

CIVIL AERONAUTICS AUTHORITY

Technical Development Report No 25

THE DEVELOPMENT OF
AN IMPROVED
IMPEDANCE MEASURING BOX

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JUNE 1940



UNITED STATES GOVERNMENT PRINTING OFFICE

WASHINGTON 1941

ACKNOWLEDGMENT

The author desires to acknowledge the assistance of D M Stuart, who offered helpful suggestions and made preliminary measurements at Indianapolis, and of R P Battle who made measurements at Washington, D C , and Richmond, Va

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SUMMARY

A technical development project was established for the purpose of providing a design specification which could be used for the purchase of a quantity of improved low-frequency impedance measuring boxes to replace the units at present being used by Authority field personnel and to provide the additional units required to serve the needs of the expanded and modernized airways system.

A specification, based on experiences with the existing types of impedance measuring boxes and including certain refinements, was prepared

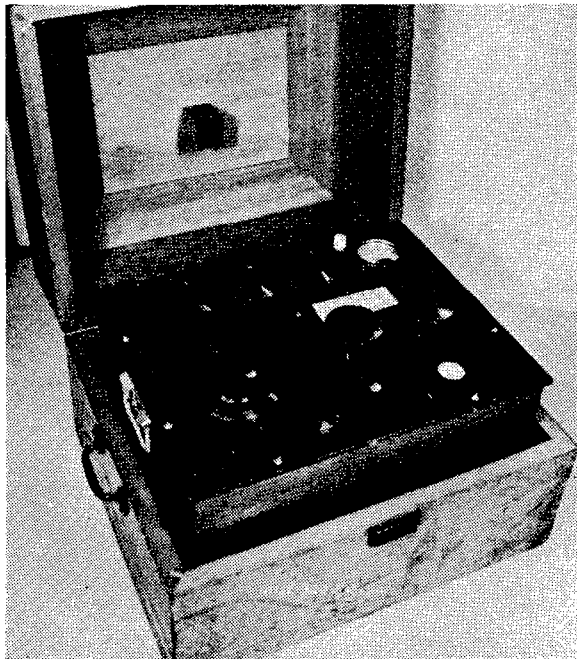


Figure 1. Type AC-46 unit.

and a sample unit was purchased. Tests were conducted on this new unit and a final specification was prepared by the Airways Engineering Division for a purchase of a quantity of these units.

INTRODUCTION

The first impedance measuring set (figs. 1 and 2) used for airways work was procured in 1933 from the Leeds & Northrup Co. under Specification 642 (Department of Commerce, Light-

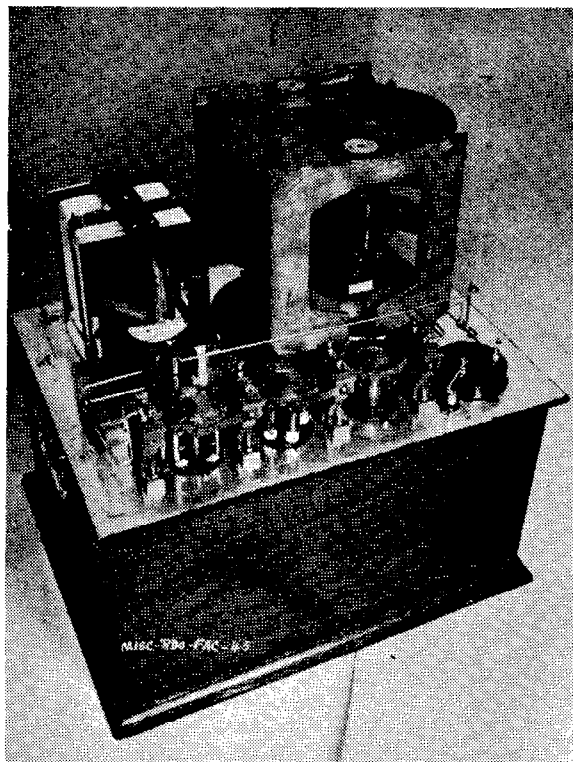


Figure 2. Internal view of type AC-46 unit.

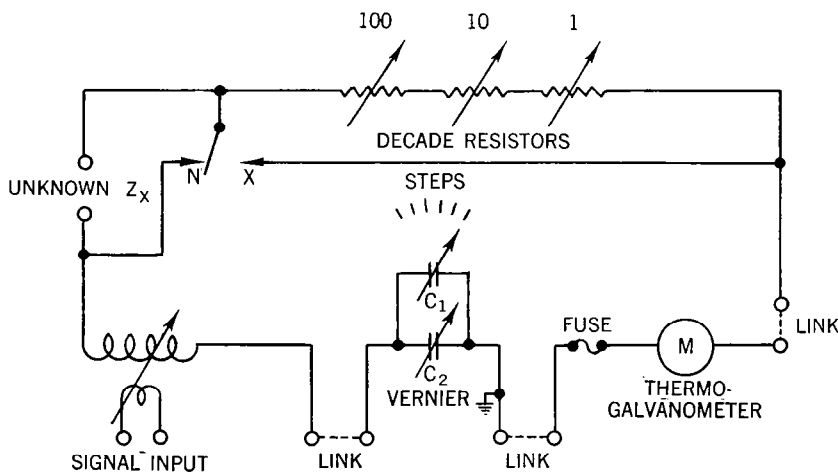


Figure 3. Diagram of type AC-46 unit

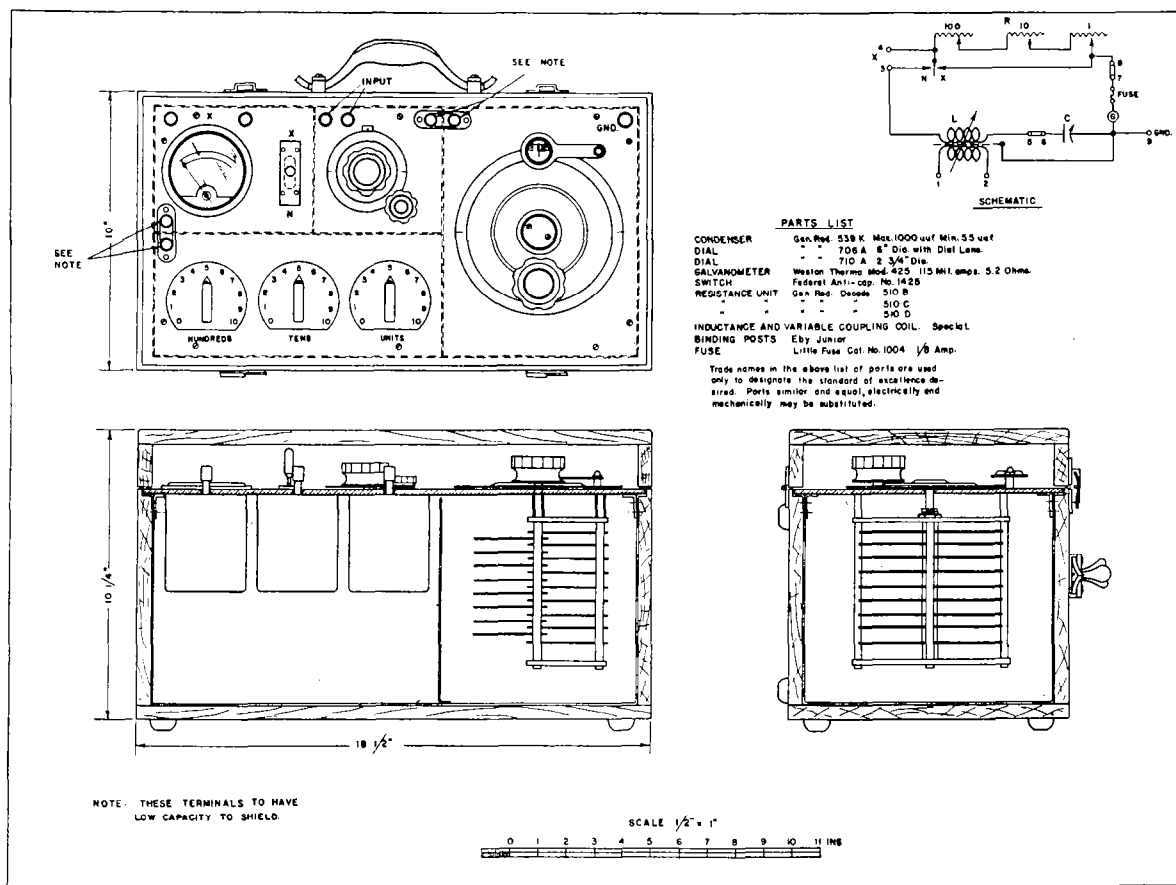


Figure 4. Sketch and outline of type AC-52 unit.

house Service, Airways Division, February 10, 1933) and was designated type AC-46. A diagram of the circuit employed is shown in figure 3.

This first set was large and heavy and was, therefore, inconvenient to carry around for field service. Only a small number of this type were purchased. The condenser, which was a large rugged unit of the variable type, had fixed stops of definite capacity. These stops were accurately calibrated and incremental capacities were obtained by operation of a vernier condenser. The condenser dials were calibrated directly in capacity.

In order to keep the errors introduced by the signal coupling system at a minimum, it was necessary to operate the coupling coil at its lowest mutual position, and in cases where a change of reading was obtained upon reversal of the unknown leads, an average reading had to be assumed as correct.

The second impedance measuring set design, known as type AC-52, was also contracted for in 1933 under the same basic specification, which had been revised to give a set with greater

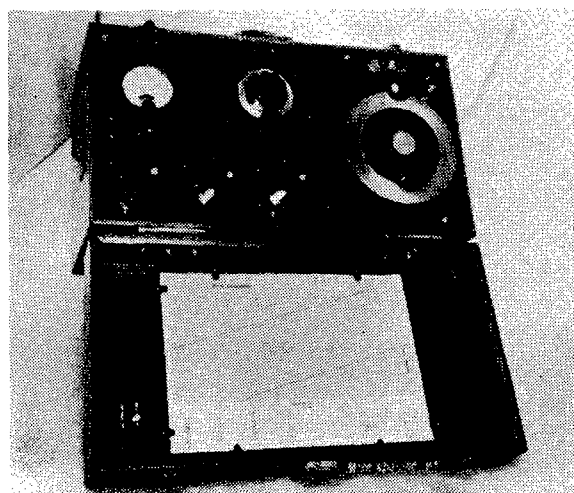


Figure 5. Type AC-52 unit.

portability. A sketch of the proposed unit and its circuit diagram is given in figure 4 (drawing RB-1001). The actual arrangement of parts and exterior items is given in figures 5 and 6. This type of measuring set has been in constant

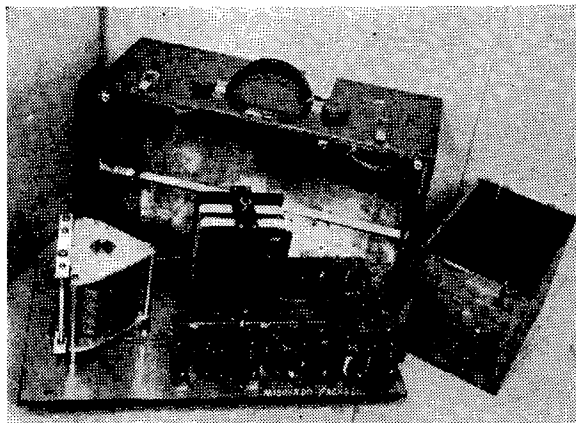


Figure 6. Internal view of type AC-52 unit.

use to date for the adjustment of airway radio facilities. Its continued use brought to light certain deficiencies which were eliminated in the new design described in this report.

DISCUSSION

Early Equipment

The experiences of engineers with the AC-46 and AC-52 types of measuring units disclosed the following deficiencies:

1. The signal source impedance introduced error in measurements in proportion to the setting of signal coupling coil.
2. The capacitance value for any setting of the condenser dial could not be determined conveniently or accurately.
3. The fuse in the measuring circuit was frequently blown during preliminary measurement. Further, the location of the fuse was inconvenient in that its replacement required removal of the panel from its case.

In the use of the type AC-52 set, it was generally noticed that if the signal input leads were reversed, or the leads to the impedance being measured were reversed, a change in the value of indicated resistance would result. This change was caused by the impedance and un-

balance of the signal source. The magnitude of this change depends upon the degree of coupling to the signal source and the true value of the unknown being measured. The unbalance continued to influence readings even though a static shield was used around the coupling coil. The latter difficulty was experienced also in the AC-46 box.

In the AC-52 unit the condenser dial was engraved with 100 uniform divisions. This scale was checked against frequency and a curve was prepared. It was necessary to refer to this condenser dial calibration curve in order to determine the capacity values for any setting of the dial. This reference invited observational errors and required more time for each measurement than was necessary when a directly calibrated dial was used.

A Weston 115-milliamperere standard r-f meter, having a resistance of 5.2 ohms, was used as an indicating instrument in the resonant circuit of the AC-46 and AC-52 units. This resistance affected, to some extent, the ability to measure accurately high values of reactance. Means of overcoming this deficiency through the use of other indicating methods were considered, such as:

(a) The use of a rectox rectifier and d-c instrument.

(b) A vacuum tube rectifier and d-c instrument.

It appeared that impedance and efficiency characteristics of rectox rectifiers at these frequencies would prevent any gain in method (a). Method (b) presented a possible solution, but the additional weight and maintenance of batteries was not favored. Allowance was made in the specification prepared for construction of the new unit, for the use of new methods, if they could be proved at least equal to the standard milliammeter method.

Comparisons of Specifications

The following tabulation, based upon the specifications for the old and the new units, illustrates the improvements contemplated for the new unit.

	New unit	Old (AC-52) unit	Old (AC-46) unit
Frequency range.....	200-500 kc.....	200-400 kc.....	
Resistance range.....	5-200 ohms.....	1,110 ohms.....	1,110 ohms
Smallest resistance steps.....	0.1 ohm.....	1 ohm.....	1 ohm
Inductance of resistance group.....	Constant.....	Proportional to resistance setting.....	Proportional to resistance setting
Resistor accuracy (d-c measurement).....	± 0.1 percent ¹	± 0.3 percent.....	± 0.3 percent
Resistor accuracy (r-f measurement).....	± 0.25 percent.....	± 1.3 percent.....	± 1.3 percent
Weight.....	35 pounds.....	None specified.....	None specified
Dimensions.....	10 by 10 $\frac{1}{4}$ by 18 $\frac{1}{2}$	10 by 10 $\frac{1}{4}$ by 18 $\frac{1}{2}$	Do
Accuracy of resistance measurements.....	{ 2.5 percent (35-150 ohms ²) 5.0 percent (any setting ²) }	{ None specified..... None specified..... }	{ Do Do }
Operating temperature range.....	-30° to +125° F.....	None specified.....	Do
Signal source power required for operation.....	5 watts maximum.....	None specified.....	Do
Resonating inductance ³ (microhenries).....	750-850.....	750-850.....	560-600
Q.....	180 min Sharpness of resonance 45 min.....	Sharpness of resonance 100 min.....	
Coupling coil design as related to errors caused by impedance or unbalance of power supply circuit.....	Maximum error shall not exceed 0.2 percent for source impedances between 70 and 300 ohms.....	Not specified.....	Not specified
Condenser.....	Similar or equal to G. R. Co 722M, 1,000 mmfd, scale calibrated to give capacity values without reference to curves (Accuracy specified by G. R. Co for No 722M is 0.1 percent.).....	1,000 mmfd, figure of merit ($2\pi fRC$) ² maximum 0.03 by 10 ⁻¹² measured at 1,000 cycles Calibration curve to be accurate to within 2 mmfd at time of delivery Dial to be marked in approximate capacity Vernier knob to have 8:1 ratio.....	1,500 mmfd, figure of merit ($2\pi fRC$) ² maximum 0.08 by 10 ⁻¹² measured at 1,000 cycles Calibration curve to be accurate to within 2 mmfd at time of delivery Dial to be marked in approximate capacity Vernier knob to have 25:1 ratio.....
Current indicating instrument.....	Weston 115 ma r-f meter, or tube, or rectov circuits with d-c instruments.....	Weston 115 ma r-f meter.....	Weston 115 ma r-f meter
Instrument fuses.....	Fuses shall be provided if the r-f milliammeter is used and they shall be replaceable without removing equipment from case.....	Fuses shall be provided.....	None specified
Panel.....	Aluminum.....	Aluminum or Bakelite.....	Aluminum or Bakelite
Shielding.....	22-gage copper.....	Copper.....	Copper
Terminal insulation.....	Bakelite BM-262 having highly polished surface.....	Bakelite.....	Bakelite

¹ For 10-ohm steps and 100-ohm resistor. 0.25 percent and 1.0 percent for 1 ohm and 0.1 ohm steps, respectively.

² Irrespective of reactive component up to $X=R$.

³ At 300 kc.

Test and Observations on New Unit

The following tests and observations were made on the unit delivered under the project contract.

Mechanical Construction—The general features of the new unit (AC-263) are shown in figures 7, 8, and 9. Better visibility of the condenser dial would have been desirable. Alteration of the condenser structure to obtain better visibility would have risked disturbance of its calibration. The metal handle supplied appeared to make the unit more difficult to

carry than the older type AC-52 unit, which has a flexible leather handle.

Weight—The weight of the unit as delivered was 37 pounds, which exceeded the specifications by 2 pounds. In general the weight is determined by the particular components called for in the specification and is not under control of the manufacturer. A change in either the cabinet or the component specification is necessary to reduce weight to the specified 35 pounds.

Electrical Features—A circuit diagram of the new unit is shown in figure 10. This conforms

in general with the diagram of the older units. The position of the parts in the new circuit, however, results in a definite improvement in the ability to measure various types of unknowns. The use of compensating inductance for each resistance as shown is in accordance with the specification, and eliminates need for correction when measuring reactance.

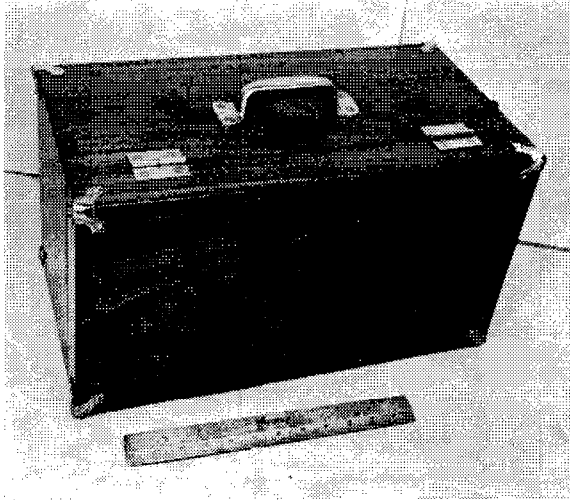


Figure 7. Closed view of type AC-263 unit.

The resistor accuracy and the accuracy of the condenser calibration were carefully determined by the subcontractor (General Radio Co.), in accordance with standard manufacturing practice.

The same type of current indicating instru-

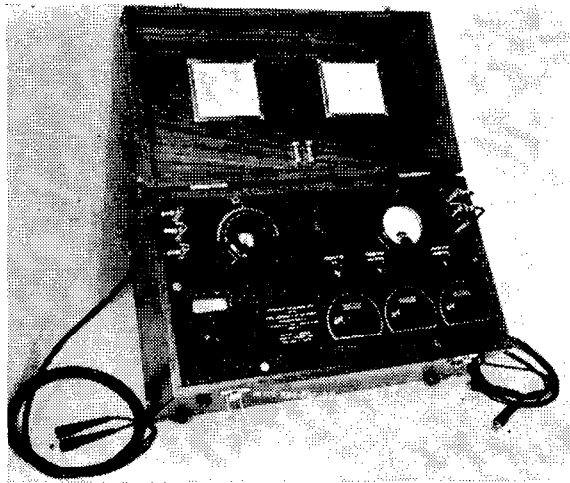


Figure 8. Open view of type AC-263 unit.

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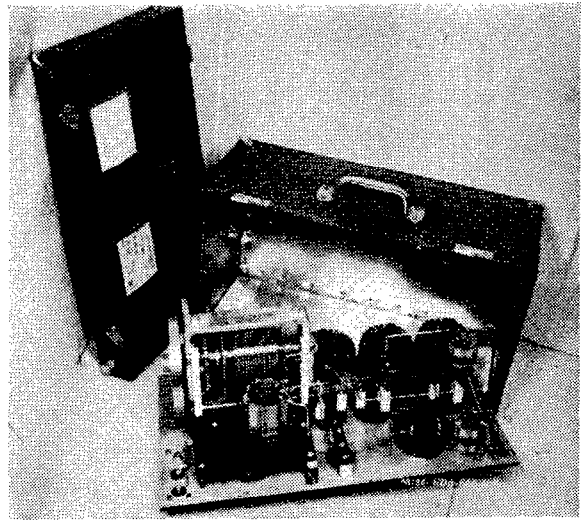


Figure 9. Internal view of type AC-263 unit.

ment (Weston Thermo-Galvanometer, 0-115 ma) as used in the older boxes was used in the new design. An analysis of the various types of indicator circuits by the contractor indicated that for greatest stability and accuracy the thermal instrument should be used. It was pointed out that aging and low efficiency at the operating frequency were points against the use

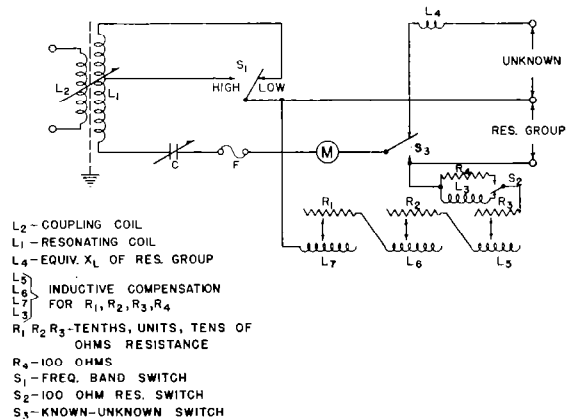


Figure 10. Diagram of type AC-263 unit.

of rectox. The use of a vacuum tube rectifier was avoided because of the disadvantage of using a battery, and because of possible disturbance of the resonant circuit by the tube element capacity. An impedance transformer was considered for coupling either the tube or the rectox to the resonant circuit. The presence of the additional elements and the weight

in contrast to the simplicity and accuracy of a thermal meter led to the final adoption of the latter. The resistance of this meter is, as in

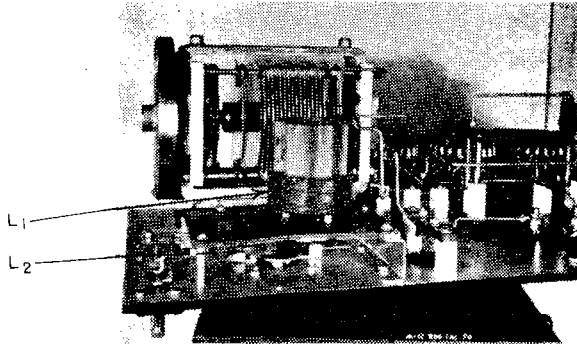


Figure 11. Type AC-263 unit showing resonating and coupling coils.

the past, 5.2 ohms. The fuse required for protection of the meter was found to have a resistance of about 8 ohms, giving a total series resistance of 13 ohms.

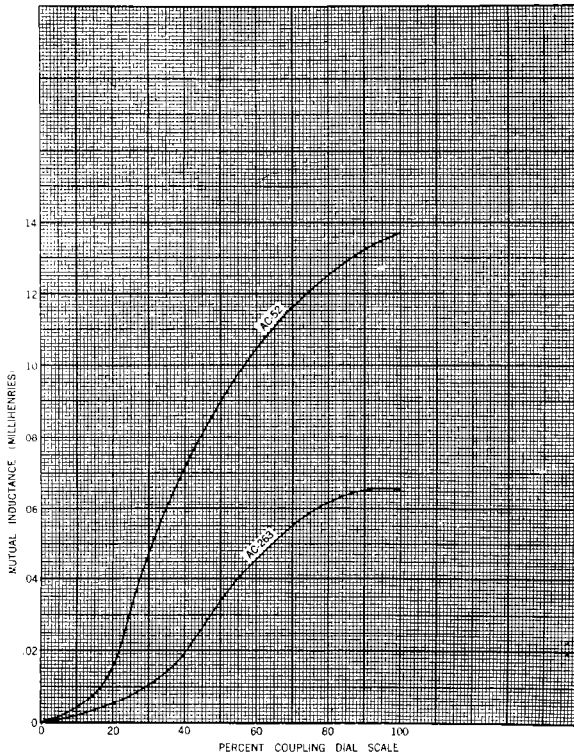


Figure 12. Curve of mutual inductance versus coupling coil setting.

The resonating inductance, L_1 , and coupling coil L_2 , shown in figure 11, were designed to have a relatively small ratio of length to diameter. By so doing it was possible to secure a

minimum effect from reversal of source or the unknown leads during measurements. The use of a grounded Faraday shield between coils also contributed toward this achievement. A deficiency in "Q" of the resonating coil over that called for in the specification was caused by the shape of coil chosen. A "Q" of 90 was observed whereas 180 was specified. The achievement of the higher "Q" would have required sacrificing some of the other desirable features.

The mutual inductance between the coupling and resonant coils for various settings of the

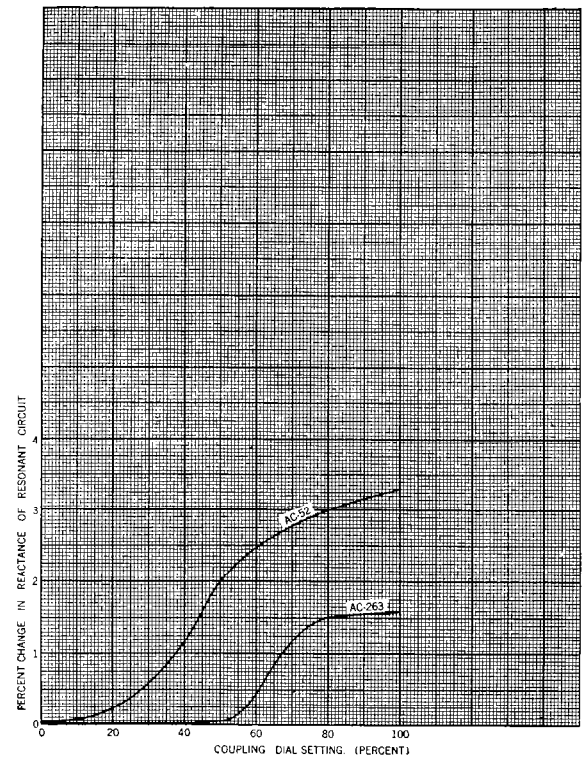


Figure 13. Curve of change in reactance of resonant circuit versus coupling coil setting.

coupling dial was computed from observations of the total series inductance of the two coils for both aiding and opposing connections:

$$M = \frac{L_A - L_B}{4} \text{ millihenries}$$

where L_A and L_B are the measured inductance values in millihenries for the aiding and opposing connections, respectively.

Coupling dial setting	$M(\text{mh})$
100-----	0 06675
80-----	06325
60-----	04575
40-----	01925
20-----	00675
10-----	00275
0-----	0

These measurements were made at 300 kc using a Boonton Radio Corp Q meter. Values of inductance were calculated from condenser scale readings of the Q meter. The coils were completely disconnected from the impedance box circuit. A curve of the results is given in figure 12.

In a test conducted at the Richmond, Va., radio range station, the variation of capacity required for resonance for various settings of

signal coupling coil was observed. The results were converted to show the change in reactance of the resonant circuit for various couplings. Similar tests on the AC-52 unit gave data to compare the two units and results of these tests are plotted on figure 13. Both were tested on the same circuit. The results of both tests show reactance changes in general conformity to their respective mutual impedance curves of figure 12. No improvement is indicated by the curves, however, experience has shown that the old AC-52 unit is unstable, being affected by unbalances in either the signal source or the unknown.

The following tabulation of the characteristics of the AC-263 unit is presented as a matter of record and is self-explanatory.

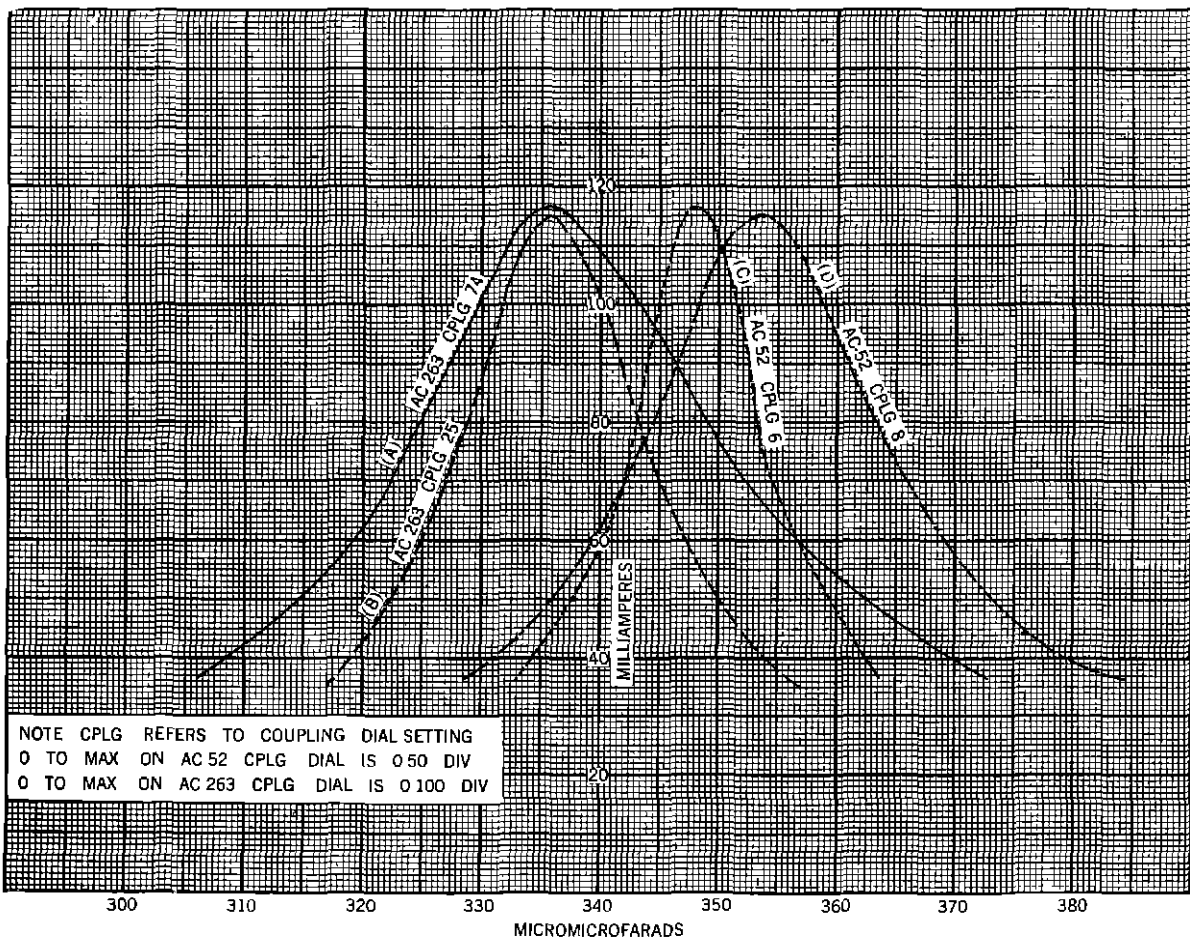


Figure 14 Resonance curves of types AC-52 and AC-263 units

SAMPLE *	COUP- LING	COND DIAL		CAPACITY		INDI- CATED R	CALC X OHMS	CORREC- TED R	TEST CIRCUIT
		X	N	Cx	Cn				
A	94	828 3	806	304 7	327	161	123	156 8	A
		826 8	806	306 2	327	165	110	159 5	B
		828	805	305	328	167	122	162	C
		831	806	302	327	168	134 5	162 5	D
		828 4	803	304 6	330	167	133	162 3	E
		828	804 8	305	328 2	166	122 5	161 5	F
B	94	805 8	804 8	327 2	328 2	5 1	4 93	5 1	G
		805 6	804 8	327 4	328 2	5 1	3 95	5 1	A
		805 4	804 2	327 6	328 8	5 1	6 33	5 1	B
		797	794	336	339	5 1	13 83	5 1	F
		794 8	794 8	338 2	338 2	5 1	0	5 1	E
		801	799 5	332	333 5	5 1	2 87	5 1	H
C	94	800 8	801	332 2	332	5 1	- 96	5 1	I
		808 2	800 5	324 8	332 5	5 5	37 8	5 45	J
		807 8	800 2	325 2	332 8	5 7	37 2	5 65	K
		811 6	804	321 4	329	5 5	38 1	5 45	E
C	94	811 6	804	321 4	329	5 7	38 1	5 65	F
		809 8	801 6	323 2	331 4	5 5	40 6	5 43	L
		809 8	801 8	323 2	331 2	5 5	39 6	5 43	G
		810 0	801 6	323	331 4	6	41 5	5 93	M
		809 6	801 8	323 4	331 2	5 6	38 6	5 53	N
		809 8	802 2	323 2	330 8	5 6	37 7	5 53	O
* A = SLIDE WIRE RHEOSTAT B = RES UNIT C = RES UNIT PLUS COIL									

Figure 15 Tabulation of test data type AC-263 unit

- 1 Q of total inductance L_1 (measured at 300 kc)..... 90
- 2 C required to resonate L_1 alone (300 kc) mmfds... 332
- 3 Resistance of resonant circuit including fuse,¹ 8 ohms and meter, 5 2 ohms (measured by use of AC-52 unit at 260 kc).....ohms... 30
- 4 Q of resonant circuit at 300 kc calculated from resonance curve B, figure 14
- $$\left(Q = \frac{f_R}{f_a - f_b} \right)$$
- where f_R = frequency at resonance, f_a and f_b are frequencies corresponding to 0.7 of resonant current peak on figure 14)..... 42 8
- 5 Circuit resistance at 300 kc $\left(\text{calc } R = \frac{\omega L}{Q} \right)$ ohms... 37 2

A typical set of measurements with the AC-263 unit made on three unknowns is given in

¹ 3/8 ampere Littelfuses "Low Res" have been measured and their resistance is 3.3 ohms at 300 kc

figure 15. Measurements made with the AC-52 unit on one of these unknowns are also given in figure 16. The circuits used in obtaining the data are shown in figure 17. From the data it will be seen that for the variety of test conditions, the maximum variation of indicated resistance was 4.6 percent for the AC-263 unit. For the 165-ohm sample, (sample A) the variation for all conditions of test was from 161 to 168 ohms, or plus or minus 2.18 percent. For the 5-ohm resistor, (sample B) there was no observable error under any method of connection or nature of source. Sample C consisted of sample B in series with a General Radio Co. wavemeter coil, and for this unknown the resistance variation was plus or minus 4.6 percent.

Sample C was tested on the AC-52 unit under a variety of conditions similar to those used on the AC-263 unit and the indicated resistance

varied from 5 ohms to 33 ohms. It should be noted that this tremendous error occurs only for the 100-percent position of the source coupling dial. For couplings under 20 percent, the AC-52 unit gave variations of 5 to 8 ohms or 60 percent. Below 15 percent coupling, the error was 20 percent. All of the AC-263 unit tests were made with 94 percent coupling so that its superior performance, when normally operating at couplings below this value, will be even more evident.

The manufacturer has provided a set of curves with the AC-203 unit to be used in determining true values of R in the unknown, when the unknown contains reactance. These curves give corrective factors ranging from 0.93 to 1.075 to be applied to the indicated resistance for values of reactance from -200 ohms to +200 ohms. No such curves were available

for the AC-52 unit. In the tabulation, figure 17, the correction factor has been applied to the "Corrected R" column.

The reactance for each unknown was calculated for each test condition ¹

$$X = -\frac{C_X - C_N}{\omega C_X \times C_N} 10^9 \text{ ohms}$$

where

C_X = capacity required to resonate the unknown in micromicrofarads

C_N = capacity required to resonate the known, or measuring circuit, in micromicrofarads

$\omega = 2\pi f$, where f is frequency in kilocycles

For the new AC-263 unit the numerator of the equation ($C_X - C_N$) becomes merely the differ-

¹ See appendix

SAMPLE *	COUP- LING	COND. DIAL		CAPACITY		INDI- CATED R	CALC X OHMS	CORREC- TED R	TEST CIRCUIT
		X	N	C _x	C _n				
C	5	95	98	332	343	5	51		P
	10	95	98	332	343	5	51		Q
	10	95	98	332	343	5	51		R
	10	95	98	332	343	5	51		S
	15	96.5	99	338	346	5	37.2		T
	15	94.5	97	331	339	5	37.9		U
	20	95	98	332	343	6	51	NO CORRECTION DATA PROVIDED	V
	20	95.3	98	333	343	6	51		L
	20	95	98	332	343	6	51		W
C	20	95.3	98	333	343	5	46.3		X
	20	95.3	97.8	333	342	8	41.8		C
	20	95.5	98	334	343	6	41.7		L
	0+	95.8	98	335	343	6	37.1		D
	20	98	101	342.5	353	6	46		D
	100	98	101	342.5	353	7	46		Y
	100	98	101	342.5	353	6	46		Z
	100	101	101	353	353	33	0		AA
	100	96	98.5	336	344.5	6	38.9		L
	100	98	98	342.5	343	33	2.25		G
	100	95	98	332	343	8	51.2		M
	100	96	98	336	343	6	32.2		L
	* C = RESISTANCE UNIT PLUS COIL								

Figure 16 Tabulation of test data type AC-52 unit

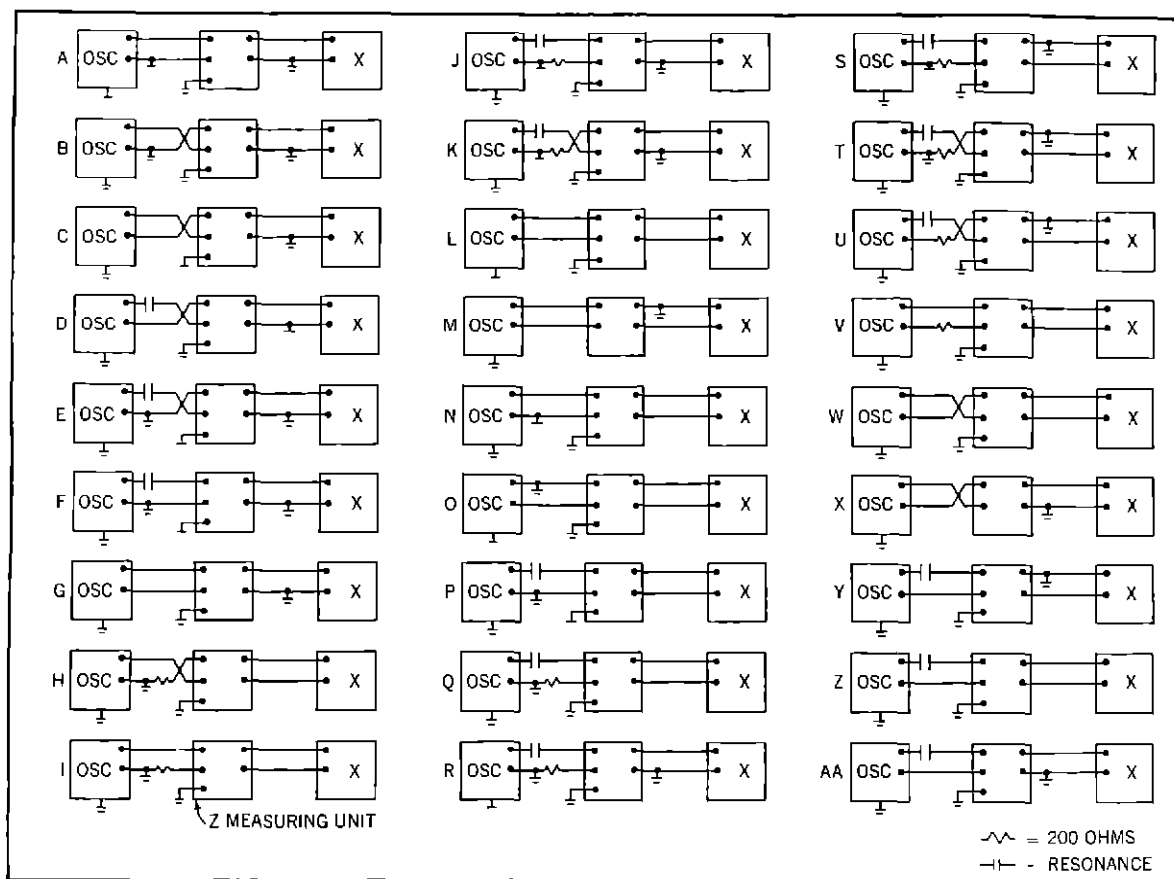


Figure 17 Diagrams of test circuits

ence in condenser dial readings since the scale is calibrated in uniform micromicrofarad steps

From the results it is observed that on low resistances having a low ratio of reactance to resistance, such as represented by sample C, the reactance is not accurately measured unless a definite procedure for connecting the unknown is always used. There was, however, consistency of reactance measurements (within 12 percent) for sample C having a relatively large ratio of reactance to resistance. Comparing the AC-52 unit with the AC-263, measurements in test circuits L, G, and M show reactance values from 2.25 to 51.2 ohms for the AC-52 and from 39.6 to 41.5 ohms for the AC-263.

It should be noted that the resistance group of the AC-263 unit is compensated, that is, its reactance is constant regardless of the resistance setting. Although this results in a large reactance present for even low resistance measurements, this reactance does not have to be con-

sidered in the calculations of reactance in the unknown because of the compensating reactance L_4 (see fig. 10) which is automatically inserted in the unknown circuit during measurements.

In order to check the correctness of the compensating inductance L_4 , the terminals of the unit marked unknown were shorted and the following readings observed:

Circuit (See fig. 17)	Coupling	Indicated R	Calc. X
H	94	0.5	98
L	94	0.5	0
C	94	5	98
B	94	4	98
A	94	3	98
D	94	3	925
W	94	3	0

The standard leads supplied with the AC-263 unit were then added to the unknown terminals and shorted at their outer end, giving the following results:

Circuit	Coupling	Indicated R	Calc X
A-----	94	-	9 4
B-----	94	-	3 94
C-----	94	-	3 94
D-----	94	-	4 4
H-----	94	-	4 9

Unknown sample B (see fig 15) was then connected directly to the AC-263 unit terminals without leads and the following observed

Circuit (See fig. 17)	Coupling	Indicated R	Calc X
W-----	94	5 0	0 453
W-----	23	5 0	0
L-----	50	5 0	0
L-----	70	5 0	0
L-----	94	5 0	0
V-----	94	5 0	0

From the foregoing it appears that because of connecting lead lengths and the inherent unbalance resulting, reactances of low value cannot be accurately determined without a definite connection procedure and a setting of coupling control always below a predetermined maximum. The discrepancies of measurements on sample B, figure 15, are evidently caused by connecting lead length and tight coupling of the resonant measuring circuit to the oscillator

CONCLUSIONS

The AC-263 unit is a definite improvement over the AC-52 for both resistance and reactance measurements. Further improvement in reactance measurement accuracy can possibly be achieved through a further study of circuit arrangement. There will always be error in measurements made by the substitution method used in the AC-52 and AC-263 units, however, the error is a function of coupling to the signal source and if this coupling can be reduced further through the development of more sensitive resonance indicators, the error can be reduced.

The use of a compensated resistance group is an advantage in measuring low values of reactance because its reactance is automatically eliminated. However, a definite procedure must be established for connecting such reactances, and the source oscillator impedance must be within certain limits, or the coupling must be below a prescribed value.

The calibrated condenser used in the AC-263 unit is a distinct advantage in both accuracy and convenience of measurements.

The three switches used in the AC-263 unit are unsatisfactory. The contacts are not of the wiping type and occasionally fail to make good contact. Another type using rotary self-cleaning contacts would be preferable.

APPENDIX

The theory involved in the use of the impedance measuring unit is as follows

(1) If a circuit consisting of a coil L_1 , condenser C_1 , and a current indicating device M_1 is coupled to a source of radio frequency energy through a coil L_2 , the current flowing in the circuit at resonance will be dependent only upon the resistance of the circuit and the electromotive force introduced by the coupling coil. If the coupling and resonating condenser are set to give a certain value of current with an unknown resistance, R_x in series with the circuit, another resistance, R_n , which is equal to R_x , may be substituted for R_x and the current will remain the same. If the resistance, R_n , is a calibrated standard variable resistance it can be used to determine the value of R_x .

(2) If I_1 is the current, E_1 the induced electromotive force, and Z_1 the circuit impedance including an unknown,

$$I_1 = \frac{E_1}{Z_1} \quad (a)$$

Similarly, if the unknown is replaced by a calibrated resistance unit, making the circuit impedance, Z_2 ,

$$I_2 = \frac{E_2}{Z_2} \quad (b)$$

If, however, the coupling to the source of energy is held constant so that $E_2 = E_1$ and if the calibrated resistance unit is adjusted to make $I_2 = I_1$ then,

$$Z_1 = Z_2$$

However, since

$$Z_1 = R_1 + jX_1$$

and

$$Z_2 = R_2 + jX_2$$

it is possible to make

$$Z_1 = R_1$$

and

$$Z_2 = R_2$$

by adjusting the circuit to resonance for each condition (a) and (b) so that jX_1 and jX_2 become zero. Then

$$R_1 = R_2 \quad (c)$$

which means that the resistance of the circuit including the unknown is equal to the resistance of the circuit including the calibrated known, or, since the coil, condenser and indicating instrument of the measuring circuit are common to both known and unknown adjustments, then the unknown is equal to the value indicated by the calibrated resistance unit.

(3) The reactance component in the unknown can be determined from the values of capacity required to resonate the circuit under conditions (a) and (b). Under the first condition

$$X_1 = \frac{1}{\omega(C_1)} \quad (d)$$

and under the second resonant condition

$$X_2 = \frac{1}{\omega(C_2)} \quad (e)$$

where X_1 , X_2 are capacitive reactances required to resonate the circuit under the two conditions respectively, C_1 and C_2 are the corresponding capacitances, and $\omega = 2\pi f$. Subtracting (e) from (d),

$$\begin{aligned} X_1 - X_2 &= \frac{1}{\omega(C_1)} - \frac{1}{\omega(C_2)} \\ &= \frac{C_2 - C_1}{\omega(C_1 C_2)} = \frac{\Delta C}{\omega(C_1 C_2)} \end{aligned}$$

$X_1 - X_2$ represents the change in capacitive reactance required for resonance between conditions of known and unknown and if the known contains no reactance this difference represents the reactance of the unknown, X . If the known contains reactance, its value must be added to the reactance determined by the given equation. If the known element has compensated or constant reactance, a reactance of the same amount may be added in series with the unknown lead, and measurements will be direct and require no correction. If ΔC , C_1 and C_2 are in micromicrofarads and f is in kilocycles,

$$X = \frac{10^9(\Delta C)}{\omega C_1 C_2} \quad \text{ohms}$$

When C_2 is greater than C_1 inductive reactance in the unknown is indicated

(4) The substitution method of measurement assumes that the electromotive force induced in the measuring circuit is constant and, since the circuit is resonated and the resistances made equal the induced voltage is generally constant and not a source of error. If the distribution of electrostatic capacity in the measuring circuit changes for different settings of the resonating condenser and for different types of unknowns, grounded or ungrounded, the current indicated may not be the true value in each case, thereby giving rise to error in measurement

The impedance of the source of oscillations which is coupled to the measuring circuit does not introduce an error in measurements because it is present for both readings. If the connection of the unknown to the measuring circuit causes an unbalance or change in the source circuit error will be introduced

REFERENCES

A detailed study of measurements of resistance at high frequencies is given in Proc I R E, Vol 26, No 12, December 1938. Additional basic data are available in Circular 74 of the National Bureau of Standards