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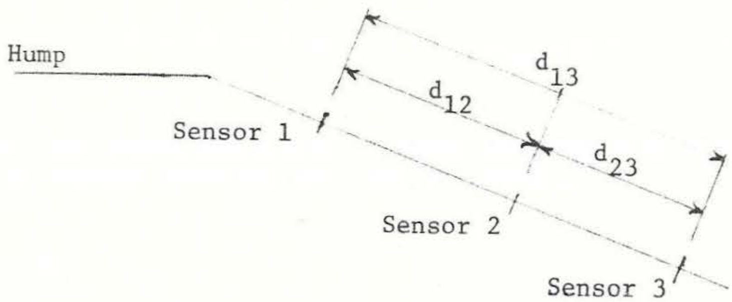
Informal Working Note #1

ACCELERATION FROM TIME AND DISTANCE MEASUREMENTS

The purpose of this informal working note is to document the derivation of an equation that determines the acceleration of a moving object when given it's time of arrival at three known points. This equation can be used to approximate the acceleration of a rolling rail car in a hump yard. This acceleration in turn gives an indication of rolling resistance. This determination assumes a constant acceleration.



Placing sensors at three points along a uniform gradient of track will yield a time of arrival as a rail car passes. These times are represented at T_1, T_2, and T_3. The distance between sensors 1, 2, and 3 are d_{12}, d_{23} and d_{13}.

Equation 1:

Average velocity between two points =

Average of the velocities at each end point =

distance traveled/time of travel.

(1) $\overline{V} = 1/2(V_{initial} + V_{final}) = d/(T_{final} - T_{initial}) = d/t$

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For points 1,2 and 1,3 the equations are:

$$\overline{V}_{12} = d_{12}/t_{12} = 1/2(V_1 + V_2) \qquad \overline{V}_{13} = d_{13}/t_{13} = 1/2(V_1 + V_3)$$

$$t_{12} = T_2 - T_1 \text{ and } t_{13} = T_3 - T_1$$

Equation 2:

Acceleration between two points = Change in Velocity/Change in time

(2) $$a = \Delta V/\Delta t = (V_{final} - V_{initial})/(T_{final} - T_{initial})$$

For points 1,2 and 1,3 the equations are:

$$a_{12} = \Delta V/\Delta t = (V_2 - V_1)/t_{12} \qquad a_{13} = \Delta V/\Delta t = (V_3 - V_1)/t_{13}$$

These four equations yield a single equation for acceleration in terms of distances traveled and time of travel.

Equations 3-6:

(1) $$\overline{V}_{12} = d_{12}/t_{12} = 1/2(V_1 + V_2) \qquad \overline{V}_{13} = d_{13}t_{13} = 1/2(V_1 + V_3)$$

(3) $$V_2 = (2d_{12}/t_{12}) - V_1$$ (4) $$V_3 = (2d_{13}/t_{13}) - V_1$$

(2) $$a_{12} = \Delta V/\Delta\ t = (V_2 - V_1)/t_{12} \qquad a_{13} = \Delta V/\Delta t = (V_3 - V_1)/t_{13}$$

(5) $$a_{12}t_{12} = V_2 - V_1$$ (6) $$a_{13}t_{13} = V_3 - V_1$$

Equations 7 and 8:

Substitute equations (3) and (4) into equations (5) and (6).

$$a_{12}t_{12} = ([2d_{12}/t_{12}] - V_1]) - V_1 \qquad a_{13}t_{13} = ([2d_{13}/t_{13}] - V_1) - V_1$$

(7) $$2V_1 = (2d_{12}/t_{12}) - a_{12}t_{12}$$ (8) $$2V_1 = (2d_{13}/t_{13}) - a_{13}t_{13}$$

Equations 9 and 10:

As a constant acceleration is assumed: $a = a_{12} = a_{13}$, therefore

(9) $(2d_{12}/t_{12}) - at_{12} = (2d_{13}/t_{13}) - at_{13}$

$a(t_{13} - t_{12}) = 2(d_{13}/t_{13} - d_{12}/t_{12})$

$= 2(d_{13}t_{12} - d_{12}t_{13})/t_{13}t_{12}$

(10) $a = 2(d_{13}t_{12} - d_{12}t_{13})/(t_{13}t_{12})(t_{13} - t_{12})$