

Association of American Railroads  
Research and Test Department

**TRAIN ENERGY MODEL  
VALIDATION USING REVENUE SERVICE  
UNIT COAL TRAIN DATA**

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## APPENDIX A: TRAIN RESISTANCE

The total train resistance is the sum of all the resistances acting on all of the vehicles in the consist.

Let

$k$  = ordinal number of vehicle in consist  
 $N$  = total number of vehicles in consist  
 $R_k$  = total resistance acting on vehicle  $k$  in lbs  
 $R_{Bk}$  = bearing resistance acting on vehicle  $k$  in lbs  
 $R_{Rk}$  = rolling resistance acting on vehicle  $k$  in lbs  
 $R_{Ck}$  = curve resistance acting on vehicle  $k$  in lbs  
 $R_{Ak}$  = aerodynamic resistance acting on vehicle  $k$  in lbs  
 $R$  = total train resistance in lbs

Then

$$R = \sum_{k=1}^{k=N} (R_k) \quad (1)$$

where

$$R_k = R_{Bk} + R_{Rk} + R_{Ck} + R_{Ak} \quad (2)$$

Each of these resistances is considered in the following pages of this appendix.

A.1 Bearing Resistance

Let

- k = ordinal number of vehicle in consist
- $R_{Bk}$  = bearing resistance in lbs
- $n_k$  = number of axles
- $C_{Bk}$  = bearing resistance coefficient in lbs/axle

Then

$$R_{Bk} = n_k C_{Bk} \quad (1)$$

Let

- $w_k$  = total vehicle weight in TONS
- T = ambient temperature in degrees Fahrenheit
- $P_k$  = empirical power-law exponent
- $Q_k$  = empirical power-law multiplier
- $b_1(i_{Bk}), b_2(i_{Bk}), b_3(i_{Bk})$   
= empirical power-law exponent coefficients
- $b_4(i_{Bk}), b_5(i_{Bk}), b_6(i_{Bk})$   
= empirical power-law multiplier coefficients
- $i_{Bk}$  = bearing type index:

$i_B$	Bearing	$b_1$	$b_2$	$b_3$	$b_4$	$b_5$	$b_6$
1	worn T	.137	.00282	-.0000275	8.70	-.1120	.001000
2	new T	.280	-.00343	.0000169	4.47	.0208	.000893
3	worn B	.106	.00518	-.0000595	10.90	-.2290	.002420
4	new B	.291	-.00207	.0000267	4.55	.0271	-.000452

Then

$$C_{Bk} = Q_k w_k^{P_k} \quad (2)$$

where

$$P_k = b_1(i_{Bk}) + b_2(i_{Bk})T + b_3(i_{Bk})T^2 \quad (3)$$

and

$$Q_k = b_4(i_{Bk}) + b_5(i_{Bk})T + b_6(i_{Bk})T^2 \quad (4)$$

## A.2 Rolling Resistance

Let

$k$  = ordinal number of vehicle in consist  
 $R_{Rk}$  = rolling resistance in lbs  
 $W_k$  = total vehicle weight in lbs  
 $C_{Rk}$  = rolling resistance coefficient in lbs/TON

Then

$$R_{Rk} = .0005W_kC_{Rk} \quad (1)$$

Let

$Y_k$  = gross rail load in lbs  
 $\tau_k$  = vehicle tare weight in lbs  
 $e(i_{Tk})$  = empty rolling resistance coefficient in lbs/TON  
 $\lambda(i_{Tk})$  = loaded rolling resistance coefficient in lbs/TON  
 where  $i_{Tk}$  = truck type index:

$i_T$	Truck	$\epsilon$ lbs/TON	$\lambda$ lbs/TON
1	three-piece worn	2.25	2.13
2	three-piece new	2.25	1.57
3	radial	1.48	1.43
4	frame-braced	1.48	1.35
5	premium two-axle	1.47	1.02
6	single-axle	4.25	1.89

Then

$$C_{Rk} = e(i_{Tk}) - [e(i_{Tk}) - \lambda(i_{Tk})] \left[ \frac{W_k - \tau_k}{Y_k - \tau_k} \right], \quad W_k < Y_k \quad (2)$$

and

$$C_{Rk} = \lambda(i_{Tk}), \quad W_k \geq Y_k \quad (3)$$

### A.3 Curve Resistance

Let

k = ordinal number of vehicle in consist  
R<sub>Ck</sub> = curve resistance in lbs  
w<sub>k</sub> = total vehicle weight in TONS  
C<sub>Ck</sub> = curve resistance coefficient in lbs/TON

Then

$$R_{Ck} = w_k C_{Ck} (i_{Tk}, c_k) \quad (1)$$

where c<sub>k</sub> = instantaneous track curve at vehicle C.G. in deg.  
i<sub>Tk</sub> = truck type index:

i<sub>T</sub> Truck Type

1	three-piece worn
2	three-piece new
3	radial
4	frame-braced
5	premium two-axle
6	single-axle

Truck Type	non-lubed curve table	lubed curve table
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three-piece worn	1	2
three-piece new	1	2
radial	3	4
frame-braced	5	5
premium two-axle	6	6
single-axle	7	7

c (deg)	TABLE: 1	2	3	4	5	6	7
0	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1	0.090	0.056	0.013	0.017	0.010	0.015	0.015
2	0.731	0.339	0.018	0.020	0.055	0.120	0.055
3	1.548	0.791	0.206	0.173	0.200	0.430	0.110
4	2.465	1.335	0.624	0.448	0.320	0.800	0.220
5	3.325	1.907	1.020	0.747	0.470	1.130	0.365
6	4.106	2.468	1.315	1.073	0.760	1.520	0.675
7	4.767	3.046	1.505	1.445	1.120	1.895	1.235
8	5.816	3.664	2.016	1.824	1.555	2.375	2.135
9	6.686	4.298	2.478	2.206	1.770	2.915	5.555
10	7.557	4.933	2.926	2.588	2.410	3.560	11.625
11	8.427	5.563	3.376	2.968	2.705	4.260	14.170
12	9.297	6.193	3.826	3.348	3.330	5.020	16.905
13	10.167	6.823	4.276	3.728	4.050	5.865	19.425
14	11.037	7.453	4.726	4.108	4.815	6.755	22.310
15	11.907	8.083	5.176	4.488	5.695	7.665	27.685

NOTE: Linear interpolation is used for non-integral values of track curve.

Exhibit A.1. Curve Resistance Coefficients.

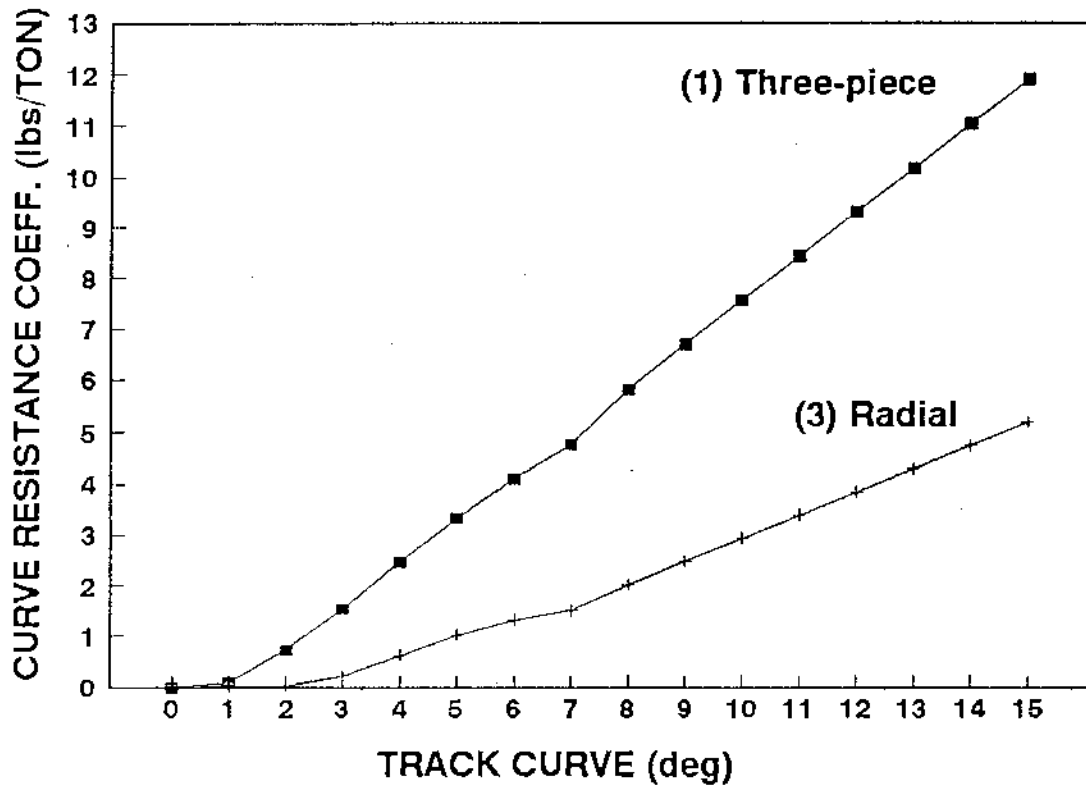


Exhibit A.2. Curve Resistance Coefficients for Unlubricated (1) Three-piece and (3) Radial Trucks.

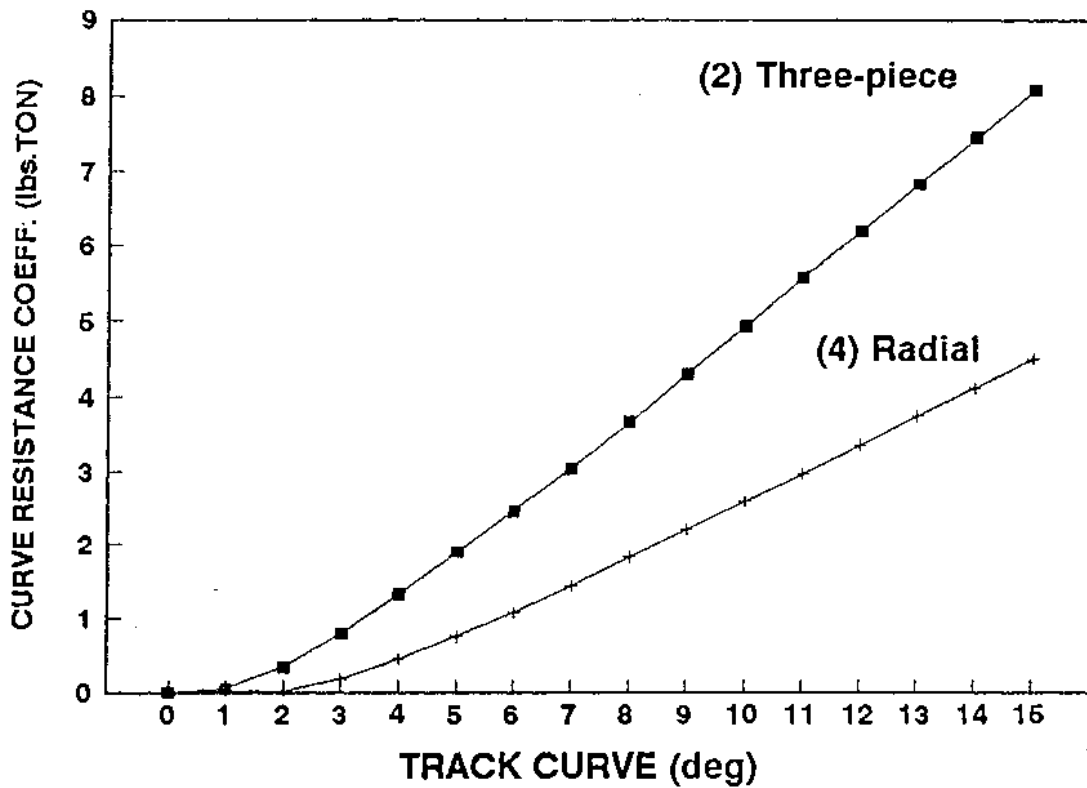


Exhibit A.3. Curve Resistance Coefficients for Lubricated (2) Three-piece and (4) Radial Trucks.



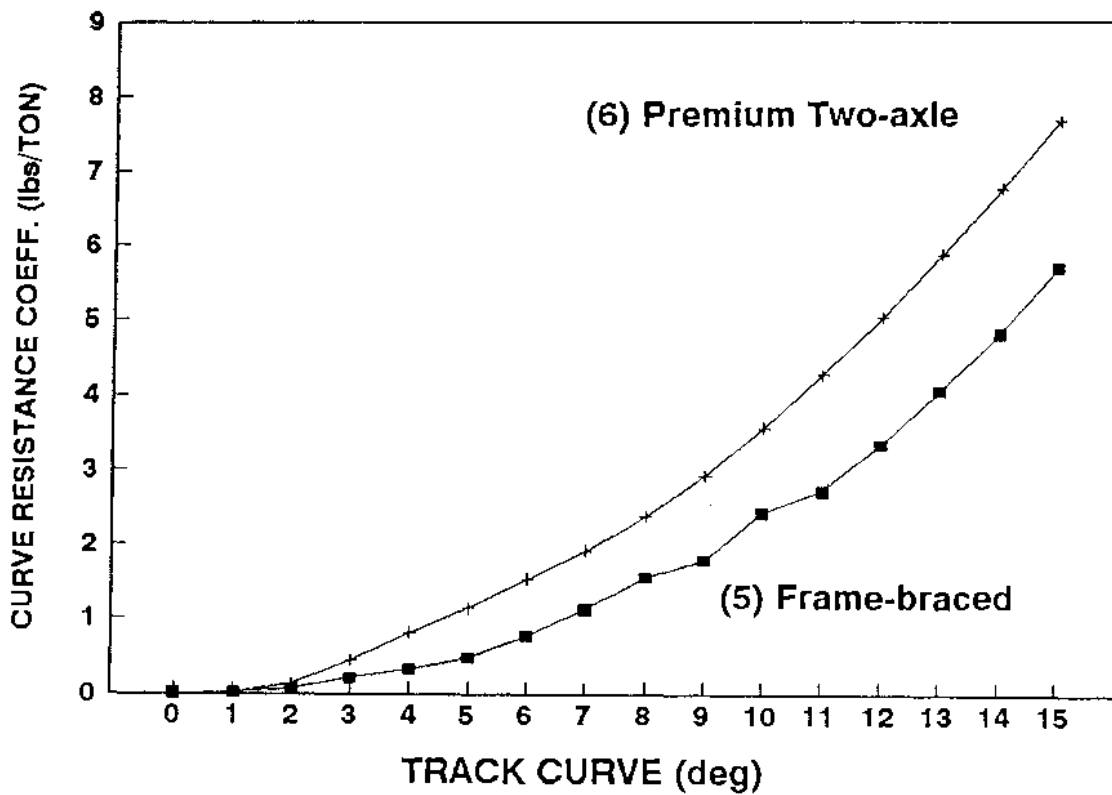


Exhibit A.4. Curve Resistance Coefficients for (5) Framed-braced and (6) Premium Two-axle Trucks.

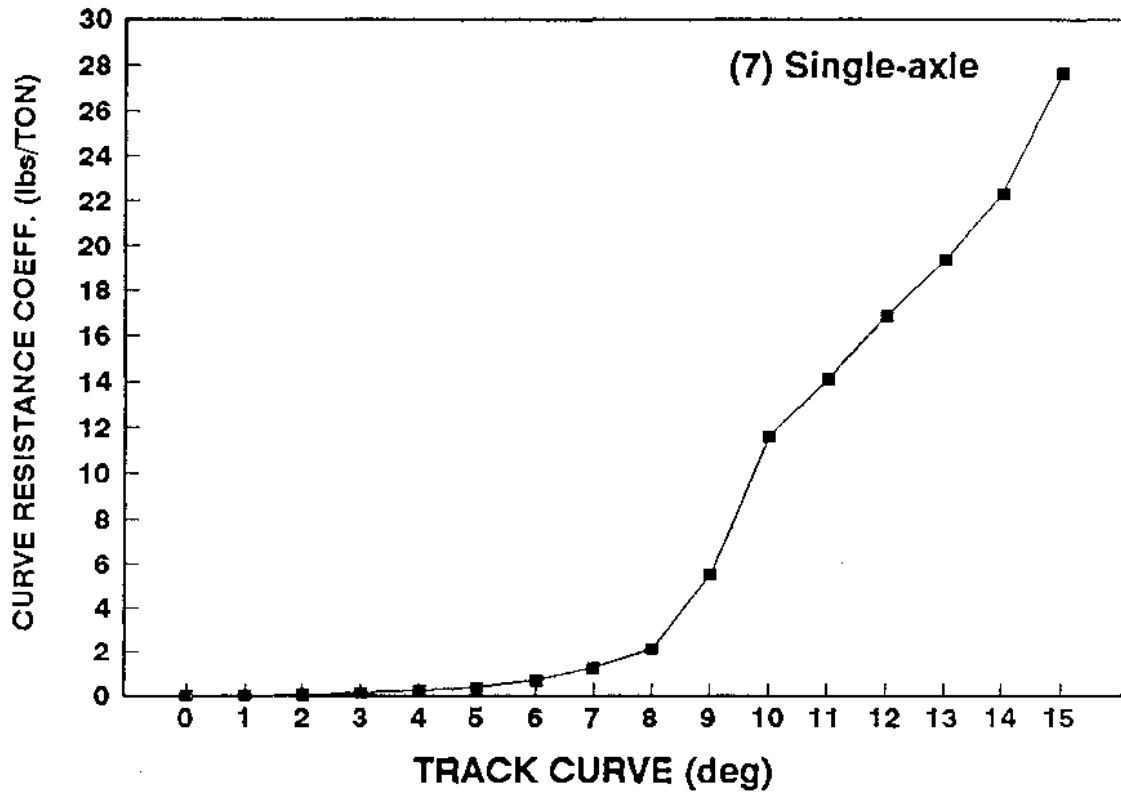


Exhibit A.5 Curve Resistance Coefficients for (7) Single-axle Truck.

#### A.4 Aerodynamic Resistance

Let  $k$  = ordinal number of vehicle in consist  
 $R_{Ak}$  = aerodynamic resistance in lbs  
 $V_k$  = vehicle velocity relative to the wind in mph  
 $C_{Ak}$  = aerodynamic resistance coefficient in lbs/mph/mph

Then

$$R_{Ak} = V_k^2 C_{Ak} \quad (1)$$

Let  $r$  = air density factor  
 $P$  = uncorrected barometric pressure in inches of Mercury  
 $T$  = ambient temperature in degrees Fahrenheit  
 $Y_k$  = yaw angle of vehicle  
 $A_k$  = drag area of vehicle in square feet

Then

$$C_{Ak} = .5r(P, T)A_k(Y_k) \quad (2)$$

where

$$r(P, T) = .02057 \frac{P}{T+460} \quad (3)$$

and  $A_k(Y_k)$  is a seventh order polynomial in  $Y_k$ .

NOTE: The coefficients in the seventh order polynomial  $A_k(Y_k)$  depend on the aerodynamic characteristics of the vehicle. However, these coefficients vary with  $k$ , so that the same type of vehicle may have different drag areas, depending on its position in the consist.

APPENDIX B: UNIT COAL TRAIN DATA

Vehicle	Zero Yaw Drag Area (sq.ft.)									
1- 10	204	68	63	71	67	64	62	61	60	59
11- 20	59	58	58	58	58	58	57	57	57	57
21- 30	57	57	57	57	57	57	57	57	57	57
31- 40	57	57	57	57	57	57	57	57	57	57
41- 50	57	57	57	57	57	57	57	57	57	71
51- 60	48	48	58	57	57	57	57	57	57	57
61- 70	57	57	57	57	57	57	57	57	57	57
71- 80	57	57	57	57	57	57	57	57	57	57
81- 90	57	57	57	57	57	57	57	57	57	57
91-100	57	57	57	57	57	57	57	57	57	57
101-103	57	57	80							

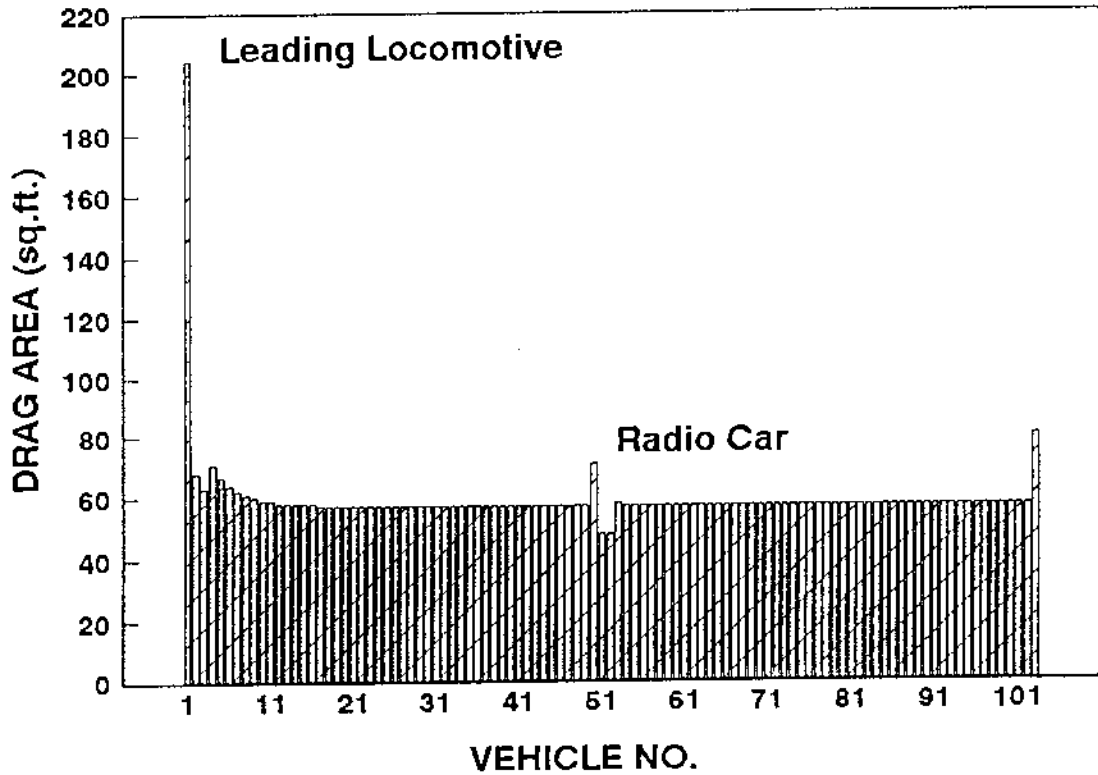


Exhibit B.1. Loaded Revenue Consist Zero Yaw Drag Areas.