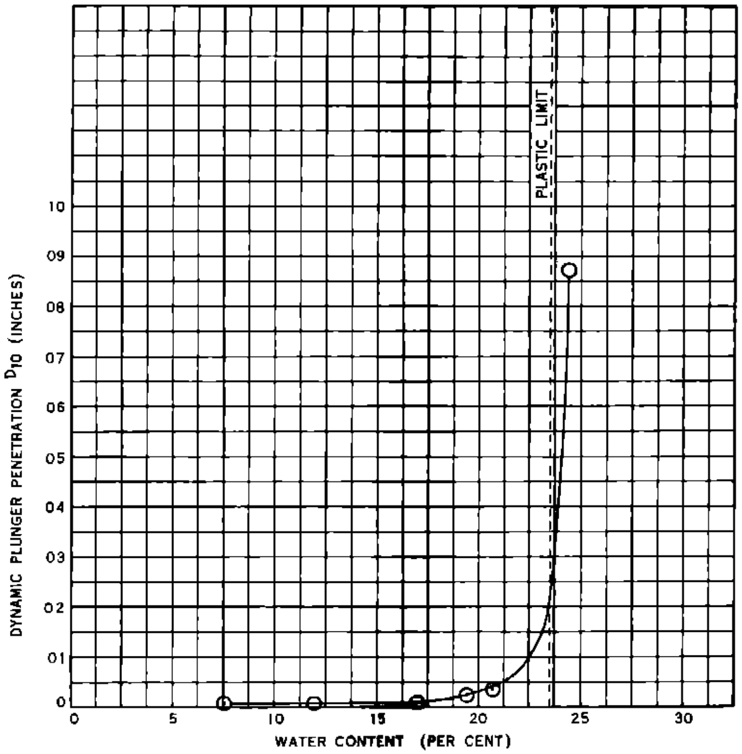
ϕ	FOR 4 DAYS	•	FOR 16 DAYS
⊠	FOR 8 DAYS	✕	FOR 32 DAYS

Figure 35 Density vs Moisture Blue Leaf Clay



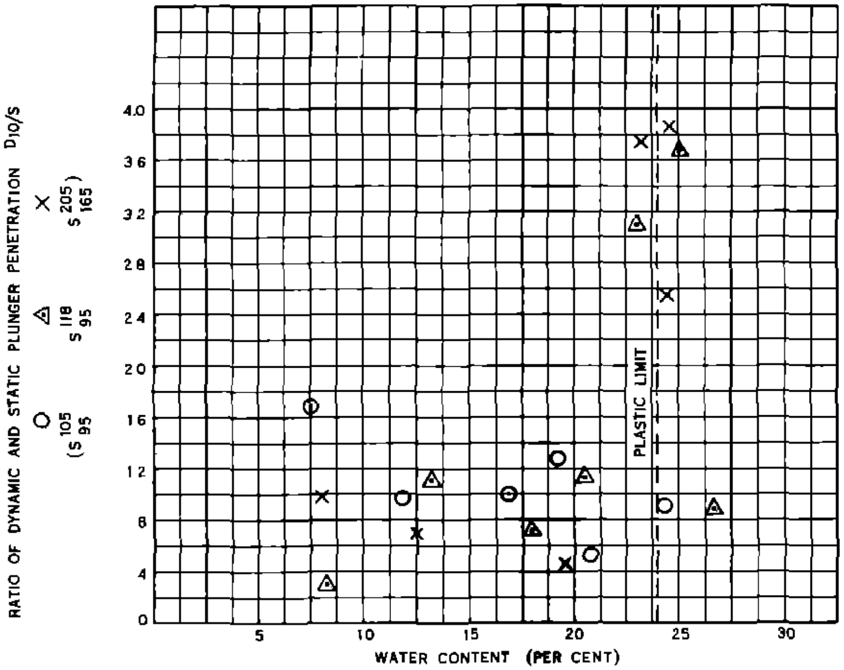
MATERIAL	RED CLAY
COMPACTION METHOD	2000 psi STATIC LOAD
TEST NUMBERS	330 332 334 336
DIAMETER OF CONTAINER	6 in
AREA OF PLUNGER	3 sq in
SURCHARGE	18 6 lb - 0 78 psi

Figure 36 Static 2 inch Plunger Penetration vs Moisture Red Clay



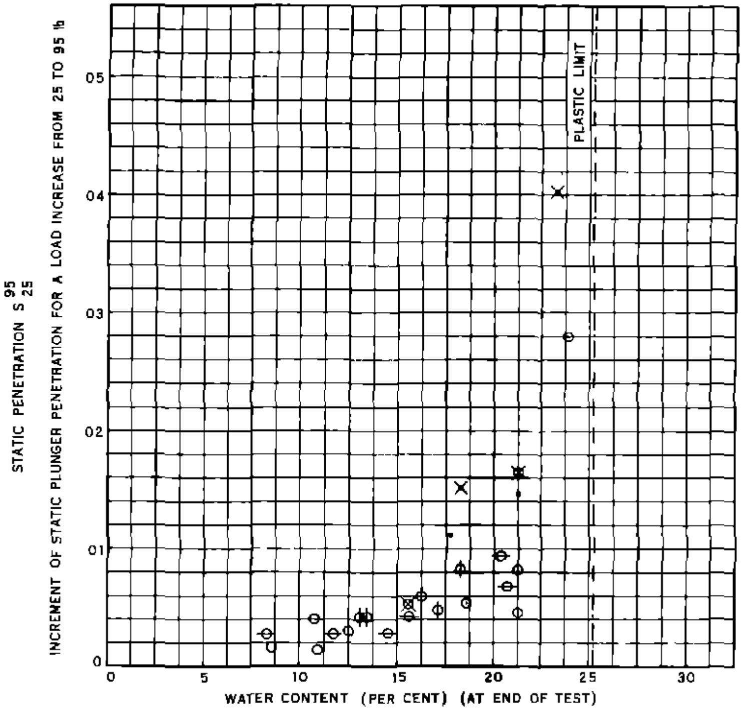
TEST NUMBERS	330 332 334 336
MATERIAL	RED CLAY
AREA OF PLUNGER	3 sq in
SURCHARGE	18 6 lb 0 78 psi
DIAMETER OF CONTAINER	6 in
STATIC LOAD	95 lb 31 7 psi
CENTRIFUGAL FORCE	10 3 lb
FREQUENCY	1000 r p m
COMPACTION METHOD	2000 psi STATIC LOAD

Figure 37 Dynamic 2 inch Plunger Penetration vs Moisture Red Clay



SYMBOL	○	△	×
MATERIAL	RED CLAY		
COMPACTION METHOD	2000 psl STATIC LOAD		
WATER CONTENT	DETERMINED AT END OF TEST		
TEST NUMBERS	330 332 334 336	441 447	448 453
DIAMETER OF CONTAINER	6 in		
AREA OF PLUNGER	3 sq in		
SURCHARGE	18 6 lb 0 78 psi	12 3 lb 0 49 psi	12 3 lb 0 49 psi
CENTRIFUGAL FORCE	10 3 lb	23 2 lb	40 4 lb
FREQUENCY	1000 r p m	1500 r p m	1500 r p m
STATIC LOAD	95 lb 31 7 psl	95 lb 31 7 psi	165 lb 55 psi

Figure 38 Ratio of Dynamic and Static 2 inch Plunger Penetrations vs Moisture Red Clay



MATERIAL	BLUE LEAF CLAY
COMPACTION METHOD	2000 psi STATIC LOAD
TEST NUMBERS	338 341 343 344 346 347 349 357 360 367
DIAMETER OF CONTAINER	6 in
AREA OF PLUNGER	3 sq in
SURCHARGE	18 6 lb 0 78 psi

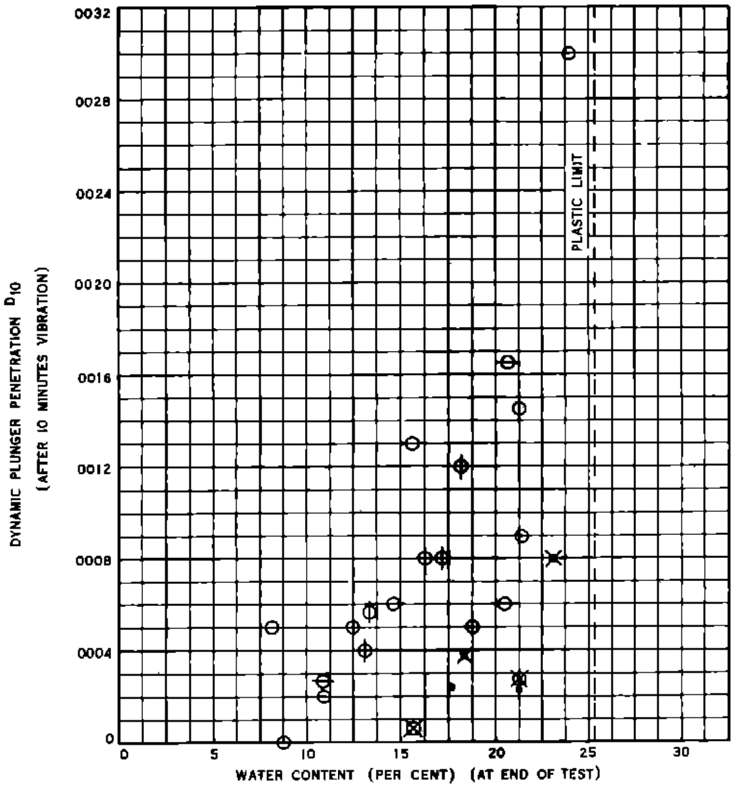
○ LOAD APPLIED BEFORE SWELLING WAS COMPLETED (WITHOUT WATER ABSORPTION)

⊖ LOAD APPLIED AFTER SWELLING WAS COMPLETED 1 DAY AFTER COMPACTION (WITHOUT WATER ABSORPTION)

LOAD APPLIED AFTER SWELLING (WITH WATER ABSORPTION THRU BOTTOM OF SAMPLE)

ϕ FOR 4 DAYS ☒ FOR 8 DAYS FOR 16 DAYS × FOR 32 DAYS

Figure 39 Static 2 inch Plunger Penetration vs Moisture Blue Leaf Clay



MATERIAL	BLUE LEAF CLAY	SURCHARGE	18 6 lb 0 78 psi
COMPACTION METHOD	2000 psi STATIC LOAD	CENTRIFUGAL FORCE	10 3 lb
TEST NUMBERS	338 341 343 344 346 347 349 357 360	FREQUENCY	1000 r p m
		STATIC LOAD	95 lb 31 7 psi
DIAMETER OF CONTAINER	6 in	VIBRATOR NUMBER	B 1
AREA OF PLUNGER	3 sq in		

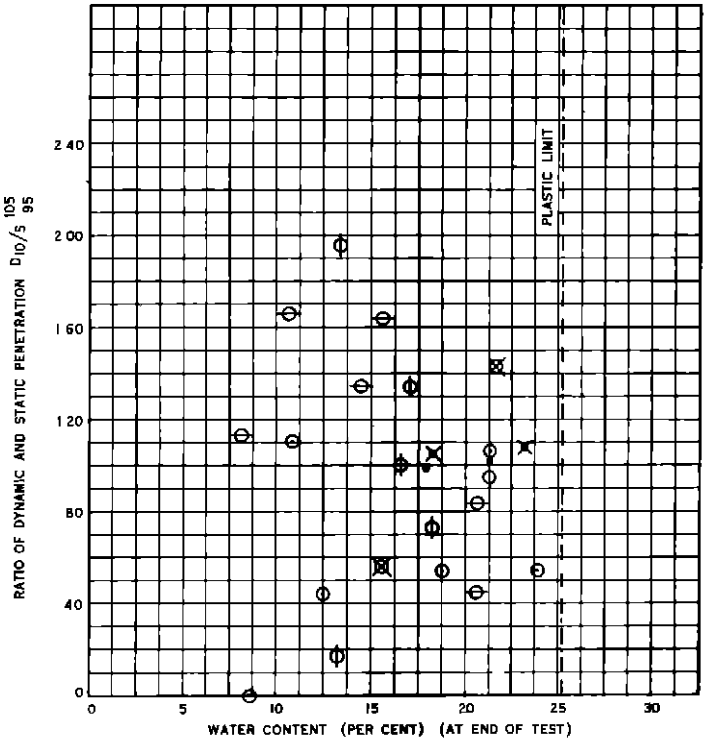
○ LOAD APPLIED BEFORE SWELLING WAS COMPLETED (WITHOUT WATER ABSORPTION)

⊖ LOAD APPLIED AFTER SWELLING WAS COMPLETED 1 DAY AFTER COMPACTION (WITHOUT WATER ABSORPTION)

LOAD APPLIED AFTER SWELLING (WITH WATER ABSORPTION THRU BOTTOM OF SAMPLE)

ɸ FOR 4 DAYS ⊠ FOR 8 DAYS • FOR 16 DAYS ✕ FOR 32 DAYS

Figure 40 Dynamic 2 inch Plunger Penetration vs Moisture Blue Leaf Clay



MATERIAL	BLUE LEAF CLAY	SURCHARGE	18 6 lb 0 78 psi
COMPACTION METHOD	2000 psi STATIC LOAD	CENTRIFUGAL FORCE	10 3 lb
TEST NUMBERS	338 341 343-344 346 347 349 357 360 367	FREQUENCY	1000 r p m
		STATIC LOAD	95 lb 31 7 psi
DIAMETER OF CONTAINER	6 in	VIBRATOR NUMBER	B 1
AREA OF PLUNGER	3 sq in		

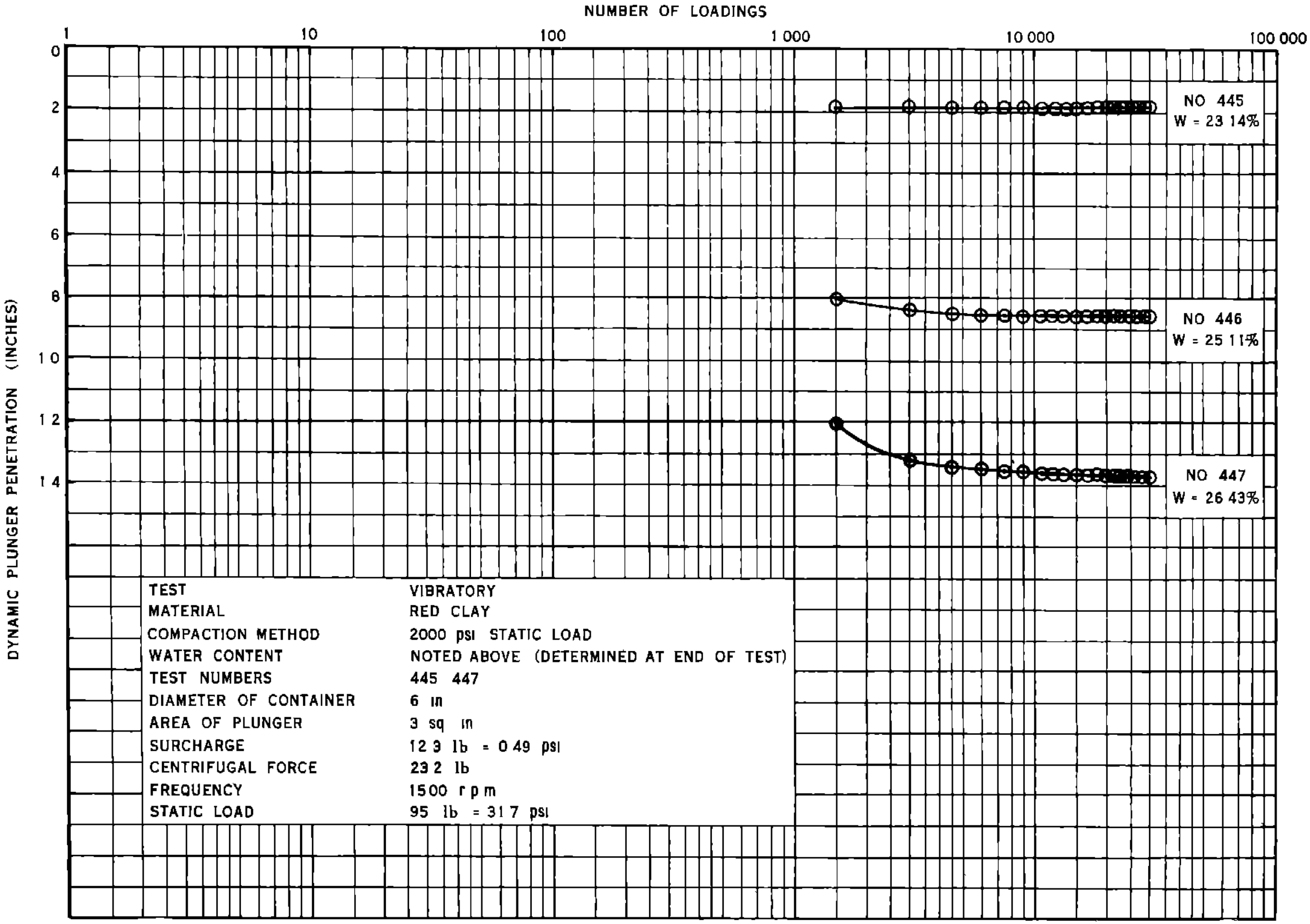
○ LOAD APPLIED BEFORE SWELLING WAS COMPLETED (WITHOUT WATER ABSORPTION)

⊖ LOAD APPLIED AFTER SWELLING WAS COMPLETED 1 DAY AFTER COMPACTION (WITHOUT WATER ABSORPTION)

LOAD APPLIED AFTER SWELLING (WITH WATER ABSORPTION THRU BOTTOM OF SAMPLE)

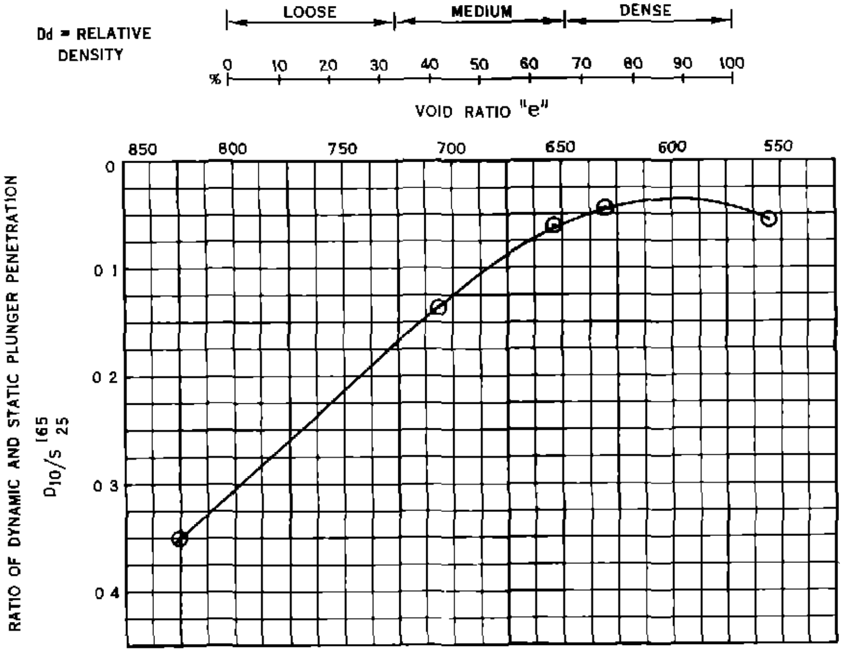
ɸ FOR 4 DAYS ⊠ FOR 8 DAYS • FOR 16 DAYS ✕ FOR 32 DAYS

Figure 41 Ratio of Dynamic and Static 2 inch Plunger Penetrations vs Moisture Blue Leaf Clay



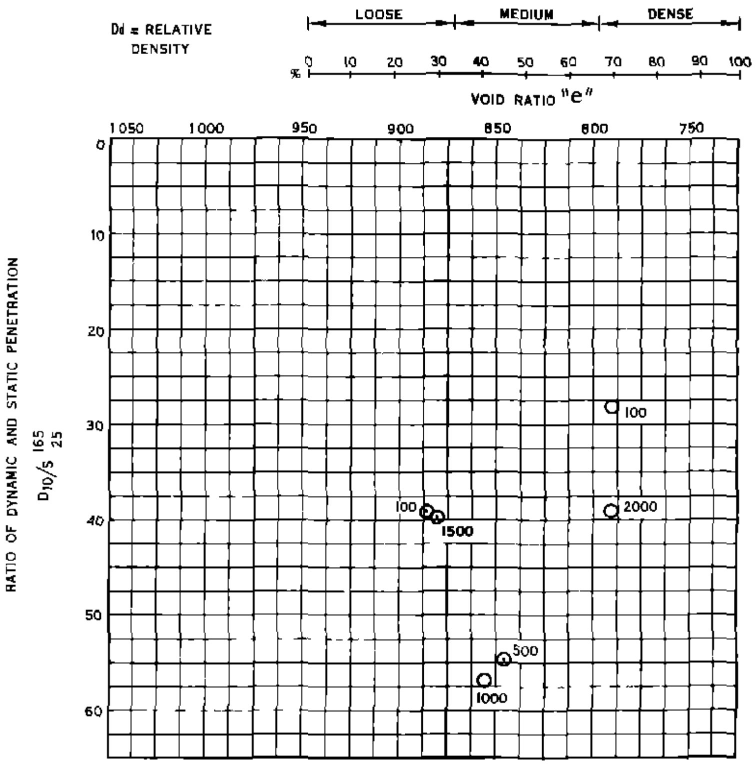
TEST	VIBRATORY
MATERIAL	RED CLAY
COMPACTION METHOD	2000 psi STATIC LOAD
WATER CONTENT	NOTED ABOVE (DETERMINED AT END OF TEST)
TEST NUMBERS	445 447
DIAMETER OF CONTAINER	6 in
AREA OF PLUNGER	3 sq in
SURCHARGE	12 3 lb = 0 49 psi
CENTRIFUGAL FORCE	23 2 lb
FREQUENCY	1500 r p m
STATIC LOAD	95 lb = 31 7 psi

Figure 42 Number of Load Repetitions vs Dynamic 2 inch Plunger Penetrations Semi-Log Scale Red Clay



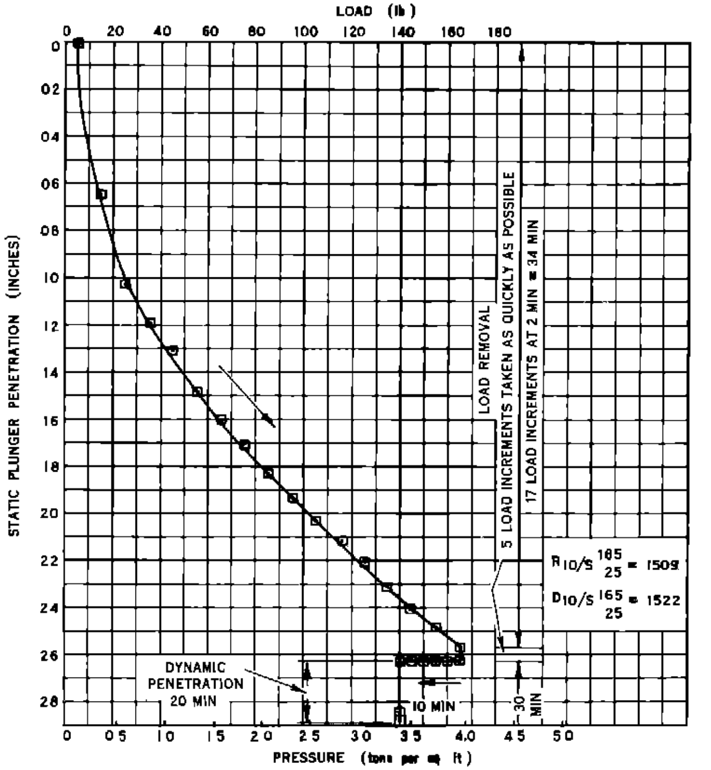
MATERIAL	SAND A
CONDITION OF SATURATION	DRY
COMPACTION METHOD	STATIC LOAD
TEST NUMBERS	487 491
DIAMETER OF CONTAINER	6 in
AREA OF PLUNGER	3 sq in
SURCHARGE	12 3 lb = 0 49 psi
CENTRIFUGAL FORCE	1 47 lb
FREQUENCY	1000 r p m
STATIC LOAD	165 lb = 55 psi

Figure 43 Density vs Ratio of Dynamic and Static 2 inch Plunger Penetration Low Value of Centrifugal Force Sand "A"



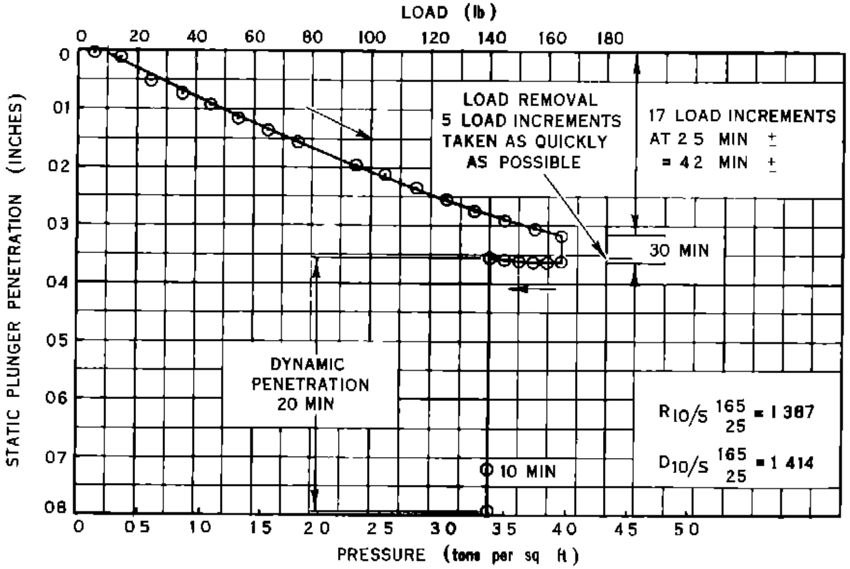
MATERIAL	SAND B
CONDITION OF SATURATION	DRY
COMPACTION METHOD	NOTED ABOVE STATIC LOAD
TEST NUMBERS	509 510 511 513 520 521
DIAMETER OF CONTAINER	6 in
AREA OF PLUNGER	3 sq in
SURCHARGE	12 3 lb = 0 49 psi
CENTRIFUGAL FORCE	1 47 lb
FREQUENCY	1000 r p m
STATIC LOAD	165 lb = 55 psi

Figure 44 Density vs Ratio of Dynamic and Static 2 inch Plunger Penetration Low Value of Centrifugal Force Sand "B"



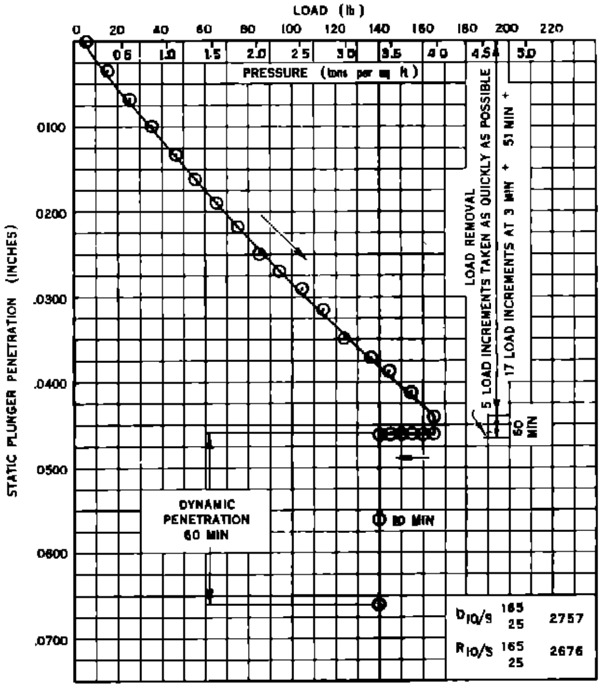
MATERIAL	SAND A
CONDITION OF SATURATION	SUBMERGED
COMPACTION METHOD	1500 psi STATIC LOAD
VOID RATIO "e"	590
TEST NUMBER	574
DIAMETER OF CONTAINER	6 in
AREA OF PLUNGER	3 sq in
SURCHARGE	12 3 lb = 0 49 psi
CENTRIFUGAL FORCE	3 31 lb
FREQUENCY	1500 r p m

Figure 45 Load vs Static and Dynamic 2 inch Plunger Penetration Low Value of Centrifugal Force and Partial Removal of Static Load Sand "A"



MATERIAL	SAND B
CONDITION OF SATURATION	DRY
COMPACTION METHOD	1000 psi STATIC LOAD
VOID RATIO "e"	847
TEST NUMBER	543
DIAMETER OF CONTAINER	6 in
AREA OF PLUNGER	3 sq in
SURCHARGE	12 3 lb = 0 49 psi
CENTRIFUGAL FORCE	3 31 lb
FREQUENCY	1500 r p m

Figure 46 Load vs Static and Dynamic 2 inch Plunger Penetration Low Value of Centrifugal Force and Partial Removal of Static Load Sand "B"



MATERIAL	SAND B
CONDITION OF SATURATION	DRY
COMPACTION METHOD	500 psi STATIC LOAD
VOID RATIO 'e'	831
TEST NUMBER	518
DIAMETER OF CONTAINER	6 in
AREA OF PLUNGER	3 sq in
SURCHARGE	12 3 lb 0 49 psi
CENTRIFUGAL FORCE	1 47 lb
FREQUENCY	1000 r p m

Figure 47 Load vs Static and Dynamic 2 inch Plunger Penetration Low Value of Centrifugal Force and Partial Removal of Static Load Sand "B"

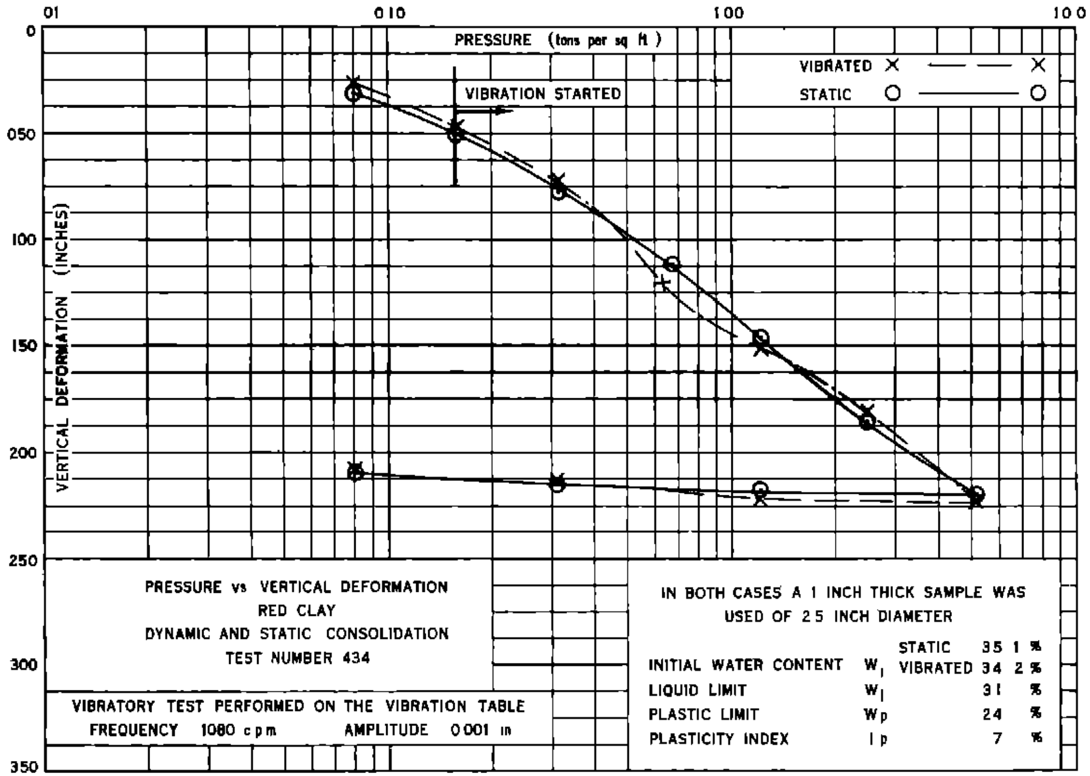


Figure 48 Pressure vs Vertical Deformation Dynamic and Static Consolidation Test Red Clay

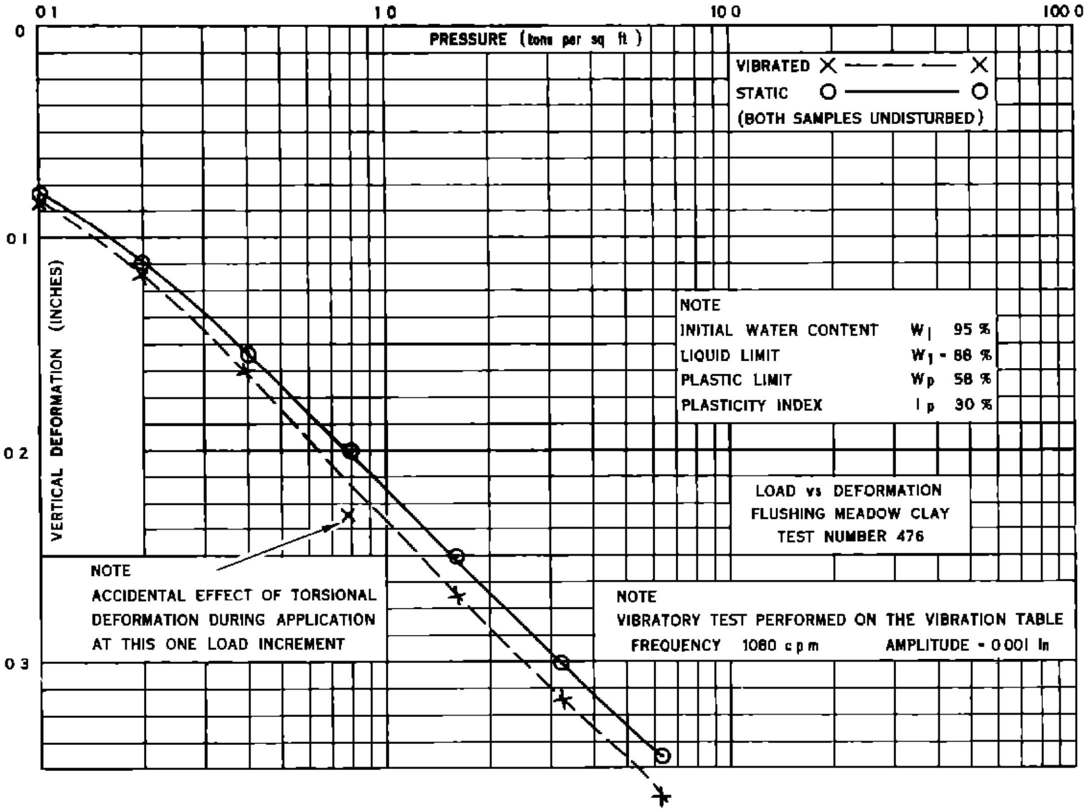


Figure 49 Pressure vs Vertical Deformation Dynamic and Static Consolidation Test Flushing Meadow Clay

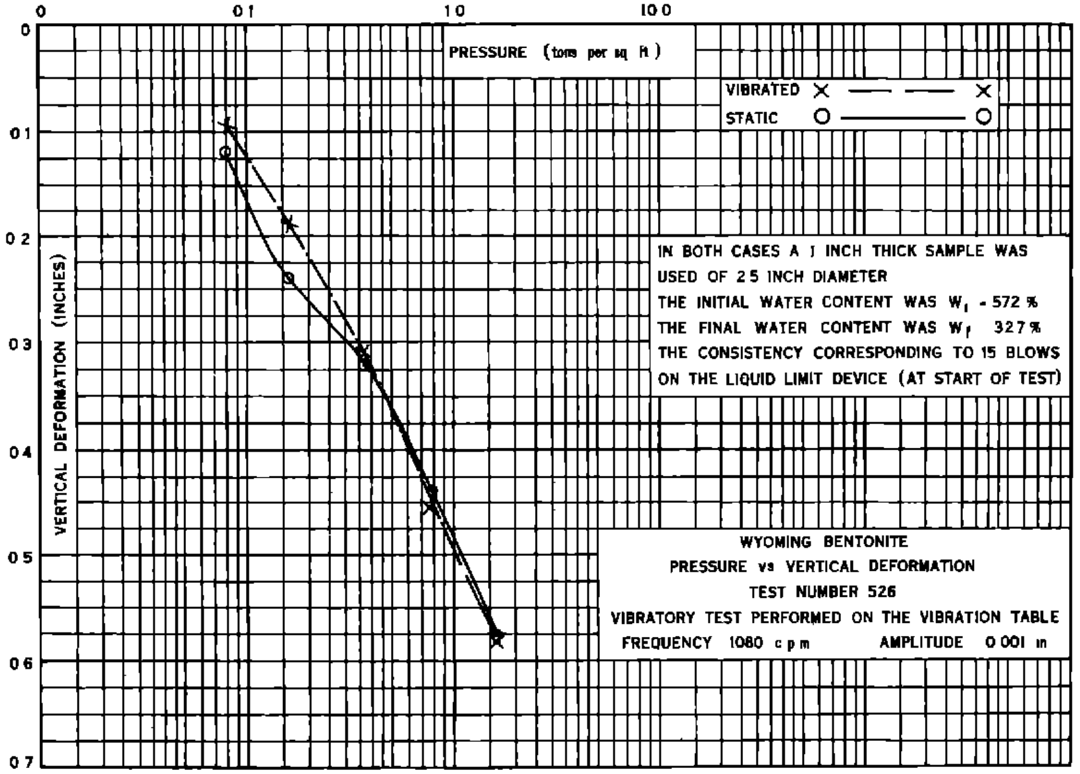
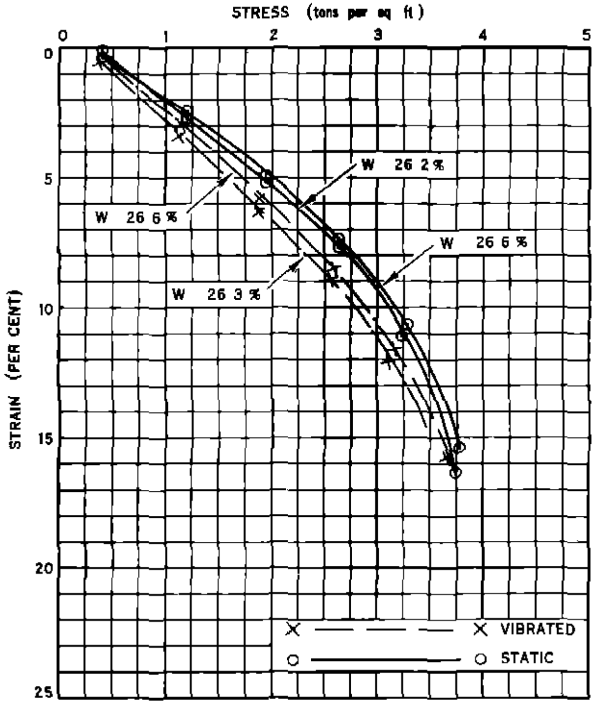


Figure 50 Pressure vs Vertical Deformation Dynamic and Static Consolidation Test, Wyoming Bentonite



TEST	UNCONFINED COMPRESSION TEST
MATERIAL	RED CLAY
COMPACTION METHOD	500 psi STATIC LOAD
WATER CONTENT	NOTED ABOVE (DETERMINED AT END OF TEST)
TEST NUMBERS	586 589
DIAMETER OF SAMPLE	3 36 in (STATIC) 3 39 in (VIBRATED)
HEIGHT OF SAMPLE	7 0 in
CENTRIFUGAL FORCE	3 31 lb
FREQUENCY	1500 r p m

Figure 51 Stress vs Strain Vibrated and Static Unconfined Compressive Strength Tests Red Clay

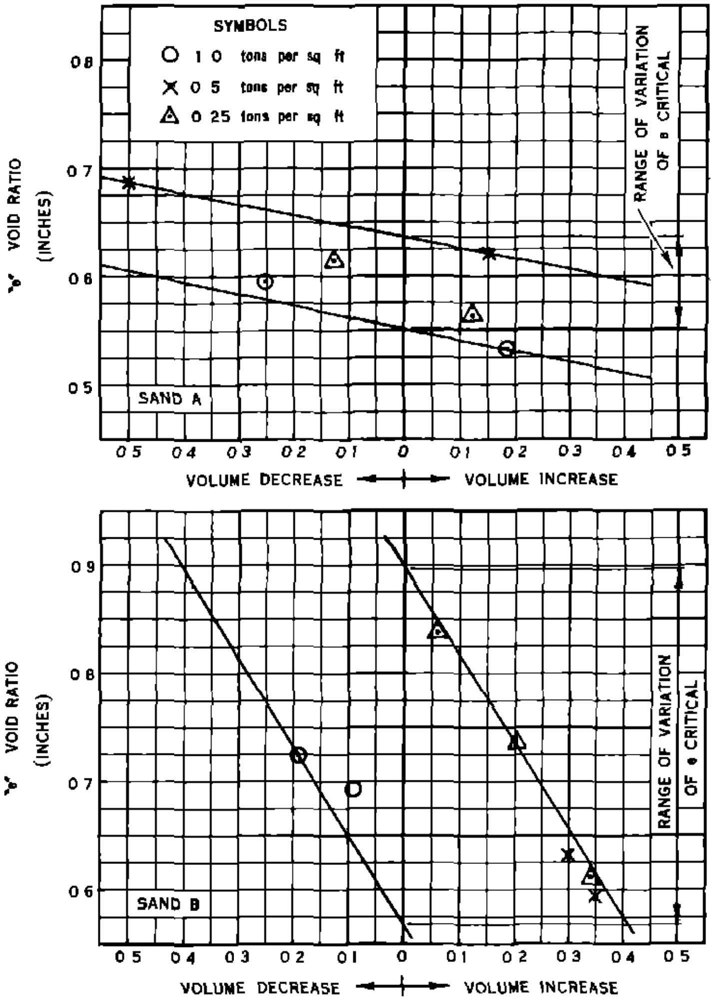


Figure 52 Void Ratio vs Volume Change During Triaxial Tests on Sand "A" and Sand "B"

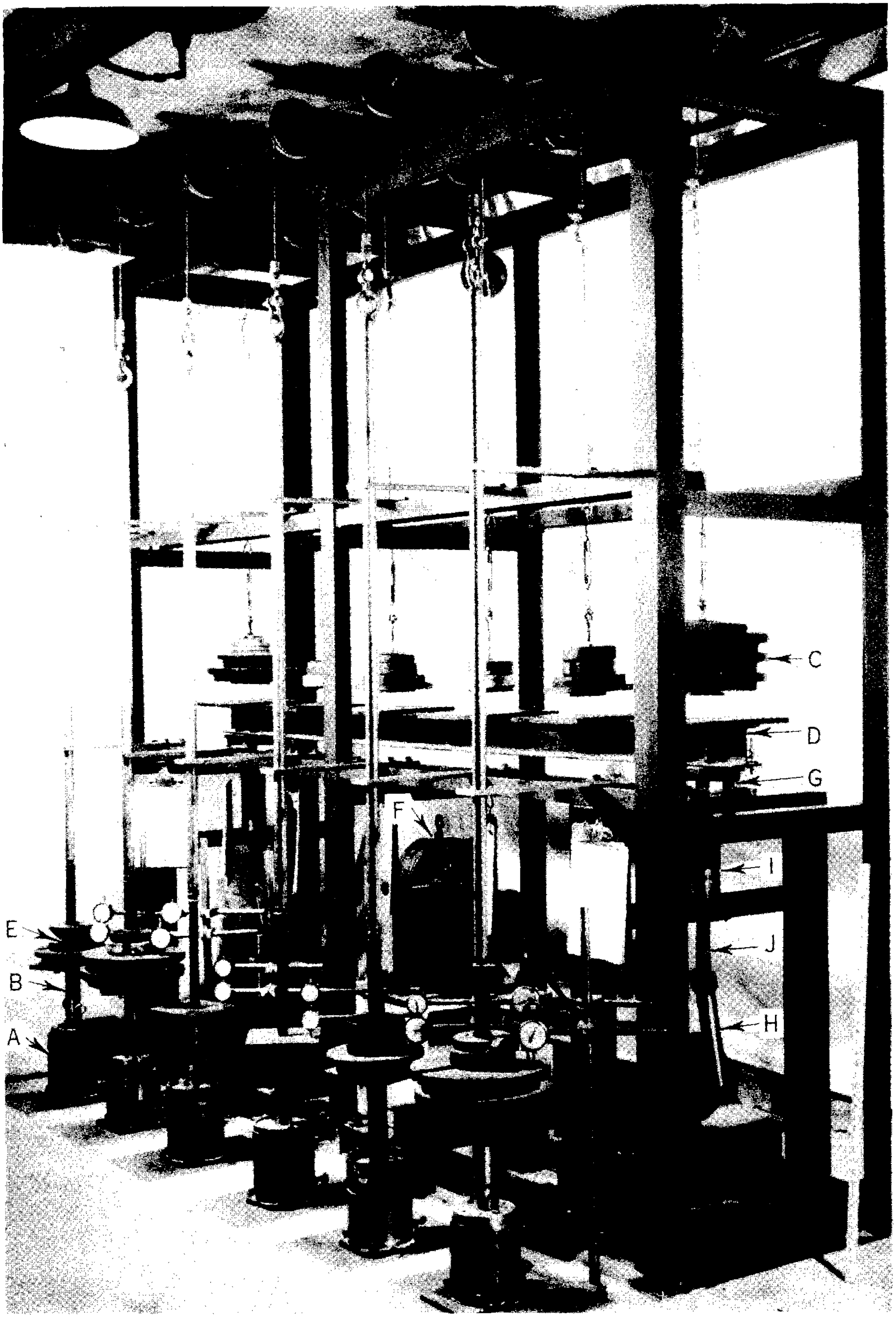
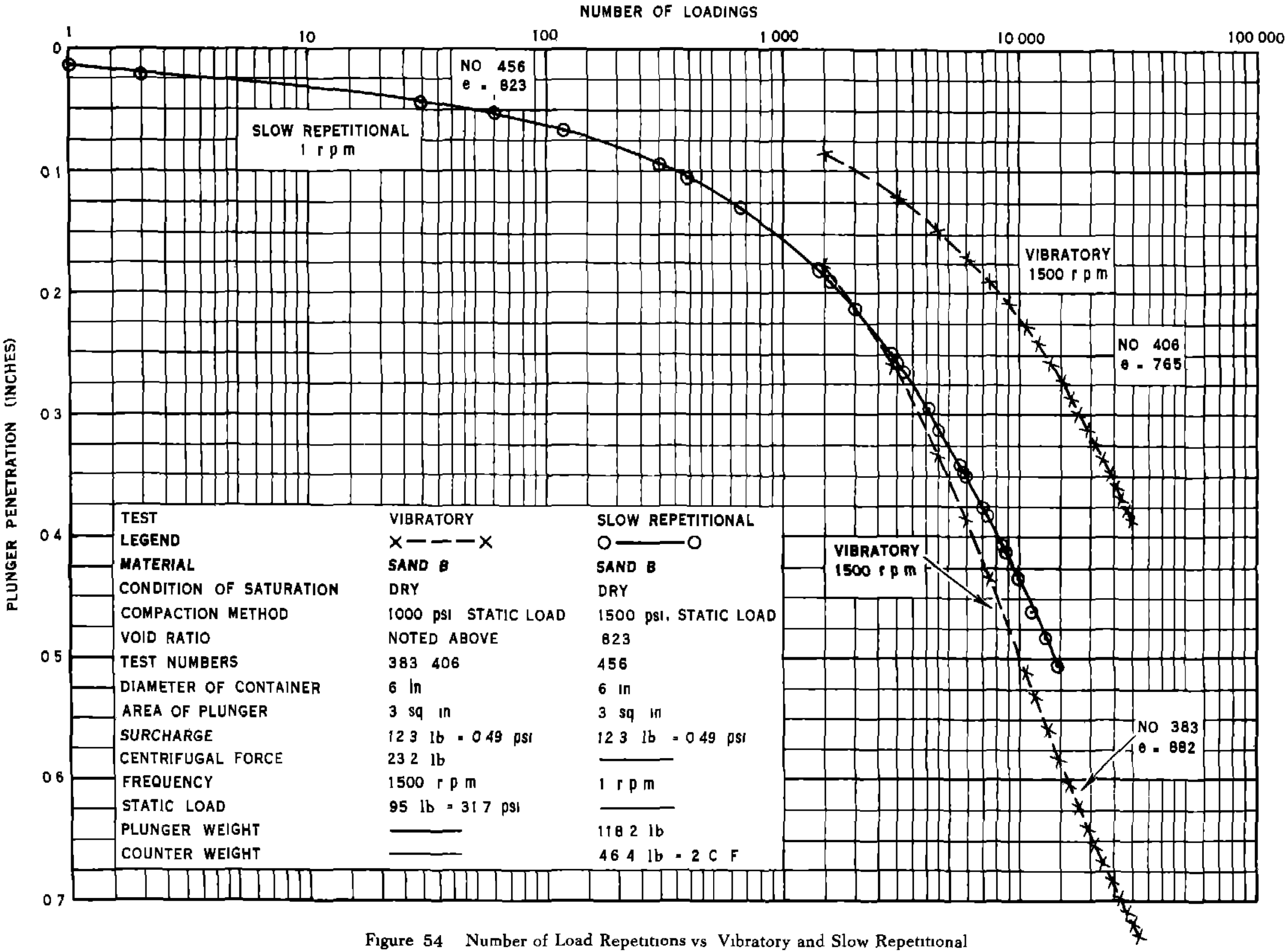
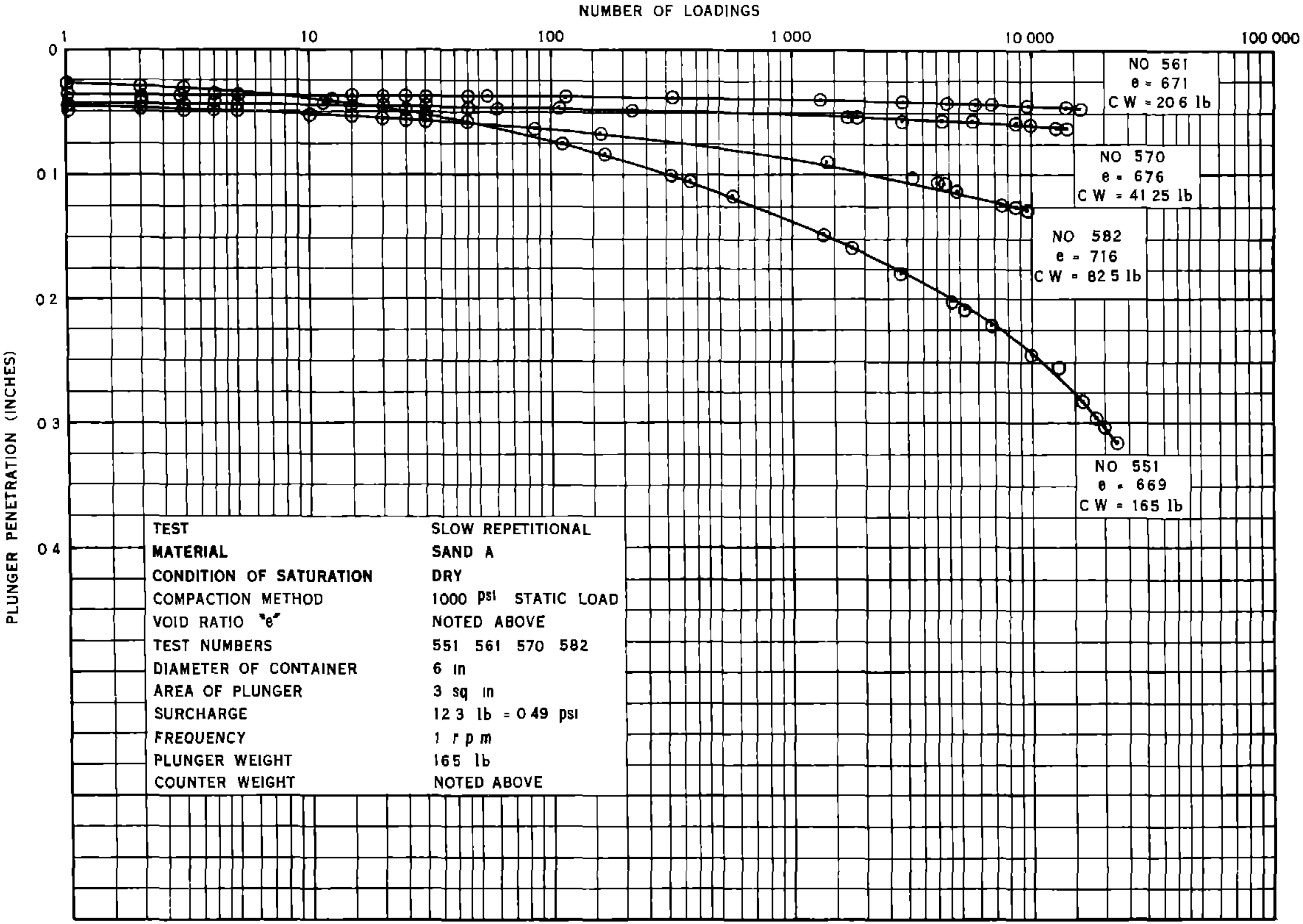


Figure 53. Photograph of Six-Bank Slow Repetitional Loading Plunger Testing Machine.



TEST	VIBRATORY	SLOW REPETITIONAL
LEGEND	X— — — —X	O————O
MATERIAL	*SAND B*	*SAND B*
CONDITION OF SATURATION	DRY	DRY
COMPACTION METHOD	1000 psi STATIC LOAD	1500 psi, STATIC LOAD
VOID RATIO	NOTED ABOVE	823
TEST NUMBERS	383 406	456
DIAMETER OF CONTAINER	6 in	6 in
AREA OF PLUNGER	3 sq in	3 sq in
SURCHARGE	*12 3 lb = 0 49 psi*	*12 3 lb = 0 49 psi*
CENTRIFUGAL FORCE	23 2 lb	———
FREQUENCY	1500 r p m	1 r p m
STATIC LOAD	95 lb = 31 7 psi	———
PLUNGER WEIGHT	———	118 2 lb
COUNTER WEIGHT	———	46 4 lb = 2 C F

Figure 54 Number of Load Repetitions vs Vibratory and Slow Repetitional Plunger Penetration Sand "B"



TEST	SLOW REPETITIONAL
MATERIAL	**SAND A**
CONDITION OF SATURATION	**DRY**
COMPACTION METHOD	1000 psi STATIC LOAD
VOID RATIO "e"	NOTED ABOVE
TEST NUMBERS	551 561 570 582
DIAMETER OF CONTAINER	6 in
AREA OF PLUNGER	3 sq in
SURCHARGE	12 3 lb = 0 49 psi
FREQUENCY	1 r p m
PLUNGER WEIGHT	165 lb
COUNTER WEIGHT	NOTED ABOVE

Figure 55 Number of Load Repetitions vs Slow Repetitional Plunger Penetration Sand "A"

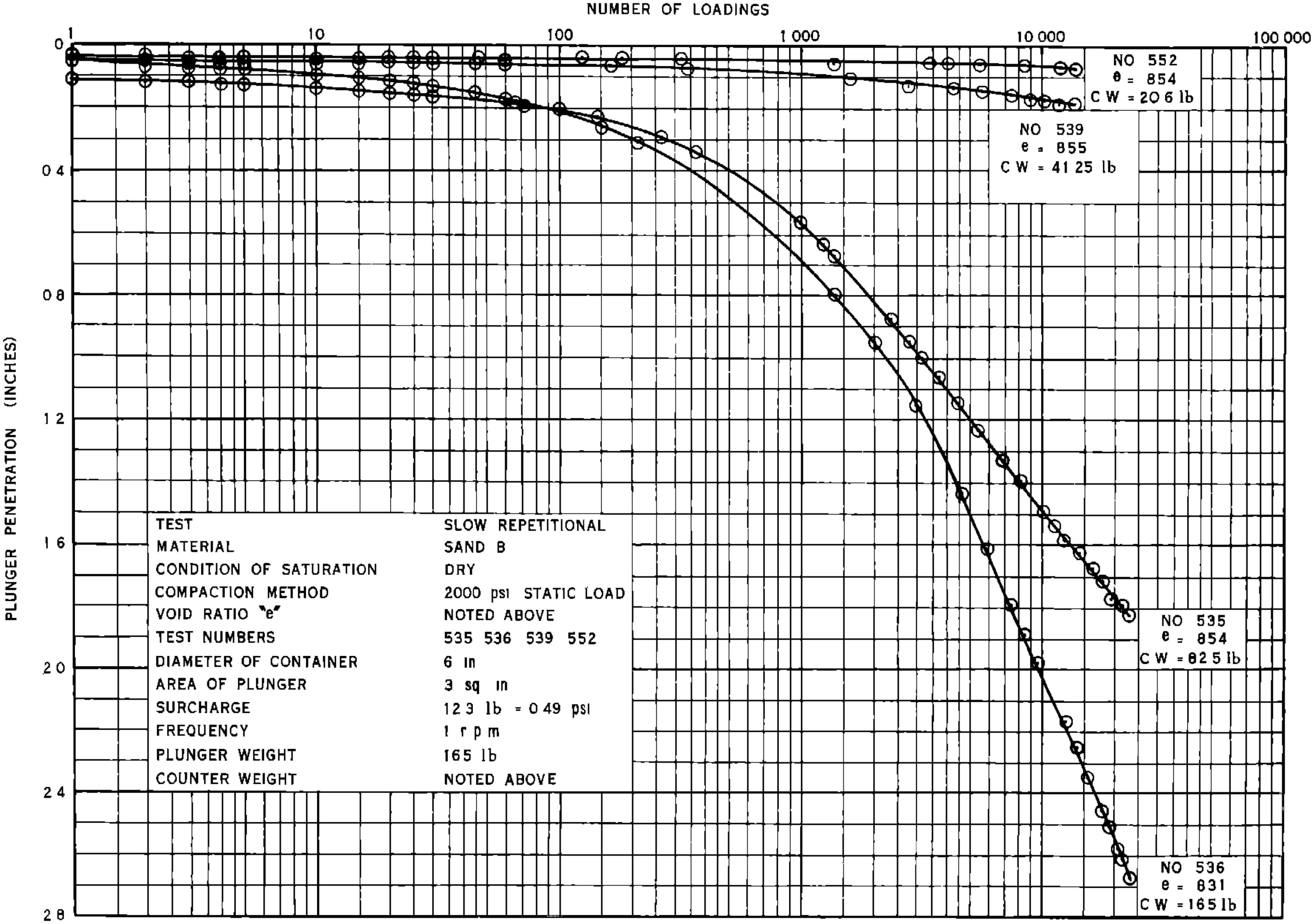


Figure 56 Number of Load Repetitions vs Slow Repetitional Plunger Penetration Sand "B"

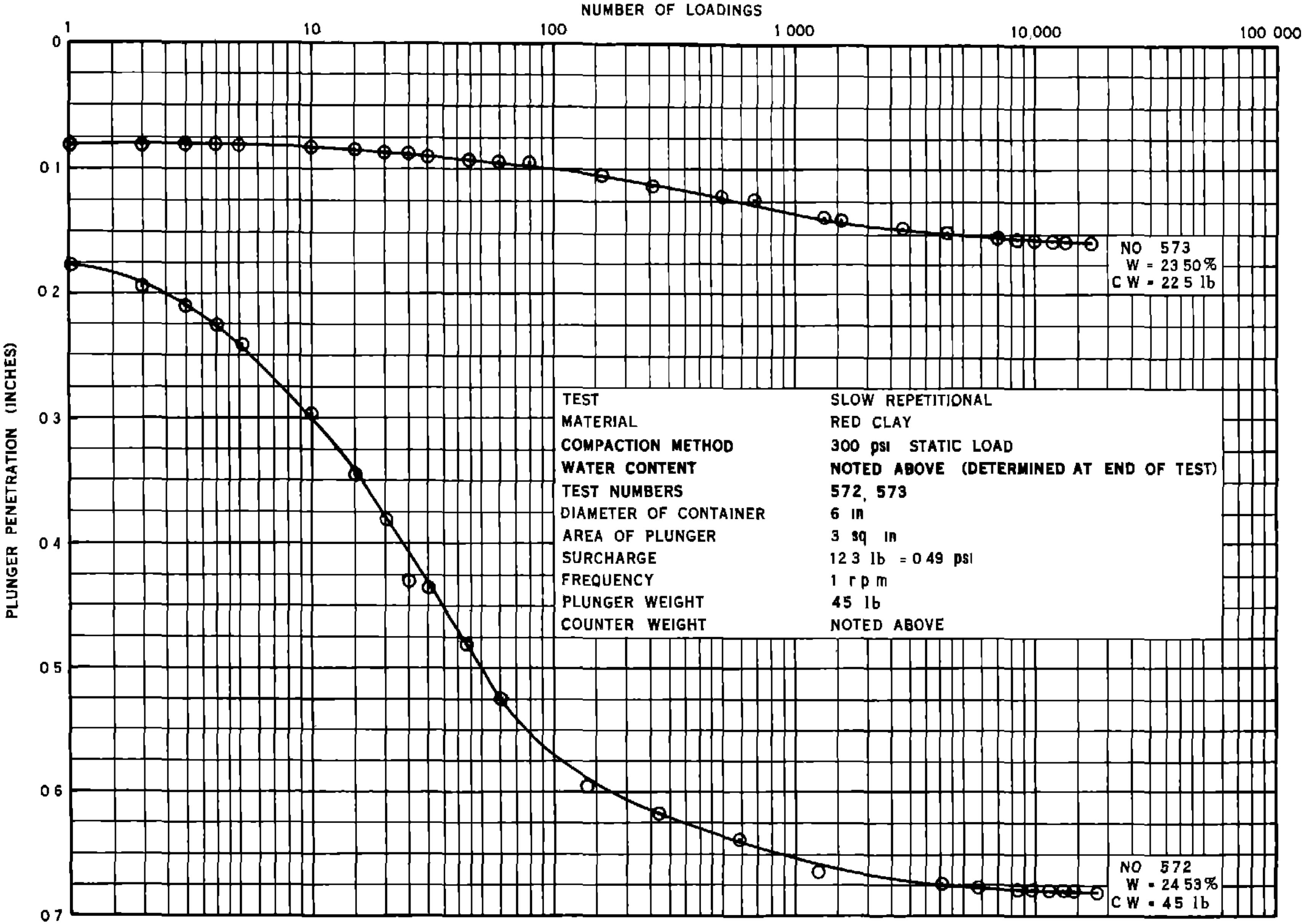


Figure 57 Number of Load Repetitions vs Slow Repetitional Plunger Penetration Red Clay

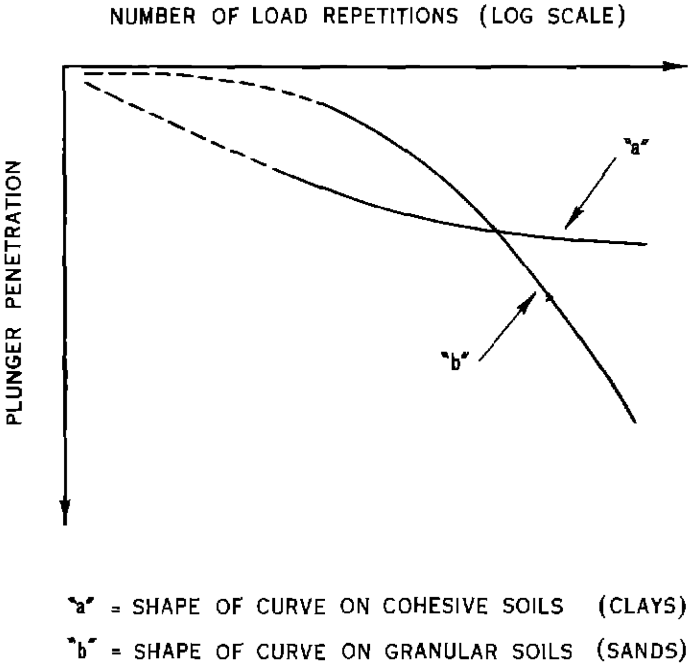
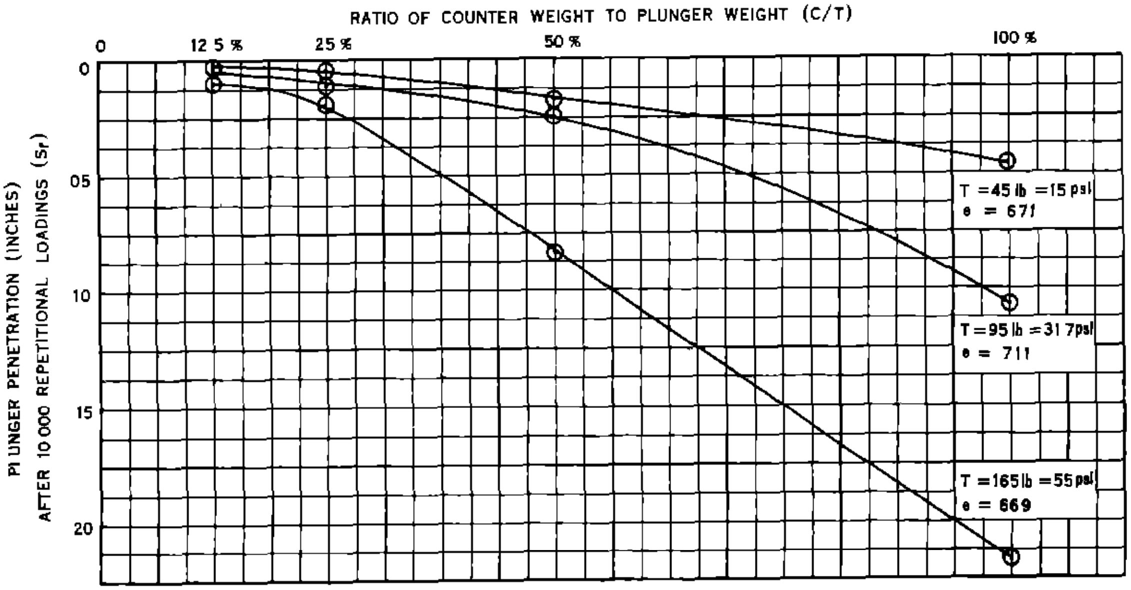
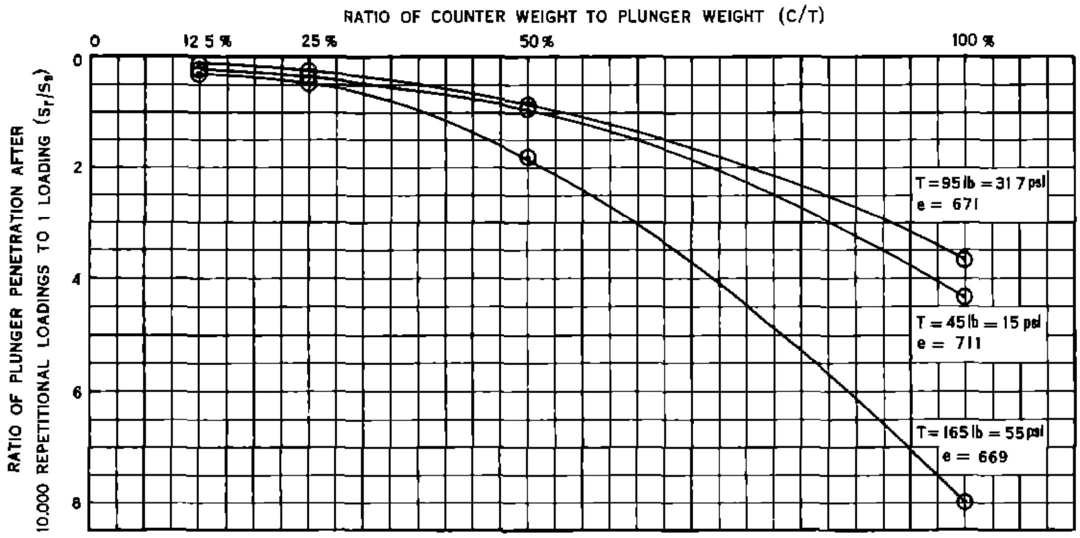


Figure 58 Different Shape of the Number of Load Repetitions vs Plunger Penetration Curves on Clays and on Sands



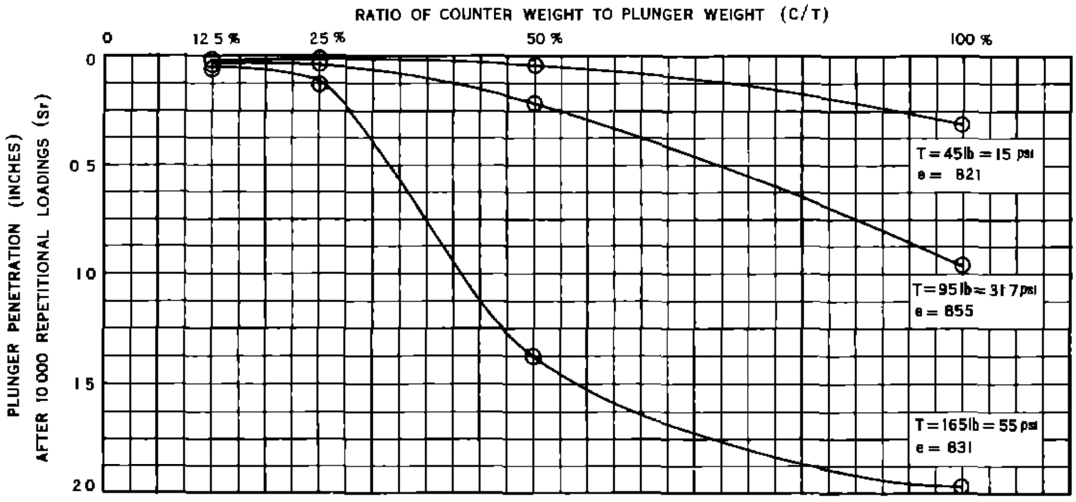
TEST	SLOW REPETITIONAL	DIAMETER OF CONTAINER	6 in
MATERIAL	SAND A	AREA OF PLUNGER	3 sq in
CONDITION OF SATURATION	DRY	SURCHARGE	12 3 lb = 0 49 psi
COMPACTION METHOD	1000 psi STATIC LOAD	FREQUENCY	1 r p m
VOID RATIO "e"	NOTED ABOVE	PLUNGER WEIGHT "T"	NOTED ABOVE
TEST NUMBERS	550 551 553 555 560 564 570 582		

Figure 59 Ratio of Counter Weight to Plunger Weight vs Plunger Penetration after 10,000 Loadings Sand "A"



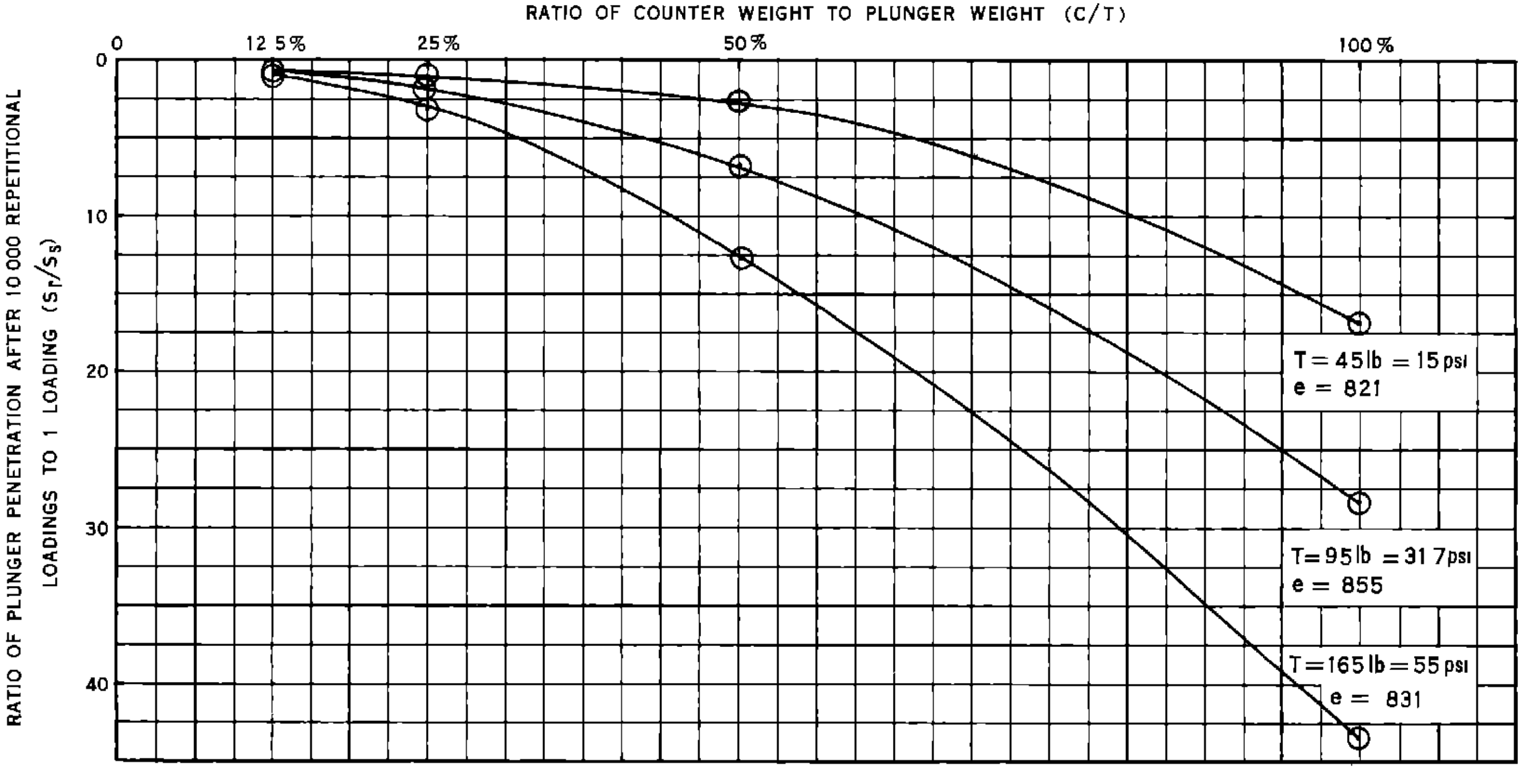
TEST	SLOW REPETITIONAL	DIAMETER OF CONTAINER	6 in
MATERIAL	SAND A	AREA OF PLUNGER	3 sq in
CONDITION OF SATURATION	DRY	SURCHARGE	12 3 lb = 0 49 psi
COMPACTION METHOD	1000 psi STATIC LOAD	FREQUENCY	1 r p m
VOID RATIO "e"	NOTED ABOVE	PLUNGER WEIGHT "T '	NOTED ABOVE
TEST NUMBERS	550 551 553 555 560 564 570 582		

Figure 60 Ratio of Counter Weight to Plunger Weight vs Ratio of Plunger Penetration after 10,000 Repetitional Loadings to Penetration after 1 Loading Sand "A"



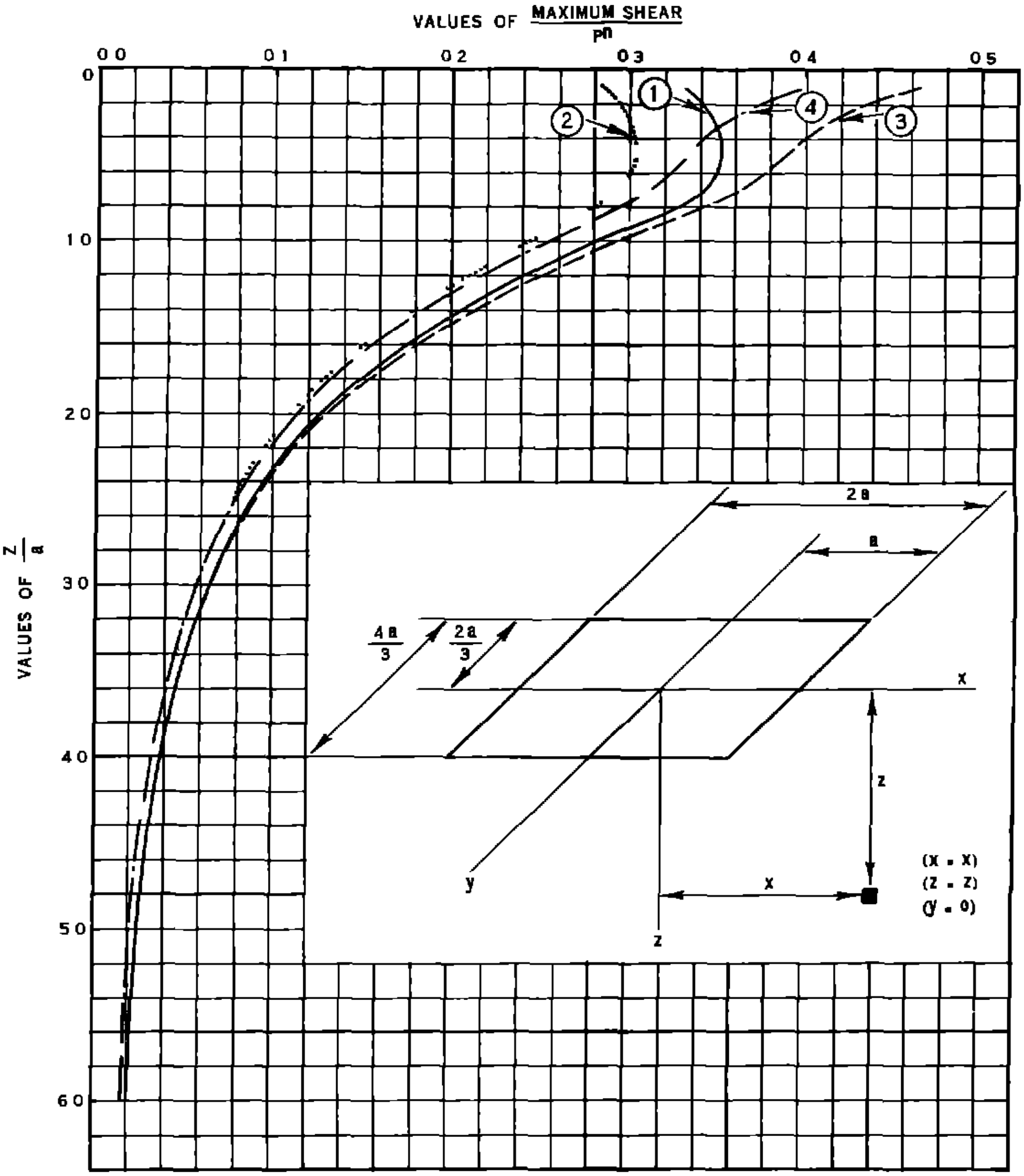
TEST	SLOW REPETITIONAL	DIAMETER OF CONTAINER	6 in
MATERIAL	SAND B	AREA OF PLUNGER	3 sq in
CONDITION OF SATURATION	DRY	SURCHARGE	12 3 lb 0 49 psi
COMPACTION METHOD	2000 psi STATIC LOAD	FREQUENCY	1 r p m
VOID RATIO "e"	NOTED ABOVE	PLUNGER WEIGHT 'T"	NOTED ABOVE
TEST NUMBERS	530 533 535 537 539 542 552		

Figure 61 Ratio of Counter Weight to Plunger Weight vs Plunger Penetration after 10,000 Loadings Sand ' B '



TEST	SLOW REPETITIONAL
MATERIAL	SAND B
CONDITION OF SATURATION	DRY
COMPACTION METHOD	2000 psi STATIC LOAD
VOID RATIO "e"	NOTED ABOVE
TEST NUMBERS	530 533 535 537 539 542 552
DIAMETER OF CONTAINER	6 in
AREA OF PLUNGER	3 sq in
SURCHARGE	12 3 lb = 0 49 psi
FREQUENCY	1 r p m
PLUNGER WEIGHT "T"	NOTED ABOVE

Figure 62 Ratio of Counter Weight to Plunger Weight vs Ratio of Plunger Penetration after 10,000 Repetitional Loadings to Penetration after 1 Loading Sand "B"



① MAXIMUM SHEAR VALUES DEVELOPED BY A VERTICAL PRESSURE "N" (CASE 1 OF PROFESSOR BARON) CORRESPONDS TO CONDITION WHEN AIRPLANE MOTORS ARE NOT RUNNING

② MAXIMUM SHEAR VALUES DEVELOPED BY A VERTICAL PRESSURE "V" ONLY (V = 0 857 N) CORRESPONDS TO CONDITION WHEN AIRPLANE MOTORS ARE RUNNING AT 2200 r p m BUT THE EFFECT OF THE HORIZONTAL FORCE IS IGNORED

③ MAXIMUM SHEAR VALUES DEVELOPED BY THE ORIGINAL VERTICAL PRESSURE "N" PLUS A HORIZONTAL FORCE H = N/3 (CASE 3 OF PROFESSOR BARON) CONDITION CANNOT ACTUALLY OCCUR

④ MAXIMUM SHEAR VALUES DEVELOPED BY A VERTICAL PRESSURE V = 0 857 N PLUS A HORIZONTAL FORCE H = V/3 ACTUAL CONDITION WHEN MOTORS ARE RUNNING AT 2200 r p m
(SEE EXPLANATIONS IN REPORT)

Figure 63 Theoretical Variation of Maximum Shearing Stresses with Depth below the Center Line of an Area Loaded by a Vertical and by a Horizontal Force

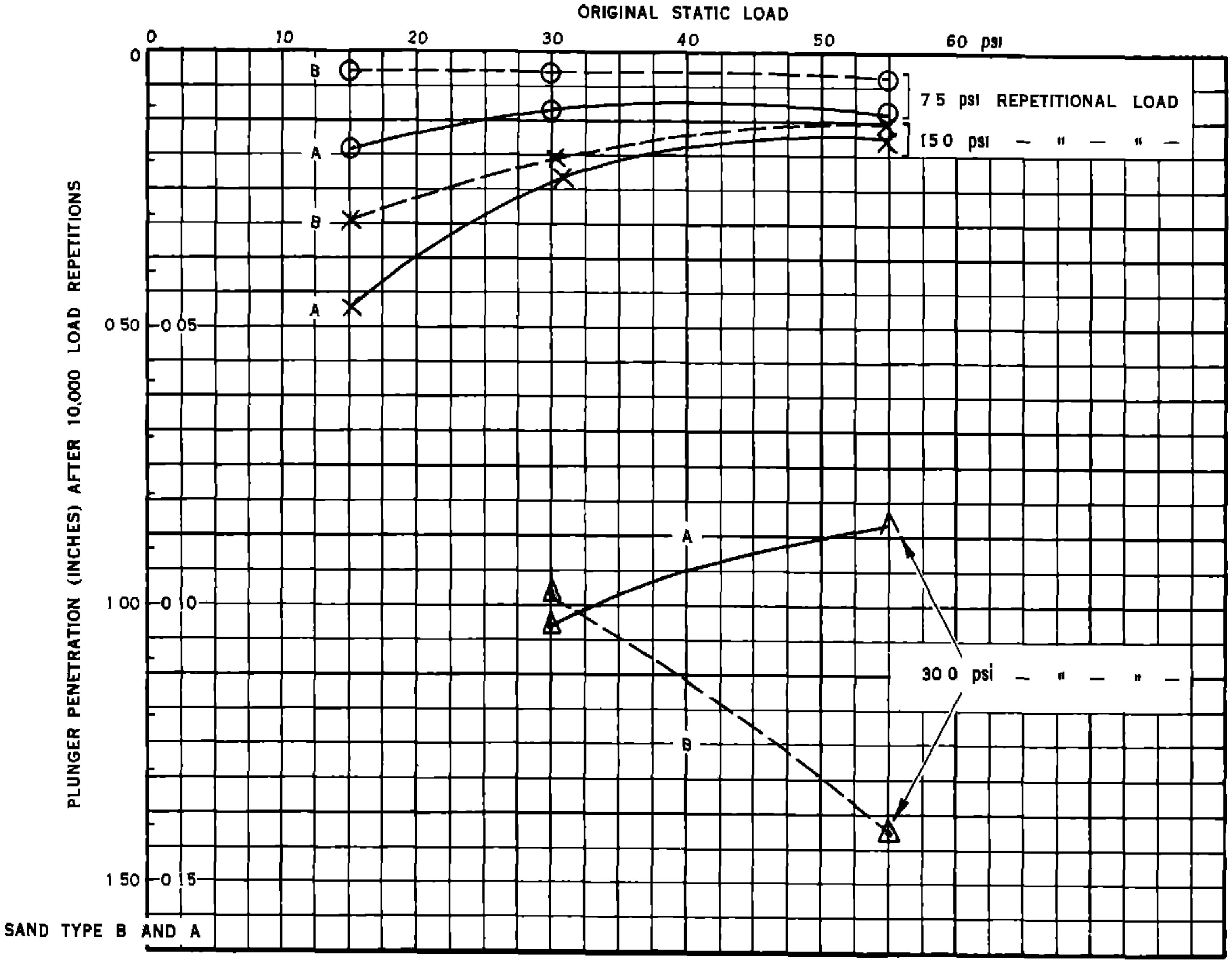


Figure 64 Effect of a Constant Slow Repetitional Force on Plunger Penetration Due to Variation in Original Static Load