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# Multi-stage Models for Dynamic Ride-Sharing in Taxi Services and Congestion Analysis

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# Disclaimer

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16. Abstract

This research introduces practical optimization model for implementing ride-sharing in taxi services and studies the effects of ride-sharing on the congestion status through the case study of Chicago. Ride-sharing combines trips into one ride-shared trip with the objective of maximizing the total mileage saving. This research proposes a multi-stage model to optimize rider matches, aiming to reduce the total travel distance and enhance the matching of multiple riders. To validate the effectiveness of the model, real taxi data from Chicago is used, demonstrating significant improvements in distance reduction. Next, this study conduct congestion analysis by investigating the differences in congestion before and after the implementation of ridesharing mode. The traffic state is assessed through the computed congestion index before being graphically represented on congestion maps. After comparing the congestion map of community areas and census tracts before and after ride-sharing, we conclude that ride-sharing can improve the overall congestion status at the city level. This is particularly crucial given the increasing public awareness of environmental issues and the need for sustainable transportation solutions in cities

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# Executive Summary

Large and growing urban areas are facing an escalation of traffic congestion and all its consequences in terms of overall delay experienced by travelers. Ridesharing is intuitively a practical solution to improve the congestion issues in our modern cities. The largely unused capacity of the private vehicles would be utilized in place of other vehicles to satisfy the mobility needs of individual passengers.

This project analyzes and quantifies the effect on congestion reduction of a large-scale application of ridesharing mode to serve metropolitan transportation demand of taxi services, which have been shown to be often unproductive in the taxi capacity. Specifically, we are evaluating the benefit of converting a sizeable portion of taxi rides from conventional direct point-to-point service to a ridesharing modality. This can potentially lead to a significant reduction of vehicles on the network and consequently ease out congestion at the expense of the service level experienced by ride-sharing participants.

This study introduces a practical and achievable model that systematically matches multiple riders, thereby enhancing taxi operation performance. The long-term goal is to attract the interest of taxi companies and government agencies to promote ride-sharing with multiple riders as a strategy. This is particularly important given the growing public awareness of environmental issues and the need for sustainable cities.

This project proposes a multi-stage ride-sharing model to pair riders. Afterward, congestion analysis is adopted to qualify the benefit from the whole city view.

## Multi-stage Ride-Sharing Models

This study uses insertion methods to identify potential matches and establish the sequence of pick-up and drop-off points, transforming the ride-sharing problem into a network optimization problem. Three elements are considered to define a feasible match, including (i) distance savings, (ii) drivers' parking time and riders' waiting time, and (iii) riders' detour tolerance. Afterward, we employ the greedy algorithm and optimization approach to determine the selection of links in the network. To accommodate more than 2 riders in a match, we propose a multi-stage model to merge unpaired riders into existing pairs. The matching process is illustrated in Figure 1.

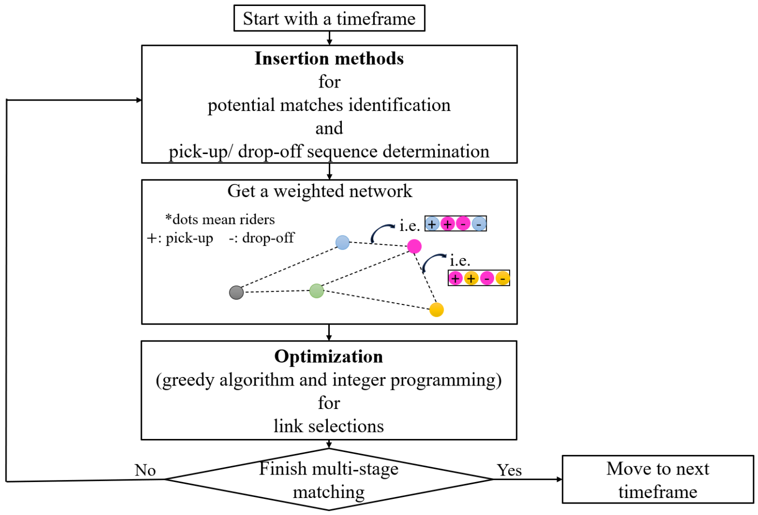



**Figure 1. The procedure of the proposed multi-stage model**

## Congestion Analysis

Through the congestion analysis, the traffic congestion status of the City of Chicago for each 15-minute interval before and after the ride-sharing is assessed. We compute the integrated congestion index based on the mean velocity and congested vehicle miles traveled (VMT) ratio. The congestion index for this project is 0 for “Heavy Congestion”, 1 for “Medium Congestion”, 2 for “Light Congestion”, and 3 for “No Congestion”.

The mean velocity of an area is computed by (i) extracting possible areas crossed by a vehicle for each trip, (ii) weighting the mean velocity of trips based on trip duration and trip distance, and (iii) adjusting the mean velocity of an area based on past mean velocity and its neighboring areas’ mean velocities through the least square model.

Unlike mean velocity, the congested VMT ratio is computed for the entire city of Chicago and for each time period (morning non-rush hour, morning rush hour, afternoon non-rush hour, afternoon rush hour, and night). As the segment with a speed greater than 20 mph is considered “no congestion” or “light congestion”, congested VMT is VMT of trips with a speed less than 20 mph. The congested VMT ratio is then the ratio of congested VMT and total VMT.

With fuzzy logic and membership function, the membership degree to each congestion index is defined based on mean velocity and congested VMT ratio separately. The combined membership degree for each congestion index is then computed as the weighted sum of the membership degrees. Since the mean velocity has a higher spatial and temporal precision, greater weight is given to the membership degree for the mean velocity. The integrated congestion index is chosen by finding the congestion index with the largest combined membership degree. Based on the integrated congestion index, the congestion map of the city of Chicago is plotted.

The promotion of ride-sharing can contribute to the creation of a sustainable transportation solution. By following these guidelines, our goal is to encourage widespread adoption, alleviate traffic congestion, and play a role in shaping efficient and equitable transportation systems.

1. Incentive Schemes: Rider matching operates on the basis of distance savings. Therefore, if these savings can be translated into monetary rewards for riders, it would likely enhance their willingness to opt for ride-sharing.

2. Public Awareness Campaigns: The government can initiate awareness campaigns, educating the public about the advantages of ridesharing and addressing concerns related to cost savings, environmental impact, and diminished traffic congestion.

3. Integration with Public Transportation: The taxi companies can promote a smooth blend of ridesharing services with public transportation, offering commuters seamless and convenient multi-modal travel choices.

# 1. Introduction

The increasing number of vehicles in cities has led to significant traffic congestion and environmental concerns (W. Chen et al., 2022; Henao & Marshall, 2017; Z. Wang et al., 2022). To address the issue, ride-sharing has emerged as a promising solution by utilizing the unused capacity of vehicles and reducing trip occupancy on roads (Gurumurthy & Kockelman, 2018; Tirachini, 2020). Ride-hailing services such as UberPOOL and Lyft Shared have introduced ride-sharing with a maximum of 2 riders in a pairing group (Schofer & Mahmassani, 2016). Nevertheless, there is potential to enhance the operational efficiency of ride-sharing services by allowing multi-rider matching, accommodating up to 4 participants in a trip (Tirachini & Gomez-Lobo, 2020). Despite this potential, the models for scheduling multiple passengers in a trip have not been extensively explored.

Insertion methods have been used in dial-a-ride problems, ride-hailing, and ride-sharing services (Coslovich et al., 2006; Diana & Dessouky, 2004; Jaw et al., 1986). Dial-a-ride problems focus on pre-booked and scheduled services with specific destinations or specialized needs while ride-hailing and ride-sharing services involve on-demand service for pairing drivers and riders. In ride-sharing services, the insertion methods can be applied to two cases regarding determined drivers:

(i) In the case of pre-determined drivers, the vehicles have specific orientations and destinations. The new riders are inserted along the driver's trip route, which resembles carpooling.
(ii) In non-predetermined cases, each demand can act as a potential driver, providing the schedulers with more flexibility.

However, in ride-sharing taxi services, there are more permutations of pick-up and drop-off sequences during a journey, as the initial and final positions may not be filled by the same rider. Moreover, the satisfaction of all the participants within a trip should be considered, adding more complexity to the matching process.

Some studies have adopted agent-based approaches to match passengers and drivers (Engelhardt et al., 2019; Fagnant & Kockelman, 2018; Fiedler et al., 2018; Zhu & Mo, 2022; Alonso-Mora et al., 2017). Simulation experiments have been conducted to observe overall ride-sharing performance, but they cannot reflect the optimal matches in systematical views. Additionally, such methods follow the matching rules specified by researchers, such as prioritizing the pick-up points close to his/her neighborhood (Guasch et al., 2015), which can lead to a more user-oriented result. Another paper studies the On-Demand Multimodal Transit system with bus and shuttle modes to serve the first/last miles to/from the hubs (Auad-Perez & Van Hentenryck, 2022). On the other hand, optimization approaches have the capability to methodically explore and identify optimal solutions by considering objectives such as reducing overall travel distance and passenger waiting time, as well as incorporating constraints like passengers' time windows and taxis' capacities.

While previous studies have focused on matching riders with drivers, there has been limited exploration of matching multiple riders within a single match. Additionally, the lack of real-world case studies has limited the testing of model efficiency. This study formulates the ride-sharing problem as a multi-stage model with the objective of grouping multiple riders while minimizing the total travel distance. User-centric factors, such as riders' waiting time and tolerable detours, are considered for potential match identification, while insertion methods and optimization techniques are both adopted to identify potential matches and determine the final matches. The proposed model is further verified by taxi data from Chicago to examine model performance and solving efficiency.

The purpose of this study is to introduce a practical and achievable model that systematically matches multiple riders, thereby enhancing taxi operation performance. In view of the growing public awareness of environmental issues and the need for sustainable cities, we hope to attract the interest of taxi companies and government agencies to promote ride-sharing with multiple riders as a strategy.

The rest of the paper is structured as follows. Section 2 reviews the methodology of rider-pairing in the relevant literature. In Section 3, we propose a multi-stage optimization model to match multiple riders within a match. In Section 4, the proposed model is examined with a case study, followed by sensitivity analysis and performance comparisons to previous models. Finally, we conclude the paper by summarizing the research findings and highlighting the contributions made by our study.

# 2. Literature Review

## Insertion Methods

Insertion methods have been widely adopted to explore potential combinations of pick-up and drop-off sequences for multiple riders in a pair. Researchers utilize these methods to select superior journeys by enumerating feasible combinations. For example, Jaw et al. (Jaw et al., 1986) proposed a heuristic algorithm to insert new requests based on the minimum incremental cost for dial-a-ride problems. To avoid unbeneficial pairing, Diana et al. (Diana & Dessouky, 2004) further introduced the concept of "regret" in insertions, prioritizing riders with a lower chance of being unplugged from current routes. However, It is important to note that the model used for dial-a-ride problems may not be directly applicable to ride-sharing problems because of different matching criteria: dial-a-ride focuses on minimizing incremental cost while ride-sharing emphasizes positive distance savings.

In the context of ride-hailing and ride-sharing, the insertion method can be employed as an initial step to create an attainable solution, and subsequently refined using various techniques (Feo & Resende, 1995). Herbawi and Weber (Herbawi & Weber, 2012) formulated dynamic ride-sharing models by considering passengers' time windows, drivers' maximum distance, and the maximum detour time for ride-sharing passengers. The insertion method was adopted to randomly generate a feasible initial solution, and the genetic algorithm was employed later to exchange the riders between matches, further improving the performance. Horn (Horn, 2002) exhaustively searched all potential combinations of pickup and drop-off points in the beginning to generate a viable solution; then, the authors used re-location and re-swap methods to enhance the result. Santos & Xavier (Santos & Xavier, 2013, 2015) randomly selected a vehicle and updated its route by inserting new requests based on the greedy algorithm, obtaining an initial result. They further adopted the local search to exchange passengers between pairs for further improvement. However, relying solely on random processes for searching and improving the solution may not ensure consistent enhancements. Additionally, practitioners may face difficulties when attempting to replicate such random processes in real-world scenarios.

In a different approach, Campbell and Savelsbergh (Campbell & Savelsbergh, 2004) decomposed the inventory problem into two stages: grouping customers in the first phase and determining routing among groups using insertion methods in the second phase. Ma et al. (Ma et al., 2013, 2015) proposed a taxi-searching algorithm that dynamically generated a list of candidate taxis for new requests and used the minimum increase in travel distance as the basis for inserting new requests into the taxis' schedules. However, either grouping riders or designating candidate taxis based on pick-up locations may limit the probabilities of different matching in ride-sharing systems.

To sum up, previous studies have not extensively investigated the use of insertion methods for matching multiple riders in ride-sharing. This study aims to fill this research gap by exploring all possible permutations of pick-up and drop-off points, specifically focusing on unique 2-rider to 2-rider pairs merging. Our objective is to assess the feasibility and effectiveness of exhausting insertion in terms of improving efficiency and performance in ride-sharing systems.

## Optimization Techniques

Optimization approaches aim to generate system-optimal solutions, where the objectives can be maximizing total distance savings, minimizing system-wide travel times, maximizing the participant number, etc. (Agatz et al., 2012; H. Wang & Yang, 2019). These approaches can also consider participants' willingness to exclude undesirable matches of riders and drivers (X. Wang et al., 2018; Yousaf et al., 2014).

To improve computational efficiency, a screening test is frequently employed to identify potential matches among riders. For instance, Agatz et al. (Agatz et al., 2011) interpreted the matching of drivers and riders by interpreting it as a bipartite graph (bigraph), where the links stand for potential matches while the weights on links symbolized distance savings. Thereafter, the authors maximize the weights without covering any node more than once based on the greedy algorithm and integer programming. Stiglic et al. (Stiglic et al., 2015) defined a feasible match as a combination of a driver, a rider, and a meeting point, limiting the matching of one driver to one rider to achieve the optimal solution. Arslan et al. (Arslan et al., 2019) applied the bigraph to the delivery problem, assigning the delivery routes to drivers. Likewise, the authors solved the problem by introducing the constraint of matching a single parcel to a single driver. Regarding the studies discussed above, the participants are often limited to one rider, while multiple riders in a match should be considered to improve operational efficiency.

In view of the lack of multiple riders, Lu et al. (Lu et al., 2022) formulated the pairing problem without pre-designating drivers to obtain one-to-one matches. Subsequently, the authors added extra passengers to the optimal result using the insertion method. Kucharski and Cats (Kucharski & Cats, 2020) introduced a complete ExMAS algorithm where the matching of riders was presented in a directed shareability multi-graph. Based on the graph, the authors looped over the riders following the directed edges to extend the passengers in a match. However, concerning the sequence of the pick-up/ drop-off positions, the joining riders are subject to FIFO or LIFO, failing to explore all potential permutations thoroughly.

To dynamically implement ride-sharing, the matching process can be implemented using the zoning approach (Lu et al., 2014; Quadrifoglio et al., 2008) and rolling horizontal framework (Sethi & Sorger, 1991). Yu and Shen (Yu & Shen, 2020) proposed a spatial and temporal decomposition heuristic method, using dynamic programming to dispatch drivers to passengers. In each time stage, the matching is based on the minimum of a weighted sum of riders’ waiting time and trip delay time. Hyland & Mahmassani (Hyland & Mahmassani, 2020) employed a dynamic framework to model shared-ride automated mobility-on-demand services, aiming to minimize curbside pickup time. The assignment decision was made in each epoch, but the formulation only allowed one user request to be assigned to a driver in a single epoch, failing to group multiple riders in a ride-sharing match.

Ride-sharing can be approached as a network problem where the goal is to select edges (representing matching) to maximize distance savings. Building upon this concept, our study aims to apply this approach to multiple riders, treating edges as pick-up/drop-off sequences and nodes as individual riders or pairs of riders. By adopting this framework, we aim to explore the potential for optimizing efficiency and maximizing the benefits of ride-sharing.

## Congestion Analysis

Many research projects predicted and estimated congestion status. The congestion status of a region, a road, or a lane can be assessed through traffic data acquired from the loop detectors (Coifman, 2003; Krause et al., 1996) or video cameras (Cho & Rice, 2006) installed on the physical roads. With technological advancement, different types of data are available to assess traffic congestion. Some past works utilized mobile phone data to estimate the congestion status (Herrera et al., 2010; Mohan et al., 2008).

Probe vehicle data or GPS trajectory data can be utilized to estimate the congestion status. Predicting and estimating congestion status with probe vehicle data has already been studied through different methods. Hofleitner et al. (Hofleitner et al., 2012) used probabilistic models, macroscopic hydrodynamic traffic theory, and Dynamic Bayesian network for the real-time estimation of the traffic congestion status. He et al. (He et al., 2017) used the "Mapping to Cells" method that constructs a spatial-temporal traffic diagram to create the maps with color based on the traffic speed. Using the statistical model and Coupled Hidden Markov Model, Herring et al. (Herring et al., 2010) examined travel time distribution and spatial distribution of vehicle locations and estimated the evolution of traffic state.

The congestion status is usually represented through the congestion index. Through congestion index, the congestion status can be assessed straightforwardly and easily even by the general public. The congestion index can be determined using different congestion metrics: travel time (Levinson & Lomax, 1996; Erdelić et al., 2021; Gore et al., 2021), travel speed (Wang et al., 2018; He et al., 2016; Toan & Wong, 2021; Zarindast, 2019), traffic flow (Wang et al., 2018; Gore et al., 2023), density (Gore et al., 2023; Toan & Wong, 2021; Saeedmanesh & Geroliminis, 2017), and other metrics. The congestion index is determined from a metric through a traffic theory concept, through fuzzy logic (Toan & Wong, 2021; Wang et al., 2018; Zhang et al., 2016), through multivariate function (Gore et al., 2023), through data-driven methodologies (Zarindast, 2019; Saeedmanesh & Geroliminis, 2017; Bao, 2019) or predetermined categories.

Some of these works considered multiple metrics to evaluate the congestion status. In other words, the congestion index is based on multiple congestion metrics. The integration of multiple congestion metrics is done based known relationship between metrics (Wang et al., 2018; Zhang et al., 2016; Bao, 2019) based on fuzzy logic approach (Toan & Wong, 2021; Wang et al., 2018), or based on clustering technique (Zhang et al., 2016; Bao, 2019).

# 3. Methodology of Multi-stage Ride-Sharing Models

## Problem Statement and Objectives

This study uses insertion methods to identify potential matches and establish the sequence of pick-up and drop-off points, transforming the ride-sharing problem into a network optimization problem. Afterward, we employ the greedy algorithm and optimization approach to determine the selection of links in the network. To accommodate more than 2 riders in a match, we propose a multi-stage model to merge unpaired riders into existing pairs. A rolling horizon framework is adopted to analyze dynamically and handle ad-hoc riders. The matching process is illustrated in Figure 1.

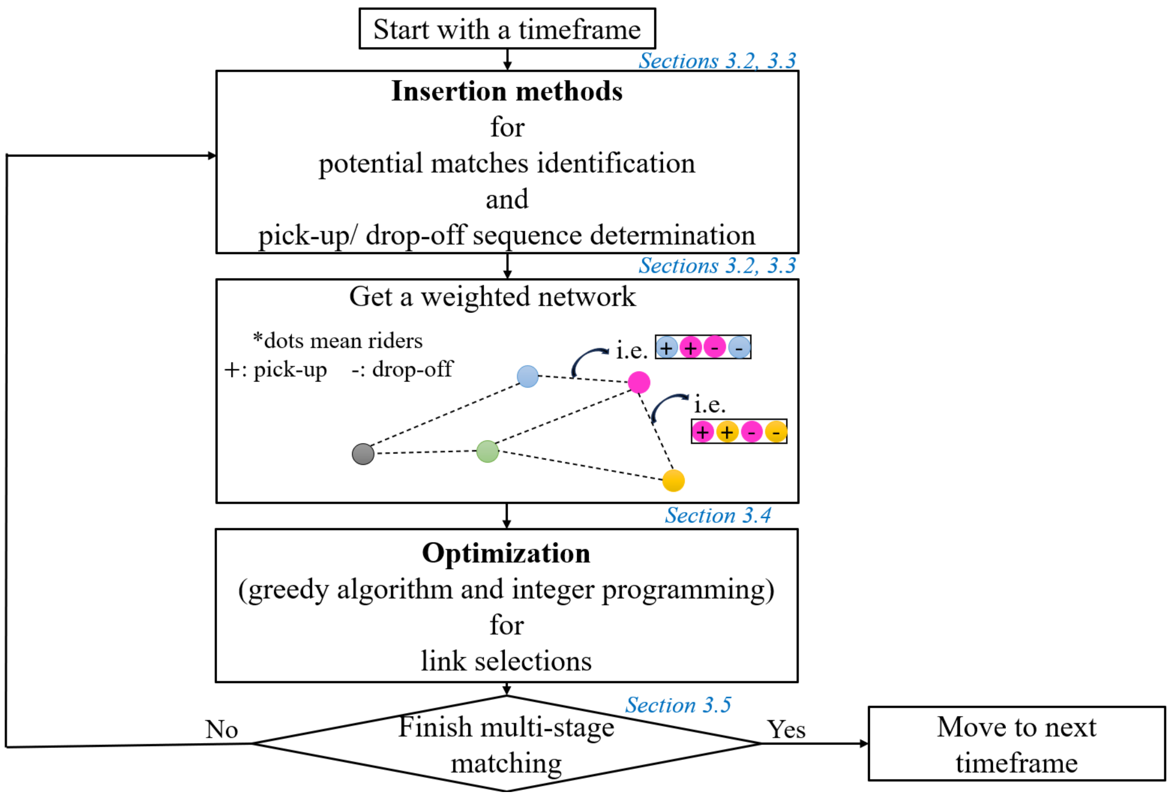



**Figure 1. The procedure of the proposed multi-stage model**

The objective of the proposed model is to reduce the overall travel distance and enhance ride-sharing matches. Three elements are considered to define a feasible match, including (i) distance savings, (ii) drivers' parking time and riders' waiting time, and (iii) riders' detour tolerance. In other words, every participant can claim their acceptable waiting time and detour degree. The assumptions adopted for the formulation are listed as follows:

(1) The supply of taxis is not considered. Taxis are assumed to be sufficient in the network and can satisfy the request of the 1st rider in a ride-sharing match within its waiting time.
(2) Participants are willing to join ride-sharing and consent to a maximum of 4 passengers in a car.
(3) If the participants cannot be paired with other people, they are assumed to travel individually.
(4) The distance is calculated based on the Manhattan distance.

## Insertion Methods and Identification of Potential Matching

A screening test is conducted to identify feasible passenger matches, and therefore, a network can be built for match selection. This study adopts insertion methods to explore all possible permutations of riders' pick-up and drop-off points. Within the proposed multi-stage model, firstly, a set of 2-rider pairs is generated. Moving on to the second stage, these 2-rider pairs are combined to form 4-rider matches. Thereafter, any remaining unpaired riders are inserted into the 2-rider matches to create 3-rider matches. Finally, those remaining unpaired riders are inserted once more to generate 4-rider matches. The potential combinations of multiple passengers are organized in Table 1.

**Table 1. The Exhaustion of Potential Pick-Up/Drop-Off Sequence for Multi-rider Matches**

| Number of riders | The potential permutation of the match |
|---|---|
| 2-rider match | Select 2 positions for one rider from 4 positions in a 2-rider match.<br>(i.e. 1+,2+,1-,2- \| 1+,2+,2-,1- \| 2+,1+,2-,1- \| 2+,1+,1-,2-)<br>( "+" means pickup and "-" means drop-off)<br>Required examined permutation numbers: $\binom{4}{2}-2=4$ |
| 4-rider match (formed by combining 2-rider pairs) | Select 4 positions for the first 2-rider pair from 8 positions in a 4-rider match. Required examined permutation numbers: $\binom{8}{4}-2=68$ |
| 3-rider match (formed by inserting unpaired riders into 2-rider pairs) | Select 2 positions for the unpaired rider from 6 positions in a 3-rider match. Required examined permutation numbers: $\binom{6}{2}-2=13$ |
| 4-rider match (formed by inserting unpaired riders into 3-rider pairs) | Select 2 positions for the unpaired rider from 8 positions in a 4-rider match. Required examined permutation numbers: $\binom{8}{2}-2=26$ |
| * For each match, two possible permutations can be eliminated where the first rider (match) occupies all the former/ latter positions and the second rider (match) takes up all the latter/ former positions. | |

Recall that three features are adopted for potential match identification: (i) distance savings, (ii) drivers' parking limitation and riders' waiting time, and (iii) riders' detour tolerance. These considerations are formulated in the following equations, where relevant notations are listed in Table 2.

$$sh\_dist_{i,j} < sg\_dist_i + sg\_dist_j \qquad \forall\, i,j \in N, i \neq j \tag{1}$$

$$w_j = Tstart_j - \left(Tstart_i + \frac{d_{i,j}}{v}\right)$$
$$if\ w_j < 0,\ \ |w_j| \leq maxW_j \qquad \forall\, i,j \in N, i \neq j \tag{2}$$
$$if w_j \geq 0,\ \ w_j \leq maxPK$$

$$k_i > \frac{sh\_ptial\_dist^{i,j}{}_i}{sg\_dist_i} \qquad \forall\, i \in N \tag{3}$$

**Table 2. Notations for Feasible Pair Identification**

| Notation | Description |
|---|---|
| $N$ | The set of riders, who are willing to join ride-sharing |
| $i, j$ | The index of riders, $i, j \in N$ |
| $sh\_dist_{i,j}$ | The total ride-sharing distance of the $i - j$ pair, $i, j \in N, i \neq j$ |
| ${sh\_ptial\_dist^{i,j}}_i$ | The partial ride-sharing distance of ride $i$ within the $i - j$ ride-sharing pair, $i, j \in N, i \neq j$ |
| $sg\_dist_i$ | The single-passenger distance of rider $i$ without ride-sharing, $i \in N$ |
| $Tstart_i$ | The expected pick-up time of ride $i, i \in N$ |
| $d_{i,j}$ | The distance between rides $i$ and $j$, $i, j \in N, i \neq j$ |
| $v$ | The taxi's speed |
| $w_j$ | The rider $j$-related waiting time, j $\in N$ |
| $maxW_j$ | The maximum waiting time for the rider $j$, j$\in N$ |
| $maxPK$ | The maximum waiting (parking) time for drivers |

Given a set of riders, $N$, we will identify the best match for riders $i$ and $j$. Equation (1) is responsible for examining beneficial pairings, meaning that the distance of ride-sharing should be shorter than the summation of individual travel distance. Equation (2) ensures no excessive waiting time in matches. In a permutation where rider $i$ is picked up first and rider $j$ is picked up later, if $w_j$ is smaller than 0 (indicating that the real pick-up time ($Tstart_j + \frac{d_{i,j}}{v}$) is later than the expected pick-up time for rider $j$), the ride $j$ should wait for the taxi coming. Hence, the waiting time, $|w_j|$, should be below his/her acceptable waiting time, $maxW_j$. On the other hand, if $w_j$ is larger than 0, the driver should wait for the rider. In general, parking space is limited in metropolitan areas, so the driver's waiting time is constrained below the maximum parking time, $maxPK$. This constraint also ensures that on-board riders do not have to wait excessively for other riders. Equation (3) handles the maximum detour a rider can endure, where ${sh\_ptial\_dist^{i,j}}_i$ is the expected travel distance of rider $i$ in ride-sharing trips while $sg\_dist_i$ is the individual travel distance without ride-sharing. The distance ratio should be smaller than a self-specified detour tolerance, $k_i$. This study recognizes the detour tolerance in spatial terms, but the detour criterion with a time-based measure is also viable (Yu & Shen, 2020).

## Waiting Time Adjustment for Multiple Riders

To extend the pairing of two-rider pairs to multiple riders in a match, Equation (2) must be further improved to calibrate the actual waiting time. As depicted in Figure 2, Riders 2 and 3 have expected pick-up times of 8:10 and 8:15, respectively. Based on Equation (2), Rider 3 anticipates that the taxi will arrive at 8:30 (8:10 + 20 min drive), resulting in a waiting time of 15 minutes. However, due to a delay in picking up Rider 2, the taxi arrives later than expected, and Rider 3 cannot be served until 8:40. As a result, Rider 3 has to wait for 25 minutes, which is longer than the initial estimation.

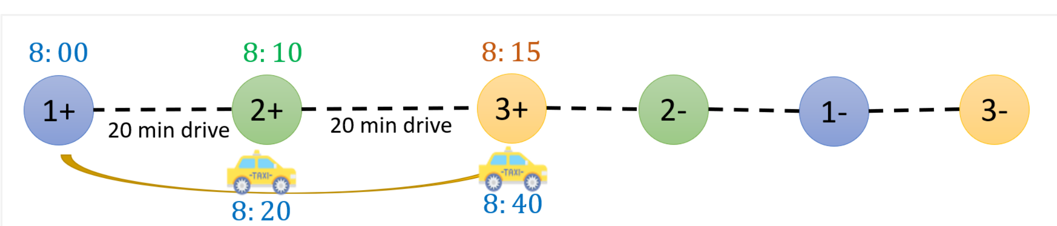



**Figure 2. The illustration of waiting time calibration**

To address the issue, a dynamic update of pick-up and drop-off timestamps is necessary. A variable, $T_{current}$, representing the current time, is introduced to examine each rider's waiting time. Figure 3 describes how $T_{current}$ is updated. Starting from the pick-up time of the first rider, if the taxi stops at a pick-up position, the algorithm will examine whether the taxi waits for the rider. Assuming the taxi arrives earlier and waits for the rider, $T_{current}$ will be set to the expected pick-up time of the rider. If the taxi does not wait, $T_{current}$ will be updated based on the arrival time of the taxi. In summary, the departure time of each point requires more precise estimation to avoid miscalculating waiting time (Figure 2), and with the proposed dynamic time tracking at each point, we can ensure that the drivers' parking time and riders' waiting time will not exceed $maxPK$and $maxW_i$ for each feasible match.

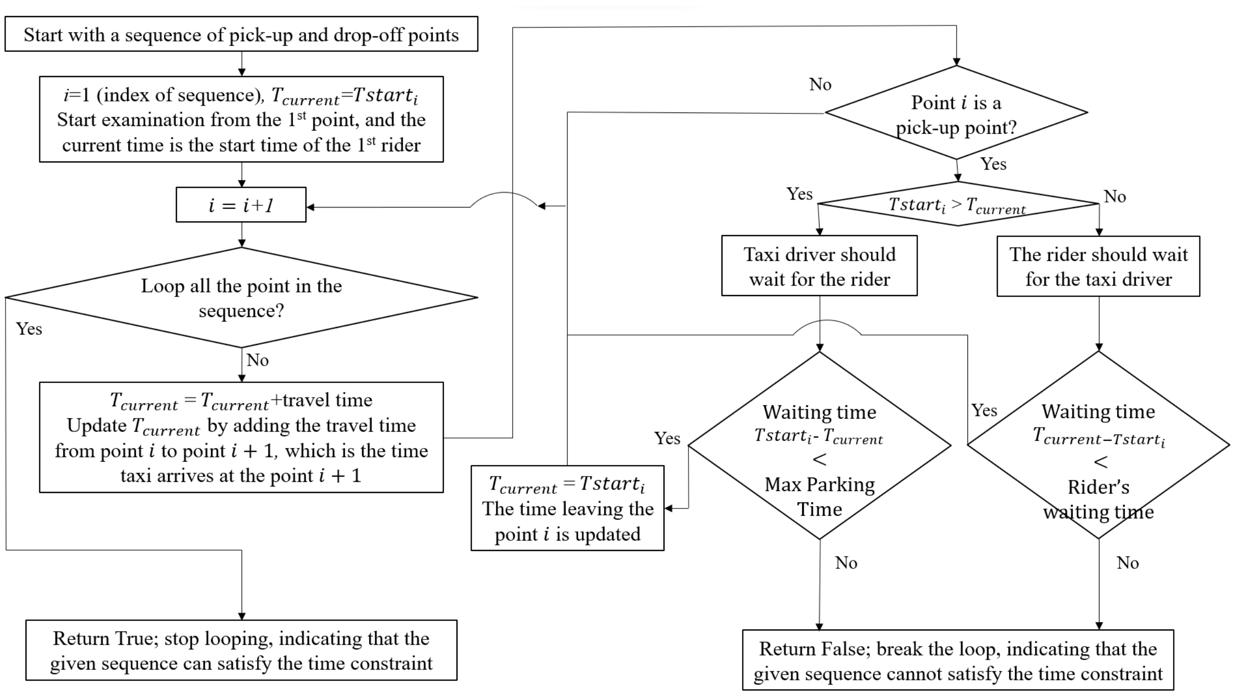



**Figure 3. The algorithm for examining time constraints for multiple riders in a journey sequence**

## Solution Methods: Greedy Algorithm and Binary Integer Programming

After the potential match identification, this study determines the matching by the greedy algorithm and binary integer programming. The greedy algorithm extracts the matches based on the hierarchy of distance savings in each matching stage.

As for the optimization approaches, a non-bipartite graph will be generated after the identification of potential matches. In the proposed model, the graph will be generated in each stage, and the node set $N$ represents all participants in a stage, which can be unpaired riders and paired matches. The link set $A$ stands for beneficial matches in the graph, consisting of a specific sequence of pick-up and drop-off points for two participants. Note that only potential matches with positive distance savings, obtained through the insertion method, are considered in set A. When multiple positive permutations exist, only the one with the largest savings is retained to represent the match of the two participants. The subset $N_i$ refers to all nodes that are matched to the participant $i$.

The distance saving for two participants is symbolized by the parameter $s_{i,j}$, $(i,j) \in A$. Additionally, the binary variables, $X_{i,j}$, are created to determine whether the link $(i,j)$ should be selected. The matching problem is formulated as the following optimization models with the objective to maximize the total weights on the links while ensuring that no participant is covered more than once.

Objective function:

$$\max Z = \sum_{(i,j)\in A} X_{i,j} s_{i,j} \tag{4}$$

Subjective to:

$$\sum_{j\in N_i} X_{i,j} \leq 1 \qquad \forall\, i \in N \tag{5}$$

$$X_{i,j} \in \{0,1\} \qquad (i,j) \in A \tag{6}$$

The objective function, Equation (4), aims to maximize the total distance savings. Constraint (5) indicates that each participant $i$ can only be connected once. Constraint (6) is the standard integrality constraint. It is worth mentioning that previous studies have used similar formulations for matching (Agatz et al., 2011; Arslan et al., 2019). However, this study advances the formulation by treating the existing pairs and matches as nodes to further improve matches. The link (i,j) does not merely represent a connection between two riders; instead, it refers to a specific pick-up/ drop-off sequence for two participants with an associated distance saving.

## Multi-stage Pairing Models with Rolling Horizontal Approaches

The multi-stage process of rider matching for taxi services is summarized in Figure 4. In Stage 0, we identify the potential pairs based on 2-rider insertion methods, where the potential pairs are marked as green links. The selection of these links is carried out in Stage 1 with the greedy algorithm or binary integer programming. Subsequently, we regard the 2-rider pairs as a single node to seek the potential matching of merging 2-rider pairs into 4-rider matches, where the potential match identification is implemented in Stage 1 and determined in Stage 2. Likewise, we investigate the feasibility of adding unpaired riders to existing 2-rider pairs in stage 2 by treating a 2-rider pair as a vertex, identifying two potential links in the matching graph. The selection of the link is made in Stage 3. Following that, we assess the matching of unpaired riders to existing 3-rider pairs in Stage 3, discovering two potential matches. In Stage 4, the final matches are determined. Overall, the process systematically explores numerous combinations and permutations to group riders as much as possible.

Additionally, the rolling horizontal approach is employed to analyze the real-time taxi request so that the model enables the inclusion of ad-hoc passengers in ride-sharing arrangements. The matching in Figure 5 occurs within a timeframe, and unpaired single riders and incomplete 2-rider and 3-rider matches are then carried over to the next timeframe to pair with new riders. If the unpaired single riders cannot find a match for more than two timeframes, they will be excluded in the pairing system. The duration of the timeframe is flexible and can be modified according to local conditions. A more extended period permits the simultaneous inclusion of a greater number of riders, although it may contribute to an increased scale in the matching graph. This adaptive approach allows for the continuous incorporation of real-time data and ensures flexibility in adjusting to evolving taxi service demands.

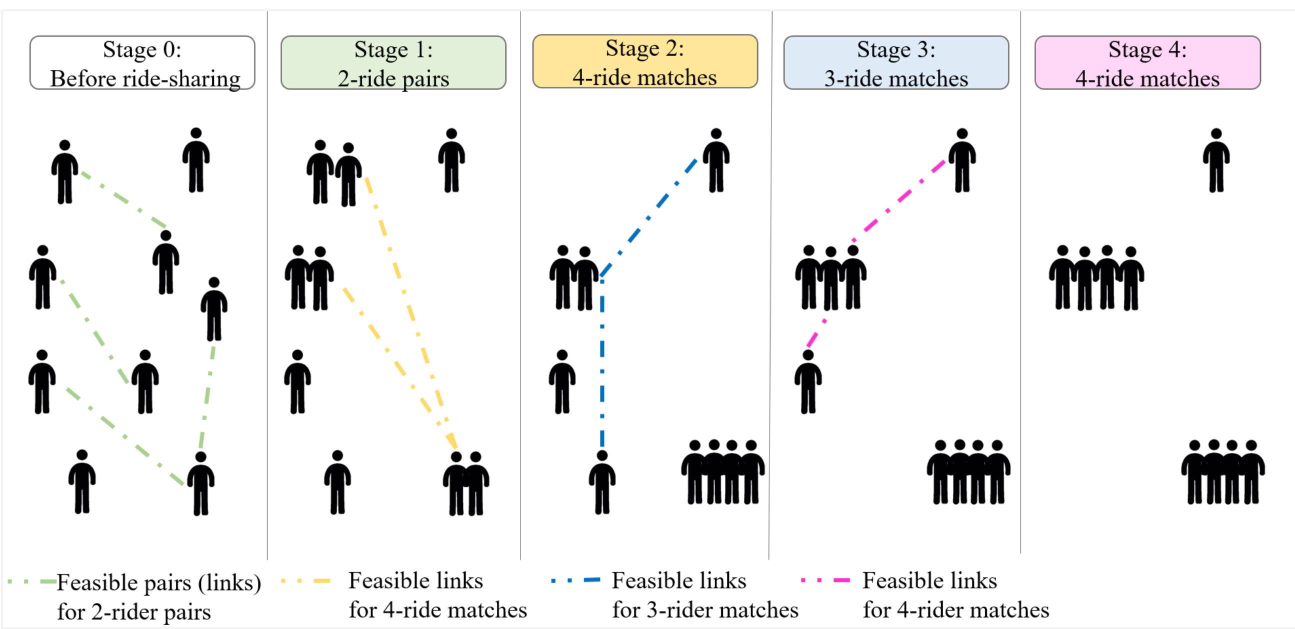



**Figure 4. Multi-stage riders matching within a timeframe**

# 4. Ride-Sharing Model: Case Study and Results

## Study Area and Input Data

The proposed multi-stage optimization model is verified using a case study in Chicago, Illinois. Chicago covers 589.82 square kilometers, with more than 2.7 million residents. Taxi has been a common transportation tool in the city (Zhou et al., 2019). In this study, we analyze the taxi trip data obtained from Chicago, focusing on a weekday - January 7th, 2019. As depicted in Figure 5, the majority of ride requests occurred between 8:00 am to 6:00 pm. Figure 6 displays the distribution of pick-up and drop-off locations in Chicago, illustrating that the primary area for taxi service is concentrated on the east side of the city. The data released by the City of Chicago utilizes neighborhood centroids to represent pick-up and drop-off points. To portray the actual situation and prevent overly generalized findings, this study introduces random dispersion around the centroid locations. The statistical analysis of the Chicago's taxi trip dataset shows that 83% of entire trips had travel duration less than 15 minutes. Also, 50% of the trips have traveled less than 1.84 miles.

The study analyzes the peak period from 8:00 am to 6:00 pm, involving a total of 16,867 riders. Within each 15-minute interval, an average of approximately 420 riders are recorded, and the average distance covered per single trip is approximately 7.2 km. The velocity of the taxi is observed to be 23 km/hr according to the released taxi data. Although our model allows each individual to specify their waiting time and detour tolerance, we assume 15 minutes and 1.5 for all riders in the case study according to Brown and LaValle's survey (Brown & LaValle, 2021). The parking limitation is 3 minutes. For the implementation of rolling horizontal approaches, we conduct the pairing process based on 15-minute intervals, and the riders will be excluded from the pairing process if remain unpaired for 2 timeframes. The pairing procedure is computed via the Python interface, and the commercial solver (Gurobi 10.0.1) is adopted to solve the binary integer programming.

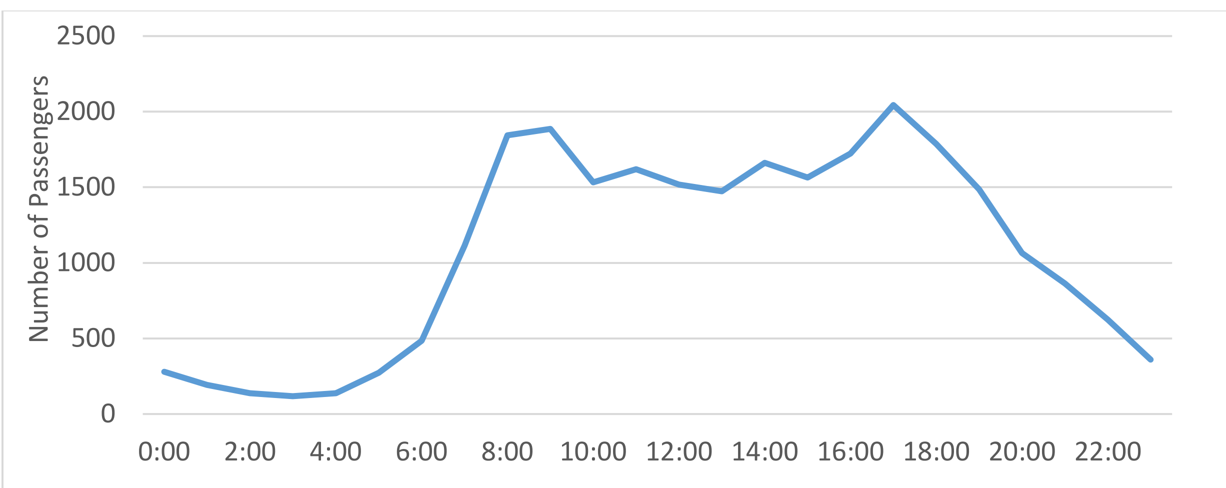
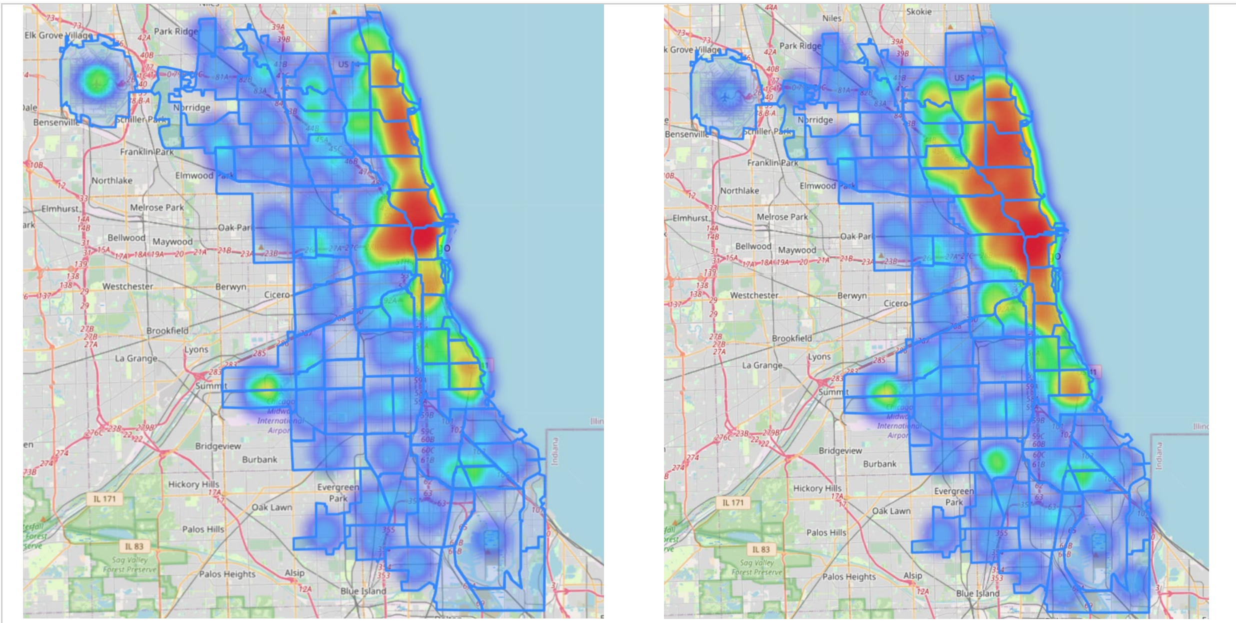



**Figure 5. The temporal distribution of taxi passengers**




Pick-up points Drop-off points

**Figure 6. The spatial distribution of passengers' pick-up and drop-off points**

## Performance of the Proposed Methods and Comparison with Other Models

This section presents the enhancement of the proposed multi-stage methods, specifically the greedy algorithm and binary integer programming. The performance of these methods is compared with the results from other models using the same network, as depicted in Figure 7. The participant models consist of the following:

*(i)* Max 2-rider —Greedy Algorithm:
This model pairs a maximum of 2 riders by selecting matches based on distance savings, continuing until no further positive savings can be achieved. The matching process is conceptually similar to the driver-to-rider pairing proposed by Agatz et al. (Agatz et al., 2011).

*(ii)* Max 2-rider —Integer Programming:
Akin to the previous model, this approach determines 2-rider pairs using binary integer programming (Agatz et al., 2011).

*(iii)* Max 4-rider —Mixing:
Based on the result of 2-rider pairs from Model (ii), we further insert the unpaired riders into existing pairs until no 4-rider matches are achieved, modifying the algorithm described by Lu et al. (Lu et al., 2022).

*(iv)* Proposed Method —Greedy Algorithm.

*(v)* Proposed Method —Integer Programming.

The comparison reveals three main findings. Firstly, both the greedy algorithm and integer programming show comparable improvements. Models (i) and (ii) from Agatz et al.'s study demonstrate reductions of 38.64% and 39.32%, respectively, while our proposed models, (iv) and (v), achieve reductions of 55.00% and 56.94%. The extent of reduction may vary depending on the network and riders' O-D trips, but the greedy algorithm has demonstrated its effectiveness in rider pairing.

Secondly, inserting two or more customers for the next insertion does not lead to significant improvement, as observed in Models (ii) and (iii). This finding aligns with Jaw et al.'s study on the dial-a-ride problem (9).

Lastly, our proposed model, which involves unique matching of 2-rider with 2-rider pairs, demonstrates notable improvement. When compared to Agatz's models (i) and (ii) and Lu's model (iii), our models show an improvement of approximately 15%. Although the level of improvement may vary with different datasets, the unique matching of 2-rider with 2-rider pairs offers greater potential for grouping riders. Although this strategy is readily implementable in real-world applications, as far as we know, previous studies have not employed the proposed insertion method.

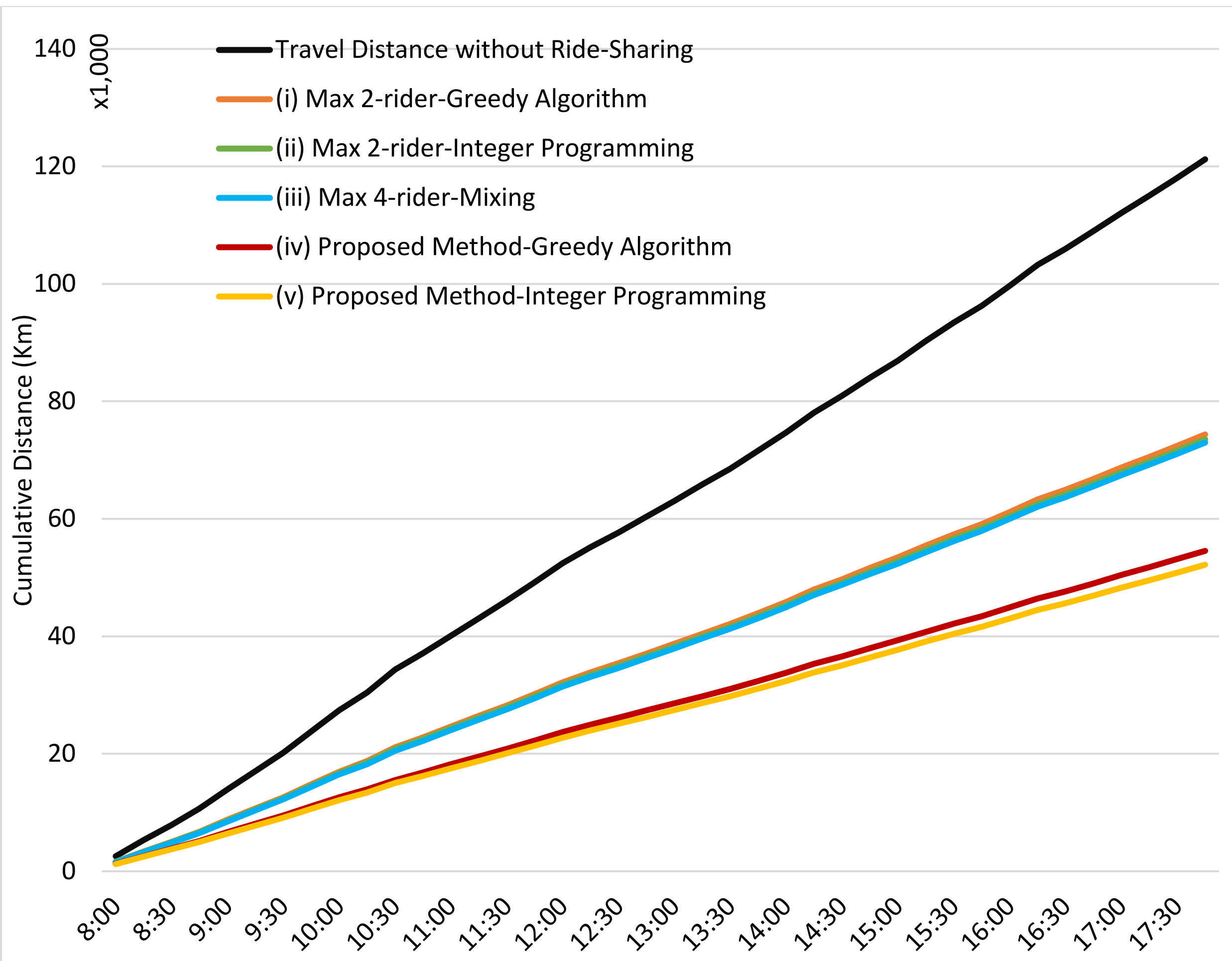



**Figure 7. The taxi cumulative distance of difference pairing models**

The number of multi-rider matches is presented in Figure 8. Our proposed methods, including the greedy algorithm and integer programming, exhibit a higher number of 4-rider matches. Since the proposed methods merge 2-rider pairs into 4-rider matches before inserting unpaired riders (leaving few 2-rider and 3-rider matches for insertion), we may have more unpaired riders compared to the Maximum Pairing of 4 Riders model. However, the higher frequency of successful 4-rider matches in our approach leads to a more substantial reduction in the overall travel distance.

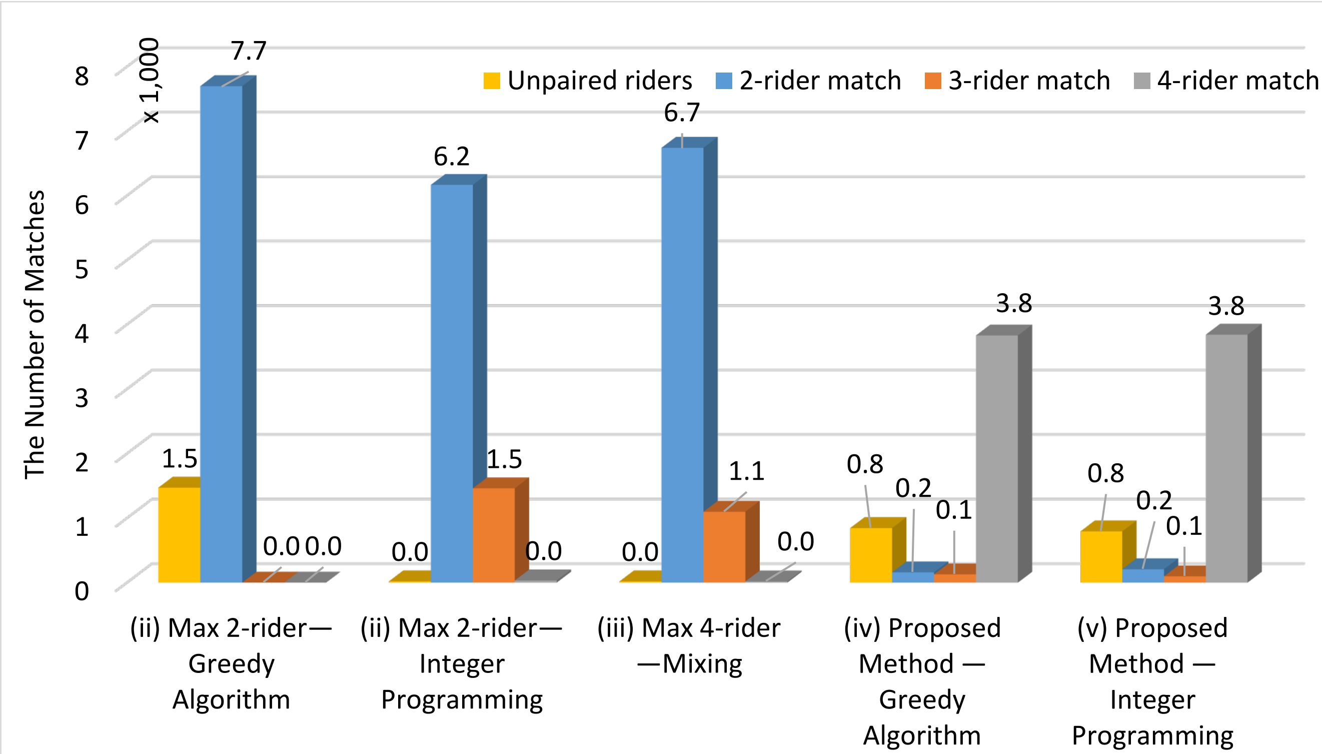



**Figure 8. The number of multiple-rider matches**

## Discussion of Match Selection Methods Regarding Performance and Computational Times

To implement the rider-matching model effectively, both performance and computational time must be taken into account. Figure 9 illustrates the relationship between computational time and the number of joining riders. From the graph, it is evident that the model utilizing integer programming becomes inefficient after 14:00 (when the number of riders starts to increase), whereas the greedy algorithm maintains manageable computational time even as more riders join the system. Consequently, it may not be practical to use integer programming for serving a large number of riders, especially considering the additional time required for the separation of multiple stages within a timeframe.

Attempting to solve the entire original scheduling problem using exact optimization methods is an intractable task; even its formulation would be very challenging, as the vehicles would be filled up with an apriori unknown number of passengers. Therefore, in consideration of the benefits of distance savings and solving efficiency, the proposed multi-stage model utilizing the greedy algorithm is better suited for practical implementation in a large network.

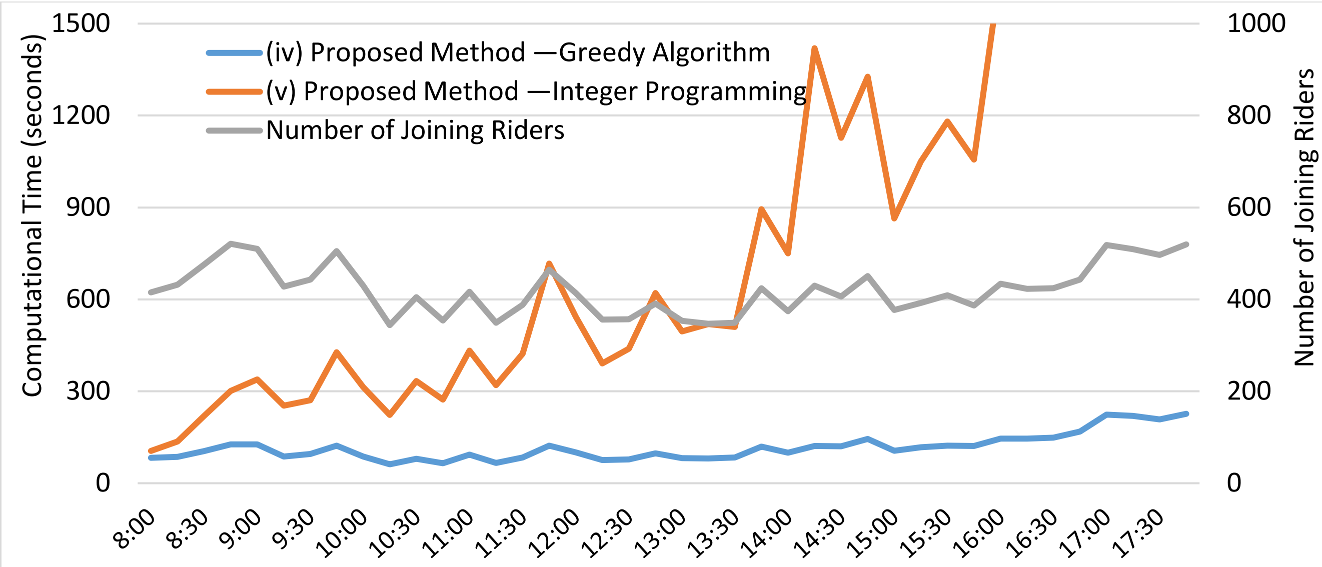



**Figure 9. The computational time of the proposed model using the greedy algorithm and integer programming**

## Sensitivity Analysis

A sensitivity analysis is conducted based on Model (iv) to assess the impact of riders' waiting time, drivers' parking time, and detour tolerance, as presented in Figure 10 and Figure 11.

Regarding the time constraints, Figure 10 indicates that the total travel distance can be decreased by extending both the waiting time for riders and the parking time for drivers. Particularly, there is a notable improvement when the riders' waiting time is increased from 5 minutes to 10 minutes. However, the degree of improvement becomes less significant once the waiting time exceeds 15 minutes, which can be attributed to the law of diminishing marginal utility. Given a shorter waiting time, the number of feasible matches is relatively low. Hence, increasing the waiting time has a more significant impact as the models can match more riders, thereby reducing the total travel distance. However, once the time is sufficient to pair the majority of riders, the benefit of increasing the waiting time is limited since most riders have already been matched. Likewise, the relaxation of parking limitation has a more improvement when the parking time increases from 1 minute to 3 minutes.

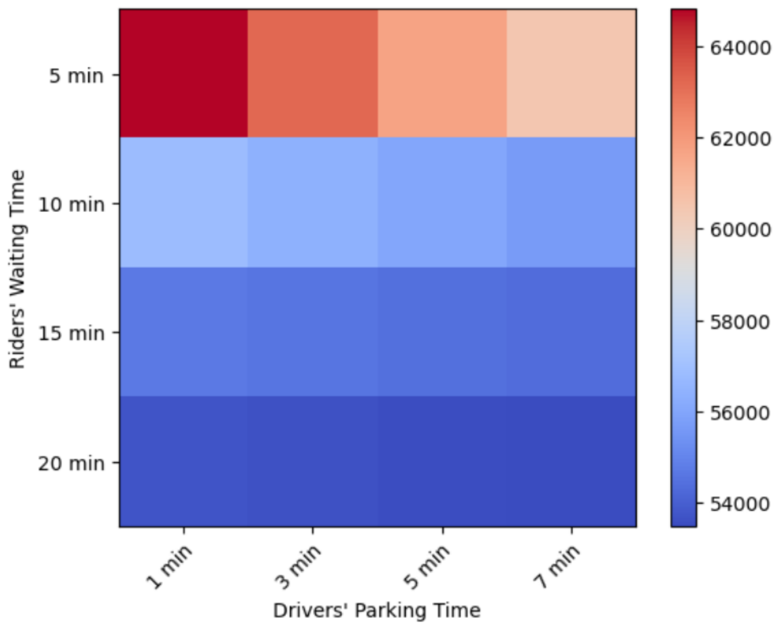



**Figure 10. The total travel distance (km) given different waiting and parking time**

Figure 11 presents distance reduction in terms of the riders' detour tolerance. As not every city has a grid network system, the Manhattan distance may not always be suitable. Therefore, to provide a broader perspective of the potential improvement through ride-sharing, this section illustrates the distance reduction using both Manhattan and Euclidean distances. This approach allows practitioners to gain a better understanding of the potential benefits in their respective cities.

The result indicates that at a detour ratio of 1.0, indicating no detour for riders, there can still be a significant 39% distance reduction in the Manhattan distance. By contrast, in the Euclidean distance, none of the riders can be matched, resulting in a 0% improvement. This discrepancy in outcomes can be attributed to the acceptable spatial deviation, where the Manhattan distance allows for a wider range of deviation compared to the Euclidean distance. As illustrated in Figure 12, ride-sharing based on Manhattan distance facilitates easier pairing of riders, even with a detour ratio of 1, as it covers the same distance for the original rider. However, there is no deviation from the original route when the distance is based on Euclidean distance.

It is noteworthy that in both cases, distance savings of up to 40% can be achieved as long as every rider allows for a 5% increase in detour. Accordingly, encouraging a small increase in detour tolerance among riders can lead to substantial distance improvements for ride-sharing services, regardless of the distance metric used.

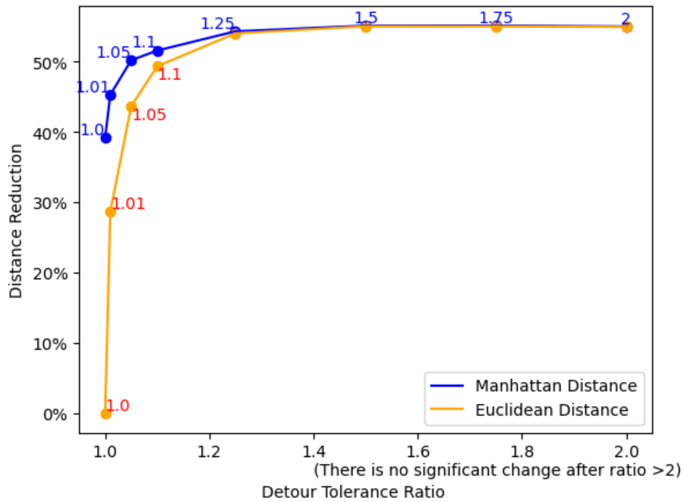



**Figure 11. The distance reduction rate given various detour tolerance**

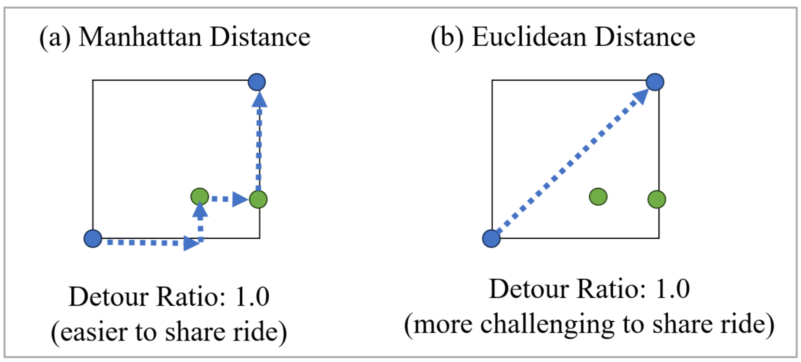



**Figure 12. The illustration of a deviation based on the Manhattan distance and Euclidean distance**

# 5. Congestion Analysis

## Problem Statement and Objectives

This study assesses the traffic congestion status of the city of Chicago for each 15-minute interval with the taxi trip and ride-shared trip data. Based on the structure and characteristics of the data, we determine and compute the congestion metrics for evaluating the congestion status. For this study, the congestion status of a community area is evaluated. Figure 13 shows the maps of community areas in the city of Chicago.

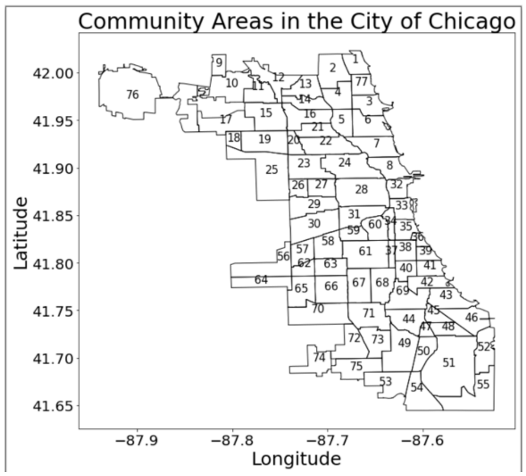



**Figure 13. Plots of community areas in Chicago**

Then, the selected congestion metrics are combined to compute the integrated congestion index using fuzzy logic and membership functions. With the integrated congestion index, the congestion status of the city of Chicago is represented graphically in the congestion map of Chicago. By comparing the congestion map of Chicago from taxi trip data and ride-shared trip data, we can evaluate the effects of the ride-sharing model on the congestion status in the city of Chicago.

## Congestion Metrics

While various metrics can assess the congestion status, the congestion metrics for this study are chosen based on the features and characteristics of the taxi trip data. We select the mean velocity and congested vehicle miles traveled ratio. The mean velocity of an area is computed by considering the trips that happen in an area. Mean velocity is chosen as the congestion metric as it can be directly interpreted to determine the congestion status. An area is uncongested if vehicles travel near the speed limit or free-flow speed in an area. If vehicles travel at a speed close to zero miles per hour, an area is extremely congested.

The congested vehicle miles traveled (VMT) ratio is also selected as the congestion metric. The congested VMT ratio is the ratio of VMT at the congested status to the total VMT (MTC, 2023), as shown in Equation (7).

$$\text{Congested VMT Ratio} = \frac{\text{VMT at Congested Status}}{\text{Total VMT}} \tag{7}$$

The process of computing mean velocity and congested VMT ratio are described in a later section with more details.

## Computation of Regional Mean Velocity

This section explains the process of computing the mean velocity of areas in the city of Chicago with taxi trip data and ridesharing data. The process breaks down into three subparts: 1. Extraction of areas a trip possibly passed through, 2. Weighted Mean Velocity, and 3. Mean Velocity Adjustment.

### Extraction of Possible Areas Crossed by a Trip

The mean velocity of community areas of the city of Chicago is computed based on the mean velocity of the trips. Therefore, we first extract the taxi trips or ride-sharing trips that occurred for each 15-minute interval. Then, we find the possible areas that each trip crossed. Figure 14 shows the process of extracting areas that a trip passed through.

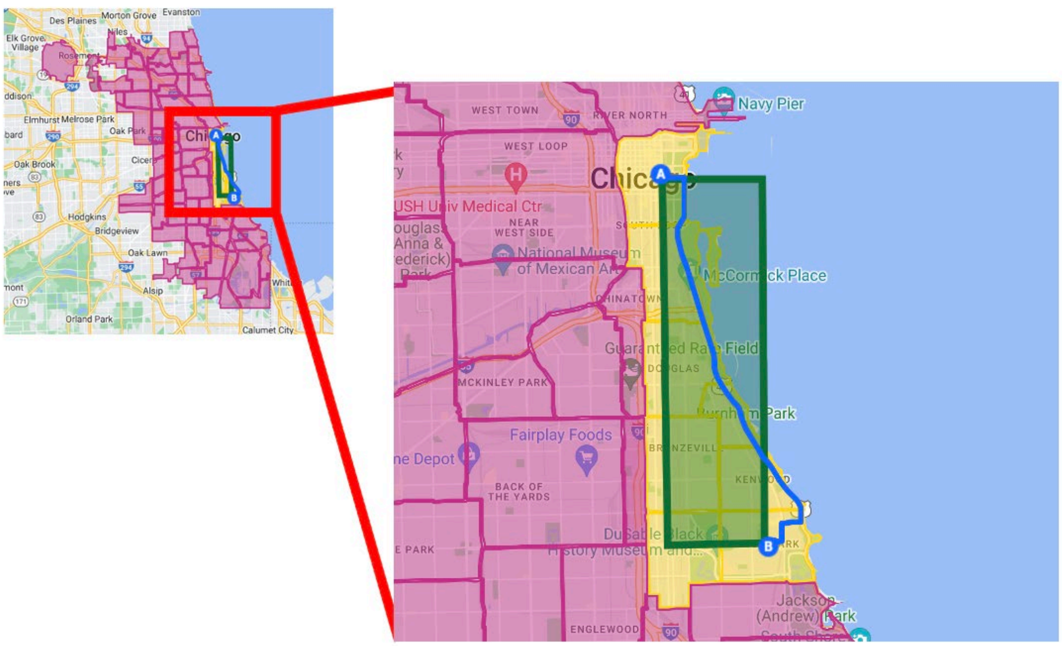



**Figure 14. Example of extracting possible community areas a trip might have passed through**

In Figure 14, The point A represents the origin coordinate of the trip, while the point B represents the destination coordinate of the trip. Based on the origin and destination coordinates, one can create a rectangular area (Green rectangle in Figure 14),. Areas that intersect with a rectangle are considered as potential areas that a trip passes through. On Figure 14, yellow areas are the potential areas that a trip might have passed through.

### Weighted Mean Velocity of Areas

Now that areas possibly crossed by each trip in a particular 15-minute time interval are found, we know the mean velocities of trips corresponding to each area at a particular time interval. The trips crossed a particular area are clustered based on the trip distance. We create 25 two-mile intervals where trips with distances between 0.5 miles and 50.5 miles are distributed. Figure 15 shows the clustering process based on a trip distance.

| | | | | | | |
|---|---|---|---|---|---|---|
| Cluster α = 1:<br><br>Trips with distance between 0.5 miles and 2.5 miles | Cluster α = 2<br><br>Trips with distance between 2.5 miles and 4.5 miles | …… | Cluster α = n<br><br>Trips with distance between 0.5+2(n-1) miles and 0.5+2n miles | …… | Cluster α = 24<br><br>Trips with distance between 46.5 miles and 48.5 miles | Cluster α = 25<br><br>Trips with distance between 48.5 miles and 50.5 miles |

**Figure 15. Clustering trips based on trip distance**

If the mean velocities of trips are used directly to compute the mean velocity of each area, we are assuming that the velocity throughout the trip is constant. However, the assumption is too strong as the velocity of a vehicle throughout the trip varies. In order to overcome the this, the mean velocities of trips are weighted when computing the mean velocity of each area.

The weight on the mean velocity of trips is based on the trip distance. As a vehicle travels for longer distances, it is more likely that a trip to span more areas. Therefore, the mean velocity of a trip traveled a longer distance is based on the velocity of more areas. Hence, the shorter the trip distance, the more representative the mean velocity of a trip to the velocity of an area. The weight of the mean velocity of a trip traveled in a shorter distance is higher than the weight of the mean velocity of a trip traveled in a longer distance. Instead of providing the weight directly based on the travel distance, it is based on what cluster a trip belongs to.

The trip duration is also considered for the weight on the mean velocity of trips. If a trip duration is longer within a 15-minute interval, a vehicle stays longer in a region within a 15-minute interval. Therefore, the velocity of an area is better represented by a trip with a longer duration within a 15-minute interval. We give more weight to the trip traveled in longer duration compared to the trip traveled in smaller duration. Equation (8) and (9) computes the weighted mean velocity of a community area based on the trip distance and trip duration.

$$v_{ca,t} = \frac{\sum_{n=1}^{25} \sum_{p=1}^{m_n} \beta_{ca,t,p,n} v_p}{\sum_{n=1}^{25} \sum_{p=1}^{m_n} \beta_{ca,t,p,n}} \tag{8}$$

$$\beta_{ca,t,p,n} = \frac{25 - \alpha_{ca,t,n} n}{25} \cdot \frac{tt_p}{15} \tag{9}$$

Table 3 below explains what each notation in Equation (8) and Equation (9) represents and means.

**Table 3. Notations for Equation (8) and Equation (9)**

| Notation | Description |
|---|---|
| $ca$ | Community Area |
| $t$ | 15-minute Time Interval |
| $n$ | Cluster index |
| $m_n$ | Number of total trips in cluster $n$ |
| $p$ | Trip index |
| $v_p$ | Mean Velocity of trip $p$ |
| $tt_p$ | Travel Time of trip $p$ within 15-minute interval $t$ |
| $\alpha_{ca,t,n}$ | Binary variable for a community area $ca$ where it equals to $\begin{cases} 0, \text{if } m_n = 0 \text{ within 15minute interval } t \text{ for a two mile interval } n \\ 1, \text{otherwise} \end{cases}$ |
| $\beta_{ca,t,p,n}$ | Weight applied to the mean velocity of trip $p$ for a cluster $n$ of a community area $ca$ at 15-minute interval $t$ |
| $v_{ca,t}$ | Weighted mean velocity of a community area at 15-minute time interval $t$ |

### Mean Velocity Adjustment

The process of adjusting the mean velocity of an area computed through Equation (8) and Equation (9) for the better estimates is explained in this section. The mean velocity of an area at a particular 15-minute time interval is adjusted based on the mean velocity of an area at a previous 15-minute time interval. It is based on the assumption that the mean velocity of an area at a particular time interval and the mean velocity of an area at its previous interval are strongly correlated. It is additionally adjusted with the mean velocity of its neighboring areas at the same 15-minute time interval. It is based on the assumption that the mean velocities of an area and its neighbors at a particular 15-minute time interval are influenced by each other.

Therefore, the final estimate of the mean velocity of a community area at a particular time interval is computed with the least square model based on three types of velocities: 1. Mean velocity of a community area at a particular 15-minute interval computed from Equation (8) and Equation (9), 2. Mean velocity of a community area at a previous 15-minute interval, and 3. Mean velocities of neighboring community areas at the same 15-minute interval.

## Computation of Congested VMT Ratio

This section describes computing the second congestion metric investigated in the study: congested vehicle miles traveled (VMT) ratio. Based on the characteristics of trip data, it is impossible to compute the VMT for each community area. To overcome this challenge, the congested VMT ratio is computed for the entire city of Chicago. Therefore, the same value of the congested VMT ratio is applied to all community areas. Also, the congested VMT ratio is going to be calculated for a time period longer than 15 minutes. Table 6 shows the five time periods determined based on the traffic pattern in the city of Chicago.

**Table 4. Five Time Periods Considered for Congested VMT Ratio**

| Description | Time Period |
|---|---|
| Morning Non-Rush Hour | 12:00 AM – 7:00 AM |
| Morning Rush Hour | 7:00 AM – 9:00 AM |
| Afternoon Non-Rush Hour | 9:00 AM – 2:00 PM |
| Afternoon Rush Hour | 2:00 PM – 6:00 PM |
| Night | 6:00 PM – 12:00 AM |

Now that we determined the five time periods, we extract the trips that happened within each time period to calculate the total VMT and congested VMT for each time period. VMT is computed based on the number of vehicles, average number of trips per vehicle, and average travel distance of trip. By extracting trips traveling slower than a threshold velocity for each time period, the congested VMT is computed. Chicago Traffic Tracker, the real-time congestion map provided by the city of Chicago, determines the congestion level based on velocity. The congestion status of an area or a road is considered “light congestion” or “no congestion” if the speed is above 20mph (CDOT, 2023). Therefore, we set the threshold mean velocity for the computation of the congested VMT to be 20mph. From the congested VMT and the total VMT, Equation (7) computes the congested VMT ratio.

## Integrated Congestion Index

For this study, the congested VMT ratio and the mean velocity of an area computed in previous sections are considered to determine the integrated congestion index. Table 7 shows the congestion indexes and congestion status each congestion index represents considered in the study.

**Table 5. Five Time Periods Considered for Congested VMT Ratio**

| Congestion Index (CI) | Congestion Status |
|---|---|
| CI = 3 | “Heavy Congestion” |
| CI = 2 | “Medium Congestion” |
| CI = 1 | “Light Congestion” |
| CI = 0 | “No Congestion” |

Congestion index can be determined with strict cutlines. However, using strict cutlines for congestion can be problematic since slight differences in a congestion metric value can lead to significant differences in congestion status. To avoid this issue, we utilize the fuzzy logic and membership functions for the congestion index. Fuzzy logic is the type of logic based on “degree of truth” instead of true or false like in traditional binary logic. While the congestion status is based on the quantitative measurements, every person might have different perception on the congestion status. Fuzzy logic can capture the ranges and nuances in congestion status. From the membership function, the membership degree in each CI is based on the value of each congestion metric. If the membership degree is 0 in a particular CI, it is not a member of that CI. If the membership degree is 1 in a particular CI, it is fully a member of a particular CI. If the value of the membership degree is between 0 and 1, it is a fuzzy member of a particular CI. Membership degrees based on mean velocities and congested VMT ratio are computed using two membership functions. Figure 16 shows the plot of a membership function for determining the membership degree based on the mean velocity value.

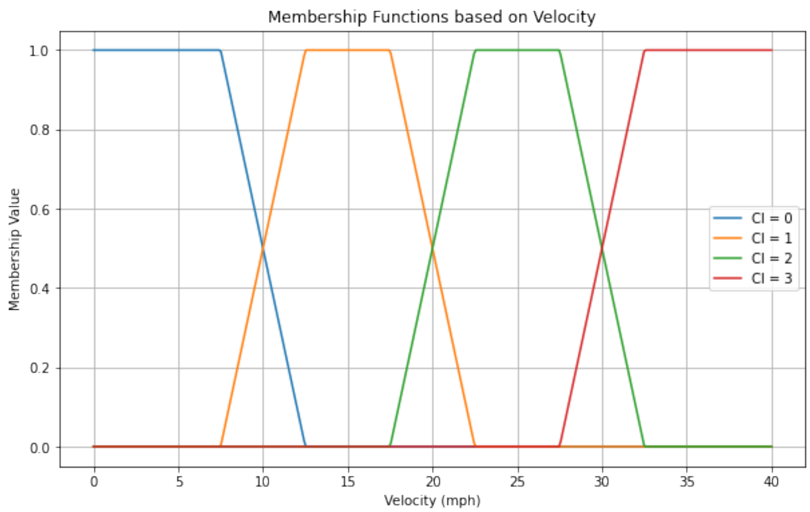



**Figure 16. Membership function based on mean velocity**

Figure 17 shows the plot of a membership function for determining the membership degree based on the congested VMT ratio.

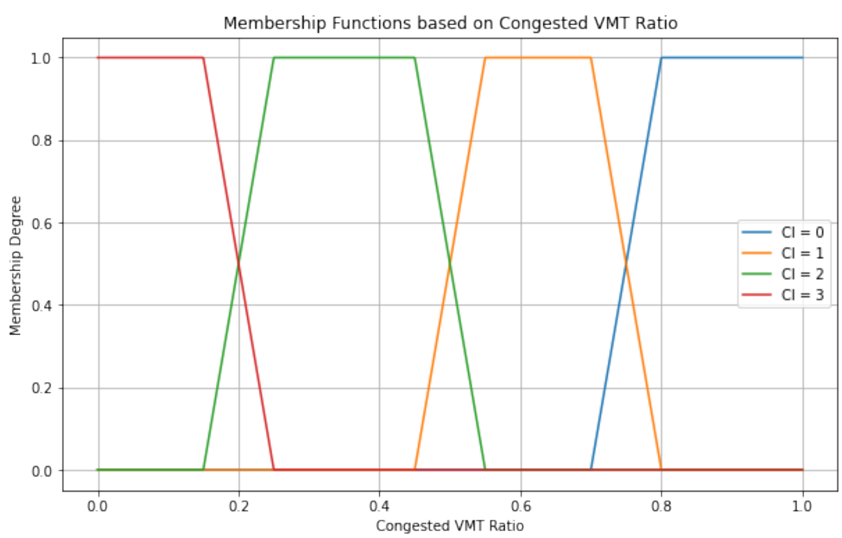



**Figure 17. Membership function based on congested VMT ratio**

After computing the membership degree for mean velocity and for congested VMT ratio separately from Figure 16 and Figure 17, we computed the combined membership degree as the weighted sum of two membership degrees. It is essential to apply greater weight on a membership degree of mean velocity than on a membership degree of VMT. The reason behind applying greater weight on a membership degree of mean velocity is due to its higher spatial and temporal precision. As previously mentioned, VMT is computed for the entire city of Chicago for the time period shown in Table 6, while the mean velocity is computed for each area for a 15-minute interval.

The integrated congestion index of an area at a particular 15-minute interval is then chosen by finding the CI with largest combined membership degree of an area at a particular 15-minute interval. Congestion map is plotted based on the integrated congestion index.

## Congestion Map: Chicago

In this section, the result of the congestion analysis is presented. The congestion maps of community areas in Chicago before ride-sharing and after ride-sharing for an 8:30 AM-8:45 AM interval (morning rush hour) and a 5:15 PM-5:30 PM interval (afternoon rush hour) on January 9th of 2019 are shown in Figure 18 and Figure 19. The congestion maps of community areas in Chicago before ride-sharing and after ride-sharing for an 8:00 AM-8:15 AM interval (morning rush hour) and a 5 PM-5:15 PM interval (afternoon rush hour) on January 15th of 2019 are shown in Figure 20 and Figure 21. On the congestion map, the areas with CI of 3, 2, 1, and 0 are colored in cyan blue, opalescent coral, coral rose, and brown, respectively.

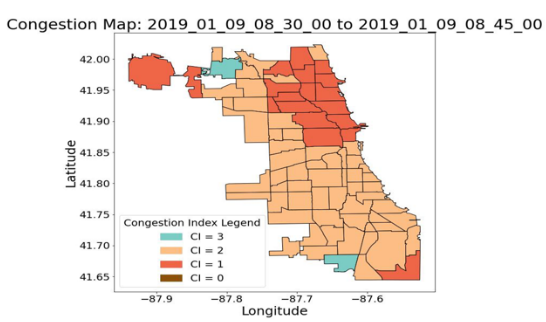
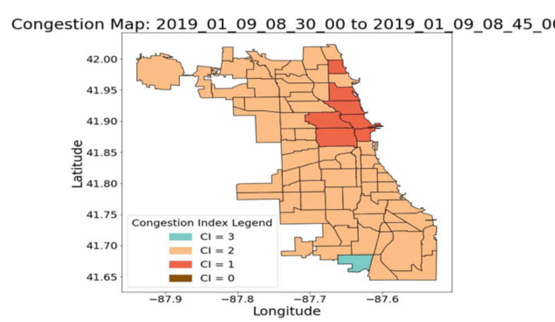
<table>
<tr><th></th><th>Before Ride-Sharing</th><th>After Ride-Sharing</th></tr>
<tr><td>Morning Rush Hour (8:30AM - 8:45AM)</td><td>
</td><td>
</td></tr>
</table>

**Figure 18. The congestion map of community areas in Chicago before and after ride-sharing from 8:30AM to 8:45AM on January 9th, 2019**

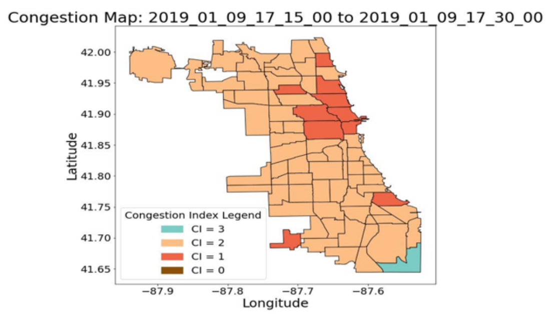
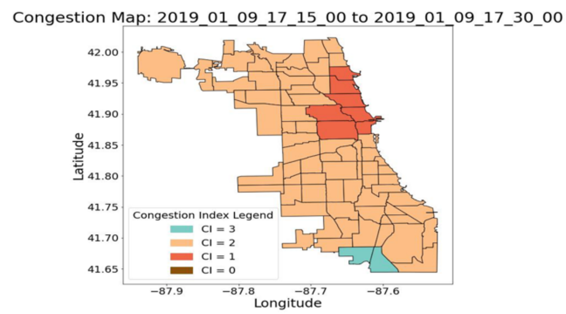
<table>
<tr><th></th><th>Before Ride-Sharing</th><th>After Ride-Sharing</th></tr>
<tr><td>Afternoon Rush Hour (5:15PM - 5:30PM)</td><td>
</td><td>
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**Figure 19. The congestion map of community areas in Chicago before and after ride-sharing from 5:15PM to 5:30PM on January 9th, 2019**

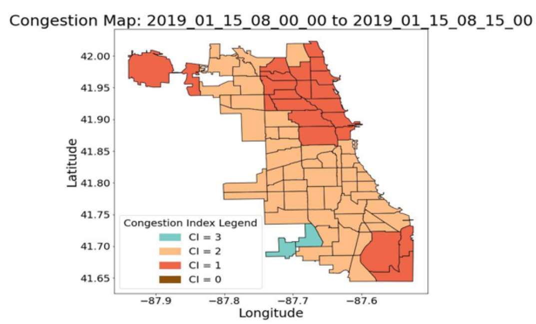
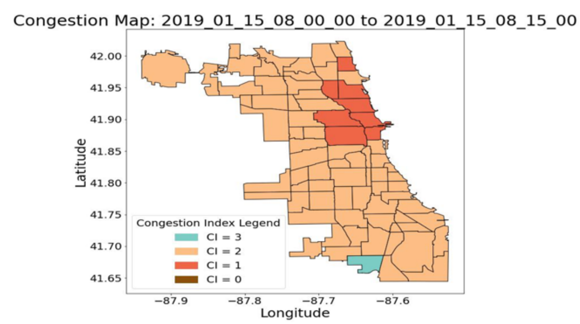
<table>
<tr><th></th><th>Before Ride-Sharing</th><th>After Ride-Sharing</th></tr>
<tr><td>Morning Rush Hour (8:00AM - 8:15AM)</td><td>
</td><td>
</td></tr>
</table>

**Figure 20. The congestion map of community areas in Chicago before and after ride-sharing from 8:00AM to 8:15AM on January 15th, 2019**

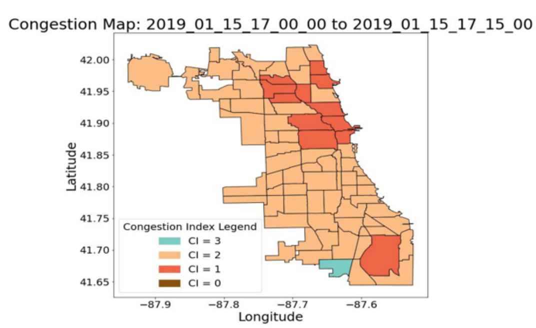
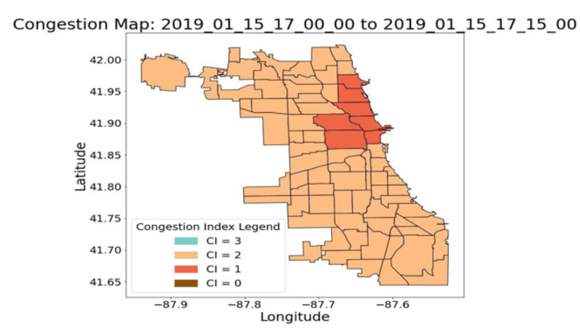
<table>
<tr><th></th><th>Before Ride-Sharing</th><th>After Ride-Sharing</th></tr>
<tr><td>Afternoon Rush Hour (5:00PM - 5:15PM)</td><td>
</td><td>
</td></tr>
</table>

**Figure 21. The congestion map of community areas in Chicago before and after ride-sharing from 5PM to 5:15PM on January 15th, 2019**

By visually comparing the congestion map of community areas before ride-sharing and after ride-sharing, there is an improvement in traffic status in some community areas where the color changed from coral rose to opalescent coral in Figure 18, Figure 19, Figure 20, and Figure 21 after the ride-sharing system is implemented.

# 6. Conclusions and Limitations

Taxis contribute significantly to city traffic, with reports indicating that half of the Chicago taxi fleet's in-service time and travel distance are unproductive (Y. Chen et al., 2018). Ride-sharing has the capacity to mitigate these issues. Additionally, increasing fuel costs serve as an added motivation for citizens to embrace ride-sharing, making it an even more appealing solution (Saranow, 2006). Hence, a comprehensive model for rider-sharing is critical to improve operational efficiency and attract more users.

This study proposes a multi-stage optimization model to match riders, aiming to minimize the total travel distance while ensuring the satisfaction of riders (waiting time and detour tolerance). Specifically, the model considers the possibility of matching multiple riders together, which is further verified by real taxi data from the city of Chicago. Some findings are observed from this study:

(1) The multi-stage model has a significant reduction in total travel distance with the overall distance decreasing by 55.00 % with the greedy algorithm and by 56.94% with the integer programming.
(2) The proposed insertion method that merges two 2-ride pairs into 4-rider matches explores more permutation probabilities, further enhancing the matching of multiple-rider groups and reducing total travel distance.
(3) Passengers' preferences, including their acceptable waiting time and detour tolerance, are critical to the overall performance. Particularly, the extension of riders' waiting time can enhance the success of ride-sharing matching.
(4) Detour tolerance has a great impact on distance reduction as it directly affects the acceptable distance of riders. When the ratio reaches 1.25, there are significant savings in distance as more riders can be paired together. However, the improvement becomes less noticeable when the value exceeds 1.5 as the riders have been paired with others nearby with nearby companions.

Previous studies have proposed various methods to pair multiple riders in a match, such as shifting riders between pairs to gradually enhance performance (Horn, 2002; Santos & Xavier, 2013). By contrast, this study introduces a standardized pairing procedure to efficiently merge more riders in successive stages. Several significant contributions are evident in this study. Firstly, we extend the existing rider matching problem formulation by treating 2-rider and 3-rider matches as individual nodes, thereby expanding the practicality of matching models. Secondly, our approach incorporates a sophisticated method to calculate waiting time, which takes into account all time lost during a journey, resulting in a more accurate and comprehensive analysis. Thirdly, we validate the proposed model using real-world data, providing empirical evidence to demonstrate its capability and effectiveness.

In order to implement the ride-sharing model effectively, the scheduler may have to consider the riders' preferences. Our model enables the riders to specify their maximum waiting time and detour tolerance, resulting in a solution that benefits both the overall system and individuals. Conceptually similar to prior studies utilizing departure and arrival time windows, the proposed approach offers the following advantages: (i) It simplifies the process for users to declare their time windows. (ii) The acceptable waiting time and detour are closely linked to their subjective satisfaction. (iii) Some users may declare a time window that cannot be achieved by ride-sharing or even a single-rider journey, which increases the workload for the pairing systems.

Some possible directions are suggested for future works. This study provides rider-to-rider matching models, and further work is needed to assign rider matches to drivers in the network (Arslan et al., 2019). Additionally, the binary integer programming model can be enhanced by a network optimization algorithm, such as Maximum Weight Matches to compute the solution more efficiently (Duan & Pettie, 2014; Edmonds, 1965). After the pairing of riders, cost allocation can also be a critical issue in ride-sharing (Kleiner et al., 2011; Lu & Quadrifoglio, 2019). A quantification of the effect of ride-sharing policies on congestion reduction would also be a desirable topic for investigation, as it would give administrators a tool to evaluate the direct results of ride-sharing policies on transportation network performance.

The promotion of ride-sharing has potential in various fields such as evacuation and emergency response (Renne & Mayorga, 2022). This study investigates the potential of taxi services, intending to gain more attention from transportation planners and taxi commercials to adopt ride-sharing of multiple riders. We acknowledge that due to the separation of multiple stages in a timeframe, we may not reach the system optimal but only the stage optimal in the specific timeframe (Santi et al., 2014). Nevertheless, the improvement of the proposed method is evident, and the ride-sharing scheduler can establish a guiding principle for matching. To put the model in practice, other contextual factors can be considered during potential ride-pair identification to better adapt to the city's environment.

# References


Agatz, N., Erera, A. L., Savelsbergh, M. W. P., & Wang, X. (2011). Dynamic ride-sharing: A simulation study in metro Atlanta. *Transportation Research Part B: Methodological*, *45* `(9), 1450–1464. https://doi.org/10.1016/j.trb.2011.05.017

Agatz, N., Erera, A., Savelsbergh, M., & Wang, X. (2012). Optimization for dynamic ride-sharing: A review. *European Journal of Operational Research*, *223*(2), 295–303. https://doi.org/10.1016/j.ejor.2012.05.028

Alonso-Mora, J., Samaranayake, S., Wallar, A., Frazzoli, E., & Rus, D. (2017). On-demand high-capacity ride-sharing via dynamic trip-vehicle assignment. *Proceedings of the National Academy of Sciences*, *114*(3), 462-467.

Arslan, A. M., Agatz, N., Kroon, L., & Zuidwijk, R. (2019). Crowdsourced delivery—a dynamic pickup and delivery problem with ad hoc drivers. *Transportation Science*, *53*(1), 222–235. https://doi.org/10.1287/trsc.2017.0803

Auad-Perez, R., & Van Hentenryck, P. (2022). Ridesharing and fleet sizing for On-Demand Multimodal Transit systems. *Transportation Research Part C: Emerging Technologies*, *138*, 103594.

Bao, D. (2019). A multi-index fusion clustering strategy for traffic flow state identification. Ieee Access, 7, 166404-166409.

Brown, A., & LaValle, W. (2021). Hailing a change: comparing taxi and ridehail service quality in Los Angeles. *Transportation*, *48*(2), 1007–1031. https://doi.org/10.1007/s11116-020-10086-z

Campbell, A. M., & Savelsbergh, M. W. P. (2004). A Decomposition Approach for the Inventory Routing Problem. *Transportation Science*, 1–32.

CDOT. (2023). Chicago Traffic Tracker: Realtime Traffic Information for Chicago. Chicago Traffic Tracker. https://chicagotraffictracker.com/

Chen, W., Chen, J., & Yin, G. (2022). Exploring side effects of ridesharing services in urban China: role of pollution–averting behavior. *Electronic Commerce Research*, *22*(4), 1007–1034. https://doi.org/10.1007/s10660-020-09443-y

Chen, Y., Hyland, M., Wilbur, M. P., & Mahmassani, H. S. (2018). Characterization of Taxi Fleet Operational Networks and Vehicle Efficiency: Chicago Case Study. *Transportation Research Record*, *2672*(48), 127–138. https://doi.org/10.1177/0361198118799165

Cho, Y., & Rice, J. (2006). Estimating velocity fields on a freeway from low-resolution videos. IEEE Transactions on intelligent transportation systems, 7(4), 463-469.

Coifman, B. (2003). Identifying the onset of congestion rapidly with existing traffic detectors. Transportation Research Part A: Policy and Practice, 37(3), 277-291.

Coslovich, L., Pesenti, R., & Ukovich, W. (2006). A two-phase insertion technique of unexpected customers for a dynamic dial-a-ride problem. *European Journal of Operational Research*, *175*(3), 1605–1615. https://doi.org/10.1016/j.ejor.2005.02.038

Diana, M., & Dessouky, M. M. (2004). A new regret insertion heuristic for solving large-scale dial-a-ride problems with time windows. *Transportation Research Part B: Methodological*, *38*(6), 539–557. https://doi.org/10.1016/j.trb.2003.07.001

Duan, R., & Pettie, S. (2014). Linear-time Approximation for Maximum Weight Matching. *Journal of the ACM*, *61*(1). https://doi.org/10.1145/2529989

Edmonds, J. (1965). Maximum Matching and a Polyhedron With O,1-Vertices 1. *JOURNAL OF RESEARCH of the National Bureau of Standards-B. Mathematics and Mathematical Physic s*, *69*(2).

Engelhardt, R., Dandl, F., Bilali, A., & Bogenberger, K. (2019). Quantifying the Benefits of Autonomous On-Demand Ride-Pooling: A Simulation Study for Munich, Germany. *2019 IEEE Intelligent Transportation Systems Conference (ITSC)*, 2992–2997.

Erdelić, T., Carić, T., Erdelić, M., Tišljarić, L., Turković, A., & Jelušić, N. (2021). Estimating congestion zones and travel time indexes based on the floating car data. Computers, Environment and Urban Systems, 87, 101604.

Fagnant, D. J., & Kockelman, K. M. (2018). Dynamic ride-sharing and fleet sizing for a system of shared autonomous vehicles in Austin, Texas. *Transportation*, *45*(1), 143–158. https://doi.org/10.1007/s11116-016-9729-z

Feo, T. A., & Resende, M. G. C. (1995). Greedy Randomized Adaptive Search Procedures. In *Journal of Global Optimization* (Vol. 6). KluwerAcademic Publishers.

Fiedler, D., Michalˇ, M., Michalčertick´y, M., Alonso-Mora, J., & Michalčáp, M. M. (2018). *The Impact of Ridesharing in Mobility-on-Demand Systems: Simulation Case Study in Prague*. https://doi.org/10.0/Linux-x86_64

Gore, N., Arkatkar, S., Joshi, G., & Antoniou, C. (2023). Developing modified congestion index and congestion-based level of service. Transport policy, 131, 97-119.

Gore, N., Pulugurtha, S. S., Arkatkar, S., & Joshi, G. (2021). Congestion index and reliability-based freeway level of service. Journal of Transportation Engineering, Part A: Systems, 147(6), 04021027.

Guasch, A., Figueras, J., Fonseca I Casas, P., Montanola-Sales, C., & Casanovas-Garcia, J. (2015). Simulation analysis of a dynamic ridesharing model. *Proceedings - Winter Simulation Conference*, *2015-January*, 1965–1976. https://doi.org/10.1109/WSC.2014.7020043

Gurumurthy, K. M., & Kockelman, K. M. (2018). Analyzing the dynamic ride-sharing potential for shared autonomous vehicle fleets using cellphone data from Orlando, Florida. *Computers, Environment and Urban Systems*, *71*, 177-185.

He, F., Yan, X., Liu, Y., & Ma, L. (2016). A traffic congestion assessment method for urban road networks based on speed performance index. Procedia engineering, 137, 425-433.

He, Z., Zheng, L., Chen, P., & Guan, W. (2017). Mapping to cells: A simple method to extract traffic dynamics from probe vehicle data. Computer-Aided Civil and Infrastructure Engineering, 32(3), 252-267.

Henao, A., & Marshall, W. (2017). *A Framework for Understanding the Impacts of Ridesourcing on Transportation*. http://www.springer.com/series/11573

Herbawi, W., & Weber, M. (2012). A genetic and insertion heuristic algorithm for solving the dynamic ridematching problem with time windows. *GECCO'12 - Proceedings of the 14th International Conference on Genetic and Evolutionary Computation*, 385–392. https://doi.org/10.1145/2330163.2330219

Herrera, J. C., Work, D. B., Herring, R., Ban, X. J., Jacobson, Q., & Bayen, A. M. (2010). Evaluation of traffic data obtained via GPS-enabled mobile phones: The Mobile Century field experiment. Transportation Research Part C: Emerging Technologies, 18(4), 568-583.

Herring, R., Hofleitner, A., Abbeel, P., & Bayen, A. (2010, September). Estimating arterial traffic conditions using sparse probe data. In 13th International IEEE Conference on Intelligent Transportation Systems (pp. 929-936). IEEE.

Hofleitner, A., Herring, R., Abbeel, P., & Bayen, A. (2012). Learning the dynamics of arterial traffic from probe data using a dynamic Bayesian network. IEEE Transactions on Intelligent Transportation Systems, 13(4), 1679-1693.

Horn, M. E. T. (2002). Fleet scheduling and dispatching for demand-responsive passenger services. *Transportation Research Part C: Emerging Technologies*, *10*(1). www.elsevier.com/locate/trc

Hyland, M., & Mahmassani, H. S. (2020). Operational benefits and challenges of shared-ride automated mobility-on-demand services. *Transportation Research Part A: Policy and Practice*, *134*, 251–270. https://doi.org/10.1016/j.tra.2020.02.017

Jaw, J., Odoni, A. R., & Psaraftis Nigel H Wilson, H. N. (1986). *A heuristic algorithm for the multi-vehicle advance request dial-a-ride problem with time windows* (Vol. 2, Issue 3).

Kleiner, A., Nebel, B., Ziparo, V. A., & Srl, A. (2011). A Mechanism for Dynamic Ride Sharing based on Parallel Auctions. *Twenty-Second International Joint Conference on Artificial Intelligence*. www.avego.com

Krause, B., Von Altrock, C., & Pozybill, M. (1996, September). Intelligent highway by fuzzy logic: Congestion detection and traffic control on multi-lane roads with variable road signs. In Proceedings of IEEE 5th International fuzzy systems (Vol. 3, pp. 1832-1837). IEEE.

Kucharski, R., & Cats, O. (2020). Exact matching of attractive shared rides (ExMAS) for system-wide strategic evaluations. *Transportation Research Part B: Methodological*, *139*, 285–310. https://doi.org/10.1016/j.trb.2020.06.006

Levinson, H. S., & Lomax, T. J. (1996). Developing a travel time congestion index. Transportation Research Record, 1564(1), 1-10.

Lu, W., & Quadrifoglio, L. (2019). Fair cost allocation for ridesharing services – modeling, mathematical programming and an algorithm to find the nucleolus. *Transportation Research Part B: Methodological*, *121*, 41–55. https://doi.org/10.1016/j.trb.2019.01.001

Lu, W., Quadrifoglio, L., Lee, D., & Zeng, X. (2022). The ridesharing problem without predetermined drivers and riders: formulation and heuristic. *Transportation Letters*, *00*(00), 1–11. https://doi.org/10.1080/19427867.2022.2116674

Lu, W., Shen, C. W., & Quadrifoglio, L. (2014). Innovative operating strategies for paratransit services with zoning. *Transportation Research Record*, *2469*(2469), 120–128. https://doi.org/10.3141/2469-13

Ma, S., Zheng, Y., & Wolfson, O. (2013). T-share: A large-scale dynamic taxi ridesharing service. *Proceedings - International Conference on Data Engineering*, 410–421. https://doi.org/10.1109/ICDE.2013.6544843

Ma, S., Zheng, Y., & Wolfson, O. (2015). Real-Time City-Scale Taxi Ridesharing. *IEEE Transactions on Knowledge and Data Engineering*, *27*(7), 1782–1795. https://doi.org/10.1109/TKDE.2014.2334313

Mohan, P., Padmanabhan, V. N., & Ramjee, R. (2008, November). Nericell: rich monitoring of road and traffic conditions using mobile smartphones. In Proceedings of the 6th ACM conference on Embedded network sensor systems (pp. 323-336).

MTC. (2017). Miles traveled in congestion. Vital Signs - SF Bay Area. https://vitalsigns.mtc.ca.gov/indicators/miles-traveled-in-congestion

Quadrifoglio, L., Dessouky, M. M., & Ordóñez, F. (2008). A simulation study of demand responsive transit system design. *Transportation Research Part A: Policy and Practice*, *42*(4), 718–737. https://doi.org/10.1016/j.tra.2008.01.018

Renne, J. L., & Mayorga, E. (2022). What has America learned Since Hurricane Katrina? Evaluating evacuation plans for carless and vulnerable populations in 50 large cities across the United States. *International Journal of Disaster Risk Reduction*, *80*. https://doi.org/10.1016/j.ijdrr.2022.103226

Saeedmanesh, M., & Geroliminis, N. (2017). Dynamic clustering and propagation of congestion in heterogeneously congested urban traffic networks. Transportation research procedia, 23, 962-979.

Santi, P., Resta, G., Szell, M., Sobolevsky, S., Strogatz, S. H., & Ratti, C. (2014). Quantifying the benefits of vehicle pooling with shareability networks. *Proceedings of the National Academy of Sciences of the United States of America*, *111*(37), 13290–13294. https://doi.org/10.1073/pnas.1403657111

Santos, D. O., & Xavier, E. C. (2013). Dynamic Taxi and Ridesharing: A Framework and Heuristics for the Optimization Problem. *Twenty-Third International Joint Conference on Artificial Intelligence*.

Santos, D. O., & Xavier, E. C. (2015). Taxi and ride sharing: A dynamic dial-a-ride problem with money as an incentive. *Expert Systems with Applications*, *42*(19), 6728–6737. https://doi.org/10.1016/j.eswa.2015.04.060

Saranow, J. (2006). Carpooling for Grown-Ups-High Gas Prices, New Services Give Ride-Sharing a Boost; Rating Your Fellow Rider. *Wall Street Journal*. www.327ride.net

Schofer, J. L., & Mahmassani, H. S. (2016). *MOBILITY 2050 A Vision for Transportation Infrastructure Prepared for The Association of Equipment Manufacturers*.

Sethi, S., & Sorger, G. (1991). A theory of rolling horizon decision making. In *Annals of Operations Research* (Vol. 29).

Stiglic, M., Agatz, N., Savelsbergh, M., & Gradisar, M. (2015). The benefits of meeting points in ride-sharing systems. *Transportation Research Part B: Methodological*, *82*, 36–53. https://doi.org/10.1016/j.trb.2015.07.025

Tirachini, A. (2020). Ride-hailing, travel behaviour and sustainable mobility: an international review. *Transportation*, *47*(4), 2011–2047. https://doi.org/10.1007/s11116-019-10070-2

Tirachini, A., & Gomez-Lobo, A. (2020). Does ride-hailing increase or decrease vehicle kilometers traveled (VKT)? A simulation approach for Santiago de Chile. *International Journal of Sustainable Transportation*, *14*(3), 187–204. https://doi.org/10.1080/15568318.2018.1539146

Toan, T. D., & Wong, Y. D. (2021). Fuzzy logic-based methodology for quantification of traffic congestion. Physica A: Statistical Mechanics and its Applications, 570, 125784.

Wang, H., & Yang, H. (2019). Ridesourcing systems: A framework and review. *Transportation Research Part B: Methodological*, *129*, 122–155. https://doi.org/10.1016/j.trb.2019.07.009

Wang, W. X., Guo, R. J., & Yu, J. (2018). Research on road traffic congestion index based on comprehensive parameters: Taking Dalian city as an example. Advances in Mechanical Engineering, 10(6), 1687814018781482.

Wang, X., Agatz, N., & Erera, A. (2018). Stable matching for dynamic ride-sharing systems. *Transportation Science*, *52*(4), 850–867. https://doi.org/10.1287/trsc.2017.0768

Wang, Z., Li, L., Tang, J., Xie, B., & Ge, J. (2022). Effect of ride sharing on air quality: evidence from Shenzhen, China. *Journal of Applied Economics*, *25*(1), 197–219. https://doi.org/10.1080/15140326.2021.2023421

Yousaf, J., Li, J., Chen, L., Tang, J., & Dai, X. (2014). Generalized multipath planning model for ride-sharing systems. *Frontiers of Computer Science*, *8*(1), 100–118. https://doi.org/10.1007/s11704-013-3021-6

Yu, X., & Shen, S. (2020). An Integrated Decomposition and Approximate Dynamic Programming Approach for On-Demand Ride Pooling. *IEEE Transactions on Intelligent Transportation Systems*, *21*(9), 3811–3820. https://doi.org/10.1109/TITS.2019.2934423

Zarindast, A. (2019). A data driven method for congestion mining using big data analytic.

Zhang, Y., Ye, N., Wang, R., & Malekian, R. (2016). A method for traffic congestion clustering judgment based on grey relational analysis. ISPRS International Journal of Geo-Information, 5(5), 71.

Zhou, X., Wang, M., & Li, D. (2019). Bike-sharing or taxi? Modeling the choices of travel mode in Chicago using machine learning. *Journal of Transport Geography*, *79*. https://doi.org/10.1016/j.jtrangeo.2019.102479

Zhu, P., & Mo, H. (2022). The potential of ride-pooling in VKT reduction and its environmental implications. *Transportation Research Part D: Transport and Environment*, *103*. https://doi.org/10.1016/j.trd.2021.103155

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