# Structural Design with UHPC Workshop Manual

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## FOREWORD

Research related to ultra-high performance concrete (UHPC) has been ongoing for the past few decades. UHPC offers enhanced mechanical and durability properties that make it an ideal material for use in the construction, repair, and retrofit of our Nation's highway bridges. Early adoption of UHPC was for connection of prefabricated bridge elements with the next phase of adoption being for preservation and repair activities. The use of UHPC for structural members is the next phase of adoption, which will be aided by the recent release of the AASHTO *Guide Specifications for Structural Design with Ultra-High Performance Concrete*.

The information presented in this manual provides background, context, and foundational knowledge to bridge owners and designers interested in using UHPC for structural applications. This manual and the accompanying workshop materials will be of interest to bridge owners and designers looking for durable and high-performance solutions for new bridge construction, replacements, and preservation.

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## Notice



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## NOTATION

| | |
|---|---|
| $A_1$ | area of section between $y_1$ from the center of gravity and the outside face of the section (inch$^2$) |
| $A_c$ | area of concrete or UHPC (inch$^2$) |
| $A_{ct}$ | area of concrete or UHPC on the flexural tension side of the member (inch$^2$) |
| $A_{cv}$ | area of UHPC considered to be engaged in interface shear transfer (inch$^2$) |
| $A_h$ | area of horizontal reinforcement (inch$^2$) |
| $A_o$ | area enclosed by shear flow path (inch$^2$) |
| $A_p$ | total area of prestressing strands (inch$^2$) |
| $A_{p,0.6in}$ | area of 0.6-inch diameter prestressing strand (inch$^2$) |
| $A_{p,0.7in}$ | area of 0.7-inch diameter prestressing strand (inch$^2$) |
| $A_{ps}$ | total area of prestressing strands on flexural tension side of member; total area of prestressing in tension tie (inch$^2$) |
| $A_{pT}$ | total area of prestressing strands (inch$^2$) |
| $A_s$ | total area of non-prestressed tension reinforcement (inch$^2$) |
| $A_{s,bar,\#4}$ | area of No. 4 bar (inch$^2$) |
| $A_{s,bar,\#11}$ | area of No. 11 bar (inch$^2$) |
| $A_{s,req}$ | total area of steel required (inch$^2$) |
| $A_{tie}$ | area of UHPC in tension tie (inch$^2$) |
| $A_v$ | area of transverse reinforcement within distance $s$ (inch$^2$) |
| $A_{vf}$ | area of interface reinforcement crossing the shear plane within the area $A_{cv}$ (inch$^2$) |
| $a_{LT}$ | distance from left support to front axle of design truck (ft) |
| $b(z)$ | width section minus interior void width as a function of $z$ (inch) |
| $b_{bf}$ | width of bottom flange (inch) |
| $b_e$ | effective flange width (inch) |
| $b_h(z)$ | width of interior void as a function of $z$ (inch) |
| $b_o(z)$ | outside width of section as a function of $z$ (inch) |
| $b_s$ | width of support (ft) |
| $b_{tf}$ | width of top flange (inch) |

| | |
|---|---|
| $b_w(y_1)$ | web width at $y_1$ from the center of gravity (inch) |
| $b_v$ | width of web for use in shear design (inch) |
| $b_{vi}$ | interface width considered to be engaged in shear transfer (inch) |
| $C$ | compression resultant force used in shear design (kips) |
| $C_1$ | normal clamping force provided by steel reinforcement (kips) |
| $C_2$ | normal clamping force provided by UHPC (kips) |
| $C_c$ | compression force in concrete or UHPC in sectional analysis (kips) |
| $C_c(c, \varepsilon_{cb})$ | compression force in UHPC as a function of neutral axis $c$ and bottom fiber strain $\varepsilon_{cb}$ (kips) |
| $c$ | neutral axis depth for flexural analysis (inch); cohesion factor for interface shear transfer (ksi) (inch) |
| $c_{cr}$ | neutral axis depth at cracking concrete or UHPC at extreme tension fiber (inch) |
| $DFD$ | distribution factor for deflection |
| $DFM$ | distribution factor for moment |
| $DFM_F$ | distribution factor for moment for fatigue |
| $DFV$ | distribution factor for shear |
| $DFV_F$ | distribution factor for shear for fatigue |
| $D_p$ | diameter of prestressing strand (inch) |
| $d$ | distance from extreme compression face to centroid of non-prestressed reinforcement (inch) |
| $d_b$ | diameter of reinforcement or prestressing strand (inch) |
| $d_{b,\#11}$ | diameter of No. 11 bar (inch) |
| $d_e$ | depth of center of gravity of steel; effective depth from extreme compression fiber to the centroid of the tensile force in the tensile reinforcement (inch) |
| $d_p$ | distance from extreme compression face to centroid of prestressing strands (inch) |
| $d_v$ | effective shear depth (inch) |
| $d_{v,1}$ | effective shear depth at Location 1 (inch) |
| $d_{v,2}$ | effective shear depth at Location 2 (inch) |
| $d_{v,3}$ | effective shear depth at Location 3 (inch) |
| $d_{y1}$ | distance between center of gravity and centroid of area $A_1$ (inch) |

| | |
|---|---|
| $E$ | modulus of elasticity (ksi) |
| $E_c$ | modulus of elasticity of concrete or UHPC (ksi) |
| $E_{ci}$ | modulus of elasticity of concrete or UHPC at transfer (ksi) |
| $E_{c,eff}$ | effective modulus of elasticity of concrete or UHPC (ksi) |
| $E_{c,eff,deck}$ | effective modulus of elasticity of concrete or UHPC using creep coefficient with loading age of $t_d$ (ksi) |
| $E_{c,eff,re}$ | effective modulus of elasticity of concrete or UHPC using creep coefficient with loading age of $t_{release}$ (ksi) |
| $E_s$ | modulus of elasticity of steel reinforcement (ksi) |
| $E_p$ | modulus of elasticity of prestressing strands (ksi) |
| $e_m$ | strand eccentricity at midspan (inch) |
| $e_{m,o}$ | strand eccentricity of outermost layer of prestressing strands at midspan (inch) |
| $e_{pg}$ | strand eccentricity at midspan (inch) |
| $f_c$ | stress in extreme compression (top) fiber; stress in concrete (ksi) |
| $f_{c,f}$ | stress in extreme compression (top) fiber due to one-half the sum of the unfactored effective prestress and permanent loads (ksi) |
| $f_{c,f,a}$ | allowable compression stress limit for fatigue (ksi) |
| $f_{c,PL}$ | stress in extreme compression (top) fiber due to effective prestress and permanent loads (ksi) |
| $f_{c,SeI}$ | stress in extreme compression (top) fiber due to effective prestress and Service I loading (ksi) |
| $f_{ci}$ | stress at time of transfer in extreme compression fiber, top (ksi) |
| $f_{ci,a}$ | allowable compressive stress at release (ksi) |
| $f_{ci,m}$ | stress in top fiber at transfer at midspan (ksi) |
| $f_{ci,tr}$ | stress in top fiber at transfer at the transfer length (ksi) |
| $f_{cgp}$ | concrete stress at the center of gravity of prestressing strands due to the prestressing force immediately at transfer and the self-weight of the member at the section of maximum moment (ksi) |
| $f_{cgp,o}$ | concrete stress at the center of gravity of the outermost layer of prestressing strands due to the prestressing force immediately at transfer and the self-weight of the member at the section of maximum moment (ksi) |

$f_{c,a}$ allowable compression stress for stresses due to effective prestress and permanent loads and transient loads (ksi)

$f_{cp,a}$ allowable compression stress for stresses due to effective prestress and permanent loads (ksi)

$f_{cpe}$ compressive stress in concrete due to effective prestress force only (after allowance for all prestress losses) at extreme fiber of section where tensile stress is caused by externally applied loads (ksi)

$f'_c$ compressive strength of concrete for use in design (ksi)

$f'_{ci}$ compressive strength of concrete at release for use in design (ksi)

$f_{min}$ minimum principal stress (maximum tension) in web (ksi)

$f_{pbt}$ stress in prestressing strands immediately before transfer (ksi)

$f_{pe}$ effective stress in prestressing (ksi)

$f_{pe2}$ effective stress in prestressing after all prestress losses ignoring elastic gains (ksi)

$f_{pe,a}$ allowable effective stress in prestressing at service (ksi)

$f_{pe,SeI}$ effective stress in prestressing due to Service I load combination (ksi)

$f_{pe,o,SeI}$ effective stress in outermost layer of prestressing strands due to Service I load combination (ksi)

$f_{pcx}$ horizontal stress in the web (ksi)

$f_{pcy}$ vertical stress in the web (ksi)

$f_{pj}$ stress in prestressing strands at jacking (ksi)

$f_{ps}$ stress in prestressing strands (ksi)

$f_{ps,i}(c, \varepsilon_{cb})$ stress in layer $i$ of prestressing strands as a function of neutral axis depth $c$ and bottom fiber strain $\varepsilon_{cb}$ (ksi)

$f_{pt}$ stress in prestressing strands immediately after transfer (ksi)

$f_{pu}$ ultimate strength of prestressing strands (ksi)

$f_{py}$ yield strength of prestressing strands (ksi)

$f_r$ modulus of rupture of concrete (ksi)

$f_s$ stress in steel (ksi)

$f_{sl}$ stress in steel at service limit (ksi)

$f_{sy}$ yield stress of steel (ksi)

| | |
|---|---|
| $f_t$ | stress in extreme tension face, bottom (ksi) |
| $f_{t,SeI}$ | stress in extreme tension (bottom) fiber due to effective prestress and Service I loading (ksi) |
| $f_{t,SeIII}$ | stress in extreme tension (bottom) fiber due to effective prestress and Service III loading (ksi) |
| $f_{t,a}$ | allowable tensile stress for stresses due to effective prestress and permanent loads and transient loads (ksi) |
| $f_{ti}$ | stress at time of transfer in extreme tension face, bottom (ksi) |
| $f_{ti,a}$ | allowable tensile stress at release (ksi) |
| $f_{ti,m}$ | stress in bottom fiber at transfer at midspan (ksi) |
| $f_{ti,tr}$ | stress in bottom fiber at transfer at the transfer length (ksi) |
| $f_{t,cr}$ | effective cracking strength for UHPC (ksi) |
| $f_{t,cri}$ | effective cracking strength for UHPC at time of transfer (ksi) |
| $f_{t,cr,min}$ | minimum allowable effective cracking strength for UHPC (ksi) |
| $f_{t,loc}$ | crack localization strength for UHPC (ksi) |
| $f_{t,loc,min}$ | minimum allowable crack localization strength for UHPC (ksi) |
| $\bar{f}_{t,cr,test}$ | average measured effective cracking strength for UHPC (ksi) |
| $\bar{f}_{t,loc,test}$ | average measured crack localization strength for UHPC (ksi) |
| $f_{v,\alpha}$ | uniaxial stress in the transverse steel reinforcement at nominal shear resistance (ksi) |
| $f_{v,\alpha,assumed}$ | assumed uniaxial stress in the transverse steel reinforcement at nominal shear resistance (ksi) |
| $f_y$ | yield strength of transverse reinforcement (ksi) |
| $H$ | average annual ambient relative humidity (percent) |
| $h$ | section height (inch) |
| $h_a$ | height of tension tie (inch) |
| $I$ | moment of inertia (inch$^4$) |
| $IM$ | impact factor for design truck |
| $IM_F$ | impact factor for design truck for fatigue |
| $I_g$ | moment of inertia for gross section (inch$^4$) |
| $J_g$ | St. Venant's torsional inertia (inch$^4$) |

| | |
|---|---|
| $K$ | limiting interface shear resistance (ksi) |
| $K_1$ | correction factor for modulus of elasticity to be taken as 1.0 unless determined by a physical test, and as approved by the owner |
| $K_3$ | correction factor for creep to be taken as 1.0 unless determined by physical tests, and as approved by the owner |
| $K_4$ | correction factor for shrinkage to be taken as 1.0 unless determined by physical tests, and as approved by the owner |
| $K_{df}$ | transformed section coefficient that accounts for time-dependent interaction between concrete and bonded steel in the section being considered for the time period between deck placement and final time |
| $K_{id}$ | transformed section coefficient that accounts for time-dependent interaction between concrete and bonded steel in the section being considered for the time period between transfer and deck placement |
| $K_L$ | factor accounting for the type of steel, taken as 30 for low relaxation strands and 7 for other types of prestressing steel |
| $k_f$ | concrete strength factor for creep and shrinkage |
| $k_{hc}$ | humidity factor for creep |
| $k_{hs}$ | humidity factor for shrinkage |
| $k_\ell$ | loading age factor for creep and shrinkage |
| $k_s$ | shape factor for creep and shrinkage |
| $k_{td}$ | time dependency factor for creep and shrinkage |
| $L$ | span length (ft) |
| $L_{beam}$ | beam length (ft) |
| $L_{max}$ | maximum design span length for given section for a typical design (ft) |
| $L_{span}$ | span length (ft) |
| $L_{vi}$ | interface length considered to be engaged in shear transfer (inch) |
| $\ell_a$ | available development length at section of interest (inch) |
| $\ell_b$ | bearing length (inch) |
| $\ell_d$ | development length for reinforcement (inch) |
| $\ell_s$ | splice length for reinforcement (inch) |
| $\ell_t$ | transfer length for reinforcement (inch) |

| | |
|---|---|
| $M$ | moment in member (kip-ft) |
| $M_b(x)$ | moment due to weight of barrier at distance $x$ from left support (kip-ft) |
| $M_{b,cr}$ | moment due to weight of barrier at critical section (kip-ft) |
| $M_{b,m}$ | moment due to weight of barrier at midspan (kip-ft) |
| $M_{b,tr}$ | moment due to weight of barrier at transfer length (kip-ft) |
| $M_{cr}$ | cracking moment (kip-inch) |
| $M_{dnc}$ | total unfactored dead load moment acting on the monolithic or noncomposite section (kip-inch) |
| $M_{FI}$ | moment due to Fatigue I load combination (kip-ft) |
| $M_g(x)$ | moment due to self-weight of member at distance $x$ from left support (kip-ft) |
| $M_{g,cr}$ | moment at critical section due to self-weight of member (kip-ft) |
| $M_{g,T,m}$ | moment due to self-weight at midspan using the beam length (kip-ft) |
| $M_{g,T,tr}$ | moment due to self-weight at transfer length using the beam length (kip-ft) |
| $M_{LL}(x)$ | moment due to lane live load at distance $x$ from left support (kip-ft) |
| $M_{LL,cr}$ | moment due to lane live load at critical section including distribution factor (kip-ft) |
| $M_{LL,m}$ | moment due to lane live load at midspan including distribution factor (kip-ft) |
| $M_{LL,tr}$ | moment due to lane live load at transfer length including distribution factor (kip-ft) |
| $M_{LT}(x)$ | moment due to design truck at distance $x$ from left support (kip-ft) |
| $M_{LT,f}(x)$ | moment due to fatigue design truck at distance $x$ from left support (kip-ft) |
| $M_{LT,cr}$ | moment due to design truck at critical section including distribution factor and dynamic load allowance (kip-ft) |
| $M_{LT,f,m}$ | moment due to fatigue design truck at midspan including distribution factor and dynamic load allowance (kip-ft) |
| $M_{LT,m}$ | moment due to design truck at midspan including distribution factor and dynamic load allowance (kip-ft) |
| $M_{LT,tr}$ | moment due to design truck at transfer length including distribution factor and dynamic load allowance (kip-ft) |
| $M_{loc}$ | moment associated with crack localization (kip-ft) |
| $M_n$ | nominal flexural resistance (kip-ft) |

| | |
|---|---|
| $M_n(c, \varepsilon_{cb})$ | total moment in section as a function of neutral axis depth $c$ and bottom fiber strain $\varepsilon_{cb}$ (kips) |
| $M_r$ | factored nominal flexural resistance (kip-ft) |
| $M_u$ | factored flexural demand (kip-ft) |
| $M_{u,m}$ | factored flexural demand at midspan (kip-ft) |
| $M_{sd,m}$ | moment due to barrier weight and future wearing surface at midspan (kip-ft) |
| $M_{sDL,m}$ | moment due to superimposed dead loads at midspan (kip-ft) |
| $M_{s\ell}$ | moment associated with the service stress limit for the reinforcement (kip-ft) |
| $M_{ws}(x)$ | moment due to weight of future wearing surface at distance $x$ from left support (kip-ft) |
| $M_{ws,cr}$ | moment due to weight of future wearing surface at critical section (kip-ft) |
| $M_{ws,m}$ | moment due to weight of future wearing surface at midspan (kip-ft) |
| $M_{ws,tr}$ | moment due to weight of future wearing surface at transfer length (kip-ft) |
| $N$ | axial force in member (kips) |
| $N_b$ | number of beams |
| $N_{lanes}$ | number of lanes |
| $N_u$ | factored axial force demand in member (kips) |
| $n_{strands,tens}$ | number of strands on flexural tension side of member |
| $n_T$ | total number of prestressing strands |
| $P_c$ | permanent net compressive force, normal to the shear plane (kips) |
| $P_{pbt}$ | force in the prestressing strands immediately before transfer (kips) |
| $P_{pe}$ | effective force in the prestressing strands after all losses (kips) |
| $P_{pe2}$ | effective force in prestressing after all prestress losses ignoring elastic gains (kips) |
| $P_n(c, \varepsilon_{cb})$ | total axial force in section as a function of neutral axis depth $c$ and bottom fiber strain $\varepsilon_{cb}$ (kips) |
| $P_r$ | factored splitting resistance of pretensioned anchorage zones (kips) |
| $P_{r,UHPC}$ | splitting resistance of pretensioned anchorage zones provided by UHPC (kips) |
| $P_{pt}$ | force in strands immediately after transfer (kips) |
| $P_{\Delta}$ | force in prestressing strands due to prestress losses occurring prior to deck casting or beam placement (kips) |

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|---|---|
| $Q(y_1)$ | first moment of the area calculated at height $y_1$ from the center of gravity ($inch^3$) |
| $Q_{gc1}$ | first moment of the area at mid-height ($inch^3$) |
| $Q_{gc2}$ | first moment of the area at top of web ($inch^3$) |
| $S$ | beam spacing (ft) |
| $S_c$ | section modulus for the extreme fiber of the composite section where tensile stress is caused by externally applied loads ($inch^3$) |
| $S_{nc}$ | section modulus for the extreme fiber of the monolithic or noncomposite section where tensile stress is caused by externally applied loads ($inch^3$) |
| $s$ | spacing of transverse reinforcement (inch) |
| $s_{max}$ | maximum permitted spacing of transverse reinforcement (inch) |
| $T$ | tension resultant force used in shear design (kips) |
| $T_c$ | tension force in concrete or UHPC in sectional analysis (kips) |
| $T_c(c, \varepsilon_{cb})$ | tensile force in UHPC as a function of neutral axis $c$ and bottom fiber strain $\varepsilon_{cb}$ (kips) |
| $T_{c,As}$ | tension force in concrete or UHPC not present because of presence of reinforcement in sectional analysis (kips) |
| $T_{ps,1}$ | force in Layer 1 of prestressing strands (kips) |
| $T_{ps,2}$ | force in Layer 2 of prestressing strands (kips) |
| $T_{ps,i}(c, \varepsilon_{cb})$ | force in layer $i$ of prestressing strands as a function of neutral axis depth $c$ and bottom fiber strain $\varepsilon_{cb}$ (kips) |
| $T_s$ | tension force in steel in sectional analysis (kips) |
| $t$ | maturity of concrete or UHPC (days) |
| $t_{B/A}$ | distance between tangent extending from deflected shape at Point A and deflected shape at Point B (inch) |
| $t_{C/A}$ | distance between tangent extending from deflected shape at Point A and deflected shape at Point C (inch) |
| $t_{bf}$ | thickness of bottom flange (inch) |
| $t_d$ | age of concrete or UHPC at time of deck placement or installation of beams (days) |
| $t_f$ | age of concrete or UHPC at final time (days) |
| $t_{fws}$ | thickness of future wearing surface (inch) |
| $t_i$ | age of concrete or UHPC at time of load application (days) |

| | |
|---|---|
| $t_{release}$ | age of concrete or UHPC at time of transfer or release of prestressing strands (days) |
| $t_{tf}$ | thickness of top flange (inch) |
| $V_b(x)$ | shear due to weight of barrier at distance $x$ from left support (kips) |
| $V_{b,cr}$ | shear at critical section due to weight of barrier (kips) |
| $V_c$ | nominal shear resistance of the concrete (kips) |
| $V_g$ | shear at section of interest due to self-weight of member (kips) |
| $V_{g,tr}$ | shear at transfer length due to self-weight of member (kips) |
| $V_{g,cr}$ | shear at critical section due to self-weight of member (kips) |
| $V_g(x)$ | shear due to self-weight of member at distance $x$ from left support (kips) |
| $V_{LL}(x)$ | shear due to lane live load at distance $x$ from left support (kips) |
| $V_{LL,cr}$ | shear at critical section due to lane live load including distribution factor (kips) |
| $V_{LT}(x)$ | shear due to design truck at distance $x$ from left support (kips) |
| $V_{LT,cr}$ | shear at critical section due to design truck including distribution factor and dynamic load allowance (kips) |
| $V_p$ | component of prestressing force in the direction of the shear force (kips) |
| $V_s$ | nominal shear resistance provided by transverse reinforcement (kips) |
| $V_u$ | factored shear demand (kips) |
| $V_{UHPC}$ | nominal shear resistance of the UHPC (kips) |
| $V_{ws}(x)$ | shear due to weight of future wearing surface at distance $x$ from left support (kips) |
| $V_{ws,cr}$ | shear at critical section due to weight of future wearing surface (kips) |
| $v_u$ | shear stress due to factored shear force (kips) |
| $W$ | width of bearing plate or pad (inch) |
| $W_R$ | clear roadway width (ft) |
| $w_a$ | unit weight of asphalt wearing surface (kcf) |
| $w_{bps}$ | barrier weight per side (kip/ft) |
| $w_b$ | barrier weight per beam (kip/ft) |
| $w_c$ | unit weight of concrete or UHPC (kcf) |
| $w_g$ | beam self-weight (kip/ft) |
| $w_{lane}$ | lane live load (kip/ft) |

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| $w_{ws}$ | future wearing surface weight (kip/ft) |
| $X_c(c, \varepsilon_{cb})$ | distance from top of section to centroid of tension force in UHPC as a function of neutral axis $c$ and bottom fiber strain $\varepsilon_{cb}$ (inch) |
| $X_u$ | clear distance between adjacent walls (inch) |
| $x$ | distance from left support (ft) |
| $x_{cr}$ | critical section for shear (ft) |
| $y_b$ | distance between bottom and center of gravity of section (inch) |
| $y_{bcb}$ | distance from bottom of section to centroid of tension tie (inch) |
| $y_{Cc}$ | distance between neutral axis and centroid of $C_C$ (inch) |
| $y_p$ | distance between bottom of section and centroid of prestressing strands (inch) |
| $y_t$ | distance between top and center of gravity of section (inch) |
| $y_{Tc}$ | distance between neutral axis and centroid of $T_C$ (inch) |
| $y_{Ts}$ | distance between neutral axis and centroid of $T_s$ and $T_{C,As}$ (inch) |
| $y_1$ | distance from center of gravity of section and height of interest for shear stress calculation (inch) |
| $Z_c(c, \varepsilon_{cb})$ | distance from top of section to centroid of compression force in UHPC as a function of neutral axis $c$ and bottom fiber strain $\varepsilon_{cb}$ (inch) |
| $z$ | distance measured from the top compression face of the section (inch) |
| $\alpha_u$ | reduction factor to account for the non-linearity of the UHPC compressive stress-strain response |
| $\beta$ | factor indicating ability of diagonally cracked conventional concrete to transmit tension and shear |
| $\gamma$ | load factor |
| $\gamma_1$ | flexural cracking variability factor |
| $\gamma_2$ | prestress variability factor |
| $\gamma_3$ | ratio of specified minimum yield strength to ultimate tensile strength |
| $\gamma_{LL}$ | live load factor |
| $\gamma_u$ | reduction factor to account for the variability in the UHPC tensile stress parameters |
| $\sigma_{ft,cr,test}$ | standard deviation of measured effective cracking strength for UHPC (ksi) |
| $\sigma_{ft,loc,test}$ | standard deviation of measured crack localization strength for UHPC (ksi) |

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| $\Delta$ | deflection (inch) |
| $\Delta_b$ | deflection due to weight of barrier (inch) |
| $\Delta_C$ | deflection at Point C (inch) |
| $\Delta_g$ | deflection due to self-weight of beam (inch) |
| $\Delta_L$ | maximum live load deflection used for optional live load deflection check (inch) |
| $\Delta_{LL}$ | live load deflection; deflection due to lane load (inch) |
| $\Delta_{LT}$ | deflection due to design truck (inch) |
| $\Delta_p$ | camber due to prestressing force (inch) |
| $\Delta_{sus,LT}$ | total long-term deflection due to prestressing force, self-weight of beam, weight of barrier, and weight of future wearing surface (inch) |
| $\Delta_{t,LT}$ | total long-term deflection (inch) |
| $\Delta_{transfer}$ | total deflection immediately after transfer (inch) |
| $\Delta_{ws}$ | deflection due to weight of future wearing surface (inch) |
| $(\Delta F)_{TH}$ | constant amplitude fatigue threshold (ksi) |
| $\Delta f$ | live load stress range due to the passage of the fatigue load (ksi) |
| $\Delta f_{c,L}$ | change in concrete stress in top face due to live loads (ksi) |
| $\Delta f_{cs,L}$ | change in concrete stress at the centroid of the prestressing strands due to live loads (ksi) |
| $\Delta f_{cs,L,o}$ | change in concrete stress at the centroid of the outermost layer of prestressing strands due to live loads (ksi) |
| $\Delta f_{c,LT,df}$ | change in concrete stress in top face due to long term effects between beam placement and final time (ksi) |
| $\Delta f_{c,LT,id}$ | change in concrete stress in top face due to long term effects between transfer and beam placement (ksi) |
| $\Delta f_{c,sDL}$ | change in concrete stress in top face due to superimposed dead loads (ksi) |
| $\Delta f_{cs,sDL}$ | change in concrete stress at the centroid of the prestressing strands due to superimposed dead loads (ksi) |
| $\Delta f_{cs,sDL,o}$ | change in concrete stress at the centroid of the outermost layer of prestressing strands due to superimposed dead loads (ksi) |
| $\Delta f_{cd}$ | change in concrete stress at centroid of prestressing strands due to long-term losses prior to deck placement and superimposed dead loads (ksi) |

$\Delta f_{p,L}$ change in prestress (prestress gain) due to live load (ksi)

$\Delta f_{p,L,o}$ change in prestress (prestress gain) at the centroid of the outermost layer of prestressing strands due to live load (ksi)

$\Delta f_{p,LT,df}$ long-term prestress losses after deck placement or beam installation (ksi)

$\Delta f_{p,LT,id}$ long-term prestress losses prior to deck placement or beam installation (ksi)

$\Delta f_{p,sDL}$ change in prestress (prestress gain) due to superimposed dead load (ksi)

$\Delta f_{p,sDL,o}$ change in prestress (prestress gain) at the centroid of the outermost layer of prestressing strands due to superimposed dead load (ksi)

$\Delta f_{pCD}$ prestress losses due to creep after deck placement or beam installation (ksi)

$\Delta f_{pCR}$ prestress losses due to creep prior to deck placement or beam installation (ksi)

$\Delta f_{pES}$ prestress losses due to elastic shortening (ksi)

$\Delta f_{pES,o}$ prestress losses due to elastic shortening in the outermost layer of prestressing strands (ksi)

$\Delta f_{pLT}$ long-term prestress losses due to time dependent effects (ksi)

$\Delta f_{pR1}$ prestress losses due to strand relaxation prior to deck placement or beam installation (ksi)

$\Delta f_{pR2}$ prestress losses due to strand relaxation after deck placement or beam installation (ksi)

$\Delta f_{pSD}$ prestress losses due to shrinkage after deck placement or beam installation (ksi)

$\Delta f_{pSR}$ prestress losses due to shrinkage prior to deck placement or beam installation (ksi)

$\Delta f_{pSS}$ prestress losses due to shrinkage of deck concrete after deck placement (ksi)

$\Delta f_{pT}$ total prestress losses (ksi)

$\Delta f_{t,L}$ change in concrete stress in bottom face due to live loads (ksi)

$\Delta f_{t,LT,df}$ change in concrete stress in bottom face due to long term effects between beam placement and final time (ksi)

$\Delta f_{t,LT,id}$ change in concrete stress in bottom face due to long term effects between transfer and beam placement (ksi)

$\Delta f_{t,sDL}$ change in concrete stress in bottom face due to superimposed dead loads (ksi)

$\Delta P_{p,LT,df}$ change in force in prestressing strands due to long-term prestress losses after deck placement or beam installation (kips)

| | |
|---|---|
| $\Delta P_{p,LT,id}$ | change in force in prestressing strands due to long-term prestress losses before deck placement or beam installation (kips) |
| $\varepsilon$ | general strain (inch/inch) |
| $\varepsilon_{bdf}$ | shrinkage strain from time of deck placement or beam installation to final time (inch/inch) |
| $\varepsilon_{bid}$ | shrinkage strain from end of curing to time of deck placement or beam installation (inch/inch) |
| $\varepsilon_{bif}$ | shrinkage strain from end of curing to final time (inch/inch) |
| $\varepsilon_{c}$ | compressive strain in extreme compression fiber of the UHPC section (inch/inch) |
| $\varepsilon_{cb}(c)$ | bottom fiber strain as a function of neutral axis depth $c$ (inch/inch) |
| $\varepsilon_{cp}$ | elastic compressive strain limit (inch/inch) |
| $\varepsilon_{cu}$ | ultimate compressive strain of conventional concrete or UHPC for use in design (inch/inch) |
| $\varepsilon_{pe}$ | effective strain in prestressed reinforcement (inch/inch) |
| $\varepsilon_{pf}$ | strain in the concrete at the level of the centroid of the prestressing strands (inch/inch) |
| $\varepsilon_{pf,1}$ | strain in the concrete at the level of the centroid of the prestressing strands in Layer 1 (inch/inch) |
| $\varepsilon_{pf,2}$ | strain in the concrete at the level of the centroid of the prestressing strands in Layer 2 (inch/inch) |
| $\varepsilon_{ps}$ | strain in prestressed reinforcement (inch/inch) |
| $\varepsilon_{ps,i}(c, \varepsilon_{cb})$ | total strain in layer $i$ of prestressing strands as a function of neutral axis depth $c$ and bottom fiber strain $\varepsilon_{cb}$ (inch/inch) |
| $\varepsilon_{ps,t}$ | strain in outermost layer of prestressing strands (inch/inch) |
| $\varepsilon_{pu}$ | ultimate strain of prestressed reinforcement, typically referred to as total elongation (inch/inch) |
| $\varepsilon_{py}$ | strain in prestressed reinforcement at yield (inch/inch) |
| $\varepsilon_{p,sl}$ | strain in prestressed reinforcement at service limit (inch/inch) |
| $\varepsilon_{s}$ | net tensile strain in the extreme tension steel (inch/inch) |
| $\varepsilon_{sh}$ | shrinkage strain (inch/inch) |
| $\varepsilon_{s\ell}$ | strain in reinforcement at service limit (inch/inch) |

$\varepsilon_{sy}$ strain in reinforcement at yield (inch/inch)

$\varepsilon_{su}$ ultimate strain of non-prestressed reinforcement, typically referred to as total elongation (inch/inch)

$\varepsilon_t$ net tensile strain in extreme tension fiber of the UHPC section (inch/inch)

$\varepsilon_{t,cr}$ elastic tensile strain limit of UHPC corresponding to a tensile stress of $\gamma f_{t,cr}$ (inch/inch)

$\varepsilon_{t,loc}$ crack localization strain of UHPC for use in design (inch/inch)

$\varepsilon_y$ yield strain for the reinforcement (inch/inch)

$\theta$ angle of inclination of diagonal compressive stresses in shear design process (degrees); rotation in elastic beam theory (radians)

$\theta_A$ rotation of beam at Point A (degrees)

$\lambda$ concrete density modification factor for conventional concrete

$\lambda_{duct}$ shear strength reduction factor accounting for the presence of a post-tensioning duct

$\mu$ curvature ductility for section for flexural analysis; friction factor for interface shear transfer

$\mu_\ell$ curvature ductility limit

$\rho_p$ prestressing ratio, $A_{pT}$ / ($b_f$ $d_p$)

$\rho_{v,\alpha}$ ratio of area of transverse shear reinforcement to gross UHCP areas of a horizontal section

$\xi$ multiplier for transfer length of prestressing strand

$\phi$ resistance factor

$\phi_c$ resistance factor for axial resistance

$\phi_f$ resistance factor for flexure

$\phi_v$ resistance factor for shear

$\phi_w$ hollow column reduction factor as defined in Article 5.6.4.7.2c of AASHTO LRFD BDS

$\tau$ shear stress (ksi)

$\tau_{max}(y_1)$ shear stress at height $y_1$ from the center of gravity (ksi)

$\Psi(t, t_i)$ creep coefficient at time $t$ for loading applied at $t_i$

$\psi$ curvature (rad/inch)

| | |
|---|---|
| $\psi_c(c, \varepsilon_{cb})$ | curvature in section as a function of neutral axis depth $c$ and bottom fiber strain $\varepsilon_{cb}$ (rad/inch) |
| $\psi_{loc}$ | curvature associated with crack localization (rad/inch) |
| $\psi_n$ | curvature associated with nominal flexural resistance (rad/inch) |
| $\psi_{s\ell}$ | curvature associated with service stress limit for reinforcement (rad/inch) |

# CHAPTER 1. INTRODUCTION

## INTRODUCTION

The general design of ultra-high performance concrete (UHPC) structural elements using the AASHTO *Guide Specification for Structural Design with UHPC*, referred to hereafter as "UHPC Structural Design Guide", will be covered in this manual and in the workshop materials. Similarities and differences between the design of UHPC and conventional concrete (CC) structural elements will be highlighted for each typical design step. Additional explanations are provided for components of structural design that are significantly different or unique with UHPC. This workshop manual expands on the explanations and examples provided in the Federal Highway Administration (FHWA) report titled *Structural Design with Ultra-High Performance Concrete*, referred to hereafter by the report number "FHWA-HRT-23-077".

## MOTIVATION FOR WORKSHOP

As UHPC was first becoming available in the U.S. in the early 2000's, owners and experts in the bridge sector initially focused on opportunities to implement UHPC in prefabricated components such as bridge girders. An early example is the first bridge built in the U.S. incorporating UHPC structural girders, shown in the left photograph of Figure 1 soon after it opened to traffic in 2006. Although the use of UHPC in a prefabricated, primary structural component showcased its potential, its implementation posed challenges due to the lack of formal design and construction guidance and the limited familiarity of the broader community with this class of concrete. The next phase of UHPC implementation involved deck-level connections and other connections between prefabricated elements; an example of a field-cast connection is shown in the middle photograph in Figure 1. Using UHPC for field-cast connections employs its superior durability and offers construction advantages by considerably shortening the development length of reinforcing bars and reducing reinforcement congestion. In addition, connections made with UHPC typically entail a smaller volume of material compared to traditional connections, which helps alleviate cost concerns. Recently, a third phase of UHPC implementation has emerged, specifically focused on bridge preservation and repair activities. These initiatives primarily targeted long-term deterioration-related challenges in bridge infrastructure by utilizing the enhanced tensile performance and durability of UHPC. Some of the prominent preservation and repair applications are deck overlays (shown in the right photograph of Figure 1), link slabs, and beam end repairs.


Source: FHWA.

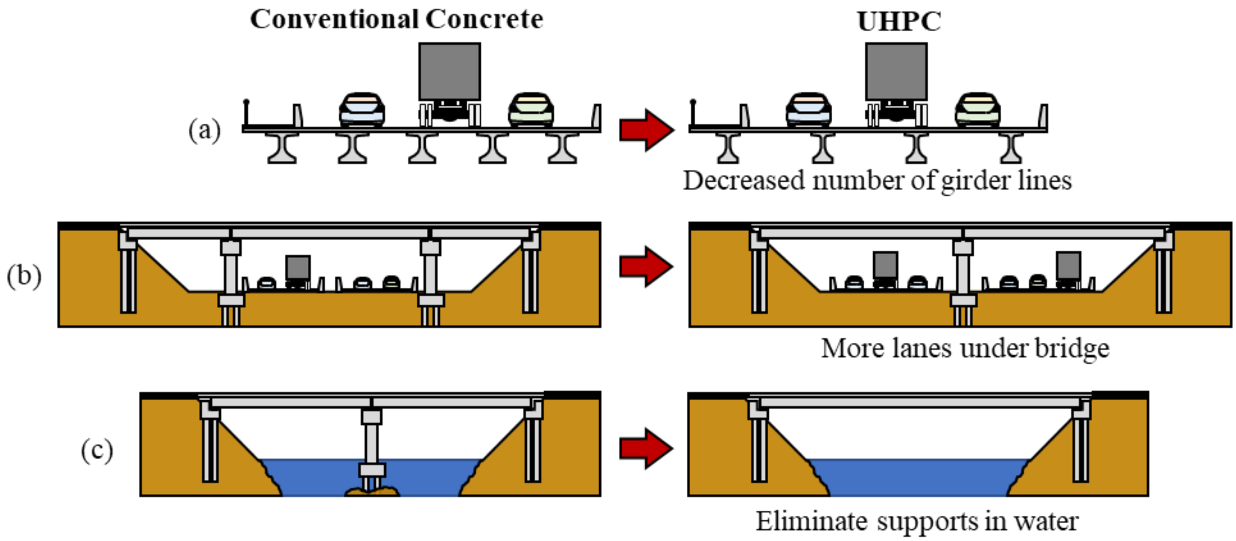
**Figure 1. Illustration. Select History of UHPC Deployment in U.S.**

As the U.S. bridge community became more familiar with the capabilities and promising applications of UHPC, a growing interest emerged to facilitate its adoption by addressing the hurdles that either formally or informally hindered its use. Both the public and private sectors directed efforts towards eliminating the primary hurdle impeding UHPC growth, which was the absence of formal structural design guidance. The release of the UHPC Structural Design Guide in 2024 marked the resolution of this issue. The UHPC Structural Design Guide enables owners and engineers to leverage the enhanced mechanical properties of UHPC, facilitating the design and construction of structural elements that offer numerous benefits over those made with conventional concrete. Some of general advantages include the following.

- **Enhanced durability**: UHPC is significantly more durable than CC. The durability of CC is primarily affected by the permeation of liquids, often bearing dissolved salts, into the concrete. With a permeability typically one to two orders of magnitude less than CC, UHPC is a considerably more durable material that is substantially more resistant to corrosion of embedded steel, scaling, and freeze-thaw effects.
- **More efficient section shapes**: The enhanced mechanical properties of UHPC enables the realization of more efficient and lighter section designs. FHWA-HRT-23-077 demonstrated that a 27 percent reduction in cross-sectional area of a CC bulb-tee section can be achieved with UHPC, resulting in an optimized girder design that can span longer distances at similar structural depths. Similarly, as demonstrated later in this workshop manual, UHPC box beams can be made 27 percent lighter than a similar CC box beam without a cast-in-place (CIP) deck and 43 percent lighter than a similar CC box beam design with a CIP deck.
- **Possible higher strength**: The enhanced material properties of UHPC and its ability to sustain larger prestressing forces can result in higher strength members compared to similar depth CC members. For example, the design example showcased in FHWA-HRT-23-077 demonstrates that a 54-inch-deep UHPC bulb-tee section with 1.5-inch haunch and 8.5-inch CIP CC deck can span 25 percent longer than a similar depth CC bulb-tee. Additionally, it has been found that piles made with UHPC can sustain more driving energy compared to CC piles without experiencing any distress at the pile ends[1].

These advantages can also lead to changes to the overall design of a bridge, thereby enabling asymmetric benefits pertaining to cost, bridge operation, environmental impact, and other topics. Some possible benefits are highlighted in Figure 2.

[1] This study refers to unpublished work by Florida Department of Transportation (FDOT) where 30-inch UHPC H-piles were either driven or tested at the FDOT Structures Research Center in Tallahassee.



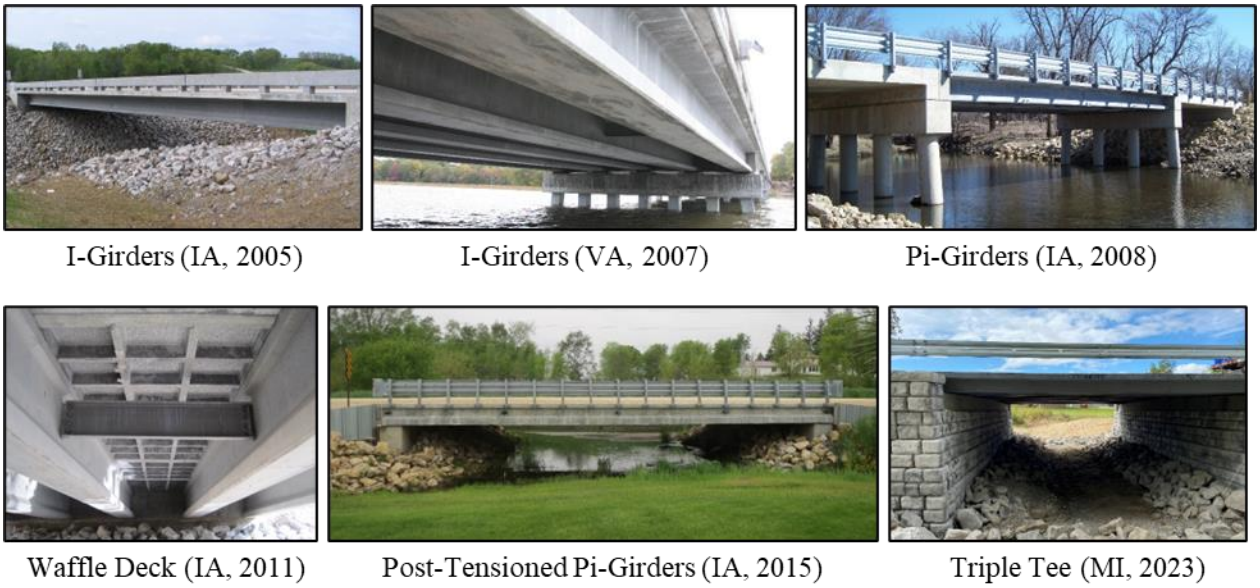

Source: FHWA.

**Figure 2. Illustration. Possible benefits to using UHPC structural elements.**

These benefits can include the following.

- **Decreased superstructure weight**: UHPC structural elements will generally be lighter than similar CC elements. The use of UHPC structural elements can also decrease the number of girder lines, eliminate the need for CIP decks (for adjacent members), and reduce the required section depths for a given span length, which will all contribute to a significant decrease in the superstructure weight. This reduction in the superstructure weight reduces the demand on substructure elements, potentially leading to cost savings or facilitating the reuse of a substructure that may not have been possible otherwise.
- **Decreased superstructure depths**: The use of UHPC in structural elements can lead to shallower sections than CC sections for similar span lengths. The UHPC box beam example presented in this workshop manual was 6 inches shallower than the CC box beam required for the same 95-foot span. The opportunity to use a shallower section can avert the need to change bridge approach grading during a superstructure replacement, can allow for an increase in the overhead clearance of the road passing under the bridge, or can increase the hydraulic capacity of a water crossing.
- **Increased span lengths**: UHPC structural elements can be designed to span longer than CC elements with the same depth. The 54-inch deep UHPC bulb-tee with CIP deck discussed in FHWA-HRT-23-077 could be designed for a 160-foot span compared to a 120-foot maximum span length for the same depth CC member. The ability to design longer spans with the same section depth could allow for the benefits shown in Figure 2 (b) and (c) in terms of possibly being able to eliminate interior supports. These benefits can decrease the overall cost of a bridge, allow for design improvements not otherwise possible (e.g., increasing number of lanes under the bridge), and improve the resilience of the bridge (e.g., by eliminating supports in water susceptible to scour concerns).

UHPC primary structural elements have been used in limited applications in the U.S. for the past few decades; some of these applications are shown in Figure 3. There have also been many successful projects completed internationally (e.g., Malaysia, France).




Source: FHWA.

**Figure 3. Photographs. Examples of in-service bridges with UHPC structural elements.**

To date, implementation of UHPC in primary structural elements of bridges has tended toward demonstration projects in the U.S. The goal of this workshop is to help owners and engineers become more comfortable with the design of UHPC structural elements, facilitating the movement toward more widespread implementation of the technology.

## WORKSHOP LEARNING OBJECTIVES

By the end of this workshop, participants should be able to:

- Identify when using UHPC will be advantageous,
- Describe the differences between CC and UHPC related to the design of structural elements, and
- Analyze and design UHPC structural elements using the UHPC Structural Design Guide.

Additional supporting learning objectives are provided for each module of this workshop.

The workshop will be limited to about 7 to 8 hours of in-person content and will contain “need-to-know” information and emphasize “what’s in it for me” and application. The objective of this workshop manual is to provide greater depth on the “need-to-know” information as well as details on “nice-to-know” and “reference” information.

## MODULES AND MODULE LEARNING OBJECTIVES

The "Structural Design with UHPC" workshop has an introductory module, eight technical modules, and a closeout module:

- Module 0: Welcome and Introduction
- Module 1: UHPC Basics and Applications
- Module 2: Material Properties and Idealizations
- Module 3: Strain-Based Design Principals
- Module 4: Design Processes and Preliminary Design
- Module 5: Design for Service and Fatigue Limit States
- Module 6: Design for Flexural Strength
- Module 7: Design for Shear
- Module 8: Miscellaneous Topics
    - Pretensioned Anchorage Zones
    - Camber and Deflections
- Module 9: Closeout

The specific learning objectives for each technical module are as follows.

### Module 1: UHPC Basics and Applications – Learning Objectives

- Define UHPC.
- List key characteristics for UHPC.
- Describe different sources of UHPC.
- Discuss different applications for UHPC.

### Module 2: Material Properties and Idealizations – Learning Objectives

- State the minimum characteristics for UHPC materials.
- Illustrate UHPC stress-strain idealizations used for compression and tension.
- Identify design values from direct tension test results.

### Module 3: Strain-Based Design Principals – Learning Objectives

- Discuss why strain-based design approaches are needed for UHPC members.
- Explain the approach for flexural analysis of UHPC members.
- Calculate the neutral axis depth and associated values for a given strain.
- Describe the procedure for developing a moment-curvature diagram.

### Module 4: Design Processes and Preliminary Design – Learning Objectives

- List the basic steps for design of a UHPC member.
- Identify some benefits of using UHPC for structural members.
- Describe ways a conventional concrete section can be modified to take advantage of UHPC.

- Compare how to calculate demand for UHPC and CC members.

**Module 5: Design for Service and Fatigue Limit States – Learning Objectives**

- Compare estimation procedures for time-dependent effects, prestress losses, and stress calculations between CC and UHPC members.
- Explain the influence of the correction factors for modulus of elasticity, shrinkage, and creep on prestress loss and stress calculations.
- Describe how to calculate principal tensile stresses in web.
- Apply UHPC Structural Design Guide stress limits (service and fatigue) to typical prestressed concrete member design.

**Module 6: Design for Flexural Strength – Learning Objectives**

- Explain basic design objective for strength limit state.
- Describe the major differences between flexural strength design for UHPC compared to CC.
- Define three failure criteria and determine which controls for a given section.
- Calculate flexural resistance factors based on curvature ductility.
- Use the basic design procedure for flexural strength.

**Module 7: Design for Shear – Learning Objectives**

- Explain the difference between B-Regions and D-Regions.
- Describe the primary differences between designing for shear for CC and UHPC members.
- Use the General Approach (iterative) and Simplified Approach (design tables) for finding $\theta$ and $f_{v,\alpha}$.
- Calculate the effective shear depth.
- Apply the longitudinal reinforcement requirement for UHPC members.

**Module 8: Miscellaneous Topics – Learning Objectives**

- Explain the benefits of UHPC in pretensioned anchorage zones.
- Detail the pretensioned anchorage zone of UHPC members.
- Describe the differences for calculating deflections for UHPC and CC members.
- Calculate long-term deflections using the effective modulus method.

## WORKSHOP EXAMPLES

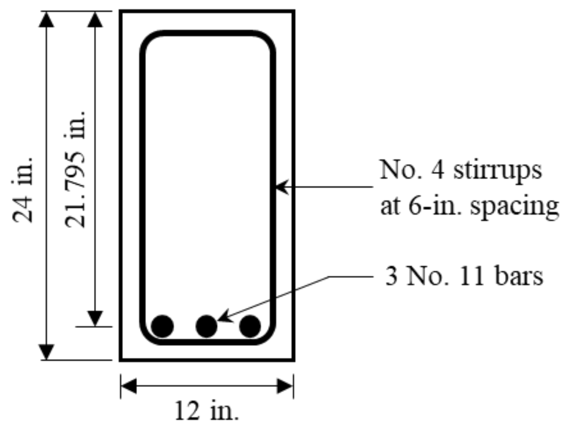
Three different examples will be referenced throughout the workshop and manual.

- Storyline Example #1: UHPC Rectangular Section with Conventional Reinforcement
- Storyline Example #2: UHPC Box Beam Bridge Design
- Storyline Example #3: UHPC H-Pile

Additional details on these examples are provided in the following sections.

**Storyline Example #1: Rectangular Section with Conventional Reinforcement**

Storyline Example #1 is the same as the example provided in FHWA-HRT-23-077 *Structural Design with Ultra-High Performance Concrete* (Appendix B). The cross section for Storyline Example #1 is shown in Figure 4.




Source: FHWA.

**Figure 4. Illustration. Cross sections for Storyline Example #1.**

The UHPC material properties for this storyline example are as follows:

- Compressive strength for use in design and analyses: $f'_c$ = 22.0 ksi
- UHPC unit weight: $w_c$ = 0.155 kcf
- Reduction factor for compression: $\alpha_u$ = 0.85
- Ultimate compression strain: $\varepsilon_{cu}$ = 0.0035
- Effective cracking strength: $f_{t,cr}$ = 1.0 ksi
- Crack localization strength: $f_{t,loc}$ = 1.0 ksi
- Crack localization strain: $\varepsilon_{t,loc}$ = 0.003
- Reduction factor for tension: $\gamma_u$ = 1.0

The material properties for the conventional steel reinforcement (Grade 60) are as follows:

- Modulus of elasticity: $E_s$ = 29,000 ksi
- Yield strength: $f_{sy}$ = 60 ksi
- Yield strain: $\varepsilon_{sy}$ = 0.00207
- Service stress limit: $f_{sl} = 0.80 f_{sy}$ = 48 ksi
- Strain at service stress limit: $\varepsilon_{s,s\ell} = f_{s\ell} / E_s$ = 0.00166
- Ultimate strain: $\varepsilon_{su}$ = 0.09
- No. 11 bar diameter: $d_{b,\#11}$ = 1.41 inch
- No. 11 bar area: $A_{s,bar,\#11}$ = 1.56 inch$^2$
- No. 4 bar area: $A_{s,bar,\#4}$ = 0.20 inch$^2$

The reinforcement provided for this example is as follows.

- Longitudinal reinforcement: (3) No. 11 bars, $A_s$ = (3)(1.56 inch$^2$) = 4.68 inch$^2$
- Transverse reinforcement: No. 4 bars spaces at 6 inches, $A_v$ = 2(0.20 inch$^2$) = 0.40 inch$^2$

**Storyline Example #2: UHPC Box Beam Bridge Design**

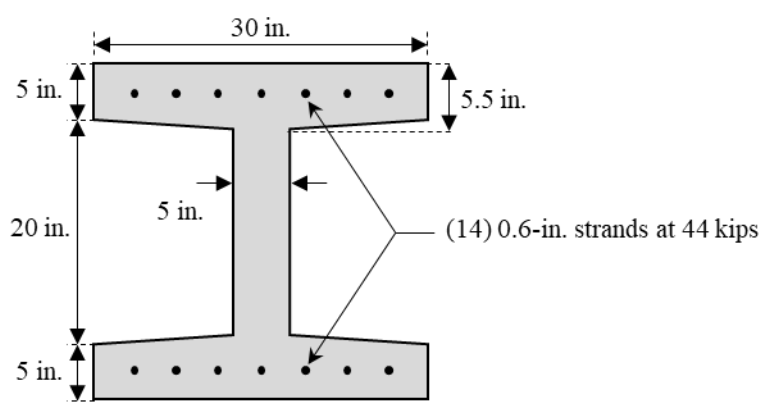
The UHPC box beam design example that will be engaged during the workshop is presented in Chapter 3.

**Storyline Example #3: UHPC H-Pile**

One of the early applications for UHPC structural elements is piles. UHPC piles may be beneficial for several different reasons as follows.

- UHPC piles can be used as a replacement for standard steel H-piles. These UHPC piles will be designed with a similar capacity, weight, and drivability to their equivalent steel H-pile.
- UHPC piles can be designed to have a higher performance than the currently available concrete or steel piles. These UHPC piles would be designed for higher stresses during driving (to accelerate the installation process) or higher moment capacity (to decrease the number of piles required).
- UHPC piles can be designed to be a replacement for square prestressed piles with high-strength stainless steel (HSSS) or carbon fiber reinforced polymer (CFRP) strands. At least one state requires the use of corrosion-resistant strands for certain applications. UHPC piles can provide improved capacity, durability, and drivability to these options.

One of the early section shapes being developed is a UHPC H-pile being investigated by the Florida Department of Transportation (FDOT) as a replacement for their current 18-inch, 24-inch, and 30-inch square prestressed piles. The approximate details for the 30-inch UHPC pile developed by FDOT are shown in Figure 5.




Source: FHWA.

**Figure 5. Illustration. Approximate details for 30-inch UHPC H-pile developed by FDOT.**

The UHPC material properties for this storyline example are as follows:

- Compressive strength at time of transfer: $f'_{ci}$ = 14.0 ksi
- Compressive strength for use in design and analyses: $f'_c$ = 17.5 ksi
- UHPC unit weight: $w_c$ = 0.155 kcf
- Reduction factor for compression: $\alpha_u$ = 0.85
- Ultimate compression strain: $\varepsilon_{cu}$ = 0.0035
- Effective cracking strength: $f_{t,cr}$ = 1.0 ksi
- Crack localization strength: $f_{t,loc}$ = 1.0 ksi
- Crack localization strain: $\varepsilon_{t,loc}$ = 0.005
- Reduction factor for tension: $\gamma_u$ = 1.0

The material properties for the prestressing strands are as follows:

- Low-relaxation
- Modulus of elasticity: $E_p$ = 28,500 ksi
- Ultimate strength: $f_{pu}$ = 270 ksi
- Yield strength: $f_{py}$ = 243 ksi
- Specified rupture strain for strands: $\varepsilon_{pu}$ = 0.035
- Diameter of prestressing strands: $D_p$ = 0.6 inch
- Area of one strand: $A_{p,0.7in}$ = 0.217 inch$^2$

This storyline example will be used to help explain concepts related to axially-loaded UHPC structural elements.

## PRIMARY WORKSHOP REFERENCES

The primary references related to the structural design of UHPC members are as follows.

- American Association of State Highway and Transportation Officials. 2020. AASHTO *LRFD Bridge Design Specifications*, 9$^{th}$ Edition. 2020. Washington, D.C.: American Association of State Highway and Transportation Officials. Referred to hereafter as "AASHTO LRFD BDS".
- American Association of State Highway and Transportation Officials. 2024. *Guide Specifications for Structural Design with Ultra-High Performance Concrete*. Washington, DC: American Association of State Highway and Transportation Officials.
- Graybeal, B. and Helou, R. 2023. *Structural Design with Ultra-High Performance Concrete*. Report No. FHWA-HRT-23-077. McLean, VA: Federal Highway Administration. Office of Infrastructure Research and Development.

Additional references used in this workshop and referenced in this manual are summarized in Chapter 5.

## LIMITATIONS ON USE OF THE UHPC STRUCTURAL DESIGN GUIDE

The UHPC Structural Design Guide (Article 1.4) states that the provisions shall not apply to:

- The non-UHPC portion of composite structural members, and
- The design of plastic hinge regions of components that are part of the earthquake resisting system in Seismic Zones 2, 3, or 4 as defined within AASHTO LRFD BDS.

The UHPC Structural Design Guide was developed based on UHPC material tests and structural tests of conventionally reinforced, unreinforced, and pretensioned members. For this reason, an additional limitation is highlighted in the UHPC Structural Design Guide:

> *The provisions in this Guide Specification do not address the provisions for specific structural components and types discussed in AASHTO LRFD Article 5.12.*

These components include:

- Deck slabs (AASHTO LRFD BDS Article 5.12.1)
- Slab superstructures (AASHTO LRFD BDS Article 5.12.2)
- Beams and girders (AASHTO LRFD BDS Article 5.12.3), including thickness limits for parts of precast beams, simple spans made continuous, and spliced precast girders
- Diaphragms (AASHTO LRFD BDS Article 5.12.4)
- Segmental concrete bridges (AASHTO LRFD BDS Article 5.12.5)
- Arches (AASHTO LRFD BDS Article 5.12.6)
- Culverts (AASHTO LRFD BDS Article 5.12.7)
- Footings (AASHTO LRFD BDS Article 5.12.8)
- Concrete piles (AASHTO LRFD BDS Article 5.12.9)

The UHPC Structural Design Guide may still be applicable to certain aspects of the design of these elements, particularly those related to UHPC behavior. However, the Guide does not provide comprehensive guidance in this regard. This provision is provided to clarify that "… *these items are not specifically addressed in these Specifications, and the guidance provided in AASHTO LRFD [BDS] Article 5.12 may not necessarily apply to UHPC*."

Post-tensioned UHPC members were not investigated at the time of the initial publication of the UHPC Structural Design Guide. Because of this, an additional note is provided.

> *The provisions in this Guide Specification were not developed to address the special considerations and detailing inherent in post-tensioned structures.*

# CHAPTER 2. BASIC DESIGN REQUIREMENTS FOR UHPC MEMBERS

The design of UHPC members has many similarities with the design of conventional concrete members. A comparison between the design steps for conventional concrete versus UHPC structural members is provided in this chapter. Additional details are provided for UHPC design components that differ from conventional concrete members.

## CALCULATING FACTORED LOADS AND DEMAND

The design components and specification references related to loads and demand are summarized in Table 1.

**Table 1. Design components related to loads and demand for conventional concrete and UHPC sections.**

| Description | Conventional Concrete | AASHTO LRFD BDS Reference | UHPC | UHPC Structural Design Guide Reference |
|---|---|---|---|---|
| Loads and Load Combinations | Load factors provided in table. | Table 3.5.1-1 | Same as for conventional concrete. | Article 1.3 |
| Material Density | $w_c$ = 0.145 kcf | Article 5.4.2.4 | $w_c$ = 0.155 kcf | Article 4.2.2 |
| Service III Live Load Factor | $\gamma_{LL}$ varies based on prestress loss method and inclusion of elastic gains. | Table 3.4.1-4 | $\gamma_{LL}$ = 1.0 | Article 9.1.1 |
| Distribution Factors (Methods of Analysis) | Method explained in referenced article. | Article 4.6.2.2 | Same as for conventional concrete. | n/a |

Loads and load combinations are the same for structural design with UHPC as with conventional concrete with two exceptions:

- **Material density (for calculating self-weight)**: UHPC has a higher density than conventional concrete, 0.155 kcf for UHPC compared to 0.145 kcf commonly found in conventional concrete. An additional 0.005 kcf is often added to this value to include the weight of the reinforcement, which would result in a total density for reinforced UHPC of 0.160 kcf.
- **Service III Load Combination**: UHPC Structural Design Guide Article 1.9.1.1 specifies that the load factor for live load ($\gamma_{LL}$) not be less than 1.0 (for all designs), while AASHTO LRFD BDS Table 3.4.1-4 allows for 0.8 to be used for prestressed components where elastic gains are not included in the design.

Distribution factors can be calculated using the same approach as provided in AASHTO LRFD BDS Article 4.6.2.2.

## MATERIAL PROPERTIES

### Concrete Material Property Definitions

Some of the key concrete material property definitions for conventional concrete and UHPC are summarized in Table 2.

**Table 2. Material property definitions and models for conventional concrete and UHPC.**

| Description | Conventional Concrete | AASHTO LRFD BDS Reference | UHPC | UHPC Structural Design Guide Reference |
|---|---|---|---|---|
| Minimum material properties | Only compressive strength specified. Minimum $f'_c$ of 4 ksi for prestressed concrete and decks. Limits $f'_c$ to 10.0 ksi only when allowed by specific articles. | Article 5.4.2.1 | $f'_c \geq 17.5$ ksi<br>$f_{t,cr} \geq 0.75$ ksi<br>$f_{t,loc} \geq f_{t,cr}$<br>$\varepsilon_{t,loc} \geq 0.0025$ | Article 1.1 |
| Compression behavior | Not specified | n/a | Elastic-plastic response specified | Article 4.2.4 |
| Modulus of rupture | $0.24\lambda f'^{0.5}_c$; may be measured using AASHTO T 97 | Article 5.4.2.6 | Not specified | n/a |
| Tension strength | May be measured by ASTM C900 (pullout strength) or AASHTO T 198 (split cylinder) | Article 5.4.2.7 | Measure using AASHTO T 397 (direct tension) | Article 4.2.5 |
| Tension behavior | Not specified | n/a | Elastic-plastic or bilinear stress-strain relationships specified based on the $f_{t,cr}$, $f_{t,loc}$, and $\varepsilon_{t,loc}$. | Article 4.2.5 |
| Coefficient of thermal expansion | $6.0 \times 10^{-6}$ /°F for normal weight concrete and $5.0 \times 10^{-6}$ /°F for lightweight concrete | Article 5.4.2.2 | $7.0 \times 10^{-6}$ /°F | Article 4.2.7 |
| Modulus of elasticity | $2{,}500 f'^{0.33}_c$ | Eqn. C5.4.2.4-1 | $2{,}500 K_1 f'^{0.33}_c$ | Eqn. 4.2.3-1 |
| Poisson's ratio | 0.2 | Article 5.4.2.5 | 0.15 | Article 4.2.6 |

There are several differences between the material definitions of conventional concrete and UHPC.

Typically, the only required property for conventional concrete is the compressive strength, at transfer ($f'_{ci}$), if applicable, and service ($f'_c$), with a minimum compressive strength of 4 ksi. UHPC has a requirement for compressive strength (measured using ASTM C1856) and the tensile response (measured using AASHTO T 397) with the following values:

- Compressive strength: $f'_c \geq 17.5$ ksi

- Effective cracking strength: $f_{t,cr} \geq 0.75$ ksi
- Crack localization strength: $f_{t,loc} \geq f_{t,cr}$
- Crack localization strain: $\varepsilon_{t,loc} \geq 0.0025$

Additional limits on design UHPC strengths at time of prestressing for pretensioned members and time of initial loading for non-prestressed members are specified in UHPC Structural Design Guide Article 9.1.2.

- Compressive strength at time of prestressing/loading: $f'_{ci} \geq 14.0$ ksi
- Effective cracking strength at time of prestressing/loading: $f_{t,cri} \leq 0.75 f_{t,cr}$ when $f'_{ci} \leq 0.90 f'_{c}$ (unless determined by physical tests, as approved by owner)

AASHTO LRFD BDS does not specify stress-strain relationships for conventional concrete in compression or tension, as these are not required for the design process of conventional concrete members. UHPC Structural Design Guide specifies idealized stress-strain responses for compression and tension that should be used in the design of UHPC members, which are required in the design process.

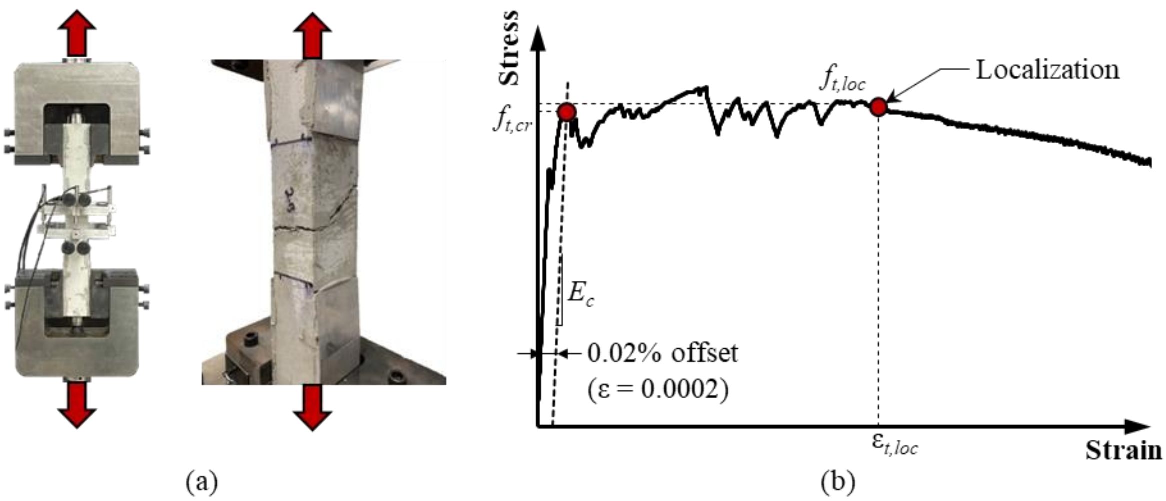
UHPC has a slightly higher coefficient of thermal expansion ($7.0 \times 10^{-6}$ /°F vs. $6.0 \times 10^{-6}$ /°F for conventional concrete) and a slightly smaller Poisson's ratio (0.15 vs. 0.2 for conventional concrete) than conventional concrete.

**Measurement of Tensile Properties of UHPC**

The following tensile properties must be specified and measured for UHPC members:

- Effective cracking strength ($f_{t,cr}$)
- Crack localization strength ($f_{t,loc}$)
- Crack localization strain ($\varepsilon_{t,loc}$)

These properties are measured using the AASHTO *Standard Method of Test for Uniaxial Tensile Response of Ultra-High Performance Concrete* (T 397-22). Tensile properties are typically measured on a 2-inch by 2-inch rectangular prism subjected to a controlled uniaxial tensile deformation within a fixed testing frame. Two photographs of a UHPC prism being tested in direct tension are shown in Figure 6 (a). Specific details of the test procedure, including specimen fabrication, testing, and results analysis, are provided in AASHTO T 397-22.




Source: FHWA.

**Figure 6. Photo and Illustration. (a) Photographs of UHPC uniaxial tensile response specimens and (b) sample UHPC tensile stress-strain response.**

A sample tensile stress versus strain response for UHPC measured using AASHTO T 397-22 is shown in Figure 6 (b) with the three key parameters needed for design highlighted: $f_{t,cr}$, $f_{t,loc}$, and $\varepsilon_{t,loc}$.

The tensile stress-strain response is linear-elastic with a slope equal to the modulus of elasticity of the UHPC, $E_c$, until the end of the elastic region. There may not be a clear transition between the initial linear elastic region and the plateau during the multi-cracking phase. For this reason, the cracking point is determined using a 0.02% offset line. To determine cracking, a line is drawn starting from a strain of 0.0002 (0.02%) at zero stress with a slope equal to the modulus of elasticity for the UHPC ($E_c$). The value of the stress where the 0.02% offset line intersects the tensile stress-strain response is considered the effective cracking strength ($f_{t,cr}$). This is a similar procedure to that used to determine the yield stress in steel reinforcement.

Crack localization is considered to occur when the stress begins to continuously decrease with increasing strain. The crack localization strength ($f_{t,loc}$) and crack localization strain ($\varepsilon_{t,loc}$) are the stress and strain values at this point. The crack localization strength needs to be greater than or equal to the effective cracking strength for a material to be considered UHPC ($f_{t,loc} \geq f_{t,cr}$, UHPC Structural Design Guide Article 1.1). The crack localization strength used for design depends on how much greater the strength is than the effective cracking strength. From UHPC Structural Design Guide Article 4.2.5.2, "*If the value of the crack localization strength is less than 1.20*$f_{t,cr}$ *(i.e.,* $f_{t,loc} < 1.20f_{t,cr}$*), it shall be taken as equal to the effective cracking strength…*"

AASHTO T 397-22 requires a minimum of three successful individual companion test specimens (Section 11.1). The average and standard deviation for key tensile properties should be used when calculating the design values for a mixture (Section 11.2). The UHPC Structural Design Guide (Article C1.1) states:

*UHPC materials should conform to the requirements for qualification and acceptance testing that consider the statistical variability of the material properties and ensure design properties that are 1.5 standard deviations below the mean values.*

The values for design can be calculated based on the values measured from AASHTO T 397-22 as follows.

Effective cracking strength for design: $f_{t,cr} = \bar{f}_{t,cr,test} - 1.5\sigma_{ft,cr,test}$

Crack localization strength for design: $f_{t,loc} = \bar{f}_{t,loc,test} - 1.5\sigma_{ft,loc,test}$

Crack localization strain for design: $\varepsilon_{t,loc} = \bar{\varepsilon}_{t,loc,test} - 1.5\sigma_{\varepsilon t,loc,test}$

Additional guidance on qualification of UHPC mixtures is currently being developed. Draft material conformance guidance is provided in Section 2 of Appendix A of FHWA-HRT-23-077.

**Time Dependent Effects**

The creep coefficient and shrinkage strain relationships for conventional concrete and UHPC are summarized in Table 3.

**Table 3. Creep coefficient and shrinkage strain for conventional concrete and UHPC sections.**

| Description | Conventional Concrete | AASHTO LRFD BDS Reference | UHPC | UHPC Structural Design Guide Reference |
|---|---|---|---|---|
| Creep Coefficient and Shrinkage Strain | Procedure and factors defined in this Article. | Article 5.4.2.3 | Different factors for creep coefficient and shrinkage strain equations | Article 4.2.8 |
| Creep coefficient | $\psi(t, t_i) = 1.9k_s k_{hc} k_f k_{td} t_i^{-0.118}$ | Eqn. 5.4.2.3.2-1 | $\psi(t, t_i) = 1.2k_s k_{hc} k_f k_{td} k_\ell K_3$ | Eqn. 4.2.8.2-1 |
| Shrinkage strain | $\varepsilon_{sh} = k_s k_{hs} k_f k_{td} 0.48\times10^{-3}$ | Eqn. 5.4.2.3.3-1 | $\varepsilon_{sh} = k_s k_{hs} k_f k_{td} K_4 0.6\times10^{-3}$ | Eqn. 4.2.8.3-1 |

The creep coefficient and shrinkage strain equations for UHPC have the same general form as those for conventional concrete. However, the factors are calibrated differently for UHPC to account for the difference in time dependent behavior. The model for the creep coefficient is only applicable for components loaded when the compressive strength at the time of loading is greater than or equal to 14.0 ksi: $f'_{ci} \geq 14.0$ ksi. The creep coefficient for components loaded when the compressive strength is less than 14.0 ksi is expected to be higher than the estimate provided by UHPC Structural Design Guide Eqn. 4.2.8.2-1.

Because of the relatively impermeable nature of UHPC, the volume-to-surface area ratio is not likely to have an effect on creep and shrinkage; this is reflected in the following shape factor.

Shape factor for UHPC: $k_s = 1.0$ UHPC Structural Design Guide Eqn. 4.2.8.2-2

The other factors can be calculated for UHPC as follows.

Humidity factor for creep: $k_{hc} = 1.12 - 0.0024H$

UHPC Structural Design Guide Eqn. 4.2.8.2-3

Humidity factor for shrinkage: $k_{hs} = 1.5 - 0.01H$

UHPC Structural Design Guide Eqn. 4.2.8.3-2

Concrete strength factor: $k_f = 18 / (1.5f'_{ci} - 3)$ UHPC Structural Design Guide Eqn. 4.2.8.2-4

Time dependent factor: $k_{td} = t / ((300/(f'_{ci} + 30)) + 0.8t^{0.98})$

UHPC Structural Design Guide Eqn. 4.2.8.2-5

Correction factor: $K_3$ = 1.0 unless determined by physical tests and approved by owner

The loading age factor, $k_\ell$, depends on the time of loading:

If $t_i < 7.0$ days, then $k_\ell = 1.0$ UHPC Structural Design Guide Eqn. 4.2.8.2-6

If $t_i \geq 7.0$ days, then $k_\ell = (t_i - 6)^{-0.15} \geq 0.5$ UHPC Structural Design Guide Eqn. 4.2.8.2-7

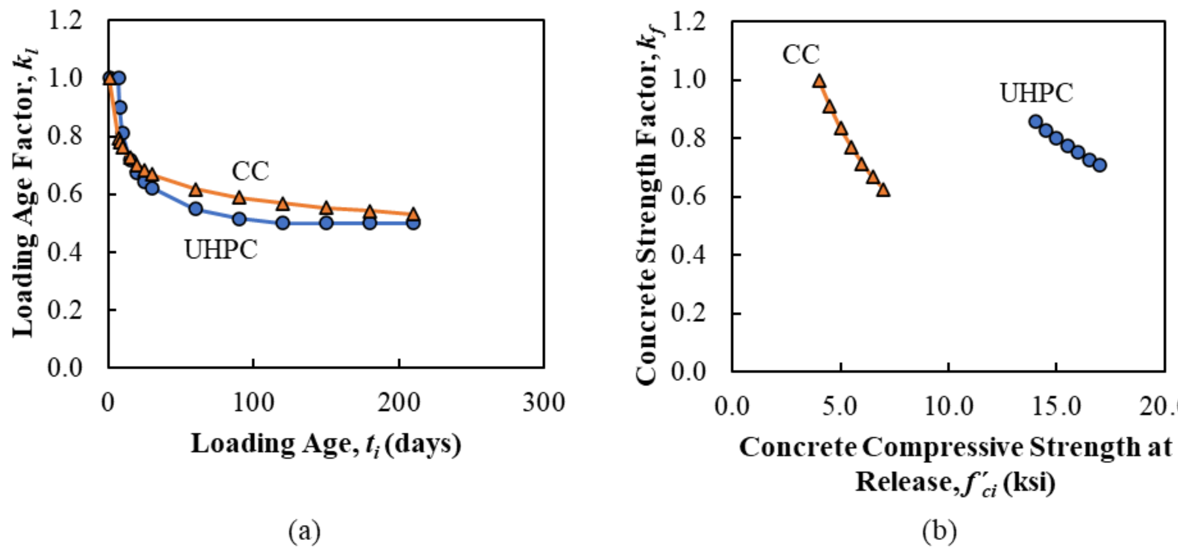
The average ambient relative humidity can be found using AASHTO LRFD BDS Figure 5.4.2.3.3-1.

***Comparison between CC and UHPC***

A comparison between the creep and shrinkage factors for UHPC and for CC is provided in Figure 7, Figure 8, Figure 9, Figure 10, Figure 11, and Figure 12 for typical design values.

The loading age factor is shown in Figure 7 (a). UHPC is sensitive to loading age; experimental results have shown that early age loading will result in two times more creep than loading at mature age (28 days), see Mohebbi et al. (2022). This behavior is captured through the loading age factor for UHPC, where $k_\ell\,(t_i = 1 \text{ day}) = 1.0$ and $k_\ell\,(t_i = 28 \text{ day}) = 0.63$. The loading age factor decreases with an increased loading age at a slightly faster rate than for CC.

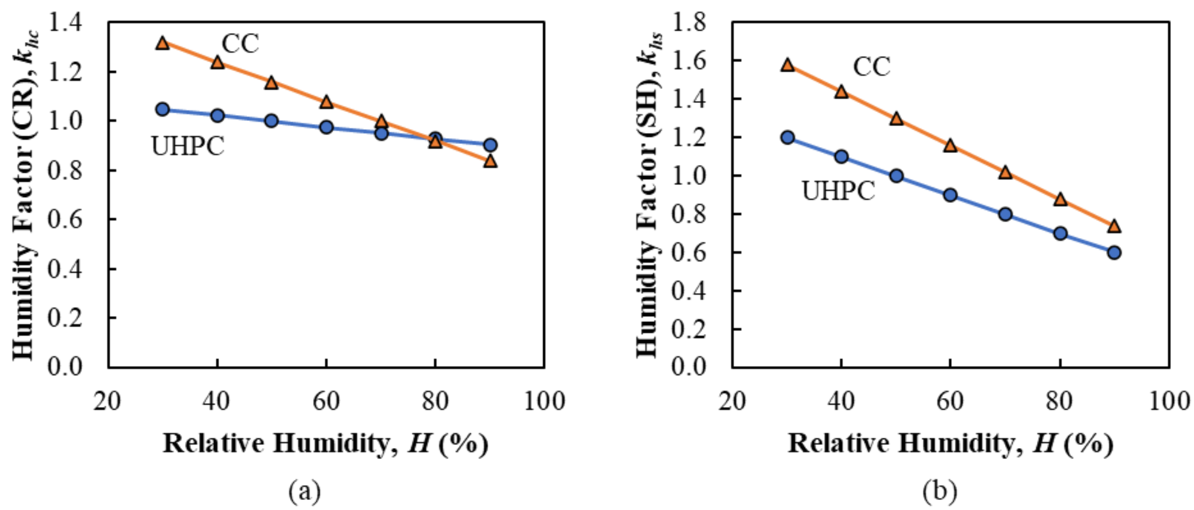
The concrete release strength factor, $k_f$, is shown for typical ranges for design in Figure 7 (b); the typical release strengths used were 4 to 7 ksi for CC and 14 to 17 ksi for UHPC. The calculated creep coefficient and shrinkage strain is less sensitive to the concrete release strength for UHPC compared to CC. The compressive strength having less of an impact on time effects in UHPC may be because the change in the UHPC modulus of elasticity over the plotted range of the compressive strengths is less than the change in the modulus of elasticity for CC with compressive strengths less than 10 ksi.




Source: FHWA.

**Figure 7. Graphs. Comparison between UHPC and CC time-dependent effect factors for (a) loading age factor and (b) concrete strength factor.**

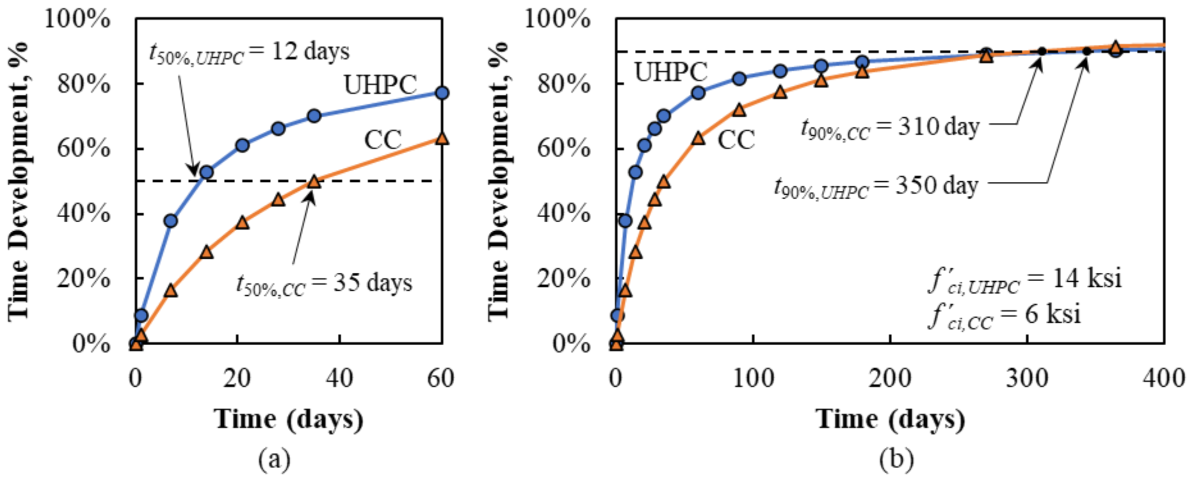
The humidity factors for creep and shrinkage for UHPC and CC are shown in Figure 8. Relative humidity has less of an effect on UHPC than on CC; see the smaller range for $k_{hc}$ and $k_{hs}$ in Figure 8.




Source: FHWA.

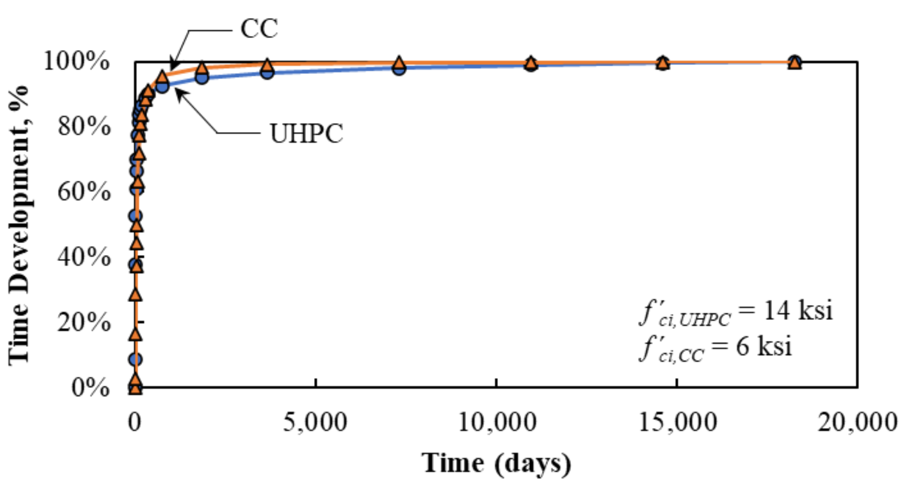
**Figure 8. Graphs. Comparison between UHPC and CC time-dependent effect factors for (a) humidity factor for creep and (b) humidity factor for shrinkage.**

The normalized time development factors for CC with a release strength of 6 ksi and UHPC with a release strength of 14 ksi over a 50-year design life are shown in Figure 9 and Figure 10. The time development factors ($k_{td}$) are normalized to the time development factor found at 50 years. Creep coefficients and shrinkage strains in UHPC are predicted to initially increase more rapidly than in CC; UHPC reaches 50 percent at 12 days compared to 35 days for CC. The creep coefficients and shrinkage strains in UHPC are then predicted to slow down quicker than CC; UHPC reaches 90 percent at 350 days compared to 310 days for CC.




Source: FHWA.

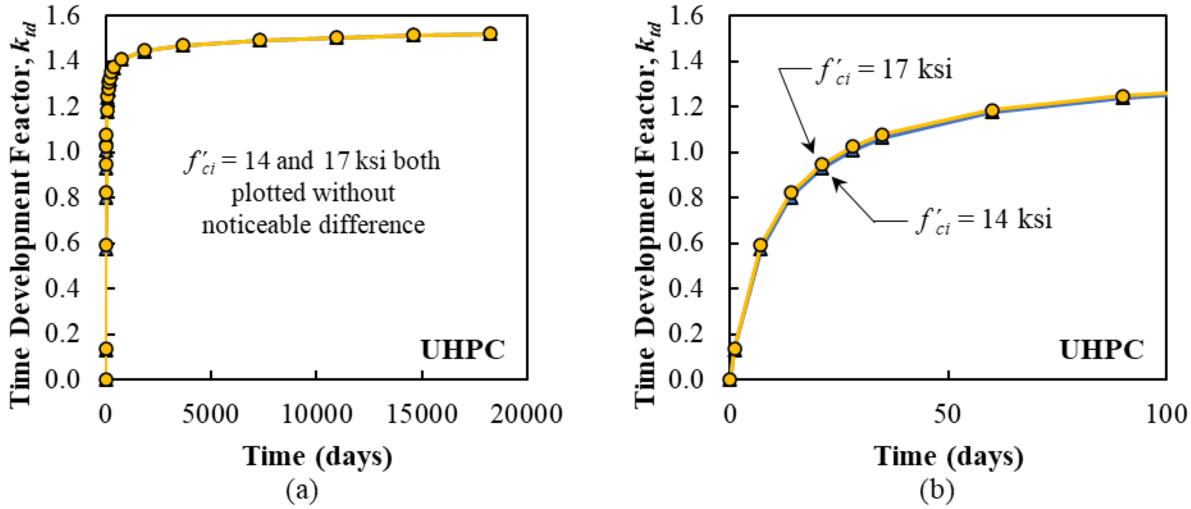
**Figure 9. Graph. Time development percentage for conventional concrete ($f'_{ci}$ = 6 ksi) and UHPC ($f'_{ci}$ = 14 ksi) for (a) time range of 0 to 60 days and (b) time range of 0 to 400 days.**




Source: FHWA.

**Figure 10. Graph. Time development percentage for conventional concrete ($f'_{ci}$ = 6 ksi) and UHPC ($f'_{ci}$ = 14 ksi) for time range of 0 to 20,000 days.**

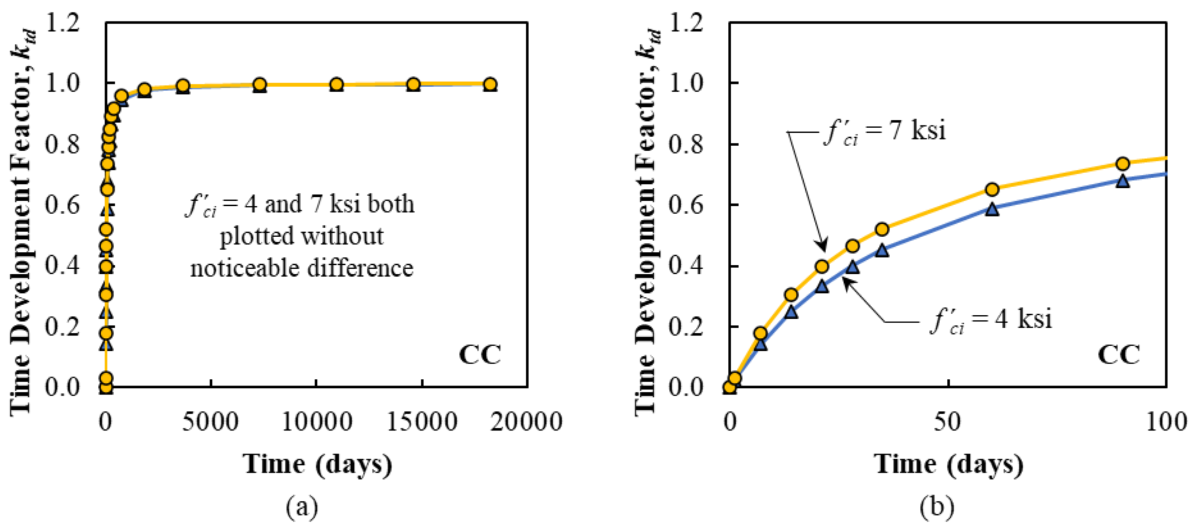
The calculated time development factors for UHPC with release strengths of 14 ksi and 17 ksi are shown in Figure 11. The time development factor is not significantly affected by the concrete release strength. There is a slight difference at early ages, e.g., $k_{td}$(t = 28 days, $f'_{ci}$ = 14 ksi) = 1.008 compared to $k_{td}$(t = 28 days, $f'_{ci}$ = 17 ksi) = 1.024, but no difference at later times, e.g., $k_{td}$(t = 7,300 days, $f'_{ci}$ = 14 ksi) = $k_{td}$(t = 7,300 days, $f'_{ci}$ = 17 ksi) = 1.491. The percent difference is estimated at 6.1 percent at 1 day, 1.6 percent at 28 days, 0.9 percent at 60 days, and decreases to 0 percent at 1,825 days.




Source: FHWA.

**Figure 11. Graphs. Time development factors for UHPC for release strengths of 14 ksi and 17 ksi.**

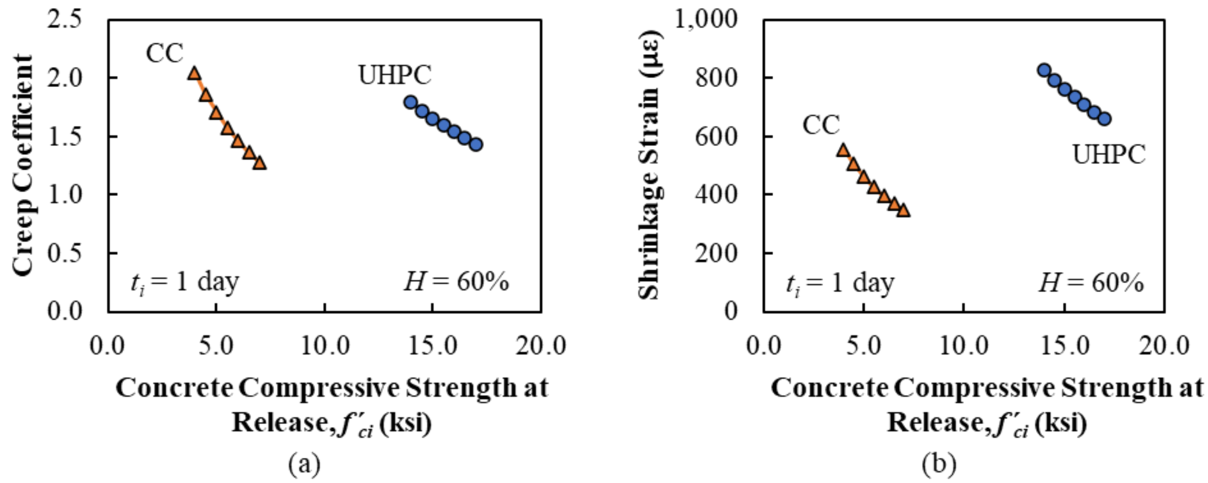
The calculated time development factors for CC with release strengths of 4 ksi and 7 ksi are shown in Figure 12. The time development factor for CC is more significantly affected by the concrete release strength than UHPC. The percent difference is estimated at 30.3 percent at 1 day, 16.7 percent at 28 days, 10.9 percent at 60 days, and decreases to 0.1 percent at 7,300 days.




Source: FHWA.

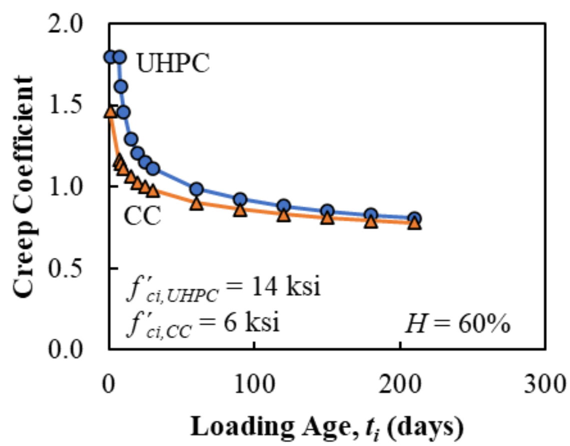
**Figure 12. Graphs. Time development factors for conventional concrete for release strengths of 4 ksi and 7 ksi.**

The creep coefficients and shrinkage strains calculated using the UHPC Structural Design Guide for UHPC and AASHTO LRFD BDS for CC for typical design ranges of concrete compressive strengths at release, loading ages, and relative humidity values are shown in Figure 13, Figure 14, and Figure 15, respectively. The typical release strengths used in Figure 13 were 4 to 7 ksi for CC and 14 to 17 ksi for UHPC



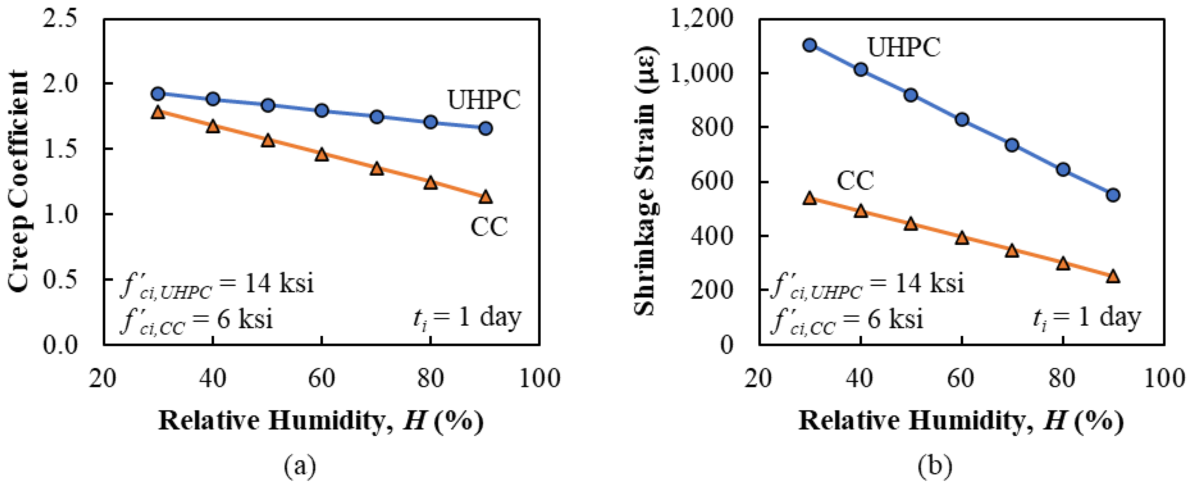

Source: FHWA.

**Figure 13. Graphs. Effect of concrete compressive strength at release on (a) creep coefficient and (b) shrinkage strain for UHPC and CC.**




Source: FHWA.

**Figure 14. Graph. Effect of loading age on creep coefficient for UHPC and CC; loading age does not affect shrinkage strain.**




Source: FHWA.

**Figure 15. Graphs. Effect of relative humidity on (a) creep coefficient and (b) shrinkage strain for UHPC and CC.**

### *Creep and Shrinkage Correction Factors*

The UHPC Structural Design Guide allows for the use of correction factors for creep ($K_3$) and shrinkage ($K_4$).

The correction factor for creep ($K_3$) can be determined two different ways: (1) based on measured results for the modulus of elasticity or (2) based on physical creep testing. Only one method should be used to calculate the correction factor.

- **$K_3$ based on modulus of elasticity testing**: The measured modulus of elasticity for the concrete at the time of loading can be used to calculate the correction factor for the modulus of elasticity, $K_1$. The correction factor for creep is the inverse of $K_1$.

  Correction factor for $E_c$: $K_1 = E_{c,measured} / E_{c,estimated}$

  Correction factor for $\Psi$: $K_3 = 1 / K_1$

- **$K_3$ based on physical creep testing**: UHPC Structural Design Guide Article C4.2.8.2 specifies that the ultimate creep coefficient can be measured using ASTM C1856 and ASTM C512 with a sustained compressive stress equal to 65 percent of the compressive strength at the time of loading. This stress should be maintained on the specimens for one year. The correction factor for creep is the ratio of the measured to estimated creep coefficient.

  Correction factor for $\Psi$: $K_3 = \Psi_{measured} / \Psi_{estimated}$

The correction factor for shrinkage ($K_4$) can be determined based on shrinkage strains measured using ASTM C1856 and ASTM C157.

Correction factor for $\varepsilon_{sh}$: $K_4 = \varepsilon_{sh,measured} / \varepsilon_{sh,estimated}$

Similar to conventional concrete, the creep coefficients and shrinkage strains for UHPC will vary between different mixtures and products. Measuring the appropriate values needed to calculate these correction factors will greatly improve the accuracy of the creep coefficient and shrinkage strain estimates.

### Prestress Losses

A summary of prestress loss information for conventional concrete and UHPC members is provided in Table 4.

**Table 4. Prestress losses for conventional concrete and UHPC sections.**

| Description | Conventional Concrete | AASHTO LRFD BDS Reference | UHPC | UHPC Structural Design Guide Reference |
|---|---|---|---|---|
| Prestress Loss | Refined and Approximate Time-Dependent Loss procedures allowed. | Articles 5.9.3.3 and 5.9.3.4 | Only Refined Estimate procedure allowed. | Article 9.3 |

Elastic shortening loss is calculated using the same methodology for both conventional concrete and UHPC sections.

The UHPC Structural Design Guide only allows the “Refined Estimates of Time Dependent Losses” procedure specified in AASHTO LRFD BDS Article 5.9.3.4 using the modified creep coefficient and shrinkage strain equations for UHPC.

The “Approximate Estimate of Time-Dependent Losses” procedure specified in AASHTO LRFD BDS Article 5.9.3.3 is not allowed for UHPC members. This equation was derived based on conventional concrete members and does not capture the time-dependent behavior of UHPC.

## SERVICE LIMIT STATE

### Concrete Stresses at Transfer

Longitudinal stresses at transfer are calculated using the same methodology as is used for conventional concrete sections. The stress limits at transfer for conventional concrete and UHPC sections are summarized in Table 5. Compression stress limits at transfer are the same for conventional concrete and UHPC sections. The tensile stress limit at release is calculated based on the specified effective cracking strength at release, $f_{t,cri}$, for UHPC sections.

**Table 5. Concrete strength and stress limits at transfer for conventional concrete and UHPC sections.**

| Description | Conventional Concrete | AASHTO LRFD BDS Reference | UHPC | UHPC Structural Design Guide Reference |
|---|---|---|---|---|
| Concrete compressive strength at release | No lower limit. | n/a | $f'_{ci} \geq 14.0$ ksi | Article 9.1.2 |
| Effective cracking strength at release | n/a | n/a | $f_{t,cri} \leq 0.75 f_{t,cr}$ when $f'_{ci} \leq 0.90 f'_c$ (unless determined by physical tests, as approved by owner) | Article 9.1.2 |
| Compressive stress limit | $0.65 f'_{ci}$ | Article 5.9.2.3.1a | Same as conventional concrete. | Articles 5.2.1.3a and 9.2.3 |
| Tensile stress limit | $-0.0948\lambda\sqrt{f'_{ci}}$ (without bonded reinforcement) | Table 5.9.2.3.1b-1 | $\gamma_u f_{t,cri}$ | Articles 5.2.1.3a and 9.2.3 |
| Principal tensile stresses in webs | Only for post-tensioned members | Article 5.9.2.3.3 | Required for all prestressed members, $\gamma_u f_{t,cri}$ | Article 5.2.1.3b |

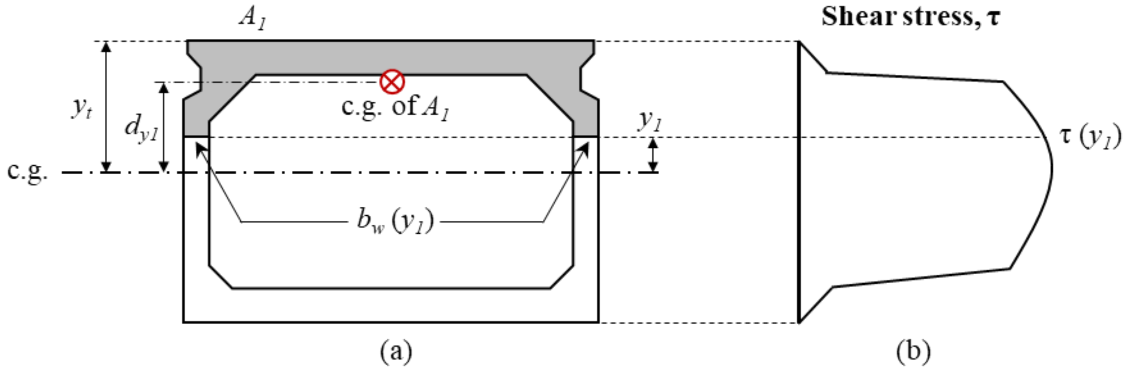
The more significant difference between conventional concrete and UHPC design checks at transfer is that principal tensile stresses in the web(s) need to be calculated and compared to the specified limit for all members. This principal tensile stress check aims to prevent cracking in webs to mitigate potential issues related to tension creep and tensile fatigue. Note that, at the present time, there is limited research or performance data on tension creep and tension fatigue of cracked UHPC. This principal tensile stress limit conservatively recognizes both the current state of knowledge and the unique mechanism through which a UHPC section may offer resistance.

The procedure for calculating the principal tensile stress in the webs is outlined in AASHTO LRFD BDS Article 5.9.2.3.3. Typically, the principal tension stress should be calculated at multiple locations along the height of the web and along the length of the member.

The first moment of the area needs to be calculated at the height of interest. The general formula for this is:

$$\mathrm{Q}(y_1) = \int_{y_1}^{y_t} z \times b(z + y_b)\,\mathrm{d}z$$

where $y_1$ is the distance from the centroid where the first moment of the area is calculated and $z$ is the distance measured from the compression face of the section. There are simplified equations available for different sections shapes. As an example, the area to use for calculating the first moment of the area for a UHPC box beam at a distance $y_1$ from the center of gravity (c.g.) of the section is shown in Figure 16 (a).




Source: FHWA.

**Figure 16. Illustration. (a) Example of parameters needed when calculating shear stress and (b) sample shear stress diagram across the depth of the section.**

The first moment of the area for the box beam in Figure 16 (a) at a distance $y_1$ from the center of gravity is calculated using the following equation.

$$Q(y_1) = A_1 \times d_{y1}$$

The maximum shear stress can be calculated using the following equation.

$$\tau_{max}(y_1) = \frac{V_g \times Q(y_1)}{I_g \times b_w(y_1)}$$

The shear stress will vary across the depth of the section with a shape like that shown in Figure 16 (b).

The vertical and horizontal normal stress in the web at the height of interest also needs to be calculated. The stress at the level of interest ($y_1$) can be calculated assuming a linear stress profile in the section.

$$f_{pcx} = f_{ti} - [(f_{ti} - f_{ci}) / h] \times (y_b + y_1)$$

There are no vertical normal stresses in the web at this section.

$$f_{pcy} = 0 \text{ ksi}$$

The principal stress in the web can be determined using Mohr's Circle, as summarized in AASHTO LRFD BDS Equation 5.9.2.3.3-4, where $\tau = \tau_{max}$.

$$f_{min} = \frac{1}{2}\left(\left(f_{pcx} + f_{pcy}\right) - \sqrt{\left(f_{pcx} - f_{pcy}\right)^2 + (2\tau)^2}\right)$$

This stress, $f_{min}$, is compared to the principal tensile stress limit, $\gamma_u f_{t,cri}$ at release. The stress, $f_{min}$, is negative if it is tension. This check is not needed if the minimum principal stress is calculated to be in compression (positive).

Principal stress in web limit (at transfer): $f_{min} \leq \gamma_u f_{t,cri}$

UHPC Structural Design Guide Article 5.2.1.3.1 and 5.2.2.4

### Concrete Stresses at Service Loads

Service stresses are calculated using the same methodology for UHPC sections as conventional concrete sections. The stress limits for conventional concrete and UHPC sections at service are summarized in Table 6. The compression stress limits for UHPC sections are the same as for conventional concrete sections. The tensile stress limit and principal tensile stress in web limit are both limited to $\gamma_u f_{t,cr}$. There is an additional tensile strain limit for non-prestressed UHPC members, the lesser of $0.25\gamma_u \varepsilon_{t,loc}$ or 0.001.

**Table 6. Stress and strain limits at service for conventional concrete and UHPC sections.**

| Description | Conventional Concrete | AASHTO LRFD BDS Reference | UHPC | UHPC Structural Design Guide Reference |
|---|---|---|---|---|
| Compressive stress limit (Service I) | $0.45f'_c$ (permanent load); $0.60\phi_w f'_c$ (all loads) | Article 5.9.2.3.2a | Same as conventional concrete. | Articles 5.2.1.3b and 9.2.3 |
| Tensile stress limit (Service III) | $-0.19\lambda\sqrt{f'_c}$ (components w/ bonded prestressing) | Table 5.9.2.3.2b-1 | $\gamma_u f_{t,cr}$ | Articles 5.2.1.3b and 9.2.3 |
| Principal tensile stresses in webs (Service I) | Only for post-tensioned members | Article 5.9.2.3.3 | Required for all members, $\gamma_u f_{t,cr}$ | Articles 5.2.1.3b and 5.2.2.4 |
| Tensile strain limit for non-prestressed components (Service I) | n/a | n/a | Lesser of $0.25\gamma_u \varepsilon_{t,loc}$ or 0.001. | Article 5.2.2 |
| Tensile stress limit for components subjected to cyclic stresses (Service I) | n/a | n/a | $0.95\gamma_u f_{t,cr}$ | Article 5.2.3 |

There is an additional tensile stress check for components subjected to cyclic stresses using the Service I (not Fatigue I) load combination. The high-cycle fatigue behavior of UHPC in tension depends on the maximum applied tensile strain instead of the stress range typically used for fatigue limit state checks. In recognition of the limited experimental data on tensile fatigue of cracked UHPC, this limit reduces the likelihood that cracked UHPC will experience cyclic loads.

## FATIGUE LIMIT STATE

A summary of the fatigue limit state provisions for conventional concrete and UHPC members is provided in Table 7.

**Table 7. Limits for fatigue limit state for conventional concrete and UHPC sections.**

| Description | Conventional Concrete | AASHTO LRFD BDS Reference | UHPC | UHPC Structural Design Guide Reference |
|---|---|---|---|---|
| Stress in embedded steel elements | Does not require check for prestressed components satisfying tensile stress limit for Service III limit state. | Article 5.5.3.1 | All embedded steel elements need to be checked for fatigue in accordance with AASHTO LRFD BDS Eqn. 5.5.3.1-1 and Articles 5.5.3.2, 5.5.3.3, and 5.5.3.4. | Article 5.3 |
| Compressive stress limit in prestressed components | 0.4 $f'_c$ for Fatigue I plus half of unfactored effective prestress and permanent loads. | Article 5.5.3.1 | Same as conventional concrete. | Article 5.3 |
| Compressive stress limit in non-prestressed components | n/a | n/a | 0.4 $f'_c$ for Fatigue I plus half of unfactored permanent loads. | Article 5.3 |

Although not required for prestressed conventional concrete members, stresses due to fatigue loads need to be checked for prestressed UHPC members. The factored live load stress range in the embedded steel elements due to the passage of the fatigue load, $\gamma\Delta f$, must be less than the constant-amplitude fatigue threshold, $(\Delta F)_{TH}$, where $\gamma$ is 1.75.

Fatigue check for embedded steel elements: $\gamma\Delta f \leq (\Delta F)_{TH}$
AASHTO LRFD BDS Eqn. 5.5.3.1-1

Fatigue threshold for straight prestressing strands: $(\Delta F)_{TH}$ = 18.0 ksi
AASHTO LRFD BDS Article 5.5.3.3

The compressive stress limit due to Fatigue I plus half of the unfactored effective prestress and permanent loads is the same for conventional concrete and UHPC members.

## STRENGTH LIMIT STATE – FLEXURE

A summary of the strength limit state provisions related to flexural capacity for conventional concrete and UHPC members is provided in Table 8.

**Table 8. Provisions for strength limit state (flexure) for conventional concrete and UHPC sections.**

| Description | Conventional Concrete | AASHTO LRFD BDS Reference | UHPC | UHPC Structural Design Guide Reference |
|---|---|---|---|---|
| Application | Valid for $f'_c \leq 15.0$ ksi | Article 5.6.2 | Valid for UHPC materials | Article 1.1 |
| Concrete compressive strain limit | 0.003; indicates concrete crushing | Article 5.6.2.1 | 0.0035, unless measured; indicates UHPC crushing | Articles 4.2.4.2 and 6.3.2.2 |
| Concrete in compression | Rectangular stress distribution, parabolic stress-strain relationship, or other shapes based on tests | Article 5.6.2.1 and 5.6.2.2 | Elastic-plastic stress-strain relationship | Article 6.3.1 |
| Concrete tensile strain limit | No specified | n/a | $\varepsilon_{t,loc}$; indicates crack localization | Article 6.3.2.2 |
| Concrete in tension | Neglect the tensile strength of concrete in tension | Article 5.6.2.1 | Included assuming elastic-plastic or bilinear stress-strain relationship | Article 4.2.5 |
| Steel tensile strain limit | Strain at rupture of reinforcement, defined by the total elongation using the appropriate AASHTO or ASTM standard (e.g., AASHTO M203, AASHTO M31). | Article 5.6.2.1 | Same as conventional concrete | Article 6.3.2.2 |
| Stress in prestressing steel at nominal flexural resistance | Simplified equations provided for calculation | Article 5.6.3.1.1 | Found using strain compatibility approach utilizing a stress-strain relationship for the prestressing steel | Article 6.3.1 |
| Nominal flexural resistance ($M_n$) | Assumed to occur when concrete crushes ($\varepsilon_c$ = 0.003) | Article 5.6.2.1 | Occurs when either of the following occurs:<br>• Concrete crushing<br>• Crack localization<br>• Rupture of reinforcement | Article 6.3.2.2 |
| Strain compatibility | Allowed | Article 5.6.3.2.5 | Required | Article 6.3.1 |
| Resistance factor | Based on strain in the outermost layer of reinforcement | Eqn. 5.5.4.2-1 | Based on curvature ductility of section (μ) | Article 5.4.2 |
| Minimum reinforcement | $\phi M_n \geq \min(1.33M_u , M_{cr})$ | Article 5.6.3.3 | Use $f_{t,cr}$ in place of $f_r$ in AASHTO LRFD BDS Eqn. 5.6.3.3-1 and Article 5.10.6 need not apply | Article 6.3.3 |

There are several differences between conventional concrete and UHPC members related to calculating the factored resistance. These differences are discussed throughout the remainder of this section.

### Strain Compatibility Approach

The nominal flexural resistance must be found using the strain compatibility approach for UHPC members. There are three fundamental concepts that are required when using the strain compatibility approach for analyzing the flexural strength of a UHPC section:

- **Constitutive relationships**: material stress is related to strain using stress versus strain relationships for each material type,
- **Compatibility**: plane sections remain plane, and
- **Equilibrium**: sum of internal force components equals externally applied forces.

Additional details on each of these concepts is provided in the following sections.

#### *Constitutive Relationships*

Idealized stress-strain curves are used for compression and tension when designing UHPC members. The stress-strain curve for compression is defined in UHPC Structural Design Guide Article 4.2.4.3 and idealized as an elastic-plastic response with a maximum stress equal to $\alpha_u f'_c$, as shown in Figure 17 (a). The strain at the end of the elastic region can be calculated using the modulus of elasticity for the UHPC, $E_c$.

Elastic compressive strain limit: $\varepsilon_{cp} = \alpha_u f'_c / E_c$

UHPC Structural Design Guide Eqn. 4.2.4.2-1

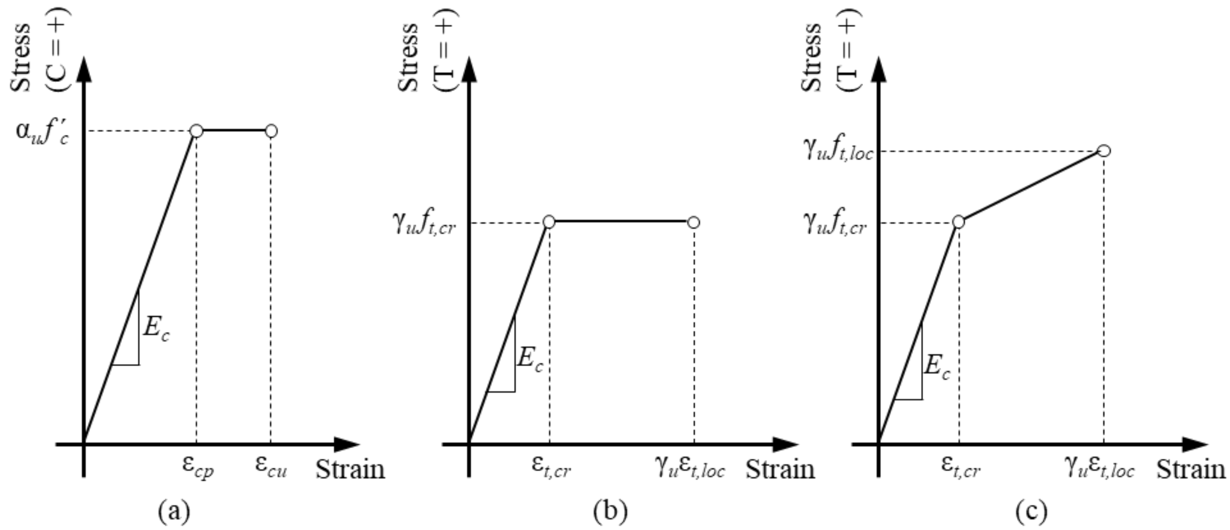
The ultimate compressive strain used in design is the greater of $\varepsilon_{cp}$ and 0.0035, unless determined by physical tests. The ultimate compressive strain is the strain corresponding to the compressive strength, $f'_c$, recorded during a compression test performed in accordance with ASTM C1856 (see UHPC Structural Design Guide Article 4.2.4.2).

The tensile stress-strain response is defined in UHPC Structural Design Guide Article 4.2.5.4 and is idealized either as elastic-plastic (if $f_{t,loc} < 1.20 f_{t,cr}$) or bilinear (if $f_{t,loc} \geq 1.20 f_{t,cr}$), as shown in Figure 17 (b) and (c), respectively. The effective cracking strain is calculated based on the effective cracking stress, $f_{t,cr}$, and the modulus of elasticity for the UHPC, $E_c$.

Elastic tensile strain limit: $\varepsilon_{t,cr} = \gamma_u f_{t,cr} / E_c$

UHPC Structural Design Guide Eqn. 4.2.5.4-1

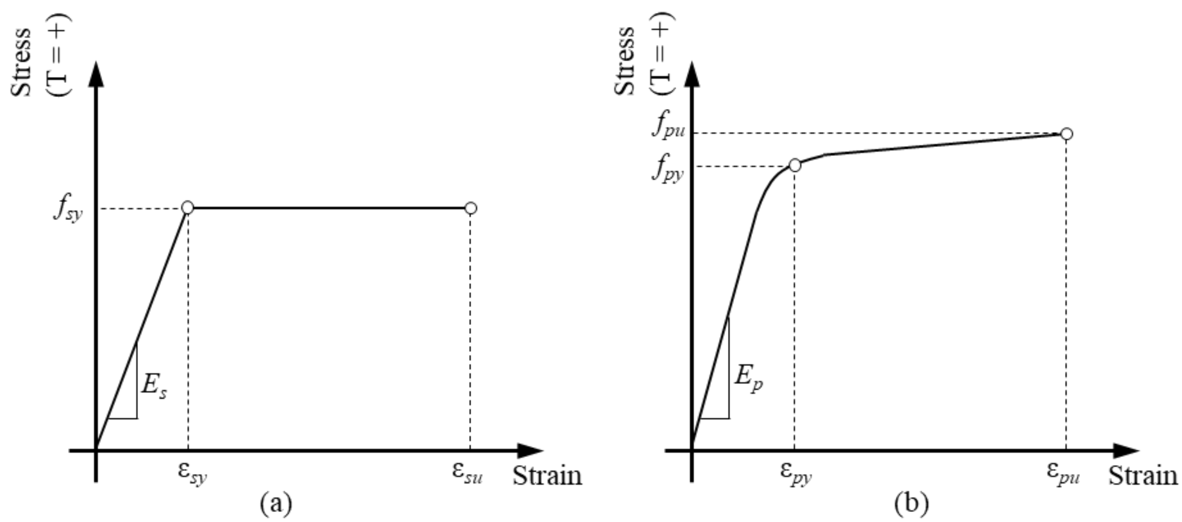
The effective cracking stress, $f_{t,cr}$, crack localization strain, $\varepsilon_{t,loc}$, and crack localization stress, $f_{t,loc}$, are measured using the direct tension test specified in AASHTO T-397 as described earlier in this chapter.




Source: FHWA.

**Figure 17. Illustration. Idealized UHPC stress-strain curves for (a) compression, (b) tension if $f_{t,loc} < 1.20 f_{t,cr}$ and (c) tension if $f_{t,loc} \geq 1.20 f_{t,cr}$.**

The stress versus strain relationships for non-prestressed steel reinforcement and prestressing strands may be assumed as shown in Figure 18 (a) and (b), respectively.




Source: FHWA.

**Figure 18. Illustration. Assumed stress-strain relationships for (a) non-prestressed steel reinforcement and (b) prestressing strands.**

The stress-strain relationship is assumed to be elastic-plastic for conventional, non-prestressed steel reinforcement.

Conventional steel reinforcement: $f_s = E_s \times \varepsilon_s \leq f_{sy}$

The stress-strain relationship for ASTM A416 Grade 270 prestressing strands can be modeled using the power formula recommended in the PCI Bridge Design Manual.

$$f_{ps} = \varepsilon_{ps}\left[887 + \frac{27{,}613}{\left(1 + \left(112.4\varepsilon_{ps}\right)^{7.36}\right)^{\frac{1}{7.36}}}\right] \leq 270 \text{ ksi}$$

Based on ASTM A416, the yield stress for prestressing strands must be at least 243 ksi and measured at a 1 percent elongation (strain of 0.01). Therefore, the strain in the prestressing steel at yield is 0.01.

Yield stress in prestressing strands: $f_{py} = 243$ ksi

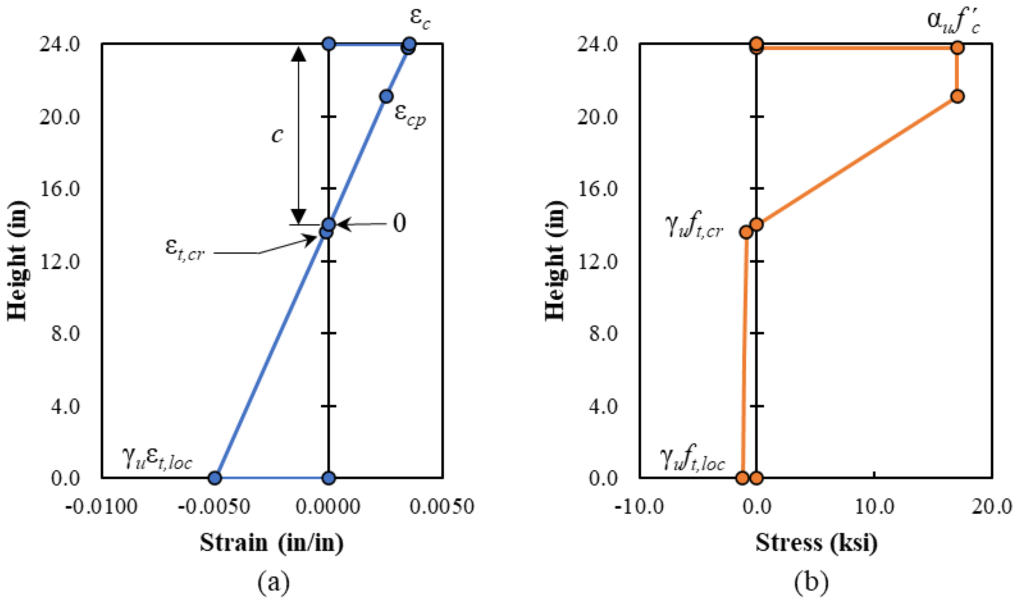
Yield strain in prestressing strands: $\varepsilon_{py} = 0.01$

The strain at the service limit ($0.8f_{py}$) can be calculated using the stress-strain relationship for the prestressing strands, by setting $f_{ps} = 0.8(243 \text{ ksi}) = 194.4$ ksi and then solving for $\varepsilon_{ps}$ as follows.

Strain in prestressing strands at service limit: $\varepsilon_{p,sl} = 0.00696$

***Compatibility***

The strain diagram is assumed to be linear across the depth of the section, i.e., plane sections remain plane. A stress diagram across the depth of the section can be developed based on the section strain profile and the assumed stress-strain relationships for the materials. A sample strain and UHPC stress diagram across the depth of the section is shown in Figure 19 with the significant strains and stresses highlighted. The section depth of 24 inches is based on Storyline Example #1.




Source: FHWA.

**Figure 19. Graphs. Sample (a) strain profile and (b) associated stress profile across the section depth.**

The strain profile can be defined by two parameters (e.g., extreme tension fiber strain and neutral axis depth).

The typical procedure for analyzing a UHPC section is to choose one strain of interest and then change the neutral axis depth until equilibrium is satisfied, as discussed in the next section. Some strains of interest used in UHPC design include:

- First cracking, extreme tension fiber equal to cracking strain: $\varepsilon_t = \varepsilon_{t,cr}$,
- Strain in steel equal to steel service stress limit: $\varepsilon_s = \varepsilon_{s,s\ell} = \varepsilon_s(0.8f_y)$,
- Strain in steel equal to yield strain for steel: $\varepsilon_s = \varepsilon_{sy} = \varepsilon_s(f_y)$,
- Strain in steel equal to ultimate strain in steel: $\varepsilon_s = \varepsilon_{su}$,
- Crack localization, extreme tension fiber equal to crack localization strain: $\varepsilon_t = \gamma_u\varepsilon_{t,loc}$, and
- Concrete crushing, extreme compression fiber equal to ultimate compressive strain: $\varepsilon_c = \varepsilon_{cu}$.

### *Equilibrium*

Equilibrium requires that at any section the stresses integrated over the depth must balance the externally applied forces. The two general equilibrium equations are shown below for axial force $N$ and moment $M$.

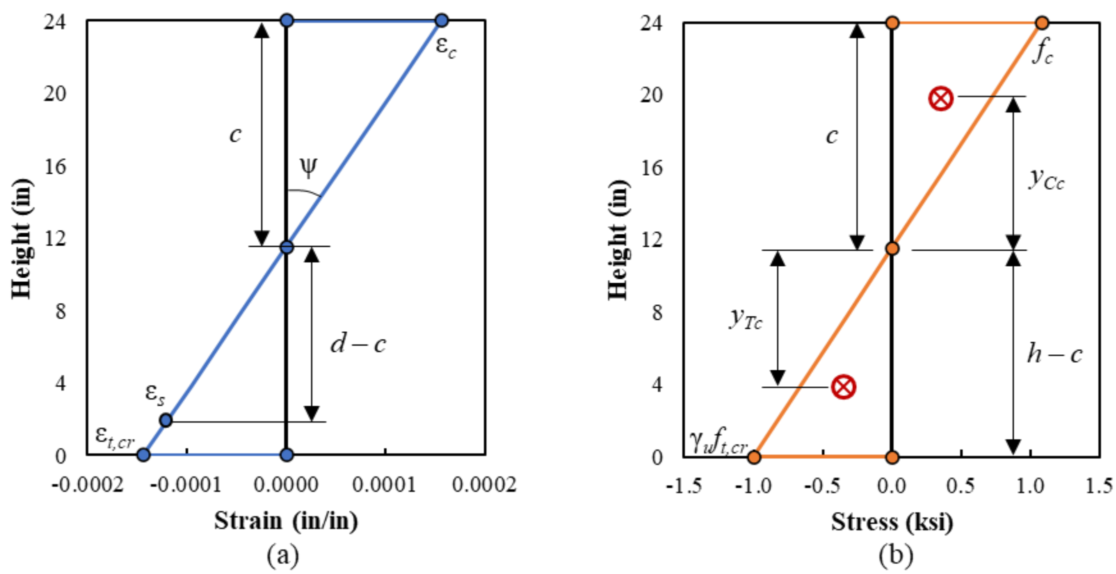
Axial force:

$$N = \int_{A_c} f_c \cdot \mathrm{d}A_c + \int_{A_s} f_s \cdot \mathrm{d}A_s + \int_{A_p} f_{ps} \cdot \mathrm{d}A_p$$

Moment:

$$M = \int_{A_c} f_c \cdot y \cdot \mathrm{d}A_c + \int_{A_s} f_s \cdot y \cdot \mathrm{d}A_s + \int_{A_p} f_{ps} \cdot y \cdot \mathrm{d}A_p$$

These integrals can be simplified for simpler stress diagrams and section shapes. The strain and stress diagrams associated with first cracking for Storyline Example #1 are shown in Figure 20. The extreme tension fiber strain, $\varepsilon_t$, is equal to the effective cracking strain, $\varepsilon_{t,cr}$, at this point.




Source: FHWA.

**Figure 20. Graphs. Sample (a) strain and (b) stress diagrams at cracking.**

The strains at the centroid of the reinforcement and at the extreme compression fiber can be calculated using the known strain in the extreme tension fiber, the neutral axis depth, and the linear strain profile as follows.

Extreme compression fiber strain: $\varepsilon_c = \varepsilon_{t,cr} \times (c / (h - c))$

Strain at centroid of reinforcement: $\varepsilon_s = \varepsilon_{t,cr} \times ((d - c) / (h - c))$

At cracking, tensile stresses in the UHPC will be linear elastic and stresses in compression will be assumed to be linear elastic, so the stresses can be calculated simply by taking the modulus of elasticity of the concrete and steel times the associated strains.

Extreme compression fiber stress: $f_c = \varepsilon_c \times E_c$

Stress in reinforcement: $f_s = \varepsilon_s \times E_s$

Assuming a constant width for the section ($b_w$), as was the case for Storyline Example #1, the force components can be calculated as follows. Both the tension and compression stress regions are triangular.

UHPC compression force: $C_c = 0.5f_c \times c \times b_w$

UHPC tension force: $T_c = 0.5\gamma_u f_{t,cr} \times (h - c) \times b_w$

Steel force: $T_s = A_s \times f_s$

Since the discrete steel reinforcement is embedded within the UHPC, a portion of the UHPC tension force needs to be subtracted. This force can be calculated as follows and should have an opposite sign to the tension forces in the equilibrium equation.

Area of UHPC not present because of steel reinforcement: $T_{c,As} = A_s \times (\varepsilon_s \times E_c)$

The area of UHPC displaced by steel reinforcement in the compression region would also need to be accounted for if compression reinforcement is present.

The equilibrium equation in this case is the summation of the internal force components equal to the externally applied axial force, $N$, which is 0 kips for this example since there is no externally applied axial force.

Equilibrium equation: $N = C_c - T_c - T_s + T_{c,As} = 0$ kips

The only unknown in this equation is the neutral axis depth, $c$, so it can be solved directly.

Neutral axis depth at cracking for Storyline Example #1: $c_{cr} = 12.48$ inches

The moment can be calculated by summing moments about the neutral axis; this is valid as long as there is no applied axial force. The lever arms to use in the moment calculation for the UHPC components are shown in Figure 20 (b). For this case with a linear stress profile, the lever arms for the UHPC can be calculated as follows.

UHPC compression force lever arm: $y_{Cc} = 2c / 3$

UHPC tension force lever arm: $y_{Tc} = 2(h - c) / 3$

The lever arm for the steel force is based on the location of the reinforcement.

Steel force lever arm: $y_{Ts} = d - c$

The moment can be calculated using these lever arms and the previously calculated force components. The component from the UHPC displaced by the presence of the reinforcement counteracts the other components in the moment calculation.

Moment: $M = (C_c \times y_{Cc}) + (T_c \times y_{Tc}) + (T_s \times y_{Ts}) - (T_{c,As} \times y_{Ts})$

The curvature is the slope of the strain profile, which can be calculated based on the tension fiber strain and distance from the neutral axis.

Curvature: $\psi = \varepsilon_t / (h - c)$

This process can be repeated for other points of interest (e.g., service limit, crack localization) by changing the assumed known strain.

***Strain and Stress for Prestressing Strands***

The stress-strain relationship for the prestressing strands was introduced above and shown in Figure 18 (b). The strain in the prestressing strands to use when calculating the stress ($\varepsilon_{ps}$) is offset by the locked-in strain differential and prestress losses that occur between transfer and service, which can be captured by calculating the effective strain in the prestressing ($\varepsilon_{pe}$).

Effective stress in prestressing strands: $f_{pe} = f_{pj} - \Delta f_{pT}$

Total prestressing loss: $\Delta f_{pT} = \Delta f_{pES} + \Delta f_{pLT}$

Effective strain in prestressing strands: $\varepsilon_{pe} = f_{pe} / E_p$

The effective strain in the prestressing strands will be added to the strain in the concrete at the level of the prestressing strands ($\varepsilon_{pf}$) to find the strain to use for calculating the stress in the prestressing strands.

Strain for calculating stress in prestressing strands: $\varepsilon_{ps} = \varepsilon_{pf} + \varepsilon_{pe}$

This strain differential is only present for prestressed reinforcement.

**Moment-Curvature Diagram**

Moment, curvature, rotation, and deflection are associated with each other through basic elastic-beam theory. If the applied moment is a function of the distance along the length of a member $x$, then rotation and deflection can be found as follows.

Curvature: $\psi = M / EI$

Rotation: $\theta = \int \psi \, dx$

Deflection: $\Delta = \iint \psi \, dx$

Elastic beam theory shows that the moment-curvature relationship is directly related to the rotation and deflection capacity of a member.

A moment-curvature diagram is developed for a cross section based on the section shape, material properties, and reinforcement. It can be developed using the strain-compatibility approach described in the previous section by calculating the moment and curvature for different top fiber strains until failure of the section occurs (as described in the following section).

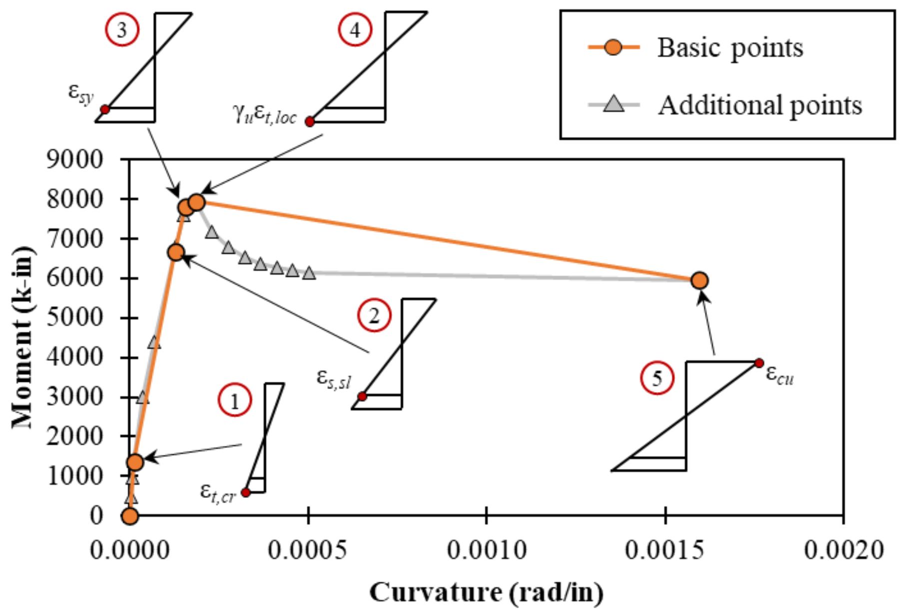
The general procedure for developing moment-curvature diagram is as follows:

- Select range of top fiber strains.
- Use the strain compatibility procedure detailed above to calculate the neutral axis, moment, and curvature associated with each top fiber strain.
- Plot points with curvature on x-axis and moment on y-axis.

There are several points of interest for a typical moment-curvature diagram for a UHPC member. These typically include:

- **First cracking**: Occurs when the strain in the extreme tension fiber is equal to the effective cracking strain ($\varepsilon_{t,cr}$).
- **Service limit**: Occurs when the strain in the outermost layer of tensile reinforcement is equal to the strain at 80 percent of the yield stress. This service limit is defined by UHPC Structural Design Guide Article 6.3.2.3 and Article 5.2 with reference to AASHTO LRFD BDS Article 5.9.2.2.
- **First yield**: Occurs when the strain in the outermost layer of tensile reinforcement is equal to the yield strain.
- **Crack localization**: Occurs when the strain in the extreme tension fiber is equal to the tensile strain limit ($\gamma_u \varepsilon_{t,loc}$).
- **Concrete crushing**: Occurs when the strain in the extreme compression fiber is equal to the ultimate compression strain ($\varepsilon_{cu}$).

A moment-curvature diagram for the Storyline Example #1 with the major points highlighted is shown in Figure 21 in the line with circular points. The moment curvature diagram with additional points is shown with triangular points behind the diagram with the circular points. The diagram with only the major points accurately represents the actual moment-curvature diagram up until the crack localization point. The moment capacity decreases more rapidly with increasing curvature after crack localization; this is not captured by assuming a linear moment-curvature relationship between the crack localization point and concrete crushing point as shown in Figure 21.




Source: FHWA.

**Figure 21. Graph. Moment-curvature diagram for Storyline Example #1 with major points highlighted.**

There may be an additional point on the diagram for the rupture of the reinforcement depending on the section shape and reinforcement type and amount. Additionally, some of the points may occur in a different order. The order of points shown in Figure 21 is typical for most UHPC members.

It is not necessary to develop a full moment-curvature diagram for UHPC sections, but several points from the diagram are used in the design of UHPC members.

**Nominal Flexural Resistance (Failure Mechanisms)**

The nominal flexural resistance occurs when one of the following occurs:

- Concrete crushes in the extreme compression fiber of the UHPC,
- Crack localization occurs in the extreme tension fiber of the UHPC, or
- The rupture strain of the tension reinforcement is reached.

The moment and curvature associated with the controlling failure mechanism are used as the nominal flexural resistance, $M_n$, and curvature at nominal flexural resistance, $\psi_n$.

All points in the moment-curvature diagram that occur after the point of failure are fictitious because the compatibility condition may no longer be valid. The failure of the section used in Storyline Example #1 is controlled by crack localization. This means that all the points in the moment-curvature diagram shown in Figure 21 after crack localization are fictitious, including the point associated with concrete crushing.

Crack localization will typically control the failure for beam elements because the crack localization strain is generally reached in the extreme tension fiber of the member prior to the ultimate compression strain in the extreme compression fiber (crushing). If crack localization controls failure, then $M_n = M_{loc}$ and $\psi_n = \psi_{loc}$.

The typical procedure for determining the nominal flexural resistance is to set the strain of interest and then change the neutral axis depth until equilibrium is satisfied using the strain compatibility approach described above.

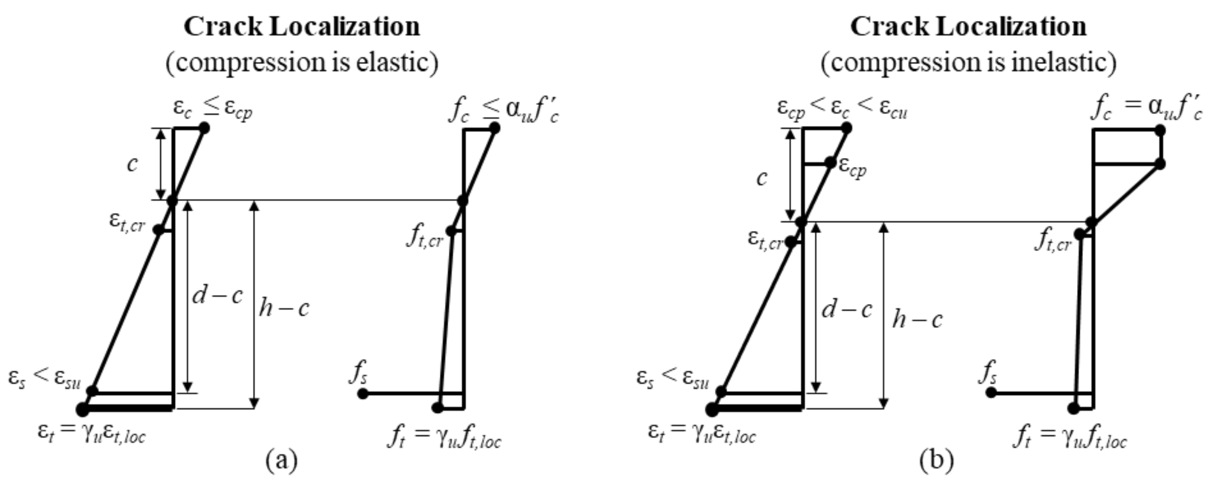
### *Crack Localization Failure ($\varepsilon_t = \gamma_u \varepsilon_{t,loc}$)*

Crack localization will typically control the failure for UHPC members. For this type of failure, the strain in the bottom fiber, $\varepsilon_t$, is equal to the crack localization strain.

For crack localization failure: $\varepsilon_t = \gamma_u \varepsilon_{t,loc}$

To check the capacity of this failure type, the extreme tension fiber strain is set equal to the crack localization strain: $\varepsilon_t = \gamma_u \varepsilon_{t,loc}$. Then, the neutral axis depth is modified until equilibrium is satisfied using the strain compatibility approach. There may not be a solution for the neutral axis depth that results in equilibrium being satisfied if crack localization does not control failure.

The compression stresses at time of crack localization may be in the linear elastic region, when $\varepsilon_c \leq \varepsilon_{cp}$, or in the inelastic region, when $\varepsilon_{cp} < \varepsilon_c < \varepsilon_{cu}$, as illustrated in Figure 22.




Source: FHWA.

**Figure 22. Illustration. Strain and stress profiles at crack localization when (a) compression stresses are in the elastic region and (b) compression stresses are inelastic.**

The strain in the extreme compression fiber and strain in the reinforcement should be checked to ensure that they are less than the ultimate compressive strain for the concrete ($\varepsilon_c \leq \varepsilon_{cu}$) and the rupture strain for the reinforcement ($\varepsilon_s \leq \varepsilon_{su}$), respectively. If these two conditions are satisfied, then crack localization controls failure. Otherwise, the one of the other conditions controls failure.

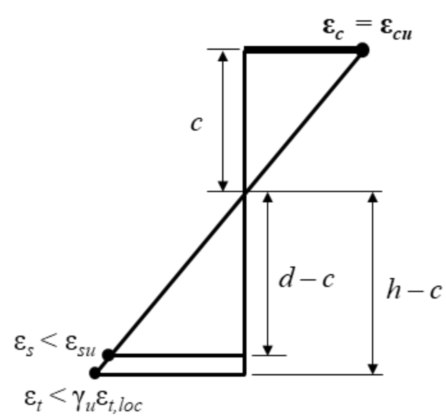
### *Concrete Crushing Failure ($\varepsilon_c = \varepsilon_{cu}$)*

The crushing of the UHPC in the extreme compression fiber, $\varepsilon_c = \varepsilon_{cu}$, may control for sections that are heavily reinforced (with high reinforcement or prestressing ratios). The strain in the bottom fiber is input as a function of the neutral axis depth, assuming a linear strain profile, as shown in Figure 23.

For concrete crushing: $\varepsilon_t = \varepsilon_{cu} \times (h - c) / c$

To check the capacity of this failure type, the extreme compression fiber strain is set equal to the strain at concrete crushing: $\varepsilon_c = \varepsilon_{cu}$. Then, the neutral axis depth is modified until equilibrium is satisfied using the strain compatibility approach. There may not be a solution for the neutral axis depth that results in equilibrium being satisfied if concrete crushing does not control failure.

The extreme tension fiber strain should be compared to the crack localization strain ($\varepsilon_t \leq \gamma_u \varepsilon_{t,loc}$) and the strain at the extreme layer of tension reinforcement compared to the rupture strain ($\varepsilon_s \leq \varepsilon_{su}$) to ensure that another failure mechanism does not control.




Source: FHWA.

**Figure 23. Illustration. Strain profile at concrete crushing.**

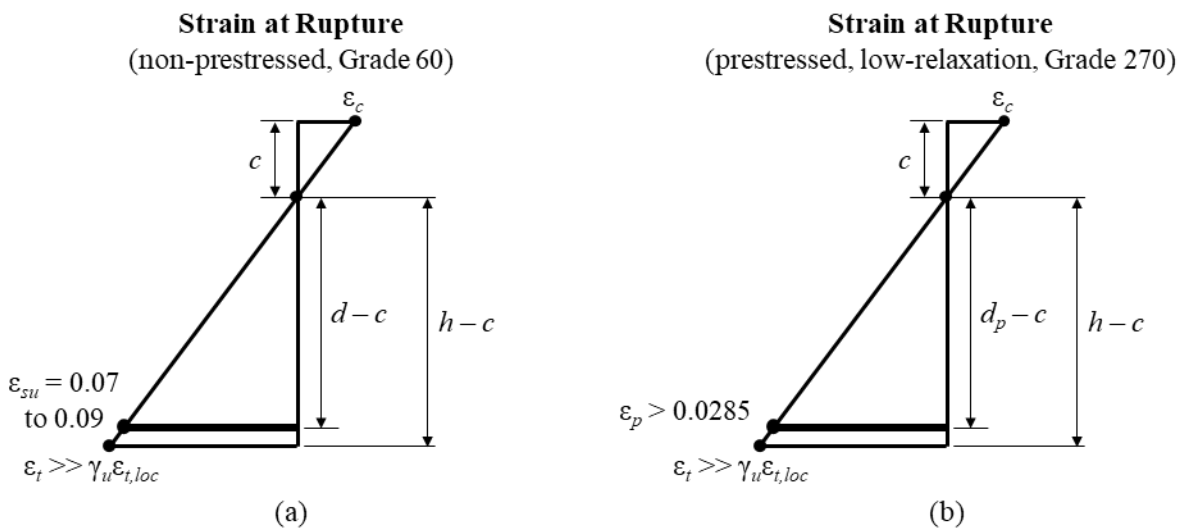
### *Rupture of Reinforcement ($\varepsilon_s = \varepsilon_{su}$ or $\varepsilon_{ps} = \varepsilon_{pu}$)*

The rupture of the reinforcement ($\varepsilon_s = \varepsilon_{su}$ or $\varepsilon_{ps} = \varepsilon_{pu}$) may control for low-strain ductility reinforcement when the UHPC has a high crack localization strain capacity. The strain in the bottom fiber is input as a function of the neutral axis depth, assuming a linear strain profile.

For rupture of non-prestressed reinforcement: $\varepsilon_t = \varepsilon_{su} \times (h - c) / (d - c)$

To check the capacity of this failure type, the strain at the centroid of the reinforcement is set equal to the rupture strain of the reinforcement, including the locked-in strain differential for prestressed reinforcement. Then, the neutral axis depth is modified until equilibrium is satisfied using the strain compatibility approach. There may not be a solution for the neutral axis depth that results in equilibrium being satisfied if the rupture of reinforcement does not control failure.

The rupture strain for conventional Grade 60 steel reinforcement is typically between $\varepsilon_{su}$ = 0.07 for larger rebar and 0.09 for smaller rebar, based on minimum elongations specified by ASTM A615, which would lead to extreme tension fiber strains much higher than the crack localization strain, as shown in Figure 24 (a). It is not anticipated that the rupture of reinforcement would control for conventional steel reinforcement.




Source: FHWA.

**Figure 24. Illustration. Strain profiles at rupture for (a) non-prestressed reinforcement (Grade 60) and (b) prestressed reinforcement (Grade 270, low-relaxation).**

The locked-in strain differential after all prestress losses, $\varepsilon_{pe}$, needs to be included when considering the rupture of prestressing strands.

For rupture of prestressed reinforcement: $\varepsilon_t = (\varepsilon_{pu} - \varepsilon_{pe}) \times (h - c) / (d - c)$

The stress in prestressing strands after all losses would typically vary between 150 ksi and 185 ksi, which corresponds to a locked-in strain differential of between 0.00526 and 0.00649, for $E_p$ = 28,500 ksi. The rupture strain of low-relaxation prestressing strands with $f_{pu}$ = 270 ksi can be assumed to be 0.035, which is the minimum elongation permissible in AASHTO M 203M. This would correspond to a minimum strain in the concrete at the level of the prestressing at rupture of 0.0285. Commonly, this would still lead to bottom fiber strains at strand rupture that are much higher than the crack localization, as shown in Figure 24 (b).

## Resistance Factors

The resistance factor in a UHPC member is calculated based on the curvature ductility ($\mu$), which is the ratio of the curvature at nominal flexural strength to the curvature at the steel service stress limit, which was described above.

Curvature ductility: $\mu = \psi_n / \psi_{s\ell}$ UHPC Structural Design Guide Eqn. 6.3.2.3-1

Resistance factor: $0.75 \leq \phi = 0.75 + 0.15(\mu - 1.0) / (\mu_\ell - 1.0) \leq 0.90$

UHPC Structural Design Guide Eqn. 5.4.2-1

Curvature ductility limit: $\mu_{\ell} = 3.0$ UHPC Structural Design Guide Articles 6.2 and C6.3.2.3

The minimum reinforcement requirement specified in AASHTO LRFD BDS Article 5.6.3.3 is also required for UHPC, except that $f_{t,cr}$ is used in place of $f_r$ in AASHTO LRFD BDS Eqn. 5.6.3.3-1 and Article 5.10.6 (on shrinkage and temperature control reinforcement) need not apply.

### Minimum Reinforcement Check

The minimum reinforcement for flexural resistance is specified in UHPC Structural Design Guide Article 6.3.3, which refers to AASHTO LRFD BDS Article 5.6.3.3 with several amendments. The factored flexural resistance, $M_r$, needs to be greater than or equal to the lesser of $1.33M_u$ and the cracking moment, $M_{cr}$.

The cracking moment is calculated using AASHTO LRFD BDS Eqn. 5.6.3.3-1 using the effective cracking strength, $f_{t,cr}$, in place of the rupture strength, $f_r$.

Cracking moment: $M_{cr} = \gamma_3[(\gamma_1 f_{t,cr} + \gamma_2 f_{cpe})S_c - M_{dnc}((S_c / S_{nc}) - 1]$

Additionally, the shrinkage and temperature reinforcement requirements specified in AASHTO LRFD BDS Article 5.10.6 do not apply for UHPC members.

### Basic Procedure for Design for Flexural Strength

The basic procedure for the design for flexural strength of UHPC members are as follows.

1. Calculate the demand ($M_u$) using Strength I load combination.
2. Calculate moment ($M_{s\ell}$) and curvature ($\psi_{s\ell}$) associated with service limit ($0.8\varepsilon_{sy}$ or $0.8\varepsilon_{py}$) as discussed in UHPC Structural Design Guide Articles 6.3.2.3 and 5.2.
3. Calculate moment ($M_n$) and curvature ($\psi_n$) associated with nominal flexural resistance as discussed in UHPC Structural Design Guide Article 6.3.2.2.
4. Calculate the curvature ductility ratio and the resistance factor ($\phi$) for flexure using UHPC Structural Design Guide Article 5.4.2.
5. Check: $M_r = \phi M_n > M_u$.
6. Check minimum reinforcement requirement using UHPC Structural Design Guide Article 6.3.3.

The moment and curvature at the service limit and associated with the nominal flexural resistance are calculated using the strain compatibility approach detailed in the preceding sections.

## STRENGTH LIMIT STATE – AXIALLY-LOADED MEMBERS

### Summary of Design Principles

A summary of the provisions related to axially-loaded members for conventional concrete and UHPC members is provided in Table 9.

**Table 9. Provisions for axially-loaded members.**

| Description | Conventional Concrete | AASHTO LRFD BDS Reference | UHPC | UHPC Structural Design Guide Reference |
|---|---|---|---|---|
| Compression members | General specifications | Article 5.6.4.1 to 5.6.4.7 | Same as conventional concrete with a few exceptions. References to Article 5.5.4.2 and 5.10.4 shall be replaced with UHPC Structural Design Guide Articles 5.4.2 and 10.4, respectively. | Article 6.4 |
| Bearing | General specifications | Article 5.6.5 | Same as conventional concrete. | Article 6.5 |
| Tension members | General specifications. | Article 5.6.6 | Several differences from conventional concrete. | Article 6.6 |
| Design objective | $P_r = \phi P_n \geq P_u$ | Article 5.6.6.1 | Same as conventional concrete. | Article 6.6.1 |
| Tensile resistance | Assume axial forces only resisted by steel elements. | Article 5.6.6.1 | Axial forces resisted by UHPC and steel elements: $P_n = P_{UHPC} + P_s$ . | Eqn. 6.6.1-2 |
| Combined flexure and tension | Use Article 5.6.2 without any modifications. | Article 5.6.6.2 | Use Article 1.6.3 with several modifications:<br>• Reduce $f_{t,cr}$ and $f_{t,loc}$ by 40%.<br>• Reduce $\varepsilon_{t,loc}$ by 50%.<br>• Reduce $f_{py}$ and $f_y$ by 20%.<br>• Reduce $f_{pu}$ by 20%. | Article 6.6.2 |
| Resistance factors | Specified based on type of component and whether the section is tension-controlled or compression-controlled. | Article 5.5.4.2 | Separate resistance factors specified for compression, tension members, and members subjected to combined tension and flexure ($\phi$ = 0.75). | Article 5.4.2 |

The provisions for UHPC compression components are the same as conventional concrete members provided in AASHTO LRFD BDS Article 5.6.4.1 through 5.6.4.6 and Article 5.6.4.7.1 with three exceptions:

- The resistance factors should be calculated according to UHPC Structural Design Guide Article 5.4.2 instead of AASHTO LRFD BDS Article 5.5.4.2,
- The limit on the design compression strength specified in AASHTO LRFD BDS Article 5.10.4.3 does not apply as specified in UHPC Structural Design Guide Article 10.4, and
- The clear spacing between bars of spiral reinforcement is modified as specified in UHPC Structural Design Guide Article 10.4.

The factored axial resistance is calculated using AASHTO LRFD BDS Eqn. 5.6.4.4-2 for members with spiral reinforcement and Eqn. 5.6.4.4-3 for members with tie reinforcement.

Factored compression resistance: $P_r = \phi P_n$ AASHTO LRFD BDS Eqn. 5.6.4.4-1

Nominal compression resistance (members with spiral reinforcement):

$$P_n = 0.85[k_c f'_c(A_g - A_{st} - A_{ps}) + f_y A_{st} - A_{ps}(f_{pe} - E_p \varepsilon_{cu})]$$

AASHTO LRFD BDS Eqn. 5.6.4.4-2

Nominal compression resistance (members with tie reinforcement):

$$P_n = 0.80[k_c f'_c(A_g - A_{st} - A_{ps}) + f_y A_{st} - A_{ps}(f_{pe} - E_p \varepsilon_{cu})]$$

AASHTO LRFD BDS Eqn. 5.6.4.4-3

The factor accounting for the ratio of the maximum concrete compressive stress to the design compressive strength of concrete, $k_c$, should be calculated as specified in AASHTO LRFD BDS Article 5.6.4.4 as follows.

$k_c$ factor: $k_c = 0.75 \leq (0.85 - 0.02(f'_c - 10\text{ ksi})) \leq 0.85$

For the minimum allowable compressive strength for UHPC ($f'_c = 17.5$ ksi), the lower limit will control.

$k_c$ factor ($f'_c = 17.5$ ksi): $k_c = 0.75 \leq (0.85 - 0.02(17.5\text{ ksi} - 10\text{ ksi})) = 0.70 \leq 0.85$

$k_c = 0.75$

The AASHTO LRFD BDS provisions on bearing in Article 5.6.5 also apply for UHPC members.

The resistance for UHPC tension members includes a contribution from the UHPC and steel elements, compared to only steel elements for conventional concrete members. The factored resistance is calculated using the following equations.

Factored tension resistance: $P_r = \phi P_n$ UHPC Structural Design Guide Eqn. 6.6.1-1

Nominal tension resistance: $P_n = P_{UHPC} + P_s$

UHPC Structural Design Guide Eqn. 6.6.1-2

UHPC tension component: $P_{UHPC} = 0.60\gamma_u f_{t,cr} A_g$

UHPC Structural Design Guide Eqn. 6.6.1-3

Steel tension component: $P_s = 0.50 E_s \gamma_u \varepsilon_{t,loc} A_s + A_{ps}[f_{pe} + 0.50 E_s \gamma_u \varepsilon_{t,loc}]$

UHPC Structural Design Guide Eqn. 6.6.1-4

There are several notes on these equations as follows.

- The gross area used in UHPC Structural Design Guide Eqn. 6.6.1-3 should be calculated deducting the area of the prestressed and non-prestressed reinforcement in the section.

- The sum of $f_{pe} + 0.50E_s\gamma_u\varepsilon_{t,loc}$ shall not be taken greater than 80 percent of the yield strength of the prestressing, $0.80f_{py}$.
- The term $0.50E_s\gamma_u\varepsilon_{t,loc}$ shall not be taken as greater than 80 percent of the yield strength of the non-prestressed longitudinal steel, $0.80f_y$.
- The nominal resistance of the steel contribution, $P_s$, shall exceed $0.80P_n$.
- The provisions of AASHTO LRFD BDS Article 5.10.8.4.4 shall apply. These provisions require splices in tie members to be made only with full-welded splices or full-mechanical connections.

The resistance factors for UHPC axially loaded members are specified in UHPC Structural Design Guide Article 5.4.2 and are dependent on the type of loading, as summarized in Table 10.

**Table 10. Summary of resistance factors for axially loaded members, based on UHPC Structural Design Guide Article 5.4.2.**

| Type of Loading | Resistance Factor |
|---|---|
| Pure compression | $\phi = 0.75$ |
| Combined axial compression and flexure | $\phi$ based on curvature ductility of section (varies between 0.75 and 0.90). |
| Pure flexure | $\phi$ based on curvature ductility of section (varies between 0.75 and 0.90). |
| Combined axial tension and flexure | $\phi = 0.75$ |
| Pure tension | $\phi = 0.75$ |

A moment-axial load interaction diagram can be developed for UHPC elements using these provisions to design of axially-loaded members.

### Moment-Axial Load Interaction Diagram

A moment-axial load interaction diagram can be developed using the strain compatibility approach with the same fundamental concepts and basic steps as described in the previous section on flexure. The basic procedure for creating a moment-axial load interaction diagram will be described using Storyline Example #3.

The basic procedure for calculating each point on the moment-axial load interaction diagram is as follows.

- **Step 1**: Select a neutral axis depth, $c$. Each neutral axis depth will be associated with a combination of moment and axial force.
- **Step 2**: If $c \leq \varepsilon_{cu} / ((\varepsilon_{cu} + \gamma_u\varepsilon_{t,loc}) \times h)$, then crack localization will control the failure and $\varepsilon_t = \gamma_u\varepsilon_{t,loc}$. Otherwise, concrete crushing controls failure and $\varepsilon_c = \varepsilon_{cu}$. This strain is used with the assumed neutral axis depth from Step 1 to define the associate strain profile.
- **Step 3**: Determine the stress profile in the UHPC and stress in each layer of non-prestressed reinforcement and prestressing strands using the defined stress-strain relationships for each material.
- **Step 4**: Calculate the axial load, $P$, and moment, $M$, using equilibrium equations. Apply the appropriate limits for compression (AASHTO LRFD BDS Eqn. 5.6.4.4-2 for members

with spiral transverse reinforcement or AASHTO LRFD BDS Eqn. 5.6.4.4-3 for members with tie reinforcement) and tension (UHPC Structural Design Guide Eqn. 6.6.1-2). This *P* and *M* are for the neutral axis depth assumed in Step 1.

- **Step 5**: Repeat this process for enough different neutral axis depths to create a complete moment-axial load interaction diagram. Varying the neutral axis depth between a range of around -2*h* to 5*h* will typically give a complete curve.

For Storyline Example #3, the compression limit is calculated using AASHTO LRFD BDS Eqn. 5.6.4.4-3 as follows.

$k_c$ factor ($f'_c = 17.5$ ksi): $k_c = 0.75 \leq (0.85 - 0.02(17.5 \text{ ksi} - 10 \text{ ksi})) = 0.70 \leq 0.85$

$k_c = 0.75$

Nominal compression resistance (members with tie reinforcement):

$$P_n = 0.80[k_c f'_c(A_g - A_{st} - A_{ps}) + f_y A_{st} - A_{ps}(f_{pe} - E_p \varepsilon_{cu})]$$

$$P_n = 0.80[(0.75)(17.5 \text{ ksi})(412.5 \text{ inch}^2 - 3.038 \text{ inch}^2) - (3.038 \text{ inch}^2)(163.6 \text{ ksi} - (28{,}500 \text{ ksi})(0.0035 \text{ ksi})] = 4{,}144 \text{ kips}$$

The tensile limit is calculated using UHPC Structural Design Guide Article 6.6.1, where the nominal resistance of the steel contribution, $P_s$, shall exceed $0.80P_n$. For this example, the contribution of each term is as follows:

UHPC tension component: $P_{UHPC} = 0.60\gamma_u f_{t,cr} A_g = 0.60(1.0)(1.0 \text{ ksi})(409.5 \text{ inch}^2)$

$P_{UHPC} = 245.7$ kips

Steel tension component: $P_s = 0.50E_s\gamma_u\varepsilon_{t,loc}A_s + A_{ps}[f_{pe} + 0.50E_s\gamma_u\varepsilon_{t,loc}]$

Limit on term: $163.6 \text{ ksi} + 0.50(29{,}000 \text{ ksi})(1.0)(0.005) \leq 0.8f_{py} = 0.8(243 \text{ ksi}) = 194.4 \text{ ksi}$

$P_s = (3.038 \text{ inch}^2)[194.4 \text{ ksi}] = 590.6$ kips

Nominal tension capacity: $P_n = P_{UHPC} + P_s = 245.7 \text{ kips} + 590.6 \text{ kips} = 836.3 \text{ kips}$

The percentage of each component to the overall capacity is as follows.

UHPC tension component percentage: $(P_{UHPC} / P_n) = (245.7 \text{ kips} / 836.3 \text{ kips}) = 29.7\%$

Steel tension component percentage: $(P_s / P_n) = (590.6 \text{ kips} / 836.3 \text{ kips}) = 70.6\%$

The percentage of contribution for the UHPC is limited to 20 percent of the nominal tension capacity, so the contribution of this component will need to be reduced. This limit can be considered using the following if/then statement.

Nominal tension capacity (considering limits on UHPC contribution to strength):

Solution for $P_n \rightarrow$ if $(P_{UHPC} / P_n) \leq 0.2P_n$, then $(P_n = P_{UHPC} + P_s)$, otherwise $(P_n = 1.25P_s)$

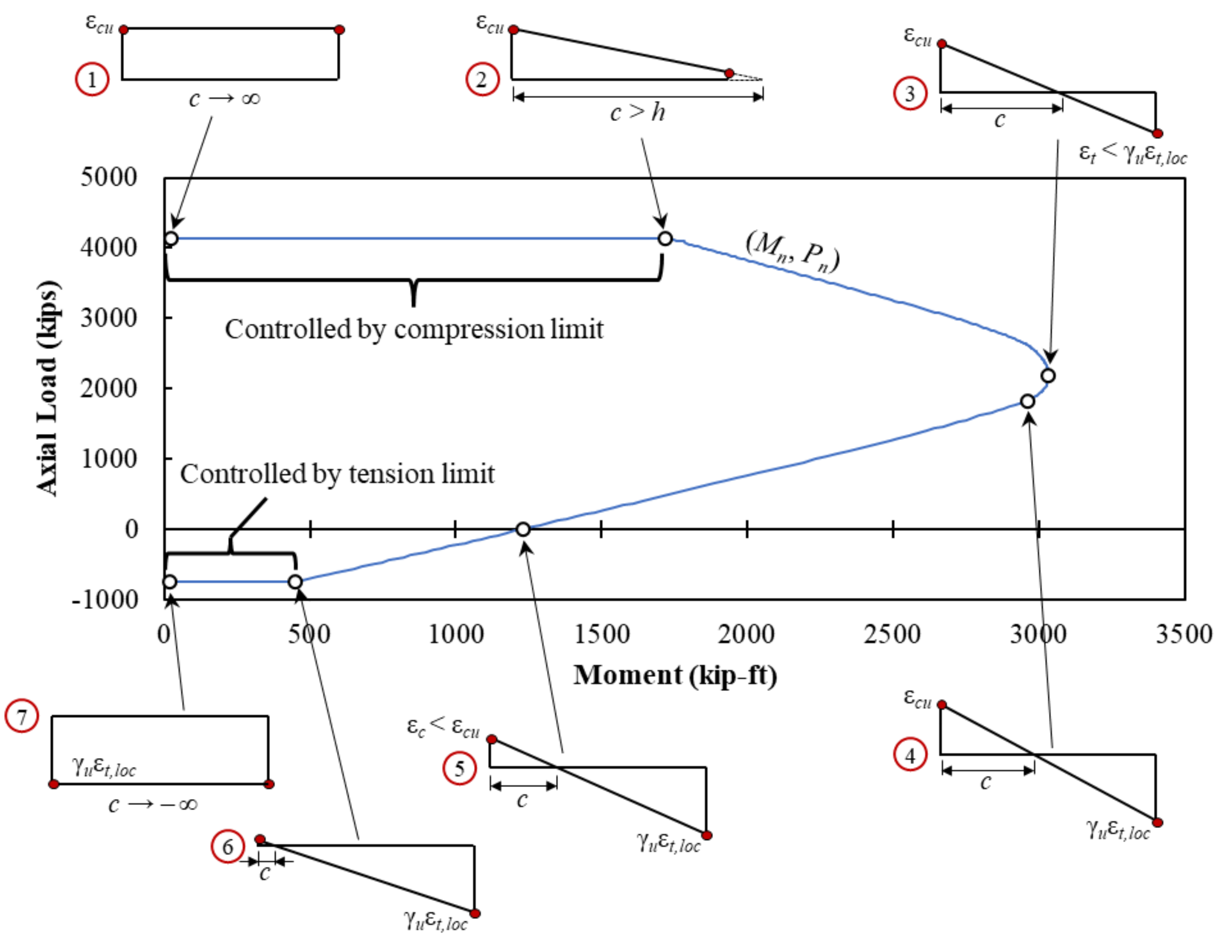
This simplifies to the following.

Nominal tension capacity (considering limits on UHPC contribution to strength):

$$P_n = P_{UHPC} + P_s \leq 1.25P_s = 1.25(590.6 \text{ kips}) = 738.2 \text{ kips}$$

This is the axial load limit in tension for Storyline Example #3.

The moment-axial load interaction diagram for Storyline Example #3 including strain diagrams at some of the key points is shown in Figure 25. This diagram includes the compression and tensile force limits.




Source: FHWA.

**Figure 25. Illustration. Example of moment-axial load interaction diagram for Storyline Example #3 with strain profiles associated with different points.**

The moment-axial load interaction diagram starts in compression with a constant strain profile across the width of the section with the strain equal to the ultimate compressive strain, $\varepsilon_c = \varepsilon_t = \varepsilon_{cu}$, as shown in Point 1 in Figure 25. The axial force is capped by the compression force limit calculated using AASHTO LRFD BDS Eqn. 5.6.4.4-3. The moment increases while having a constant axial load until the compression force limit no longer controls the axial load, at Point 2. The compression strain in the extreme compression face is still equal to the ultimate compressive strain, $\varepsilon_c = \varepsilon_{cu}$, while the compression in the other face decreases with increasing moment.

After Point 2, the axial load begins to decrease with increasing moment until the maximum moment is reached at Point 3. For Storyline Example #3, the maximum moment point is associated with the strain in the compression face equal to the ultimate compressive strain, $\varepsilon_c = \varepsilon_{cu}$; the strain in the extreme tension fiber is less than the crack localization strain, $\varepsilon_t < \gamma_u \varepsilon_{t,loc}$. After this point, the moment begins to decrease as the axial force decreases.

Point 4 is associated with a strain profile where the strain in the extreme compression face is equal to the ultimate compressive strain at the same time the strain in the extreme tension face is equal to the crack localization strain, $\varepsilon_c = \varepsilon_{cu}$ and $\varepsilon_t = \gamma_u \varepsilon_{t,loc}$. This may be considered the balanced point but will not necessarily be associated with the maximum moment. Point 5 is the pure moment point where the capacity of the section is controlled by the strain in the extreme tension fiber being equal to the crack localization strain, $\varepsilon_t = \gamma_u \varepsilon_{t,loc}$.

The moment will continue to decrease as the axial load increases in tension until the tension limit in the member is reached, Point 6 in Figure 25. The axial load will remain constant as the moment is decreased to 0 kip-feet at Point 7; this cap on the tensile capacity is based on the tensile force limit calculated using UHPC Structural Design Guide Eqn. 6.6.1.2. The strain profile at Point 7 will have a constant strain equal to the crack localization strain, $\varepsilon_c = \varepsilon_t = \gamma_u \varepsilon_{t,loc}$.

The factored moment-axial load interaction diagram for Storyline Example #3 is shown in Figure 26.

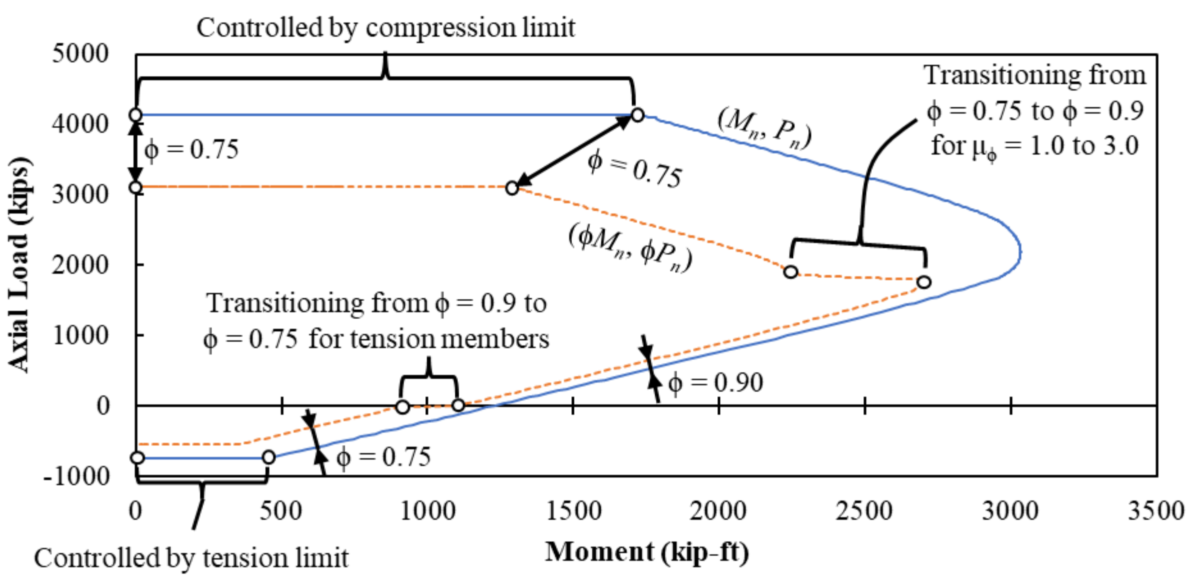



**Figure 26. Graph. Example of factored moment-axial load interaction diagram for Storyline Example #3.**

The factored moment-axial load interaction diagram generally has the same shape as for conventional concrete. A few of the key aspects of the factored diagram are highlighted in Figure 26 and summarized as follows.

- The diagram has a cap on the compressive strength based on AASHTO LRFD BDS Eqn. 5.6.4.4-3 (for members with tie reinforcement) and a strength reduction factor of 0.75 in this region; these design caps are identical to those used for conventional concrete.
- The strength of the axially-loaded member is reduced by 0.75 for the compression region where $\mu_\phi$ is less than 1.0. The transition between $\phi = 0.75$ to 0.90 for $\mu_\phi = 1.0$ to 3.0 creates the nose of the diagram that is typical for convention concrete members.
- The strength of the axially loaded member is reduced by 0.90 for $\mu_\phi \geq 3.0$ until the point of pure flexure (axial load equal to zero). At this point there is an immediate change in the strength reduction factor from $\phi = 0.90$ to 0.75 when axial tension is present in the member.
- The strength reduction factor for UHPC elements with any axial tension is $\phi = 0.75$.
- There is a cap on the tensile strength of the axially loaded member based on UHPC Structural Design Guide Eqn. 6.6.1-2 with a strength reduction factor of 0.75 in this region.

The factored moment-axial load interaction diagram shown in Figure 26 would be used in the design process for the UHPC H-pile from Storyline Example #3.

## STRENGTH LIMIT STATE – SHEAR (B REGIONS)

A summary of the strength limit state provisions related to shear capacity for conventional concrete and UHPC members is provided in Table 11.

**Table 11. Provisions for strength limit state (shear) for conventional concrete and UHPC sections.**

| Description | Conventional Concrete | AASHTO LRFD BDS Reference | UHPC | UHPC Structural Design Guide Reference |
|---|---|---|---|---|
| Regions requiring transverse reinforcement | Transverse reinforcement shall be provided where: $V_u > 0.5\phi(V_c + V_p)$ | Article 5.7.2.3 | Transverse reinforcement shall be provided where: $V_u > \phi(V_{UHPC} + V_p)$ | Article 7.2.3 |
| Minimum transverse reinforcement | $A_v \geq 0.0316\lambda\sqrt{f'_c} \times (b_v s/f_y)$ | Article 5.7.2.5 | Need not be provided where not required for strength. | Article 7.2.5 |
| Maximum spacing of transverse reinforcement | Varies based on shear stress:<br>$s_{max} = 0.8d_v \leq 24.0$ inches if $v_u < 0.125f'_c$<br>$s_{max} = 0.4d_v \leq 12.0$ inches if $v_u \geq 0.125f'_c$ | Article 5.7.2.6 | When transverse reinforcement required:<br>$s_{max} = 0.25d_v \cot\theta \leq 24.0$ inches | Article 7.2.6 |
| Shear stress | Determined based on:<br>$v_u = \text{abs}(V_u - \phi V_p) / (\phi b_v d_v)$ | Article 5.7.2.8 | Same as for conventional concrete. | Article 7.2.8 |
| Shear depth | Distance between resultants of tensile and compressive force, need not taken less than the greater of $0.9d_e$ and $0.72h$ | Article 5.7.2.8 | Same as for conventional concrete; except $d_v$ should be measured from the bottom of the conventional concrete deck for composite sections. | Article 7.2.8 |
| Nominal shear resistance | $V_n = V_c + V_s + V_p$ where $V_n \leq 0.25f'_c\, b_v\, d_c + V_p$ | Article 5.7.3.3 | $V_n = V_{UHPC} + V_s + V_p$ where $V_n \leq 0.25f'_c\, b_v\, d_v + V_p$ | Article 7.3.3 |
| Concrete contribution | $V_c = 0.0316\beta\lambda\sqrt{f'_c} \times b_v\, d_v$ | Eqn. 5.7.3.3-3 | $V_{UHPC} = \gamma_u\, f_{t,loc}\, b_v\, d_v \cot\theta$ | Eqn. 7.3.3-3 |
| Steel contribution (for $\alpha = 90°$) [a] | $V_s = (A_v\, f_y\, d_v \cot\theta / s)\, \lambda_{duct}$ | Eqn. C5.7.3.3-1 | $V_s = A_v\, f_{v,\alpha}\, d_v \cot\theta / s$ | Eqn. C7.3.3-1 |
| General approach for finding $\beta$ and $\theta$ | Solve for $\beta$ and $\theta$ directly based on the net longitudinal tensile strain ($\varepsilon_s$). Assume that shear reinforcement yields ($f_v = f_y$). | Article 5.7.3.4.2 | Iterative procedure to solve for $\theta$ and $f_{v,\alpha}$ or simplified approach using provided design tables. | Article 7.3.4.1 |
| Longitudinal reinforcement (inside edge of bearing) | $A_s f_y + A_{ps} f_{ps} \geq (V_u / \phi_v - 0.5V_s - V_p) \cot\theta$<br>Equation assumes yielding of non-prestressed reinforcement. | Eqn. 5.7.3.5-2 | $A_{ps} f_{ps} + A_s E_s \gamma_u \varepsilon_{t,loc} + 0.6A_{ct}\gamma_u f_{t,cr} \geq (V_u / \phi_v - 0.5V_s - V_p) \cot\theta$, where $A_s E_s \gamma_u \varepsilon_{t,loc} \leq A_s f_y$ | Eqn. 7.3.5-2 |

[a] The UHPC Structural Design Guide is currently not intended to be used for post-tensioned members, so $\lambda_{duct}$ is currently not included in UHPC Structural Design Guide Eqn. C7.3.3-1.

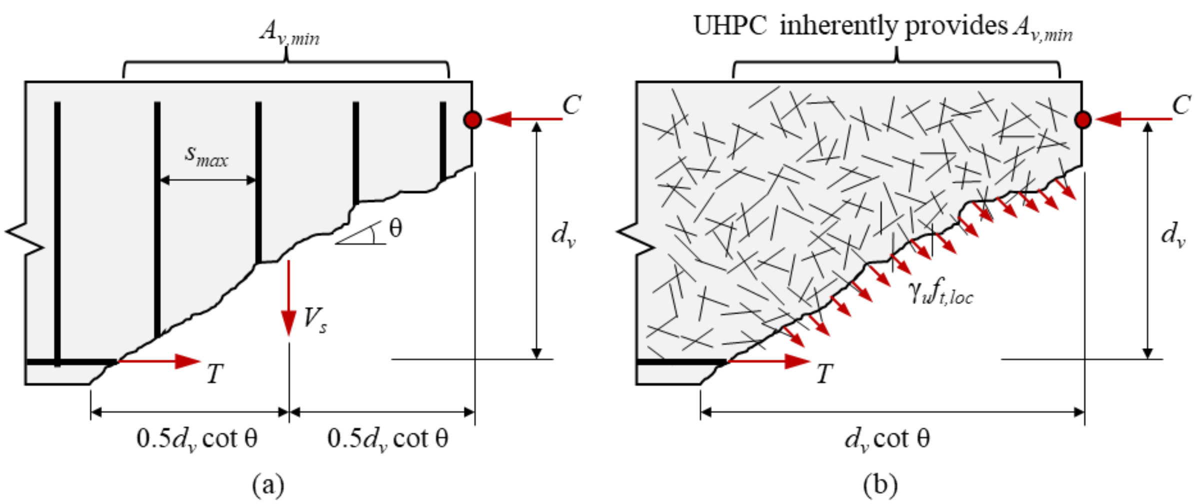
Some of the more significant differences between shear design of conventional concrete and UHPC members are described in more detail in the following sections.

**Minimum Reinforcement and Maximum Spacing**

The first difference is related to when transverse reinforcement is required. Conventional concrete members require a minimum amount of transverse reinforcement to limit crack widths and ensure distributed shear cracking, as shown in Figure 27 (a). For UHPC members, transverse reinforcement is only required when needed to provide shear strength because the distribution of steel fiber addresses crack control.

Min. reinforcement required if: $V_u > \phi(V_{UHPC} + V_p)$

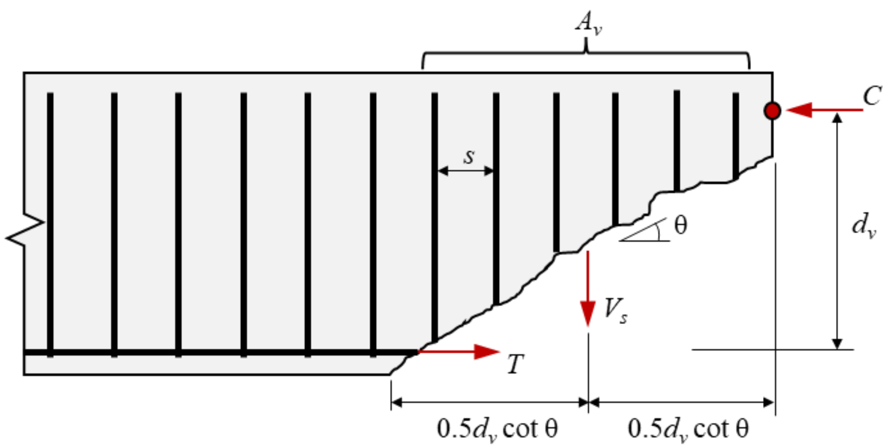
Otherwise, the post-cracking tensile resistance resulting from the steel fibers in the UHPC provides sufficient resistance in shear, illustrated in Figure 27 (b). This post-cracking resistance for UHPC is also the reason behind eliminating the minimum transverse reinforcement requirements for UHPC members.




Source: FHWA.

**Figure 27. Illustration. Free body diagram at shear crack with (a) typical minimum reinforcement provided by transverse reinforcement for conventional concrete members and (b) equivalent minimum reinforcement provided by steel fibers.**

The maximum spacing requirement in the UHPC Structural Design Guide is calculated based on the crack angle θ to help ensure that transverse reinforcement crosses a shear crack when it is required. Some of the key parameters used for calculating the contribution of transverse reinforcement to the shear strength are illustrated in Figure 28. The horizontal length of the shear crack is $d_v \cot\theta$. Providing transverse reinforcement at a maximum spacing of $s_{max} = 0.25d_v \cot\theta$ ensures that four legs of reinforcement cross the shear crack.




Source: FHWA.

**Figure 28. Illustration. Key parameters for calculating contribution of shear reinforcement.**

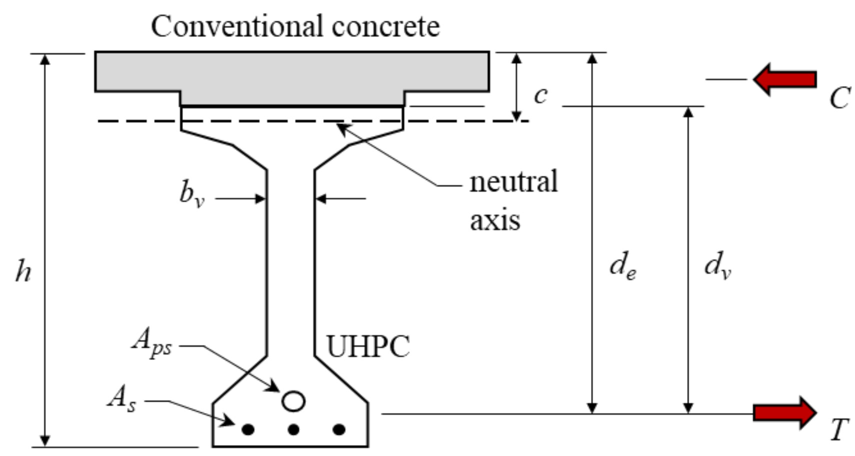
For a common crack angle (e.g., 28 degrees), the maximum spacing requirement is calculated as follows.

Max. spacing for typical crack angle:

$$s_{max} = 0.25d_v \cot(28°) \leq 24.0 \text{ inches} \rightarrow 0.47d_v \leq 24.0 \text{ inches}$$

This is a tighter maximum spacing than is required for conventional concrete members with shear stress less than $0.125f'_c$.

The shear depth for UHPC members is calculated using a similar methodology as conventional concrete members. The effective shear depth $d_v$ is calculated as the distance measured between the resultants of the tensile and compressive forces due to flexure, shown in Figure 28, and it need not be less than the greater of $0.9d_e$ and $0.72h$. The effective shear depth is limited to the bottom of the composite deck (i.e., only within the UHPC section) for composite members with conventional concrete decks on UHPC members, as shown in Figure 29. This is because the assumptions related to the shear resistance of UHPC are not valid for the conventional concrete deck.




Source: FHWA.

**Figure 29. Illustration of terms $b_v$, $d_v$, and $d_e$ for composite sections with UHPC beams and conventional concrete decks showing the maximum allowable $d_v$ in composite sections, based on UHPC Structural Design Guide Figure C7.2.8-3.**

## Nominal Shear Resistance

The nominal shear resistance for UHPC members is the summation of the shear contributions of the UHPC, transverse reinforcement, and the vertical resultant of inclined prestressing strands.

UHPC Shear Resistance: $V_n = V_{UHPC} + V_s + V_p$ UHPC Structural Design Guide Eqn. 7.3.3-1

The total shear resistance is limited by the following equation, which is the same as conventional concrete.

Upper limit for shear resistance: $V_n \leq 0.25 f'_c\, b_v\, d_v + V_p$
UHPC Structural Design Guide Eqn. 7.3.3-2

The UHPC and steel contributions to the shear resistance are dependent on the angle of inclination of the compressive stresses, θ, and the stress in the transverse reinforcement, $f_{v,\alpha}$, which are found using an iterative procedure.

UHPC contribution to shear resistance: $V_{UHPC} = \gamma_u\, f_{t,loc}\, b_v\, d_v \cot\theta$
UHPC Structural Design Guide Eqn. 7.3.3-3

Transverse reinforcement contribution to shear resistance (for α = 90°):
$V_s = A_v\, f_{v,\alpha}\, d_v \cot\theta / s$ UHPC Structural Design Guide Eqn. C7.3.3-1

## Iterative Process for Calculating θ and $f_{v,\alpha}$

The process for calculating θ and $f_{v,\alpha}$ is as follows.

- **Step 1**: Calculate the net longitudinal tensile strain in the section at the centroid of the tension reinforcement using UHPC Structural Design Guide Eqn. 7.3.4.1-6 if $\varepsilon_s > \varepsilon_{t,cr}$ or UHPC Structural Design Guide Eqn. 7.3.4.1-7 if $\varepsilon_s \le \varepsilon_{t,cr}$.

If $\varepsilon_s > \varepsilon_{t,cr}$: UHPC Structural Design Guide Eqn. 7.3.4.1-6

$$\varepsilon_s = \frac{\left(\frac{|M_u|}{d_v} + 0.5N_u + |V_u - V_p| - A_{ps} f_{po} - \gamma_u f_{t,cr} A_{ct}\right)}{E_s A_s + E_p A_{ps}}$$

If $\varepsilon_s \le \varepsilon_{t,cr}$: UHPC Structural Design Guide Eqn. 7.3.4.1-7

$$\varepsilon_s = \frac{\left(\frac{|M_u|}{d_v} + 0.5N_u + |V_u - V_p| - A_{ps} f_{po}\right)}{E_s A_s + E_p A_{ps} + E_c A_{ct}}$$

- **Step 2**: Assume details for transverse reinforcement (e.g., $A_v$, $s$, $E_s$).
- **Step 3**: Assume value for stress in transverse reinforcement $f_{v,\alpha,assumed}$. A good initial assumption is $f_{v,\alpha,assumed} = f_y$. This step is skipped if $A_v$ = 0 inch$^2$.
- **Step 4**: Solve for the principal angle direction, θ, using UHPC Structural Design Guide Eqn. 7.3.4.1-1 and Eqn. 7.3.4.1-5 and $f_{v,\alpha} = f_{v,\alpha,assumed}$.

UHPC Structural Design Guide Eqn. 7.3.4.1-1:

$$\gamma_u \varepsilon_{t,loc} = \frac{\varepsilon_s}{2}(1 + \cot^2\theta) + \frac{2f_{t,loc}}{E_c}\cot^4\theta + \frac{2\rho_{v,\alpha} f_{v,\alpha}}{E_c}\cot^2\theta\,(1 + \cot^2\theta)$$

UHPC Structural Design Guide Eqn. C7.3.4.1-4 (for α = 90°): $\rho_{v,\alpha} = A_v / (b_v s)$

- **Step 5**: Check the assumed stress in the transverse reinforcement by calculating $f_{v,\alpha}$ using UHPC Structural Design Guide Eqn. 7.3.4.1-2, Eqn. 7.3.4.1-3, and Eqn. 7.3.4.1-4. This step is skipped if $A_v$ = 0 inch$^2$.

UHPC Structural Design Guide Eqn. 7.3.4.1-2:

$$\varepsilon_2 = -\frac{2f_{t,loc}}{E_c}\cot^2\theta - \frac{2\rho_{v,\alpha} f_{v,\alpha}}{E_c}(1 + \cot^2\theta)$$

UHPC Structural Design Guide Eqn. 7.3.4.1-3: $\varepsilon_v = \gamma_u \varepsilon_{t,loc} - 0.5\varepsilon_s + \varepsilon_2$

UHPC Structural Design Guide Eqn. C7.3.4.1-3 (for α = 90°): $f_{v,\alpha} = E_s \varepsilon_v \le f_y$

- **Step 6**: Check that calculated $f_{v,\alpha}$ in Step 5 equals $f_{v,\alpha,assumed}$. Return to Step 3 if they are not equal using the calculated $f_{v,\alpha}$ from Step 5 as the new assumed value. This step is skipped if $A_v$ = 0 inch$^2$.
- **Step 7**: Calculate resistance ($V_r$) using the calculated θ and $f_{v,\alpha}$ to find $V_{UHPC}$ and $V_s$. Calculate $V_n$ using UHPC Structural Design Guide Eqn. 7.3.3-1 and Eqn. 7.3.3-2. Check

the factored resistance ($\phi V_n$) against the demand ($V_u$). Return to Step 2 and modify the transverse reinforcement provided as needed.

### Simplified Approach for Calculating θ and $f_{v,\alpha}$

A simplified approach for finding θ and $f_{v,\alpha}$ is provided in UHPC Structural Design Guide Article 7.3.4.2. The simplified approach can only be used for members with orthogonal transverse reinforcement ($\alpha$ = 90°) or no transverse reinforcement, $E_c \geq$ 6,500 ksi, $f_{t,loc} \leq$ 1.80 ksi, and $f_y \leq$ 75.0 ksi. The simplified approach was calibrated to always be more conservative than the detailed iterative method.

- UHPC Structural Design Guide Table B1.2-1 – Sections without transverse reinforcement ($\rho_{v,\alpha} = 0$)
- UHPC Structural Design Guide Table B1.3-1 – Section with $\rho_{v,\alpha} \leq$ 0.005
- UHPC Structural Design Guide Table B1.3-2 – Section with $\rho_{v,\alpha} \leq$ 0.010
- UHPC Structural Design Guide Table B1.3-3 – Section with $\rho_{v,\alpha} \leq$ 0.015
- UHPC Structural Design Guide Table B1.3-4 – Section with $\rho_{v,\alpha} \leq$ 0.020
- UHPC Structural Design Guide Table B1.3-5 – Section with $\rho_{v,\alpha} \leq$ 0.025
- UHPC Structural Design Guide Table B1.3-6 – Section with $\rho_{v,\alpha} \leq$ 0.030

The following steps are used for finding θ and $f_{v,\alpha}$ using the simplified approach.

- **Step 1**: Calculate the net longitudinal tensile strain in the section at the centroid of the tension reinforcement using UHPC Structural Design Guide Eqn. 7.3.4.1-6 if $\varepsilon_s > \varepsilon_{t,cr}$ or UHPC Structural Design Guide Eqn. 7.3.4.1-7 if $\varepsilon_s \leq \varepsilon_{t,cr}$.
- **Step 2**: Assume details for transverse reinforcement (e.g., $A_v$, $s$, $E_s$)
- **Step 3**: Calculate the transverse reinforcement ratio using UHPC Structural Design Guide Eqn. C7.3.4.1-4 (for $\alpha$ = 90°).

  Transverse reinforcement ratio: $\rho_{v,\alpha} = A_v / (b_v\, s)$

  UHPC Structural Design Guide Eqn. C7.3.4.1-4

- **Step 4**: Use $\varepsilon_s$ and $\gamma_u \varepsilon_{t,loc}$ to find θ and $f_{v,\alpha}$ using the appropriate design table. A brief example of how to use the design tables is provided below.

  Example known values: $\varepsilon_s$ = – 0.00023, $\gamma_u \varepsilon_{t,loc}$ = 0.0048, $\rho_{v,\alpha}$ = 0.010, $f_y$ = 60 ksi

  Use UHPC Structural Design Guide Table B1.2-1 for this example. An excerpt from the appropriate design table is shown in Table 12.

**Table 12. Excerpt from UHPC Structural Design Guide Table B1.3-2 for finding θ.**

| $\varepsilon_s \times 1{,}000$ | Parameter | … | $\gamma_u \varepsilon_{t,loc} \geq 0.0045$ | $\gamma_u \varepsilon_{t,loc} \geq 0.0050$ | … |
|---|---|---|---|---|---|
| … | … | … | … | … | … |
| $\leq -0.5$ | θ (deg) | … | 32.4 | 31.8 | … |
| | $f_{v,\alpha}$ (ksi) | … | $\leq 74.8$ | $\leq 75.0$ | … |
| $\leq 0.0$ | θ (deg) | … | 33.6 | 32.9 | … |
| | $f_{v,\alpha}$ (ksi) | … | $\leq 72.9$ | $\leq 75.0$ | … |
| … | … | … | … | … | … |

**Linear interpolation is not permitted for these tables.**

For this example, $\varepsilon_s = -0.00023 \leq 0.0 \times 1{,}000$ and $\gamma_u \varepsilon_{t,loc} = 0.0048 \geq 0.0045$, so the following values for θ and $f_{v,\alpha}$ would be used.

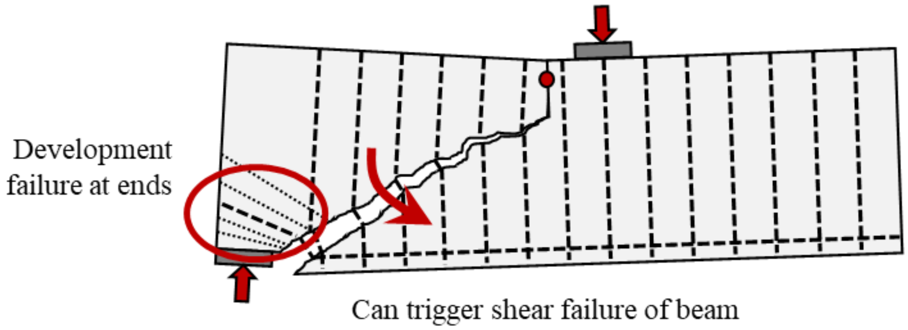
$\theta = 33.6°$

$f_{v,\alpha} \leq 72.9$ ksi → $f_{v,\alpha} = 60$ ksi (since $f_y = 60$ ksi for this example)

- **Step 5**: Calculate resistance using the calculated θ and $f_{v,\alpha}$ to find $V_{UHPC}$ and $V_s$. Calculate $V_n$ using UHPC Structural Design Guide Eqn. 7.3.3-1 and Eqn. 7.3.3-2. Check the factored resistance ($\phi V_n$) against the demand ($V_u$). Return to Step 2 and modify the transverse reinforcement provided as needed.

## Longitudinal Reinforcement

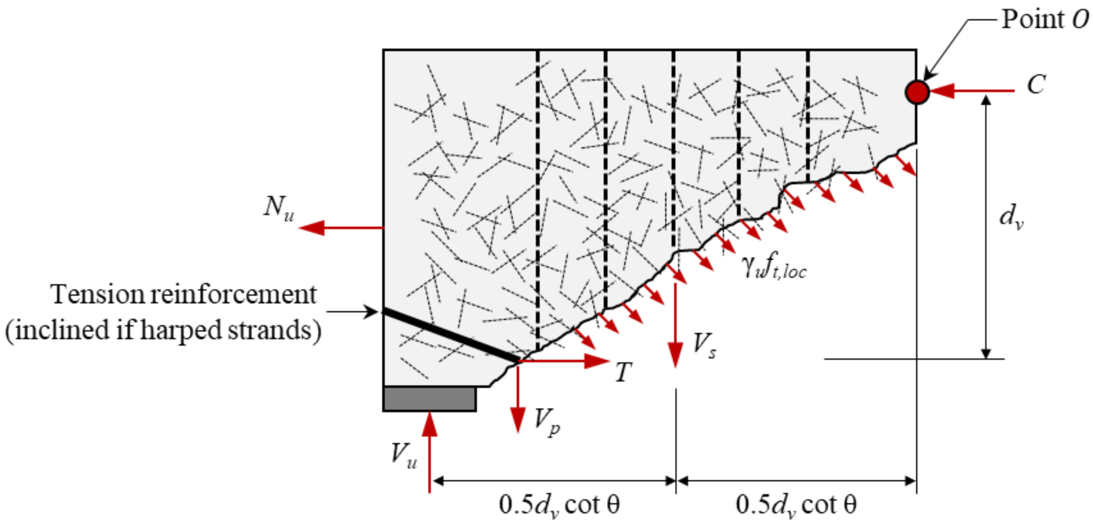
A longitudinal reinforcement check for UHPC is provided in UHPC Structural Design Guide Article 7.3.5, which is similar to the check for conventional concrete members in AASHTO LRFD BDS Article 5.7.3.5. This design check ensures that the formation of a shear crack does not trigger a development failure of the reinforcement in the flexural tension zone, illustrated in Figure 30.




Source: FHWA.

**Figure 30. Illustration. Failure mechanism prevented by the longitudinal reinforcement design check.**

The resistance to this failure mechanism is calculated based on the free body diagram of the member at a shear crack, shown in Figure 31 where the design equation is derived by summing the moments about Point O.




Source: FHWA.

**Figure 31. Illustration. Forces assumed in resistance model for longitudinal reinforcement, based on AASHTO LRFD BDS Figure C5.7.3.5-1.**

The general longitudinal reinforcement check is specified in UHPC Structural Design Guide Eqn. 7.3.5-1, which is derived based on the free-body diagram in Figure 31. The contribution of the tensile strength of the UHPC along the shear crack is conservatively ignored on the right side of the equation. The tensile resistance consists of contributions from the prestressing ($A_{ps}f_{ps}$), non-prestressed reinforcement ($A_sE_s\gamma_u\varepsilon_{t,loc}$), and UHPC on the flexural tension side of the member ($A_{ct}\gamma_u f_{t,cr}$). The non-prestressed reinforcement is limited by the yield stress, $f_s = E_s\gamma_u\varepsilon_{t,loc} \leq f_y$.

$$A_{ps} f_{ps} + A_s E_s \gamma_u \varepsilon_{t,loc} + A_{ct}\gamma_u f_{t,cr} \geq \frac{|M_u|}{d_v \phi_f} + 0.5\frac{N_u}{\phi_c} + \left(\left|\frac{V_u}{\phi_v} - V_p\right| - 0.5V_s\right)\cot\theta$$

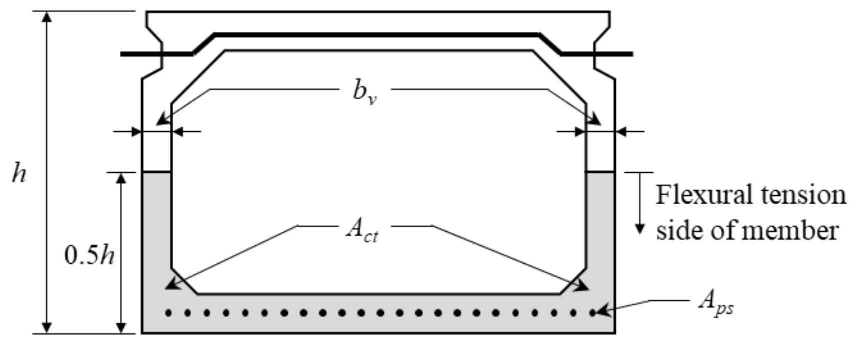
where: $A_sE_s\gamma_u\varepsilon_{t,loc} \leq A_sf_y$

The longitudinal reinforcement check will typically be most critical toward the end of the beam, where the shear is typically the highest and shorter available strand development lengths will lead to a lower calculated stress in the prestressing strands on the resistance side of the equation. The UHPC Structural Design Guide provides a separate equation for the longitudinal reinforcement check at the inside edge of the bearing area of a simple end support in Eqn. 7.3.5-2.

$$A_{ps}f_{ps} + A_sE_s\gamma_u\varepsilon_{t,loc} + 0.6A_{ct}\gamma_u f_{t,cr} \geq (V_u / \phi_v - 0.5V_s - V_p)\cot\theta$$

where: $A_sE_s\gamma_u\varepsilon_{t,loc} \leq A_sf_y$

The tensile resistance includes the UHPC and non-prestressed and prestressed reinforcement on the flexural tension side of the member. The definition of the flexural tension side of a member is illustrated in Figure 32.




Source: FHWA.

**Figure 32. Illustration. Area of UHPC and prestressing on the flexural tension side of a member for shear calculations.**

There are two primary differences for checking the longitudinal reinforcement requirement for UHPC members:

- The tension contribution of the UHPC is included in the tensile resistance force calculation. The area of the steel should be deducted from $A_{ct}$ to calculate this force.
- The stress in the conventional reinforcement may not be equal to yield at the time of failure.

The concrete area within the flexural tension zone does not affect the capacity for conventional concrete members, as the tensile contribution of conventional concrete is not included in this design check. In contrast, the area within the flexural tension zone will affect the resistance provided by the UHPC; therefore, it is important to conservatively estimate the area of the UHPC contributing to the tensile resistance. The flexural tension zone height is assumed to be calculated as the cross-sectional area of the section within 0.5$h$ from the extreme tension layer (per the definition for $A_{ct}$) in areas other than the inside edged of bearing. This assumption will overestimate the area at the inside edge of the bearing, so a reduction factor equal to 0.6 is introduced to the $A_{ct}$ to account for the reduced area of the tensile zone at this location. In addition, the area of the reinforcement within the flexural tension zone must be deducted from $A_{ct}$. Even though 0.6$A_{ct}$ accounts for the effective UHPC area within the flexural tension zone contributing to the longitudinal reinforcement resistance, the area of the tensile reinforcement contributing to the resistance is taken as the area of reinforcement located within 0.5$h$ from the extreme tension layer (per the definition for $A_{ct}$).

An additional level of conservatism is built into UHPC Structural Design Guide Eqn. 7.3.5-1 and 7.3.5-2 when the tensile behavior of UHPC can be described by the bilinear relationship of Figure 4.2.5.4-2 of the UHPC Structural Design Guide (i.e. $f_{t,loc} \geq 1.20 f_{t,cr}$). The stress in the UHPC is taken equal to the reduced effective cracking strength, $\gamma_u f_{t,cr}$, instead of the reduced crack localization strength, $\gamma_u f_{t,loc}$, while assuming a crack localization failure mode as is evident by limiting the strain in the non-prestressed reinforcement to reduced crack localization strain, $\gamma_u \varepsilon_{t,loc}$.

## STRENGTH LIMIT STATE – SHEAR (D REGIONS)

The design of D-regions with UHPC is discussed in UHPC Structural Design Guide Article 8.

*Refined analysis, strut-and-tie, and elastic stress analysis methods may be used to determine the internal force effects in disturbed regions, such as those near supports, and the points of application of concentrated loads at strength and extreme limit states. The design method shall be approved by the owner and shown in the contract documents.*

There are several limitations and recommendations provided:

- Internal strains in the UHPC and reinforcement at strength and extreme limit states shall use material properties in UHPC Structural Design Guide Article 1.4. Tensile strain in the UHPC shall be limited to $\gamma_u \varepsilon_{t,loc}$.
- Calculated resistances for complex D-regions are recommended to be justified by performance testing a prototype.

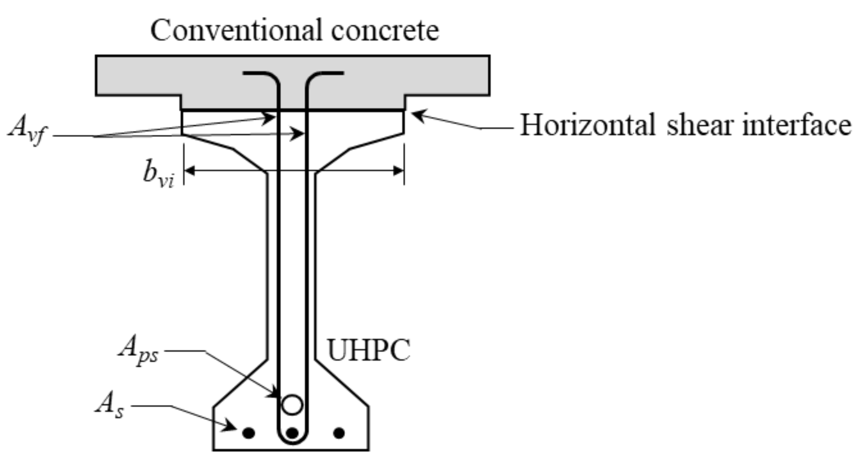
No additional guidance is provided in the UHPC Structural Design Guide at this time.

More research is needed in this area, but an initial framework for using the strut-and-tie method (STM) with UHPC members is provided in FHWA-RC-24-0004 *Possible Framework for Using the Strut-and-Tie Method (STM) with Ultra-High Performance Concrete (UHPC)*. This report provides some initial recommendations and limitations for using STM with UHPC members.

## OTHER DESIGN CONSIDERATIONS

### Interface Shear Transfer

The interface shear transfer needs to be checked at any interfaces where shear friction or horizontal shear may control. One common location where the interface shear must be checked is the horizontal shear plane between precast UHPC members and conventional concrete cast-in-place decks, as shown in Figure 33.




Source: FHWA.

**Figure 33. Illustration. Parameters to use for interface shear check at horizontal shear interface between precast UHPC member and convention concrete cast-in-place deck.**

The nominal shear resistance for UHPC interfaces includes a cohesion and friction component, similar to the approach adopted in AASHTO LRFD BDS Eqn. 5.7.4.3-3. Two different equations are provided in the UHPC Structural Design Guide for calculating the nominal shear resistance of the interface plane, one for a monolithic interface and one for all other cases.

Nominal shear resistance of monolithic UHPC interface:

$V_{ni} = cA_{cv} + \mu(C_1 + C_2 + P_c)$ UHPC Structural Design Guide Eqn. 7.4.3-1

In which:

$C_1 = A_{vf} f_s$ UHPC Structural Design Guide Eqn. 7.4.3-2

$C_2 = A_{cv} \gamma_u f_{t,loc}$ UHPC Structural Design Guide Eqn. 7.4.3-3

$f_s = E_s \gamma_u \varepsilon_{t,loc} \leq f_y$

The equations used to calculate the nominal shear resistance for all interfaces other than monolithic UHPC is the same as for conventional concrete.

$V_{ni} = cA_{cv} + \mu(A_{vf} f_y + P_c)$ UHPC Structural Design Guide Eqn. 7.4.3-4

The nominal shear resistance is limited by the following upper limit.

$V_{ni} \leq KA_{cv}$ UHPC Structural Design Guide Eqn. 7.4.3-5

The shear interface area can be found using UHPC Structural Design Guide Eqn. 1.7.4.3-6 based on the width of the interface ($b_{vi}$), shown in Figure 33, and either a unit length (typically per inch) or actual interface length.

Interface area: $A_{cv} = b_{vi} L_{vi}$ UHPC Structural Design Guide Eqn. 7.4.3-6

The area of the interface shear reinforcement, $A_{vf}$, includes all reinforcement crossing the shear interface, as shown in Figure 33. This can also be found based on a unit length (typically per inch) by dividing the area of the bars crossing the interface at one plane by the spacing of the reinforcement along the length of the interface.

Many different cohesion and friction factors are included in the UHPC Structural Design Guide for a monolithic UHPC interface and other common UHPC interfaces. A sample of those most applicable to structural design with UHPC is provided in the following list.

- Case 1 – UHPC placed monolithically.
  - $c$ = 1.40 ksi
  - $\mu$ = 1.0
  - $K$ = 4.5 ksi
- Case 7 – Conventional concrete placed against a clean UHPC substrate surface, free of laitance, with surface cast to have 0.25-inch-deep by 0.25-inch-wide formed flutes.
  - $c$ = 0.24 ksi

 - $\mu = 1.0$
 - $K = 1.8$ ksi
- Case 8 – Conventional concrete placed against a clean UHPC substrate surface, free of laitance, but not cast to have keys or formed flutes.
 - $c = 0.025$ ksi
 - $\mu = 0.6$
 - $K = 0.8$ ksi

The minimum area of interface shear reinforcement is specified in UHPC Structural Design Guide Article 7.4.2. Minimum interface shear reinforcement is not required for monolithic UHPC interfaces where $\gamma_u f_{t,loc} A_{cv} \geq 0.05 A_{cv}$. The area of UHPC considered to be engaged in interface shear transfer, $A_{cv}$, can be removed from each side of this equation leaving $\gamma_u f_{t,loc} \geq 0.05$. This criterion is always satisfied by definition for UHPC, where $f_{t,loc,min} \geq f_{t,cr,min} = 0.75$ ksi, so the minimum area of interface shear reinforcement is not required for monolithic UHPC interfaces.

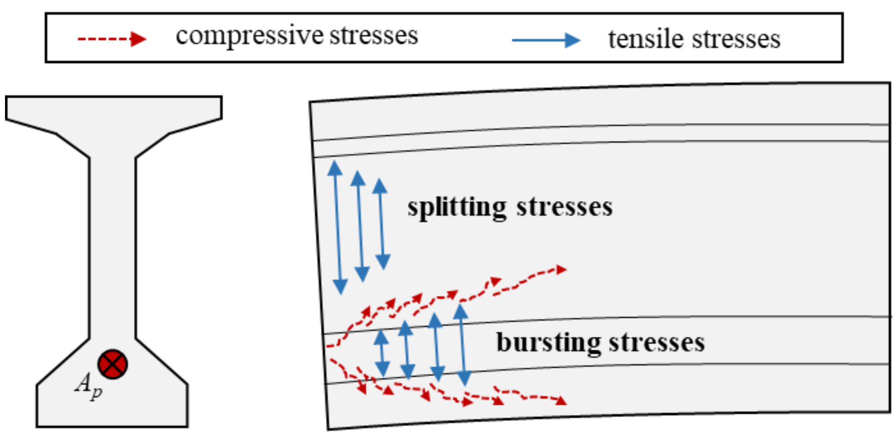
Minimum interface shear reinforcement per AASHTO LRFD BDS Article 5.7.4.2 is always required for non-monolithic interfaces.

Minimum interface shear reinforcement: $A_{vf} \geq 0.05 A_{cv} / f_y$

AASHTO LRFD BDS Eqn. 5.7.4.2-1

**Anchorage Zone and Confinement Reinforcement**

Pretensioned girders typically have a large amount of prestressing force applied in the end region from the anchoring of the prestressing strands. Splitting stresses develop in the web of the member as the eccentrically applied force from the prestressing strands pulls the bottom flange away from the rest of the member, as shown in Figure 34. The anchoring of the pretensioned strands also result in radial stresses in the concrete around the strands (Hoyer's Effect); these stresses are typically called bursting stresses as illustrated in Figure 34.




Source: FHWA.

**Figure 34. Illustration. Development of splitting and bursting stresses in the end region of pretensioned concrete members.**

These splitting and bursting stresses also occur in pretensioned UHPC members, consequently, end region reinforcement is often required to resist the two different mechanisms. A summary of the splitting and confinement reinforcement provisions for conventional concrete and UHPC is provided in Table 13.

**Table 13. Provisions for anchorage zone and confinement reinforcement for conventional concrete and UHPC sections.**

| Description | Conventional Concrete | AASHTO LRFD BDS Reference | UHPC | UHPC Structural Design Guide Reference |
|---|---|---|---|---|
| Splitting reinforcement required | The resistance shall not be less than 4 percent of the total prestressing force at transfer. | Article 5.9.4.4.1 | Same as for conventional concrete. | Article 9.4.4.1 |
| Splitting resistance calculation | $P_r = f_s A_s$ | Eqn. 5.9.4.4.1-1 | $P_r = f_s A_s + P_{r,UHPC}$ | Eqn. 9.4.4-1 |
| Confinement reinforcement | For the distance of 1.5*d* from the end of the beams other than box beams, reinforcement shall be placed to confine the prestressing steel in the bottom flange. The reinforcement shall not be less than No. 3 deformed bars, with spacing not exceeding 6.0 in. and shaped to enclose the strands.<br><br>For box beams, transverse reinforcement shall be provided and anchored by extending the leg of stirrup into the web of the girder. | Article 5.9.4.4.2 | Same as for conventional concrete. | Article 9.4.4.2 |

Like conventional concrete, the resistance provided by the splitting reinforcement shall not be less than 4 percent of the prestressing force just before transfer, $P_{pbt}$.

Required splitting resistance: $P_r \geq 0.04P_{pbt}$

The resistance component for UHPC members also includes the contribution of the UHPC.

Factored resistance provided by discrete reinforcement and UHPC in end region:

$$P_r = f_s A_s + P_{r,UHPC}$$ UHPC Structural Design Guide Eqn. 9.4.4-1

Contribution of UHPC:

$$P_{r,UHPC} = 0.25\gamma_u f_{t,cri} b_v h$$ UHPC Structural Design Guide Eqn. 9.4.4-2

The design UHPC effective cracking strength, $f_{t,cri}$, is defined by UHPC Structural Design Guide Article 9.1.2: "Unless determined by physical tests and as approved by the owner, the value of $f_{t,cri}$ shall not be taken as greater than $0.75f_{t,cr}$ when $f'_{ci}$ is less than or equal to $0.90f'_c$."

Effective cracking strength at release: $f_{t,cri} \leq 0.75 f_{t,cr}$    UHPC Structural Design Guide 9.1.2

The stress in the reinforcement is still limited to 20.0 ksi, i.e., $f_s \leq 20$ ksi.

The amount of splitting reinforcement required can be calculated as follows.

Required splitting reinforcement: $A_{s,req} = (0.04P_{pbt} - P_{r,UHPC})$ / 20 ksi

This reinforcement shall be provided within a distance $h$/4 from the end of the beam. The reinforcement to include in $A_s$ depends on the member type, as shown in Table 14.

**Table 14. Area of reinforcement to include when calculating $A_s$ for splitting reinforcement, based on AASHTO LRFD BDS Article 5.9.4.4.1.**

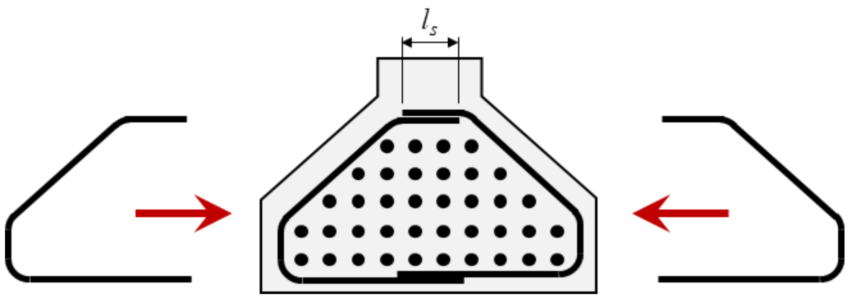
| **Member Type** | **Area of Steel to Count** |
|---|---|
| Pretensioned I-Girders | Total area of the vertical reinforcement located within a distance of $h$/4 from the end of the member; $h$ is the overall height of the member. |
| Pretensioned Solid or Voided Slabs | Total area of the horizontal reinforcement located within a distance of $h$/4 from the end of the member; $h$ is the overall width of the member. |
| Pretensioned Box or Tub Girders | Total area of vertical or horizontal reinforcement located within a distance $h$/4 from the end of the member; $h$ is the lesser of the overall width or height of the member. |
| Pretensioned Members with Multiple Stems | Total area of vertical reinforcement divided evenly among the webs and located within a distance $h$/4 from the end of each web. |

Reinforcement used for splitting resistance can be used to satisfy other design requirements (such as shear reinforcement), see last paragraph in AASHTO LRFD BDS Article 5.9.4.4.1.

Confinement reinforcement is provided to resist the bursting stresses caused by the prestressing force. The same confinement reinforcement required for conventional concrete members is also required for UHPC members. AASHTO LRFD BDS Article 5.9.4.4.2 states:

> *For the distance of 1.5d from the end of the beams other than box beams, reinforcement shall be placed to confine the prestressing steel in the bottom flange. The reinforcement shall not be less than No. 3 deformed bars, with spacing not exceeding 6.0 in. and shaped to enclose the strands.*
>
> *For box beams, transverse reinforcement shall be provided and anchored by extending the leg of stirrup into the web of the girder.*

Thought should be given to the installation of the confinement reinforcement while still providing sufficient splice lengths for overlapping reinforcement. A sample detail for confinement reinforcement that can be installed after the stressing of the prestressing strands is shown in Figure 35.




Source: FHWA.

**Figure 35. Illustration. Installation of confinement reinforcement around strands in bottom of pretensioned member.**

The splice length for the No. 3 bars in the UHPC is calculated using UHPC Structural Design Guide Article 10.8.4, which will be as follows.

Splice length for No. 3 bar: $\ell_s = \ell_d = 10(0.375 \text{ inch}) = 3.75 \text{ inch}$

UHPC Structural Design Guide Article 10.8.4 states that lap splices in UHPC may be classified as Class A splices for No. 8 bars and smaller embedded in UHPC with $f'_c$ greater than 14 ksi and a minimum cover of $2d_b$.

## Deflection and Camber

One of the benefits of UHPC structural elements is that they can be used to create a shallower bridge profile, i.e., the use of a UHPC structural element can lead to a shallower section depth compared to the conventional concrete alternative. This means that deflection checks can become the controlling aspect of design for longer spans if the owner decides to invoke the optional live load deflection criteria.

Provisions related to deformations for conventional concrete and UHPC are summarized in Table 15.

**Table 15. Provisions related to deformations for conventional concrete and UHPC sections.**

| Description | Conventional Concrete | AASHTO LRFD BDS Reference | UHPC | UHPC Structural Design Guide Reference |
|---|---|---|---|---|
| Deformations | General specifications | Article 5.6.3.5 | Same as conventional concrete with a few exceptions | Article 6.3.5 |
| Coefficient of thermal expansion | $6.0 \times 10^{-6}$ /°F for normal weight concrete | Article 5.4.2.2 | $7.0 \times 10^{-6}$ /°F | Article 4.2.7 |
| Creep and Shrinkage | Procedure and factors defined in this Article. | Article 5.4.2.3 | Different factors for creep coefficient and shrinkage strain equations | Article 4.2.8 |
| Modulus of elasticity | $2{,}500 f_c'^{0.33}$ | Eqn. C5.4.2.4-1 | $2{,}500 K_1 f_c'^{0.33}$ | Eqn. 4.2.3-1 |
| Modulus of rupture / tensile strength | $M_{cr} = f_r \times (I_g / y_t)$ | Eqn. 5.6.3.5.2-2 | Use $\gamma_u f_{t,cr}$ instead of $f_r$ in AASHTO LRFD BDS Eqn. 5.6.3.5.2-2. References to AASHTO LRFD BDS Article 5.4.2.6 shall not apply. | Article 6.3.5 |
| Deflection Limit | Live load deflection limit = $L_{span}$ / 800 | Article 2.5.2.6.2 | Same as for conventional concrete. | Article 6.3.5 |
| Distribution factor | $DF = N_{lanes} / N_{beams}$ | Article C2.5.2.6.2 | Same as for conventional concrete. | Article 6.3.5 |

Deflections are generally found using a procedure like that for conventional concrete with a few differences, which will be discussed in the following sections.

Non-prestressed and prestressed components will be uncracked if designed to meet UHPC Structural Design Guide Article 5.2.3 (for components subjected to cyclic stresses), i.e., $f_t \leq 0.95\gamma_u f_{t,cr}$.

***Long-Term Deflection Calculations***

UHPC Structural Design Guide Article 6.3.5 refers to AASHTO LRFD BDS Article 5.6.3.5 for deformation calculations. AASHTO LRFD BDS Article 5.6.3.5.2 states that:

> *Deflection and camber calculations shall consider dead load, live load, prestressing, erection loads, concrete creep and shrinkage, and steel relaxation.*

Deflections and camber are calculated assuming linear elastic material properties (AASHTO LRFD Article 4.5.2.2). No specific procedure for calculating long-term deflections is specified, but AASHTO LRFD BDS C5.6.3.5.2 states that long-term deflections may be calculated based on creep and shrinkage calculations or using other methods that consider different types of loads and sections such as the PCI Multiplier Method.

The Precast/Prestressed Concrete Institute (PCI) provides a simplified method for calculating the long-term deflections of prestressed concrete girders, see Table 8.7.1-1 in the PCI *Bridge Design Manual* and Table 5.8.2 in the PCI Design Handbook. These multipliers are based on Martin (1977) and include the long-term effects of creep, shrinkage, and strand relaxation. All deflection multipliers in the derivation are based on a base creep coefficient of 2.0 for long-term deflection in a non-prestressed concrete member from an equation in ACI 318-71 (Section 9.5.2.3) and an assumed 15 percent prestress losses. Given that these PCI multipliers are based on conventional concrete, they may not accurately predict the deflections of UHPC members.

Creep coefficients, shrinkage strain, and prestress losses in UHPC members were developed based on several research efforts by FHWA. In one study examining various materials, Mohebbi et al. (2022) found that the ultimate creep coefficients of UHPC ranges between 0.91 and 1.53 for early-age loading, which is less than typical creep coefficients for conventional concrete between 1.5 and 3.0. Specimens loaded at a more mature age resulted in creep coefficients about half of those observed in the specimens loaded at early-age. Loading at mature ages would decrease the creep effects on the deflections from superimposed dead loads (e.g., deck, barrier, wearing surface).

The effective modulus method may be used with the calculated creep coefficient and long-term prestress losses to better estimate the long-term deflections in UHPC structural elements. Creep coefficients can be calculated using the procedure specified in UHPC Structural Design Guide Article 4.2.8.2 and prestress losses can be calculated using the procedure in UHPC Structural Design Guide Article 9.3. The effective modulus of elasticity can be calculated as follows.

Effective UHPC modulus of elasticity: $E_{c,eff} = E_c / [1 + \Psi(t, t_i)]$

The effective modulus of elasticity for the UHPC should be used in place of the normal modulus of elasticity when calculating deflections due to sustained loads. Different creep coefficients and effective moduli should be calculated for each sustained load based on the age of the UHPC at the time of loading. The normal modulus of elasticity of the UHPC should be used for any transient loads (e.g., live loads).

The prestressing force should be reduced by the appropriate prestress losses at the time when deflection is being calculated and used to calculate the deflections due to eccentric prestressing strands.

Gross and composite section properties can be used for these deflection calculations. Gross section properties should be used for any loads applied prior to the hardening of the composite deck. Composite section properties should be used for any loads applied after the deck has hardened. Transformed non-composite and composite section properties may also be used to calculate deflections. The appropriate effective modulus can be used to calculate the modular ratios for transforming the steel and conventional concrete to an equivalent UHPC area.

As an example, the deflections at final time for a simply supported girder from prestressing (straight strands), self-weight ($w_{sw}$ applied at $t_{release}$ = 1 day), deck ($w_{deck}$ applied at $t_d$ = 180 days), and a distributed live load ($w_{LL}$) would be the following.

$$\Delta = \frac{P_f e_p L_{span}^{\ 2}}{8E_{c,eff,re} I_g} + \frac{5w_{sw} L_{span}^{\ 4}}{384E_{c,eff,re} I_g} + \frac{5w_{deck} L_{span}^{\ 4}}{384E_{c,eff,deck} I_g} + \frac{5w_{LL} L_{span}^{\ 4}}{384E_c I_c}$$

where:

$P_f$ = prestressing force after all prestress losses (kips)

$E_{c,eff,re}$ = effective modulus of elasticity of UHPC using creep coefficient with loading age of $t_{release}$

$E_{c,eff,deck}$ = effective modulus of elasticity of UHPC using creep coefficient with loading age of $t_d$

This procedure can be used to estimate the deflections for placement of the girders and final rideability of the bridge.

***Live Load Deflection Checks (at Owner's Discretion)***

The live load deflection checks are at the owner's discretion (AASHTO LRFD Article 2.5.2.6.2). If the owner decides to invoke the optional live load deflection criteria, the live load deflections need to be less than $L_{span}$ / 800, which may control the design for longer spans. Different deflection criteria may be present in different jurisdictions and for different types of bridges (e.g., pedestrian bridges). The live load deflection should be calculated based on the maximum of the deflection from the design truck alone ($\Delta_{LT}$) and that from 25 percent of the design truck and the design lane load ($\Delta_{LL}$), see AASHTO LRFD BDS Article 3.6.1.3.2.

Live load deflection: $\Delta_L$ = maximum of $\Delta_{LT}$ and $(0.25\Delta_{LT} + \Delta_{LL})$

AASHTO LRFD BDS Article 3.6.1.3.2

AASHTO LRFD BDS Article 2.5.2.6.2 states that "all design lanes should be loaded and all supporting members should be assumed to deflect equally," which is equivalent to the distribution factor for deflections (DFD) being the ratio of design lanes to number of beams.

Distribution factor for deflections: $DFD$ = (number of lanes) / (number of beams)

AASHTO LRFD BDS Article 2.5.2.6.2

This distribution factor should be applied to the lane load and design truck loads. The impact factor (0.33) should also be applied to the truck load when calculating defection.

The live load deflection limit can be checked as follows.

Live load deflection limit: $\Delta_L \leq L_{span}$ / 800 AASHTO LRFD BDS Article 2.5.2.6.2

As previously mentioned, live load deflection limits may control the design of longer span UHPC members. Deflections are inversely proportional to the modulus of elasticity for the UHPC, therefore, longer spans may be achievable for UHPC with a larger modulus of elasticity. The modulus of elasticity is calculated using UHPC Structural Design Guide Eqn. 4.2.3-1.

Modulus of elasticity: $E_c = 2500K_1 f'_c{}^{0.33}$ UHPC Structural Design Guide Eqn. 4.2.3-1

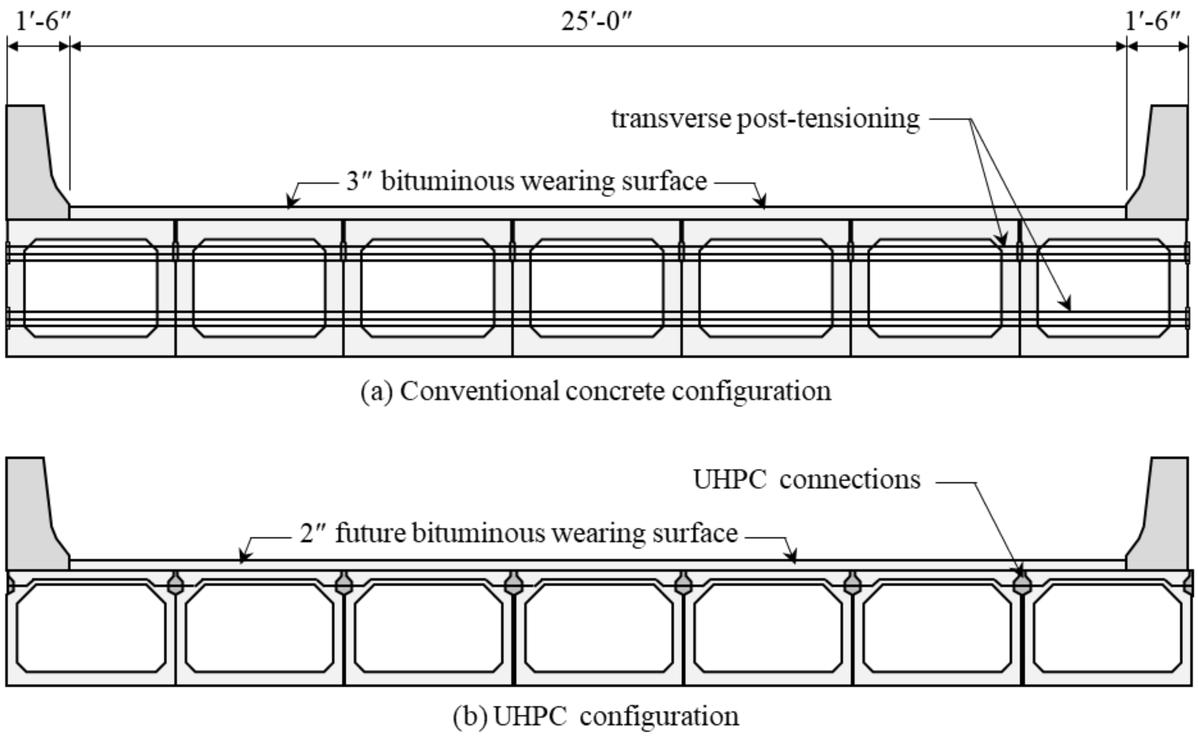
Increasing the compressive strength would lead to a higher modulus. However, the compressive strength is raised to the third power, which means an increase in the compressive strength will result in a smaller increase in the modulus. As an example, it would take about a 34 percent increase in the compressive strength for a 10 percent increase in the modulus of elasticity.

UHPC Structural Design Guide Article 4.2.3 allows for the modulus of elasticity of UHPC be modified based on physical tests, which would likely occur during the material qualification stage, using the $K_1$ correction factor. Some UHPC materials will have a $K_1$ greater than 1.0 (see Mohebbi and Graybeal, 2022). Using a UHPC with a higher modulus of elasticity, i.e., with a $K_1$ greater than 1.0, may allow for longer spans to be achieved with shallower sections without reaching the deflection threshold.

# CHAPTER 3. BOX BEAM DESIGN EXAMPLE

## INTRODUCTION

The primary design example for this workshop is based on Example 9.4 from the PCI *Bridge Design Manual* (BDM). The cross section of the bridge with the conventional concrete box beam configuration is shown in Figure 36 (a). The design for conventional concrete adjacent box beams this bridge can be found in the PCI BDM. The design for UHPC adjacent box beams is summarized in this chapter; the cross section for the UHPC design is shown in Figure 36 (b).




Source: FHWA.

**Figure 36. Illustration. Bridge cross section for primary design example with (a) conventional concrete and (b) UHPC box beams.**

The span length and other properties for the bridge are summarized below.

Span length: $L = 95$ ft

Beam length: $L_{beam} = 96$ ft

Width of support: $b_s = 1$ ft

Clear roadway width: $W_R = 25$ ft

The beam spacing is equal to the beam width for an adjacent box beam configuration.

Beam spacing: $S = 4$ ft

Number of girders: $N_b = 7$

The number of design lanes is calculated from AASHTO LRFD BDS Article 3.6.1.1.1.

Number of design lanes: $N_{lanes} = (W_R$ / 12 ft) rounded down to nearest integer = 2

No wearing surface is initially needed for the UHPC superstructure. The thickness of a future wearing surface was included in the design.

Thickness of future wearing surface: $t_{fws} = 2$ inch

Density of wearing surface (asphalt): $w_a = 0.145$ kcf

## DEFINITIONS

### Material Definitions

The UHPC material properties for this example are as follows:

- Compressive strength at transfer: $f'_{ci} = 14.0$ ksi
- Compressive strength for use in design and analyses: $f'_c = 17.5$ ksi
- Correction factor for modulus of elasticity: $K_1 = 1.1$
- UHPC unit weight (including reinforcement): $w_c = 0.160$ kcf
- Reduction factor for compression: $\alpha_u = 0.85$
- Ultimate compression strain: $\varepsilon_{cu} = 0.0035$
- Effective cracking strength: $f_{t,cr} = 1.0$ ksi
- Effective cracking strength at transfer: $f_{t,cri} = 0.75 f_{t,cr} = 0.75$ ksi
- Crack localization strength: $f_{t,loc} = 1.0$ ksi
- Crack localization strain: $\varepsilon_{t,loc} = 0.005$
- Reduction factor for tension: $\gamma_u = 1.0$

The modulus of elasticity of the concrete at transfer and for use in design are found using UHPC Structural Design Guide Eqn. 4.2.3-1.

Modulus of elasticity at transfer: $E_{ci} = 2500 K_1 f'^{0.33}_{ci} = 6{,}570$ ksi

Modulus of elasticity for use in design: $E_c = 2500 K_1 f'^{0.33}_{c} = 7{,}072$ ksi

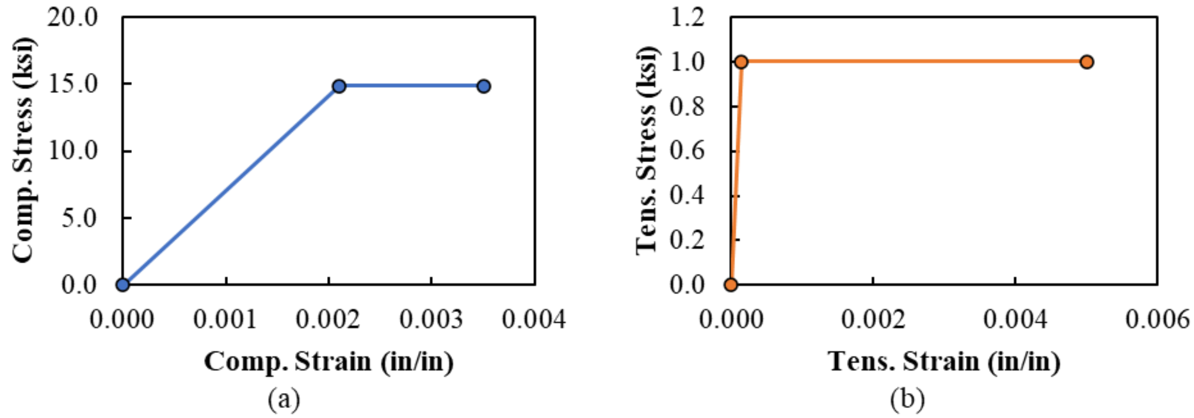
The elastic compression strain is found using UHPC Structural Design Guide Eqn. 4.2.4.2-1 assuming a linear elastic response up to the reduced compression strength.

Elastic compression strain: $\varepsilon_{cp} = (\alpha_u f'_c) / E_c = 0.0021$

The cracking strain is found assuming a linear elastic response up to the effective cracking strength.

Effective cracking strain: $\varepsilon_{t,cr} = (\gamma_u f_{t,cr}) / E_c = 0.000141$

The stress-strain relationships used for design are shown in Figure 37. An elastic-plastic response is assumed for the UHPC in both compression and tension.




Source: FHWA.

**Figure 37. Graphs. UHPC stress versus strain responses for (a) compression and (b) tension.**

The material properties for the conventional steel reinforcement (Grade 60) are as follows:

- Modulus of elasticity: $E_s = 29{,}000$ ksi
- Yield strength: $f_{sy} = 60$ ksi

The material properties for the prestressing strands are as follows:

- Low-relaxation
- Modulus of elasticity: $E_p = 28{,}500$ ksi
- Ultimate strength: $f_{pu} = 270$ ksi
- Yield strength: $f_{py} = 243$ ksi
- Specified rupture strain for strands: $\varepsilon_{pu} = 0.035$

The following stress-strain relationship was used for the prestressing strands:

$$f_{ps} = \varepsilon_{ps}\left[887 + \frac{27{,}613}{\left(1 + \left(112.4\varepsilon_{ps}\right)^{7.36}\right)^{\frac{1}{7.36}}}\right] \le 270 \text{ ksi}$$

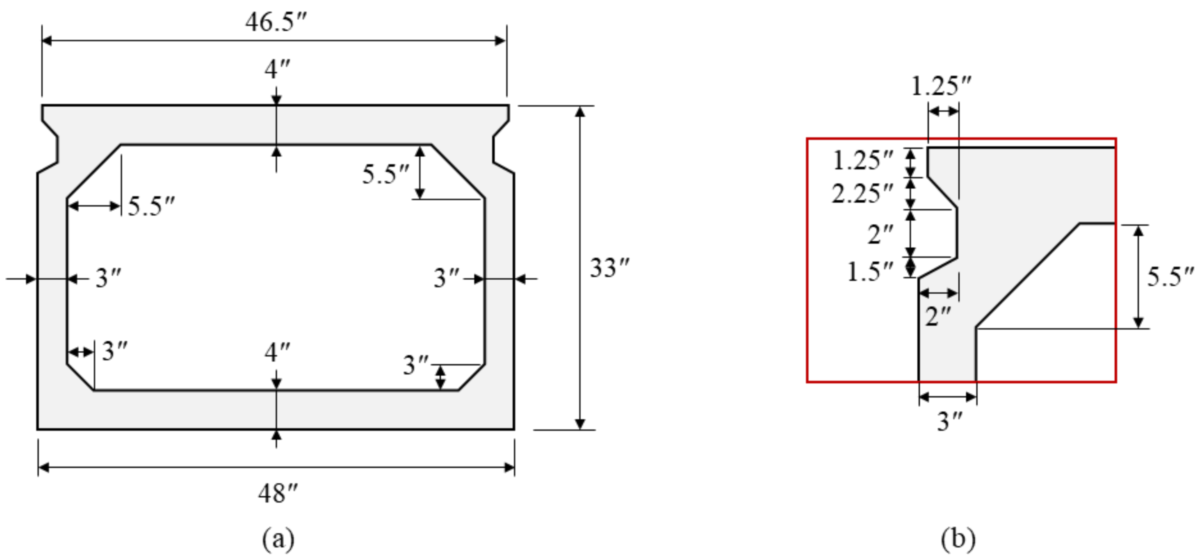
The area and diameter of the prestressing strands used in this example are as follows.

- Diameter of prestressing strands: $D_p = 0.7$ inch
- Area of one strand: $A_{p,0.7in} = 0.294$ inch$^2$

A smaller diameter strand could also be used for this example (e.g., 0.6-inch diameter).

**Section Definition**

A Modified BII-48 section is used for this example; cross section dimensions for this section are shown in Figure 38. The section shape was optimized to take advantage of the enhanced properties of UHPC by reducing the web and flange widths.




Source: FHWA.

**Figure 38. Illustration. Details for Modified BII-48, (a) cross section and (b) joint dimensions.**

Some of the basic section properties used in the design are as follows.

- Height of non-composite section: $h$ = 33 inch
- Effective flange width: $b_e$ = 48 inch
- Width of top flange: $b_{tf} = b_e$ = 48 inch
- Thickness of top flange: $t_{tf}$ = 4 inch
- Width of girder web: $b_w$ = 6 inch (includes 3 inches for each web of the box section)
- Width of bottom flange: $b_{bf}$ = 48 inch
- Thickness of bottom flange: $t_{bf}$ = 4 inch

The section geometry was defined as a function to facilitate the strain compatibility analysis for the strength limit state. Three functions were defined for the section geometry: $b_o(z)$ for the outside geometry, $b_h(z)$ for the geometry of the void, and $b(z)$ for the box section, where $z$ is measured from the compression face of the section.

The outside geometry was defined as follows. The joint region is based on the adjacent box beam joint developed by FHWA-HRT-17-093.

$b_o(z)$ = if 0 inch < $z$ < 1.25 inch, then 46.5 inch

else if 1.25 inch ≤ $z$ < 3.5 inch, then [46.5 inch – 2.5 inch × ($z$ – 1.25 inch) / 2.25 inch]
else if 3.5 inch ≤ $z$ < 5.5 inch, then 44 inch
else if 5.5 inch ≤ $z$ < 7 inch, then [44 inch + 4 inch × ($z$ – 5.5 inch) / 1.5 inch]
else if 7 inch ≤ $z$ ≤ 33 inch, then 48 inch
else 0 inch

The interior void was defined by the following function.

$b_h(z)$ = if 0 inch < $z$ < 4 inch, then 0 inch
else if 4 inch ≤ $z$ < 9.5 inch, then [31 inch + 11 inch × ($z$ – 4 inch) / 5.5 inch]
else if 9.5 inch ≤ $z$ < 26 inch, then 42 inch
else if 26 inch ≤ $z$ < 29 inch, then [42 inch – 6 inch × ($z$ – 26 inch) / 3 inch]
else 0 inch

The section geometry is defined based on the functions for the outside width of the section and the width of the void.

Section geometry function: $b(z) = b_o(z) - b_h(z)$

The section properties can be calculated based on the section geometry functions as follows.

Area:

$$A_g = \int_{0\text{ in.}}^{h} b(z)\ \mathrm{dz} = 554.1\ \text{in}^2$$

Center of gravity (measured from compression face):

$$y_t = cg = \frac{\int_{0\text{ in.}}^{h} b(z) \cdot z \cdot \mathrm{dz}}{A_g} = 16.55\ \text{in}$$

Distance from tension face to center of gravity: $y_b = h - y_t = 16.45$ inch

Moment of inertia about the centroid:

$$I_g = \int_{0\text{ in.}}^{h} b(z) \cdot (cg - z)^2\ \mathrm{dz} = 90{,}567\ \text{in}^4$$

Section modulus to bottom of section: $S_b = I_g / y_b = 5{,}504\ \text{inch}^3$

Section modulus to top of section: $S_t = I_g / y_t = 5{,}474\ \text{inch}^3$

First moment of inertia for shear stress ($y_1$ is the distance from mid-height):

$$Q(y_1) = \int_{y_1}^{cg} z \cdot b(cg - z)\ \mathrm{dz}$$

The St. Venant's torsional inertia, $J_g$, for the box beam needs to also be calculated. The area enclosed by the centerlines of the outside elements of the box beam, $A_o$, and the St. Venant's torsional inertia, $J_g$, are calculated as follows.

$A_o = (b_e - 0.5b_w) \times (h - 0.5t_{tf} - 0.5t_{bf}) = 1{,}305 \text{ in}^2$

$$J_g = \frac{4A_o^{\,2}}{\left(\dfrac{b_e - 0.5b_w}{t_{bf}}\right) + \left(\dfrac{b_e - 0.5b_w}{t_{tf}}\right) + 2\left(\dfrac{h - \left(0.5t_{tf} + 0.5t_{bf}\right)}{0.5b_w}\right)} = 162{,}800 \text{ in}^4$$

Gross section properties were used for this example.

### Strand Profile

The strand profile includes (13) 0.7-inch diameter strands in the bottom layer of strands and (2) 0.7-inch diameter strands in the top layer. Details on the strand profile are shown in Table 16.

**Table 16. Strand profile for box beam design example.**

| Layer # | Number of strands | Distance from bottom to centroid of strands |
|---|---|---|
| 1 | 13 | 2 inches |
| 2 | 2 | 31 inches |

All strands were fully stressed to a jacking stress, $f_{pj}$, of 202.5 ksi.

The centroid and eccentricity of all the prestressing strands is calculated using the number of strands in each layer and the height of the layers.

Total number of strands: $n_T = \sum(n_i) = 15$

Distance between strand centroid and bottom of section:

$y_p = \sum(y_{p,i} \times n_i) / \sum(n_i) = 5.867$ inch

Strand eccentricity at midspan: $e_m = y_b - y_p = 10.588$ inch

The total area of the prestressing strands is found as follows.

Total area of prestressing: $A_{pT} = A_{p,0.7in} \times n_T = (0.294 \text{ inch}^2)(15) = 4.41 \text{ inch}^2$

## SHEAR FORCES AND BENDING MOMENTS

### Distribution Factors and Dynamic Load Allowance

The distribution factors can be calculated using AASHTO LRFD BDS Article 4.6.2.2. Based on AASHTO LRFD Table 4.6.2.2.1-1, the adjacent box beam configuration with UHPC joints is a Type (g) superstructure. The distribution factors were found using the tables and procedures in AASHTO LRFD BDS Article 4.6.2.2 for an interior beam.

Distribution factor for moment (interior beam): $DFM = 0.286$

Distribution factor for shear (interior beam): $DFV = 0.442$

Distribution factor for moment (fatigue, interior beam): $DFM_F = 0.150$

Distribution factor for shear (fatigue, interior beam): $DFV_F = 0.352$

The design in general is for an interior beam. Calculations were not performed for an exterior beam.

The dynamic load allowance is found from AASHTO LRFD BDS Article 3.6.2. A dynamic load allowance of 33 percent in general ($IM = 0.33$) and 15 percent for fatigue ($IM_F = 0.15$) was used in this example.

**Unfactored Bending Moments due to Dead Load and Live Loads**

The distributed loads applied to the superstructure in this example include the following components.

- Beam self-weight: $w_g = 0.616$ kip/ft
- Barrier weight (per side): $w_{bps} = 0.3$ kip/ft
- Barrier weight (per beam): $w_b = (2w_{bps}) / N_b = 0.0857$ kip/ft
- Future wearing surface weight (per beam): $w_{ws} = (t_{fws}\, w_a\, W_R) / N_b = 0.0863$ kip/ft
- Lane live load: $w_{lane} = 0.64$ kip/ft

The shear and moment for each type of load were defined as functions dependent on the distance along the length of the beam.

Beam weight: $V_g(x) = w_g \times (0.5L - x)$

$M_g(x) = 0.5w_g \times x \times (L - x)$

Barrier weight: $V_b(x) = w_b \times (0.5L - x)$

$M_b(x) = 0.5w_b \times x \times (L - x)$

Future wearing surface weight: $V_{ws}(x) = w_{ws} \times (0.5L - x)$

$M_{ws}(x) = 0.5w_{ws} \times x \times (L - x)$

The simplified equations provided in the PCI BDM Article 8.11.1 for calculating the shear and moment along the length of a simply supported beam due to HL-93 loading were used in this example.

Lane live load: $V_{LL}(x) = w_{lane} \times (L - x)^2 / (2L)$ if $0 \le x \le 0.5L$

$V_{LL}(x) = -w_{lane} \times (L - (L - x))^2 / (2L)$ if $0.5L < x \le L$

$M_{LL}(x) = 0.5w_{lane} \times x \times (L - x)$

Truck load without impact:

$V_{LT}(x)$ = (72 kips) × (($L$ – $x$) – 9.33 ft) / $L$ if $0 \le x \le 0.5L$

$V_{LT}(x)$ = – (72 kips) × (($L$ – ($L$ – $x$)) – 9.33 ft) / $L$ if $0.5L < x \le L$

$M_{LT}(x)$ = (72 kips) × $x$ × [($L$ – $x$) – 9.33 ft] / $L$ if $0 \le x < 0.333L$

$M_{LT}(x)$ = (72 kips) × $x$ × [($L$ – $x$) – 4.67 ft] / $L$ – 112 kip-ft if $0.333L \le x \le 0.5L$

$M_{LT}(x)$ = (72 kips) × ($L$ – $x$) × [($L$ – ($L$ – $x$)) – 4.67 ft] / $L$ – 112 kip-ft if $0.5L < x \le 0.667L$

$M_{LT}(x)$ = (72 kips) × ($L$ – $x$) × [($L$ – ($L$ – $x$)) – 9.33 ft] / $L$ if $0.667L < x \le L$

The fatigue truck load without impact was found based on PCI BDM Article 8.11.3.

Fatigue truck load without impact:

$M_{LT,f}(x)$ = (72 kips) × $x$ × [($L$ – $x$) – 18.22 ft] / $L$ if $0 \le x < 0.241L$

$M_{LT,f}(x)$ = (72 kips) × $x$ × [($L$ – $x$) – 11.78 ft] / $L$ – 112 kip-ft if $0.241L \le x \le 0.5L$

$M_{LT,f}(x)$ = (72 kips) × ($L$ – $x$) × [($L$ – ($L$ – $x$)) – 11.78 ft] / $L$ – 112 kip-ft
if $0.5L < x \le 0.759L$

$M_{LT,f}(x)$ = (72 kips) × ($L$ – $x$) × [($L$ – ($L$ – $x$)) – 18.22 ft] / $L$ if $0.759L < x \le L$

The appropriate distribution and impact factors were added to the live loads. As an example, the lane load and truck load with impact at midspan are as follows.

Lane load with DFM at midspan: $M_{LL,m} = DFM \times M_{LL}(0.5L)$ = 206.6 kip-ft

Truck load with IM and DFM at midspan:

$$M_{LT,m} = DFM \times M_{LT}(0.5L) \times (1 + IM) = 544.3 \text{ kip-ft}$$

These shear and moment functions were used to calculate the demand needed for service, fatigue, and strength limit state checks.

## PRESTRESS LOSSES

The prestress losses were calculated according to UHPC Structural Design Guide Article 9.3, which specifies the use of AASHTO LRFD BDS Article 5.9.3.4 (Refined Estimates for Time-Dependent Losses) where shrinkage strains and creep coefficients are calculated using UHPC Structural Design Guide Eqn. 4.2.8.2-1 and Eqn. 4.2.8.3-1.

The construction and field conditions used in this design example are as follows.

UHPC age at transfer: $t_i$ = 1 day

UHPC age at beam installation and joint casting: $t_d$ = 90 days

UHPC age at final stage: $t_f$ = 20,000 days

Relative humidity: $H$ = 73 percent (input as 73 in shrinkage and creep equations)

There is no cast-in-place composite deck cast on the UHPC box beams, so the before and after deck placement loss components will be divided at the time of beam installation and joint casting at 90 days.

### Shrinkage Strains

The strain due to shrinkage is calculated using UHPC Structural Design Guide Article 4.2.8.3. The factors required to calculate the shrinkage strain are as follows.

Volume-to-surface area ratio factor: $k_s = 1.0$

Humidity factor for shrinkage: $k_{hs} = 1.5 - 0.01H = 1.5 - 0.01(73) = 0.770$

Strength factor: $k_f = 18 / (1.5f'_{ci} - 3) = 18 / (1.5(14\text{ ksi}) - 3) = 1.00$

UHPC Structural Design Guide Article 4.2.8.3 allows for a correction factor ($K_4$) to incorporate the results from physical tests. It was assumed for this example that physical tests were conducted to measure the shrinkage on the UHPC used for these beams.

Correction factor for shrinkage (from physical tests): $K_4 = 0.41$

Two different time development factors are required: between the end of curing and time of beam placement and between the time of beam placement and final time.

Time development factor (curing to beam placement):

$$k_{td,id} = (t_d - t_i) / [300 / (f'_{ci} + 30) + 0.8(t_d - t_i)^{0.98}] = 1.238$$

Time development factor (curing to final time):

$$k_{td,if} = (t_f - t_i) / [300 / (f'_{ci} + 30) + 0.8(t_f - t_i)^{0.98}] = 1.523$$

These factors can be used to calculate the shrinkage strains between the end of curing to the time of beam placement and final time using UHPC Structural Design Guide Eqn. 4.2.8.3-1.

Shrinkage strain (curing to beam placement): $\varepsilon_{bid} = k_s\, k_{hs}\, k_f\, k_{td,id}\, K_4 \times (0.6 \times 10^{-3}) = 0.000234$

Shrinkage strain (curing to final time): $\varepsilon_{bif} = k_s\, k_{hs}\, k_f\, k_{td,if}\, K_4 \times (0.6 \times 10^{-3}) = 0.000288$

The shrinkage between the time of beam placement and final time is the difference between $\varepsilon_{bif}$ and $\varepsilon_{bid}$.

Shrinkage strain (beam placement to final time): $\varepsilon_{bdf} = \varepsilon_{bif} - \varepsilon_{bid} = 0.000054$

These shrinkage strains are used in the prestress loss estimation equations.

### Creep Coefficients

The creep coefficients needed for calculating prestress losses are calculated using UHPC Structural Design Guide Article 4.2.8.2. The volume-to-surface area ratio factor ($k_s$), strength factor ($k_f$), and time development factor ($k_{td}$) are the same for creep as for shrinkage. The additional factors needed for calculating creep coefficients are as follows.

Humidity factor for creep: $k_{hc} = 1.12 - 0.0024H = 1.12 - 0.0024(73) = 0.945$

Loading age factor (for load applied at $t_i$ = 1 day):

$k_\ell = 1.0$ if $t_i < 7.0$ days

$k_\ell = (t_i - 6)^{-0.15} \geq 0.5$ if $t_i \geq 7.0$ days

For $t_i$ = 1 day, $k_\ell = 1.0$

UHPC Structural Design Guide Article 4.2.8.2 allows for a correction factor ($K_3$) to incorporate the results from physical tests. It was assumed for this example that physical tests were conducted to measure the creep on the UHPC used for these beams.

Correction factor for creep (from physical tests): $K_3 = 0.62$

The creep coefficients at time of beam placement and final time are found using UHPC Structural Design Guide Eqn. 4.2.8.2-1 as follows.

Creep coefficient at time of beam placement due to load placed at $t_i$:

$\Psi(t_d, t_i) = 1.2k_s\, k_{hc}\, k_f\, k_{td,id}\, k_\ell\, K_3 = 0.870$

Creep coefficient at final time due to load placed at $t_i$:

$\Psi(t_f, t_i) = 1.2k_s\, k_{hc}\, k_f\, k_{td,if}\, k_\ell\, K_3 = 1.071$

The creep coefficient for loads applied at time of deck placement to final time are as follows.

Loading age factor (for load applied at $t_i$ = 1 day):

$k_\ell = 1.0$ if $t_i < 7.0$ days

$k_\ell = (t_i - 6)^{-0.15} \geq 0.5$ if $t_i \geq 7.0$ days

For $t_d$ = 90 days, $k_{\ell,d} = [(90 - 6)^{-0.15} \geq 0.5] = 0.514$

Time development factor (curing to final time):

$k_{td,df} = (t_f - t_d) / [300 / (f'_c + 30) + 0.8(t_f - t_d)^{0.98}] = 1.523$

Creep coefficient at final time due to load placed at $t_d$:

$\Psi(t_f, t_d) = 1.2k_s\, k_{hc}\, k_f\, k_{td,df}\, k_{\ell,d}\, K_3 = 0.551$

These creep coefficients are used in the prestress loss estimation equations.

## Prestress Losses

### *Elastic Shortening Loss*

The elastic shortening loss was calculated using AASHTO LRFD BDS Eqn. C5.9.3.2.3a-1. The midspan moment due to self-weight of the girder being supported by the ends of the beam is also needed to calculate elastic shortening loss.

Unfactored self-weight moment at midspan:

$$M_{g,m} = 0.5w_g \times 0.5L_{beam} \times (L_{beam} - 0.5L_{beam}) = 709.3 \text{ kip-ft}$$

Elastic shortening loss:

$$\Delta f_{pES} = \frac{A_{ps}f_{pbt}\left(I_g + e_m{}^2A_g\right) - e_mM_{g,m}A_g}{A_{ps}\left(I_g + e_m{}^2A_g\right) + \dfrac{A_gI_gE_{ci}}{E_p}} = 7.059 \text{ ksi}$$

The stress and force in the prestressing steel immediately after transfer is as follows.

Stress in strands immediately after transfer: $f_{pt} = f_{pbt} - \Delta f_{pES} = 202.5 \text{ ksi} - 7.1 \text{ ksi} = 195.4 \text{ ksi}$

Force in strands immediately after transfer: $P_{pt} = A_{pT}\, f_{pt} = (4.41 \text{ inch}^2)(195.4 \text{ ksi})$

$P_{pt} = 861.9 \text{ kips}$

The stress in the UHPC at the center of gravity of the prestressing strands, $f_{cgp}$, was found as follows.

$$f_{cgp} = P_{pt}\left(\frac{1}{A_g} + \frac{e_m{}^2}{I_g}\right) - \frac{M_{g,m}e_m}{I_g} = 1.627 \text{ ksi}$$

### *Prestress Loss due to Shrinkage of UHPC before Beam Placement*

The prestress loss due to the shrinkage of the UHPC before the placement of the beams is calculated using AASHTO LRFD BDS Eqn. 5.9.3.4.2a-1. The transformed section coefficient is found using AASHTO LRFD BDS Eqn. 5.9.3.4.2a-2 as follows.

Transformed section coefficient:

$$K_{id} = \frac{1}{\left[1 + \dfrac{E_p}{E_{ci}}\dfrac{A_{pT}}{A_g}\left(1 + \dfrac{A_ge_m{}^2}{I_g}\right)\left(1 + 0.7\Psi_b\left(t_f, t_i\right)\right)\right]} = 0.908$$

Prestress loss due to shrinkage between curing and beam placement:

$$\Delta f_{pSR} = \varepsilon_{bid}\, E_p\, K_{id} = (0.000234)(28{,}500 \text{ ksi})(0.908) = 6.064 \text{ ksi}$$

***Prestress Loss due to Creep of UHPC before Beam Placement***

The prestress loss due to creep of the UHPC before beam placement is found using AASHTO LRFD BDS Eqn. 5.9.3.4.2b-1.

Prestress loss due to creep between curing and beam placement:

$$\Delta f_{pCR} = (E_p \,/\, E_{ci}) \times f_{cgp} \times \Psi(t_d,\, t_i) \times K_{id} = 5.574 \text{ ksi}$$

***Prestress Loss due to Relaxation of Strands before Beam Placement***

The prestress loss due to the relaxation of the prestressing strands before beam placement is found using AASHTO LRFD BDS Eqn. 5.9.3.4.2c-1. The factor for the type of steel, $K_L$, is 30 for low relaxation strands.

Prestress loss due to strand relaxation between curing and beam placement:

$$\Delta f_{pR1} = (f_{pt} \,/\, K_L) \times (f_{pt} \,/\, f_{py} - 0.55) = 1.657 \text{ ksi}$$

***Prestress Loss due to Shrinkage of UHPC after Beam Placement***

The prestress loss due to the shrinkage of the UHPC after the placement of the beams is calculated using AASHTO LRFD BDS Eqn. 5.9.3.4.3a-1. The transformed section coefficient is found using AASHTO LRFD BDS Eqn. 5.9.3.4.3a-2. There is no composite cast-in-place deck, so the composite section properties are equal to the gross section properties. This leads to the transformed section coefficient after deck placement being equal to that before deck placement.

Transformed section coefficient (no CIP deck): $K_{df} = K_{id} = 0.908$

Prestress loss due to shrinkage between curing and beam placement:

$$\Delta f_{pSD} = \varepsilon_{bdf} E_p K_{df} = (0.000054)(28{,}500 \text{ ksi})(0.908) = 1.398 \text{ ksi}$$

***Prestress Loss due to Creep of UHPC after Beam Placement***

The prestress loss due to creep of the UHPC after beam placement is found using AASHTO LRFD BDS Eqn. 5.9.3.4.3b-1.

The change in UHPC stress at the centroid of the prestressing strands due to long-term losses between transfer and beam placement combined with superimposed dead loads is as follows.

Long-term losses prior to beam placement:

$$P_\Delta = -(\Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR1})A_{pT} = -58.632 \text{ kips}$$

Moment at midspan from barrier weight and wearing surface:

$$M_{sd,m} = M_{b,m} + M_{ws,m} = 194.1 \text{ kip-ft}$$

Change in concrete stress at centroid of prestressing strands:

$$\Delta f_{cd} = \frac{P_\Delta}{A_g} + \frac{P_\Delta e_m^{\,2}}{I_g} - \frac{M_{sd} e_m}{I_g} = -0.451 \text{ ksi}$$

Prestress loss due to creep after beam placement:

$$\Delta f_{pCD} = (E_p / E_{ci})\, f_{cgp}\, [\Psi(t_f, t_i) - \Psi(t_d, t_i)] K_{id} + (E_p / E_c)\, \Delta f_{cd}\, \Psi(t_f, t_d)\, K_{df} = 0.377 \text{ ksi}$$

### *Prestress Loss due to Relaxation of Strands after Beam Placement*

The prestress loss due to the relaxation of the prestressing strands after beam placement is found using AASHTO LRFD BDS Eqn. 5.9.3.4.3c-1.

Prestress loss due to strand relaxation after beam placement: $\Delta f_{pR2} = \Delta f_{pR1} = 1.657$ ksi

### *Prestress Loss due to Shrinkage of Deck Concrete*

There is no cast-in-place deck in this example, so there is no prestress loss due to the shrinkage of the deck concrete. The effects of the shrinkage of the joints between the adjacent box beams will be neglected.

Prestress loss due to shrinkage of deck concrete: $\Delta f_{pSS} = 0$ ksi

### *Total Prestress Losses*

The total long-term prestress losses are found using AASHTO LRFD BDS Eqn. 5.9.3.4.1-1, which is the summation of all the long-term loss components.

Total time-dependent losses:

$$\Delta f_{pLT} = (\Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR1}) + (\Delta f_{pSD} + \Delta f_{pCD} + \Delta f_{pR2} + \Delta f_{pSS}) = 16.73 \text{ ksi}$$

## Force in Prestressing Strands after Losses

Strand stress and force after elastic shortening and after all losses is needed for the service limit state checks.

Strand stress after elastic shortening: $f_{pt} = f_{pbt} - \Delta f_{pES} = 195.44$ ksi

Strand force after elastic shortening: $P_{pt} = f_{pt} \times A_{pT} = 861.9$ kips

This stress and force after elastic shortening loss will be used in the stress calculations since gross section properties will be used. The stress in the strands before transfer should be used when calculating stresses if transformed section properties are used.

The strand stress after all losses including the gains from superimposed dead loads is calculated as follows.

Total pressing loss: $\Delta f_{pT} = \Delta f_{pES} + \Delta f_{pLT} = 23.79$ ksi

Strand stress after all losses (including permanent gains):

$$f_{pe} = f_{pbt} - \Delta f_{pT} + [(M_{b,m} + M_{ws,m})\, e_m / I_g] \times (E_p / E_c) = 179.81 \text{ ksi}$$

Force after all losses (including permanent gains): $P_{pe} = f_{pe} \times A_{pT} = 792.97$ kips

## SERVICE LIMIT STATE

Stresses need to be checked at the time of prestress transfer and at service (due to permanent loads and total loads). The sign convention for the stress checks provided in this section is positive for compression and negative for tension.

### UHPC Stresses at Prestress Transfer

#### *UHPC Stress Limits at Transfer*

The stress limits at transfer are specified in UHPC Structural Design Guide Article 9.2 which references UHPC Structural Design Guide Article 5.2. For prestressed components, the temporary stresses in the UHPC are limited to the following.

Compressive stress limit (AASHTO LRFD BDS Article 5.9.2.3.1a): $f_{ci,a} = 0.65f'_{ci} = 9.1$ ksi

Tensile stress limit (UHPC Structural Design Guide Article 5.2.1.3a): $f_{ti,a} = \gamma_u f_{t,cri} = -0.75$ ksi

#### *UHPC Stresses at Transfer Length at Transfer*

Stresses at the time of transfer are typically critical at the transfer length. For UHPC members, the transfer length is defined by UHPC Structural Design Guide Eqn. 9.4.3.1-1. The $\xi$ factor in the equation depends on if a shorter ($\xi = 0.75$) or longer ($\xi = 1.0$) transfer length is more critical. A shorter transfer length is more critical for checking stresses at the time of prestress transfer, so $\xi = 0.75$.

Transfer length: $\ell_t = \xi\, 24 d_b = (0.75)(24)(0.7 \text{ inch}) = 12.6$ inch

The self-weight of the beam is the only distributed load to consider at this point. The span length to use for calculating the moment and shear is the entire length of the beam, as the beam will rest on its ends as it cambers upward during transfer.

Moment at transfer length due to self-weight: $M_{g,T,tr} = 0.5 w_g\, \ell_t\, (L_{beam} - \ell_t) = 30.7$ kip-ft

The prestressing force and bending moment are applied to the gross section at transfer in this example. Elastic shortening loss needs to be subtracted from the jacking stress to find the prestress force at this time.

The prestressing force at the transfer length should include only strands that are developed at this point. Some prestressing strands can be debonded to reduce end region stresses. These strands should not be considered in total prestressing force. There are no debonded strands in this design example, so the prestressing force in the end region equals the prestressing force at midspan.

Stress at extreme top fiber of beam at transfer at transfer length:

$$f_{ci,tr} = \frac{f_{pt}A_{pT}}{A_g} - \frac{f_{pt}A_{pT}e_{pg}}{S_t} + \frac{M_{g,T,tr}}{S_t} = -0.044 \text{ ksi}$$

Stress at extreme bottom fiber of beam at transfer at transfer length:

$$f_{ti,tr} = \frac{f_{pt}A_{pT}}{A_g} + \frac{f_{pt}A_{pT}e_{pg}}{S_b} - \frac{M_{g,T,tr}}{S_b} = 3.146 \text{ ksi}$$

The stress in the top fiber and bottom fiber at the transfer length are less than the allowable stresses at transfer.

### *UHPC Stresses at Midspan at Transfer*

The moment at midspan from the self-weight of the beam is calculated as follows.

Moment at midspan due to self-weight:

$$M_{g,T,m} = 0.5w_g\,(0.5L_{beam})(L_{beam} - 0.5L_{beam}) = 709 \text{ kip-ft}$$

The stress at the top and bottom fiber are as follows.

Stress at extreme top fiber of beam at transfer at midspan:

$$f_{ci,m} = \frac{f_{pt}A_{pT}}{A_g} - \frac{f_{pt}A_{pT}e_{pg}}{S_t} + \frac{M_{g,T,m}}{S_t} = 1.443 \text{ ksi}$$

Stress at extreme bottom fiber of beam at transfer at midspan:

$$f_{ti,m} = \frac{f_{pt}A_{pT}}{A_g} + \frac{f_{pt}A_{pT}e_{pg}}{S_b} - \frac{M_{g,T,m}}{S_b} = 1.667 \text{ ksi}$$

The stress in the top fiber and bottom fiber are less than the allowable stresses at transfer.

### *Principal UHPC Stress in Web of Beam at Transfer Length at Transfer*

The principal stress in the web at the time of transfer also needs to be checked versus the allowable tensile stress at release. The principal stress at transfer should be checked at the mid-height of the section and at the top of the web.

The first moment of area at the centroid of the gross UHPC area ($y_1$ = 0 inch) is calculated using the function defined above.

First moment of area (at mid-height): $Q_{gc1} = Q(y_1 = 0 \text{ inch}) = 3{,}345 \text{ inch}^3$

The shear force at the transfer length due to the self-weight of the beam and the associated shear stress at the mid-height of the beam is calculated as follows.

Shear force at transfer length due to beam self-weight: $V_{g,tr} = w_g(0.5L_b - l_t) = 28.9$ kips

Maximum shear stress in web at centroid of gross UHPC section:

$\tau_{max} = (V_{g,tr} \times Q_{gc1}) / (I_g \times b_w) = 0.178$ ksi

The longitudinal stress in the web at the centroid of the gross UHPC section at the time of transfer is found assuming a linear stress profile across the depth of the section, where the stress in the top fiber is $f_{ci,tr}$ and the stress in the bottom fiber is $f_{ti,tr}$.

Longitudinal stress at beam centroid at transfer:

$f_{pcx} = f_{ti,tr} – [(f_{ci,tr} – f_{ti,tr}) / h] \times y_b = 1.555$ ksi

There are assumed to be no vertical stresses in the section at the transfer length.

Vertical stress at beam centroid at transfer: $f_{pcy} = 0$ ksi

The principal tensile stress is calculated using AASHTO LRFD BDS Eqn. 5.9.2.3.3-4.

Principal tensile stress at beam centroid at transfer:

$f_{min} = 0.5[(f_{pcx} + f_{pcy}) – \sqrt{}((f_{pcx} – f_{pcy})^2 + (2\tau_{max})^2) = –0.020$ ksi

This value is less than the allowable tensile stress at transfer ($f_{ti,a} = \gamma_u f_{t,cri} = –0.75$ ksi).

The first moment of area at the top of the web of the UHPC section ($y_1 = y_t – t_{tf} – 5.5$ inch) is calculated using the function defined above. The top of the web is assumed to be at the bottom of the 5.5-inch chamfer; the web width, $b_w$, at this location is still 6 inches.

First moment of area (at top of web): $Q_{gc2} = Q(y_1 = y_t – t_{tf} – 5.5 \text{ inch}) = 3{,}196 \text{ inch}^3$

The shear force at the transfer length is the same as above. The shear stress at the top of the web is as follows.

Maximum shear stress in web at top of the web:

$\tau_{max} = (V_{g,tr} \times Q_{gc2}) / (I_g \times b_w) = 0.170$ ksi

The longitudinal stress at the top of the web is found assuming the same linear stress profile as above with the distance between the bottom of the beam and top of the web as follows.

Distance from bottom of beam to top of web: $y_{tw} = h – t_{tf} – 5.5 \text{ inch} = 23.5$ inch

Longitudinal stress at beam centroid at transfer:

$f_{pcx} = f_{ti,tr} – [(f_{ci,tr} – f_{ti,tr}) / h] \times y_{tw} = 0.874$ ksi

There are assumed to be no vertical stresses in the section at the transfer length.

Vertical stress at beam centroid at transfer: $f_{pcy} = 0$ ksi

The principal tensile stress is calculated using AASHTO LRFD BDS Eqn. 5.9.2.3.3-4.

Principal tensile stress at beam centroid at transfer:

$f_{min} = 0.5[(f_{pcx} + f_{pcy}) – \sqrt{}((f_{pcx} – f_{pcy})^2 + (2\tau_{max})^2) = –0.032$ ksi

The tensile stress is slightly higher than the mid-height of the section, but still less than the allowable tensile stress at transfer ($f_{ti,a} = \gamma_u f_{t,cri} = –0.75$ ksi).

***Summary of UHPC Stress Checks at Transfer***

A summary of the actual versus allowable stresses in the UHPC at transfer is provided in Table 17.

**Table 17. Actual versus allowable stresses in UHPC at transfer.**

| Design Check | Allowable Value | Service Value | Check? |
|---|---|---|---|
| Top fiber at $l_t$ | $f_{ti,a} = –0.75$ ksi | $f_{ci,tr} = –0.044$ ksi | OK |
| Top fiber at $0.5L_{beam}$ | $f_{ci,a} = 9.1$ ksi | $f_{ci,m} = 1.443$ ksi | OK |
| Bottom fiber at $l_t$ | $f_{ci,a} = 9.1$ ksi | $f_{ti,tr} = 3.146$ ksi | OK |
| Bottom fiber at $0.5L_{beam}$ | $f_{ci,a} = 9.1$ ksi | $f_{ti,m} = 1.667$ ksi | OK |
| Principal web stress (mid-height) | $f_{ti,a} = –0.75$ ksi | $f_{min} = –0.020$ ksi | OK |
| Principal web stress (top of web) | $f_{ti,a} = –0.75$ ksi | $f_{min} = –0.032$ ksi | OK |

All stresses at transfer are less than the allowable stresses specified by the UHPC Structural Design Guide.

**Concrete Stresses at Service Loads**

***UHPC Stress Limits at Service***

The stress limits at service are specified in UHPC Structural Design Guide Article 9.2 which references UHPC Structural Design Guide Article 5.2.

The reduction factor $\phi_w$ is found according to AASHTO LRFD BDS Article 5.6.4.7.1 based on the wall slenderness ratio $\lambda_w$. The clear length of the constant thickness portion of between walls, $X_u$, is as follows.

Clear distance between adjacent walls: $X_u = b_e – b_w = 48$ inch – 6 inch = 42 inch

Wall thickness (assume minimum): $t = 0.5b_w = (0.5)(6$ inch$) = 3$ inch

Wall slenderness ratio: $\lambda_w = X_u / t = 42$ inch / 3 inch = 14

The reduction factor $\phi_w$ may be taken as 1.0 when the slenderness ratio is less than or equal to 15.0. The slenderness ratio was 14 for this example; a wider section with the same web width may lead to a slenderness ratio greater than 15, which would lead to a reduction factor less than 1.0.

Reduction factor: $\phi_w = 1.0$

For prestressed components, the service stresses in the UHPC are limited to the following.

Compressive stress due to effective prestress and permanent loads (AASHTO LRFD BDS Table 5.9.2.3.2a-1): $f_{cp,a} = 0.45f'_c = 0.45(17.5 \text{ ksi}) = 7.875 \text{ ksi}$

Compressive stress due to effective prestress and permanent loads, and transient loads (AASHTO LRFD BDS Table 5.9.2.3.2a-1): $f_{c,a} = 0.60\phi_w f'_c = 10.5 \text{ ksi}$

Tensile stress limit (UHPC Structural Design Guide Article 5.2.1.3b): $f_{t,a} = \gamma_u f_{t,cr} = -1.0 \text{ ksi}$

There is an additional stress limit for UHPC components subjected to cyclic stresses (UHPC Structural Design Guide Article 5.2.3). The stresses for this check are found using the Service I load combination. This limit will currently supersede the other tensile stress limit when it applies.

Tensile stress limit for UHPC components subjected to cyclic stresses (UHPC Structural Design Guide Article 5.2.3): $f_{t,a} = 0.95\gamma_u f_{t,cr} = -0.95 \text{ ksi}$

***Stress Components at Midspan at Service***

The concrete stresses at midspan at service must be calculated at the top and bottom of the UHPC section. Each component of the stresses can be calculated and then combined by superposition. The primary stress components are as follows. The first letter in the subscript "*t*" symbolizes the tension face, "*c*" the compression face, and "*s*" at the centroid of the tension reinforcement.

- Instantaneous (elastic) stresses at transfer ($f_{ti}$ and $f_{ci}$)
- Long-term effects between transfer and beam placement ($\Delta f_{t,LT,id}$ and $\Delta f_{c,LT,id}$)
- Instantaneous (elastic) stresses due to superimposed dead loads ($\Delta f_{t,sDL}$ and $\Delta f_{c,sDL}$)
- Long-term effects between beam placement and final time ($\Delta f_{t,LT,df}$ and $\Delta f_{c,LT,df}$)
- Instantaneous (elastic) stresses due to live loads ($\Delta f_{t,L}$ and $\Delta f_{c,L}$)

The stresses at transfer were previously calculated. These are summarized again as follows.

Stress at transfer at top of UHPC girder: $f_{ci} = 1.443 \text{ ksi}$

Stress at transfer at bottom of UHPC girder: $f_{ti} = 1.667 \text{ ksi}$

Jacking stress: $f_{pbt} = 202.5 \text{ ksi}$

Elastic shortening loss: $\Delta f_{pES} = 7.059 \text{ ksi}$

The long-term effects between transfer and beam placement are based on the prestress losses that occur during this period.

Long-term loss of steel stress: $\Delta f_{p,LT,id} = \Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR1} = 13.295 \text{ ksi}$

Long-term loss of steel force: $\Delta P_{p,LT,id} = \Delta f_{p,LT,id} \times A_{pT} = 58.632 \text{ kips}$

This force is applied at the centroid of the prestressing strands as a negative value (loss) to find the stress change due to these long-term effects.

Change in stress at top of UHPC beam between transfer and beam placement:

$\Delta f_{c,LT,id} = (-\Delta P_{p,LT,id} / A_g) - (-\Delta P_{p,LT,id} \times e_m) / S_t = 0.008$ ksi

Change in stress at bottom of UHPC beam between transfer and beam placement:

$\Delta f_{t,LT,id} = (-\Delta P_{p,LT,id} / A_g) + (-\Delta P_{p,LT,id} \times e_m) / S_b = -0.219$ ksi

The stress due to superimposed dead loads include the barrier weight and the future wearing surface weight.

Midspan moment due to superimposed dead loads: $M_{sDL,m} = M_{b,m} + M_{ws,m} = 194.1$ kip-ft

This moment results in the following stresses.

Change in stress at top of UHPC beam due to superimposed dead load:

$\Delta f_{c,sDL} = M_{sDL,m} / S_t = 0.425$ ksi

Change in stress at bottom of UHPC beam due to superimposed dead load:

$\Delta f_{t,sDL} = -M_{sDL,m} / S_b = -0.423$ ksi

Change in stress at centroid of reinforcement due to superimposed dead load:

$\Delta f_{cs,sDL} = -M_{sDL,m} \times e_m / I_g = -0.272$ ksi

Prestress gain due to superimposed dead load:

$\Delta f_{p,sDL} = \Delta f_{cs,sDL} \times (E_p / E_c) = -1.097$ ksi

The long-term effects between beam placement and final time are based on the prestress losses that occur during this period.

Long-term loss of steel stress: $\Delta f_{p,LT,df} = \Delta f_{pSD} + \Delta f_{pCD} + \Delta f_{pR2} = 3.431$ ksi

Long-term loss of steel force: $\Delta P_{p,LT,df} = \Delta f_{p,LT,df} \times A_{pT} = 15.132$ kips

This force is applied at the centroid of the prestressing strands as a negative value (loss) to find the stress change due to these long-term effects.

Change in stress at top of UHPC beam between beam placement and final time:

$\Delta f_{c,LT,df} = (-\Delta P_{p,LT,df} / A_g) - (-\Delta P_{p,LT,df} \times e_m) / S_t = 0.00196$ ksi

Change in stress at bottom of UHPC beam between beam placement and final time:

$\Delta f_{t,LT,df} = (-\Delta P_{p,LT,df} / A_g) + (-\Delta P_{p,LT,df} \times e_m) / S_b = -0.0560$ ksi

The stress due to live loads include the lane load and design truck. These moments include the appropriate distribution factors and impact factors.

Midspan moment due to superimposed dead loads: $M_{L,m} = M_{LL,m} + M_{LT,m} = 750.9$ kip-ft

This moment results in the following stresses.

Change in stress at top of UHPC beam due to live load:

$\Delta f_{c,L} = M_{L,m} / S_t = 1.646$ ksi

Change in stress at bottom of UHPC beam due to live load:

$\Delta f_{t,L} = M_{L,m} / S_b = –1.637$ ksi

Change in stress at centroid of reinforcement due to live load:

$\Delta f_{cs,L} = M_{L,m} \times e_m / I_g = –1.053$ ksi

Prestress gain due to live load:

$\Delta f_{p,L} = \Delta f_{cs,L} \times (E_p / E_c)= –4.245$ ksi

### *Net UHPC Girder Stresses at Midspan*

The stress components calculated in the previous section can be combined to calculate the stresses (1) due to the effective prestress and permanent loads and (2) due to the sum of effective prestress, permanent loads, and transient loads.

Stress at top fiber of beam due to effective prestress and permanent loads:

$f_{c,PL} = f_{ci} + \Delta f_{c,LT,id} + \Delta f_{c,sDL} + \Delta f_{c,LT,df} = 1.878$ ksi

Stress at top fiber of beam due to Service I load combination:

$f_{c,SeI} = f_{ci} + \Delta f_{c,LT,id} + \Delta f_{c,sDL} + \Delta f_{c,LT,df} + \Delta f_{c,L} = 3.524$ ksi

Stress at bottom fiber of beam due to Service I load combination:

$f_{t,SeI} = f_{ti} + \Delta f_{t,LT,id} + \Delta f_{t,sDL} + \Delta f_{t,LT,df} + \Delta f_{t,L} = –0.668$ ksi

The live load factor ($\gamma_{LL}$) is always equal to 1.0 for UHPC, so the stresses due to the Service III load combination are equal to those due to the Service I load combination.

Stress at bottom fiber of beam due to Service III load combination: $f_{t,SeI} = f_{t,SeIII} = –0.668$ ksi

These stresses are all less than the stress limits.

### *Principal Tensile Stress in Webs*

The principal stress at the centroid of the UHPC section needs to also be calculated and checked versus the specified limits using UHPC Structural Design Guide Article 5.2.1.3b. The critical section for shear is taken as the effective shear depth $d_v$ from the internal face of the support (UHPC Structural Design Guide Article 7.3.2 referring to AASHTO LRFD BDS Article 5.7.3.2). The effective shear depth $d_v$ need not be taken to be less than the greater of $0.9d_e$ and $0.72h$ (UHPC Structural Design Guide Article 7.2.8); this limit will be used to calculate the effective shear depth.

Effective shear depth: $d_v$ = maximum of $0.9d_e$ and $0.72h = 2.035$ ft

Critical section for shear: $x_{cr} = d_v + 0.5b_s = 2.535$ ft

The shear and moments at the critical section are summarized in Table 18. These were calculated using the previously discussed functions and include the appropriate distribution factors and impact factors for the live loads.

**Table 18. Shear and moments at critical section for shear.**

| Load | Shear | Moment |
|---|---|---|
| Beam self-weight | $V_{g,cr}$ = 27.7 kips | $M_{g,cr}$ = 72.2 kip-ft |
| Barrier weight | $V_{b,cr}$ = 3.9 kips | $M_{b,cr}$ = 10.0 kip-ft |
| Future wearing surface weight | $V_{ws,cr}$ = 3.9 kips | $M_{ws,cr}$ = 10.1 kip-ft |
| Lane load | $V_{LL,cr}$ = 12.7 kips | $M_{LL,cr}$ = 21.5 kip-ft |
| Design truck | $V_{LT,cr}$ = 37.0 kips | $M_{LT,cr}$ = 60.8 kip-ft |

The shear force at the critical section using the Service I load combination is calculated as follows.

Shear force at critical section at service:

$$V_{cr,SeI} = V_{g,cr} + V_{b,cr} + V_{ws,cr} + V_{LL,cr} + V_{LT,cr} = 85.1 \text{ kips}$$

The first moment of area at the centroid of the gross UHPC area ($y_1$ = 0 inch) is calculated using the function defined above.

First moment of area (at mid-height): $Q_{gc1} = Q(y_1 = 0 \text{ inch}) = 3{,}345 \text{ inch}^3$

The maximum shear stress in the web at the centroid of the gross section is calculated as follows.

Maximum shear stress in web at centroid of gross UHPC section:

$$\tau_{max} = (V_{cr,SeI} \times Q_{gc1}) / (I_g \times b_w) = 0.524 \text{ ksi}$$

The top and bottom fiber stresses in the UHPC at the critical section need to be calculated. Since gross section properties are being used, the total moment at the critical section can be used to calculate stresses.

Total moment at critical section:

$$M_{cr} = M_{g,cr} + M_{b,cr} + M_{sw,cr} + M_{LL,cr} + M_{LT,cr} = 174.6 \text{ kip-ft}$$

Top fiber stress at the critical section:

$$f_{c,cr} = (f_{pt} A_{pT}) / A_g - (f_{pt} A_{pt}\, e_{pg}) / S_t + \Delta f_{c,LT,id} + \Delta f_{c,LT,df} + M_{cr} / S_t = 0.281 \text{ ksi}$$

Bottom fiber stress at the critical section:

$$f_{t,cr} = (f_{pt} A_{pT}) / A_g + (f_{pt} A_{pt}\, e_{pg}) / S_b + \Delta f_{t,LT,id} + \Delta f_{t,LT,df} + M_{cr} / S_b = 2.646 \text{ ksi}$$

The longitudinal stress in the web at the centroid of the gross UHPC section at service is found assuming a linear stress profile across the depth of the section, where the stress in the top fiber is $f_{c,cr}$ and the stress in the bottom fiber is $f_{t,cr}$.

Longitudinal stress at beam centroid at transfer:

$$f_{pcx} = f_{t,cr} – [(f_{c,cr} – f_{t,cr}) / h] \times y_b = 1.466 \text{ ksi}$$

There are assumed to be no vertical stresses in the section at the transfer length.

Vertical stress at beam centroid at transfer: $f_{pcy} = 0$ ksi

The principal tensile stress is calculated using AASHTO LRFD BDS Eqn. 5.9.2.3.3-4.

Principal tensile stress at beam centroid at transfer:

$$f_{min} = 0.5[(f_{pcx} + f_{pcy}) – \sqrt{((f_{pcx} – f_{pcy})^2 + (2\tau_{max})^2)} = –0.168 \text{ ksi}$$

This is less than the allowable tensile stress at transfer ($f_{t,a} = \gamma_u f_{t,cr} = –1.0$ ksi).

***Summary of UHPC Stress Checks at Service***

A summary of the actual versus allowable stresses in the UHPC at service is provided in Table 19.

**Table 19. Actual versus allowable stresses in UHPC at service.**

| Design Check | Allowable Value | Service Value | Check? |
|---|---|---|---|
| UHPC compression (sustained) | $f_{cp,a}$ = 7.875 ksi | $f_{c,PL}$ = 1.878 ksi | OK |
| UHPC compression (Service I) | $f_{c,a}$ = 10.5 ksi | $f_{c,SeI}$ = 3.524 ksi | OK |
| UHPC tension (Service III) | $f_{t,a}$ = –1.0 ksi | $f_{t,SeIII}$ = –0.668 ksi | OK |
| UHPC tension (cyclic, Service I) | $f_{t,a}$ = –0.95 ksi | $f_{t,SeI}$ = –0.668 ksi | OK |
| Principal tension in web | $f_{t,a}$ = –1.0 ksi | $f_{min}$ = –0.168 ksi | OK |

All stresses at service are less than the allowable stresses specified by the UHPC Structural Design Guide.

**Steel Stress at Service Loads**

The prestress limit for bonded prestressing tendons for the Service I load combination is specified in UHPC Structural Design Guide Article 5.2.1.2, which references AASHTO LRFD BDS Article 5.9.2.2. Based on AASHTO LRFD BDS Table 5.9.2.2-1, the stress limit for low relaxation prestressing strands at the service limit state after losses ($f_{pe}$) is $0.80f_{py}$.

Stress limit for bonded prestressing strands at service: $f_{pe,a} = 0.80f_{py} = 194.4$ ksi

This stress limit should be in the outermost layer of prestressing strands, which will have the highest tensile stress.

Eccentricity of outermost layer of prestressing strands: $e_{m,o} = y_b – 2$ inch = 14.454 inch

The change in strand stress at transfer, $\Delta f_{pES,o}$, due to superimposed dead loads, $\Delta f_{p,sDL,o}$, and due to live loads, $\Delta f_{p,L,o}$, will be calculated at the centroid of the outermost layer of prestressing stands as follows.

Stress in concrete at centroid of prestressing strands at time of transfer:

$$f_{cgp,o} = P_{pt}\left(\frac{1}{A_g} + \frac{e_m\, e_{m,o}}{I_g}\right) - \frac{M_{g,m}e_{m,o}}{I_g} = 1.653 \text{ ksi}$$

Elastic shortening loss at centroid of outermost layer of prestressing strands:

$$\Delta f_{pES,o} = f_{cgp,o}\,(E_p / E_{ci}) = 7.173 \text{ ksi}$$

Change in stress at outermost layer of reinforcement due to superimposed dead load:

$$\Delta f_{cs,sDL,o} = -M_{sDL,m} \times e_{m,o} / I_g = -0.372 \text{ ksi}$$

Prestress gain due to superimposed dead load:

$$\Delta f_{p,sDL,o} = \Delta f_{cs,sDL,o} \times (E_p / E_c) = -1.498 \text{ ksi}$$

Change in stress at outermost layer of reinforcement due to live load:

$$\Delta f_{cs,L,o} = M_{L,m} \times e_{m,o} / I_g = -1.438 \text{ ksi}$$

Prestress gain due to live load:

$$\Delta f_{p,L,o} = \Delta f_{cs,L,o} \times (E_p / E_c) = -5.796 \text{ ksi}$$

The long-term prestress losses will be assumed to be the same in the outermost layer of strands as at the centroid of the strands.

Stress in prestressing strands due to Service I load combination after all losses and gains at the outermost layer of prestressing strands:

$$f_{pe,o,SeI} = f_{pbt} - \Delta f_{pES,o} - \Delta f_{p,LT,id} - \Delta f_{p,sDL,o} - \Delta f_{p,LT,df} - \Delta f_{p,L,o} = 185.9 \text{ ksi}$$

This value is less than the stress limit.

Many published design examples check the strand stress at the centroid of the prestressing strands. This stress can be calculated using the stress components described in the previous concrete stresses at service loads section.

Stress in prestressing strands due to Service I load combination after all losses and gains at the centroid of prestressing strands:

$$f_{pe,SeI} = f_{pbt} - \Delta f_{pES} - \Delta f_{p,LT,id} - \Delta f_{p,sDL} - \Delta f_{p,LT,df} - \Delta f_{p,L} = 184.1 \text{ ksi}$$

This value is slightly less than the stress at the centroid of the outermost layer of prestressing strands and less than the stress limit.

## FATIGUE LIMIT STATE

The moment at midspan due to the fatigue truck is calculated based on the function presented earlier. The impact factor for fatigue, $IM_f$, is 15 percent based on AASHTO LRFD BDS Article 3.6.2.

Fatigue truck moment at midspan: $M_{LT,f,m} = DFM_f\, M_{LT,f}\,(0.5L)(1 + IM_f) = 202.7$ kip-ft

The Fatigue I load combination only includes the factored live load as follows with a live load factor ($\gamma_{LL}$) of 1.75.

Fatigue I load combination: $M_{FI} = \gamma_{LL} M_{LT,f,m} = (1.75)(202.7 \text{ kip-ft}) = 354.8$ kip-ft

The fatigue limit state should be checked for UHPC compressive stress and tension stress in embedded steel elements.

### UHPC Compression Stress for Fatigue Limit State

The UHPC compressive stress limit for fatigue is defined by UHPC Structural Design Guide Article 5.3. The compressive stress due to Fatigue I load combination and one-half the sum of the unfactored effective prestress and permanent load shall not exceed $0.40f'_c$.

Fatigue compressive stress limit: $f_{c,f,a} = 0.40f'_c = 7.0$ ksi

The compressive stress due to one-half of the effective prestress and permanent loads includes the following components.

One-half effective prestress and permanent loads:

$$0.5f_{c,PL} = 0.5(f_{ci} + \Delta f_{c,LT,id} + \Delta f_{c,sDL} + \Delta f_{c,LT,df})$$
$$0.5f_{c,PL} = 0.5(1.443 \text{ ksi} + 0.008 \text{ ksi} + 0.425 \text{ ksi} + 0.002 \text{ ksi}) = 0.939 \text{ ksi}$$

The total compressive stress for the fatigue limit state requirement is calculated as follows.

Total compressive stress for fatigue:

$$f_{c,f} = 0.5f_{c,PL} + (M_{FI} / S_t) = 1.717 \text{ ksi} \leq f_{c,f,a} = 0.40f'_c$$
$$f_{c,f} = 0.939 \text{ ksi} + (354.8 \text{ kip-ft})(12 \text{ inch/ft}) / (5{,}474 \text{ inch}^3)$$
$$f_{c,f} = 1.717 \text{ ksi} \leq f_{c,f,a} = 0.40f'_c = (0.40)(17.5 \text{ ksi}) = 7.0 \text{ ksi}$$

This value is less than the stress limit for fatigue for the UHPC in compression.

### Tensile Stress for Embedded Steel Elements

The constant-amplitude fatigue threshold is defined by AASHTO LRFD BDS Article 5.5.3.1 in general and Article 5.5.3.3 for prestressing strands.

Constant-amplitude fatigue threshold (for prestressing): $(\Delta F)_{TH} = 18$ ksi

The tensile stress is calculated using the Fatigue I load combination at the outermost layer of the tension steel. The live load stress range at the bottom of the prestressing strands from the fatigue load is calculated using AASHTO LRFD BDS Article 5.5.3.1.

Live load stress range for reinforcement fatigue:

$\Delta f = [(M_{LT,f,m} \times (y_b - 2 \text{ inch})) / I_g] \times (E_p / E_c) = 1.565$ ksi

The factored stress is compared to the constant-amplitude threshold by AASHTO LRFD BDS Eqn. 5.5.3.1-1.

For fatigue considerations: $\gamma_{LL}(\Delta f) = 1.75(1.565 \text{ ksi}) = 2.74 \text{ ksi} \leq (\Delta F)_{TH} = 18$ ksi

The fatigue stress is less than the constant-amplitude threshold.

## STRENGTH LIMIT STATE – FLEXURE

The factored flexural resistance, $M_r$, must be greater than or equal to the factored demand due to the Strength I load combination.

The factored midspan moment demand is calculated using the Strength I load combination from AASHTO LRFD BDS Table 3.4.1-1.

Factored midspan moment demand:

$M_{u,m} = 1.25[M_g(0.5L) + M_b(0.5L)] + 1.5M_{ws}(0.5L) + 1.75(M_{LL,m} + M_{LT,m})$

$M_{u,m} = 2{,}449$ kip-ft

The factored flexural resistance is calculated according to UHPC Structural Design Guide Article 6.3.2 using the strain compatibility approach outlined UHPC Structural Design Guide Article 6.3.1. The following sections detail the process for calculating the moment capacity.

### Functions for Calculating Moment Capacity

Several different functions were defined to perform the strain compatibility analysis for different strain profiles. These functions were generally defined in terms of neutral axis depth, $c$, and bottom fiber strain, $\varepsilon_{cb}$.

The bottom fiber strain was sometimes defined in terms of $c$, so that other strain values in the section depth could be input. As an example, for the point where reinforcement reached the service limit, the bottom fiber strain was defined using the following function.

Bottom fiber strain at service limit function: $\varepsilon_{cb}(c) = (\varepsilon_{p,sl} - \varepsilon_{pe}) \times (h - c) / (d_t - c)$

The curvature function was defined assuming a linear strain profile based on the bottom fiber strain and the distance between the neutral axis and bottom fiber.

Curvature function: $\psi_c(c, \varepsilon_{cb}) = \varepsilon_{cb}(c) / (h - c)$

The compression force in the UHPC and distance from the top of the section to the centroid of the force were defined as follows.

UHPC compression force function:

$$C_c(c, \varepsilon_{cb}) = \int_{0\text{ in.}}^{c} b(c-y) \cdot f_c(\psi(c, \varepsilon_{cb}) \cdot y) \cdot \mathrm{d}y$$

Compression force centroid (distance from top of section) function:

$$Z_c(c, \varepsilon_{cb}) = \frac{\int_{0\text{ in.}}^{c} b(c-y) \cdot f_c(\psi(c, \varepsilon_{cb}) \cdot y) \cdot (c-y) \cdot \mathrm{d}y}{C_c(c, \varepsilon_{cb})}$$

Similarly, the tension force in the UHPC and distance from the top of the section to the centroid of the force were defined as follows.

UHPC tension force function:

$$T_c(c, \varepsilon_{cb}) = \int_{0\text{ in.}}^{h-c} b(c+y) \cdot f_t(\psi(c, \varepsilon_{cb}) \cdot y) \cdot \mathrm{d}y$$

Tension force centroid (distance from top of section) function:

$$X_c(c, \varepsilon_{cb}) = \frac{\int_{0\text{ in.}}^{h-c} b(c+y) \cdot f_t(\psi(c, \varepsilon_{cb}) \cdot y) \cdot (c+y) \cdot \mathrm{d}y}{T_c(c, \varepsilon_{cb})}$$

The strain in each layer of prestressing strands ($\varepsilon_{ps}$) was calculated based on the curvature ($\psi$), depth of the prestressing strand layer ($d_p$), neutral axis depth ($c$), and effective strain in the prestressing strands after all prestress losses ($\varepsilon_{pe}$). This strain was calculated for each layer, $i$, of prestressing strands.

Total strain in prestressing strands function: $\varepsilon_{ps,i}(c, \varepsilon_{cb}) = \psi_c(c, \varepsilon_{cb}) \times (d_{p,i} - c) + \varepsilon_{pe}$

The stress in each layer of prestressing was calculated based on the total strain in the prestressing. The stress in the UHPC at the level of the prestressing was subtracted from the total stress to account for the voids created in the UHPC by the prestressing strands. The tensile stress was deducted if the strands are located on the tension side of the neutral axis.

Stress in prestressing strands function: $f_{ps,i}(c, \varepsilon_{cb}) = f_p(\varepsilon_{ps,i}(c, \varepsilon_{cb})) - f_t(\varepsilon_{ps}(c, \varepsilon_{cb}) - \varepsilon_{pe})$

The force in the prestressing strands was this stress times the total area of all the strands in that specific layer.

Force in prestressing strands function: $T_{ps,i}(c, \varepsilon_{cb}) = A_{ps,i} \times f_{ps,i}(c, \varepsilon_{cb})$

The strain, stress, and force were calculated in each layer, *i*, of prestressing. All layers of prestressing strands should be included in this analysis, even those on the flexural compression side of the section.

The total axial force function was the summation of the UHPC and prestressing force components. Compression is positive in the following function.

Axial force function: $P_n(c, \varepsilon_{cb}) = C_c(c, \varepsilon_{cb}) - T_c(c, \varepsilon_{cb}) - \sum T_{ps,i}(c, \varepsilon_{cb})$

The moment were calculated by taking each force component times the respective lever arm.

Moment: $M_n(c, \varepsilon_{cb}) = C_c(c, \varepsilon_{cb}) \times [cg - Z_c(c, \varepsilon_{cb})] + T_c(c, \varepsilon_{cb}) \times [X_c(c, \varepsilon_{cb}) - cg]$
$+ \sum [T_{ps,i}(c, \varepsilon_{cb}) \times (d_{p,i} - \mathrm{cg})]$

These functions can be used to find the sectional response for any strain profile where the top fiber is in compression and bottom fiber in tension.

### Basic Points for Moment-Curvature Diagram

The only points on the moment-curvature diagram needed to calculate the factored flexural resistance are the service limit and the nominal moment capacity (typically controlled by crack localization).

The moment and curvature associated with four basic points were calculated for this example.

- Point 1: First Cracking
- Point 2: Service Limit
- Point 3: Yielding of Reinforcement
- Point 4: Crack Localization

The strain functions associated with these points are as follows.

Defined strain for Point 1 (cracking): $\varepsilon_{cb}(c) = \varepsilon_{t,cr} = 0.000156$

Defined strain for Point 2 (service limit): $\varepsilon_{cb}(c) = (\varepsilon_{p,sl} - \varepsilon_{pe}) \times (h - c) / (d_t - c)$

Defined strain for Point 3 (yield): $\varepsilon_{cb}(c) = (\varepsilon_{py} - \varepsilon_{pe}) \times (h - c) / (d_t - c)$

Defined strain for Point 4 (crack localization): $\varepsilon_{cb}(c) = \gamma_u \varepsilon_{t,loc} = 0.005$

These strain functions were used with the functions defined in the previous section to determine the associate neutral axis depths, moments, and curvatures. Calculations were completed separately to determine the neutral axis depth required for an axial load equal to 0 kips. The associated neutral axis depths, moments and curvatures for each point are summarized in Table 20.

**Table 20. Summary of neutral axis depths, moments, and curvatures for key points for Storyline Example 2.**

| Point | *c* (inch) | *M* (kip-inch) | ψ (rad / inch) |
|---|---|---|---|
| Point 1: Cracking | 26.16 | 21,737 | $2.069 \times 10^{-5}$ |
| Point 2: Service Limit | 16.39 | 27,540 | $4.712 \times 10^{-5}$ |
| Point 3: Yielding | 7.93 | 34,683 | $1.619 \times 10^{-4}$ |
| Point 4: Crack Localization | 7.16 | 35,483 | $1.935 \times 10^{-4}$ |

The nominal moment is associated with Point 4, which is where the section reaches the crack localization strain at the extreme tension fiber, which typically controls the response.

Nominal moment capacity: $M_n$ = 35,483 kip-inch

The other possible failure mechanisms are crushing of the concrete at the extreme compression fiber and rupture of the reinforcement at the outermost layer. The compression fiber strain at the time of crack localization is as follows.

Compression fiber strain at crack localization: $\varepsilon_c = 0.00139 < \varepsilon_{cu} = 0.0035$

This strain is less than the strain when the UHPC crushes, $\varepsilon_{cu} = 0.0035$, so crushing of the UHPC in compression does not control the failure.

The strain in the outermost layer of tension steel at the time of crack localization is as follows. This includes the strain in the concrete at this level and the locked-in strain differential, $\varepsilon_{pe}$.

Tensile strain in outermost layer of strands at crack localization: $\varepsilon_{ps,t} = 0.011$

This is less than the assumed ultimate strain for the prestressing strands, $\varepsilon_{pu} = 0.035$, so rupture of the strands does not control the failure.

The controlling failure mechanism for Point 4 would need to be modified if the extreme compression fiber strain were calculated as greater than the strain at which the UHPC crushes or if the strain in outermost layer of reinforcement were calculated as greater than the rupture strain for the reinforcement.

There are many different variables that will affect the controlling failure mechanism. Crack localization will control most of the time. Crushing of the concrete may control for high reinforcement or prestressing ratios, where the ratio would account for narrow compression faces and larger amounts of reinforcement.

Prestressing ratio: $\rho_p = A_{pT} / (b_f\, d_p)$

The rupture of the reinforcement may control for low-strain materials (e.g., carbon fiber reinforced polymer reinforcement) when the UHPC has a higher crack localization strain capacity in combination with a wide top flange (compression region).

### Strength Limit State Design Values

The nominal flexural resistance is calculated using UHPC Structural Design Guide Article 6.3.2.2. The moment associated with the section curvature values are calculated when:

- Compressive strain in UHPC at extreme compression fiber is equal to the compression strain limit, $\varepsilon_{cu}$,
- Tensile strain in UHPC at extreme tension fiber is equal to crack localization strain, $\gamma_u\varepsilon_{t,loc}$, or
- Strain in extreme tension steel is equal to rupture strain for the reinforcement.

For this example, the failure is controlled by the strain in the extreme tension fiber reaching the crack localization strain.

Nominal flexural resistance: $M_n = M_{loc} = 2{,}957$ kip-ft

Curvature at nominal moment capacity: $\psi_n = \psi_{loc} = 1.94 \times 10^{-4}$

The resistance factor is calculated using UHPC Structural Design Guide Article 5.4.2 and the curvature ductility ratio calculated using UHPC Structural Design Guide Article 6.3.2.3. The curvature at the steel service stress limit is needed to calculate the curvature ductility; this was calculated above as associated with Point 2.

Curvature at steel service stress limit: $\psi_{s\ell} = 4.71 \times 10^{-5}$

Curvature ductility ratio: $\mu = (\psi_n / \psi_{s\ell}) = (1.94 \times 10^{-4}) / (4.71 \times 10^{-5}) = 4.106$

The curvature ductility ratio limit, $\mu_\ell$, is defined in UHPC Structural Design Guide Article 6.2 and in the commentary of Article 6.3.2.3.

Curvature ductility ratio limit: $\mu_\ell = 3.0$

The resistance factor is calculated using UHPC Structural Design Guide Eqn. 5.4.2-1.

Resistance factor: $0.75 \leq \phi = 0.75 + 0.15(\mu - 1.0) / (\mu_\ell - 1.0) \leq 0.90$

Resistance factor when $\mu \geq \mu_\ell = 3.0$: $\phi = 0.9$

The factored nominal flexural resistance is as follows.

Factored nominal flexural resistance: $M_r = \phi M_n = 2{,}661$ kip-ft $\geq M_u = 2{,}449$ kip-ft

The factored nominal flexural resistance is greater than the factored demand.

### Minimum Reinforcement Check

The minimum reinforcement for flexural resistance is specified in UHPC Structural Design Guide Article 6.3.3, which refers to AASHTO LRFD BDS Article 5.6.3.3 with several amendments.

The factored flexural resistance, $M_r$, needs to be greater than or equal to the lesser of $1.33M_u$ and the cracking moment, $M_{cr}$. The cracking moment is calculated using AASHTO LRFD BDS Eqn. 5.6.3.3-1 using the effective cracking strength, $f_{t,cr}$, in place of the rupture strength, $f_r$.

There are several factors that are needed to calculate the cracking moment.

For structures other than precast segmental: $\gamma_1 = 1.6$

For bonded tendons: $\gamma_2 = 1.1$

For prestressing steel: $\gamma_3 = 1.0$

The cracking moment includes the compressive stress due to the effective prestressing force. The effective prestressing force is calculated ignoring elastic gains. Elastic shortening losses must also be subtracted from the jacking stress since gross section properties are used in the stress calculation.

Effective prestress in strands after allowance for all losses: $f_{pe2} = f_{pbt} - \Delta f_{pT}$

$$f_{pe2} = 202.5 \text{ ksi} - 23.785 \text{ ksi} = 178.7 \text{ ksi}$$

Effective force in strands after allowance for all losses: $P_{pe2} = A_{pT} \times f_{pe2}$

$$P_{pe2} = (4.41 \text{ inch}^2)(178.7 \text{ ksi}) = 788.1 \text{ kips}$$

The compressive stress caused by the effective prestress force at the extreme tension fiber is as follows.

Compressive stress in extreme tension fiber from effective prestress:

$$f_{cpe} = P_{pe2} / A_g + (P_{pe2}\, e_{pg}) / S_b$$

$$f_{cpe} = (788.1 \text{ kips} / 554.1 \text{ inch}^2) + (788.1 \text{ kips})(10.588 \text{ inch}) / 5{,}504 \text{ inch}^3 = 2.938 \text{ ksi}$$

For this example, there is no composite, cast-in-place deck, so the section modulus for the composite section, $S_c$, and non-composite section, $S_{nc}$, are equal. This allows AASHTO LRFD BDS Eqn. 5.6.3.3-1 to be simplified and calculated as follows.

Cracking moment (no CIP deck): $M_{cr} = \gamma_3[(\gamma_1 f_{t,cr} + \gamma_2 f_{cpe})S_b]$

$$M_{cr} = (1.0)[((1.6)(1.0 \text{ ksi}) + (1.1)(2.938 \text{ ksi}))(5{,}504 \text{ inch}^3)] = 2{,}216 \text{ kip-ft}$$

Demand limit component: $1.33M_{u,m} = 1.33(2{,}449 \text{ kip-ft}) = 3{,}257 \text{ kip-ft}$

The factored flexural resistance must be greater than the minimum of $M_{cr}$ and $1.33M_u$, which is $M_{cr} = 2{,}216$ kip-ft for this example. The factored flexural resistance in this example, $M_r = 2{,}661$ kip-ft is greater than the limit.

## STRENGTH LIMIT STATE – SHEAR

The factored shear resistance, $V_r = \phi V_n$, calculated using AASHTO LRFD BDS Eqn. 5.7.2.1-1 must be greater than or equal to the factored demand $V_u$.

The nominal shear resistance is calculated based on UHPC Structural Design Guide Article 7.3.3.

Nominal shear resistance: $V_n = V_{UHPC} + V_s + V_p$

There are no harped prestressing strands, so $V_p = 0$ kips.

UHPC contribution to shear resistance: $V_{UHPC} = \gamma_u f_{t,loc} b_v d_v \cot\theta$

Transverse reinforcement contribution to shear resistance (for $\alpha = 90°$): $V_s = A_v f_{v,\alpha} d_v \cot\theta / s$

The UHPC and steel contributions to the shear resistance are dependent on the angle of inclination of the compressive stresses θ and the stress in the transverse reinforcement, $f_{v,\alpha}$, which can be calculated either using an iterative procedure through the General Approach (UHPC Structural Design Guide Article 7.3.4.1) or design tables through the Simplified Approach (UHPC Structural Design Guide Article 7.3.4.2).

### Critical Section and Effective Shear Depth

The critical section and effective shear depth have similar definitions between the UHPC Structural Design Guide and AASHTO LRFD BDS. The critical section is defined as the distance $d_v$ from the internal face of the support for beams where the reaction force is in the direction of the applied shear (UHPC Structural Design Guide Article 7.3.2 referring to AASHTO LRFD BDS Article 5.7.3.2). The effective shear depth is defined in UHPC Structural Design Guide Article 7.2.8 as the distance between the resultants of the tensile and compressive forces due to flexure.

The distance between the resultant forces is typically straightforward to calculate for a conventional concrete section, because conventional concrete is neglected in tension and the compression stresses are idealized as a rectangular stress block. Additionally, the compression stress block region for conventional concrete members typically remains in a rectangular portion of the section (e.g., top flange, deck). This allows for the effective shear depth for conventional concrete members to be simply $d_v = M_n / (A_s f_y + A_{ps} f_{ps})$.

Determining the resultant forces for UHPC sections is not as straightforward. Strains, stresses, and associated forces at crack localization for the example are shown in Figure 39. The UHPC compression and tension stresses are across the entire depth of the section and steel reinforcement forces would commonly be toward the top and bottom of the section.

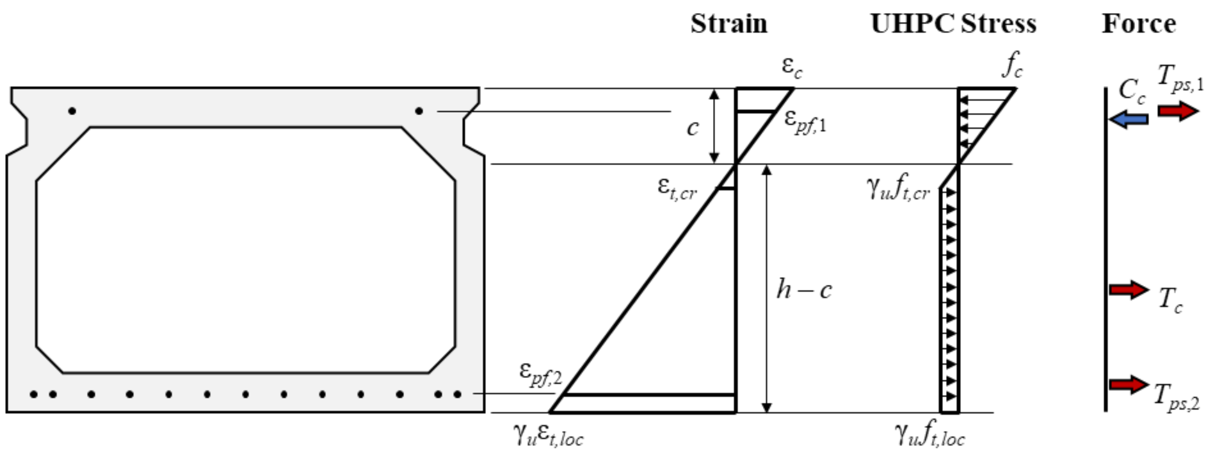



**Figure 39. Illustration. Strains, stresses, and forces at crack localization for Storyline Example 2.**

The definition for the effective depth includes a lower-bound value of the greater of $0.9d_e$ and $0.72h$. The simplest option for the effective shear depth is to assume it is equal to this lower bound.

Simplified assumption for effective shear depth: $d_v$ = maximum of $0.9d_e$ and $0.72h$

The shear depth can also be determined using the functions defined in the previous section. The centroid of the resultant compression force is simply $Z_c(c, \varepsilon_{cb})$, measured from the top of the section.

Centroid of compression resultant: $y_C(c, \varepsilon_{cb}) = Z_c(c, \varepsilon_{cb})$

The centroid of the tension resultant is a weighted average of the distances for the three forces.

Centroid of tension resultant: $y_T(c, \varepsilon_{cb}) = [X_c(c, \varepsilon_{cb}) \times T_c(c, \varepsilon_{cb}) + \sum (T_{ps,i}(c, \varepsilon_{cb}) \times d_{p,i})] / [T_c(c, \varepsilon_{cb}) + \sum T_{ps,i}(c, \varepsilon_{cb})]$

The effective shear depth is the distance between the centroids of these resultant forces.

Effective shear depth: $d_v(c, \varepsilon_{cb}) = y_T(c, \varepsilon_{cb}) - y_C(c, \varepsilon_{cb})$

The centroids of the resultant forces and therefore also the effective shear depth will depend on the strain and stress distribution in the section, as evidenced by being a function of the neutral axis depth and the bottom fiber strain. The actual strain and stress profiles in a section will change along the length of the beam depending on the applied moment; this is illustrated in Figure 40.

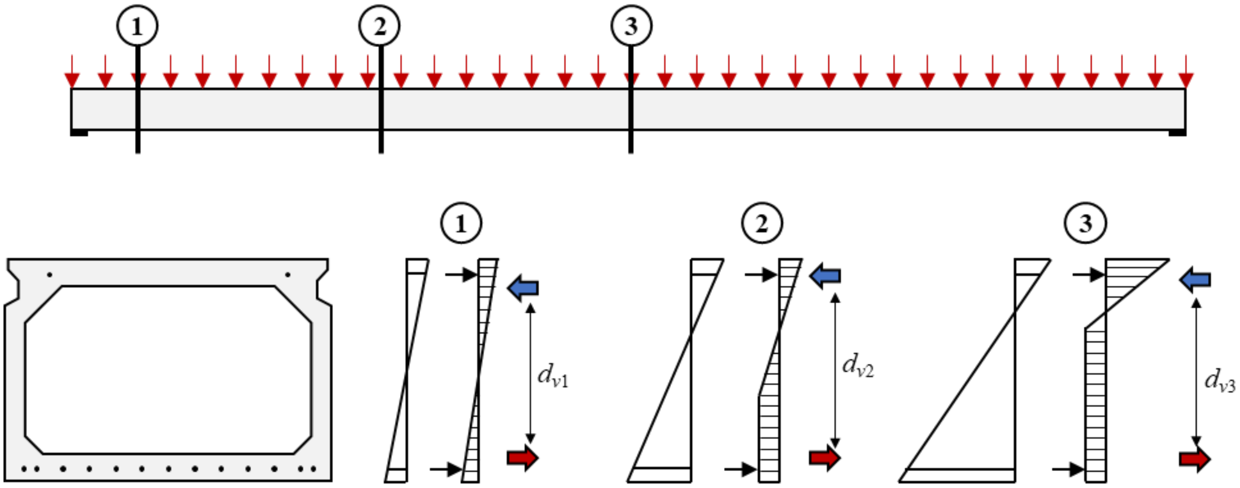



**Figure 40. Illustration. Changing strain and stress profile along length of beam and corresponding effective shear depth.**

The distances from the top of the section to the centroids of the resultant compression and tension forces, $y_C$ and $y_T$ respectively, and the effective shear depths associated with the key points from the moment-curvature diagram previously discussed are summarized in Table 21.

**Table 21. Summary of neutral axis depths, bottom fiber strain, and effective shear depth for key points on the moment-curvature diagram for Storyline Example 2.**

| Point | $c$ (inch) | $y_C$ (inch) | $y_T$ (inch) | $d_v$ (inch) |
|---|---|---|---|---|
| Point 1: Cracking | 26.14 | 4.413 | 27.996 | 23.583 |
| Point 2: Service Limit | 16.39 | 3.023 | 28.056 | 25.032 |
| Point 3: Yielding | 7.93 | 2.012 | 27.899 | 25.887 |
| Point 4: Crack Localization | 7.16 | 1.905 | 27.847 | 25.941 |

The effective shear depth is typically calculated assuming the strain and stress profile associated with the nominal flexural capacity; not the actual strain and stress profiles caused by the applied moment at a section along the length of the beam. Finding the effective shear depth based on the strain and stress profiles at the nominal flexural capacity is also a reasonable assumption for UHPC members.

For Storyline Example 2 at the crack localization point, the resultant force centroids and effective shear depth are as follows.

Centroid of compression resultant: $y_C(c = 7.16 \text{ inch}, \varepsilon_{cb} = \gamma_u \varepsilon_{t,loc}) = 1.91$ inch

Centroid of tension resultant: $y_T(c = 7.16 \text{ inch}, \varepsilon_{cb} = \gamma_u \varepsilon_{t,loc}) = 27.85$ inch

Effective shear depth: $d_v(c = 7.16 \text{ inch}, \varepsilon_{cb} = \gamma_u \varepsilon_{t,loc}) = 25.94$ inch

The lower-bound limit for the effective shear depth will be used in this example.

Effective shear depth used for Storyline Example 2:

$d_v$ = maximum of $0.9d_e$ and $0.72h$ = 24.42 inch

The demand was found using this simplified assumption for the effective shear depth.

Critical section: $x_{cr} = 0.5\ell_b + d_v$ = 0.5(12 inch) + 24.42 inch = 30.42 inch

### Demand at Critical Section

The factored shear demand and factored bending moment are calculated at the section along the length of the beam where the shear resistance is being calculated, which is the critical section for this example. The shear and moments at the critical section from the different applied loads are summarized in Table 18. The Strength I load combination is used to calculate the factored shear demand and bending moment.

Factored shear demand due to dead load and live load at critical section:

$$V_{u,cr} = 1.25(V_{g,cr} + V_{b,cr}) + 1.5V_{ws,cr} + 1.75(V_{LL,cr} + V_{LT,cr}) = 132.2 \text{ kips}$$

Factored moment demand due to dead load and live load at critical section:

$$M_{u,cr} = 1.25(M_{g,cr} + M_{b,cr}) + 1.5M_{ws,cr} + 1.75(M_{LL,cr} + M_{LT,cr}) = 261.9 \text{ kip-ft}$$

### Transfer and Development Length for Shear

Shear is often critical near the supports and ends of the beam. The transfer length and development length need to be calculated and compared to the available strand bond length at the critical section.

For UHPC members, the transfer length is defined by UHPC Structural Design Guide Eqn. 9.4.3.1-1. The $\xi$ factor in the equation depends on if a shorter ($\xi = 0.75$) or longer ($\xi = 1.0$) transfer length is more critical. A longer transfer length is more critical when designing for shear, so $\xi = 1.0$.

Transfer length: $\ell_t = \xi\, 24d_b$ = (1.0)(24)(0.7 inch) = 16.8 inch

The development length is calculated using UHPC Structural Design Guide Eqn. 9.4.3.2-1. The average stress in the prestressing steel at the nominal flexural resistance can be calculated using the strain compatibility approach at the same time the nominal flexural resistance is calculated. The following stress was calculated at the point of crack localization for this example.

Average stress at nominal flexural resistance: $f_{ps}$ = 248.5 ksi

The effective prestress in the prestressing steel after all losses, not including gains, was also previously calculated.

Effective prestress in strands after allowance for all losses: $f_{pe2} = f_{pbt} - \Delta f_{pT}$ = 178.7 ksi

The development length will be as follows.

Development length: $\ell_d = \ell_t + 0.30(f_{ps} - f_{pe2})d_b$ = 31.5 inch

The available development length for the prestressing strands extends from the end of the beam to the critical section, $d_v$ from the face of the support.

Available development length: $\ell_{d,avail} = d_v + b_s = 36.4$ inch

For this example, the available development length is greater than the required transfer and development lengths.

**General Approach**

The steps for calculating θ and $f_{v,\alpha}$ using the General Approach are outlined in Chapter 3.

**Step 1**: Calculate the net longitudinal tensile strain in the section at the centroid of the tension reinforcement using UHPC Structural Design Guide Eqn. 7.3.4.1-6 if $\varepsilon_s > \varepsilon_{t,cr}$ or UHPC Structural Design Guide Eqn. 7.3.4.1-7 if $\varepsilon_s \leq \varepsilon_{t,cr}$.

The definition for $M_u$ under UHPC Structural Design Guide Eqn. 7.3.4.1-7 specifies that $M_u$ need not be taken less than the absolute value of $(V_u - V_p)d_v$.

Factored moment demand (previously calculated): $M_{u,cr} = 261.9$ kip-ft

Lower limit for $M_u$: $M_{u,cr} \geq (V_u - V_p)d_v = 269.1$ kip-ft

Factored moment demand at critical section (lower limit controls):

$$M_{u,cr} = 269.1 \text{ kip-ft} = 3{,}229 \text{ kip-inch}$$

The area of prestressing on the flexural tension side of the member includes the strands in the bottom flange, but not the top strands.

Number of strands on flexural tension side: $n_{strand,tens} = 13$

Area of strands on flexural tension side: $A_{ps} = A_{p,0.7in} \times n_{strand,tens}$

$$A_{ps} = (0.294 \text{ inch}^2)(13) = 3.822 \text{ inch}^2$$

The area of concrete on the flexural tension side of the member can be calculated through integration since the section was defined by a function. The function was defined based on section depth $z$ measured from the compression face.

Area of UHPC on flexural tension side of member:

$$A_{ct,cr} = \int_{0.5h}^{h} b(z)\,\mathrm{d}z - A_{ps} = 272.1 \text{ inch}^2$$

The parameter taken as the modulus of elasticity of prestressing steel multiplied by the locked-in strain differential, $f_{po}$, can be taken as $0.7f_{pu}$ for usual prestressing level.

Locked-in strain differential parameter: $f_{po} = 0.7f_{pu} = 189$ ksi

Assuming initially that $\varepsilon_s \leq \varepsilon_{t,cr}$, UHPC Structural Design Guide Eqn. 7.3.4.1-7 is used to calculate $\varepsilon_s$. There is no applied axial force ($N_u$ = 0 kips), no harped strands ($V_p$ = 0 kips), and no non-prestressed reinforcement on the flexural-tension side of the member ($A_s$ = 0 kips), so the equation simplifies to the following.

Net longitudinal tensile strain at centroid of tension reinforcement:

$$\varepsilon_s = \frac{\left(\frac{|M_u|}{d_v} + |V_u| - A_{ps} f_{po}\right)}{E_p A_{ps} + E_c A_{ct}} = \frac{\left(\frac{|3{,}229 \text{ kip-in}|}{24.42 \text{ in}} + |132.2 \text{ kip}| - (3.822 \text{ in}^2)(189 \text{ ksi})\right)}{(28{,}500 \text{ ksi})(3.822 \text{ in}^2) + (7{,}072 \text{ ksi})(272.1 \text{ in}^2)} = -0.00023$$

**Step 2**: Assume details for transverse reinforcement (e.g., $A_v$, $s$, $E_s$).

The beam is initially assumed to not required any transverse reinforcement.

Area of transverse reinforcement: $A_v$ = 0 inch$^2$

**Step 3**: Assume $f_{v,\alpha} = f_y$ (will check this assumption at the end).

This step is not required, because there is no transverse reinforcement.

**Step 4**: Solve for the principal angle direction, θ, using UHPC Structural Design Guide Eqn. 7.3.4.1-1 and Eqn. 7.3.4.1-5.

UHPC Structural Design Guide Eqn. C7.3.4.1-1 can be used to solve for the principal angle direction for sections without transverse reinforcement.

UHPC Structural Design Guide Eqn. C7.3.4.1-1:

$$\gamma_u \varepsilon_{t,loc} = \frac{\varepsilon_s}{2}(1 + \cot^2\theta) + \frac{2f_{t,loc}}{E_c}\cot^4\theta = \frac{-0.00023}{2}(1 + \cot^2\theta) + \frac{2(1.0 \text{ ksi})}{7{,}072 \text{ ksi}}\cot^4\theta = (1.0)(0.005)$$

A solver was used to solve for θ in UHPC Structural Design Guide Eqn. C7.3.4.1-1.

Principal angle direction: θ = 25.35°

**Step 5**: Check the assumed stress in the transverse reinforcement using UHPC Structural Design Guide Eqn. 7.3.4.1-2, Eqn. 7.3.4.1-3, and Eqn. 7.3.4.1-4.

This step is not required since there is no transverse reinforcement.

**Step 6**: Return to Step 3 and modify the assumed $f_{v,\alpha}$ as needed.

This step is not required since there is no transverse reinforcement.

**Step 7**: Calculate resistance ($V_r$) using the calculated θ and $f_{v,\alpha}$ to find $V_{UHPC}$ and $V_s$. Check the factored resistance ($\phi V_n$) against the demand ($V_u$). Return to Step 2 and modify the transverse reinforcement provided as needed.

UHPC contribution to shear resistance: $V_{UHPC} = \gamma_u f_{t,loc} b_v d_v \cot\theta$

$$V_{UHPC} = (1.0)(1.0 \text{ ksi})(6 \text{ inch})(24.42 \text{ inch})\cot(25.35°) = 309.3 \text{ kips}$$

Transverse reinforcement contribution to shear resistance (for $\alpha$ = 90°):

$$V_s = A_v f_{v,\alpha} d_v \cot\theta / s = 0 \text{ kips}$$

Nominal shear resistance: $V_n = V_{UHPC} + V_s + V_p = 309.3$ kips

The upper limit for shear resistance is defined by UHPC Structural Design Guide Eqn. 7.3.3-2.

Upper limit for shear resistance: $V_n \leq 0.25 f'_c b_v d_v + V_p$

$$V_n \leq 0.25(17.5 \text{ ksi})(6 \text{ inch})(24.42 \text{ inch}) + 0 \text{ kips} = 641.0 \text{ kips}$$

The nominal shear resistance is less than this upper limit.

The resistance factor for shear is defined by UHPC Structural Design Guide Article 5.4.2 for shear and torsion in reinforced and unreinforced section as 0.90.

Factored nominal shear resistance: $\phi V_n = 0.9(309.3 \text{ kips}) = 278.3 \text{ kips} \geq V_{u,cr} = 132.2$ kips

The capacity is greater than the demand at the critical section using the General Approach.

**Simplified Approach**

The simplified approach may only be used when certain conditions are met. The conditions from UHPC Structural Design Guide Article 7.3.4.2 for using the simplified approach for members without transverse reinforcement are summarized in Table 22. All conditions are met, so the simplified approach may be used for this example.

**Table 22. Requirements for using UHPC Structural Design Guide Table B1.2-1**

| Requirement for Simplified Approach | Example Value | OK? |
|---|---|---|
| $E_c \geq 6{,}500$ ksi | $E_c = 7{,}072$ ksi | OK |
| $f_{t,loc} \leq 1.80$ ksi | $f_{t,loc} = 1.0$ ksi | OK |
| $\rho_{v,\alpha} = 0$ percent | $\rho_{v,\alpha} = 0$ percent | OK |

The modulus of elasticity associated with the minimum compressive strength of UHPC ($f'_c = 17.5$ ksi) is $E_c = 6{,}429$ ksi when $K_1 = 1.0$. This means that the simplified approach cannot be used for UHPC only meeting the minimum qualifications.

The following steps are used for finding $\theta$ and $f_{v,\alpha}$ using the simplified approach.

**Step 1**: Calculate the net longitudinal tensile strain in the section at the centroid of the tension reinforcement using UHPC Structural Design Guide Eqn. 7.3.4.1-6 if $\varepsilon_s > \varepsilon_{t,cr}$ or UHPC Structural Design Guide Eqn. 7.3.4.1-7 if $\varepsilon_s \leq \varepsilon_{t,cr}$.

The net longitudinal tensile strain was calculated above.

Net longitudinal tensile strain at centroid of tension reinforcement: $\varepsilon_s = -0.00023$

**Step 2**: Assume details for transverse reinforcement (e.g., $A_v$, $s$, $E_s$)

The beam is initially assumed to not required any transverse reinforcement.

Area of transverse reinforcement: $A_v = 0$ inch$^2$

**Step 3**: Calculate the transverse reinforcement ratio using UHPC Structural Design Guide Eqn. C7.3.4.1-4 (for $\alpha = 90°$).

There is no transverse reinforcement for this example.

Transverse reinforcement ratio: $\rho_{v,\alpha} = 0$ percent

**Step 4**: Use $\varepsilon_s$ and $\gamma_u \varepsilon_{t,loc}$ to find $\theta$ and $f_{v,\alpha}$ using the appropriate design table. A brief example of how to use the design tables is provided below.

The known values for this example are as follows.

Example known values: $\varepsilon_s = -0.00023$, $\gamma_u \varepsilon_{t,loc} = 0.005$, and $\rho_{v,\alpha} = 0.000$

UHPC Structural Design Guide Table B1.2-1 should be used for this example. An excerpt from this table is shown in Table 23.

**Table 23. Excerpt from UHPC Structural Design Guide Table B1.2-1 for finding θ.**

| $\varepsilon_s \times 1{,}000$ | … | $\gamma_u \varepsilon_{t,loc} \geq 0.0045$ | $\gamma_u \varepsilon_{t,loc} \geq 0.0050$ | … |
|---|---|---|---|---|
| … | … | … | … | … |
| ≤ – 0.5 | … | 29.3 | 28.8 | … |
| ≤ 0.0 | … | 30.6 | 30.0 | … |
| … | … | … | … | … |

Linear interpolation is not permitted for these design tables. The principal angle direction, θ, will be the following.

Principal angle direction: $\theta = 30.0°$

**Step 5**: Calculate resistance using the calculated θ and $f_{v,\alpha}$ to find $V_{UHPC}$ and $V_s$. Check the factored resistance ($\phi V_n$) against the demand ($V_u$). Return to Step 2 and modify the transverse reinforcement provided as needed.

UHPC contribution to shear resistance: $V_{UHPC} = \gamma_u f_{t,loc} b_v d_v \cot\theta$

$$V_{UHPC} = (1.0)(1.0 \text{ ksi})(6 \text{ inch})(24.42 \text{ inch})\cot(30.0°) = 253.8 \text{ kips}$$

Transverse reinforcement contribution to shear resistance (for $\alpha = 90°$):

$$V_s = A_v f_{v,\alpha} d_v \cot\theta / s = 0 \text{ kips}$$

Nominal shear resistance: $V_n = V_{UHPC} + V_s + V_p = 253.8$ kips

Factored nominal shear resistance: $\phi V_n = 0.9(253.8 \text{ kips}) = 228.4 \text{ kips} \geq V_{u,cr} = 132.2$ kips

The capacity is greater than the demand at the critical section using the Simplified Approach.

**Minimum Longitudinal Reinforcement Requirement**

The minimum longitudinal reinforcement required to prevent a development failure of the tension reinforcement is defined by UHPC Structural Design Guide Article 7.3.5. The end region will typically control this requirement, so UHPC Structural Design Guide Eqn. 7.3.5-2 is used to check this requirement.

Minimum longitudinal reinforcement check (at the inside edge of bearing support in a simply-supported beam):

$$A_{ps}f_{ps} + A_sE_s\gamma_u\varepsilon_{t,loc} + 0.6A_{ct}\gamma_uf_{t,cr} \geq (V_u / \phi_v - 0.5V_s - V_p) \cot \theta$$

The right side of the equation represents that capacity and the left side the demand at the section. The following assumptions were made for this design check.

- Only the reinforcement and prestressing strands on the flexural tension side of the section should be included in $A_s$ and $A_{ps}$.
- Demand at the inside edge of the bearing is taken as the demand at the critical section (AASHTO LRFD BDS Article C5.7.3.5), i.e., use $V_u$, $V_s$, $V_p$, and $\theta$ calculated at $d_v$ from the face of the support.

Many of the values needed for this design check were previously calculated or discussed. These include the following for the demand side of the equation.

- Resistance factor for shear: $\phi_v = 0.9$
- No transverse reinforcement: $V_s = 0$ kips
- No harped strands: $V_p = 0$ kips
- Factored shear demand at critical section: $V_u = 132.2$ kips
- Angle of inclination of diagonal compressive stresses: $\theta = 25.35°$

The demand side of the equation is equal to the following value.

Demand side of longitudinal reinforcement check:

$$(V_u / \phi_v - 0.5V_s - V_p) \cot \theta = (132.2 \text{ kips} / 0.9 - 0.5(0 \text{ kips}) - (0 \text{ kips})) \cot(25.35°)$$
$$= 310.2 \text{ kips}$$

The capacity side of the equation includes contribution from prestressing strands, non-prestressed reinforcement, and UHPC. Some of the values needed for the capacity were previously calculated or discussed.

- Area of strands on flexural tension side: $A_{ps} = 3.822$ inch$^2$
- No non-prestressed tension reinforcement: $A_s = 0$ inch$^2$

- Area of UHPC on flexural tension side of member: $A_{ct}$ = 272.1 inch$^2$
- Effective cracking strength: $\gamma_u f_{t,cr}$ = 1.0 ksi

The stress in the prestressing strands at this location will depend on the available development length and required transfer and development lengths. The transfer and development lengths were previously calculated with $\xi$ = 1.0 as a longer transfer and development length will be more critical to the design.

- Transfer length ($\xi$ = 1.0): $\ell_t$ = 16.8 inch
- Development length ($\xi$ = 1.0): $\ell_d$ = 31.5 inch

The available development length will depend on the length of the bearing, if there is any overhang between the end of the beam and outside edge of the bearing, and the distance between the inside edge of the bearing and the point when the centroid of the tension prestressing strands leaves the assumed failure crack, shown in Figure 41.

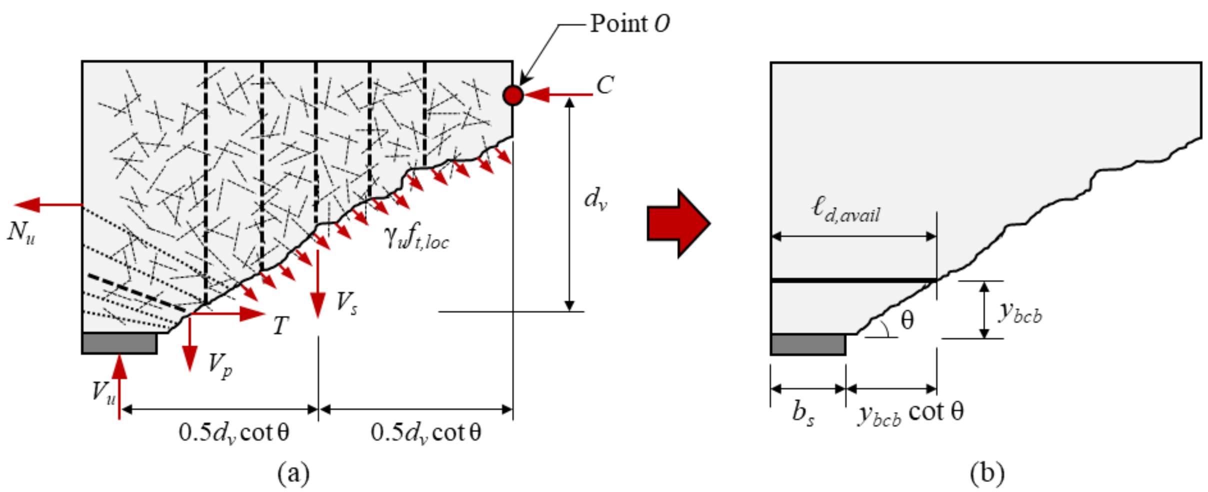



**Figure 41. Illustration. (a) Assumed failure crack in resistance model development and (b) available development length for reinforcement at failure crack.**

The available development length can be calculated as follows.

- Width of support: $b_s$ = 12 inch
- Distance from bottom face to centroid of strands: $y_{bcb}$ = 2 inch
- Available development length: $\ell_{d,avail} = b_s + y_{bcb} \cot\theta$ = 12 inch + (2 inch)cot(25.35°) = 16.95 inch

The stress in the prestressing strands considering the available development is calculated using UHPC Structural Design Guide Article 9.4.3.2.

Stress in strands if $\ell_{d,avail} \leq \ell_t$: $f_{px} = f_{pe}\, \ell_{d,avail} / \ell_t$

Stress in strands if $\ell_t < \ell_{d,avail} \leq \ell_d$: $f_{px} = f_{pe} + (\ell_{px} - \ell_t) / (0.30 d_b)$

The effective prestress and stress at the nominal flexural strength were previously calculated as the following.

- Effective stress in strands (no gains): $f_{pe2}$ = 178.7 ksi
- Strand stress at nominal flexural strength: $f_{ps}$ = 248.5 ksi

In this example, the available development length is greater than the transfer length, but less than the development length. The stress in the strands is as follows.

Stress in strands if $\ell_t < \ell_{d,avail} \leq \ell_d$: $f_{px} = f_{pe2} + (\ell_{px} - \ell_t) / (0.30d_b)$

$$f_{px} = 178.7 \text{ ksi} + (16.95 \text{ inch} - 16.8 \text{ inch}) / (0.30(0.7 \text{ inch})) = 179.4 \text{ ksi}$$

The capacity side of the longitudinal reinforcement requirement check can now be calculated as follows.

Capacity side of longitudinal reinforcement check:

$$A_{ps}f_{ps} + A_sE_s\gamma_u\varepsilon_{t,loc} + 0.6A_{ct}\gamma_uf_{t,cr} = (3.822 \text{ inch}^2)(179.4 \text{ ksi}) + 0.6(272.1 \text{ inch}^2)(1.0 \text{ ksi})$$
$$= 849.1 \text{ kips}$$

The capacity (849.1 kips) is greater than the demand (310.2 kips) for the longitudinal reinforcement check.

## PRETENSIONED ANCHORAGE ZONE

Splitting and confinement reinforcement are required for pretensioned anchor zone regions as specified by UHPC Structural Design Guide 9.4.4.1 with references to AASHTO LRFD BDS 5.9.4.4.1. Based on AASHTO LRFD BDS 5.9.4.4.1, the resistance shall not be less than 4 percent of the total prestressing force at transfer.

Force in strands just before transfer: $P_{pbt} = A_{pT}\, f_{pbt} = (4.41 \text{ inch}^2)(202.5 \text{ ksi}) = 893.0$ kips

Required resistance: $P_r \geq 0.04P_{pbt} = 0.04(893.0 \text{ kips}) = 35.72$ kips

The splitting resistance in UHPC members is provided by splitting reinforcement and the tensile strength of the UHPC.

Contribution of UHPC: $P_{r,UHPC} = 0.25\gamma_u\, f_{t,cri}\, b_v\, h$

$$P_{r,UHPC} = 0.25(1.0)(0.75 \text{ ksi})(6 \text{ inch})(33 \text{ inch}) = 37.13 \text{ kips}$$

No splitting reinforcement is required because $P_{r,UHPC}$ is greater than $0.04P_{pbt}$. Some reinforcement will be provided to satisfy the confinement reinforcement requirement that will also contribute to splitting resistance. For box beams, $A_s$ is taken as the total area of the vertical or horizontal reinforcement located within $h$/4 from the end of the beam.

Confinement reinforcement shall include No. 3 bars spaced at less than 6.0 inches within a distance 1.5$d$ from the ends of the beam.

End region for confinement reinforcement: $1.5d_p = 1.5(27.1 \text{ inch}) = 40.7$ inch

The minimum amount of reinforcement will be provided in the end of the box beam.

End region reinforcement: No. 3 bars spaced at 6.0 inches

The total area of reinforcement in the end region considered for splitting reinforcement ($h/4 = 8.25$ inches) is as follows.

Splitting reinforcement provided: $A_s = 2(2 \text{ legs})(0.11 \text{ inch}^2) = 0.44 \text{ inch}^2$

The stress in the splitting reinforcement is defined as not to exceed 20 ksi; the upper limit is typically assumed for this resistance calculation.

Stress in splitting reinforcement (assume maximum allowed): $f_s = 20$ ksi

The factored splitting resistance is as follows.

Factored splitting resistance: $P_r = f_s A_s + P_{r,UHPC} = (20 \text{ ksi})(0.44 \text{ inch}^2) + 37.1 \text{ k} = 45.9$ kips

The factored resistance is greater than the required resistance. No. 3 bars spaced at 6.0 inches should be provided within the first $1.5d_p = 40.7$ inches of the beam ends.

## DEFLECTION AND CAMBER

Camber and long-term deflections will be estimated using the effective modulus method. These deflection estimates can be used for construction and rideability. The immediate deflections due to live loads can be checked versus provided live load deflection limits at the owner's discretion.

### Camber and Long-Term Deflection Estimates

The immediate camber due to the effect of prestressing and beam self-weight is calculated using the strand stress immediately after transfer and the modulus of elasticity at release for the concrete. The beam will rest on its ends as it cambers upward in the prestressing bed, so the total beam length should be used in these deflection estimates.

The previously calculated values needed for calculating the deflection at transfer include the following.

- Force in prestressing immediately after transfer: $P_{pt} = 861.9$ kips
- Modulus of elasticity of concrete at transfer: $E_{ci} = 6{,}570$ ksi
- Total beam length: $L_T = 96 \text{ ft} = 1{,}152$ inch
- Strand eccentricity (straight strand profile): $e_{pg} = 10.588$ inch
- Gross moment of inertia: $I_g = 90{,}567 \text{ inch}^4$
- Self-weight: $w_g = 0.616$ kip/ft

The camber and deflection at transfer can be calculated as follows.

Camber due to prestressing force at transfer: $\Delta_p = (P_{pt}\, e_{pg}\, L_T^2) / (8E_{ci}\, I_g) = 2.54$ inch (up)

Deflection due to self-weight at transfer: $\Delta_g = (5w_g L_T^4) / (384E_{ci} I_g) = 1.98$ inch (down)

Deflection at transfer: $\Delta_{transfer} = \Delta_g - \Delta_p = -0.567$ inch (negative is upward deflection)

Long-term deflections can be calculated using the effective modulus method. For this method, creep coefficients are used to modify the modulus of elasticity of the concrete. The creep coefficients were previously calculated.

- Creep coefficient (transfer to final time): $\Psi(t_f, t_i) = 1.070$
- Creep coefficient (placement to final time): $\Psi(t_f, t_d) = 0.551$
- Modulus of elasticity of concrete: $E_c = 7{,}072$ ksi

The effective modulus of elasticity of the concrete to use with loads placed at the time of transfer and at the time of beam placement are as follows.

Effective modulus (transfer to final time): $E_{c,eff,tr} = E_c / [1 + \Psi(t_f, t_i)] = 3{,}415$ ksi

Effective modulus (placement to final time): $E_{c,eff,d} = E_c / [1 + \Psi(t_f, t_d)] = 4{,}560$ ksi

The other superimposed dead loads in this example are the barrier weight and the weight of the future wearing surface. The span length should be used in these long-term deflection estimates. The prestressing force after all losses should be used to estimate the camber due to the prestressing force.

- Barrier weight (per beam): $w_b = 0.0857$ kip/ft
- Future wearing surface weight (per beam): $w_{ws} = 0.0863$ kip/ft
- Span length: $L = 95$ ft = 1,140 inch
- Prestressing force (after all losses, no gains): $P_{pe} = 788.1$ kip

The long-term deflections due to the prestressing and sustained loads are as follows.

Long-term deflection due to prestressing: $\Delta_p = (P_{pe}\, e_{pg} L^2) / (8E_{c,eff,tr} I_g) = 4.48$ inch (up)

Long-term deflection due to self-weight: $\Delta_g = (5w_g L^4) / (384E_{c,eff,tr} I_g) = 3.65$ inch (down)

Long-term deflection due to barrier weight: $\Delta_b = (5w_b L^4) / (384E_{c,eff,d} I_g) = 0.38$ inch (down)

Long-term deflection due to weight of future wearing surface:

$$\Delta_{ws} = (5w_{ws} L^4) / (384E_{c,eff,d} I_g) = 0.38 \text{ inch (down)}$$

The total long-term deflection from the sustained loads is the summation of these four components, with positive downward.

Total long-term deflection: $\Delta_{t,LT} = -\Delta_p + \Delta_g + \Delta_b + \Delta_{ws} = -0.0641$ inch (negative is upward deflection)

### Comparison with PCI Multiplier Method

The deflections calculated using the effective modulus method are compared to those calculated using the PCI Multiplier Method in this section. The PCI Multiplier Method uses the deflections at the time of transfer and at the time of loading (for superimposed dead loads).

Camber due to prestressing force at transfer: $\Delta_p = (P_{pt}\, e_{pg}\, L_T^2) / (8E_{ci}\, I_g) = 2.54$ inch (up)

Deflection due to self-weight at transfer: $\Delta_g = (5w_g\, L^4) / (384E_{ci}\, I_g) = 1.90$ inch (down)

Deflection due to barrier weight: $\Delta_b = (5w_b\ L^4) / (384E_c\, I_g) = 0.25$ inch (down)

Deflection due to future wearing surface: $\Delta_{ws} = (5w_{ws}\ L^4) / (384E_c\, I_g) = 0.25$ inch (down)

These deflection components are multiplied by different deflection multipliers to account for long-term effects.

Total long-term deflections: $\Delta_{t,LT} = -2.45\Delta_p + 2.70\Delta_g + 3.0\Delta_b + 3.0\Delta_{ws} = 0.364$ inch (down)

This is a slightly larger deflection than that calculated using the effective modulus method, $\Delta_{t,LT} = -0.0641$ inch.

### Live Load Deflection Checks

AASHTO LRFD BDS Article 2.5.2.6.2 specifies an optional live load deflection check where the live load deflection is calculated as the maximum of the deflection from the design truck alone ($\Delta_{LT}$) and that from 25 percent of the design truck and the design lane load ($\Delta_{LL}$), see AASHTO LRFD BDS Article 3.6.1.3.2.

Live loads are transient loads, so the modulus of elasticity, $E_c$, should be used to calculate the corresponding deflections.

The distribution factor for deflections due to live loads is calculated based on the ratio of the number of design lanes ($N_{lanes}$) to the number of beams ($N_b$).

Distribution factor for deflections: $DFD = N_{lanes} / N_b = 0.286$

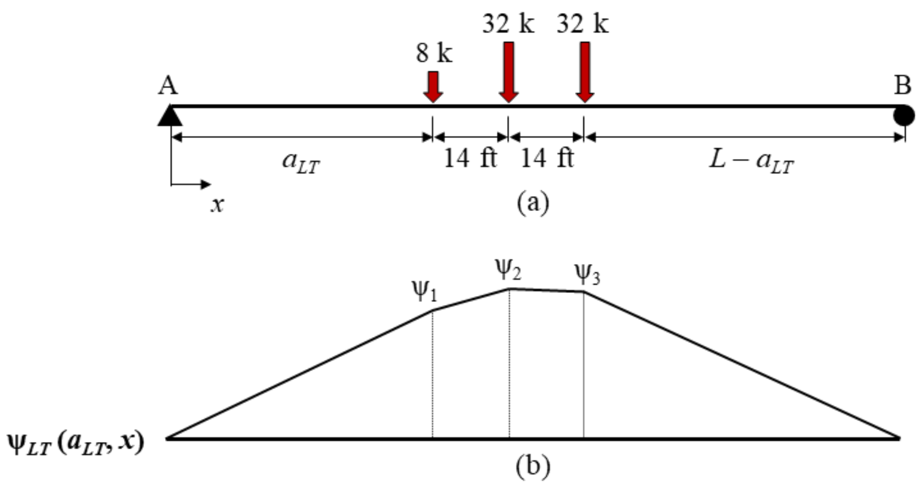
The deflection due to the lane live load ($w_{lane} = 0.64$ kip/ft = 0.0533 kip/inch) is as follows.

Deflection due to lane live load: $\Delta_{LL} = DFD \times (5w_{lane}\, L^4) / (384E_c\, I_g)$

$$\Delta_{LL} = (0.286)(5)(0.0533\ \text{kip/inch})(1{,}140\ \text{inch})^4) / [(384)(7{,}072\ \text{ksi})(90{,}567\ \text{inch}^4)]$$

$$\Delta_{LL} = 0.52\ \text{inch}$$

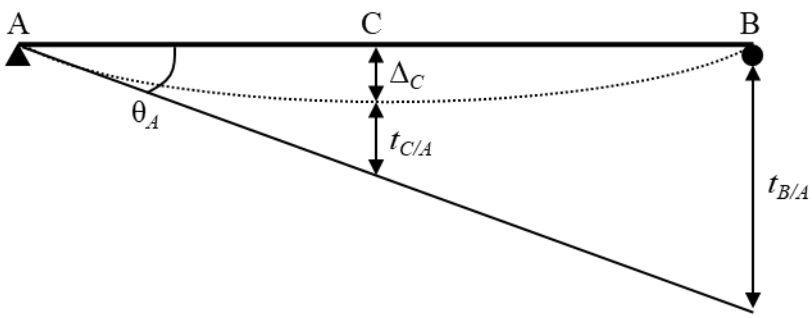
The deflection from the design truck can be found using a design software or other structural analysis method. The moment-area theorem was used to calculate the deflection in this example. A function was developed for the curvature along the length of the beam based on the distance between the left support and the first truck axle, $a_{LT}$. The design truck loading and a schematic of the associated curvature diagram are shown in Figure 42.




Source: FHWA.

**Figure 42. Illustration. (a) Location of design truck and (b) associated curvature diagram.**

The deflection at midspan can be calculated based on the tangents extending from the left support and the rotation of the left support, as shown in Figure 43.




Source: FHWA.

**Figure 43. Illustration. Parameters needed for calculating midspan deflection using moment-area theorem.**

These parameters can be defined by integration as follows.

$$t_{B/A}(a_{LT}) = \int_0^L (L - x_1) \cdot \psi_{LT}(a_{LT}, x_1) \cdot \mathrm{d}x_1$$

$$\theta_A(a_{LT}) = t_{B/A}(a_{LT}) / L$$

$$t_{C/A}(a_{LT}) = \int_0^{0.5L} (0.5L - x_1) \cdot \psi_{LT}(a_{LT}, x_1) \cdot \mathrm{d}x_1$$

The midspan deflection can be calculated as follows.

$$\Delta_{LT}(a_{LT}) = [\theta_A(a_{LT}) \times (0.5L) - t_{C/A}(a_{LT})] \times (1 + IM) \times DFD$$

For a truck positioned with the middle axle 2.33 ft to the left of midspan ($a_{LT}$ = 31.2 ft), the midspan deflection was calculated as the following.

$\Delta_{LT}$ ($a_{LT}$ = 31.2 ft) = 1.24 inch

$0.25\Delta_{LT}$ ($a_{LT}$ = 31.2 ft) + $\Delta_{LL}$ = 0.83 inch

The live load deflection to compare to the optional live load deflection limit is the maximum of these two values.

Live load deflection: $\Delta_L$ = maximum of $\Delta_{LT}$ and ($0.25\Delta_{LT} + \Delta_{LL}$) = 1.24 inch

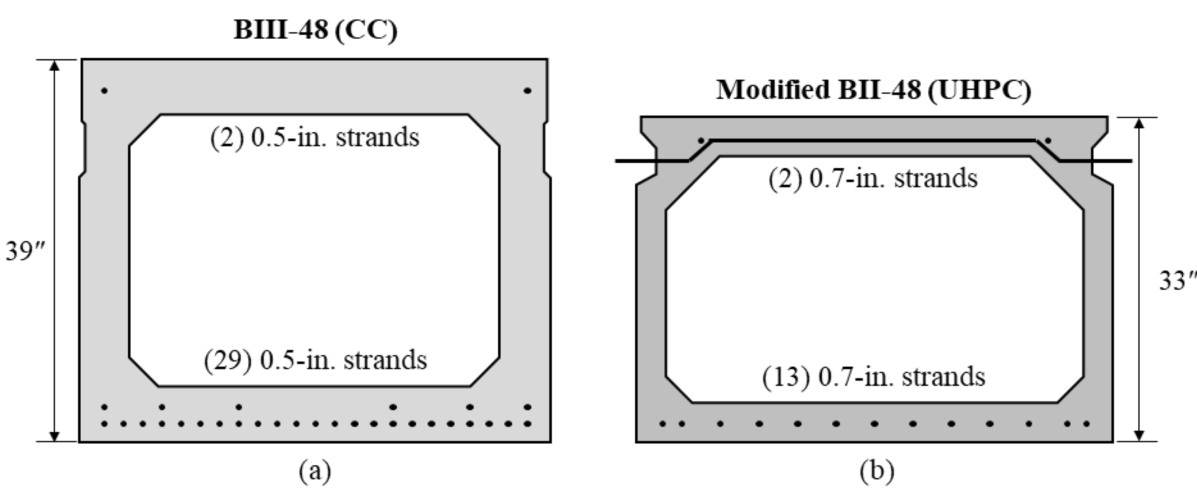
This can be compared to the live load deflection limit specified in AASHTO LRFD BDS Article 2.5.2.6.2.

Live load deflection limit: $\Delta_L = 1.24 \text{ inch} \leq L / 800 = 1.43$ inch

The live load deflection is less than the deflection limit.

## SUMMARY OF FINAL DESIGNS

A summary of the final designs for the conventional concrete and UHPC box beam sections required for this bridge is shown in Figure 44. The final design for the conventional concrete section is taken from Example 9.4 of the PCI *Bridge Design Manual*. The convention concrete box beam design required a 39-inch-deep section with a total area of prestressing on the flexural tension side of the member of 4.437 square inches. The UHPC box beam section is six inches shallower (33 inches deep) with 13.9 percent less total area of prestressing on the flexural tension side (3.822 square inches).




Source: FHWA.

**Figure 44. Illustration. Final design for (a) conventional concrete and (b) UHPC sections.**

A summary of the section properties and weight of the section is shown in Table 24. The density of the UHPC material is specified in UHPC Structural Design Guide Article 4.2.2 as 0.155 kcf. The density used in this example is assumed to include 0.005 kcf for the reinforcement and prestressing strands.

Density of UHPC: $\rho_{UHPC}$ = 0.155 kcf + 0.005 kcf = 0.160 kcf

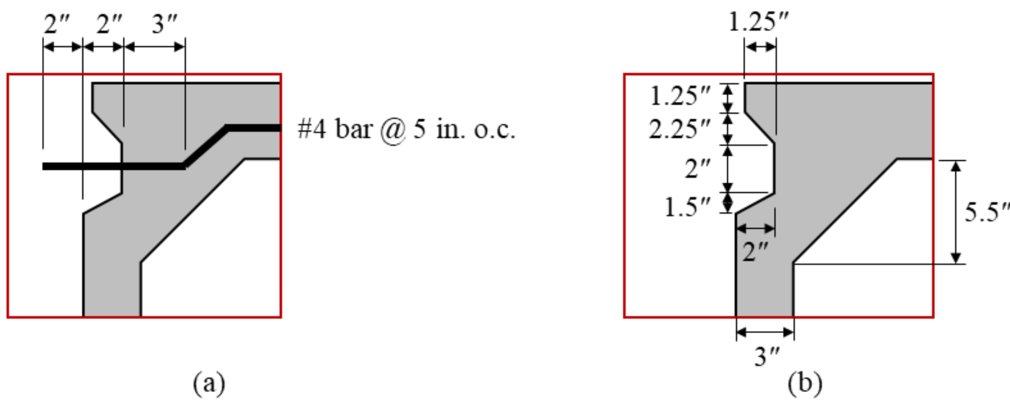
Density of CC: $\rho_{CC}$ = 0.145 kcf + 0.005 kcf = 0.150 kcf

The UHPC box beam section for this example is 27.3 percent lighter than the conventional concrete section required for the design.

**Table 24. Summary of section properties for final designs.**

| Section Type | $h$ (in) | $A_g$ (in$^2$) | $I_g$ (in$^4$) | $y_b$ (in) | $w_g$ (kip/ft) |
|---|---|---|---|---|---|
| BIII-48 (CC) | 39 | 813 | 168,367 | 19.29 | 0.847 |
| Mod. BII-48 (UHPC) | 33 | 554.1 | 90,567 | 16.45 | 0.616 |

The UHPC joint detail described in FHWA-HRT-19-011 can be used with the UHPC box beam sections. Details for the joint geometry and the proposed reinforcement, Number 4 bars spaced at 5 inches on center, are shown in Figure 45.

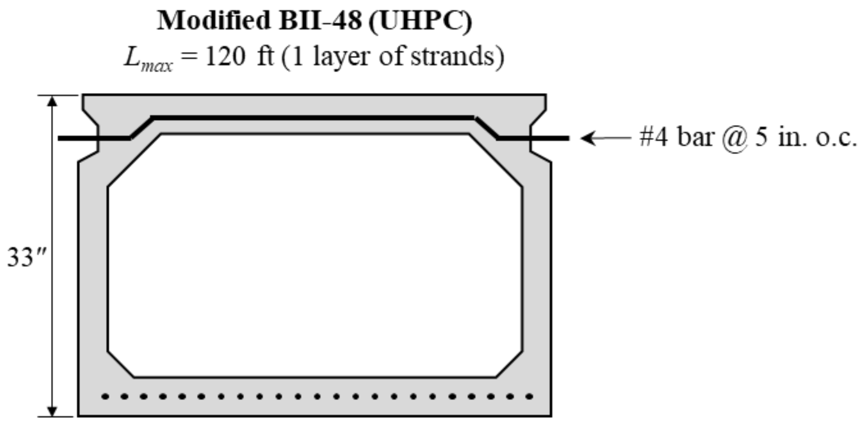



Source: FHWA.

**Figure 45. Illustration. Recommended joint details based on FHWA-HRT-19-011 for (a) reinforcement and (b) joint geometry.**

## DESIGN GUIDES FOR UHPC BOX BEAM SECTIONS

Design details were developed for this modified UHPC box beam section, shown in Figure 46. The bridge configuration for the provided design tables is the same as that shown in Figure 36 (a).




Source: FHWA.

**Figure 46. Illustration. Proposed modified UHPC box beam sections, (a) Modified BII-48 and (b) Modified BIII-48.**

Some of the details on the span configuration are as follows:

- Total beam length: $L_T = L + 1$ ft
- Support width: $b_s = 0.5$ ft

The UHPC material properties for the design guides are as follows:

- Compressive strength at transfer: $f'_{ci} = 14.0$ ksi
- Compressive strength for use in design and analyses: $f'_c = 17.5$ ksi
- Correction factor for modulus of elasticity: $K_1 = 1.0$
- UHPC unit weight (including reinforcement): $w_c = 0.160$ kcf
- Reduction factor for compression: $\alpha_u = 0.85$
- Ultimate compression strain: $\varepsilon_{cu} = 0.0035$
- Effective cracking strength: $f_{t,cr} = 1.0$ ksi
- Effective cracking strength at transfer: $f_{t,cri} = 0.75 f_{t,cr} = 0.75$ ksi
- Crack localization strength: $f_{t,loc} = 1.0$ ksi
- Crack localization strain: $\varepsilon_{t,loc} = 0.005$
- Reduction factor for tension: $\gamma_u = 1.0$

The material properties for the conventional steel reinforcement (Grade 60) are as follows:

- Modulus of elasticity: $E_s = 29{,}000$ ksi
- Yield strength: $f_{sy} = 60$ ksi

The material properties for the prestressing strands are as follows:

- Low-relaxation
- Modulus of elasticity: $E_p = 28{,}500$ ksi
- Ultimate strength: $f_{pu} = 270$ ksi
- Yield strength: $f_{py} = 243$ ksi

- Specified rupture strain for strands: $\varepsilon_{pu} = 0.035$

The following stress-strain relationship was used for the prestressing strands:

$$f_{ps} = \varepsilon_{ps}\left[887 + \frac{27{,}613}{\left(1 + \left(112.4\varepsilon_{ps}\right)^{7.36}\right)^{\frac{1}{7.36}}}\right] \leq 270 \text{ ksi}$$

Two different strand diameters were used to develop the design guides. Strands with a 0.6-inch diameter were used when possible. 0.7-inch diameter strands were used for longer span lengths requiring a larger amount of prestressing.

- Area of one 0.6-inch diameter strand: $A_{p,0.6in} = 0.217$ inch$^2$
- Area of one 0.7-inch diameter strand: $A_{p,0.7in} = 0.294$ inch$^2$

The parameters related to creep coefficients and shrinkage strains used to develop the design tables are as follows.

- UHPC age at transfer: $t_i = 1$ day
- UHPC age at field placement: $t_d = 90$ days
- UHPC age at final time: $t_f = 36{,}500$ days
- Average ambient relative humidity: $H = 70$ percent
- Correction factor for creep: $K_3 = 1.0$
- Correction factor for shrinkage: $K_4 = 1.0$

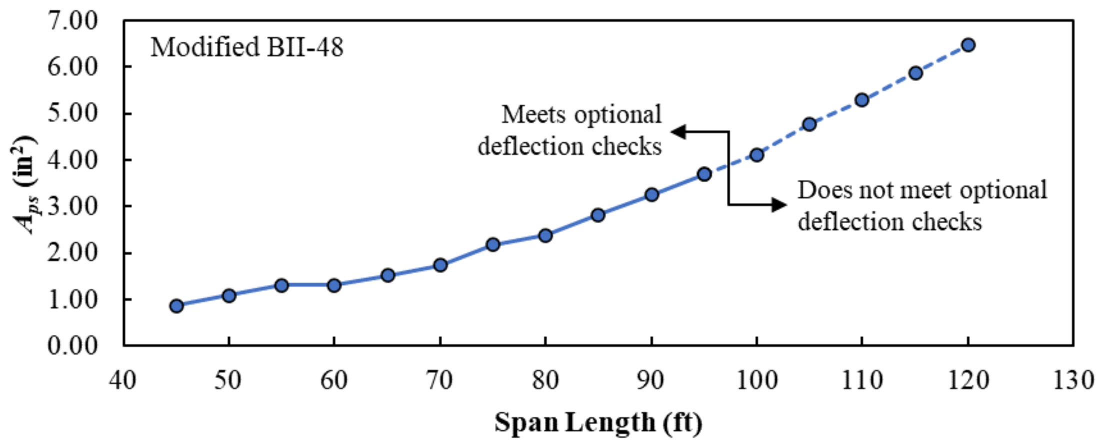
The section properties for the two box beam sections are summarized in Table 25.

**Table 25. Summary of section properties for UHPC box beam sections for provided design tables.**

| **Section Type** | ***h*** **(inch)** | **$A_g$ (inch$^2$)** | **$I_g$ (inch$^4$)** | **$y_b$ (inch)** | **$w_g$ (kip/ft)** |
|---|---|---|---|---|---|
| Mod. BII-48 (UHPC) | 33 | 554.1 | 90,567 | 16.45 | 0.616 |

Gross section properties were used in prestress loss, stress, and deflection calculations. Elastic gains were included in calculation of stress in the prestressing strands for stress calculations.

The total area of prestressing on the flexural tension side of the member ($A_{ps}$) required for different span lengths for the modified box beam section is shown in Figure 47. The point at which the design no longer satisfies the optional live load deflection criteria specified in AASHTO LRFD BDS Article 2.5.2.6.2 is highlighted in Figure 47.




Source: FHWA.

**Figure 47. Graph. Design guide for proposed modified UHPC box beam sections.**

Details on the designs for each of the span lengths are provided in Table 26 for the Modified BII-48 box beam section. The details in the table include the following. The stresses due to the Service I load combination are provided.

- Stress in extreme tension fiber due to Service I load combination: $f_{t,SeI,m}$
- Stress in extreme compression fiber due to Service I load combination: $f_{c,SeI,m}$

These stresses are compared to the stress limits at service.

- Compressive stress limit due to Service I load combination: $f_{c,a} = 0.60\phi_w f'_c = 10.5$ ksi
- Tension stress limit due to Service III load combination: $f_{t,a} = \gamma_u f_{t,cr} = -1.0$ ksi
- Tension stress limit for members exposed to cyclic loads due to Service I load combination: $f_{t,a} = 0.95\gamma_u f_{t,cr} = -0.95$ ksi

The cracking moment ($M_{cr}$), demand due to the Strength I load combination at midspan ($M_{u,m}$), and factored resistance at midspan ($M_{r,m}$) are also provided in the table. The two associated design checks related to these are as follows.

- Flexural strength check: $M_{r,m} = \phi M_{n,m} \geq M_{u,m}$
- Minimum flexural reinforcement: $M_{r,m} = \phi M_{n,m} \geq$ minimum of ($1.33M_u$ and $M_{cr}$)

Two deflections are provided in the design tables. The total long-term deflection due to sustained loads ($\Delta_{sus,LT}$) includes the deflection due to the prestressing, self-weight, barrier weight, and weight of the future wearing surface. A positive deflection is downward, negative upward. The live load deflection is calculated based on AASHTO LRFD BDS Article 3.6.1.3.2.

Live load deflection: $\Delta_L$ = maximum of $\Delta_{LT}$ and $(0.25\Delta_{LT} + \Delta_{LL})$

The live load deflection can be compared to the optional live load deflection limit in AASHTO LRFD BDS Article 2.5.2.6.2.

Live load deflection limit: $\Delta_L \leq L / 800$

The live load deflections exceeding the optional limit are highlighted in the design tables.

The details in the design tables are provided assuming only one layer of 0.6-inch or 0.7-inch diameter prestressing strands on the flexural tension side of the box beam. A note is provided beneath the table on the longer span lengths possible if the bottom flange of the box beam is made thick enough for two layers of prestressing strands.

**Table 26. Design table for Modified BII-48 Box Beam Section (33-inches deep)**

| Span (ft) | $A_{ps}$ [a] (inch$^2$) | $f_{t,SeI,m}$ (ksi) | $f_{c,SeI,m}$ (ksi) | $M_{u,m}$ (k-ft) | $M_{r,m}$ (k-ft) | $M_{cr}$ (k-ft) | $\Delta_{sus,LT}$ [b] (inch) | $\Delta_L$ [b] (inch) | Controlling |
|---|---|---|---|---|---|---|---|---|---|
| 45 | 0.868 | -0.409 | 0.958 | 759 | 1,089 | 1,071 | -0.073 | 0.120 | Min. Flex. Reinf. |
| 50 | 1.085 | -0.449 | 1.131 | 898 | 1,204 | 1,153 | -0.112 | 0.173 | Min. Flex. Reinf. |
| 55 | 1.302 | -0.501 | 1.314 | 1,043 | 1,319 | 1,235 | -0.150 | 0.238 | Min. Flex. Reinf. |
| 60 | 1.302 | -0.718 | 1.539 | 1,195 | 1,319 | 1,238 | -0.020 | 0.317 | Min. Flex. Reinf. |
| 65 | 1.519 | -0.791 | 1.745 | 1,354 | 1,435 | 1,320 | -0.010 | 0.412 | Strength I |
| 70 | 1.736 | -0.876 | 1.962 | 1,519 | 1,550 | 1,402 | 0.023 | 0.523 | Strength I |
| 75 | 2.170 | -0.824 | 2.163 | 1,691 | 1,778 | 1,558 | -0.148 | 0.651 | Service I (Cyclic) |
| 80 | 2.387 | -0.934 | 2.404 | 1,870 | 1,892 | 1,639 | -0.068 | 0.799 | Service I (Cyclic) |
| 85 | 2.821 | -0.912 | 2.630 | 2,056 | 2,118 | 1,792 | -0.228 | 0.967 | Service I (Cyclic) |
| 90 | 3.255 | -0.907 | 2.868 | 2,249 | 2,343 | 1,942 | -0.381 | 1.157 | Service I (Cyclic) |
| 95 | 3.689 | -0.918 | 3.119 | 2,449 | 2,555 | 2,091 | -0.518 | 1.369 | Service I (Cyclic) |
| 100 | 4.123 | -0.943 | 3.382 | 2,657 | 2,748 | 2,237 | -0.628 | 1.606 [c] | Service I (Cyclic) |
| 105 | 4.774 | -0.858 | 3.634 | 2,871 | 3,026 | 2,446 | -1.091 | 1.868 [c] | Service I (Cyclic) |
| 110 | 5.292 [d] | -0.948 | 4.331 | 3,092 | 3,236 | 2,572 | 0.133 | 2.156 [c] | Service I (Cyclic) |
| 115 | 5.880 [d] | -0.934 | 4.611 | 3,321 | 3,488 | 2,757 | -0.065 | 2.473 [c] | Service I (Cyclic) |
| 120 [e] | 6.468 [d] | -0.937 | 4.904 | 3,557 | 3,737 | 2,938 | -0.217 | 2.818 [c] | Service I (Cyclic) |

[a] 0.6-inch diameter strands were used up to 105-foot spans; 0.7-inch diameter strands were used for spans 110 feet and longer.
[b] Positive deflection is downward; negative deflection indicates upward deflection.
[c] Deflections do not meet the optional live load deflection checks for span lengths of 110 feet and longer.
[d] Two fully stressed top strands at 31 inches from bottom are required to meet stress checks at time of transfer. These are not included in the $A_{ps}$ shown.
[e] Span lengths of up to 160 feet are possible with two layers of 0.7-inch diameter prestressing strands on the flexural tension side of the member. A higher effective cracking strength at transfer ($f_{t,cri}$) is required for the concrete at longer spans.

## CHAPTER 4. ADDITIONAL INFORMATION ON WORKSHOP ACTIVITIES

### INTRODUCTION

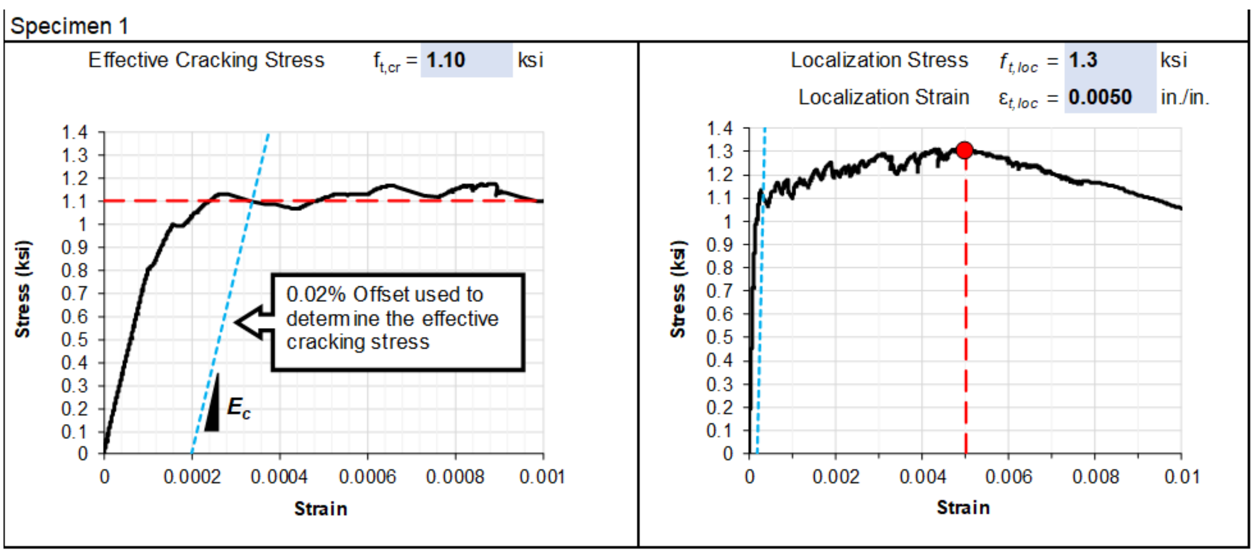
Several activities have been designed to help participants better understand some of the key components of structural design with UHPC that differ from design with conventional concrete. Additional details on these activities are included in this chapter.

### ACTIVITY #1: MEASURING DIRECT TENSILE STRENGTH

This activity is a guided walk-through of how to take UHPC direct tension test data and determine the appropriate stress-strain model to use in design.

The stress versus strain relationship for three direct tension tests is shown in the activity. Although the actual number of tests required to qualify a UHPC mixture will exceed three, three is used here as an example for determining the statistical properties used to calculate design values.

Two different plots are provided for each test, as shown in Figure 48. These plots are of the same data but have different ranges for the strain values on the x axis. The left plot, Figure 48 (a), has a range of 0 to 0.001 strain, which allows for easier selection of the intersection of the 0.02 percent offset line and the stress-strain curve. The right plot, Figure 48 (b), includes a larger strain range, 0 to 0.01 for Specimen #1, which allows for selection of the crack localization stress and strain.




(a) (b)

Source: FHWA.

**Figure 48. Illustration. Two stress-strain plots shown for Specimen #1 in Activity #1.**

A dashed diagonal line in both plots displays the measured modulus of elasticity based on the slope from a linear regression between 0.01 ksi and 0.4 ksi. For the data set for Specimen #1, the slope of the data in this stress range is $E_c$ = 7,964 ksi. The diagonal line is based on the zero-stress point at a strain of 0.0002 (i.e., the strain offset) and the stress at a strain of 0.0004, as shown in Table 27.

**Table 27. Points for 0.02% offset line for Specimen #1.**

| Strain (inch/inch) | Stress |
|---|---|
| 0.0002 | 0 ksi |
| 0.0004 | (0.0004)(7,964 ksi) = 1.59 ksi |

The effective cracking stress should be the stress where this blue dashed line intersects the measured stress-strain response. The input for the effective cracking stress in Figure 48 (a) will create a horizontal red dashed line that will change with the entry.

| | | | |
|---|---|---|---|
| Effective Cracking Stress | $f_{t,cr}$ = | **1.10** | ksi |

The crack localization stress and localization strain inputs in Figure 48 (b) create a vertical dashed line and point in the plot.

| | | | |
|---|---|---|---|
| Crack Localization Stress | $f_{t,loc}$ = | **1.30** | ksi |
| Crack Localization Strain | $\varepsilon_{t,loc}$ = | **0.0050** | inch/inch |

The crack localization stress and strain can be modified until the stress decreases with increasing strain after the point in the plot.

A summary table is provided for the effective cracking stress, crack localization stress, and crack localization strain for each of the specimen. The average and standard deviation of the measured values is also provided.

Based on these, the values for design can be calculated based on the values measured from AASHTO T 397-22 as follows.

Effective cracking strength for design: $f_{t,cr} = \bar{f}_{t,cr,test} - 1.5\sigma_{ft,cr,test}$

Crack localization strain for design: $\varepsilon_{t,loc} = \bar{\varepsilon}_{t,loc,test} - 1.5\sigma_{\varepsilon t,loc,test}$

The ratio of the average measured crack localization stress to the average measured effective cracking stress ($\bar{f}_{t,loc,test} / \bar{f}_{t,cr,test}$) is used to determine the tensile stress-strain model idealization that should be used in design. If the ratio is less than 1.2, then $\gamma_u f_{t,loc} = \gamma_u f_{t,cr}$.

Crack localization strength for design:

If $\bar{f}_{t,loc,test} < 1.20\bar{f}_{t,cr,test}$ then $f_{t,loc} = f_{t,cr}$,

Else $f_{t,loc} = \bar{f}_{t,loc,test} - 1.5\sigma_{ft,loc,test}$

The design values for effective cracking stress, crack localization stress, and crack localization strain are calculated in the spreadsheet based on the values input for Specimen #1, Specimen #2, and Specimen #3.

## ACTIVITY #2: FLEXURAL ANALYSIS

This activity is a guided walk-through of the strain compatibility analysis procedure for a reinforced UHPC section. The key inputs to be varied for this activity are the total area of the reinforcement, $A_s$, and the strain in the extreme tension face of the member, $\varepsilon_t$, as shown in Table 28. Different combinations for these values are given in the activity handout to see how strain compatibility can be used to calculate the moment and curvature related to different extreme tension fiber strains.

**Table 28. Primary inputs for Activity #2.**

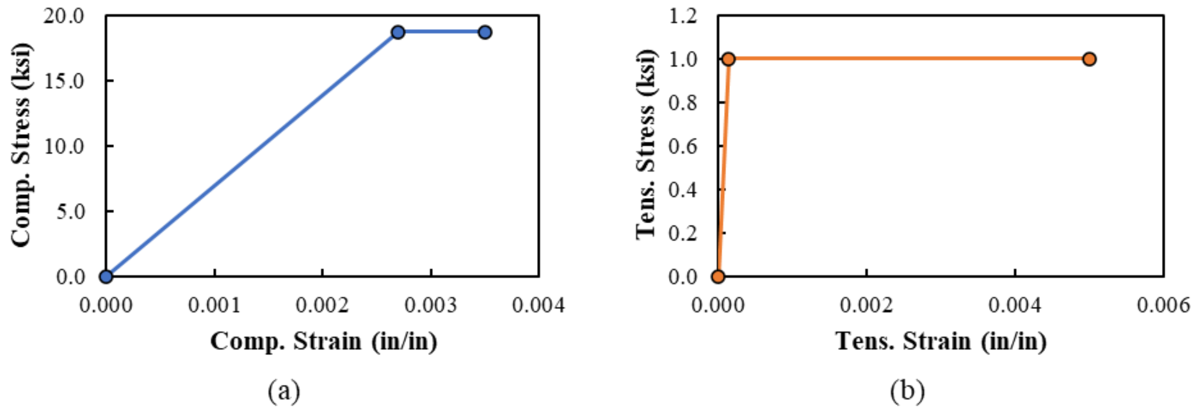
| Variable | Value | Definition |
|---:|:---:|---|
| $A_s$ (inch$^2$) = | 9.360 | total area of reinforcement |
| $\varepsilon_t$ = | 0.00500 | strain in extreme tensile face of UHPC |

The general UHPC material properties and section properties are summarized in Table 29. An elastic-plastic tensile stress-strain is used in this activity.

**Table 29. Basic spreadsheet inputs for UHPC material properties in Activity #2.**

| Variable | Value | Definition |
|---:|:---:|---|
| $\alpha_u$ = | 0.85 | reduction factor on compressive strength ($\leq$ 0.85) |
| $f'_c$ [ksi] = | 22.0 | compressive strength ($\geq$ 17.5 ksi) |
| $E_c$ [ksi] = | 6,933 | modulus of elasticity (calculated if not measured) |
| $\varepsilon_{cp}$ = | 0.00270 | elastic compression strain limit |
| $\varepsilon_{cu}$ = | 0.00350 | ultimate compression strain limit (calculated if not measured) |
| $\gamma_u$ = | 1.00 | reduction factor on tensile strength properties ($\leq$ 1.0) |
| $f_{t,cr}$ [ksi] = | 1.00 | effective cracking stress ($\geq$ 0.75 ksi) |
| $\varepsilon_{t,cr}$ = | 0.000144 | cracking strain |
| $f_{t,loc}$ [ksi] = | 1.0 | localization stress ($\geq f_{t,cr}$) |
| $\varepsilon_{t,loc}$ = | 0.005 | localization strain |

A stress-strain plot is generated for compression and tension based on these UHPC material property inputs, as shown in Figure 49.




Source: FHWA.

**Figure 49. Graphs. Stress-strain plots for UHPC in (a) compression and (b) tension based on input UHPC material properties.**

The section height, $h$, and distance between the compression face and centroid of the tension reinforcement, $d$, are also inputs in the spreadsheet.

The strain profile across the section depth, UHPC stress profile across the section depth, and strain in the UHPC at the centroid of the reinforcement are summarized in the spreadsheet.

Several key points of interest are calculated first. The following points are on the tension side of the neutral axis.

Distance from neutral axis to height where cracking strain occurs (if applicable):

$$y_{t,cr} = [(h - c) / \varepsilon_t] \times \varepsilon_{t,cr}$$

Distance from neutral axis to height where crack localization strain occurs (if applicable):

$$y_{t,loc} = [(h - c) / \varepsilon_t] \times \gamma_u \varepsilon_{t,loc}$$

The strain at the extreme compression fiber is calculated based on the strain at the extreme tension fiber and the neutral axis depth.

Strain at extreme compression fiber: $\varepsilon_c = \varepsilon_t \times [c / (h - c)]$

Distance from neutral axis to height where elastic region in compression ends (if applicable):

$$y_{c,cp} = (c / \varepsilon_c) \times \varepsilon_{cp}$$

Distance from neutral axis to height where concrete crushes in compression (if applicable):

$$y_{c,cu} = (c / \varepsilon_c) \times \varepsilon_{cu}$$

These points are used to create the strain and stress profiles plotted in the spreadsheet. The spreadsheet is set up to plot these significant points if they fall within the height of the cross section, e.g., a point is plotted at the height of the end of the elastic region in compression if $y_{c,cp} < c$.

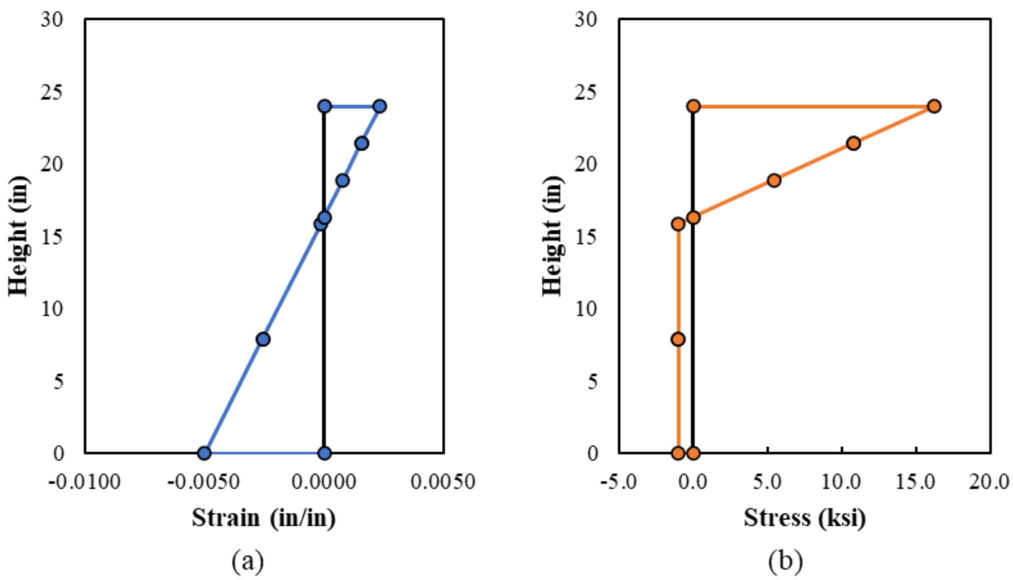
A sample of the data to plot strain and stress across the section depth in the activity is provided in Table 30.

**Table 30. Sample of strain and stress data used to create strain and stress profiles across the depth of the section for Activity #2 using the input values from Table 28 and Table 29.**

| *Description of Point* | $x$ (inch) | ε (inch/inch) | $f_c$ (ksi) |
|---:|:---:|:---:|:---:|
| Zero point on tension face | 0.0 | 0.00000 | 0.00 |
| **Extreme tension fiber** | **0.0** | **-0.00500** | **-1.00** |
| Row allows for drop in stress if crack localization occurs | 7.9 | -0.00257 | -1.00 |
| Halfway between cracking and tension fiber[1] | 7.9 | -0.00257 | -1.00 |
| Effective cracking strain | 15.9 | -0.00014 | -1.00 |
| Neutral axis (zero strain) | 16.3 | 0.00000 | 0.00 |
| End of elastic region in compression | 18.9 | 0.00078 | 5.41 |
| Concrete crushes in compression | 21.4 | 0.00156 | 10.82 |
| Row allows for drop in stress if concrete crushes | 21.4 | 0.00156 | 10.82 |
| **Extreme compression fiber** | **24.0** | **0.00234** | **16.23** |
| Zero point on compression face | 24.0 | 0.00000 | 0.00 |

[1] This row will be the crack localization strain if it occurs in the cross section depth, i.e., when $y_{t,loc} < (h - c)$.

The stress is calculated from the strain based on the stress-strain relationship for the UHPC in compression and tension. Plots for the strain and stress profiles across the depth of the section are provided; sample plots for the input values from Table 28 and Table 29 are shown in Figure 50.

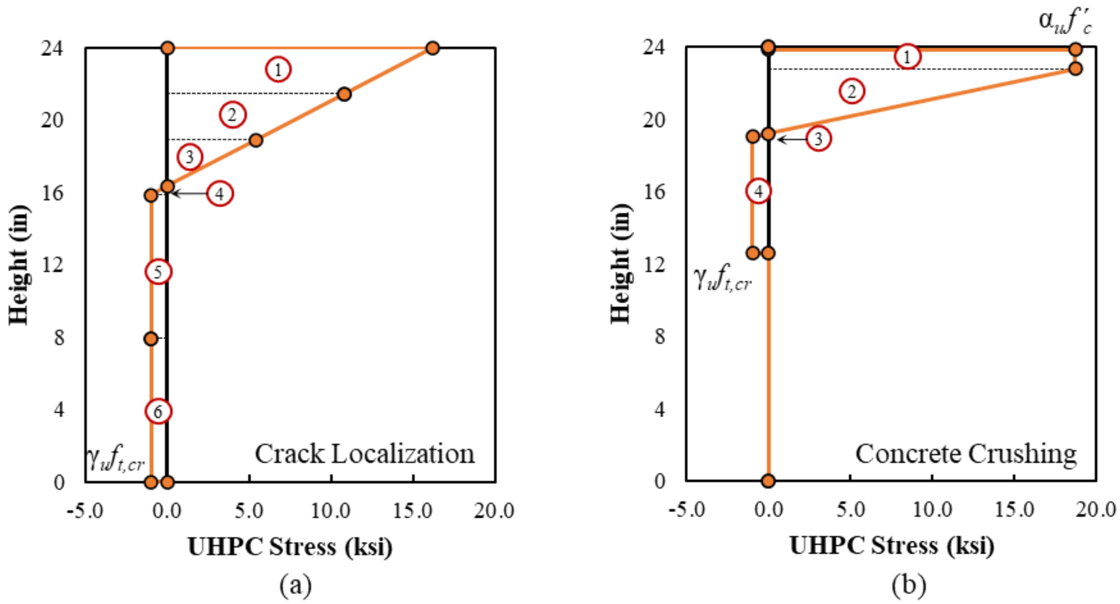



Source: FHWA.

**Figure 50. Graphs. (a) Strain and (b) stress profiles across the depth of the section for Activity #2 using the input values from Table 28 and Table 29.**

The strain in the UHPC at the centroid of the reinforcement is also shown in the spreadsheet; this strain can be compared to the yield strain of reinforcement (0.00207 for Grade 60 reinforcement).

The force is calculated for each of the segments based on the stress diagram. Plots of the stress over the depth of the section at the time of crack localization and concrete crushing are shown in Figure 51.




Source: FHWA.

**Figure 51. Graphs. Stress over section depth at time of (a) crack localization and (b) crushing of concrete at compression face.**

The areas used for summations for finding the neutral axis depth and moment through equilibrium at the time of crack localization are highlighted in Figure 51 (a). Enough points were included in the section depth to accommodate the most possible points of interest on the compression and tension side of the neutral axis (e.g., section depth at cracking strain on tension side, section depth at the end of the elastic region on compression side).

The force for each UHPC layer is calculated based on the area of a trapezoid times the constant section width. The "top" is the side of the trapezoid furthest from the neutral axis, the stress at this face is $f_{c,top,i}$, and the distance from the neutral axis to the top side of the layer is defined as $y_{top,i}$ in the following expression.

Force for single UHPC layer: $F_{UHPC,i} = b_w \times 0.5(f_{c,top,i} - f_{c,bot,i})(y_{top,i} - y_{bot,i})$

An example of the UHPC force components when the extreme tension fiber equals the crack localization strain of 0.005 is shown in Table 31.

**Table 31. Sample of strain, stress, force, and moment components for Activity #3 when extreme tension fiber equals the crack localization strain; layer column associated with Figure 51 (a).**

| *x* (inch) | ε (inch/inch) | $f_c$ (ksi) | *Layer* | $F_c$ (kips) | $y_{ci}$ (inch) | $F_c \cdot y_{ci}$ (k-inch) |
|---|---|---|---|---|---|---|
| 0.0 | 0.00000 | 0.00 | | | | |
| **0.00** | **-0.005000** | **-1.00** | 6 | -95.24 | 12.38 | 1178.8 |
| 7.94 | -0.002572 | -1.00 | | 0.00 | 8.41 | 0.0 |
| 7.94 | -0.002572 | -1.00 | 5 | -95.24 | 4.44 | 422.9 |
| 15.87 | -0.000144 | -1.00 | 4 | -2.83 | 0.31 | 0.9 |
| 16.35 | 0.000000 | 0.00 | | -- | -- | -- |
| 18.90 | 0.000780 | 5.41 | 3 | 82.84 | 1.70 | 140.9 |
| 21.45 | 0.001561 | 10.82 | 2 | 248.52 | 3.97 | 986.4 |
| 21.45 | 0.001561 | 10.82 | | 0.00 | 5.10 | 0.0 |
| **24.00** | **0.002341** | **16.23** | 1 | 414.20 | 6.46 | 2677.3 |
| 24.0 | 0.00000 | 0.00 | | | | |

The force in the prestressing strands and non-prestressed reinforcement is also calculated in Table 32 and Table 33, respectively. There are no prestressing strands in the example shown, so the force from the prestressing strands is equal to zero.

**Table 32. Sample of prestressing force for Activity #3 when extreme tension fiber equals the crack localization strain.**

| Type | *x* (inch) | ε (inch/inch) | $f_p$ (ksi) | $F_p$ (kips) | $y_{pi}$ (inch) | $F_p \cdot y_{pi}$ (k-inch) |
|---|---|---|---|---|---|---|
| Prestressing | 2.2 | -0.004326 | 245.6 | 0.0 | 14.1 | 0.0 |
| UHPC | 2.2 | -0.004326 | -1.0 | 0.0 | 14.1 | 0.0 |

The force in the UHPC that is displaced by the reinforcement is also shown. This force is subtracted from the tension force provided by the reinforcement.

**Table 33. Sample of non-prestressed reinforcement force for Activity #3 when extreme tension fiber equals the crack localization strain.**

| Type | *x* (inch) | ε (inch/inch) | $f_s$ (ksi) | $F_s$ (kips) | $y_{si}$ (inch) | $F_s \cdot y_{si}$ (k-inch) |
|---|---|---|---|---|---|---|
| Steel | 2.2 | -0.004326 | 60.00 | -561.6 | 14.1 | 7941.4 |
| UHPC | 2.2 | -0.004326 | -1.00 | 9.4 | 14.1 | -132.4 |

The force from all the UHPC layers is added to the force from the steel and prestressing layers for the equilibrium check, where compression layers are positive and tension layers are negative.

Equilibrium equation: $N = \sum F_{UHPC,i} - T_s + T_{C,As} = 0$ kips

The neutral axis depth input is shown next to the summation of the internal force components in the spreadsheet, shown in Table 34. A solver (e.g., Goal Seek) can be used to modify the neutral axis depth until the summation of the internal forces is equal to zero kips.

**Table 34. Example of neutral axis depth input and resulting summation of internal forces for Activity #3 when extreme tension fiber equals the crack localization strain.**

| Variable | Value | Definition |
|---|---|---|
| $c$ (inch) = | 7.65 | neutral axis depth (**solve either manually or using solver**) |
| $N$ (kips) = | 0.00 | summation of internal forces (positive is compression) |

The lever arm for each trapezoidal region is calculated for the moment calculation.

Lever arm for single UHPC layer:

$$y_{UHPC,i} = y_{bot,i} + (y_{top,i} - y_{bot,i}) \times (2f_{c,bot,i} + f_{c,top,i}) / [3(f_{c,bot,i} + f_{c,top,i})]$$

The moment can be calculated about the neutral axis since there is no externally applied axial force.

The moment is calculated by summing the contribution of each UHPC layer with the steel and prestressing layers.

Moment: $M = \sum(F_{UHPC,i} \times y_{UHPC,i}) + (T_s \times y_{Ts}) - (T_{C,As} \times y_{Ts})$

The curvature is calculated based on the strain profile.

Curvature: $\psi = \varepsilon_c / c$

The calculated values for the example when the extreme tension fiber equals the crack localization strain are shown in Table 35.

**Table 35. Summary of outputs for moment and curvature for Activity #3 when extreme tension fiber equals the crack localization strain.**

| Variable | Value | Definition |
|---|---|---|
| $M$ (kip-inch) = | 13,216 | moment |
| $M$ (kip-ft) = | 1,101 | moment |
| $\psi$ (rad/inch) = | 3.06E-04 | curvature |

The neutral axis depth, moment, and curvature can be compared for different amounts of reinforcement and extreme tension fiber strains.

## ACTIVITY #3: SERVICE AND FATIGUE LIMIT STATE

This activity goes through the design of the modified BII-48 UHPC box beam with different span lengths and correction factors for service and fatigue limit states.

The initial inputs in this spreadsheet are based on Storyline Example #2 with the exception of $K_1 = K_3 = K_4 = 1.0$. The key input that will be varied in this activity is the span length, $L_{span}$. The secondary inputs that may be varied depending on time are the correction factors for modulus of elasticity, creep, and shrinkage. These are summarized in Table 36.

**Table 36. Key inputs for Activity #3.**

| Variable | Input | Definition |
|---:|:---:|---|
| $L_{span}$ [ft] = | 95 | span length |
| $K_1$ = | 1.00 | correction factor for modulus of elasticity |
| $K_3$ = | 1.00 | correction factor for creep coefficient |
| $K_4$ = | 1.00 | correction factor for shrinkage strain |

The additional basic inputs needed for the design for service and fatigue limit states in this activity are input in the UHPC material properties, section properties, details on span, and loads for example sections. The initial inputs for these values based on Storyline Example #2 are summarized in Table 37 through Table 40.

Part of the activity is to design the prestressed concrete box beam section. The sample calculations and sample values provided in this section are based on a 95-foot span length with 17 0.6-inch diameter strands, as shown in Table 39.

**Table 37. Summary of initial inputs related to material properties from Storyline Example #2 for Activity #3.**

| Variable | Value | Definition |
|---:|:---:|---|
| $\alpha_u$ = | 0.85 | reduction factor on compressive strength (≤ 0.85) |
| $f'_c$ [ksi] = | 17.5 | compressive strength (≥ 17.5 ksi) |
| $K_1$ = | 1.0 | correction factor for modulus of elasticity |
| $E_c$ [ksi] = | 6,429 | modulus of elasticity (calculated if not measured) |
| $f'_{ci}$ [ksi] = | 14.0 | compressive strength (≥ 17.5 ksi) |
| $E_{ci}$ [ksi] = | 5,973 | modulus of elasticity (calculated if not measured) |
| $\gamma_u$ = | 1.00 | reduction factor on tensile strength properties (≤ 1.0) |
| $f_{t,cr}$ [ksi] = | 1.00 | effective cracking stress (≥ 0.75 ksi) |
| $\varepsilon_{t,cr}$ = | 0.000156 | cracking strain |
| $f_{t,loc}$ [ksi] = | 1.00 | localization stress (≥ $f_{t,cr}$) |
| $\varepsilon_{t,loc}$ = | 0.005 | localization strain |
| $f_{t,cri}$ [ksi] = | 0.75 | effective cracking stress at release |

**Table 38. Summary of initial inputs related to section properties from Storyline Example #2 for Activity #3.**

| Variable | Value | Definition |
|---|---|---|
| $h$ [inch] = | 33 | section height |
| $b$ [inch] = | 48 | compression block width (top flange) |
| $A_g$ [inch$^2$] = | 554.1 | gross area |
| $y_t$ [inch] = | 16.55 | distance from top to centroid |
| $y_b$ [inch] = | 16.45 | distance from bottom to centroid |
| $I_g$ [inch$^4$] = | 90,567 | gross moment of inertia |
| $S_t$ [inch$^3$] = | 5,472 | section modulus to top of section |
| $S_b$ [inch$^3$] = | 5,506 | section modulus to bottom of section |

**Table 39. Summary of initial inputs related to prestressing strand properties from Storyline Example #2 for Activity #3.**

| Variable | Value | Definition |
|---|---|---|
| $y_p$ [inch] = | 2.00 | distance from bottom of section to centroid of prestressing strands |
| $d_p$ [inch] = | 31.00 | depth of prestressed tension reinforcement |
| $e_p$ [inch] = | 14.45 | strand eccentricity |
| $A_p$ [inch$^2$] = | 3.689 | total area of prestressing strands |
| $\rho_p$ = | 0.002 | prestressing ratio ($A_p$ / $bd_p$) |
| $E_p$ [ksi] = | 28,500 | modulus of elasticity for prestressing strands |
| $f_{py}$ [ksi] = | 243 | yield stress for prestressing |
| $f_{pu}$ [ksi] = | 270 | ultimate stress for prestressing |
| $f_{pbt}$ [ksi] = | 202.5 | jacking stress for prestressing (before transfer) |

**Table 40. Summary of initial inputs related to span and loading from Storyline Example #2 for Activity #3.**

| Variable | Value | Definition |
|---|---|---|
| $L_b$ [ft] = | 96 | beam length |
| $L = L_{span}$ [ft] = | 95 | span length |
| $b_s$ [ft] = | 0.5 | bearing length |
| $\rho_c$ [kcf] = | 0.160 | density of concrete |
| $w_g$ [k/ft] = | 0.616 | self-weight of girder |
| $M_{g,m}$ [k-in] = | 8,511 | midspan moment from girder self-weight (using beam length) |
| $w_{sDL}$ [k/ft] = | 0.172 | other superimposed dead load |
| $M_{sDL,m}$ [k-in] = | 2,328 | midspan moment from superimposed dead loads (using span length) |

Several time parameters are also needed as inputs, as summarized in Table 41. There is no cast-in-place composite deck for Storyline Example #2, so the time when the beams are erected in the field is used for $t_d$.

**Table 41. Time and humidity related initial inputs from Storyline Example #2 for Activity #3.**

| Variable | Value | Definition |
|---|---|---|
| $t_i$ [days] = | 1 | time of transfer (loading age for prestressing) |
| $t_d$ [days] = | 90 | time of beam placement in the field |
| $t_f$ [days] = | 20,000 | final time (75-year design life) |
| $H$ [%] = | 73 | ambient relative humidity |

Prestress losses are calculated using UHPC Structural Design Guide Article 9.3. These provisions specify the use of the Refined Estimates of Time-Dependent Losses (AASHTO LRFD BDS Article 5.9.3.4) using creep coefficients and shrinkage strains calculated using UHPC Structural Design Guide Articles 4.2.8.2 and 4.2.8.3, respectively. The Approximate Estimate of Time-Dependent Losses (AASHTO LRFD BDS Article 5.9.3.3) are not permitted to be used for UHPC members.

The shrinkage strains are calculated using UHPC Structural Design Guide Eqn. 4.2.8.3-1. Shrinkage strains are calculated for three different times: end of curing to beam placement ($\varepsilon_{bid}$), end of curing to final time ($\varepsilon_{bif}$), and beam placement to final time ($\varepsilon_{bdf}$).

Shrinkage strain (curing to beam placement): $\varepsilon_{bid} = k_s k_{hs} k_f k_{td,id} K_4 \times (0.6 \times 10^{-3})$

Shrinkage strain (curing to final time): $\varepsilon_{bif} = k_s k_{hs} k_f k_{td,if} K_4 \times (0.6 \times 10^{-3})$

Shrinkage strain (beam placement to final time): $\varepsilon_{bdf} = \varepsilon_{bif} - \varepsilon_{bid}$

Creep coefficients are found for loading at $t_i$ to time of beam placement $t_d$ ($\Psi(t_d, t_i)$) and to final time $t_f$ ($\Psi(t_f, t_i)$). The creep coefficient was also calculated for loading at $t_d$ to final time $t_f$ ($\Psi(t_f, t_d)$).

Creep coefficient at time of beam placement due to load placed at $t_i$:

$$\Psi(t_d, t_i) = 1.2 k_s k_{hc} k_f k_{td,id} k_\ell K_3$$

Creep coefficient at final time due to load placed at $t_i$:

$$\Psi(t_f, t_i) = 1.2 k_s k_{hc} k_f k_{td,if} k_\ell K_3$$

Creep coefficient at final time due to load placed at $t_d$:

$$\Psi(t_f, t_d) = 1.2 k_s k_{hc} k_f k_{td,df} k_{\ell,d} K_3$$

An example of the calculated shrinkage strains and creep coefficients from the spreadsheet for the base inputs is provided in Table 42 and Table 43.

**Table 42. Shrinkage strains based on initial inputs based on Storyline Example #2 for Activity #3.**

| Variable | Value | Definition |
|---:|:---:|---|
| $K_4$ = | 1.00 | correction factor for shrinkage strain |
| $k_s$ = | 1.000 | volume-to-surface area ratio factor |
| $k_{hs}$ = | 0.770 | humidity factor for shrinkage |
| $k_f$ = | 1.000 | strength factor |
| $k_{td,id}$ = | 1.238 | time development factor (curing to beam placement) |
| $k_{td,if}$ = | 1.523 | time development factor (curing to final time) |
| $\varepsilon_{bid}$ = | 5.72E-04 | shrinkage strain (curing to beam placement) |
| $\varepsilon_{bif}$ = | 7.04E-04 | shrinkage strain (curing to final time) |
| $\varepsilon_{bdf}$ = | 1.32E-04 | shrinkage strain (placement to final time) |

**Table 43. Creep coefficients based on initial inputs based on Storyline Example #2 for Activity #3.**

| Variable | Value | Definition |
|---:|:---:|---|
| $K_3$ = | 1.00 | correction factor for creep coefficient |
| $k_{hc}$ = | 0.945 | humidity factor for creep |
| $k_{\ell,i}$ = | 1.000 | loading age factor for transfer |
| $k_{\ell,d}$ = | 0.514 | loading age factor for loads at beam placement |
| $\Psi(t_d, t_i)$ = | 1.403 | creep coefficient ($t_i$ loading, curing to beam placement) |
| $\Psi(t_f, t_i)$ = | 1.727 | creep coefficient ($t_i$ loading, curing to final time) |
| $k_{td,df}$ = | 1.523 | time development factor (deck to beam placement) |
| $\Psi(t_f, t_d)$ = | 0.888 | creep coefficient ($t_d$ loading, deck to final time) |

These shrinkage strains and creep coefficients are used in the following section of the spreadsheet to calculate the prestress losses. The procedure for calculating the losses is the same as that described in Chapter 3 for Storyline Example #2. A sample of the calculated prestress losses provided by the spreadsheet for this span length are summarized in Table 44.

**Table 44. Significant outputs related to prestress loss to compare for Activity #3.**

| Variable | Value | Definition |
|---:|:---:|---|
| $\Delta f_{pES}$ [ksi] = | 7.6 | elastic shortening loss |
| $\Delta f_{pLT}$ [ksi] = | 29.4 | total time-dependent prestress losses |
| $\Delta f_{pT}$ [ksi] = | 37.0 | total prestress losses |

These values will change as the design is modified and if the correction factors for modulus of elasticity, creep, and shrinkage are varied.

The service and fatigue limit state checks in this spreadsheet follow the calculations provided for the Storyline Example #2 in Chapter 3. Calculated stresses are checked at transfer, service, and under fatigue loading. The stresses at the bottom fiber of the beam due to Service I and Service III load combinations ($f_{t,Se.III}$) can be compared for the different input variables, shown in Table 45. Service I and Service III load combinations are equal for UHPC members because the live load factor ($\gamma_{LL}$) is always equal to 1.0.

**Table 45. Significant outputs to compare for Activity #3.**

| Variable | Value | Definition |
|---:|:---:|---|
| $f_{t,Se.III}$ [ksi] = | -0.90 | stress at bottom fiber of beam due Service III load combination |
| $f_a$ [ksi] = | -1.00 | allowable tensile stress for Service III load combination |
| $f_a$ [ksi] = | -0.95 | allowable tensile stress for Service I load combination for cyclic loads |

The calculated stresses can be compared to the allowable stress limits at different design stages (e.g., transfer, sustained loads, service, cyclic).

The area of the prestressing and location of the prestressing strands will change with the input design values; more prestressing strands will be required for longer spans. The diameter of prestressing strands can be modified to use either 0.6-inch diameter strands with an area of 0.217 inch$^2$ or 0.7-inch diameter strands with an area of 0.294 inch$^2$. Up to 23 bottom fiber strands can be added at a height of 2 inches from the bottom. The prestressing-related properties that will change as the design is modified are summarized in Table 46.

**Table 46. Inputs related to area and location of prestressing for Activity #3.**

| Variable | Value | Definition |
|---:|:---:|---|
| $y_p$ [inch] = | 2.00 | distance from bottom of section to centroid of prestressing strands |
| $d_p$ [inch] = | 31.00 | depth of prestressed tension reinforcement |
| $e_p$ [inch] = | 14.45 | strand eccentricity |
| $A_p$ [inch$^2$] = | 3.689 | total area of prestressing strands |

Two top fiber strands can be added at a height of 31 inches to help decrease top fiber stresses at transfer if needed. Adding top fibers will increase $y_p$ and decrease $d_p$ and $e_p$, which will decrease the top fiber stresses.

## ACTIVITY #4: DESIGN FOR FLEXURAL STRENGTH

This spreadsheet used for this activity is based on the Activity #2 spreadsheet, but allows for calculating the factored nominal flexural resistance for rectangular sections with different amounts of reinforcement, $A_s$, and crack localization strains, $\varepsilon_s$.

The nominal flexural resistance occurs when one of the following occurs:

- Concrete crushes in the extreme compression fiber,
- Crack localization occurs in the extreme tension fiber, or
- Rupture of the tension reinforcement.

The strain compatibility approach, described in Chapter 2, is used checking each of these possible failure mechanisms. Calculation of the resistance factor also requires calculating the curvature at the service limit, so this is also calculated in this spreadsheet.

The initial inputs in this spreadsheet are based on Storyline Example #1. The key inputs that will be varied in this activity are total area of prestressing strands and crack localization strain, shown in Table 47.

**Table 47. Key inputs for Activity #4.**

| Variable | Input | Definition |
|---:|:---:|---|
| $A_s$ [inch$^2$] = | 4.680 | total area of prestressing strands |
| $\gamma_u \varepsilon_{t,loc}$ = | 0.003 | crack localization strain |

The process for calculating the moment and curvature associated with the service limit is discussed in Chapter 2. The service limit is associated with the moment and curvature when the strain at the centroid of the reinforcement is equal to the service limit strain (when the stress in the reinforcement is at 80 percent of its yield stress). For conventional reinforcement, the service limit strain, $\varepsilon_{s\ell}$, is equal to $0.8\varepsilon_y$.

For Grade 60 reinforcement: $\varepsilon_{s\ell} = 0.8\varepsilon_y = 0.8(0.0021) = 0.00166$

The spreadsheet is based on an input bottom fiber strain. The linear strain profile is used to relate the bottom fiber strain to the strain at centroid of the reinforcement. The strain in the bottom fiber when the strain in the reinforcement is equal to the service limit strain is as follows.

Strain in bottom fiber at service limit: $\varepsilon_{t,s\ell} = \varepsilon_{s\ell}\,(h - c) / (d - c)$

The procedure described in Activity #2 is used with this bottom fiber strain to calculate the moment and curvature associated with the service limit, $M_{s\ell}$ and $\psi_{s\ell}$, respectively.

The process for using the spreadsheet to determine the controlling failure mechanism and the associated nominal flexural resistance is as follows.

1. Assume that crack localization controls failure first.

| | | |
|---|---|---|
| $\varepsilon_t =$ | $\gamma_u \varepsilon_{t,loc}$ | strain in extreme tensile face of UHPC (T = +) |

2. Solve for the neutral axis depth using a solver.

| | | |
|---|---|---|
| c (inch) = | 7.78 | neutral axis depth (**solve either manually or using solver**) |
| N (kips) = | 0.00 | summation of internal forces (positive is compression) |

**Note: The answer may not converge if another failure mechanism controls.**

3. Check the strain in the extreme compression face and the strain at the centroid of the reinforcement to see if crushing or reinforcement rupture already occurred.

| | | |
|---|---|---|
| $\varepsilon_c =$ | 0.00144 | strain in extreme compression face of UHPC (C = +) |
| $\varepsilon_{cu} =$ | 0.0035 | ultimate compression strain limit |
| $\varepsilon_s =$ | 0.00259 | strain in non-prestressed steel (T = +) |
| $\varepsilon_{su} =$ | 0.100 | ultimate strain for non-prestressed reinforcement |

If $\varepsilon_c \leq \varepsilon_{cu}$ and $\varepsilon_s \leq \varepsilon_{su}$, then crack localization controls the failure. Otherwise, go to the next step.

4. **Concrete Crushing**: To find the capacity when the concrete crushes, set the strain in the bottom fiber equal to $\varepsilon_{cu} \times (h - c) / c$. Then solve for the neutral axis using the solver.

| | | |
|---|---|---|
| $\varepsilon_t =$ | $\varepsilon_{cu} \times (h - c) / c$ | strain in extreme tensile face of UHPC (T = +) |

Check the strain the extreme tension fiber and the strain at the centroid of the reinforcement to see if another failure mechanism controls.

5. **Rupture of Reinforcement**: To find the capacity when the reinforcement ruptures, set the strain in the bottom fiber equal to $\varepsilon_{su} \times (h - c) / (d - c)$. Then solve for the neutral axis using the solver.

| | | |
|---|---|---|
| $\varepsilon_t =$ | $\varepsilon_{su} \times (h - c) / (d - c)$ | strain in extreme tensile face of UHPC (T = +) |

**Note: The spreadsheet is not setup for having a neutral axis depth outside of the section, so the results are not valid if *c* < 0 inches.**

Check the strain the extreme tension fiber and the strain at the centroid of the reinforcement to see if another failure mechanism controls.

The moment and curvature calculated using this procedure is considered the nominal flexural resistance and curvature at nominal flexural resistance, $M_n$ and $\psi_n$, respectively.

The curvatures at the service limit and at nominal flexural resistance are used to calculate the curvature ductility, $\mu$, and associated resistance factor, $\phi$, as follows.

Curvature ductility: $\mu = (\psi_n / \psi_{s\ell})$ UHPC Structural Design Guide Eqn. 6.3.2.3-1

Resistance factor: $0.75 \leq \phi = 0.75 + 0.15(\mu - 1.0) / (\mu_\ell - 1.0) \leq 0.90$
UHPC Structural Design Guide Eqn. 5.4.2-1

Curvature ductility ratio limit: $\mu_\ell = 3.0$ UHPC Structural Design Guide Article 6.2

This resistance factor is used to calculate the factored nominal flexural resistance, $M_r = \phi M_n$.

## ACTIVITY #5: DESIGN FOR SHEAR

This activity and the accompanying spreadsheet go through the design procedure for shear using the Storyline Example #2 as the initial input values. The initial inputs in this spreadsheet are based on Storyline Example #2. The key inputs that will be varied in this activity are the crack localization strain ($\varepsilon_{t,loc}$), spacing of the transverse reinforcement ($s$) and area of transverse reinforcement ($A_v$), as shown in Table 48. The shear demand is also modified to provide designs with and without shear reinforcement.

**Table 48. Key inputs for Activity #5.**

| Variable | Input | Definition |
|---|---|---|
| $\varepsilon_{t,loc}$ = | 0.005 | localization strain |
| $s$ [inch] = | 6.00 | spacing of transverse reinforcement |
| $A_v$ [inch$^2$] = | 0.00 | area of transverse reinforcement |

The process for using the spreadsheet follows the procedure for calculating the nominal shear resistance using the General Approach for calculating θ and $f_{v,\alpha}$ described in Chapter 2 and 3.

1. Select values for the transverse reinforcement.

| | | |
|---|---|---|
| $s$ [inch] = | 6.00 | spacing of transverse reinforcement |
| $A_v$ [inch$^2$] = | 0.00 | area of transverse reinforcement |

2. Assume a value for the stress in the transverse reinforcement.

| | | |
|---|---|---|
| $f_{v,\alpha}$ (ksi) = | 60 | assumed stress in transverse reinforcement |

3. Use Goal Seek to solve for the principal angle direction. Select the "Difference =" "difference between crack localization strain" cell. Use Goal Seek to solve for the principal angle direction (θ) such that the difference equals 0.

| | | |
|---|---|---|
| θ [rad] = | 25.85 | principal angle direction |
| Difference = | 0.00 | difference between crack localization strains |

4. Check the difference between the calculated and assumed $f_{v,\alpha}$.

| | | |
|---|---|---|
| $f_{v,\alpha}$ (ksi) = | 60 | assumed uniaxial stress in transverse steel reinforcement |
| $f_{v,\alpha}$ (ksi) = | 60 | uniaxial stress in transverse steel reinforcement |
| Difference = | 0.00 | difference between assumed and actual stress |

If there is a difference between these values, then change the assumed $f_{v,\alpha}$ and repeat Step 3.

5. Compare resistance to demand: The resistance can be compared to the demand for the shear strength.

| | | |
|---|---|---|
| $\phi_v V_n$ [kips] = | 281.6 | factored nominal shear resistance at critical section |
| $V_{u,cr}$ [kips] = | 132.2 | factored shear demand at critical section |

The minimum longitudinal reinforcement requirement can also be checked.

The uniaxial stress in transverse steel reinforcement ($f_{v,\alpha}$), principal angle direction ($\theta$), UHPC contribution to shear resistance ($V_{UHPC}$), and contribution of transverse reinforcement to the shear resistance ($V_s$) will vary for different combinations of the input variables.

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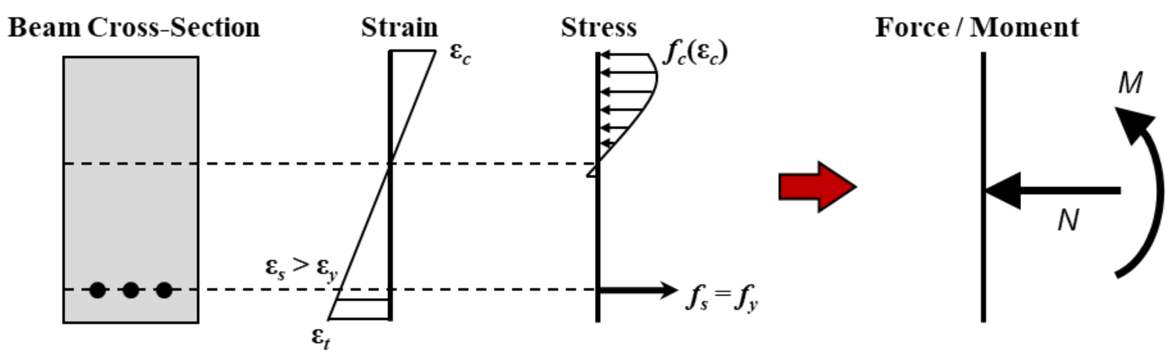
# APPENDIX: ADDITIONAL WORKSHOP MATERIALS

The other materials for the *Structural Design with UHPC* workshop include the following:

- Background knowledge check worksheet for attendees to assess their knowledge before and after the workshop,
- Presentation slides for the eight modules and an introduction and closeout module,
- Editable spreadsheets to accompany the activities described in Chapter 4, and
- Worksheets to accompany the activities described in Chapter 4.

The background self-assessment questions are as follows.

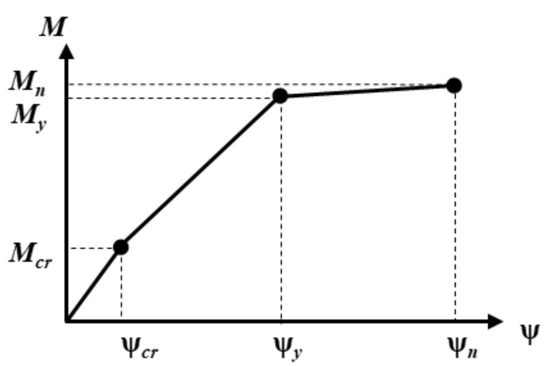
- Background Knowledge Check #1: What is UHPC
- Background Knowledge Check #2: What are the key characteristics and minimum material properties for UHPC?
- Background Knowledge Check #3: What is your level of comfort with Figure 52?




Source: FHWA.

**Figure 52. Illustration. Figure associated with Background Knowledge Check #3.**

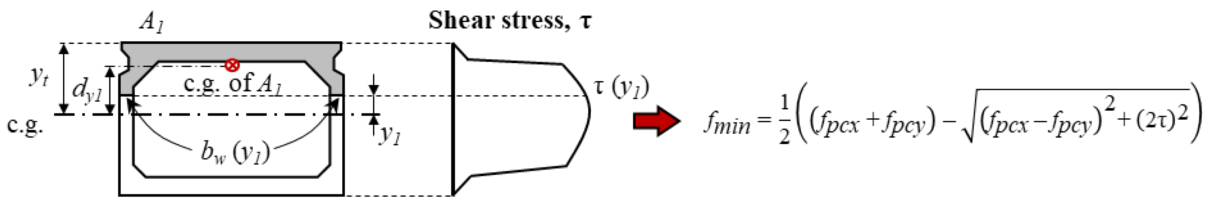
- Background Knowledge Check #4: What is your level of comfort with Figure 53?




Source: FHWA.

**Figure 53. Illustration. Figure associated with Background Knowledge Check #4.**

- Background Knowledge Check #5: When may it be advantageous to use UHPC for structural elements? Include possible examples.
- Background Knowledge Check #6: What is your level of comfort with AASHTO LRFD BDS Article 5.9.2.3.3 (Principal Tensile Stresses in Webs) and Figure 54?




Source: FHWA.

**Figure 54. Illustration. Figure associated with Background Knowledge Check #6**

- Background Knowledge Check #7: What are the three possible failure modes in flexure for UHPC members?
- Background Knowledge Check #8: Describe the basic procedure for designing a UHPC member for shear. Include any key differences between shear design for CC and UHPC members.

The presentation slides, editable spreadsheets, and activity worksheets are provided to workshop attendees, but not publicly available at the time of publication of this manual.