

ROLLING RESISTANCE REVISITED

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SUMMARY

Resistance to motion is an important input for train performance simulations, and the Davis equation is a well-known resistance formula, widely used for freight trains. It is known to give conservative values, and has numerous variants such as Modified Davis and Adjusted Davis with factors of e.g. 0.7 being applied to the formula. There is confusion about the right constants to use, and frequently specifications for equipment contain values which are inappropriate.

This paper begins with a review of some of the published formulae, and then presents a methodology for calculating the overall resistance of trains of known weight and composition operating on known track geometry, using tractive effort, GPS location and speed records downloaded from locomotive data loggers. A simulation program was modified to take the time history of tractive effort as an input, and to calculate velocity, which is then compared with the recorded speed. The "Davis" formula parameters are adjusted iteratively until the calculated speed approximately matches the actual speed. This approach is used to benchmark values of published constants and to suggest appropriate values for trains operating in Australia.

1. INTRODUCTION

Train performance simulations are used for planning new track alignments, for predicting the maximum loads which can be hauled, or the speed or fuel consumption for proposed train configurations on existing track. Resistance to motion is an important input for any such simulation, and the Davis equation, which is a well-known resistance formula, is widely used for freight trains.

The Davis equation is a quadratic in velocity of the form $R = A + Bv + Cv^2$, with constant coefficients based on experimental work done by Davis in 1926 [1]. It is known to give conservative resistance values, and the formula has numerous variants such as Modified Davis and Adjusted Davis. There is confusion about the right constants to use.

2. DAVIS EQUATION

The original form of the Davis equation for freight cars was:

$$R' = 1.3 + 29/w + 0.045v + 0.0005av^2/wn \quad (1)$$

where R' is resistance in lb/ton
 w is axle load in tons
 (here ton= short ton = 2000 lb)
 n is the number of axles
 a is the frontal area in sq.ft

Note my (non-standard) use of R' , to distinguish per unit resistance from total R , in units of force.

The "Modified" Davis equation for Freight Cars published in AAR RP-548 [2] takes the form:

$$R' = 1.3 + 72.5/wn + 0.015v + 0.055v^2/wn \quad (2)$$

Note the second term is a constant by the number of axles. In Davis' work, when most rollingstock had journal bearings, it was 29lb/axle. The constant in RP-548 is based on 18lb/axle, for a four axle wagon with roller bearings, which is not immediately obvious in the formula above.

In RP-548, the last (aerodynamic drag) term incorporates a cross-sectional area of 110 sq.ft. Other variants of the equation have drag coefficients according to wagon type.

Converting to metric units, (with $g = 10 \text{ m/s}^2$):

$$R' = 6.5 + 320/wn + 0.046v + 0.096 v^2/wn \text{ N/t} \quad (3)$$

Note that these constants have an accuracy of less than two decimal places, hence the use of 10 m/s^2 in the conversions for convenience.

3. COMPONENTS IN THE EQUATION

Table 1 following compares the contribution of each component of resistance, for a wagon when loaded (80 tonne gross) and empty (20 tonne tare). R is the total resistance, and R' the Davis resistance in N/tonne.

V (km/h)	Mass (t)	A	A0	B	C	R (N)	R'(N/t)
20	80	520	320	73.6	37.8	951	11.9
	20	130	320	18.4	37.8	506	25.3
80	80	520	320	294	605	1740	21.7
	20	130	320	73.6	606	1128	56.4
100	80	520	320	368	946	2154	26.9
	20	130	320	92	946	1488	74.4

Table 1 Resistance components for 80 tonne wagon as per AAR RP-548

At 20 km/h, the rolling friction (A+ A0) terms make up 88% of the total resistance for the wagon at gross, and 47% at tare. The minimum continuous speed of a locomotive is typically around 20 km/h, so only the A and A0 terms come into play when a loaded train crawls up a steep grade. At higher speeds, the C (air drag) term may be more than half of the total resistance.

The A terms give resistance equivalent to a 0.12% or 1/824 grade for the loaded wagon, and 0.26% or 1/388 for the empty wagon above. Small variations in grade may therefore cause significant errors when trying to measure resistance.

Calculating the resistance R' in the units of the A term (lb/ton) can be confusing as adding mass reduces unit resistance, since the air resistance term is independent of axle load. It is only useful for calculations using equivalent grades. For the cases in this table, the total resistance R varies by a factor of 4, while R' as calculated by Davis varies by a factor of 6, and yet the value (26.9) for the loaded wagon at 100 km/h is almost the same (25.3) as the empty wagon at 20 km/h. R' therefore provides little insight into the train's behaviour. It is clearer to write the equation for the resistance per wagon of mass m and n axles as:

$$R = 6.5m + 80n + 0.046vm + 0.096 v^2 \text{ (N)} \quad (4)$$

4. IS B BOGUS?

It is questionable whether the B term has any validity. As mentioned in the AREMA Manual [3], some work by AAR in 1988 suggested this term should be zero, and I have seen evidence to support this. In 2000, when the 4000 Class locomotive was introduced in Queensland, some tests to determine rolling resistance were performed by measuring deceleration while coasting to rest, and curve fitting the data. The constant term was found to be 1262N, which was close to the AAR formula (albeit for a wagon). The air drag term also agreed reasonably well with the RP548 value, but oddly, the B term was negative, i.e -265 N/km/h. It was assumed this was a mistake.

We were not alone in finding this anomalous result. A 2006 paper by Kim, Kwon, Kim and Park [4] describes resistance values derived from coasting tests during the introduction of the KTX high speed train in Korea. The initial result found a negative value for B, of -19.5 N/km/h for a train weighing 326 tonnes. The tests were intended to compare the aerodynamic drag of the KTX with the TGV, from which it was derived. The Koreans assumed the French coefficients were authoritative, and re-analysed their data accordingly, which changed the values determined for the air drag coefficient.

Forces proportional to velocity are associated with damping, or laminar flow. The B term is often described as "flanging", but unless a vehicle is hunting badly, the flanges do not make contact on tangent track. Even if they did, sliding friction would not give resistance increasing linearly with velocity. In fact, Coulomb friction is high at near zero velocity, and then decreases, which might explain a negative value. Other "dynamic" effects such as draft gear action are considered to be covered by this term, but there seems to be no theoretical basis for this. Based on the above, I believe the velocity coefficient is very small, and can be taken as zero.

5. OTHER FORMULATIONS

In 1990, Canadian National (quoted in [3]) published a version of the Davis formula with A = 1.5 lb/ton. AAR also suggested that A can vary from 0.7 to 2.13 lb/ton [3]. There are many factors which affect the "constant".

A different approach is often used in passenger train formulae for the velocity independent term, with the product of mass and number of axles n under a square root function. For example, SNCF uses the following formula for double deck EMUs

$$R = 14 \times \sqrt{10 \times n \times M} \text{ (N)} \quad (5)$$

Where n is number of axles
M is mass in tonnes

Surprisingly, this non-linear function gives values which are not significantly different from Davis, as shown below in Table 2 for the 80 tonne wagon case:

Case	Mass	SCNF	Davis	Diff
Loaded	80 T	792 N	840 N	6%
Half	50 T	626 N	645 N	3%
Empty	20 T	396 N	450 N	12%

Table 2 Resistance as per Davis vs SCNF

While my initial intention was to use experimental data to check which approach is more valid, the small differences in the results might preclude this. Nevertheless, both formulas will give double the resistance if the number of wagons (and hence axles) is doubled. Being linear, Davis is the simpler formulation, which passes Ockham's Razor.

6. AERODYNAMIC DRAG

As above, the formula in RP-548 has a "one size fits all" approach, and combines an area of 110sqft with a drag coefficient of 0.0005 into one term, but there are other published values for various wagons. Table 3 following compares the RP-548 value with some values published by Canadian National [3]:

	Coeff	A sq.ft	CxA	Metric
AAR RP548	0.0005	110	0.055	0.096
Coal gondola	0.00042	105	0.044	0.077
Empty gondola	0.0012	105	0.126	0.219
Lead loco	0.0024	160	0.384	0.670
Trailing loco	0.00055	160	0.088	0.154

Table 3 Air Drag as per Canadian National

The Canadian work suggests that the resistance for an empty hopper wagon is nearly 3 times that for a loaded wagon, due to turbulence inside the hopper. Also, resistance for the lead locomotive is quite high, but it is generally found that trailing locomotives have lower resistance, similar to that of wagons.

Total resistance for the train is calculated by summing the drag for each vehicle. However, Lai and Barkan [5] report studies indicating that drag decreases until about the 10th unit, and then remains approximately constant per unit.

Again it should be stated that it is incorrect to include the air resistance in a lb/ton value for each vehicle, and then to sum over R' and then multiply by the mass of the train. The aerodynamic drag is

independent of mass and should be calculated separately.

The drag value calculated in the coasting tests for the 4000 class locomotive was $0.24v^2$ [N]. Using a cross-sectional area of 11 m² with the RP-548 value of 0.0024 in the table gives $0.54 v^2$ which is at bit more than double. However, the following assumes the AAR value for the locomotive, and only considers wagon resistance.

7. EXPERIMENTAL COMPARISONS

The following section uses data obtained from locomotive event recorders to benchmark Davis formula results. Note that these were not special, instrumented runs, but trains in normal service. It would be more accurate to measure resistance by hauling trains on flat track, and measuring the forces in the coupler of the lead wagon using strain gauges, but this has the disadvantage that it requires a great deal of preparation and it is hard to find perfectly flat track of sufficient length. The method following tries to make use of data which is easily obtained, but this necessarily includes many variables such as wind conditions, which must be accounted for.

Event recorder data typically includes tractive effort, speed and GPS data for the locomotive, recorded at one second intervals. GPS co-ordinates allow the instantaneous location on the track to be determined. Simulation programs calculate the acceleration of a train from the force balance based on the track geometry, the tractive effort curve, and the instantaneous resistance. This is integrated to give velocity and distance. In this case, the instantaneous tractive effort time history is known and becomes an input to the simulation. By trialling different values for the resistance coefficients, the aim is to match the recorded instantaneous velocity.

Using this methodology to verify Davis coefficients relies on accurate data for both track geometry and the mass for the vehicles in the consist. It was found that in practice there may be variations in all of these. Loadings on coal and ore trains vary by a few percent from the nominal, and while the exact weight may be known if the payload is accurately weighed, it is sometimes a problem to match the locomotive logs to that particular train. Also, published track geometry can be surprisingly inaccurate. Gradients are subject to small changes as track is maintained and re-laid, and this can be significant. This method uses data from multiple runs to average out the variations, rather than trying to get very accurate data for each run.

To perform a run, it is necessary to choose a fairly straight section of track, or one with generous curve radii, as resistance increases in tight curves. It is necessary to locate the starting point at a clearly defined feature such as a passing loop, where the GPS co-ords from the event recorder (EVR) can be matched to a point in the track geometry. There should be no friction braking through the run, as it is not possible to accurately calculate the actual brake effort from the EVR data. Dynamic braking, however, can be handled if the DB Effort is recorded as a force. The section should also contain no steep grades requiring very high adhesion, as high creep values will affect the results.

8. IRON ORE TRAIN

The first case considered is for a long train carrying ore in the Pilbara. Data was obtained for two locomotives hauling a train with 244 wagons, approximately 2500m in length. Wagon mass was 23 tonnes tare, and 160 tonnes gross. The A and A0 terms largely determine the fit at lower speeds, and then the air drag term can then be adjusted to fit higher speeds. The very large variation in axle load presents an opportunity to check the linear fit over a wide range, with the aim of finding A and A0 values which are truly constant, and not 'adjusted' for empty and loaded conditions as in some publications. Note that for low to medium axle loads, the A (weight) and A0 (axle) components are similar in magnitude, which makes it harder to distinguish their relative contributions.

By trial and error, the A value was varied from the RP-548 value of 6.5 N/tonne, while keeping A0 fixed at 80 N/axle. Figures 1 and 2, below show plots of tractive effort and speed versus time, and show the fit between the simulated speed and the actual speed (from EVR data). A good fit at low speeds was found with $A = 5$ N/t for the loaded case, and 7.5 N/t for the empty case, as in Table 4 below. Recalculating from these data points to get constants which work for both the empty and loaded cases gives $A = 4.5$ N/tonne ($= 0.9$ lb/ton $\sim 65\%$ of the RP-548 value), and $A0 = 100$ N/axle ($= 22$ lb/axle, slightly higher than the RP-548 value).

Car Mass Tonne	A N/t	A0 N/axle	R (N)
160	5	80	1120
23	7.5	80	492.5
Both	4.5	100	

Table 4 Constants for iron ore train

While there is no reason to expect the SNCF double-deck EMU formula (5) to be valid at this axle load, it surprisingly gives the exact resistance

for the loaded case, i.e. 1120 N for 160 tons. However, at 23 tonnes, it gives 424 N, which is below the measured value.

Using the above for A and A0, the air drag values were found to be 0.045 for the empty case, and 0.030 for the loaded case (for 10m^2 cross-sectional area). These are significantly lower than the RP-548 value (.0096) and the Canadian National values (0.077 and 0.219 – see Table 3). This would support the theory that drag mostly affects the front dozen vehicles and decreases along the train, so that average resistance decreases for longer trains. Especially significant is the result that, although the air drag is higher on empty wagons, it is only by around 50%, while the Canadian National figures suggest it should be higher than the loaded case by a factor of three.

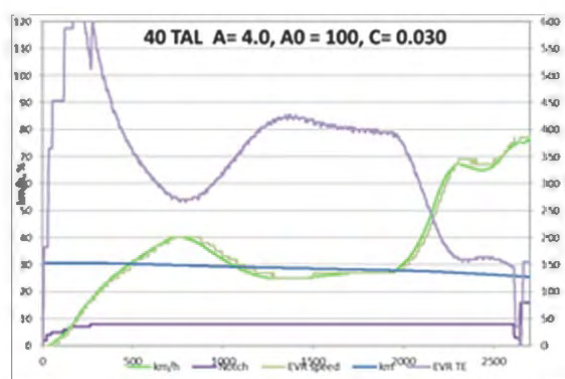


Fig 1 Loaded 244 wagon ore train

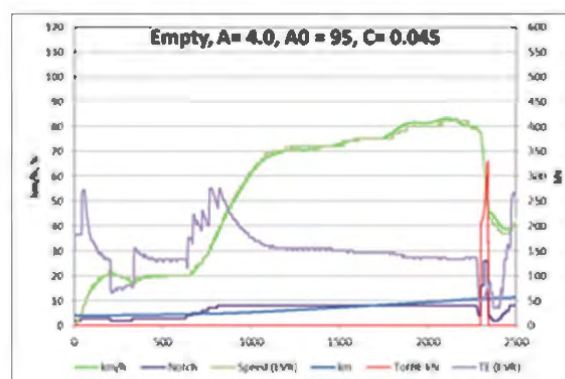


Fig 2 Empty 244 wagon ore train

9. HUNTER VALLEY COAL

The second case study is for coal trains in the Hunter Valley. Train length is about half of the Pilbara case, with wagons which are longer, but similar in tare weight. There are two lengths of trains – 82 and 96 wagons, with loaded weights of 100 tonnes and 120 tonnes per wagon respectively. Both are hauled by three GT46C-ACE

locomotives. The difference in length might be expected to show up in the air drag value.

The section from Kooragang Island to Hexham is relatively flat and straight, and it was expected that this would give good results. However, the published track data for this section is very inconsistent, to the extent that the sign of some grades is wrong.

Figure 3 below shows the results for an empty train of 82 wagons running from near Hexham to Metford, a distance of 14 km. The track is almost straight, with one large radius curve. Using the previous values for A and A0, it was found that C= 0.09. The curve fit is not as good as for the iron ore trains.

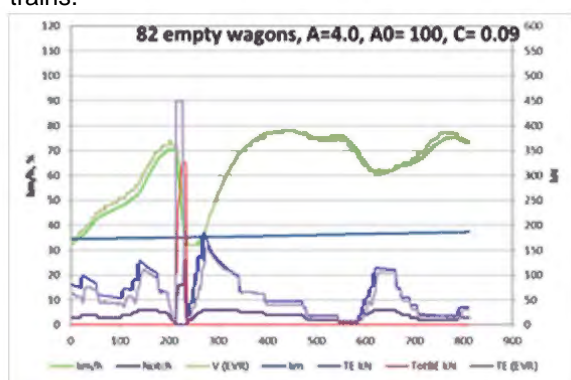


Fig 3 82 empty wagons Hexham to Metford

Figure 4 below shows results for an empty 96 wagon train. The coefficient values are low, but they are somewhat meaningless as the correlation between the speed curves is rather poor. This may be due to inaccuracies in the track data, but serves as an example of poor correlation – the method does not always give reliable results.

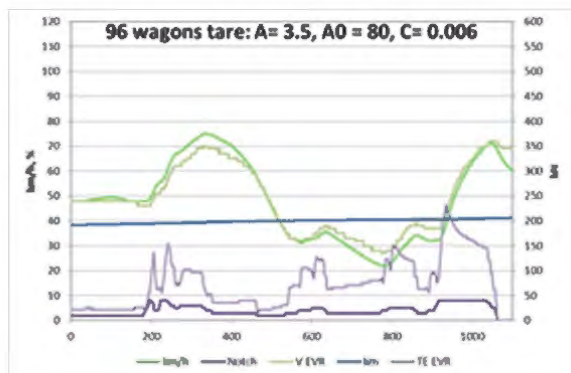


Fig 4 96 empty wagons Maitland to Greta

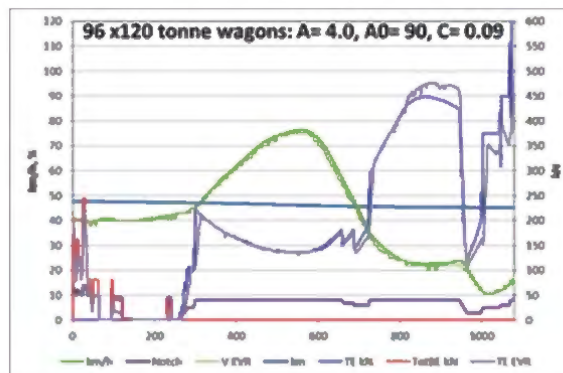


Fig 5 Singleton-Belford loaded, 96 wagons

Figure 5 shows results for a loaded 96 wagon train in the section from Singleton to Belford. It should be noted that for this section, the published gradient data in Manual TS 0002 TI v2 July 1999, [6], appears to be very inaccurate, and it was impossible to correlate the results. After wasting many hours, I found that the data in M-Train file TS21304 allowed good correlation. M-Train is a simulation program developed by the PTC NSW Mechanical Branch in the 1970s – so old that the curve radii are in chains. I had no expectation that this data would still be valid or accurate. However, at several sections where I have tried to compare simulation results and actual data, the M-Train data appears to be more accurate in terms of where the gradients begin and end, and even the magnitude of the slope.

The results for the Hunter Valley coal trains confirm that the rolling resistance components are lower than given by RP-548, but that the air drag terms are close to RP-548. A summary comparing the experimentally derived coefficients with RP-548 follows in Table 5.

	A	A0	C
	N/t	N/axle	
RP-548	6.5	80	0.096
Ore E/L	4.5	100	0.030/0.045
Coal	4.0	100	0.09

Table 5 Comparison of Constants

10. CONCLUSION

Based on the limited number of cases presented for coal and ore trains, the results consistently show that the constant for the weight dependent term in the Davis equation is around 4 to 4.5 N/tonne, which is lower than in RP-548 (6.5 N/t = 1.3 lb/t). However, the axle term which is more dominant for empty wagons is 90-100 N/axle, 10-20% higher than the 80 N/axle (18 lb/axle) in RP-548.

Having said that, these are not the “right” values to use for all cases. Many factors influence rolling resistance, including track stiffness, wheel diameter, temperature of grease and lozenging of bogie frames. The cases presented are for track and rolling stock in good condition, typical of heavy haul operations in Australia. Resistance may well be higher in more general conditions.

While by no means proving that the velocity term in the Davis equation is zero, my results support the contention that it is negligible.

Results for the aerodynamic drag term are somewhat more inconclusive. For the limited number of coal trains examined, the drag seems to be close to the RP-548 figure, but much lower for long ore trains. It would be necessary to check many more cases to reach a conclusion.

Part of the problem is that the apparent aerodynamic drag is affected by wind speed, so it is necessary to use data from many runs on different days to account for this effect.

As a methodology, it is shown that deriving resistance coefficients from event recorder data is viable. However, it is fairly labour intensive. I was intending to include many more cases in this paper. Picking track sections and the corresponding sections from event recorder data was all done manually, and I am giving some thought to how this process can be automated. Also, having erroneous data for track geometry is a major problem.

Future work is aimed at getting better results for the aerodynamic drag coefficient, and especially in

comparing the effects of train length. The intent is to look at intermodal trains too, but this task will be much more difficult due to the varying weights, lengths and cross-sectional areas of vehicles making up a general freight train.

11. REFERENCES

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