NATIONAL INSTITUTE FOR CONGESTION REDUCTION

FINAL REPORT
APRIL 2024

Optimizing Service Areas to Reduce Congestion and Enhance Equity in Access to Transportation Systems

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National Institute for Congestion Reduction
University of South Florida
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Technical Report Documentation Page

<table>
<tr><td>1. Report No.</td><td>2. Government Accession No.</td><td>3. Recipient's Catalog No.</td></tr>
<tr><td colspan="2" rowspan="2">4. Title and Subtitle
Optimizing Service Areas to Reduce Congestion and Enhance Equity in Access to Transportation Systems</td><td>5. Report Date
April 2024</td></tr>
<tr><td>6. Performing Organization Code</td></tr>
<tr><td colspan="2">7. Author(s)
Daniel Rodriguez-Roman, José López-Martínez, Alberto M. Figueroa-Medina, Carlos del Valle-González</td><td>8. Performing Organization Report No.</td></tr>
<tr><td colspan="2" rowspan="2">9. Performing Organization Name and Address
University of Puerto Rico at Mayagüez
Department of Civil Engineering and Surveying
Call Box 9000
Mayagüez, PR 00681-9000</td><td>10. Work Unit No. (TRAIS)</td></tr>
<tr><td>11. Contract or Grant No.
69A3551947136, 79075-00-SUB A</td></tr>
<tr><td colspan="2" rowspan="2">12. Sponsoring Organization Name and Address
U.S. Department of Transportation
University Transportation Centers
1200 New Jersey Avenue, SE
Washington, DC 20590
United States

National Institute for Congestion Reduction
4202 E. Fowler Avenue
Tampa, FL 33620-5375
United States</td><td>13. Type of Report and Period Covered
Final Report,
June 2021 – September 2022</td></tr>
<tr><td>14. Sponsoring Agency Code</td></tr>
<tr><td colspan="3">15. Supplementary Notes</td></tr>
<tr><td colspan="3">16. Abstract
This report presents optimization-based methods for the design of transportation service areas. The contribution of this report is twofold. First, optimization models are presented for service area design problems that simultaneously account for the goals of system operators and users, with particular emphasis on the objectives of reducing congestion and enhancing equity-in-access to transportation services. The models account for spatial coverage constraints that are specified in real-world situations to ensure equitable access to transportation services. Two service area design applications are considered. In the first application, the problem of designing transit routes and the accompanying paratransit service area is examined. This problem is of practical interest to US transit agencies given federal regulations that set minimum spatial coverage of paratransit services based on fixed transit route alignments. In the second application, the problem of defining the service area for dockless micromobility services is considered. This design problem is also of practical interest given the goal of cities to ensure equitable spatial access for emerging travel alternatives and the need for these alternatives to be economically viable. The second contribution of the report is a set of genetic algorithm-based heuristics designed to search for solutions to the proposed problems. In the heuristics, service areas are represented as polygons that are modified using easy-to-implement geometric operations. Numerical tests were performed to illustrate the application of the proposed design models and heuristics.</td></tr>
<tr><td colspan="2">17. Key Words
Service area, optimization, equity, transit, paratransit, micromobility</td><td>18. Distribution Statement</td></tr>
<tr><td>19. Security Classification (of this report)
Unclassified.</td><td>20. Security Classification (of this page)
Unclassified.</td><td>21. No. of Pages
40

22. Price</td></tr>
</table>

Acknowledgements

The research team would like to thank Skootel for providing insights on the planning and operations of micromobility systems. The authors also want to express their gratitude for the administrative support provided by the staff of the Civil Infrastructure Research Center of the University of Puerto Rico at Mayagüez.

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Abbreviations and Acronyms

ADA	Americans with Disabilities Act
BNDP	Bus Network Design Procedure
GA	Genetic Algorithm
GA-MSA	Genetic Algorithm – Micromobility Service Area
GA-PSA	Genetic Algorithm – Paratransit Service Area
GIS	Geographic Information System
IT	Information Technology
MSA	Micromobility Service Area
PSA	Paratransit Service Area
SJMA	San Juan Metropolitan Area
US	United States

Executive Summary

Communities, transportation service providers, and planning agencies are often required to identify service areas within which transportation vehicles can operate. The selection of which neighborhoods are included in or excluded from a service area can have significant implications on the level of accessibility, economic sustainability, and congestion reduction potential of a transportation system. This report presents optimization-based methods for the design of transportation service areas. The optimization-based approaches proposed in the report can be used to guide the service area design process given multiple conflicting objectives and constraints.

The main contribution of this report is twofold. First, optimization models are presented for service area design problems that simultaneously account for the goals of system operators and users, with particular emphasis on the objectives of reducing congestion and enhancing equity-in-access to transportation services. The models account for spatial coverage constraints that are specified in real-world situations to ensure equitable access to transportation services. Two service area design applications are considered in this report. In the first application, the problem of designing transit routes and the accompanying paratransit service area is examined. This problem is of practical interest to transit agencies in the United States given federal regulations that set minimum spatial coverage of paratransit services based on fixed transit route alignments. In the second application, the problem of defining the service area for dockless micromobility services is considered. This design problem is also of practical interest given the goal of cities to ensure equitable spatial access for emerging travel alternatives and the need for these alternatives to be economically viable.

Second, this report contributes a set of genetic algorithm-based heuristics that are designed to search for solutions to the proposed problems. In the heuristics, service areas are represented as polygons that are modified using easy-to-implement geometric operations. The performance of the heuristics was examined in numerical tests that also illustrated the application of the design models. The San Juan Metropolitan Area was used as a case study for the application setting of the methodology for the design of paratransit service areas and transit route networks, while the city of Mayagüez was used as the application setting of the methodology for the design of micromobility service areas.

Chapter 1. Introduction

As new travel alternatives emerge and existing services are redesigned, communities, transportation service providers, and planning agencies are often required to identify service areas in which transportation services can operate. The selection of which neighborhoods are included in or excluded from a service area can have significant implications on the level of accessibility, economic sustainability, and congestion reduction potential of a transportation service. As is often the case in transportation planning processes, the various stakeholders concerned with service area design issues have objectives that do not necessarily align with each other, which is why deliberative processes are often required before a design is operationalized. In response to this practical planning problem, the primary goal of this report is to present a quantitative approach that can be used in decision-making processes to generate alternative service area designs given multiple objectives and spatial coverage constraints. In this report, a spatial coverage constraint refers to a requirement that a set of zones in the study area is contained by a service area design. A prime example of this type of constraint is observed in the laws (e.g., Americans with Disabilities Act) and regulations governing paratransit services in the United States, which mandate that any agency that operates a fixed route transit system must provide paratransit services within at least three-fourths of a mile on each side of the transit routes.

The contribution of this report is twofold. First, spatial optimization models are presented for multi-objective service area design problems, with particular emphasis on the operational and equity-in-access objectives of service operators and users. The proposed models account for spatial coverage constraints that are present in real-world planning situations. Second, this report contributes genetic algorithm (GA)–based heuristics to solve the proposed design problems. Optimization models are presented for two service area design problems:

i. *Paratransit Service Area Design Problem*: The first problem considers the design of paratransit service areas in the context of Americans with Disabilities Act (ADA) regulations that transit operators in the United States must satisfy. Given the relationship between paratransit service areas and the alignment of fixed transit routes, a combined transit route network and paratransit service area design model is proposed. The design problem is formulated as a multi-objective, mixed integer optimization problem with constraints. Three design objectives are considered: maximizing the operational effectiveness of the fixed route transit service, maximizing the operational effectiveness of the paratransit service, and maximizing equity-in-access for both.
ii. *Micromobility Service Area Design Problem*: Cities are interested in micromobility service areas (MSA) as they are one of the system features that can be designed to ensure that historically disadvantaged communities have access to micromobility services (e.g., shared e-scooters, bicycles) (Blickstein et al., 2019; Price et al., 2021). Naturally, micromobility operators are also interested in the design of their service area as it has a direct impact on the demand for their vehicles. In this report, a bi-objective optimization problem is proposed for the design of micromobility service areas considering the operators and planning agencies' perspectives.

The report is organized as follows. A literature review is presented in Chapter 2, which focuses on the optimization methods developed to design service areas, as well as other works that consider the related concepts of coverage areas and shape constraints. The design problems are discussed in Chapter 3, and the heuristics in Chapter 4. Illustrative applications of the models and the heuristics are presented in Chapter 5, followed by a summary of the developed methods, as well as opportunities for future research, in Chapter 6.

Chapter 2. Literature Review

A *transportation service area* is defined in this project as a region within which a vehicle (e.g., bus, e-scooter) can operate to provide a service to population groups. A related concept is a *coverage area*, a term usually used in the literature to refer to areas of influence around a system component, such as a bus stop, from which it is reasonable to expect that demand for a service could originate. Coverage areas are often incorporated in transit network design optimization models (Ibarra-Rojas et al., 2015), particularly to account for accessibility objectives. For example, Murray (2003) proposed a set covering problem to find the stop locations that maximize access to a transit service. Wei et al. (2017) extended this work by presenting a problem that considered both operational effectiveness and access equity in the selection of transit routes; access equity was measured in terms of the disadvantaged populations living within the transit stop coverage areas. Set covering problems and coverage area considerations are also present in location problems for vehicle charging stations, as in the work of Tang et al. (2011) who used Voronoi diagrams to define charging coverage areas.

As in the case of coverage problems, service area problems have also been approached from an optimization perspective. Chang and Schonfeld (1991) proposed analytical models to compare fixed route conventional bus and flexible route subscription bus systems for providing feeder services to a single point. Their analytical models can be used to optimize rectangular service areas for the flexible route service. Li and Quadrifoglio (2009) also proposed an analytical model that can be used to compare the operations of fixed route and flexible route feeder services and, in particular, to determine the optimal number of zones for each type of operation. Kim and Schonfeld (2013) proposed nonlinear mixed integer optimization problems for the design of a mixed-fleet transit system composed of conventional and flexible bus services; in this model, multiple rectangular service areas can be optimized. Additional analytical models to optimize rectangular service areas for flexible route bus service, among other service features, can also be found in the work by Nourbakhsh and Ouyang (2012), Kim et al. (2019), and Kim and Schonfeld (2012, 2014). Although these analytical models only consider rectangular service areas and make other significant assumptions regarding, for example, the physical space of operation and the spatial distribution of demand, they offer valuable, computational, inexpensive insight about the operations and service quality of different types of bus services, and they provide general guidance regarding their design. The work by Pan et al. (2015) extends previous work on flexible feeder bus services by proposing a mixed integer linear programming model to optimize irregularly shaped service areas and transit route planning. The service area in this model is defined by the sequence of vehicle visits to designated pickup points (i.e., vehicle tours) in predefined blocks around the transfer station.

Liang et al. (2016) and Li and Szeto (2019) proposed integer and mixed integer programming models, respectively, to define the service area of taxi services. Liang et al. considered a profit maximization design objective, while Li and Szeto considered social welfare maximization. Research on the taxi service region problem has used models with binary decision variables that define which nodes in a transportation network are within the service area (Zhou & Chow, 2021). This modeling approach is also present in models proposed for the optimization of congestion pricing boundaries; in these problems, the nodes that must be within the charging region are identified (Sumalee, 2004; Zhang & Yang, 2004). The cut-set heuristic proposed by Zhang and Yang (2004) for the congestion pricing problem has been adapted to identify optimal autonomous vehicle zones (Chen et al., 2017) and vehicle restriction areas (Shi et al., 2014). In contrast to these graph-based models, Bischoff et al. (2018) proposed a simulation-based model to optimize the service areas of pooled ride-hailing operators. In their model, an initial service area composed of equally sized polygons is reduced (by excluding component polygons) based on the simulated performance of the service. Liu and Ouyang (2021) extended this design problem to consider fixed transit routes; the service region of interest in this model is a square service region around each transit station in which last-mile services operate.

The actual geometric shape of the service region is not a direct concern in the reviewed literature. In practice, however, the geometry of the service area can be of critical importance, as exemplified by the previously discussed service area regulations in the United States (Government Accountability Office, 2012). The geometric shape of an area can also be of relevance when communicating a scheme to the public, as in the case of cordon or area pricing schemes (Maruyama et al., 2014; Rodriguez-Roman & Allahviranloo, 2019; Rodriguez-Roman & Ritchie, 2019).

The topic of service area design is of practical importance. This is particularly true in the context of ongoing efforts to redesign bus networks and/or introduce new information technology (IT)–based transit services. As noted by TCRP Report 221, agencies redesigning bus networks must also consider the impact to their paratransit service area, and some have decided to grandfather “in either specific users or geographic areas to ensure continuity of paratransit service” (National Academies of Sciences, Engineering, and Medicine, 2021). The methods presented in this report provide agencies with quantitative tools to jointly make these redesign decisions.

This report advances previous research by presenting new methods to solve service area design problems given potentially conflicting social and operational objectives. In contrast to previous work, coverage constraints of practical interest are incorporated in the design process. In addition, flexible methods for defining service areas are proposed.

Chapter 3. Service Area Design Problems

This chapter presents two service area design problems that are special cases of the general optimization problem presented next. Consider a set of decision-makers interested in designing a service area such that a set of objectives are minimized. Let $\boldsymbol{F} = \left(F_1(\boldsymbol{s},\boldsymbol{x}), F_2(\boldsymbol{s},\boldsymbol{x}), \ldots, F_{\boldsymbol{O}}(\boldsymbol{s},\boldsymbol{x})\right)$ be the set of objectives of interest, $\boldsymbol{s}$ be the boundary of the service area, and $\boldsymbol{x}$ represents other decision variables that might be connected to the design problem. Also, define $\boldsymbol{\Omega}$ as the set of acceptable service area boundaries given the spatial coverage constraints and $\boldsymbol{\Xi}$ as the set of values that $\boldsymbol{x}$ can assume. The general design problem of interest can be formulated as:

$$\min \ \boldsymbol{F}(\boldsymbol{s},\boldsymbol{x}) \tag{1}$$

$$subject\ to$$

$$\boldsymbol{h}(\boldsymbol{s},\boldsymbol{x}) \leq \boldsymbol{b} \tag{1.1}$$

$$\boldsymbol{s} \in \boldsymbol{\Omega} \tag{1.2}$$

$$\boldsymbol{x} \in \boldsymbol{\Xi} \tag{1.3}$$

where Equation 1.1 represents a set of generic functional constraints (e.g., with $\boldsymbol{b}$ representing threshold values such as a budget level). In the next subsections, the generic design problem is extended to consider paratransit (Section 3.1) and micromobility (Section 3.2) service area design. For illustrative and context purposes, specific mathematical formulations are given to the objectives considered in presented design models. However, the heuristics proposed to solve the design problems do not depend on the type of formulation used. Therefore, the optimization models and heuristics contained in this report can be easily adapted to other types of service area design situations.

3.1. Paratransit Service Area Design Problem

A transit agency's budget is usually divided across the different types of operations and services that are designed to address different user needs. US transit agencies that operate fixed route bus services are required by the ADA and related regulations to also operate complementary paratransit systems that serve the travel needs of individuals with disabilities. A complementary paratransit system must, at a minimum, provide service to trips that originate and end within three-fourths of a mile on each side of the fixed routes that constitute the bus network (Code of Federal Regulations, 1991). Therefore, when an agency is designing or redesigning a fixed route network, it is also selecting the minimum paratransit service area (PSA) and, to a degree, implicitly deciding on the minimum demand and cost levels of that complementary system. Extending the PSA beyond the regulatory minimum has the positive effect of increasing a paratransit service's accessibility. However, this can be economically challenging as paratransit costs can be significant relative to the number of trips served by these systems. Besides the shape of the service area, the objectives, or criteria, used in the process of designing or redesigning a transit service is closely regulated in the US context. Among these are Federal Transit Administration directives that require equity analysis for the redesign or change of transit service features to avert disparate impact or disproportionate burden, particularly to minorities and low-income population groups (FTA, 2012). Therefore, transit design methods are required to account for equity considerations.

In this section, an optimization problem is proposed that can be used by transit agencies to jointly plan a fixed route transit network and its complementary PSA considering operational and social objectives and practical constraints related to the operation of these services. The problem focuses on the design of a centralized service area's PSA, in which the entire PSA is treated as a single zone as opposed to a decentralized PSA composed of multiple, independently managed PSAs (Shen & Quadrifoglio, 2013).

Consider a transit operator that is interested in designing the layout of routes ($\boldsymbol{R}$) for a fixed route bus network, determining the service frequency of each route in the network ($\boldsymbol{f}$), and defining the service area ($\boldsymbol{s}$) for the complementary paratransit service. Assume that the design objectives of interest are to minimize the service ineffectiveness of the bus service ($M_b(\boldsymbol{R},\boldsymbol{f})$), the service ineffectiveness of the paratransit service ($M_p(\boldsymbol{s})$), and a measure of inequality in access to the transit services ($E(\boldsymbol{R},\boldsymbol{f},\boldsymbol{s})$), which is used as a proxy for the goal of increasing equity-in-access. As is standard for bus network design problems, assume that there is a set of design constraints $\boldsymbol{h_R}(\boldsymbol{R})$ that control the characteristics of the routes in the bus network (e.g., number of routes, minimum length of a route), a set of constraints $\boldsymbol{h_f}(\boldsymbol{f})$ that restrain the possible values of the route service frequencies (e.g., maximum and minimum headway values), and that $\boldsymbol{b_R}$ and $\boldsymbol{b_f}$, respectively, are the threshold values for these sets of constraints. Additionally, let $\boldsymbol{\Omega}$ represent all spatial constraints that the PSA must satisfy. Define $C_b(\boldsymbol{R},\boldsymbol{f})$ and $C_p(\boldsymbol{s})$ as the cost of operating the bus and paratransit services, respectively, and H as the design budget. Given this notation, the design problem can be defined in general terms as:

$$\min\ \boldsymbol{Z} = \left(M_b(\boldsymbol{R},\boldsymbol{f}), M_p(\boldsymbol{s}), E(\boldsymbol{R},\boldsymbol{f},\boldsymbol{s})\right) \tag{2}$$

$$subject\ to$$

$$\boldsymbol{h_R}(\boldsymbol{R}) \le \boldsymbol{b_R} \tag{2.1}$$

$$\boldsymbol{h_f}(\boldsymbol{f}) \le \boldsymbol{b_f} \tag{2.2}$$

$$C_b(\boldsymbol{R},\boldsymbol{f}) + C_p(\boldsymbol{s}) \le H \tag{2.3}$$

$$\boldsymbol{s} \in \boldsymbol{\Omega} \tag{2.4}$$

Definitions for the bus effectiveness objective M_b and the sets of constraints 2.1 and 2.2 are the subject of many studies found in the literature. In the numerical experiments performed for this report and the discussion that follows, the formulation of these bus network design components is based on the work of Fan (2004). In practice, the specific formulation of the objective functions and constraints in design problem 2 would depend on a transit agency's goals, the performance measures stemming from these goals, and the modeling tools available in the planning process, among other factors. $M_b(\boldsymbol{R},\boldsymbol{f})$ could be used to directly account for the goal of reducing congestion or travel delays in the road network. In this report, it is assumed that an effective bus service will attract travelers away from the auto mode, and thus help reduce road congestion. However, if a more direct quantification of congestion reduction was desired, an additional congestion objective could be introduced to Equation 2, as discussed in Section 3.1.3. Note that the solution approach proposed in Chapter 4 would not be significantly affected by introducing an additional congestion-specific objective.

To ground the discussion in Section 5.1, the functions used in the simulations are discussed next. In this discussion, it is assumed that the design for the bus route network is performed using a discrete network with a set of nodes $\boldsymbol{N_b}$, that demand is fixed, and that there is a single design period (e.g., peak hour). The demand for the paratransit system is also assumed to be fixed; trips are assumed to start and end in a set of zones represented by nodes $\boldsymbol{N_p}$.

3.1.1. Design Objectives for PSA Problem

The bus effectiveness objective is defined as the weighted sum of the total bus user cost and the unmet bus demand cost. Let d_{ij} be bus travel demand from node i to node j, d_{ijr} be trips on route $r \in \boldsymbol{R}_{ij}$

(where $\boldsymbol{R}_{ij}$ refers to the set of routes that connect i and j, including routes with transfers), and t_{ijr} be the corresponding travel time, which could be assumed to depend on $\boldsymbol{f}$. Also, let δ_{ij} be a binary variable that equals 1 if i and j are not connected, either directly or by transfers between routes, by $\boldsymbol{R}$, and 0 otherwise. The parameters ω_1 and ω_2 denote the weights of the user cost and unmet demand cost terms. Then, the bus effectiveness objective can be computed using:

$$M_b(\boldsymbol{R}, \boldsymbol{f}) = \omega_1 \sum_{i \in \boldsymbol{N_b}} \sum_{j \in \boldsymbol{N_b}} \sum_{r \in \boldsymbol{R}_{ij}} d_{ijr} t_{ijr}(\boldsymbol{f}) + \omega_2 \sum_{i \in \boldsymbol{N_b}} \sum_{j \in \boldsymbol{N_b}} \delta_{ij} d_{ij} \tag{3}$$

The paratransit effectiveness objective used in this study accounts for the unmet paratransit demand. It is assumed that only trips with origins and destinations within the service area are served by the paratransit system. Let q_{ij} be the paratransit travel demand from node i to node j and γ_{ij} be a binary variable that equals 1 if i or j are not within service area $\boldsymbol{s}$, and 0 otherwise. The paratransit ineffectiveness objective that is minimized can be stated as:

$$M_p(\boldsymbol{s}) = \sum_{i \in \boldsymbol{N}_p} \sum_{j \in \boldsymbol{N}_p} \gamma_{ij} q_{ij} \tag{4}$$

The previous objective function could include terms that account for travel times and aggregate measures of paratransit user cost, but to include these terms would require explicit consideration of additional decision variables (e.g., paratransit fleet size), which is out of the scope of this report. Note that, in contrast to most of the flexible-route transit studies considered in the literature review, here space is discretized (i.e., demand originates from discrete points). However, given that the heuristic presented in Chapter 4 does not depend on the type of objective function formulations (e.g., the heuristic does not use derivatives), there is no obstacle to using a continuum approximation approach, like the one proposed by Rahimi et al. (2014) to specify $M_p(\boldsymbol{s})$.

The inequality objective is defined in terms of the spatial access that different population groups have to the two transit services being considered. Let σ_{gm} represent an aggregate measure of the level of access that population group g ($g \in \boldsymbol{G}$) has to mode m (where $m \in \boldsymbol{M} = \{\text{bus, paratransit}\}$). Given this measure, the inequality in the distribution of access levels can be quantified using an inequality index, of which there are several (Ramjerdi, 2006). Here, the Atkinson index (Ramjerdi, 2006) is used as the inequality objective:

$$E(\boldsymbol{R}, \boldsymbol{f}, \boldsymbol{s}) = 1 - \left[\frac{1}{|\boldsymbol{G}||\boldsymbol{M}|} \sum_{g \in \boldsymbol{G}} \sum_{m \in \boldsymbol{M}} \left(\frac{\sigma_{gm(\boldsymbol{R},\boldsymbol{f},\boldsymbol{s})}}{\bar{\sigma}} \right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \tag{5}$$

where ε is a parameter set by the analyst that indicates the level of aversion to inequality and $\bar{\sigma}$ is the average value of the σ_{gm} values. The value of E ranges from zero (perfect equality) to one. In the numerical tests, $\sigma_{g,bus}$ is computed as the proportion of the population of group g that is within the coverage area of the bus service, as defined by a distance parameter from the bus routes, and $\sigma_{g,paratransit}$ as the proportion of the population of group g that is within the PSA.

3.1.2. Constraints for PSA Problem

In this study, the constraint sets 2.1 and 2.2 are defined using the formulations proposed by Fan (2004) and are therefore omitted here for the sake of brevity. To formulate the cost of operating the bus service that appears in Equation 2.3, denote T_r as the round-trip time of route r and f_r as the service frequency of route r. Also, define the number of days considered in the budget analysis as τ, the operating hours per analysis day as O_b, and the cost per hour of operating a bus as c_b. Then, $C_b(\boldsymbol{R},\boldsymbol{f})$ term can be defined as:

$$C_b(\boldsymbol{R},\boldsymbol{f}) = \tau c_b O_b \sum_{r\in\boldsymbol{R}} f_r T_r \tag{6}$$

The paratransit service cost is computed as a function of the number of paratransit trips that must be served by the system. Defining c_p as the cost per paratransit trip, the $C_p(\boldsymbol{s})$ is computed by:

$$C_p(\boldsymbol{s}) \ = c_p \sum_{i\in\boldsymbol{N}_p} \sum_{j\in\boldsymbol{N}_p} (1-\gamma_{ij}) q_{ij} \tag{7}$$

In the numerical tests, Equation 2.4 will be specified to reflect the minimum PSA coverage regulation in the United States, but naturally it could be used to reflect other types of spatial considerations. Let $B(\boldsymbol{R},\Delta_{min}) \in \boldsymbol{\Omega}$ represent the paratransit service space within the minimum regulatory distance Δ_{min} of all the routes that compose $\boldsymbol{R}$; that is, $B(\boldsymbol{R},\Delta_{min})$ is a buffer, in geographic information systems (GIS) terminology, and $B(\cdot)$ is the buffer function that generates the space. Using this space object, constraint 2.4 can be operationalized by:

$$\boldsymbol{s} \cap B(\boldsymbol{R},\Delta_{min}) = B(\boldsymbol{R},\Delta_{min}) \tag{8}$$

Equation 8 states that the spatial intersection between $\boldsymbol{s}$ and $B(\boldsymbol{R},\Delta_{min})$ must equal $B(\boldsymbol{R},\Delta_{min})$; that is $\boldsymbol{s}$ must contain the minimum PSA set by regulations for $\boldsymbol{R}$. In Figure 1, the concept of the $B(\boldsymbol{R},\Delta_{min})$ buffer space (the minimum PSA) for a bus route is illustrated, along with a PSA that satisfies constraint (8) as it contains in its entirety the minimum PSA. Note that mathematically the $\boldsymbol{s}$ can be treated as a polygon (as it will be done in the solution approach discussed in Chapter 4) and that by extension the function B produces buffer polygons.

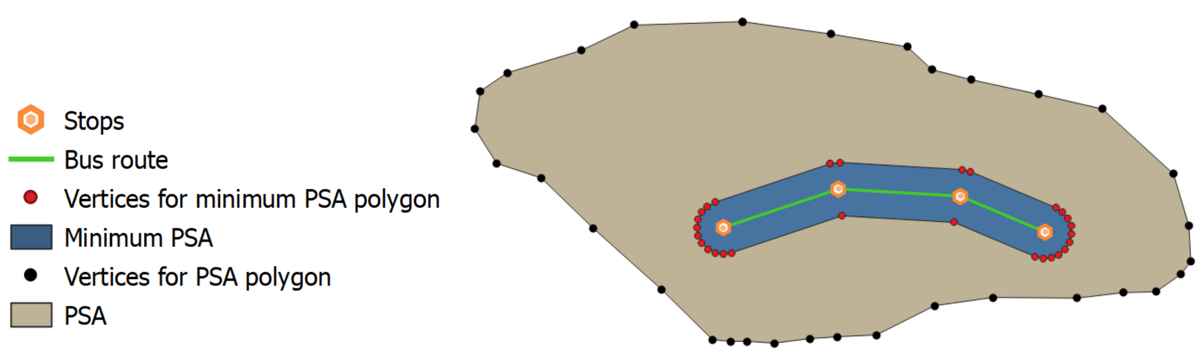

Figure 1. Example of PSA polygons.

3.1.3. *Models Systems for Direct Computation of Congestion Reduction Objectives*

In practice, transportation planners use regional travel demand models to forecast the impacts of new transit service configurations on all components of the transportation network, including impacts on vehicle traffic that uses the road network. These computer-based model systems are usually sequential trip-based models, such as four-step models, or to a lesser extent, activity-based models. The heuristic presented in Chapter 4 can interact with these models as if they were black-box computer models that receive as input the tuple $(\boldsymbol{R}, \boldsymbol{f}, \boldsymbol{s})$, plus the additional transportation network and sociodemographic data, and return as output standard travel estimates such as the travel time (t_a) and vehicle flow (x_a) on road network link $a \in A$. These outputs could be used to compute congestion reduction objectives such as minimizing total vehicle travel time in the road network (i.e., $\sum_{a\in A} x_i t_i$). These modeling considerations were not incorporated in the numerical tests and sample applications reported in Chapter 5 given the high computation cost (i.e., computer model run times) that would result from modeling auto travel demand. However, a modeler could easily include an additional congestion objective as part of design problem 2 and use the proposed heuristic to find solutions to the expanded problem.

3.2. Dockless Micromobility Service Area Design Problem

As in the PSA design problem, the micromobility service area (MSA) design problem is formulated as a multi-objective problem with constraints. Assume that the MSA is collaboratively designed by a dockless micromobility service operator and city planners. The service operator's goal is to maximize profit, while the city planners' goals are to reduce congestion and ensure equitable access to the micromobility service. The goal of ensuring equitable access is operationalized in the form of a spatial coverage constraint: a set of city zones that must be included in the final MSA design is defined. These zones could represent, for example, areas where the residences of historically disadvantaged populations concentrate. Let $M_o(\boldsymbol{s})$ be the operator's objective, $M_c(\boldsymbol{s})$ denote the city's objectives, and $\boldsymbol{\Psi}$ represent the set of zones that must be included in the MSA. The MSA design problem can be formulated as:

$$\min\ \boldsymbol{Z} = (M_o(\boldsymbol{s}), M_c(\boldsymbol{s})) \tag{9}$$

$$subject\ to$$

$$\boldsymbol{s} \cap \boldsymbol{\Psi} = \boldsymbol{\Psi} \tag{9.1}$$

3.2.1. *Design Objectives for the MSA Problem*

Given the relative novelty of micromobility services, there are no standard approaches to simulate the demand for micromobility and, therefore, to compute the $M_o(\boldsymbol{s})$ and $M_o(\boldsymbol{s})$ objectives. A promising alternative is to model micromobility demand using an activity-based travel demand model, as in the work of Rodriguez-Roman et al. (2021). A challenge with activity-based models is that they are relatively computationally expensive, which limits the number of design evaluations that could be performed in the process of finding good MSA alternatives. For simplicity, it is assumed in the simulation experiments that the decision-makers have access to simple-to-evaluate models (i.e., surrogate models (Forrester & Keane, 2009)) that can provide good estimates of the objective function values.

In the simulation experiments for the MSA problem (Section 5.2), $M_o(\boldsymbol{s})$ is defined as the total number of micromobility trips per unit area of the MSA (e.g., number of trips per km^2 of service area) and it is assumed that the operator is interested in maximizing this metric, which serves as a simple proxy for profit. The $M_c(\boldsymbol{s})$ objective is defined as the total number of car trips reduced per unit area of the MSA. Note that an MSA with a large extension would require more resources to, for example, search for and relocate the vehicles as they spread and concentrate in suboptimal locations in the service area. Given that Equation 9 and its heuristic assume that the objectives will be minimized, the reciprocal of the previously discussed objectives was used in simulation experiments. As before, these objective function specifications are only examples; the heuristics presented in Chapter 4 do not depend on these assumptions.

Chapter 4. Solution Approach

In this section, solution approaches are presented for the PSA and MSA design problems. The heuristic for the PSA problem includes procedures to search for designs for the fixed route network. Naturally, these procedures are not relevant for the MSA design problem. However, in both heuristics the service area s is treated as a polygon and the same strategies are used to iteratively evolve a set of candidate design solutions (or population, in genetic algorithm terminology).

4.1. Heuristic for PSA Problem

The proposed PSA design model is a multi-objective, non-linear mixed integer optimization problem with constraints. A GA-based solution approach is proposed to search for solutions to the proposed problem. The main steps of the heuristic are illustrated in Figure 2. The heuristic consists of two main steps: (i) a bus network design procedure (BNDP) that generates bus networks, their minimum PSA, and the route frequencies, and (ii) a GA procedure to search for paratransit service area improvements (GA-PSA). Several procedures can be found in the literature to generate bus networks, determine route frequencies, and select the best set of solutions in multi-objective problems (Gunantara, 2018; Ibarra-Rojas et al., 2015). As the focus of this report is not on developing new methods for these tasks, existing methods were used in the simulation tests for generating bus networks, setting route frequencies, and selecting the best set of solutions in multi-objective problems. The general solution framework does not depend on the methods used to accomplish these specific tasks. In the following sections, the notation and steps of the solution approach are presented.

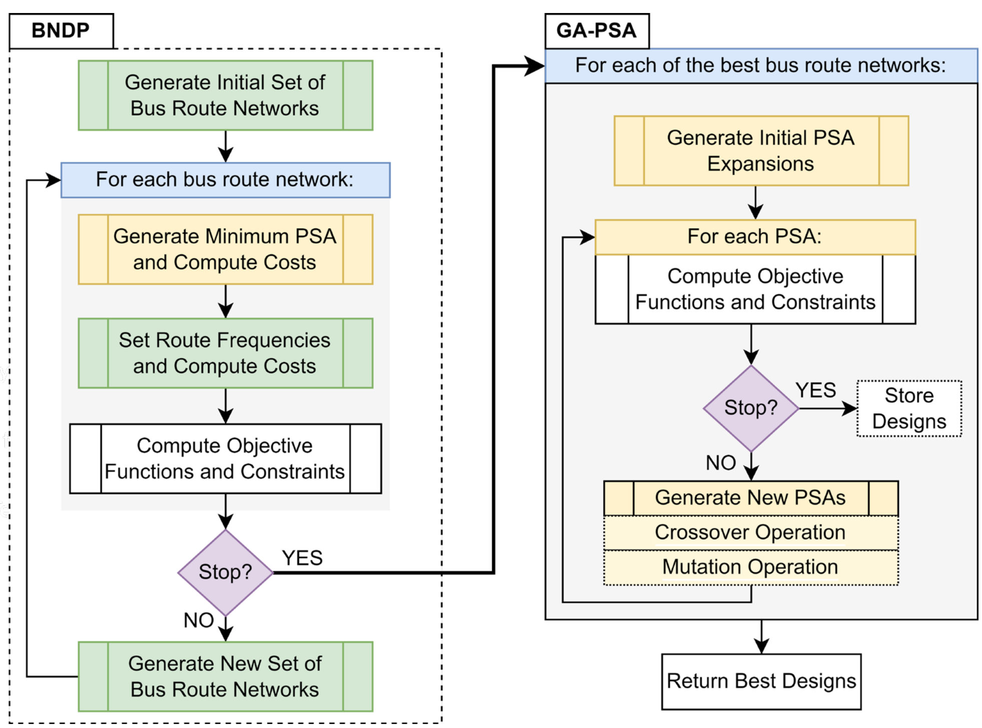

Figure 2. Main steps of the solution method.

Notation

Counters and Indices

n	:	iteration counter
k	:	candidate design index in BNDP
u	:	candidate design index in GA-PSA
ι	:	grid column index
a	:	index designs produced by BNDP and improved by GA-PSA

Parameters

n_{max}	:	BNDP's maximum number of iterations
n_{psa}	:	GA-PSA's maximum number of iterations
η_b	:	population size of BNDP
η_p	:	number of BNDP solutions to be improved by GA-PSA
η_s	:	PSA population size in GA-PSA
η_{IP}	:	initial population size in GA-PSA
η_{gp}	:	number of grip points
η_{m1}, η_{m2}	:	lower and upper bound on the number of vertices to include in mutation segment
α_g	:	perpendicular distance of grid used in crossover operation
H	:	budget
$\Delta_{min}, \Delta_{max}$	:	minimum and minimum buffer distance, respectively
ξ	:	buffer increment
ρ_c	:	probability of a crossover operation
ρ_{cm}	:	probability of a mutation operation given that a crossover operation occurred
ρ_e	:	probability of expansion in mutation operation
ρ_v	:	probability that a new vertex is created
θ_1, θ_2	:	randomly generated angles that define the swapped segments in crossover operation
ς	:	angle increment in crossover operation
l_u	:	perimeter of a polygon u
v_1, v_2	:	randomly generated vertex indices
$\ddot{\boldsymbol{\beta}}, \boldsymbol{\beta}_1, \boldsymbol{\beta}_2$	:	randomly generated vertex based on grid vertices $\boldsymbol{\beta}_1$ and $\boldsymbol{\beta}_2$
w	:	random number in the interval [0,1]

Decision Variables

$\boldsymbol{R}$	:	bus route networks
$\boldsymbol{f}$	:	service route frequencies
$\boldsymbol{s}$	:	PSA polygon for network

Sets

$\widetilde{\mathbf{X}}, \mathbf{X}, \boldsymbol{\chi}_n$	:	contains decision variable values for all evaluated designs, the best-known designs, and the designs generated in iteration n, respectively
$\widetilde{\boldsymbol{\Lambda}}, \boldsymbol{\Lambda}, \overline{\boldsymbol{\Lambda}}, \boldsymbol{\lambda}_n$	:	objective function values for all evaluated designs, the best-known designs, all designs generated by GA-PSA, and the designs generated in iteration n, respectively

$\widetilde{\mathbf{\Phi}}, \mathbf{\Phi}, \overline{\mathbf{\Phi}}, \boldsymbol{\phi}_n$	:	remaining budget values for all evaluated designs, the best-known designs, all designs generated by GA-PSA, and the designs generated in iteration n, respectively
$\overline{\boldsymbol{P}}, \boldsymbol{P}, \boldsymbol{P}_n$	:	contains all polygons, the best polygons, and the polygons generated in iteration n, respectively
$\boldsymbol{v}$	:	vertex index set for a polygon
$\boldsymbol{p}_u, \dot{\boldsymbol{p}}_u, \ddot{\boldsymbol{p}}_u$	:	vertex set for polygon, a subset ($\dot{\boldsymbol{p}}_u \subseteq \boldsymbol{p}_u$), and a set of new vertices
$\boldsymbol{A}$	:	indices of the η_p-best designs

Functions

B	:	buffer function
M_b, M_p	:	bus and paratransit effectiveness objectives, respectively
E	:	equity objective
C_b, C_p	:	bus and paratransit service cost, respectively

4.1.1. Main Steps of Integrated Fixed Transit – PSA Design Approach

The general steps of the proposed procedure are described next. The specific methods used in the numerical tests of this study can be found in Chapter 5. The structure of the solution approach assumes that an algorithm that works on iteratively optimizing a population of solutions (i.e., designs) is being used. In the numerical tests performed in this study, a GA based on the work by Fan (2004) was used.

Steps

1. Initialization: Read initial model inputs, including demand data and transit operational parameters, and set values for algorithmic parameters (e.g., maximum number of iterations).
2. BNDP:
2.1. Set $n := 0$ and initiate $\widetilde{\mathbf{X}}, \widetilde{\mathbf{\Lambda}}, \widetilde{\mathbf{\Phi}}, \mathbf{X}, \mathbf{\Lambda}, \mathbf{\Phi}, \boldsymbol{\chi}_n, \boldsymbol{\lambda}_n$, and $\boldsymbol{\phi}_n$ as empty sets.
2.2. Generate initial set of candidate bus network designs $\boldsymbol{R}_n$ using initial bus network generation procedure.
2.3. For each candidate design k with bus network $\boldsymbol{R}_k \in \boldsymbol{R}_n$:
2.3.1. Determine $\boldsymbol{s}_k = B(\boldsymbol{R}_k, \Delta_{min})$ and the corresponding paratransit values $M_p(\boldsymbol{s}_k)$ and $C_p(\boldsymbol{s}_k)$.
2.3.2. Compute remaining budget: $H - C_p(\boldsymbol{s}_k)$.
2.3.3. Given remaining budget, determine $\boldsymbol{f}_k$ using route frequency setting procedure. Typically, the bus system objective $M_b(\boldsymbol{R}_k, \boldsymbol{f}_k)$ and the cost $C_b(\boldsymbol{R}_k, \boldsymbol{f}_k)$ can be computed at this stage.
2.3.4. Compute remaining budget: $H - C_p(\boldsymbol{s}_k) - C_b(\boldsymbol{R}_k, \boldsymbol{f}_k)$.
2.3.5. Compute equity objective $E(\boldsymbol{R}_k, \boldsymbol{f}_k, \boldsymbol{s}_k)$.
2.3.6. Store $(\boldsymbol{R}_k, \boldsymbol{f}_k, \boldsymbol{s}_k)$ in $\boldsymbol{\chi}_n$ and $\widetilde{\mathbf{X}}$, $\left(M_b(\boldsymbol{R}_k, \boldsymbol{f}_k), M_p(\boldsymbol{s}_k), E(\boldsymbol{R}_k, \boldsymbol{f}_k, \boldsymbol{s}_k)\right)$ in $\boldsymbol{\lambda}_n$ and $\widetilde{\mathbf{\Lambda}}$, and the remaining budget information in $\boldsymbol{\phi}_n$ and $\widetilde{\mathbf{\Phi}}$.
2.4. If $n > n_{max}$ continue to step 3; otherwise, continue to step 2.5.
2.5. Select set of η_b best-known solutions: Based on a solution quality criterion, pick the η_b best designs from the set $\boldsymbol{\chi}_n \cup \mathbf{X}$ given $\boldsymbol{\lambda}_n \cup \mathbf{\Lambda}$ and $\boldsymbol{\phi}_n \cup \mathbf{\Phi}$ (for example, in the numerical tests performed in this study, the Pareto ranking and crowding distance procedure proposed by Deb et al. [2002] was applied to compute the solution quality criterion). Reset $\mathbf{X}, \mathbf{\Lambda}$, and $\mathbf{\Phi}$ to contain the corresponding values of the η_b best designs. Set $\boldsymbol{\chi}_n, \boldsymbol{\lambda}_n$ and $\boldsymbol{\phi}_n$ to be empty sets.

2.6. Set $n \coloneqq n + 1$.
2.7. Based on the information in $\mathbf{X}$, generate η_b new bus network designs using existing procedures (e.g., [Fan, 2004]) and store them in $\boldsymbol{R}_n$, and return to step 2.3.

3. Select set of η_p best-known solutions: Based on a solution quality criterion, pick the η_p best designs from the set $\widetilde{\mathbf{X}}$ given $\widetilde{\mathbf{\Lambda}}$ and $\widetilde{\mathbf{\Phi}}$.
4. Search for PSA improvements: The GA-PSA is applied to the η_p best-known solutions (see next subsection for description). This procedure adds new designs to $\widetilde{\mathbf{X}}$, along with corresponding values to $\widetilde{\mathbf{\Lambda}}$ and $\widetilde{\mathbf{\Phi}}$.
5. Return set of non-dominated $\widetilde{\mathbf{X}}$ designs given $\widetilde{\mathbf{\Lambda}}$ and $\widetilde{\mathbf{\Phi}}$.

4.1.2. Genetic Algorithm for PSA Design Problem

GA-PSA is applied to search for improvements to the minimum PSAs of the η_p best designs identified by the BNDP. The search for better PSA designs occurs, independently, for each of the η_p best designs; let $\boldsymbol{A}$ be a set that contains the indices of the η_p best designs and $a \in \boldsymbol{A}$. In GA-PSA, a service area is represented as a polygon. Let S be a simple polygon generating function and $\boldsymbol{p}$ be geometric information used by S to generate the polygon. Then, a service area is generated by $\boldsymbol{s} = S(\boldsymbol{p})$. In this study, $\boldsymbol{p}$ is specified as an ordered vertex set, which is connected by S via straight lines that define the polygon edges. An alternative to the approach used here is to define $\boldsymbol{p}$ as the set of control vertices that determine the shapes of spline curves generated by S. The spline curve approach was not used in this study as initial tests suggested that spline curves are more computationally expensive than the parsimonious PSA polygon representation selected here. Three PSA generation procedures are described in this section: the procedure to generate an initial set of PSA expansions, and the crossover and mutation operators that modify the population of PSA designs. After these procedures are described, the main steps of GA-PSA are presented.

4.1.2.1. Initial Population Generation Procedure for PSA Problem

The initial population generation procedure (IP-PSA) uses the buffer function to generate an initial set $\boldsymbol{P}$ of PSA designs (i.e., the population). Let Δ_{max} denote the maximum buffer distance from bus routes, ξ a distance interval, and η_{IP} the initial population size. For each value u in $\{1, \dots, \eta_{IP}\}$, a candidate PSA design is generated using $\boldsymbol{s}_u = B(\boldsymbol{R}_a, \Delta_u)$ and added to $\boldsymbol{P}$. In this process, the value of Δ_u begins at $\Delta_{min} + \xi$ and it iteratively increases until Δ_{max}, with increments of ξ; the value of ξ is determined such that η_{IP} values are ultimately generated.

4.1.2.2. Crossover and Mutation Procedures

GA-PSA uses crossover and mutation procedures to iteratively generate new solutions based on the population of solutions in $\boldsymbol{P}$. With probability ρ_c, in each iteration of GA-PSA, the crossover operation is performed, and with probability $1 - \rho_c$, only the mutation operation is performed. If the crossover operation is performed, the PSA designs generated by it are modified by the mutation operation with probability ρ_{cm}. An illustration of the crossover operations in presented in Figure 3. In the crossover operation, two distinct $\boldsymbol{s}_{u1}$ and $\boldsymbol{s}_{u2}$ designs are randomly selected from $\boldsymbol{P}$ and contiguous subsets of vertices are swapped between these two parent solutions. To identify the vertex subsets to be swapped, a reference point is first defined by finding the centroid of the spatial intersection $\boldsymbol{s}_{u1} \cap \boldsymbol{s}_{u2}$. Second, an angle θ_1 is randomly generated in the interval $[0,2\pi]$, after which a second angle θ_2 is randomly generated in the interval $[\theta_1, \theta_1 + \varsigma]$, where ς is an angle increment. Third, assuming the reference point as the origin of the coordinate system, the angle with respect to the X-axis is measured for each vertex in $\boldsymbol{s}_{u1}$ and $\boldsymbol{s}_{u2}$. Finally, for each parent $\boldsymbol{s}_{u1}$ and $\boldsymbol{s}_{u2}$, the vertex with an angle nearest to θ_1 and the vertex with an angle nearest to θ_2 are selected as the starting and ending vertices in the ordered subset of vertices to be swapped between parents, and the subsets are swapped to create new offspring solutions. In Figure 3, the dash lines represent the (θ_1, θ_2) angles that are used to select the vertex subset that is swapped from polygon (b) to polygon (a) to generate the new polygon (c).

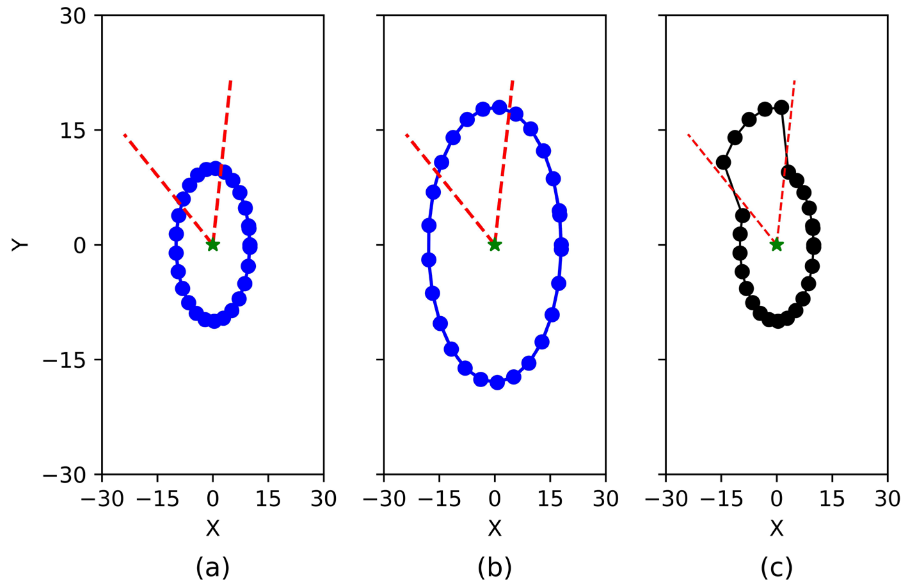

Figure 3. Vertices of two polygons, (a) and (b), swapped to create a new PSA polygon, (c).

The mutation operation either expands or contracts segments of a PSA boundary. For each segment to be modified, a squared grid of points is generated. The points are used to identify the space outside and inside the $\boldsymbol{s}_u$ and guide the expansion or contraction of the boundary. Figure 4 illustrates the procedure described next. Let $\boldsymbol{p}_u$ be the vertex set of the $\boldsymbol{s}_u$ polygon and define the corresponding set of vertex indices as $\boldsymbol{v} = \{1, \ldots, |\boldsymbol{p}_u|\}$. The first step of the mutation operation consists of selecting the segment to modify. A segment is a subset $\dot{\boldsymbol{p}}_u \subseteq \boldsymbol{p}_u$ of contiguous vertices. $\dot{\boldsymbol{p}}_u$'s starting vertex v_1 is randomly selected from set $\boldsymbol{v}$ by computing the length l_u of the $\boldsymbol{s}_u$ perimeter, generating a random number in the interval $[0, l_u]$, and finding the vertex whose distance to the first vertex in $\boldsymbol{p}_u$ is closest to the random number. The final vertex in the ordered subset $\dot{\boldsymbol{p}}_u$is defined as $v_2 = v_1 + \ell$, where ℓ is a randomly generated integer in the range $[\eta_{m1}, \eta_{m2}]$. The value of $[\eta_{m1}, \eta_{m2}]$ can be determined, for example, so that a percentage range of the vertices in $\boldsymbol{p}_u$ are part of the mutated segment. If $v_2 > |\boldsymbol{p}_u|$, then $v_2 := v_2 - |\boldsymbol{p}_u|$; in this case $\dot{\boldsymbol{p}}_u$ contains all vertices from v_1 to $|\boldsymbol{p}_u|$ and from $\boldsymbol{p}_u$'s initial vertex to v_2.

The second step of the mutation process is to generate a grid of η_{gp} with equally spaced points that extend perpendicularly for a distance α_g from both sides of the line that connects vertices v_1 and v_2. A point-in-polygon algorithm is then applied to define which grid points are inside and outside of $\boldsymbol{s}_u$. With probability ρ_e, the boundary is expanded based on the information of the points outside $\boldsymbol{s}_u$, and with probability $1 - \rho_e$, the boundary is contracted using the points inside $\boldsymbol{s}_u$. For every grid column $\iota \in \boldsymbol{I}$, a new boundary vertex is created with probability ρ_v. A new vertex $\ddot{\boldsymbol{\beta}}_\iota = [x_\iota, y_\iota]$ is created using the expression $\ddot{\boldsymbol{\beta}}_\iota = \boldsymbol{\beta}_{1\iota} + w \times (\boldsymbol{\beta}_{2\iota} - \boldsymbol{\beta}_{1\iota})$ and stored in $\ddot{\boldsymbol{p}}_u$. $\boldsymbol{\beta}_{1\iota}$, and $\boldsymbol{\beta}_{2\iota}$ ($\boldsymbol{\beta}_{2\iota} \succ \boldsymbol{\beta}_{1\iota}$) are the extreme points (lowest and highest order points) that satisfy the mutation condition (expansion or contraction) of interest in the column ι, and w is a random number in the interval [0,1]. The new vertices in set $\ddot{\boldsymbol{p}}_u$ are embedded in $\boldsymbol{p}_u$ in replacement of the vertices in $\dot{\boldsymbol{p}}_u$.

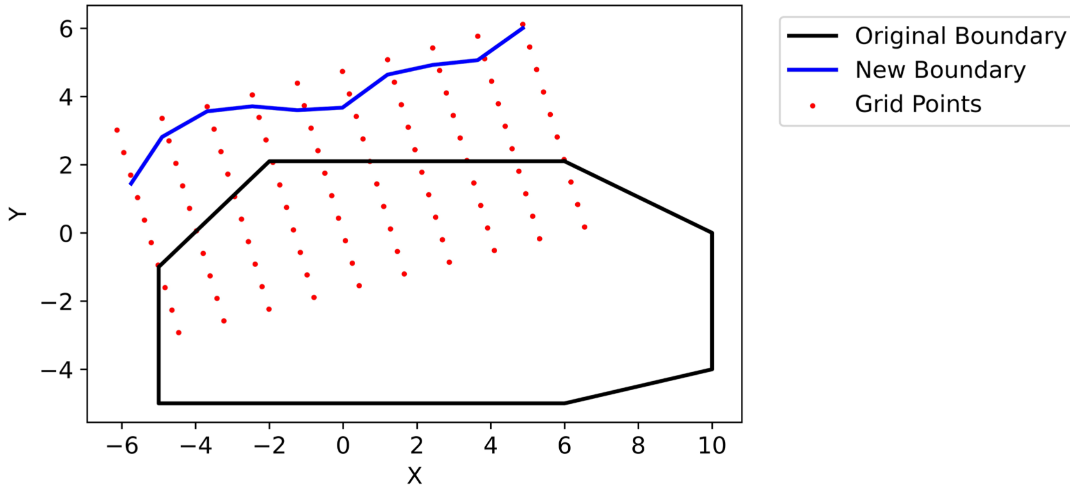

Figure 4. Example of PSA boundary expansion using the grid-based mutation operation.

4.1.2.3. Steps of GA-PSA

4. For each $a \in \boldsymbol{A}$:
4.1. Set $n \coloneqq 0$ and initiate $\overline{\boldsymbol{P}}, \boldsymbol{P}, \overline{\boldsymbol{\Lambda}}, \boldsymbol{\Lambda}, \overline{\boldsymbol{\Phi}}, \boldsymbol{\Phi}$, $\boldsymbol{\lambda}_n$, and $\boldsymbol{\phi}_n$ as empty sets.
4.2. Generate initial population $\boldsymbol{P}_n$ using IP-PSA. Store $\boldsymbol{P}_n$ in $\overline{\boldsymbol{P}}$.
4.3. For each candidate design $\boldsymbol{s}_u \in \boldsymbol{P}_n$:
4.3.1. Compute $M_p(\boldsymbol{s}_u)$, $C_p(\boldsymbol{s}_u)$, and $E(\boldsymbol{R_a}, \boldsymbol{f}_a, \boldsymbol{s}_u)$
4.3.2. Compute remaining budget: $H - C_p(\boldsymbol{s}_u) - C_b(\boldsymbol{R_a}, \boldsymbol{f}_a)$
4.3.3. Store $\left(M_p(\boldsymbol{s}_u), E(\boldsymbol{R}_a, \boldsymbol{f}_a, \boldsymbol{s}_u)\right)$ in $\boldsymbol{\lambda}_n$ and $\overline{\boldsymbol{\Lambda}}$, and the remaining budget in $\boldsymbol{\phi}_n$ and $\overline{\boldsymbol{\Phi}}$.
4.4. If $n > n_{psa}$, continue to step 4.9; otherwise, continue to step 4.5.
4.5. At most, select set of η_s best-known solutions: Based on a solution quality criterion, select, at most, the η_s best designs from the set $\boldsymbol{P_n} \cup \boldsymbol{P}$ given $\boldsymbol{\lambda}_n \cup \boldsymbol{\Lambda}$ and $\boldsymbol{\phi}_n \cup \boldsymbol{\Phi}$. Reset $\boldsymbol{P}, \boldsymbol{\Lambda}$, and $\boldsymbol{\Phi}$ to contain values corresponding of the η_s best designs. Set $\boldsymbol{P}_n, \boldsymbol{\lambda}_n$ and $\boldsymbol{\phi}_n$ to be empty sets.
4.6. Set $n \coloneqq n + 1$.
4.7. Based on the designs in $\boldsymbol{P}$, use the PSA crossover and mutation operators to generate η_s new designs, and store the designs in $\boldsymbol{P}_n$.
4.8. Remove from $\boldsymbol{P}_n$ all $\boldsymbol{s}_u$ that are not simple polygons or that do not satisfy Equation 8. Store remaining designs in $\overline{\boldsymbol{P}}$ and return to step 4.3.
4.9. Find the set of non-dominated designs in $\overline{\boldsymbol{P}}$ given $\overline{\boldsymbol{\Lambda}}$ and $\overline{\boldsymbol{\Phi}}$ and add their information to $\widetilde{\mathbf{X}}$, $\widetilde{\boldsymbol{\Lambda}}$, and $\widetilde{\boldsymbol{\Phi}}$. The $\boldsymbol{R}_a$, $\boldsymbol{f}_a$, and $\boldsymbol{M}_b(\boldsymbol{R}_a, \boldsymbol{f}_a)$ information is appended to the new designs before adding their information to the sets.

The discussion so far has assumed that the PSA is a simple polygon, that is, a polygon that has no holes and whose edges do not intersect. However, it is possible for a buffer polygon to have holes (i.e., areas surrounded by the PSA boundary but not inside it). If an agency wants to allow the possibility for service holes, the proposed procedures can be applied for each of the vertex sets that define the holes. In the experiments discussed in Chapter 5, only simple polygons are considered.

4.2. Heuristic for MSA Design Problem

The heuristic for the micromobility service area design problem (GA-MSA) has the same geometric representation of service area boundaries and the same steps of the GA-PSA heuristic. The main difference between the GA-PSA and GA-MSA is the procedure used to generate the initial population of candidate solutions. This procedure is explained next, and for the sake of completeness, the notation used in the discussion and the steps of the GA-MSA heuristic are also presented.

Notation

Counters and Indices

n	:	iteration counter
u	:	candidate design index in GA-MSA

Parameters

n_{msa}	:	GA-MSA's maximum number of iterations
η_{IM}	:	initial population size per IP-MSA procedure
η_{pop}	:	population size in GA-MSA
α_b	:	side length for box polygons that define random zones
η_r	:	maximum number of random zones that can be generated

Decision Variable

$\boldsymbol{s}$	:	MSA polygon

Sets

$\boldsymbol{\Lambda}, \overline{\boldsymbol{\Lambda}}, \boldsymbol{\lambda}_n$	:	objective function values for the best-known designs, all designs generated by GA-MSA, and the designs generated in iteration n, respectively
$\overline{\boldsymbol{P}}, \boldsymbol{P}, \boldsymbol{P}_n$	:	contains all polygons, the best polygons, and the polygons generated in iteration n, respectively

Functions

M_b, M_p	:	objectives of the operator and the city, respectively

4.2.1. Initial Population Generation Procedure for MSA Problem

The initial population generation procedure (IP-MSA) uses the convex hull operator to create polygons that contain the city zones that must be included in the MSA (i.e., the $\boldsymbol{\Psi}$ set). Generally speaking, a convex hull is the intersection of all convex sets containing a set of points (Weisstein, n.d.). In the context of the MSA problem, the points can be the vertices of the polygons representing the $\boldsymbol{\Psi}$ city zones. The convex hull operator returns a set of points that can be used to create a polygon that, at a minimum, contains the $\boldsymbol{\Psi}$ city zones. Figure 5 illustrates the concept of a convex hull.

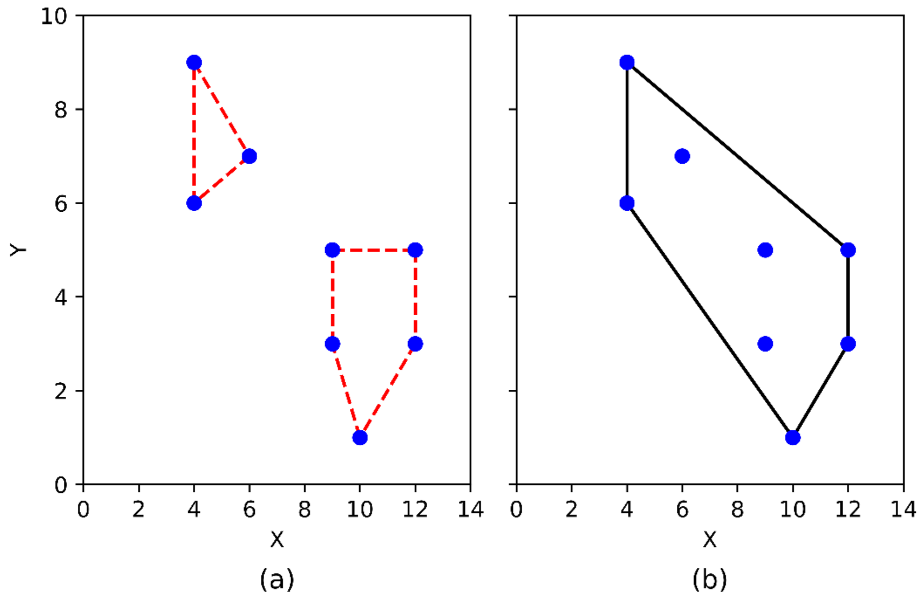

Figure 5. Set of points representing city zones (a) and their convex hull polygon (b).

Two procedures are used to generate the initial population using the convex hull operator. In the first procedure, a base convex hull polygon that contains the $\boldsymbol{\Psi}$ city zones is generated. Then, η_{IM} feasible solutions are generated by applying the mutation procedure described in Section 4.1.2.2 to the base polygon η_{IM} times. Recall that a feasible service area polygon must entirely contain the $\boldsymbol{\Psi}$ city zones and be a simple polygon; mutated polygons that are infeasible are replaced by new ones until η_{IM} feasible solutions are identified. The second procedure iteratively uses the convex hull operator to create polygons that contain the $\boldsymbol{\Psi}$ city zones and randomly generated zones. In each iteration of this procedure:

i. One to η_r points are randomly generated in the study region space.
ii. For each randomly generated point, a box zone is created by generating the vertices for a box polygon with side length of α_b and a center located at the random point.
iii. The convex hull operator is used to generate a polygon that contains the $\boldsymbol{\Psi}$ city zones and randomly generated zones created in step ii.

η_{IM} feasible solutions are also generated by applying the second IP-MSA procedure.

4.2.2. Genetic Algorithm for MSA Design Problem

The steps of the GA-MSA are:

1. Set $n \coloneqq 0$ and initiate $\overline{\boldsymbol{P}}, \boldsymbol{P}, \overline{\boldsymbol{\Lambda}}, \boldsymbol{\Lambda}, \overline{\boldsymbol{\Phi}}, \boldsymbol{\Phi}$, $\boldsymbol{\lambda}_n$, and $\boldsymbol{\phi}_n$ as empty sets.
2. Generate initial population $\boldsymbol{P}_n$ using IP-MSA. Store $\boldsymbol{P}_n$ in $\overline{\boldsymbol{P}}$.
3. For each candidate design $\boldsymbol{s}_u \in \boldsymbol{P}_n$:

3.1. Compute $M_o(\boldsymbol{s}_u)$ and $M_c(\boldsymbol{s}_u)$

3.2. Store $\left(M_o(\boldsymbol{s}_u), M_c(\boldsymbol{s}_u)\right)$ in $\boldsymbol{\lambda}_n$ and $\overline{\boldsymbol{\Lambda}}$.

4. If $n > n_{msa}$ continue to step 9; otherwise, continue to step 5.

5. At most, select set of η_{pop} best-known solutions: Based on a solution quality criterion, select, at most, the η_{pop} best designs from the set $\boldsymbol{P_n} \cup \boldsymbol{P}$ given $\boldsymbol{\lambda}_n \cup \boldsymbol{\Lambda}$. Reset $\boldsymbol{P}$ and $\boldsymbol{\Lambda}$ to contain values corresponding of the η_{pop} best designs. Set $\boldsymbol{P}_n$ and $\boldsymbol{\lambda}_n$ to be empty sets.
6. Set $n \coloneqq n + 1$.
7. Based on the designs in $\boldsymbol{P}$, use the crossover and mutation operators (see Section 4.1.2.2) to generate η_{IM} new designs, and store the designs in $\boldsymbol{P}_n$.
8. Remove from $\boldsymbol{P}_n$ all $\boldsymbol{s}_u$ that are not simple polygons or that do not completely contain the $\boldsymbol{\Psi}$ city zones. Store remaining designs in $\overline{\boldsymbol{P}}$ and return to step 3.
9. Find and return the set of non-dominated designs in $\overline{\boldsymbol{P}}$ given $\overline{\boldsymbol{\Lambda}}$, along with the respective objective function values.

Chapter 5. Numerical Tests

Numerical tests were performed to illustrate the application of the proposed design models and their heuristics. Sections 5.1. and 5.2. present the experiments performed for the PSA design problem and MSA design problem, respectively.

5.1. Numerical Experiments for PSA Design Problem

The GA-PSA experiments explored the impact that the ρ_c (crossover probability) and ρ_{cm} (crossover-and-mutation probability) parameters have on the rate at which the dominated objective function space expands with each evaluation of the candidate designs generated by GA-PSA (the objective function space is three-dimensional as three objective functions are considered). The application setting, the demand and operational models, key parameter values, and the results from the simulation results are described in the next sections. The complete set of programs, parameters values, and data files, including network and demand information, can be found in an online repository (Rodriguez-Roman, 2022).

5.1.1. Application Setting: Zonal and Network Systems

The San Juan Metropolitan Area (SJMA) was selected as the application region. The Metropolitan Bus Authority (MBA), the largest bus service in Puerto Rico with 23 fixed routes, operates in this region. The SJMA was divided into 100 zones for the simulation of the bus service demand. The region's 951 US Census block zones were used in the paratransit demand model; as they were not essential to illustrate the study's contribution, fewer zones were used in the bus zonal system to reduce the computational time of the simulations. A road network composed of 2,457 links and 1,067 nodes was created for the bus network design problem. Demographic and network data were extracted and adapted from US Census sources. Figure 6 presents the application region, zonal systems, and road network used in the simulation.

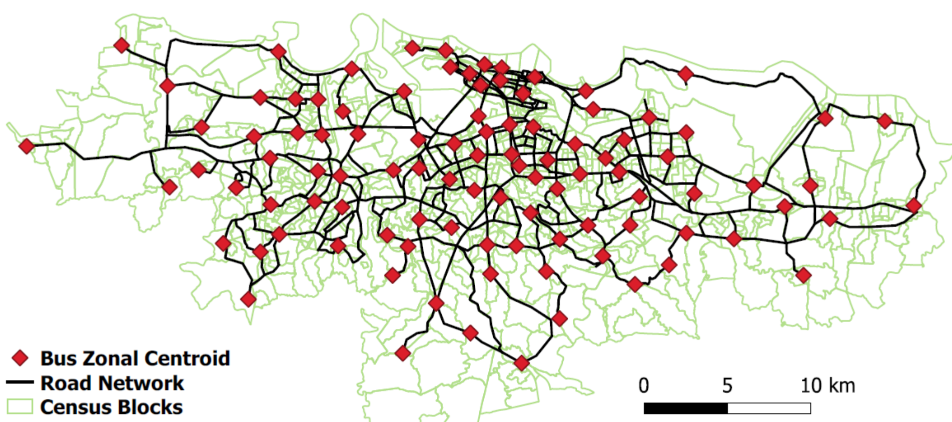

Figure 6. Zones and road network.

5.1.2. Demand, Supply, and Equity Models and Parameters

Fixed demand models were used in the simulations. A total of 3,825 origin-destination (OD) pairs were considered in the bus demand model; the total daily travel demand for the design period was set to 25,000 trips. For each OD pair connected by a bus network, bus travel demand was distributed across the two shortest network paths using a logit model. The logit model only accounted for the travel times on each path. For simplicity, the route frequencies were not considered in the logit model. The service frequencies on the bus network were set to equal the upper bound of each route's load factor constraint (Fan, 2004). Computed frequencies that were less than the minimum service frequency or greater than the maximum service frequency were set to the minimum or maximum frequency treshhold value, respectively. The total annual demand for the paratransit service was set at 250,000 trips, which were distributed across 112,010 OD pairs. Following MBA's service regulations, service holes were eliminated from PSAs and only OD trips that started and ended within the PSA were considered as feasible in the simulations. The demand matrices for both modes were generated using simulations. It was assumed that the budget for operational expenses was $33 million.

The populations covered by the PSA and the bus service were needed to compute the σ_{gm} terms of the inequality objective function. Covered population totals were computed using the GIS-based areal interpolation method (O'Neill et al., 1994). The coverage area of a bus route network was defined to be the same as its minimum PSA; the population within this area was considered in the $\sigma_{g,bus}$ terms. The population groups were defined as the people within each municipality that composes the SJMA and that have incomes of less than $25,000. The GIS operations were performed using the US Census block shapefile for the region. The ε parameter in Equation 5 was set to 0.75.

5.1.3. Procedures and Key Algorithmic Parameters

The Initial Candidate Route Set Generation Procedure proposed by Fan (2004) was used in the simulations to generate an initial set of candidate bus network designs. As previously stated, Fan's bus network design procedures were adapted and applied in the BNDP. The selected solution quality criteria were based on Deb et al.'s (2002) non-dominated sorting genetic algorithm II, which was modified to consider the budget constraint. Designs that violated the budget constraint were assigned the worst Pareto rank obtained from the feasible solutions, plus one. In Table 1, key algorithmic parameter values are reported.

Table 1. Key GA-PSA Parameter Values

Parameter	Value
n_{max}	30
n_{psa}	30
η_b	64
η_p	32
η_{IP}	64
η_{gp}	30
η_{m1}, η_{m2}	2.5% and 5%, respectively, of the number of vertices in polygon
α_g	1207 meters (3/4 miles)
$\Delta_{min}, \Delta_{max}$	1207 meters, 82021 meters
ρ_c	$\{0.25, 0.75\}$
ρ_{cm}	$\{0.75, 0.25\}$
ρ_e	0.8
ρ_v	0.5
ς	$\pi/6$ radians

5.1.4. Test Results and Discussion

Two sets of parameters were considered: Set 1, defined as $\{\rho_c = 0.25, \rho_{cm} = 0.50\}$, and Set 2, defined as $\{\rho_c = 0.75, \rho_{cm} = 0.25\}$. Note that parameter Set 1 ensures that GA-PSA relies more on the mutation operation to generate candidate designs than Set 2. The BNDP was first applied to generate an initial set of bus route network designs, and from this initial set of designs the 32-best designs were selected (i.e., $\eta_p = 32$) for improvement of their PSAs. For each parameter set, 10 trial runs of the GA-PSA (i.e., the simulation) were performed, and each trial run had the same set of 32-best designs. The quality of solutions was assessed using the hypervolume improvement metric, which summarizes the percent change in the dominated objective function space. Figure 7 presents the average hypervolume percent improvement for the Set 1 and Set 2 trials. On average, GA-PSA expanded the dominated objective function space faster when Set 1 parameters were used, which emphasizes mutation-only. However, for both sets of trials, the largest improvement occurred in the design evaluation range of 15,000 to 20,000 after a particular design in the set of 32-best designs was reached. This result suggests that incorporating greedy search techniques within GA-PSA could improve its performance, particularly when evaluating a design is computationally expensive and it is impractical to expect thousands of design evaluations. The greedy search techniques could consist of, for example, a limited series of initial buffer expansions for each of the 32-best designs, followed by a focused exploration of new buffer design expansions that resulted in the greatest performance improvements.

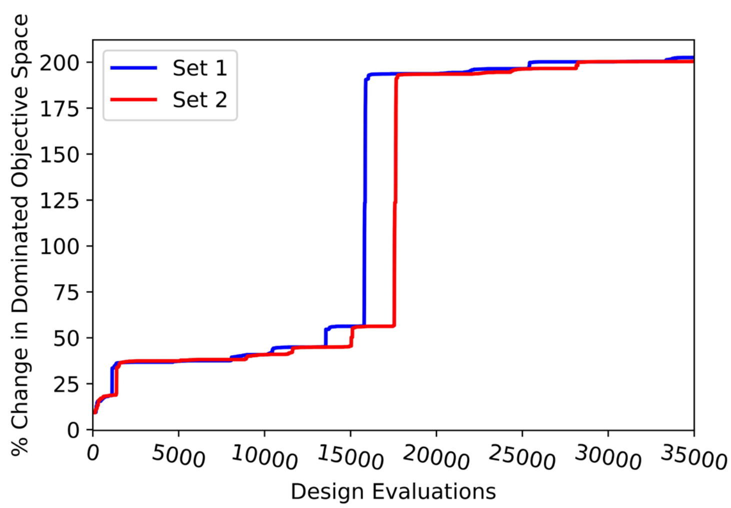

Figure 7. Average percent change in dominated objective function space for GA-PSA tests.

Figure 8 presents a parallel coordinate plot of the scaled objective function values for a set of non-dominated designs. These designs were generated in a trial run of GA-PSA with Set 1 parameters. In this run, 12,519 feasible PSA designs were generated, but only 33 non-dominated designs were identified (Figure 8 presents the objective function values for the 33 designs). Ultimately, after the PSA improvements, five bus network designs formed the basis of the non-dominated solutions, as can be seen by the concentration of solutions on five points in the M_b column. As expected, given budget constraints, there are trade-offs between optimizing the effectiveness of the bus service, the effectiveness of the paratransit service, and the access to the services. For example, the designs with the lowest M_b values had the highest M_p and E objective function values in the set. Provided that there is sufficient budget, the heuristic will attempt to expand the service area as much as possible at the GA-PSA step in search of lower M_p and E values.

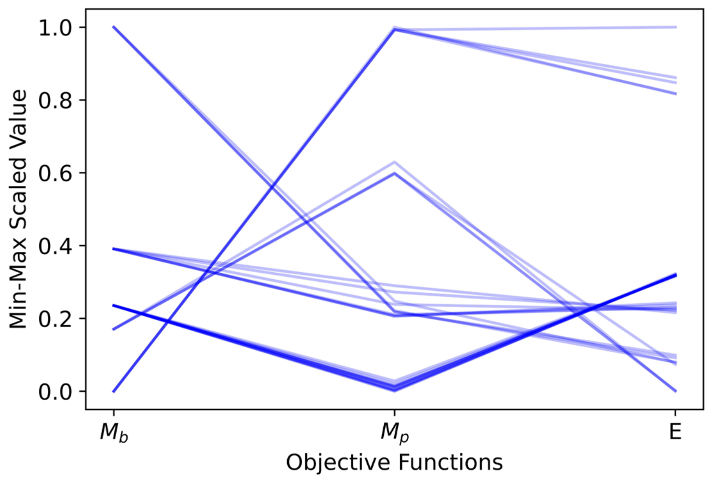

Figure 8. Scaled objective function values for non-dominated solutions for GA-PSA tests.

As an example, Figure 9 presents the PSA for the solution with the lowest M_p value in the 33-non-domnated solution set, along with the minimum PSA coverage from which it was generated. This PSA covers 98% of the study region and it resulted in an $M_p = 1968$ (99% of the demand met) and an $E = 0.02$, with only $2,451 in remaining budget (out of $33 million). In this case, GA-PSA would have been able to generate a PSA that covered the study region if the budget was $86,100 higher ($c_p$ was set to $45/trip).

(a)

(b)

Figure 9. Minimum PSA (a) and its best expansion PSA (b) according to the M_p objective.

5.2. Numerical Experiments for MSA Design Problem

Besides illustrating the application of the MSA design problem, the primary objective of the GA-MSA experiments was to examine the solutions generated by the heuristic given a hypothetical problem with a simple solution space. In addition, the performance of the GA-MSA heuristic with the Set 1 ($\{\rho_c = 0.25, \rho_{cm} = 0.50\}$) and Set 2 ($\{\rho_c = 0.75, \rho_{cm} = 0.25\}$) parameters was examined. The key algorithm parameters used in the numerical tests are reported in Table 2.

Table 2. Key GA-PSA Parameter Values

Parameter	Value
n_{msa}	60
η_{IM}	40
η_{pop}	64
η_r	1
η_{m1}, η_{m2}	5% and 10%, respectively, of the number of vertices in polygon
α_b	108 meters
ρ_c	$\{0.25, 0.75\}$
ρ_{cm}	$\{0.50, 0.25\}$
ρ_e	0.8

5.2.1. Application Setting, Models, and Parameters

The region selected for the application of the MSA design problem was the municipality of Mayagüez, Puerto Rico. Specifically, the study area presented in Figure 10 was selected. Located in this area is the historical urban center of Mayagüez, along with office and apartment buildings, the Mayagüez campus of the University of Puerto Rico, and various other trip attractors. Figure 10 also presents three polygons that represent communities categorized as "special" (in Spanish, "comunidades especiales") by the Puerto Rican government given their historical economic disadvantage. The three special communities represented the $\boldsymbol{\Psi}$ city zones in the tests.

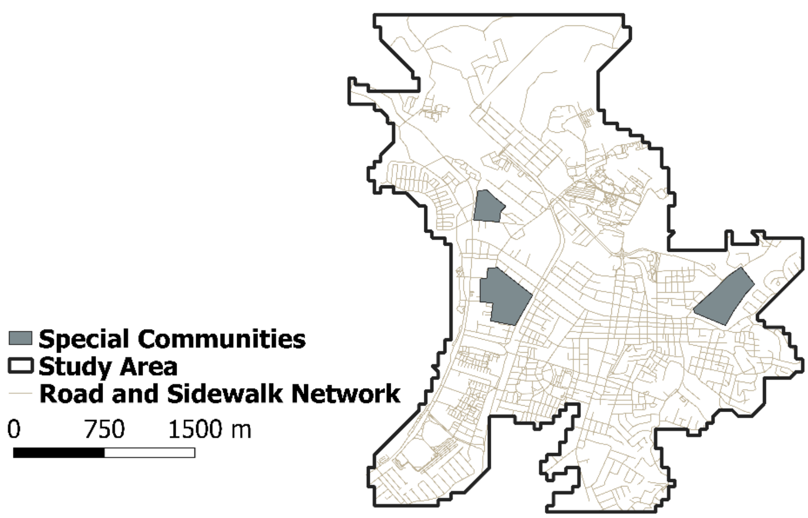

Figure 10. Study area for MSA tests.

To create a simple solution space, the study area was divided using a grid system, with each grid cell having 53-by-55-meter dimensions, and the objective functions were defined with the general form:

$$M_z = \frac{\sum_{j \in \boldsymbol{L}} I(j, \boldsymbol{s}) m_{zj}}{\vartheta(\boldsymbol{s})} \tag{10}$$

where z refers to the general objective index ($z = \{o, c\}$), j is a cell index, $\boldsymbol{L}$ is the set of cells, m_j is the objective function contribution obtained by including cell j in the MSA, $\vartheta(\boldsymbol{s})$ is the surface area of the MSA, and $I(j, \boldsymbol{s})$ is an indicator function that has the value of 1 if cell j is within the MSA, and 0 otherwise. By predefining each m_{zj} value, this objective function specification allows for easy a priori demarcation of the study area regions that should be included in the MSA to maximize each objective function value. In Figure 11, the m_j value surfaces for the M_o and M_c objectives are presented.

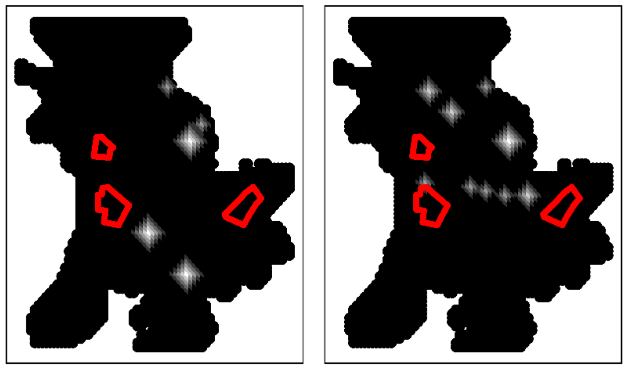

Figure 11. Objective function value surfaces (white areas have higher m_j values).

5.2.2. Test Results and Discussion

For each parameter set, 10 trial runs of the GA-MSA were performed, and each trial run had the same initial population of candidate MSA designs. As before, the quality of solutions was assessed using the hypervolume improvement metric. Figure 12 presents the average hypervolume percentage improvement for the Set 1 and Set 2 trials. In contrast to the GA-PSA tests, the heuristic expanded the dominated objective function space faster in the GA-MSA tests when Set 2 parameters were used, which emphasizes the crossover operation.

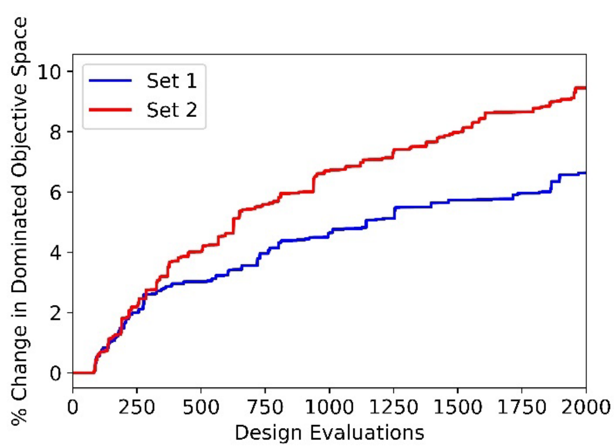

Figure 12. Average percentage change in dominated objective function space for GA-MSA tests.

In Figure 13, the objective function values for the MSA design generated in a trial run are plotted. The observed trade-off between the objective function values in the Pareto front is the result of the predefined objective function surfaces presented in Figure 11. The best MSA designs generated in the trial run for the M_o and M_c objectives, independently, are illustrated in Figure 14. The GA-MSA found solutions that contained the areas with greater m_{zj} values, as predefined in the objective function surfaces presented in Figure 11. Interestingly, the best MSA solution according to the M_c objective exhibits an impractical spike in the top left area of the MSA. Given that micromobility vehicles generally can only operate within the MSA, the spike in the MSA polygon is impractical as there is probably no path that a micromobility vehicle could use to access this area. There are at least two possible approaches to prevent this type of design. The most direct approach is to consider the road and sidewalk network of the study region and use the cut-set concept to ensure that the network within the MSA is connected (Zhang & Yang, 2004); this would introduce an additional constraint to the design problem. Another more indirect approach is to introduce a constraint to ensure a degree of smoothness in the polygon using, for example, a convex hull-based indicator (Maruyama et al., 2014). Note, however, that in practice the designs generated by the GA-MSA should only be a step within a more comprehensive planning process in which the algorithmically generated polygons would be polished and modified in discussions with the various stakeholders.

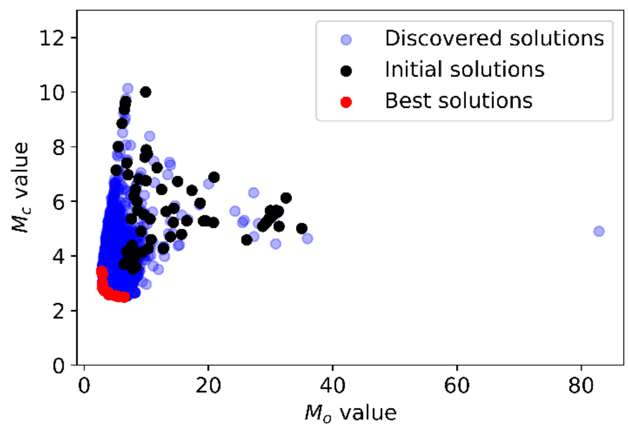

Figure 13. Example of objective function values for a GA-MSA trial run.

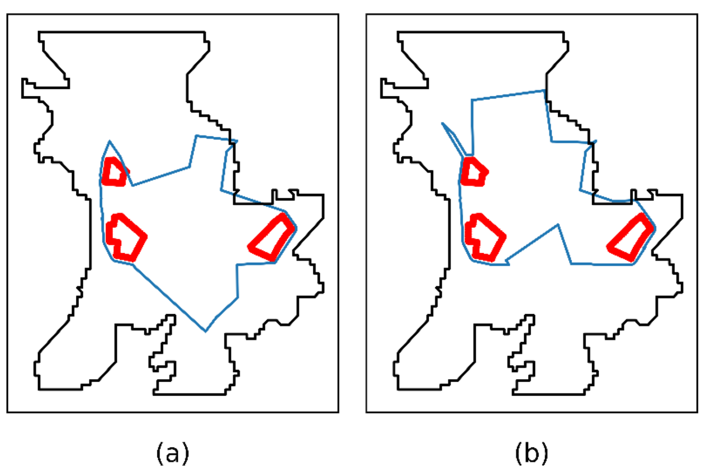

Figure 14. Example of best MSA designs (blue polygons) based on M_o (a) and M_c (b) objectives for a trial run.

Chapter 6. Closing Remarks

An optimization-based approach was proposed that incorporates the design of transportation service areas. Models and heuristics were presented for the joint design of fixed route transit networks and their complementary PSA, and for the design of micromobility service areas. The proposed methodologies can be used to generate design alternatives that would, in theory, be discussed and modified in planning processes involving analysts, stakeholders, and decision-makers. The views of communities and other stakeholders could directly be reflected as inputs in the GA heuristics proposed. For example, naïve initial population procedures were presented that generate candidate designs based on mathematical procedures. However, designs generated as part of conversations with communities and service operators could be directly fed into the heuristics as initial designs that the algorithm tries to improve.

As the GA-PSA and GA-MSA procedures do not depend on the mathematical characteristics of the design problem's objective function or constraints, the proposed approaches can be adapted with ease to account for other agency considerations beyond the ones considered in this report. For example, in the PSA design problem, the equity objective considered here could be substituted by a function that reflects the PSA modification problem faced by agencies redesigning their services due to budget cuts. In this context, PSA reductions, subject to regulatory constraints, are a common cost reduction strategy (Government Accountability Office, 2012). The proposed design approach can be used to objectively search for PSA designs that equitably distribute the cost of the PSA reduction across different population groups. The proposed approach could also be applied for the opposite problem when the budget increases and the agency must decide how to expand their PSA so that the benefits from the budget increase are equitably distributed. In both situations, PSA coverage constraints could be introduced to ensure, for example, that only a percent of the residents living within the existing PSA are negatively affected by its redesign.

Emerging IT-enabled transit services (e.g., microtransit) offer the opportunity to provide expanded or targeted services to particular groups (e.g., poor communities, people with certain types of disabilities) and develop other strategies to provide equitable mobility options (National Academies of Sciences, Engineering, and Medicine, 2016). Given the variety of services enabled by IT, a possible extension to the work presented in this report is a design problem that jointly defines multiple types of service areas for different types of service types that could be distinguished, for example, by their hours of operation, spatial coverage, and user group.

There are several opportunities for future research. For example, besides the greedy search strategies mentioned in Section 5.1.4, the GA-PSA can also be improved by including local search mechanisms that direct the evolution of PSA boundaries based on the heuristic's performance on preceding iterations, an idea that could also be applied to the GA-MSA. For example, parameters could be adapted given repeated failures (or successes) in generating non-dominated solutions. This could include changing which boundary segments can be considered in the search process or modifying the magnitude of the boundary expansions or contractions. Additionally, the GA-PSA can be modified by adding a procedure that explores if the polygons generated by the GA-PSA can substitute and improve the PSA of designs that were not among the η_p-best designs considered in Step 4. Naturally, this process would need to ensure that the minimum $B(\boldsymbol{R}_k, \Delta_{min})$ of the network is contained by the substitute PSA and that the budget constraint is not violated by the swap. A potential challenge of this modification, and the GA-PSA in general, is the computational cost associated with the model systems used by a transit agency to predict demand for its services.

In this study, simple demand models were used, but in practice, an agency could use more time-consuming models to compute the objective functions and constraints of interest, which could limit the number of designs that could be evaluated. This is particularly true if more direct and comprehensive measures of congestion reductions are desired. For example, in the PSA problem, congestion is tackled by searching for designs that maximize the usage of transit. However, one could imagine a regional agency would be interesting in searching for transit network designs with the explicit objective of reducing total travel delays in the road network ($M_c(\boldsymbol{R}, \boldsymbol{f}, \boldsymbol{s})$), and then the decision maker's objective would be:

$$\min\ \boldsymbol{Z} = \left(M_b(\boldsymbol{R}, \boldsymbol{f}), M_p(\boldsymbol{s}), M_c(\boldsymbol{R}, \boldsymbol{f}, \boldsymbol{s})), E(\boldsymbol{R}, \boldsymbol{f}, \boldsymbol{s})\right) \tag{2b}$$

As discussed in Section 3.1.3, to compute $M_c(\boldsymbol{R}, \boldsymbol{f}, \boldsymbol{s})$), however, would require complex and time-consuming travel demand models, and the total run time of the proposed procedures would not be reasonable in a real-world context. This problem could be addressed by developing greedy GA-PSA/GA-MSA and/or by new surrogate-based optimization techniques that provide computationally inexpensive approximations to the computationally expensive model systems.

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