

## Development of a Computational Model for Predicting Fracture in Rails Subjected to Long-Term Cyclic Fatigue Loading

David H. Allen  
Professor  
Director, Center for Railway Research  
Ocean Engineering Department  
Texas A&M University/Texas A&M Transportation Institute

Yong-Rak Kim  
Professor  
Associate Director, Center for Railway Research  
Civil Engineering Department  
Texas A&M University

A Report on Research Sponsored by

University Transportation Center for Railway Safety (UTCRS)

Texas A&M University

September 30, 2024

## Technical Report Documentation Page

1. Report No. UTCRS-TAMU-I2CY23	2. Government Accession No.	3. Recipient's Catalog No.	
4. Title and Subtitle  Development of a Computational Model for Predicting Fracture in Rails Subjected to Long-Term Cyclic Fatigue Loading		5. Report Date September 30, 2024	
		6. Performing Organization Code UTCRS-TAMU	
7. Author(s) D.H. Allen, G. Dorsett, and Y.-R. Kim		8. Performing Organization Report No. UTCRS-TAMU-I2CY23	
9. Performing Organization Name and Address University Transportation Center for Railway Safety (UTCRS) Texas A&M Transportation Institute (TTI) Texas A&M University (TAMU) College Station, TX 77843		10. Work Unit No. (TRAIS)	
		11. Contract or Grant No. 69A3552348340	
12. Sponsoring Agency Name and Address U.S. Department of Transportation (USDOT) University Transportation Centers Program 1200 New Jersey Ave. SE Washington, DC, 20590		13. Type of Report and Period Covered Project Report June 1, 2023-August 31, 2024	
		14. Sponsoring Agency Code USDOT UTC Program	
15. Supplementary Notes			
16. Abstract It is well-known that one of the most significant causes of train derailments within the U.S. is rail fracture. Despite this fact, a reliable model for predicting fatigue fracture in rails has not yet been deployed within the U.S. We have recently been developing a two-way coupled multiscale computational algorithm for predicting crack evolution in ductile solids subjected to long-term cyclic loading. In this UTCRS project, we will adapt this model to the prediction of crack growth in rails. Concomitantly, with funding provided by TTCI (Now MxV), we have for nearly a decade performed long-term laboratory cyclic crack growth experiments on rails. We possess the ability to both predict crack growth due to cyclic fatigue in rails and utilize our experimental results to validate our predictive methodology. It is therefore our intention to: (1) modify our multi-scale computational model to predict crack growth due to cyclic fatigue in rails; (2) validate our model against our own previously obtained experimental results; and (3) develop a procedure based on our model for railway engineers to utilize to determine when rails should be inspected and potentially removed from service for cause, thereby enhancing rail safety.			
17. Key Words Construction, Maintenance, Civil Engineering		18. Distribution Statement This report is available for download from <a href="https://www.utrgv.edu/railwaysafety/research/infrastructure/index.htm">https://www.utrgv.edu/railwaysafety/research/infrastructure/index.htm</a>	
19. Security Classification (of this report) None	20. Security Classification (of this page) None	21. No. of Pages 15	22. Price

## Tables of Contents

<b>List of Figures</b> .....	3
<b>List of Abbreviations</b> .....	4
<b>Disclaimer</b> .....	4
<b>Acknowledgments</b> .....	4
<b>Research Overview</b> .....	5
<b>Modeling Approach</b> .....	8
<b>Results to Date</b> .....	9
<b>References</b> .....	15

## List of Figures

Figure 1: Rail head in experimental apparatus.....	5
Figure 2: Results of multiaxial fatigue tests performed on rails .....	6
Figure 3: Photo of fracture surface of Rail#7, fractured after 18M load cycles .....	7
Figure 4: Image depicting our cohesive zone model .....	8
Figure 5: Relation between CZ-model material parameter-A and the number of cycles to failure of a single element within rail steel .....	9
Figure 6: Relation between CZ-model material parameter-m and the number of cycles to failure of a single element within rail steel .....	10
Figure 7: Typical evolution of the traction-displacement curve during cyclic fatigue loading .....	11
Figure 8: Depiction of two-way coupled computational model for predicting crack growth under cyclic fatigue loading .....	11
Figure 9: (a) Crack propagation for different angles, (b) Crack growth rate for different angles .....	13
Figure 10: Crack propagation for different loadings.....	13
Figure 11: Crack propagation is illustrated for various initial crack sizes.....	14

## **List of Abbreviations**

RVE	Representative Volume Element
USDOT	U.S. Department of Transportation
UTC	University Transportation Centers

## **Disclaimer**

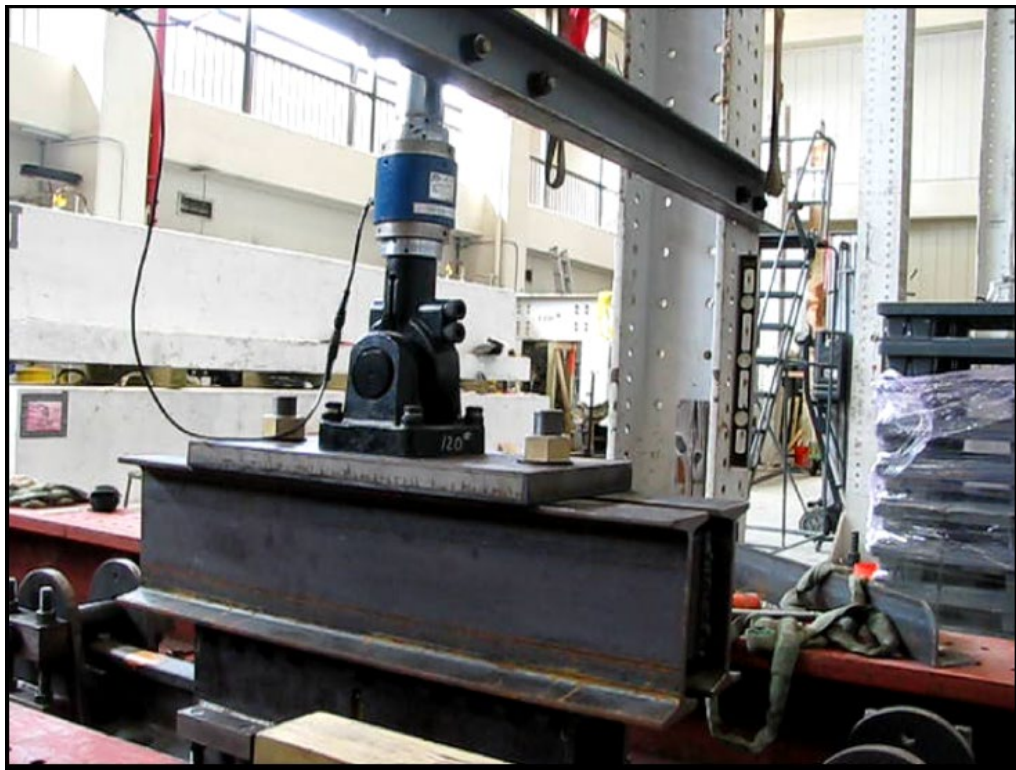
The contents of this report reflect the views of the authors, who are responsible for the facts and the accuracy of the information presented herein. This document is disseminated under the sponsorship of the U.S. Department of Transportation's University Transportation Centers Program, in the interest of information exchange. The U.S. Government assumes no liability for the contents or use thereof.

## **Acknowledgments**

The authors wish to acknowledge the University Transportation Center for Railway Safety (UTCRS) for funding this project under the USDOT UTC Program Grant No 69A3552348340.

## Research Overview

Given that an accurate model for predicting rail fracture due to cyclic fatigue loading has thus eluded the research community, we proposed to 1) modify and deploy our robust multi-scale model for predicting crack growth in rails due to cyclic fatigue loading; 2) validate that model against experimental results previously obtained within our own labs; and 3) make our model accessible to on-site track engineers, thereby mitigating the likelihood of train derailments caused by track fracture within the U.S.

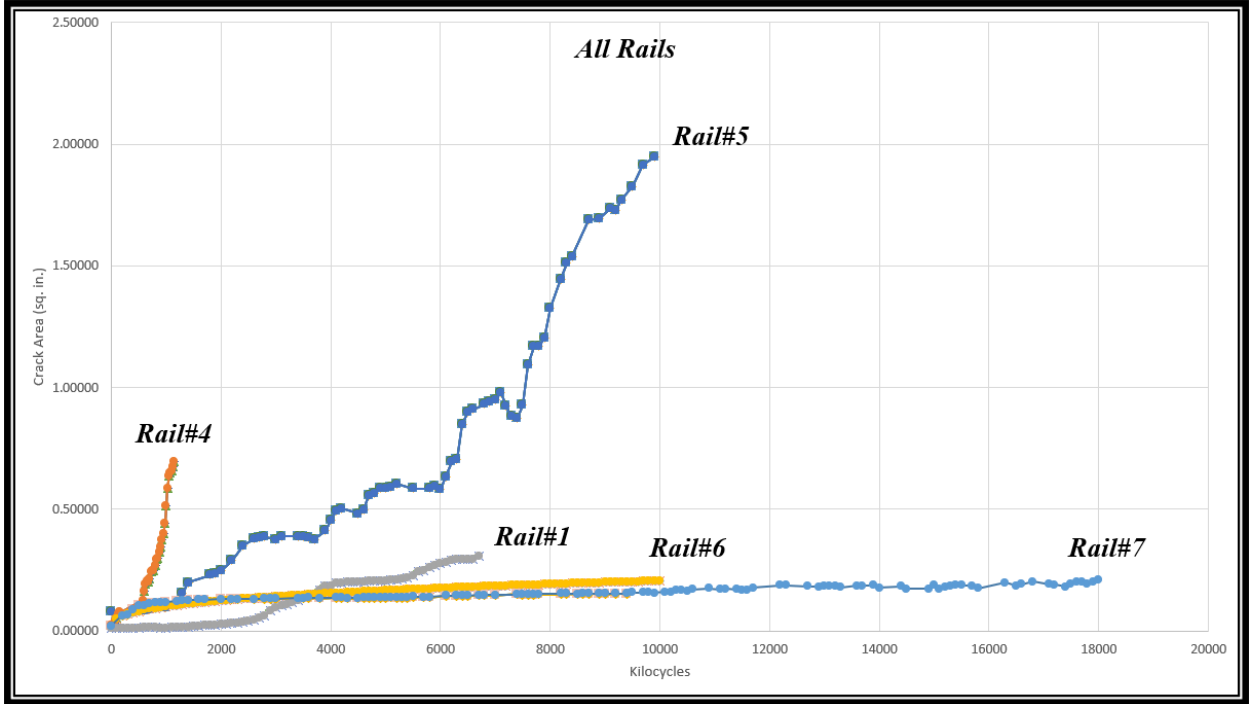


**Figure 1:** Rail head in experimental apparatus

We note herein that our model, which deploys a multiscale finite element method previously developed by our research group, together with a nonlinear viscoelastic cohesive zone model, to predict fatigue-induced crack growth in rails. Furthermore, we have over the previous decade performed the most complex set of multiaxial laboratory fatigue experiments on previously in-service rails with internal defects ever undertaken within the U.S. (Fig. 1). Our objective then is to develop via a newly constructed set of simplified railhead experiments to determine the material

properties necessary to deploy our multiscale computational algorithm and determine the accuracy of our computational approach versus the complex multiaxial rail experiments we performed over the previous decade (funded by MxV Rail).

We have now been performing these tests for nine years, during which we have successfully tested five rails over several million cycles [Whetstone et al., 2023], thereby resulting in the crack evolution diagram shown in Fig. 2.

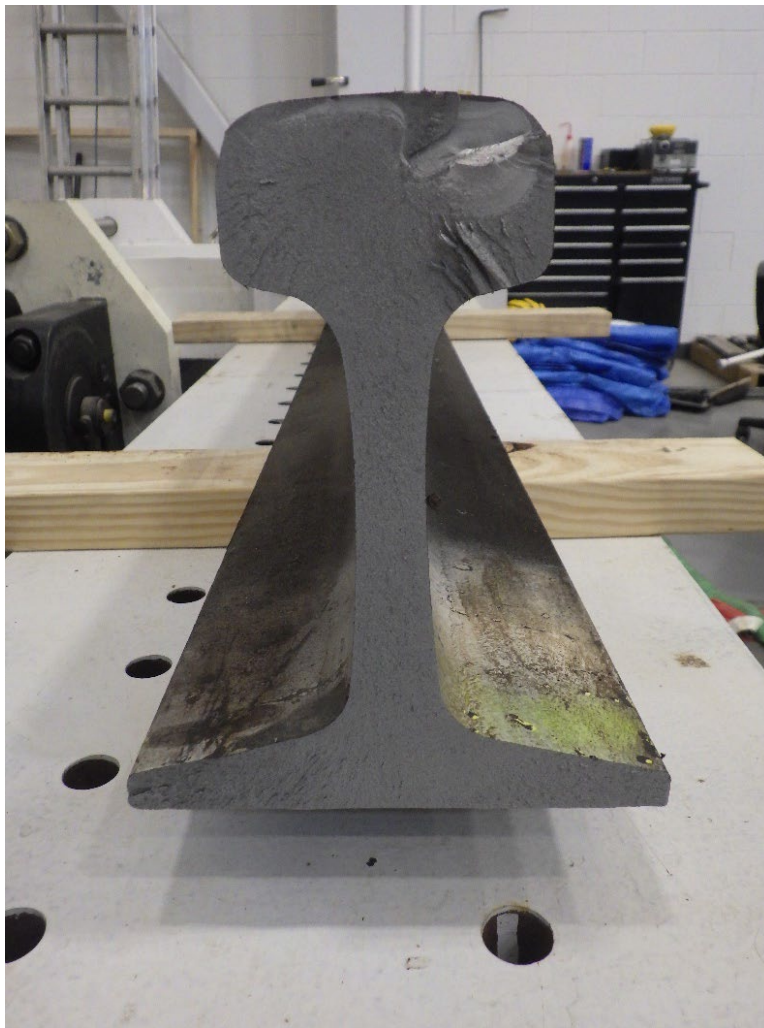


**Figure 2:** Results of multiaxial fatigue tests performed on rails

If our model proves to be accurate compared to the previously performed experiments, then we believe that we will have significant evidence of its veracity as a tool for predicting fracture in rails subjected to long term cyclic fatigue.

We will then utilize our model to predict crack growth due to fatigue loading in rails as a function of three primary variables: initial crack location within the railhead; initial crack size within the railhead; and initial crack orientation within the rail head. We note that these three geometric parameters are readily determined by field engineers who perform nondestructive evaluations of in-service rails using acoustic emitters.

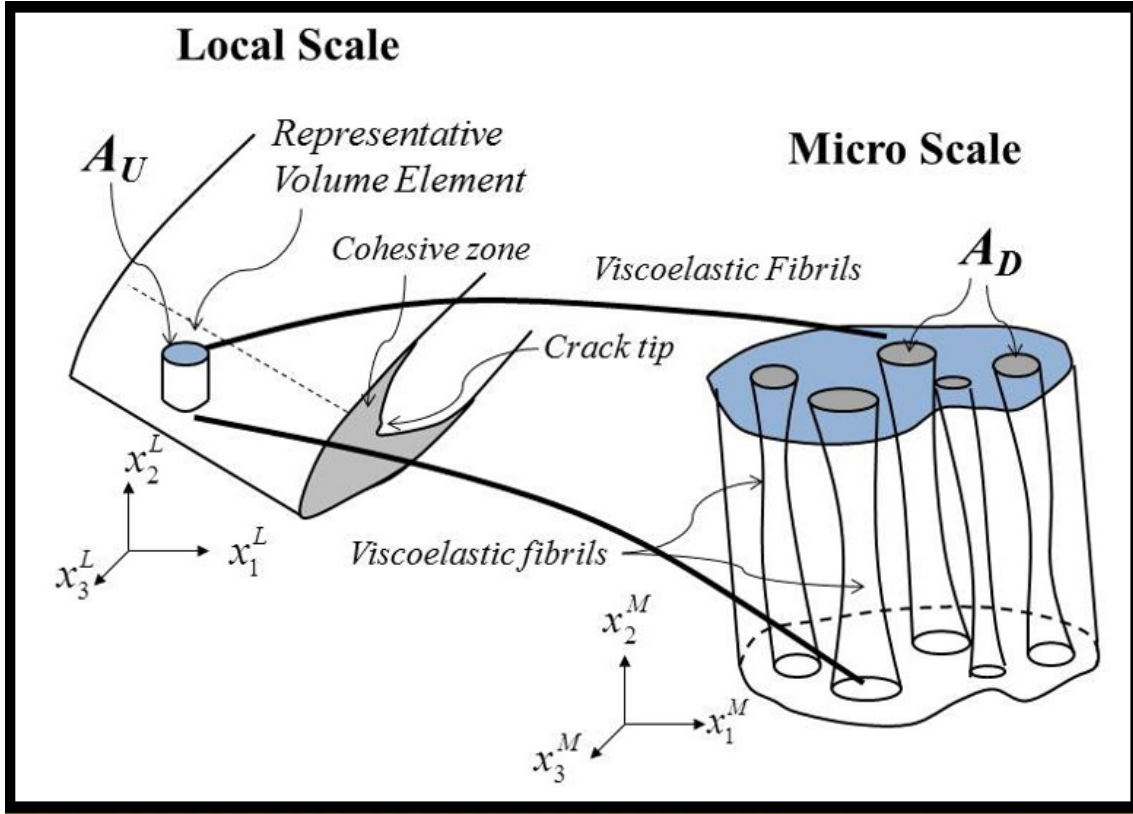
Currently, when a rail engineer detects an internal flaw within the railhead it is standard practice to remove the rail from service immediately. However, the rail may actually have significantly more life before the rail needs to be removed from service, and our model is intended to supply that information ‘on the fly’ to field engineers, thereby oftentimes allowing the rail to remain in service. Such an approach has previously been developed within commercial aviation, we might add – to great success – thereby making commercial air travel the safest means of transport in history. This then is our objective within the rail industry. If successful, our model will then simultaneously increase safety and decrease the cost of rail transport.



**Figure 3:** Photo of fracture surface of Rail#7, fractured after 18M load cycles



## Modeling Approach



**Figure 4:** Image depicting our cohesive zone model

Our nonlinear cohesive zone model [1,2] is described by the following nonlinear traction-displacement law:

$$t_i(t) = \frac{u_i}{\delta_i} [1 - \alpha(t)] \int_0^t D(t - \tau) \frac{\partial \lambda}{\partial \tau} d\tau \quad (1)$$

where

- $t_i$  are the components of the crack-opening traction vector
- $u_i$  are the components of the crack opening displacement vector
- $D(t)$  is the cohesive zone relaxation modulus
- $\alpha(t)$  is the current value of the interfacial damage parameter, which is modeled by a damage evolution law.
- $\lambda(t)$  is the Euclidean norm of the cohesive zone interfacial displacement vector



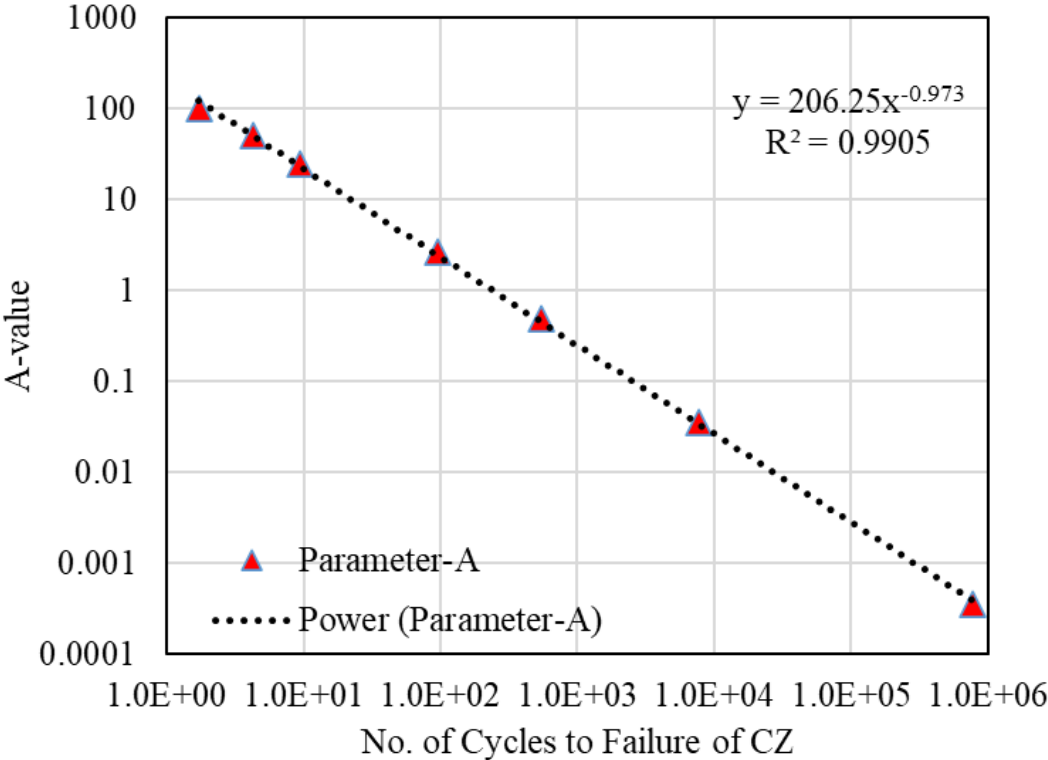
The evolution of damage,  $\alpha(t)$ , is modeled via the following damage evolution law:

$$\frac{d\alpha}{dt} = A\lambda^m \tag{2}$$

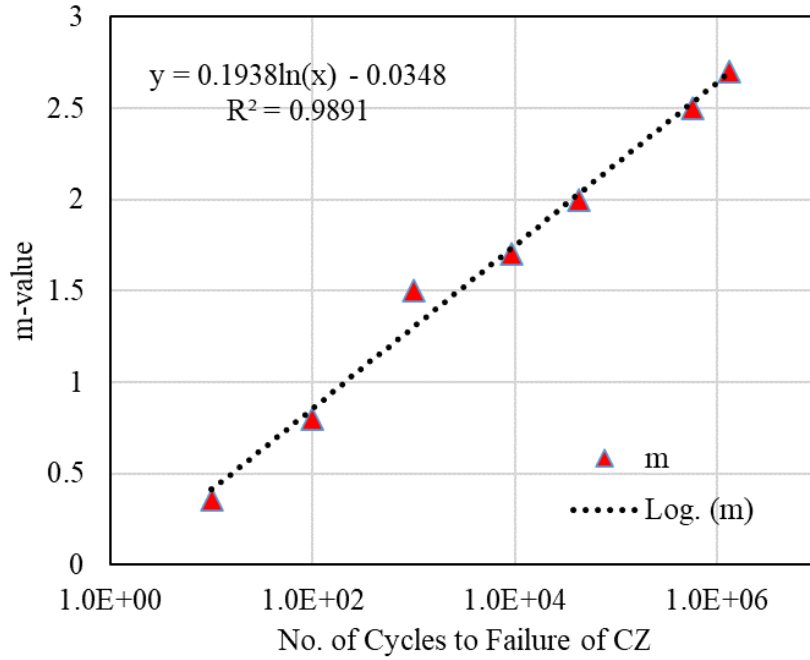
where A and m are material properties to be determined by the experimental program in our companion research effort [CRR-2024-1].

**Results to Date**

To demonstrate the sensitivity of the cohesive zone model to the material parameters imbedded within the model, we show two results below. In Fig. 5, the relation between the parameter A and number of cycles to failure of the cohesive zone is depicted. In Fig. 6, the relation between the parameter m and number of cycles to failure of the cohesive zone is depicted



**Figure 5:** Relation between CZ-model material parameter-A and the number of cycles to failure of a single element within rail steel



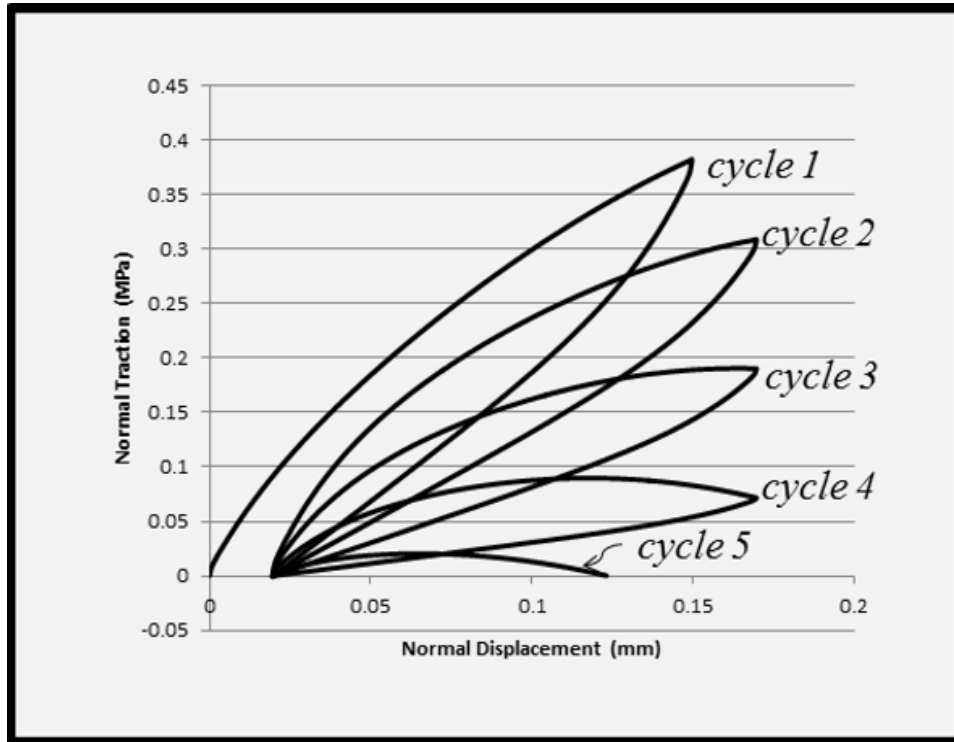
**Figure 6:** Relation between CZ-model material parameter- $m$  and the number of cycles to failure of a single element within rail steel

The above fracture model will be deployed within our multiscale nonlinear finite element computational algorithm [3-10] to predict crack growth in rail subjected to complex long-term fatigue loading, as described in our companion annual report [CRR-2024-1]. As an example, Fig. 7 shows a depiction of a typical evolution of the traction-traction graph, with crack extension predicted on the fifth cycle of loading.

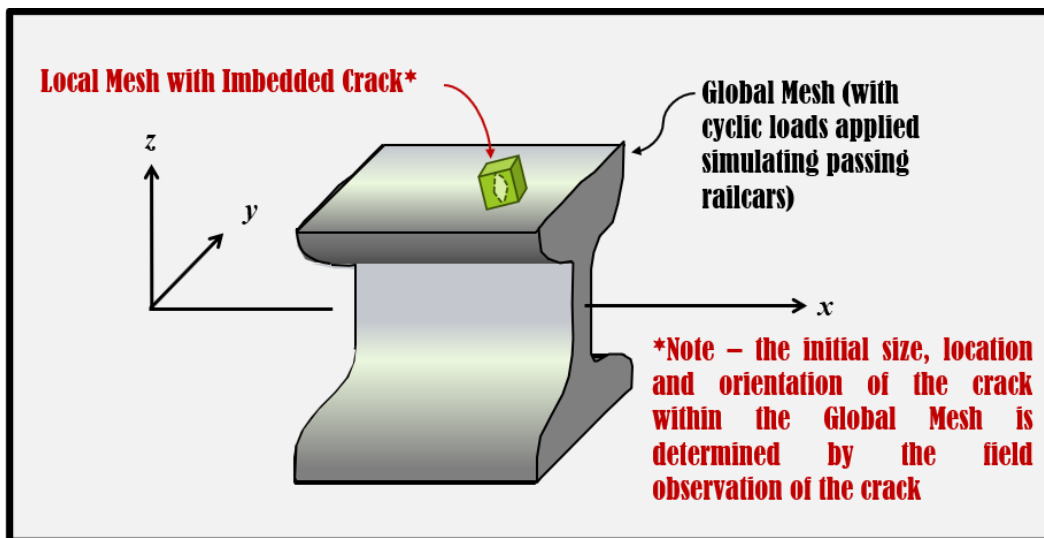
The above fracture model has been deployed within our multiscale nonlinear finite element computational algorithm [3-10], and we are now utilizing it to predict crack growth in rails subjected to complex long-term fatigue loading.

The two-way coupled multiscale computational algorithm that is deployed herein is based on the prior works of Allen and co-workers [Souza et al., 2008, Souza et al., 2009, Souza and Allen, 2009, Souza and Allen, 2012, Allen et al., 2017a,b,c, Little et al., 2018]. In this approach, we model the response of the rail head on two simultaneous length scales (Fig. 8), wherein the larger scale is the scale of the rail cross-section, and the smaller scale is on the length-scale of the metal grains. These two length scales are coupled to one another both ways, thereby rendering a two-way coupled multiscale model. Furthermore, explicit evolutionary cracking is predicted on the

smaller length scale using a cohesive zone model [Allen and Searcy 2000, Foulk et al. 2000, Allen and Searcy 2001a, 2001b].



**Figure 7:** Typical evolution of the traction-displacement curve during cyclic fatigue loading

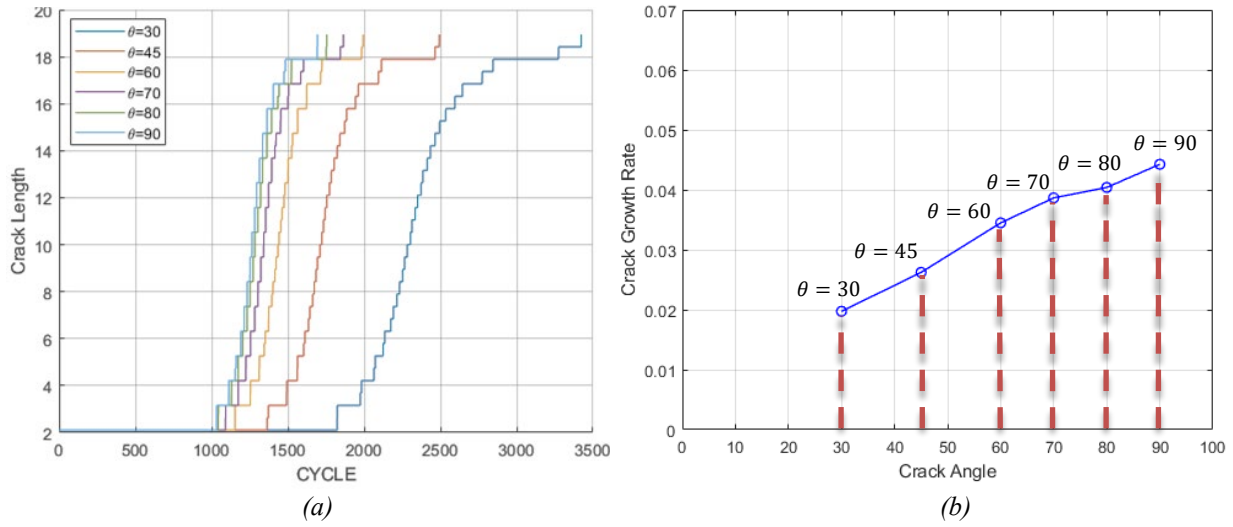


**Figure 8:** Depiction of two-way coupled computational model for predicting crack growth under cyclic fatigue loading

To account for this two-way coupling between the local and global length scales, the multi-scale algorithm utilizes a time marching scheme. On the first time-step, it is assumed that the structural component is undamaged, and a global analysis is performed for an increment of loading, wherein it is assumed that the component is initially spatially homogeneous at the global length scale. The globally calculated stresses obtained from this first loading increment are then applied to each and every local scale representative volume element (RVE), thereby producing spatially varying predicted states in the evolutionary cracking in each RVE, with greater cracking occurring in those RVE's that undergo greater global stresses or contain pre-existing defects. This process causes the local analyses to be dependent on (coupled to) the global analyses due to the spatial variation in the global scale stresses, thereby accounting for the differences caused by spatially varying wheel distributions, as well as spatially varying crack locations within the rail head.

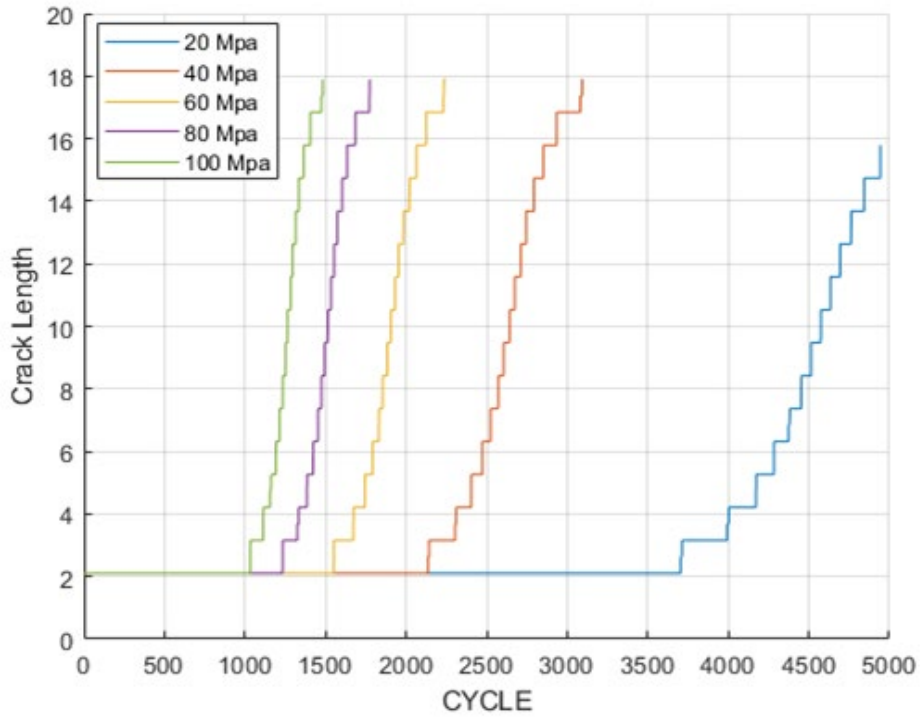
Once the local analyses have been performed within each local RVE on the given time step, the predicted results are then utilized to construct the homogenized tangent modulus within each RVE, and this modulus varies spatially due to the different state of cracking predicted within each RVE. Note that the solution for each RVE at the local scale is obtained using the finite element method, thus resulting in one finite element mesh for each RVE at the local scale. The total number of finite element meshes is thus one plus the number of local RVE's. Applying this procedure repeatedly within a time marching scheme produces the desired two-way coupled multi-scaling algorithm. The evolving nonlinearity due to the evolution of cracking in the calculations is accounted for using Newton iteration (Little et al. 2018).

The two-way coupled multiscale finite element method is used to model the effect of the imposed loading profile on the spatial and temporal variation in fatigue crack nucleation at the microstructural scale. As described above, in the calculations the transverse section of the rail is modeled at the global scale while at the local scale, a multiple grain microstructure characterized by the distribution of grain boundaries (or cohesive surfaces) is constructed.



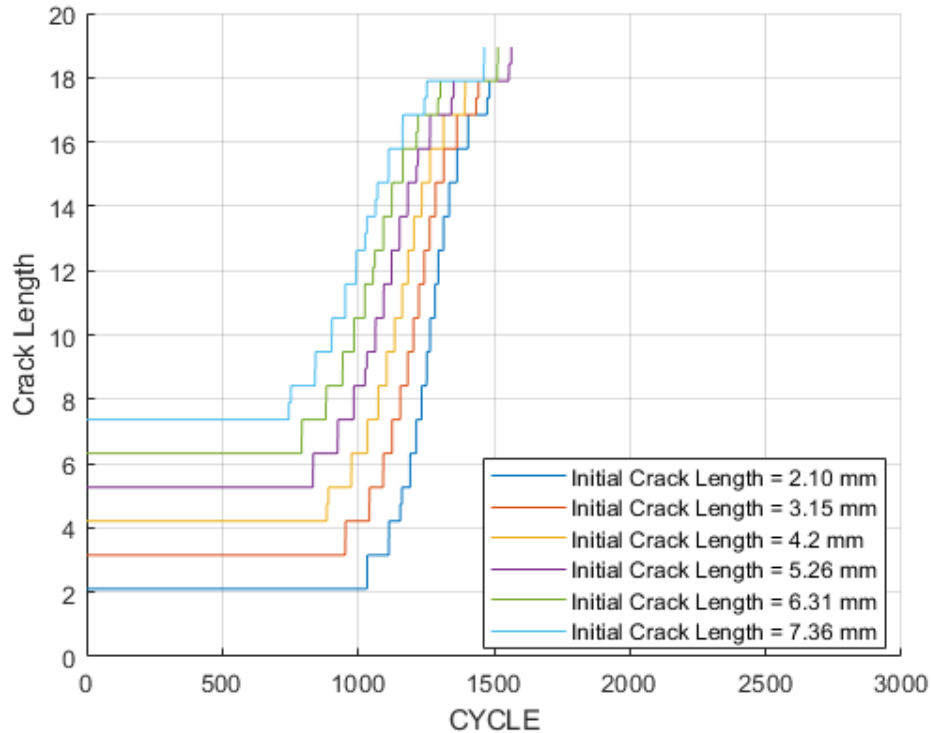
**Figure 9:** (a) Crack propagation for different angles, (b) Crack growth rate for different angles

Figure 9 illustrates the relationship between crack length and cyclic load (Figure 9(a)) as well as the crack growth rate as a function of crack angle (Figure 9(b)) for different angles.



**Figure 10:** Crack propagation for different loadings

Figure 10 illustrates the relationship between crack length and the number of cycles under different cyclic loading magnitudes, with each curve representing how crack length evolves as the material is subjected to cyclic loading at specific stress levels. We have vertical cracks ( $\theta = 90^\circ$ ) in all cases. The progression of crack propagation is illustrated for various initial crack sizes in Fig. 11. We have vertical cracks ( $\theta = 90^\circ$ ) in all cases.



**Figure 11:** Crack propagation is illustrated for various initial crack sizes

Based on these initial results, we are optimistic that this relatively new and unique approach to modeling rail head fracture due to long-term cyclic fatigue will produce a tool capable of dramatically improving our ability to assess track worthiness of rails with internal imperfections.

## References

- Federal Railroad Administration (2022) Railroad Accident Statistics, downloaded at:<http://safetydata.fra.dot.gov/officeofsafety/publicsite/Query/TrainAccidentsFYCYWithRates.aspx>
- D Allen and C Searcy, (2000) Numerical Aspects of a Micromechanical Model of a Cohesive Zone, *Journal of Reinforced Plastics and Composites*, Vol. 19, No. 3, pp. 240-248.
- D Allen and C Searcy, (2001) A Micromechanical Model for a Viscoelastic Cohesive Zone, *International Journal of Fracture*, Vol. 107, pp. 159-176.
- D Allen and C Searcy, (2001) A Micromechanically Based Model for Predicting Dynamic Damage Evolution in Ductile Polymers, *Mechanics of Materials*, Vol. 33, pp. 177-184.
- D Allen, D Little, R Soares and C Berthelot, (2017) Multi-Scale Computational Model for Design of Flexible Pavement – Part I: Expanding Multi-Scaling, *Int. Journal of Pavement Engineering*, Vol. 18, pp. 309-320.
- D Allen, D Little, R Soares, and C Berthelot, (2017) Multi-Scale Computational Model for Design of Flexible Pavement – Part II: Contracting Multi-Scaling, *Int. Journal of Pavement Engineering*, Vol. 18, pp. 321-334.
- D Allen, D Little, R Soares, and C Berthelot, (2017) Multi-Scale Computational Model for Design of Flexible Pavement – Part III: Two-Way Coupled Multi-Scaling, *Int. Journal of Pavement Engineering*, Vol. 18, pp. 335-348.
- J Foulk, D Allen and K Helms, (2000) Formulation of a Three-Dimensional Cohesive Zone Model for Application to a Finite Element Algorithm, *Computational Methods in Applied Mechanics and Engineering*, Vol. 183, pp. 51-66.
- D Little, D Allen, and A Bhasin, (2018) Modeling and Design of Flexible Pavements, Springer.
- F Souza, D Allen and Y Kim, (2008) Multiscale Model for Predicting Damage Evolution in Composites Due to Impact Loading, *Composites Science and Technology*, Vol. 68, pp. 2624-2634.
- F Souza and D Allen, (2009) Model for predicting multiscale crack growth due to impact in heterogeneous viscoelastic solids, *Mechanics of Composite Materials*, Vol. 45, pp. 145-152.
- F Souza and D Allen, (2012) Modeling the Transition of microcracks into macrocracks in Heterogeneous Viscoelastic Media Using a Two-way Coupled Multiscale Model, *International Journal of Solids and Structures*, Vol. 48, pp. 3160-3175.
- G Whetstone, T Liu, P Fudlailah and D Allen, (2023) Evaluation of Phased Array Data Reading for Purposes of Crack Growth Modeling in Rail Heads, in review.