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Informal Note #12

CAN WE STUDY CARS WITH V^2 ROLLING RESISTANCE TERMS?

The current PROFILE simulation and its "offshoot" SPEEDCON (used to study speed control devices) can model cars whose rolling resistance can be represented by a constant term plus a term which is proportional to the velocity (V). However, it would be desirable in some situations to add a term which is proportional to the square of the velocity (V^2).

The rolling behavior of a car <u>without</u> the V^2 term is a <u>linear</u> differential equation with constant coefficients; the closed form solution for velocity and position as a function of time is well known and consists of exponentials and powers of t. Using the closed form solution in PROFILE and SPEEDCON allows them both to be an order of magnitude faster in running time as compared to using a numerical-integration procedure (e.g., Runge-Kutta) to solve the associated differential equation (whose parameters change with each track section).

Because the rolling behavior of a car <u>with</u> V^2 term is described by a <u>non-linear</u> differential equation, it had been assumed that no closed form solution existed and that we would have to use a numerical-integration procedure to solve the associated differential equation. Since this was considered impractical, both PROFILE and SPEEDCON could not accomodate a rolling resistance model with a V^2 term.

However, it now appears that even with a V^2 term, there does exist a closed form solution for the associated differential equation. The derivation of velocity and position as a function of time is attached. This implies that a simple modification to PROFILE and SPEEDCON would allow them both to accomodate a more complex rolling resistance model.

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DERIVATION: EQUATIONS WITH V^2 ROLLING RESISTANCE

DIFFERENTIAL EQUATION OF CAR WITH V^2 TERM

(1) $$\frac{dV}{dt} = C + BV + AV^2$$

SEPARATION OF VARIABLES

(2) $$\int \frac{dV}{C + BV + AV^2} = \int dt$$

L.H.S. OF (2) IS IN INTEGRAL TABLES

FIRST CONSTANT OF INTEGRATION

(3) $$\frac{1}{\sqrt{B^2-4AC}} \ln\left[\frac{2AV+B-\sqrt{B^2-4AC}}{2AV+B+\sqrt{B^2-4AC}}\right] = t + K_1$$

LET $\sqrt{\ } \triangleq \sqrt{B^2-4AC}$

TAKE EXPONENTIAL OF BOTH SIDES OF (3)

(4) $$\frac{2AV+B-\sqrt{\ }}{2AV+B+\sqrt{\ }} = K_1 e^{\sqrt{\ }\, t}$$

(5) $$V = \frac{K_1(B+\sqrt{\ })e^{\sqrt{\ }\, t} - (B-\sqrt{\ })}{2A(1-K_1 e^{\sqrt{\ }\, t})}$$

EQUATION (5) GIVES V AS A FUNCTION OF t, WHERE $\sqrt{\ } = \sqrt{B^2-4AC}$ AND K_1 IS CONSTANT OF INTEGRATION

WE ALSO WANT X AS A FUNCTION OF t,

WE REWRITE $V = \frac{dx}{dt}$, SO THAT (5)

BECOMES

(6) $$\int dx = \int \frac{K_1(B+\sqrt{\ })e^{\sqrt{\ }t} - (B-\sqrt{\ })}{2A\ (1-K_1 e^{\sqrt{\ }t})}\,dt + K_2$$

SECOND CONSTANT OF INTEGRATION

(7) $$x = \underbrace{\int \frac{K_1(B+\sqrt{\ })e^{\sqrt{\ }t}}{2A(1-K_1 e^{\sqrt{\ }t})}\,dt}_{\text{TERM 1}} - \underbrace{\int \frac{(B-\sqrt{\ })}{2A(1-K_1 e^{\sqrt{\ }t})}\,dt}_{\text{TERM 2}} + K_2$$

TERM 2 IS IN INTEGRAL TABLES

(8) $$\text{TERM 2} = (B-\sqrt{\ })\left[\frac{t}{2A} - \frac{1}{2A\sqrt{\ }}\ln\left\{2A(1-K_1 e^{\sqrt{\ }t})\right\}\right]$$

HOWEVER TERM 1 NOT IN INTEGRAL TABLES

(9) $$\text{TERM 1} = K_1(B+\sqrt{\ })\int \frac{e^{\sqrt{\ }t}}{2A\ (1-K_1 e^{\sqrt{\ }t})}\,dt$$

LET $u = e^{\sqrt{\ }t}$ $\quad du = \sqrt{\ }e^{\sqrt{\ }t}dt$

$$dt = \frac{du}{\sqrt{\ }e^{\sqrt{\ }t}}$$

(10) $$\text{TERM } 1 = K_1(B+\sqrt{\ }) \int \frac{du}{2A\sqrt{\ }\,(1-K_1 u)}$$

WITH A SUBSITUTION OF VARIABLES (10) IS FOUND IN INTEGRAL TABLES

(11) $$\text{TERM } 1 = -K_1(B+\sqrt{\ })\left[\frac{1}{2AK_1\sqrt{\ }}\ln\left\{2A\sqrt{\ }(1-K_1 u)\right.\right.$$

SUBSTITUTING $u = e^{\sqrt{\ }t}$ INTO (11)

(12) $$\text{TERM } 1 = -K_1(B+\sqrt{\ })\left[\frac{1}{2AK_1\sqrt{\ }}\ln\left\{2A\sqrt{\ }(1-K_1 e^{\sqrt{\ }}\right.\right.$$

SUBSTITUTING (12) AND (8) INTO (7)

$$X = -K_1(B+\sqrt{\ })\left[\frac{1}{2AK_1\sqrt{\ }}\ln\left\{2A\sqrt{\ }(1-K_1 e^{\sqrt{\ }t})\right\}\right.$$

$$-\ (B-\sqrt{\ })\left[\frac{t}{2A} - \frac{1}{2A\sqrt{\ }}\ln\left\{2A(1-K_1 e^{\sqrt{\ }t})\right\}\right.$$

$$+\ K_2$$

EQUATION (12) GIVES X AS A FUNCTION OF t, WHERE $\sqrt{\ } = \sqrt{B^2-4AC}$ AND K_1 AND K_2 ARE CONSTANTS OF INTEGRATION.