Report No. FHWA-RD-75-49

IMPACT OF ECONOMIC RISKS ON BOX CULVERT DESIGNS--AN APPLICATION TO 22 VIRGINIA SITES

G. K. Young and M. R. Childrey

August 1974

Final Report

This document is available to the public through the National Technical Information Service, Springfield, Virginia 22161

Prepared for

FEDERAL HIGHWAY ADMINISTRATION

Offices of Research & Development

Washington, D.C. 20590

NOTICE

This document is disseminated under the sponsorship of the Department of Transportation in the interest of information exchange. The United States Government assumes no liability for its contents or use thereof.

The contents of this report reflect the views of Water Resources Engineers, which is responsible for the facts and the accuracy of the data presented herein. The contents do not necessarily reflect the official views or policy of the Department of Transportation. This report does not constitute a standard, specification, or regulation.

FHWA DISTRIBUTION NOTICE

Sufficient copies of this report are being distributed by FHWA memorandum to provide a minimum of two copies to each regional office. Copies are not being distributed to division offices because a condensed version of the report was published in the July 1974 issue of the ASCE Hydraulics Journal.

TECHNICAL REPORT STANDARD TITLE PAGE

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
FHWA-RD-75-49		PB 244 107

4. Title and Subtitle
Impact of Economic Risks on Box Culvert Designs--an Application to 22 Virginia Sites

5. Report Date
August 1974

6. Performing Organization Code

7. Author(s)
G. K. Young
M. R. Childrey

8. Performing Organization Report No.
WRE 20810

9. Performing Organization Name and Address
Water Resources Engineers
8001 Forbes Place
Springfield, Virginia 22151

10. Work Unit No.

11. Contract or Grant No.
DOT-FH-11-7669

12. Sponsoring Agency Name and Address
Offices of Research and Development
Federal Highway Administration
U.S. Department of Transportation
Washington, D. C. 20590

13. Type of Report and Period Covered
Final Report

14. Sponsoring Agency Code
E0098

15. Supplementary Notes
FHWA Contract Administrator: Gilbert Trainer
FHWA Contract Manager: J. Sterling Jones

16. Abstract

This report demonstrates the results of a preliminary effort to incorporate economic risks into culvert design and to relate this economic design to conventional design practice for 22 culvert sites in Virginia. Economic design is defined as the minimum annual construction cost plus the expected flood-related loss or risk. Each design is evaluated in a five-step procedure: (1) calculate annual construction cost, (2) perform dynamic flow routings for a series of flood hydrographs, (3) estimate embankment erosion, (4) calculate losses and (5) weigh losses with flood probabilities to derive risks.

PRICES SUBJECT TO CHANGE

17. Key Words
Culvert Design, Economic Flood Risk Evaluation

18. Distribution Statement
No restrictions. This document is available to the public through the National Technical Information Service, Springfield, Virginia 22161

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price
Unclassified	Unclassified	52	4.25 - 2.25

Form DOT F 1700.7 (8-69)

ACKNOWLEDGEMENTS

This report was originally authored by G. K. Young and M. R. Childrey under the same title in 1970. Since that time, the report has been reviewed and revised by both Federal Highway Administration and Water Resources Engineers (WRE) staff.

Contributors to the original report were Dr. H. W. Parker, F. K. Stovicek and J. M. Normann of the Federal Highway Administration and A. L. Pond, Jr., L. H. Love and E. C. Cochran, Jr., of the Virginia Department of Highways, who furnished the basic data for the study. Contributions were also made by WRE staff L. S. Costello, R. S. Taylor, G. F. Tierney, J. L. Matticks and J. J. Hanks. Final internal WRE review of the report was made by M. T. Tseng and P. P. Weiner.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

I. INTRODUCTION

This report is the first volume in a series of six comprising the final report for the study entitled *Evaluation of Flood Risk Factors in the Design of Highway Stream Crossings*, authorized by the Federal Highway Administration (FHWA), Department of Transportation, under Contract No. DOT-FH-11-7669. The overall objective of the study as a whole is to develop an engineering systems analysis technique to reduce flood damage to highway stream crossings on a sound, probabilistic basis by including economic risk analysis in the hydraulic and hydrologic design of bridge waterways

This volume demonstrates the results of a preliminary effort to incorporate economic risks into culvert design and to relate this economic design (1)[1] to conventional design practice for 22 culvert sites in Virginia. Present culvert design is based on hydrologic and hydraulic considerations. Although economic risks have been indirectly considered, they have not been directly used in the design of culverts. Economic risks are expected losses and can be divided into three general loss categories: direct damage to roadway and culvert, traffic-related losses and losses due to flood damage in the upstream flood plain. These losses are converted into yearly flood risks by applying the appropriate probability for each flood.

A balanced design would include both construction costs and economic flood risks. Actual data show that, to a close approximation, construction costs vary linearly with culvert area; i.e., as the area of the opening increases, the cost increases. Flood risks vary inversely with the culvert

---

[1]Numbers in parentheses refer to corresponding items in the List of References.

area; small areas yield high risks, whereas larger areas yield lower risks. The sum of the annual construction cost and risk yields a total cost with a discrete minimum as shown in Figure 1; it is assumed that alignment, balancing of cut and fill volumes, and traffic volumes are resolved.

The procedure as it is reported here is limited primarily by:

1. The scarcity of flood damage statistics, particularly in monetary terms, for evaluating economic factors.
2. The lack of understanding of the precise mechanism of progressive erosion along the downstream fill slope. This part of the hydraulic computation can be improved when better information becomes available.
3. The lack of data describing small watershed hydrology. Here also such data are expected to be available in the near future and may then be applied to the program.

The major disadvantage of the economic design procedure itself is that extensive data and considerable computation are required to obtain the minimum economic design; consequently, the application of the procedure will probably be limited to medium and major installations. In order to overcome this complexity, a general relationship between the conventional design and economic design is investigated by analyzing a sample of conventionally designed culverts using the optimum economic design procedure. The sample consists of 22 culvert sites located in Virginia. All study sites are located on rural four-lane highways in order to make the sample homogeneous. The spatial distribution of the sample is shown in Figure 2.

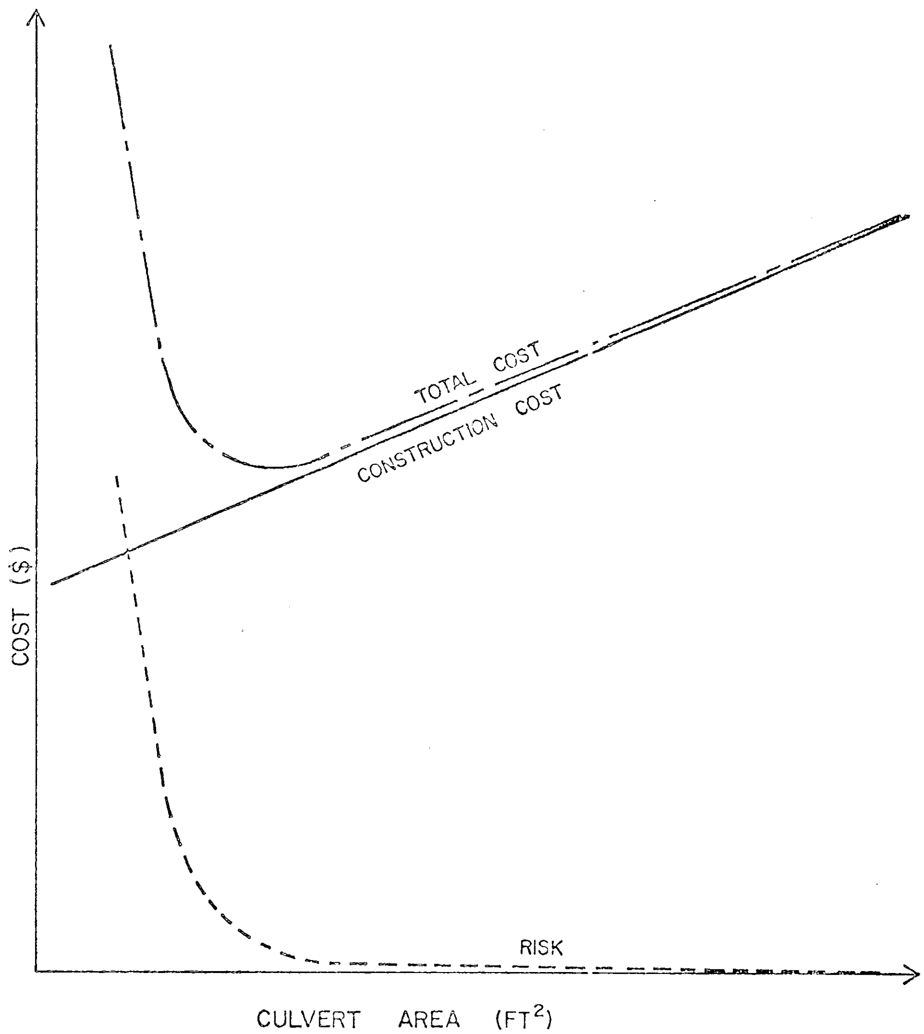


FIGURE I
CULVERT COST RESPONSE CURVE

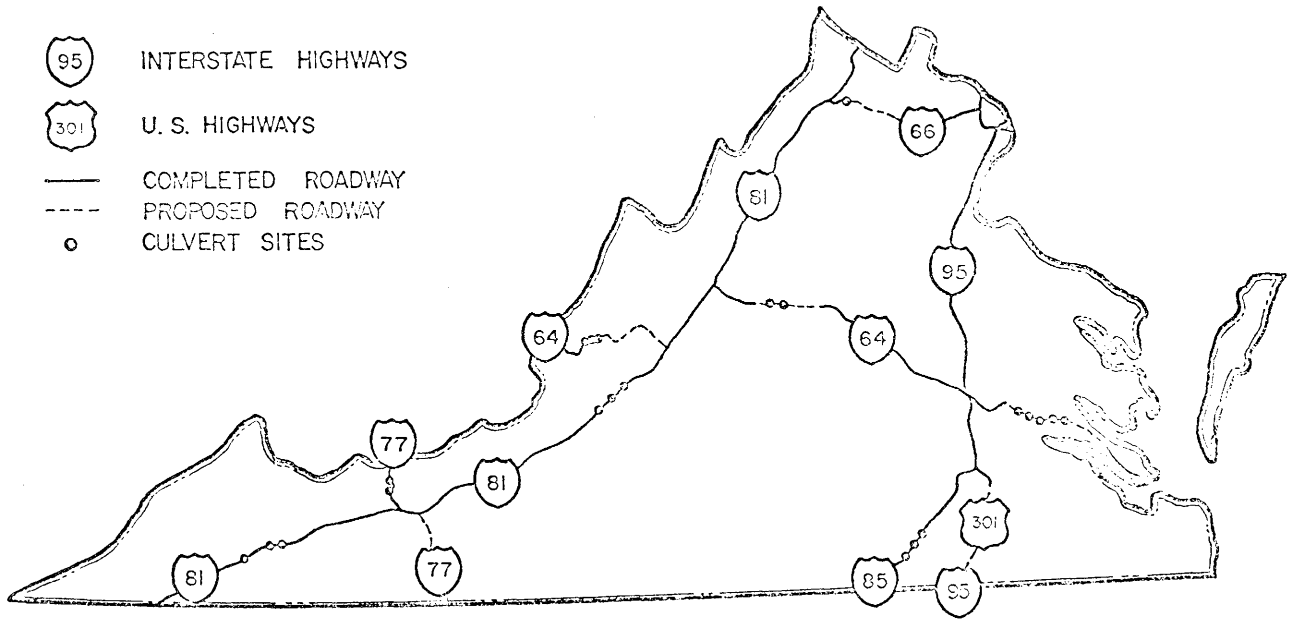


FIGURE 2
SPATIAL DISTRIBUTION OF CULVERT CASE SITES

II. ECONOMIC DESIGN

The analysis procedure for deriving the economic design is automated by use of a digital computer. The analysis calculates the economic design of the stream crossings with its expected total tangible costs. The definition of the economic design is the minimum annual construction costs plus the expected flood-related loss or risk. The procedure uses five steps to evaluate each design:

1. Calculate annual construction cost,
2. Perform dynamic flow routings for a series of flood hydrographs,
3. Estimate embankment erosion,
4. Calculate losses and
5. Weigh losses with flood probabilities to derive risks.

The logical flow of the procedure is given in Figure 3.

CALCULATE ANNUAL CONSTRUCTION COST

An estimated annual construction cost is used to compute the risk component of the economic design. Construction cost includes the cost of the structural excavation, culvert, embankment and roadway.

Structural excavation costs per cubic yard are:

$$C_X = \frac{U_X L \frac{D}{2} (NB + 2)}{27} \tag{1}$$

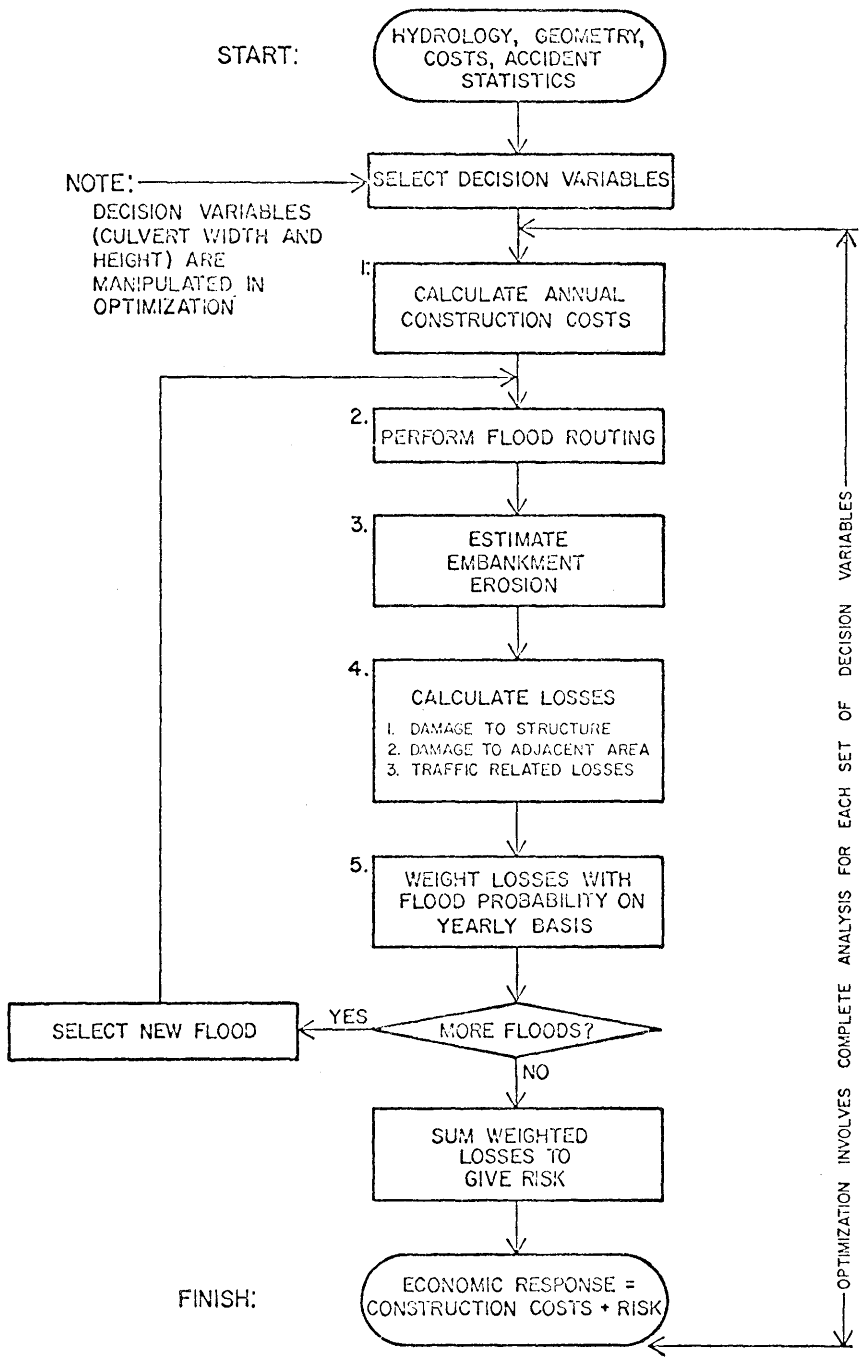


FIGURE 3
ANALYSIS PROCEDURE

where C_x = total cost of structural excavation for culvert,[2] \$,
U_x = unit cost of structural excavation, \$/yd^3,
L = culvert length, ft,
D = culvert depth, ft,
N = number of culvert barrels, and
B = width of barrel, ft.

Culvert costs include the cost of concrete and reinforcing steel for the culvert barrel. For program demonstration, unit quantities for concrete and steel cost estimating are derived from a statistical fit to data taken from California standard plans (2).[3] Figures 4 and 5 show the steel and concrete curves for 104 standard plans for single barrel culverts; unit amounts for multiple barrel culverts are obtained by applying the appropriate multiplier. Sensitivity analysis (3) indicates that the appropriate multiplier is simply the number of barrels. The functional form used in the statistical least squares fit, derived from analysis of bending moments in the culvert associated with soil pressure and traffic loads, displays a high degree of correlation (0.96 for steel and 0.98 for concrete). For a given width and depth (B and D), the cost of the culvert barrel is

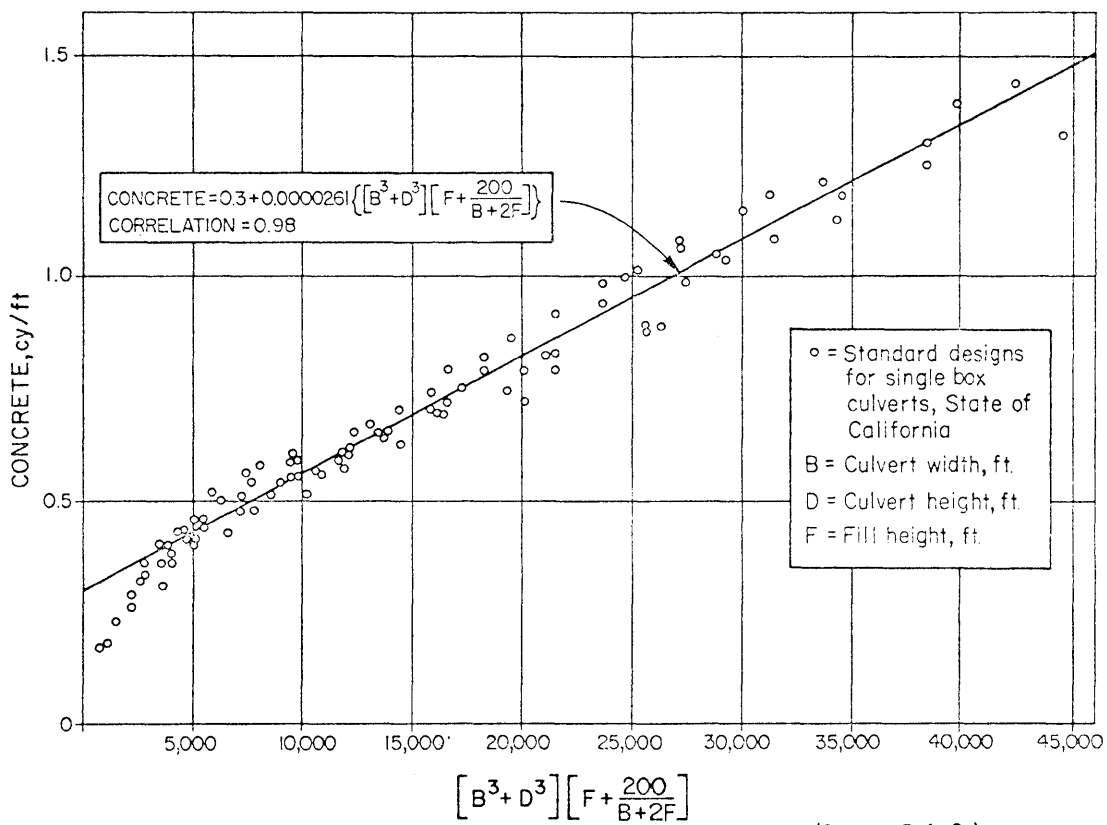
$$C_c = U_c C_b L + U_s S_b L \qquad (2)$$

where C_c = cost of culvert barrel, \$,
U_c = unit cost of concrete, \$/yd^3,
C_b = unit quantity of concrete, yd^3/ft,
U_s = unit cost of reinforcing steel, \$/lb, and
S_b = unit quantity of steel, lb/ft.

---

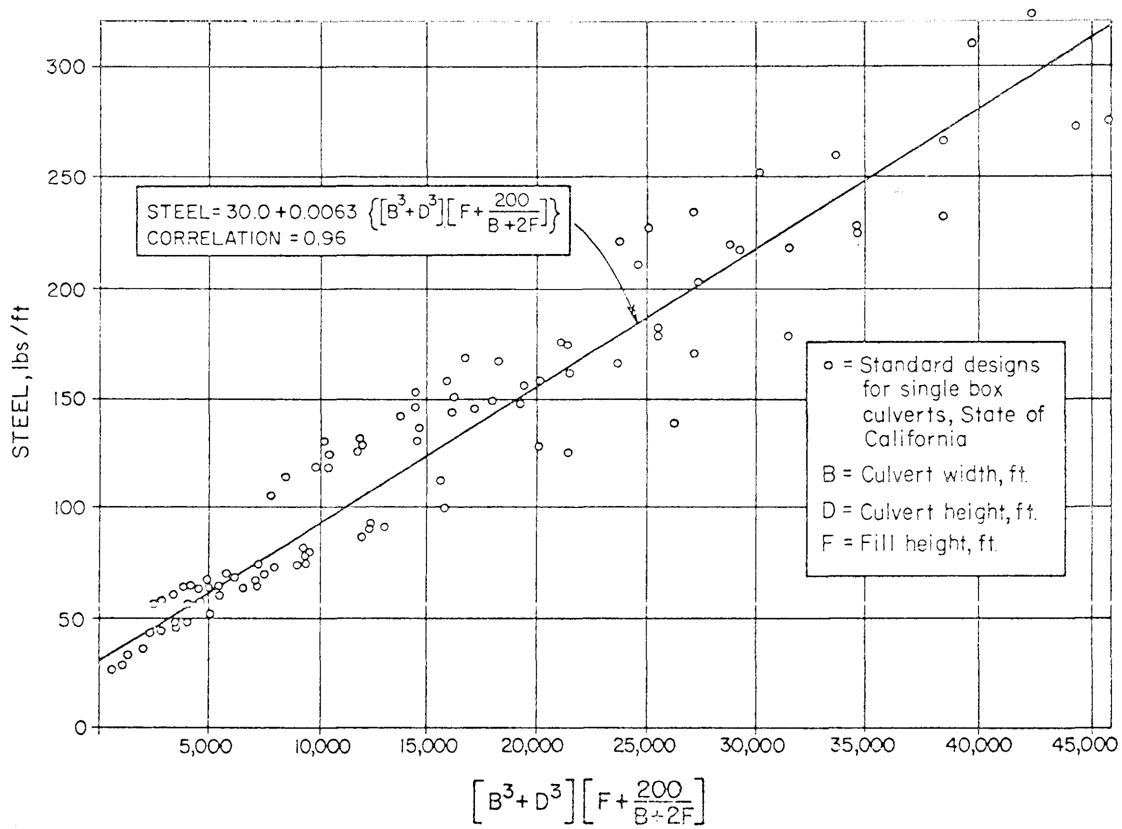
[2]Letter symbols are defined where they first appear and are arranged alphabetically in the Appendix.

[3]It would be difficult to develop standardized cost plans for the nation as a whole. However, program inputs on construction costs may be modified in such a way that unit price of materials and labor vary with culvert site, provided that the culvert design procedure can be standardized.



(Source: Ref. 2)

FIGURE 4
CONCRETE UNIT QUANTITY CURVE



(Source : Ref. 2)

FIGURE 5
STEEL UNIT QUANTITY CURVE

The cost of the embankment is

$$C_f = E_f U_f \qquad (3)$$

where C_f = total cost of embankment, \$,
E_f = volume of embankment, yd^3, and
U_f = unit cost of "in place" fill, $\$/yd^3$.

The volume of the embankment is obtained from the plan and profile maps at the site.

Finally, the cost of the roadway is

$$C_r = L_r U_r \qquad (4)$$

where C_r = cost of roadway (guardrail to guardrail), \$,
L_r = actual roadway length, ft, and
U_r = unit cost of roadway (guardrail to guardrail), \$/ft.

Therefore, the annual construction cost is

$$C_t = (C_x + C_c + C_f)\ CRF \qquad (5)$$

where C_t = annual construction cost, \$/yr, and
CRF = capital recovery factor, applied to obtain a series of annual costs using, in this case, an interest rate of 6.5 percent over 100 years.

The estimated annual *repair* cost for structural damage caused by large floods is weighted by the probability of flood occurrence.

The unit costs of materials were obtained from the individual project bid sheet and converted to a 1970 base using a construction cost index (4). Where bid sheets were not available, the unit costs were taken as those of the geographically closest sample point in which bid sheets were available.

PERFORM FLOOD ROUTING

Flood routing is used in this procedure to obtain the upstream stage-discharge curve for risk factor analysis. From a technical point of view dynamic routing rather than steady state flood peak discharge is preferred since dynamic routing gives the duration of overtopping, which in turn determines the amount of embankment erosion and duration of traffic interruption. However, possible future expansion and adaptation of the risk analysis procedure should include making flood routing an optional feature to allow more flexibility in those cases where good hydrologic data exist.

In this study, as an approximation for design purposes, runoff hydrographs are assumed to be triangular in shape. Three basic parameters describe a unique triangular hydrograph: time to peak (T_p), flood duration (T_b), and peak flow (Q_p). From Reference 5, T_p is approximated by

$$T_p = \frac{L_w}{3600\ V_o} \tag{6}$$

where L_w = hydraulic length of watershed, ft, and
V_o = overland velocity, ft/sec.

The velocity value used in Equation 6 is obtained from a Soil Conservation Service (SCS) graph (5) which expresses velocity as a function of slope and ground cover of the watershed. The Bureau of Reclamation guidelines (6) are used to determine a value of T_r. the length of time between the peak flow and the end of the hydrograph. For a watershed, the relationship between T_r and T_p is approximately

$$T_r/T_p = \text{constant} \tag{7}$$

The SCS analysis (5) shows a value of 1.67 as a general average for ungaged watersheds.

The third parameter required to describe the triangular shape of the hydrographs is the peak flow (Q_p). The range of flows associated with different recurrence intervals is determined by fitting runoff data of the sites to a Gumbel plot (7, 8). The peak flow of a storm of any recurrence interval can be determined from the plots. An example is given in Figure 6.

In the culvert analysis, it is necessary to consider a wide range of hydrologic possibilities. A number of floods, each having an associated yearly probability of occurrence, are analyzed to assess the risks. The probability, P, of a flow equaling or exceeding a given peak and flow is the reciprocal of the recurrence interval, R. For the purpose of this analysis, the peak flows are divided into classes based on return period and the midpoint discharge of the class is taken as the representative discharge. The probability of a representative flow occurring in a class interval is the difference in the cumulative probabilities of the class boundary values. An example is given in Table 1.

The flood routing procedure routes the representative set of flood hydrographs through a culvert site using the following procedure:

1. Start with the first flood to be routed.
2. Compute the inflow from the flood hydrograph:

$$I = \frac{Q_p T}{T_p} \qquad \text{for } 0 \leq T < T_p \text{, and} \tag{8a}$$

$$I = \frac{Q_p \, (T_b - T)}{T_r} \qquad \text{for } T_p \leq T \leq T_b \tag{8b}$$

where I = inflow from hydrograph, cfs, and
T = elapsed time, hr.

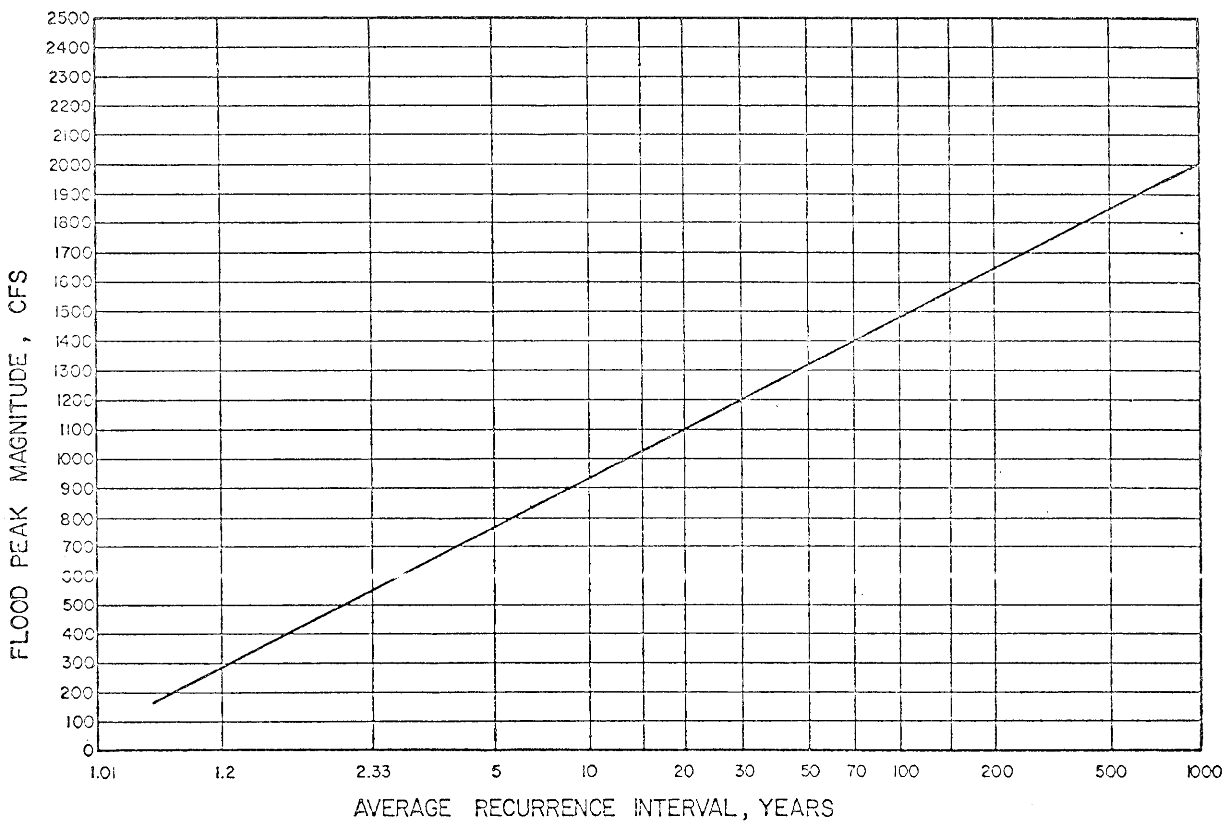


FIGURE 6

ESTIMATED FLOOD PEAK RECURRENCE

Table 1. Flood Probability Analysis

1 Q Class Interval (cfs)	2 Return Period (yrs)	3 Class Midpoint[1] (cfs)	4 Σp (Inverse of Col. 2)	5 Probability of Col. 3 Values[2] (Δ Col. 4)
293	1.11	-	0.9100	-
-	-	439	-	0.4800
584	2.33	-	0.4300	-
-	-	730	-	0.3300
875	10	-	0.1000	-
-	-	1003	-	0.0600
1130	25	-	0.0400	-
-	-	1220	-	0.0200
1310	50	-	0.0200	-
-	-	1395	-	0.0100
1480	100	-	0.0100	-
-	-	1565	-	0.0050
1650	200	-	0.0050	-
-	-	1995	-	0.0035
2340	650	-	0.0015	-
				Σ = 0.9085

[1]These values are used as the apex of the triangular hydrograph which is used in the flood routing.

[2]These values are assumed to be the probability of yearly occurrence of their respective flood hydrographs (whose peaks are given in Col. 3 and computed from the differences in Col. 4).

3. Solve for discharge and the headwater depth assuming inlet control. This assumption is based on a side calculation which indicates outlet control[4] to be in effect a small percentage (less than 10 percent) of the time during the passage of floods through culverts. The relationship of headwater depth to storage is determined from a topographic map of the watershed upstream from the culvert. This relationship is fitted by the functional formula for a straight line on log-log paper for headwater depth (y axis) versus storage (x axis):

$$H = \Theta S^{\Omega} \tag{9}$$

where H = headwater depth, ft,

S = storage, ft^3, and

Θ, Ω = empirical constants obtained by the use of topographic maps.

The time step used in the flood routing calculations is $T_b/50$, which yields an interval small enough to assure accurate computations for the sites investigated.

The outflow is divided into two categories: flow through the culvert and flow over the roadway. The culvert equations for $H > 1.5D$ are

$$A = NBD \text{ and} \tag{10a}$$

$$V_c = \sqrt{2g(H - D)} \tag{10b}$$

The culvert equations for $H < 1.5D$ are

---

[4]The calculation of outlet control is based on a method described in Reference 3. The procedure may be adapted to incorporate the improved inlet design described in "Hydraulic Design of Improved Inlets for Culverts," *FHWA Hydraulic Engineering Circular No. 13,* August 1972 (9).

$$A = NB(.67H) \tag{11a}$$

$$V_c = \sqrt{2g(.33H)} \tag{11b}$$

where A = area of flow in culvert, ft^2, and
V_c = culvert flow velocity, ft/sec.

The weir flow over the roadway is found by

$$Q_t = 3.03(H - F)^{1.5} \ell \tag{12}$$

where $H > F$,
Q_t = overtopping flow, cfs,
F = fill height above invert, ft, and
ℓ = equivalent roadway length, ft.

The equivalent roadway length used in the weir equation is computed in such a way that the actual profile area is equal to a rectangular section of height, F, and length, ℓ, as shown in Figure 7.

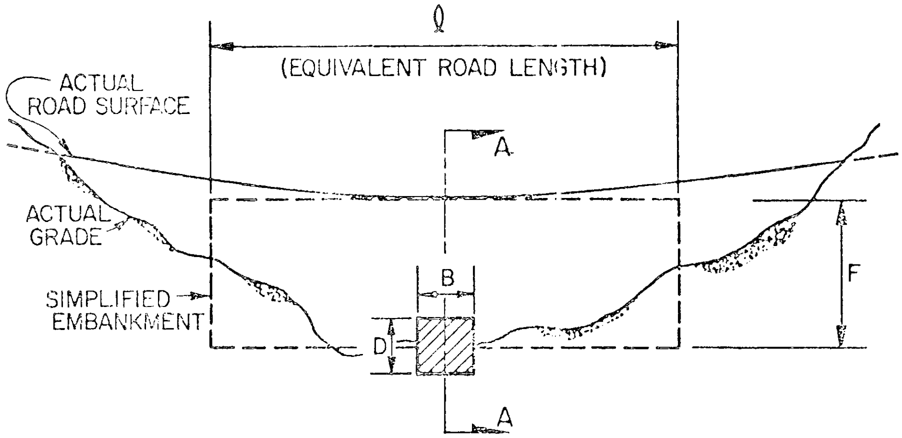
4. Compute the end-of-period storage, which is

$$S_n = S_o + (I + O)\ \Delta T \tag{13}$$

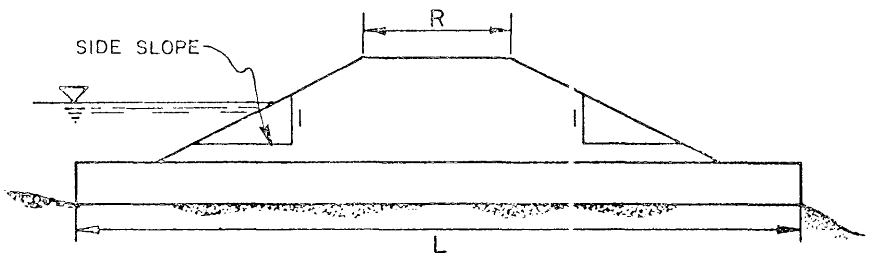
where S_n = end-of-period storage, ft^3,
S_o = end-of-period storage in previous time step, ft^3, and
O = total outflow (culvert + weir).

5. Increment the time interval and repeat steps 2 through 5 until the entire inflow hydrograph is routed.

6. Select the next flood and repeat steps 2 through 6 until all hydrographs are routed through the site.



Typical Elevation



Section A-A

FIGURE 7
EMBANKMENT SCHEMATIC

ESTIMATE EMBANKMENT EROSION

It is assumed that damage and ultimate failure of culverts are linked to embankment failure (see References 10, 11, 12, 13, 14, 15, 16 and 17); specifically, erosion on the downstream slope caused by overtopping, and scour and erosion caused by high culvert velocities. Culvert failure due to piping along the outside walls of the culvert and embankment saturation is not likely to occur in the relatively short duration of upstream ponding and can be ignored safely in this analysis.

It is also assumed that the fill is undamaged until overtopping occurs and material is eroded from the downstream shoulder. For damage estimation, the amount of material eroded is related to the total material in the embankment. The extent of erosion is related to the velocity on the downstream embankment. The velocity on the downstream embankment is computed by employing Manning's Equation. It is assumed that erosion does not begin until a specified threshold velocity is attained (18). The routing calculations yield velocity of overtopping flow as a function of time, and sediment carrying capacity is related empirically to velocity. The relationship is of the form

$$E = \alpha V_t^{\beta} \tag{14}$$

where E = erosion (tons/ft/day)

V_t = mean velocity at toe of downstream embankment, ft/sec,

α, β = empirical constants representing soil type erodability.

Comparison of the two extremes of a cohesive and cohesionless soil condition indicates that values of $\alpha = 0.25$ and $\beta = 3.8$ provide reasonable estimates. A summation over time is performed to calculate the volume of eroded fill. Culvert velocities in excess of 17 ft/sec are assumed to wash out endwalls and cut back into the fill (19). Velocities higher than 30 ft/sec are assumed to completely destroy the embankment and culvert.[5] Accordingly, the maximum culvert velocity for each flood routing is noted in order to estimate damages associated with high culvert velocities.

CALCULATE LOSSES

Damages associated with different flood magnitudes may be divided into three main categories: damage to the site, traffic-related damage and upstream property damage.

For the first category the roadway may be damaged due to overtopping and the fill may erode due to high velocity flows. Figure 8 relates percent erosion to percent of roadway failure and percent of damage to box culvert. It is assumed that until 90 percent of the volume of fill is eroded, no damage is done to the culvert. In extreme cases, the entire site may "wash out" causing the replacement of the entire roadway, fill volume and the box culvert.

Figure 8 shows that damage is also assumed to occur due to high velocities leaving the culvert. This damage may occur at sites which have a relatively large fill height and modest backwater storage. In such cases, the computations assume the damage to begin when velocities entering the culvert exceed 17 ft/sec and, following a linear relation, to reach 100 percent of total construction cost at a velocity of 30 ft/sec. This damage is multiplied by the cost adjustment factor (C_a) to estimate the losses. The economic loss incurred by damage to the site, L_S, is computed as

$$L_S = (P_1C_f + P_2C_r + P_3C_c + P_4C_t)\, C_a \qquad (15)$$

---

[5]This velocity is considered conservative. Actually, damage may reach 100 percent of costs at velocities somewhat lower than 30 ft/sec.

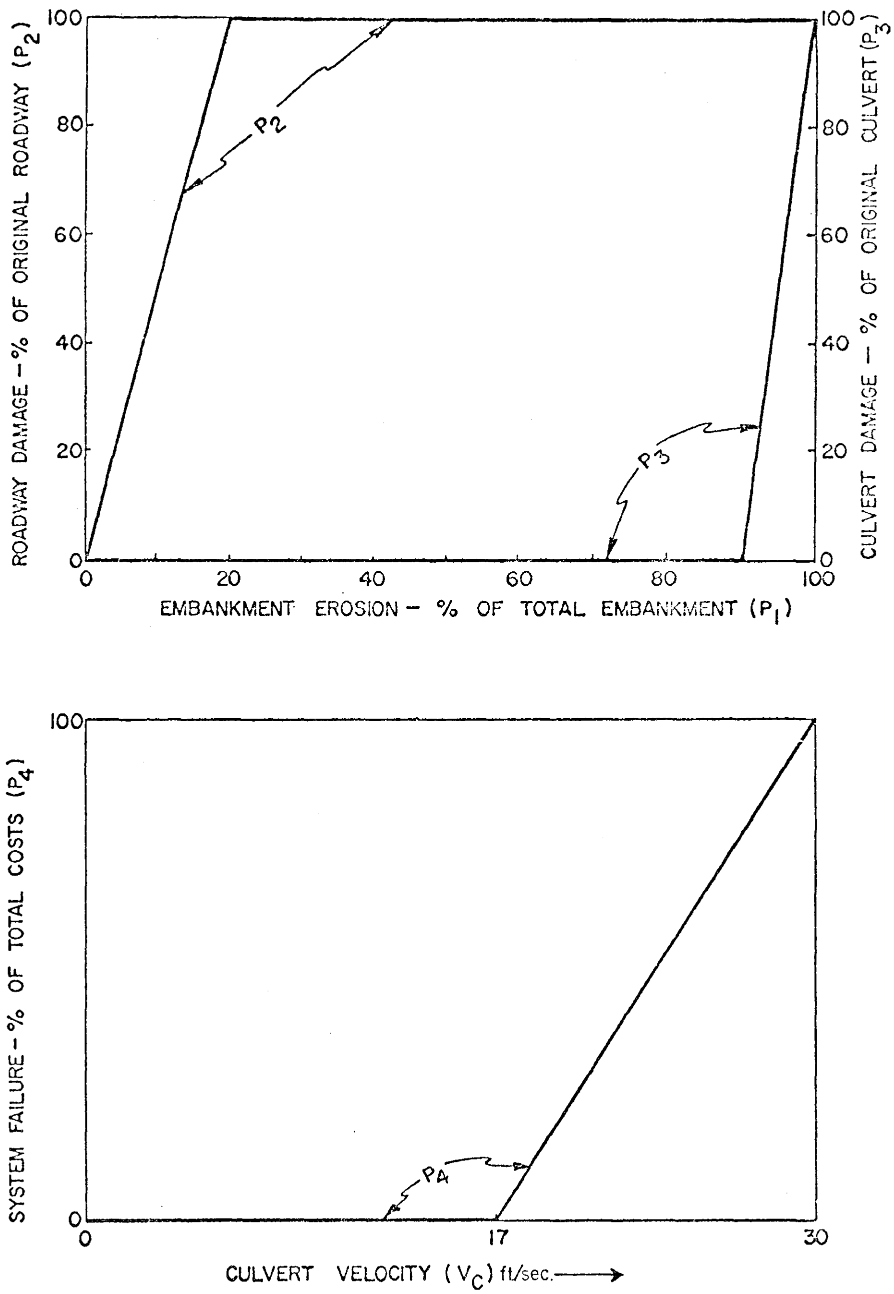


FIGURE 8
DAMAGE FUNCTIONS

where P_1 = percent of original embankment volume eroded due to overtopping,

P_2 = percent of roadway damage due to overtopping,

P_3 = percent of culvert damage due to overtopping,

P_4 = percent total damage due to excessive culvert velocities and

C_a = cost adjustment factor (typical value of 2.0).

The cost adjustment factor derived for this study allows for an increase in contracting cost to have a site quickly repaired.

The second loss component is the traffic-related loss. An estimate of the total time that the traffic is unable to travel at its normal rate over the crossing is required. This time is assumed to be equal to the sum of the duration of the flood overtopping the road and additional time required to repair significant damage to the site. The duration of overtopping is computed in the flood routing procedure; the time of repair is estimated from Figure 9. Representative figures and, in some cases, empirical data (20) are used for the distribution and magnitude of the average daily traffic in the case studies. There are four sub-categories of traffic-related losses:

1. Additional running cost,
2. Lost time of vehicle occupants,
3. Expected accidents on additional detour miles and
4. Expected accidents due to the unexpected obstacle.

The parameters necessary to evaluate these losses are:

X_1 = Duration of detour = duration of overtopping + repair time (hours),

X_2 = Average daily traffic (ADT--vehicles/day),

X_3 = Passenger cars (fraction of ADT),

X_4 = Commercial delivery vehicles (fraction of ADT),

X_5 = Single unit trucks (fraction of ADT),

X_6 = Semi-trailers (fraction of ADT),

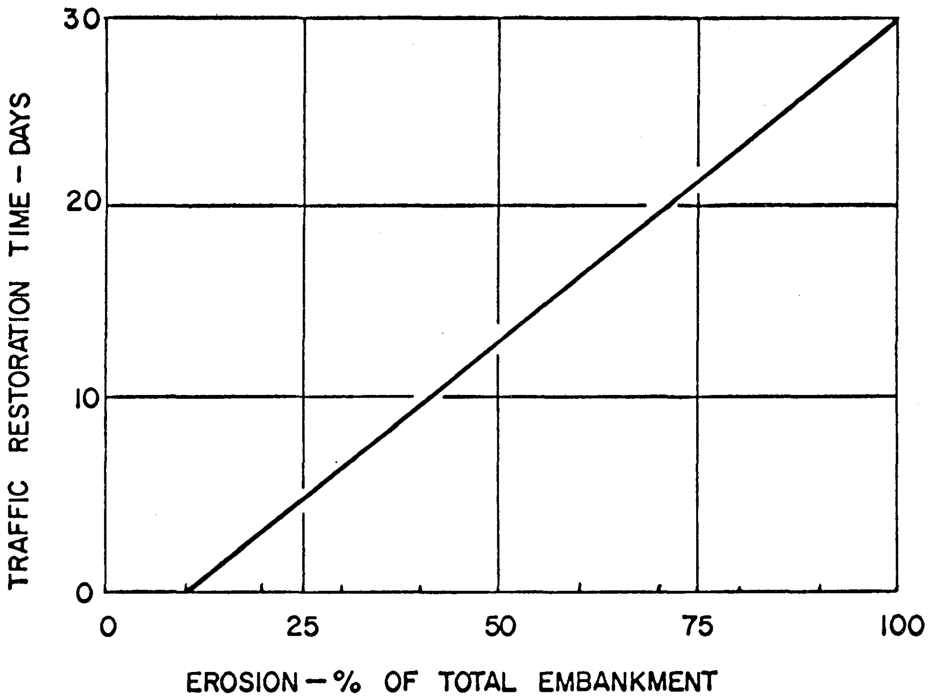


FIGURE 9

ASSUMED TRAFFIC RESTORATION FUNCTION

X_8 = Length of detour (miles),

XA_8 = Length of original route (miles),

X_9 = Speed on detour (miles/hr),

X_{10} = Occupancy rate (people/vehicle, typical value = 1.7),

X_{11} = Accident distribution ratio - normal conditions (personal injuries/death, typical value = 30),

X_{12} = Accident distribution ratio - normal conditions (property damage/death, typical value = 300),

X_{13} = Accident distribution ratio - unexpected obstacle (personal injuries/death, typical value = 15),

X_{14} = Accident distribution ratio - unexpected obstacle (property damage/death, typical value = 150),

X_{15} = Death rate (people/100 million miles, typical value = 5.5),

X_{16} = Death rate factor for unexpected obstacle (multiplier to X_{15}, typical value = 1000),

C_1 = Cost of a death (typical value = \$50,000),

C_2 = Cost of a personal injury (typical value = \$2,000),

C_3 = Cost of property damage (typical value = \$400),

C_4 = Value of time (typical value = \$2/hr).

Parameters X_1 through XA_9 are different for each site considered, but X_{10} through X_{15} represent national statistics (21). X_{16} is a multiplier that is applied to the death rate to increase it due to the increased hazard of an unexpected obstacle. Parameters C_1 through C_4 are given in a study published by the Stanford Research Institute (22) on flood damage. The value of each parameter varies depending on the site conditions and must be evaluated by engineering techniques. The estimated running cost of a passenger car (C_5) in dollars per 1000 vehicle miles is a function of speed, as shown in Figure 10 (23). The equation of the curve in Figure 10 is

$$C_5 = 42.5 - .455X_9 + .0068X_9^2 \tag{16}$$

To adjust this passenger car running cost for varying types of vehicle distributions, the equations become

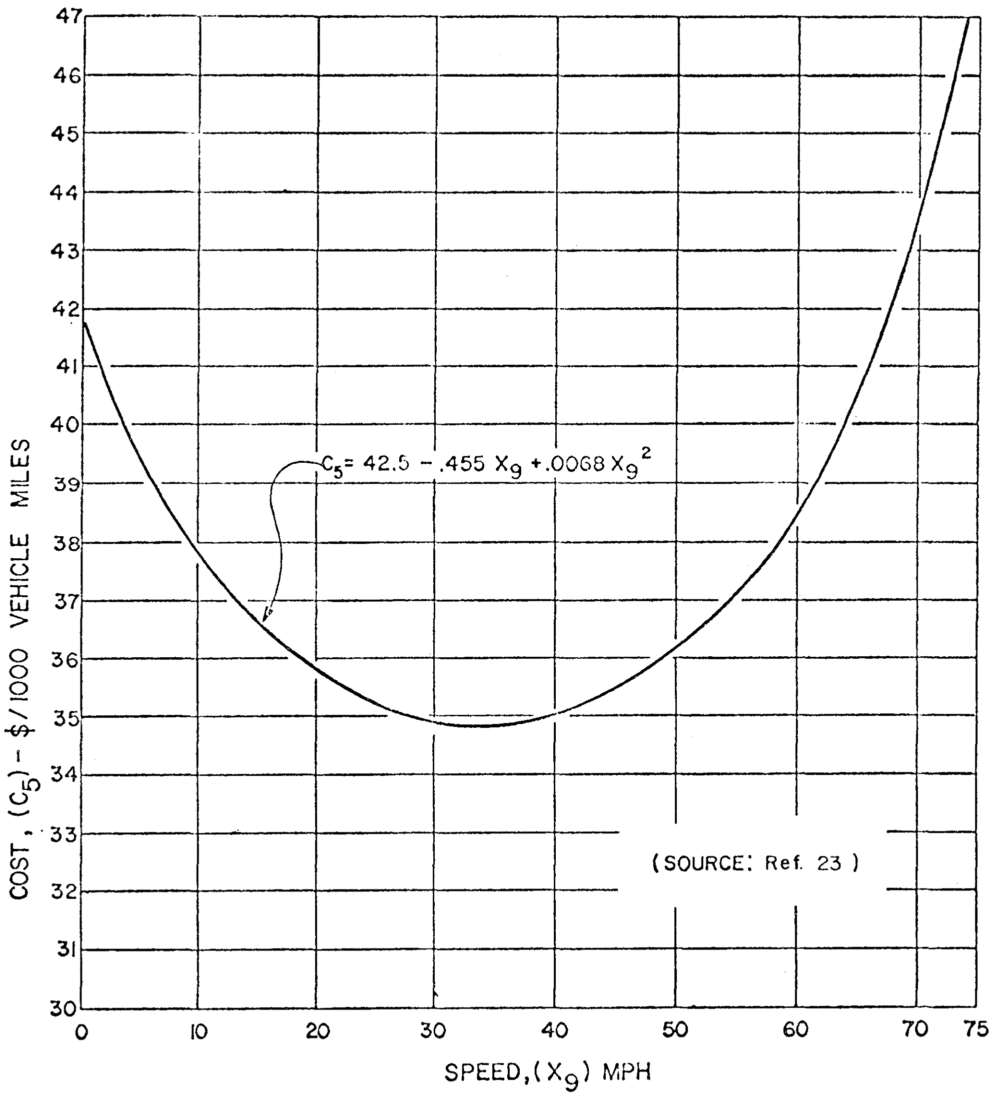


FIGURE 10
PASSENGER CAR RUNNING COSTS

$$C_5 = (42.5 - .455X_9 + .0068X_9^2)\,(X_3 + 1.2X_4 + 2.0X_5 + 3.15X_6) \quad (17a)$$

for the detour route, and

$$C_6 = (42.5 - .455XA_9 + .0068XA_9^2)\,(X_3 + 1.2X_4 + 2.0X_5 + 3.15X_6) \quad (17b)$$

for the normal route.

To estimate the losses associated with running costs, it is necessary to compute the running cost over the normal route and the detour. The difference in these two cost values represents the additional cost to the user of having to detour due to the failure of the culvert. The equations for computing the running costs (XL_1, \$) are

$$XL_{1a} = (X_1\ X_2\ X_8\ C_5)/24{,}000 \quad , \quad (18a)$$

$$XL_{1b} = (X_1\ X_2\ XA_8\ C_6)/24{,}000 \quad \text{and} \quad (18b)$$

$$XL_1 = XL_{1a} - XL_{1b} \quad (18c)$$

Time lost by the vehicle occupants is the delay or the additional time it takes to detour the site. The value of lost time (XL_2, \$) is computed by calculating the difference of the time value of the detour and the original route. The equations used for this calculation are

$$XL_{2a} = (X_1\ X_2\ X_8\ X_{10}\ C_4)/(X_9\ 24.0) \quad , \quad (19a)$$

$$XL_{2b} = (X_1\ X_2\ XA_8\ X_{10}\ C_4)/(XA_9\ 24.0) \quad \text{and} \quad (19b)$$

$$XL_2 = XL_{2a} - XL_{2b} \quad (19c)$$

The expected accident cost (XL_3, $) due to the difference in the dollar value of accidents on the detour and on the original route is calculated by

$$XL_3 = \frac{X_1 X_2 (X_8 - XA_8)\ X_{15}}{2.4 \times 10^9} (C_1 + X_{11}C_2 + X_{12}C_3) \qquad (20)$$

The expected accident cost due to an unexpected obstacle is computed by assuming one mile of road in the vicinity of the damaged culvert has a one-hour exposure to a higher death rate, defined as the death rate, X_{15}, times a death rate multiplier for unexpected obstacles, X_{16}. Thus, the higher rate is $X_{15}X_{16}$. The accident distribution ratios are X_{13} and X_{14} which may vary from those for the normal death rate, X_{11} and X_{12}. The equation for calculating this loss is

$$XL_4 = \frac{X_2 X_{15} X_{16}}{2.4 \times 10^9} (C_1 + X_{13}C_2 + X_{14}C_3) \qquad (21)$$

By applying these equations to a set of data describing a given site, it is possible to compute the total dollar value of the traffic loss due to flooding at the culvert site.

The third loss component, upstream property damage, depends largely on land use. The land uses on rural highways are usually one or a combination of the following:

1. Agriculture,
2. Single family residences and/or animal and other shelters and
3. Other highway systems.

The damages done to agricultural land use are developed for selected counties in Virginia for increasing flood depths. It is assumed that crops are completely destroyed when the flood depth reaches three feet. These data are presented in Table 2, with references used for data development, representing various types of agriculture in Virginia. The damage-depth relationship for each case study is taken from the appropriate data in Table 2, selecting that site in the table which is most representative of the case study.

Damage to single family residences as a function of flood depth is shown in Table 3. The value of single family property units upstream of the culvert is taken to be the median for the county in which it is located.

Damage to secondary and primary highways located in the upstream flood zone of the highway being studied is assumed to be a linear function of inundation depth, reaching 100 percent destruction at five feet.

The types of property units (agricultural acres, residences, etc.) and their location with respect to the culvert invert elevation are obtained from a topographic map of the area and site inspection. This information with (1) the relationships of flood damage versus flood depth for the types of property units and (2) a maximum headwater for each flood are used to assess upstream flood damage for each flood.

WEIGH LOSSES TO DERIVE RISKS

An estimate of the total cost for an economic culvert design requires one complete pass through the analysis shown in Figure 3. Construction costs are computed as a function of geometry and unit cost. Losses are computed as a function of geometry, hydrology, accident statistics, and flood stage versus damage at the site. Probabilities of annual occurrence of runoff hydrographs (see Table 1) are multiplied by associated losses to determine annual risks. The total annual cost is the sum of the annual construction cost and the annual risk.

Table 2. Estimated Flood Loss Data for Agriculture in Selected Counties in Virginia,[1] 1970

County	Direct and Indirect Damages[2] ($/Ac) at Various Water Depths			
	0.5 ft	1 ft	2 ft	3+ ft
Fairfax	3.33	7.88	10.50	13.13
Roanoke	5.51	14.27	18.99	23.72
Brunswick	60.22	144.60	192.83	240.97
Caroline	13.92	33.26	44.38	55.49
Nelson	4.20	9.98	13.04	16.72
Tazewell	2.36	5.60	7.44	9.28
Virginia Beach	21.35	51.29	68.41	85.51

[1]Source: Ref. 24, 25, 26.

[2]Includes damages to all field crops, including vegetables, but excludes tree-grown crops. Costs are based on 1964 figures adjusted to 1970 dollars by applying an agricultural inflation index multiplier of 1.173 (Ref. 27).

Table 3. Estimated Flood Loss Data for Single Family Residences,[1] 1970

Estimated Market Value of Property	Direct and Indirect Damages[2] to Structure and Contents for Inside Water Depths 1 ft	2 ft	3 ft	4+ ft
< $8,000	$ 1,550	$ 1,990	$ 2,580	$ 4,010
8,000 - 12,000	2,410	3,040	4,000	6,230
12,000 - 16,000	3,320	4,270	5,520	8,590
16,000 - 20,000	4,230	5,450	7,040	10,960
20,000 - 24,000	5,100	6,649	8,490	12,900
24,000 - 29,000	5,930	7,630	9,870	15,360
29,000 - 34,000	6,790	8,730	11,300	17,580
34,000 - 43,000	8,020	10,310	13,340	20,770
43,000 - 57,000	10,350	13,330	17,240	26,820
> 57,000	13,110	16,880	21,840	33,980

[1]Source: Ref. 22, 28.

[2]These classifications do not include land values. Costs are based on 1960 figures adjusted to 1970 dollars by applying the Boeck single family residence construction cost index (29).

III. RESULTS

An optimum design for a culvert site is reached when the total annual cost (construction cost plus risk) is minimized. In order to find the minimum total cost, many culvert sizings must be analyzed. This is done by use of a discrete gradient search technique (30) to find the minimum total cost in the B, D plane on a digital computer.

OPTIMUM DESIGNS

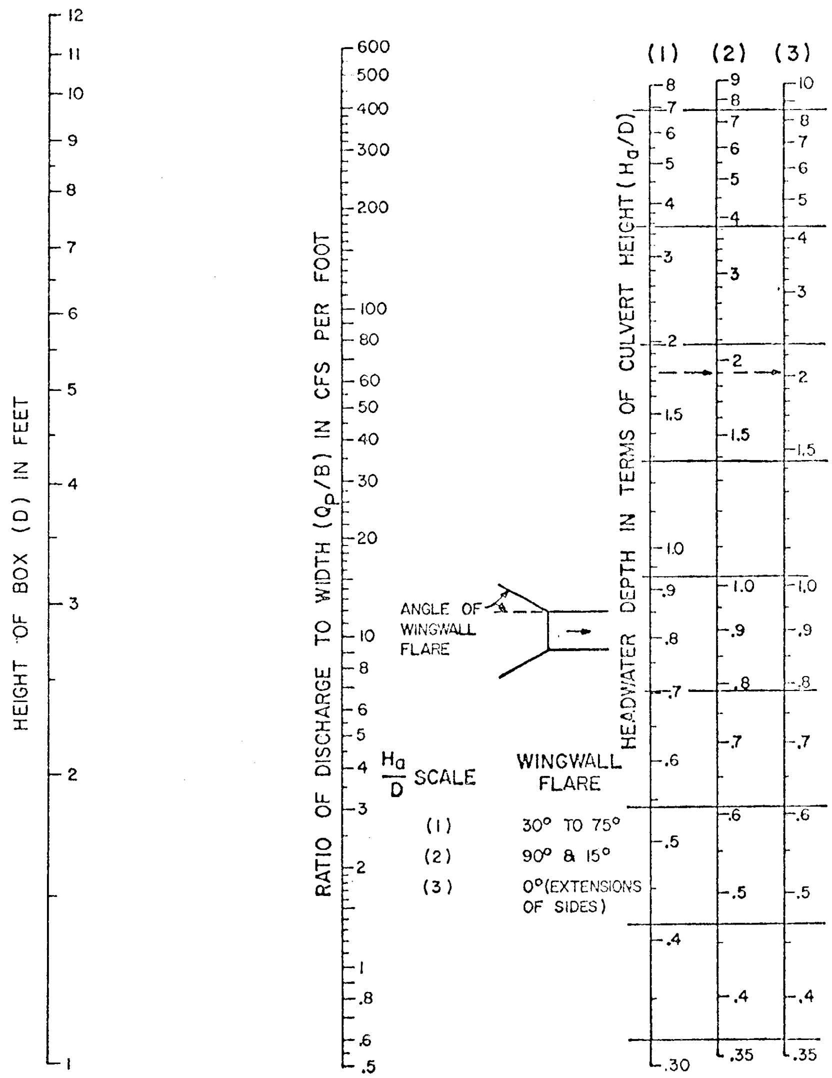
Each of the 22 case study sites is analyzed using the procedure described herein. The optimum design for each site is given in Table 4, with the fill height and drainage area.

CRITERIA

In conventional culvert design practice (19, 31) a flood of specified frequency (return period) is used to determine culvert size. For major four-lane highways, the flood with a return period of 50 years or the highest flood of record usually is used. The allowable headwater depth is then determined by establishing acceptable upstream flooding and culvert velocities. Maximum headwater depth is limited to the height of the shoulder of the roadway. Assuming the hydraulics of the inlet govern flow, the peak discharge of the specified flood and the allowable headwater depth are used with a nomograph (19) to find culvert size. This nomograph is shown in Figure 11.

Table 4. Case Study:
Optimum Culvert Designs for Virginia

Site No.	No. of Barrels	Width (ft)	Height (ft)	Fill Height (ft)	Drainage Area (mi^2)
1	3	6	6	62	3.19
2	1	8	8	24	1.34
3	1	6	7	49	1.08
4	2	7	6	31	1.92
5	1	8	7	24	1.11
6	1	6	7	19	0.69
7	3	5	6	55	2.59
8	1	7	7	23	0.67
9	1	5	5	35	0.39
10	1	5	7	45	2.29
11	1	7	8	51	2.03
12	1	6	7	39	2.00
13	4	7	7	35	14.34
14	2	6	7	49	3.70
15	1	8	9	23	3.31
16	2	7	7	27	4.93
17	4	9	9	30	15.00
18	2	5	5	43	2.10
19	4	6	6	73	7.58
20	1	6	6	25	1.99
21	1	8	9	24	2.97
22	1	8	7	44	2.15



(SOURCE: REF. 30)

FIGURE 11

HEADWATER DEPTH FOR BOX CULVERTS WITH INLET CONTROL

Table 5 presents a comparison of conventional design versus optimum design for the 22 sites analyzed and shows that optimum design procedures result in smaller culverts than conventional design procedures for the cases studied. This indicates that a substantial reduction in culvert cost could be realized by application of the optimum design procedure. The construction cost savings for the 22 sites averaged approximately 15 percent.

The two design methods are further compared for general relationships which would eliminate the need for extensive sets of data and calculations. For example, is it possible to determine that flood return period which would translate the conventional design into the optimal design. One would first determine (1) the conventional allowable headwater/culvert depth ratios for each case study and (2) the computed optimal size (see Table 5) for each case study (which is less than the size corresponding to the conventional design using the 50-year flood). Then Figure 11 is used to determine the design flow for the optimal design. Using the optimal design flow in conjunction with a frequency curve such as that in Figure 6, the return period for the optimal design can be determined.

The return periods obtained in this manner for the case studies ranged from approximately two to 16 years. A design based on these return periods, using Figure 11 in the manner for which it was designed, will produce an optimum design. The variance in these optimum return periods is too great to assume that the return period is constant; therefore, a statistical analysis is undertaken to explain this variance.

A correlation analysis is performed on optimum return period versus other pertinent design variables. Two of the design variables show a rather high degree of correlation to return period: (1) allowable headwater/culvert depth ratio and (2) area of the upstream poind at 15 feet above the culvert invert. A multiple correlation analysis of return period versus these two variables shows a correlation coefficient of 0.93. The expression

Table 5. Comparison of Culvert Designs

	Conventional			Optimal		
Site No.	N	B	D	N	B	D
1	3	7	10	3	6	6
2	3	5	7	1	8	8
3	2	6	6	1	6	7
4	2	7	8	2	7	6
5	2	6	7	1	8	7
6	2	6	6	1	6	7
7	2	8	8	3	5	6
8	1	8	10	1	7	7
9	1	7	10	1	5	5
10	2	8	8	1	5	7
11	2	8	10	1	7	8
12	2	8	10	1	6	7
13	3	9	12	4	7	7
14	1	10	12	2	6	7
15	2	8	8	1	8	9
16	2	7	10	2	7	7
17	4	10	10	4	9	9
18	1	10	12	2	5	5
19	3	8	8	4	6	6
20	1	10	12	1	6	6
21	2	7	10	1	8	9
22	1	10	8	1	8	7

for return period or recurrence interval is

$$R = \frac{10.91\left(\frac{H_a}{D}\right)^{2.825}}{A_L^{0.313}} \tag{22}$$

where A_L = watershed area at 15 ft above culvert invert (acres) and
H_a = allowable headwater depth.

The plot of this equation is presented in Figure 12, using the actual data points.

Because of the small sample, it is suggested that two standard deviations be added to the least squares fit for actual design for rural highway culverts. Thus, the return period expression becomes

$$R = 4.0 + \frac{10.91\left(\frac{H_a}{D}\right)^{2.825}}{A_L^{0.313}} \tag{23}$$

A family of curves of return period versus acres flooded by 15 feet of water at the site for various allowable headwater/culvert depth ratios is shown in Figure 13. The values of allowable headwater/culvert depth ratios are limited to those found in the case studies.

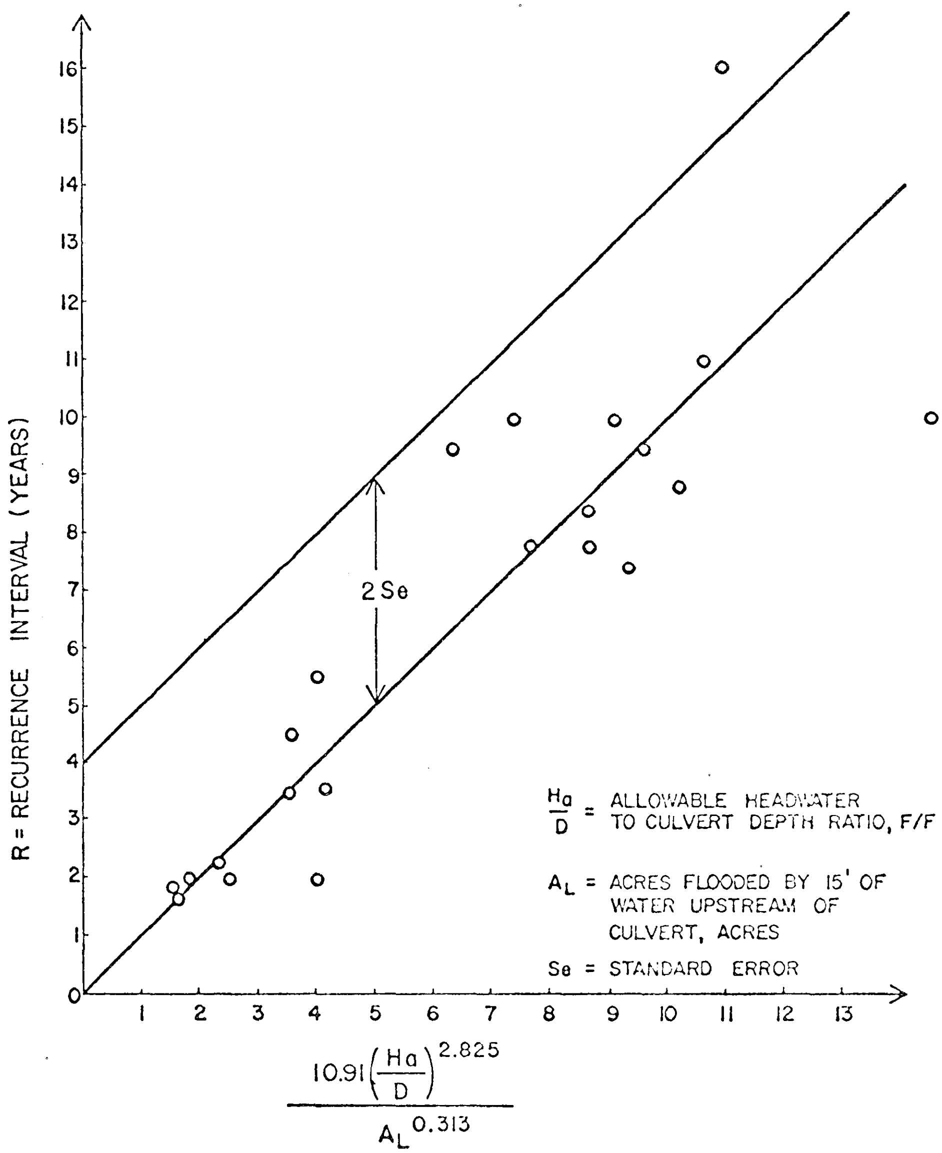


FIGURE 12
RETURN PERIOD EQUIVALENCE GRAPH

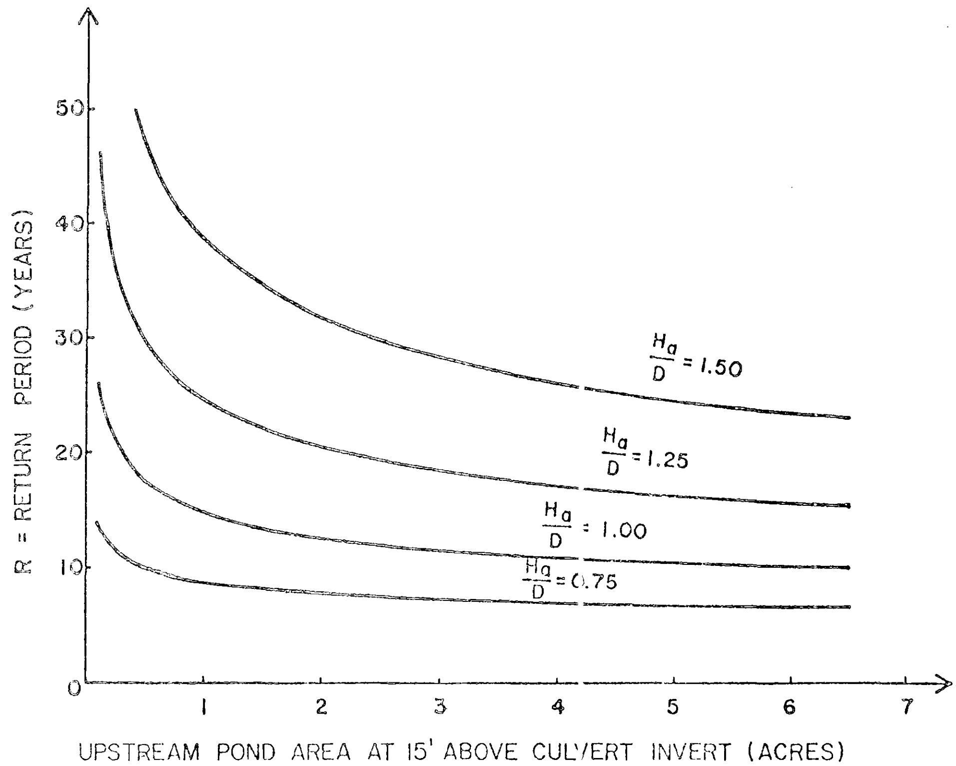


FIGURE 13
OPTIMUM RETURN PERIOD DESIGN CURVES

IV. CONCLUSIONS

Optimal culvert designs for rural four-lane highways, based on economic criteria and data from 22 cases, appear to yield smaller cross-section area than designs based upon the conventional 50-year flood. The smaller culverts are about 15 percent less in cost than culverts designed to the 50-year criteria.

It is a relatively simple matter to obtain an optimum culvert design using conventional design procedure and the graph shown in Figure 13. Such a procedure is merely suggested as a possibility, with a warning that additional data are required prior to a change in design standards. The following is an illustration of the suggested procedure. Example data:

1. Flood-frequency curve (see Figure 6).
2. Area at 15 feet above invert = 4.5 acres.
3. Headwater/depth ratio = 1.0.
4. Fill height = 24.0 feet.
5. Wingwall flare angles = 45°.

Step 1:

Using the given data for area and allowable headwater/culvert depth ratio, obtain from Figure 13 a return period of 11 years.

Step 2:

Select from the flood frequency curve (Figure 6) the peak flow corresponding to a return period of 11 years: Q_p = 950 cfs.

Step 3:

Using the nomograph in Figure 11, select a trial size of B (culvert width), compute Q_p/B. The culvert depth is obtained by connecting the points of Q_p/B and H_a/D with a straight line and extending the line to culvert depth scale. Repeat this procedure until several sizings of standard box culverts are obtained. For this example, sizings are:

1. a single 8 x 12,
2. a double 6 x 9, and
3. a double 7 x 8.

Step 4:

Cost comparisons should be made on the preliminary sizes and the most economical design selected.

LIST OF REFERENCES

1. Pritchett, D. C., "An Economy Approach to Selecting Highway Culverts," *Rural and Urban Roads*, April 1966.

2. "Manual of Culvert Design Practice," State of California Division of Highways, 1960.

3. Young, G. K., R. S. Taylor and L. S. Costello, *Evaluation of the Flood Risk Factor in the Design of Box Culverts*, WRE Final Report, Vol. I, to the Federal Highway Administration, Contract No. F-11-7324, September 1970.

4. "Engineering News Record Index," *Engineering News Record*.

5. *National Engineering Handbook - Hydrology*, U. S. Department of Agriculture, Soil Conservation Service, Washington, D. C.

6. *Design of Small Dams*, U. S. Bureau of Reclamation, Washington, D. C., 1960.

7. Gumbel, E. J., "On the Plotting of Flood Discharges," *Trans. AGU*, Vol. 24, 1943, pp. 669-719.

8. "Statistical Theory of Extreme Values and Some Practical Applications," *National Bureau of Standards Applied Mathematics Series 33*, February 1954.

9. "Hydraulic Design of Improved Inlets for Culverts," *Hydraulic Engineering Circular No. 13*, Federal Highway Administration, Bridge Division, U. S. Department of Transportation, August 1972.

10. ASCE Committee on Sedimentation, "Abstracted Bibliography on Erosion of Cohesive Materials," *J. Hydraulics Div., ASCE*, Vol. 92, No. HY2, March 1966.

11. ASCE Committee on Erosion of Cohesive Materials, "Erosion of Cohesive Sediments," *J. Hydraulics Div., ASCE*, Vol. 94, No. HY4, July 1968.

12. Colby, B. R., "Practical Computations of Bed-Material Discharge, *J. Hydraulics Div., ASCE*, Vol. 9, No. HY2, March 1964.

13. Dunn, Irving S., "Tractive Resistance of Cohesive Channels," *J. Soil Mechanics and Foundations Div., ASCE*, Vol. 85, No. SM3, June 1959.

14. Haynie, R. M., and D. B. Simons, "Design of Stable Channels in Alluvial Materials," *J. Hydraulics Div., ASCE,* Vol. 94, No. HY6, November 1968.

15. Miller, C. R., and W. M. Borland, "Stabilization of Five-Mile and Muddy Creeks," *J. Hydraulics Div., ASCE,* Vol. 89, No. HY1, January 1963.

16. Moore, W. L., and F. D. Mash, "Experiments on the Scour Resistance of Cohesive Sediments," *J. of Geophysical Research,* Vol. 67, No. 4, April 1962.

17. Partheniades, E., "Erosion and Deposition of Cohesive Soils," *J. Hydraulics Div., ASCE,* Vol. 91, No. HY1, January 1965.

18. Seelye, E. E., *Data Book for Civil Engineers,* Vol. 1, May 1966.

19. *Drainage Manual,* Virginia Department of Highways, Richmond, Virginia, 1967.

20. *Average Daily Traffic Volumes on Interstate, Arterial and Primary Routes,* Virginia Department of Highways, Richmond, Virginia, 1970.

21. *Accident Facts,* National Safety Council, Chicago, Illinois, 1968.

22. *A Study of Procedures in Estimating Flood Damage to Residential, Commercial, and Industrial Properties in California,* Stanford Research Institute, January 1960.

23. Winfrey, R., *Motor Vehicle Running Costs for Highway Economic Studies,* Published by Author, November 1963.

24. *1964 United States Census of Agriculture*, Vol. 1, Part 24, "Statistics for Counties," U. S. Department of Commerce, Bureau of the Census.

25. *Economics Guide for Watershed and Flood Prevention,* Soil Conservation Service, U. S. Department of Agriculture, March 1964.

26. James, L. D., *A Time-Dependent Planning Process for Combining Structural Measures, Land Use and Flood Proofing to Minimize the Economic Cost of Floods,* Stanford University, August 1964.

27. *Economic Report to the President,* Council of Economic Advisors, January 1972.

28. *United States Census of Housing, 1960 - Virginia, State and Small Areas,* U. S. Department of Commerce, Bureau of the Census.

29. *Construction Index, October - November 1971*, Bureau of Domestic Commerce, U. S. Department of Commerce, 1971

30. Wilde, D. J., *Optimum Seeking Methods*, Prentice-Hall, 1965.

31. *Road Designs and Standards*, Virginia Department of Highways, Richmond, Virginia, 1968.

32. "Hydraulic Charts for the Selection of Highway Culverts," *Hydraulic Engineering Circular No. 5*, Federal Highway Administration, Bridge Division, U. S. Department of Transportation, December 1965.

APPENDIX

NOTATION USED IN THIS STUDY

A = area of flow, ft^2

A_L = watershed area at 15 feet above culvert invert, acres

B = width of culvert, ft

C_1 = cost of a death

C_2 = cost of a personal injury

C_3 = cost of property damage

C_4 = value of a person's time, $/hr

C_5 = running cost of a passenger car, $/1000 miles

C_a = cost adjustment multiplier for quick repairs

C_b = unit quantity of concrete, yd^3/ft of length

C_c = cost of culvert barrel

C_f = total cost of embankment

C_r = cost of roadway (guardrail to guardrail)

C_t = annual construction cost

C_x = total cost of structural excavation for culvert

cfs = cubic feet per second

CRF = capital recovery factor

D = depth of culvert barrel, ft

E = erosion, tons/ft/day

E_f = volume of embankment, yd^3

F = fill height above invert, ft

g = acceleration due to gravity, ft/sec^2

H = headwater depth (from invert), ft

H_a = allowable headwater depth (from invert), ft

I = inflow to culvert, cfs

ℓ = equivalent roadway length, ft
L = culvert length, ft
L_r = actual roadway length, ft
L_w = hydraulic length of watershed, ft
N = number of culvert barrels
O = total outflow (culvert + weir), cfs
P = probability of a storm occurring in any year
P_1 = percent of original embankment eroded due to overtopping
P_2 = percent of roadway damage due to overtopping
P_3 = percent of culvert damage due to overtopping
P_4 = percent of total system damage due to excessive culvert velocities
Q_p = peak flood flow, cfs
Q_t = overtopping flow, cfs
R = recurrence interval, yr
S = storage in backwater pond, ft^3
S_b = unit cost of steel, lb/ft
S_n = end of period storage, ft^3
S_o = end of period storage in previous time step, ft^3
T = time recorded from start of flood, hr
T_b = flood duration, hr
T_p = time to hydrograph peak, hr
T_r = flood recession duration, hr
U_c = unit cost of concrete, \$/yd^3
U_f = unit cost of "in place" fill, \$/yd^3
U_r = unit cost of roadway (guardrail to guardrail), \$/ft
U_s = unit cost of reinforcing steel, \$/lb
U_x = unit cost of structural excavation, \$/yd^3
V_c = culvert flow velocity, ft/sec
V_o = overland velocity, ft/sec
V_t = mean overtopping velocity at toe of downstream embankment, ft/sec

X_1 = duration of detour = duration of overtopping + repair time, hr

X_2 = average daily traffic (ADT), vehicles/day

X_3 = passenger cars, fraction of ADT

X_4 = commercial delivery vehicles, fraction of ADT

X_5 = single unit trucks, fraction of ADT

X_6 = semi-trailers, fraction of ADT

X_8 = length of detour, miles

XA_8 = length of original route, miles

X_9 = speed on detour, mi/hr

XA_9 = speed on original route, mi/hr

X_{10} = occupancy rate, people/vehicle

X_{11} = accident distribution ratio - normal conditions

X_{12} = accident distribution ratio - normal conditions, property damage/death

X_{13} = accident distribution ratio - unexpected obstacle

X_{14} = accident distribution ratio - unexpected obstacle, property damage/death

X_{15} = death rate, people/100 million miles

X_{16} = death rate factor for unexpected obstacle, multiplier to X_{15}

XL_1 = vehicular running cost

XL_2 = value of lost time

XL_3 = expected accident cost

XL_4 = expected accident cost for obstacles

α = erosion-velocity coefficient (multiplier)

β = erosion-velocity coefficient (exponent)

Θ = headwater-storage coefficient (multiplier)

Ω = headwater-storage coefficient (exponent)

57198

FHWA

R&D