DESIGN CRITERIA FOR OVERHANGING ENDS OF BENT CAPS

By

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SYNOPSIS

A program of 36 bent cap sections of 36 in. depth were tested in an investigation primarily directed to the bond and shear strengths which should be used in design. Intermediate, A432, and A431 grade steels were used. Even though the final failures were often classified as bond or shear only 8% of the specimens failed at less than the calculated f_y steel stress. The average f_s developed was between $1.15f_y$ and $1.20f_y$.

In shear it was found that an ultimate v much higher than allowed by current specifications was feasible for loads placed between 0.5d and 1.2d from the support. Vertical stirrups added no perceptible strength but horizontal web steel was effective.

In bond it was found that the nominal bond stress in the length between load and support was not important, but that an end anchorage distance beyond the load was essential. With an end anchorage of 15 in. for #11 bars or 12 in. for #8 bars, there seemed to be no problem in developing a 75 ksi steel.

The width of web cracks was only slightly less than that of flexural cracks on the tension face and suggested the desirability of using horizontal stirrups in these cantilever ends.

STUDY OF DESIGN CRITERIA FOR OVERHANGING ENDS OF BENT CAPS

The Problem of Designing Bent Cap Cantilever Ends

The overhanging end of a bent cap is a short cantilever beam with large depth relative to its projection. Elastic methods of analysis indicate that such cantilever members must be classified as deep beams with flexural stress distributions far from linear. In such elastic beams the resultant of compressive stress rises above middepth as the length decreases to provide an internal couple with a lever arm much less than 0.5d rather than the usual arm of around 0.9d for ordinary concrete beams. For a uniformly loaded member, the horizontal unit flexural stresses would be as shown in Fig. 1a. There is very little information on how the usual flexural cracking of reinforced concrete modifies this elastic stress distribution.

It is customary to design such short cantilever members at the support as though ordinary flexure and bond formulas applied. However, it is usually realized that these steel stresses will not decrease as rapidly as does the moment at sections closer to the load. Instead, the resultant compression tends to slope from the bottom of the beam at the support and trend towards the load point at the top (Fig. 1b), much the same as if the cantilever were somewhat more triangular in shape. This implies considerable tension in the bars at the load point (Fig. 1c) and thus much smaller flexural bond stresses ($u = V/\Sigma o\ jd$) between the support and the load than this formula indicates. Likewise it requires that the tension at the load be anchored by bond stress beyond the load, that is, by what is usually designated as end anchorage.

In the calculation of shear stress near the support there are traditional methods which can include the effect of the sloping bottom of a bracket or short cantilever and which result in lower calculated critical stresses. These assume that the resultant compression is sloping, parallel to the compression face. The compression thus has a vertical component which balances part of the external shear and only the remaining remnant of the shear causes diagonal tension, which is the real critical stress for design. As already indicated in Fig. 1 it is certain, even with no bottom slope, that the resultant compression does slope and thereby reduces the critical shear stress. With any reasonable proportions (even for a sloping bottom) experience shows that shear failure always involves the total depth at support. This investigation was planned to establish, in part, proper numerical values of such shear strength for a load applied on top of the cap and a reaction applied to the bottom of the cap, that is, for the typical

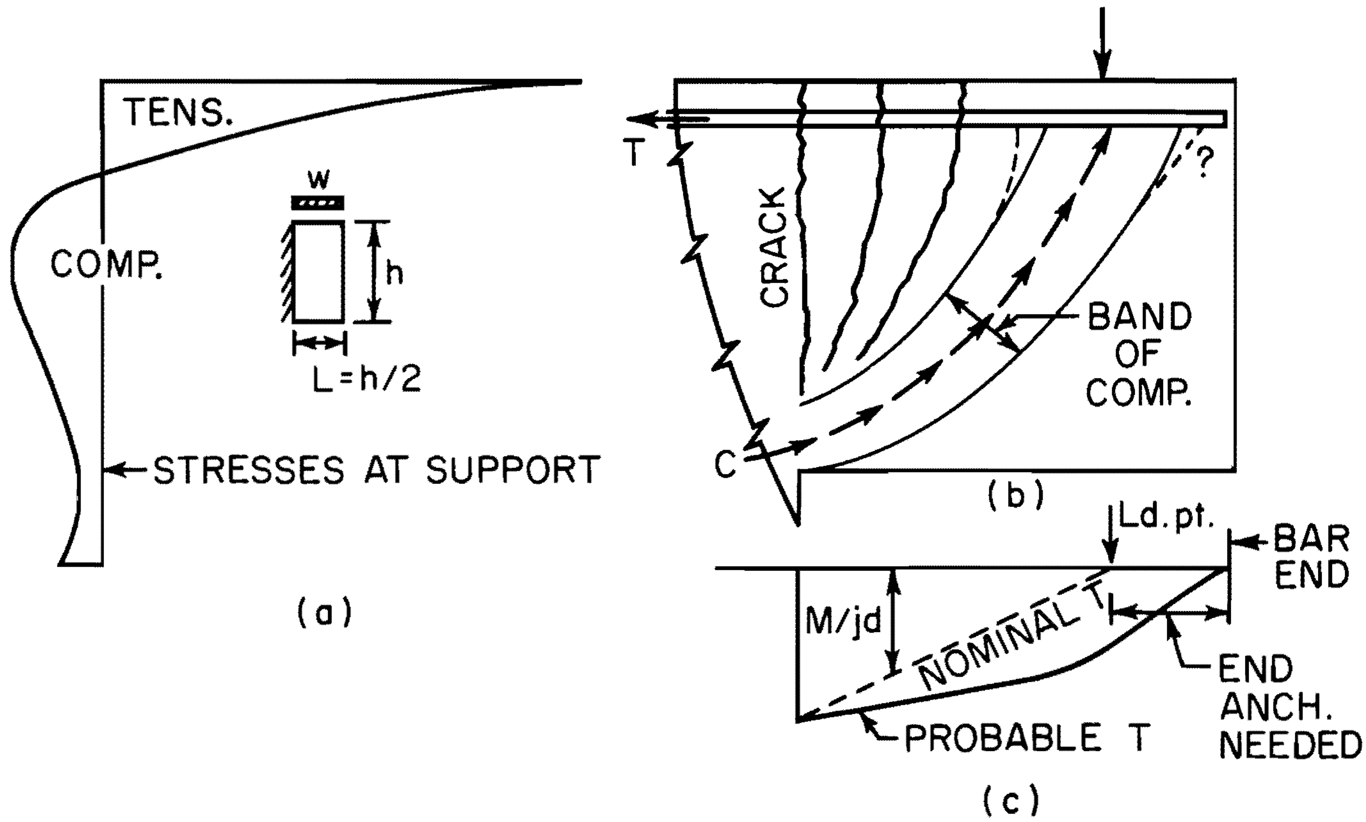

FIG. 1. (a) ELASTIC STRESS DISTRIBUTION BY DEEP BEAM THEORY. (b) THE PRACTICAL CASE. (c) PROBABLE DISTRIBUTION OF BAR TENSION.

bent cap design situation.

Bottom slopes on bent caps are usually small and compression capacity normally exceeds tension capacity. Since these bottom slopes probably have no influence on the member capacity and very little influence on the stresses developed by a given loading, this entire study has been presented on the basis of nominal stresses calculated as though the member were of uniform depth and of considerable length.

Since the critical behavior of the cantilever always follows the usual cracking in reinforced concrete, the cracking invalidates all the assumptions and conclusions of elastic deep beam theory. At no part of the investigation would the use of deep beam theory serve to explain member behavior better than ordinary beam calculations. Hence no further mention of deep beam theory will be made.

Objective of Investigation

The purpose of this investigation was to establish reasonable and safe design procedures and stresses, especially shear and bond stresses, which could be applied in designing overhanging cantilever ends having proportions typical for highway structures. Although the conclusions are not intended to be limited to a particular member depth, all the members tested were 36 in. deep overall and all were reinforced with moment steels having pf_y values in a narrow range of some 380 to 450 psi, i.e., p values approximately 0.0094 to 0.0110 for an intermediate grade steel. All loads were applied to the top of the members and the investigation is not intended to relate to cases where bridge girders might be monolithic with bent caps and thus deliver their loads through a shear surface.

Scope of Investigation

A total of 36* overhanging ends were made and tested with the following variables represented.

1. Distance of load from support.
2. Member length beyond center of load.
3. Grades of steel.
4. Type of web reinforcement, and with no web reinforcement.

*The contract called for 35 tests.

5. A slight variation in bar size.
6. A limited comparison between circular and rectangular column supports.

It was originally intended to investigate also the effect of neoprene pads and separated bearing plates on bond, but there was no evidence which seemed to make these as significant as the matter of type of web steel and additional variations in item 2 above.

Test Specimens

All specimens had the same depth at the critical section, as shown in Fig. 2a. The distance "a" out to the load (called the shear span) and the extension B beyond the load (called the end anchorage length) were varied when casting the specimen merely by omitting any unwanted portion of the outer end. The bent proportions matched one of the Texas Highway Department standards.*

Other than in length, there were many variations based on this standard. Among these variations were the following (Fig. 2a and 2b):

1. The shaft or column support was round (Type 1) with diameter equal to cap thickness (30 in.) or rectangular of the same cross-sectional area. (Type 2)
2. The cap thickness, which was initially 30 in., was reduced to about 12.5 in. (Type 3) with many of the rectangular columns, and for a few to 8.5 in. (Type 4) without changing the reinforcement ratio. This was chiefly for convenience in testing.
3. The grade of main tension steel was varied between intermediate grade, ASTM A432, and ASTM A431 steels, in each case maintaining nearly the same effective steel ratio pf_y/f_c'.
4. Bar size of main steel was usually #11, but #8 bars were also used with higher strength steels, and in a few cases a #5 bar was added with #8 bars to make up the desired tension capacity.
5. Stirrups were varied slightly in their ratio r, but primarily by complete omission or by placing horizontal steel in lieu of vertical stirrups, as in Fig. 2b.

For convenience two specimens were cast as a single unit. An attempt was made to keep concrete strength constant, but transit mixed concrete was used in all cases and controls were not adequate at times.

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*Interior Bents for Use with Prestressed Concrete Beam Spans, BGp-28-65.

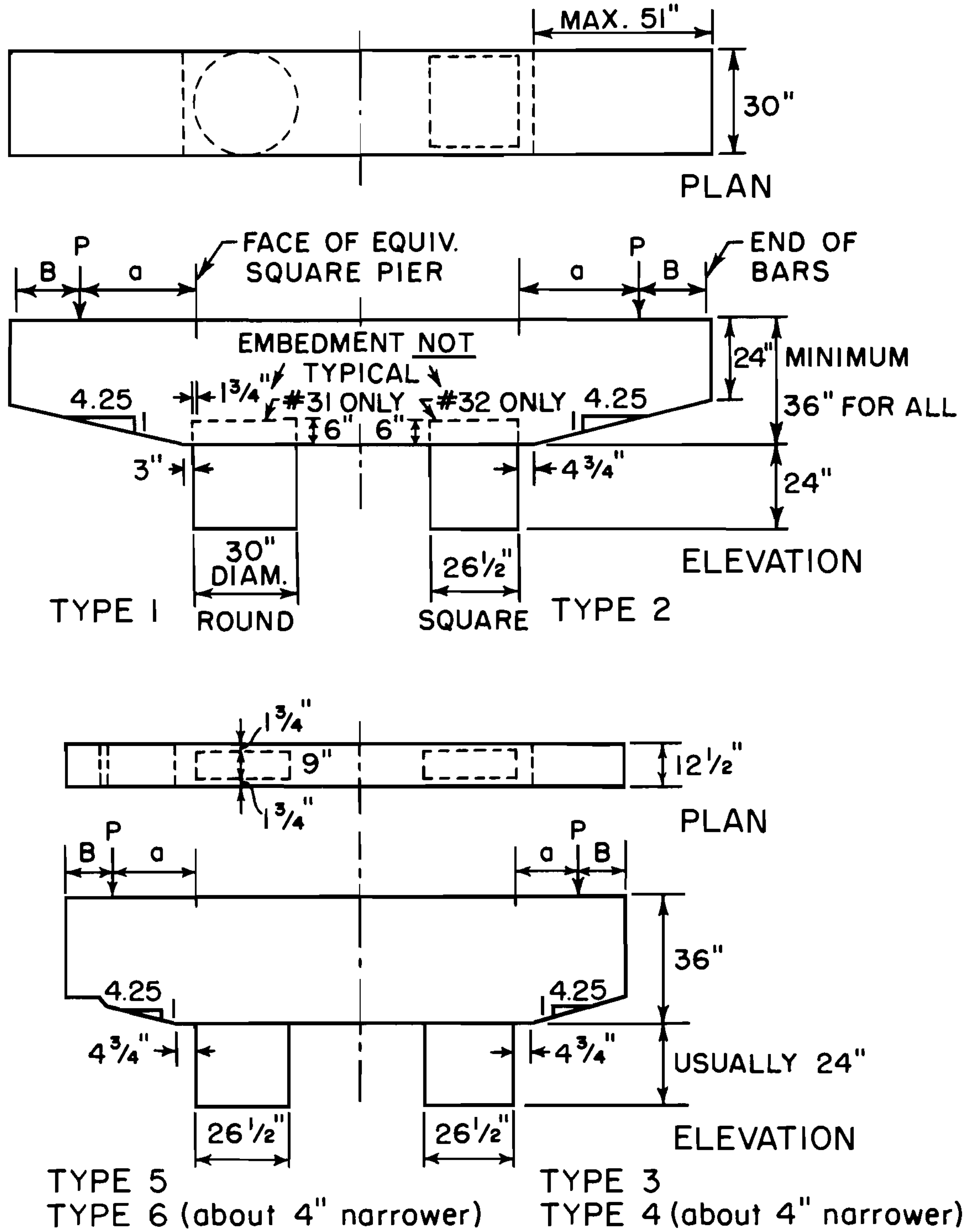

FIG. 2a. TYPES OF SPECIMENS AND NOMINAL DIMENSIONS.

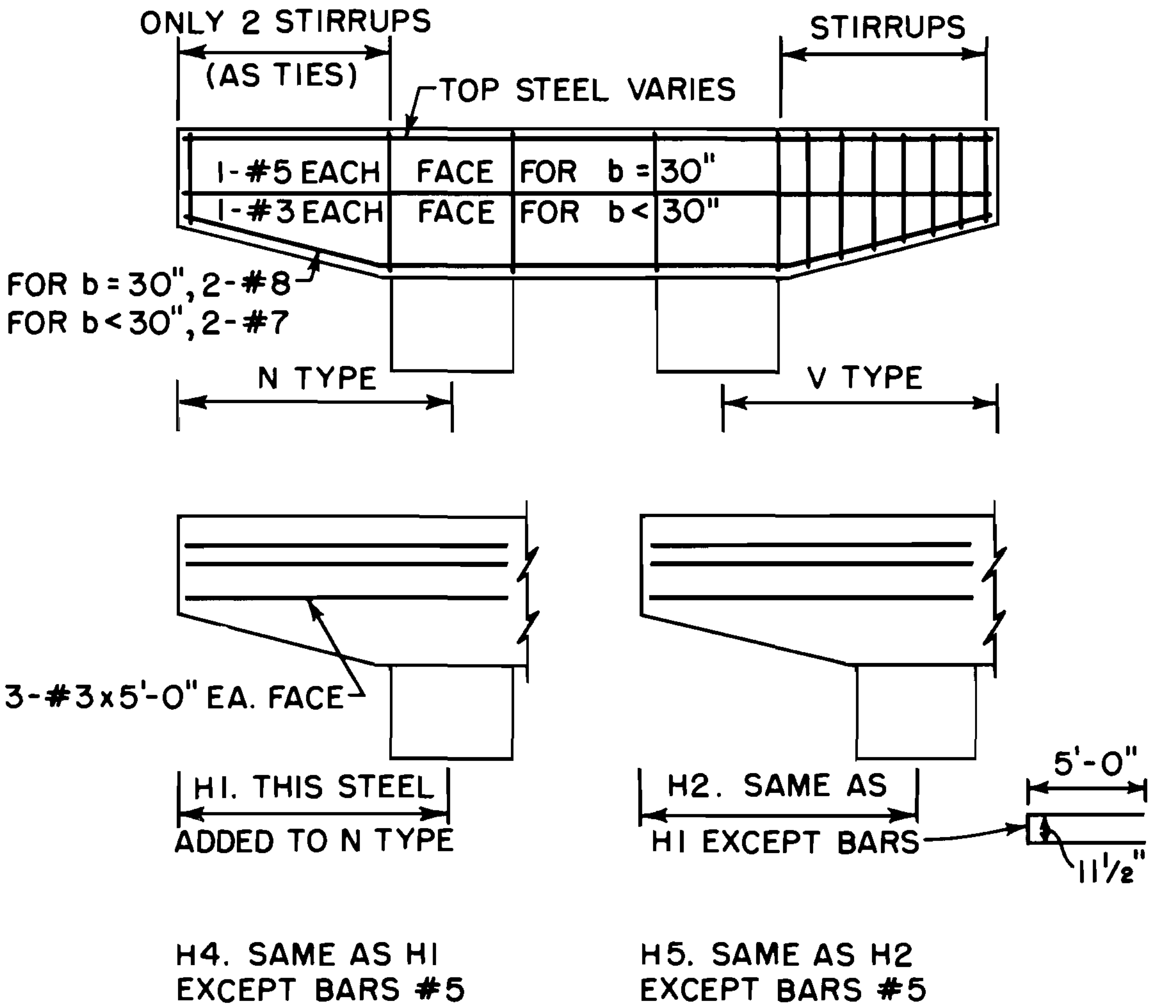

FIG. 2b. REINFORCEMENT PATTERNS.

The specimens are listed in Table 1 in the Appendix with exact dimensions and reinforcing. The specimens are numbered in the sequence tested, but they are grouped according to type of specimen and reinforcement, which are coded according to the following plan.

First - the test serial number

Second - type of specimen (Fig. 2a)

Third - pattern of reinforcing (Fig. 2b)

Fourth - quality of steel (4 for intermediate, 6 for A432, 8 for A431)

Thus 31-1-V-4 indicates the 31st specimen tested, 30 in. wide with a round column, vertical stirrups, and main steel of intermediate grade.

Test Procedure

Specimens and cylinders were cast from the mix given in the Appendix in wooden forms with concrete made with high-early-strength cement, cured in the forms under plastic covering until the day before the test (usually until the sixth day), and then tested the following day. The specimen, shown to the left in Fig. 3, was placed on its side on rollers and jacked against an anchor beam using a steel yoke. The active jack and yoke is shown in the foreground. Since the cantilever end was statically determined, the anchor yoke and jack was not critical and was usually placed over the other column. Load was applied in increments and was carried to failure except when failure required more than the 400k jack capacity. Crack lengths and widths were marked as the test progressed.

Data

The ultimate loads and mode of failure are indicated in Table 2 in the Appendix. Also in this table are values of stresses calculated by standard ultimate strength methods as though for members of uniform depth:

$$f_s = M_u/(A_s 0.9d)$$
$$v = V_u/bd$$
$$u = V_u/(\Sigma o\ 0.9d)$$

These relations for f_s and for u were also used for beams with horizontal bars for web steel, thus ignoring the help this horizontal web steel should contribute towards f_s and u stresses.

Since there was some scattering of f_c' values, corrections of v and u values were made to stresses corresponding to $f_c' = 4500$ psi by use of the relations

(a)

(b)

Fig. 3. Test setup. Load is applied at the near end. The jack position in (b) is the more stable. The load is applied to specimen through a 1-in. square rod between bearing plates in lower left corner.

$v_{4.5} = v\sqrt{4500/f_c'}$ and $u_{4.5} = u\sqrt{4500/f_c'}$, which both assume that v and u vary as $\sqrt{f_c'}$.

Comparison of Round and Square Column Effects

In this investigation, as often is done in design, it was assumed that a round support might be replaced by a square support of the same cross-sectional area. The first variation introduced into the tests was an attempt to check the validity of this engineering assumption. Specifically Specimens 1 and 3 with round columns were compared with Specimen 4 with a square column; and similarly Specimen 5 with Specimen 6.

Unfortunately in these early tests conclusive evidence of yielding of the main longitudinal steel was taken as evidence of flexural failure and the loading was not carried to secondary compression failure. This was only in part a technical decision, since the test equipment established a 400k limit. Specimens 1, 3, and 5 reached 399k, 402k, and 391k, respectively, before the tests were stopped.

These comparisons were not conclusive but they showed no difference greater than 6% and the differences were in opposite directions for the two sets, with developed steel stresses of 110 to 120% of yield strength. Accordingly, the later specimens generally used rectangular columns and a reduced width which brought most specimens well within the capacity of the test equipment, even for loads closer to the support.

It was observed that both round and square columns introduced a cracking phenomenon different from that which would have accompanied a square support extending all across the cap. In the latter, flexure cracks would tend to run to the support edge and very little inside it. In the tests here reported, as shown in Fig. 4, these cracks extended 6 in. to 8 in. inside the support, and this was more apparent with the circular columns than with the square columns. This suggests that with both types the crack surface inside the cap is probably not a plane surface from side face to side face.

Flexural Strength

In only three out of 36 specimens did members fail in any fashion before reaching the calculated yield point of the longitudinal steel. In only five cases did failure occur at less than $1.09f_y$. Thus in the following discussion of ultimate shear and bond it must be kept in mind that the failures were

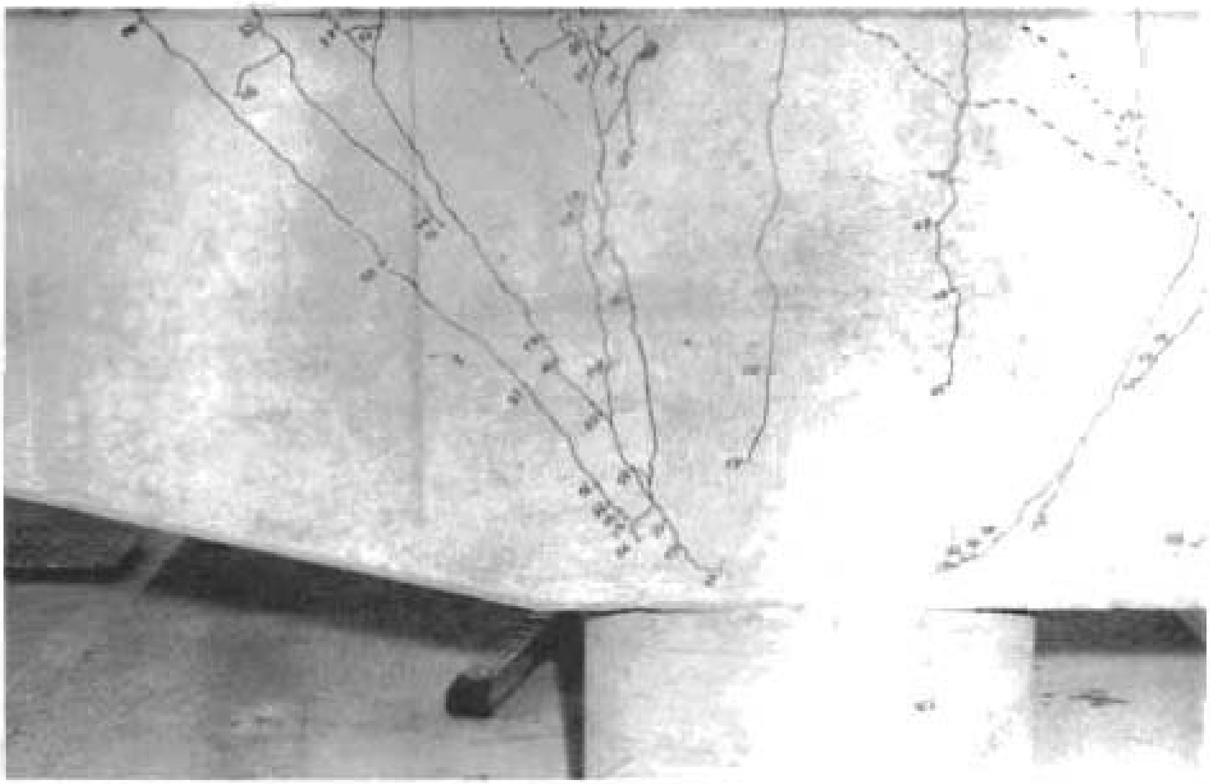

Fig. 4. Side cracks tended to turn so as to intrude back into support area.

chiefly in members already well past their computed flexural capacity. This is illustrated by the bar charts in Fig. 5 showing the distribution of calculated f_s/f_y for the three separate steel grades. These charts show that the concrete strains associated with high-strength steels constituted no handicap insofar as flexural strength was concerned, the low f_s/f_y values being for specimens failing in bond or shear. Crack width is discussed separately in a later section.

Shear Strength

When all failure points or maximum load points are plotted in terms of shear, as in Fig. 6, there is considerable scatter with a general trend of higher values showing for small shear spans (the distance a shown in Fig. 2a). All shear values are much higher than ordinarily used in design. Each of these points constitutes a fair "proof load" measure of capacity in shear, but the bond failures and flexural failures cannot properly be used in setting the lower limit for shear capacity. When such failures are culled out of the lower regions, as in Fig. 7, the number of points is more limited but is adequate to support the dotted curve as a safe lower boundary. Within this limited a/d range this curve can be defined by the equation

$$v_u = 320 + 140d/a \qquad (\text{for } 0.5 < a/d < 1.2)$$

These shear stresses are quite large in terms of the usual accepted design values.

The Joint ACI-ASCE Committee on Shear and Diagonal Tension reported that it would be safe to design beams on the basis of shear stresses at a distance d from the face of the support and then to use the same web reinforcement (if any) back to the support. The next section in the present report indicates that vertical stirrups as actually used contributed little or no strength although their contribution to the calculated ultimate strength was from 135 psi to 160 psi for the various specimens. The Joint Committee recommended that shear on the concrete itself be limited to the diagonal cracking stress which is approximately $2\sqrt{f_c'}$, or 136 psi for an f_c' of 4500 psi. Thus the sum of these two components might be calculated as 136 plus 160 (maximum) for a total of 296 psi. In contrast, the value proposed here (without stirrups) becomes 460 psi for an a/d of 1.00. This is no discredit to the Joint Committee Report which was not aimed at the short a/d values of importance in the design of the overhanging end of a bent cap.

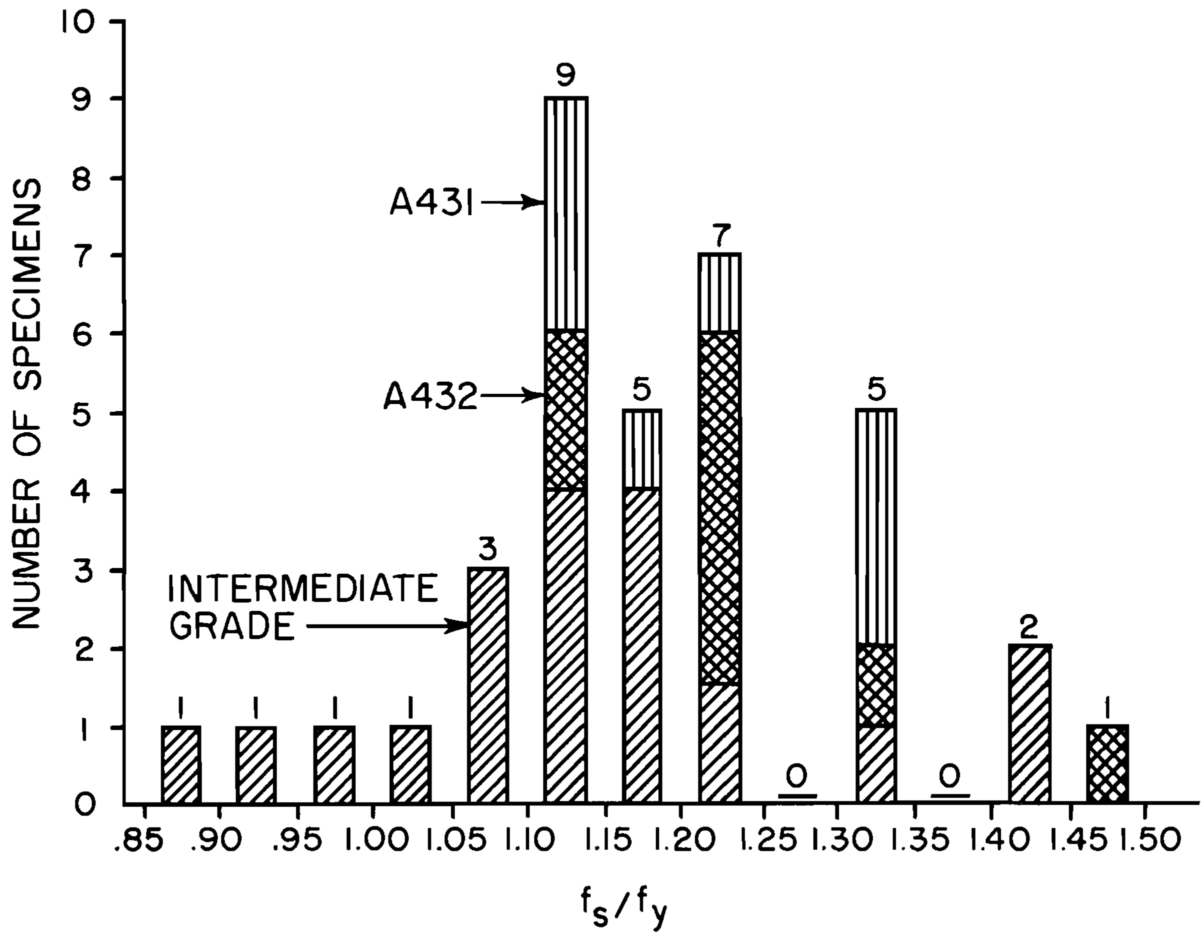

FIG. 5. HISTOGRAM OF ULTIMATE f_s ATTAINED.

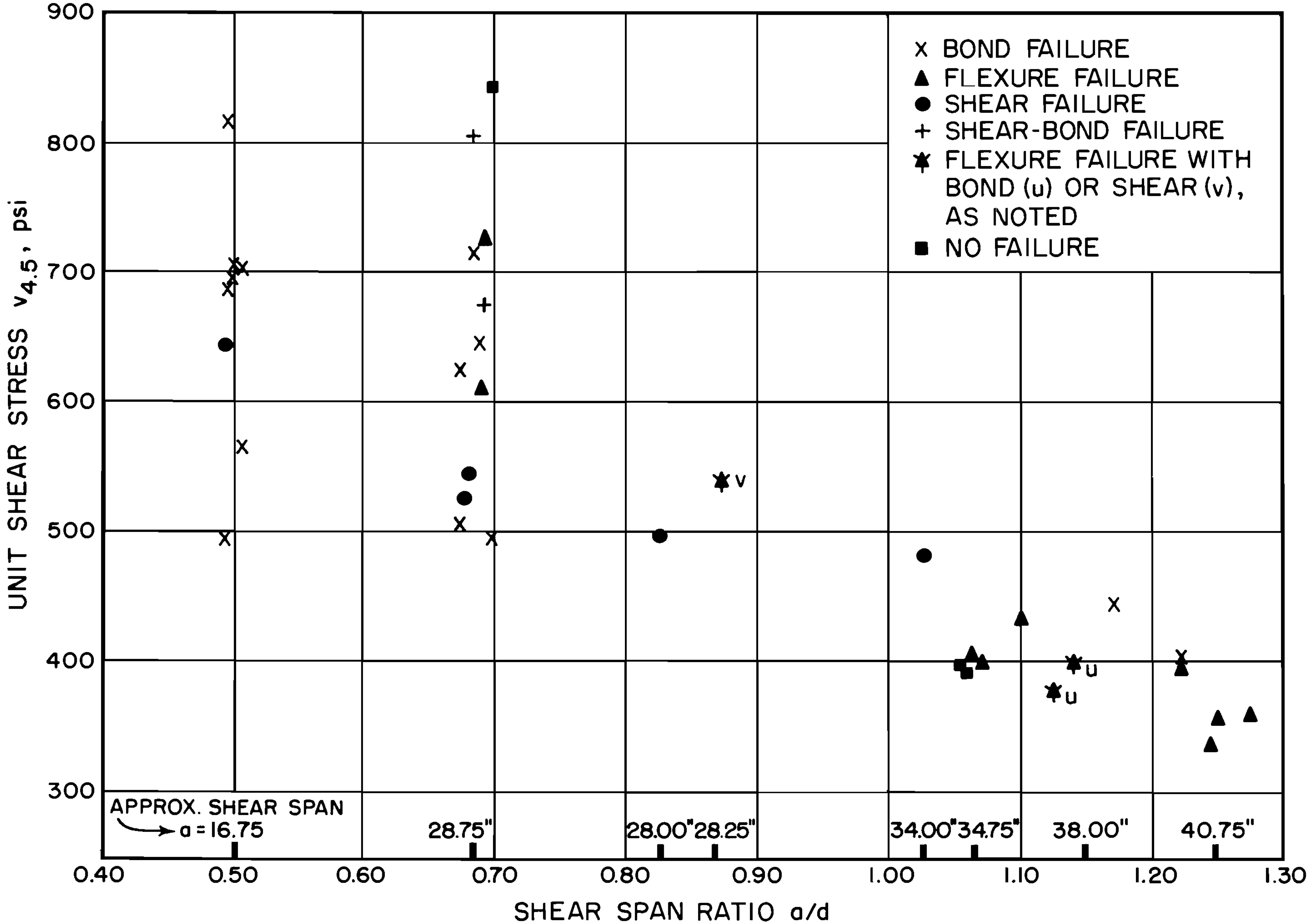

FIG. 6. ULTIMATE SHEAR STRESS, INCLUDING ALL TYPES OF FAILURE.

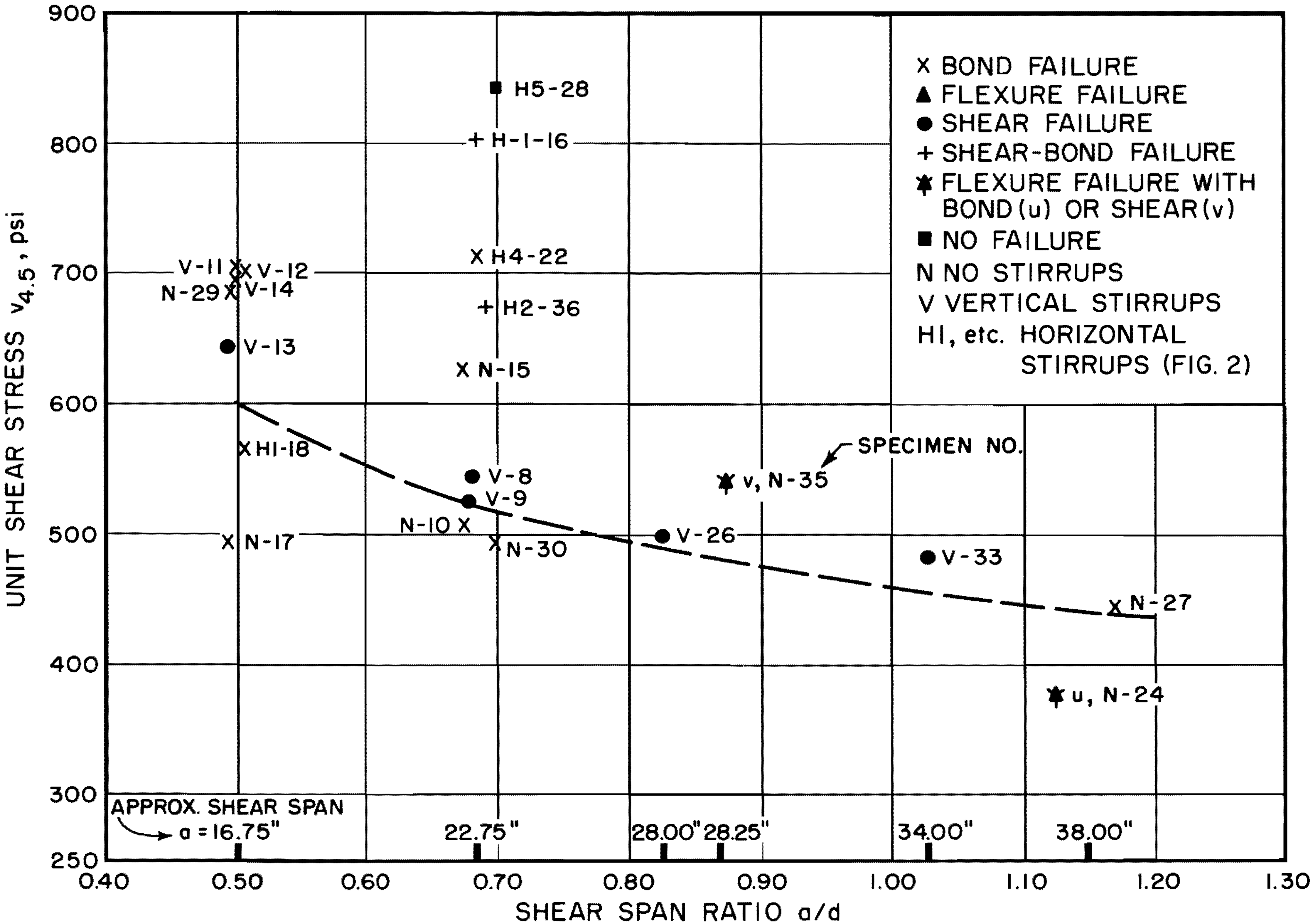

FIG. 7. ULTIMATE $v_{4.5}$ FOR SHEAR FAILURES AND FOR BOND FAILURES WHICH ALSO SERVE AS PROOF LOADS.

Influence of Web Reinforcement

The data can be analyzed further for the influence of horizontal or vertical web reinforcing. To simplify the comparison, Fig. 8 has been prepared from Fig. 7 to show only a comparison between vertical stirrups (V) and no stirrups at all (N). The x-marks represent bond failures and thus the three low points on the left are shown only to account for all members made without stirrups. In the upper left are four bond failures which do have significance as proof loads, to show at least this much shear capacity.

The direction of cracks in early tests had indicated that very few stirrups were crossed by the critical cracks. Hence it was suspected that the stirrups were nearly ineffective. In Fig. 8 a detailed comparison at each given a/d indicates that the vertical stirrups either add nothing to the strength, or, more probably, that the amount they add is so small that it is lost in the ordinary scatter of the experimental data.

The steepness of the observed "diagonal" cracks suggested that horizontal steel would be more effective than vertical steel in improving shear strength.* Unfortunately all horizontal steel was used in specimens having a small end anchorage beyond the load (B in Fig. 2a) which probably failed to show the full potential of this shear reinforcement.

The effect of U-shaped horizontal steel is shown by points H4-22 and H5-28. The first of these, of #3 bars, failed in bond but with a shear strength 35 percent above the lower bounding curve. The second, of #5 bars, did not fail in the cap because of a premature column failure, but it reached a shear stress 60 percent above the bounding curve. Of the four specimens made with horizontal side bars not having either hooks or end loops, two definitely failed in bond at shears 4 to 6 percent below the curve and two failed in combined shear and bond at very satisfactory shear stresses.

These data strongly support the idea that vertical stirrups are of little value and that looped horizontal stirrups can add substantial shear strength. The scope of this investigation did not permit evaluating the horizontal bars completely. It is even possible that the high shear capacity shown without any web reinforcement makes the higher value with horizontal web reinforcement of little practical importance.

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*In spite of the fact that German tests with highly stressed webs in I-shapes have shown that horizontal web bars lowered the compression "diagonal" strength and resulted in lower shear strengths because of diagonal compression failures.

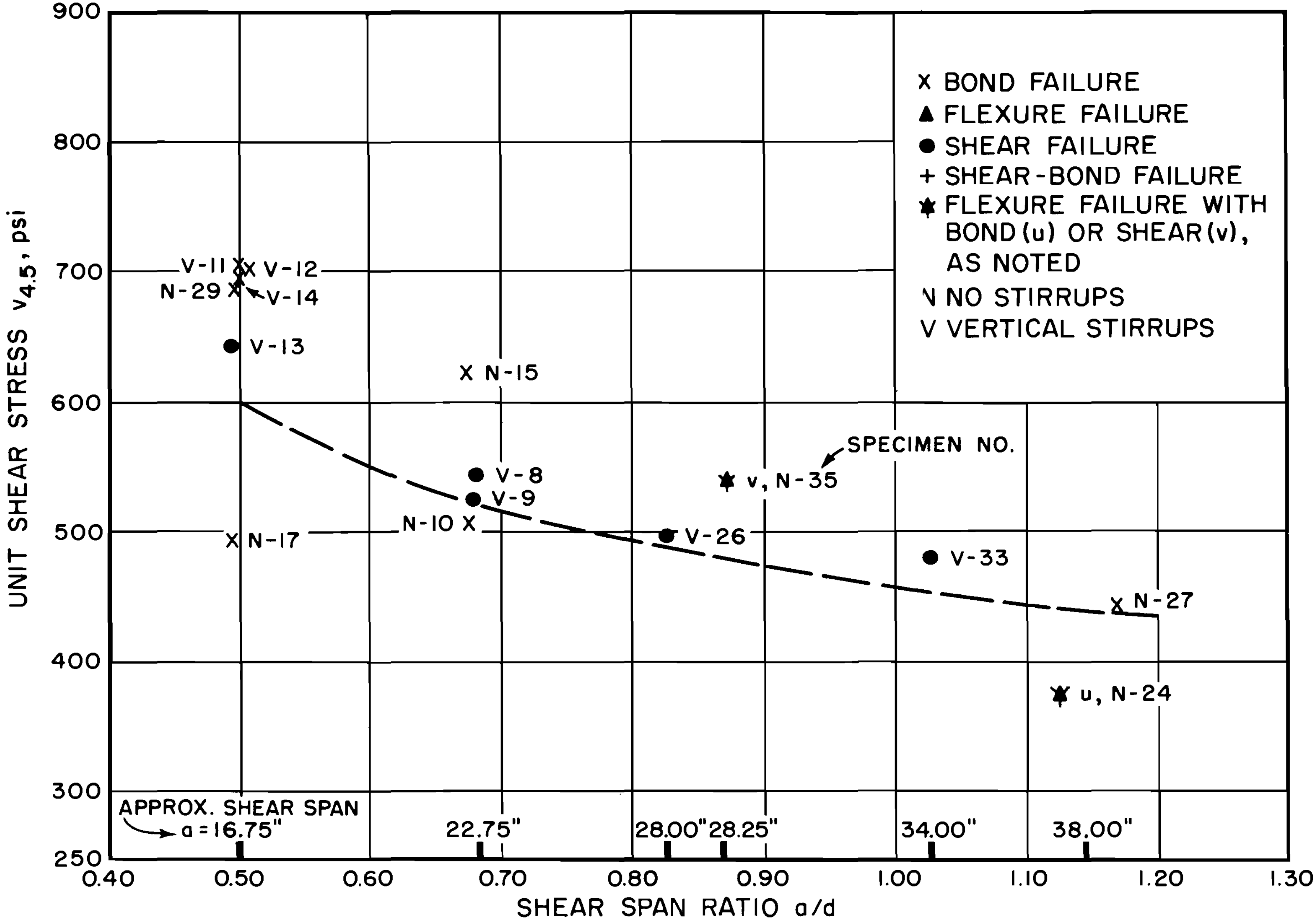

FIG. 8. ULTIMATE $v_{4.5}$, COMPARING VERTICAL STIRRUPS WITH NO WEB STEEL.

Flexural Bond Stress

When all the calculated bond stresses at failure are plotted, excluding definite flexure failures and shear failures as in Fig. 9, the overall picture is still somewhat confusing. The very fact that bond stresses of 1500 psi to 1900 psi were calculated on #11 bars is evidence that these are not real stresses, because #11 bars do not have this bond capacity. The calculated values emphasize the comment in the opening paragraphs of this report to the effect that steel tensile stress increases at a less rapid rate than moment such that $u = V/(\Sigma o\ jd)$ is a purely nominal calculation for these short cantilevers. At the same time many calculated values are quite reasonable, around 600 to 800 psi, and a couple are in the 400 to 500 psi range, a little low.

Closer investigation indicated that not a single beam having an end anchorage (B in Fig. 2a) in excess of 9" failed in bond and only one beam having an end anchorage as small as 5.5" (Specimen No. 16*) escaped without a bond failure. Thus it appears that bond failure can be avoided by the simple expedient of extending the bars more than 9" beyond the center of the applied load. The tests show that 14" or 15" is adequate end anchorage even though the developed steel stress is quite high. It is probable that even a 12" end anchorage is adequate, based on the trend of the data, although there are no tests of this length. With adequate end anchorage there seems to be no reason to calculate the ordinary bond stress at all.

In spite of the fact that it appears the best design technique is to avoid bond problems by using end anchorage, there were some interesting trends indicated by the bond failures. It was first suspected that the high bond values might relate to #8 bars and the low ones to #11 bars, but it was found that both the highest and the lowest values applied to #11 bars and the #8 bars fell in between. The larger shear span ratios showed lower minimum bond values and the smaller shear spans showed higher minimum values, all within the 400 to 800 psi bond stress range.

It must be noted that the only three recorded values of ultimate $f_s/f_y < 1.0$ were caused by bond failures and represented anchorage lengths of 4 to 5.5 in. In general all other bond failures came after the calculated steel stress was much greater than f_y, a condition generally thought to be adverse to bond strength. This situation was explored further. Figure 10 plots the developed bond stress

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*Specimen 16 had to be classified as a combined shear and bond failure, which means it could have been a shear failure.

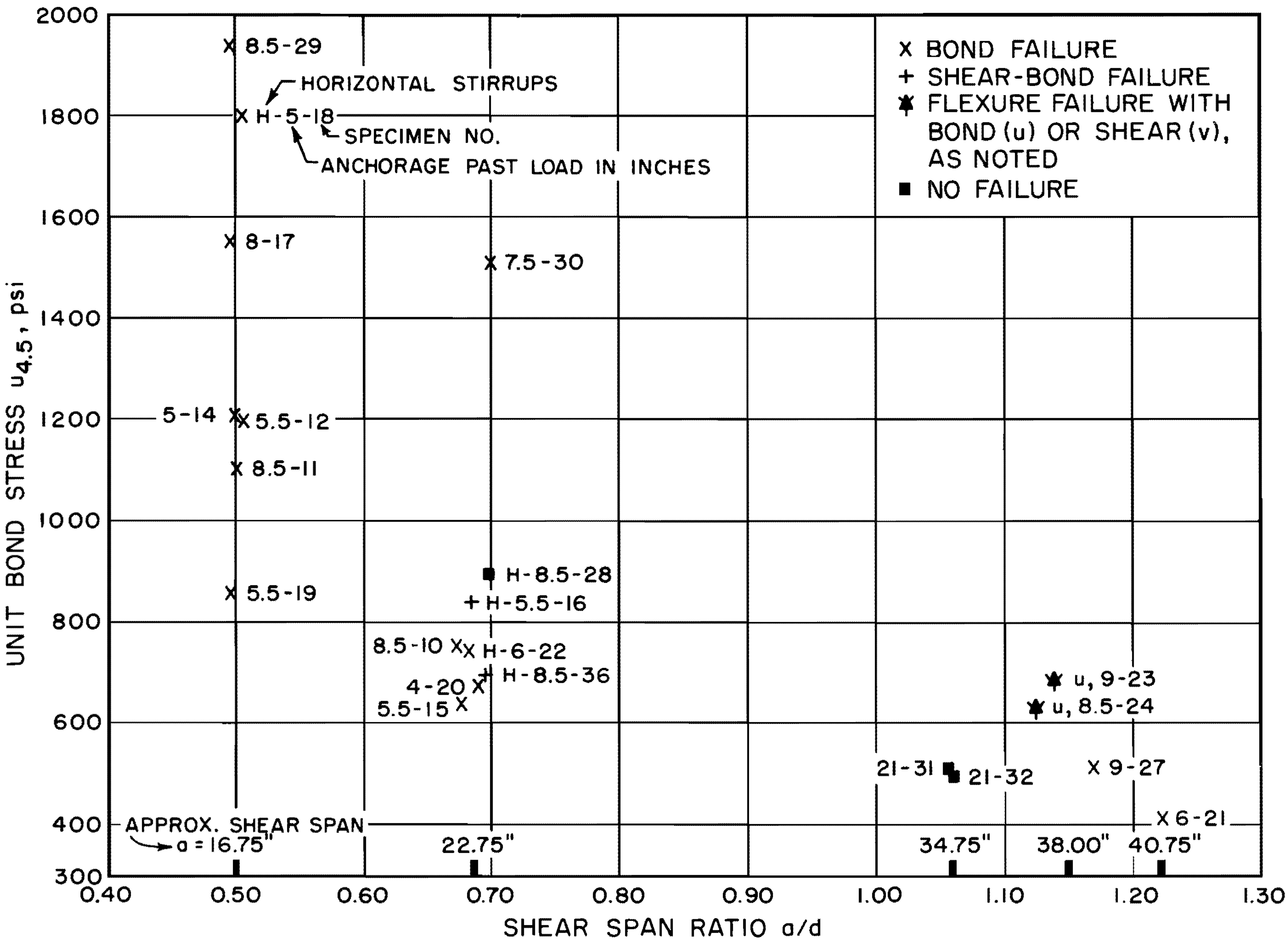

FIG. 9. BOND STRENGTH RELATED TO SHEAR SPAN.

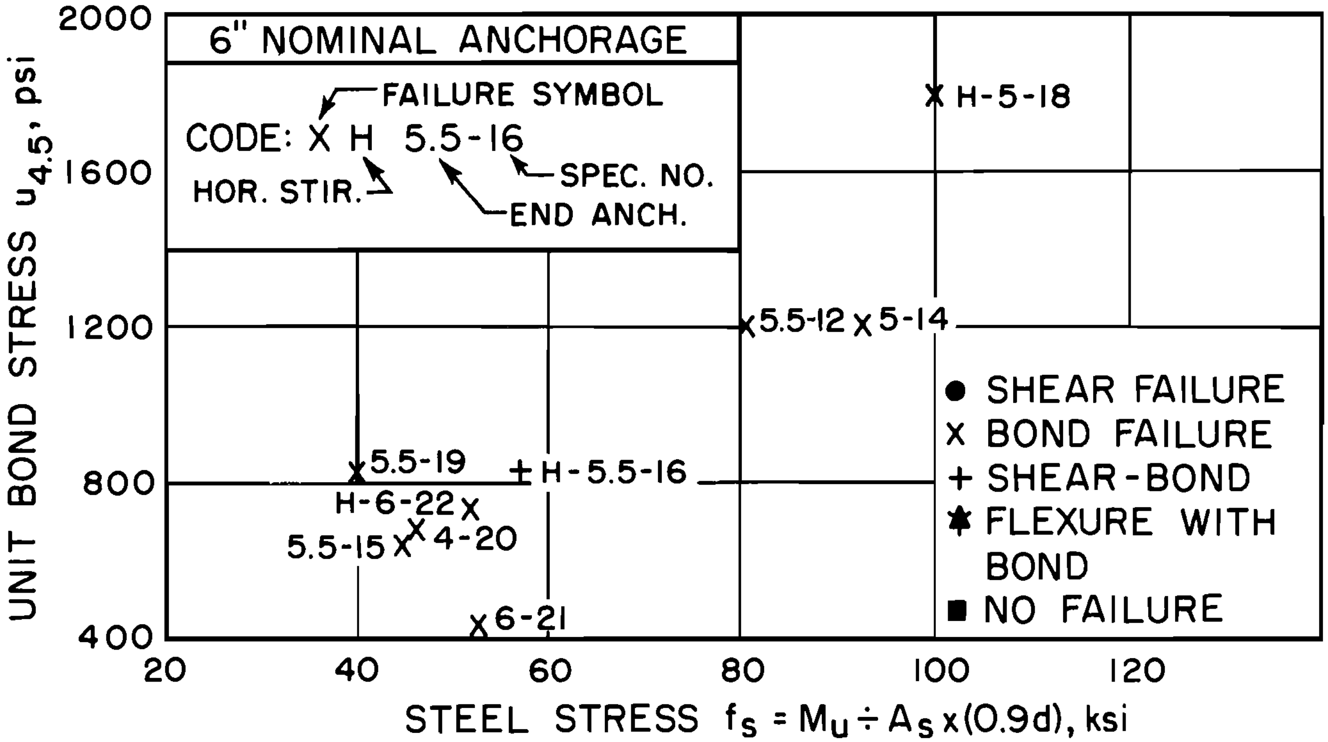

FIG. 10. VARIATION OF BOND STRENGTH WITH STEEL STRESS FOR 6" ANCHORAGE.

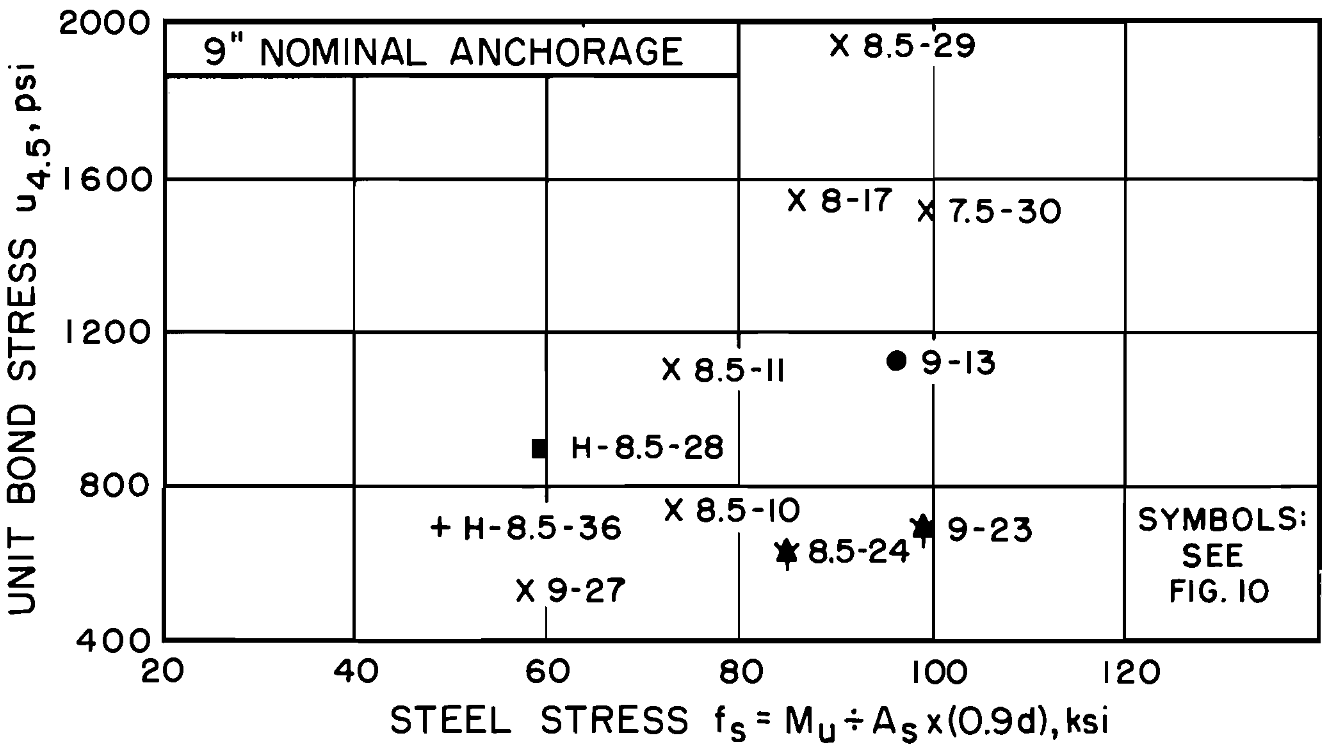

FIG. 11. VARIATION OF BOND STRENGTH WITH STEEL STRESS FOR 9" ANCHORAGE.

against ultimate f_s for all end anchorages of 6" or less; it shows much higher bond stresses for higher steel stresses. The values for nominal anchorages of 9" in Fig. 11 show a similar trend, although there are also two low specimens* in the 85-100 ksi steel stress range. These figures may be taken as further evidence that it is not really calculated bond stress which caused failure, but something else, such as end anchorage.

Examination of these bond values in excess of 1000 psi, eight specimens in all, showed that six represented A431 steel and two A432 steel. The A431 steels developed from 1.14 to 1.34 times their nominal yield stress before failing in bond stress. The A432 steels developed 1.13 to 1.24 times the measured yield point. Thus it could be said, even for these short end anchorages, that all these eight cases were primary flexural failures with a secondary failure in bond. The interpretation which is here preferred is rather that, even with fairly small end anchorages, the steel can develop to high values of strain (and high stress in high strength steels) before failing at the end anchorage. Thus a 12" or 15" end anchorage beyond the center of load could develop almost any commercial reinforcing steel and still not fail in bond.

The bond stress data are presented in another manner in Fig. 12 which plots the ultimate bond stress against the ratio of end anchorage to the shear span a. This shows high bond strengths attainable for an anchorage-arm ratio from 0.30 to 0.55, but there are also lower values included in the ratio range of 0.30 to 0.40. It is interesting to note that all bond values in excess of 1000 psi were in beams with calculated f_s values of 73 to 100 ksi.

The most serviceable conclusion that can be noted from the study of bond stress is that in only a few cases (and then for very high bond stresses) did splitting seem to work along the bars all the way to their end. In most cases the failure was a sudden one suggesting that the end of the anchorage slipped and permitted an entirely new diagonal crack to develop as evidence of this failure (Fig. 13). Empirically the data seem to indicate that, if end anchorage of as much as 12" or 15" is provided, one can forget bond stress completely in evaluating the strength of the cantilever. A number of specimens with a 9" end anchorage did not fail in bond, indicating that for the #8 and #11 bars used here this was about the breakpoint between bond and other types of failure.

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*These two showed rather minor evidence pointing toward the possibility of bond failure and possibly do not belong in this figure.

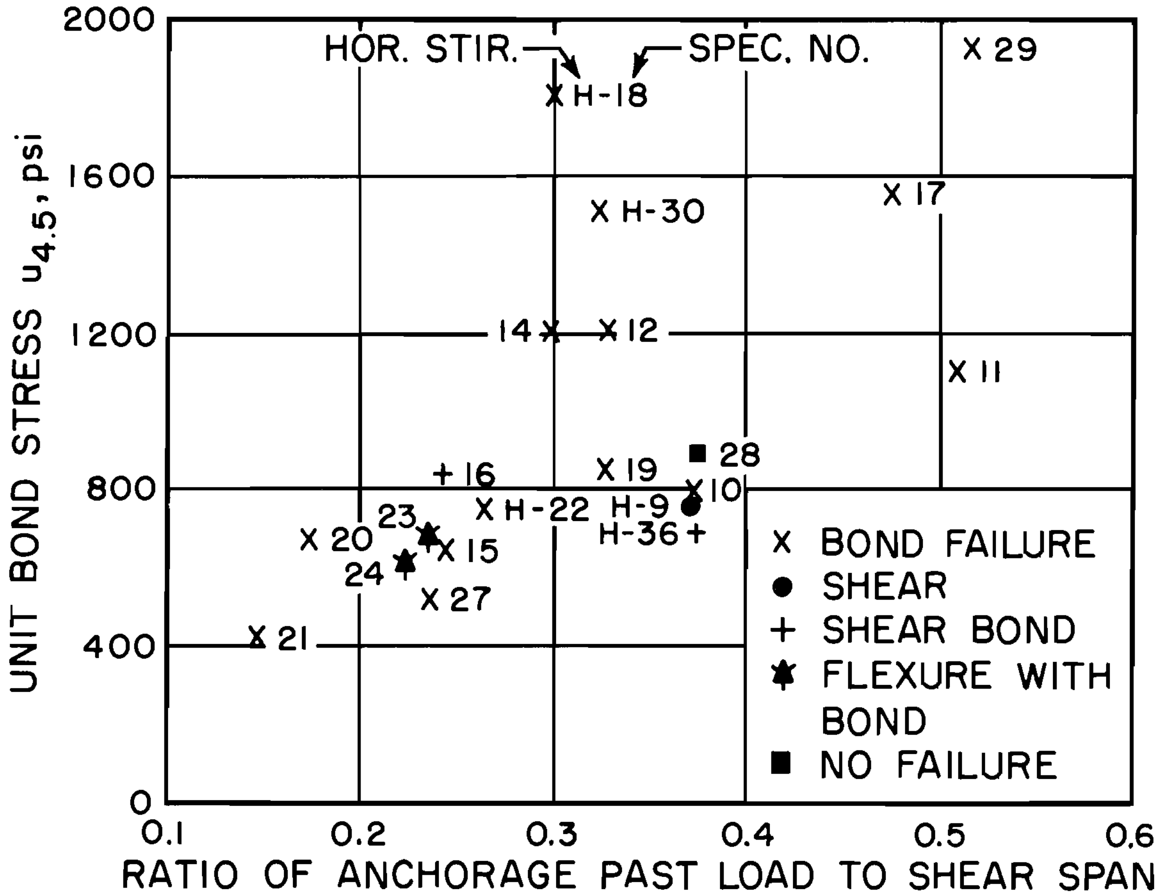

FIG. 12. VARIATION OF BOND STRESS WITH POSITION OF LOAD.

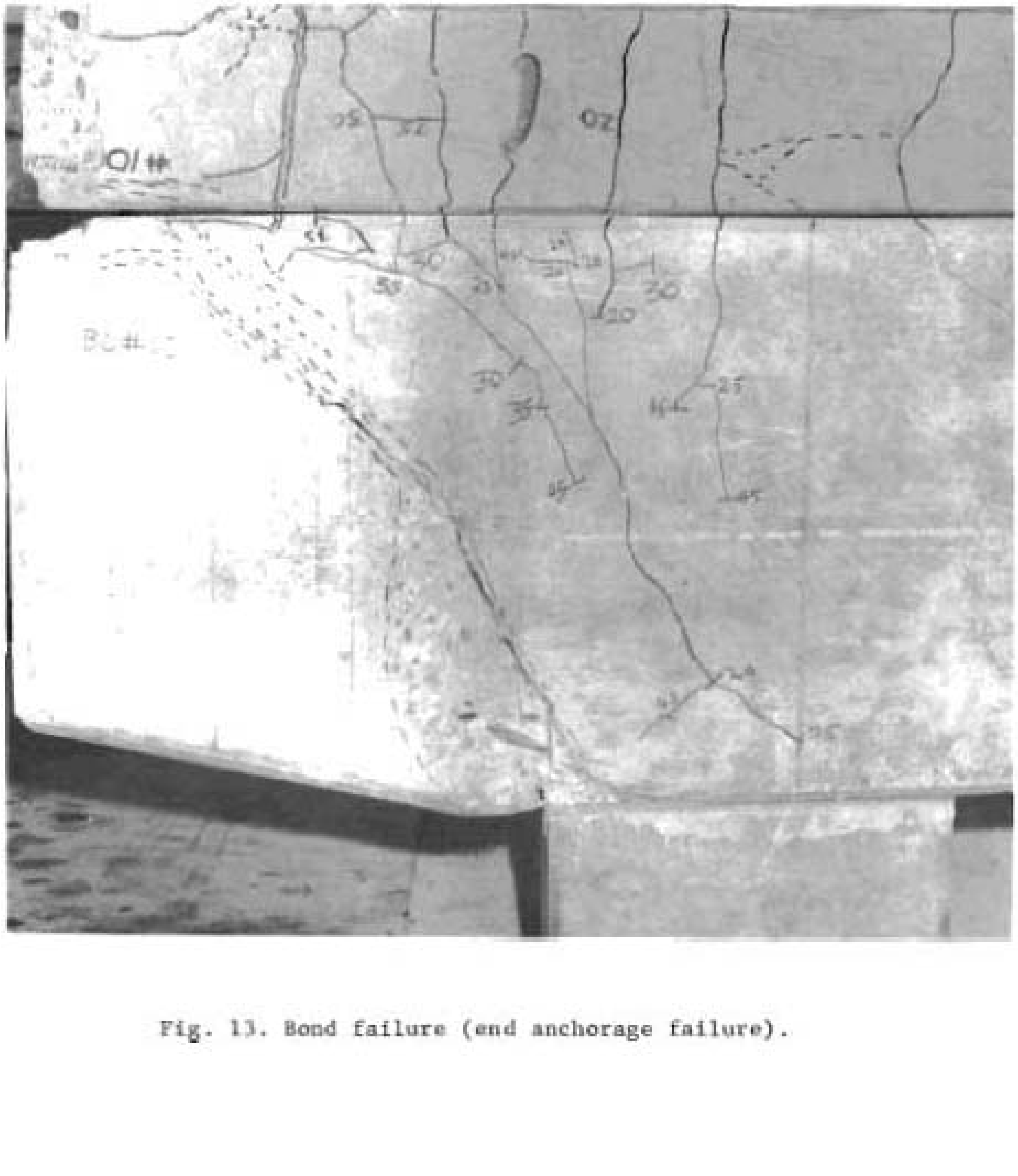

Fig. 13. Bond failure (end anchorage failure).

<u>Effect of Embedding Column Shaft in Cap</u>

Specimens 31 and 32 were made identical with the early full size specimens except that the precast round and square shafts were each extended 6 in. into the cap form before the cap was cast. It was thought that this type of specimen might show some sign of weakness or a reduced effective depth. These two tests were only partially effective. The concrete strength was some 8 percent greater than on the original specimens and the full capacity of the loading frame and jacks was not enough to bring the specimen to failure. However, crack widths of 0.5 mm and 0.6 mm were attained and it was estimated that flexure failure would have occurred with not over 5 or 10 percent more load. Although the earlier specimens had been marked as flexure failures, they were not actually carried to their secondary failure in compression.

Up to the 400 k load the specimens with the 6" embedment of the shafts performed quite similarly to those where the cap was cast just in contact with the top of the shaft. The shear $v_{4.5}$ sustained was 399 psi and 393 psi at a shear span of 34.75 in., which can be compared with a calculated v_u of 454 psi suggested in Fig. 7 as a minimum for design. The embedded specimens thus came within 12 percent of the expected shear strength and showed no evidence of impending shear distress. A complete check on shear strength of embedded specimens would require more specimens, including short shear spans, and the present equipment would not be adequate for this large a load. In other words, the length actually tested would normally lead to a flexure failure, as indicated for these specimens.

<u>Loads on Cap Between Shafts</u>

Two tests were made with the load inside the span between the shafts to see if behavior on a short shear span inside was similar to behavior on the overhanging end. The test setup and final loading is indicated in Fig. 14a. While the overhanging end is statically determined the loading between shafts is actually not determinate because of the uncertain reaction location within the column or shaft. The first specimen tested, Spec. 25-4-V-4, in retrospect, was not well planned. In an attempt to provide a large negative moment over the support, the load assigned to the outside end was a little too large and it caused the cap to act almost as a balanced cantilever, which was intended. However, the plan failed to take into account the fact that the large negative moment between loads would produce curvature sufficient to cause tilt over the support and a very off-centered reaction. By appearance the reaction was confined totally within the length FG (Fig. 14a) and there was about 6.5 in. of

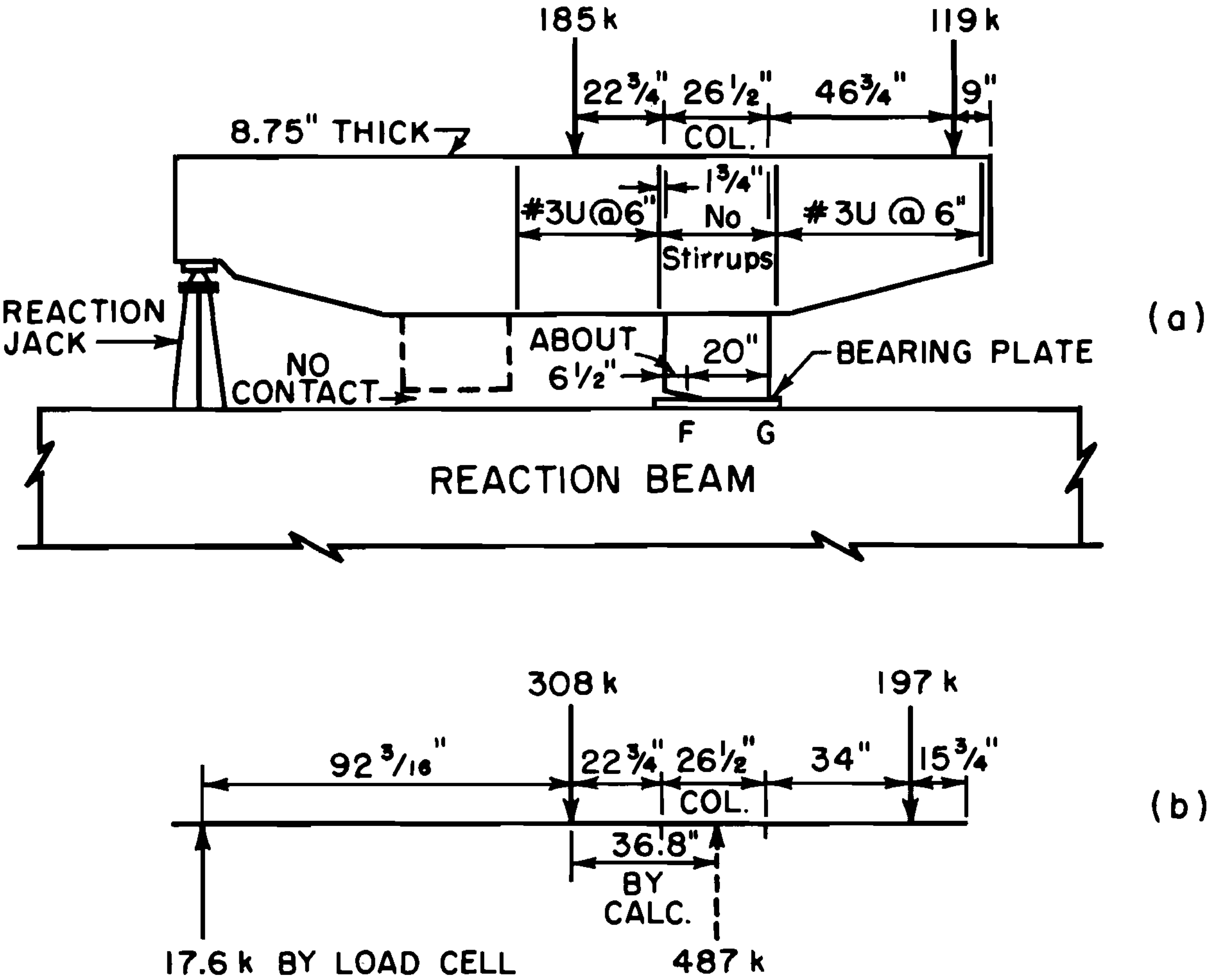

FIG. 14. TEST WITH INTERIOR LOAD. (a) SPECIMEN 25-4-V-4. (b) SPECIMEN 34-3-V-4.

the shaft not in contact with the reaction beam. How much load went into the reaction jack is indeterminate but it seemed from the cap behavior that it was almost negligibly small. Accordingly the shear (or moment) carried is only an estimate. The most serious crack opened near the middle of the support (which reflected the displaced reaction) and the failure was in flexure at the outside face. There was no distress in shear in the inside span and it was assumed to carry the entire 185k load, probably slightly in excess of its real shear.

The test was repeated with modifications in Spec. 34-3-V-4. A load cell was introduced at the far reaction (which made the system statically determinate) and the cantilever load was placed nearer the reaction shaft. This case is shown schematically in Fig. 14b and the loads gave a calculated location of the main reaction just 0.8 in. outside the center of the 26.5 in. reaction shaft. The failure was in flexure at the cantilever face of the support. The test was stopped after the flexure crack was 0.16 in. wide, prior to the secondary compression failure.

This specimen carried a shear $v_{4.5}$ inside the support of 724 psi at an a/d ratio of 0.70, which compares to the 520 psi by the strength equation established earlier in this report.

Crack Widths in Flexure

In the use of steels stronger than intermediate grade, the crack width at service load becomes important. Crack widths were carefully measured and recorded, but must be interpreted somewhat cautiously. Crack width always varies from point to point. Occasionally a crack splits into two branches and occasionally it is joined by a meandering neighboring crack. To add to these problems, although measuring devices are subdivided to 0.2 mm and distances can easily be estimated to 0.04 mm (0.0016 in.), the cracks have ragged or jagged edges to such an extent it seemed useless to record cracks this closely. Records were made in terms of "small" (less than 0.05mm), 0.1 mm, 0.2 mm, etc. Crack widths were recorded at points where a crack crossed one of the grid lines, which were 6 by 6 in. on the sides and top for the 30-in. width, and 6 by 4 in. centered on the top of the 12.5 and 8.5 in. widths.

In Table 3 in the Appendix the steel stress calculated for the loading which first produced a surface crack width of 0.2 mm (0.008 in.) at any one point is tabulated for each specimen, grouped by grade of steel, size of bar, and the presence of horizontal web steel. This crack width is approximately the maximum that would be universally acceptable for construction not protected from the weather. Since some might accept a wider crack width the same table shows comparable data for a crack width of 0.4 mm and also the steel stress at initial cracking. The average values of stress are plotted separately for each classification in Fig. 15 and the curves for #11 bars are grouped together in Fig. 16. The higher a curve the more favorable it is, because it means the member sustained a higher f_s before developing a crack of the width plotted. Average concrete strengths are marked for A431 and A432 steel specimens. For the intermediate grade steel the strengths averaged nearly the same, 5030 psi for the specimens with horizontal web steel, 4750 psi for the others.

It was somewhat of a surprise to find that the A431 steel sustained a higher stress before reaching each crack width. It is easy to see that the calculated stress at initial cracking should be higher for this steel. The cracking moment itself is influenced only slightly by the area of steel present. At cracking the total tension originally carried in the concrete (about the same for all steel grades) must move into the steel. Thus the smaller the steel area, the higher will be the calculated steel stress at the cracking load. However, it is not evident why A431 #11 bars stressed to a nominal 40 ksi produced a crack width (at 0.008 in.) no greater than intermediate grade #11 bars developed at 26 ksi. At the 0.008 in. crack width only one intermediate bar specimen had reached a calculated stress as high as 37.6 ksi and no A431 bar showed less than 37.4 ksi.

In terms of design it is probable that average crack width should be considered. In these data the widest measured point on each specimen is reported and the average reported is an average including only these worst points. Hence it is a little severe to use these data. Nevertheless, on this basis Fig. 15 indicates the following unit stresses would give reasonable crack widths with #11 bars for a service load equal to half the ultimate design load:

	#11 bars	#8 bars
Intermediate grade	$f_s = 25$ ksi $= 0.62f_y$	------
A432 grade	$f_s = 26$ ksi $= 0.43f_y$	29 ksi $= 0.48f_y$
A431 grade	$f_s = 40$ ksi $= 0.53f_y$	40 ksi $= 0.53f_y$

In general these data support the use of a working stress design based on half the minimum yield point, although this is somewhat high for the A432 data (especially for #11 bars) and somewhat conservative for the A431 grade. With intermediate grade steel, flexural crack width is apparently no problem.

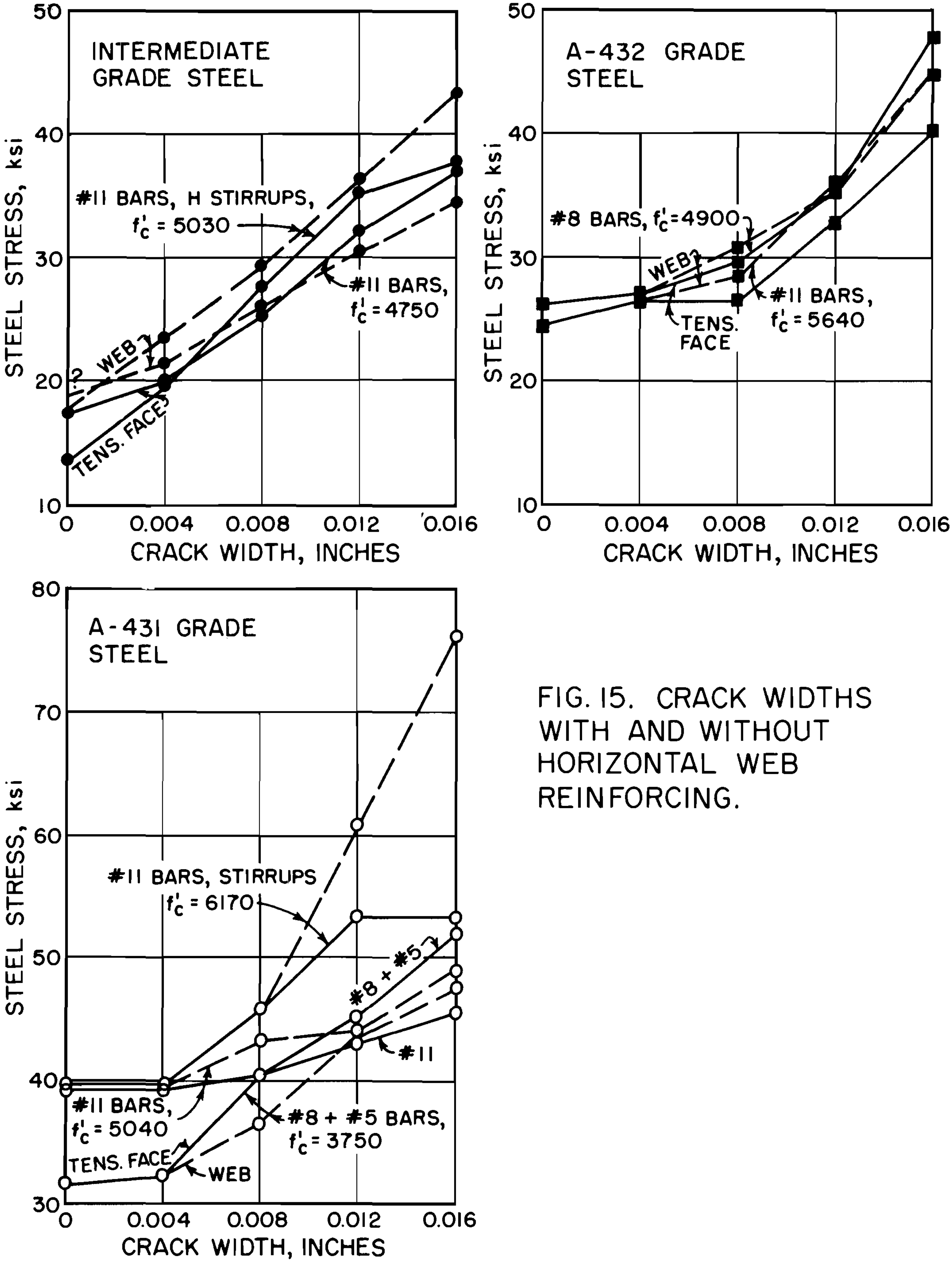

FIG. 15. CRACK WIDTHS WITH AND WITHOUT HORIZONTAL WEB REINFORCING.

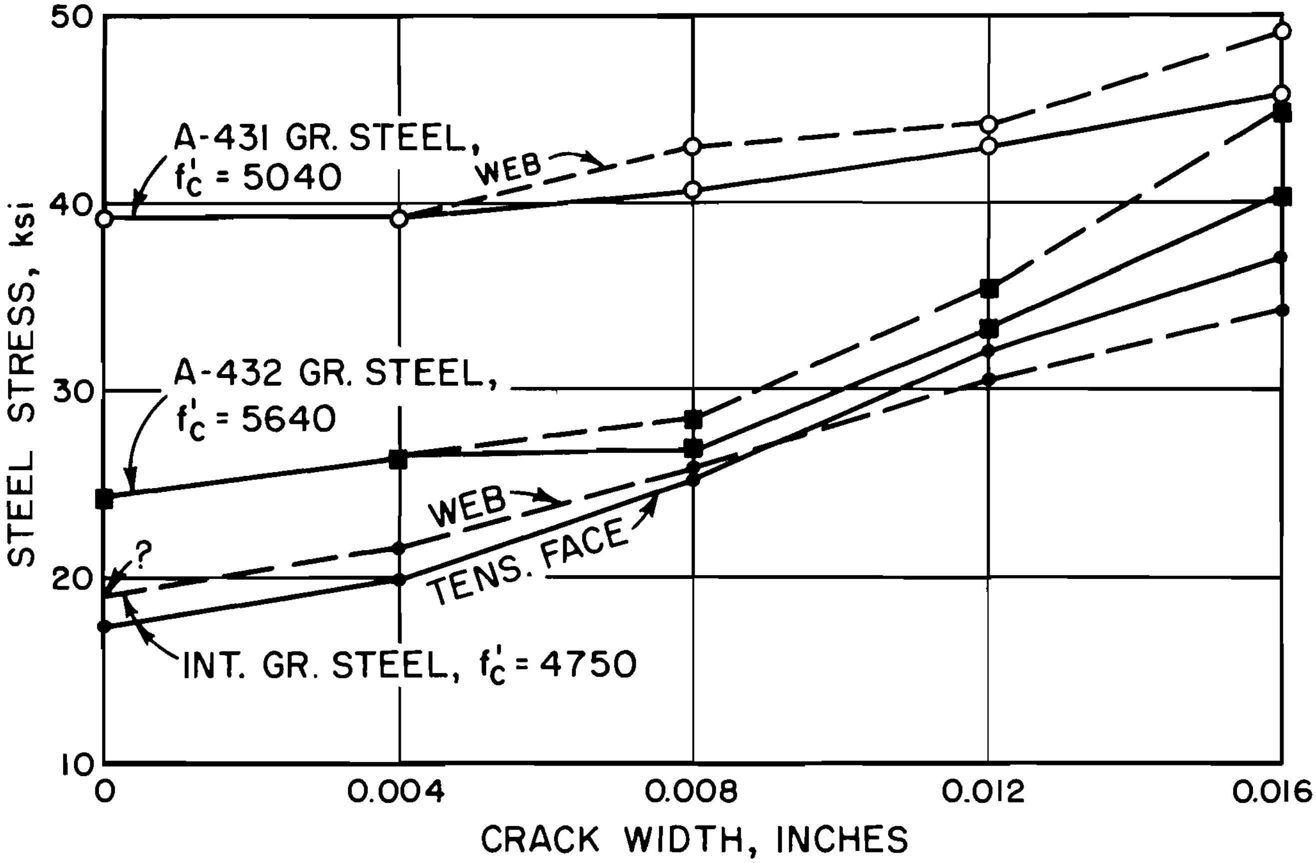

FIG. 16. CRACK WIDTHS FOR VARIOUS GRADES OF STEEL, #11 BARS.

Crack Widths on Sides of Cap

In advanced cases of shear distress it is commonplace to observe in laboratory research diagonal cracks which are wider than flexural cracks, especially on small shear spans. However, it was noticeable in these deep beams that side crack width near middepth was from the beginning nearly as wide as the cracks on the tension face, and this is not commonly observed. This was in spite of the standard use of the equivalent of one horizontal #5 bar on each face at midheight (for a 30" width specimen). This excessive side cracking suggested the need for more horizontal side steel.

Five specimens were further reinforced with horizontal side bars and these did show more favorably, that is, higher calculated steel stresses were reached in these specimens before developing any given crack width. The increase in nominal steel stress averaged upwards from 16% and was greater when #5 bars were used than for #3 bars. Part of the increase was purely nominal; part was the effect of a higher f_c' (for A431 steel); but part must represent the effect of better directed reinforcement. The calculated steel stress was based solely on the nominal area of steel, ignoring the horizontal web steel. While actual steel stresses would be lower than these calculated values, a more exact calculation would account for not over half of the benefit observed.

DESIGN CONCLUSIONS

Behavior of Present Designs

The bent cap from which the tests were modeled behaved excellently in the tests, its only deficiency being in wider side cracks than were expected. Even these side cracks were not wider than commonly accepted on the tension face in flexure.

Flexure

In flexure the use of ordinary beam theory, without any correction for variable depth, gave f_s values which seemed to be in good agreement with test results. The moment was calculated from the face of a square support having the same area as the circular column. It appears that no modifications in ordinary flexural theory (for uniform depth members) are needed for these short span cantilevers so long as they are designed to fail in tension rather than compression. This should be a normal procedure since ultimate strength design theory shows that a beam balanced by working stress methods will always fail in tension. Furthermore, beams designed for compression failure are expensive and are undesirable because of their sudden failure mode.

Beams designed in flexure with A432 steel (minimum f_y of 60 ksi) would be more economical than beams using intermediate grade steel. While crack widths with such steel may not make a 30 ksi design stress (working stress) desirable, certainly 24 ksi or 26 ksi appears entirely feasible. The upper boundary of usable stress might be raised further by additional studies with this grade of steel.

Shear Strength

Within a shear span-depth ratio of 0.5 to 1.2, the ultimate shear strength may be conservatively evaluated much higher than used in the past, for f_c' of 4500 psi as

$$v_u = V/bd = 320 + 140\ d/a$$

For working stress design with a factor of safety, say, of 2.25 this becomes

$$v = V/bd = 142 + 62.5\ d/a \text{ for } f_c' = 4500 \text{ psi}$$

$$v = V/bd = 116 + 51\ d/a \text{ for } f_c' = 3000 \text{ psi}$$

The latter assumes that v varies as the $\sqrt{f_c'}$, as is now commonly accepted. These values compare with AASHO allowable values of 90 psi for V/(bjd), or 103 psi for V/bd, for either grade of concrete without stirrups, as plotted in Fig. 17.

The more recent Joint ACI-ASCE Committee recommendation was more conservative, for $d/a \approx 1$ an ultimate shear of

$$v_u = 1.9\sqrt{f_c'} + 2500\ p^*$$

For f_c' = 4500 psi and 6-#11 bars in a 30" width (p = 0.00945) this becomes

$$v_u = 1.9\sqrt{4500} + 2500 \times 0.00945 = 127 + 24 = 151 \text{ psi}$$

With the same 2.25 factor used above this reduces to 67 psi for f_c' of 4500 psi and 55 psi for f_c' of 3000 psi, substantially lower than the AASHO allowable values. However, the Joint Committee was not primarily interested in these small a/d ratios while these recommendations are only for $0.5 < a/d < 1.2$.

The recommendation from these tests would thus substantially raise the presently specified shear capacity without stirrups for $a/d < 1.2$, as Fig. 17 shows. The test data for loads inside the column indicate the same high capacity, v reaching 610 psi and 742 psi with failures in flexure, for a/d = 0.69 and

*The last term is $2500\ p\frac{Vd}{M}$ for $a/d > 1$.

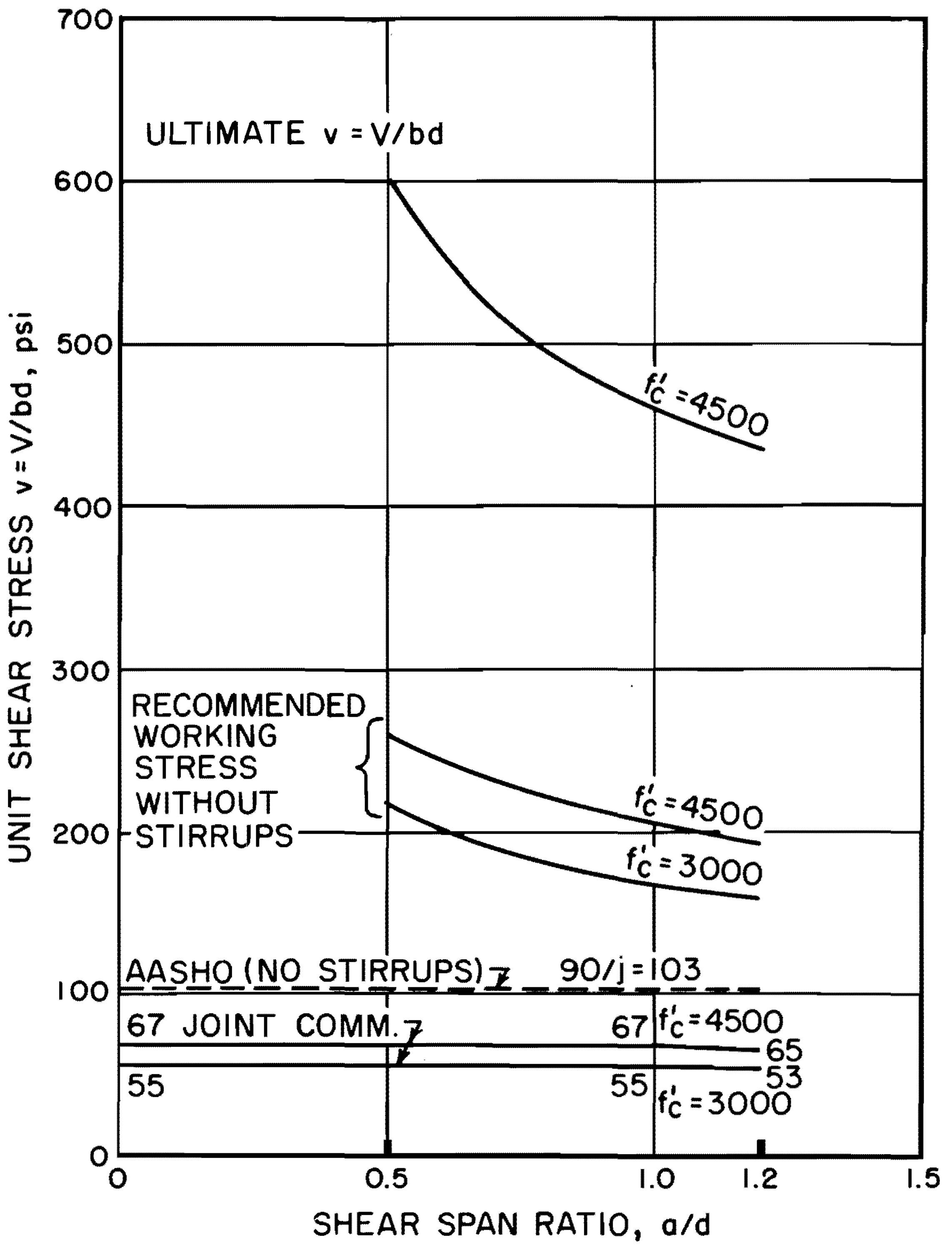

FIG. 17. RECOMMENDED SHEAR STRESS COMPARISON.

$f_c' = 4500$ psi. Two tests constitute meager data but there is also no apparent factor to suggest that behavior should be poorer on one side of the column than the other. In neither case should any bars be cut off within the shear span. Cutting off bars complicates behavior and was not investigated; it is presumed to have an adverse effect. In neither case can these shear values be recommended without end anchorage beyond the load, as later discussed.

Stirrups

It is noted that the above comparisons have been made without stirrups. Vertical stirrups did little good insofar as the test observations showed. Specimens without any stirrups were on the whole as good as those with vertical stirrups (which ordinary working stress theory would value at 67 psi). Horizontal web reinforcing with one closed end did reduce crack width and raise the strength in shear. The data here are inadequate to do more than support a recommendation that if web reinforcement is used, as probably is desirable, the same metal placed horizontally over the upper 60 percent of the effective depth will serve a more useful purpose than vertical stirrups. The greatly increased shear values recommended make web reinforcement much less necessary.

Bond Strength

The tests indicate that bond stress between load and support constitutes no problem for these small a/d ratios, even with the small perimeter furnished by high strength steel. On the other hand end anchorage beyond the center of load is essential. All specimens with a 6" end anchorage failed in the anchorage in bond. Since no bond failure occurred with a 15" end anchorage and only about half of the 9" end anchorages showed distress even at high f_s values, it appears amply safe to specify a 15" end anchorage for #11 bars and a 12" end anchorage for #8 bars. This assumes that no bar is cut off within the shear span and that all bars extend for the stated end anchorage. On long interior spans the moment diagram could require some bars to be extended further than the end anchorage minimum.

If the above conditions, that is, both small a/d and adequate end anchorage are met, it appears that no limit needs to be set on allowable u calculated from $u = V/(\Sigma o\ jd)$. The calculation is not an indication of whether the member is safe or unsafe; safety depends on the end anchorage.

This end anchorage distance could probably be reduced by a cross bar welded across the end or some other type of end anchor. Such devices should be tested before their use is recommended. The Portland Cement Association has had good success with the welded cross bar in their research.

APPENDIX

TABLE 1 SPECIMEN PROPERTIES

Spec. Code	b in.	d in.	Clear cov., in.	f_c' psi	f_y ksi	A_s	Arm A, in.	Bar ext. B, in.	Stirrups
Flexure failures - full width specimens									
1-1-V-4	30.1	31.6	3.69	4200	45.4	6-#11	34.75	21	#5@6"
2-1-V-4	30.1	32.0	3.35	4340	45.4	6-#11	40.75	15	#5@6"
3-1-V-4	30.2	32.7	2.74	4470	46.4	6-#11	34.75	21	#5@6"
4-2-V-4	30.1	32.5	2.98	4470	46.4	6-#11	34.75	21	#5@6"
5-1-V-6	30.2	32.7	2.99	5520	66.5	4-#11	40.75	15	#5@6"
6-2-V-6	30.2	32.6	2.93	5750	66.5	4-#11	40.75	15	#5@6"
Flexure failures - narrow specimens									
7-3-V-6	12.25	33.3	2.65	4700	64.9	3-#8	40.75	7	#3@4¼"
Flexure failures - narrow specimens, interior load									
25-4-V-4	8.75	32.9	2.59	5000	45.7	2-#11	22.75	--	#3@6"
34-3-V-4	12.62	32.7	2.82	4250	42.5	3-#11	22.75	--	#3@4¼"
Combined flexure + shear - narrow specimens									
35-3-N-4	12.25	32.4	3.17	5050	39.8	3-#11	28.25	8.7	None
Combined flexure + bond - narrow specimens									
23-3-V-8	12.62	33.8	2.28	3290	75.4	2-#8,1-#5	38.50	9	#3@4¼"
24-3-N-8	12.44	33.9	2.19	2860	75.4	2-#8,1-#5	38.00	8.5	None
Flexure failure approached (loading stopped) - 6" embedment									
31-1-V-4	30.25	33.0	2.83	4820	43.8	6-#11	34.75	21	#5@6"
32-2-V-4	30.25	32.8	2.54	4820	43.8	6-#11	34.75	21	#5@6"
Shear failures - narrow specimens									
9-3-N-6	12.87	33.5	2.45	5330	64.9	3-#8	22.75	14	None
8-3-V-6	13.25	33.4	2.51	4980	64.9	3-#8	22.75	15	#3@4¼"
13-3-V-8	12.94	33.8	2.55	4470	75.4 47.8	2-#8 1-#5	16.75	9	#3@4¼"
26-6-V-4	8.62	33.8	2.03	5000	45.7	2-#11	28.0	27.75	#3@6"
33-5-V-4	12.75	33.2	2.59	4320	42.5	3-#11	34.0	15.75	#3@4¼"
Combined shear + bond - narrow specimens*									
16-3-HI-4	12.50	33.2	2.29	5100	47.1	3-#11	22.75	5.5	3-#3ea.fa.
36-3-H2-4	12.31	32.7	3.07	5370	39.8	3-#11	22.75	8.5	3-#3ea.fa.
Bond failures - narrow specimens**									
12-3-V-6	14.50	33.2	2.55	4540	64.9	3-#8	16.75	5.5	#3@4¼"
14-3-V-8	12.88	33.6	2.50	4390	75.4 47.8	2-#8 1-#5	16.75	5	#3@4¼"
15-3-N-4	12.37	33.6	2.40	5100	47.1	3-#11	22.75	5.5	None
18-3-HI-8	12.62	33.3	2.44	6170	75.8	1-#11	16.75	5	3-#3ea.fa.

*See also #35 under combined flexure and shear.
**See also #23, 24 under combined flexure and bond.

Spec. Code	b in.	d in.	Clear cov., in.	f_c' psi	f_y ksi	A_s	Arm a, in.	Bar ext. B, in.	Stirrups
19-3-V-4	12.56	33.9	1.98	4340	45.0	3-#11	16.75	5.5	#3@4¼"
20-3-V-4	12.50	33.2	2.43	4800	45.0	3-#11	22.87	4	#3@4¼"
21-3-V-4	12.38	33.5	2.14	5330	46.0	3-#11	40.94	6	#3@4¼"
22-3-H4-4	12.50	33.3	2.49	5160	46.0	3-#11	22.75	6	3-#5ea.fa.
10-3-N-6	12.62	33.8	2.14	5330	64.9	3-#8	22.75	8.5	None
11-3-V-6	13.25	33.5	2.50	4540	64.9	3-#8	16.75	8.5	#3@4¼"
17-3-N-8	12.50	33.9	2.00	6170	75.8	1-#11	16.75	8	None
27-3-N-4	14.00	32.5	3.02	4820	41.0	3-#11	38.0	9	None
29-3-N-8	11.25	33.4	2.38	4480	75.8	1-#11	16.5	8.5	None
30-3-N-8	12.12	33.3	2.25	4480	75.8	1-#11	23.25	7.5	None
Loading stopped short of failure+									
28-3-H5-4	12.62	32.4	3.15	4820	41.0	3-#11	22.62	8.5	3-#5ea.fa.

+See also #31, 32 under flexure failure approached (loading stopped).

TABLE 2 ULTIMATE LOADS AND CALCULATED STRESSES

Spec. Code	f_c'	f_y	Bar ext. B	P_u	M_u	$f_s = \frac{M_u}{A_s(.9)d}$	$v = \frac{V}{bd}$	$v_{4.5}$	$u = \frac{V}{\Sigma o(.9)d}$	$u_{4.5}$
	psi	ksi	in.	kips	k-in.	ksi	psi	psi	psi	psi
Flexure failures - full width specimens										
1-1-V-4	4200	45.4	21	399	13,870	52.1	420	435	528	547
2-1-V-4	4340	45.4	15	340	13,860	51.4	353	360	444	453
3-1-V-4	4470	46.4	21	402	13,970	50.7	407	408	514	515
4-2-V-4	4470	46.4	21	391	13,590	49.6	400	401	503	504
5-1-V-6	5520	66.5	15	370	15,080	82.1	375	338	709	639
6-2-V-6	5750	66.5	15	393	16,010	87.5	399	353	756	668
Flexure failures - narrow specimens										
7-3-V-6	4700	64.9	7	165	6,730	94.7	405	396	584	571
Flexure failures - narrow specimens, interior load										
25-4-V-4	5000	45.7	--	185	5,560	60.2	643	610	705	668
34-3-V-4	4250	42.5	--	290	5,080	36.9	703	724	742	765
Combined flexure + shear - narrow specimens										
35-3-N-4	5050	39.8	8.7	227	6,420	47.0	572	540	585	552
Combined flexure + bond - narrow specimens										
23-3-V-8	3290	75.4	9	147	5,640	98.2	343	401	584	683
24-3-N-8	2860	75.4	8.5	127	4,840	83.9	302	379	505	634
Flexure failure approached (loading stopped) - 6" embedment										
31-1-V-4	4820	43.8	21	412	14,320	51.5	413	417	522	527
32-2-V-4	4820	43.8	21	404	14,040	50.8	407	377	515	477
Shear failures - narrow specimens										
9-3-N-6	5330	64.9	14	246	5,600	78.4	570	524	866	796
8-3-V-6	4980	64.9	15	254	5,780	81.1	574	545	897	852
13-3-V-8	4470	75.4 47.8***	9	280	4,690	86.4	640	642	1118	1120
26-6-V-4	5000	45.7	27.75	153	4,280	45.1	525	498	568	538
33-5-V-4	4320	42.5	15.75	200	6,780	48.5	471	481	503	513
Combined shear + bond - narrow specimens*										
16-3-HI-4	5100	47.1	5.5	356	8,100	57.9	858	774	898	810
36-3-H2-4	5370	39.8	8.5	296	6,750	49.0	735	673	757	693
Bond failures - narrow specimens**										
12-3-V-6	4540	64.9	5.5	340	5,700	80.5	706	703	1207	1201
14-3-V-8	4390	75.4 47.8***	5	298	4,990	93.4	689	697	1195	1210
15-3-N-4	5100	47.1	5.5	276	6,280	44.4	664	651	687	673

*See also #35 under combined flexure and shear.

**See also #23, 24 under combined flexure and bond.

***Through error the #5 bar of Specimens 13 and 14 was of intermediate grade.

TABLE 2 (con't)

Spec. Code	f_c'	f_y	Bar ext. B	P_u	M_u	$f_s = \frac{M_u}{A_s(.9)d}$	$v = \frac{V}{bd}$	$v_{4.5}$	$u = \frac{V}{\Sigma o(.9)d}$	$u_{4.5}$
	psi	ksi	in.	kips	k-in.	ksi	psi	psi	psi	psi
18-3-HI-8	6170	75.8	5	280	4,700	100.5	665	567	2110	1800
19-3-V-4	4340	45.0	5.5	340	5,700	40.1	799	813	839	854
20-3-V-4	4800	45.0	4	276	6,310	45.1	665	644	695	673
21-3-V-4	5330	46.0	6	183	7,490	53.1	442	406	457	420
22-3-H4-4	5160	46.0	6	318	7,250	51.7	764	713	799	745
10-3-N-6	5330	64.9	8.5	234	5,320	73.8	548	504	817	751
11-3-V-6	4540	64.9	8.5	314	5,260	73.6	708	705	1105	1100
17-3-N-8	6170	75.8	8	246	4,130	86.8	580	495	1818	1550
27-3-N-4	4820	41.0	9	210	7,980	58.3	462	446	540	521
29-3-N-8	4480	75.8	8.5	258	4,260	90.8	686	687	1936	1939
30-3-N-8	4480	75.8	7.5	200	4,650	99.5	495	496	1508	1510
Loading stopped short of failure+										
28-3-H5-4	4820	41.0	8.5	357	8,080	59.2	873	843	921	890

+See also #31, 32 under flexure failure approached (loading stopped).

TABLE 3 CALCULATED STEEL STRESSES (ksi)
AT VARIOUS CRACK WIDTHS

Spec. Code	Tension face crack			Web cracks		
	Initial crack	0.008"	0.016"	0.008"	0.012"	0.016"
Intermediate grade steel						
#11 bars						
1-1-V-4	19.4	22.2	30.5	27.7	30.5	33.3
2-1-V-4	19.3	22.5	35.3	22.5	28.9	35.3
3-1-V-4	18.7	29.4	40.2	24.1	29.4	34.8
4-2-V-4	18.9	24.3	37.8	29.7	37.8	37.8
15-3-N-4	13.7	20.5	----	20.5	27.3	30.7
19-3-V-4	10.5	22.6	----	22.6	22.6	27.6
20-3-V-4	17.3	29.5	39.9	20.8	22.5	24.3
21-3-V-4	16.1	26.0	----	26.0	40.8	50.6
26-6-V-4	16.3	37.6	----	31.3	37.6	37.6
27-3-N-4	18.3	23.6	30.6	28.3	28.3	33.0
31-1-V-4	18.6	26.6	42.5	26.6	31.9	37.2
32-2-V-4	21.4	24.1	42.8	32.1	32.1	33.5
33-5-V-4	20.6	25.8	41.3	25.8	36.1	36.1
35-3-N-4	13.2	22.0	28.6	22.0	22.0	26.4
Ave. f_s	17.3	25.5	37.0	25.7	30.6	34.2
#11 with H-stirrups						
16-3-HI-4	15.2	24.1	----	20.7	31.0	37.9
22-3-H4-4	17.3	31.0	41.4	41.4	44.8	48.3
28-3-H5-4	12.7	26.1	33.1	26.1	36.6	43.7
36-3-H2-4	10.5	28.1	38.6	28.1	31.6	35.1
Ave. f_s	13.9	27.3	37.7	29.1	36.0	41.3
A432 grade steel						
#8 bars						
7-3-V-6	26.7	30.4	42.5	30.4	36.4	48.5
8-3-V-6	25.7	35.2	51.4	29.8	40.6	55.5
9-3-N-6	28.4	33.8	51.4	33.8	37.8	51.4
10-3-N-6	26.9	26.9	60.5	33.6	33.6	33.6
11-3-V-6	21.9	24.9	49.8	24.9	29.9	44.8
12-3-V-6	27.2	27.2	32.2	32.2	32.2	37.2
Ave. f_s	26.1	29.7	48.0	30.8	35.1	45.2

Spec. Code	Tension face crack			Web cracks		
	Initial crack	0.008"	0.016"	0.008"	0.012"	0.016"
#11 bars						
5-1-V-6	24.6	24.6	42.5	28.3	33.1	47.3
6-2-V-6	24.6	28.4	37.8	28.4	37.8	42.5
Ave. f_s	24.6	26.5	40.2	28.4	35.4	44.9
A431 grade steel						
#8 + #5 bars						
13-3-V-8	26.2	32.7	50.0	32.7	45.8	50.0
14-3-V-8	30.0	40.0	50.3	40.0	46.7	46.7
23-3-V-8	37.0	48.4	62.6	37.0	42.7	48.4
24-3-N-8	32.2	39.2	44.7	35.6	39.2	44.7
Ave. f_s	31.4	40.1	51.9	36.3	43.6	47.5
#11 bars						
17-3-N-8	33.7	37.4	52.4	44.9	44.9	56.2
29-3-N-8	41.7	41.7	41.7	41.7	44.7	47.7
30-3-N-8	42.4	42.4	42.4	42.4	42.4	42.4
Ave. f_s	39.3	40.5	45.5	43.0	44.0	48.8
#11 with H-stirrups						
18	39.6	45.7	53.3	45.7	60.9	76.2

CONCRETE MIX

Quantities per cubic yard

High-early strength cement	5 sacks
Puzzolith	5 quarts
Darex (air entraining)	2 oz.
Coarse aggregate, gravel	2130 lbs.
Fine aggregate	1150 lbs.
Water	21.4 gal.

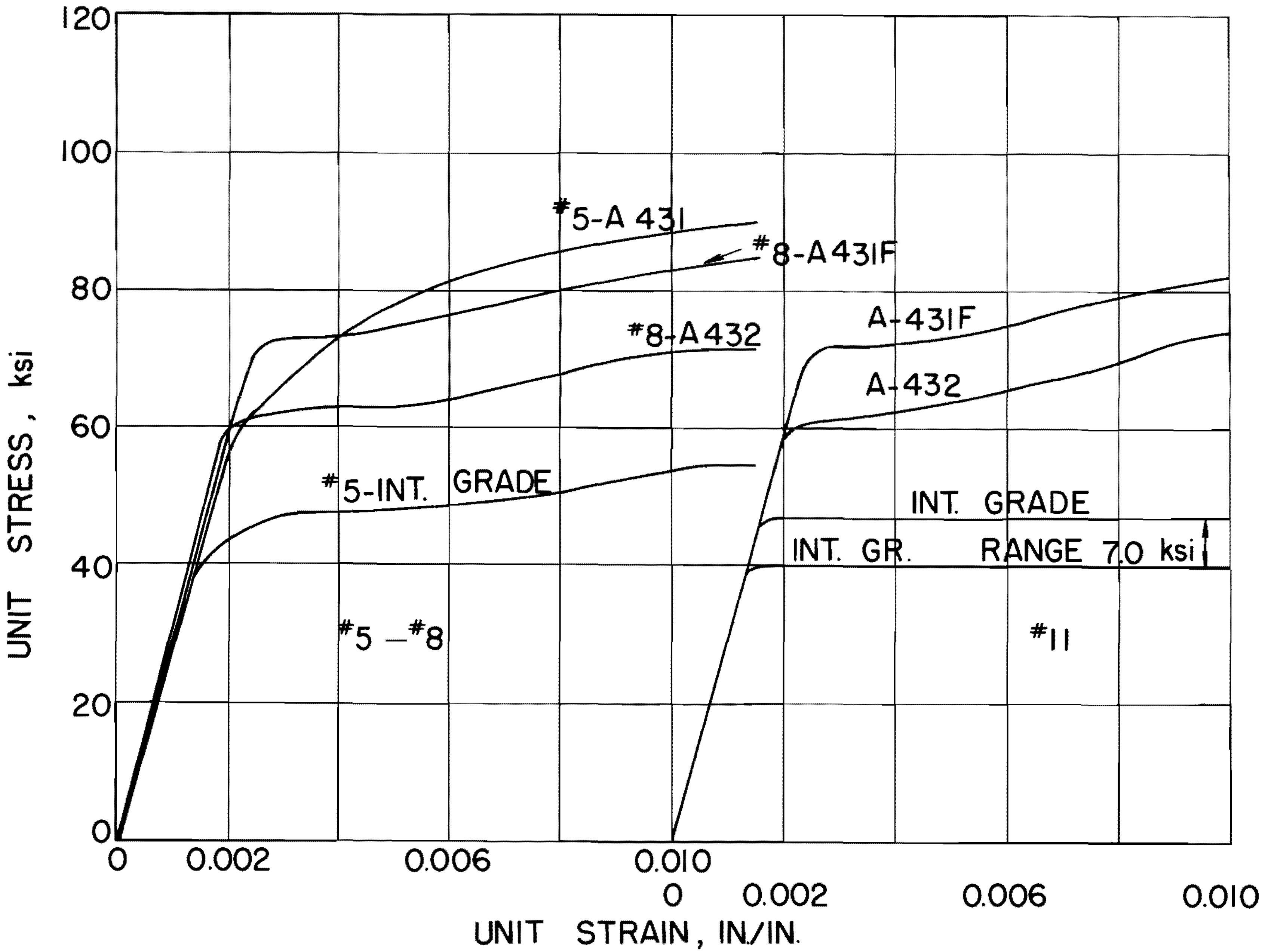

FIGURE 18. REINFORCING STEELS USED