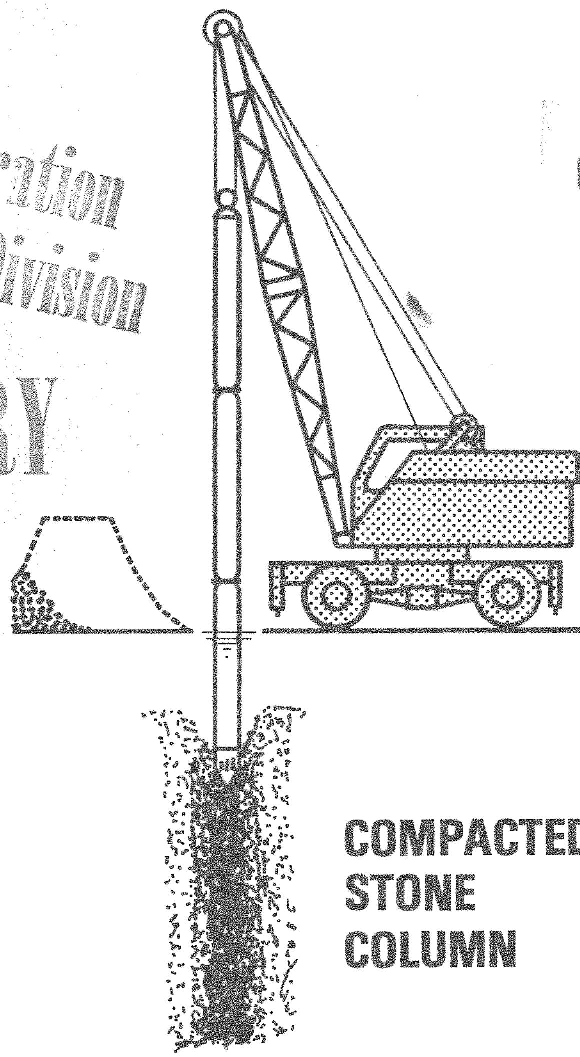
027

STONE COLUMNS
A FOUNDATION TREATMENT
(Insitu Stabilization of Cohesive Soils)



**COMPACTED
STONE
COLUMN**

DEMONSTRATION PROJECT NO. 46

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U.S. DEPARTMENT OF TRANSPORTATION
FEDERAL HIGHWAY ADMINISTRATION
REGION 15
DEMONSTRATION PROJECTS DIVISION
ARLINGTON, VA. 22201
JUNE 1978

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NOMENCLATURE

$(q)_{stone}$	=	Vertical stress on stone at top of column
$(q)_{clay}$	=	Vertical stress on clay at level of column top
$(\Delta p)_{vc}$	=	Vertical stress increment in clay at any depth
$(\Delta p)_{rc}$	=	Radial stress increment in clay at any depth
$(\Delta p)_{t}$	=	Tangential stress increment in clay at any depth
$(\Delta p)_{vs}$	=	Vertical stress increment in stone at any depth
$(\Delta p)_{rs}$	=	Radial stress increment in stone at any depth
$(p_o)_{vc}$	=	Vertical overburden on clay at any depth
$(p_o)_{vs}$	=	Vertical overburden on stone at any depth
$(p_c)_{vc}$	=	Vertical preconsolidation stress on clay at any depth
$(p_c)_{rc}$	=	Radial preconsolidation stress on clay at any depth
K_o	=	Coefficient of earth pressure in clay = $\frac{(p_o)_{rc}}{(p_o)_{vc}}$ for condition $\epsilon_r = 0$ & $\epsilon_v > 0$
K	=	Incremental coefficient of earth pressure = $\frac{(\Delta p)_{rc}}{(\Delta p)_{vc}}$ in clay for condition $\epsilon_r > 0$ & $\epsilon_v > 0$
$(A)_{stone}$	=	Cross sectional area of stone column

NOMENCLATURE (Continued)

$(A)_{clay}$	=	Cross sectional area of clay
$(A)_{total}$	=	Cross sectional area of "unit cell" = $(A)_{stone} + (A)_{clay}$
ϵ_r	=	Radial strain in clay at any depth
ϵ_v	=	Vertical strain at any depth (same for clay and stone)
ΔH	=	Thickness under consideration
ΔS	=	Settlement (compression) in ΔH
L_t	=	Total load on "unit cell"
e	=	Void ratio of the clay
M	=	Proportionality constant in stress tensor equation
*f	=	Modified Boussinesq (or other) factor
$(\gamma)_{stone}$	=	Stone Column unit weight
CS	=	Shear force per Stone Column
S	=	Stone Column center to center spacing
ϕ	=	Angle of internal friction
V_o	=	Initial volume of the unit cell
V	=	Instantaneous volume of unit cell
$(V_o)_c$	=	Initial volume of the clay
$(\Delta V)_c$	=	Change in the volume of the clay
V_v	=	Volume of voids
V_s	=	Volume of solids

NOMENCLATURE (Continued)

E	=	Young's modulus
Loading	=	Uniform stress applied to the foundation area
C_r	=	Radial coefficient of consolidation
C_v	=	Vertical coefficient of consolidation
C_c	=	Compression index
C_α	=	Coefficient of secondary consolidation
U	=	Degree of consolidation
T_r	=	Radial time factor
tc	=	Time period of interest secondary consolidation
tp	=	Time period for primary consolidation
D_e	=	Unit cell diameter
D_s	=	Stone Column diameter
C	=	Cohesion
S*	=	Ratio of smear zone radius to Stone Column radius
$(r)_{smear}$	=	Smear zone radius
$(r)_{stone}$	=	Stone Column radius
$(K)_{ru}$	=	Radial permeability of undisturbed soil
$(K)_{rs}$	=	Radial permeability within smear zone

NOMENCLATURE (Continued)

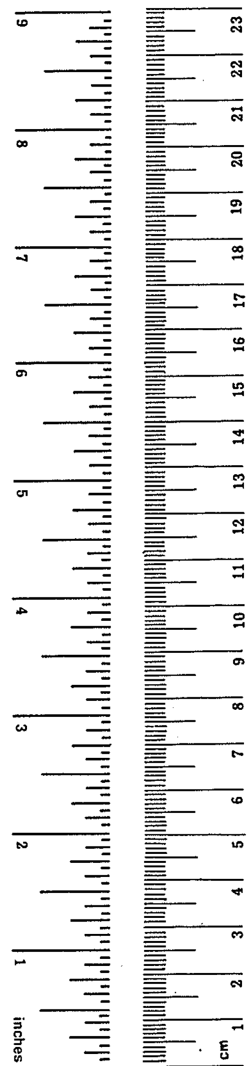
re	=	Unit cell radius
$t_{\%U}$	=	Time required for percent consolidation
w	=	Stone Column width for stability analysis

METRIC CONVERSION FACTORS

Approximate Conversions to Metric Measures

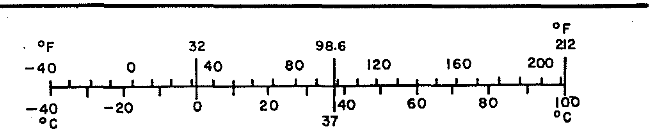
Symbol	When You Know	Multiply by	To Find	Symbol
	LENGTH			
in	inches	*2.5	centimeters	cm
ft	feet	30	centimeters	cm
yd	yards	0.9	meters	m
mi	miles	1.6	kilometers	km
	AREA			
in^2	square inches	6.5	square centimeters	cm^2
ft^2	square feet	0.09	square meters	m^2
yd^2	square yards	0.8	square meters	m^2
mi^2	square miles	2.6	square kilometers	km^2
	acres	0.4	hectares	ha
	MASS (weight)			
oz	ounces	28	grams	g
lb	pounds	0.45	kilograms	kg
	short tons (2000 lb)	0.9	tonnes	t
	VOLUME			
tsp	teaspoons	5	milliliters	ml
Tbsp	tablespoons	15	milliliters	ml
fl oz	fluid ounces	30	milliliters	ml
c	cups	0.24	liters	l
pt	pints	0.47	liters	l
qt	quarts	0.95	liters	l
gal	gallons	3.8	liters	l
ft^3	cubic feet	0.03	cubic meters	m^3
yd^3	cubic yards	0.76	cubic meters	m^3
	TEMPERATURE (exact)			
°F	Fahrenheit temperature	5/9 (after subtracting 32)	Celsius temperature	°C

*1 in = 2.54 (exactly). For other exact conversions and more detailed tables, see NBS Misc. Publ. 286, Units of Weights and Measures, Price $2.25, SD Catalog No. C13.10:286.



Approximate Conversions from Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
	LENGTH			
mm	millimeters	0.04	inches	in
cm	centimeters	0.4	inches	in
m	meters	3.3	feet	ft
m	meters	1.1	yards	yd
km	kilometers	0.6	miles	mi
	AREA			
cm^2	square centimeters	0.16	square inches	in^2
m^2	square meters	1.2	square yards	yd^2
km^2	square kilometers	0.4	square miles	mi^2
ha	hectares (10,000 m^2)	2.5	acres	
	MASS (weight)			
g	grams	0.035	ounces	oz
kg	kilograms	2.2	pounds	lb
t	tonnes (1000 kg)	1.1	short tons	
	VOLUME			
ml	milliliters	0.03	fluid ounces	fl oz
l	liters	2.1	pints	pt
l	liters	1.06	quarts	qt
l	liters	0.26	gallons	gal
m^3	cubic meters	35	cubic feet	ft^3
m^3	cubic meters	1.3	cubic yards	yd^3
	TEMPERATURE (exact)			
°C	Celsius temperature	9/5 (then add 32)	Fahrenheit temperature	°F



I. INTRODUCTION

The densification of cohesionless granular soils with vibratory equipment is a well-known construction procedure. The development of a special probe, called a Vibroflot, to densify such soils at depth was the beginning of the method known as Vibroflotation. The original Vibroflot was developed and patented in Germany more than 45 years ago.

The mechanism of densifying cohesionless soils with vibrators can be described in brief terms as follows: mechanical vibrations and simultaneous application of water nullify effective stresses which exist between adjacent soil grains. The grains in a unconstrained and unstressed configuration are rearranged to the densest possible state under the continued application of vibrations and jetted induced stress reduction. This process has been economically applied as a foundation solution since the latter part of the 1930's with great success throughout the world.

The application of this method in cohesive soil does not produce the same results. In cohesive soils, contact forces between individual particles cannot be eliminated by vibration and, therefore, these soil particles are not separated, even temporarily, during the above mentioned process. Similarly, in soils such as fine grained silts with low permeability that exhibit "apparent cohesion," the particles are difficult to separate by the vibration process. For purposes of this discussion, therefore, these fine grained silts will be included in the category of cohesive soils.

Although the Vibroflotation process does not materially improve the consistency of cohesive soils, a variant method was developed in Germany about 25 years ago to strengthen such soils, in situ. This method, a construction technique called Stone Columns, strengthens cohesive soils to a point where they are able to sustain considerably larger bearing stresses without developing detrimental or excessive settlements, or bearing capacity failures.

II. EQUIPMENT

The Vibroflot consists of a 12- to 16-inch diameter, hollow cylindrical body, which can be 7 to 15 feet in length and which is connected over a special elastic coupling to follower tubes of a slightly smaller outside diameter (figure 1). Eccentric weights in the lower part of the Vibroflot are driven by an electric or hydraulic motor operating at 3000 revolutions per minute at 50 Hertz or 1800 revolutions at 60 Hertz to create vibrations in a horizontal plane. Eighteen to 28 tons of centrifugal force can be generated, creating commensurate amplitudes of 0.2 to .5 inches at the tip of an unconstrained Vibroflot. The total weight of the Vibroflot is adjustable by the addition of heavy or light weight follower tubes which can produce a total weight of approximately 4 to 8 tons for a 45-foot long Vibroflot. All electric cables or hydraulic hoses and water hoses are connected to the uppermost extension tube. Two sets of water outlets are located along the Vibroflot's length. The lower set, located at the probe's tip, aid in probe penetration, while the upper set assists in the removal of displaced cohesive material which lies within the probe's path.

The complete assembly can be supported from a commercial crane. Special supporting rigs have also been developed which can exert a downward hydraulic thrust to force the Vibroflot into the ground. Other additional supporting equipment normally consist of: high capacity-high pressure water pump, a portable energy source to provide power for the Vibroflot, and a front-end loader to feed the required backfill material.

SECTION THROUGH VIBROFLOT

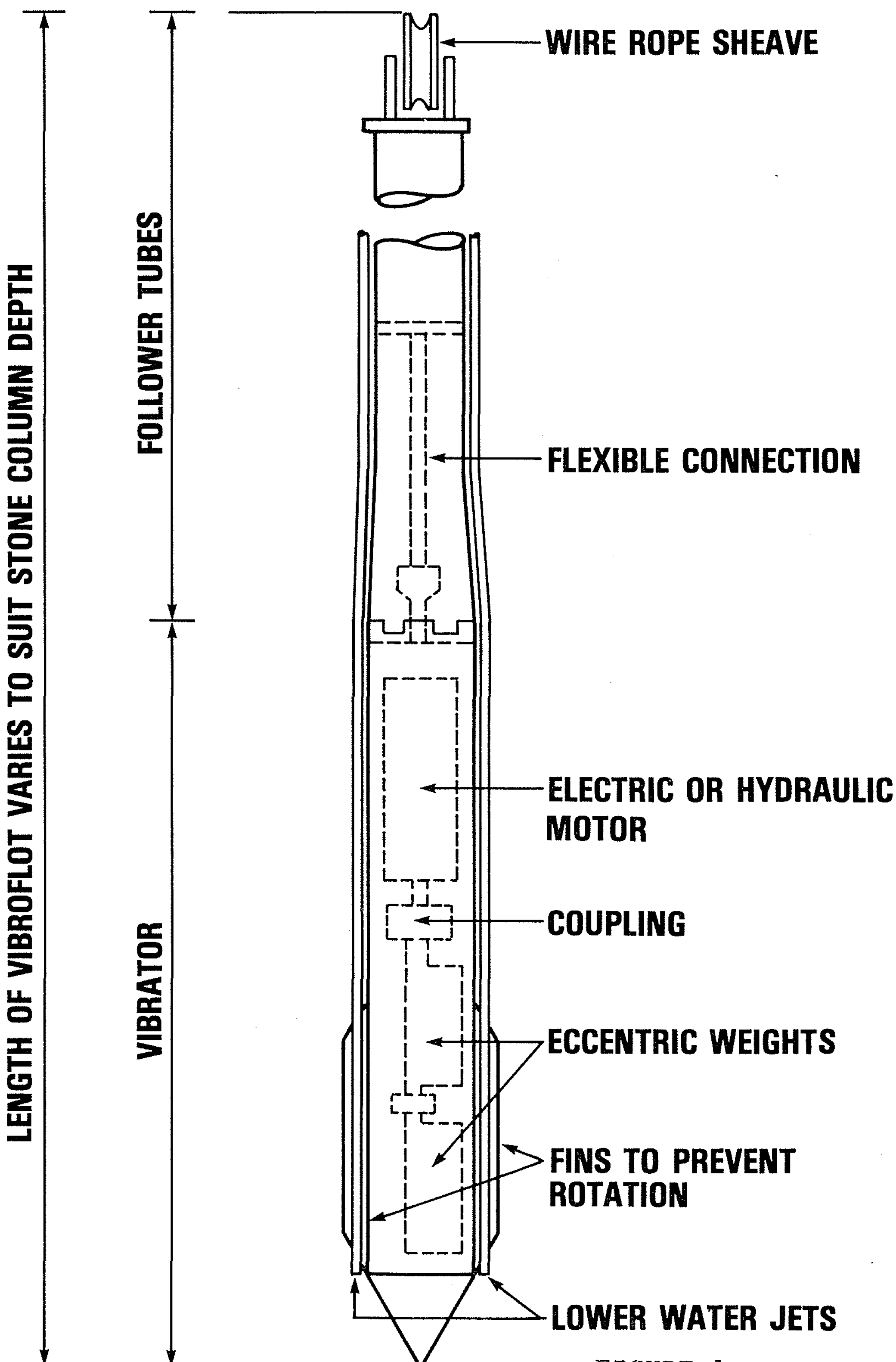


FIGURE 1

III. CONSTRUCTION PROCESS

The Vibroflot penetrates soft cohesive soil to a pre-established depth, under its own weight with (figure 2) vibration and with the assistance of a jetting medium. The jetting medium may be water or compressed air. During the penetration process, the soil immediately surrounding the vibrator is disturbed or remolded to a nominal extent. When water is used during the jetting process, the disturbed material is flushed from the hole, however, true displacement of the in situ soil occurs when compressed air is utilized as a jetting medium. After penetration to the full depth, the Vibroflot is withdrawn while the jetting medium prevents the hole from collapsing. By using water as a jetting medium, a difference in hydrostatic head between the water filled hole and the natural ground-water table assists in the stabilization of the cylindrical hole created by the Vibroflot. Generally, water should be used when the natural material is fully saturated. Air is preferred in cases where the existing soil is only partially saturated. The use of compressed air prevents the creation of a vacuum beneath the vibrating point when the vibrator is extracted. The process of penetration may be repeated to insure that the hole remains open over its entire depth and that most of the disturbed material has been removed.

At sites where hole stability is not a problem the following procedures may be utilized. After the desired depth has been developed, approximately 1/2 to 1 cubic yard of coarse granular backfill is dumped into the hole. This backfill material should consist of coarse gravel, crushed stone or slag, sized 1/2 to 3 inches. The Vibroflot is then lowered into the hole and under its own weight and with the assistance of vibration, it compacts the backfill material. The specially shaped point of the Vibroflot enables it to displace the granular backfill radially into the soft in situ soil.

Repetition of the process of incremental feeding and compacting produces a very dense granular column which is imbedded with the native cohesive soil. Depending on the consistency of the natural soil, columns of 2 to 3 1/2 feet in diameter are formed. It should be noted that the system is self-compensating in that the softer the in situ material, the larger the column diameter. The use of the term Column throughout this narrative is not meant to imply Stone Columns are rigid elements. They can perhaps be thought of as piles with a rather low factor of stiffness in their initial state.

CONSTRUCTING A STONE COLUMN

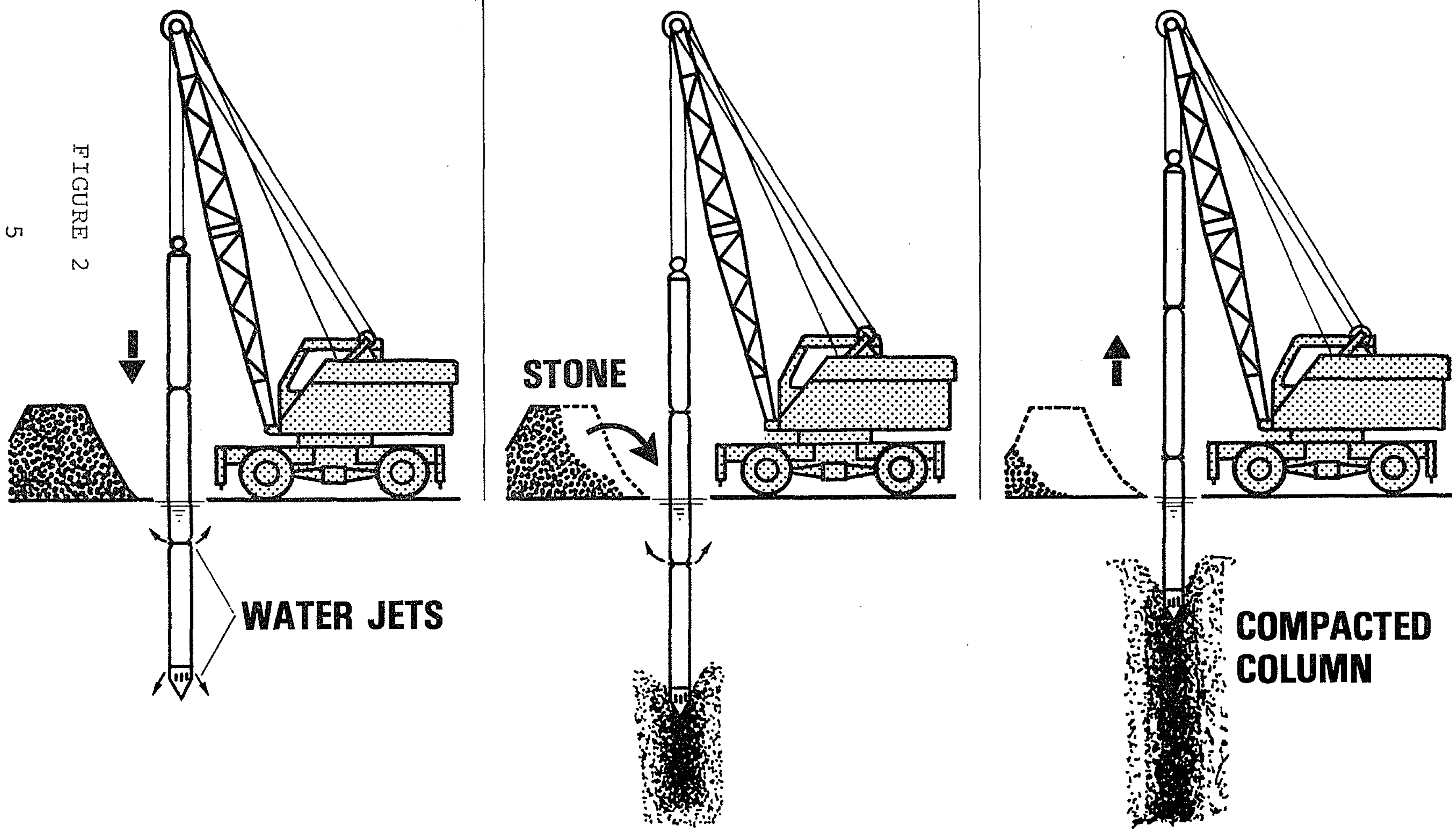


FIGURE 2

Quite often the natural material is so soft that the aforementioned techniques do not produce a stabilized cavity for Stone Column construction. When this condition occurs, the Vibroflot is left at the bottom of the cavity and backfill material is dumped from the surface to fill the void between the probe and the walls of the hole. Under the action of horizontal vibration, the backfill material is forced into the walls and stabilizes them. Once the wall has been stabilized, the Vibroflot is withdrawn a few feet and the center of the column is constructed by dumping additional granular material into the cavity. This procedure is repeated until the entire length of column is completed. Other aspects of the construction procedure under these conditions are identical to those discussed in the previous method.

By constructing Stone Columns in a square or triangular grid pattern, the originally soft cohesive ground is transformed into a composite mass of vertical, compacted granular cylinders with intervening native soil (Figure 3 and 4). The triangular grid pattern is the most efficient, i.e., greatest area coverage for the least number of columns and is most often used.

In addition, the triangular pattern permits a less complex theoretical analysis of the load transfer interaction between the column and in situ soil. The volume relationship of native and replaced material depends on the diameter of each individual Stone Column and therefore, on the consistency of the in situ soil. Depending upon the engineering characteristics of the native soil and the loading to be applied, center to center column spacings may vary between 4.0 to 9.0 feet. The pattern covers the entire foundation area with additional coverage around the foundation perimeter to include any induced stress influence from the applied loads.

PLAN VIEW - STONE COLUMN FOUNDATION PATTERN

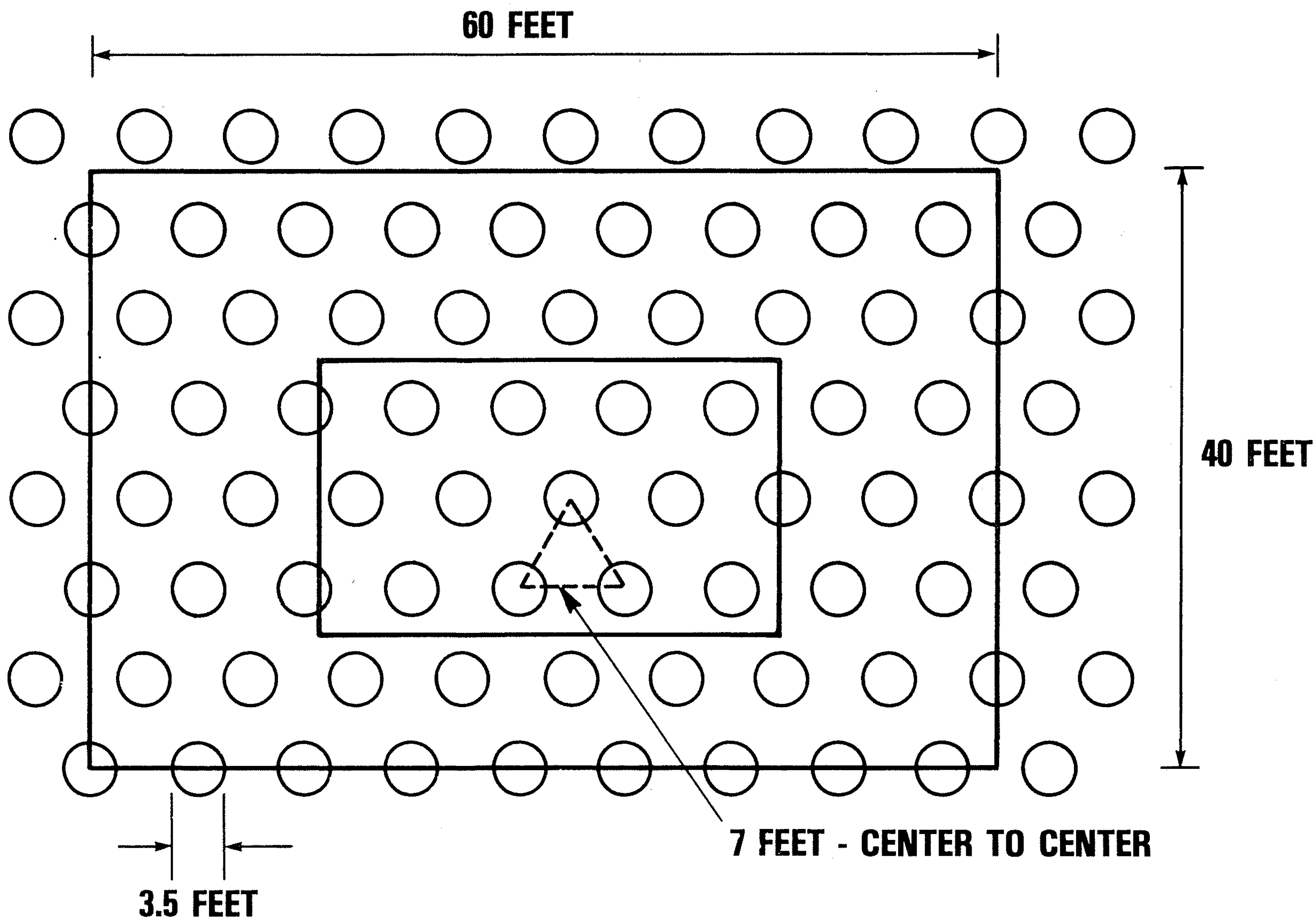


FIGURE 3

STONE COLUMN FOUNDATION PATTERN AND AREA DISTRIBUTION

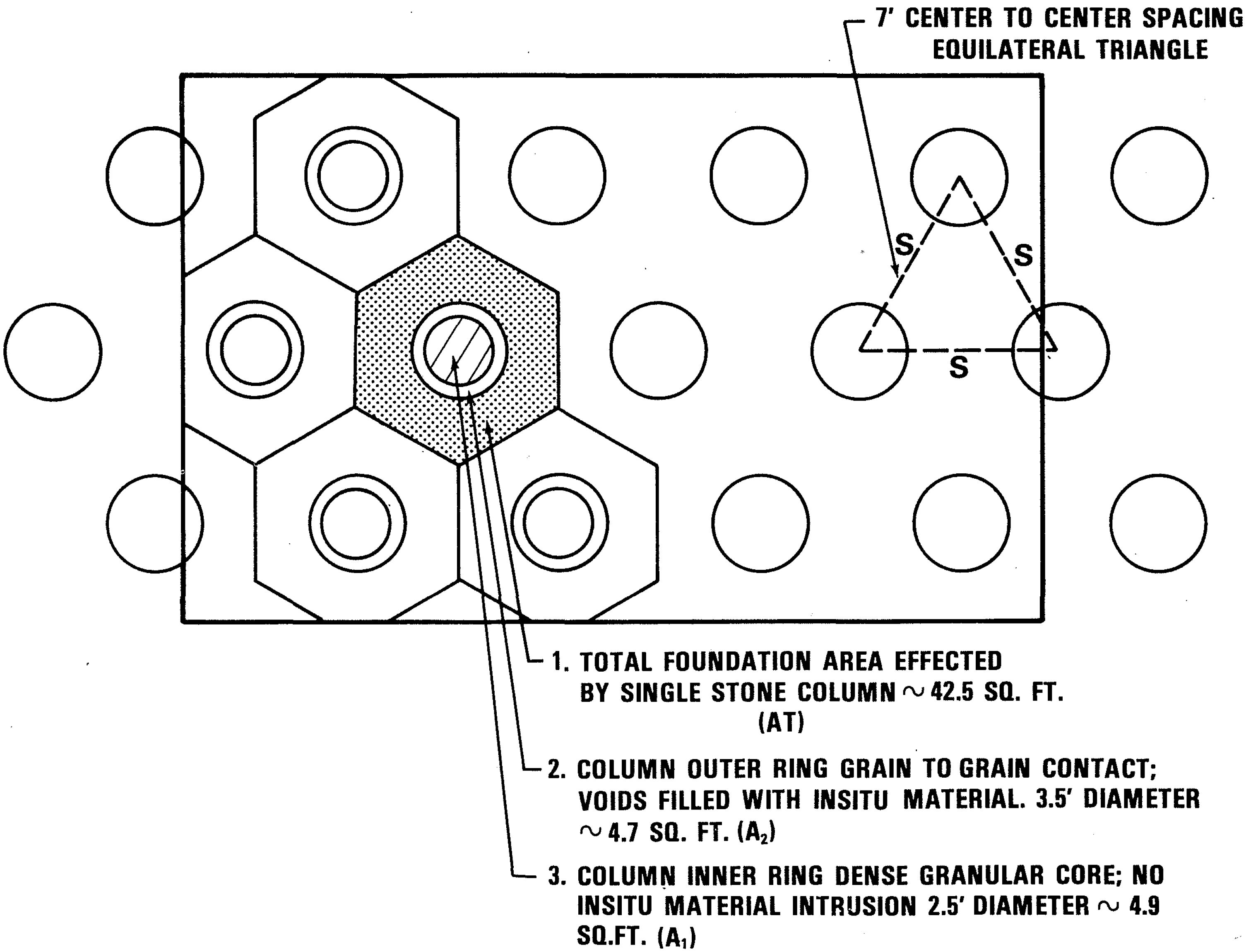


FIGURE 4

IV. APPLICATIONS

A. Slope Stability

Weak subsurface deposits encountered along a highway alinement threaten the stability of any proposed facility. Instability may exist in either cut or fill slopes and may result in either rotational or translational movements.

Preventive or corrective designs which resist these movements have generally consisted of one or more of the following: excavation of the weak subsurface material followed by replacement with select backfill, construction of a resisting berm, or provision of internal drainage systems.

Present day environmental considerations and high construction costs may make a Stone Column design alternative a feasible solution to such stability problems (figure 5).

B. Embankment Settlements

The installation of Stone Columns beneath bridge approach embankments underlain with cohesive material can, in many cases, eliminate "on to" and "off of" bumps. By increasing the column spacing as the distance from the bridge abutment increases, a smooth transition from the bridge to the adjacent embankment can be achieved (figure 6).

C. Structural Foundations

The utilization of Stone Columns beneath footings or abutments, in many cases, offers an alternate to conventional foundation treatments. Depending on a site's subsurface profile the applications may vary. Combined utilization of Stone Columns and a backfill densifing process may make a shallow foundation treatment feasible (figure 7). Installation of Stone Columns rather than conventional foundation piles sometimes offers the most economical foundation treatment (figure 8). Stone Columns do not develop negative skin friction and, therefore, can eliminate structural problems associated with embankment settlement (figure 9).

SLOPE STABILITY

Stone Columns
Economical solution for embankment stability problems

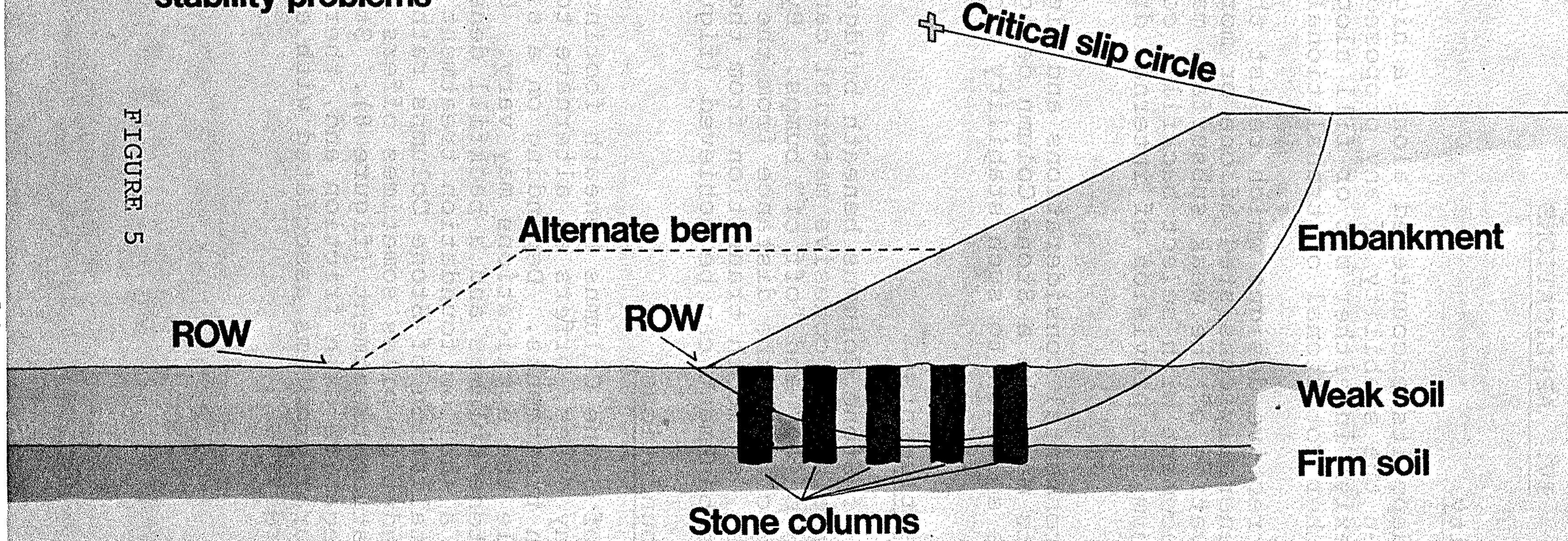


FIGURE 5

SMOOTH SETTLEMENT TRANSITION

Stone Columns provide smooth settlement transition

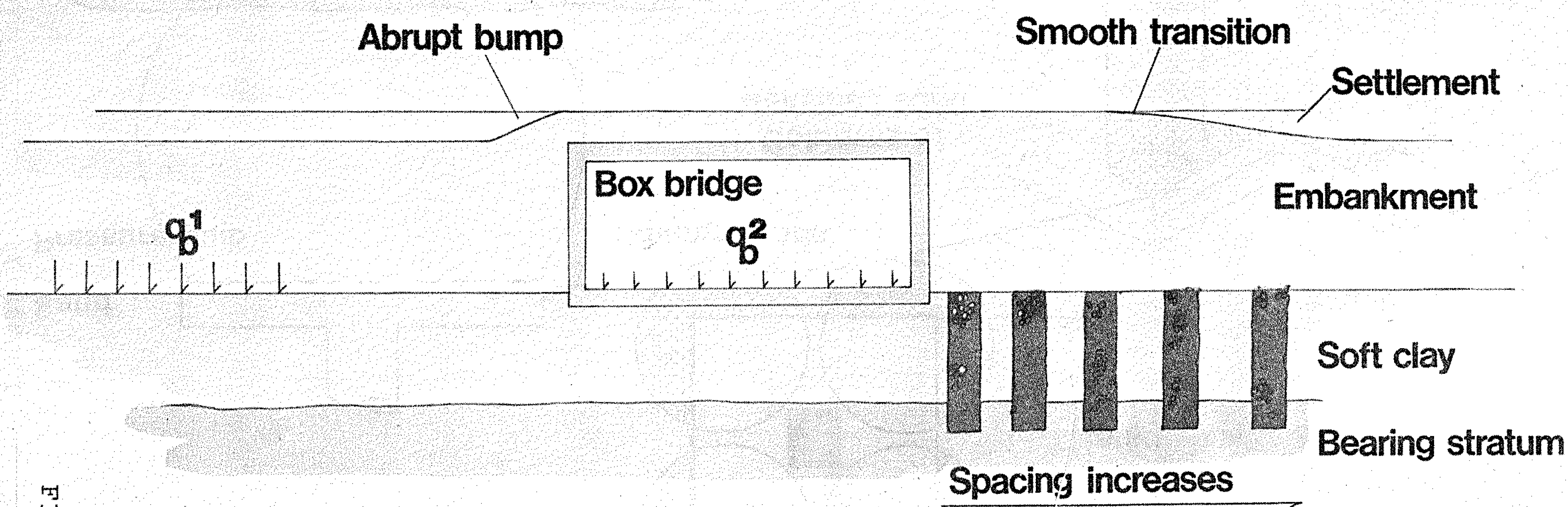


FIGURE 6

COMBINE STONE COLUMNS WITH DENSIFIED SAND COLUMNS

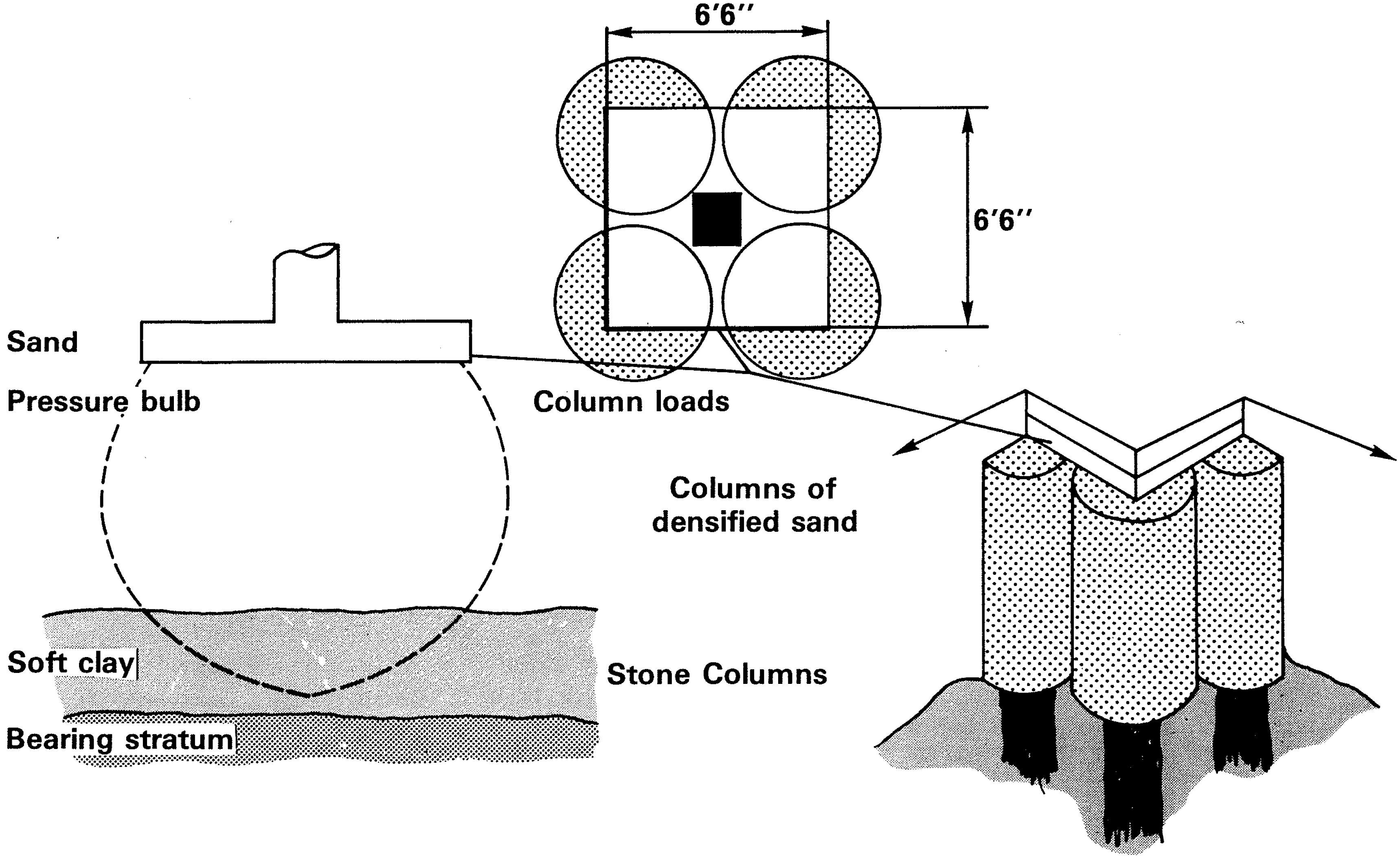


FIGURE 7

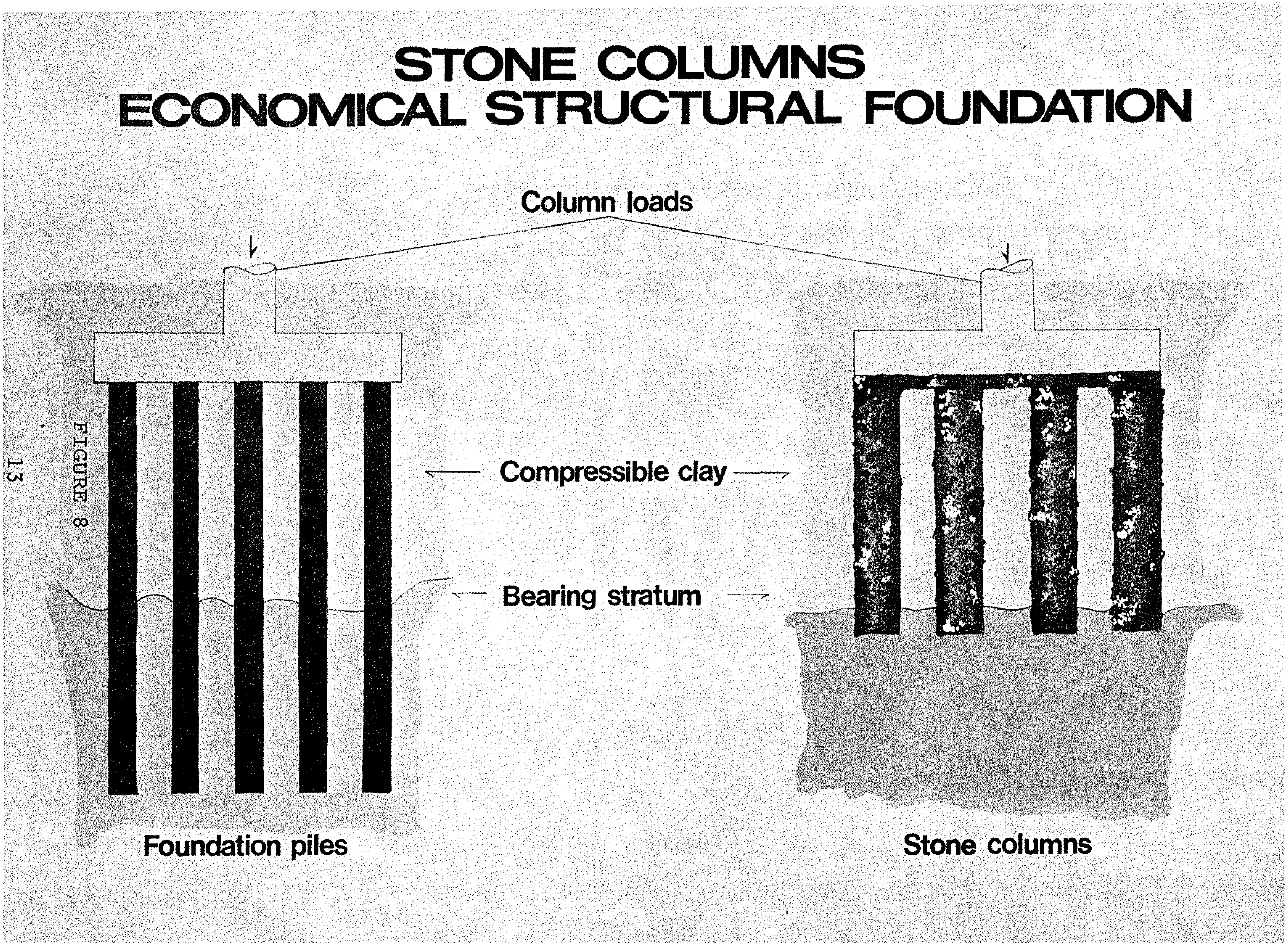

STONE COLUMNS
ECONOMICAL STRUCTURAL FOUNDATION
Column loads
Compressible clay
Bearing stratum
Foundation piles
Stone columns

FIGURE 8

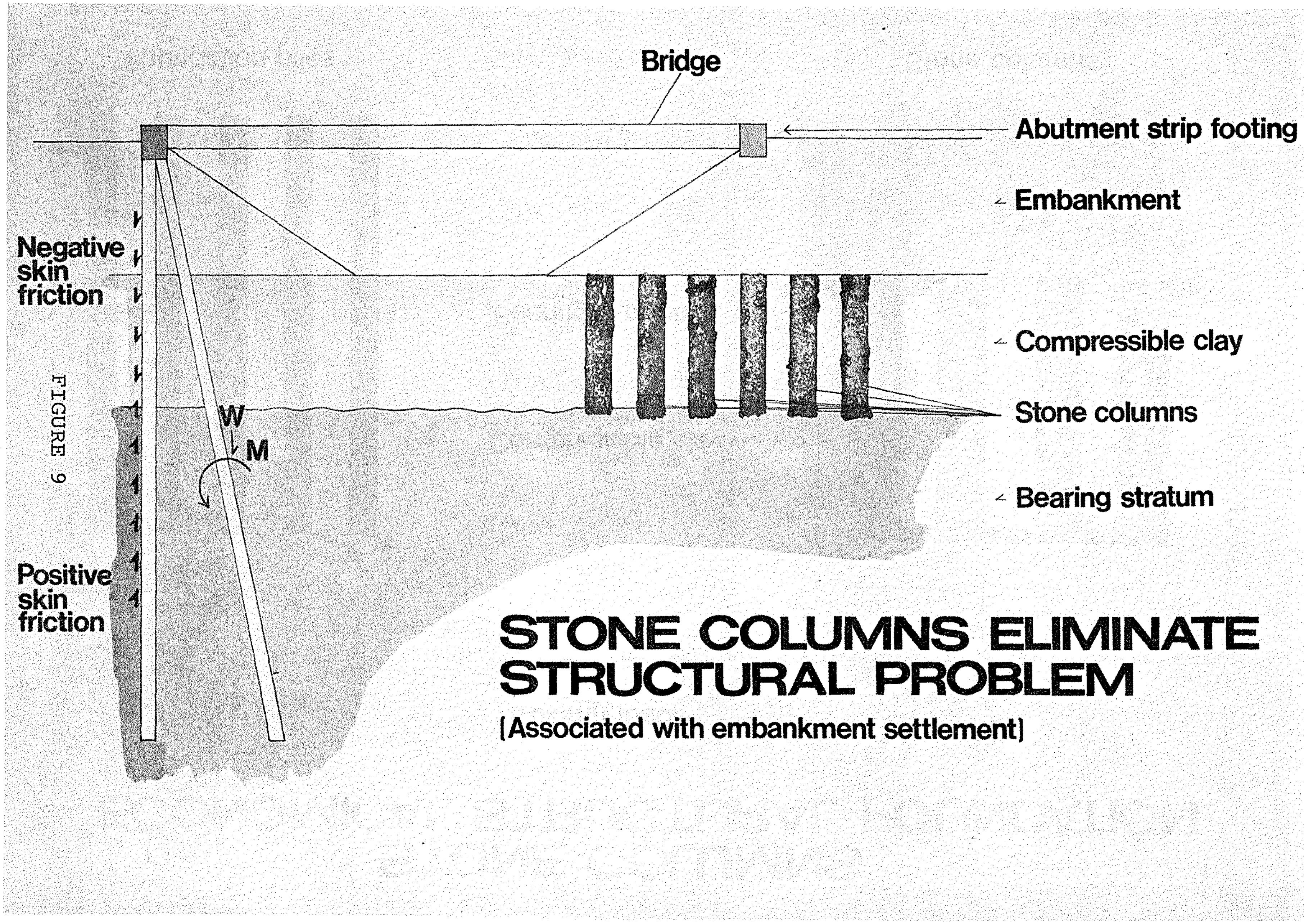


FIGURE 9

STONE COLUMNS ELIMINATE STRUCTURAL PROBLEM

(Associated with embankment settlement)

D. Summary

The previously mentioned applications illustrate a few project conditions where use of Stone Columns may be an effective and economic solution to stability, bearing capacity or settlement problems. The dual advantage of increasing the average shear strength of the composite soil mass and decreasing the overall compressibility of the foundation area can be effectively applied to a number of problem areas. As development and widespread use of this method occur, the experience gained should expand the variety of applications related to highway construction considerably.

V. ANALYSIS OF STONE COLUMNS

A. Introduction

The success of every foundation system can be evaluated by its ability to meet two basic requirements. These requirements are:

1. The foundation system must be safe from a bearing capacity or stability failure.

2. The foundation system must prevent settlement or deflection which would damage the structure or impair its usefulness.

This section presents the development of the design concepts which are utilized in Stone Column analysis and provides a discussion concerning their assumptions and limitations. These procedures are illustrated in the Example Problems section.

The present methods of analysis for the design of a Stone Column foundation treatment are semi-empirical. Stone Column behavior under loading conditions has been observed in both model studies and full scale field testing programs. Results of these tests have prompted the development of the design approaches outlined within this manual. Additional testing and evaluation of this construction method will undoubtedly result in the development of more exact design models.

The enclosed design approach is considered to be a conservative method of analysis; it may be visualized as an interim method of design which will be expanded and refined with time and experience.

B. Load-Settlement Relationship

The development of a theoretical model which addresses Stone Column deformational behavior under vertical load has been attempted by a number of authors (Baumann and Bauer; Hughes and Withers). The equations and design procedures suggested by these and similar efforts agree very well with field tests conducted at various locations on individual Stone Columns.

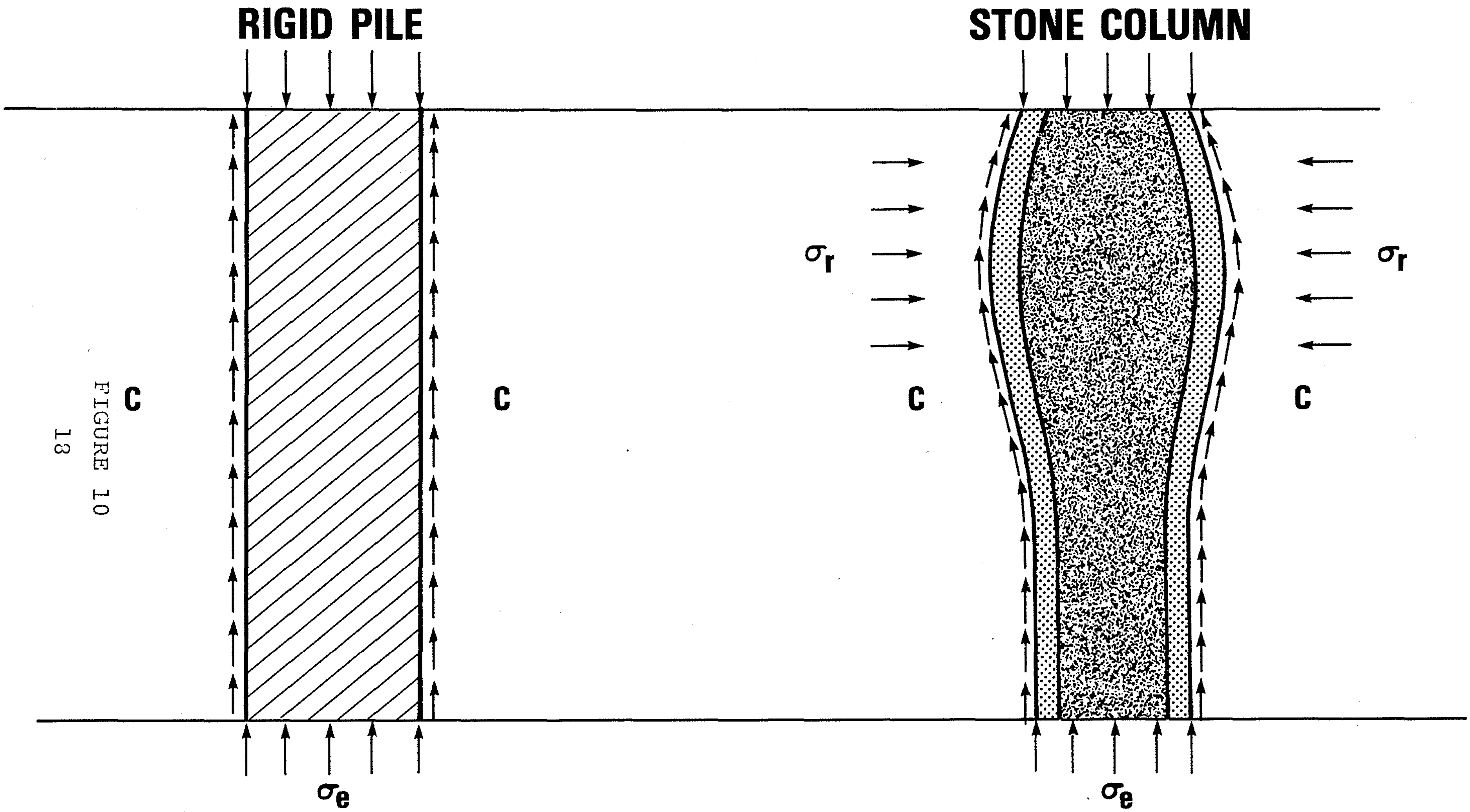
Upon reviewing the existing state of the art of Stone Column analysis, it has been noted that several important factors related to column capacity must be considered. These factors include: shear strength of the in situ soil, lateral stress within the soil, radial pressure/deformation characteristics of the soil, angle of internal friction of the column and diameter of the column. However, within the development of design computations, the application of these factors is based entirely on the assumption that they represent Stone Column behavior under field loading conditions. The test procedures utilized for the above mentioned model and field study comparisons have failed to reproduce actual field conditions for the following reasons:

1. Results of vertical field tests conducted in various studies represent undrained testing conditions. In most cases loading periods were approximately one or two hours per increment. Results from quick tests such as this, are of limited value when developing a design procedure because: first, the rate of loading on most projects occurs over a much longer time period and second, these tests cannot provide any information pertinent to ultimate anticipated settlements and time rate of settlement.

2. The loading arrangement of most tests, applied vertical load directly to the column and left the in situ soil surrounding the column in its original state of vertical stress. This arrangement forces the column down into the in situ soil. The relative motion between the column and the in situ soil develops shear stresses along the periphery of the column, with the effect that the largest load on the Stone Column occurs at the top. By these shear stresses, vertical load is transferred out of the column into the surrounding soil and at some depth sufficient load is transferred so that from that depth down, the column does not "bulge." Figure 10 illustrates this approach to stress distribution by comparing a Stone Column loaded in this manner to a conventional pile.

   Under actual loading conditions however, the applied load is distributed between the Stone Column and the in situ soil between adjacent columns. This loading condition creates an entirely different picture regarding stress distribution along a column's length.

LOADING DISTRIBUTION STONE COLUMN VS. RIGID PILE

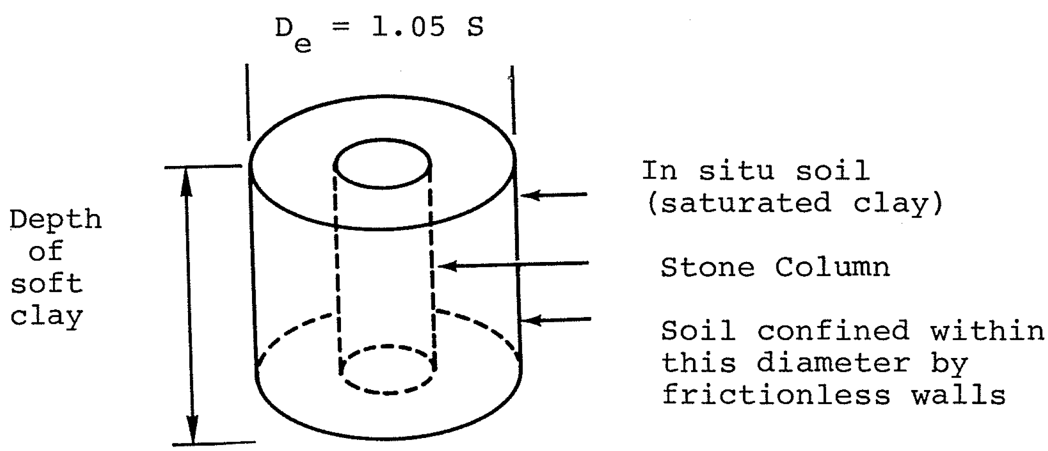


ON THE LEFT A RIGID PILE IN A COHESIVE SOIL IS LOADED, IT SETTLES, DEVELOPING END BEARING AND COHESIVE RESISTING STRESSES. A STONE COLUMN SIMILARLY DEVELOPS THESE TYPE STRESSES. HOWEVER THE STONE COLUMN ALSO BULGES AND SO IT IS SUPPORTED BY LATERAL STRESSES EXERTED BY THE ADJACENT IN SITU SOIL.

FIGURE 10

For these reasons, the development of a new design approach which models actual column-soil loading conditions appears to be appropriate. The development of such an approach follows directly.

Consider the situation where the surrounding soil is loaded along with the Stone Column. For simplicity consider a limiting case where the loaded area is relatively wide and the thickness of the clay layer is relatively thin so that the vertical stress increase is uniform throughout the depth of the clay layer. Assume also that Stone Columns are placed on a triangular pattern. Under these conditions one could represent the behavior of a single Stone Column by the following "unit cell."



D_e = effective diameter = 1.05 S

S = center to center column spacing

FIGURE 11 UNIT CELL

Assume that the load is applied to this "unit cell" in such a way that the Stone Column and in situ soil must deform equally and the load is applied quickly.

If the in situ soil behaves ideally as saturated clay, and the Stone Column is incompressible, no immediate settlement can occur, and no shearing stress can be developed between the Stone Column and the surrounding soil initially.

As the clay consolidates (undoubtedly with predominately radial drainage for normal Stone Column spacings and diameters), vertical load will gradually transfer to the Stone Column. If the Stone Column is designed with sufficient strength, equilibrium will be reached without plastic deformation occurring in the Stone Column. This design may be overly extravagant for many purposes.

If the load on the Stone Column becomes sufficiently large, bulging or plastic deformation will occur, and the Stone Column is in a state of plastic equilibrium. <u>This condition does not necessarily indicate failure, however, since this may be a contained state of plastic equilibrium. Eventually consolidation will be complete and settlement will stop.</u> The Stone Column will be left with internal stresses entrapped such that an inpending state of plastic equilibrium exists.

Note that if the stone within the column is incompressible, all of the volumetric strain must be accomodated by the in situ soil. This volumetric strain is the result of both vertical and radial consolidation of the in situ soil. Note also that, in effect, all of the strength of the stone-clay system is provided by the clay--without lateral support the Stone Column would collapse.

During the consolidation process shear stresses will in general be induced between the Stone Column periphery and the in situ soil, as a result of unequal vertical strains. If relative vertical movement between the Stone Column and in situ soil is considered, these shear stresses would be very difficult to evaluate. Therefore, two limiting conditions could be considered.

1. ignore shear stress between the column and soil.

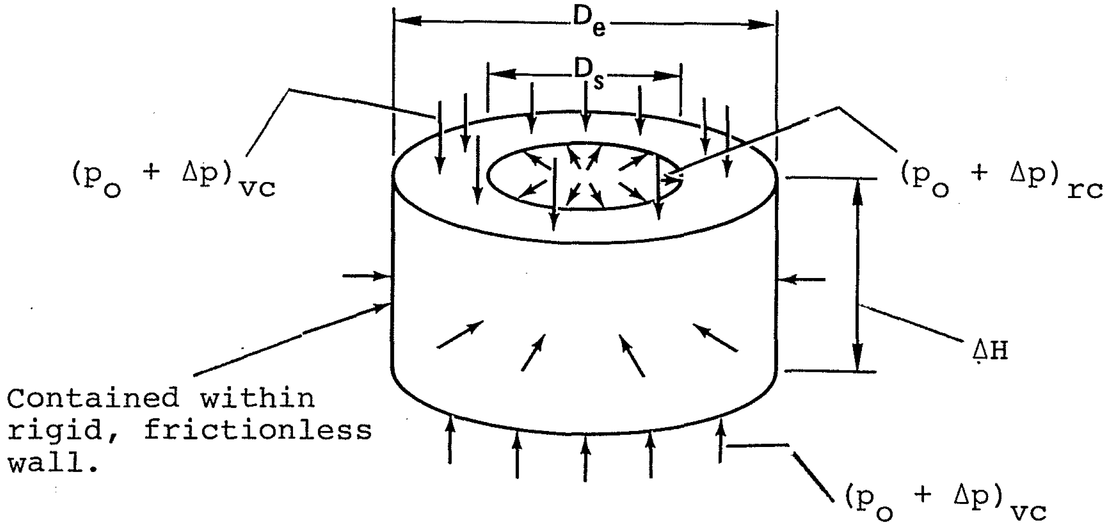
2. assume zero relative vertical movement between the column and soil (equal vertical strain).

The first assumption would probably be fairly accurate for relatively "short" Stone Columns. However, in longer columns the accumulated load transfer from this shear stress could be large. Subsequent calculations indicate that the magnitude of this shear stress is seldom greater than 20 to 30 psf.

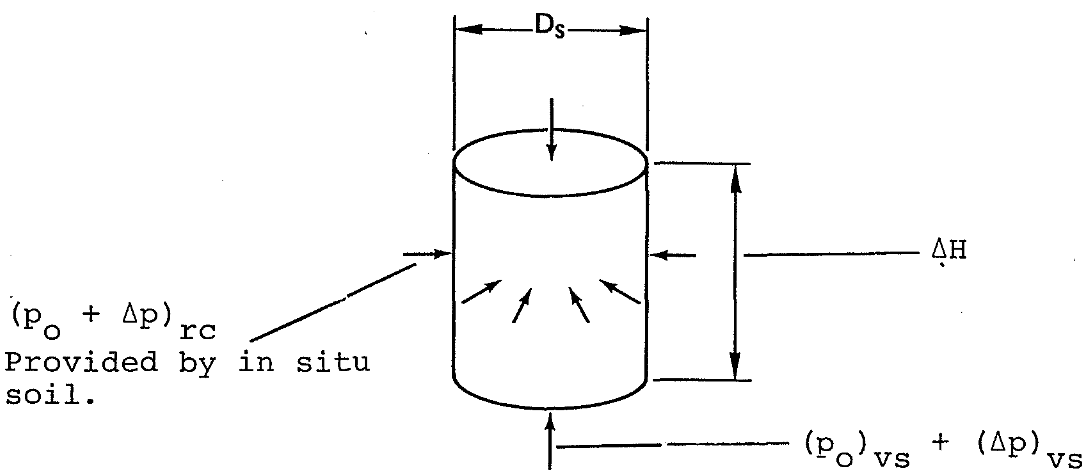
and so would not violate the yield condition of most soils, but the total magnitude of force can be significant. Consequently, the following derivation follows the equal vertical strain assumption.

Since the magnitude of this shear stress is relatively small, it is assumed that the major and minor principal stress directions are not altered by the shear stress. Principal stress directions are assumed to be vertical, radial, and tangential.

The stress analysis will be performed on an incremental basis working down from the top of the column.



In Situ Soil



Stone Column

Where:

D_e = Effective diameter of the "unit cell"

D_s = Effective diameter of a Stone Column

ΔH = Thickness under consideration

$(p_o + \Delta p)_{vc}$ = Vertical overburden plus vertical stress increment in the clay at any depth

$(p_o + \Delta p)_{rc}$ = Radial overburden plus radial stress increment in the clay at any depth

$(p_o + \Delta p)_{vs}$ = Vertical overburden plus vertical stress increment in the stone at any depth

FIGURE 12 INCREMENTAL STRESS ANALYSIS

PLASTIC ANALYSIS:

If the stone in the column approaches shear failure with an angle of internal friction of ϕ stone, the limiting vertical stress in the column is given by

$$(p_o + \Delta p)_{vs} = \tan^2 (45 + \frac{\phi stone}{2}) (p_o + \Delta p)_{rc}$$

$$(\Delta p)_{vs} = (p_o + \Delta p)_{rc} \tan^2 (45 + \frac{\phi stone}{2}) - (p_o)_{vs}$$

where: $\tan^2 (45 + \frac{\phi stone}{2})$ = Coefficient of passive earth pressure

By equilibrium, the total load L_t on the "unit cell" is given by:

$$L_t = (\Delta p)_{vs} (A)_{stone} + (\Delta p)_{vc} (A)_{clay}$$

or

$$L_t = (p_o + \Delta p)_{rc} \tan^2 (45 + \frac{\phi stone}{2}) (A)_{stone} - (p_o)_{vs} (A)_{stone} + (\Delta p)_{vc} (A)_{clay}$$

where: $(p_o + \Delta p)_{rc}$ = Radial overburden plus radial stress increment in the clay at any depth

$(A)_{stone}$ = Area of Stone Column per unit cell

$(A)_{clay}$ = Area of clay per unit cell

$$\therefore (p_o + \Delta p)_{rc} = \frac{L_t + (p_o)_{vs} (A)_{stone} - (\Delta p)_{vc} (A)_{clay}}{(A)_{stone} \tan^2 (45 + \frac{\phi stone}{2})}$$

EQUATION 1

The next step is to establish another relationship between $(\Delta p)_{rc}$ and $(\Delta p)_{vc}$. (The vertical and radial stress increments in the clay.)

Assume that K and K_o exist in the clay

where: K_o = Coefficient of earth pressure in the clay = $\frac{(p_o)_{rc}}{(p_o)_{vc}}$ for the condition

$$\epsilon_r = 0 \text{ and } \epsilon_v > 0$$

K = incremental coefficient of earth pressure = $\frac{(\Delta p)_{rc}}{(\Delta p)_{vc}}$ in the clay for the condition

$\epsilon_r > 0$ and $\epsilon_v > 0$

It is reasonable to assume the following relationships based on the above definitions.

1. if $\epsilon_v > 0$ and $\epsilon_r = 0$ then $K_o = \frac{(\Delta p)_{rc}}{(\Delta p)_{vc}}$

2. if $\epsilon_r > 0$ and $\epsilon_v = 0$ then $\frac{1}{K_o} = \frac{(\Delta p)_{rc}}{(\Delta p)_{vc}}$ and

3. if $\epsilon_r = \epsilon_v$ then $\frac{(\Delta p)_{rc}}{(\Delta p)_{vc}} = 1$

Where K_o can be approximated by: $^*K_o = .95 - \sin \overline{\phi}$

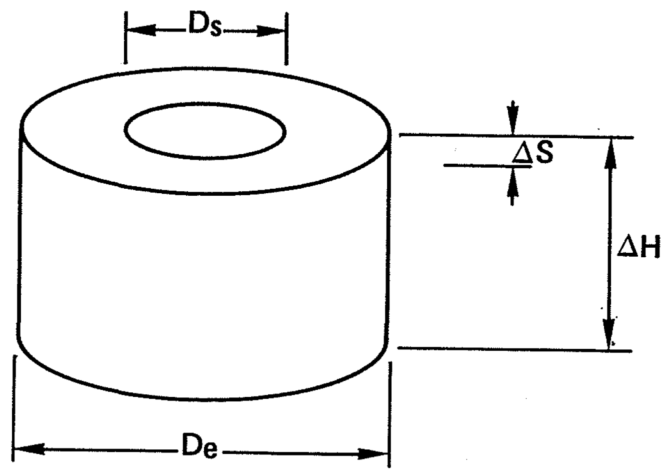
*Brooker, E. Q. and Ireland, H. O. "Earth Pressure at Rest Related to Stress History" Canadian Geotechnical Journal, Volume 2, No. 1 1965 pp. 1-15.

These three relationships can be satisfied by the equation:

$$\frac{K_o\epsilon_v + \epsilon_r}{\epsilon_v + K_o \epsilon_r} (\Delta p)_{vc} = (\Delta p)_{rc}$$

or $$K = \frac{K_o \epsilon_v + \epsilon_r}{\epsilon_v + K_o \epsilon_r}$$ <u>EQUATION 2</u>

By the problem's geometry, we can establish a relationship between ϵ_v and ϵ_r for the clay.



D_e = 1.05 x Column Spacing

FIGURE 13 VERTICAL DEFLECTION OF A "UNIT CELL"

<u>Volume of Stone</u> (before and after loading must be equal)

$$V_s = \frac{\pi D_s^2}{4} \Delta H = \pi \frac{(D_s + \Delta D_s)^2}{4} (\Delta H - \Delta S)$$

$$\frac{\cancel{\pi} D_s^2 \Delta H}{\cancel{4}} = \frac{\cancel{\pi} (D_s^2 + 2D_s \Delta D_s + \Delta D_s^2) (\Delta H - \Delta S)}{\cancel{4}}$$

$$\cancel{D_s^2} \cancel{\Delta H} = \cancel{D_s^2} \cancel{\Delta H} - D_s^2 \Delta S + 2D_s \Delta D_s (\Delta H - \Delta S) + \Delta D_s^2 (\Delta H - \Delta S)$$

$$D_s^{2}\Delta S = (2D_s\Delta D_s + \Delta D_s^{2})\quad(\Delta H-\Delta S)$$

$$\frac{D_s^{2}\Delta S}{\Delta H-\Delta S} = 2D_s\Delta D_s + \Delta D_s^{2}$$

$$\Delta D_s^{2} + 2D_s\,\Delta D_s - \frac{D_s^{2}\,\Delta S}{\Delta H-\Delta S} = 0$$

Solving this quadratic equation by the general solution

$$x = \frac{-b\pm\sqrt{b^{2} - 4AC}}{2A}$$

Yields:

$$\Delta D_s = \frac{-2D_s \pm \sqrt{4D_s^{2} + \frac{4D_s^{2}\quad\Delta S}{\Delta H - \Delta S}}}{2}$$

$$\Delta D_s = \frac{-\cancel{2}D_s \pm \cancel{2}D_s\sqrt{1 + \frac{\Delta S}{\Delta H - \Delta S}}}{\cancel{2}}$$ $\pm$ meaningless - yields $\Delta D_s > D_s$

But

$$1 + \frac{\Delta S}{\Delta H-\Delta S} = \frac{\Delta H-\Delta \cancel{S} + \Delta \cancel{S}}{\Delta H-\Delta S} = \frac{1}{\frac{\Delta H-\Delta S}{\Delta H}} = \frac{1}{1-\frac{\Delta S}{\Delta H}} = \frac{1}{1-\epsilon_v}$$

$$\therefore\quad \Delta R_s = \frac{\Delta D_s}{2} = \frac{D_s\sqrt{\frac{1}{1-\epsilon_v}} - D_s}{2}$$

$$\epsilon_r = \Delta R_s \div \left(\frac{D_e - D_s}{2}\right) = \left[\frac{D_s\sqrt{\frac{1}{1-\epsilon_v}} - D_s}{2}\right]\left(\frac{2}{D_e-D_s}\right)$$

$$\epsilon_r = \left[\sqrt{\frac{1}{1-\epsilon_v}} -1\right]\frac{D_s}{D_e-D_s}$$ <u>EQUATION 3</u>

From the theory of consolidation, we also have the following relationship:

$$\Delta e = - C_c \log \left(\frac{p_o + \Delta p}{p_c} \right) v \qquad \underline{\underline{\text{EQUATION 4}}}$$

Δe = Change in the void ratio "e"

C_c = Compression Index, which is the slope of the e-logP plot

p_o = Overburden stress on the clay

p_c = Preconsolidation stress on the clay

Δp = Stress increment on the clay

But this equation applies only for $\epsilon_r = 0$,

$$\text{or } (\Delta p)r = K_o(\Delta p)_v = (\Delta p)_t$$

$$\text{and } (\Delta p_c)_r = K_o (\Delta p_c)_v$$

We must now establish some relationships which reflect the strain conditions as they actually occur; this is $\epsilon_v > \epsilon_r$ or $\epsilon_v < \epsilon_r$.

It is reasonable to assume that the value of Δe is some function of the hydrostatic component of the stress tensor which exists at that incremental depth.

Then, for the expression for Δe established in Equation 4

$$\Delta p = \left[(\Delta p)_v + K_o(\Delta p)_v + K_o(\Delta p)_v \right] M$$

where Δp = stress increment in the clay as presented in Equation 4.

$(\Delta p)_v$ = major principal stress component in the vertical direction

$(K_o\Delta p)_v$ = intermediate and minor principal stresses in the radial and tangential directions

M = proportionality constant

$$\Delta p = \left[(1 + 2K_o) \ (\Delta p)_v \right] M$$

$$M = \frac{1}{1+2K_o}$$

and by applying the same relationship to the preconsolidation stress:

$$p_c = \left[(1 + 2K_o)\ (p_c)_v \right] M$$

There are two cases in general:

<u>CASE I: $\epsilon_v > \epsilon_r$</u>

$$\Delta p = M \left[(\Delta p)_v + \left(\frac{K_o \epsilon_v + \epsilon_r}{\epsilon_v + K_o \epsilon_r} \right) (\Delta p)_v + K_o (\Delta p)_v \right]$$

By applying the stress tensor equation established earlier:

Δp_v = is the major principal stress provided by the vertical component.

$\dfrac{K_o \epsilon_r + \epsilon_r}{\epsilon_v + K_o \epsilon_r} (\Delta p)_v$ = is the intermediate stress provided by the radial component.

$K_o (\Delta p)_v$ = is the minor principal stress provided by the tangential component. In this case we assume the tangential strain is zero so the K_o function is applied.

and

$$P_c = (p_c)_v$$

<u>CASE II: $\epsilon_r > \epsilon_v$</u>

$$\Delta p = M \left[(\Delta p)_v + \left(\frac{\epsilon_v K_o + \epsilon_r}{\epsilon_v + K_o \epsilon_r} \right) (\Delta p)_v + K_o \left[\left(\frac{\epsilon_v K_o + \epsilon_r}{\epsilon_v + K_o \epsilon_r} \right) (\Delta p)_v \right] \right]$$

In this case the vertical and radial components exchange major and intermediate principal stress roles. This exchange also modifies the contribution of the tangential stress component, by substituting the radial component as the major principal stress in place of the vertical component as in Case I. The tangential strain is still assumed to be zero, so K_o still applies.

Returning to the geometry of the problem, we can establish a relationship between the change in void ratio, Δe and the vertical strain, ϵ_v.

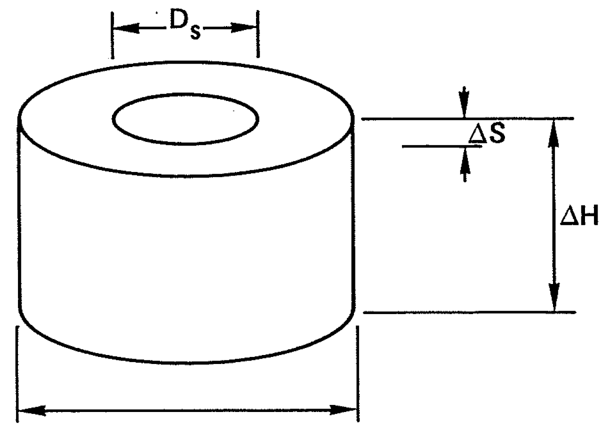


FIGURE 14 VERTICAL DEFLECTION OF A UNIT CELL

Initial Volume of the Unit Cell

$$V_o = \frac{\pi D_e^2}{4} \Delta H$$

Instantaneous Volume of the Unit Cell

$$V = \frac{\pi D_e^2}{4} (\Delta H - \Delta S)$$ (This reduction is due to consolidation)

Change in Volume of the Unit Cell

$$V_o - V = (\Delta V)_{\text{unit cell}} = \frac{\pi D_e^2 \Delta S}{4}$$

For the clay:

$$e = \frac{Vv}{Vs} \qquad Vv = eVs$$

$$(V_o)_{clay} = V_s + e_o V_s = V_s (1 + e_o) = \text{initial volume of clay}$$

$$(\Delta V)_{clay} = \Delta V_v = -\Delta e V_s$$

(volume decrease is defined as positive)

$$\therefore \quad (\Delta V)_{clay} = \frac{-\Delta e \; (V_o)_{clay}}{1 + e_o}$$

But $(\Delta V)_{clay} = (\Delta V)_{unit\ cell}$

$$\therefore \quad \frac{-\Delta e \; (V_o)_{clay}}{1 + e_o} = \frac{\pi D_e^{\,2}}{4} \Delta S$$

$$\Delta S = \frac{-4\Delta e \; (V_o)_{clay}}{(1+e_o) \; \pi D_e^{\,2}} = \frac{-\Delta e (V_o)_{clay}}{(1+e_o) \; (A)_{total}}$$

And

$$(V_o)_{clay} = (A)_{total} \; \Delta H - (A)_{stone} \; \Delta H$$

$$\therefore \quad \Delta S = \frac{\left[(A)_{total} - (A)_{stone} \right] \Delta H \; \Delta e}{(1 + e_o) \; (A)_{total}}$$

or $$\Delta S = \frac{-\Delta H \Delta e}{1 + e_o} \; \frac{(A)_{clay}}{(A)_{total}}$$

$$\epsilon_v = \frac{\Delta S}{\Delta H} = \frac{-\Delta e}{1+e_o} \; \frac{(A)_{clay}}{(A)_{total}} \qquad \underline{\text{EQUATION 5}}$$

We now have five equations which must be combined as much as possible.

EQUATIONS 1 and 2

Define K_{comp} as the counterpart of K_o, but taking into account the increased $(p_o)_{rc}$ because of forcing stone into the in situ material during the construction procedure.

We can now establish equation A

$$\text{(A)}\quad K_{comp}\ (p_o)_{vc} + \frac{K_o\ \epsilon_v + \epsilon_r}{\epsilon_v + K_o\ \epsilon_r}\ (\Delta p)_{vc} =$$

$$\frac{L_t + (p_o)_{vs}\ (A)_{stone} - (\Delta p)_{vc}\ (A)_{clay}}{(A)_{stone}\ \tan^2\left(45 + \frac{\phi stone}{2}\right)}$$

EQUATION 4 and 5 can be combined to produce equations B and C.

$$\text{(B)}\quad \epsilon_v = \frac{(A)_{clay}}{(A)_{total}}\ \frac{C_c}{1+e_o}\ \frac{\log\ (p_o)_v + \Delta p}{(p_c)_v}$$

Δp is given by cases I and II, developed above.

$$\text{(C)}\quad \epsilon_r = \left(\sqrt{\frac{1}{1-\epsilon_v}} - 1\right)\frac{D_s}{D_e - D_s}$$

Solve equations A B and C simultaneously to obtain values for

$(\Delta p)_{vc}$ and $(p_o + \Delta p)_{vc}$

Then we know:

$$(p_o + \Delta p)_{rc} = (p_o)_{vc}\ K_{comp} + \frac{K_o\ \epsilon_v + \epsilon_r}{\epsilon_v + K_o\ \epsilon_r}\ (\Delta p)_{vc}$$

$$(p_o)_{vs} + (\Delta p)_{vs} = (p_o + \Delta p)_{rc} \tan^2 (45 + \frac{\phi stone}{2})$$

$(p_o)_{vs} + (\Delta p)_{vs}$ represents the vertical unit stress in the stone at the elevation under consideration, based on rigid-plastic behavior for the Stone Column.

We can now solve for ϵ_v by equation B. This represents the vertical strain in that increment when the Stone Column behaves rigid-plastically.

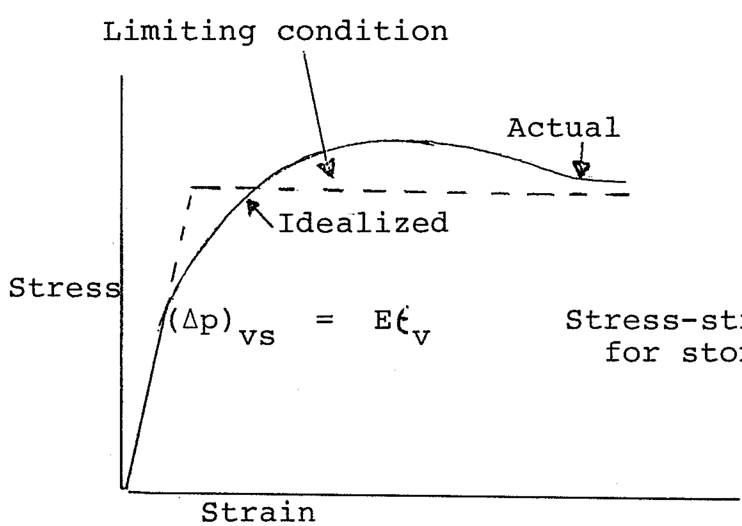
ELASTIC ANALYSIS:

$$L_t = (\Delta p)_{vs} (A)_{stone} + (\Delta p)_{vc} (A)_{clay}$$

The limiting condition of elastic behavior is given by:

$$(p_o + \Delta p)_{vs} = \tan^2 (45 + \frac{\phi stone}{2}) (p_o + \Delta p)_{rs}$$

The stress-strain behavior of the stone is shown below:



Stress-strain behavior for stone

$$L_t = \epsilon_v E (A)_{stone} + (\Delta p)_{vc} (A)_{clay} \qquad \text{EQUATION 1}$$

$$\epsilon_v = \frac{(A)_{clay}}{(A)_{total}} \frac{C_c}{1+e_o} \log \frac{(p_o + \Delta p)}{(p_c)_v} \qquad \text{EQUATION 2}$$

Equation 2 is the same as Equation B from plastic analysis and is independent of the yield condition of the stone.

$(p_o+\Delta p)$ is the same as that derived in plastic analysis.

Solve these equations simultaneously to obtain values for

$(p_o+\Delta p)_{vc}$ and ϵ_v

Then $(\Delta p)_{vs} = E\epsilon_v$

and

Then $(\Delta p)_{vs} + (p_o)_{vs}$ represents the vertical unit stress in the stone at the elevation under consideration for elastic behavior of the stone.

And ϵ_v is the vertical strain if the stone behaves elastically.

* The larger value ϵ_v (plastic) or ϵ_v (elastic) represents *
* the true ϵ_v and determines whether the stone is in the *
* elastic or plastic range. *

<u>Depth Effect on Stress</u>

The distribution of total load with depth must be considered as we evaluate various column depth increments along the total length.

Examining the extreme conditions:

1. If Stone Columns are not present, assume that Boussinesq or Westergaard analysis is correct.

2. If Stone Columns are completely rigid, and end bearing; the entire load will be transmitted through the Stone Columns and there would be no decrease in L_t with depth.

We assume an intermediate condition:

$$L_t = \left[f^* \frac{(A)_{clay}}{(\cancel{A})_{total}} + \frac{(A)_{stone}}{(\cancel{A})_{total}} \right] \text{Loading} \times (\cancel{A})_{total}$$

$$L_t = f^* \ (A)_{clay} \times \text{Loading} + (A)_{stone} \times \text{Loading}$$

f^* = Modified Boussinesq (or other) factor

Loading = STRESS applied to foundation area

* Use $f^* = \frac{1}{1 + Qd^2}$

Find Q by

$$f^* + f^* Qd^2 = 1$$

$$f^* Qd^2 = 1 - f^*$$

$$Q = \frac{1-f^*}{f^* d^2}$$

* The above exercise was used to establish an automated method to account for dissipation of L_t with depth.

If the entire analysis is performed by hand calculations an "f" factor based on a standard stress distribution may be used for each increment.

Using the automated method the following procedure is used.

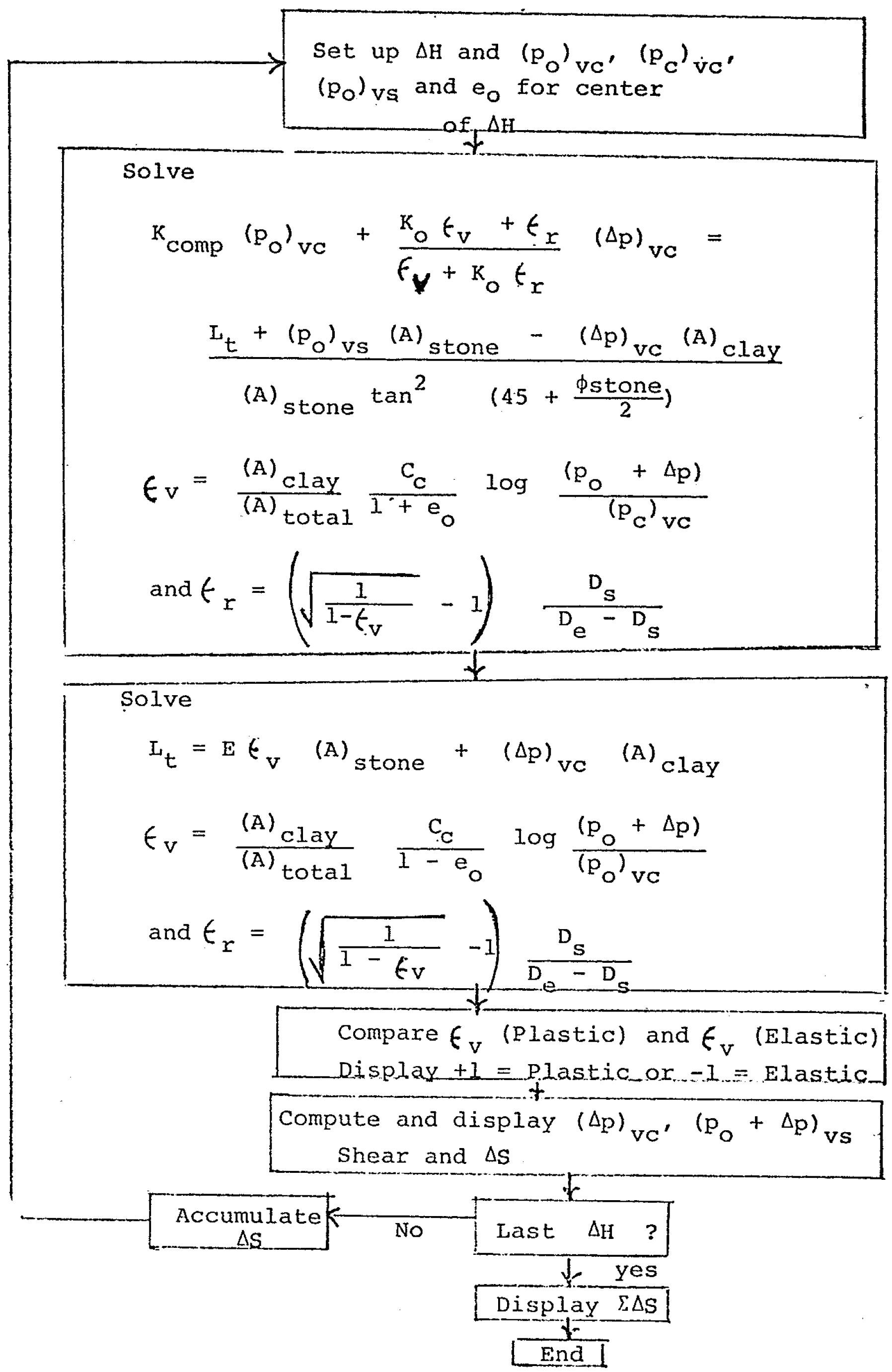
1. Pick some column depth, approximately 2/3 to 3/4 of the total column length.
2. Find the "f" factor by Boussinesq or Westergaard method for a homogeneous soil.
3. Substitute "f" and "d" in the above equation to find "Q."

Then:

$$L_t = \left[\frac{(A)_{clay}}{1 + Qd^2} + (A)_{stone} \right] \text{Loading}$$

The method of Stone Column analysis described within this section is very complex to perform by hand calculation. This complex approach to analyzing a single column increment must be repeated a number of times for a complete analysis of a single column. The total number of increments selected will vary from project to project and primarily depends on the judgement of the designer and the quantity of soil testing information which is available.

The flow chart presented below summarizes the equation solving procedure required to analyze a single Stone Column.



The results provided by performing a complete settlement analysis are illustrated in the Example Problem Section.

C. Time-Rate of Settlement and Secondary Consolidation

Field test results have indicated that Stone Columns behave similar to sand drains, that is; they provide a network of vertical drainage paths which accelerate the rate of consolidation within the clay. It is beyond the scope of this text to provide a detailed explanation of sand drain design and behavior. The example problems will illustrate how time-rate of settlement values may be calculated.

Secondary settlements must be considered when installing Stone Columns. The conventional method for calculating the quantity of secondary settlement in areas without Stone Columns is:

$$\Delta Hs = \Sigma HC\alpha \log \frac{tc}{tp}$$

$C\alpha$ = coefficient of secondary consolidation

H = height of the layer considered

tp = time required for primary consolidation

tc = time period of interest; this is usually the design life of the facility

ΔHs = settlement due to secondary consolidation

We suggest that the quantity of secondary consolidation be computed using the above equation and then, a reduction factor of $\frac{(A)_{clay}}{(A)_{total}}$ be applied to determine the secondary effect which must be accounted for after Stone Column installation. The computed quantity of secondary consolidation can then be addressed according to accepted precompression procedures of the design agency.

Admittedly, this approach to secondary consolidation appears to be conservative. This aspect of design, as well as time-rate of settlement, are two areas where additional field testing and project evaluations could provide a great deal of benefit for refinement of design approaches. Designers should recognize that the accuracy of the calculated values are highly dependent on the soil parameters used. Values of C_v (vertical), C_r (radial), and $C\alpha$ are often estimated quantities, therefore, the accuracy of these values is extremely important as to whether or not the suggested methods provide consistent results.

D. Slope Stability

At locations where Stone Columns are applied as a correction to stability problems, column shear strength within the critical zone is a primary concern. When first considering the shear strength of a column, one might tend to envision the column as an isolated unit. That is, shear strength of a column at a depth (d) below the surface might be computed by:

$$CS = \gamma_{stone}\ d\ Tan\phi\ A_{sc}$$

CS = shear per column

γ_{stone} = unit weight of Stone Column

ϕ = Stone Column angle of internal friction (this is usually assumed to be 38 degrees)

A_{sc} = area of the Stone Column

The resultant shearing force would then be incorporated into the stability calculations as an additional resisting force. This approach is very similar to a direct shear test.

Under field conditions, this approach is unrealistic because of the additional factors which must be considered:

1. Below the ground-water table, the submerged unit weight of the Stone Column must be used.

2. During the column construction process; a volume of in situ soil equal to the Stone Column volume is removed. Therefore, the existing safety factor is reduced by a certain percentage and this must be accounted for in the analysis.

3. The assumed horizontal failure plane is actually curvilinear in shape and depends on the circle which is analyzed and its position within the cross section.

4. The correct normal force, at the shear plane elevation will probably be less than the above equation would imply. That is, it does not account for any reduction of normal stress with depth.

In order to eliminate the effect of these factors and provide a consistent method of analysis, we recommend using a computerized solution which applies Bishop's Method of Slices. A computerized solution will also provide a versatile design tool, with which column spacing, diameter, and location within the cross section can be changed easily.

While there are a variety of stability programs available or in use, they all allow for soil input by parameters of: unit weight; friction angle, and cohesion. Using this approach, the effect of a particular Stone Column pattern on stability may be determined by one of two procedures:

1 - Columns may be incorporated as individual soil units within the input data. The resulting cross section is a repetition of areas which reflects separately the Stone Column areas and areas of in situ soil. While this approach permits an exact duplication of the as constructed section; it is cumbersome to develop. In addition, variables such as column spacing, diameter, and location within the section are difficult to comprehensively examine because of the large quantity of input data required for each cross section.

2 A simple approach is to average shear strength and unit weight parameters within the treated section. The Stone Column treated area could therefore be easily incorporated as a homogeneous soil unit. This method could be accomplished as follows:

Returning to our unit cell:

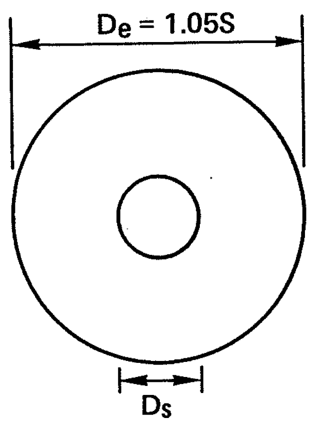


FIGURE 15 AREA RATIO

Total Area of Unit Cell

$$(A)_{total} = \frac{\pi (D_e)^2}{4}$$

$$(A)_{stone} = \frac{\pi (D_s)^2}{4}$$

$$(A)_{clay} = \frac{\pi}{4} (D_e^2 - D_s^2)$$

Given: γclay, γstone, Cclay, ϕclay, and ϕstone

Find: Average values for γ, C, and Tanϕ

$$C\ ave = Cclay\left(1-\frac{(A)_{stone}}{(A)_{total}}\right)$$

$$\gamma\ ave = \gamma_{stone}\ \frac{(A)_{stone}}{(A)_{total}} + \gamma\ clay\ \frac{(A)_{clay}}{(A)_{total}}$$

$$\tan\phi_{ave} = \tan\phi_{clay}\ \frac{(A)_{clay}}{(A)_{unit\ cell}} + \tan\phi_{stone}\ \frac{(A)_{stone}}{(A)_{unit\ cell}}$$

Differences between these two procedures will vary according to the computer analysis used and the location of the Stone Column treated area within the cross section. In cases where the columns are symmetrically located with regard to the center of the circle analyzed, the difference will be minimal. However, when the centroid of the Stone Column treated mass is positioned to the right or left of the circle's center, the difference becomes greater and may be conservative or unconservative. For design purposes, the individual Stone Column unit approach is most exact and should always be used as a reference. The average parameter procedure should be used only when correlated with the more exact analysis.

VI. EXAMPLE PROBLEMS

Settlement Problem ①

Calculate the anticipated settlement beneath the proposed Reinforced Earth wall which is shown in figure 16. Analyze this problem first without Stone Columns; then with Stone Columns placed in the following pattern: diameter = 3.5 feet and triangular center to center spacing of 5.0 feet.

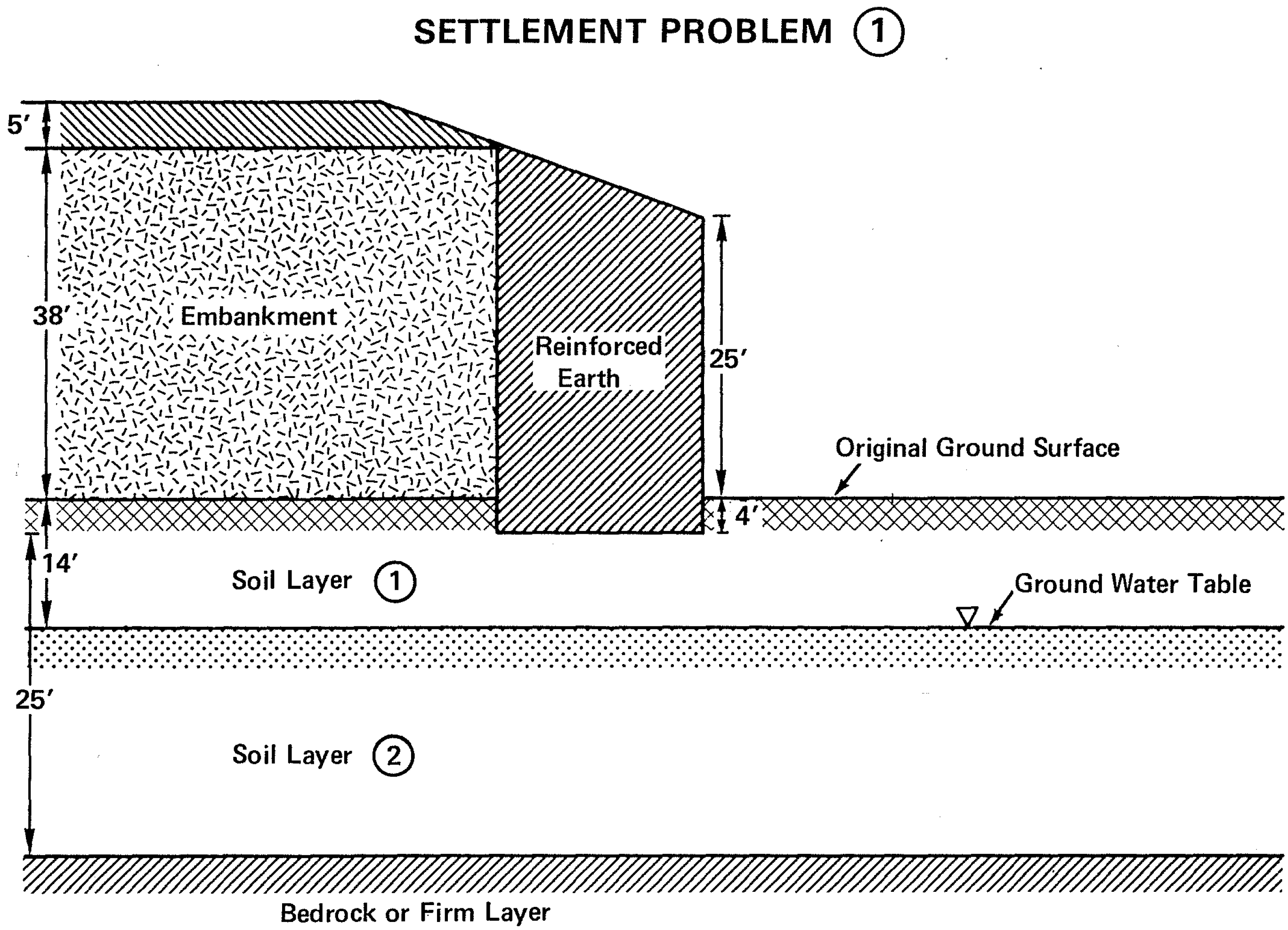


FIGURE 16 SETTLEMENT PROBLEM ①

Soil Parameters

Parameters	Reinforced Earth	Soil Layer 1	Soil Layer 2
Unit Weight p.c.f.	130	115	55 (submerged)
Friction Angle ϕ °	Not Required	6	8.5
Cohesion psf	Not Required	400	400
Compression Index C_c	Not Required	0.37	0.17
Earth Pressure at Rest K_o	Not Required	0.6	0.6
Void Ratio at Layer Surface e_o	Not Required	0.767	0.548

Average vertical stress imposed at the foundation of the Reinforced Earth wall

$$\Delta p = \left(\frac{42' + 29'}{2} \right) 130 \text{ p.c.f.} = 4615 \text{ p.s.f.}$$

Assume the following design values for the Stone Columns analysis

$K_{comp} = K_o = 0.6$

(γ) stone (submerged) = 75 p.c.f.

$E_{(stone)}$ = 600 t.s.f.

f = 0.9 at a column depth of 20 feet assuming a Boussinesq distribution.

Column diameter, D_s = 3.5 feet

Column spacing, S = 5.0 feet (Triangular Spacing)

$$Q = \frac{1-f}{fd^2} = .000278$$

(1) Using the settlement procedure outlined in section V, calculate the anticipated settlement without columns.

ΔH (feet)	Depth (feet)	(p_o) at Depth (p.s.f.)	$(\Delta p)_{vc}$ at Depth (p.s.f.)	ΔS (feet)
4.0	2.0	230.0	4610.2	0.717
4.0	6.0	690.0	4569.0	0.622
2.0	9.0	1035.0	4513.0	0.280
4.0	12.0	1260.0	4437.0	0.229
4.0	16.0	1480.0	4309.0	0.210
4.0	20.0	1700.0	4153.0	0.193
3.0	23.5	1892.0	4001.0	0.134

Total Settlement = $\Sigma\Delta S$ = 2.384 feet

(2) Calculate the anticipated settlement with Stone Columns installed as described previously.

ΔH (feet)	Depth (feet)	(p_o) at Depth (p.s.f.)	$(\Delta p)_{vc}$ (p.s.f.)	$(p_o+\Delta p)_{vs}$ (p.s.f.)	Shear (p.s.f.)	Plastic+1 Elastic-1	ΔS (feet)
4.0	2.0	230.0	1829.0	8366.0	0.0	+1	0.256
4.0	6.0	690.0	1713.0	9010.0	-21.0	+1	0.196
2.0	9.0	1035.0	1619.0	9470.0	-14.0	+1	0.081
4.0	12.0	1260.0	1557.0	9741.0	5.0	+1	0.063
4.0	16.0	1480.0	1481.0	9974.0	15.0	+1	0.055
4.0	20.0	1700.0	1399.0	10183.0	20.0	+1	0.047
3.0	23.5	1892.0	1324.0	10349.0	24.0	+1	0.031

Total Settlement = $\Sigma\Delta S$ = 0.728 feet

Settlement Problem (2)

Calculate the anticipated settlement for the load-soil configuration shown in figure 17. Analyze the problem under existing conditions and with Stone Columns as described within the problem. Analyze the problem for time-rate of settlement after the columns have been installed.

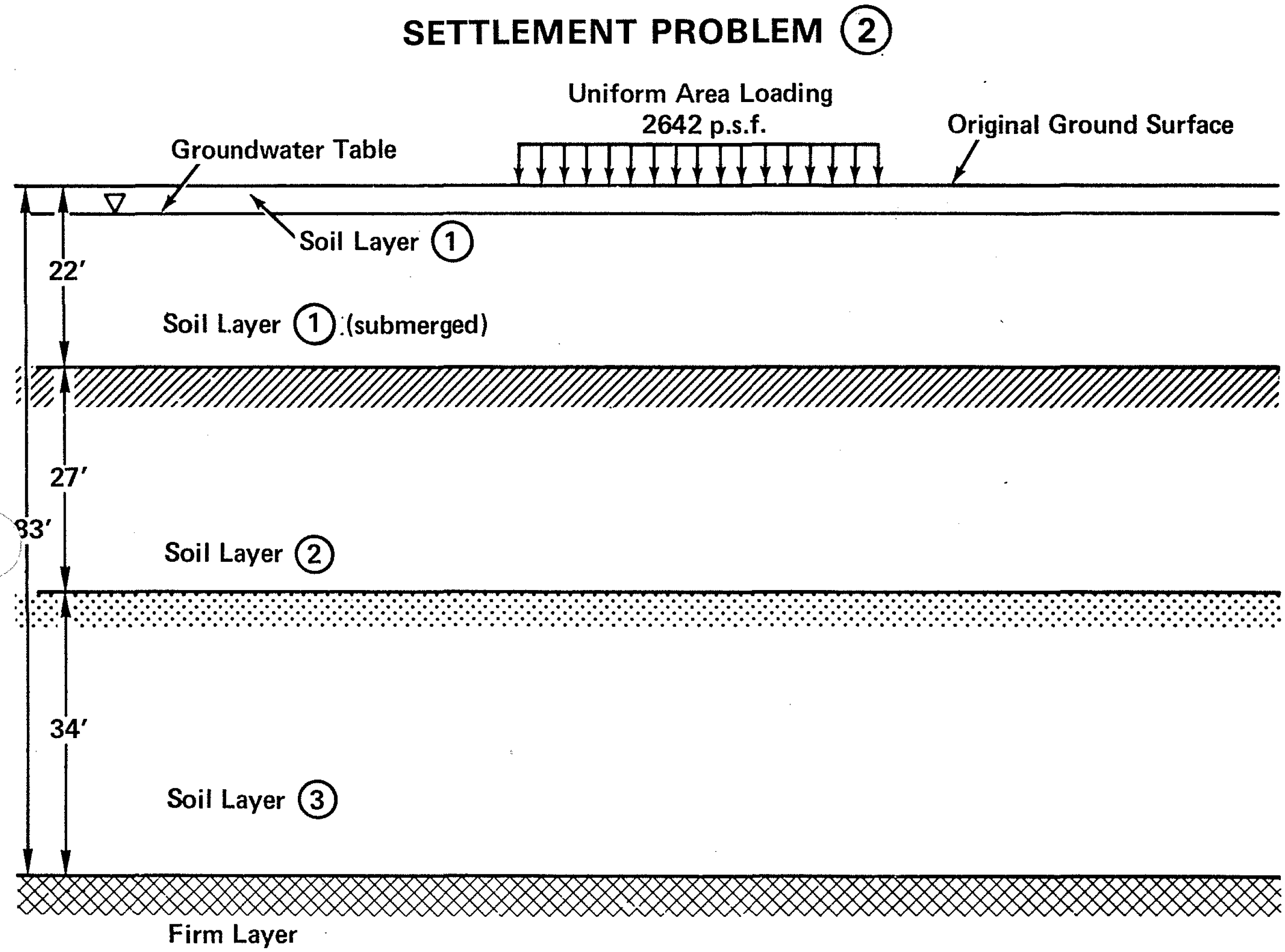


FIGURE 17 SETTLEMENT PROBLEM (2)

Soil Parameters

Parameters	Soil Layer 1	Soil Layer 2	Soil Layer 3
Thickness (feet)	22.0	27.0	34.0
Unit Weigth (p.c.f.)	90.0	115.0	98.0
Void Ratio at Layer Surface e_o	2.57	1.32	1.83
Compression Index, C_c	0.57	0.50	0.64
Earth Pressure at Rest, K_o	0.60	0.60	0.60
Coefficient of Vertical Consolidation Cv (feet^2)/day	0.2438	1.9068	0.1644

Assume the following design values for the Stone Column Analysis

$\gamma_{stone(submerged)}$ = 75 p.c.f.

$E_{(stone)}$ = 600 t.s.f.

Column diameter, D_s = 3.3 feet

Column spacing, S = 5.0 feet (Triangular Spacing)

Friction angle, ϕstone = 38°

f = 0.950 at a column depth of 13.8 feet assuming a Boussinesq distribution.

$$Q = \frac{1-f}{fd^2} = .000276$$

1 Using the settlement procedure outlined in section V, calculate the anticipated settlement without columns.

ΔH (feet)	Depth (feet)	(p_o) at Depth (p.s.f.)	(Δp)$_{vc}$ at Depth (p.s.f.)	ΔS (feet)
2.0	1.0	90.0	2641.0	0.422
1.0	2.5	225.0	2637.0	0.168
2.0	4.0	298.0	2630.0	0.309
2.0	6.0	353.0	2616.0	0.291
2.0	8.0	408.0	2596.0	0.276
2.0	10.0	463.0	2571.0	0.262
2.0	12.0	518.0	2541.0	0.249
2.0	14.0	574.0	2506.0	0.238
2.0	16.0	629.0	2467.0	0.227
2.0	18.0	684.0	2425.0	0.217
2.0	20.0	739.0	2379.0	0.208
1.0	21.5	781.0	2343.0	0.100
2.0	23.0	847.0	2305.0	0.233
2.0	25.0	952.0	2253.0	0.218
2.0	27.0	1057.0	2199.0	0.204
2.0	29.0	1163.0	2144.0	0.191
2.0	31.0	1268.0	2088.0	0.179
2.0	33.0	1373.0	2031.0	0.169
2.0	35.0	1478.0	1974.0	0.159
2.0	37.0	1583.0	1917.0	0.149
2.0	39.0	1689.0	1860.0	0.141
2.0	41.0	1794.0	1804.0	0.132
2.0	43.0	1899.0	1749.0	0.125
2.0	45.0	2004.0	1694.0	0.118
2.0	47.0	2109.0	1640.0	0.111
1.0	48.5	2188.0	1601.0	0.053
2.0	50.0	2250.0	1562.0	0.101
2.0	52.0	2321.0	1512.0	0.097
2.0	54.0	2393.0	1463.0	0.092
2.0	56.0	2464.0	1415.0	0.088
2.0	58.0	2535.0	1369.0	0.084
2.0	60.0	2606.0	1324.0	0.080
2.0	62.0	2677.0	1281.0	0.076
2.0	64.0	2749.0	1239.0	0.073
2.0	66.0	2820.0	1199.0	0.070
2.0	68.0	2891.0	1160.0	0.066
2.0	70.0	2962.0	1122.0	0.063
2.0	72.0	3033.0	1086.0	0.061
2.0	74.0	3105.0	1051.0	0.058
2.0	76.0	3176.0	1018.0	0.055
2.0	78.0	3247.0	985.0	0.053
2.0	80.0	3318.0	954.0	0.050
2.0	82.0	3389.0	924.0	0.048

Total Settlement = $\Sigma \Delta S$ = 6.36 feet

(2) Calculate the anticipated settlement with Stone Columns installed as described previously. Assume $K_{comp} = K_o = 0.6$. Analyze also the time-rate of settlement behavior of this column design.

ΔH (feet)	Depth (feet)	(p_o) at Depth (p.s.f.)	$(\Delta p)_{vc}$ (p.s.f.)	$(p_o+\Delta p)_{vs}$ (p.s.f.)	Shear (p.s.f.)	Plastic+1 Elastic-1	ΔS (feet)
2.0	1.0	90.0	1168.0	5035.0	0.0	+1	0.208
1.0	2.5	225.0	1148.0	5266.0	-14.0	+1	0.077
2.0	4.0	298.0	1140.0	5411.0	0.0	+1	0.139
2.0	6.0	353.0	1139.0	5541.0	8.0	+1	0.129
2.0	8.0	408.0	1136.0	5665.0	11.0	+1	0.121
2.0	10.0	463.0	1132.0	5783.0	13.0	+1	0.114
2.0	12.0	518.0	1126.0	5895.0	15.0	+1	0.107
2.0	14.0	574.0	1119.0	6003.0	17.0	+1	0.101
2.0	16.0	629.0	1111.0	6107.0	19.0	+1	0.096
2.0	18.0	684.0	1102.0	6206.0	21.0	+1	0.092
2.0	20.0	739.0	1091.0	6301.0	22.0	+1	0.087
1.0	21.5	781.0	1083.0	6371.0	24.0	+1	0.042
2.0	23.0	847.0	1062.0	6458.0	14.0	+1	0.097
2.0	25.0	952.0	1028.0	6580.0	11.0	+1	0.089
2.0	27.0	1057.0	993.0	6701.0	12.0	+1	0.081
2.0	29.0	1163.0	958.0	6821.0	13.0	+1	0.074
2.0	31.0	1268.0	922.0	6939.0	13.0	+1	0.068
2.0	33.0	1373.0	886.0	7057.0	13.0	+1	0.063
2.0	35.0	1478.0	850.0	7175.0	13.0	+1	0.058
2.0	37.0	1583.0	814.0	7293.0	13.0	+1	0.053
2.0	39.0	1689.0	778.0	7411.0	13.0	+1	0.049
2.0	41.0	1794.0	742.0	7530.0	13.0	+1	0.045
2.0	43.0	1899.0	707.0	7650.0	12.0	+1	0.042
2.0	45.0	2004.0	671.0	7771.0	12.0	+1	0.038
2.0	47.0	2109.0	636.0	7893.0	12.0	+1	0.035
1.0	48.5	2188.0	610.0	7985.0	11.0	+1	0.016
2.0	50.0	2250.0	591.0	8066.0	17.0	+1	0.031
2.0	52.0	2321.0	572.0	8168.0	20.0	+1	0.029
2.0	54.0	2393.0	553.0	8272.0	19.0	+1	0.028
2.0	56.0	2464.0	535.0	8377.0	19.0	+1	0.026
2.0	58.0	2535.0	517.0	8484.0	18.0	+1	0.025
2.0	60.0	2606.0	500.0	8592.0	17.0	+1	0.024
2.0	62.0	2677.0	482.0	8702.0	17.0	+1	0.023
2.0	64.0	2749.0	466.0	8814.0	16.0	+1	0.021
2.0	66.0	2820.0	449.0	8927.0	15.0	+1	0.020
2.0	68.0	2891.0	433.0	9042.0	15.0	+1	0.019
2.0	70.0	2962.0	418.0	9158.0	14.0	+1	0.018
2.0	72.0	3033.0	402.0	9276.0	13.0	+1	0.017
2.0	74.0	3105.0	388.0	9395.0	13.0	+1	0.016
2.0	76.0	3176.0	373.0	9516.0	12.0	+1	0.015
2.0	78.0	3247.0	359.0	9639.0	11.0	+1	0.015
2.0	80.0	3318.0	345.0	9762.0	11.0	+1	0.014
2.0	82.0	3389.0	331.0	9887.0	10.0	+1	0.013

Total Settlements = $\Sigma\Delta S$ = 2.478 feet

If we perform a similar analysis and change the K_{comp} value from .6 to 1.0 the total calculated settlement is 1.812 feet.

The following assumptions were made for an analysis of time settlement.

1. $K_{comp} = K_o = 0.6$. Therefore, the total anticipated settlement is 2.478 feet.

2. $C_r = 3C_v$ where C_r is radial coefficient of consolidation and C_v is the vertical coefficient of consolidation. This assumption should be conservative, but it will vary depending on the soil characteristics. Further information concerning this relationship can be obtained from texts and technical papers which address the design of sand drains.

3. Vertical drainage within the clay is not considered. The spacing between columns is such that radial drainage predominates during consolidation.

4. $S^* = {}^{(r)_{smear}}/_{(r)_{stone}} = 1.38$

   Where $(r)_{smear}$ = Radius of the smear zone

   $(r)_{stone}$ = Radius of Stone Column

5. $\dfrac{(K)_{ru}}{(K)_{rs}} = 2.0$

   Where $(K)_{ru}$ = permeability of the undisturbed soil in the horizontal direction

   $(K)_{rs}$ = permeability of the soil within the smeared zone in the horizontal direction

   The ratios of assumptions 4 and 5 vary according to soil type and construction disturbance. Once again, texts and technical papers addressing sand drains will provide more information concerning these subjects.

For a detailed discussion of the procedures used for this analysis refer to R. F. Scott "Principles of Soil Mechanics," pages 198-203, Addison-Wesley Publishing Company, Inc.

Other Design Values

Column Spacing, $S = 5.0$ feet

Stone Column Radius, $(r)_{stone} = 1.65$ feet

Unit Cell Radius, $re = \frac{1.05(S)}{2} = 2.62$ feet

Using the information above and figure 18, which is taken from Scott, as referenced earlier, the following time-rates of settlement were computed.

Where: $n = {}^{re}/rs = 1.59$

$$(K)_{ru}/{}_{r_{stone}}K = \frac{(K)_{ru}\ (S^{*}-1)}{(K)_{rs}}$$

$\frac{1}{K}$ = surface resistance at the well due to smeared zone

$m = 1.2$ (from figure 18b)

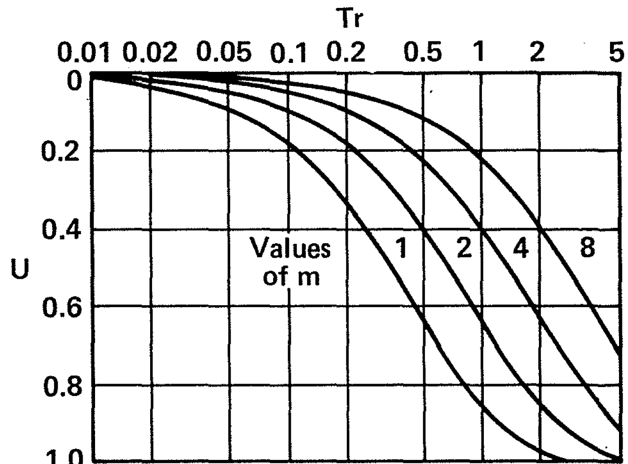
U = degree of consolidation (measured in percent) figure 18a

T_r = radial time factor (figure 18a)

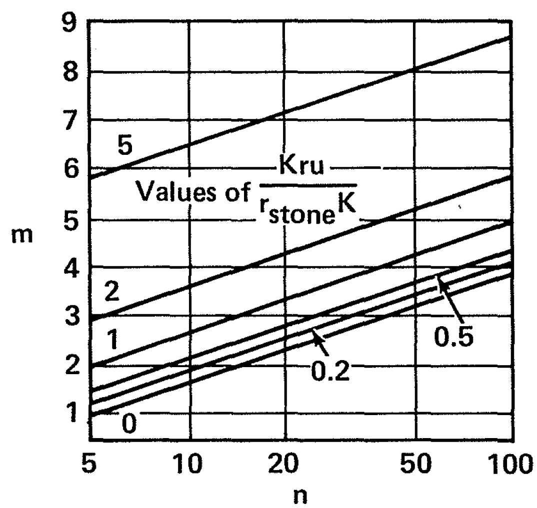
$t_{\%U}$ = time it will require for a selected percentage of consolidation to take place

$$T_r = \frac{C_r t_{\%U}}{(re)^2}$$

TIME SETTLEMENT PARAMETERS
RADIAL DRAINAGE WITH SMEAR; EQUAL STRAINS



(a)



(b)

FIGURE 18 TIME SETTLEMENT PARAMETERS RADIAL DRAINAGE WITH SMEAR; EQUAL STRAINS

Time Settlement

Soil	Ultimate	3 (days)	7 (days)	15 (days)	30 (days)	60 (days)
		Settlement (feet) - Time (days)				
1	1.313	.515	.902	1.204	1.304	1.313
2	0.810	.793	.810	0.810	0.810	0.810
3	0.355	.101	.193	0.289	0.343	0.355
TOTAL	2.478	1.409	1.904	2.303	2.457	2.477

Stability Problem

*Calculate the factor of safety for the design section shown below under the following conditions:

(a) Design section without Stone Columns

(b) Stone Column design included - columns simulated by averaging soil parameters

(c) Stone Column design included - columns simulated by vertical stone strips

*This entire exercise was performed by using a computer analysis which applied the simplified Bishop Method of slices for a solution.

(a) Design Section Without Stone Columns

STABILITY PROBLEM 1

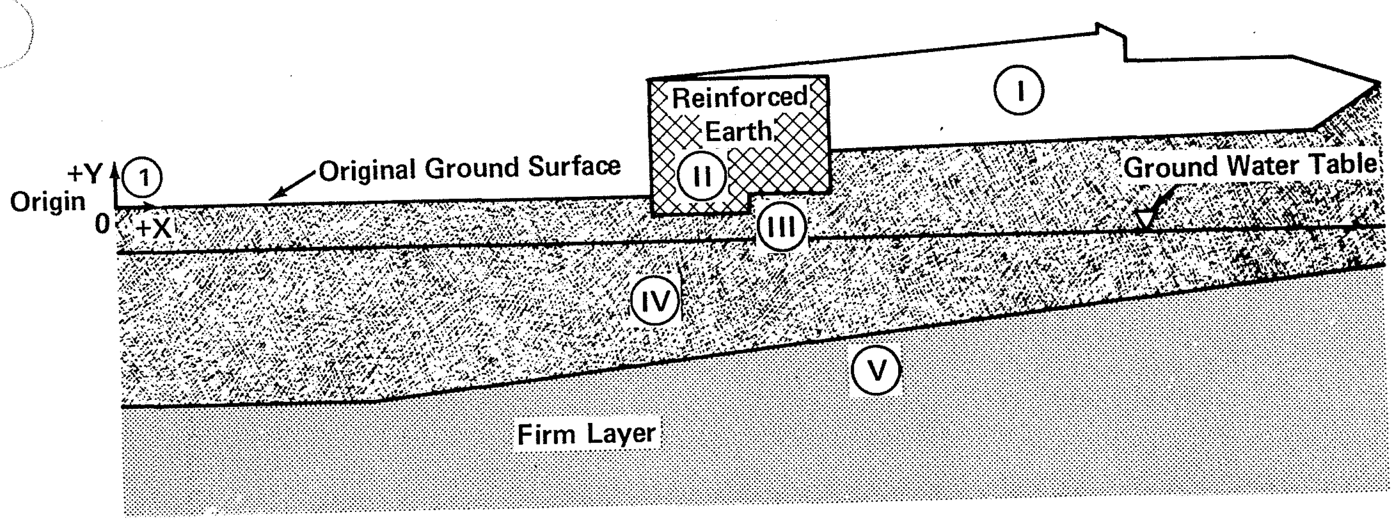


FIGURE 19 STABILITY PROBLEM

Soil Parameters Given:

Material	γwet p.c.f.	Cp.s.f.	ϕ^o
I	130.0	150	31.0
II	130.0	0	45.0
III	115.0	400	6.0
IV	117.0	400	8.5
V	150.0	5000	0.0

The minimum factor of safety for the design section shown in Figure 19 was found to be 1.004. The critical circle had the coordinates: Radius = 68.93 feet, x =+128.0 feet and y = +42.5 feet.

(b) Columns Simulated by Averaging Soil Parameters

Stone Column Design Given

(γ) stone = 125 p.c.f.

S = 5.0 feet

Ds = 3.0 feet

11 Rows of Columns

(ϕ) stone = 38^o

This design provides for a Stone Column treated area which extends 47.63 feet within the cross-section. This value was calculated as follows:

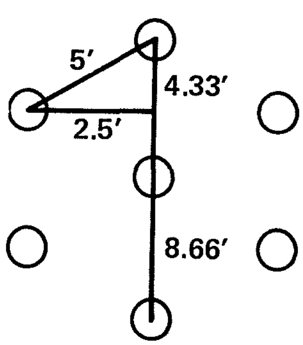
Row 1

Row 2

Row 3

Row 4

Row 5



Length of Treatment

Rows 2 thru 10 =
10 x 4.33 = 43.30

End Rows 1 and 11 =
$2 \times \frac{4.33}{2} = 4.33$

Total = 47.63 feet

Assume the location of the columns is such that they are placed symmetrically beneath the Reinforced Earth Wall. We can calculate the shearing resistance provided by the columns by using average values of γ, ϕ and C.

<u>Soil VI</u> Average Values - Area above the Groundwater Table

$$(\gamma)\text{ave} = (\gamma)\text{ clay } \frac{(A)\text{ clay}}{(A)\text{ total}} + (\gamma)\text{ stone } \frac{(A)\text{ stone}}{(A)\text{ total}}$$

$$(\gamma)\text{ave} = (115)\ \frac{14.58}{21.65} + (125)\ \frac{7.07}{21.65} = 118.26 \text{ p.c.f.}$$

$$(C)\text{ave} = (400)\ \frac{14.58}{21.65} = 269.4 \text{ p.s.f.}$$

$$\text{Tan}(\phi)\text{ave} = (\text{Tan } 6^{\circ})\frac{14.58}{21.65} + (\text{Tan } 38^{\circ})\ \frac{7.07}{21.65} = .325$$

$$\phi^{\circ}\text{ ave} = 18.0^{\circ}$$

<u>Soil VII</u> Average Values - Area below the Groundwater Table

$$(\gamma)\text{ave} = (\gamma)\text{ clay } \frac{(A)\text{ clay}}{(A)\text{ total}} + (\gamma)\text{ stone } \frac{(A)\text{ stone}}{(A)\text{ total}}$$

$$= (117)\ \frac{14.58}{21.65} + (125)\ \frac{7.07}{21.65} = 119.68 \text{ p.c.f.}$$

$$(C)\text{ave} = 269.5 \text{ p.s.f.}$$

$$\text{Tan}(\phi)\text{ ave} = (\text{Tan})\,8.5^{\circ}\ \frac{14.58}{21.65} + (\text{Tan } 38^{\circ})\ \frac{(7.07)}{21.65} = .355$$

$$\phi^{\circ}\text{ave} = 19.57^{\circ}$$

Using the same coordinate system established in Part (a), the limits of new soil layers are: $y = -5 \quad x = 117.84$
$y = -5 \quad x = 165.47$

The safety factor of the critical circle found in Part (a) is now recomputed and is equal to <u>1.455.</u>

(c) Stone Columns Simulated as Individual Soil Strips

Given: S = 5.0 feet center to center spacing

Ds = 3.0 feet Stone Column diameter

The column spacing results in a staggered array of shear pins rather than a continuous treatment across an unstable area. Therefore, in order to perform a conventional stability analysis which deals with a one-foot width of failure, we must simulate the column effect by reducing the column area to an equivalent area per linear foot of failure. This can be performed as follows:

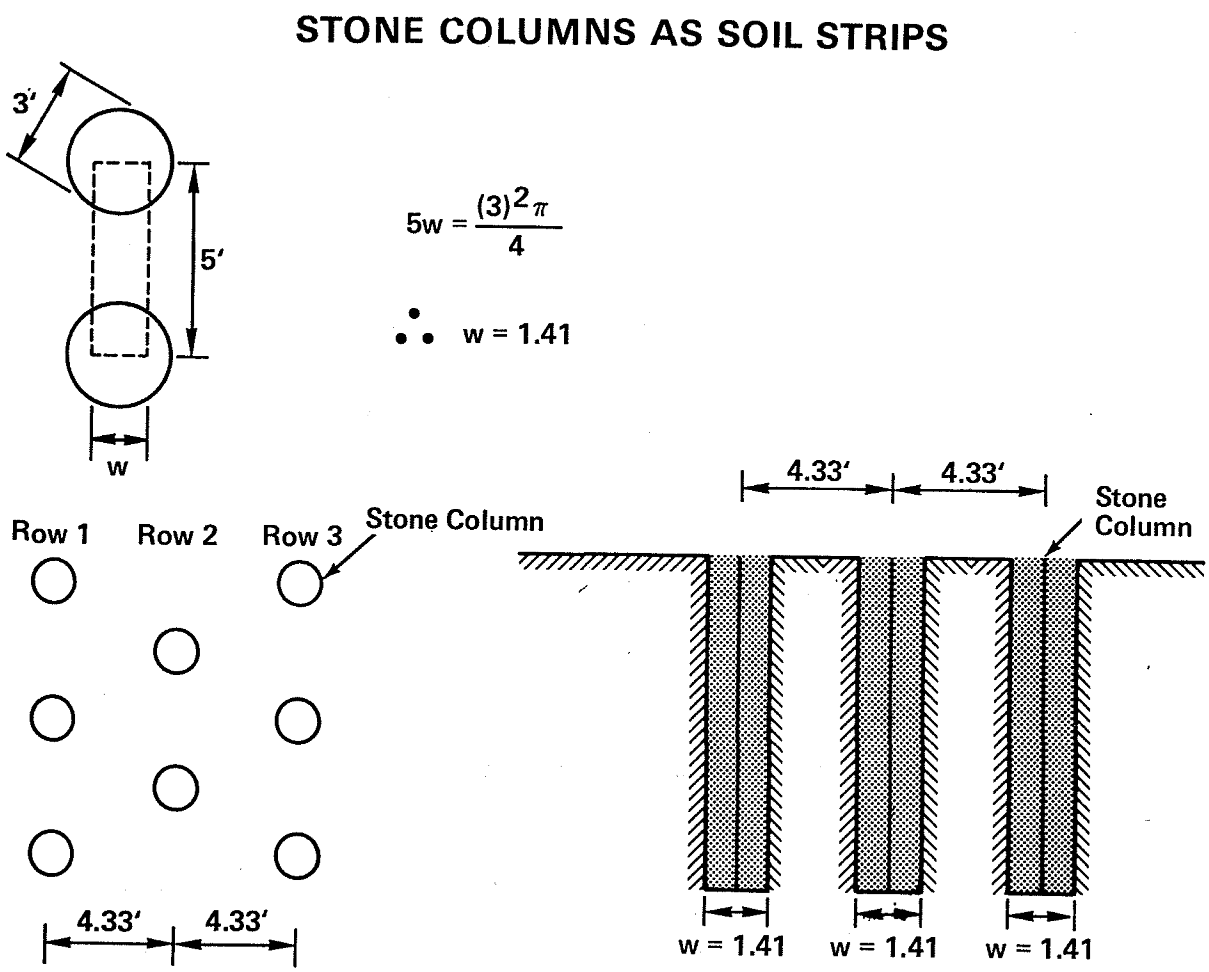


FIGURE 20 STONE COLUMNS AS SOIL STRIPS

The widths calculated on the preceding page are then input as the Stone Columns at the appropriate locations. Using this procedure the resulting safety factor was 1.449. The differences between an analysis as shown on the preceding page and in part (b) are minimal. The exact variation will vary depending on the design section and the computer program used.

The completed section is shown below.

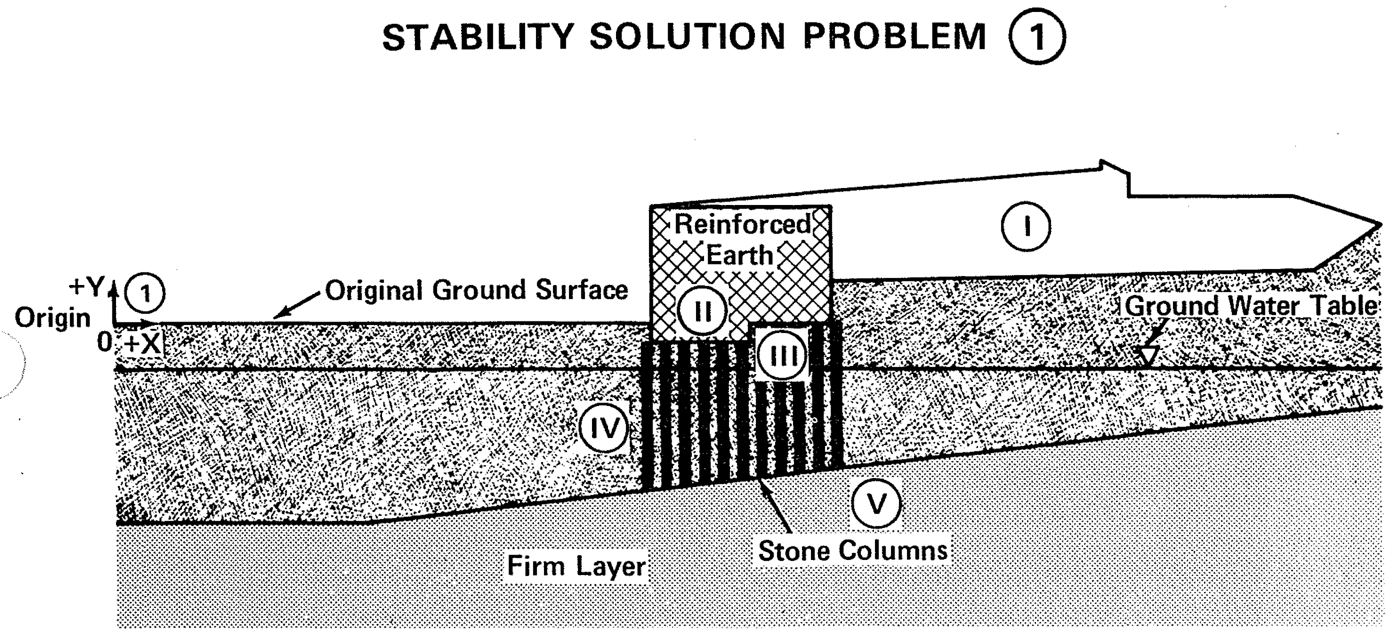


FIGURE 21 STABILITY SOLUTION

VII. FIELD TESTING, INSTRUMENTATION

A. Field Testing

As a means to verify design assumptions prior to construction, it is often advantageous to perform one or a number of field load tests. Both vertical and lateral load tests can be performed on Stone Column projects.

1) Lateral Load Tests

Horizontal shear tests should be conducted when the ultimate shear strength of the columns is questioned. This situation may occur in areas of seismic activity or when the backfill material proposed to be used is of questionable shear strength.

Lateral load testing of columns is performed similar to a direct shear test. A typical test setup is shown in Figures 22 through 24. At one project site in Santa Barbara, California, two types of test were conducted, one used a 3.5-foot diameter shearing ring and the other, a ring 5.8 feet in diameter. The two different sizes were selected to demonstrate the shear capacity of an individual Stone Column and the Stone Column and surrounding soil combined, respectively.

In order to develop a plot of ultimate shear capacity as a function of normal stress, tests are performed at different vertical loads. The load-deflection curves and shear strength parameters for a 3.5-foot and 5.8-foot diameter test are illustrated in Figures 25 and 26. It should be mentioned that, prior to soil improvement, the angle of internal friction for the soft cohesive soil was 18 to 20 degrees. After installing Stone Columns, the average angle of internal friction was increased to 27 degrees.

2) Vertical Load Tests

Meaningful vertical load testing can only be accomplished when settlements are monitored over an extended period of time. The imposed loading should be uniformly applied over the entire unit cell area and not just the Stone Column. As discussed within Section V, tests which apply load solely to the Stone Column are misrepresenting actual field behavior. There is a great need for test sections in order to verify design assumptions. The additional expense required for a test section is easily justified on larger projects.

LATERAL LOAD TEST

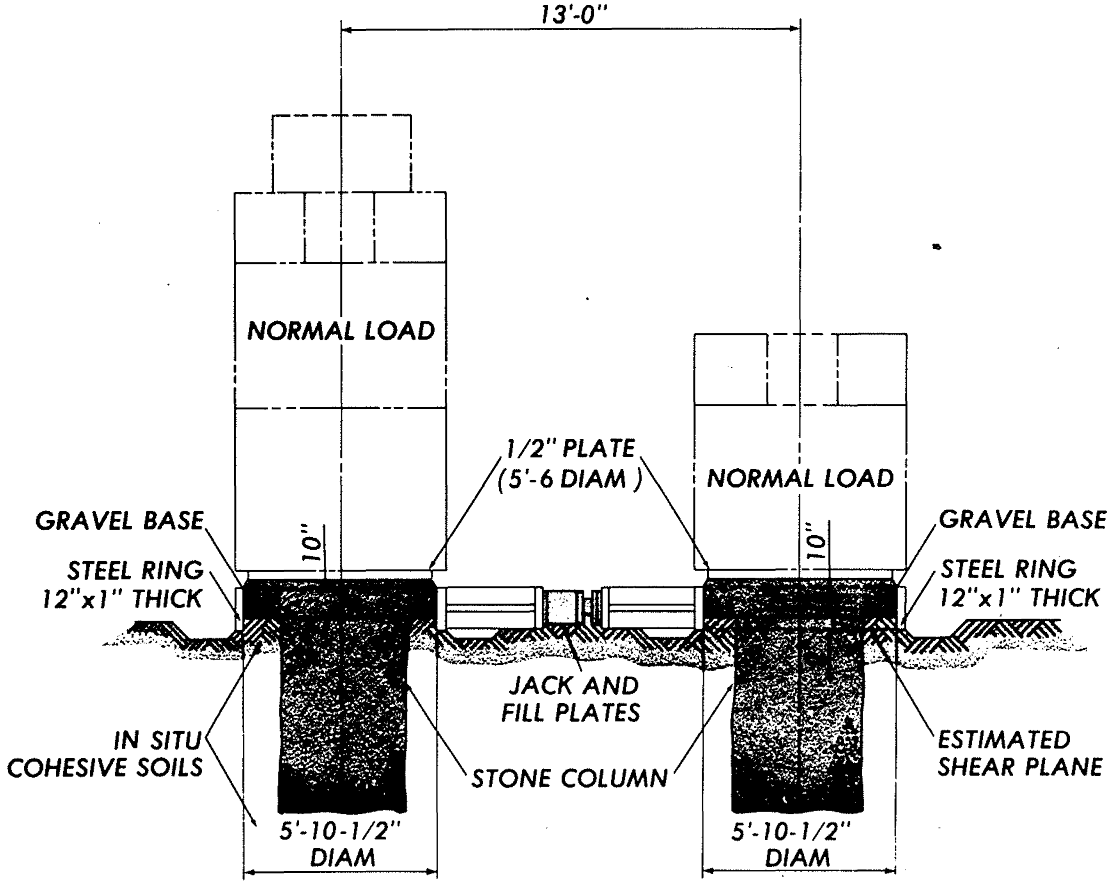


FIGURE 22

LATERAL LOAD TEST 3'-6" DIA. RING

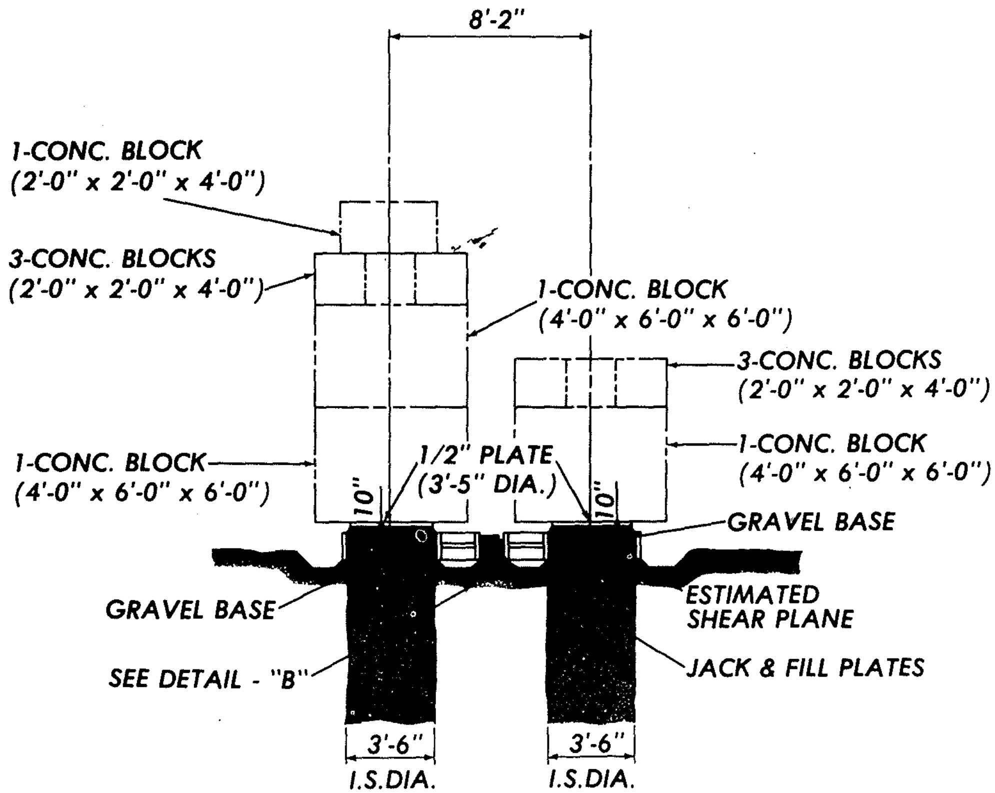


FIGURE 23

DETAIL - "B"

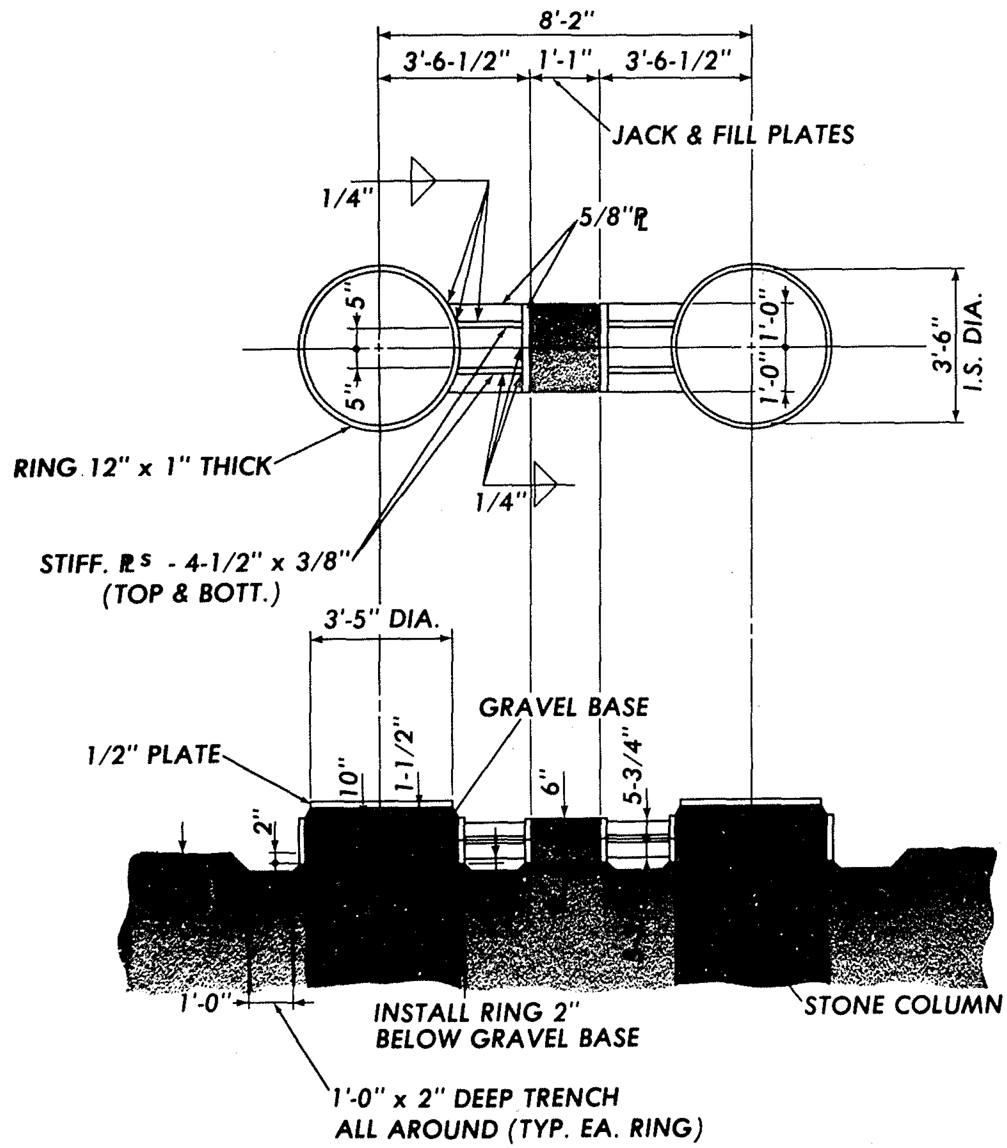


FIGURE 24

LATERAL LOAD TEST

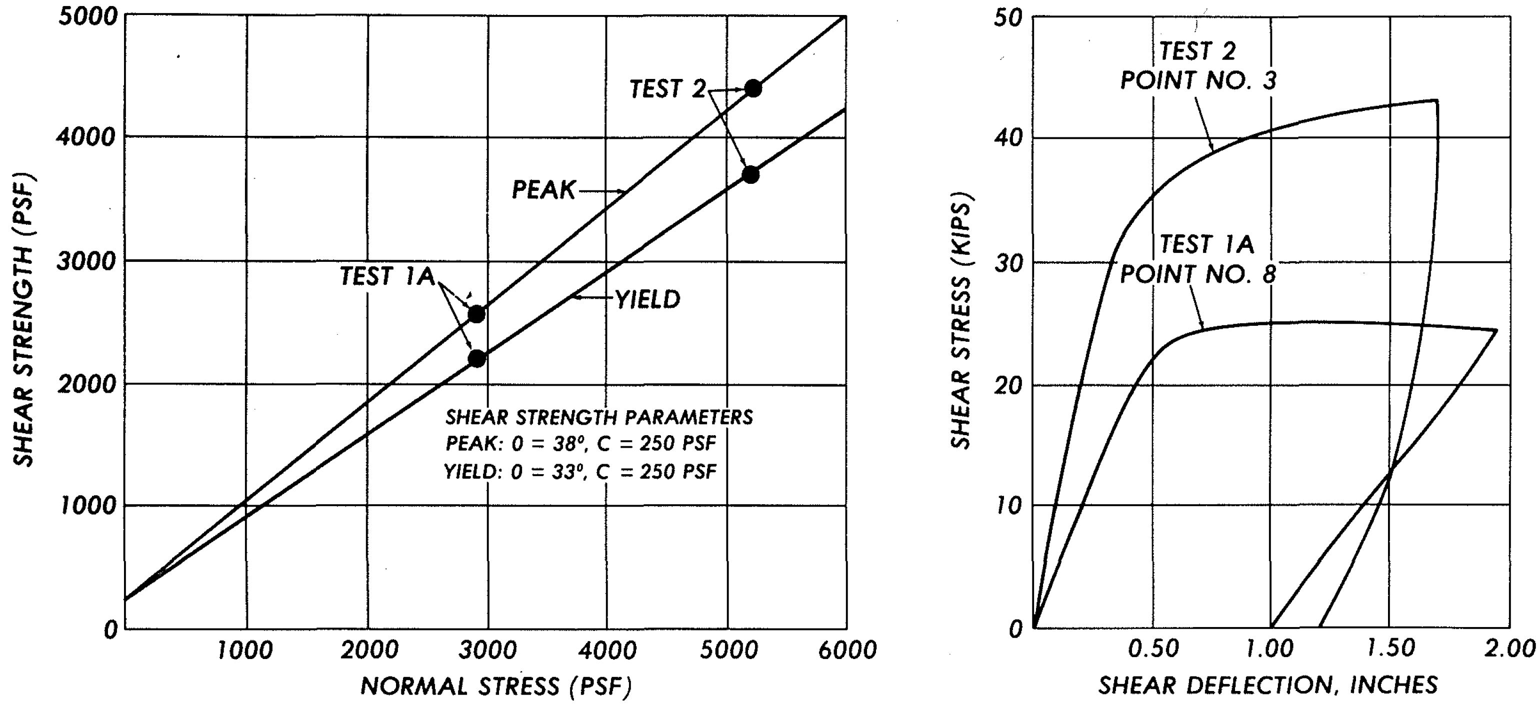


FIGURE 25

LATERAL LOAD TESTS
Ring Diam = 5.87 ft, Area = 27.0 sq ft

NOTE: PLOTTED FOR AVERAGE OF 4 DIAL READINGS

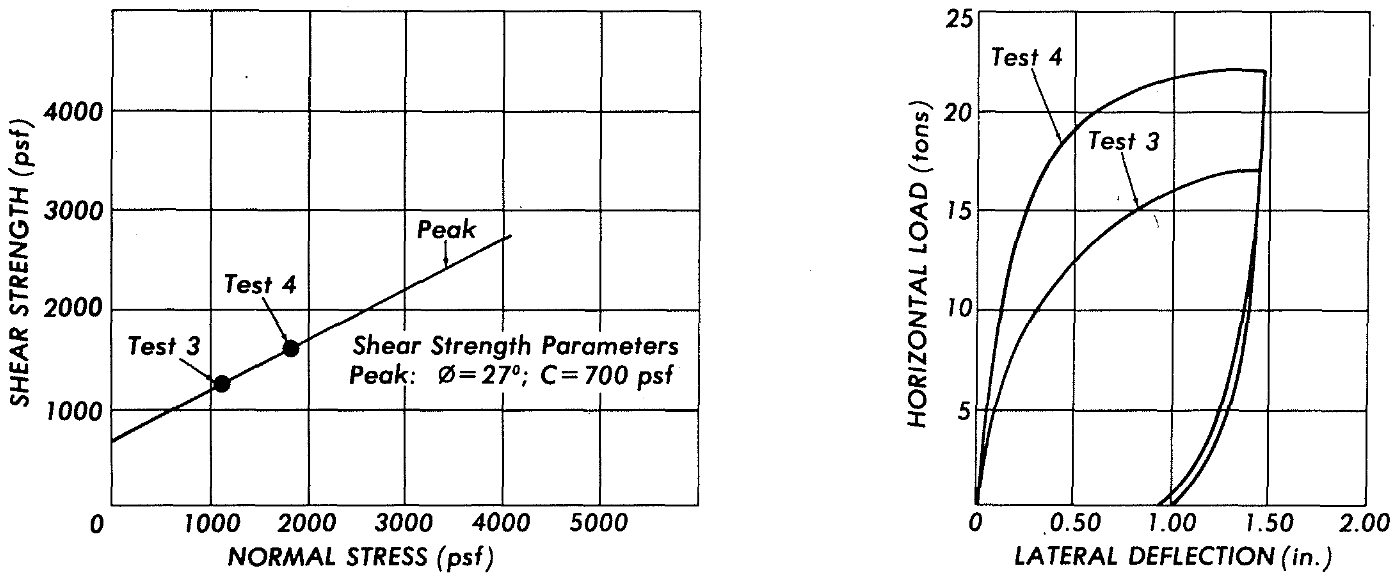


FIGURE 26

B. Instrumentation

To insure the proposed facility is performing as designed an adequate instrumentation program should be conducted at all projects. Slope indicators should be employed at locations where stability is of concern, to monitor lateral movement. The use of Stone Columns provides some benefits toward improved settlement and rate of settlement characteristics, but only limited documentation of these benefits exists. It is important to measure settlements and monitor pore pressure, both during and after construction, in order to better document these improvements.

VIII. BACKFILL REQUIREMENTS AND WORKING PLATFORM

This section provides designers with some insight regarding the requirements and functions of the backfill material and working platform used in column construction.

A. Backfill Requirements

1) Hardness and Durability

The coarse aggregate used must not be susceptible to deterioration under intense vibrational forces imposed during the construction process. In addition, backfill material should pass soundness tests to insure long-term frictional support under seasonal environmental changes. Both of these requirements may be satisfied by applying local pertinent specifications related to coarse concrete aggregates.

2) Angularity

The intergranular frictional resistance, within the central core is an important parameter in the Stone Column concept. It is, therefore, important to maximize shearing resistance by using highly angular backfill material whenever possible. The angularity will have an effect on the core's modulus of elasticity and anticipated settlement under loading conditions.

3) Gradation

The general specification for backfill gradation requires a properly graded material, which has 100 percent passing the 3.0-inch sieve and not more than 10 percent passing 0.5-inch sieve. Gradation variation within these upper and lower limits may be changed according to local specifications and individual project requirements. Designers should consider the following backfill material functions when making final gradation selection:

First - The backfill provides support for the imposed vertical load; it is, therefore, important to maximize its shear strength parameters.

Second - The presence of this highly permeable material assists in the dissipation of pore pressures which are generated within the in situ material under load.

4) <u>General</u>

Project designers may use .50 tons per linear foot of column as an estimate of the backfill quantity required for column construction. The above discussions stress the importance of developing high frictional resistance within the column. It is equally important for designers to perform cost comparisons that examine economic benefits that might be gained from using local material rather than importing higher quality materials from longer distances. Use of local material may require more columns per treated area but in turn, may be more economical in the final analysis.

B. <u>Working Platform</u>

Prior to beginning construction a working platform must be placed over the area to be treated. This platform is usually 1 to 3 feet in thickness and should be composed of granular, free draining material. The platform performs the following functions:

1. Permits heavy construction equipment to move across the area more easily.

2. It is an important factor in the transfer of load from the structure to the Stone Column.

3. Assists in the consolidation process by providing a drainage path for pore pressure dissipation.

4. Assists in the rapid removal of the jetting fluid during the penetrating process and filters this fluid before it is returned to adjacent waterways.

IX. SPECIFICATIONS AND PROCESS LIMITATIONS

Specifications

A. Description

This work shall consist of subsurface soil reinforcement by Stone Columns in accordance with these specifications and in reasonably close conformity with the lines, grades, design, and dimensions shown on the plans or established by the engineer.

Soil reinforcement is performed by constructing compacted Stone Columns within the in situ soil or soils.

The contractor will furnish all supervision, labor, equipment, materials and services necessary to perform all subsurface soil reinforcement work and field tests related thereto as described hereinafter.

B. Construction Requirements

Construction of the Stone Columns shall be in accordance with the details shown on the plans or elsewhere in the contract. The location and length of each compacted Stone Column shall be as shown on the plans or as determined by the engineer.

Stone Columns shall be installed by jetted vibratory probes capable of: 1) creating a hole by penetrating the in situ soils to the specified elevations, and 2) compacting the well-graded backfill stone which has been incrementally added into the hole and forcing the stone radially into the surrounding in situ soils.

The placing and compaction of the stone shall be performed in such increments and with such compactive energy that each completed Stone Column will be continuous throughout its length and will have an average effective diameter of Number feet.

Aggregate used in Stone Columns shall be 100 percent minus 3.0-inch material and not contain more than 10 percent passing 0.5-inch sieve. This material shall be well-graded and meet all abrasive and/or durability requirements which are generally utilized when specifying coarse concrete aggregates. A further discussion on backfill requirements can be found in the section entitled, "Backfill Requirements and Working Platform."

C. Field Tests

Prior to the start of project work, (Number) Test Stone Columns shall be installed at locations determined by the engineer for the purpose of establishing quality control procedures for the project work.

For purposes of verifying design assumptions, the engineer will select (Number) Test Stone Columns on which (Type) load tests are to be performed. It shall be the responsibility of the contractor to submit to the engineer for his approval, shop drawings and procedures for the load tests.

D. Method of Measurement

The quantity of Stone Columns to be paid for will be the actual number of linear feet of columns in place, completed and accepted, including test Stone Columns. Measurements will be made from the bottom of each column to the top of the working surface. Measurements will be to the nearest one-half (0.5) foot.

The quantity of backfill to be paid for will be the actual number of tons (dry unit weight basis) placed in the Stone Columns completed and accepted.

E. Basis of Payment

The quantities, determined as provided above, will be paid for at the contract price per unit of measurement, respectively, for each of the particular pay items listed below and shown in the bid schedule, which prices and payment will be full compensation for the work prescribed in this section.

Stone Column	-	per linear foot
Backfill for Stone Column	-	per ton
Load Test	-	per each

F. <u>Special Notes</u>

1) Each bidder shall submit with his bid a statement of his work experience similar to that proposed. This statement shall include the dates between which the contract was enforced, the extent of the work and the manner of its execution, and any other information tending to prove his ability to prosecute vigorously the work required by these specifications.

Any bidder that fails to demonstrate satisfactory experience or ability to construct Stone Columns as specified in this contract, will be disqualified and his bid rejected.

2) Each bidder shall include, within the above statement of work experience, a description of a suitable means for the evaluation of consistent compaction efforts for all columns. Any bid which does not contain such a description, will be rejected.

<u>Process Limitations</u>

In organic soils, the state of decomposition has to be considered as well as the details of stratification. Intermittent layers of peat 5.0 to 7.0 feet in thickness or less have been removed by attaching cutting edges to the vibrator and floating the material to the surface. For thicker organic deposits a conservative design may be necessary. If the organic deposit is such that adequate restraint to support the column cannot be provided, complete replacement of the deposit may occur during column construction.

Experience in sensitive soils is limited to clays having sensitivities not exceeding five and remolded shear strengths not less than 150 p.s.f. Higher sensitive soils might be easy to replace by liquifying the soil with the vibrator and dumping stones into the remolded area, however, the extent of the disturbance can only be determined in the field. Under these conditions there is the possibility of the vibrator decreasing the shear strength to such an extent that a stable working platform cannot be maintained or adequate lateral support to construct a column will not be provided.

At locations where thick organic deposits and/or sensitive clays exist, the economic feasibility of using a Stone Column solution should be thoroughly examined.

X. References

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☆U.S. GOVERNMENT PRINTING OFFICE: 1978-261-264/198