A COMPARISON OF THE POLAR STEREOGRAPHIC, GNOMONIC, AND TRANSVERSE MERCATOR PROJECTIONS FOR POLAR AERONAUTICAL CHARTS

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A COMPARISON OF THE POLAR STEREOGRAPHIC, GNOMONIC, AND TRANSVERSE MERCATOR PROJECTIONS FOR POLAR AERONAUTICAL CHARTS

INTRODUCTION

The following note will be devoted to a comparison of the various properties of the polar stereographic, gnomonic, and transverse Mercator projections, for the purpose of determining their relative suitabilities for polar aeronautical charts. Where the proofs of certain mathematical formulae involved are short, they will be included, along with certain data derived from these formulae. It will be the aim of this note to state the facts regarding each projection, rather than to argue in favor of one or another. After all, the facts, if stated correctly, will speak for themselves.

SCALE

Uniformity of scale, or a close approach thereto, is one of the more important desideratums in a navigational chart. The writer believes that in any discussion of projection properties, including scale distortion, the picture may be presented more clearly if, in addition to stating facts and figures, the mathematical development of the formulae used to arrive at these facts and figures is shown. Since the formulae for scale distortion in these projections can be developed very simply, their proofs will be given here. These formulae will then be used to investigate the scale distortion in each projection over the latitude range commonly covered by polar projections. In developing all formulae, the earth will be regarded as a sphere rather than as an oblate spheroid, the eccentricity being considered negligible for the purposes of this discussion.

Taking first the stereographic projection, consider Fig. 1. Let PE'P'E be a meridional section of the reduced earth, with PP' the polar axis, EE' the equator, AA' the trace of the plane of projection, tangent at the pole, and P' the origin of the rays of projection. Then any point G on the earth, at latitude ϕ, will be projected on the chart in point M, and its distance from the pole, PG, will be represented on the chart by the distance PM.

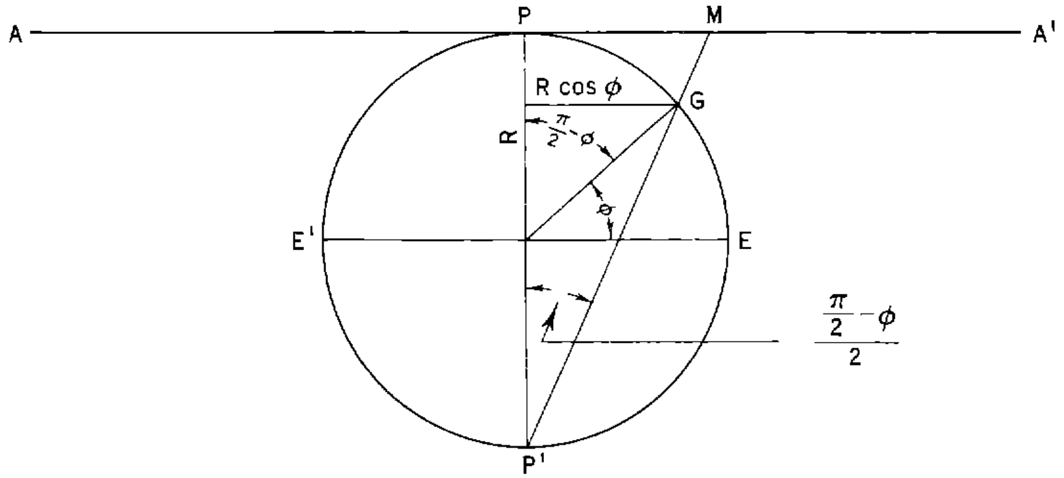

Figure 1 Meridional Section, Polar Stereographic Projection

Let $S_e = PG$ = distance on the earth along the meridian

$S_c = PM$ = distance on the chart along the meridian

R = radius of the reduced earth

ϕ = latitude

then $S_e = R \left(\frac{\pi}{2} - \phi\right)$

$$S_c = 2R \tan \left(\frac{\frac{\pi}{2} - \phi}{2}\right)$$

and scale along meridian $= \dfrac{dS_c}{dS_e} = \dfrac{2R \sec^2 \left(\frac{\frac{\pi}{2}-\phi}{2}\right)\left(-\frac{d\phi}{2}\right)}{-R\, d\phi}$

$$= \frac{-R\, d\phi}{-R\, d\phi \cos^2 \left(\frac{\frac{\pi}{2}-\phi}{2}\right)}$$

$$= \frac{1}{\cos^2 \left(\frac{\frac{\pi}{2}-\phi}{2}\right)} = \frac{1}{\frac{1 + \cos\left(\frac{\pi}{2}-\phi\right)}{2}}$$

$$= \frac{2}{1 + \sin\phi} \qquad (1)$$

Consider now the scale along the parallel Using the same notations as before, except that S_e and S_c now represent distance along the arc of parallel on the reduced earth and the chart respectively and $d\lambda$ represents difference in longitude, we have

$$S_e = R \cos \phi\, d\lambda$$
$$S_c = PM \times d\lambda = 2R \tan \left(\frac{\frac{\pi}{2}-\phi}{2}\right) d\lambda$$

$$\text{Scale along the parallel} = \frac{S_c}{S_e} = \frac{2R \tan\left(\dfrac{\frac{\pi}{2} - \phi}{2}\right) d\lambda}{R \cos\phi \, d\lambda}$$

$$= \frac{2 \tan\left(\dfrac{\frac{\pi}{2} - \phi}{2}\right)}{\cos\phi} = \frac{\dfrac{2 \sin\left(\frac{\pi}{2} - \phi\right)}{1 + \cos\left(\frac{\pi}{2} - \phi\right)}}{\cos\phi}$$

$$= \frac{2 \sin\left(\frac{\pi}{2} - \phi\right)}{\left[1 + \cos\left(\frac{\pi}{2} - \phi\right)\right]\cos\phi} = \frac{2\cos\phi}{\left(1 + \sin\phi\right)\cos\phi}$$

$$= \frac{2}{1 + \sin\phi} \qquad (2)$$

Since the meridians of the projection are radii of the concentric circular arcs of the parallels, all meridians and parallels intersect at right angles, also, from the above two proofs, the scales along both meridian and parallel, at any particular point, are equal, therefore the projection satisfies the definition of conformality

We may now substitute values of sin ϕ in the above formula for scale, selecting the particular latitudes in which we are interested, which, for polar projections, would be from the pole to about latitude seventy degrees

Comparing the scale thus computed for any selected latitude with the true scale at the pole gives the percent of scale distortion. Table I gives the scale distortion at certain latitudes.

TABLE I

SCALE DISTORTION IN A POLAR STEREOGRAPHIC PROJECTION

Latitude	Percent Distortion
Pole	0
80°	+0 8
75°	+1.7
70°	+3.1

Turning now to the polar gnomonic projection, consider Fig. 2.

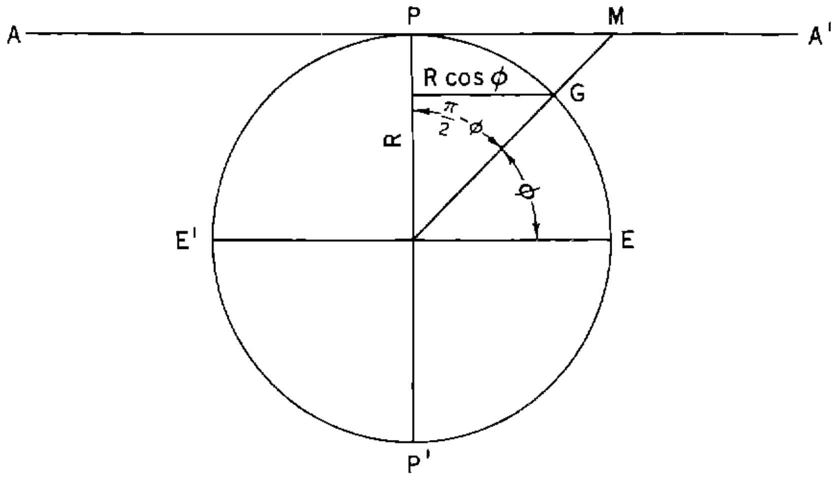

Figure 2. Meridional Section, Polar Gnomonic Projection

Let PE'P'E be a meridional section of the reduced earth, with notation similar to Fig 1 Then any point G on the earth will be projected on the chart in point M, and its distance from the pole, PG, will be represented on the chart by the distance PM

Again letting S_e = PG = distance on the earth along the meridian

S_c = PM = distance on the chart along the meridian

R = radius of the reduced earth

ϕ = latitude

then $S_e = R\left(\frac{\pi}{2} - \phi\right)$

$$S_c = R \tan\left(\frac{\pi}{2} - \phi\right) = R \cot \phi$$

$$\text{Scale along meridian} = \frac{dS_c}{dS_e} = \frac{-R \csc^2 \phi \, d\phi}{-R \, d\phi}$$

$$= \csc^2 \phi$$

$$= \frac{1}{\sin^2 \phi} \qquad (3)$$

Consider now the scale along the parallel Letting S_e and S_c represent distance along the arc of parallel of the reduced earth and the chart respectively, and $d\lambda$ the difference in longitude, we have

$$S_c = R \cot\phi \, d\lambda$$

$$S_e = R \cos\phi \, d\lambda$$

$$\text{Scale along parallel} = \frac{S_c}{S_e} = \frac{R \cot\phi \, d\lambda}{R \cos\phi \, d\lambda} = \frac{\cot\phi}{\cos\phi}$$

$$= \frac{1}{\sin\phi} \qquad (4)$$

The meridians in this projection are obviously radii of the concentric circular arcs of the parallels and therefore intersect all parallels at right angles, but the above proofs show that at any point the scale along the meridian does not necessarily equal the scale along the parallel and the projection is, therefore, not conformal.

We may now use these two formulae for scale along the meridian and parallel to determine the percent of scale distortion in both directions at any latitude in which we are interested. The results are tabulated below.

TABLE II

SCALE DISTORTION IN A POLAR GNOMONIC PROJECTION

Latitude	Percent Distortion	
	Along Meridian	Along Parallel
Pole	0	0
80°	+3.1	+1.5
75°	+7.2	+3.5
70°	+13.2	+6.4

The ordinary Mercator projection is a mathematical projection with the scale true along the equator and increasing along the meridians from the equator toward the pole in such a manner as to represent on the projection, by parallel lines perpendicular to the equator, the converging meridians on the earth. Since, on the earth, distance along a parallel between two meridians varies as the cosine of the latitude, when this distance is held constant on the projection the scale varies as the secant of the latitude. The scale along the meridian is arbitrarily increased to hold it equal to the scale along the parallel and parallels of latitude are drawn, parallel to the equator, at the proper distances. We then have a conformal projection, since meridians and parallels intersect at right angles and the scale along each is the same at any specific point.

In the transverse Mercator, the projection has been rotated ninety degrees and a meridian replaces the equator as a line of true scale. Imagine, on the earth, a network of false meridians and parallels, bearing the same relationship to this meridian as the actual meridians and parallels bear to the equator. If the projection scale is then varied with respect to these false meridians and parallels in the identical manner that it is varied with respect to the true meridians and parallels on the ordinary Mercator projection, we will have a transverse Mercator projection. The scale of this projection obviously varies as the secant of the arc distance from the meridian of true scale. Since the arc distance from any point on a given parallel of latitude may vary from 0 (where the parallel crosses the meridian of true scale) to $90° - \phi$ (where it crosses a meridian 90° removed in longitude), it is evident that scale along any parallel of latitude may vary from 1 to a maximum of $\sec(90° - \phi)$. From the expression $\sec(90° - \phi)$, or $\frac{1}{\sin\phi}$, we may now compute the maximum scale distortion at any latitude. Table III shows maximum scale distortions for latitudes from the pole to 70°.

TABLE III

MAXIMUM SCALE DISTORTION IN A TRANSVERSE MERCATOR PROJECTION

Latitude	Maximum Distortion - Percent
Pole	0
80°	+ 1.5
75°	+ 3.5
70°	+ 6.4

The results of the foregoing investigations immediately suggest certain facts as to the relative usability of these projections for the measurement of distances.

Since the scale of the stereographic is, at any point, the same in all directions and changes only with latitude, it is a function of the latitude and the latitude alone. It has been shown in the table for stereographic scale distortions that, considering the scale at the pole as correct, the maximum scale distortion on such a projection extending to latitude 70° occurs at the edge of the sheet and is about 3% A parallel of mean distortion may arbitrarily be assumed as having correct scale, and distance errors will be further reduced, since, in such a case, the scale error at the pole would become about minus 1½% and at latitude 70° about plus 1½% Still further refinement in distance measurement may be attained by establishing a latitude scale and using mean latitude in the measurement of distances

Considering now the polar gnomonic projection, we find from Table II that the scale at latitude 70° is 13 2% greater along the meridian and 6.4% greater along the parallel than at the pole A mean scale, somewhere in between that at the pole and that along the meridian at latitude 70°, might arbitrarily be assumed as correct, but it is obvious that a scale error of more than 6% would still exist at the two extreme latitudes of the projection. However, a more serious disadvantage than this is the fact that the scale at any point is not the same in all directions We have one scale along the meridian, another along the parallel, and an infinite number of intermediate scales at directions in between. The scale, therefore, is a function not only of latitude but also of direction Hence it is clearly impossible to establish a latitude scale, or, in fact, any scale dependent upon one variable, as in the case of the stereographic projection.

A comparison of the previous tables shows that within the latitude limits of pole to 70° the transverse Mercator has about twice the scale distortion of the stereographic and about half the distortion of the gnomonic. As in the case of the stereographic and gnomonic, it is always possible to assume the mean scale of the chart as true, thus approximately halving the extreme variation from true scale As previously shown, the scale on this projection is the same at any point in all directions but is not constant for a given latitude It is, however, dependent upon one variable alone; namely, the arc distance from the meridian of true scale It is, therefore, possible to establish a varying scale, dependent upon the distance from the meridian of true scale, as an aid to greater accuracy in distance measurement.

MEASUREMENT OF BEARINGS

The ability of a projection to maintain on the projection the same angular relationship between intersecting lines as that which exists between the same lines on the earth is obviously a valuable property in the measurement of bearings It is proposed to develop here a formula for the determination of the error in bearing, if any, on these projections, and then apply that formula to each projection at various latitudes and with various bearings, in order to obtain a comparison of their merits in this respect

Consider Fig 3.

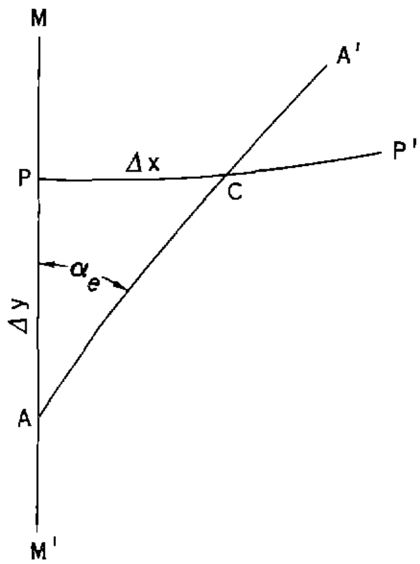

Figure 3 Bearing of Any Line on Earth's Surface

Let AA' be any line on the earth, having a bearing of α_e at meridian MM' Draw any parallel PP', intersecting AA' in C. Denote AP by Δy and PC by Δx. The average bearing, α_a, of the line AA' from A to C may be defined as the angle whose tangent is $\frac{\Delta x}{\Delta y}$, or increment along the parallel divided by increment along the meridian, or

$$\tan \alpha_a = \frac{\Delta x}{\Delta y}$$

Now let Δy approach 0, Δx will also approach 0, and α_a will approach α_e, the bearing at A, as a limit The tangent of the bearing at A will be the limit of the ratio $\frac{\Delta x}{\Delta y}$ as Δy approaches 0, or $\frac{dx}{dy}$.

Hence $\tan \alpha_e = \frac{dx}{dy}$

Now on the chart dx and dy will be represented by $S_p dx$ and $S_m dy$ respectively, where S_p = scale along the parallel and S_m = scale along the meridian at the point in question Therefore, letting the bearing on the chart be represented by α_c, we have

$$\tan \alpha_c = \frac{S_p\, dx}{S_m\, dy} = \frac{S_p}{S_m} \tan \alpha_e \qquad (5)$$

or, in words, tangent of bearing of any line on the chart equals tangent of bearing of the same line on the earth multiplied by the ratio of the scale along the parallel to the scale along the meridian at the point in question. Knowing the scales at any point by the formulae developed under the paragraph on scales, we can now investigate each projection in which we are interested as to error in bearing at any point.

Considering first the polar stereographic and transverse Mercator projections, we already have seen that at any point the scales along the meridian and parallel are equal, therefore $\frac{S_p}{S_m}$ equals unity, the bearing on the chart equals the bearing on the earth, and there is no error in bearing

In the polar gnomonic projection

$$\text{scale along the parallel} = S_p = \frac{1}{\sin \phi}$$

$$\text{scale along the meridian} = S_m = \frac{1}{\sin^2 \phi}$$

Therefore $$\frac{S_p}{S_m} = \frac{\frac{1}{\sin \phi}}{\frac{1}{\sin^2 \phi}} = \sin \phi$$

and $$\tan \alpha_c = \sin \phi \tan \alpha_e \qquad (6)$$

The error, E, in chart bearing for any given bearing α_e on the earth may then be expressed as follows

$$E = \arc \tan \quad (\sin \phi \tan \alpha_e) - \alpha_e \qquad (7)$$

By differential calculus we may now derive an expression for the bearing at which maximum error occurs for any given latitude, as follows

$$\frac{dE}{d\alpha_e} = \frac{\sin \phi \sec^2 \alpha_e}{1 + \sin^2 \phi \tan^2 \alpha_e} - 1 = 0$$

$$\sin \phi \sec^2 \alpha_e - 1 - \sin^2 \phi \tan^2 \alpha_e = 0$$

$$\sin \phi (1 + \tan^2 \alpha_e) - 1 - \sin^2 \phi \tan^2 \alpha_e = 0$$

$$\tan^2 \alpha_e (\sin \phi - \sin^2 \phi) + \sin \phi - 1 = 0$$

$$\tan^2 \alpha_e = \frac{1 - \sin \phi}{\sin \phi \ (1 - \sin \phi)} = \frac{1}{\sin \phi}$$

$$\tan \alpha_e = \pm \sqrt{\frac{1}{\sin \phi}} \qquad (8)$$

The table immediately following shows the error in bearing on the polar gnomonic chart at selected latitudes and with selected bearings from 0° to 180° either east or west from the polar direction While ordinarily the lowest latitude used on a polar chart would not be less than 70°, figures are given for latitude 60° in order to portray the rapid increase in error of bearing as distances from the pole increase. Fig 4 gives a graphical representation of the data in the table

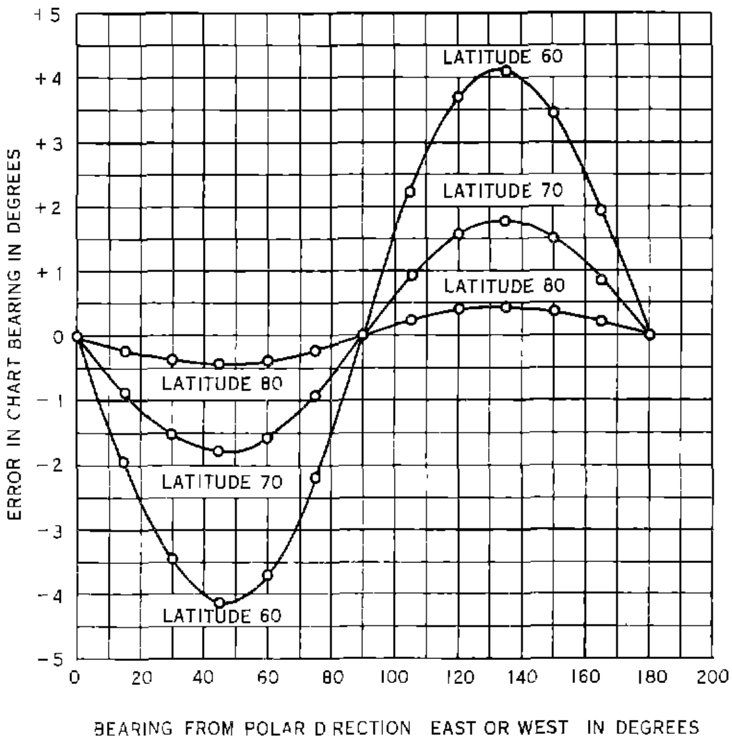

Figure 4 Error in Bearing on a Polar Gnomonic Projection

TABLE IV

ERRORS IN BEARING ON A POLAR GNOMONIC CHART

	True Bearing on Earth - East or West From Polar Direction												
	0°	15°	30°	45°	60°	75°	90°	105°	120°	135°	150°	165°	180°
Latitude	Minus Error							Plus Error					
Pole	No error												
80°	0	13'	23'	26'	23'	13'	0	13'	23'	26'	23'	13'	0
70°	0	52'	1°-31'	1°-47'	1°-34'	55'	0	55'	1°-34'	1°-47'	1°-31'	52'	0
60°	0	1°-56'	3°-26'	4°-06'	3°-41'	2°-12'	0	2°-12'	3°-41'	4°-06'	3°-26'	1°-56'	0

If formula (8) is used to compute the direction of bearing for maximum error and formula (7) the amount of maximum error, the results in Table V are obtained

TABLE V

MAXIMUM ERRORS IN BEARING ON A POLAR GNOMONIC CHART

Latitude	Bearing of Maximum Error	Amount of Error
80°	45° - 13' & 134° - 47'	-26' & +26'
70°	45° - 53' & 134° - 07'	-1°-47' & +1°-47'
60°	47° - 04' & 132° - 56'	-4°-07' & +4°-07'

These results will be found to agree with results obtained from the curves plotted from selected bearings

SPECIAL PROPERTIES

Both polar stereographic and polar gnomonic projections have the desirable features that the meridians are straight lines emanating from the pole, the parallels are circles with a common center at the pole, and all meridians and parallels intersect at right angles The transverse Mercator has neither straight line meridians nor circles for parallels, although, within the limits of a polar projection, both are close approximations thereto The meridians and parallels do intersect at right angles, as is true in any conformal projection.

The gnomonic projection also has the unique property that every great circle on the earth is projected as a straight line on the chart and, conversely, every straight line on the chart is the arc of a great circle on the earth This property is of particular value in determining the great circle route between distant points, since it is necessary only to join the two points by a straight line on the chart to have the exact great circle track plotted However, this property is of value chiefly in the location of great circle tracks for transfer by latitude and longitude to other charts, where the great circle is not projected as a straight line It has already been shown that, on the polar gnomonic projection, bearings are inaccurate and scale is distorted in such a manner as to make the accurate measurement of distances impracticable.

The stereographic projection has the unique property that all circles on the earth, both great and small, are projected as circles on the chart The straight line meridians of the polar stereographic are no exception to this rule since they may be considered a family of circles with a radius equal to infinity Moreover, within the limits of a polar projection, all great circles on the stereographic closely approximate straight lines. It is proposed to develop here a formula for the curvature of the great circle on the polar stereographic projection, in order to determine exactly the amount of deviation of a great circle from a straight line

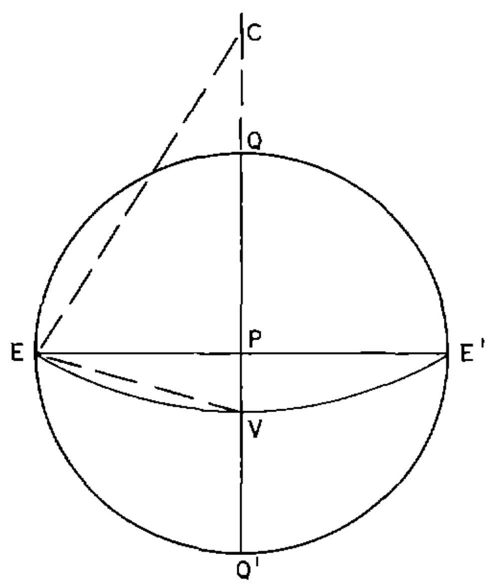

Figure 5 Radius of Curvature of the Great Circle on a Polar Stereographic Projection.

Referring to Fig 5, let EQE'Q' be the equator of the polar stereographic projection of a hemisphere, where

P = pole

PQ and PQ' = meridians 180° apart

V = vertex of any great circle

PE and PE' = meridians at 90° to the line QQ'

Now any great circle on the earth with its vertex on the meridian PQ' must pass through the equator in points separated in longitude by 90° from Q', namely, points E and E', and the circle on the projection must therefore be represented by a circular arc passing through points E, V, and E' It is required to find the radius of curvature of this circular arc

Since, as has previously been shown, in a stereographic projection all lines maintain their true angles of intersection, the line QQ' is perpendicular to the circular arc at V and the center of that arc must lie somewhere along line QQ'

Now let C be the center of the circular arc Draw CE = CV (since both are radii of the same circle), and EV Now let

R = radius of the earth at the true projection scale at the pole

ϕ = latitude of the vertex, V

R_c = radius of curvature of the arc EVE'

In the paragraph on scales it was shown that the distance on the chart, S_c, from the pole along any meridian could be expressed as follows

$$S_c = 2R \tan\left(\frac{\frac{\pi}{2} - \phi}{2}\right)$$

Reducing

$$S_c = \frac{2R \sin\left(\frac{\pi}{2} - \phi\right)}{1 + \cos\left(\frac{\pi}{2} - \phi\right)} = \frac{2R \cos\phi}{1 + \sin\phi}$$

Hence, the distance PV to any vertex V in latitude ϕ is equal to $\dfrac{2R \cos\phi}{1 + \sin\phi}$

and the distance PE to point E where ϕ is 0 is equal to 2R.

In triangle EPV

$$\tan PVE = \frac{PE}{PV} = \frac{2R}{\dfrac{2R \cos\phi}{1 + \sin\phi}} = \frac{1 + \sin\phi}{\cos\phi}$$

and $$EV = \sqrt{\overline{PE}^2 + \overline{PV}^2} = \sqrt{4R^2 + \frac{4R^2 \cos^2\phi}{(1 + \sin\phi)^2}}$$

$$= \sqrt{4R^2 + \frac{4R^2 \ (1 - \sin^2\phi)}{(1 + \sin\phi)^2}} \quad = \quad \sqrt{4R^2 + \frac{4R^2 \ (1 - \sin\phi)}{1 + \sin\phi}}$$

$$= \sqrt{\frac{8R^2}{1 + \sin\phi}} \quad = \quad 2R\sqrt{\frac{2}{1 + \sin\phi}}$$

In the isosceles triangle ECV

$$\sec \ PVE = \frac{CV}{\frac{1}{2}\ EV}$$

or $\sqrt{1 + \tan^2 PVE} = \dfrac{R_c}{\frac{1}{2}\ EV}$

Substituting for tan PVE and EV their above determined values, we have

$$\sqrt{1 + \left(\frac{1 + \sin\phi}{\cos\phi}\right)^2} = \frac{R_c}{\frac{1}{2}\left(2R\sqrt{\frac{2}{1 + \sin\phi}}\right)}$$

$$R_c = R\sqrt{\frac{2}{1 + \sin\phi}\left[1 + \left(\frac{1 + \sin\phi}{\cos\phi}\right)^2\right]}$$

$$= R\sqrt{\frac{2 \ (2 + 2\sin\phi)}{(1 + \sin\phi)\cos^2\phi}}$$

$$= R\sqrt{\frac{4 \ (1 + \sin\phi)}{(1 + \sin\phi)\cos^2\phi}}$$

$$= \frac{2R}{\cos\phi} \qquad (9)$$

By inspection of this formula it is evident that, when $\phi = 0$ and the great circle is the equator the radius of curvature on the projection is 2R, when $\phi = 90°$ and the great circle is a meridian the radius of curvature is ∞ and the great circle is represented by a straight line, and, between these two values of ϕ, R_c varies as the reciprocal of the cosine of the vertex latitude. It is thus established that the more nearly the vertex of the great circle approaches the pole, the greater the radius of curvature and the more nearly the great circle on the chart approaches a straight line. Since the only great circles applying to a polar chart are those whose vertices lie in high latitudes, all great circles that can be drawn on a polar chart will closely approximate straight lines - how closely is indicated in Table VI, where, for various vertex latitudes, the radius of curvature and the maximum departure of the great circle from a straight line on a 2000 mile course is tabulated.

TABLE VI

GREAT CIRCLE PROPERTIES ON POLAR STEREOGRAPHIC CHARTS

Latitude of Vertex	R_c Expressed as a Coefficient of R	Maximum Departure from a Straight Line in Nautical Miles on a 2000 Mile Course
90°	∞	0
85°	23 0	6
80°	11 5	13
75°	7.7	19
70°	5 8	26

It should now be apparent that if great refinement is desired in plotting a great circle track on a polar stereographic projection, it can be secured by means of a series of circular curves of properly varying radii Each curve would apply to a great circle of a particular vertex latitude, so that it would be necessary to know only the vertex latitude of the great circle in question to choose the curve to be used No computation for vertex latitude would be necessary, since the circle so closely approximates a straight line in polar latitudes, it would be necessary only to join points of departure and destination by a straight line and use the vertex latitude thus indicated - the difference between this latitude and the actual vertex latitude would be so slight as to be negligible so far as choosing the proper curve is concerned Once the proper curve is chosen, the exact great circle may be as easily drawn on this projection as with a straight edge on the polar gnomonic projection It should be noted here that when changing from a projection on one scale to that on another the value of R and hence of R_c changes Therefore, while a series of curves might be varied enough to apply to a number of differently scaled projections, different curvatures would necessarily be used for the same vertex latitude, depending upon the base scale of the projection.

The great circle on the transverse Mercator projection is not a straight line (except in a few special instances), nor is it a circular curve. It is a curve of constantly varying radius, the amount of curvature at any point being a function of its direction with reference to the previously mentioned series of false meridians. The transverse Mercator also loses the straight line rhumb property of the ordinary Mercator projection

CONSTRUCTION

The polar stereographic and gnomonic are both geometric projections and may be constructed either by geometric construction or by computing the formulae for S_c (chart distance from the pole along the meridian), given in the paragraph on scales There is little to choose between the two in ease of construction The straight line meridians and series of concentric circles for parallels are ideally suited for rapid and accurate construction

The transverse Mercator is considerably more difficult to construct It is first necessary to construct the ordinary Mercator projection (or at least assume such a graticule of meridians and parallels as would obtain in the ordinary Mercator), and then use this coordinate system as a basis for plotting the meridian and parallel intersection points of the transverse Mercator The values to be used in plotting may be obtained from the table "Transformation from Geographical to Azimuthal Coordinates," found in Special Publication No. 67 of the U S Coast and Geodetic Survey. After sufficient points have been plotted, the meridians and parallels may be drawn by joining appropriate points with a smooth curve

RESUME

It may be helpful, for purposes of comparison, to tabulate, in juxtaposition, the facts regarding these three projections Following is such a tabulation.

PROPERTIES BETWEEN POLE AND LATITUDE SEVENTY
of

Polar Stereographic Projection	Polar Gnomonic Projection	Transverse Mercator Projection
Meridians are straight lines and parallels concentric circles, intersecting at right angles.	Meridians are straight lines and parallels concentric circles, intersecting at right angles.	Meridians and parallels are curves of varying radius. Within polar limits, meridians approximate straight lines and parallels approximate circles Intersect at right angles
Conformal	Not Conformal	Conformal
Scale varies with latitude only. Scale distortion varies from 0 at pole to +3.1% at latitude 70°	Scale varies with both latitude and direction. Scale distortion varies from 0 at pole to a maximum of +13.2% at latitude 70°.	Scale varies with arc distance from meridian of true scale only. Scale distortion varies from 0 along this meridian to +6.4% at arc distance of 20° from this meridian.
Bearings of all lines are correct.	Bearings are inaccurate Errors increase with polar distance and also depend on direction. Maximum error of 1°47' occurs in certain directions at latitude 70°.	Bearings of all lines are correct
All circles, both great and small, projected as circles. Great circle closely approximates straight line in polar regions. Radius of curvature is function of vertex latitude and may be easily obtained for great circles on polar charts No great circle touching latitude 70° or above departs from straight line by more than 26 miles in a 2000 mile course.	Great circle is a straight line and all straight lines are arcs of great circles	Great circle is a curve of varying radius (except where a straight line in a few special instances). Transverse Mercator loses the property of the straight line rhumb of the ordinary Mercator
Easy to Construct	Easy to Construct	Difficult to Construct