THE STATISTICAL APPROACH TO QUALITY CONTROL IN HIGHWAY CONSTRUCTION



RESEARCH • GUIDES

The Statistical Quality Control Task Force Office of Research and Development

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PREFACE

This publication is intended primarily to assist State highway departments in planning research programs to establish quantitative values of statistical parameters for highway materials and processes. These parameters are needed to apply statistical concepts to quality control in highway construction.

This publication also includes background information concerning the development of an overall plan for application of the statistical approach to quality control in highway construction.

To avoid misunderstanding, it must be emphasized that the relatively large amount of sampling and testing required in this research program to establish the significant parameters involved will not be required when specifications are based on statistical concepts. The methods of application of statistical principles to specific control problems in highway construction have not, as yet, been selected. In certain cases, it is likely that present control practices will be continued, whereas in other cases, a different system of control employing considerably reduced numbers of tests may be adequate. However, a new concept of interpretation of test results then will be necessary.

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Chapter 1

GENERAL INFORMATION

1-1 BACKGROUND AND OBJECTIVES

The construction of quality highways and the processes that assure achievement of such quality have always been of primary concern to highway engineers. They are also primary concerns of the U.S. Bureau of Public Roads.

The purpose of any quality control program is control of the uniformity of the materials and products that make up the highway structure. The standards of such control and the methods to be used are set forth in the specifications governing construction.

A. Present Types of Specifications

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Current highway specifications, are basically of three types:

- (1) The specification that calls for 100 percent compliance with a definite limit or limits, usually including prescribed tolerances.
- (2) The specification that bases acceptance on the satisfaction of the engineer.
- (3) The specification that states definite limits but specifies substantial compliance.

Excellent highways can be and are being built under these specifications. A conscientious, experienced engineer can supervise construction under these specifications and obtain excellent results. However, he may have a very difficult task in documenting or proving that such is the case.

It has been the custom in the past, when a project has been constructed under the Type (1) specification, for the engineer to use his judgment as to the acceptance or rejection of an item, when test results fall outside the specified limits. He bases this judgment both upon his knowledge of materials and tests, and on how critical the particular process or material will be to the performance of the highway. For the most part the judgment has been good. However, for post-construction inspection and literal interpretation of the specifications, this method of control cannot be legally defended. Any deviation from the stated limits of the specification technically constitutes a violation of the terms of the contract.

The Type (2) specification, which calls for the construction to be to the satisfaction of the engineer, can be complied with, and this compliance can be proved. Although many excellent highways have been constructed under such specifications, again there is no way of determining or expressing the true quality of the construction, except through performance with time.

The Type (3) specification is now coming into widespread use. Actually, it is essentially the Type 1 specification, modified by a phrase calling for substantial compliance. This type of specification can be complied with, can be enforced, and does recognize engineering judgment. However, it lacks one essential: a method of evaluating and expressing compliance. This type of specification serves well during the interim period necessary for research and development of new control methods that will define substantial compliance in terms of statistical parameters and agreed upon quality levels.

B. What Specifications Should Do

What is needed in specifications?

A specification should be designed to fulfill four basic functions. These are:

- (1) To provide a standard guide for the contractor to use in preparing his bid.
- (2) To outline the required characteristics of materials, products, or completed work.
- (3) To provide guidelines for construction methods necessary to the production of desired results.
- (4) To state the basis for acceptance. This basis should include sampling and testing methods.

To fulfill its purpose, a specification must be both complete and enforceable. To be complete, a specification must contain both a detailed listing of the identifying or qualitycontrol characteristics and the acceptable limits of variation for measurable properties. It must state definitely the method of test, the point and method of sampling, and the sampling frequency or pattern. To be enforceable, the specification must state definitely the basis of acceptance or rejection of the material or work performed.

C. Statistical Concepts Applicable to Quality Control

The use of statistical concepts in writing specifications is a rational method of setting numerical limits so that the average quality of all construction will be satisfactory. Under this concept the specification limits take into account the normal variations in test results that occur from random causes--the small inadvertent deviations in the materials or in the process itself and the random errors of sampling and testing. The quality control program is then designed to detect, locate, and correct the more serious deviations resulting from assignable causes. (An example is a mistake in weighing or a hole in a screen.) Such a control program is usually based on a specific number of samples taken in a predetermined randomized manner.

However, the inspector is not relieved of the responsibility for vigilant visual observations to detect trouble from assignable causes. Obviously defective material must be rejected and excluded from the project whether or not such material would be sampled by the random selection.

A second concept that must be accepted for the successful application of statistics to quality control is that is is impracticable to test all of the material or all of the end product on any project; thus, it is obviously impossible to be 100 percent certain that all the material is good. However, random sampling enables the inspector to describe the probability or odds that the properties of the incorporated materials are within some range. The more nearly uniform or perfect a product is specified, the more difficult will be its manufacture and the higher the cost. Also, the more certain one wants to be that some material outside of the specified range has not been incorporated in the project, the more costly will be both the testing program and the product.

It follows from these considerations that to be realistic and to provide for optimum economy, the need for near perfect materials having higher costs must be weighed against the use of less perfect materials having lower initial cost but acceptable performance characteristics.

D. Task Group Program

During the past year (1964), the Bureau of Public Roads' Task Group on Quality Control has been working on a five-step program:

- (1) Awakening the highway industry's interest to the utility of statistical quality control.
- (2) Developing guides for research to establish the statistical parameters to be used in writing specifications.
- (3) Planning and correlating a nationwide program of statistical quality control research for highway materials and construction.
- (4) Gathering and analyzing data and disseminating research findings.
- (5) Designing and implementing experimental projects by which the findings of the research program will be evaluated.

Until future research shows otherwise, the Task Group will assume that present construction is good construction, worthy of being reproduced. The plan is to measure this construction, determine the statistical parameters, the average value of each characteristic \overline{X}^{\dagger} , and the standard deviation of the measurements (a measure of the spread of the values, σ^{\dagger}). These measurements will provide knowledge of the average level of present construction and of the control now being exercised under present methods. The ultimate goal of this program is to devise methods for developing specifications and sampling and testing programs which will assure that the new construction will approximate in quality the good construction being used as a pattern.

1-2 GENERAL RESEARCH PLAN

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The initial tasks under the outlined program for the use of statistical methods are:

- (1) Measurement of present quality in a statistically valid manner.
- (2) Determination of the portions of the total variability attributable to: actual variability in the materials, variability caused by sampling, and variability caused by testing.

From the quantitative values of the parameters so determined, the proper specification tolerances based on statistical concepts can be calculated.

Measurement of Present Construction Quality

To obtain statistically valid data, measurements of construction quality must be made in accordance with basic statistical concepts that are based upon the laws of probability. It is, therefore, essential that the measurements be made under conditions that will allow the effective operation of these laws. Adherence to the guides set forth in this publication will provide satisfactory data.

The research proposed in these guides is based on the following general steps to be taken by the participating State:

(1) Select a material or an item of construction for which statistical parameters are desired.

- (2) For each material or process selected, choose the significant quality control characteristics.
- (3) Design a sampling plan that will assure randomness of sampling. Sampling and testing methods should correspond with AASHO and ASTM standards where possible.
- (4) Apply the sampling plan to the material or item of construction under study in a completed project, or in a project under construction, after normal control procedures have been exercised. A minimum of 50 duplicate samples from each of 3 projects is recommended.
- (5) Divide each duplicate sample, where possible, into two portions.
- (6) Measure selected characteristics of each sample portion by routine standard test methods.
- (7) Compute the average value, \overline{X} , for each characteristic, and perform an analysis of variance on the data for each characteristic to determine overall variance, materials or operation variance, sampling variance, and testing variance.

1-3 RANDOM SAMPLING PROCEDURE

It is essential in the measurement of construction quality that the basic laws of probability be allowed to function. Any influence of personal choice of samples must therefore be eliminated by the selection of truly random samples.

By definition, randomization in the statistical sense involves two basic principles:

- (1) Choice of when and where to take a sample must not be left to the person taking the sample.
- (2) Each element of the lot being sampled must have a known chance to be included in the sample.

To comply with these principles, each sample location must be established by a specific randomization procedure, such as use of tables of random numbers. A complete explanation of random numbers and their use is outside the scope of this document, but such explanations can be found in statistical texts and quality control manuals. For this research program, a random number table like that given in the table of appendix A is recommended for locating sampling sites or selecting samples.

To use the table of random numbers, the sampling unit of the material or construction first must be determined. (See appendix B for definitions of this and other statistical terms.) This unit should be related to the quantity that will actually affect performance, such as a square yard of pavement, a mixer load of concrete, or a batch of asphaltic paving mixture.

Random samples may be selected on the basis of units of time, location, or quantity. Selection of the specific sampling basis in any given situation must comply with the second principle (above) of randomness.

All sampling units in the project or portion of a project to be sampled, referred to as the lot, are then considered to be collected in time or space in a frame or grid.

Select a column of numbers from the table for each coordinate needed--one number for each coordinate for each sample to be selected. Enter the table at any point, but use a different entry point for each project. Select consecutive numbers for all samples. If the frame is one unit wide (trucks, batches, or time), use only one column of numbers. If the sample is to be selected from a unit in a plane, use two columns of numbers. If a sampling unit is to be selected in space, such as a cubic foot of embankment material, use three columns of numbers.

1-4 DEFINITIONS AND SYMBOLS

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Fundamental definitions and symbols of all the standard statistical parameters are not within the scope of this document, but definitions of the chief terms used and the sense in which the terms are applied herein, as well as a brief discussion on statistics, appear in appendixes B and C.

1-5 DATA RECORDS AND COMPUTATIONS

The data should be in such form that the machine methods of computation outlined in appendix D may be used.

Chapter 2

PROGRAM FOR ASPHALTIC CONCRETE CONSTRUCTION

2-1 GENERAL

As the total program is based on measuring the variations existing in normally good construction, the project for study should be carefully selected, taking into account:

- (1) The type and quality of the contractor's equipment and his willingness to cooperate.
- (2) The quality level of the designs for the project.
- (3) The size of the project.

To preclude personal bias that unconsciously can result from advance knowledge, those involved in the actual construction should not be permitted to know in advance where or when a sample is to be taken. Normal inspection and control practices of the State should be continued in the usual manner throughout the course of the sampling for a study.

Each project selected for study should be large enough to provide the 50 samples at a ratio of 1 sample for each 90-110 tons of mix (4,500-5,500-ton job). To maintain the intensity of sampling per job at an approximately constant level, this ratio of samples to tonnage sampled should be maintained consistently. For jobs larger than 5,500 tons, only a fractional part of the job should be sampled.

All the test characteristics needed for the program are indicated in figure 1. However, States not adequately equipped or staffed to make determinations of all characteristics may limit their studies to those that are within their capabilities. A participant may set up a study that includes both mixtures and compacted pavements, or either alone.

2-2 SAMPLING PROCEDURE

A. Basis of Randomization

For asphaltic concrete mixtures, the sampling unit used as the basis for randomization should be a truckload. For pavements, the sampling unit should be a square yard. The units of mixtures (truckloads) to be sampled and tested should be determined by use of the table of random numbers, appendix A, in the following manner:

- (1) Estimate the total number of truckloads of asphaltic concrete to be supplied to the project or portion of the project to be studied (the lot).
- (2) Select 50 consecutive numbers from the table of random numbers (appendix A) after selecting the first number by any means desired.
- (3) Multiply each number selected by the total number of truckloads. Round the product to the nearest whole number. The resultant numbers designate the truckload number to be sampled.



Figure 1 Test Characteristics, Asphaltic Concrete

For pavement samples for which a square yard has been designated as sampling unit, a two-coordinate grid system for randomization must be used. In this case, proceed as follows:

- (1) Assume the sampling unit to be a square yard of surface, but for convenience, use the length and width in feet of the pavement area in locating the sampling unit.
- (2) Divide the length of the project or portion of the project to be sampled into approximately 10 equal sections. Select 5 sampling units randomly in each of the 10 sections.
- (3) Select 5 consecutive pairs of random numbers from the random number table for each of the 10 sections. Each group of 5 random numbers should be different.
- (4) Multiply the first number of each pair of the 5 random numbers by the length of the section in feet. The products, rounded to the nearest whole number, will be the longitudinal coordinates in feet of the 5 sampling unit locations within the section.
- (5) Multiply the second number of each pair of the 5 random numbers by the width of the section in feet. The rounded products will be the transverse coordinates of the 5 sampling unit locations within the sections.
- (6) The 50 duplicate samples should be taken within the square yard area so located. The coordinates may be used as the location of the center or a corner of the area.

B. Point of Sampling

In studies of asphaltic concrete mixtures, the samples may be taken from trucks either at the plant or at the job site.

In studies of compacted pavements, the samples should be taken before the sections to be sampled are opened to traffic.

C. Size of Samples

Table 1 shows the estimated size of each sample.

Approximate Required Size of Duplicate					
Mixtures		Pavements			
		Uncompacted	Compacted		
Extracted	Stability	Extracted	Gradation		
Gradation	Density	Gradation	Density		
(pounds)			-		
8					
10					
15	25 pounds	l square foot	7 by 7		
18			inches		
21					
25					
	Ap Miz Extracted Gradation (pounds) 8 10 15 18 21 25	Approximate ReqMixturesExtractedStabilityGradation (pounds)Density8101525 pounds18212525	Approximate Required Size of DuplicMixturesPaveUncompactedExtractedStabilityExtractedDensityGradationDensity(pounds)Gradation8101525 pounds18212525		

Table 1 Estimated Sample Sizes, Asphaltic Concrete

D. Sampling Technique

1. Mixtures

From the pile formed by the last batch to go into the truck, take three portions for each sample--one near the bottom, one near the middle, and one near the top. The three portions should be taken along an imaginary line from the bottom of the pile to the apex. Experience indicates that a sturdy handscoop with a bowl about 8 inches long, 5 inches wide, and 3 inches deep is a convenient device for taking the portions.

Take a sufficient number of scoops from each level at a location in the pile to provide the total sample size (combined material from the three levels) indicated in table 1.

2. Compacted Pavement Courses

a. <u>Sampling by Sawing</u>. From the square yard designated as the sampling unit, cut full-depth samples approximately 7 by 21 inches in size representing a complete wearing course or base course, whichever is under study. Divide each sample of the compacted course into three approximately equal portions (7 by 7 inches). Use two of the three portions in making the two tests of each characteristic as required for the subsequent analysis of variance. Hold the third portion in reserve for possible check or repeat determinations, if required because of accident or personal error.

When density is not to be determined immediately after dividing the sample, secure each portion of the sample to a piece of plywood of adequate size and store the portions until the test is to be made.

b. <u>Sampling by Core Drill</u>. Samples may be taken by coring, if desired. Three pairs of 6-inch diameter cores, cut to the full depth of the course, should be taken instead of the 7-by 21-inch sawed section suggested under <u>Sampling by Sawing</u>, above. A pair of 6-inch cores will correspond to a 7-by 7-inch portion of a sawed specimen for a test. Use two of the three pairs of cores to make the two tests desired on each duplicate for the subsequent analysis of variance. Retain the third pair for a check test, if required.

E. Reduction of Samples to Test Size

The procedure described below is applicable to all tests except those for density determinations on compacted pavement samples. Reduction of compacted pavement samples to size has already been described.

In reducing samples to test size, take every precaution to insure that each test specimen is representative of the sample from which it is obtained. Sample splitters are recommended for this operation.

If the sample is not sufficiently soft to be separated for splitting, it may be softened by warming. Make as many consecutive splits as will provide a test speciment with ± 10 percent of the amount required by the applicable test method. This amount should be a <u>catch weight</u>. Do not bring it to the required weight by handpicking.

2-3 TEST METHODS

A. Temperature

Temperature determinations on mixtures will be feasible only immediately after the mixtures are discharged from the mixer, or immediately before they are dumped into the hopper of the paver-finisher.

Make two temperature determinations for each sample, using thermometers calibrated within an accuracy of + 2 degrees F. around the expected temperatures.

The recommended procedure for measuring temperature is to insert the thermometers near the locations in the pile where the samples are to be taken. Use two thermometers at each sample location, positioning one about one-third of the distance from the bottom of the pile and the other about one-third of the distance from the top. Experience has indicated that about five minutes are required for dial-type thermometers to reach equilibrium. Place the thermometers before sampling of the pile is started and leave them in place during the sampling operation. Equilibrium probably will be reached by the time sampling is completed and the readings may then be taken.

B. Asphalt Content

Determine asphalt content by any of the standard extraction methods described in the <u>Method of Test for Quantitative Extraction of Bitumen from Bituminous Paving Mixtures</u>, ASTM D 2172-63T, except that the water determination may be omitted for mixtures.

In addition to the standard method, it is recommended that asphalt content also be determined by any rapid method, such as the nuclear method, if the participant has the necessary equipment and experience to make such a determination. It is hoped that the information obtained will be useful for determining whether the general use of such methods is feasible in future practice.

C. Gradation of Extracted Aggregate

Determine the gradation of the extracted aggregate in accordance with the Standard Method of Test for Mechanical Analysis of Extracted Aggregate, AASHO T-30-55.

D. Density

Determine density by the Standard Method of Test for Effect of Water on Cohesion of Compacted Bituminous Mixtures, AASHO T-165-55, section 4, except that the specimen may be of any convenient size or weight.

In determinations on compacted pavement specimens that may contain moisture, it is advisable to measure dry weight in air after weights in water have been determined.

Dry specimens by any suitable method: for example, by heating to constant weight in an oven at 220-240 degrees F. If theoretical maximum density (see paragraph F below) is to be determined on the same specimen, dry-weight measurement should be the last step of the theoretical maximum density procedure.

E. Stability

Compact the mixture and determine stability in accordance with appropriate sections of ASTM D-1559-62T, <u>Method of Test for Resistance to Plastic Flow of Bituminous Mixtures</u> Using Marshall Apparatus.

F. Theoretical Maximum Density

Determine theoretical maximum density (specific gravity) by the method described in appendix I (pp. 53-54) of the paper <u>Maximum Specific Gravity of Bituminous Mixtures by</u> <u>Vacuum Saturation Procedure</u>, ASTM Special Technical Publication No. 191, <u>Symposium on</u> <u>Specific Gravity of Bituminous Coated Aggregates</u>. Use either compacted pavement specimens or samples of uncompacted mixtures for the determination. If maximum theoretical density is to be determined on uncompacted mixtures, the samples should correspond closely with the compacted specimens used for density determination, paragraph D above. A suggested method is to take the sample before compaction in the unit area of pavement from which the compacted pavement specimen is to be taken for density determination, paragraph D above.

G. Air Voids

Calculate the percentage air voids in the compacted material from the following formula:

Voids (air) =
$$\frac{D-d}{D}$$

where D is the theoretical maximum density (paragraph F above)

and d is the actual density (paragraph D above).

2-4 REPORTING RESULTS

Since the test results are to be subjected to statistical treatment, report to one more significant figure than is normally reported for the particular test.

In addition to the test results, include the following information in the report:

- (l) Job or project identification.
- (2) Type of plant producing the mixture--continuous, manual-batch, automatic-batch, etc.
- (3) Type of mixture, including types of aggregate components, e.g., gravel + sand + filler, etc.
- (4) Job-mix formula, or specification, if job-mix formula is not used for job.
- (5) Proportioning of mixture components at plant, including changes in proportioning (when and if made) during sampling period.
- (6) Total tonnage of material sampled.
- (7) Location of tonnage sampled in project, if total job is not sampled.
- (3) Detailed description of, or reference to, test method, if a method other than that recommended is used.

Chapter 3

PROGRAM FOR PORTLAND CEMENT CONCRETE--PAVING AND STRUCTURES

3-1 GENERAL

The program in this chapter is intended for obtaining data on the variations of the results of tests on plastic and hardened concrete. Two samples from each of 50 locations on a concrete paving or structural job will be taken and each sample will be divided for tests. Thus, for each type of test on plastic concrete, 200 determinations and 400 cylinders for use in the statistical study will result from each job.

Studies should be avoided on projects or jobs having inexperienced contractors or in which unusual materials are being used. Studies can be made on jobs with site-mixed concrete from paving or structural mixers, with central-mixed concrete hauled to the job in agitating or nonagitating trucks, or with truck-mixed concrete mixed at the batching plant, in transit, or at the jobsite. To establish the statistical parameters, the studies should be made on a series of three similar projects.

3-2 SAMPLING PROCEDURE

A. Basis of Randomization

For pavements, define the lot as the total area in square yards of concrete to be placed in the project or portion of the project studied. For structures, define the lot as the volume in cubic yards of concrete to be placed.

To determine the sample location, proceed as follows:

- Select 50 consecutive numbers from the random table (appendix A), after choosing the first number in any manner. Do not use the same starting point for all projects.
- (2) Multiply each random number by the total number of square yards (for pavements) or cubic yards (for structures) of concrete to be placed in the project (lot).
- (3) Determine the mixerload or truckload that will contain the square yard or cubic yard of concrete indicated by the random selection in step 2. Take samples from this mixerload or truckload in the standard manner. For example, if a total of 5000 square yards of concrete are to be placed in a pavement project and one of the numbers selected from the table is .470, then .470 x 5000 = 2350. Thus, a sample would be taken from the first truckload or mixer load that would bring the total production of the project to over 2350 square yards.

B. Project Size

The minimum quantity in cubic yardage or square yardage of concrete for a project study should be large enough to allow sufficient time on any one day to obtain the maximum number of location samples and to make the required tests. No minimum values for volume or area of concrete are given for a selected project because the rate of placement can vary widely. However, it is recommended that any project studied should extend over a period of 7 days or more. In sampling at each location, follow the Standard Method of Sampling Fresh Concrete, ASTM C172-54 (AASHO T 141-53), as amplified in this chapter.

Do not take samples from concrete that has been placed in forms or has been compacted, puddled, spaded, screeded, or manipulated by the application of force after it has been discharged from the mixer or truck. Section 1, Scope, of ASTM Method C-172, applies only to fresh concrete from stationary and paving mixers, truck mixers, agitators, or trucks.

C. Sampling Techniques

1. Sampling from Stationary Mixers

When sampling from stationary mixers as described in the first procedure, 3(a) of ASTM Method C-172, do not fill to overflowing the receptacle passed through the discharging stream. Overfilling can result in a segregated or nonrepresentative sample. It is difficult to fill and lift manually through a discharging stream of concrete a receptacle that is larger than one-half cubic foot in capacity. Consequently, when an amount greater than one-half cubic foot is needed for a location sample, take two or more portions from several receptacles and mix them thoroughly before dividing each location sample for the two tests to check the variance of testing.

Obtain the two samples from one location (stationary mixer) in an identical manner, either with a receptacle or by diverting the discharge stream into a container. Obtain the two samples from about the middle or first part of the third quarter of the same discharging batch of concrete.

2. Sampling from Paving Mixers

Concrete on the subgrade discharged from a paving mixer is usually in the form of a conical pile having an elliptical or circular base-cross-section. Halfway between the base line and apex of the pile, take each of the five or more portions for the first sample at points equally spaced around the pile (5 portions would be taken every 72° , 6 portions every 60° , etc.). To determine the variance of testing, mix the five or more portions before dividing the sample for tests. Obtain the second sample according to the procedure used for the first sample, taking the five or more portions from the same approximate horizontal plane used in the first sample, but extracting them equidistantly from between the points where the portions for the first samples were taken.

3. Sampling from Revolving-Drum Mixers or Agitators

ASTM Method C-172, section 3(c), requires that samples of concrete from revolvingdrum truck mixers or agitators shall be taken from three or more regular intervals throughout the discharge of the entire batch. Ideally, the first sample of concrete should consist of portions taken at about the one-eighth, three-eighth, and five-eighth points of discharge and the second sample of portions taken at the three-eighth, five-eighth, and seven-eighth points of discharge; but, to simplify operations, take both samples at about the one-quarter, one-half, and three-quarter points of discharge. Thoroughly mix the three portions before dividing the the location sample into the two samples required for tests. The precautions that apply to sampling from stationary mixers by use of a receptacle, section 3(a) of ASTM Method C-172, also apply to the sampling by receptacle described in section 3(c) of ASTM Method C-172.

Test Designation	Title of Test or Sampling Method	Applicability	Concrete Sample Size (cubic feet)			
ASTM C143-58 AASHO T 119-60	Slump of Portland Cement Concrete	l Slump Test	0.20			
		l Chace air meter test	Negligible			
ASTM C 231-62 AASHO T 152-57	Air Content of Freshly Mixed Concrete by the Pressure Method	l Pressure air meter test	0.25			
ASTM C 138-63 AASHO T 121-56	Test for Weight Per Cubic Foot, Yield, and Air Con- tent (Gravimetric) of Concretè	l Unit weight test	0.50			
ASTM C 31-62 T AASHO T 23-60	Making and Curing Concrete Compression and Flexure Test Specimens in the Field	2 cylinders,	0.40			
AASHO T 22-60	Compressive Strength of Molded Concrete Cylinders	28 days	0.40			
TOTAL VOLUME OF CONCRETE - 1.35 Cubic Feet						

 Table 2 Tests to be Performed on Concrete Samples

4. Sampling from Open-Top Truck Mixers, Agitators, Dump Trucks, or Other Types of Open-Top Containers

The choice of sampling method (1, 2, 3) should be the one most likely to afford a representative sample. Segregation of non-air-entrained concrete occurs frequently in nonagitating trucks of flat body design. Sampling by use of scoops or shovels from the concrete in opentop mixers or trucks is unsatisfactory, and the samples should always be taken from the discharging stream, from a diverted stream, or from the completely discharged batch.

3-3 MIXING OF CONCRETE SAMPLE

For making tests or for molding specimens, section 4 of ASTM method C-172, entitled Remixing Sample, requires that the portions of a sample receive a minimum of mixing and that a sample receive a minimum of remixing. Overworking of the plastic concrete can cause a change in the air content or consistency.

Divide each of the two samples from a location into two samples for tests to determine variance of testing by placing alternate shovelfuls or scoopfuls in two containers and then remixing each.

3-4 TESTS

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Make the tests listed in table 2 on each of the 200 portions of samples (50 locations x 2 samples each location x 2 divided portions of each duplicate).

Cylinders must be cured in accordance with the requirements of the Method of Making and Curing Concrete Compression and Flexure Test Specimens in the Field, ASTM C 31-62 T, sections 9(a) and 9(b). The importance of maintaining the temperature range at 60-80 F. during the first 24 hours after molding and at 70.4 - 76.4 F. during the remaining moistcuring period must be emphasized to those in charge of making the tests. Sheet metal molds of the can type or of machined metal are preferable to other types. Both paraffined-cardboard and can-type molds must meet the requirements of the Specification for Single-Use Molds for Forming 6 by 12-In. Concrete Compression Test Cylinders, ASTM C 470-61T.

When capping the cylinders, ASTM Method C-31, and testing in compression, ASTM C-39-64, Method of Test for Compressive Strength of Molded Concrete Cylinders, do not deviate from the specified requirements. All machines must meet the requirements of ASTM Method C-39.

Chapter 4

PROGRAM FOR THICKNESS OF PAVEMENT SURFACING

4-1 GENERAL

The program outlined in this chapter is designed to provide data for establishing statistical parameters peculiar to thickness variations of surface layers. Often the thickness of such layers is a major specification item and may be the criterion for acceptance or rejection of the project.

4-2 SAMPLING PROCEDURES

A. Basis of Randomization

The total project to be studied should be divided into 10 approximately equal sections of 2000 to 5000 feet in length. Five sampling locations in each section then shall be determined by means of the table of random numbers, appendix A, as follows:

- (1) Select 5 consecutive pairs of numbers from the table for each of the 10 sections. Enter the table at any point, but use a different point of entry for each section.
- (2) Multiply the first of the pair of numbers by the length of the section, in feet. Round the product to the nearest whole number. The resulting number designates longitudinal position of the sampling unit within the section.
- (3) Multiply the second of the pair of numbers by the width of the project, in feet. Round the product to the nearest whole number. This number designates the transverse position of the sampling unit (measured from the left edge of the section) within the section.
- (4) Repeat the operation for each pair of numbers to locate the positions of all the sampling units within each section.
- (5) Consider the points so located as the centers of 1 square-foot areas, termed sampling units, and obtain two cores (for duplicate sampling) from within each sampling unit. Extract the first core by drilling at a point located 0.5 feet longitudinally from the indicated center of the sampling unit. Extract the second core by drilling at a point located 0.5 feet transversely from the center of the sampling unit.

B. Thickness Determinations

Identify the cores as to their location in the pavement. Clean away loose sand, chips, or soil material, and measure core thickness to the nearest 1/16 of an inch at three points about 120 degrees apart. Record the average of the three measurements as the thickness. Independently repeat the three measurements for each core, and average the results as a second determination of thickness. This should be done by a different operator or at a different time without reference to the original values or to the original measurement points. Make the measurements with calipers of suitable size and accuracy or in conformance with the Standard Method of Measuring Length of Drilled Concrete Cores, AASHO T 148-49 (ASTM C 174-49).

If the cores are to be used for tests in addition to those for thickness measurements, store them in a manner such that they will not be damaged or affected by weather.

C. Nondestructive Testing

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Participants who have the necessary experience and equipment are urged also to make thickness determinations with a suitable nuclear device, sonic device, or other modern apparatus that has more recently become acceptable in this field of work. Information on the device or apparatus should be included in the report, as the data might be valuable in establishing a more universal acceptance of the device for future use.

Chapter 5

PROGRAM FOR AGGREGATES

5-1 GENERAL

The program in this chapter outlines a research project to provide data for establishing the statistical parameters peculiar to aggregate variability. Because of the large volumes of materials handled and the inspection and testing practices, the study of aggregate variability introduces problems not associated with other materials.

The basis for taking samples may be units of time, such as time of operation of a conveyor belt; units of location, such as a point in a stockpile; or units of quantity, such as truckloads, tons, etc. The selection of the specific sampling basis in any given situation is governed by the second principal of randomness--that all units of a lot have an opportunity to be chosen. For example, it is not feasible to use location as a basis for taking random samples of coarse aggregate from a railroad car since the material at the bottom of the car is not readily accessible. However, random selection on the basis of units of time during the loading or unloading of the car by means of a conveyor belt would be applicable.

The lot, to be represented by 50 random sampling locations, may be the source, that is, the quarry, pit, or scene of a river dredging operation, or the lot may be defined as the total material supplied to a project. In some cases, where a number of quarries supply similar stone within a State, it may be desirable to define the population as all such quarries in the State and to take only 10 to 20 randomly selected samples from each to obtain the 150 (or more) sample locations necessary for the statistical analysis.

Source sampling is applicable to those inherent quality characteristics of an aggregate, such as soundness or hardness, over which the producer has little or no control. Often it is practical to certify a source of aggregate, such as a quarry, pit, or river deposit, as acceptable with regard to inherent quality characteristics, rather than to determine these characteristics in the particular lot of aggregate actually delivered. For this situation, the lot of aggregate to be randomly sampled would be the quantity produced over a considerable period of time, such as several months or an entire production season. Typical examples of sources of this type which in statistical terms can be considered a single population are:

- (1) Quarries or pits of only one geologic formation.
- (2) Quarries or pits of more than one geologic formation, where operations are so conducted that the final product represents the various formations in proportion to their occurrence over the time being considered.
- (3) A river deposit for which the material obtained by dredging or pumping does not vary appreciably over the area being worked.

Examples of sources which cannot be considered <u>single population</u>, and which therefore are not suitable for source sampling in this program, are materials obtained by dredging operations conducted over widely separated portions of a river (sometimes as much as 100 miles), or products dredged from several rivers and marketed individually without differentiation as to source.

The aggregate actually delivered to each project should be sampled to determine the acceptability of the material with respect to any specification item that is controlled by the

producer or that may be appreciably changed by processing or handling--for example, the grading or deleterious particle content. If desired, the inherent aggregate characteristics, usually determined at the source, may be determined from the aggregate provided for each project.

5-2 SAMPLING PROCEDURE

A. Intensity of Sampling

For sampling at sources where the variability of the product is not affected by seasonal changes and only three sources are involved, the minimum quantity of material represented by the 50 random samples from each source should be at least 5000 tons for fine aggregates and 10,000 tons for coarse aggregates. However, for sources subject to seasonal effects such as waterborne deposits, the 50 samples should be randomly selected from an entire season of production, regardless of the quantity involved. A minimum of three sources producing materials under the same specifications and conditions should be sampled to establish the parameters of the population.

B. Point of Sampling

1. Sampling at Source

Samples for quality testing should be taken after the material has been completely processed. The truck or conveyor system delivering aggregate to the producer's stockpile, or removing aggregate from the stockpile, are suitable sampling points.

2. Sampling at Project

Samples taken at a project are intended to indicate acceptability of the aggregate with respect to grading as well as to other characteristics not previously determined. Thus, the point of sampling must give all portions of the aggregate a chance of being included in the sample. This requirement precludes sampling of coarse aggregates from a carload, barge, or other container in which some elements of the lot are not accessible. Although stockpile sampling is possible under some conditions, it is not recommended for this program because of the difficulties involved.

Coarse aggregate should be sampled at some point during delivery of the material to the project--such as from a conveyor belt, truck, or dragline bucket. Fine aggregate, provided it is moist, can be sampled satisfactorily by means of sampling tubes from any point in the delivery process, as well as from stockpiles, barges, or carloads.

A minimum program is considered to be the sampling and testing of the material supplied under the same specifications to three projects.

C. Size of Samples

Table 3 shows minimum weight for each sample of coarse aggregate. These weights are based on the minimum amounts of material specified in the Method of Test for Sieve or Screen Analysis of Fine and Coarse Aggregate, ASTM C 136-63. They provide sufficient material for two determinations of grading. Except as noted, these quantities suffice for the quality test, which can be made on the sample after the grading has been performed.

Nominal Maximum Size of Particle (inches)	Maximum Weight of Sample (pounds)
$\begin{array}{c} 3/8\\ 1/2\\ 3/4\\ 1\\ 1^{\frac{1}{2}}\\ 2\\ 2^{\frac{1}{2}}\\ 3\\ 3^{\frac{1}{2}} \end{array}$	10 (*) 20 (*) 30 (*) 50 70 90 110 200 300
* Increase to 50 pounds if a	Los Angeles test is to be made

Table 3 1	Minimum	Sample	Weight for	Coarse	Aggregate
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Each sample of fine aggregate should weigh at least 10 pounds, except that 15-pound samples or larger are required for the Method of Test for Effect of Organic Impurities in Fine Aggregate on Strength of Mortar, ASTM C 87-63T.

D. Sampling Techniques

The sampling procedure must conform with the purpose for taking the sample. For example, if grading is the characteristic being determined, sampling must be so conducted that errors from obvious segregation are not introduced. On the other hand, if only the quality of the material is of interest, as is usual for source sampling, it is necessary only to obtain enough of each sieve size for performing the necessary tests.

For quality tests, therefore, sampling is satisfactory from the most accessible point in the truck, conveyor, belt, or stockpile location that has been randomly selected. The manner of taking the sample is immaterial, provided that the sampler does not make preferential selections of individual particles.

When samples are taken at the project for the purposes of establishing the variability of a characteristic such as grading, the sampling techniques described below are applicable.

1. Conveyor Belt or Delivery Chute

When sampling from a conveyor belt or delivery chute, divert the entire width of the stream for the sample being collected. Diverting only a portion of the stream is a question-able procedure. For a conveyor belt, the best procedure is to stop the belt during sampling. On bucket-type conveyors, take all the material from a sufficient number of buckets to provide the sample.

The random determination of the 50 sampling locations is most conveniently based on the time of operation of the delivery system. If the approximate amount of aggregate to be delivered to a project and the rate at which the aggregate is to be handled is known, the approximate total time of operation can be estimated. Specific sampling times are calculated by multiplying the total time by each of 50 random numbers taken from the table. Under this system, duplicate samples are obtained by taking two samples within a short time--the first 15 to 30 seconds before the calculated sampling time and the second 15 to 20 seconds after the calculated time.

2. Drag-line Bucket, Shovel, or Truck

In sampling aggregate handled by bucket, shovel, or truck, the unit to be sampled is the amount delivered in each load of the equipment being used. Randomization may be on the basis of time of equipment operation, or units such as truckloads.

Obtain the sample by depositing the material from each unit in a pile. Then take shovelfuls of material at regular intervals along two straight lines across the pile at right angles to each other.

Take duplicate samples by repeating the sampling procedure on the same units of material along lines oriented at 45° to those along which the first samples were taken.

E. Reduction of Samples to Test Size

Samples to be tested for grading, or for any property that is likely to change with the grading of the material, should be brought to test size by quartering or sample splitting.

For quality testing, where the grading is given by the test procedure, as in the Los Angeles test, or where average grading can be assigned, as in test for soundness or deleterious particles, it is necessary only that the required weight be taken in an unbiased manner from each size fraction after the sample has been separated into sizes.

5-3 TEST METHODS

NOTE

Not all of the following tests are expected to be included in this research program. The methods are described in this section to complete the information concerning the testing of aggregates to determine statistical parameters.

A. Grading

Use the Method of Test for Sieve or Screen Analysis of Fine and Coarse Aggregates, ASTM C136-63.

When sieving coarse aggregates, do not guide the aggregate particles through the sieve openings with the fingers. Do not extend the sieving any more than the ASTM method calls for. It directs sieving to continue only ". . . until not more than 1 percent by weight of the residue passes any sieve in one minute."

With fine aggregates, it is important to check sieving efficiency on sieves No. 50 and finer to be certain that they are not clogged.

B. Material Passing No. 200 Sieve

Use the Method of Test for Materials Finer Than No. 200 Sieve in Mineral Aggregates by Washing, ASTM C 117-61T.

Perform this test on the aggregate before it has been dry-sieved for grading. The entire sample to be graded can be used for the washing test, or a smaller portion may be used if it is obtained by quartering or sample splitting to provide the weight of sample called for in ASTM Method C 117.

C. Los Angeles Loss

Use the Method of Test for Resistance to Abrasion of Small Size Coarse Aggregate by Use of the Los Angeles Machine, ASTM C 131-64T.

D. Soundness

Use the Method of Test for Soundness of Aggregates by Use of Sodium Sulfate or Magnesium Sulfate, ASTM C 88-63.

The samples for this test must be sieved before the initial sample weight is determined. Sieving must be more thorough than that specified by ASTM Method C 136. For coarse aggregate, sieving must continue to refusal--the point at which no additional particles will pass the sieve event though sieving is continued. To check for thoroughness of sieving, try to pass doubtful particles by fingering. For fine aggregates, some particles will continue to pass through the sieve regardless of how long the shaking continues; therefore, for this type mateerial, continue only until the rate of particle passage remains constant with continued sieving.

After the samples have been subjected to sulfate cycling, do not continue final sieving longer than is necessary to pass the particles affected by sulfate actions. For this part of the test, hand sieving is preferable to machine sieving, because the operator can better determine when the particle fall is complete.

The grading used to weight the loss for each test size should be the average grading of the 50 samples from the source, rather than the grading of the sample being tested.

E. Deleterious Particles

Control of deleterious-particle content is generally a localized problem brought about by the natural occurrence of certain types of particle detrimental to concrete. Not only does the type of particle vary in different locations, but the criteria by which deleterious particles are identified are different. Often, no standard test methods are available, and identification must depend on visual examination. Such a procedure can lead to considerable disparity between the results obtained by different operators, particularly when they are in different laboratories. Frequently, the difference between the results obtained by two operators on the same sample can be of the same order of magnitude as the specification limit. This leads to considerable difficulty in administering specifications for deleterious particles.

Although personal judgment probably can never be entirely eliminated from visual determinations of deleterious particles, methods for minimizing differences between inspectors are available. The greatest need is to understand what type of particle is limited by the specification. Particles of a deleterious type may be difficult to describe in writing, but may be readily recognizable by the trained eye. Thus, the best way of minimizing differences is to personally instruction the operators using samples containing the types of deleterious particles concerned. An alternative to personal instruction is the use of hand samples established by the specification-writing agency as typical of the deleterious particle type being excluded. Such hand samples may be maintained for reference at the headquarters laboratory or may be supplied to such interested parties as field laboratories, commercial laboratories, and aggregate producers.

The types of particle usually considered deleterious in concrete aggregates are identified in paragraphs 1 through 5 below.

1. Chert

Since many types of chert are not deleterious, a qualifying word such as lightweight, unsound, or deleterious is generally used. Lightweight chert normally is material having a bulk specific gravity (saturated surface-dry) less than some value in the range of 2.35 to 2.45, as determined by the Method of Test for Lightweight Pieces in Aggregate, ASTM C 123-64. Unsound or deleterious chert may be defined by its specific gravity or its resistance to a soundness test, or it may be identified by visual examination.

2. Shale

Shales grade imperceptibly from very soft, highly porous, or unsound, to better consolidated and reasonably sound types. Definitely unsound shales normally are soft enough to be detected by the scratch hardness test (ASTM Method C 235). If this test is not considered adequate, use visual identification.

3. Soft Pieces

To identify soft pieces, use the Method of Test for Scratch Hardness of Coarse Aggregate Particles, ASTM C 235-62T.

4. Coal and Lignite

To identify coal and lignite, use the Method of Test for Lightweight Pieces in Aggregate, ASTM C 123-64, with a liquid of 2.00 specific gravity. Only brownish-black or black material should be considered coal or lignite. Coke should be classed as coal or lignite.

5. Other Types

Use visual identification for other types of deleterious particle.

F. Unit Weight

For unit weight tests, use the <u>Method of Test for Unit Weight of Aggregate</u>. ASTM C 29-60. This test measures a characteristic that is partly inherent in the aggregate but can be affected by the grading. Thus, while it is feasible to test for unit weight of coarse aggregate at the source, the test must be performed on material that has been sampled in the same manner as for a grading test.

G. Sand Equivalent Test

To perform sand-equivalent tests, use the <u>Standard Method for Plastic Fines in Graded</u> Aggregates and Soils by Use of the Sand Equivalent Tests, AASHO T 176-56.

Chapter 6

PROGRAM FOR CHARACTERISTICS OF COMPACTED BASES AND SUBBASES

6-1 GENERAL

The program outlined in this chapter is designed to provide data for establishing the statistical parameters on compacted base or subbase courses.

The research is to be conducted on base or subbase courses as they are constructed under normal operating conditions and control methods. The statistical lot will be projects or portions of projects constructed of like materials and in accordance with the same specifications. Three projects or lots should be studied. The sampling unit will be a square yard of compacted material of specific thickness representing one or more lifts. The total depth should not be more than what can be conveniently sampled by methods applicable to normal control.

6-2 SAMPLING PROCEDURES

Duplicate samples should be taken or measurements made at 50 random locations in each of the 3 projects. Each of these projects or portions of a project should contain a minimum of 10,000 cubic yards of material, the characteristics of which are being evaluated.

Divide each project into 10 equal or nearly equal sections. Take 5 duplicate samples or measurements in each of the 10 sections for each project.

Select the units to be sampled within each section as follows:

(1) Starting at any line in the random number table, appendix A, select 5 consecutive pairs of random numbers.

NOTE

In steps (2) and (3), be sure to use the random numbers as decimal numbers.

- (2) Multiply the first number of each pair by the width, w, of the base. The resultant numbers will be the transverse coordinates within the square-yard unit to be sampled.
- (3) Multiply the second number of each pair by the length, l, of the section. The resultant numbers will be the longitudinal coordinates within the square-yard unit to be sampled. The origin of the coordinate system will be at one corner of the section.

Take duplicate density tests within the square-yard unit so located. As there is no sampling procedure, as such, the duplicate tests will suffice to establish the testing error.

Split the duplicate samples from the test holes and test each portion for moisture and percentage of course material. Be certain to use care in splitting samples and in preserving moisture conditions of the samples.

6-3 TEST METHODS

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The following AASHO and ASTM standard test methods should be used throughout this program:

Standard Test Methods for Density --

AASHO T 99-611, Moisture-Density Relations of Soils Using A 5.5-lb. Rammer and A 12-in. Drop

AASHO T 180-611, Moisture-Density Relations of Soils Using a 10-1b. Rammer and an 18-in. Drop

AASHO T 181-60, In-Place Density of Compacted Base Courses Containing Large Sizes of Coarse Aggregate

ASTM D 1556-64, Density of Soil in Place by the Sand-Cone Method

ASTM D 2167-63T, Density of Soil in Place by the Rubber-Balloon Method

Standard Test Methods for Gradation--

AASHO T 88-57, Mechanical Analysis of Soils

ASTM D 422-63, Grain-Size Analysis of Soils

The samples obtained in this program may be used in the measurement of characteristics other than those above, i.e., liquid limit, plastic limit, etc., if the State so desires.

The density data should be analyzed and reported as percent of laboratory compaction.

Chapter 7

PROGRAM FOR CHARACTERISTICS OF COMPACTED EMBANKMENTS

7-1 GENERAL

The program outlined in this chapter is designed to provide data for establishing the statistical parameters pertaining to density, percent compaction, and moisture content of compacted embankments and to related material characteristics.

The research is to be conducted on embankments as they are constructed under normal operating conditions and control methods. The lot will be projects or portions of projects constructed in accordance with the same specifications. Three projects should be chosen for study. The sampling unit will be a square yard of compacted material one lift in thickness.

7-2 SAMPLING PROCEDURES

Duplicate samples should be taken or measurements made at 50 random locations in each of the 3 projects. Each project should contain a minimum of 100,000 cubic yards of embank-ment.

A three-dimensional sampling plan is recommended for this research. Divide each project into separate embankments, not necessarily of equal size but large enough to contain at least 5,000 cubic yards each. Take 5 duplicate samples in each of 10 embankments in each of the 3 projects, as the work progresses.

Select the 10 embankments to be sampled within a project by multiplying the total number of embankment sections by 10 consecutive random numbers from the random number table, appendix A. Use the random numbers as whole numbers and round each product to the nearest whole number. The products then will be the numbers of the consecutively numbered embankments to be sampled.

Select the units in each embankment to be sampled in the following manner:

- (1) For each test section, start at any point in the random number table and select 5 consecutive groups of three numbers, Z, X, and Y.
- (2) Multiply the first random number, Z, of each group by the maximum thickness or or height, h, of each section at the centerline. This can be determined directly on the profile. The products, Zh, added to the ground elevation of the embankment base at the thickest or deepest part of the embankment will establish the elevation of the sampling plane parallel to the roadway surface.
- (3) Multiply the second random number, X, of each group by the length, 1, of the plane at the centerline. The resultant length, XI, measured on the centerline from one end of the sampling plane, establishes the longitudinal position of a transverse line extending across the width of the embankment on the sampling plane. The unit will be located on this line at point established by Yw (step 4).
- (4) Multiply the third random number, Y, of each group by the width, w, of the sampling plane at the transverse line established in step 3. The resultant length, Yw, measured from the left or right of the embankment centerline, locates the unit on the transverse line.

Duplicate density tests should be taken within the square-yard unit so located. As there is no sampling procedure, as such, for the density determinations, the duplicate tests will suffice to establish the testing error.

Duplicate samples from the test holes should be split and each portion tested for moisture and any other characteristics of interest to the study.

In cases where there is doubt as to the correlation between the test soil and the laboratory compaction curve, one point (or more) on the density curve should be determined to help in correlation.

7-3 TEST METHODS

The following AASHO and ASTM standard test methods should be used throughout this study:

AASHO T 147-54, The Field Determination of Density of Soil In-Place

ASTM D 2167-63T, Density of Soil in Place by the Rubber-Balloon Method

ASTM D 1556-64, Density of Soil in Place by the Sand-Cone Method

Rapid nondestructive test methods should also be used in conjunction with this research where practical.

7-4 DATA RECORDS AND COMPUTATIONS

Perform the following calculations and record the results in the project records:

- (1) Calculate the percent of standard density, moisture content, and the values of other statistics, if other statistics are desired.
- (2) Compute the mean and the variances.
- (3) Determine the relationships between specified values and measured values.

APPENDIX A

TABLE	OF	RANDOM	NUMBERS

· · · · · · · · · · · · · · · · · · ·				
.576 .730	.430 .754	.271 .870	.732 .721	.998 .239
.892 .948	.858 .025	.935 .114	.153 .508	.749 .291
.669 .726	.501 .402	.231 .505	.009 .420	.517 .858
.609 .482	.809 .140	.396 .025	.937 .310	.253 .761
.971 .824	.902 .470	.997 .392	.892 .957	.640 .463
.053 .899	.554 .627	.427 .760	.470 .040	.904 .993
.810 .159	.225 .163	.549 .405	.285 .542	.231 .919
.081 .277	.035 .039	.860 .507	.081 .538	.986 .501
.982 .468	.334 .921	.690 .806	.879 .414	.106 .031
.095 .801	.576 .417	.251 .884	.522 .235	.398 .222
.509 .025	.794 .850	.917 .887	.751 .608	.698 .683
.371 .059	.164 .838	.289 .169	.569 .977	.796 .996
.165 .996	.356 .375	.654 .979	.815 .592	.348 .743
.477 .535	.137 .155	.767 .187	.579 .787	.358 .595
.788 .101	.434 .638	.021 .894	.324 .871	.698 .539
.566 .815	.622 .548	.947 .169	.817 .472	· .864 .466
.901 .342	.873 .964	.942 .985	.123 .086	.335 .212
.470 .682	.412 .064	.150 .962	.925 .355	.909 .019
.068 .242	.667 .356	.195 .313	.396 .460	.740 .247
.874 .420	.127 .284	.448 .215	.833 .652	.601 .326
.897 .877	.209 .862	.428 .117	.100 .259	.425 .284
.875 .969	.109 .843	.759 .239	.890 .317	.428 .802
.190 .696	.757 .283	.666 .491	.523 .665	.919 .146
.341 .688	.587 .908	.865 .333	.928 .404	.892 .696
.846 .355	.831 .218	.945 .364	.673 .305	.195 .887
.882 .227	$\begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$.454 .731	.716 .265	.058 .075
.464 .658		.069 .998	.917 .217	.220 .659
.123 .791		.659 .463	.994 .307	.631 .422
.116 .120		.263 .176	.798 .879	.432 .391
.836 .206		.870 .390	.104 .755	.082 .939
.636 .195	.614 .486	.629 .663	.619 .007	.296 .456
.630 .673	.665 .666	.399 .592	.441 .649	.270 .612
.804 .112	.331 .606	.551 .928	.830 .841	.602 .183
.360 .193	.181 .399	.564 .772	.890 .062	.919 .875
.183 .651	.157 .150	.800 .875	.205 .446	.648 .685

APPENDIX B

DEFINITIONS OF STATISTICAL TERMS

(As Applied in this Program)

- ASSIGNABLE CAUSE--A factor that contributes to variation, usually due to error or process change, which can be identified and corrected within economic limits. A cause that disrupts the normal pattern.
- ANALYSIS OF VARIANCE -- A mathematical method of isolating causes of variation.
- BIAS--A constant error, in one direction, that causes the mean to be offset from the true value.
- CHARACTERISTIC -- A measurable property of a material, product, or item of construction.
- COEFFICIENT OF VARIATION--A measure of relative precision found by dividing the <u>Stand-</u> <u>ard Deviation</u> of a set of values by their average and multiplying by 100 to give an expression in percent.
- LOT (Or Population)--A measured amount of material or construction produced by the same process. A project or selected portion of a project constructed under the same specifications.
- NORMAL CURVE--A curve of the distribution of individual values of measured characteristics about their average having a symmetrical bell-shaped form dependent upon values of the average, \overline{X}^{\dagger} , and the standard deviation, σ^{-1} , of the population.
- RANDOM ERRORS--Differences from the true value, due to chance, that will cancel out when a large number of measurements are averaged.
- RANDOM SAMPLE--A sample taken by the use of a sampling plan in which each unit of the lot must have some known chance of being chosen. For this program, the plan is based on a table of random numbers, Appendix A.
- RANGE, R--The difference between the highest and lowest measured values of a characteristic in the group of samples from a lot.

- SAMPLE--That portion of the sampling unit taken from a single location at a single time. Thus, in this program, two duplicate samples are taken from each sampling unit. When <u>Samples</u> are split, reference is made to the <u>a</u> and <u>b</u> portion of the sample. (Note: This usage is not in accordance with the usual statistical definition.)
- STATISTIC--A summary value such as average \overline{X} , standard deviation σ , or range R computed from a group of measurements.
- UNIT (Or Sampling Unit)--A portion of the material, product, or process feasible for use as a sampling base. The sampling unit is the basis upon which the sampling plan is formulated.
- VARIATION--Differences in measured values of a measurable <u>Characteristic</u> within a stable pattern due to chance, or outside this normal pattern due to assignable causes.
- MEAN, \overline{X} --The sum of the measured values of a <u>Characteristic</u> divided by the number of measurements performed.
- \overline{X}^{I} --The true mean of the lot.
- STANDARD DEVIATION, σ --An estimate of the true dispersions of the measurements. The root-mean-square deviation of the observed values from their mean.

 σ^{-1} --A measure of the true dispersion of the measurements of the lot.

VARIANCE, σ^2 --The mean-square deviation of the observed values of the Sample.

APPENDIX C

STATISTICAL CONCEPTS

The long-range contribution of statistics to highway construction depends not so much upon getting highly trained statisticians into the industry as it does upon creating a statistically minded generation of engineers, inspectors, and others who will have a hand in developing and directing the production processes of tomorrow.

The power of statistics lies in its ability to separate assignable causes of quality variation from inherent variations. Statistical methods make possible the diagnosis and correction of many production troubles and often bring about substantial improvements in product quality and in reduction of rework. Improvement of conventional acceptance procedures often provides better quality assurance at lower inspection cost. Better comparisons between alternate designs and production methods are also possible.

A statistic is a particular measure or a function of a group of measurements, such as an average or a coefficient of correlation, made on suitable samples representing a lot. Such a function is used to provide information regarding certain properties or characteristics of the lot from which the samples were taken. The statistic most commonly used in engineering work is the average, which is the arithmetic mean.

Group "A"	Group ''B''
5.7	5.0
9.3	11.7
10.7	8.8
7.1	9.0
11,3	6.9
· 8,1	14.0
6.1	7.4
6.5	13.8
8,7	1.7
11.8	6.0
7.4	9.8
9.4	10.8
10.2	9.2
7.8	13.2
9.0	5.7
9.7	3.0
10.4	17.7
8,6	7.4
13.2	9,9
Av. 9.0	Av. 9.0
$\sigma^2 = 2$	σ = 4
	1

Table C-1 Numerical Data for Figure C-1

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Figure C-1 Sigma as a Slope of a Line

A statistical approach to the development of highway specifications requires the use of another statistic, the standard deviation, symbolized by the Greek letter σ (sigma), which is a measure of the variation of the individual measurements of a group from their average.

The following explanation shows the significance of the sigma. There are 19 numbers in each group in table C-l. The numbers in the two groups appear to be much alike and have the same average. However, if the numbers in each group are arranged in order of their rank, with the lowest values first, and a plot is prepared on probability paper, it will be noted that the slopes of the lines drawn through the points are quite different. The reason for this difference is that the numbers in group "B" are more variable than those in group "A"; that is, the values are not so close to the average, and the range between the highest and lowest values is greater. As shown in figure C-l, group "B" has a larger σ , and the plotted line has a flatter slope.

Thus, a group of numbers can be characterized by two values. One is its average, and the other is its σ , which may be visualized as the slope of a plotted line. If all the numbers in the group are the same, σ is zero and the line plotted on probability paper is vertical. As the spread is increased by adding more and more high and low values, the line becomes more nearly horizontal. This indicates loss of uniformity.

In this discussion, the standard deviation, σ^2 , is defined mathematically as the square root of the sum of the squares of the deviations of the measurements from their average, divided by one less than the total number of measurements. Sigma, σ^2 , is the second essential statistic, in addition to the average, for the development of realistic acceptance limits by the use of statistical methods. Usually the numerical value of σ is calculated. For example, consider the group consisting of the four numbers 2, 8, 4, 6. If X is used to represent the individual numbers and \overline{X} denotes their average, the calculations for σ follow:

$$X = 2, 8, 4, 6$$

$$\overline{X} = \frac{\sum X}{n}$$

$$\overline{X} = \frac{2+8+4+6}{4} = \frac{20}{4} = 5$$

$$\sigma = \sqrt{\frac{\sum (X - \overline{X})^2}{(n-1)}}$$

$$\sigma = \frac{(2-5)^2 + (8-5)^2 + (4-5)^2 + (6-5)^2}{4-1} = \frac{9+9+1+1}{3} = \frac{20}{3}$$

$$\sigma = \sqrt{\frac{20}{3}} = 2.58$$

Another statistic of a group of numbers is the range, which is simply the numerical difference between the smallest and largest values in the group.

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In table C-2, each group of numbers in table C-l is arranged so that the numbers are divided into subgroups called intervals, each of which has a spread of 3 units. It is seen that the numbers in group "A" are closely grouped near the average, while the numbers in group "B" are more widely dispersed.

Intervals	1,1-3,0	3.1-6.0	6.1-9.0	9.1-12.0	12.1-15.0	15,1-18,0
Group ''A'' σ = 2		5.7	6.1 6.5 7.1 7.4 7.8 8.1 8.6 8.7 9.0	9.3 9.4 9.7 10.2 10.4 10.7 11.3 11.8	13.2	
Group "B" σ = 4	1.7 3.0	5.0 5.7 6,0	6.9 7.4 7.4 8.8 9.0	9.2 9.8 9.9 10.7 11.7	13.2 13.8 14.0	17.7

Table C-2 Grouping of Numbers

A distribution chart, called a histogram, can be plotted by taking each occurrence of a number in an interval as one scale unit of height for the ordinate of a bar chart. Histograms for the groups of numbers in table C-1 and C-2 are shown in figure C-2. If desired, a smooth curve, which characterizes each group of numbers, may be drawn through the points located by the ends of the ordinates. This curve is called the frequency distribution curve.

The normal frequency distribution curve is bell-shaped, as shown in figure C-2. One of the basic assumptions made in this study is that the normal distribution curve will adequately describe the random properties of the material to be sampled.

Thus, normal distribution curves, drawn using the computed values \overline{X} and σ , will be used to predict the frequency of occurrence of measurements and to provide a basis for developing specification tolerances related to actual variability.



Figure C-2 Typical Bar Charts and Normal Distribution Curves

NOTE

It is usually assumed that numbers representing observations are distributed around the average in a pattern conforming to the normal distribution curve, which is symmetrical. This is not always the case in highway work, particularly when some limit is approached. A curve representing the distribution then is not exactly symmetrical and is called a skewed distribution. Application of the ordinary statistical methods to such a curve will not produce entirely accurate results. However, this inaccuracy is not generally serious. Under the worst conditions, at least 89 percent of the observations will lie between the limits $\overline{X} + 3 \sigma$ and $\overline{X} - 3 \sigma$, in comparison with 99.7 percent for an entirely normal distribution.

Since the normal distribution curve may be used to describe data, standard tables have been prepared so that the percentage of the area under the curve between any two points can be readily found. The method of determining the relative frequency of values larger or smaller than any given number, such as X_1 , is as follows:

The horizontal distance from the ordinate at the centerline of the distribution curve, the average, \overline{X} , to the ordinate for any given number, X_1 , is expressed in terms of sigma units and is denoted by z. As shown in figure C-3, the value of z is found by taking the difference between X_1 and \overline{X} and dividing by σ . For example, if $\sigma = 7$ for the distribution curve in figure C-3, and $X_1 = \overline{X} + 9$, then z = 9/7 = 1.282. A table giving the area under the normal distribution curve for various values of z will show that when z = 1.282, the area (shown shaded in figure C-3) under the part of the curve for all values lower than X_1 is about 90 percent of the total area. This means for a distribution in which σ is 7, the value of X_1 will be less than $\overline{X} + 9$ about 90 percent of the time, and will be greater than $\overline{X} + 9$ only about 10 percent of the time. If a decimal is used instead of a percentage, it may be said that the probability of X_1 being greater than $\overline{X} + 9$ is 0.10.



Figure C-3 Relation of Percentage of Total Area Under Curve to z

The fact that the percentage or probability is predictable is the basis for setting numerical acceptance limits which take into consideration the ever-present risks of accept-ing poor material or rejecting good material.

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APPENDIX D

COMPUTER PROGRAM FOR ANALYSIS OF VARIANCE

This program is written for the purpose of analyzing data obtained in statisticalquality-control projects wherein random measurements of characteristics of present construction have been made.

The assumption is made that the measurements will have a normal distribution.

The program is written in FORTRAN to insure versatility of its use.

INPUT DATA

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The input data is coded to provide two types of information: identification of the data and actual test results.

The first item of input data for the computer program is identification of the data. The format of the first item is as follows:

Column 1 of first card:

The number 1

Columns 2 through 72 of first and second card:

Any identifying information describing the data to be analyzed, such as, the person submitting the data and the date.

The second item of input data is the actual test results. The test results must be data obtained by taking duplicate samples, D_1 and D_2 , from random units of the lot, and each duplicate sample must be split, or divided, into two portions, a and b, as nearly identical as possible. The format for the test results input data is as follows:

Columns 1 through 15:

The a portion of duplicate sample No. 1.

Columns 16 through 30:

The b portion of duplicate sample No. 1.

Columns 31 through 45:

The a portion of duplicate sample No. 2.

Columns 46 through 60:

The b portion of duplicate sample No. 2.

The data is entered with the decimal point in columns 11, 26, 41, and 56. One data card is coded for each set of duplicate samples.

1 2 3 4 5	6 7 8 9 10 1		5 27 28 29 30 31 32 33 34 35 36 37 38 39 40 4	1 42 43 44 45 46 47 48 49 50 51 52 53 54 55 5	6 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72
1 ANAL	YSISI	DIF VARIANCE OF I	ATA FROM SUBGR	aup 11 11 11 11 11 11	
	ISTIC	AL APPROACH TO C	UALITY CONTROL	MATERIAL PASSI	NG 3, 8 INCH SIJEVE
	117 1/1	.6	.9		
	69	<u>372</u>	7	7	5
1	65	A	7	7,3	6
	, 17 131	9	9	3	7
	76	3	,/,,,,,,,,,,,,,,,,,,,,,,,,7, <i>S</i> ,	P	R
	7,1	4	1	2	<i>B</i>
	17 ₁ 3	6	9	9	3
	72	2	4.111111111111111	2	5
	7,6	5	8 114	7 66	9
	1 7 4	2	R	11 1 1 1 1 1 1 1 1 1 1 6 6 6	B
	76	131111116B	191111111111111111111111111111111111111	9	2
	17.4	9.1.1.1.1.1.66	5	7	2
	7,2	3	5	7	5
	65	1.1.1.1.1.1.1.1.1.7.2	3	9	2
	68	8	6	A	7
	1 76	1	2	H	H
	170	17	6	7	9
	7,0	3	6	3	7
	165	5	PILLING TR	η_{1}	0
	1.170	17	181 I I I I I I I I I 7131	B	4
	7,2	4	7	5 67	4
		<u></u>	<u></u>		

Figure D-1 Sample Input

ANALYSIS OF VARIANCE OF DATA FROM SUBGROUP 1 STATISTICAL APPROACH TO QUALITY CONTROL MATERIAL PASSING 3/8 INCH SIEVE

ARITHMET	IC MEAN	=	71.09285
MATERIAL	VARIANCE	=	33194
SAMPLING	VARIANCE	=	2.48500
TESTING	VARIANCE	=	6.99833
OVERALL	VARIANCE	Ξ	9.15138
OVERALL	SIGMA	=	3.02512

Figure D-2 Sample Output

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A zero test result is used as the last piece of data to show that the test result input data for a specific problem is complete. This last piece of data indicates to the computer program that all data of the specific problem has been read and stored. The zero value is recorded on the data sheet as the a portion of duplicate sample No. 1.

An example of the input-data format is shown in figure D-l.

OUTPUT DATA

The results of the analysis of variance of a set of test data are:

- The arithmetic mean
- The inherent or material variance, σ^2
- The variance between samples, σ_{s}^{2}
- The variance due to inaccuracies of testing, σ^2
- The overall variance of the measurement, σ'^2
- The overall standard deviation, σ^{-1} .

An example of the output is shown in figure D-2.

PROGRAMING PROCEDURES

The following steps suggest a method that may be used to put this program into production on a FORTRAN programing system.

- (1) Review the FORTRAN program listing figure D-3, and note changes to the program that appear to be necessary. In many cases changes will have to be made so that the program can be compiled and run on the user's equipment. Normally, the changes that are needed will be limited to the input and output statements. This means that changes may be required for those operations that bring information into the computing system, or transfer results out of the system. As an example, it may be necessary for one user to change all PRINT statements included in the attached listing to PUNCH statements. This change is needed because the computing system to be used produces results in punched card form in place of printing them directly by means of a printing device. The FORMAT statements included in the listing that control the arrangement of input and output information must be checked carefully. When computer controlled line printers are to be used, special attention should be given to the printer carriage control codes.
- (2) Keypunch or type the FORTRAN source program. The source program is then compiled on the user's system by means of the compiler to obtain the object computer program. The compilation of a FORTRAN source program is a routine operation for most computing centers equipped to handle FORTRAN compilation. Any special instructions needed to perform this step can readily be found with the operating instructions that are furnished with the FORTRAN compiler programs.

C		ANALYSIS OF VARIANCE COMPUTER PROGRAM
		THE PROGRAM IS FOR THE ANALYSIS OF DATA OBTAINED IN THE SAMPLING AND TESTING OF CONSTRUCTION MATERIALS. THIS IS FOR ORGANIZATIONS USING THE STATISTICAL CONCEPTS OF MATERIAL ACCEPTANCE TESTING.
		FIXED POINT WORD SIZE = 04 FLOATING POINT WORD SIZE = 10 + 2
		INITIALIZE ACCUMULATORS. SUMA, IS THE ACCUMULATOR OF ALL VALUES SUMB, IS THE ACCUMULATOR OF SQUARED DIFFERENCES OF TEST PORTIONS SUMC, IS THE ACCUMULATOR OF SQUARED DIFFERENCES OF DUPLICATES SUMD, IS THE ACCUMULATOR OF SQUARED SUM OF DUPLICATES
	1	SUMA = 0.0 SUMB = 0.0 SUMC = 0.0 SUMD = 0.0
C C		INITIALIZE COUNTER FOR NUMBER OF UNITS OF A LOT
c c		N = 0
с С		READ AND STORE PROBLEM IDENTIFICATION INFORMATION
		READ 900
		READ AND STORE PROBLEM DATA
		DIA IS THE A PORTION OF DUPLICATE SAMPLE NO. 1 DIB IS THE B PORTION OF DUPLICATE SAMPLE NO. 1 D2A IS THE A PORTION OF DUPLICATE SAMPLE NO. 2 D2B IS THE B PORTION OF DUPLICATE SAMPLE NO. 2
	10	READ 910, D1A, D1B, D2A, D2B
		TEST FOR END OF COMPUTER RUN OR END OF DATA OR ADDITIONAL DATA
		IF (D1A)15,30,20
C C		WHEN DIA IS SUBMITTED AS A MINUS VALUE THIS INDICATES END OF RUN
c	15 20	TYPE 920 STOP 111 N = N + 1 SUM1 = D1A + D1B SUM2 = D2A + D2B
С		ACCUMULATE NECESSARY SUMS

Figure D-3 FORTRAN Program Listing (Sheet 1 of 3)

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```
С
              = SUMA + SUM1 + SUM2
      SUMA
             = SUMB + ( D1A - D1B )**2 + ( D2A - D2B )**2
= SUMC + ( SUM1 - SUM2 )**2
      SUMB
      SUMC
             = SUMD + ( SUM1 + SUM2 )**2
      SUMD
      GO TO 10
С
С
       ALL DATA HAS BEEN READ, CONTINUE SUM OF SQUARES CALCULATIONS
C.
   30 XN
              = N
      CF
              = SUMA * SUMA / 4.0 / XN
С
С
       CALCULATE UNIT SUM OF SQUARES
С
      USOFS = SUMD / 4.0 - CF
С
       CALCULATE DUPLICATE SAMPLE SUM OF SQUARES
С
С
      DSSOFS = SUMC / 4.0
С
       CALCULATE SUM OF SQUARES FOR TEST PORTIONS
С
С
      SOFSTP = SUMB / 2.0
С
С
       CALCULATE MEAN SQUARES
С
              = USOFS / ( XN - 1.0 )
      XMS1
      XMS2
              = DSSOFS / XN
      XMS3
              = SOFSTP / 2.0 / XN
С
С
       CALCULATE MATERIAL VARIANCE
С
      SIGA2 = ( XMS1 - XMS2 ) / 4.0
С
С
       CALCULATE SAMPLING VARIANCE
С
      SIGS2 = (XMS2 - XMS3) / 2.0
С
       CALCULATE TESTING VARIANCE
С
С
      SIGT2 = XMS3
С
       CALCULATE OVERALL VARIANCE
С
С
      OVVAR = SIGA2 + SIGS2 + SIGT2
С
       CALCULATE OVERALL SIGMA, STANDARD DEVIATION
С
С
      OVSIG = SQRTF ( OVVAR )
С
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Figure D-3 FORTRAN Program Listing (Sheet 2 of 3)

```
CALCULATE ARITHMETIC MEAN
С
С
             = SUMA / XN / 4.0
      XBAR
С
С
       PRINT RESULTS
С
      PRINT 900
      PRINT 930, XBAR, SIGA2, SIGS2, SIGT2, DVVAR, DVSIG
      GO TO 1
с
с
                    INPUT AND OUTPUT FORMATS
Ċ
  900 FORMAT ( 72H
                         / 72H
     1
                                     }
     2
  910 FORMAT ( 4F15.4 )
  920 FORMAT ( 12H END OF RUN )
                                              =,F10.5 / 10X,9HMATERIAL ,
  930 FORMAT ( 1HK,9X,20HARITHMETIC MEAN
                11HVARIANCE =, F10.5 / 10X, 20HSAMPLING VARIANCE =, F10.5,
     1
                / 10X,20HTESTING VARIANCE =,F10.5 / 10X,9HDVERALL
     2
                                                                       .
               11HVARIANCE =, F10.5 / 10X, 20HOVERALL SIGMA
                                                                  =,F10.5 )
     3
      END
```

Figure D-3 FORTRAN Program Listing (Sheet 3 of 3)

Each data set consists of the items that were discussed previously. Any number of data sets can be assembled and submitted for processing at one time. For a clearer understanding of the data set items and their order, the reader should carefully study the separate READ instructions and the data lists that are included in the FORTRAN listing together with the example problem.

An example of the arrangement of the object deck and input data cards is shown in figure D-4.

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Figure D-4 Object Deck and Data Cards

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