RMS VALUE OF HORIZONTAL FIELD PATTERNS OF UHF TWO-COURSE RADIO RANGE

By

H W Kohler

Technical Development Division

Technical Development Note No 37

June 1945

U S DEPARTMENT OF COMMERCE

CIVIL AERONAUTICS ADMINISTRATION

WASHINGTON, D C

UNITED STATES DEPARTMENT OF COMMERCE

HENRY A WALLACE, *Secretary*

CIVIL AERONAUTICS ADMINISTRATION

THEODORE P WRIGHT, *Administrator*

TABLE OF CONTENTS

FIGURE INDEX

CA INFORMATION AND STATISTICS

RMS VALUE OF HORIZONTAL FIELD PATTERN OF
UHF TWO-COURSE RADIO RANGES

INTRODUCTION

Most ultra-high-frequency two-course range antenna arrays consist of three loop antennas located in a horizontal plane The centers of the loop antennas lie on a straight line, and the outside antennas are spaced S degrees from the center antenna The currents in the outside antennas are equal in magnitude and opposite in phase The current in the center antenna is k times the current in each side antenna and is given the phase angle 0° The phase angles of the currents in the outside antennas thus are +90° and −90°, respectively With this layout the horizontal field pattern is a function of two variables, antenna spacing S and current ratio k The field strength in a given direction θ can be plotted as a function of one of the variables, the other being a parameter Of special interest are the field strengths on-course, at 90° off-course, and the maximum field strength plotted versus S or k Field strengths obtained on this basis are not directly comparable since different amounts of power are radiated

The purpose of this Note is to determine the factors by which the field strengths, obtained with the above method, have to be multiplied if the radiated power in the horizontal plane is to remain constant as k and S are varied *

Determination of the total radiated power would involve integration of Poynting's vector over the hemisphere above ground, centered with the array This difficult operation can be avoided by considering the field strength in the horizontal plane only

What may appear to be a rough method of arriving at the total radiated power is, from a practical point of view, quite satisfactory Due to the counterpoise, relatively little high-angle radiation is obtained in the radio ranges considered, and for aircraft more than a few miles distant from the range station, the horizontal field strength is a fair approximation to the prevailing field strength The variation of field strength due to the interaction of direct and reflected waves does not need to be considered here

DETERMINATION OF RMS VALUE OF HORIZONTAL FIELD PATTERN

The rms value of field intensity F_{rms} taken around a complete circle in the horizontal plane is defined as

$$1)\quad F_{rms} = \left[\frac{1}{2\pi}\int_{-\pi}^{\pi} F^2(\theta)\, d\theta\right]^{1/2}$$

where θ is the azimuth angle and $F(\theta)$ is the field strength in the direction θ It is readily seen that F_{rms} is proportional to the area of the horizontal field pattern, and, of course, to the power radiated through a cylindrical surface of small height normal to the horizontal plane One method of finding F_{rms} would be to plot the horizontal field patterns for various value of S and k and to determine the

---

* It is recognized that from a physical point of view it would be better to speak of the radiated power near the horizontal plane By "radiated power in the horizontal plane" is meant the power passing through a cylindrical surface of unit height extending at right angles above and below the horizontal plane

area of the patterns with a planimeter The method for finding F_{rms} proposed here is much faster since it merely involves evaluation of a simple formula

The derivation given below includes the effect of parasitic currents (of magnitude p in each side antenna and phase angle δ with respect to the current in the center antenna) on the rms value of the horizontal field strength

The relative field strength in the horizontal plane in the direction θ is given by equation 2 below The azimuth angle θ is 0° or 180° on-course, the course is at right angles to the line connecting the centers of the three loop antennas

2) $F(\theta) = k + 2 \sin (S \sin\theta) + 2p \cos (S \sin\theta) \cos\delta$

$= F_1(\theta) + F_2(\theta) \cos\delta$

where $F_1(\theta) = k + 2 \sin (S \sin\theta)$

$F_2(\theta) = 2p \cos (S \sin\theta)$

3) $F^2(\theta) = F_1^2(\theta) + F_2^2(\theta) + 2F_1(\theta)F_2(\theta) \cos\delta$

4) $F_1^2(\theta) = k^2 + 2 - 2 \cos (2 S \sin\theta) + 4 k \sin (S \sin\theta)$

5) $F_2^2(\theta) = 2p^2 + 2p^2 \cos (2 S \sin\theta)$

6) $F_1(\theta)F_2(\theta) = 2p k \cos (S \sin\theta) + 2p \sin (2 S \sin\theta)$

Introducing equations 4, 5, and 6 into 3 and 1 we find integrals of the types

7) $\frac{1}{2\pi}\int_{-\pi}^{\pi} \sin (S \sin\theta) \, d\theta = 0$

and

8) $\frac{1}{2\pi}\int_{-\pi}^{\pi} \cos (S \sin\theta) \, d\theta = J_0(S)$

where $J_0(S)$ is the Bessel function of the first kind, of order zero.

The rms value of horizontal field strength is found to be

9) $F_{rms} = \left[k^2 + 2(1 + p^2) - 2(1 - p^2) J_0 (2S) + 4 p k \cos\delta \, J_0 (S)\right]^{1/2}$

In the absence of parasitic currents p = 0 and

10) $F_{rms} = \left[k^2 + 2 \left\{ 1 - J_0 (2S) \right\}\right]^{1/2}$

The limiting cases are of interest in checking equations 9 and 10 Let the spacing S be zero

$J_0 (2S) = 1$ $\qquad$ $J_0 (S) = 1$

From equation 10 we obtain

$F_{rms} = k$

since the equal and opposite currents in the side antennas cancel each other as S becomes zero

In the case with parasitic currents, and with spacing $S = 0$ (equation 9)

$$F_{rms} = \left[k^2 + (2p)^2 + 4\ p\ k \cos \delta \right]^{1/2}$$

This is the rms value of a current k plus a current 2p in the same antenna, having phase angle δ between each other

Assume that F_{rms} has been calculated for two cases of antenna spacings and current ratios, as given below (assume parasitic currents zero)

11) $F_{1\ rms} = \left[k_1^2 + 2 - 2\ J_o\ (2S_1) \right]^{1/2}$

12) $F_{2\ rms} = \left[k_2^2 + 2 - 2\ J_o\ (2S_2) \right]^{1/2}$

To find the scale factor m by which the field pattern equation for case 2 has to be multiplied so that the radiated power in the horizontal plane is equal to that of case 1, take the ratio

13) $$\frac{F_{1\ rms}}{F_{2\ rms}} = \left[\frac{k_1^2 + 2 - 2\ J_o\ (2S_1)}{k_2^2 + 2 - 2\ J_o\ (2S_2)} \right]^{1/2} = m$$

and use the equation

14) $F_2'(\theta) = m\ F_2\ (\theta) = m \left[k_2 + 2 \sin\ (S_2 \sin\ \theta) \right]$

where $F_2'\ (\theta)$ is the "adjusted" field strength in the direction θ, and $F_2\ (\theta)$ is the field strength for case 2 prior to adjustment for equal radiated power

If $F_{1\ rms}$ and $F_{2\ rms}$ in equations 11 and 12 are set equal, the following relation for patterns having the same radiated power in the horizontal plane is found

15) $1/2\ (k_1^2 - k_2^2) = J_o(2S_1) - J_o(2S_2)$

APPLICATION OF THEORY

Assume a reference pattern with spacing $S = 120°$ and current ratio $k = 2$. Keeping the same antenna spacing, vary k from 1 0 to 2 4 and plot maximum field strength, field strength on course and at 90° off-course for constant radiated power in the horizontal plane For comparison, show the same quantities not adjusted for constant radiated power

In figure 1 the calculated field strengths are shown The dashed lines are for the radiated power in the horizontal plane when the current in the side antennas is held constant and the current in the center antenna is varied and are given by

$F_{max} = k + 2$

$F(0°\ ) = k$

$F(90°) = k + 1\ 732$

The "adjusted" field strengths (solid curves) are calculated from equations 13 and 14

For $S = 120°$

$2S = 240° = 4\ 19$ radians

$J_o\ (2S) = J_o\ (4\ 19) = -0\ 378$

$J_0(x)$ is plotted in the range from $x = 0$ to 7 0 in figure 2

For the reference pattern ($k = 2$ 0)

$F_{rms} = 2.6$ and $m = 1$ 0

Consideration of the curves shows that with constant radiated power in the horizontal plane, the maximum field strength and the field strength at right angles to the course remain rearly constant in the practical range of values of k The field strength on-course, however, decreases as k decreases, but less than in direct proportion.

CONCLUSIONS

It is interesting to note that when keeping the power radiated in the horizontal plane constant, the maximum field strength and the field strength at 90° off-course remain constant to within 2 per cent in the practical range of k values On the contrary, the field strength on-course decreases appreciably as k decreases In the example considered in figure 1, the field strength on-course varies from 1 34 when $k = 1$, to 2 12 when $k = 2$ 4 with constant radiated power However, if the radiated power is not held constant, the field strength on-course varies from 1 0 to 2 4 for the same values of k

Therefore, for proper comparison of field strengths on-course with various values of S and k, the calculations must be made for constant radiated power in the manner described above

It will be noted that the scale factor m does not affect course sharpness or clearance

REFERENCE Karl Spangenberg, "Charts for the Determination of the Root-mean-square Value of the Horizontal Radiation Pattern of Two-element Broadcast Antenna Arrays "

Proc IRE 30, 237, 1942 (May)

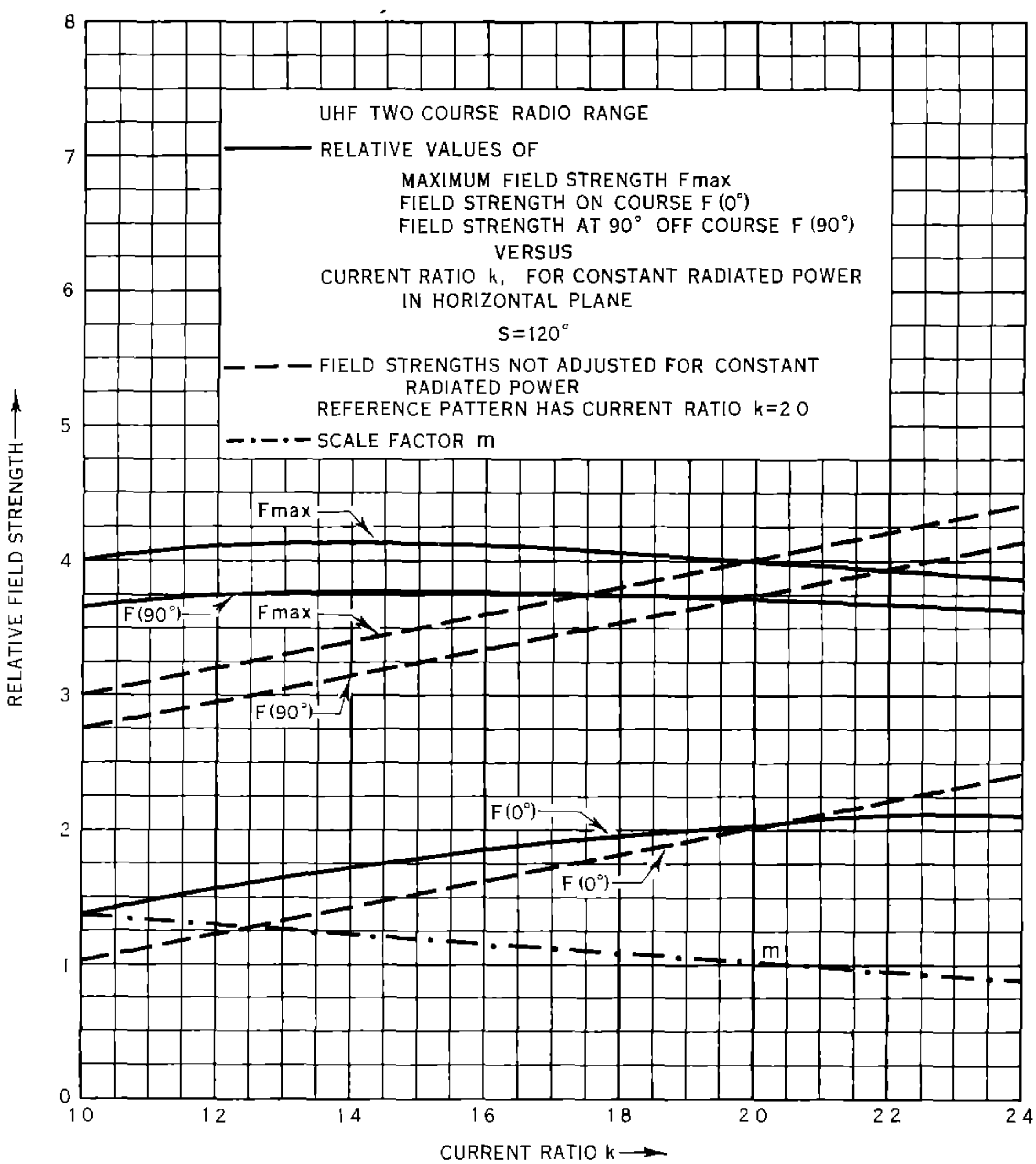

Figure 1 Relative Value of Maximum Field Strength F max , Field Strength on Course F (0°), Field Strength at 90° Off Course F (90°) versus Current Ratio k for Constant Radiated Power in Horizontal Plane.

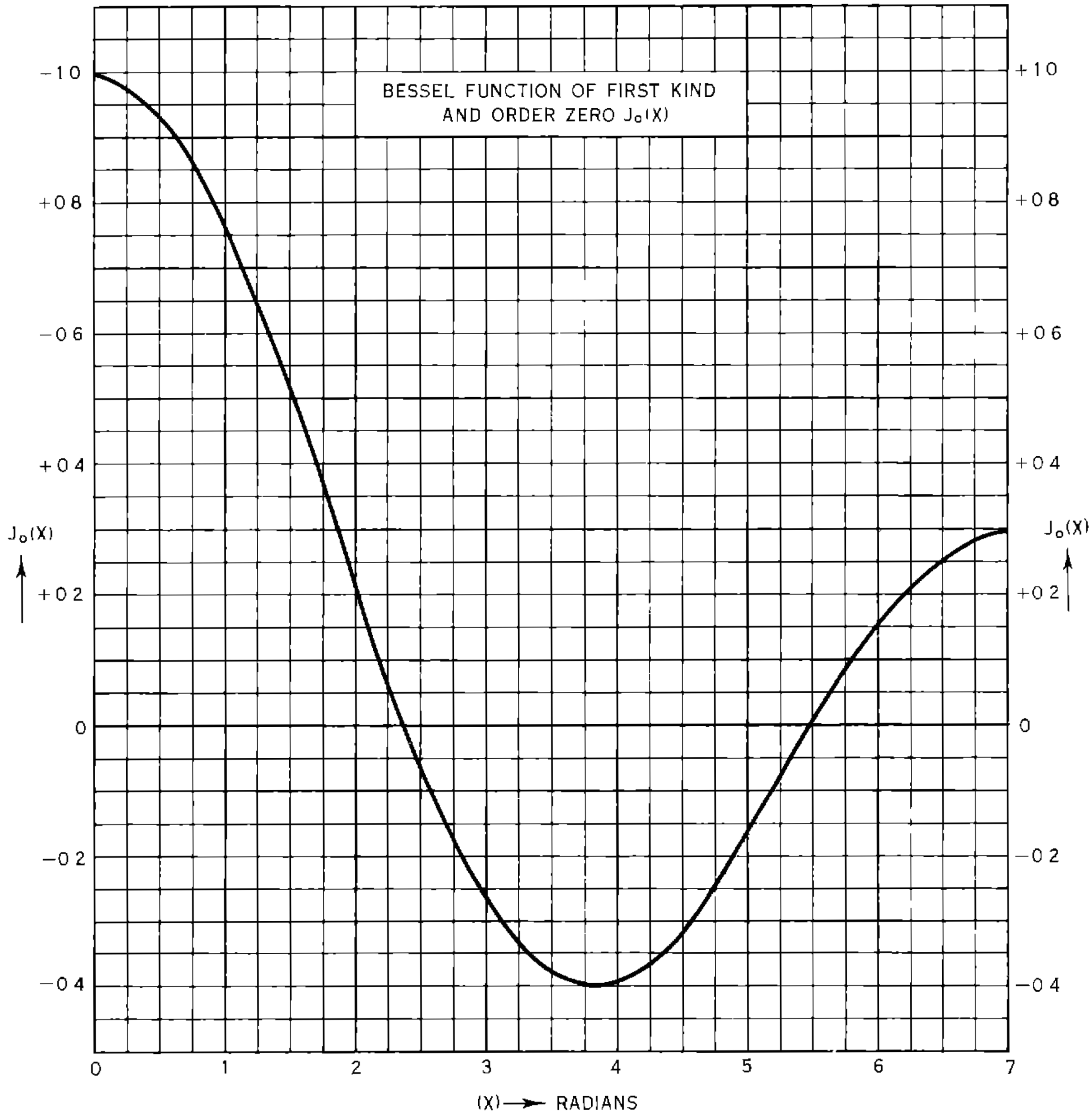

Figure 2 Bessel Function of First Kind and Order Zero $J_0(x)$