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THE EFFECT OF AIR TEMPERATURE UPON THE RATE OF CLIMB OF AN AIRPLANE EQUIPPED WITH A CONSTANT SPEED PROPELLER

FLIGHT ENGINEERING & FACTORY INSPECTION DIVISION
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REFERENCES

a. Wright Aeronautical Corporation "Engineering Notes" - (Second Edition)

b. Pratt and Whitney Specification No 5854, Curve No T-461

c NACA Technical Report No. 642

d. Flight Engineering Report No 10

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INTRODUCTION

This report covers the derivation of equations by means of which to calculate the effect of air temperature upon the rate of climb, applies the equations to a typical airplane, and discusses the significance of the results of this application.

The derivation is based upon the fundamental equation for rate of climb, the generally accepted relationship between the brake horsepower output of an engine and the temperatures of the air involved in the carbureting, cooling and supercharging processes, and upon a rational treatment of the effect of outside air temperature upon propeller aerodynamic characteristics No new experimental data are contained in this report

The equations are believed useful for the purpose of calculating the effect of temperature upon a rate of climb determined by testing at a given temperature and require for their application the establishment of certain engine characteristics normally available in the standard engine power output and certain propeller characteristics.

DERIVATION

General

The rate of climb of an airplane in feet per minute may be expressed by means of the following equations:

$$C = \frac{33,000}{W} (THP_a - THP_r) \qquad (1)$$

where

W = Airplane weight in pounds

THP_a = thrust horsepower available

THP_r = thrust horsepower required

Now: $$THP_a = BHP \, \eta \qquad (2)$$

where:

BHP = brake horsepower output of engine(s)

η = propulsive efficiency

and:

$$THP_r = \frac{f \rho V^3}{1100} + \frac{2W^2}{550 \pi \rho e b^2 V} \qquad (3)$$

where:

ρ = Mass density of air in slugs/cu ft

f = Airplane parasite area in sq ft

V = Airplane airspeed (true) in ft/sec

e = Airplane efficiency factor = $\frac{C_L^2}{\pi A R C_{D_1}}$

b = Airplane wing span in feet

The above equations indicate that, in order to establish the effect of air temperature upon the rate of climb, it is necessary to consider the effect upon the terms involved in equations (2) and (3), viz: BHP, η, $\frac{f\rho V^3}{1100}$, and $\frac{2W^2}{550 \pi \rho e b^2 V}$, separately. This is done immediately hereunder.

Brake Horsepower - BHP

The experimental data available indicate there are at least two separate effects of air temperature upon the power output of an engine operating at a given manifold pressure, rpm, and mixture strength. One of these manifests itself in a reduction of power with increase in the temperature of the air entering the carburetor. The magnitude of the effect is ordinarily expressed by means of the following relation:

$$BHP = BHP_o \sqrt{\frac{T_o}{T_c}}$$

where

T_o = the absolute temperature of the carburetor air under the conditions producing BHP_o (ie- it is the temperature in degrees Fahrenheit plus 459.4)

T = the absolute temperature of the carburetor air under the conditions producing BHP.

The second such effect is an observed reduction in power with increase in the temperature of the air entering the cooling process for an air cooled engine. Although not so well supported by the experimental evidence as in the case of carburetor air, the magnitude of this effect is ordinarily expressed:

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$$BHP = BHP_o\sqrt{\frac{To}{T}}$$

where:

To and T = the absolute temperature of the cooling air but are otherwise as defined above

The primary effect of temperature upon the brake horsepower is then

$$BHP = BHP_o\left(\sqrt{\frac{To}{T}}\right)_c\left(\sqrt{\frac{To}{T}}\right)_r \quad \text{-------------------------------} \quad (4)$$

Propulsive Efficiency = η

If propulsive efficiency, η, be plotted as contours upon the standard power coefficient versus advance ratio chart (C_p vs $\frac{V}{ND}$ @ β), the resulting diagram (see Fig 1 for example, which has been prepared from the data of Ref (C)) will indicate, in the region representative of climbing conditions (ie; comparatively high values of C_p and low values of $\frac{V}{ND}$), that, at a constant value of $\frac{V}{ND}$, an increase of C_p is accompanied by a decrease in efficiency and also that at a constant value of C_p, an increase in $\frac{V}{ND}$ is accompanied by an increase in efficiency. This situation may be summarized by the following equations

$$\left.\begin{aligned} \left.\frac{d\eta}{dC_p}\right)_{\frac{V}{ND} = \text{a constant}} &= A \\ \left.\frac{d\eta}{d\frac{V}{ND}}\right)_{C_p = \text{a constant}} &= B \end{aligned}\right\} \quad \text{-------------------------------} \quad (5)$$

If it can be shown that the values of A and B, so defined, remain reasonably constant over the ranges of C_p and $\frac{V}{ND}$ respectively resulting from any temperature change which must be dealt with and also the value of A is substantially independent of any value of $\frac{V}{ND}$ and that of B substantially independent of any value of C_p either of which must be considered, then the rate at which efficiency changes with both C_p and $\frac{V}{ND}$ is substantially

$$d\eta = AdC_p + Bd\frac{V}{ND} \quad \text{-------------------------------} \quad (6)$$

In order to indicate the nature, magnitude and variation of the values of A and B with C_p and $\frac{V}{ND}$, Figures 2 and 3 have been prepared from the data of Figure 1 These data indicate the assumption of appropriate constant values of A and B to be a reasonably close approximation to the actual variation of η. (This is further discussed below)

Now

$$C_p = \frac{P}{\rho N^3D^5}$$

and:

$$\frac{V}{ND} = \frac{88V}{N_D}$$

Also:

$$\rho \backsim \frac{P}{T}$$

where:

P = atmospheric pressure

T = absolute air temperature

ie:

$$\frac{\rho}{\rho_o} = \frac{To}{T}$$

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At a constant indicated airspeed:

$$\frac{V}{V_o} = \sqrt{\frac{\rho_o}{\rho}} = \sqrt{\frac{T}{T_o}}$$

Now D is a constant for a given airplane and with a constant speed propeller, N is also constant The effect of temperature upon P has been separately considered above Since P = BHP, the effect of temperature upon C_p and $\frac{V}{ND}$ is therefore as follows:

$$\frac{C_p}{C_{p_o}} = \frac{BHP}{BHP_o} \times \frac{\rho_o}{\rho} = \frac{T}{T_o}\sqrt{\frac{T_o}{T}}_c\sqrt{\frac{T_o}{T}}_r$$

and:

$$\frac{\frac{V}{ND}}{\left.\frac{V}{ND}\right)_o} = \frac{V}{V_o} = \sqrt{\frac{T}{T_o}}$$

or,

Assuming $\left.\frac{T_o}{T}\right)_c = \left.\frac{T_o}{T}\right)_r = \frac{T_o}{T}$, which appears likely always to be nearly true:

$$C_p = C_{p_o}\frac{T}{T_o}\sqrt{\frac{T_o}{T}}\sqrt{\frac{T_o}{T}} = C_{p_o}$$

and:

$$\frac{V}{ND} = \left.\frac{V}{ND}\right)_o\sqrt{\frac{T}{T_o}}$$

also:

$$dC_p = 0d\left(\frac{T}{T_o}\right) = 0$$

and:

$$d\frac{V}{ND} = \frac{\left.\frac{V}{ND}\right)_o}{2\sqrt{\frac{T}{T_o}}}d\left(\frac{T}{T_o}\right) = \frac{\left.\frac{V}{ND}\right)_o}{2}\sqrt{\frac{T_o}{T}}\,d\left(\frac{T}{T_o}\right) \qquad (7)$$

Substituting these values of dC_p and $d\frac{V}{ND}$ into equation (6), the rate of change of propulsive efficiency with absolute temperature ratio is obtained as follows:

$$d\eta = \frac{\left.B\frac{V}{ND}\right)_o}{2}\sqrt{\frac{T_o}{T}}\,d\left(\frac{T}{T_o}\right)$$

and efficiency as a function of the absolute temperature ratio may be obtained by integration thus:

$$= \int \frac{B\left.\frac{V}{ND}\right)_o}{2\sqrt{\frac{T}{T_o}}}\,d\frac{T}{T_o}$$

$$= \left.B\frac{V}{ND}\right)_o\sqrt{\frac{T}{T_o}} + k$$

where:

k = a constant

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When:

$$\frac{T}{T_o} = 1.000, \eta = \eta_o$$

Therefore:

$$\eta_o = B\frac{V}{ND}\Big)_o + k$$

ie:

$$k = \eta_o - B\frac{V}{ND}\Big)_o$$

Substituting this value of k into the above expression for η:

$$\eta = B\frac{V}{ND}\Big)_o\sqrt{\frac{T}{T_o}} + \eta_o - B\frac{V}{ND}\Big)_o$$

$$= \eta_o + B\frac{V}{ND}\Big)_o\left(\sqrt{\frac{T}{T_o}} - 1\right) \quad \text{------------------------------ (8)}$$

Thrust Horsepower Required $= \frac{f V^3}{1100} + \frac{2W^2}{550 \quad eb^2 V}$

It has been shown above that:

$$\rho = \rho_o \frac{T_o}{T}$$

and, at constant indicated airspeed, that:

$$V = V_o\sqrt{\frac{T}{T_o}}$$

Substituting these values into the expression for thrust horsepower required (equation (3)):

$$THP_r = \frac{\rho_o \frac{T_o}{T} f V_o^3 \left(\frac{T}{T_o}\right)^{\frac{3}{2}}}{1100} + \frac{2W^2}{550 \pi \rho_o \frac{T_o}{T} eb^2 V_o\sqrt{\frac{T}{T_o}}}$$

$$= \frac{\rho_o f V_o^3}{1100}\sqrt{\frac{T}{T_o}} + \frac{2W^2}{550 \pi \rho_o eb^2 V_o}\sqrt{\frac{T}{T_o}}$$

ie:

$$THP_r = THP_{r_o}\sqrt{\frac{T}{T_o}} \quad \text{------------------------------ (9)}$$

However, when $THP_r = THP_{r_o}$, $C = C_o$, $BHP = BHP_o$, and $\eta = \eta_o$. Substituting these values into equation (1) and solving for THP_{r_o} gives:

$$THP_{r_o} = BHP_o\eta_o - \frac{C_o W}{33,000}$$

and, therefore from equation (9):

$$THP_r = BHP_o\eta_o\sqrt{\frac{T}{T_o}} - \frac{C_o W}{33,000}\sqrt{\frac{T}{T_o}} \quad \text{------------------------------ (10)}$$

Rate of Climb = C

All of the elements involved in the basic climb equation (1) have been related to temperature by equations (4), (8), and (10). When these are substituted into equation (1) the following results:

$$C = \frac{33,000}{W}\left[BHP_o\sqrt{\frac{T_o}{T}}_c\sqrt{\frac{T_o}{T}}_r\left\{\eta_o + B\frac{V}{ND}\Big)_o\left(\sqrt{\frac{T}{T_o}} - 1\right)\right\} - \left(BHP_o\,\eta_o\sqrt{\frac{T}{T_o}} - \frac{C_o W}{33,000}\sqrt{\frac{T}{T_o}}\right)\right]$$

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$$= \frac{33{,}000}{W}\left[BHP_o\eta_o\sqrt{\left(\frac{T_o}{T}\right)_c}\sqrt{\left(\frac{T_o}{T}\right)_r} + B\,BHP_o\left(\frac{V}{ND}\right)_o\left(\sqrt{\frac{T}{T_o}}-1\right)\sqrt{\left(\frac{T_o}{T}\right)_c}\sqrt{\left(\frac{T_o}{T}\right)_r} - BHP_o\eta_o\sqrt{\frac{T}{T_o}} + \frac{C_oW}{33{,}000}\sqrt{\frac{T}{T_o}}\right]$$

$$= \frac{33{,}000BHP_o\eta_o}{W}\left[\left\{\sqrt{\left(\frac{T_o}{T}\right)_c}\sqrt{\left(\frac{T_o}{T}\right)_r} - \sqrt{\frac{T}{T_o}}\right\} + \frac{B\frac{V}{ND_o}}{\eta_o}\left(\sqrt{\frac{T}{T_o}}-1\right)\sqrt{\left(\frac{T_o}{T}\right)_c}\sqrt{\left(\frac{T_o}{T}\right)_r} + \frac{C_oW}{33{,}000BHP_o\eta_o}\sqrt{\frac{T}{T_o}}\right]$$

$$= C_o\sqrt{\frac{T}{T_o}} + \frac{33{,}000BHP_o\eta_o}{W}\sqrt{\left(\frac{T_o}{T}\right)_c}\sqrt{\left(\frac{T_o}{T}\right)_r}\left[1 - \sqrt{\frac{T}{T_o}}\sqrt{\left(\frac{T}{T_o}\right)_c}\sqrt{\left(\frac{T}{T_o}\right)_r} - \frac{B\frac{V}{ND}_o}{\eta_o}\left(1-\sqrt{\frac{T}{T_o}}\right)\right] \quad \text{------ (11)}$$

DISCUSSION

General

Equation (11) is strictly applicable to the rate of climb for an airplane equipped with an air-cooled engine and a constant speed propeller when operated at a constant indicated airspeed and under such powerplant operating conditions as to make possible the maintenance of constant manifold pressure and engine rpm and also when and only when the ratio of initial and final values of outside, carburetor, and cooling air temperatures (absolute) are identical and the rate of change of propulsive efficiency with advance ratio at constant power coefficient is constant over the range of advance ratio produced by any temperature change for which it is desired to apply the equation

It cannot be applied in the case of an airplane not equipped with a constant speed propeller

The remainder of the limitations upon the general applicability of equation (11) are discussed below where certain modifications are derived or suggested in order to obviate these limitations or to simplify the equation to improve its practicability.

Operation At a Fixed Throttle Setting

For an unsupercharged engine operating at a fixed throttle setting the manifold pressure is substantially independent of Carburetor air temperature. The limitation stated above, that the manifold pressure remain constant, is met and equation (11) is applicable

For a supercharged engine at fixed throttle, however, the effect of carburetor air temperature is such that an increase in temperature is accompanied by a reduction in manifold pressure Page 31 of Reference a indicates the nature of the relation, of the ratio of supercharger outlet to inlet pressure to the temperature of the carburetor air. The outlet pressure is substantially the manifold pressure and the inlet pressure, the atmospheric pressure The slope of the lines appearing in the referenced diagram may therefore be taken to be the rate of change with respect to carburetor air temperature of the ratio of manifold to atmospheric pressures. ie

$$\text{slope} = \frac{d\frac{MP}{P}}{dT_c}$$

The diagram indicates that this slope is a function of both the pressure ratio and t_c Values of this slope have been read at values of $t_c = 0^\circ$, 50°, and $100^\circ F$ and at the corresponding values of the pressure ratio and appear in Figure 4 as:

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$$\frac{d\frac{MP}{p}}{dT_c} \text{ vs. } \frac{MP}{p} \text{ @ } T_c$$

If the magnitude of the changes in the temperature and the resultant changes in the pressure ratio which must be dealt with be assumed small enough that the above rate remains sensibly constant when the temperature changes, then:

$$\frac{d\frac{MP}{p}}{dT_c} = \text{a constant} = E$$

or:

$$dMP = pEdT_c \quad \text{------------------------------------} \quad (12)$$

The above described change in manifold pressure is of course accompanied by a corresponding change in the brake horsepower output of the engine. The standard sea level power chart of BHP vs MP @ N indicates that the rate at which this change in power takes place is, at constant engine rpm, a constant which may be read directly from the chart and the altitude chart assumes this rate a constant, ie:

$$\frac{dBHP}{d\ MP} = \text{A constant} = F$$

or:

$$dBHP = FdMP$$

Substituting this value of dMP into equation (12) gives:

$$dBHP = pEFdT_c$$

Integrating:

$$BHP = pEFT_c + k_1 \quad \text{------------------------------------} \quad (13)$$

When $Tc = T_{c_o}$, $BHP = BHP_o$, and:

$$BHP_o = pEFTc_o + k_1$$

or:

$$k_1 = BHP_o - pEFTc_o$$

Substituting this value of k_1 into equation (13) gives:

$$BHP = pEFTc + BHP_o - pEFT_{c_o}$$

$$= BHP_o + pEF(T-T_o)_c$$

and combining this with equation (4) previously derived, the total effect of temperature upon BHP becomes, for the case under consideration:

$$BHP = \left[BHP_o + pEF(T-T_o)_c\right] \sqrt{\frac{T_o}{T}}_c \sqrt{\frac{T_o}{T}}_f \quad \text{------------------------} \quad (4a)$$

where:

p = atmospheric pressure in inches of mercury

$E = \frac{d\frac{MP}{p}}{dT_c}$ which may be read from Figure 4 at the initial values of MP, p, T_c.

$F = \left.\frac{dBHP}{dMP}\right)_{N = \text{constant}}$ which may be read from the engine power chart for the initial values of M.P. and N.

If equation (4a) be substituted for equation (4) in the development of equation (11), the following results:

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$$C = C_o\sqrt{\frac{T}{T_o}} + \frac{33{,}000 BHP_o \eta_o}{W}\sqrt{\frac{T_o}{T}}_c\sqrt{\frac{T_o}{T}}_r\left[1-\sqrt{\frac{T}{T_o}}\sqrt{\frac{T}{T_o}}_c\sqrt{\frac{T}{T_o}}_r - \frac{B\frac{V}{ND}_o}{\eta_o}\left(1-\sqrt{\frac{T}{T_o}}\right)\left\{1 + \frac{pKF}{BHP_o}(T_o-T)_c\right\} - \frac{pKF}{BHP_o}(T_o-T)_c\right] \quad (11a)$$

Liquid Cooled Engine Installation

The temperature of the air entering the cooling process in a liquid cooled engine affects the power output of the engine only in so far as it affects the temperature of the coolant The temperature of the coolant is, however, usually thermostatically controlled and remains substantially constant once the engine is warmed up and operating equilibrium established. Under such conditions the power output would be independent of cooling air temperature and the term in equation (4) to account for this temperature must be eliminated, leaving:

$$BHP = BHP_o\sqrt{\frac{T_o}{T}}_c \quad (4b)$$

This term must also be eliminated from the derivation of equations (7) and (8), ie: (see page 4)

$$\frac{C_P}{C_{P_o}} = \frac{BHP}{BHP_o} \times \frac{\rho_o}{\rho} = \frac{T}{T_o}\sqrt{\frac{T_o}{T}}_c$$

or, making the same assumptions concerning $\frac{T}{T_o}$ and $\frac{T}{T_o}\Big)_c$

$$C_P = C_{P_o}\sqrt{\frac{T}{T_o}}$$

and equation (7) becomes:

$$\left.\begin{aligned} dC_P &= \frac{C_{P_o}}{2\sqrt{\frac{T}{T_o}}}\, d\left(\frac{T}{T_o}\right) = \frac{C_{P_o}}{2}\sqrt{\frac{T_o}{T}}\, d\left(\frac{T}{T_o}\right) \\ d\frac{V}{ND} &= \frac{\frac{V}{ND_o}}{2}\sqrt{\frac{T_o}{T}}\, d\left(\frac{T}{T_o}\right) \end{aligned}\right\} \quad (7b)$$

Then:

$$d\eta = \frac{A C_{P_o} + B\frac{V}{ND}\Big)_o}{2}\sqrt{\frac{T_o}{T}}\, d\left(\frac{T}{T_o}\right)$$

and equation (8):

$$\eta = \eta_o + \left[A C_{P_o} + B\frac{V}{ND}\Big)_o\right]\left(\sqrt{\frac{T}{T_o}} - 1\right) \quad (8b)$$

Finally, equation (11) becomes:

$$C = C_o\sqrt{\frac{T}{T_o}} + \frac{33{,}000 BHP_o \eta_o}{W}\sqrt{\frac{T_o}{T}}_c\left[1 - \sqrt{\frac{T}{T_o}}\sqrt{\frac{T}{T_o}}_c - \frac{A C_{P_o} + B\frac{V}{ND}\Big)_o}{\eta_o}\left(1-\sqrt{\frac{T}{T_o}}\right)\right] \quad (11b)$$

Propeller Characteristics

In order to indicate a region on Figures 2 and 3 appropriate to climbing flight a point has been located on each of the figures to represent for each of the 35 airplanes of Table I the values of C_P, $\frac{V}{ND}$ and η corresponding with climb at sea level at METO power at best rate of climb speed Some of these airplanes are equipped with two blade propellers and in such cases the value of C_P used in these figures is the actual value divided by 0 70 to make it correspond with the three blade characteristics of the figures. It may be noted that all these points lie within the following limits

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$$\eta = .660 \text{ to } 785$$

$$p = .030 \text{ to } 110$$

$$\frac{V}{ND} = .400 \text{ to } .800$$

Appropriate lines bounding the area contained within these limits have been drawn on Figures 1 and 2. The corners of this boundary have then been relocated to correspond with departures of outside air temperature from initial conditions such that $\sqrt{\frac{T}{T_o}} = 1\ 10$ and $\sqrt{\frac{T}{T_o}} = \ 90$. These would represent temperatures of 167°F and -39°F at sea level or 125°F and -68°F at an altitude of 10,000 feet respectively when T_o is assumed to be the standard temperature and are probably in excess of any temperature variation likely to be encountered. The variation of the slopes, $A = \frac{d\eta}{dC_p}$ and $B = \frac{d\eta}{d\frac{V}{ND}}$ within the original area is representative of that likely to be due to differences among types of airplanes while the variation between boundaries of the two extreme areas is representative of that due to the extremes of temperature variation likely to be encountered. These variations lie within the following limits:

Slope	Airplane to Airplane	Temperature to Temperature
$A = \frac{d\eta}{dC_p} =$	- 1 35 to -2 85	+ 30% to - 30%
$B = \frac{d\eta}{d\frac{V}{ND}} =$	+ 0 50 to + 1 20	+ 17% to - 17%

These limits indicate that in order to establish values of A and B with very much precision, it is necessary to use Figures 2 and 3 or their equivalent and in any event, this may as well be done since Cp_o and $\frac{V}{ND}_o$ must be known in order to apply any of the three alternative forms of equation (11) and these values locate an appropriate point on the figures at which the corresponding values of A and B may be determined.

The extent to which the characteristics of the 6101 blade three blade propeller is representative of those of any constant speed propeller likely to be encountered is a question upon which it is very difficult to shed much light. It appears desirable, however, to use the characteristics of the actual propeller if these are available particularly if great accuracy is necessary or desirable.

Carburetor, Cooling, and Outside Air Temperature

The temperature of the air entering the carburetor is ordinarily above that of the outside air due either to the heat absorbed from the engine or to a certain amount of compression in the intake process. It may also be higher due to the deliberate application of heat to prevent the formation of ice in the intake system In most of the cases with which the Civil Aeronautics Administration has had experience the carburetor air temperature is of the order 0°F to 15°F above outside air temperature when the heater control is "Full Cold" Also when full heat is applied the carburetor air temperature rarely exceeds the outside air temperature by more than 120°F, and this heat rise tends to decrease with increasing outside air temperature Under most operating conditions the heater is not used

For conventional installation of an air cooled engine, the temperature of the air entering the cooling process is substantially that of the outside air For engines buried deeply inside the airplane structure it appears possible that the temperature of the cooling air might be elevated by compression in the intake duct.

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Possible Simplifications

The only simplification of the various alternative forms of equation (11) which appears warranted under any likely circumstances is that made possible by the assumption already made in the derivation of equation (7) above, viz:

$$\frac{T}{T_o} = \left.\frac{T}{T_o}\right)_c = \left.\frac{T}{T_o}\right)_r$$

Upon this basis, equations (11), (11a) and (11b) become:

$$C = C_o\sqrt{\frac{T}{T_o}} + \frac{33{,}000 BHP_o \eta_o}{W}\ \frac{T_o}{T}\left[1-\left(\frac{T}{T_o}\right)^{3/2} - \frac{\left.B\frac{V}{ND}\right)_o}{\eta_o}\left(1-\sqrt{\frac{T}{T_o}}\right)\right] \quad \text{-------------(11')}$$

$$C = C_o\sqrt{\frac{T}{T_o}} + \frac{33{,}000 BHP_o \eta_o}{W}\ \frac{T_o}{T}\left[1-\left(\frac{T}{T_o}\right)^{3/2} - \frac{\left.B\frac{V}{ND}\right)_o}{\eta_o}\left(1-\sqrt{\frac{T}{T_o}}\right)\left\{1+\frac{pEF}{BHP_o}(T_o-T)\right\} - \frac{pEF}{BHP_o}(T_o-T)\right] \quad \text{-------------(11a')}$$

$$C = C_o\sqrt{\frac{T}{T_o}} + \frac{33{,}000 BHP_o \eta_o}{W}\sqrt{\frac{T_o}{T}}\left[1-\frac{T}{T_o} - \frac{ACp_o + \left.B\frac{V}{ND}\right)_o}{\eta_o}\left(1-\sqrt{\frac{T}{T_o}}\right)\right] \quad \text{-------------(11b')}$$

EXAMPLE

In order to illustrate the application of the equations as well as the effect of outside air temperature upon the rate of climb, the variation of two typical rates of climb for the Douglas DC3-S1C3G airplane at two altitudes will be calculated over the range of outside air temperature likely to be encountered

For this purpose the airplane will be assumed to operate at a weight of 25,200# and METO power on the operating engine(s) The best rates of climb with both engines operating and with one engine inoperative and its propeller feathered with, in each case, the airplane in the "clean" configuration (ie: wing flaps and landing gear fully retracted), each at sea level and at 10,000 feet are selected for the purpose. Under standard atmospheric conditions these rates of climb are as follows

Configuration	Sea Level	10,000'
Both Engines	940	690
One Engine	295	109

The engine actually installed in this airplane is an air cooled engine and at sea-level will be assumed to operate at constant manifold pressure while at 10,000 feet altitude the operation will be assumed to be at fixed (full) throttle. The sea level variation of rate of climb will also be calculated assuming a liquid cooled engine at constant manifold pressure to illustrate that case The propeller gear ratio is 16:9 and the propeller diameter 11'-6"

The values of the terms in the various forms of equation (11') which will be used for the purpose and the calculations are as follows:

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Seà Level

T_o = 518.4°F (59° + 459 4°)

p = 29.92 "Hg.

BHP_o = 1050 per engine

MP = manifold pressure = 41.5" Hg.

$$E = \frac{\frac{dMP}{p}}{dT_c} = -.00058 \text{ (see Fig.4)}$$

$$F = \frac{dBHP}{dMP} = +29.0 \text{ BHP/"Hg. (see Ref.b)}$$

	Both Engines	One Engine
V =	130 MPH	110 MPH
$\frac{V}{ND}_o$ =	.694	.587
C_{p_o} =	.089	.089
η_o =	.760	685
$A = \frac{d\eta}{dC_p}$ =	-1.90	-2.60
$B = \frac{d\eta}{d\frac{V}{ND}}$ =	+.650	+ 650

For the air-cooled engine actually installed, equation 11' applies as follows:

	t	-20	0	+20	+40	+60	+80	+100	+120
	T	439.4	459.4	479.4	499.4	519 4	539 4	559.4	579.4
	$\frac{T_o}{T}$	1 181	1 130	1.082	1.040	1 000	.962	.928	.896
	$\sqrt{\frac{T}{T_o}}$	.920	940	.961	.980	1 000	1.020	1 039	1.057
	$\left(\frac{T}{T_o}\right)^{\frac{3}{2}}$	780	.832	.886	942	1 000	1 060	1 119	1.182
Both Engines	$\frac{33{,}000 BHP_o \eta_o}{W} \frac{T_o}{T}$	2,476	2,362	2,260	2,174	2,088	2,012	1,940	1,872
	$\frac{B\frac{V}{ND})_o}{\eta_o}\left(1-\sqrt{\frac{T}{T_o}}\right)$	+.0475	+ 0356	+.0231	+.0119	0	-.0119	-.0231	.0338
	$\frac{\Delta C}{C_o\sqrt{\frac{T}{T_o}}}$	+426	+313	+205	+122	0	-98	-186	-278
		+865	+883	+903	+921	+940	+958	+976	+992
	C	+1,291	+1,196	+1,108	+1,043	+ 940	+860	+790	+714
One Engine	$\frac{33{,}000 BHP_o \eta_o}{W} \frac{T_o}{T}$	1,238	1,181	1,130	1,087	1,044	1,006	970	936
	$\frac{B\frac{V}{ND})_o}{\eta_o}\left(1-\sqrt{\frac{T}{T_o}}\right)$	+.0445	+.0334	+.0217	+.0112	0	-.0112	-.0217	-.0318
	$\frac{\Delta C}{C_o\sqrt{\frac{T}{T_o}}}$	+217	+159	+104	+51	0	-49	-94	-141
		+272	278	284	290	295	301	306	312
	C	+489	+437	+388	+341	+295	+252	+212	+171

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For the equivalent liquid cooled engine, equation (11b') applies as follows:

	t	-20°	0°	+20	+40	+60	+80	+100	+120
	T	439.4	459.4	479.4	499.4	519.4	539.4	559.4	579.4
	$\frac{T}{T_o}$	.847	.885	.923	.962	1 000	1.040	1.079	1.117
	$\sqrt{\frac{T}{T_o}}$	.920	.940	.960	.980	1.000	1.020	1.039	1.057
Both Engines	AC_{p_o}	-.169	- 169	-.169	-.169	-.169	-.169	-.169	-.169
	$B\frac{V}{ND}\Big)_o$	+.450	+ 450	+.450	+.450	+.450	+.450	+.450	+.450
	$\frac{33{,}000 BHP_o \eta_o}{W}\sqrt{\frac{T_o}{T}}$	2,270	2,220	2,180	2,135	2,095	2,050	2,015	1,980
	$\frac{AC_{p_o} + B\frac{V}{ND}\Big)_o}{\eta_o}\left(1-\sqrt{\frac{T}{T_o}}\right)$	+ 0296	+.0222	+.0148	+.0074	0	-.0074	-.0144	-.0211
	ΔC	+280	+220	+136	+65	0	-67	-130	-190
	$C_o\sqrt{\frac{T}{T_o}}$	865	883	902	921	940	959	976	993
	C	1,145	1,103	1,038	986	940	892	846	803
One Engine	AC_{p_o}	- 232	-.232	- 232	-.232	-.232	-.232	-.232	-.232
	$B\frac{V}{ND}\Big)_o$	+.382	+.382	+.382	+.382	+ 382	+.382	+ 382	+ 382
	$\frac{33{,}000 BHP_o \eta_o}{W}\sqrt{\frac{T_o}{T}}$	1,150	1,125	1,090	1,067	1,047	1,025	1,007	990
	$\frac{AC_{p_o} + B\frac{V}{ND}\Big)_o}{\eta_o}\left(1-\sqrt{\frac{T}{T_o}}\right)$	+ 0197	+ 0131	+.0088	+ 0044	0	- 0044	- 0085	-.0125
	ΔC	+153	+113	+75	+36	0	-36	-71	-103
	$C_o\sqrt{\frac{T}{T_o}}$	268	277	284	289	295	301	306	312
	C	421	391	359	325	295	265	235	209

10,000 ft altitude

$T_o = 482.8°F$

$p = 20.6"$ Hg

$BHP_o = 960$ per engine

$MP = 36"$ Hg.

$$E = \frac{\frac{dMp}{p}}{dT_c} = -.00135 \text{ (see Fig. 4)}$$

$$F = \frac{dBHP}{dMP} = 31\ 0\ \frac{BHP}{"Hg} \text{ (See Ref. b)}$$

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	Both Engines	One Engine
$V =$	145 MPH	124 MPH
$\left.\frac{V}{ND}\right)_o =$	.773	.660
$C_{P_o} =$	.110	110
$\eta_o =$	750	690
$A = \frac{d\eta}{dC_p} =$	-2.5	-2.5
$B = \frac{d\eta}{d\frac{V}{ND}} =$	510	.641

For the air cooled engine actually installed, equation (11a') applies

		-20	0	+20	+40	+60	+80	+100	+120
	t	-20	0	+20	+40	+60	+80	+100	+120
	T	439.4	459.4	479.4	499.4	519 4	539 4	559 4	579.4
	$T_o - T$	+43.4	+23 4	+3.4	-16.6	-36 6	-56 6	-76 6	-96 6
	$\frac{T_o}{T}$	1 100	1 051	1 008	.967	930	896	864	834
	$\sqrt{\frac{T}{T_o}}$	.954	975	996	1 018	1 038	1 057	1 077	1.094
	$\left(\frac{T}{T_o}\right)^{3/2}$	.868	926	.989	1 055	1 118	1 181	1.249	1.310
Both Engines	$\frac{33{,}000 BHP_o \eta_o}{W} \frac{T_o}{T}$	2,075	1,985	1,902	1,823	1,753	1,690	1,630	1,571
Both Engines	$\frac{B \left.\frac{V}{ND}\right)_o}{\eta_o}\left(1 - \sqrt{\frac{T}{T_o}}\right)$	+ 0242	+ 0131	+ 0021	- 0095	- 0200	- 0300	- 0405	- 0495
Both Engines	$\frac{pEF}{BHP_o}(T_o - T)$	- 0195	- 0105	- 0015	+ 0075	+ 0164	+ 0254	+ 0344	+.0434
Both Engines	ΔC	+264	+132	+20	-78	-189	-296	-394	-475
Both Engines	$C_o \sqrt{\frac{T}{T_o}}$	658	672	687	702	716	728	743	755
Both Engines	C	922	804	707	624	527	432	349	280
One Engine	$\frac{33{,}000 BHP_o \eta_o}{W} \frac{T_o}{T}$	1,038	993	951	912	877	845	815	786
One Engine	$\frac{B \left.\frac{V}{ND}\right)_o}{\eta_o}\left(1 - \sqrt{\frac{T}{T_o}}\right)$	+.0282	+ 0153	+ 0025	+ 0110	+.0233	+.0350	+ 0471	+.0576
One Engine	$\frac{pEF}{BHP_o}(T_o - T)$	-.0390	- 0210	- 0031	+ 0149	+ 0328	+ 0509	+ 0689	+ 0869
One Engine	ΔC	+149	+ 80	+11	-54	-111	-165	-218	-255
One Engine	$C_o \sqrt{\frac{T}{T_o}}$	104	106	109	111	113	115	117	119
One Engine	C	+253	+186	+120	+57	+2	-50	-101	-136

The results of these calculations appear as curves of rate of climb versus air temperature in Figure 5 weight upon the rate of change of climb with temperature therefore depends upon the relation between these two rates of climb and upon the effect of power and/or weight upon the actual rate of climb If, for example, a ten percent increase in power increases the actual rate of climb by 25%, the rate of change of climb with temperature is unaffected If the 10% increase in power increases C_0 less than 25%, the temperature effect is increased, if C_0 is increased by more than 25%, the temperature effect is reduced The relation between weight and the actual rate of climb C_0 is established and discussed in Ref (d)

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CONCLUDING REMARKS

The rate at which climb is affected by a change in air temperature, ie; the feet per minute reduction in climb per degree Fahrenheit increase in temperature, may be obtained by differentiation of the appropriate version of equation (11). The derivative of equation (11') is;

$$\frac{dC}{dt} = \frac{dC}{dT} = \frac{C_o}{2\sqrt{T_oT}} - \frac{33{,}000 BHP_o \eta_o}{W}\left[\frac{1}{2\sqrt{T_oT}} + \frac{T_o}{T^2} + \frac{\left.\frac{BV}{ND}\right)_o}{\eta_o}\left(\frac{\sqrt{T_o}}{2T^{\frac{3}{2}}} - \frac{T_o}{T^2}\right)\right]$$

When secondary effects are eliminated by calling $T = T_o = 500°F$ and substituting the following typical value;

$$B = 0.65$$

$$\left.\frac{V}{ND}\right)_o = 0.60$$

$$\eta_o = 0.70$$

this becomes;

$$\frac{dC}{dt} = 001C_o - \frac{33{,}000 BHP_o \eta_o}{W}\left[.001 + .002 + .557\ (001 - 002)\right] = 001C_o - 56.5\frac{BHP_o}{W} \text{ - - - - - - - -(14)}$$

These equations indicate the following:

1. For a given airplane the magnitude of the effect of a given change of temperature upon the rate of climb decreases with increase in the general level of the two temperatures between which the change occurs; ie; the effect is smaller at comparatively high than at comparatively low temperatures This fact is also indicated by the concavity upward of the curves of Fig. 5. The effect of a temperature rise of 20°F from -20°F to 0°F upon the rate of climb with both engines operating at sea level is to reduce the climb by 90 feet per minute, whereas the reduction in climb due to the same temperature rise from 100°F to 120°F is only 70 feet per minute

2. For an airplane climbing at a given power loading, any increase in rate of climb such as might obtain from greater aerodynamic refinement of the airplane, thus reducing the power required for level flight at the climbing speed, will also reduce the magnitude of the effect of a given change of air temperature upon the rate of climb. This is clearly indicated by equation (14)

3. As a corollary of 2 above, for an airplane operating in climb at a given power loading the greater the resulting rate of climb, the smaller the effect of a given temperature change upon the rate of climb. Consider for example the curve labeled "sea level" with one engine inoperative The rate of climb at a temperature of 60°F is 295 Ft /Min. From equation (14) the rate of change of climb with temperature is; + .295 – 2.355 = -2 06 ft. per min. per degree, Fahrenheit. If, however, the initial rate of climb were 600 ft /min rather than 295 ft /min., the rate of change would become +.600 – 2.355 = -1 755 ft /min /°F.

4. For a given airplane, the effect of changes in power or weight upon the rate of change of climb with temperature is less simple. It may be noted from the derivation above that equation (14) states the rate of change of climb is equal to one thousandth of the rate of climb actually available minus approximately two and one half thousandths of the rate of climb which would exist if no power were required for level flight and all power therefore available for climb. This latter fictitious climb is directly proportional to power and inversely to weight. The effect of a change in power or

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TABLE I

PROPELLER OPERATING CONDITIONS AT BEST RATE OF CLIMB AT SEA LEVEL

	AIRPLANE	C_p	$\frac{V}{ND}$	η
1.	Beech C-17-B	.097	.656	.711
2.	Beech D-17-S	.0707	.553	.714
3.	Beech 18-S	0707	61	.746
4.	Bellanca 31-50	.0670	.499	.688
5	Boeing 247-D	.0873	.67	.750
6.	Boeing S-307	.0649	.637	.772
7.	Boeing SA-307-B	0649	.637	.772
8.	Boeing A-314	.0332	.568	.787
9.	Douglas DC-2 (WF2B)	.0601	593	759
10.	Douglas DC-2 (WF3A)	.0664	614	.759
11.	Douglas DC-3 (WG102)	.0627	590	.753
12.	Douglas DC-3 (WG102A)	.0547	.565	.754
13.	Douglas DC-3 (WG202A)	.0691	.605	.749
14.	Douglas DC-3 (P&W S1C3G)	.0890	.694	.760
15.	Douglas DC-5	.0565	.601	.771
16.	Grumman G-21-A	.0591	.546	.737
17.	Lockheed 10-A	.0444	.479	.685
18.	Lockheed 10-B	0470	.485	727
19.	Lockheed 14-H2	.0931	.777	.784
20.	Lockheed 18-07	.0931	.717	.757
21.	Lockheed 18-08	0836	.685	.759
22.	Lockheed 18-50	.1087	.777	.755
23.	Martin 130	.0487	.532	.746
24.	Sikorsky S-40-A	.0466	.417	680
25.	Sikorsky S-42-B	.0519	.553	.755
26.	Sikorsky S-43-B	.0565	.578	.756
27.	Sikorsky VS-44-A	.0580	.620	780
28.	Stinson SR-5B	.0371	.434	.681
29.	Stinson SR-8B	0308	.420	.721
30.	Stinson SR-8C	.0340	.403	705
31.	Stinson SR-8E	0536	.419	.660
32.	Stinson SR-9F	.0588	.462	.682
33.	Stinson SR-10C	.0350	.409	.705
34.	WACO EGC-7	.0406	.397	.678
35.	WACO EGC-8	0406	397	.678

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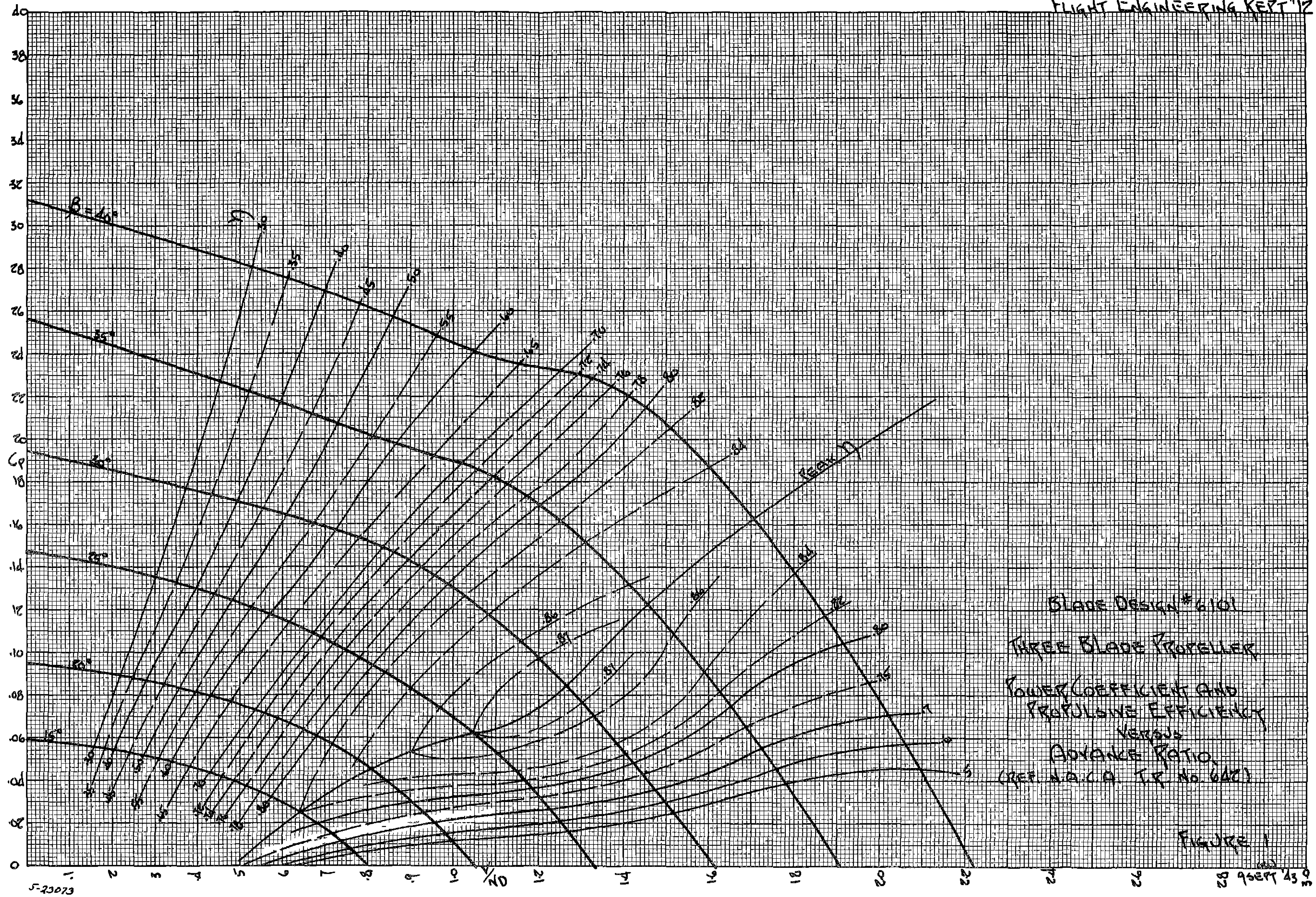

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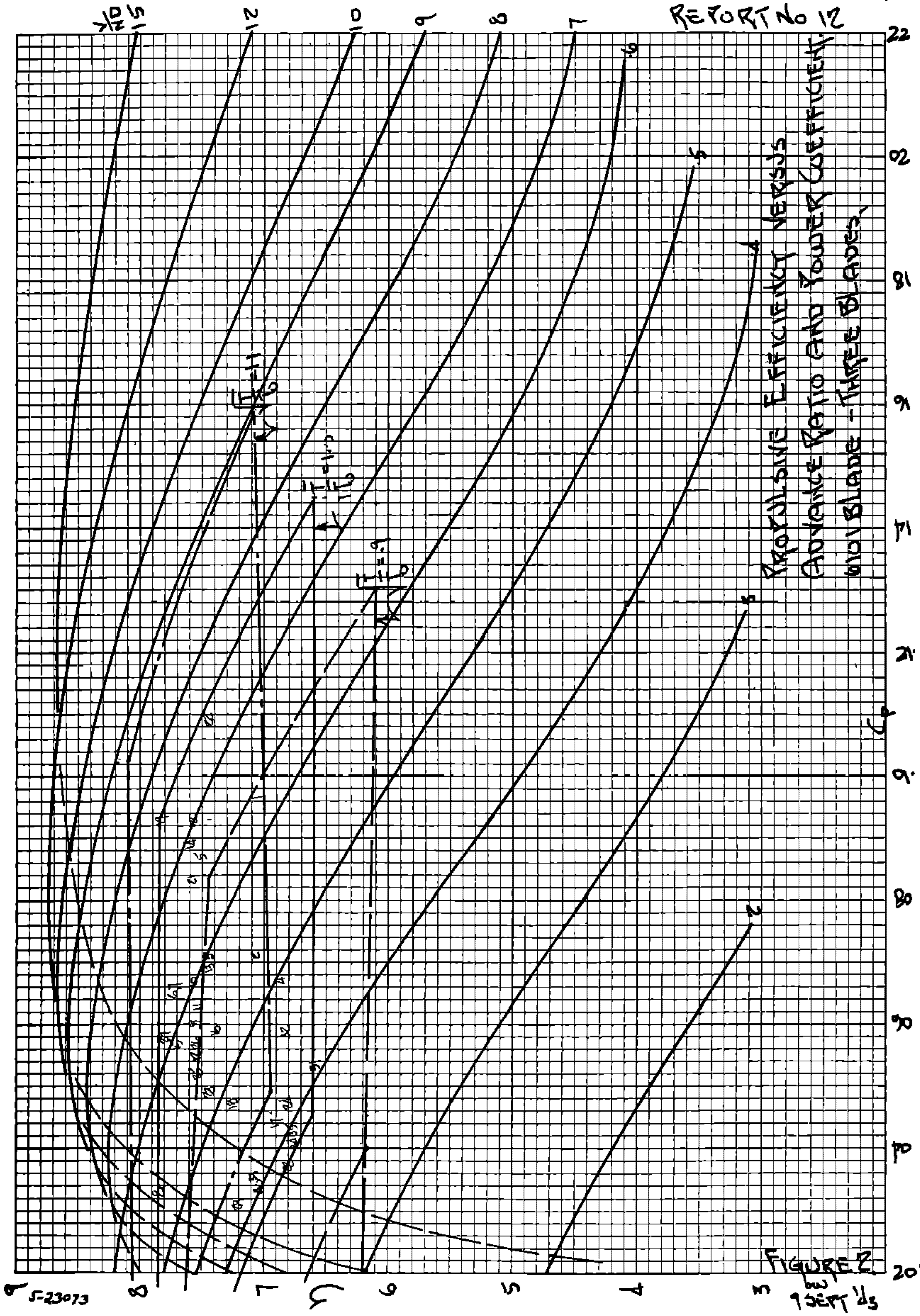

FIGURE 2
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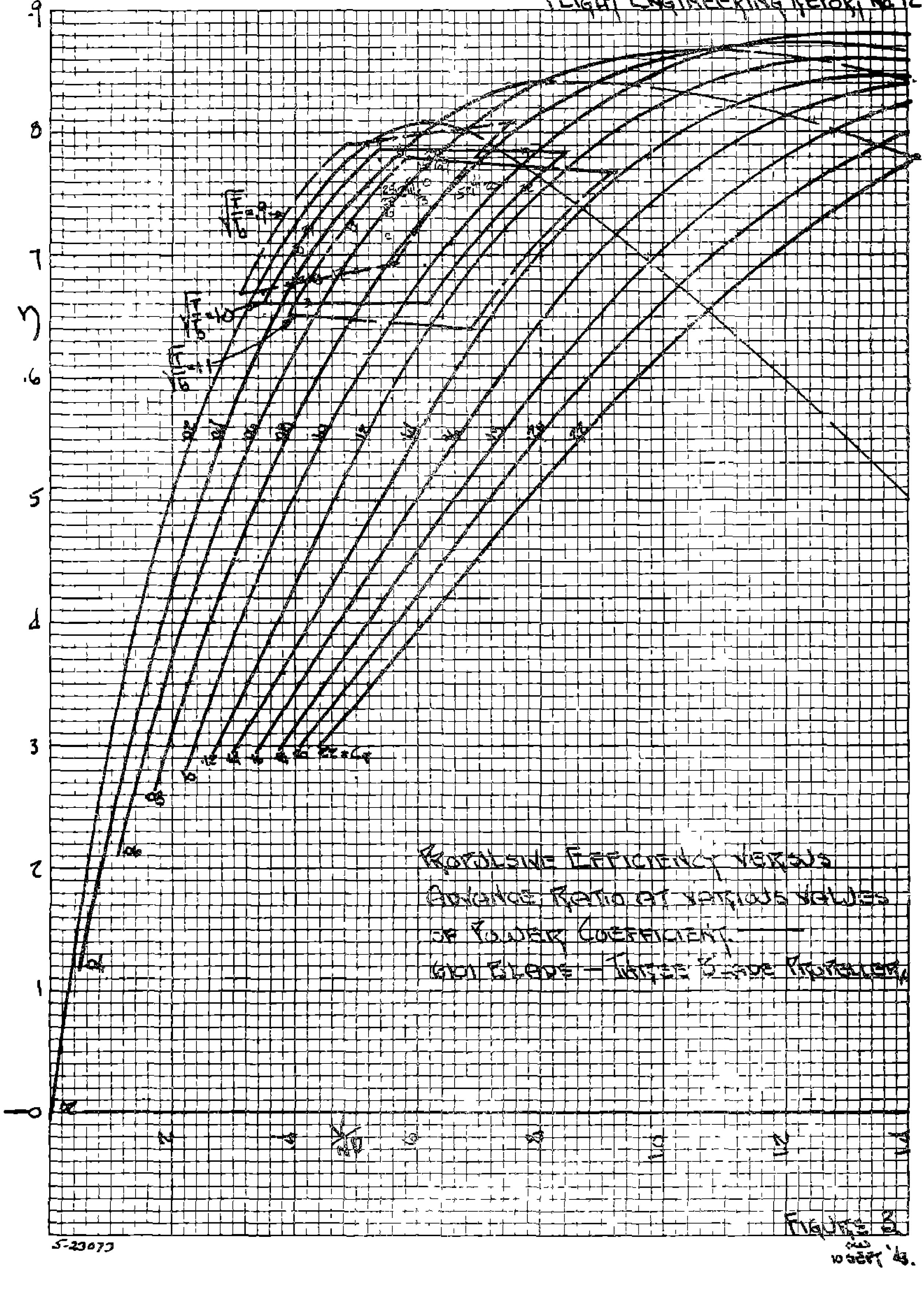

PROPULSIVE EFFICIENCY VERSUS ADVANCE RATIO AT VARIOUS VALUES OF POWER COEFFICIENT.
6101 BLADE — THREE BLADE PROPELLER.

FIGURE 3.

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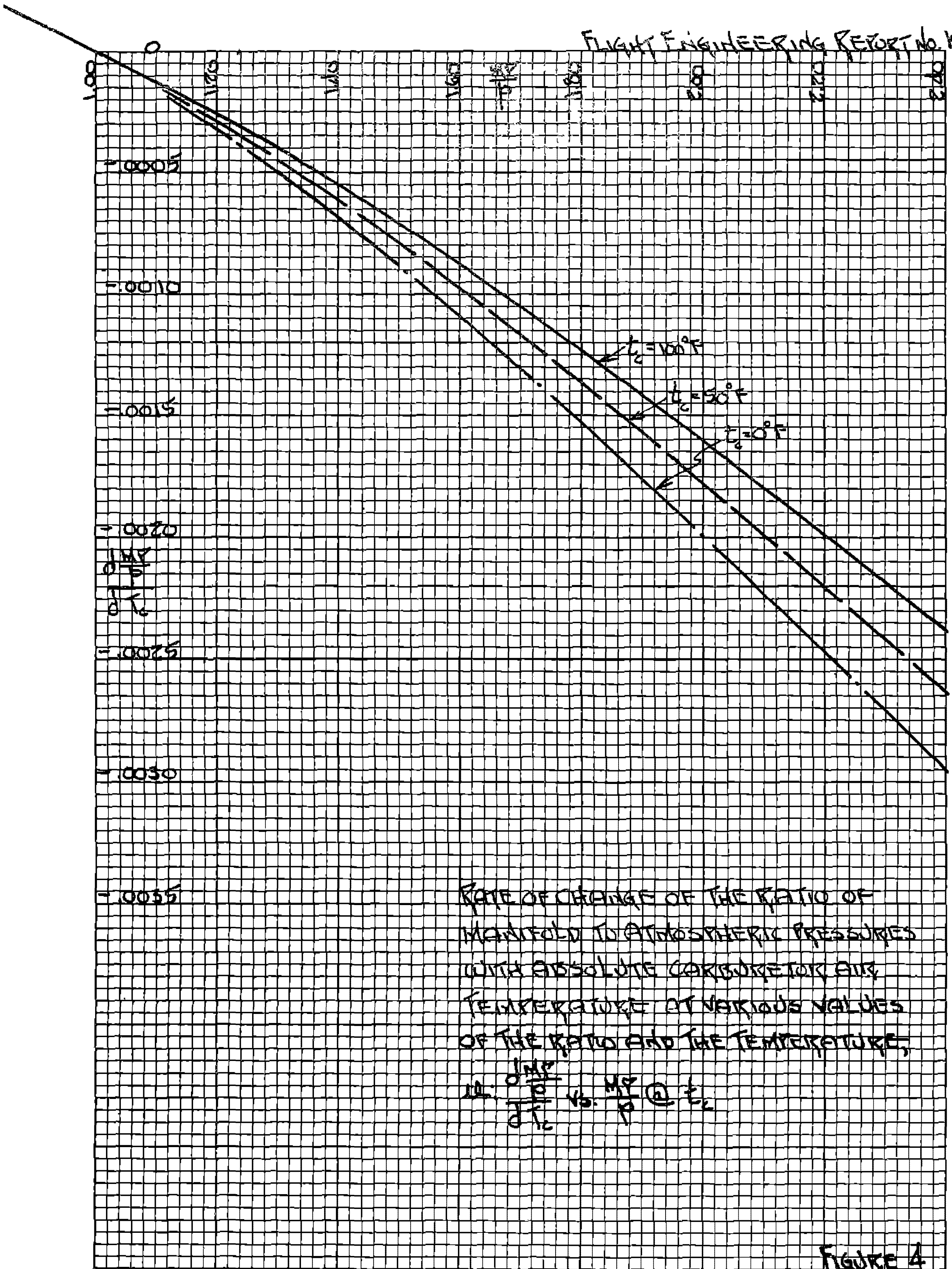

RATE OF CHANGE OF THE RATIO OF MANIFOLD TO ATMOSPHERIC PRESSURES WITH ABSOLUTE CARBURETOR AIR TEMPERATURE AT VARIOUS VALUES OF THE RATIO AND THE TEMPERATURE, i.e. $\frac{d\frac{MP}{P}}{dT_c}$ vs. $\frac{MP}{P}$ @ t_c

FIGURE 4

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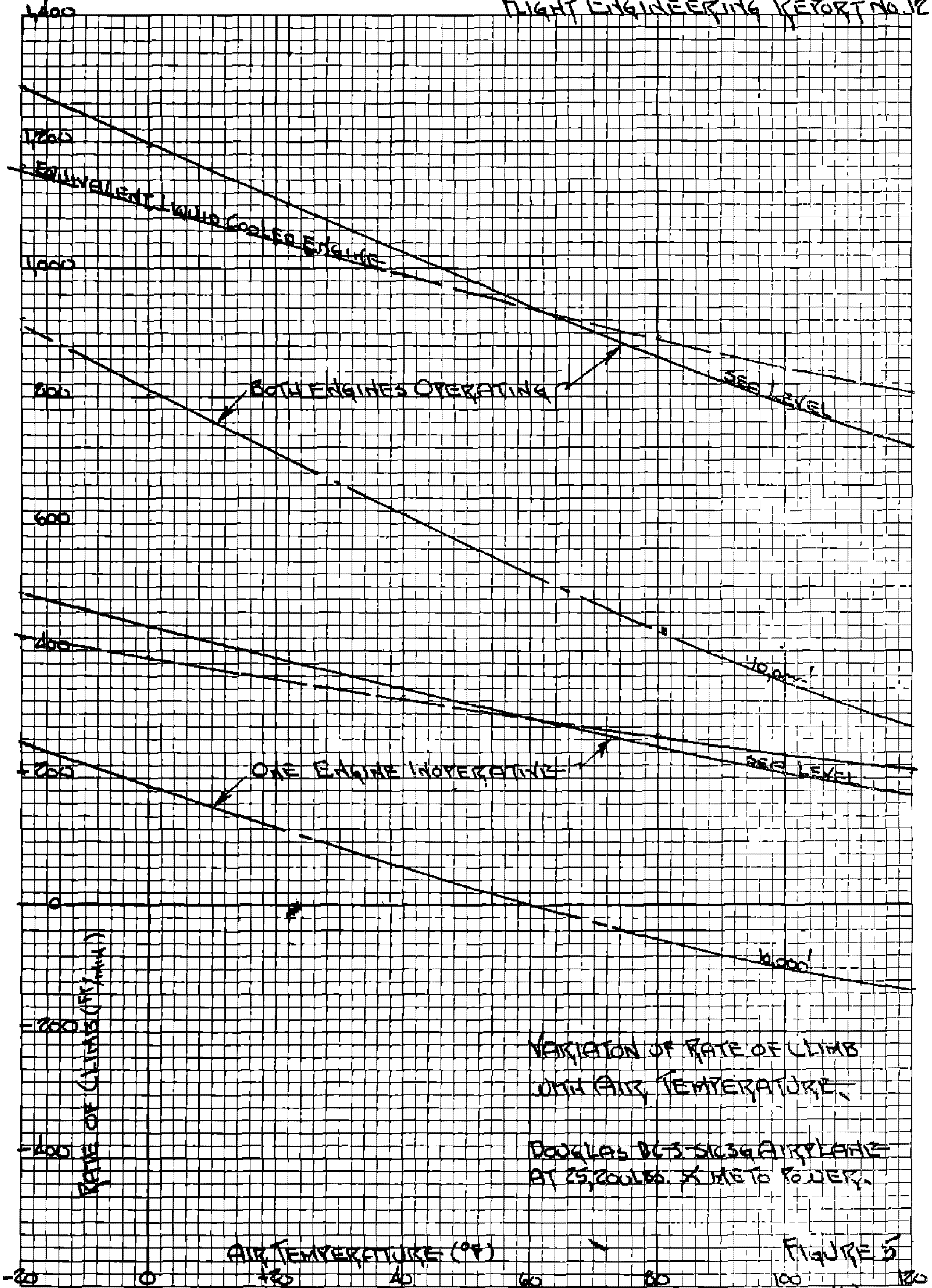

FIGURE 5

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