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Load and Resistance Factor Design for Highway Bridges

Participant Notebook

Volume II
(Version 3.01)

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V.2

National Highway Institute

Conversion Factors English to SI			
English		SI Metric	
1 IN		25.4 mm	
1 FT		304.8 mm	
1 IN^2		645 mm^2	
1 IN^2/FT		2.12 mm^2/mm	
1 cubic Yard = 1 CY		0.764 m^3	
1 MPH = 1.467 FT/SEC		1.609 km/hr = 0.447 m/s	
1 FT/SEC		0.3048 m/s = 1.097 km/hr	
g = 32.2 FT/SEC^2		9.81 m/s^2	
1 lb (pound)		4.448 N (0.4536 kg)	
1 lb·IN (moment)		113.0 N·mm	
1 lb·FT (moment)		1356 N·mm	
1 lb/FT		1.488 kg/m	
1 lb/FT^2		4.883 kg/m^2	
1 PSF/FT		$1.57x10^{-7}$ MPa/mm = 0.157 Pa/mm	
1 FT·lb (energy)		1.356 J (joule) or N·m	
1 PSI (lb/IN^2)		$6.895x10^{-3}$ MPa (mega·pascal)	
1 PSF (lb/FT^2)		$4.79x10^{-5}$ MPa = 47.9 Pa	
1 PSF/FT		$4.79X10^{-5}$ MPa/mm	
1 PCF (lb/FT^3) (unit weight)		16.02 kg/m^3 = γ (density)	
1 PCY ($lb/yard^3$)		0.593 kg/m^3 = γ (density)	
1 KIP (kilo pounds)		4448 N = 4.448 kN	
1 K·IN	1 $K·IN^2$	$1.13x10^5$ N·mm	$2.87x10^6$ $N·mm^2$
1 K/IN		175.2 N/mm	
1 KLF (KIP/FT) or KPF		14.6 N/mm	
1 KSI (KIP/IN^2)		6.895 MPa	
1 KSF (KIP/FT^2)		0.0479 MPa	
1 KCF (KIP/FT^3) (unit weight)		$1.602x10^4$ kg/m^3 = γ (density)	
1 TSF (ton/FT^2)		0.0958 MPa	
1 TCF (ton/FT^3) (unit weight)		$3.204x10^4$ kg/m^3 = γ (density)	
Ships			
1 knot = 1.152 MPH		1.852 km/hr	
1 knot		0.514 m/s = 1.85 km/hr	
1 tonne (2.205 KIP)		1.00 metric ton (1000 kg)	

To convert 55 KIPs to newtons: 55 KIPs*4.448 kN/KIP = 244.64 = 245 kN

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LECTURE 12 - STEEL DESIGN - I

12.1 OBJECTIVE

12.2 NEW PROVISIONS IN LRFD SPECIFICATION NOT CONTAINED IN LFD SPECIFICATION

12.2.1 Deflection Limitations

Deflection requirements for other than orthotropic decks are now optional (see Article S2.5.2.6.2), as are requirements for limits on span-to-depth ratios (see Article S2.5.2.6.3).

However, flange stress controls of Articles S6.10.3, summarized below, and S6.10.11.2, which are similar in intent, are mandatory and provide limits on permanent deformation in lieu of deflection limitations. Load Combination Service II in Table S3.4.1-1 applies to this check. For compact members, the investigation of permanent deflection may be based on moment redistribution.

Flange stresses in positive and negative bending are not to exceed:

- for both steel flanges of composite sections:

$$f_f \le 0.95 R_h F_{yf} \qquad (12.2.1\text{-}1)$$

- for both flanges of non-composite sections:

$$f_f \le 0.80 R_h F_{yf} \qquad (12.2.1\text{-}2)$$

where:

R_h = flange-stress reduction factor specified in Article 12.7.3

f_f = elastic flange stress caused by the factored loading (MPa)

F_{yf} = yield stress of the flange (MPa)

A resistance factor is not applied since the specified limit is a serviceability criterion for which the resistance factor is 1.0.

These provisions are intended to apply to the design live load specified in Article S3.6.1.1. If this criterion were to be applied to a permit load situation, a reduction in the load factor for live load should be considered.

This limit state check is intended to prevent objectionable permanent deflections due to expected severe traffic loadings which would impair rideability. It corresponds to the overload check in the 1992 AASHTO Standard Specification and is merely an indicator of successful past practice, the development of which is described in Vincent (1969).

Under the load combinations specified in Table S3.4.1-1, the criterion for control of permanent deflections does not govern for composite non-compact sections, therefore, it need not be checked for those sections. This may not be the case under a different set of load combinations.

Article S6.10.11.2 provides comparable controls for beams and girders designed by inelastic procedures, such as the mechanism method or the unified autostress method.

12.2.2 Fatigue

12.2.2.1 GENERAL

The fatigue provisions of the Steel Structures Section of the AASHTO LRFD Specification for Highway Bridge Design combine aspects of both the AASHTO Standard Specification for Highway Bridges (AASHTO 1992) and the Guide Specification for Fatigue Design of Steel Bridges (AASHTO 1989).

These provisions are based upon two principles of fatigue of welded steel details:

- If all of the stress ranges that a welded steel detail experiences in its lifetime are less than the constant-amplitude fatigue threshold (i.e., the maximum stress range is less than the threshold), the detail will not experience fatigue crack growth; otherwise

- the fatigue life of the detail can be estimated considering an effective (weighted average of sorts) stress range, which represents all of the varying magnitudes of stress range experienced by the detail during its lifetime (Fisher, et al, 1983).

These two principles result in two branches in the flow of fatigue design, infinite life design and finite life design.

Further, an explicit check of whether any of the tensile live load stresses experienced by the detail overcome the compressive dead load stresses is specified in Article S6.6.1.2.1 prior to the consideration of the fatigue provisions.

Fatigue details for bridges with higher truck traffic volumes are designed for infinite life. This practice is carried over from both the Standard Specifications and the Guide Specifications.

Bridges with lower truck traffic volumes are designed for the fatigue life required by the estimated site-specific traffic volumes projected for their lifetimes.

12.2.2.2 FATIGUE LOAD

On the load side of the LRFD equation, the specified load condition for fatigue is a single truck, the current HS20 truck with a fixed rear axle spacing of 9.0 m. The load effects (e.g., moments, shear, etc.) caused by this truck are of lesser magnitudes than those caused by the combination of a truck and a uniformly distributed load applied for strength design. Further, the truck occupies a single lane on the bridge at a time, not multiple lanes as in strength design.

This fatigue load produces a lower calculated stress range than that of the Standard Specifications. This reduction in calculated stress range is offset by an increase in the number of cycles to be considered. The number of cycles to be considered is the number of cycles due to the trucks anticipated to cross the bridge in the most heavily traveled lane during its design life, as calculated by Equation 12.2.2.2-1. This will be a large number of cycles compared to the design cycles in the 15th Edition of AASHTO. Both the lower stress range and the increased number of cycles are more reflective of the actual conditions experienced by the bridge. In this consideration, these fatigue provisions resemble the Guide Specifications.

$$N = (365)\,(75)\,n\,(ADTT)_{SL} \qquad (12.2.2.2\text{-}1)$$

where:

n = number of stress range cycles per truck passage given in Table 12.2.2.2-1

$(ADTT)_{SL}$ = single-lane ADTT

Table 12.2.2.2-1 - Cycles per Truck Passage, n

Longitudinal Members	Span Length	
	> 12 000 mm	≤ 12 000 mm
Simple-Span Girders	1.0	2.0
Continuous Girders		
1) near interior support	1.5	2.0
2) elsewhere	1.0	2.0
Cantilever Girders	5.0	
Trusses	1.0	
Transverse Members	Spacing	
	> 6000 mm	≤ 6000 mm
	1.0	2.0

In the absence of better information, the single-lane average daily truck traffic shall be taken as:

$$ADTT_{SL} = p \times ADTT \qquad (12.2.2.2\text{-}2)$$

where:

ADTT = the number of trucks per day in one direction averaged over the design life

$ADTT_{SL}$ = the number of trucks per day in a single-lane averaged over the design life

p = taken as specified in Table 12.2.2.2-2

Table 12.2.2.2-2 - Fraction of Truck Traffic in a Single Lane, p

Number of Lanes Available to Trucks	p
1	1.00
2	0.85
3 or more	0.80

The single-lane ADTT is that for the traffic lane in which the majority of the truck traffic crosses the bridge. On a typical bridge with no nearby entrance/exit ramps, the shoulder lane carries most of the truck traffic.

Since future traffic patterns on the bridge are uncertain, the frequency of the fatigue load for a single lane is assumed to apply to all lanes.

Research has shown that the Average Daily Traffic, ADT, including all vehicles, i.e., cars and trucks, is physically limited to about 20 000 vehicles per lane per day under normal conditions. This limiting value of traffic should be considered when estimating the ADTT. The ADTT can be determined from the ADT by multiplying by the fraction of trucks in the traffic. In lieu of site-specific fraction of truck traffic data, the values of Table 12.2.2.2-3 may be applied for routine bridges.

Table 12.2.2.2-3 - Fraction of Trucks in Traffic

Class of Highway	Fraction of Trucks in Traffic
Rural Interstate	0.20
Urban Interstate	0.15
Other Rural	0.15
Other Urban	0.10

In the LRFD Specification, 75% of the stress range due to the passage of the fatigue load (the current HS20 truck with a fixed rear axle spacing) is considered representative of the effective stress range. One-hundred-fifty percent of the stress range of the fatigue load is considered representative of the

maximum stress range. Only the 75% is specified in the load combinations table of the Loads and Load Factors Section of the Specification. The 150% is accounted for by modifying the resistance side of the equation.

12.2.2.3 FATIGUE RESISTANCE

On the resistance side of the equation, while the provisions of the LRFD Specification appear quite different than those of the Standard Specifications, they are little changed in terms of effect.

The nominal fatigue resistance is taken as:

$$(\Delta F)_n = \left(\frac{A}{N}\right)^{\frac{1}{3}} \geq \frac{1}{2}(\Delta F)_{TH} \qquad (12.2.2.3\text{-}1)$$

where:

A = constant taken from Table 12.2.2.3-1

$(\Delta F)_{TH}$ = constant-amplitude fatigue threshold taken from Table 12.2.2.3-1

Table 12.2.2.3-1 - Detail Category Constant, A, and Thresholds

DETAIL CATEGORY	CONSTANT, A TIMES 10^{11} (MPa^3)	THRESHOLD (MPa)
A	82.0	165
B	39.3	110
B′	20.0	82.7
C	14.4	69.0
C′	14.4	82.7
D	7.21	48.3
E	3.61	31.0
E′	1.28	17.9
M164 (A325M) Bolts in Axial Tension	5.61	214
M253 (A490M) Bolts in Axial Tension	10.3	262

If the maximum stress range is less than the constant-amplitude fatigue threshold, then all of the stress ranges experienced by the detail are less than the threshold and fatigue crack growth is not expected. In this case, the detail is said to have infinite life. The maximum stress range is assumed to be twice the effective stress range. Therefore, since Equation 12.2.2.3-1 is a check on the effective stress range, the constant-amplitude threshold in the equation is divided by two. The constant-amplitude fatigue threshold of the LRFD Specification is the allowable stress range for over two million cycles of the Standard Specifications.

If the maximum stress range is greater than the constant-amplitude fatigue threshold, fatigue crack growth is expected and the detail must be checked to insure that the necessary finite life can be achieved. Since an infinite number of different finite fatigue lives or numbers of design cycles are now possible, the tables of limited numbers of allowable stress ranges based upon the several numbers of design cycles (i.e., 100 000; 500 000; 2 000 000 and over 2 000 000 cycles) have been replaced by equations for each design category. These equations are represented by Equation 12.2.2.3-1 above and its accompanying tables. The resistances represented by these equations are, however, identical to those of the tables of the Standard Specifications as shown below.

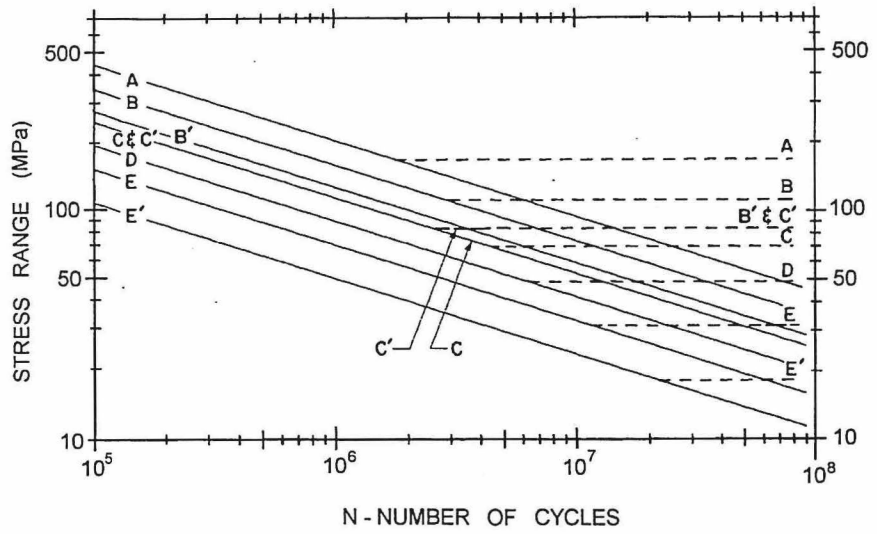


Figure 12.2.2.3-1 - Stress Range Versus Number of Cycles

In the past, the decision was made to design bridges with higher daily truck traffic volumes to insure infinite life. In the Standard Specifications, infinite life for such bridges is mandated by footnote "c" to Table 10.3.2A. This decision was partially based upon the notion that high daily traffic volumes

are difficult to predict accurately. Twenty-five hundred trucks per day was arbitrarily chosen to represent the volume beyond which infinite life is desirable.

The philosophy of infinite life for higher truck traffic volume bridges will be maintained in the LRFD Specification.

12.2.2.4 SPECIAL REQUIREMENTS FOR GIRDER WEBS

New provisions are provided to control out-of-plane flexing and fatigue cracking of the web under repeated live loading.

Webs without longitudinal stiffeners shall satisfy the following requirements:

- If $\frac{2D_c}{t_w} \le 5.76\sqrt{\frac{E}{F_{yc}}}$, then:

$$f_{cf} \le R_h F_y \qquad (12.2.2.4\text{-}1)$$

- If $5.76\sqrt{\frac{E}{F_{yc}}} < \frac{2D_c}{t_w} \le 6.43\sqrt{\frac{E}{F_{yc}}}$, then:

$$f_{cf} \le R_h F_{yc}\left[3.58 - 0.448\left(\frac{2D_c}{t_w}\right)\sqrt{\frac{F_{yc}}{E}}\right] \qquad (12.2.2.4\text{-}2)$$

- If $\frac{2D_c}{t_w} > 6.43\sqrt{\frac{E}{F_{yc}}}$, then: $f_{cf} \le 28.9 R_h E\left(\frac{t_w}{2D_c}\right)^2$

(12.2.2.4-3)

where:

f_{cf} = maximum compressive elastic flexural stress in the compression flange due to the unfactored permanent load and the fatigue loading, taken as being indicative of the maximum flexural stress in the web (MPa)

F_{yc} = yield strength of the compression flange (MPa)

D_c = depth of the web in compression (mm)

R_h = flange-stress reduction factor specified in Article 12.7.3

t_w = web thickness (mm)

The elastic bend-buckling capacity of the web given by Equation 12.2.2.4-3 is based on an elastic buckling coefficient, k, equal to 32.0. This value is approximately halfway between the theoretical k value for bending-buckling of 23.9 for simply-supported boundary conditions at the flanges and the theoretical k value of 39.6 for fixed boundary conditions at the flanges, Timoshenko and Gere (1961). This intermediate k value is used to reflect the rotational restraint offered by the flanges. Equation 12.2.2.4-2 represents a straight line estimate of the inelastic buckling strength between a bending resistance of R_hF_{yc} given by Equation 12.2.2.4-1 and $0.7R_hF_{yc}$. The specified web slenderness limit of $6.43(E/F_{yc})^{1/2}$ is derived by setting the critical buckling stress given by Equation 12.2.2.4-3 equal to $0.7R_hF_{yc}$. The specified web slenderness limit of $5.76(E/F_{yc})^{1/2}$ is equivalent to the allowable web slenderness given in the allowable stress design provisions of AASHTO (1992) for a section to reach the yield strength.

Theoretically, longitudinal stiffeners prevent bend-buckling of the web; thus, the provisions in this article do not apply to sections with longitudinally stiffened webs.

For the loading and load combination applicable to this limit state, it is assumed that the entire cross-section will remain elastic and, therefore, D_c can be calculated using superposition of stresses.

The shear force in unstiffened webs and in webs of hybrid sections is already limited to either the shear yielding or the shear buckling force at the strength limit state by provisions which are summarized in Article 12.7.6. Consequently, these requirements need not be checked for those sections.

Webs of homogeneous sections with transverse stiffeners and with or without longitudinal stiffeners shall be proportioned to satisfy:

$$v_{cf} \leq 0.58\, C\, F_{yw} \qquad (12.2.2.4\text{-}4)$$

where:

F_{yw} = yield strength of the web (MPa)

C = ratio of shear buckling stress to shear yield strength as specified in Article 12.7.6.2

v_{cf} = maximum elastic shear stress in the web due to the unfactored permanent load and the fatigue loading (MPa)

12.2.3 Resistance Factors

The resistance factors, ϕ, for the strength limit state are:

- for flexure . . . . . $\phi_f = 1.00$
- for shear . . . . . $\phi_v = 1.00$
- for axial compression, steel only . . . . . $\phi_c = 0.90$
- for axial compression, composite . . . . . $\phi_c = 0.90$
- for tension, fracture in net section . . . . . $\phi_u = 0.80$
- for tension, yielding in gross section . . . . $\phi_y = 0.95$
- for bearing on pins, in reamed, drilled or bolted holes and milled surfaces . . . . . $\phi_b = 1.00$
- for bolts bearing on material . . . . . $\phi_{bb} = 0.80$
- for shear connectors . . . . . $\phi_{sc} = 0.85$
- for A325M and A490M bolts in tension . $\phi_t = 0.80$
- for A307 bolts in tension . . . . . $\phi_t = 0.80$
- for A307 bolts in shear . . . . . $\phi_s = 0.65$
- for A325M and A490M bolts in shear . . . $\phi_s = 0.80$
- for block shear . . . . . $\phi_{bs} = 0.80$
- for weld metal in complete penetration welds:
  - shear on effective area . . . . . $\phi_{e1} = 0.85$
  - tension or compression normal to effective area . . . . . ϕ = base metal ϕ
  - tension or compression parallel to axis of the weld . . . . . ϕ = base metal ϕ
- for weld metal in partial penetration welds:
  - shear parallel to axis of weld . . . . $\phi_{e2} = 0.80$
  - tension or compression parallel to axis of weld . . . . . ϕ = base metal ϕ
  - compression normal to the effective area . . . . . ϕ = base metal ϕ
  - tension normal to the effective area $\phi_{e1} = 0.80$
- for weld metal in fillet welds:
  - tension or compression parallel to axis of the weld . . . . . ϕ = base metal
  - shear in throat of weld metal . . . . $\phi_{e2} = 0.80$

All resistance factors for the extreme event limit state, except for bolts, shall be taken as 1.0.

12.2.4 Diaphragm Spacing

The 7620 mm spacing limit in the current AASHTO Specification has been eliminated. A structural investigation and analysis to determine need and spacing of diaphragms is now required.

12.2.5 Pins

New requirements are provided for pins subjected to combined flexure and shear.

12.3 TENSILE RESISTANCE

The net area, A_n, of a member is the sum of the products of thickness and the smallest net width of each element. The width of each standard bolt hole shall be taken as the nominal diameter of the bolt plus 3.2 mm. The width deducted for oversize and slotted holes, where permitted in Article 6.13.2.4.1, shall be taken as 1.6 mm greater than the hole size specified in Article 6.13.2.4.2. The net width shall be determined for each chain of holes extending across the member along any transverse, diagonal, or zigzag line.

The net width for each chain shall be determined by subtracting from the width of the element the sum of the widths of all holes in the chain and adding the quantity $s^2/4g$ for each space between consecutive holes in the chain, where:

s = pitch of any two consecutive holes (mm)

g = gage of the same two holes (mm)

For angles, the gage for holes in opposite adjacent legs shall be the sum of the gages from the back of the angles less the thickness.

The development of the "$s^2/4g$" rule for estimating the effect of a chain of holes on the tensile resistance of a section is described in McGuire (1968). Although it has theoretical shortcomings, it has been used for a long time and has been found to be adequate for ordinary connections.

In designing a tension member, it is conservative and convenient to use the least net width for any chain together with the full tensile force in the member. It is sometimes possible to achieve an acceptable, slightly less conservative, design by checking each possible chain with a tensile force

obtained by subtracting the force removed by each bolt ahead of that chain, i.e., closer to mid-length of the member, from the full tensile force in the member. This approach assumes that the full force is transferred equally by all bolts at one end.

The resistance to fracture at a net section now includes a reduction factor, U, to account for shear lag.

Members and splices subjected to axial tension shall be investigated for two conditions:

- yield on the gross section, i.e., Equation 12.3-1, and
- fracture on the net section, i.e., Equation 12.3-2.

The determination of the net section requires consideration of:

- the gross area from which deductions will be made, or reduction factors applied, as appropriate
- deductions for all holes in the design cross-section,
- correction of the bolt hole deductions for the stagger rule,
- application of the reduction factor U, specified below to account for shear lag, and
- application of an 85% maximum area efficiency factor for splice plates and other splicing elements.

The factored tensile resistance, P_r, shall be taken as the lesser of the values given by Equations 12.3-1 and 12.3-2.

$$P_r = \phi_y P_{ny} = \phi_y F_y A_g \qquad (12.3\text{-}1)$$

$$P_r = \phi_u P_{nu} = \phi_u F_u A_n U \qquad (12.3\text{-}2)$$

where:

P_{ny} = nominal tensile resistance for yielding in gross section (N)

F_y = yield strength (MPa)

A_g = gross cross-sectional area of the member (mm^2)

P_{nu} = nominal tensile resistance for fracture in net section (N)

F_u = tensile strength (MPa)

A_n = net area of the member as described above (mm^2)

U = reduction factor to account for shear lag; 1.0 for components in which force effects are transmitted to all elements, and as specified below for other cases

ϕ_y = resistance factor for yielding of tension members

ϕ_u = resistance factor for fracture of tension members

The reduction factor, U, does not apply when checking yielding on the gross section because yielding tends to equalize the non-uniform tensile stresses caused over the cross-section by shear lag.

Due to strain hardening, a ductile steel loaded in axial tension can resist a force greater than the product of its gross area and its yield strength prior to fracture. However, excessive elongation due to uncontrolled yielding of gross area not only marks the limit of usefulness, it can precipitate failure of the structural system of which it is a part. Depending on the ratio of net area to gross area and the mechanical properties of the steel, the component can fracture by failure of the net area at a load smaller than that required to yield the gross area. General yielding of the gross area and fracture of the net area both constitute measures of component strength. The relative values of the resistance factors for yielding and fracture reflect the different reliability indices deemed proper for the two modes.

The part of the component occupied by the net area at fastener holes generally has a negligible length relative to the total length of the member. As a result, the strain hardening is quickly reached and, therefore, yielding of the net area at fastener holes does not constitute a strength limit of practical significance, except, perhaps, for some built-up members of unusual proportions.

For welded connections, A_n is the gross section less any access holes in the connection region.

In the absence of more refined analysis or tests, the reduction factors below may be used to account for shear lag in connections.

The reduction factor, U, for rolled I-shaped and tees cut from I-shapes, subjected to load transmitted directly to some, but not all, elements may be taken as:

- for connections with only transverse end welds:

$$U = \frac{A_{ne}}{A_{gn}} \qquad (12.3\text{-}3)$$

The reduction factor, U, for all other members subject to load transmitted to some, but not all, elements through bolted connections with three or more bolts per line in the direction of load, or welded connections, except as noted below may be taken as:

$$U = 0.85$$

The reduction factor, U, for connections with longitudinal welds along both edges of the connected part may be taken as:

If $L > 2W$, then $U = 1.0$
If $2W > L > 1.5W$, then $U = 0.87$
If $1.5W > L > W$, then $U = 0.75$

where:

A_{ne} = net area receiving the load within the connected length of the elements (mm^2)

A_{gn} = least net area of the member outside the connected length (mm^2)

W = width of connected element (mm)

L = weld length (mm)

A component subjected to tension and flexure must satisfy Equations 12.3-4 and 12.3-5.

$$If \frac{P_u}{P_r} < 0.2, \ then$$

$$\frac{P_u}{2.0\,P_r} + \left(\frac{M_{ux}}{M_{rx}} + \frac{M_{uy}}{M_{ry}}\right) \le 1.0 \qquad (12.3\text{-}4)$$

$$If \frac{P_u}{P_r} \ge 0.2, \ then$$

$$\frac{P_u}{P_r} + \frac{8.0}{9.0}\left(\frac{M_{ux}}{M_{rx}} + \frac{M_{uy}}{M_{ry}}\right) \le 1.0 \qquad (12.3\text{-}5)$$

where:

P_r = factored tensile resistance (N)

M_{rx}, M_{ry} = factored flexural resistances about the x and y axes, respectively (N·mm)

M_{ux}, M_{uy} = moments about the x and y axes, respectively, resulting from factored loads (N·mm)

P_u = axial force effect resulting from factored loads (N)

Interaction equations in tension and compression members are a design simplification. Such equations involving exponents of 1.0 on the moment ratios are usually conservative. More exact, nonlinear interaction curves are also available and are discussed in Galambos (1988). If these interaction equations are used, additional investigation of service limit state stresses is necessary to avoid premature yielding.

The stability of a flange subjected to a net compressive stress due to the tension and flexure shall be investigated for local buckling.

12.4 COMPRESSIVE RESISTANCE

The axial compressive load, P_u, and concurrent moments, M_{ux} and M_{uy}, calculated for the factored loadings by elastic analytical procedures must satisfy the following relationships:

$$If \frac{P_u}{P_r} < 0.2, then$$

$$\frac{P_u}{2.0P_r} + \left(\frac{M_{ux}}{M_{rx}} + \frac{M_{uy}}{M_{ry}}\right) \le 1.0$$

(12.4-1)

$$If \frac{P_u}{P_r} \ge 0.2, then$$

$$\frac{P_u}{P_r} + \frac{8.0}{9.0}\left(\frac{M_{ux}}{M_{rx}} + \frac{M_{uy}}{M_{ry}}\right) \le 1.0$$

(12.4-2)

P_r = factored compressive resistance (N)

M_{rx} = factored flexural resistance about the x axis (N·mm)

M_{ry} = factored flexural resistance about the y axis (N·mm)

M_{ux} = factored flexural moment about the x axis calculated as specified below (N·mm)

M_{uy} = factored flexural moment about the y axis calculated as specified below (N·mm)

M_{ux} and M_{uy}, moments about axes of symmetry, may be determined by either:

- a second order elastic analysis that accounts for the magnification of moment caused by the factored axial load, or
- the approximate single step adjustment specified in Article S4.5.3.2.2b.

These equations are identical to the provisions in the AISC LRFD Specification (1986). They were selected for use in that Specification after comparing it and a number of alternative formulations with the results of refined inelastic analyses of 82 frame sidesway cases, Kanchanalai (1977). P_u, M_{ux} and M_{uy} are simultaneous axial and flexural forces on cross-sections determined by analysis under factored loads. The maximum calculated moment in the member in each direction should be considered, including the second order effects.

Where maxima occur on different cross-sections, each should be checked.

12.5 NON-COMPOSITE COMPRESSION MEMBERS

New provisions for compressive resistance, adopted from AISC LRFD Specification, are contained in Article S6.9.4.1.

For members that satisfy the width/thickness requirements specified in Table 12.5-1, the nominal compressive resistance, P_n, is taken as:

$$If\ \lambda \le 2.25,\ then\ P_n = 0.66^{\lambda} F_y A_s \qquad (12.5\text{-}1)$$

$$If\ \lambda > 2.25,\ then\ P_n = \frac{0.88 F_y A_s}{\lambda} \qquad (12.5\text{-}2)$$

for which:

$$\lambda = \left(\frac{K\ell}{r_s \pi}\right)^2 \frac{F_y}{E}$$

where:

A_s = gross cross-sectional area (mm^2)

F_y = yield strength (MPa)

E = modulus of elasticity (MPa)

K = effective length factor

ℓ = unbraced length (mm)

r_s = radius of gyration about the plane of buckling (mm)

For steel piles under axial load only, P_n shall not exceed the following:

- for H-Piles . . . . . $0.78 F_y A_s$
- for pipe piles . . . . . $0.87 F_y A_s$

The limits on P_n for steel piles under only axial load are intended to account for unintended eccentricities. The use of

these reduced axial capacities is consistent with the resistance factors specified in Section S12.

These equations are identical to the column design equations of the AISC LRFD Specification (1986). Both are essentially the same as column strength curve 2P of SSRC (1988). They incorporate an out-of-straightness criterion of L/1500. The development of the mathematical form of these equations is described in Tide (1985), and the structural reliability they are intended to provide is discussed in SSRC (1988).

Width/Thickness ratios for plates, tubes and built-up members are to satisfy:

$$\frac{b}{t} \le k\sqrt{\frac{E}{F_y}} \qquad (12.5\text{-}3)$$

where:

k = plate buckling coefficient as specified in Table 12.5-1

b = width of plate as specified in Table 12.5-1 (mm)

t = plate thickness (mm)

Wall thickness of tubes shall satisfy:

- for circular tubes: $\frac{D}{t} \le 2.8\sqrt{\frac{E}{F_y}}$ (12.5-4)

- for rectangular tubes: $\frac{b}{t} \le 1.7\sqrt{\frac{E}{F_y}}$ (12.5-5)

where:

D = diameter of tube (mm)

b = width of face (mm)

t = thickness of tube (mm)

Table 12.5-1 - Limiting Width-Thickness Ratios

Plates Supported Along One Edge	k	b
Flanges and Projecting Legs or Plates	0.56	• Half-flange width of I-sections
		• Full-flange width of channels
		• Distance between free edge and first line of bolts or welds in plates
		• Full-width of an outstanding leg for pairs of angles in continuous contact
Stems of Rolled Tees	0.75	• Full-depth of tee
Other Projecting Elements	0.45	• Full-width of outstanding leg for single angle strut or double angle strut with separator
		• Full projecting width for others
Plates Supported Along Two Edges	**k**	**b**
Box Flanges and Cover Plates	1.40	• Clear distance between webs minus inside corner radius on each side for box flanges
		• Distance between lines of welds or bolts for flange cover plates
Webs and Other Plate Elements	1.49	• Clear distance between flanges minus fillet radii for webs of rolled beams
		• Clear distance between edge supports for all others
Perforated Cover Plates	1.86	• Clear distance between edge supports

12.6 COMPOSITE COMPRESSION MEMBERS

New provisions for axially loaded composite columns are contained in Article S6.9.5.1. The provisions of Article S6.12.2.3 apply to composite columns in flexure. These types of members are used infrequently in bridges and, hence, are not covered here in detail.

12.7 I-SECTIONS IN FLEXURE

12.7.1 General

These provisions apply to flexure of rolled or fabricated straight steel I-sections, symmetrical about the vertical axis in the plane of the web.

Hybrid sections consisting of a web with a specified minimum yield strength lower than one or both flanges may be designed under these specifications. Sections with a higher strength steel in the web than in the flanges are permitted, but should not be considered hybrid sections.

The provisions of these articles apply to:

- composite and non-composite compact sections,
- composite and non-composite non-compact sections.

Flexural members are to be designed for:

- the strength limit state,
- the service limit state control of permanent deflections summarized in Article 12.2.1,
- the fatigue and fracture limit state requirements for details and the fatigue requirements for webs as summarized in Article 12.2.2,
- constructibility as specified in Article S6.10.10.

Provisions now require that, generally, the yield strength of steel may not exceed 485 MPa or, where plastic capacity is used, 345 MPa. For steels exceeding 345 MPa, moment redistribution provisions of Article S6.10.2.2 and inelastic analysis procedures of Article S6.10.11 are not permitted.

New limits on weak axis moment of inertia of the compression flange of a steel section are provided. Flexural components shall be proportioned such that:

$$0.1 \leq \frac{I_{yc}}{I_y} \leq 0.9 \qquad (12.7.1\text{-}1)$$

where:

I_y = moment of inertia of the steel section about the vertical axis in the plane of the web (mm^4)

I_{yc} = moment of inertia of the compression flange of the steel section about the vertical axis in the plane of the web (mm^4)

The ratio of I_{yc}/I_y determines the location of the shear center of a singly symmetric section. Girders with ratios outside of the limits specified behave more like a "T" section with the shear center located at the intersection of the larger flange and the web.

Provisions for web slenderness, compression flange slenderness and compression flange bracing requirements are essentially the same as contained in the AASHTO Load Factor Design Specification.

Web slenderness ratios for compact-composite members are now based on the depth of the web in compression at the plastic moment rather than the total web depth.

The yield moment, M_y, of a composite section is needed only for the strength limit state investigation of the following types of composite sections:

- Compact positive bending sections in continuous spans - Article S6.10.5.2.2a.
- Negative bending sections - designed by the alternative formula Article S6.10.5.6.2.
- Hybrid negative bending sections for which the neutral axis is more than 10% of the web depth from mid-depth of the web - Article S6.10.5.4.1c.
- Compact homogeneous sections with stiffened webs subjected to combined moment and shear values exceeding specified limits - Article S6.10.7.3.3a.

- Non-compact sections used at the last plastic hinge location in inelastic designs - Article S6.10.11.1.2c.

The yield moment, M_y, of a composite section is determined as the sum of the moments applied separately to the steel, short-term and long-term composite sections to cause first yielding in either steel flange when any web yielding in hybrid sections is disregarded.

A procedure for calculating the yield moment is presented in Appendix A of the LRFD Specification.

M_p shall be calculated as the first moment of plastic forces about the plastic neutral axis. Plastic forces in steel portions of a cross-section shall be calculated using the yield strengths of the flanges, the web and reinforcing steel, as appropriate. Plastic forces in concrete portions of the cross-section which are in compression may be based on the rectangular stress block as specified in Article S5.7.2.2. Concrete in tension shall be neglected.

The position of the plastic neutral axis is determined by the equilibrium condition that there is no net axial force. The plastic moment of a composite section in positive flexure can be calculated by the following procedure:

- Calculate the element forces and use them to determine whether the plastic neutral axis is in the web, top flange, or slab,
- Calculate the location of the plastic neutral axis within the element determined in the first step,
- Sum moments about the plastic neutral axis to calculate M_p.

Equations for the five cases most likely to occur in practice are given in Appendix A of the LRFD Specification.

The forces in the longitudinal reinforcement may be conservatively neglected. To do this, set P_{rb} and P_{rt} equal to 0 in the equations in Appendix A of the LRFD Specification.

The plastic moment of a composite section in negative flexure can be calculated by an analogous procedure.

The yield moment, M_y, of a non-composite section is determined as the moment required to cause first yielding in

either flange when any web yielding in hybrid sections is disregarded.

The plastic moment, M_p, of a non-composite section is determined as the resultant moment of the fully plastic stress distribution acting on the section.

The plastic moment of non-composite sections may be calculated by eliminating the terms pertaining to the concrete slab and longitudinal reinforcement from the equations in the Specification, Appendix A, for composite sections.

The depth of web in compression at the plastic moment shall be determined as:

If: $F_{yw}A_w \geq |F_{yc}A_c - F_{yt}A_t|$, then

$$D_{cp} = \frac{D}{2A_wF_{yw}}(F_{yt}A_t + F_{yw}A_w - F_{yc}A_c) \qquad (12.7.1\text{-}2)$$

otherwise:

$$D_{cp} = D \qquad (12.7.1\text{-}3)$$

where:

D = web depth (mm)

A_t = area of the tension flange (mm^2)

A_c = area of the compression flange (mm^2)

A_w = area of the web (mm^2)

F_{yt} = specified minimum yield strength of the tension flange (MPa)

F_{yc} = specified minimum yield strength of the compression flange (MPa)

F_{yw} = specified minimum yield strength of the web (MPa)

If the inequality is satisfied, the neutral axis is in the web. If it is not, the neutral axis is in the flange and D_{cp} is equal to the depth of the web.

For continuous spans with compact sections in positive moment areas and non-compact sections at interior supports, new restrictions on the positive moment resistance are contained in Article S6.10.5.2.2a and are summarized below.

12.7.2 Compact Composite Sections

The design provisions for compact composite sections are summarized in Table 12.7.2-1.

Compact section provisions are not applicable to steel with F_y greater than 345 MPa since their inelastic rotation capacity is not fully established. For steels with F_y greater than 345 MPa, the provisions for non-compact sections are applicable.

Table 12.7.2-1 - Strength Limit State - Compact Composite I-Sections

	Positive Flexure	Negative Flexure
Nominal Flexural Resistance	For simple spans and continuous spans with compact interior support sections: ● If $D_p \leq D'$, then: $M_n = M_p$ ● If $D' < D_p \leq 5D'$, then: $M_n = \frac{5M_p - 0.85M_y}{4} + \frac{0.85M_y - M_p}{4}\left(\frac{D_p}{D'}\right)$ For continuous spans with non-compact interior support sections: M_n is as defined above, but not taken greater than $1.3\ R_h\ M_y$, or the value from Equation 6.10.5.2.2a-4 if that equation is used. Sections must satisfy the ductility requirement in Article 6.10.5.2.2b.	$M_n = M_p$
Web Slenderness	$\frac{2D_{cp}}{t_w} \leq 3.76\sqrt{\frac{E}{F_{yc}}}$ If $2D_{cp}/t_w$ is not met, the section is non-compact, go to Article S6.10.5.3.2. See also Article S6.10.10.2.2	$\frac{2D_{cp}}{t_w} \leq 3.76\sqrt{\frac{E}{F_{yc}}}$ If $2D_{cp}/t_w$ is not met, the section is non-compact, go to Article S6.10.5.3.3 or Article S6.10.5.6

Compression Flange Slenderness	No requirement at strength limit state	$\frac{b_f}{2t_f} \le 0.382\sqrt{\frac{E}{F_{yc}}}$ If $b_f/2t_f$ is not met, the section is non-compact, go to Article S6.10.5.3.3 or Article S6.10.5.6
Compression Flange Bracing	No requirement at strength limit state	$L_b \le \left[0.124 - 0.0759\left(\frac{M_l}{M_p}\right)\right]\left[\frac{r_y E}{F_{yc}}\right]$ If L_b is not met, the section is non-compact, go to Article S6.10.5.3.3

Some further explanation of the requirements for positive flexure is needed. For simple spans and continuous spans with compact interior support sections, the nominal positive flexural resistance is:

- If $D_p \leq D'$, then:

$$M_n = M_p \qquad (12.7.2\text{-}1)$$

- If $D' < D_p \leq 5D'$, then:

$$M_n = \frac{5M_p - 0.85M_y}{4} + \frac{0.85M_y - M_p}{4}\left(\frac{D_p}{D'}\right) \qquad (12.7.2\text{-}2)$$

where:

β = 0.90 where F_y = 250 MPa
= 0.70 where F_y = 345 MPa

D_p = distance from the top of the slab to the neutral axis at the plastic moment (mm)

D' = depth at which a composite section reaches its theoretical plastic moment capacity when the maximum strain in the concrete slab is at its theoretical crushing strain (mm) which are to be approximated as $\beta\,(d + t_s + t_h)/7.5$

M_y = moment capacity at first yield of the long-term composite positive moment section, N·mm

For the purpose of the calculation of M_n using Equation 2, M_y may be taken as product of the long-term composite section modulus with respect to the tension flange and the yield strength of the steel.

For continuous spans with non-compact interior support sections, the nominal flexural resistance may be determined by either Method "A" or Method "B", specified herein, but shall not be taken greater than the applicable value of M_n computed from either Equation 1 or Equation 2.

- Method "A" - Approximate Analysis:

$$M_n = 1.3R_hM_y \qquad (12.7.2\text{-}3)$$

- Method "B" - More Refined Analysis:

$$M_n = R_hM_y + A\,|M_{np} - M_{cp}| \qquad (12.7.2\text{-}4)$$

for which:

M_{np} - M_{cp} for interior spans shall be taken as the smaller value at either end of the span.

where:

M_{np} = nominal flexural resistance at an interior support (N·mm)

M_{cp} = moment due to the factored loadings at an interior support concurrent with the maximum positive flexural moment at the cross-section under consideration (N·mm)

A = for end spans, the distance from the end support to the location of the cross-section in the span divided by the span length. For interior spans, A shall be taken as 1.0

M_y = yield moment (N·mm)

R_h = flange-stress reduction factor from Article 12.7.3

The ductility requirement herein is equivalent to the requirement given in the AASHTO (1995). The ratio of D_p to D' is limited to a value of 5.0 to ensure that the tension flange of the steel section reaches strain hardening prior to crushing of the concrete slab. D' is defined as the depth at which the composite section reaches its theoretical plastic moment capacity M_p when the maximum strain in the concrete slab is at its theoretical crushing strain. The term $(d + t_s + t_h)/7.5$ in the definition of D', hereafter referred to as D*, was derived by assuming that the concrete slab is at the theoretical crushing strain of 0.3% and the tension flange is at the assumed strain-hardening strain of 1.2%.

From the results of a parametric analytical study of over 400 different composite steel sections, including unsymmetrical, as well as symmetrical steel sections. It was determined that sections utilizing 250 MPa steel reached M_p at a ratio of D_p/D^* equal to approximately 0.9, and sections utilizing 345 MPa steel reached M_p at a ratio of D_p/D^* equal to approximately 0.7. Thus, 0.9 and 0.7 are specified as the values to use for the β factor, which is multiplied by D* to compute D' for 250 MPa and 345 MPa yield strength steels.

Compression flange slenderness and bracing should not be investigated for the strength limit state of composite sections in positive flexure.

Where Equation 12.7.2-5 is used, the concurrent positive flexural moment shall not exceed $R_h M_y$ for the factored loading which produces the maximum negative flexural moment at the adjacent support.

Alternatively, the actual effects of the positive flexural yielding can be accounted for by using the inelastic procedures specified in Article S6.10.11.1.

The shape factor (M_p/M_y) for composite sections in positive bending can be as high as 1.5. Therefore, a considerable amount of yielding is required to get from M_y to M_p, and this yielding reduces the effective stiffness of the section. In continuous spans, the reduction in stiffness can shift moment from positive flexural regions to negative flexural regions. Therefore, the actual moments in negative flexural regions may be higher than those predicted by an elastic analysis. Negative flexural sections would have to have the capacity to sustain these higher moments, unless some limits were placed on the extent of yielding of the positive moment section. This latter approach is used in the Specification.

The live loading patterns causing the maximum elastic moments in negative flexural sections are different from those causing maximum moments in positive flexural sections. When the loading pattern causing maximum positive flexural moments are applied, the concurrent negative flexural moments are usually well below the flexural resistance of the sections in those regions. Therefore, the specifications conservatively allow additional moment above M_y to be applied to positive flexural sections of continuous spans with non-compact interior support sections. Compact interior support sections have sufficient capacity to sustain the higher moments caused by the reduction in stiffness of the positive flexural region. Thus, the nominal flexural resistance of positive flexural sections in members with compact interior support sections is specified to be M_p with no additional limit.

New alternative formulas for flexural resistance given below apply to constant depth members without longitudinal stiffeners in negative flexure that exceed either the compression flange slenderness requirements specified in Articles S6.10.5.2.3c or S6.10.5.3.3c, or S6.10.6.2 or S6.10.6.3, or the web slenderness requirements specified in Article S6.10.5.2.3b or S6.10.6.2.

The nominal flexural resistance, M_n, of sections satisfying the requirements of Articles S6.10.5.6.3 for compression flange slenderness, S6.10.5.2.3d for compression flange bracing, and S6.10.5.3.2b for slenderness of webs without longitudinal stiffeners with the compression depth of web taken as D_{cp} and f_c taken as F_{yc} is taken as the lesser of:

$M_n = M_p$, or (12.7.2-5)

$$M_n = \left[1 - \left(1 - \frac{0.7}{(M_p/M_y)}\right)\left(\frac{Q_p - Q_{fl}}{Q_p - 0.7}\right)\right] M_p \qquad (12.7.2\text{-}6)$$

for which:

Q_p = 5.47(M_p/M_y)-3.13 for unsymmetrical sections

Q_p = 3.0 for symmetrical sections

If $\frac{b_f}{2t_f} \le 0.382\sqrt{\frac{E}{F_{yc}}}$, then

$$Q_{fl} = \frac{30.5}{\sqrt{\frac{2D_{cp}}{t_w}}} \qquad (12.7.2\text{-}7)$$

Otherwise:

$$Q_{fl} = \frac{4.45}{\left(\frac{b_f}{2t_f}\right)^2\sqrt{\frac{2D_{cp}}{t_w}}}\frac{E}{F_{yc}} \qquad (12.7.2\text{-}8)$$

where:

M_p = plastic moment (N·mm)

M_y = yield moment (N·mm)

F_{yc} = specified minimum yield strength of the compression flange (MPa)

The nominal flexural resistance given by Equation 12.7.2-6 is based on the inelastic buckling strength of the compression flange, and results from a fit to available experimental data. The equation considers the interaction of

the web and compression flange slenderness in the determination of the resistance of the section by using a flange buckling coefficient, $k_f = 4.92/(2D_{cp}/t_w)^{1/2}$, in computing the Q_{fl} parameter in Equation 12.7.2-8. Q_{fl} is the ratio of the buckling capacity of the flange to the yield strength of the flange. The buckling coefficient given above was based on the test results reported in Johnson (1985), as well as data from other available composite and non-composite steel beam tests. A similar buckling coefficient is given in Section B5.2 and Appendix B5.2 of AISC (1989). Equation 12.7.2-7 is specified to compute Q_{fl} if the compression flange slenderness is less than the value specified in Article S6.10.5.2.3c to effectively limit the increase in the bending resistance at a given web slenderness with a reduction in the compression flange slenderness below this value. Equation 12.7.2-7 is obtained by substituting the compression flange slenderness limit from Article S6.10.5.2.3c in Equation 12.7.2-8.

Equation 12.7.2-6 represents a linear fit of the experimental data between a flexural resistance of M_p and 0.7 M_y. The Q_p parameter, defined as the web and compression flange slenderness to reach a flexural resistance of M_p, was derived to ensure the equation yields a linear fit to the experimental data. Equation 12.7.2-6 was derived to determine the maximum flexural resistance and does not necessarily ensure a desired inelastic rotation capacity. Sections in negative flexure which are required to sustain plastic rotations may be designed according to the procedures specified in Article S6.10.11. If elastic procedures are used and Equation 12.7.2-6 is not used to determine the nominal flexural resistance, the resistance shall be determined according to the procedures specified in Article S6.10.5.3.3 for negative moment regions of non-compact composite sections.

Compression flanges of sections providing the flexural resistance above must satisfy:

$$\frac{b_f}{2t_f} \le 2.52\sqrt{\frac{E}{F_{yc}\sqrt{\frac{2D_{cp}}{t_w}}}} \qquad (12.7.2\text{-}9)$$

12.7.3 Non-Compact Composite Sections

New and expanded provisions for web and compression flange slenderness, and for compression flange bracing are contained in Article S6.10.5.3 which are summarized below.

Table 12.7.3-1 - Strength Limit State - Non-Compact Composite I-Sections

	Positive Flexure	Negative Flexure
Nominal Flexural Resistance for Each Flange	$F_n = R_b R_h F_{yf}$	
Web Slenderness • without longitudinal stiffeners • with longitudinal stiffeners	$\frac{2D_c}{t_w} \le 6.77\sqrt{\frac{E}{f_c}}$ $\frac{2D_c}{t_w} \le 11.63\sqrt{\frac{E}{f_c}}$ See also Article S6.10.10.2.2	
Compression Flange Slenderness	No requirement at strength limit state	$\frac{b_f}{2t_f} \le 1.38\sqrt{\frac{E}{f_c\sqrt{\frac{2D_c}{t_w}}}}$ If $b_f/2t_f$ not satisfied, go to Article S6.10.5.6
Compression Flange Bracing	No requirement at strength limit state	For a uniform moment: $L_b \le 1.76 r_t\sqrt{\frac{E}{F_{yc}}}$ If L_b not satisfied, go to Article S6.10.5.5

The specified web slenderness limit corresponds to the upper limit for transversely stiffened webs in the AASHTO (1992). This limit defines an upper bound below which fatigue due to excessive lateral web deflections is not a consideration, Yen and Mueller (1966), and Mueller and Yen (1968).

The specified web slenderness limit for longitudinally stiffened webs is retained from the Allowable Stress Design portion of the AASHTO (1992). Since the flange-stress reduction factor, R_b, specified in Article S6.10.5.4.2a is assumed to be 1.0 for longitudinally stiffened sections, local bend-buckling of the web needs to be prevented. Using an

elastic plate buckling coefficient, k, equal to 129.4 for longitudinally stiffened webs yields an allowable web slenderness slightly below that specified in Equation 12.7.3-2, Dubas (1948). However, flanges provide additional rotational restraint to the web so the use of the higher value is justified.

The limit on compression flange slenderness includes the effect of the web slenderness.

The limit on $b_f/2t_f$ is the largest ratio for which the flanges can be expected to reach yield, subject to various factors, before local buckling.

The compression flange bracing requirement defines the maximum unbraced length for which a composite negative flexural section can reach the specified minimum yield strength times the applicable flange stress reduction factors, under a uniform moment, before the onset of lateral torsional buckling. Under a moment gradient, sections with larger unbraced lengths can still reach the yield strength. This larger allowable unbraced length may be determined by setting Equation 12.7.3-9 equal to $R_b\ R_h\ F_{yc}$, and solving the equation for L_b using the calculated value of the moment gradient factor C_b given herein.

Hybrid Factor

The hybrid factor accounts for the nonlinear variation of stresses caused by yielding of the lower strength steel in the web of a hybrid beam. The formulas defining this factor are the same as those given in the AASHTO (1992), and are based on experimental and theoretical studies of composite and non-composite beams and girders, ASCE (1968), Schilling (1968), and Schilling and Frost (1964). The factor applies to non-compact sections in both shored and unshored construction.

For homogeneous sections, R_h shall be taken as 1.0.

For hybrid sections in which the stress in both flanges under the factored loading does not exceed the yield strength of the web, the hybrid factor R_h shall be taken as 1.0.

The reduction factor should not be applied to compact sections because the effect of the lower-strength material in the web is accounted for in calculating the plastic moment as shown below.

For positive flexural resistance of composite hybrid sections in positive flexure, the hybrid reduction factor shall be taken as:

$$R_h = 1 - \left[\frac{\beta\psi(1 - \rho)^2(3 - \psi + \rho\psi)}{6 + \beta\psi(3 - \psi)}\right] \qquad (12.7.3\text{-}1)$$

where:

ρ = F_{yw}/F_{yb}

β = A_w/A_{fb}

ψ = d_n/d

d_n = distance from outer fiber of bottom flange to neutral axis of transformed short-term composite section (mm)

d = depth of steel section (mm)

F_{yb} = specified minimum yield strength of bottom flange (MPa)

F_{yw} = specified minimum yield strength of web (MPa)

A_w = web area (mm^2)

A_{fb} = bottom flange area (mm^2)

For negative flexure, where the neutral axis of composite hybrid sections, determined as specified in Article S6.10.5.1.4a, is located within 10% of the web depth from mid-depth of the web, the hybrid factor is:

$$R_h = \frac{12 + \beta(3\rho - \rho^3)}{12 + 2\beta} \qquad (12.7.3\text{-}2)$$

where:

ρ = F_{yw}/f_{fl}

β = $2A_w/A_{tf}$

f_{fl} = lesser of either the specified minimum yield strength, or the stress due to the factored loading in either flange (MPa)

A_{tf} = total area of both steel flanges and the longitudinal reinforcement included in the section (mm^2)

For other composite hybrid sections in negative flexure, the hybrid factor shall be taken as:

where:

$$R_h = \frac{M_{yr}}{M_y} \qquad (12.7.3\text{-}3)$$

M_y = yield resistance in terms of moment, when web yielding is disregarded (N·mm)

M_{yr} = yield resistance in terms of moment, when web yielding is accounted for (N·mm)

The approximate method illustrated in Figure 12.7.3-1 may be used in determining the yield moment resistance, M_{yr}, when web yielding is accounted for. The solid line connecting F_{yf} with f_r represents the distribution of stress at M_y if web yielding is neglected. For unshored construction, this distribution can be obtained by first applying the proper permanent load to the steel section, then applying the proper permanent load and live load to the composite section, and combining the two stress distributions. The dashed lines define a triangular stress block whose moment about the neutral axis is subtracted from M_y to account for the web yielding at a lower stress than the flange. M_y may be determined as specified earlier. Thus,

$$M_{yr} \cong M_y - aP \qquad (12.7.3\text{-}4)$$

for which:

$$P \cong \frac{(F_{yf} - F_{yw})}{2} t_w h_w \qquad (12.7.3\text{-}5)$$

where:

F_{yf} = specified minimum yield strength of the bottom flange (MPa)

F_{yw} = specified minimum yield strength of the web (MPa)

t_w = web thickness (mm)

a = distance from the elastic neutral axis to the centroid of the stress block (mm)

f_r = stress in reinforcing steel at M_y (MPa)

h_w = depth of yielded web (mm)

Figure 12.7.3-1 is specifically for the case where the elastic neutral axis is above mid-depth of the web and web yielding occurs only below the neutral axis. However, the same approach can be used if web yielding occurs both above and below the neutral axis or only above the neutral axis. The moment due to each triangular stress block due to web yielding must be subtracted from M_y.

This approach is approximate because web yielding causes a small shift in the location of the neutral axis. The effect of this shift on M_{yr} is almost always small enough to be neglected. The exact value of M_{yr} can be calculated from the stress distribution by accounting for yielding, Schilling (1968).

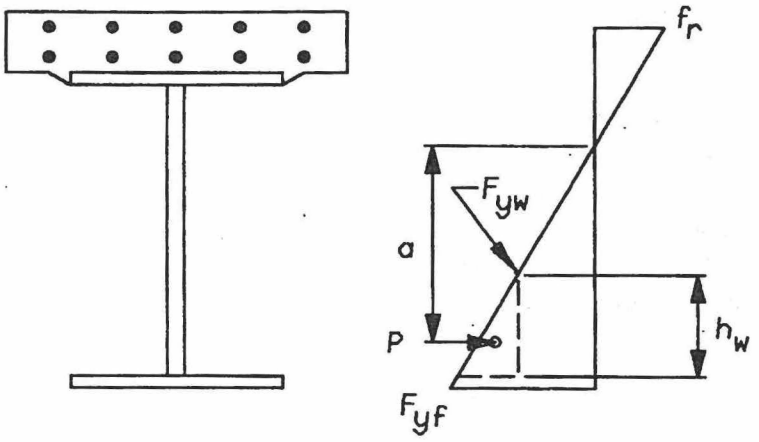


Figure 12.7.3-1 - Stress Distribution in Hybrid Girder

Load Shedding Factors, R_b for Compression Flanges

This factor accounts for the nonlinear variation of stresses caused by local buckling of slender webs subjected to flexural stresses. It does not apply if a suitable longitudinal stiffener is provided to control this local buckling, or if the web slenderness is below $\lambda_b\ (E/f_c)^{1/2}$, which corresponds to the

theoretical elastic bend-buckling stress of the web. The formula defining this factor is based on extensive experimental and theoretical studies, SSRC (1988). Equation 12.7.3-7 represents the exact formulation for the factor R_b given by Basler (1961).

The value of λ_b reflects different assumptions of support provided to the web by the flanges. The value for members with a compression flange area less than the tension flange area is based on the theoretical elastic bend-buckling coefficient, k, of 23.9 for simply-supported boundary conditions at the flanges. The value for members with a compression flange area equal to or greater than the tension flange area is based on a value of k between the value for simply-supported boundary conditions and the theoretical k value of 39.6 for fixed boundary conditions at the flanges, Timoshenko and Gere (1961).

If either a longitudinal stiffener is provided, or Equation 12.7.3-6 is satisfied, then R_b is taken as 1.0.

$$\frac{2D_c}{t_w} \le \lambda_b \sqrt{\frac{E}{f_c}} \qquad (12.7.3\text{-}6)$$

If the requirements above are not satisfied, R_b is:

$$R_b = 1 - \left(\frac{a_r}{1200 + 300a_r}\right)\left(\frac{2D_c}{t_w} - \lambda_b\sqrt{\frac{E}{f_c}}\right) \qquad (12.7.3\text{-}7)$$

for which:

$$a_r = \frac{2D_c t_w}{A_{fc}} \qquad (12.7.3\text{-}8)$$

where:

λ_b = 5.76 for members with a compression flange area equal to or greater than the tension flange area

λ_b = 4.64 for members with a compression flange area less than the tension flange area

f_c = stress in the compression flange due to the factored loading (MPa)

A_{fc} = compression flange area (mm^2)

R_b is 1.0 for tension flanges because the increase in flange stresses due to web buckling occurs primarily in the compression flange and the tension flange stress is not significantly increased by the web buckling, Basler (1961).

Lateral-Torsional Buckling

The nominal flexural resistance of sections in negative flexure in terms of stress, F_n, is determined as:

If $1.76\, r_t \sqrt{\frac{E}{F_{yc}}} < L_b \le 4.44\, r_t \sqrt{\frac{E}{F_{yc}}}$, then:

$$F_n = C_b R_b R_h F_{yc} \left[1.33 - 0.187 \left(\frac{L_b}{r_t} \right) \sqrt{\frac{F_{yc}}{E}} \right] \le R_b R_h F_{yc} \qquad (12.7.3\text{-}9)$$

If $L_b > 4.44\, r_t \sqrt{\frac{E}{F_{yc}}}$, then:

$$F_n = C_b R_b R_h \left[\frac{9.86E}{(L_b/r_t)^2} \right] \le R_b R_h F_{yc} \qquad (12.7.3\text{-}10)$$

for which:

for unbraced cantilevers or for members where the moment within a significant portion of the unbraced segment exceeds the larger of the segment end moments:

C_b = 1.0 (12.7.3-11)

for all other cases:

C_b = $1.75 - 1.05\,(P_\ell/P_h) + 0.3(P_\ell/P_h)^2 \le 2.3$ (12.7.3-12)

where:

C_b = moment gradient correction factor

P_ℓ = force in the compression flange at the brace point with the lower force due to the factored loading (N)

P_h = force in the compression flange at the brace point with the higher force due to the factored loading (N)

L_b = distance between points bracing the compression flange (mm)

r_t = radius of gyration of a notional section comprised of the compression flange of the steel section, plus one-third of the web in compression, taken about the vertical axis (mm)

F_{yc} = specified minimum yield strength of the compression flange (MPa)

R_b, R_h = flange-stress reduction factors discussed above

(P_ℓ/P_h) is negative if P_ℓ is a tensile force.

The provisions for lateral-torsional buckling of composite sections in this article differ from those specified for non-composite sections because they attempt to handle the complex general problem of lateral-torsional buckling of a constant or variable depth section with stepped flanges constrained against lateral displacement at the top flange by the composite concrete slab.

The equations provided in this article are based on the assumption that only the flexural stiffness of the compression flange will prevent the lateral displacement of that element between brace points, which ignores the effect of the restraint offered by the concrete slab, Basler and Thurlimann (1961). As such, the behavior of a compression flange in resisting lateral buckling between brace points is assumed to be analogous to that of a column. These simplified equations, developed based on this assumption, are felt to yield conservative results for composite sections under the various conditions listed above.

The effect of the variation in the compressive force along the length between brace points is accounted for by using the factor C_b, which has a minimum value of 1.0 when the flange compressive force, and corresponding moment, is constant over the unbraced length. As the force at one of the brace points is progressively reduced, C_b becomes larger, and is taken as 1.75 when this force is 0.0. If the force at the end

is then progressively increased in tension, C_b continues to increase until it reaches the maximum permitted value of 2.3.

If the cross-section is constant between brace points, M_ℓ/M_h is expressed in terms of P_ℓ/P_h, and may be used in calculating C_b. It is conservative and convenient to use the maximum moments from the moment envelope at both brace points in this ratio, or in computing P_ℓ/P_h, although the actual behavior depends on the concurrent moments at these points. Under a moment gradient, P_ℓ will become a tensile force beyond the point of contraflexure. For this case, the ratio of (P_ℓ/P_h) shall be taken as negative in computing C_b.

An alternative formulation for C_b is given by the following formula, AISC (1991):

$$C_b = \frac{12.5 P_{max}}{2.5 P_{max} + 3P_A + 4P_B + 3P_c} \qquad (12.7.3\text{-}13)$$

where:

P_{max} = absolute value of the maximum compression flange force in the unbraced segment (N)

P_A = absolute value of the compression flange force at the quarter point of the unbraced segment (N)

P_B = absolute value of the compression flange force at the centerline of the unbraced segment (N)

P_c = absolute value of the compression flange force at the three-quarter point of the unbraced segment (N)

This formulation gives improved results for the cases of non-linear moment gradients and moment reversal.

The effect of a variation in the lateral stiffness properties, r_t, between brace points can be conservatively accounted for by using the minimum value that occurs anywhere between the brace points. Alternatively, a weighted average r_t could be used to provide a reasonable, but somewhat less conservative, answer.

The alternative formulas for flexural resistance of Article S6.10.5.6 applies to constant depth members without longitudinal stiffeners in negative flexure and which exceed the

compression flange slenderness requirements of Article S6.10.5.3.3c.

12.7.4 Compact Non-Composite Sections

Requirements regarding yield strength of steel, and provisions for web and compression flange slenderness and for compression flange bracing are the same as for the negative flexure provisions of compact composite sections.

The restriction on positive moment resistance of continuous spans with non-compact support sections is not applicable because of the lower non-composite shape factor.

12.7.5 Non-Compact Non-Composite Sections

Requirements regarding yield strength of steel, and provisions for web and compression flange slenderness and for compression flange bracing are the same as for the negative flexure provisions of compact non-composite sections with modifications as noted below.

The provisions of this article apply to sections in either positive or negative flexure satisfying the web slenderness, compression flange slenderness and compression flange bracing requirements in Articles S6.10.5.3.2b, S6.10.5.3.3c and S6.10.5.3.3d for non-compact sections, and which are summarized in Table 12.7.3-1 under the heading negative flexure.

The following interpretation and exceptions shall apply:

- r_t shall be taken as the minimum radius of gyration of the compression flange about the vertical axis.
- R_h and R_b shall be determined, as described above, neglecting the longitudinal reinforcement.
- If the flange bracing requirements of Table 12.7.3-1, with the exception related to r_t noted above, are not satisfied, M_n is calculated as described below.
- References to the "Q-formulas" apply.

Requirements to control lateral-torsional buckling require that if the unbraced length of a compression flange exceeds L_b in Table 12.7.3-1, the nominal flexural resistance based on the stability of the compression flange is determined from Equations 12.7.5-1, 12.7.5-2 or 12.7.5-3, as applicable.

If either a longitudinal stiffener is provided or:

If $\frac{2D_c}{t_w} \le \lambda_b \sqrt{\frac{E}{F_{yc}}}$, then:

$$M_n = 3.14 E C_b R_h \left(\frac{I_{yc}}{L_b}\right) \sqrt{0.772\left(\frac{J}{I_{yc}}\right) + 9.87\left(\frac{d}{L_b}\right)^2}$$
$$\le R_h M_y$$

(12.7.5-1)

If a longitudinal stiffener is not provided, and if:

$\frac{2D_c}{t_w} > \lambda_b \sqrt{\frac{E}{F_{yc}}}$, then

If $L_p < L_b \le L_r$, then:

$$M_n = C_b R_b R_h M_y \left[1 - 0.5\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le R_b R_h M_y$$

(12.7.5-2)

If $L_b > L_r$, then:

$$M_n = C_b R_b R_h \frac{M_y}{2}\left(\frac{L_r}{L_b}\right)^2 \le R_b R_h M_y$$

(12.7.5-3)

for which:

$$J = \frac{D t_w^3}{3} + \Sigma \frac{b_f t_f^3}{3}$$

$$L_p = 1.76 r' \sqrt{\frac{E}{F_{yc}}}$$

$$L_r = \sqrt{\frac{19.71 I_{yc} d}{S_{xc}} \frac{E}{F_{yc}}}$$

where:

λ_b = 5.76 for members with a compression flange area equal to or greater than the tension flange area

λ_b = 4.64 for members with a compression flange area less than the tension flange area

C_b = moment gradient correction factor given above

I_{yc} = moment of inertia of the compression flange about the vertical axis in the plane of the web (mm^4)

S_{xc} = section modulus about the horizontal axis of the section to the compression flange (mm^3)

M_y = yield moment for the compression flange (N·mm)

R_b and R_h = flange stress reduction factors as specified

r' = minimum radius of gyration of the compression flange about the vertical axis (mm)

t_w = thickness of web (mm)

t_f = thickness of a flange (mm)

F_{yc} = yield strength of compression flange (MPa)

Much of the discussion of the lateral buckling formulas in Article 12.7.3 also applies to this article. The formulas of this article are simplifications of the formulas presented in AISC (1986), and Kitipornchai and Trahair (1980) for the lateral buckling capacity of unsymmetrical girders.

The formulas predict the lateral buckling moment within approximately 10% of the more complex Trahair equations for sections satisfying the proportions specified in Article S6.10.1.1. The formulas treat girders with slender webs differently than girders with stocky webs. For sections with stocky webs with a web slenderness less than or equal to $\lambda_b(E/F_{yc})^{1/2}$, or with longitudinally stiffened webs, bend-buckling of the web is theoretically prevented. For these sections, the St. Venant torsional stiffness and the warping torsional

stiffness are included in computing the elastic lateral buckling moment given by Equation 12.7.5-1. For sections with thinner webs or without longitudinal stiffeners, cross-sectional distortion is possible; thus, the St. Venant torsional stiffness is ignored for these sections. Equation 12.7.5-3 is the elastic lateral torsional buckling moment given by Equation 12.7.5-1 with J taken as 0.0.

Equation 12.7.5-2 represents a straight line estimate of the inelastic lateral buckling resistance between $R_b R_h M_y$ and $0.5 R_b R_h M_y$. A straight line transition similar to this is not included for sections with stocky webs or longitudinally stiffened webs because the added complexity is not justified.

A discussion of the derivation of the value of λ_b may be found in Article SC6.10.5.4.2a.

12.7.6 Shear Resistance

12.7.6.1 GENERAL

A flow chart for shear capacity of I-sections is shown below:

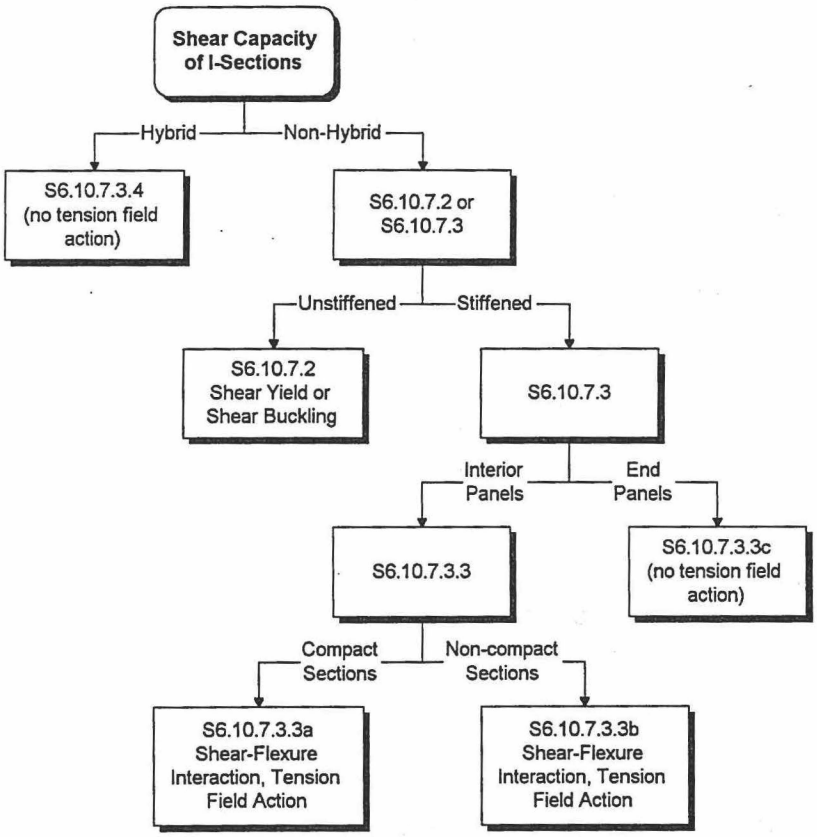


Figure 12.7.6.1-1 - Flow Chart for Shear Design of I-Sections

12.7.6.2 UNSTIFFENED WEBS

The nominal shear resistance of unstiffened webs of hybrid and homogeneous girders shall be taken as:

If $\frac{D}{t_w} \leq 2.46\sqrt{\frac{E}{F_{yw}}}$, then:

$$V_n = V_p = 0.58\, F_{yw}\, Dt_w \qquad (12.7.6.2\text{-}1)$$

If $2.46\sqrt{\frac{E}{F_{yw}}} < \frac{D}{t_w} \leq 3.07\sqrt{\frac{E}{F_{yw}}}$, then:

$$V_n = 1.48 t_w^2 \sqrt{EF_{yw}} \qquad (12.7.6.2\text{-}2)$$

If $\frac{D}{t_w} > 3.07\sqrt{\frac{E}{F_{yw}}}$, then:

$$V_n = \frac{4.55 t_w^3 E}{D} \qquad (12.7.6.2\text{-}3)$$

where:

F_{yw} = specified minimum yield strength of the web (MPa)

D = web depth (mm)

t_w = thickness of web (mm)

Equation 12.7.6.2-1 is equal to the web area times the assumed shear yield strength of $F_{yw}/3^{1/2}$. The D/t_w limit was determined by setting the shear buckling stress, calculated assuming a buckling coefficient, k, equal to 5.0, equal to the yield strength of the web, F_{yw}, in Formula 35 of Cooper, et al, (1978).

When $D/t_w > 3.07(E/F_{yw})^{1/2}$, the nominal shear resistance of the web is based on elastic shear buckling. Basler (1961) suggests taking the proportional limit as 80% of F_{yw}. This corresponds to $D/t_w = (2.46/0.8)\,(E/F_{yw})^{1/2}$. Thus, when $D/t_w > 3.07(E/F_{yw})^{1/2}$, the nominal shear resistance is determined from Equation 12.7.6.2-3, which is equal to the web area times

the elastic shear buckling stress given by Formula 6 of Cooper, et al, (1978), calculated assuming a buckling coefficient, k, equal to 5.0.

Equation 12.7.6.2-2 represents a straight-line transition between the two web slenderness limits.

12.7.6.3 STIFFENED WEBS

The nominal shear resistance of transversely or transversely and longitudinally stiffened interior and end web panels is specified herein for homogeneous and hybrid sections. The total web depth, D, shall be used in determining the nominal shear resistance of web panels with longitudinal stiffeners. Transverse stiffeners shall be spaced using the maximum shear in a panel.

Longitudinal stiffeners divide a web panel into subpanels. The shear resistance of the entire panel can be taken as the sum of the shear resistance of the subpanels, Cooper (1967). However, the contribution of the longitudinal stiffener at a distance of $2D_c/5$ from the compression flange is relatively small. Thus, it is conservatively recommended that the influence of the longitudinal stiffener be neglected in computing the nominal shear resistance of the web plate.

For web panels without longitudinal stiffeners, transverse stiffeners are required if:

$$\frac{D}{t_w} > 150 \qquad (12.7.6.3\text{-}1)$$

The spacing of transverse stiffeners, d_o, must satisfy:

$$d_o \le D\left[\frac{260}{(D/t_w)}\right]^2 \qquad (12.7.6.3\text{-}2)$$

Transverse stiffeners are required on web panels with a slenderness ratio greater than 150 in order to facilitate handling of sections without longitudinal stiffeners during fabrication and erection. The spacing of the transverse stiffeners is arbitrarily limited by Equation 12.7.6.3-2, Basler (1961). Substituting a web slenderness of 150 into Equation 12.7.6.3-2 results in a maximum transverse stiffener spacing of 3D, which corresponds to the maximum spacing permitted for web panels without longitudinal stiffeners. For higher web slenderness

ratios, the maximum allowable spacing is reduced to less than 3D.

The requirement in Equation 12.7.6.3-2 is not needed for web panels with longitudinal stiffeners since maximum transverse stiffener spacing is already limited to 1.5D.

Stiffened interior web panels of homogeneous sections may develop post-buckling shear resistance due to tension-field action, Basler (1961). The action is analogous to that of the tension diagonals of a Pratt truss. The nominal shear resistance of these panels can be computed by summing the contributions of beam action and of the post-buckling tension-field action. The resulting expression is given in Equation 12.7.6.3-1, where the first term in the bracket relates to either the shear yield or shear buckling force and the second term relates to the post-buckling tension-field force.

The nominal shear resistance of interior web panels of compact sections complying with the requirements above is:

If $M_u \leq 0.5\phi_f M_p$, then:

$$V_n = V_p\left[C + \frac{0.87(1-C)}{\sqrt{1+\left(\frac{d_o}{D}\right)^2}}\right] \qquad (12.7.6.3\text{-}3)$$

If $M_u > 0.5\phi_f M_p$, then:

$$V_n = RV_p\left[C + \frac{0.87(1-C)}{\sqrt{1+\left(\frac{d_o}{D}\right)^2}}\right] \geq CV_p \qquad (12.7.6.3\text{-}4)$$

for which:

$$R = \left[0.6 + 0.4\left(\frac{M_r - M_u}{M_r - 0.75\phi_f M_y}\right)\right] \leq 1.0 \qquad (12.7.6.3\text{-}5)$$

$$V_p = 0.58\, F_{yw} D t_w \qquad (12.7.6.3\text{-}6)$$

where:

M_u = maximum moment in the panel under consideration due to the factored loads (N·mm)

V_n = nominal shear resistance (N)

V_p = plastic shear force (N)

M_r = factored flexural resistance (N·mm)

ϕ_f = resistance factor for flexure

M_y = yield moment (N·mm)

D = web depth (mm)

d_o = stiffener spacing (mm)

C = ratio of the shear buckling stress to the shear yield strength

When both shear and flexural moment are high in a stiffened interior panel under tension-field action, the web plate must resist the shear and also participate in resisting the moment. Panels whose resistance is limited to the shear buckling or shear yield force are not subject to moment-shear interaction effects. Basler (1961) shows that stiffened web plates in non-compact sections are capable of resisting both moment and shear, as long as the shear force due to the factored loadings is less than $0.6\phi_v V_n$ or the flexural stress in the compression flange due to the factored loading is less than $0.75\phi_f F_y$.

For compact sections, flexural resistances are expressed in terms of moments rather than stresses. For convenience, a limiting moment of $0.5\phi_f M_p$ is defined rather than a limiting moment of $0.75\phi_f M_y$ in determining when the moment-shear interaction occurs by using an assumed shape factor (M_p/M_y) of 1.5. This eliminates the need to compute the yield moment to simply check whether or not the interaction effect applies. When the moment due to factored loadings exceeds $0.5\phi_f M_p$, the nominal shear resistance is taken as V_n, given by Equation 12.7.6.3-2, reduced by the specified interaction factor, R.

Both upper and lower limits are placed on the nominal shear resistance in Equation 12.7.6.3-2 determined by applying the interaction factor, R. The lower limit is either the shear yield or shear buckling force. Sections with a shape factor below 1.5 could potentially exceed V_n according to the interaction equation at moments due to the factored loadings slightly above the defined limiting value of $0.5\phi_f M_p$. Thus, for compact sections, an upper limit of 1.0 is placed on R.

To avoid the interaction effect, transverse stiffeners may be spaced so that the shear due to the factored loadings does not exceed the larger of:

- $0.60\,\phi_v V_n$ where V_n is given by Equation 12.7.6.3-3,
- the factored shear buckling or shear yield resistance equal to $\phi_v C V_p$.

The coefficient, C, is equal to the ratio of the elastic shear buckling stress of the panel, computed assuming simply-supported boundary conditions, to the shear yield strength assumed to be equal to $F_{yw}/(3)^{0.5}$. Equation 12.7.6.3-7 is applicable only for C values not exceeding 0.8, Basler (1961). Above 0.8, C values are given by Equation 12.7.6.3-6 until a limiting slenderness ratio is reached where the shear buckling stress is equal to the shear yield strength and C = 1.0. Equation 12.7.6.3-8 for the shear buckling coefficient is a simplification of two exact equations for k that depend on the panel aspect ratio.

The ratio, C, is given below:

If $\frac{D}{t_w} < 1.10\sqrt{\frac{Ek}{F_{yw}}}$, then

$$C = 1.0 \qquad (12.7.6.3\text{-}5)$$

If $1.10\sqrt{\frac{Ek}{F_{yw}}} \le \frac{D}{t_w} \le 1.38\sqrt{\frac{Ek}{F_{yw}}}$, then:

$$C = \frac{1.10}{\frac{D}{t_w}}\sqrt{\frac{Ek}{F_{yw}}} \qquad (12.7.6.3\text{-}6)$$

If $\frac{D}{t_w} > 1.38\sqrt{\frac{Ek}{F_{yw}}}$, then:

$$C = \frac{1.52}{\left(\frac{D}{t_w}\right)^2}\left(\frac{Ek}{F_{yw}}\right) \qquad (12.7.6.3\text{-}7)$$

for which:

$$k = 5 + \frac{5}{\left(\frac{d_o}{D}\right)^2} \qquad (12.7.6.3\text{-}8)$$

The nominal shear resistance of interior web panels of non-compact sections complying with the requirements for maximum stiffener spacing is:

If $f_u \leq 0.75\phi_f F_y$, then:

$$V_n = V_p\left[C + \frac{0.87(1-C)}{\sqrt{1+\left(\frac{d_o}{D}\right)^2}}\right] \qquad (12.7.6.3\text{-}9)$$

If $f_u > 0.75\phi_f F_y$, then:

$$V_n = RV_p\left[C + \frac{0.87(1-C)}{\sqrt{1+\left(\frac{d_o}{D}\right)^2}}\right] \geq CV_p \qquad (12.7.6.3\text{-}10)$$

for which:

$$R = \left[0.6 + 0.4\left(\frac{F_r - f_u}{F_r - 0.75\phi_f F_y}\right)\right] \leq 1.0 \qquad (12.7.6.3\text{-}11)$$

where:

f_u = maximum stress in the compression flange in the panel under consideration due to the factored loading (MPa)

C = ratio of shear buckling stress to the shear yield strength

F_r = factored flexural resistance of the compression flange for which f_u was determined (MPa)

The nominal shear resistance of an end panel is limited to the shear buckling or shear yield force given by:

$$V_n = CV_p \qquad (12.7.6.3\text{-}12)$$

for which:

$V_p = 0.58 F_{yw} D t_w$ (12.7.6.3-13)

where:

C = ratio of the shear buckling stress to the shear yield strength

V_p = plastic shear force (N)

The transverse stiffener spacing for end panels without a longitudinal stiffener is not to exceed 1.5D. The transverse stiffener spacing for end panels with a longitudinal stiffener is not to exceed 1.5 times the maximum subpanel depth.

The transverse stiffener spacing requirements are applicable to hybrid sections.

Tension-field action is not permitted for hybrid sections. Thus, the nominal shear resistance is limited to either the shear yield or the shear buckling force given by:

$V_n = CV_p$ (12.7.6.3-14)

12.7.7 Stiffeners

Provisions for transverse intermediate, bearing and longitudinal stiffeners are summarized below.

12.7.7.1 TRANSVERSE INTERMEDIATE STIFFENERS

Transverse stiffeners consist of plates or angles welded or bolted to either one or both sides of the web.

Stiffeners not used as connection plates are required to have a tight fit at the compression flange, but need not be in bearing with the tension flange.

Stiffeners used as connecting plates for diaphragms or cross-frames should be connected by welding or bolting to both flanges.

The distance between the end of the web-to-stiffener weld and the near edge of the web-to-flange fillet weld should not be less than $4t_w$ or more than $6t_w$.

The requirements in this article are intended to prevent local buckling of the transverse stiffener.

The width, b_t, of each projecting stiffener element must satisfy:

$$50 + \frac{d}{30} \le b_t \le 0.48 t_p \sqrt{\frac{E}{F_{ys}}} \qquad (12.7.7.1\text{-}1)$$

and

$$16.0 t_p \ge b_t \ge 0.25 b_f \qquad (12.7.7.1\text{-}2)$$

where:

d = steel section depth (mm)

t_p = thickness of the projecting element (mm)

F_{ys} = specified minimum yield strength of the stiffener (MPa)

b_f = full-width of compression or tension flange (mm)

The moment of inertia of any transverse stiffener must satisfy:

$$I_t \ge d_o t_w^3 J \qquad (12.7.7.1\text{-}3)$$

for which:

$$J = 2.5\left(\frac{D_p}{d_o}\right)^2 - 2.0 \ge 0.5 \qquad (12.7.7.1\text{-}4)$$

where:

I_t = moment of inertia of the transverse stiffener taken about the edge in contact with the web for single stiffeners and about the mid-thickness of the web for stiffener pairs (mm^4)

t_w = web thickness (mm)

d_o = transverse stiffener spacing (mm)

D_p = web depth for webs without longitudinal stiffeners or maximum subpanel depth for webs with longitudinal stiffeners (mm)

For the web to adequately develop the tension field, the transverse stiffener must have sufficient rigidity to cause a node to form along the line of the stiffener. For ratios of (d_o/D_p) less than 1.0, much larger values of I_t are required as discussed in Timoshenko (1961). Each subpanel of a subdivided panel behaves as a separate panel. Therefore, the depth of the subpanel is used in this requirement. This requirement should also be investigated for web panels not required to develop a tension field.

Transverse stiffeners used in conjunction with longitudinal stiffeners must also satisfy:

$$I_t \geq \left(\frac{b_\ell}{b_t}\right)\left(\frac{D}{3.0d_o}\right)I_\ell \qquad (12.7.7.1\text{-}5)$$

where:

b_t = projecting width of transverse stiffener (mm)

b_ℓ = projecting width of longitudinal stiffener (mm)

I_ℓ = moment of inertia of the longitudinal stiffener taken about the edge in contact with the web, based on the effective section as specified in Article S6.10.8.3.3 (mm^4)

D = web depth (mm)

Lateral loads along the length of a longitudinal stiffener are transferred to the adjacent transverse stiffeners as concentrated reactions, Cooper (1967). Equation 12.7.7.1-3 gives a relationship between the moments of inertia of the longitudinal and transverse stiffeners to ensure the latter does not fail under the concentrated reactions. Equation 12.7.7.1-3 is equivalent to Equation 10-111 in the 1992 AASHTO.

Transverse stiffeners need sufficient area to resist the vertical component of the tension field. The formula for the required stiffener area below can give a negative result. In that case, the required area is 0.0. A negative result indicates that the web alone is sufficient to resist the vertical component of the tension field. The stiffener then need only be proportioned for stiffness and satisfy the projecting width requirements. For

web panels not required to develop a tension field, this requirement need not be investigated.

Transverse intermediate stiffeners required to carry the forces imposed by tension-field action of the web must satisfy:

$$A_s \geq \left[0.15BDt_w(1.0 - C)\frac{V_u}{V_r} - 18.0t_w^2\right]\left(\frac{F_{yw}}{F_{ys}}\right) \qquad (12.7.7.1\text{-}6)$$

where:

V_r = factored shear resistance (N)

V_u = shear due to factored loads at the strength limit state (N)

A_s = stiffener area; total area of both stiffeners for pairs (mm^2)

B = 1.0 for stiffener pairs

B = 1.8 for single angle stiffeners

B = 2.4 for single plate stiffeners

C = ratio of the shear buckling stress to the shear yield strength

F_{yw} = specified minimum yield strength of the web (MPa)

F_{ys} = specified minimum yield strength of the stiffener (MPa)

12.7.7.2 BEARING STIFFENERS

Bearing reactions and other concentrated loads, either in the final state or during construction, shall be resisted by bearing stiffeners.

Inadequate provision to resist concentrated loads has resulted in failures, particularly in temporary construction.

Bearing stiffeners should be placed on webs of rolled beams at all bearing locations and other points of concentrated loads where:

$$V_u > 0.75\phi_b V_n \qquad (12.7.7.2\text{-}1)$$

where:

ϕ_b = resistance factor for bearing

V_u = shear due to the factored loads (N)

V_n = nominal shear resistance (N)

Bearing stiffeners should be placed on the webs of plate girders at all bearing locations and at all locations supporting concentrated loads.

Bearing stiffeners consist of one or more plates or angles welded or bolted to both sides of the web. The connections to the web are to be designed to transmit the full bearing force due to the factored loads.

The stiffeners should extend the full-depth of the web and, as closely as practical, to the outer edges of the flanges.

Each stiffener should be either milled to fit against the flange through which it receives its reaction, or attached to that flange by a full penetration groove weld.

If an owner chooses not to utilize bearing stiffeners where specified in this article, the web crippling provisions of AISC (1986) should be used to investigate the adequacy of the component to resist a concentrated load.

The width, b_t, of each projecting stiffener element must satisfy:

$$b_t \le 0.48 t_p \sqrt{\frac{E}{F_{ys}}} \qquad (12.7.7.2\text{-}2)$$

where:

t_p = thickness of projecting element (mm)

F_{ys} = specified minimum yield strength of the stiffener (MPa)

To bring bearing stiffener plates tight against the flanges, part of the stiffener must be clipped to clear the web-to-flange fillet weld. Thus, the area of direct bearing is less than the gross area of the stiffener. The bearing resistance is

based on this bearing area and the yield strength of the stiffener.

The factored bearing resistance, B_r, shall be taken as:

$$B_r = \phi_b A_{pn} F_{ys} \qquad (12.7.7.2\text{-}3)$$

where:

F_{ys} = specified minimum yield strength of the stiffener (MPa)

A_{pn} = area of the projecting elements of the stiffener outside of the web-to-flange fillet welds, but not beyond the edge of the flange (mm^2)

ϕ_b = resistance factor for bearing

The factored axial resistance, P_r, is determined as specified for compression members. The radius of gyration shall be computed about the mid-thickness of the web. The end restraint against column buckling provided by the flanges allows for the use of a reduced effective length, therefore, the effective length may be 0.75D, where D is the web depth.

The web of hybrid girders is not included in the computation of the radius of gyration because the web may be yielding due to longitudinal flexural stress. At end supports where the moment is NIL, the web may be included.

For stiffeners bolted to the web, the effective column section shall consist of the stiffener elements only.

A portion of the web is assumed to act in combination with the bearing stiffener plates.

For stiffeners welded to the web, the effective column section shall consist of all stiffener elements, plus a centrally located strip of web extending not more than $9t_w$ on each side of the outer projecting elements of the group, if more than one pair of stiffeners is used.

The strip of web is not included in the effective section at interior supports of continuous span hybrid members if:

$$\frac{F_{yw}}{F_{yf}} < 0.70 \qquad (12.7.7.2\text{-}4)$$

where:

F_{yw} = specified minimum yield strength of the web (MPa)

F_{yf} = higher of the specified minimum yield strengths of the flanges (MPa)

12.7.7.3 LONGITUDINAL STIFFENERS

Where required, longitudinal stiffeners should consist of either a plate welded longitudinally to one side of the web, or a bolted angle, and shall be located a distance of $2D_c/5$ from the inner surface of the compression flange, where D_c is the depth of the web in compression at the section with the maximum compressive flexural stress.

Theoretical and experimental studies have indicated that the optimum location of one longitudinal stiffener is $2D_c/5$ for bending and D/2 for shear. Tests have also shown that longitudinal stiffeners located at $2D_c/5$ can effectively control lateral web deflections under flexure, Cooper (1967). The distance $2D_c/5$ is specified because shear is always accompanied by moment and because a properly proportioned longitudinal stiffener reduces lateral web deflections caused by shear.

Because D_c may vary along the length of the span, it is specified here that the stiffener be located based on the D_c computed at the section with the largest compressive flexural stress. Thus, the stiffener may not be located at its optimum location at other sections with a lower stress and a different D_c. Research is presently underway to attempt to better define the resistance for sections with longitudinal stiffeners at locations other than the optimum location.

Continuous longitudinal stiffeners placed on the opposite side of the web from the transverse intermediate stiffeners are preferred. If longitudinal and transverse stiffeners must be placed on the same side of the web, it is preferable that the longitudinal stiffener not be interrupted for the transverse stiffener. If the longitudinal stiffener must be interrupted, it should only be interrupted in compression regions. Furthermore, if the longitudinal stiffener is interrupted, its area should not be included when calculating section properties.

Where the transverse stiffener is interrupted, copes should be provided for the longitudinal stiffener-to-web welds to avoid intersecting welds, and the transverse stiffener should be fillet or groove welded to the longitudinal stiffener to assure

it will function properly in resisting web buckling and supporting the longitudinal stiffener. All interruptions must be carefully designed with respect to fatigue, Schilling (1986).

The projecting width, b_t, of the stiffener must satisfy:

$$b_t \le 0.48 t_s \sqrt{\frac{E}{F_{yc}}} \qquad (12.7.7.3\text{-}1)$$

where:

t_s = thickness of stiffener (mm)

F_{yc} = specified minimum yield strength of the adjacent compression flange (MPa)

This requirement is intended to prevent local buckling of the longitudinal stiffener. The yield strength of the adjacent compression flange is conservatively used rather than the yield strength of the stiffener to prevent overstressing the stiffener if it has a lower yield strength than the compression flange.

The section properties of the stiffener shall be based on an effective section consisting of the stiffener and a centrally located strip of the web not exceeding $18t_w$.

The moment of inertia requirement is to ensure the stiffener will have adequate rigidity to force a horizontal line of nil deflection in the web panel. The radius of gyration requirement is to ensure the longitudinal stiffener will be rigid enough to withstand the axial compressive stress without lateral buckling. A partially restrained end condition is assumed for the stiffener acting as a column. It is also assumed in the development of Equation 12.7.7.3-2 that the eccentricity of the load and initial out-of-straightness cause a 20% increase in stress in the stiffener.

Longitudinal stiffeners must satisfy:

$$I_t \ge D t_w^3 \left[2.4 \left(\frac{d_o}{D} \right)^2 - 0.13 \right] \qquad (12.7.7.3\text{-}2)$$

$$r \ge 0.234 d_o \sqrt{\frac{F_{yc}}{E}} \qquad (12.7.7.3\text{-}3)$$

where:

I_ℓ = moment of inertia of the longitudinal stiffener and the web strip about the edge in contact with the web (mm^4)

r = radius of gyration of the longitudinal stiffener and the web strip about the edge in contact with the web (mm)

D = web depth (mm)

d_o = transverse stiffener spacing (mm)

t_w = web thickness (mm)

F_{yc} = specified minimum yield strength of the adjacent compression flange (MPa)

A longitudinal stiffener meeting the requirements above will have sufficient area to anchor the tension field. Therefore, no additional area requirement is given for longitudinal stiffeners.

12.7.8 Constructibility

New provisions for flexural and shear resistance of composite sections which are non-composite during construction are contained in Article S6.10.10.

12.7.9 Inelastic Analysis Procedures

Provisions similar to the Alternate Load Factor Design Guide Specifications of AASHTO are contained in Article S6.10.11 for both the mechanism and unified autostress methods. Deflection limitations at the service limit state are also imposed.

12.7.10 Steel Plate Girder Design Example

12.7.10.1 BRIDGE DESCRIPTION

Bridge Geometry:

Total length	= 72 000 mm (distance between centerlines of bearing at abutments)
Number of spans	= 2
Length of spans	= 36 000 mm - 36 000 mm
Number of lanes	= 2
Number of girders	= 4

Lane width	=	3600 mm
Shoulder width	=	1560 mm
Overhang	=	800 mm
Total deck width	=	11 200 mm
Girder type	=	Straight, constant depth plate girder, homogeneous
Skew angle	=	No skew
Cross-frame spacing	=	7200 mm

Cross-section Geometry:

Girder:

Spacing	=	3200 mm
Web depth	=	1425 mm

Slab/Wearing Surface:

Slab Thickness	=	210 mm (10 mm of which is considered non-structural)
Haunch Thickness	=	0 mm
Reinforcing (top)	=	4700 mm^2 (within effective slab width for negative moment)
Reinforcing (bottom)	=	2400 mm^2 (within effective slab width for negative moment)
SIP Forms	=	0.72 kN/m^2
FWS	=	1.44 kN/m^2

Parapets (PL-3):

Width at base	=	440 mm
Weight	=	7.61 kN/m

Materials:

Structural steel	=	345 MPa
Reinforcing steel	=	400 MPa
E_s	=	200 000 MPa
Slab concrete	=	28 MPa
E_c	=	25 400 MPa
n	=	8
Steel Density	=	7850 kg/m^3
Concrete Density	=	2400 kg/m^3

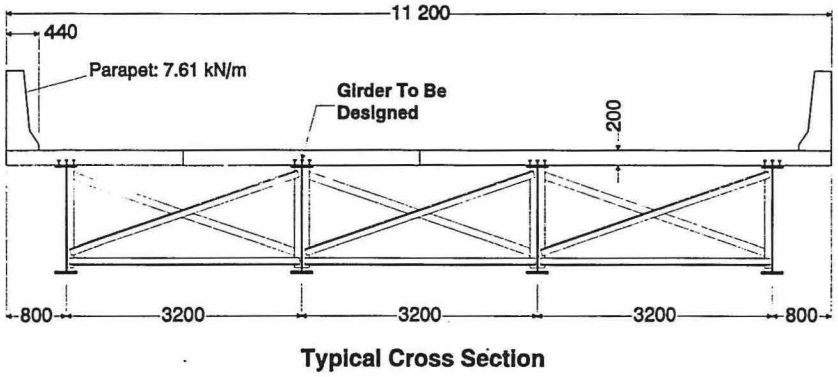


Figure 12.7.10.1-1 - Bridge Cross-Section Geometry

12.7.10.2 ASSUMPTIONS

- Interior girder design
- ADTT = 500; $ADTT_{SL}$ = 425
- Weight of attachments (stiffeners, cross-frames, etc.) taken into account by increasing girder weight by 5 percent
- Live load applied to "n" section, superimposed dead loads applied to "3n" section
- Composite, positive flexural stiffness properties used for entire length of girder for superimposed dead load and live load analysis (SC6.10.1.3)
- Ignore contribution of slab reinforcing, except for negative flexure
- Strength II loading case (permit live load) not considered
- Strength IV loading case will not control (DL/LL << 7.0; see C3.4.1)
- Unshored construction
- Slab pouring sequence will not be considered

12.7.10.3 LIVE LOAD VEHICLES

- Design Vehicle (Application per Article S3.6.1.3.1)
- Fatigue Vehicle (Article S3.6.1.4.1)
- Deflection Vehicle (Article S3.6.1.3.2)
- Live and permanent load analysis will be performed using PennDOT's *Continuous Beam Analysis* (CBA) program

12.7.10.4 PLATE SIZES

Table 12.7.10.4-1 - Plate Sizes (all dimensions in mm)

Section	A	B	C	D	E	F
Length	6000	17400	2400	4800	2400	3000
Web Depth	1425					
Web Thickness	12			10		
Top Flange Width	300			460		
Top Flange Thickness	13			16	25	42
Bottom Flange Width	300			460		
Bottom Flange Thickness	18	30	21	23	31	48

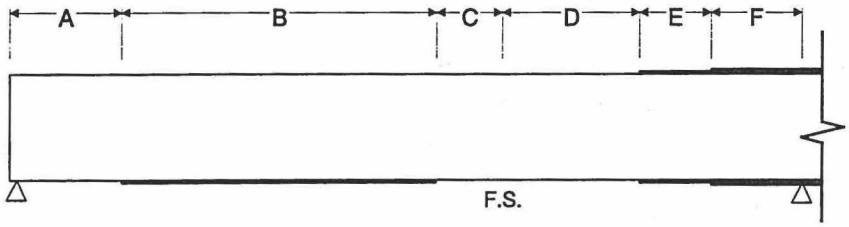


Figure 12.7.10.4-1 - Cross-section References

Calculate the effective flange width, b_e (S4.6.2.6)

The effective flange width, b_e (effective slab width for composite action), can be a function of the effective span length which is taken as the actual span length for simple spans and as the distance between points of permanent load contraflexure for continuous spans, as appropriate for either positive or negative flexure.

Therefore, for a two-span continuous girder the effective span length L_{eff} is:

For positive moment regions:

- $L_{eff} \approx 0.7$(Span length) = 0.7(36 000) = 25 200 mm

For negative moment regions:

- $L_{eff} \approx 2(0.3)$(Span length) = 0.6(36 000) = 26 100 mm

For an interior girder, the effective slab width is then taken as the least of the following:

- $\frac{1}{4}(L_{eff})$,
- $12t_s + MAX(t_w, b_{ft}/2)$,
- Average beam spacing, S

So, in the positive moment region (Sections A, B and C):

- $\frac{1}{4}(L_{eff}) = 0.25(25\ 200) = 6300$
- $12t_s + MAX(t_w, b_{ft}/2) = 12(200) + MAX(12, 300/2)$
  $= 2550$ (Controls)
- $S = 3200$

And, in the negative moment region (Sections D, E and F):

- $\frac{1}{4}(L_{eff}) = 0.25(21\ 600) = 5400$
- $12t_s + MAX(t_w, b_{ft}/2) = 12(200) + MAX(12, 460/2)$
  $= 2630$ (Controls)
- $S = 3200$

12.7.10.5 SECTION PROPERTIES

Table 12.7.10.5-1 - Permanent Loads and Section Properties
(dimensions in mm unless stated otherwise)

Section	A	B	C	D	E	F
End of Range	6000	23 400	25 800	30 600	33 000	36 000
Steel Area	26 400	30 000	27 300	32 190	40 010	55 650
Steel + Details	2.13 kN/m	2.43 kN/m	2.21 kN/m	2.60 kN/m	3.24 kN/m	4.50 kN/m
Slab, Haunch, SIP	18.12 kN/m	18.12 kN/m	18.12 kN/m	18.12 kN/m	18.12 kN/m	18.12 kN/m
Total DL1	20.25 kN/m	20.55 kN/m	20.33 kN/m	20.72 kN/m	21.36 kN/m	22.62 kN/m
DL2 Parapet	3.81 kN/m					
DL2 FWS, Util	3.72 kN/m					
b_{eff}	2550			2630		
I_{DL1}	7.676e+09	9.207e+09	8.091e+09	1.161e+10	1.591e+10	2.471e+10
I_{DL2+}	1.659e+10	2.051e+10	1.762e+10	2.228e+10	2.716e+10	3.730e+10
I_{LL+}	2.192e+10	2.784e+10	2.343e+10	2.940e+10	3.577e+10	4.892e+10
K_g	2.201e+11	2.904e+11	2.376e+11	3.022e+11	3.800e+11	5.520e+11
S_{DL1bf}	1.114e+07	1.490e+07	1.209e+07	1.752e+07	2.298e+07	3.418e+07
S_{DL1tf}	1.001e+07	1.083e+07	1.025e+07	1.448e+07	2.018e+07	3.120e+07
S_{DL2bf+}	1.543e+07	2.027e+07	1.664e+07	2.168e+07	2.698e+07	3.825e+07
S_{DL2bf-}	1.376e+07	1.817e+07	1.487e+07	1.995e+07	2.517e+07	3.624e+07
S_{DL2tf+}	4.364e+07	4.495e+07	4.402e+07	5.105e+07	5.730e+07	6.907e+07
S_{DL2tf-}	2.079e+07	2.192e+07	2.112e+07	2.599e+07	3.199e+07	4.341e+07
S_{DL2c+}	2.860e+07	3.125e+07	2.935e+07	3.501e+07	4.030e+07	5.040e+07
S_{DL2r-}	1.652e+07	1.789e+07	1.691e+07	2.103e+07	2.601e+07	3.565e+07
S_{LLbf+}	1.683e+07	2.203e+07	1.813e+07	2.319e+07	2.873e+07	4.056e+07
S_{LLbf-}	1.376e+07	1.817e+07	1.487e+07	1.995e+07	2.517e+07	3.624e+07
S_{LLtf+}	1.425e+08	1.364e+08	1.405e+08	1.497e+08	1.515e+08	1.583e+08
S_{LLtf-}	2.079e+07	2.192e+07	2.112e+07	2.599e+07	3.199e+07	4.341e+07
S_{LLc+}	6.194e+07	6.890e+07	6.389e+07	7.416e+07	8.201e+07	9.612e+07
S_{LLr-}	1.652e+07	1.789e+07	1.691e+07	2.103e+07	2.601e+07	3.565e+07
M_{p+}	8123	9988	8590	10139	12134	16389
M_{p-}	6478	7648	6787	8406	10376	14465
D_{cp+}	0	0	0	0	0	0
D_{cp-}	993	843	955	963	986	986
$I_{y\ steel}$	6.996e+07	9.696e+07	7.671e+07	3.165e+08	4.544e+08	7.301e+08
I_{ybf}	4.050e+07	6.750e+07	4.725e+07	1.866e+08	2.515e+08	3.893e+08
I_{ytf}	2.925e+07	2.925e+07	2.925e+07	1.298e+08	2.028e+08	3.407e+08

Note: The member proportions should be checked per Article S6.10.1.1 which requires that the ratio of I_{yc}/I_y (moment of inertia of compression flange divided by moment of inertial of the steel section, both taken about a vertical axis) be between 0.1 and 0.9. For Sections A through F above, this ratio ranged from 0.30 to 0.59.

12.7.10.6 DISTRIBUTION FACTORS

Use of The Multiple Presence Factors

Longitudinal Stiffness, K_g:

K_g, was based on a weighted average for the entire length of the bridge. The resulting value of K_g was computed as 3.05 E11 mm^4.

Single Lane Loaded:

Flexure:

$$DFM_{SL} = 0.06 + \left(\frac{S}{4300}\right)^{0.4} + \left(\frac{S}{L}\right)^{0.3}\left(\frac{K_g}{Lt_s^3}\right)^{0.1} = 0.49$$

Shear:

$$DFV_{SL} = 0.36 + \frac{S}{7600} = 0.78$$

Multiple Lanes Loaded:

Flexure:

$$DFM_{ML} = 0.075 + \left(\frac{S}{2900}\right)^{0.6} + \left(\frac{S}{L}\right)^{0.2}\left(\frac{K_g}{Lt_s^3}\right)^{0.1} = 0.73$$

Shear:

$$DFV_{ML} = 0.2 + \frac{S}{3600} - \left(\frac{S}{10700}\right)^{2.0} = 1.0$$

Fatigue:

Flexure:

$DFM_{fatigue} = DFM_{SL}/1.2 = 0.41$

Shear:

$DFV_{fatigue} = DFV_{SL}/1.2 = 0.65$

Deflection (assuming all girders deflect equally; see Article S2.5.2.6.2):

$DF_{\Delta} = N_L/N_B = 2/4 = 0.5$

Dynamic Load Allowance (IM) - See Table S3.6.2.1-1:

Fatigue and Fracture Limit State:

IM = 1.15

Other Limit States:

IM = 1.33

12.7.10.7 GENERAL LOAD FACTORS, η (Articles S1.3.3, S1.3.4, S1.3.5)

The LRFD Specification allows the owner/engineer to account for several parameters that, in the past, were not generally considered when factoring the design loads. These include ductility, redundancy and operational importance.

A. Ductility, η_D = 1.00 for Strength Only
B. Redundancy, η_R = 1.00 for Strength Only
C. Importance, η_I = 0.95 for Strength Only
   (would also apply to extreme event if seismic loads were to be investigated)
D. Total ($\eta_D\eta_R\eta_I$):
   Strength: $\eta = 0.95 \geq 0.95$ (OK)
   Fatigue, Service: $\eta = 1.00 \geq 0.95$ (OK)

12.7.10.8 LOAD CALCULATIONS (Permanent and Live)

Permanent Loads:

Non-Composite Loads:

- Steel Girder, Section A:

  A = 26 400 mm^2

$$(26400\,mm^2)\left(\frac{m^2}{1E6\,mm^2}\right)(7850\,kg/m^3)(9.81\,m/s^2)\left(\frac{1\,kN}{1000\,N}\right)$$
$$= 2.03\,kN/m$$

Use "detail factor" of 1.05 to account for stiffeners, cross-frames, etc.:

2.03 (1.05) = 2.13 kN/m

Calculations should be repeated for Sections B through F, but are not shown here. See the summary of dead loads and section properties in Table 12.7.10.5-1.

- Slab:

  $A = S\,t = (3200\ mm)(210) = 672\,000\ mm^2$

  $$(67200\,mm^2)\left(\frac{m^2}{1E6\,mm^2}\right)(2400\,kg/m^3)(9.81\,m/s^2)\left(\frac{1\,kN}{1000\,N}\right) = 15.82\ kN/m$$

- SIP Forms:

  $(3.2\ m)\ (0.72\ kN/m^2) = 2.30\ kN/m$

- Haunch:

  Assumed zero haunch

Composite Loads:

- Parapet (equally distributed to all girders):

  $(7.61\ kN/m)(2\ parapets)/(4\ girders) = 3.81\ kN/m$

- Future Wearing Surface (FWS equally distributed to all girders):

  $(10.32\ m)(1.44\ kN/m^2)/(4\ girders) = 3.72\ kN/m$

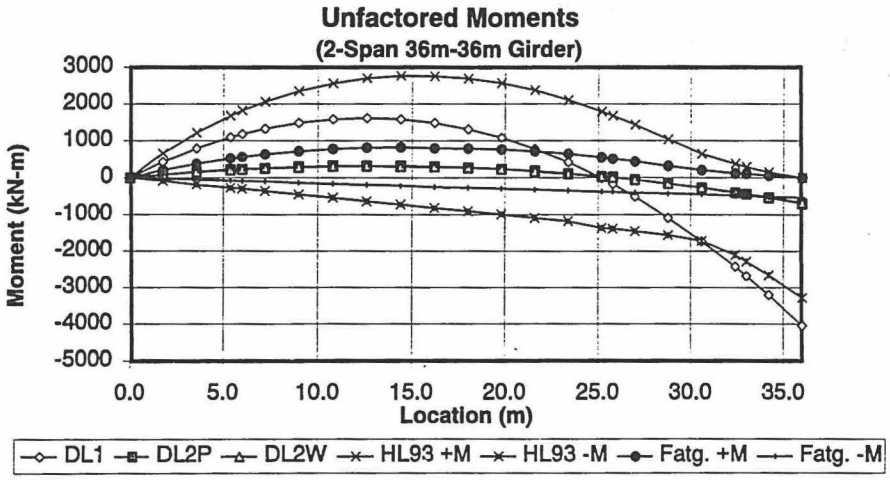
Utilities:

Assumed no utility loads

PennDOT's Continuous Beam Analysis Program (CBA) was used to compute the unfactored moments, shears and deflections due to the permanent loads. The results are tabulated in Table 1 and shown in Figure 1.

Live Loads:

PennDOT's Continuous Beam Analysis Program (CBA) was used to compute the LRFD live loads for the HL93 loading and the fatigue live loading. The results are tabulated in Table 2 and shown in Figure 1. The deflections shown adjacent to the LRFD live load moments and shears correspond to the deflection vehicle loading given in Article S3.6.1.3.2 rather than the design vehicle.



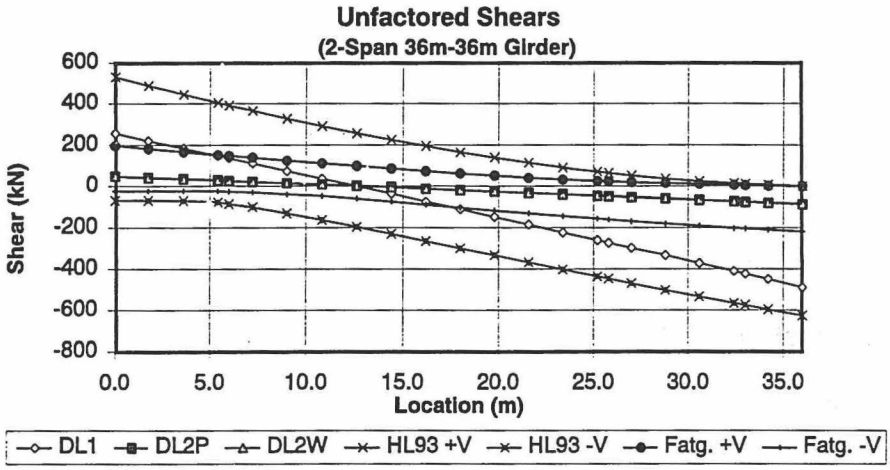


Figure 12.7.10.8-1 - Unfactored Moment and Shear Diagrams

Unfactored Load Effects

Table 12.7.10.8-1 - Permanent Loads (Moments in kN·m, Shears in kN, Deflections in mm):

X (m)	DL1			DL2P			DL2W		
	M	V	Defl	M	V	Defl	M	V	Defl
0.0	0.0	256.4	0.0	0.0	49.3	0.0	0.0	48.1	0.0
1.8	428.8	220.0	16.5	82.5	42.4	1.5	80.5	41.4	1.5
3.6	792.0	183.5	32.1	152.6	35.5	2.9	149.0	34.7	2.9
5.4	1089.6	147.1	46.1	210.4	28.7	4.2	205.4	28.0	4.1
6.0	1174.1	134.8	50.3	226.9	26.4	4.6	221.5	25.8	4.5
7.2	1321.3	110.3	57.9	255.8	21.8	5.3	249.8	21.3	5.2
9.0	1486.5	73.3	67.3	288.9	15.0	6.2	282.1	14.6	6.0
10.8	1585.2	36.3	74.1	309.7	8.1	6.8	302.4	7.9	6.6
12.6	1617.2	-0.7	78.2	318.1	1.2	7.2	310.6	1.2	7.0
14.4	1582.7	-37.7	79.4	314.1	-5.6	7.4	306.7	-5.5	7.2
16.2	1481.6	-74.7	77.8	297.9	-12.5	7.3	290.8	-12.2	7.1
18.0	1313.9	-111.7	73.7	269.2	-19.3	6.9	262.9	-18.9	6.8
19.8	1079.6	-148.7	67.3	228.3	-26.2	6.4	222.9	-25.6	6.2
21.6	778.8	-185.6	59.0	174.9	-33.1	5.7	170.8	-32.3	5.5
23.4	411.3	-222.6	49.3	109.3	-39.9	4.8	106.7	-39.0	4.7
25.2	-22.3	-259.2	38.9	31.3	-46.8	3.9	30.5	-45.7	3.8
25.8	-181.5	-271.6	35.3	2.5	-49.1	3.5	2.5	-47.9	3.5
27.0	-522.1	-296.3	28.4	-59.1	-53.6	2.9	-57.7	-52.4	2.8
28.8	-1089.0	-333.6	18.7	-161.8	-60.5	2.0	-158.0	-59.1	1.9
30.6	-1723.0	-370.9	10.6	-276.8	-67.3	1.2	-270.3	-65.8	1.1
32.4	-2425.2	-409.3	4.7	-404.2	-74.2	0.5	-394.7	-72.4	0.5
33.0	-2674.5	-422.6	3.3	-449.4	-76.5	0.4	-438.8	-74.7	0.4
34.2	-3197.5	-449.3	1.2	-543.9	-81.1	0.1	-531.1	-79.1	0.1
36.0	-4042.8	-490.0	0.0	-696.0	-87.9	0.0	-679.6	-85.8	0.0

Table 12.7.10.8-2 - Unfactored Live Loads (Moments in kN·m, Shears in kN, Deflections in mm):

X (m)	LRFD Live Load (including DF and IM)					LRFD Fatigue (including DF and IM)				
	PosM	NegM	PosV	NegV	Defl	PosM	NegM	PosV	NegV	Defl
0.0	0.0	0.0	532.1	-68.9	0.0	0.0	0.0	196.9	-23.8	0.0
1.8	655.0	-90.9	488.5	-69.4	4.4	206.2	-27.0	181.8	-23.8	3.5
3.6	1216.3	-181.7	446.3	-71.0	8.7	378.7	-54.1	166.9	-23.8	7.0
5.4	1685.2	-272.6	405.4	-78.1	12.6	518.4	-81.1	152.3	-23.8	10.2
6.0	1821.5	-302.8	392.3	-85.7	13.9	557.9	-90.1	147.5	-26.0	11.1
7.2	2063.8	-363.4	366.1	-100.9	16.2	626.4	-108.1	138.0	-30.3	13.0
9.0	2354.0	-454.3	328.3	-130.6	19.3	716.2	-135.2	124.1	-38.7	15.4
10.8	2565.5	-545.1	292.1	-163.2	21.8	778.7	-162.2	110.6	-47.9	17.3
12.6	2708.4	-636.0	257.7	-196.4	23.7	812.9	-189.2	97.6	-61.3	19.0
14.4	2770.5	-726.8	224.9	-230.2	25.0	820.5	-216.3	85.1	-75.6	20.0
16.2	2755.7	-817.7	194.1	-264.3	25.6	804.1	-243.3	73.2	-90.2	20.4
18.0	2698.6	-908.5	165.1	-298.6	25.5	792.4	-270.3	61.9	-104.5	20.3
19.8	2570.2	-999.4	138.1	-333.1	24.7	764.7	-297.4	51.3	-118.4	19.8
21.6	2373.4	-1090.2	113.2	-367.6	23.3	712.7	-324.4	41.4	-131.9	18.7
23.4	2113.3	-1181.1	90.4	-402.0	21.3	638.9	-351.4	32.4	-145.0	17.2
25.2	1796.9	-1354.9	69.7	-436.0	18.9	546.3	-378.5	24.5	-157.5	15.2
25.8	1679.1	-1386.4	63.6	-447.1	17.9	511.8	-387.5	22.7	-161.5	14.4
27.0	1431.1	-1451.7	51.2	-469.4	15.9	438.2	-405.5	19.2	-169.4	12.8
28.8	1027.8	-1554.8	35.7	-502.3	12.7	316.1	-432.5	14.2	-180.7	10.3
30.6	648.1	-1712.5	24.2	-534.5	9.4	197.2	-459.6	9.8	-191.4	7.6
32.4	370.4	-2108.5	14.1	-565.8	6.1	122.5	-486.6	5.9	-201.4	5.0
33.0	285.8	-2281.4	11.2	-575.9	5.0	102.1	-495.6	4.9	-204.4	4.1
34.2	144.1	-2652.9	5.3	-596.3	3.0	61.2	-513.6	2.8	-210.6	2.4
36.0	0.0	-3274.1	0.0	-626.0	0.0	0.0	-540.7	0.0	-219.1	0.0

12.7.10.9 LIVE LOAD DEFLECTION (Article S2.5.2.6)

For a steel girder bridge without pedestrian loads, the recommended deflection limit is the span length/800. Thus, for this example the maximum live load deflection is:

$$\Delta_{max} = \frac{Span}{800} = \frac{36000}{800} = 45\ mm$$

The maximum computed deflection has been determined to be 26 mm which is less than the recommended allowable deflection.

12.7.10.10 SPECIFICATION CHECKS

1. Flexural capacity
   - Critical Sections: **14.4 m, 36.0 m** (points of maximum positive and negative moment)
   - Web fatigue stress limits (flexure and shear)

2. Shear (stiffened web) - Show a check 0 and 36.0 meters (distance measured centerline of bearing at the abutment)

3. Fatigue (at selected points for various detail categories)

4. Shear connectors

5. Transverse stiffeners and bearing stiffeners

6. Wind loads

12.7.10.10.1 Specification Check: Flexure

Compute Flexural Capacity at Point of Maximum Positive Moment (x = 14.4 m)

Check Compactness:

Web Slenderness (S6.10.5.2.2c)

$$\frac{2D_{cp}}{t_w} \le 3.76\sqrt{\frac{E}{F_{yc}}} \qquad \text{(S6.10.5.2.2c-1)}$$

D_{cp} = 0, therefore, the section is compact

Compression flange slenderness and compression flange bracing: For the final stage of loading, i.e., at the strength limit state, the compression flange is assumed to be fully braced by the concrete slab. However, the compression flange bracing and slenderness requirements should be checked during a construction analysis using the appropriate loading condition.

Compute the Flexural Capacity of a Compact Section:

Compactness must be checked at the interior support to determine the applicable flexural capacity equation for the section of interest.

Check Compactness at the Pier (x = 36 m):

Web Slenderness (S6.10.5.2.3b)

$$\frac{2D_{cp}}{t_w} \le 3.76\sqrt{\frac{E}{F_{yc}}} \qquad \text{(S6.10.5.2.3b-1)}$$

where:

D_{cp} = 986
t_w = 10

$$\frac{2D_{cp}}{t_w} = \frac{2(986)}{10} = 197$$

$$3.76\sqrt{\frac{E}{F_{yc}}} = 3.76\sqrt{\frac{200000}{345}} = 90.5 < 197$$

therefore, the section at the interior support is non-compact.

Ductility Check:

Because the factored stress in the tension flange exceeds the yield stress of the flange, the ductility check of Article S6.10.5.2.2b applies:

$$\left(\frac{D_p}{D'}\right) \le 5 \qquad \text{(S6.10.5.2.2b-1)}$$

in which:

$$D' = \beta\frac{(d + t_s + t_h)}{7.5}$$

where:

β = 0.9 for F_y = 250 MPa
= 0.7 for F_y = 345 MPa
d = $D + t_{fc} + t_{ft}$ = 1425 + 13 + 30
= 1468
t_s = 200
t_h = 0 (no haunch assumed)

Calculate the depth to the plastic neutral axis from the top of the slab, D_p: (see Appendix SA6.1, assuming PNA is in the slab above P_{rb})

$$D_p = \bar{Y} = t_s\left[\frac{P_{rb} + P_c + P_w + P_t - P_{rt}}{P_s}\right]$$

The longitudinal reinforcing steel in the slab was neglected for positive flexural resistance in this example (as is the practice by some DOT's), therefore, the forces P_{rt} and $P_{rb} = 0$.

$$D_p = \bar{Y} = 200\left[\frac{0 + 345(13)(300) + 345(12)(1425) + 345(30)(300) - 0}{0.85(28)(2550)(200)}\right]$$
$$= 170$$

And the upper limit on the depth of web in compression at the plastic moment is calculated as:

$$\beta\frac{(d + t_s + t_h)}{7.5} = (0.7)\left[\frac{1468 + 200 + 0}{7.5}\right] = 156 \quad (OK)$$

$$\frac{D_p}{D'} = \frac{170}{156} = 1.09 < 5 \quad (OK)$$

The Specification allows one of two methods for computing the flexural capacity in such cases, Method "A" and Method "B." In this case, the simpler Method "A" has been selected. Thus, the flexural capacity shall be determined by the following:

- If $D_p \leq D'$, then:

  $M_N = M_p$ (S6.10.5.2.2a-2)

- If $D' < D_p \leq 5D'$, then:

$$M_n = \frac{5M_p - 0.85M_y}{4} + \frac{0.85M_y - M_p}{4}\left(\frac{D_p}{D'}\right)$$

Because the section is non-compact, M_n shall not be greater than:

$$M_n = 1.3R_hM_y \quad \text{(S6.10.5.2.2a-3)}$$

where:

R_h = 1.0 (non-hybrid section; see S6.10.5.4.1a)

Compute the yield moment, M_y:

Table 12.7.10.10.1-1 - Factored Moment (values in kN·m; Strength I):

Force Effect	Unfactored Values	$\eta\gamma$	Factored Values
M_{DL1}	1582.7	(0.95)(1.25)	1879
M_{DL2P}	314.1	(0.95)(1.25)	373
M_{DL2W}	306.7	(0.95)(1.50)	437
M_{LL}	2770.5	(0.95)(1.75)	4606

$$M_y = M_{DL1} + M_{DL2} + S_{LL}\left(F_{yf} - \frac{M_{DL1}}{S_{DL1}} - \frac{M_{DL2}}{S_{DL2}}\right)$$

(See SA6.2)

Compute the yield moment for each flange and the minimum yield moment controls:

$$M_{ycf} = 1879 + 810 + 1.364E8\left(345 - \frac{1879(1E6)}{1.083E7} - \frac{810(1E6)}{4.495E7}\right)1E6$$
$$= 23622\ kN{\cdot}m$$

$$M_{ytf} = 1879 + 810 + 2.203E7\left(345 - \frac{1879(1E6)}{1.490E7} - \frac{810(1E6)}{2.027E7}\right)1E6$$
$$= 6631\ kN{\cdot}m$$

$M_{y\ min}$ = 6631 kN·m
M_u = 1879 + 810 + 4606 = 7295 kN·m
D_p = 170
D′ = 156
5D′ = 780

156 < 170 < 780

Therefore:

$$M_n = \frac{5(9988) - 0.85(6631)}{4} + \frac{0.85(6631) - 9988}{4}\left(\frac{170}{156}\right)$$
$$= 9890\ \text{kN·m}$$

$1.3\ R_h\ M_y$ = (1.3)(1.0)(6631) = 8620 kN·m
9890 kN·m > 8620 kN·m, therefore, M_n = 8620 kN·m

8620 kN·m > M_u = 7295 kN·m (OK)

Check Service Limit State Control of Permanent Deflections at Location of Maximum Service II Flexural Stress (x = 14.4 m).

Article S6.10.3:

The Service II load combination corresponds to the "overload check" in the 1992 AASHTO Standard Specifications. It was determined that for this problem, the maximum flexural stress for the Service II load combination was located in the bottom flange at a distance 14.4 meters from the support at the abutment. Figure 1 shows the Service II flexural stress in each of the flanges due to the positive and negative live load effects.

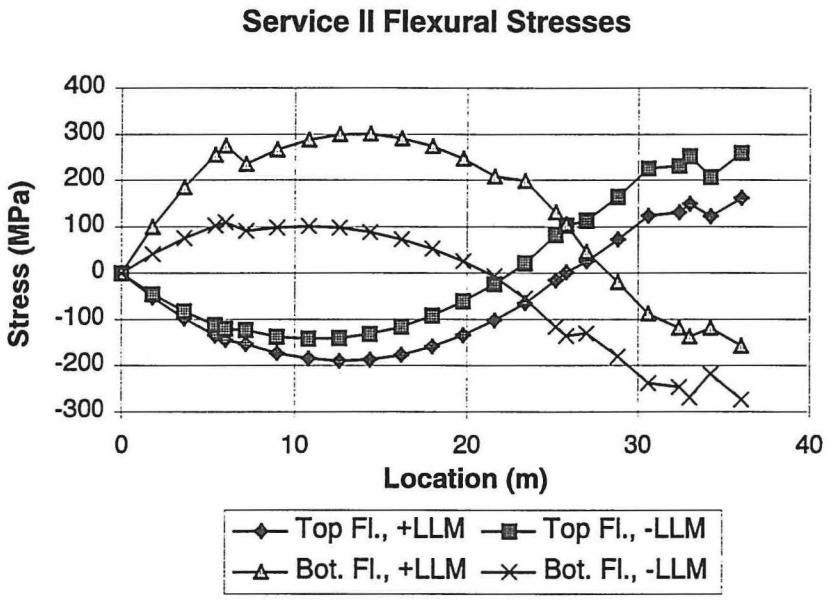


Figure 12.7.10.10.1-1 - Service II Stresses (tension is positive, compression is negative)

For a composite section, the stress in each flange is limited by the following equation.

$$f_f \le 0.95 R_h F_{yf} \qquad \text{(S6.10.3.2-1)}$$

So,

$f_f \le 0.95(1.0)(345) = 328$ MPa

and the maximum stress at 14.4 meters is 300 MPa, therefore, the girder satisfies the Service II limit state requirements. It should also be noted that the live load factor for the Service II limit state is 1.30. However, if the Service II limit state is to be

checked for a permit vehicle, the Owner may wish to select a reduced live load factor (SC6.10.3.1).

Check the Web Fatigue Stresses at the Point of Maximum Positive Moment (x = 14.4 m), Article S6.10.4.3:

This provision requires the calculation of the depth of web in compression resulting from the maximum fatigue loading (including the unfactored permanent load stresses). Note: the load factor for the fatigue live load is 0.75 and the general load factor, η, does not apply for the fatigue limit state. Also, Article S6.10.4.2 requires that twice the factored fatigue be used for checking the web fatigue stress. This applies to both shear and flexural fatigue stresses due to the fatigue vehicle.

Calculate the stress in each flange due to the unfactored dead load plus twice the factored fatigue live load.

$$f_c = \left(\frac{M_{DL1}}{S_{cDL1}} + \frac{M_{DL2}}{S_{cDL2}} + \frac{2.0\,M_{LLf}}{S_{cLL}} \right) = 169\ MPa$$

$$f_t = \left(\frac{M_{DL1}}{S_{tDL1}} + \frac{M_{DL2}}{S_{tDL2}} + \frac{2.0\,M_{LLf}}{S_{tLL}} \right) = 193\ MPa$$

$$D_c = \frac{df_c}{(f_c + f_t)} - t_{fc} = \frac{1468(169)}{(169 + 193)} - 13 = 672\ mm$$

Check:

$$\frac{2D_c}{t_w} \le 5.76\sqrt{\frac{E}{F_{yc}}}$$

$$\frac{2D_c}{t_w} = \frac{2(672)}{12} = 112$$

$$5.76\sqrt{\frac{E}{F_{yc}}} = 5.76\sqrt{\frac{200000}{345}} = 139 > 112$$

Therefore,

$f_{cf} \le R_h F_{yc}$ (S6.10.5.4.3-1)

$R_h = 1.0$
$F_{yc} = 345$ MPa

So,

$f_{cf} \le 345$ MPa (OK)

Web Fatigue Shear Stress at X = 14.4 m (S6.10.4.4)

In addition to the web fatigue check for flexure, Article S6.10.4.4 contains a limit on web fatigue shear stresses. Because this is the point of maximum moment, the shear stress is low and will not control. This check will be illustrated at the point of maximum negative moment.

Compute Flexural Capacity at Point of Maximum Negative Moment (x = 36.0 m)

Table 12.7.10.10.1-2 - Factored Moments (Strength I; values in kN-m):

Force Effect	Unfactored Values	$\eta\gamma$	Factored Values
M_{DL1}	-4043	(0.95)(1.25)	-4801
M_{DL2P}	-696	(0.95)(1.25)	-827
M_{DL2W}	-680	(0.95)(1.50)	-968
M_{LL}	-3274	(0.95)(1.75)	-5443

Compactness:

The section at the interior support was already determined to be non-compact (see calculations for positive moment section).

Compute the stresses in the flanges due to the factored loads (Strength I):

Compression flange:

$$f_c = \left(\frac{M_{DL1}}{S_{cDL1}} + \frac{M_{DL2}}{S_{cDL2}} + \frac{M_{LL}}{S_{cLL}}\right) = 340\ MPa$$

Tension flange:

$$f_t = \left(\frac{M_{DL1}}{S_{tDL1}} + \frac{M_{DL2}}{S_{tDL2}} + \frac{M_{LL}}{S_{tLL}}\right) = 321\ MPa$$

Depth of Web in Compression at Elastic Moment, D_c:

Article S6.10.5.4.1a provides some guidance for the calculation of the depth of web in compression at the elastic design moment. The Specification and the accompanying commentary suggests that the composite dead loads are assumed to be applied to the short-term composite section. This was intended to be a simplification to the process. However, because the stresses, applied to the appropriate sections for dead and live load, need to be computed for other reasons, this "simplification" will not be used.

If the section at the location under consideration reaches the yield moment under the factored loads, the value of D_c will be based on the yield moment, i.e., the depth of the web in compression in the elastic range. The stress in the compression flange is taken as the least of the yield stress and the stress based on the factored loads.

$$M_y = M_{DL1} + M_{DL2} + M_{AD}$$

$$M_y = M_{DL1} + M_{DL2} + S_{LL}\left(F_{yf} - \frac{M_{DL1}}{S_{DL1}} - \frac{M_{DL2}}{S_{DL2}}\right)$$

(computed for each flange, the minimum M_y controls)

$$f_c = \left(\frac{M_{DL1}}{S_{cDL1}} + \frac{M_{DL2}}{S_{cDL2}} + \frac{M_{ADmin}}{S_{cLL}}\right)$$

$$f_t = \left(\frac{M_{DL1}}{S_{tDL1}} + \frac{M_{DL2}}{S_{tDL2}} + \frac{M_{ADmin}}{S_{tLL}}\right)$$

The depth of the web in compression, D_c, can be calculated based on a linear stress distribution between the compression and tension flanges at the yield moment. Thus, D_c is calculated by the following expression:

$$D_c = \frac{df_c}{(f_c + f_t)} - t_{fc}$$

Because the compression flange has <u>not</u> yielded, the depth of web in compression, D_c, is based on the <u>factored</u> stresses:

Compute D_c:

$$D_c = \frac{df_c}{(f_c + f_t)} - t_{fc} = \frac{(1425 + 42 + 48)(340)}{(340 + 321)} - 48 = 731\ mm$$

Check the web slenderness as required by S6.10.5.3.2b:

$$\frac{2D_c}{t_w} \le 6.77\sqrt{\frac{E}{f_c}} \qquad \text{(S6.10.5.3.2b-1)}$$

If the above slenderness is not met, longitudinal stiffeners would be required and the web slenderness would be limited to $11.63(E/f_c)^{0.5}$.

$$\frac{2D_c}{t_w} = \frac{2(731)}{10} = 146$$

$$6.77\sqrt{\frac{E}{f_c}} = 6.77\sqrt{\frac{200000}{340}} = 164 > 146 \quad (OK)$$

Check compression flange slenderness:

$$\frac{b_f}{2t_f} \le 1.38\sqrt{\frac{E}{f_c\sqrt{\frac{2D_c}{t_w}}}} \qquad \text{(S6.10.5.3.3c-1)}$$

$$\frac{b_f}{2t_f} = \frac{460}{(2)(48)} = 4.79$$

$$1.38\sqrt{\frac{E}{f_c\sqrt{\frac{2D_c}{t_w}}}} = 1.38\sqrt{\frac{200000}{340\sqrt{\frac{2(731)}{10}}}} = 9.63 \quad (OK)$$

Check compression flange bracing: (S6.10.5.3.3d)

$$L_b \le 1.76 r_t\sqrt{\frac{E}{F_{yc}}} \qquad \text{(S6.10.5.3.3d-1)}$$

where r_t is the minimum (within the braced length) radius of gyration of the compression flange plus one third of the depth of web in compression, D_c. Aside from the analysis point being considered, the nearest adjacent brace point for the compression flange is located at a distance of 28.8 meters (Section "D") from the centerline of bearing at the abutment. The needed data for that point is as follows:

Table 12.7.10.10.1-3 - Factored Moments at X = 28.8 m
(Strength I, values in kN·m):

Force Effect	Unfactored Values	$\eta\gamma$	Factored Values
M_{DL1}	1089	(0.95)(1.25)	1293
M_{DL2P}	162	(0.95)(1.25)	192
M_{DL2W}	158	(0.95)(1.50)	225
M_{LL}	1555	(0.95)(1.75)	2585

Stresses and section dimensions at X = 28.8 m (Section "D"):

f_c = 224 MPa
f_t = 205 MPa
D_c = 741 mm (based on factored stresses above)
t_{fc} = 23 mm
b_{fc} = 460 mm
t_w = 10 mm

$$r_t = \sqrt{\frac{I_{cf+1/3\,D_c}}{A_{cf+1/3\,D_c}}}$$

The moment of inertia of 1/3 of the depth of web in compression about the vertical axis is typically negligible, therefore:

$$r_t = \sqrt{\frac{\frac{23(460)^3}{12}}{23(460) + \frac{741}{3}(10)}} = 120\ mm$$

$$1.76\, r_t \sqrt{\frac{E}{F_{yc}}} = 1.76(120)\sqrt{\frac{200000}{345}} = 5085\ mm$$

But, the unbraced length, L_b, was assumed to be 7200 mm. Therefore, lateral torsional buckling controls.

Lateral Torsional Buckling of Compression Flanges (S6.10.5.5.2):

Check

$$L_b \le 4.44 r_t \sqrt{\frac{E}{F_{yc}}} :$$

$$4.44 r_t \sqrt{\frac{E}{F_{yc}}} = 4.44(120)\sqrt{\frac{200000}{345}} = 12828\ mm \quad (OK)$$

Therefore, Equation S6.10.5.5.2-1 applies:

$$F_n = C_b R_b R_h F_{yc}\left[1.33 - 0.187\left(\frac{L_b}{r_t}\right)\sqrt{\frac{F_{yc}}{E}}\right] \le R_b R_h F_{yc}$$

where:

$$C_b = 1.75 - 1.05\left(\frac{P_l}{P_h}\right) + 0.3\left(\frac{P_l}{P_h}\right)^2 \le 2.3$$

At X = 28.8 m:
f_c = 224 MPa
A_{cf} = (23)(460) = 10 580 mm^2 (Section "D")

thus,

P_l = (224 N/mm^2) (10 580 mm^2)
= 2.37E6 N

For the brace point located at the analysis point (x = 36 m; Section "F"):

f_c = 340 MPa
A_{cf} = (48)(460) = 22 080 mm^2

thus,

P_l = (340 N/mm^2) (22 080 mm^2)
= 7.50E6 N

$$C_b = 1.75 - 1.05\left(\frac{2.37}{7.50}\right) + 0.3\left(\frac{2.37}{7.50}\right)^2 = 1.45 \le 2.3$$

Therefore, use C_b = 1.45

Compute the flange stress reduction factor, R_b:

Check $\frac{2D_c}{t_w} \le \lambda_b\sqrt{\frac{E}{f_c}}$ (S6.10.5.4.2a-1)

$$\frac{2D_c}{t_w} = \frac{2(731)}{10} = 146$$

$\lambda_b = 5.76$ *if* $A_{cf} \ge A_{tf}$
$\lambda_b = 4.64$ *if* $A_{cf} < A_{tf}$
therefore, $\lambda_b = 5.76$

$$\lambda_b\sqrt{\frac{E}{f_c}} = 5.76\sqrt{\frac{200000}{345}} = 139 < 146$$

Therefore, $R_b \neq 1.0$

So,

$$R_b = 1 - \left(\frac{a_r}{1200 + 300\,a_r}\right)\left(\frac{2D_c}{t_w} - \lambda_b\sqrt{\frac{E}{f_c}}\right) \quad \text{(S6.10.5.4.2a-2)}$$

where:

$$a_r = \frac{2D_c t_w}{A_{fc}} = \frac{2(731)(10)}{22080} = 0.66 \quad \text{(S6.10.5.4.2a-3)}$$

$$R_b = 1 - \left(\frac{0.66}{1200 + 300(0.66)}\right)(146 - 139) = 0.997$$

for the compression flange only. R_b for the tension flange is 1.0 (S6.10.5.4.2b).

$$F_n = (1.45)(0.997)(1.0)(345)\left[1.33 - 0.187\left(\frac{7200}{120}\right)\sqrt{\frac{345}{200000}}\right]$$
$$= 431\ MPa \le R_b R_h F_{yc}$$

So,

$R_b R_h F_{yc} = (0.997)(1.0)(345) = 344\ MPa$ controls.

$F_r = \phi_f F_n = (1.0)(344) = 344$ MPa

and the required flexural resistance, F_u, is 340 MPa which is approximately one percent less than the actual resistance of 344 MPa, therefore, this section is adequate for flexure at the pier.

Check the Web Fatigue Stresses at the Point of Maximum Negative Moment (x = 36 m), Article S6.10.4.3:

Flexural Fatigue Stress:

Table 12.7.10.10.1-4 - Stress Due to the Unfactored Dead Load Plus Twice the Factored Fatigue Live Load

Force Effect	Unfactored Values	γ	Factored Values
M_{DL1}	-4043	(1.0)	-4043
M_{DL2P}	-696	(1.0)	-696
M_{DL2W}	-680	(1.0)	-680
M_{LL}	-541	(2)(0.75)	-812

$$f_c = \left(\frac{M_{DL1}}{S_{cDL1}} + \frac{M_{DL2}}{S_{cDL2}} + \frac{2.0\,M_{LLf}}{S_{cLL}}\right) = 179\ MPa$$

$$f_t = \left(\frac{M_{DL1}}{S_{tDL1}} + \frac{M_{DL2}}{S_{tDL2}} + \frac{2.0\,M_{LLf}}{S_{tLL}}\right) = 180\ MPa$$

$d = 42 + 1425 + 48 = 1515$

$t_{fc} = 48$

$$D_c = \frac{df_c}{(f_c + f_t)} - t_{fc} = \frac{(1515)(179)}{(179 + 180)} - 48 = 707\ mm$$

Check:

$$\frac{2D_c}{t_w} \le 5.76\sqrt{\frac{E}{F_{yc}}}$$

$$\frac{2D_c}{t_w} = \frac{2(707)}{10} = 141$$

$$5.76\sqrt{\frac{E}{F_{yc}}} = 5.76\sqrt{\frac{200000}{345}} = 139 < 141$$

Therefore,

$$f_{cf} \le R_h F_{yc}\left[3.58 - 0.448\left(\frac{2D_c}{t_w}\right)\sqrt{\frac{F_{yc}}{E}}\right] \qquad \text{(S6.10.4.3-2)}$$

$R_h = 1.0$

$F_{yc} = 345$ MPa

So,

$$f_{cf} \le R_h F_{yc}\left[3.58 - 0.448\left(\frac{2D_c}{t_w}\right)\sqrt{\frac{F_{yc}}{E}}\right] = 1.0(345)$$

$$\left[3.58 - 0.448(141)\sqrt{\frac{345}{200000}}\right] = 0.956(345) = 330\ MPa$$

330 MPa > 179 MPa (OK)

Check Web Fatigue Shear Stress at X = 36.0 m (S6.10.4.4)

Transversely stiffened homogeneous sections with or without longitudinal stiffeners must satisfy:

$v_{cf} \le 0.58\ C\ F_{yw}$ (S6.10.4.4-1)

The shear stress, v_{cf}, is calculated on the basis of the shear due to the unfactored permanent load and twice the factored fatigue load (per Article S6.10.4.2):

V_{DL1} = -490 kN
V_{DL2} = -88 kN - 86 kN = -174 kN
$V_{fatigue}$ = 0.75(2.0)(-219) = -329 kN

therefore, V_{Total} = -993 kN

$$v_{cf} = \frac{V_{Total}}{A_{web}} = \frac{(993\,kN)(1000\,N/kN)}{(1425)(10)} = 69.7\ MPa$$

The maximum stiffener spacing, d_o, is assumed for the calculation of the web buckling coefficient, C. d_o, therefore, is taken as the least of:

$$d_o \le D\left[\frac{260}{(D/t_w)}\right]^2$$ (S6.10.7.3.2-2)

and,

$d_o \le 3D$ (see S6.10.7.1)

where:

D = web depth = 1425 mm
t_w = web thickness = 10 mm

So,

$$d_o \le 1425\left[\frac{260}{(1425/10)}\right]^2 = 4744$$

But, the maximum allowable stiffener spacing for an interior panel is controlled by three times the depth of the web or 4275 mm

Calculate the stiffener spacing coefficient, k:

$$k = 5 + \frac{5}{\left(\frac{d_o}{D}\right)^2} \qquad \text{(S6.10.7.3.3a-8)}$$

$$k = 5 + \frac{5}{\left(\frac{4275}{1425}\right)^2} = 5.56$$

To determine the appropriate equation for the ratio of shear buckling stress to shear yield strength, C, we need to compare the web slenderness to the limiting values:

Check $\frac{D}{t_w} \le 1.10\sqrt{\frac{Ek}{F_{yw}}}$?

$$\frac{D}{t_w} = \frac{1425}{10} = 142.5$$

$$1.10\sqrt{\frac{Ek}{F_{yw}}} = 1.10\sqrt{\frac{200000(5.56)}{345}} = 62.4$$

Check $1.10\sqrt{\frac{Ek}{F_{yw}}} \le \frac{D}{t_w} \le 1.38\sqrt{\frac{Ek}{F_{yw}}}$?

$$1.38\sqrt{\frac{Ek}{F_{yw}}} = 1.38\sqrt{\frac{200000(5.56)}{345}} = 78.3 < 142.5$$

Therefore,

$$C = \frac{1.52}{\left(\frac{D}{t_w}\right)^2\left(\frac{Ek}{F_{yw}}\right)} = \frac{1.52}{\left(\frac{1425}{10}\right)^2\left(\frac{200000(5.56)}{345}\right)} = 0.241$$

(S6.10.7.3.3a-7)

And the fatigue web shear stress limit is then:

$$v_{cfmax} = 0.58CF_{yw} \qquad \text{(S6.10.4.4-1)}$$

$= 0.58(0.241)(345)$
$= 48.3 \text{ MPa} < 69.7 \text{ MPa (No Good)}$

Thus, this provision must be checked with a more realistic stiffener spacing which has been determined as a part of the shear design (further along in this example) to be 1000 mm at this analysis point. So,

$$k = 5 + \frac{5}{\left(\frac{1000}{1425}\right)^2} = 15.15$$

$$1.10\sqrt{\frac{Ek}{F_{yw}}} = 1.10\sqrt{\frac{200000(15.15)}{345}} = 103.1 < 142.5$$

Check $1.10\sqrt{\frac{Ek}{F_{yw}}} \le \frac{D}{t_w} \le 1.38\sqrt{\frac{Ek}{F_{yw}}}$?

$$1.38\sqrt{\frac{Ek}{F_{yw}}} = 1.38\sqrt{\frac{200000(15.15)}{345}} = 129 < 142.5$$

Therefore,

$$C = \frac{1.52}{\left(\frac{D}{t_w}\right)^2}\left(\frac{Ek}{F_{yw}}\right) = \frac{1.52}{\left(\frac{1425}{10}\right)^2}\left(\frac{200000(15.15)}{345}\right) = 0.657$$

And the fatigue web shear stress limit is then:

$$v_{cfmax} = 0.58CF_{yw} \qquad \text{(S6.10.4.3-1)}$$

$= 0.58(0.657)(345)$
$= 132 \text{ MPa} > 69.7 \text{ MPa (OK)}$

12.7.10.10.2 Specification Check: Shear

Compute and Check the Shear Capacity, V_r, at the abutment:
(x = 0 m)

Table 12.7.10.10.2-1 - Factored Shear Load (Strength I; values in kN):

Force Effect	Unfactored Values	$\eta\gamma$	Factored Values
V_{DL1}	256.4	(0.95)(1.25)	304.5
V_{DL2P}	49.3	(0.95)(1.25)	58.5
V_{DL2W}	48.1	(0.95)(1.50)	68.5
V_{LL}	532.1	(0.95)(1.75)	884.6
			1316 kN

Shear Capacity of an Exterior Panel:

$V_n = CV_p$ (S6.10.7.3.3c-1)

where:

$V_p = 0.58F_{yw}Dt_w$ (S6.10.7.3.3c-2)

for which:

F_{yw} = 345 MPa
D = 1425 mm
t_w = 12 mm

Therefore, the web yield strength is:

$V_p = 0.58(345)(12)(1\text{ kN}/1000\text{ N})$
$= 3422$ kN

Per Article S6.10.7.3.3c, the maximum stiffener spacing for an exterior panel is:

$1.5D = 1.5(1425) = 2138$ mm

Assume a stiffener spacing of 2100 mm

Calculate C:

$$k = 5 + \frac{5}{\left(\frac{d_o}{D}\right)^2} = 5 + \frac{5}{\left(\frac{2100}{1425}\right)^2} = 7.30$$

Check $\frac{D}{t_w} \le 1.10\sqrt{\frac{Ek}{F_{yw}}}$?

$$\frac{D}{t_w} = \frac{1425}{12} = 119$$

$$1.10\sqrt{\frac{Ek}{F_{yw}}} = 1.10\sqrt{\frac{200000(7.30)}{345}} = 71.6 < 119$$

Therefore, check $\frac{D}{t_w} \le 1.38\sqrt{\frac{Ek}{F_{yw}}}$?

$$1.38\sqrt{\frac{Ek}{F_{yw}}} = 1.38\sqrt{\frac{200000(7.30)}{345}} = 90 < 119$$

therefore,

$$C = \frac{1.52}{\left(\frac{D}{t_w}\right)^2}\left(\frac{Ek}{F_{yw}}\right) = \frac{1.52}{\left(\frac{1425}{12}\right)^2}\left(\frac{200000(7.30)}{345}\right) = 0.45$$

Finally,

$V_n = CV_p = 0.45(3422\ kN) = 1540\ kN$

and,

$V_r = \phi V_n = 1.0(1540)$
$= 1540\ kN > 1316\ kN$ (OK)

Compute the Factored Shear Resistance at the Pier (x = 36 m):

Table 12.7.10.10.2-2 - Factored Shear Load (Strength I; values in kN):

Force Effect	Unfactored Values	$\eta\gamma$	Factored Values
V_{DL1}	-490	(0.95)(1.25)	-582
V_{DL2P}	-88	(0.95)(1.25)	-105
V_{DL2W}	-86	(0.95)(1.50)	-123
V_{LL}	-626	(0.95)(1.75)	-1041
			-1851 kN

Shear Capacity of an Interior Panel of a Non-Compact Section (S6.10.7.3.3b):

Because $f_u > 0.75\ \phi_f F_y$ (340 MPa > 259 MPa):

$$V_n = RV_p\left[C + \frac{0.87(1 - C)}{\sqrt{1 + \left(\frac{d_o}{D}\right)^2}}\right] \geq CV_p \qquad \text{(S6.10.7.3.3b-2)}$$

where:

$$R = \left[0.6 + 0.4\left(\frac{F_r - f_u}{F_r - 0.75\phi_f F_y}\right)\right] \leq 1.0 \qquad \text{(S6.10.7.3.3b-3)}$$

$$V_p = 0.58\ F_{yw}\ D\ t_w \qquad \text{(S6.10.7.3.3a-4)}$$

Calculate R:

From previous calculations for flexure:

F_r = 344 MPa
F_u = 340 MPa
F_y = 345 MPa
ϕ_f = 1.0

Therefore,

$$R = \left[0.6 + 0.4\left(\frac{344 - 340}{344 - 0.75(1.0)345}\right)\right] = 0.619 \leq 1.0$$

Calculate V_p:

$V_p = 0.58\ F_{yw}\ D\ t_w$
$= 0.58(345)(1425)(10)(1\ kN/1000\ N)$
$= 2851\ kN$

Compute the maximum stiffener spacing, d_o, as the lesser of:

- $d_{omax} = D\left[\frac{260}{(D/t_w)}\right]^2$ (S6.10.7.3.2-2)
- 3.0 D (for interior panels without longitudinal stiffeners)

So,

$$d_{omax} = 1425\left[\frac{260}{(1425/10)}\right]^2 = 4744$$
$$3.0D = 3(1425) = 4275 \quad (controls)$$

Try a spacing of 4200 mm:

Compute C:

$$k = 5 + \frac{5}{\left(\frac{d_o}{D}\right)^2} = 5 + \frac{5}{\left(\frac{4200}{1425}\right)^2} = 5.58$$

Check $\frac{D}{t_w} \le 1.38\sqrt{\frac{Ek}{F_{yw}}}$?

$$\frac{D}{t_w} = \frac{1425}{10} = 143$$

$$1.38\sqrt{\frac{Ek}{F_{yw}}} = 1.38\sqrt{\frac{200000(5.58)}{345}} = 78.5 < 143$$

Therefore,

$$C = \frac{1.52}{\left(\frac{D}{t_w}\right)^2}\left(\frac{Ek}{F_{yw}}\right) = \frac{1.52}{\left(\frac{1425}{10}\right)^2}\left(\frac{200000(5.58)}{345}\right) = 0.242$$

$$V_n = RV_p\left[C + \frac{0.87(1 - C)}{\sqrt{1 + \left(\frac{d_o}{D}\right)^2}}\right] \ge CV_p$$

$$V_n = (0.619)(2851)\left[0.242 + \frac{0.87(1 - 0.242)}{\sqrt{1 + \left(\frac{4200}{1425}\right)^2}}\right]$$

$= (0.619)(2851)(0.453)$
$= 801 \text{ kN} > 0.242(2851) = 690 \text{ kN (OK)}$

$V_r = \phi V_n = (1.0)(801) = 801 \text{ kN}$

But, 801 kN < V_u = 1851 kN so the section fails in shear.

Try decreasing the stiffener spacing to 1000 mm:

$$k = 5 + \frac{5}{\left(\frac{d_o}{D}\right)^2} = 5 + \frac{5}{\left(\frac{1000}{1425}\right)^2} = 15.15$$

Check $\frac{D}{t_w} \leq 1.38\sqrt{\frac{Ek}{F_{yw}}}$?

$$\frac{D}{t_w} = \frac{1425}{10} = 143$$

$$1.38\sqrt{\frac{Ek}{F_{yw}}} = 1.38\sqrt{\frac{200000(15.15)}{345}} = 129 < 143$$

Therefore,

$$C = \frac{1.52}{\left(\frac{D}{t_w}\right)^2}\left(\frac{Ek}{F_{yw}}\right) = \frac{1.52}{\left(\frac{1425}{10}\right)^2}\left(\frac{200000(15.15)}{345}\right) = 0.657$$

$$V_n = RV_p\left[C + \frac{0.87(1 - C)}{\sqrt{1 + \left(\frac{d_o}{D}\right)^2}}\right] \geq CV_p$$

$$V_n = (0.619)(2851)\left[0.242 + \frac{0.87(1 - 0.657)}{\sqrt{1 + \left(\frac{1000}{1425}\right)^2}}\right]$$

$= (0.619)(2851)(0.901)$
$= 1590 \text{ kN} < 0.657(2851) = 1873 \text{ kN}$
(1873 kN controls)

$V_r = \phi V_n = (1.0)(1873)$
$= 1873 \text{ kN} > 1851 \text{ kN}$ (OK)

12.7.10.10.3 Specification Check: Fatigue

For a plate girder being examined in this example, fatigue is a concern for the following types of details:

Category B Details:

Article S6.6.1.2.3 of the LRFD Specification states that for typical design problems, only details of Category C or worse need to be checked. Experience has shown that detail Categories A through B' rarely govern other than, perhaps, for an unusual design problem.

- Base and weld metal of full penetration groove welded splices with transitions in width or thickness ground to provide slopes no greater than 1.0 to 2.5 (for base metal other than Grades 690/690W would be category B'). See Figure S6.6.1.2.3-1(11).

- Base and weld metal in components without attachments connected by continuous fillet welds parallel to the direction of the applied stress (flange to web fillet welds) See Figure S6.6.1.2.3-1(4).

Category C and C' Details:

- Base metal adjacent to stud-type shear connectors. See Figure S6.6.1.2.3-1(18).

- Base metal at toe of transverse stiffener-to-flange and transverse stiffener-to-web welds (this is a Category C' detail which has a higher value of the constant amplitude fatigue threshold). See Figure S6.6.1.2.3-1(6).

Fatigue stresses in the tension flange in negative moment regions will be controlled by Category "C" detail of the stud connectors. The allowable fatigue stress range may be further limited by the allowable fatigue stress range in the longitudinal reinforcing in the slab as given in Article S5.5.3.1.

In positive moment regions, the fatigue stresses at the toe of the transverse stiffener or cross-frame connection plate welds are limited to those appropriate for Category "C'". Locations of the flange transitions should also be checked for Category "B" where the flange can undergo reversal or is in tension.

The allowable fatigue stress is a function of not only the detail Category, but also the number of cycles of loading which is now calculated based on the ADTT rather than assumed for a particular class of highway.

For an actual design, the fatigue should be checked at the critical locations for all of the appropriate detail categories. In this particular example, however, the fatigue checks will only be shown for the following selected details:

- The toe of the transverse stiffener/cross-frame connection plate at the point of maximum positive moment (14 400 mm from the centerline of bearing at the abutment).
- The shear connectors at the point of maximum negative moment over the interior support.
- The fatigue stress range in the slab reinforcing at the point of maximum negative moment.

Fatigue Load - Resistance: Basic Equation

Because ϕ and η are taken as 1.0 at the fatigue limit state the base equation that must be satisfied for fatigue is as follows:

$$\gamma(\Delta f) \leq (\Delta F)_n \qquad \text{(S6.6.1.2.2-1)}$$

Check fatigue stress for a transverse stiffener/cross-frame connection plate located at the point of maximum positive moment (x = 14.4 m)

Typically, the stiffeners can be terminated at a distance 4 to 6 times the flange thickness from the inside face of the bottom flange to facilitate painting and to reduce the possibility of a fatigue crack forming in the weld during shipping and handling. Stiffeners which also act as cross-frame connection plates, however, should be welded to both flanges. The neutral axis for the live load section (i.e., the "n-section") is located at a distance of 1264 mm from the bottom of the beam. Because the connection is located on the inside face of the flange, this value is reduced by the flange thickness although some may wish to take the slightly more conservative approach of computing stresses at the bottom of the beam. Therefore:

$c = 1264 - t_{fb} = 1234$ mm

$I_{LL} = 2.784E10$ mm^4

The range of unfactored fatigue moments is:

ΔM_{LLf} = 820.5 - (-216.3) = 1037 kN·m

$\gamma \Delta M_{LLf}$ = (0.75)(1037) = 778 kN·m

$$\gamma \Delta f = \frac{778(1E6)(1234)}{2.784E10} = 34.5\ MPa$$

Note that the "positive moment" section properties were used to compute the stress due to the range of fatigue moments. Although the lower moment is negative, it was determined that the stress in the slab due to the composite dead load moment and the negative fatigue live load moment is compressive. As a result, the net effect is that the section is in positive flexure for the entire range of fatigue moments. In some locations along the girder, it would be appropriate to use the positive section properties for the larger fatigue moment and the negative section properties for the smaller fatigue moment.

Compute the fatigue resistance (Category C'):

Fatigue Constants (Tables S6.6.1.2.5-1 and 3):

A = 14.4 E11 MPa^3

ΔF_{TH} = 82.7 MPa

Determine the number of cycles per truck passage, n (Table S6.6.1.2.5-2):

For a continuous girder with a span length greater than 12 000 mm, at a location greater than ten percent of the span length away from an interior support, the "equivalent" number of cycles per truck passage, n = 1.0.

Compute the number of cycles in the design life of the bridge (75 years):

$ADTT_{SL} = p(ADTT)$ (S3.6.1.4.2-1)

where:

p = percentage of trucks in a single design lane
= 0.85 (Table S3.6.1.4.2-1)

ADTT = 500 (assumed)

Therefore,

$ADTT_{SL} = (0.85)(500) = 425$

So,

$N = 365(75)(n)(ADTT_{SL})$ (S6.6.1.2.5-2)
$= 365(75)(1)(425) = 11.6$ million cycles

Compute the resistance:

$$(\Delta F_n) = \left(\frac{A}{N}\right)^{\frac{1}{3}} \geq \frac{1}{2}(\Delta F)_{TH} \quad \text{(S6.6.1.2.5-1)}$$

$$\left(\frac{A}{N}\right)^{\frac{1}{3}} = \left(\frac{14.4E11}{11.6E6}\right)^{\frac{1}{3}} = 49.9\ MPa\ \ (controls)$$

$$\frac{1}{2}(\Delta F)_{TH} = 41.4\ MPa$$

49.9 MPa > 34.5 MPa (OK)

Check a shear connector for fatigue at the interior support

(Category C):

The live load section modulus for the tension flange, S_{tLL}, is 4.341E7 mm^3.

Calculate the factored fatigue range of moments:

$\gamma\Delta M_{LLf} = (0.75)(540.7) = 406$ kN·m

So,

$$\gamma\Delta f = \frac{406(1\,E6)}{4.341\,E7} = 9.4\ MPa$$

Fatigue Constants for Category C:

A = 4.4 E11 MPa^3
ΔF_{TH} = 69.0 MPa

Determine the number of cycles per truck passage, n (S6.6.1.2.5):

For a continuous girder with a span length greater than 12 000 mm, at a location greater within ten percent of the span length away from an interior support, the "equivalent" number of cycles per truck passage, n = 1.5.

Compute the number of cycles in the design life of the bridge (75 years):

$ADTT_{SL} = 425$

So,

$N = 365(75)(n)(ADTT_{SL})$
$= 365(75)(1.5)(425) = 17.5$ million cycles

Compute the resistance:

$$(\Delta F_n) = \left(\frac{A}{N}\right)^{\frac{1}{3}} \geq \frac{1}{2}(\Delta F)_{TH} \qquad \text{(S6.6.1.2.5-1)}$$

$$\left(\frac{A}{N}\right)^{\frac{1}{3}} = \left(\frac{14.4\,E11}{17.5\,E6}\right)^{\frac{1}{3}} = 43.5\ MPa\ \ (controls)$$

$$\frac{1}{2}(\Delta F)_{TH} = 34.5\ MPa$$

43.5 MPa > 9.4 MPa (OK)

Check fatigue stresses in the slab reinforcement at the pier (X = 36 m):

Because the slab steel is always in tension at this location, a fatigue check is required. The maximum stress in the slab steel is given by the following equation:

$$f_f = 145 - 0.33 f_{min} + 55\left(\frac{r}{h}\right) \qquad \text{(S5.5.3.2-1)}$$

The unfactored moments at the pier are as follows:

M_{DL2P} = -696.0
M_{DL2W} = -679.6
M_{LLf} = -540.7

(570.7 is the range of fatigue moments because there is no positive fatigue moment at the support)

Thus,

M_{DL2} = (-696) + (-680) = -1376 kN·m

The section modulus for the slab steel is 3.565E7 mm^3 (for both dead and live load).

Compute the stress due to the unfactored permanent load:

$$f_{perm} = \frac{1376(1\,E6)}{3.565\,E7} = 38.6\ MPa \quad \text{(tension)}$$

Compute the stress due to the factored fatigue live load:

$$f_f = \frac{(0.75)(540.7)(1\,E6)}{3.565\,E7} = 11.4\ MPa \quad \text{(tension)}$$

Compute f_{min} which is the stress due to the unfactored permanent plus the factored fatigue moment which causes the most compression. Because the range of factored fatigue moments for fatigue is 0 to 406 kN·m, the zero value is selected because it induces the least tension.

Therefore,

f_{min} = 38.6 + 0 (positive because tensile)

As a result, the allowable fatigue stress range in the slab steel can be computed as:

$$f_f = 145 - 0.33f_{min} + 55\left(\frac{r}{h}\right) = 145 - 0.33(38.6) + 55(0.3)$$

$$f_f = 149\ MPa > 11.4\ MPa\ \ (OK)$$

12.7.10.10.4 Specification Check: Shear Connectors

This girder being designed in this example is continuous for the entire length of the bridge. Thus, shear connectors are required in both the positive and negative flexural regions. Figure 1 below shows the regions for shear connector placement for the strength limit state as required by Article S6.10.7.4.4.

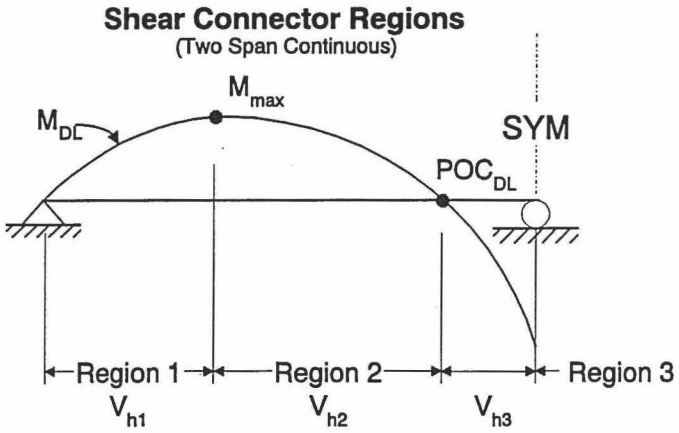


Figure 12.7.10.10.4-1 - Shear Connector Regions

For this problem, the point of maximum live load moment is at X = 14.4 meters and the point of dead load contraflexure for composite dead load is around X = 25.8 meters.

Determine number of shear connectors required for the strength limit state:

At the strength limit state, the number of shear connectors required in each region is determined by the following equations:

$$Q_r = \phi_{sc} Q_n \qquad \text{(S6.10.7.4.4a-1)}$$

and,

$$n = \frac{V_h}{Q_r} \qquad \text{(S6.10.7.4.4a-2)}$$

where the nominal horizontal shear force, V_h, is taken as:

Positive Moment Regions; V_h is the least of:

$V_h = 0.85\ f_c'\ b\ t_s$ (S6.10.7.4.4b-1)

or

$V_h = F_{yw}Dt_w + F_{yt}b_t t_t + F_{yc}b_c t_c$ (S6.10.7.4.4b-2)

Negative Moment Regions:

$V_h = A_r F_{yr}$ (S6.10.7.4.4b-3)

Region 1 (Sections A, B):

$$0.85 f_c' b_{ts} = \frac{0.85(28)(2550)(200)}{1000} = 12138\ kN$$

$$F_{yw}Dt_w + F_{yt}b_t t_t + F_{yc}b_c t_c\ (Use\ Section\ B - most\ critical)$$

$$= \frac{[(345)(1425)(12) + (345)(300)(30) + (345)(300)(13)]}{1000} = 10350\ kN$$

V_{h1} = 10 350 kN (controls)

Region 2 (Sections B, C):

Same as for region 1, therefore, V_{h2} = 10 350 kN

Region 3 (Sections D, E, F):

$A_r F_{yr}$ = (4700 + 2400)(400)/1000
= 2840 kN

Determine the nominal capacity of a single shear connector:

$$Q_n = 0.5 A_{sc}\sqrt{f_c' E_c} \le A_{sc} F_u$$ (S6.10.7.4.4c-1)

where:

d_{sc} = 22

$$A_{sc} = \frac{\pi(22)^2}{4} = 380\ mm^2$$

f'_c = 28 MPa
E_c = 25 400 MPa
F_u = 400 MPa (see S6.4.4)

So,

$0.5A_{sc}\sqrt{f'_c E_c} = 0.5(380)\sqrt{(28)(25400)}/1000 = 160.2\ kN$
$A_{sc}F_u = (380)(400)/1000 = 152\ kN\ (controls)$

Then,

$Q_r = \phi_{sc}Q_n = (0.85)(152)$
$= 129.2$ kN per connector

Compute the number of shear connectors in each region:

$$n_1 = n_2 = \frac{V_h}{Q_r} = \frac{10350}{129.2} = 80.1 \text{ (round to 81)}$$

$$n_3 = \frac{V_{h3}}{Q_r} = \frac{2840}{129.2} = 22 \text{ (round to 24 for even rows of 3)}$$

Check the transverse spacing assuming three shear connectors per row (Article S6.10.7.4.1c). Figure 2 shows the minimum clearances for a cross-section with three rows of shear connectors.

Shear Connector
Minimum Clearances

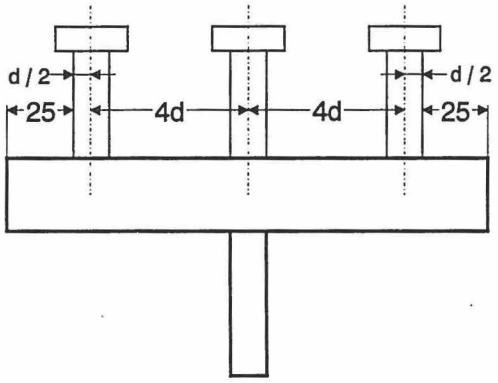


Figure 12.7.10.10.4-2 - Minimum Clearances for Shear Connectors

$$b_{fmin} = 2(25) + 2\left(\frac{22}{2}\right) + 2(4(22)) = 248 > 300\ (OK)$$

Compute the required pitch for the strength limit state:

Region 1 (x = 0 to 14.4 m)

$$p = \frac{3(14400)}{81} = 533$$

Region 2 (x = 14.4 to 25.8 m)

$$p = \frac{3(10000)}{81} = 370$$

Region 3 (x = 25.8 to 36.0 m)

$$p = \frac{3(11600)}{24} = 1450$$

But the limits on the pitch are given in Article S6.10.7.4.1b as follows:

$6d \le p \le 600$ *mm*, *So*:
$6(22) \le p \le 600$
$132 \le p \le 600$

Determine the maximum pitch for fatigue resistance as given by Equation S6.10.7.4.1b-1:

$$p \le \frac{nZ_r I}{V_{sr} Q}$$

where:

n = 3 connectors per cross-section

$Z_r = \alpha d^2 \ge 19d^2$ (S6.10.7.4.2-1)

for which:

α = 238 - 29.5 log (N)

N = 11.6 million cycles away from support

N = 17.5 million cycles near the support
(see fatigue calculations)

Compute α for sections away from the support:

α = 238 - 29.5 log(11.6E6) = 29.6
αd^2 = 14 326 N (Sections A-D)

Compute α for sections near the support:

α = 238-29.5 log (17.5 E6) = 24.3
αd^2 = 11 776 N (Sections E, F)

Check the fatigue threshold value for Z_r:

$Z_r = 19d^2$
$= 19(22)^2 = 9196$ N (does not control)

Compute the maximum pitch for Section "A":

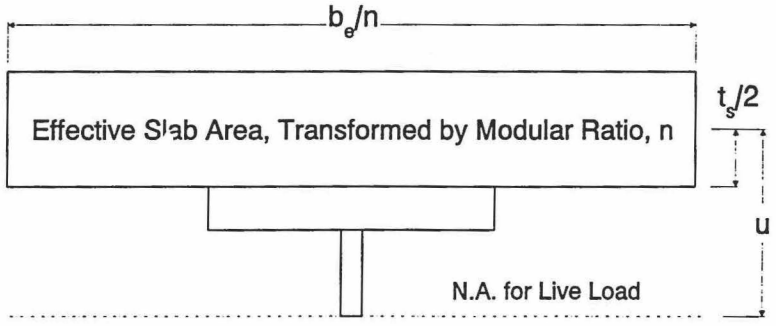


Figure 12.7.10.10.4-3 - Section for Computing First Moment of Transformed Area, Q

Section properties for the short-term section:

I = 2.192E10
Q = Au

$$u = \frac{t_s}{2} + c_{tf} = \frac{t_s}{2} + \frac{I_{LL}}{S_{LLtf}} = 100 + \frac{2.192E10}{1.425E8} = 254$$

A = $b_e/n\ (t_s)$ = (2550/8)(200) = 63 750

So,

Q = (63 750)(254) = 1.62E7 mm^3

The maximum shear force range, V_{sr}, within Section A (at X = 0) is:

V_{sr} = (0.75)(196.9 - (-23.8)) = 166 kN

So,

$$p \leq \frac{nZ_r I}{V_{sr} Q} = \frac{3(14326)(2.192\,E10)}{(166000)(1.62\,E7)} = 350\ mm$$

At Section "B":

Section properties for the short-term section:

I = 2.784E10
Q = Au

$$u = \frac{t_s}{2} + c_{tf} = \frac{t_s}{2} + \frac{I_{LL}}{S_{LLtf}} = 100 + \frac{2.784\,E10}{1.364\,E8} = 304$$

A = b_e/n (t_s) = (2550/8)(200) = 63 750

So,

Q = (63 750)(304) = 1.94E7 mm^3

The maximum shear force range, V_{sr}, within Section B (at X = 23.4) is:

V_{sr} = (0.75)(177.4) = 133 kN

So,

$$p \leq \frac{nZ_r I}{V_{sr} Q} = \frac{3(14326)(2.784\,E10)}{(133000)(1.94\,E7)} = 463\ mm$$

At Section "C":

Section properties for the short-term section:

I = 2.343E10
Q = Au

$$u = \frac{t_s}{2} + c_{tf} = \frac{t_s}{2} + \frac{I_{LL}}{S_{LLtf}} = 100 + \frac{2.343\,E10}{1.405\,E8} = 267$$

A = b_e/n (t_s) = (2550/8)(200) = 63 750

So,

Q = (63 750)(267) = 1.70E7 mm^3

The maximum shear force range, V_{sr}, within Section C (at X = 25.8) is:

V_{sr} = (0.75)(184.2) = 138 kN

So,

$$p \le \frac{nZ_r I}{V_{sr} Q} = \frac{3(14326)(2.343\,E10)}{(138000)(1.70\,E7)} = 429\ mm$$

At Section "D":

Section properties for the short-term section:

I = 1.652E10
Q = Aū
c = 786 mm (live load N.A. to the top layer of steel)
ū = c = 786
A = A_r = (4700 + 2400) = 7100 mm^2

So,

Q = (7100)(786) = 5.58E6 mm^3

The maximum shear force range, V_{sr}, within Section D (at X = 30.6) is:

V_{sr} = (0.75)(201.1) = 151 kN

So,

$$p \le \frac{nZ_r I}{V_{sr} Q} = \frac{3(14326)(1.652\,E10)}{(151000)(5.58\,E6)} = 842\ mm$$

At Section "E":

Section properties for the short-term section:

I = 2.086E10
Q = Aū
c = 802 mm (live load N.A. to the top layer of steel)
ū = c = 802
A_r = (4700 + 2400) = 7100 mm^2

So,

Q = (7100)(802) = 5.69E6 mm^3

The maximum shear force range, V_{sr}, within Section E (at X = 33.0) is:

V_{sr} = (0.75)(209.3) = 157 kN

So,

$$p \le \frac{nZ_r I}{V_{sr} Q} = \frac{3(11776)(2.992\,E10)}{(157000)(5.69\,E6)} = 825\ mm$$

At Section "F":

Section properties for the short-term section:

I = 2.992E10
Q = Au
c = 839 mm (live load N.A. to the top layer of steel)
$u = c = 839$
A_r = (4700 + 2400) = 7100 mm^2

So,

Q = (7100)(839) = 5.96E6 mm^3

The maximum shear force range, V_{sr}, within Section F (at X = 36.0) is:

V_{sr} = (0.75)(219.1) = 164 kN

So,

$$p \le \frac{nZ_r I}{V_{sr} Q} = \frac{3(11776)(2.992\,E10)}{(164000)(5.96\,E6)} = 1081\ mm$$

Table 12.7.10.10.4-1 - Summary of Maximum Pitch Values (based on three shear connectors per cross-section):

Section	Maximum Pitch	Limited By
A	350	Fatigue
B_{start} to M_{max}	460	Fatigue
M_{max} to D_{start}	370	Strength
D	600	Maximum Allowed
E	600	Maximum Allowed
F	600	Maximum Allowed

12.7.10.10.5 Specification Check: Transverse Stiffener Details

Assumed dimensions:

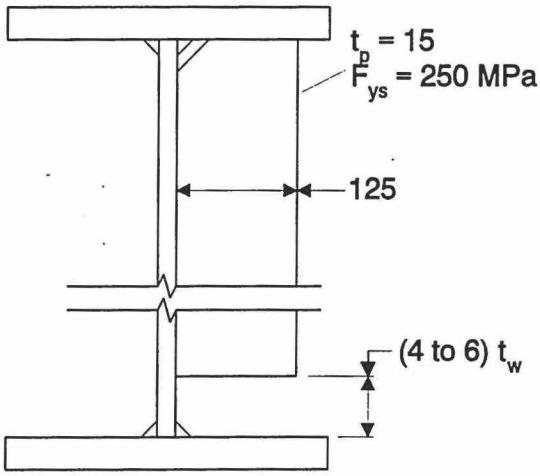


Figure 12.7.10.10.5-1 - Assumed Transverse Stiffener Dimensions

Check the projecting width:

$$50 + \frac{d}{30.0} \le b_t \le 0.48 t_p \sqrt{\frac{E}{F_{ys}}} \qquad \text{(S6.10.8.1.2-1)}$$

and,

$$16.0 t_p \ge b_t \ge 0.25 b_f \qquad \text{(S6.10.8.1.2-2)}$$

d_{max} = 1425 + 42 + 48
= 1515 (at X = 36 m)

$$50 + \frac{d}{30.0} = 50 + \frac{1515}{30} = 101 < 125 \quad (OK)$$

$$0.48 t_p \sqrt{\frac{E}{F_{ys}}} = 0.48(15)\sqrt{\frac{200000}{250}} = 204 > 125 \quad (OK)$$

$16.0t_p = 16(15) = 240 > 125$ (OK)
$0.25b_f = 0.25(460) = 115 < 125$ (OK)

Check the required moment of inertia:

$$I_t \ge d_o t_w^3 J \qquad \text{(S6.10.8.1.3-1)}$$

where:

$$J = 2.5\left(\frac{D_p}{d_o}\right)^2 - 2.0 \geq 0.5 \qquad \text{(S6.10.8.1.3-2)}$$

I_t = the moment of inertia about the edge of web in contact with the stiffener for single stiffeners, and about the mid-thickness of the web for stiffener pairs

D_p = web depth (for girders without longitudinal stiffeners)

Assume a single stiffener, therefore, the actual I_t is:

$$I_t = \frac{15(125)^3}{12} + (15)(125)\left(\frac{125}{2}\right)^2 = 9.77\,E6\ mm^4$$

The maximum stiffener spacing is 4200 mm (approximately 3D) and the maximum web thickness is 12 mm. The required moment of inertia will first be checked against the "required" value.

$$J = 2.5\left(\frac{D_p}{d_o}\right)^2 - 2.0 = 2.5\left(\frac{1425}{4200}\right)^2 - 2.0 = -1.71 \quad (0.5\ \textit{controls})$$

$d_o t_w^3 J = 4200(12)^3(0.5) = 3.63\,E6\ mm^4$

$9.77\,E6 \geq 3.63\,E6$ *(OK)*

Check Transverse Stiffener Area:

The shear resistance formulas used in this example were based on the assumption that the transverse stiffeners would have sufficient area to resist the vertical component of the tension field that exists in the web. Transversely stiffened webs designed under this assumption must satisfy the following minimum area requirement:

$$A_s \geq \left[0.15BDt_w(1.0 - C)\frac{V_u}{V_r} - 18t_w^2\right]\left(\frac{F_{yw}}{F_{ys}}\right) \qquad \text{(S6.10.8.1.4-1)}$$

Although this check should be performed at all analysis points along the girder, only a single check of this requirement will be shown herein for brevity. The minimum stiffener area requirement will be checked at the pier where the following data was calculated during the shear design:

$d_{o\ reqd.}$ = 1000 mm
C = 0.657
V_u = 1851 kN
V_r = 1873 kN
t_w = 10 mm

And, for a single plate stiffener, welded to the web, B = 2.4. Therefore,

$$A_s \geq \left[0.15BDt_w(1.0 - C)\frac{V_u}{V_r} - 18t_w^2\right]\left(\frac{F_{yw}}{F_{ys}}\right)$$

$$= \left[0.15(2.4)(1425)(10)(1.0 - 0.657)\frac{1851}{1873} - 18(10)^2\right]\left(\frac{345}{250}\right)$$

$$= -84.3\ mm.^2$$

$A_{s\ prov}$ = (125)(15) = 1875 mm^2 (OK)

Because the required area was calculated as a negative value, the stiffener need only satisfy the projecting width and stiffness requirements given in Articles S6.10.8.1.2 and S6.10.8.1.3, respectively. The negative value was obtained because the web alone is sufficient to resist the vertical component of the tension field.

Bearing Stiffener Design (S6.10.8.2)

Design the stiffeners at the abutment (X = 0)

Table 12.7.10.10.5-1 - Factored Reactions (Strength I; values in kN):

Force Effect	Unfactored Values	$\eta\gamma$	Factored Values
R_{DL1}	256	(0.95)(1.25)	304
R_{DL2P}	49	(0.95)(1.25)	58
R_{DL2W}	48	(0.95)(1.50)	68
R_{LL}	532	(0.95)(1.75)	884
			1314 kN

Try a single pair of 17 mm thick bearing stiffeners (one on each side of the web). The yield stress of the bearing stiffener is assumed to be 345 MPa (Note that for the transverse stiffeners, the yield stress was assumed to be 250 MPa). Compute the maximum width:

$$b_t \le 0.48\, t_p \sqrt{\frac{E}{F_{ys}}} = 0.48(17)\sqrt{\frac{200\,000}{345}} = 196 \quad \text{(S6.10.8.2.2-1)}$$

Try a bearing stiffener width of 140 mm. The assumed clip width is 13 mm and the bearing stiffener width should be reduced by this amount when checking bearing resistance. This reduction is not required for the axial resistance check, however.

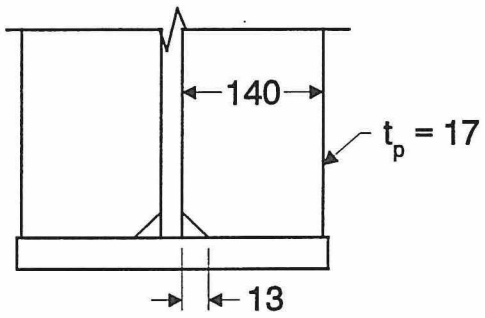


Figure 12.7.10.10.5-2 - Bearing Stiffener Geometry

Compute the net area:

$A_{pn} = (140 - 13)(17) = 2154 \text{ mm}^2$

Compute the bearing resistance:

$B_r = \phi_b A_{pn} F_{ys}$ (S6.10.8.2.3-1)

$= 1.0(2159)(345)(2 \text{ stiffeners})/1000$
$= 1490 \text{ kN} > 1316 \text{ kN (OK)}$

Determine the axial resistance (S6.10.8.2.4):

Effective Length:

$kl = 0.75\, D$ (S6.10.8.2.4a)
$= 0.75(1425) = 1069 \text{ mm}$

Determine effective section:

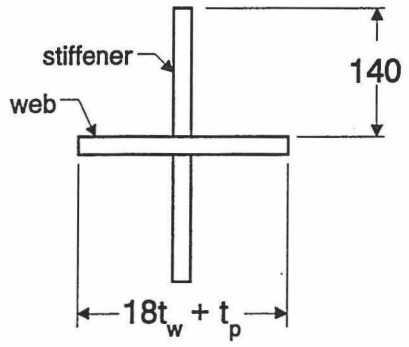


Figure 12.7.10.10.5-3 - Effective Section for a Single Pair of Bearing Stiffeners

$$A_s = 2(17)(140) + 10[18(10) + 17] = 6730\ mm^2$$

$$I_s = \frac{17(2(140) + 10)^3}{12} = 3.455E7\ mm^4$$

$$r = \sqrt{\frac{I_s}{A_s}} = \sqrt{\frac{3.455E7}{6730}} = 71.7\ mm$$

Axial Resistance (Article S6.9.4.1):

Compute slenderness factor, λ:

$$\lambda = \left(\frac{kl}{r_s\pi}\right)^2 \frac{F_y}{E} = \left(\frac{1069}{(71.7)3.14}\right)^2 \frac{345}{200000} = 0.039$$

Because $\lambda \leq 2.25$,

$$P_n = 0.66^{\lambda} F_y A_s$$

$$= 0.66^{0.039}(345)(6730)/1000$$
$$= 339(6730)/1000$$
$$= 2285\ kN$$

$$\phi_c P_n = 0.90(2285) = 2057$$

$2057 > 1316$ (OK)

12.7.10.10.6 Specification Check: Wind Loads

Although the effects of wind loads on steel girder is only applicable to exterior girders, these effects will be investigated herein to illustrate the procedure.

Table 12.7.10.10.6-1 - Load Combinations and Load Factors for Wind

Load Combination	γ_{LL}	γ_{WS}
Strength III	0.0	1.40
Strength V	1.35	0.40

The load factors for permanent loads are the same as those used for the Strength I Load Combination.

Determine the Wind Load:

Assumptions:

Height of superstructure = 5500 mm
Basic wind speed, V_B = 160 km/hr (see 3.8.1.1)

Compute the exposed height of superstructure:

$H_{exp} = H_{parapet} + t_{slab} + t_{haunch} + d_{beam}$
$H_{exp} = 1070 + 210 + 0 + 1468 = 2748$ mm

Compute the design wind load, V_{DZ} (3.8.1.1):

Because the superstructure height is less than 10 000 mm, the design wind speed is taken as the basic wind speed V_B.

Determine the design wind pressure, P_D:

$$P_D = P_B\left(\frac{V_{DZ}}{V_B}\right)^2 = P_B\frac{V_{DZ}^2}{25600} \qquad \text{(S3.8.1.2-1)}$$

where,

P_B = 0.0024 MPa (Table S3.8.1.2-1)

So,

$$P_B\frac{V_{DZ}^2}{25600} = 0.0024\left(\frac{160^2}{25600}\right) = 0.0024\ MPa$$

Compute the load per lineal mm:

$w_{wind} = P_D H_{exp} = 0.0024(2748)$
$= 6.60$ N/mm

And the minimum for a girder component is 4.4 N/mm (OK).

Calculate the wind moment, M_w (SC4.6.2.7.1):

Compute the pressure to be carried by the bottom flange:

$$W = \frac{\gamma P_D d}{2} \qquad \text{(SC4.6.2.7.1-1)}$$

Strength III:

$$W = \frac{\gamma P_D d}{2} = \frac{(1.4)(0.0024)(1468)}{2} = 2.47 \ N/mm$$

Strength V:

$$W = \frac{\gamma P_D d}{2} = \frac{(0.4)(0.0024)(1468)}{2} = 0.70 \ N/mm$$

Compute M_w assuming the wind is transmitted by frame action of the cross-frames transmitting forces to the deck (second load path in Article S4.6.2.7.1).

$$M_w = \frac{wL_b^2}{10} \qquad \text{(S4.6.2.7.1-2)}$$

Strength III:

$$M_w = \frac{wL_b^2}{10} = \frac{2.47(7200)^2}{10} = 12.8 \ kN{\cdot}m$$

Strength V:

$$M_w = \frac{wL_b^2}{10} = \frac{0.70(7200)^2}{10} = 3.6 \ kN{\cdot}m$$

Determine the wind effects at point of maximum moment (X = 14.4 m; See Article S6.10.5.7.1):

Because the section was determined to be compact, Equation S6.10.5.7.1-1 applies:

The reduction of the flange width at each edge of the bottom flange is computed by the following:

$$b_w = \frac{b_f - \sqrt{b_f^2 - \frac{4M_w}{t_f F_{yb}}}}{2} \le \frac{b_f}{2} \qquad \text{(S6.10.5.7.1-1)}$$

Strength III:

$$b_w = \frac{b_f - \sqrt{b_f^2 - \frac{4M_w}{t_f F_{yb}}}}{2} = \frac{300 - \sqrt{300^2 - \frac{4(12.8E6)}{(30)(345)}}}{2} = 4.2\ mm$$

Strength V:

$$b_w = \frac{b_f - \sqrt{b_f^2 - \frac{4M_w}{t_f F_{yb}}}}{2} = \frac{300 - \sqrt{300^2 - \frac{4(3.6E6)}{(30)(345)}}}{2} = 1.16\ mm$$

Compute the capacity of the compact section using method "A":

$$M_n = 1.3 R_h M_y \le M_p \qquad \text{(S6.10.5.2.2a-2)}$$

Compute the factored loads for Strength III and Strength V:

Table 12.7.10.10.6-2 - Factored Moment (values in kN·m; Strength III):

Force Effect	Unfactored Values	$\eta\gamma$	Factored Values
M_{DL1}	1582.7	(0.95)(1.25)	1879
M_{DL2P}	314.1	(0.95)(1.25)	373
M_{DL2W}	306.7	(0.95)(1.50)	437
M_{LL}	2770.5	(0.95)(0)	0

Table 12.7.10.10.6-3 - Factored Moment (values in kN·m; Strength V):

Force Effect	Unfactored Values	$\eta\gamma$	Factored Values
M_{DL1}	1582.7	(0.95)(1.25)	1879
M_{DL2P}	314.1	(0.95)(1.25)	373
M_{DL2W}	306.7	(0.95)(1.50)	437
M_{LL}	2770.5	(0.95)(1.35)	3553

Reduced bottom flange widths:

Strength III:

$b'_f = b_f - 2(b_w)$
$b'_f = 300 - 2(4.4) = 291.2$ mm

Strength V:

$b'_f = 300 - 2(1.16) = 297.7$ mm

Compute the new section properties and yield moment, M_y, based on the reduced section:

Table 12.7.10.10.6-4 - Section Moduli for Wind Load Case (units of mm^4):

Section Properties	DL1		DL2		LL	
	Strength III	Strength V	Strength III	Strength V	Strength III	Strength V
S_{cf}	1.078E7	1.082E7	4.488E7	4.494E7	1.368E8	1.365E8
S_{tf}	1.483E7	1.483E7	1.991E7	2.018E7	2.164E7	2.193E7

Compute M_y, M_p, M_u for Strength III:

$$M_y = M_{DL1} + M_{DL2} + S_{LL}\left(F_{yf} - \frac{M_{DL1}}{S_{DL1}} - \frac{M_{DL2}}{S_{DL2}}\right) \quad \text{(See SA6.2)}$$

$M_y = 1879 + 810 + 3804 = 6494\ kN{\cdot}m$

$M_p = 9853$ kN·m (See SA6.1)
$M_u = 2689$ kN·m

Compute M_y, M_p, M_u for Strength V:

$M_y = 1879 + 810 + 3906 = 6595\ kN{\cdot}m$

$M_p = 9953$ kN·m
$M_u = 6242$ kN·m

Compute the factored resistance, M_r:

Strength III:

$M_n = 1.3\ R_h\ M_y = 1.3(1.0)(6494)$
$= 8442$ kN·m $\leq M_p$ (OK)
$M_r = \phi M_n = (1.0)(8442)$
$= 8442$ kN·m $>$ 2689 kN·m (OK)

Strength V:

$M_n = 1.3\ R_h\ M_y = 1.3(1.0)(6595)$
$= 8574$ kN·m $\leq M_p$ (OK)
$M_r = \phi M_n = (1.0)(8574)$
$= 8574$ kN·m $>$ 6242 kN·m (OK)

Therefore, this section is sufficient to resist the wind load cases of Strength III and Strength V.

REFERENCES

Fisher, J. W., Mertz, D. R., and Zhong, A. (1983), "Steel bridge members under variable amplitude long life fatigue loading, "National Cooperative Highway Research Program Report 267, Transportation Research Broad, National Research Council, Washington, DC

Guide Specifications for Fatigue Design of Steel Bridges (1989), American Association of State Highway and Transportation Officials, Washington, DC

Standard Specifications for Highway Bridges (1992), American Association of State Highway and Transportation Officials, Washington, DC

Quiz: Steel Girder Fatigue

Given:

- 40 m long, two-lane simple span plate girder bridge
- estimated ADTT over design life = 3000 trucks per day one way
- plate girder cross-section

top flange	=	305 mm x 19 mm
web	=	1829 mm x 13 mm
bottom flange	=	356 mm x 35 mm
girder spacing	=	3600 mm
slab thickness	=	200 mm
modular ratio	=	7

Find:

1. Check fatigue at the toe of a transverse stiffener-to-flange fillet weld at mid-span of an interior girder.

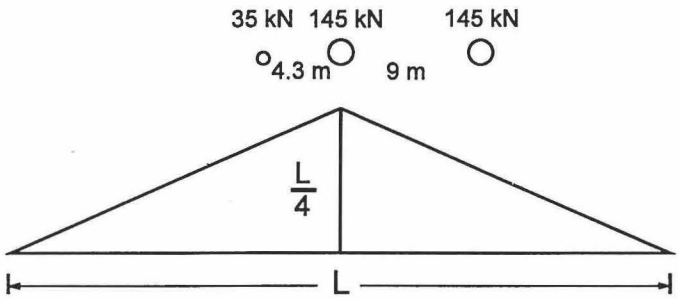
2. Check fatigue at the end of a long gusset plate (< 25 mm thick) groove-welded to flange with no transition radius at mid-span of an interior girder.

Part 1

The stress due to permanent loads in the bottom flange at mid-span is tensile, therefore, the fatigue provisions of Article 6.6.1.2 must be applied.

Fatigue Load

Estimate the live load moment per lane due to the fatigue truck.



Influence line for mid-span moment of simple span beam

Influence line ordinate at front axle = 1/2 (20-4.3) = 7.85 m
Influence line ordinate at drive axle = 1/2 (20) = 10.0 m
Influence line at rear axle = 1/2 (20-9.0) = 5.50 m

Live load moment per lane:

$$M_{LL} = 35\ kN\ (7.85) + 145\ kN\ (10.0) + 145\ kN\ (5.50)$$
$$= 2522\ kN\text{-}m$$

Live load plus impact moment per lane:

$$M_{LL}+I = (1+0.15)\ (2522\ kN\text{-}m)$$
$$= 2900\ kN\text{-}m$$

Estimate the live load plus impact moment per girder using the distribution factor for moment to interior girders for single-lane loading.

$$g = 0.06 + \left(\frac{S}{4300}\right)^{0.4}\left(\frac{S}{L}\right)^{0.3}\left(\frac{k_g}{Lt_s^3}\right)^{0.1}$$

S = girder spacing = 3600 mm

L = span length = 40 000 mm

$k_g = n\,(I + Ae_g^2)$

n = modular ratio = 7

I = girder moment of inertia = 2.13×10^{10} mm^4

A = girder area = 4.22×10^4 mm^2

e_g = eccentricity of girder

$$= \bar{y}_{top} + \frac{t_s}{2}$$

$$= 1{,}083 + \frac{200}{2} = 1{,}183\ mm$$

$k_g = 7\,(2.13 \times 10^{10} + (4.22 \times 10^4)\,(1183)^2)$

$= 5.63 \times 10^{11}$

$$g = 0.06 + \left(\frac{3660}{4300}\right)^{0.4}\left(\frac{3660}{40\ 000}\right)^{0.3}\left(\frac{5.63 \times 10^{11}}{(40\ 000)\,(200)^3}\right)^{0.1}$$

$= 0.06 + (0.851)^{0.4}\,(0.092)^{0.3}\,(1.759)^{0.1}$

$g = 0.06 + 0.938\ (0.489)\ (1.058) = 0.545$

For fatigue design, the multiple presence factor must be removed.

$$D_M = \frac{g}{1.2} = 0.454$$

Live load plus impact moment per girder

$M_{LL+I} = 0.454\ (2900\ kN\text{-}m)$

$= 1317\ kN\text{-}m$

Live load plus impact stress in bottom flange

$$\sigma = \frac{M_{LL+I}}{S_b} = \frac{1317\ kN\text{-}m}{3.80\ x10^7\ mm^3}$$

$$= 34\ 660\ \frac{kN}{m^2}$$

$$= 3.47\ x10^7\ \frac{N}{m^2}\ or\ Pa$$

$= 34.7\ MPa$

In a simple span, the maximum live load plus impact stress is the stress range.

$(\Delta f) = 34.7\ MPa$

$\gamma\ (\Delta f) = 0.75\ (34.7\ MPa) = 26.0\ MPa$

γ = load factor specified in Table 3.4.1-1 for fatigue

Fatigue Resistance

Toe of transverse stiffener-to-flange fillet weld is Category C′ as per Table 6.6.1.2.3-1

Nominal fatigue resistance

$$(\Delta F)_n = \left(\frac{A}{N}\right)^{1/3} \geq \frac{1}{2}(\Delta F)_{TH}$$

First, always check the lower bound of nominal fatigue resistance.

$$(\Delta F)_n \geq \frac{1}{2}(\Delta F)_{TH} = \frac{1}{2}(82.7\ MPa) = 41.4\ MPa$$

Constant amplitude threshold from Table 6.6.1.2.5-3

Since the lower bound of fatigue resistance is greater than the fatigue load, the LRFD equation for fatigue, Equation 6.6.1.2.2-1, is satisfied.

$\gamma(\Delta f) \leq (\Delta F)_n$

$26.0\ MPa = \gamma(\Delta f) \leq (\Delta F)_n \geq 41.4\ MPa$

∴ Theoretically, the Category C′ detail will experience infinite life.

Part 2

End of long gusset plate, less than 25 mm thick, groove-welded to flange with no transition radius is Category E as per Table 6.6.1.2.3-1.

$$(\Delta F)_n = \left(\frac{A}{N}\right)^{1/3} \geq \frac{1}{2}(\Delta F)_{TH}$$

For Category E, $\frac{1}{2}(\Delta F)_{TH} = \frac{1}{2}(31.0\ MPa) = 15.5\ MPa$ (from Table 6.6.1.2.5-3)

∴ Detail will not exhibit infinite life! Check finite life.

$N = (365)(75)\, n\, (ADTT)_{SL}$ Equation 6.6.1.2.5-2

$(ADTT)_{SL} = P\,(ADTT)$ Equation 3.6.1.4.2-1
$= 0.85\,(3000) = 2550$ (from Table 3.6.1.4.2-1)]

$n = 1.0$ (from Table 6.6.1.2.5-2)

$N = (365)(75)(1.0)(2550)$
$= 70 \times 10^6$ cycles

$$(\Delta F)_n = \left(\frac{A}{N}\right)^{1/3} = \left(\frac{3.61 \times 10^{11}}{70 \times 10^6}\right)$$ (from Table 6.6.1.2.5-1)

$= 17.3$ MPa

$26.0\ MPa = \gamma\,(\Delta f) \leq (\Delta F)_n = 17.3\ MPa$

∴ The detail is not acceptable!

Try the same detail, but with a 150 mm transition radius with end welds ground smooth.

Now detail is Category C, just as the detail in Part 1, and, therefore, it is acceptable.

LECTURE 13 - BOLTED CONNECTIONS

13.1 GENERAL

Provisions regarding bolts and bolted connections have been updated and consolidated in Article S6.13.2.3.

In bolted slip-critical connections subject to shear, the load is transferred between the connected parts by friction up to a certain level of force which is dependent upon the total clamping force on the faying surfaces and the coefficient of friction of the faying surfaces. The connectors are not subject to shear, nor is the connected material subject to bearing stress. As loading is increased to a level in excess of the frictional resistance between the faying surfaces, slip occurs, but failure in the sense of rupture does not occur. As a result, slip-critical connections are able to resist even greater loads by shear and bearing against the connected material. The strength of the connection is not related to the slip load. These Specifications require that the slip resistance, and the shear and bending resistance be computed separately. Because the combined effect of frictional resistance with shear or bearing has not been systematically studied and is uncertain, any potential greater resistance due to combined effect is ignored.

Joints subject to stress reversal, heavy impact loads, severe vibration or where stress and strain due to joint slippage would be detrimental to the serviceability of the structure are to be designated as slip-critical. They include:

- joints subject to fatigue loading;
- joints in shear with bolts installed in oversized holes;
- joints in shear with bolts installed in short- and long-slotted holes where the force on the joint is in a direction other than perpendicular to the axis of the slot, except where the Engineer intends otherwise and so indicates in the contract documents,
- joints subject to significant load reversal;
- joints in which welds and bolts share in transmitting load at a common faying surface;
- joints in axial tension or combined axial tension and shear;

- joints in axial compression only, with standard or slotted holes in only one ply of the connection with the direction of the load perpendicular to the direction of the slot, except for connections designed according to the provisions specified in Article S6.13.6.1.3;

- joints in which, in the judgment of the Engineer, any slip would be critical to the performance of the joint or the structure and which are so designated in the contract documents.

Slip-critical connections are proportioned to prevent slip under Load Combination Service II, as specified in Table S3.4.1-1, and to provide bearing, shear and tensile resistance at the applicable strength limit state load combinations.

Bearing-type connections shall be permitted only for joints subjected to axial compression, or for joints on bracing members, and shall satisfy the factored resistance, R_r, at the strength limit state.

In bolted bearing-type connections, the load is resisted by shear in the fastener and bearing upon the connected material, plus some uncertain amount of friction between the faying surfaces. The final failure will be by shear failure of the connectors, or by tear out of the connected material, or by unacceptable ovalization of the holes. Final failure load is independent of the clamping force provided by the bolts.

13.2 FACTORED RESISTANCE

13.2.1 General

For slip-critical connections, the factored resistance, R_r, of a bolt at the Service II Load Combination is taken as:

$$R_r = R_n \qquad (13.2.1\text{-}1)$$

where:

R_n = nominal resistance as specified in Article S6.13.2.8 and summarized in Article 13.2.3

Equation 13.2.1-1 applies to a service limit state for which the resistance factor is 1.0, and, hence, is not shown in the equation.

The factored resistance, R_r or T_r, of a bolted connection at the strength limit state is taken as either:

$$R_r = \phi R_n \qquad (13.2.1\text{-}2)$$

$$T_r = \phi T_n \qquad (13.2.1\text{-}3)$$

where:

R_n = nominal resistance of the bolt, connection or connected material as follows:

- for bolts in shear, R_n is taken as specified in Article 13.2.2
- for the connected material in bearing joints, R_n is taken as specified in Article 13.2.4
- for connected material in tension or shear, R_n is taken as specified in Article S6.13.5

T_n = nominal resistance of a bolt as follows:

- for bolts in axial tension, T_n is taken as summarized in Article 13.2.5
- for bolts in combined axial tension and shear, T_n is taken as summarized in Article 13.2.6

ϕ = resistance factor for bolts specified in Article S6.5.4.2 and given in Article 12.2.3, taken as:

- ϕ_s for bolts in shear,
- ϕ_t for bolts in tension,
- ϕ_{bb} for bolts bearing on material,
- ϕ_y or ϕ_u for connected material in tension, as appropriate, or
- ϕ_v for connected material in shear.

13.2.2 Slip Resistance

The current AASHTO LFD Specifications and the LRFD Specifications for slip-critical connections are identical insofar as the surface conditions of the faying surfaces and hole size factors are concerned.

In the LFD Specifications, the bolt tension, T_b (MPa), is constant for all bolt sizes in a given faying surface condition and hole type, whereas in the LRFD Specifications, the bolt tension, P_t (Kn), is given for each bolt size and is independent of the surface condition and hole type.

For A325 bolts up to 25 mm, the LFD and LRFD Specifications provide approximately the same slip resistance, however, in the larger bolt sizes, the LRFD Specifications have about 10% less resistance than the LFD Specifications.

For A490 bolts in all bolt sizes, the LRFD Specifications have 10-15% greater slip resistance than the LFD Specifications.

The nominal slip resistance of a bolt in a slip-critical connection is taken as:

$$R_n = K_h K_s N_s P_t \qquad (13.2.2\text{-}1)$$

where:

N_s = number of slip planes per bolt

P_t = minimum required bolt tension specified in Table 13.2.2-1 (N)

K_h = hole size factor specified in Table 13.2.2-2

K_s = surface condition factor specified in Table 13.2.2-3

Extensive data developed through research has been statistically analyzed to provide improved information on slip probability of connections in which the bolts have been preloaded to the requirements of Table 13.2.2-1. Two principal variables, coefficient of friction, i.e., the surface condition factor of the faying surfaces, and bolt pretension were found to have the greatest effect on the slip resistance of connections.

Hole size factors less than 1.0 are provided for bolts in oversize and slotted holes because of their effects on the induced tension in bolts using any of the specified installation methods. In the case of bolts in long slotted holes, even though the slip load is the same for bolts loaded transverse or parallel to the axis of the slot, the values for bolts loaded parallel to the axis have been further reduced, based upon judgment, because of the greater consequences of slip.

The criteria for slip resistance are for the case of connections subject to a coaxial load. For cases in which the load tends to rotate the connection in the plane of the faying surface, a modified formula accounting for the placement of bolts relative to the center of rotation should be used.

Table 13.2.2-1 - Minimum Required Bolt Tension

Bolt Diameter, mm	Required Tension-P_t ($x10^3$N)	
	M164 (A325M)	M253 (A490M)
16	94.2	130
20	147	203
22	182	251
24	212	293
27	275	381
30	337	466
36	490	678

Table 13.2.2-2 - Values of K_h

for standard holes	1.0
for oversize and short-slotted holes	0.85
for long-slotted holes with the slot perpendicular to the direction of the force	0.70
for long-slotted holes with the slot parallel to the direction of the force	0.60

Table 13.2.2-3 - Values of K_s

for Class A surface conditions	0.33
for Class B surface conditions	0.50
for Class C surface conditions	0.33

The following descriptions of surface condition shall apply to Table 3:

- Class A Surface: unpainted clean mill scale, and blast cleaned surfaces with Class A coatings,
- Class B Surface: unpainted blast cleaned surfaces, and blast cleaned surfaces with Class B coatings, and
- Class C Surface: hot-dip galvanized roughened by wire brushing after galvanizing.

The effect of ordinary paint coatings on limited portions of the contact area within joints and the effect of overspray over the total contact area have been investigated experimentally. The tests demonstrated that the effective area for transfer of shear by friction between contact surfaces was concentrated in an annular ring around and close to the bolts. Paint on the contact surfaces approximately 25 mm, but not less than the bolt diameter, away from the edge of the hole did not reduce the slip resistance. On the other hand, bolt pretension might not be adequate to completely flatten and pull thick material into tight contact around every bolt. Therefore, these Specifications require that all areas between bolts also be free of paint.

On clean mill scale, this research found that even the smallest amount of overspray of ordinary paint, i.e., a coating not qualified as Class A, within the specified paint-free area, reduced the slip resistance significantly. On blast-cleaned surfaces, the presence of a small amount of overspray was not as detrimental. For simplicity, these Specifications prohibit any overspray from areas required to be free of paint in slip-critical joints, regardless of whether the surface is clean mill scale or blast cleaned.

The contract documents shall specify that joints having painted faying surfaces be blast cleaned and coated with a

paint which has been qualified by test as a Class A or Class B coating.

The mean value of slip coefficients from many tests on clean mill scale, blast cleaned steel surfaces and galvanized and roughened surfaces were taken as the basis for the three classes of surfaces. As a result of research, a test method was developed and entitled "Test Method to Determine the Slip Coefficient for Coatings Used in Bolted Joints", AISC (1988). The method includes long-term creep test requirements to ensure reliable performance for qualified paint coatings. The method, which requires requalification if an essential variable is changed, is the sole basis for qualification of any coating to be used under these Specifications. Further, normally only two categories of surface conditions for paints to be used in slip-critical joints are recognized: Class A for coatings which do not reduce the slip coefficient below that provided by clean mill scale, and Class B for paints which do not reduce the slip coefficient below that of blast cleaned steel surfaces.

Tests on surfaces that were wire brushed after galvanizing have indicated an average value of the slip coefficient equal to 0.35. Untreated surfaces with normal zinc have much smaller slip coefficients. Even though the slip coefficient for Class C surfaces is the same as for Class A surfaces, a separate class is retained to avoid potential confusion. The higher value of the slip coefficient equal to 0.40 in previous specifications assumes that the surface has been blast cleaned after galvanizing, which is not the typical practice.

13.2.3 Shear Resistance

In the LFD Specifications, the values of shear strength (ϕR) are 25% lower than the LRFD Specifications. The reductions in shear strength for the larger bolt diameters and for length between extreme fasteners are the same in either Specification.

The nominal shear resistance of a high-strength bolt or an ASTM A307 bolt at the strength limit state in joints whose length between extreme fasteners measured parallel to the line of action of the force is less than 1270 mm is taken as:

- where threads are excluded from the shear plane:

$$R_n = 0.48\, A_b\, F_{ub} N_s \qquad (13.2.3\text{-}1)$$

- where threads are included in the shear plane

$$R_n = 0.38\ A_b\ F_{ub} N_s \qquad (13.2.3\text{-}2)$$

where:

A_b = area of the bolt corresponding to the nominal diameter (mm^2)

F_{ub} = specified minimum tensile strength of the bolt specified in Article S6.4.3 (MPa)

N_s = number of shear planes per bolt

The nominal shear resistance of a bolt in connections greater than 1270 mm in length shall be taken as 0.80 times the value given by Equations 13.2.3-1 or 13.2.3-2.

In determining whether the bolt threads are excluded from the shear planes of the contact surfaces, the thread length of the bolt shall be determined as two thread pitch lengths greater than the specified thread length.

If the threads of a bolt are included in the shear plane in the joint, the shear resistance of the bolt in all shear planes of the joint shall be the value for threads included in the shear plane.

The nominal resistance in shear is based upon the observation that the shear strength of a single high-strength bolt is about 0.6 times the tensile strength of that bolt. However, in shear connections with more than two bolts in the line of force, deformation of the connected material causes nonuniform bolt shear force distribution so that the strength of the connection in terms of the average bolt strength decreases as the joint length increases. Rather than provide a function that reflects this decrease in average fastener strength with joint length, a single reduction factor of about 0.80 was applied to the 0.6 multiplier. Studies have shown that the allowable stress factor of safety against shear failure ranges from 3.3 for compact, i.e., short, joints to approximately 2.0 for joints with an overall length in excess of 1270 mm. It is of interest to note that the longest, and often the most important, joints had the lowest factor, indicating that a factor of safety of 2.0 has proven satisfactory in service.

The average value of the nominal resistance for bolts with threads in the shear plane has been determined by a series of tests to be 0.833 F_{ub} with a standard deviation of 0.03. A value of 0.80 times the nominal resistance, when threads are excluded, was selected for the specification formula based

upon the area corresponding to the nominal body area of the bolt.

The shear strength of bolts is not affected by pretension in the fasteners provided the connected material is in contact at the faying surfaces.

The factored resistance equals the nominal shear resistance multiplied by a resistance factor less than that used to determine the factored resistance of a component. This ensures that the maximum strength of the bridge is limited by the strength of the main members rather than by the connections.

13.2.4 Bearing Resistance

The effective bearing area of a bolt shall be taken as its diameter multiplied by the thickness of the connected material on which it bears. The effective thickness of connected material with countersunk holes shall be taken as the thickness of the connected material, minus one-half the depth of the countersink.

Bearing stress produced by a high-strength bolt pressing against the side of the hole in a connected part is important only as an index to behavior of the connected part. Thus, the same bearing resistance applies regardless of bolt shear strength or the presence or absence of threads in the bearing area. The critical value can be derived from the case of a single bolt at the end of a tension member.

It has been shown using finger-tight bolts, that a connected plate will not fail by tearing through the free edge of the material if the distance L, measured parallel to the line of applied force from a single bolt to the free edge of the member toward which the force is directed, is not less than the diameter of the bolt multiplied by the ratio of the bearing stress to the tensile strength of the connected part.

The criterion for nominal bearing strength is

$$\frac{L}{d} \geq \frac{r_n}{F_u} \qquad (13.2.4\text{-}1)$$

where:

r_n = nominal bearing pressure (MPa)

F_u = specified minimum tensile strength of the connected part (MPa)

In these Specifications, the nominal bearing resistance of an interior hole is based on the clear distance between the hole and the adjacent hole in the direction of the bearing force. The nominal bearing resistance of an end hole is based on the clear distance between the hole and the end of the member. The nominal bearing resistance of the connected member may be taken as the sum of the resistances of the individual bolts. The clear distance is used to simplify the computations for oversize and slotted holes.

Bolts may be spaced at clear distances less than the specified values, as long as the lower value specified by Equation 13.2.4-2 or Equation 13.2.4-4, as applicable, is used for the nominal bearing resistance.

For standard holes, oversize holes, short-slotted holes loaded in any direction, and long-slotted holes parallel to the applied bearing force, the nominal bearing resistance of interior and end bolt holes at the strength limit state, R_n, are taken as:

- with bolts spaced at a clear distance between holes not less than 2.0d and with a clear end distance not less than 2.0d:

$$R_n = 2.4dtF_u \qquad (13.2.4\text{-}2)$$

- if either the clear distance between holes is less than 2.0d, or the clear end distance is less than 2.0d:

$$R_n = 1.2L_ctF_u \qquad (13.2.4\text{-}3)$$

For long-slotted holes perpendicular to the applied bearing force:

- with bolts spaced at a clear distance between holes not less than 2.0d and with a clear end distance not less than 2.0d:

$$R_n = 2.0dtF_u \qquad (13.2.4\text{-}4)$$

- if either the clear distance between holes is less than 2.0d, or the clear end distance is less than 2.0d:

$$R_n = L_ctF_u \qquad (13.2.4\text{-}5)$$

where:

d = nominal diameter of the bolt (mm)

t = thickness of the connected material (mm)

F_u = tensile strength of the connected material specified in Table S6.4.1-1 (MPa)

L_c = clear distance between holes or between the hole and the end of the member (mm)

The spacing limits for center-to-center of bolts and edge distance in the LFD Specifications are 3d and 1-1/2d respectively, whereas these limits are 2d for either situation in the LRFD Specifications.

The LRFD Specifications also recognize oversize holes, whereas the LFD Specifications do not.

The bearing resistance in the LRFD Specifications are 33% higher than the LFD Specifications.

13.2.5 Tensile Resistance

The nominal tensile resistance of a bolt, T_n, independent of any initial tightening force shall be taken as:

$$T_n = 0.76\, A_b\, F_{ub} \qquad (13.2.5\text{-}1)$$

where:

A_b = area of bolt corresponding to the nominal diameter (mm^2)

F_{ub} = specified minimum tensile strength of the bolt specified in Article S6.4.3 (MPa)

The recommended design strength is approximately equal to the initial tightening force; thus, when loaded to the service load, high-strength bolts will experience little, if any, actual change in stress. For this reason, bolts in connections, in which the applied loads subject the bolts to axial tension, are required to be fully tensioned.

The lower value specified for A307 bolts in tension reflects the greater variability of the test data, since tensile loading of mild steel bolts may often result in the thread stripping before development of the tensile strength.

Where high-strength bolts in axial tension are subject to fatigue, the stress range, Δ_f, in the bolt, due to the fatigue design live load, plus the dynamic load allowance for fatigue loading specified in Article S3.6.1.4, plus the prying force resulting from cyclic application of the fatigue load, shall satisfy Equation S6.6.1.2.2-1.

The nominal diameter of the bolt shall be used in calculating the bolt stress range. In no case shall the calculated prying force exceed 60% of the externally applied load.

Properly tightened A325M and A490M bolts are not adversely affected by repeated application of the recommended service load tensile stress, provided that the fitting material is sufficiently stiff such that the prying force is a relatively small part of the applied tension. The provisions covering bolt tensile fatigue are based upon study of test reports of bolts that were subjected to repeated tensile load to failure.

Low carbon ASTM A307 bolts shall not be used in connections subjected to fatigue.

The tensile force due to prying action shall be taken as:

$$Q_u = \left[\frac{3b}{8a} - \frac{t^3}{328\,000}\right] P_u \qquad (13.2.5\text{-}2)$$

where:

Q_u = prying tension per bolt due to the factored loadings taken as 0 when negative (N)

P_u = direct tension per bolt due to the factored loadings (N)

a = distance from center of bolt to edge of plate (mm)

b = distance from center of bolt to the toe of fillet of connected part (mm)

t = thickness of thinnest connected part (mm)

13.2.6 Resistance to Combined Shear and Tension

The nominal tensile resistance of bolts subject to combined axial tension and shear is provided by elliptical interaction curves, which account for the connection length effect on bolts loaded in shear, the ratio of shear strength to tension strength of threaded bolts, and the ratios of root area to nominal body area and tensile stress area to nominal body area.

The nominal tensile resistance of a bolt subjected to combined shear and axial tension, T_n, shall be taken as:

If $\frac{P_u}{R_n} \leq 0.33$, then

$T_n = 0.76 A_b F_{ub}$

Otherwise:

$$T_n = 0.76 A_b F_{ub} \sqrt{1 - \left(\frac{P}{\phi_s R_n}\right)^2} \qquad (13.2.6\text{-}1)$$

where:

A_b = area of the bolt corresponding to the nominal diameter (mm^2)

F_{ub} = specified minimum tensile strength of the bolt specified in Article S6.4.3 (MPa)

P_u = shear force on the bolt due to the factored loads (N)

R_n = nominal shear resistance of a bolt specified in Article 13.2.3 (N)

Equation 13.2.6-1 is a conservative simplification of the set of elliptical curves, and represents the case for A325M bolts where threads are not excluded from the shear plane. Curves for other cases may be found in AISC (1988). No reduction in the nominal shear resistance is required when applied tensile stress is equal to or less than the nominal tensile resistance.

The nominal resistance of a bolt in slip-critical connections under Load Combination Service II, specified in Table S3.4.1-1, subjected to combined shear and axial tension, shall not exceed the nominal slip resistance specified in Article 13.2.2 multiplied by:

$$1 - \frac{T_u}{P_t} \qquad (13.2.6\text{-}2)$$

where:

T_u = tensile force due to the factored loads under Load Combination Service II (N)

P_t = minimum required bolt tension specified in Table 13.2.2-1 (N)

13.3 BOLTED SPLICE DESIGN EXAMPLE

The design of a bolted splice is presented in this example. The section dimensions of the composite plate girder and the section forces at the splice location were obtained from a preliminary design of a continuous girder bridge with two spans 55 000 mm each. The splice was located about 15 000 mm from the intermediate support. The splice plates were designed to satisfy the requirements of the strength and fatigue limit states, and the bolts were designed to satisfy the strength and service limit states.

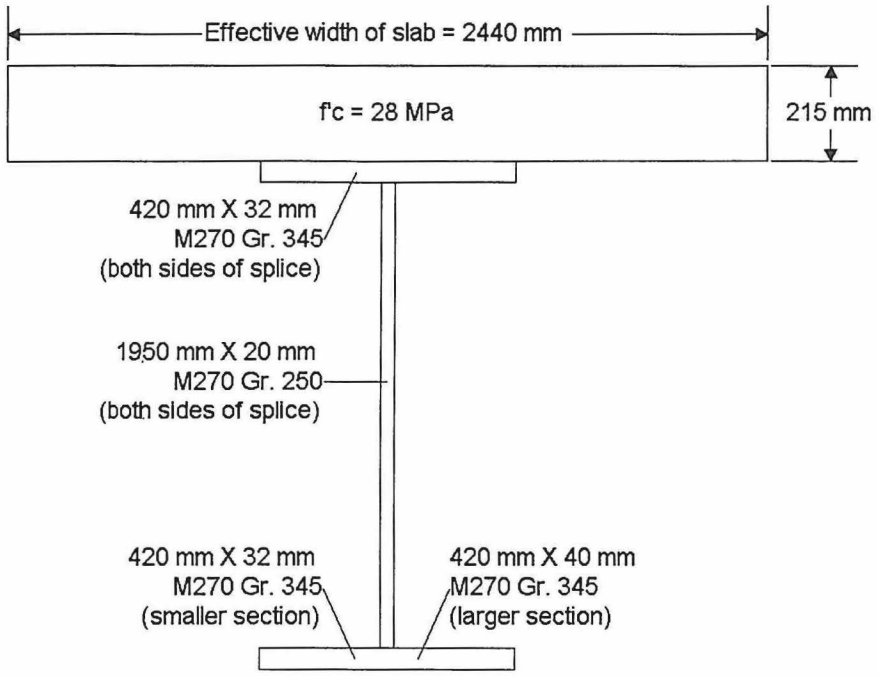


Figure 13.3-1 - Girder Dimensions at Splice Location

Assumptions

- All bolts are high-strength, slip-critical in standard holes.
- For all limit states, assume combined factor for ductility, redundancy and importance, $\eta = 1.0$ (S1.3.2.1).
- Only the forces produced by the notional design loads were considered in this example. For a complete design, the forces produced by the permit vehicles need to be checked using the same procedures.
- The longitudinal steel in the slab was ignored in the analysis of both the positive and negative moment sections. However, the designer may chose to consider such reinforcement as a part of the section resisting negative moments (S6.10.5.1.1c).
- When the combined effect of the moments and shears is applied to a component, the maximum moment and the maximum shear at the section are applied simultaneously. The designer may chose to check the component under the effect of the maximum moment and the concurrent shear or the maximum shear and the concurrent moment.

- When calculating the bolt edge distance in the girded web and flanges, a 6 mm gap is assumed to exist between the two segments of the spliced girder.

Steel Plate Dimensions

The design is controlled by the smaller of the two girder sections connected by the splice (S6.13.6.1.1). All given plate dimensions, cross-section properties and load resistance are calculated for this section.

D_w = 1950 mm, depth of girder web

T_w = 20 mm, thickness of girder web

W_{tf} = 420 mm, width of girder top flange

T_{tf} = 32 mm, thickness of girder top flange

W_{bf} = 420 mm, width of girder bottom flange

T_{bf} = 32 mm, thickness of girder bottom flange

Composite Concrete Slab

For the purpose of this example, no deck slab haunch was considered in the analysis. See Figure 13.3-1 for the dimensions of the effective flange width of the composite girder.

f'_c = 28 MPa, 28-day compressive strength of slab concrete

n = 8, modular ratio (S6.10.5.1.1b)

Unfactored Loads at the Strength and Service Limit States

M_{DL1} = -6.274E08 N·mm, moment due to noncomposite dead load

V_{DL1} = -4.496E05 N, shear due to noncomposite dead load

M_{DL2a} = -2.29E07 N·mm, moment due to weight of parapet

V_{DL2a} = -5.29E04 N, shear due to weight of parapet

M_{DL2b} = -2.90E07 N·mm, moment due to weight of future wearing surface

V_{DL2b} = -6.69E04 N, shear due to weight of future wearing surface

$M_{LL,pos}$ = 2.916E09 N·mm, maximum positive live load moment

$M_{LL,neg}$ = -2.677E09 N·mm, minimum negative live load moment

$V_{LL,pos}$ = 6.45E04 N, maximum positive live load shear

$V_{LL,neg}$ = -4.912E05 N, minimum negative live load shear

Unfactored Moments for the Fatigue Limit States

M_{pos} = 1.233E09 N·mm

M_{neg} = -8 867E08 N·mm

Steel Plate Materials

M270 web and web splice plates:

F_y = 250 MPa

F_u = 400 MPa

M270 flange and flange splice plates:

F_y = 345 MPa

F_u = 450 MPa

Allowable Fatigue Stress Range (All Steel Plate Components)

$F_{r,fat} \geq 55.0$ MPa (S6.6.1.2.5)

Bolts

For the purpose of this example, all bolts are assumed to be AASHTO M164 bolts with no threads included in the shear planes (S6.13.2.7). It is worth noting that having several, e.g., 3-5, threads in the grip (not necessarily in the shear plane) improves the ductility of the connection.

Dia_{bolt} = 24 mm (S6.13.2.4.2-1)

Dia_{hole} = 27 mm (S6.13.2.4.2-1)

F_{ubolt} = 830 MPa (S6.4.3)

The hole size used in this example conforms to the provisions of the 1994 AASHTO-LRFD Specifications. However, due to the continuous revisions of the SI conversions, future refinements to the hole sizes in the SI edition of the Specifications are possible. The user of this example is advised to refer to any applicable addendums to the Specifications.

See Figure 13.3-2 for splice plate dimensions and for bolt pattern, pitch, gage spacing and end and edge distances. These distances satisfy the spacing limits of Article S6.13.2.6.

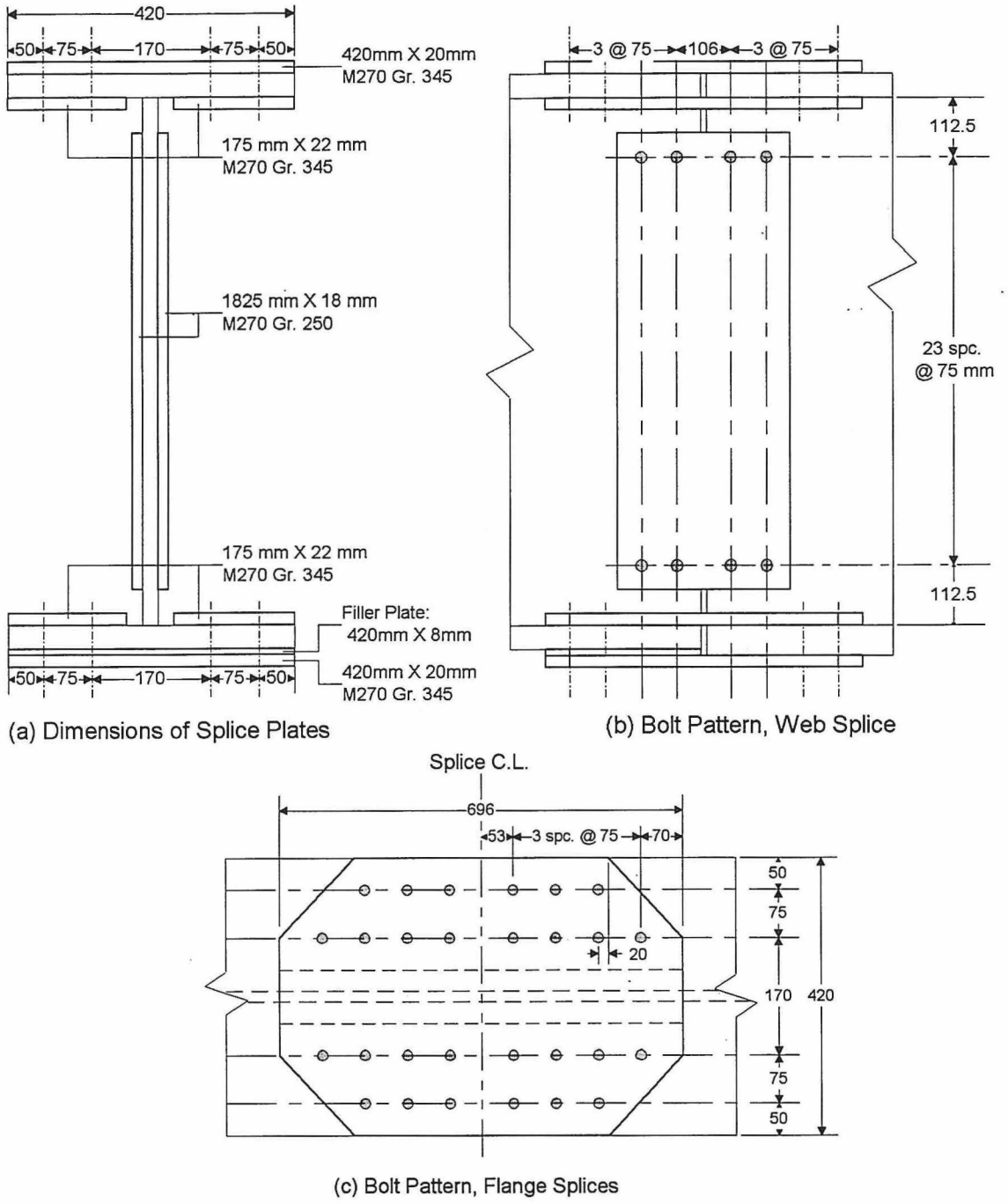


Figure 13.3-2 - Dimensions of Splice Plates and Configuration of Bolt Patterns

Section Properties

The gross section properties are required to be used in determining the stresses at the fatigue limit state (S6.6.1.2.3 and S6.13.6.1.4a). To facilitate the analysis, the gross section properties were also used to determine the section moment resistance, the portion of the design moment resisted by the web and the design force in the flanges at all limit states. In

checking the stresses in the flange splice plates at the strength limit state, both the yield on the gross section and the fracture on the net section are required to be investigated. In determining the stresses in the web splice plates, gross and net section properties of the web splice plates were used to determine the stresses at the fatigue and strength limit states, respectively.

The great majority of splices are located near the point of dead load countraflexure points. In this case, live loads produce most of the moments at the splice location. Therefore, it is customary to use the following approximate procedure to calculate the applied stresses and bolt forces under the effect of live loads or the combined effect of live loads plus dead loads:

- In case of positive total factored moments, the short-term composite section properties are used.

- In case of negative total factored moments, the noncomposite section properties are used.

- The fatigue check requires the calculation of the stresses under the effect of permanent loads. In this case, the noncomposite section properties were considered.

In general, following this procedure reduces the amount of calculations and does not cause a significant error. However, in some unique situations, the designer is forced to locate field splices at a significant distance from the dead load countraflexure points. An example of these situations is the case of long-span bridges where the location of the field splices may be controlled by the maximum available plate length and the maximum transportable segment length. In such situations, a more accurate approach to calculate the stresses in the splice plates and forces on the bolts may be required. One such approach is to apply each of the moment components to the appropriate section properties, i.e, DL1, DL2 and live load moments are applied to the noncomposite section, long-term composite section and short-term composite section, respectively.

Since the splice in this example is located near the point of dead load countraflexure point, it was appropriate to use the approximate procedure.

Noncomposite section properties:

I_{nc} = 3.876E10 mm^4

Distance to Section N.A., from bottom of bottom flange:

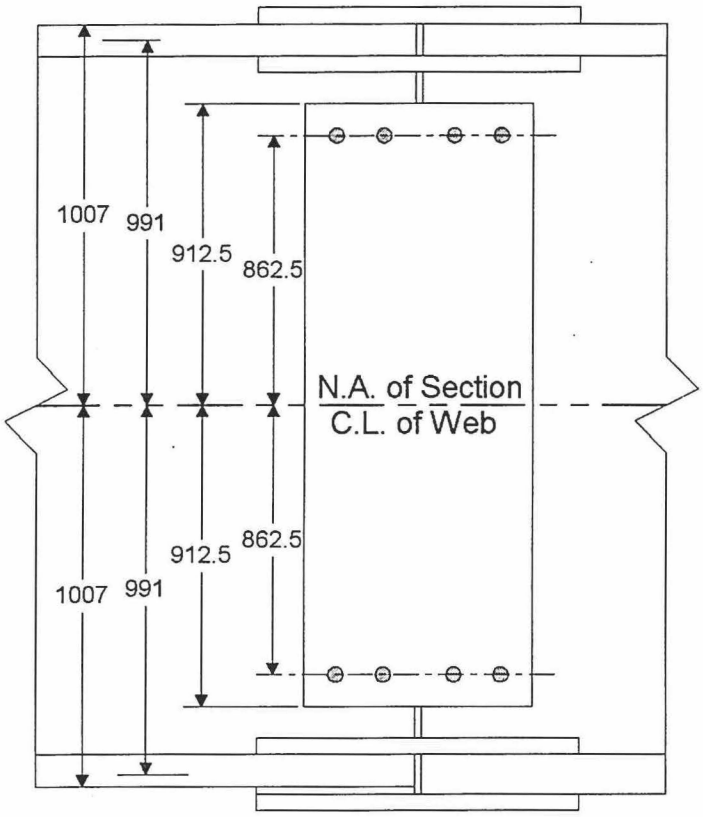
Y_{nc} = 1007.0 mm

Composite short-term section properties:

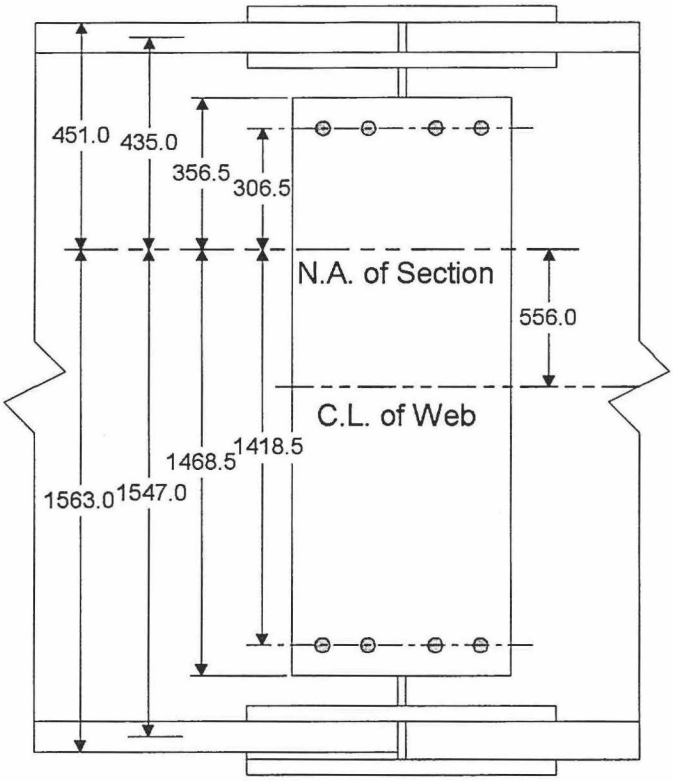
$I_{c\text{-}n}$ = 7.983E10 mm^4

Distance to Section N.A., from bottom of bottom flange:

$Y_{c\text{-}n}$ = 1563.0 mm



(a) Non-Composite Section



(b) Short-Term Composite Section

Figure 13.3-3 - Location of Neutral Axis for Composite and Noncomposite Sections

Section Resistance

Using the provisions of Articles S6.10.5 and S6.10.6, the resistance of the smaller of the two girder sections at the splice location was determined as:

Positive Flexure:

$M_{r,pos}$ = 2.043E10 N·mm (Compact,Composite,Plastic)

Negative Flexure:

$M_{r,neg}$ = -1.394E10 N·mm (Compact,Plastic)

For shear:

V_r = 3.733E06 N (Unstiffened Web)

Maximum Factored Loads (S3.4)

Apply the maximum load factor to the components of the load effect that has the same sign as the force effect being maximized. For the load effect components that have the opposite sign, apply the minimum load factor to the effect of noncomposite dead loads and the weight of the parapet and ignore the effects of the future wearing surface and the live loads.

Factored Loads, Strength I Limit State

Maximum factored positive moments:

$M_{u,pos} = 0.9M_{DL1} + 0.9M_{DL2a} + 1.75M_{LL,pos}$

$= -(0.90)(6.274E08) - (0.90)(2.29E07) + (1.75)(2.916E09)$

$= 4.518E09$ N·mm

Minimum factored negative moments:

$M_{u,neg} = 1.25M_{DL1} + 1.25M_{DL2a} + 1.5M_{DL2b} + 1.75M_{LL,neg}$

$= -(1.25)(6.274E08) - (1.25)(-2.29E07) - (1.50)(2.90E07) - (1.75)(2.677E09)$

$= -5.541E09$ N·mm

Maximum factored positive shear:

$V_{u,pos} = 0.9V_{DL1} + 0.9V_{DL2a} + 1.75V_{LL,pos}$

$= -(0.90)(4.496E05) - (0.90)(5.29E04) + (1.75)(6.45E04)$

$= -3.35E05$ N

Minimum factored negative shear:

$V_{u,neg} = 1.25V_{DL1} + 1.25V_{DL2a} + 1.5V_{DL2b} + 1.75V_{LL,neg}$

$= -(1.25)(4.496E05) - (1.25)(5.29E04) - (1.50)(6.69E04) - (1.75)(4.912E05)$

$= -1.588E06$ N

The minimum negative shear has a larger absolute value and, hence, controls the design.

Maximum Factored Loads, Service II Limit State

Maximum factored positive moments:

$M_{u,pos} = M_{DL1} + M_{DL2a} + 1.3M_{LL,pos}$

$= -6.274E08 - 2.29E07 + (1.3)(2.916E09)$

$= 3.141E09$ N·mm

Minimum factored negative moments:

$M_{u,neg} = M_{DL1} + M_{DL2a} + M_{DL2b} + 1.3M_{LL,neg}$

$= -6.274E08 - 2.29E07 - 2.90E07 - (1.3)(2.677E09)$

$= -4.159E09$ N·mm

Maximum factored positive shear:

$V_{u,pos} = V_{DL1} + V_{DL2a} + 1.3V_{LL,pos}$

$= -4.496E05 - 5.29E04 + (1.3)(6.45E04)$

$= -58.64$ N

Minimum factored negative shear:

$V_{u,neg} = V_{DL1} + V_{DL2a} + V_{DL2b} + 1.3V_{LL,neg}$

$= -4.496E05 - 5.29E04 - 6.69E04 - (1.3)(4.912E05)$

$= -1.21E06$ N

The minimum negative shear has a larger absolute value and, hence, controls the design.

Determination of Splice Design Loads (S6.13)

Positive Flexure:

Average of section moment resistance and the applied factored moment: (S6.13.1)

$M_{ave} = (M_{u,pos} + M_{r,pos})/2$

$= (4.518E09 + 2.043E10)/2 = 1.247E10$ N·mm (1)

75% of section moment resistance: (S6.13.1)

$M_{r,75\%} = 0.75M_{r,pos} = 0.75 \times 2.043E10 = 1.532E10$ N·mm (2)

Splice design negative moment = Larger of (1) and (2) = 1.532E10 N·mm

Negative Flexure:

Average of section moment resistance and the applied factored moment (S6.13.1):

$M_{ave} = (M_{u,neg} + M_{r,neg})/2$

$= (-5.541E09 - 1.394E10)/2 = -9.74E09$ N·mm (3)

75% of section moment resistance: (S6.13.1)

$M_{r,75\%} = 0.75M_{r,neg} = -0.75 \times 1.394E10 = -1.046E10$ N·mm (4)

Splice design negative moment = Larger of (3) and (4) = 1.046E10 N·mm

Shear:

Average of applied factored shear and shear resistance of the section:

$V_{ave} = (|V_{u,neg}| + V_r)/2$

$= (|-1.588E06| + 3.733E06)/2 = 2.660E06$ N (5)

75% of the shear resistance of the section: (S6.13.1)

$V_{r,75\%} = 0.75V_r = 0.75 \times 3.733E06 = 2.800E06$ N (6)

Notional Shear = the applied factored shear multiplied by the ratio of the design moment divided by the applied factored moment

Case of positive moment: Notional shear = $|-1.588E06| \times 1.532E10 / 4.518E09 = 5.385E06$ N (7)

Case of negative moment: Notional shear = $|-1.588E06| \times 1.046E10 / 5.541E09 = 2.998E06$ N (8)

For the design of web splice plates and their bolts, the design shear = Largest of (5) through (8) = 5.385E06 N

Notice that the value of the notional shear exceeds the calculated shear resistance of the web plate. The designer may chose to use an upper bound to the notional shear value equal to the calculated shear resistance of the web plate, i.e., 3.733E06 N. However, the post-buckling shear resistance of the web plate is actually higher than that calculated above. In addition, for this example, the web shear resistance was determined assuming unstiffened web. The presence of the splice plates will provide some stiffening effect to the web and the actual shear resistance may exceed the calculated value. Using the notional shear for design allows the bridge owner to upgrade the bridge load capacity without disturbing the field splice. Therefore, the notional shear was used in the design without any adjustment to account for the smaller web capacity.

In addition to the above design loads, the web splice plates and their bolts are designed to resist an additional moment due to the eccentricity of the notional shear relative to the center of the bolt group.

Notional shear	=	5.385E06 N
Eccentricity	=	distance between the center of the bolt group at one side of the splice to the center of the splice = 90.5 mm
Moment of eccentricity of notional shear	=	5.385E06 x 90.5 = 4.875E08 N·mm

The entire moment due to the eccentricity of the notional shear is resisted by the web splice plates and their bolts.

Design of Web Splice Plates

The portion of the design moment resisted by the web plate at the splice location cannot be determined accurately. The most common method to determine the portion of the design moment resisted by the web plate is to multiply the splice design moment by the ratio of the moment of inertia of the web to the moment of inertia of the girder section. The following are some of the sources of the inaccuracy of this assumption:

- The ratio of the moment of inertia of the web to the moment of inertia of the section at a certain location differs if the properties of the net or gross section were used. In addition, the ratio changes based on if the section was in positive or negative flexure due to the change from composite to noncomposite section. Even if the sign of the moment does not change, more than one value of this ratio may be obtained for the same section. For sections in positive flexure, the ratio of the moment of inertia of the web to the moment of inertia of the section differs for the noncomposite section, long-term composite section and short-term composite section. For sections in negative flexure, if the longitudinal reinforcement of the slab were considered active, the ratio would differ before and after the slab hardens. In addition, a separate set of ratios may be obtained for each of the two sections of the girder connected at the splice location.

- The design moment for the splice may exceed the moment that initiates yield in the webs of hybrid sections. In this case, the portion of the moment resisted by the web is not proportionate to the ratio of

the moment of inertia of the web to the moment of inertia of the section.

For the purpose of this example, in case of positive total factored moments, the properties of the gross short-term composite section were used to determine the portion of the total moment resisted by the web. Similarly, in case of negative total factored moments, the portion of the total moment resisted by the web was calculated using the properties of the gross noncomposite steel section.

Web Plate Section Properties

Gross area of the web plate:

$A_w = D_w T_w = (1950)(20) = 3.90E04\ mm^2$

Gross moment of inertia of the web plate about its horizontal centroidal axis:

$I_w = T_w(D_w)^3/12 = (20)(1950)^3/12 = 1.236E10\ mm^4$

Gross moment of inertia of the web plate, about the neutral axis of the gross noncomposite section (see Figure 13.3.3-a):

$I_{w,nc} = I_w + A_w \times (0.0)^2 = 1.236E10\ mm^4$

Ratio of web stiffness to section stiffness, R_{nc} = 1.236E10/3.876E10 = 0.319

Gross moment of inertia of the web plate, about the neutral axis of the gross short-term composite section (see Figure 13.3.3-b):

$I_{w,c\text{-}n} = I_w + A_w \times (556)^2$

$= 1.236E10 + (3.90E04)(556)^2$

$= 2.442E10\ mm^4$

Ratio of web stiffness to section stiffness, R_c = 2.442E10/7.983E10 = 0.306

Design of Web Splice Plates

Gross section properties:

The gross section properties of the web splice plates are required to be used in the calculations of fatigue stresses (S6.13.6.1.4a).

Gross area of both web splice plates:

$A_{wspl} = 2(1825)(18) = 6.57E04\ mm^2$

Moment of inertia of both web splice plates about their horizontal centroidal axis:

$I_{wspl} = 2(18)(1825)^3/12 = 1.824E10\ mm^4$

Moment of inertia of both web splice plates about the neutral axis of different sections:

For noncomposite section (see Figure 13.3.3-a):

$I_{nc} = I_{wspl} + A_{wspl} \times (0.0)^2$

$= 1.824E10\ mm^4$

$S_{t,nc} = 1.824E10/912.5 = 1.998E07\ mm^3$

$S_{b,nc} = 1.824E10/912.5 = 1.998E07\ mm^3$

For short-term composite section (see Figure 13.3.3-b):

$I_{c-n} = I_{wspl} + A_{wspl} \times (556)^2$

$= 1.824E10 + 6.57E04 \times (556)^2$

$= 3.855E10\ mm^4$

$S_{t,c-n} = 3.855E10/356.5 = 1.081E08\ mm^3$

$S_{b,c-n} = 3.855E10/1468.5 = 2.625E07\ mm^3$

Net Section Properties

The net section properties of the web splice plates are required to be used in the calculations of stresses at the strength and service limit states.

Diameter of standard holes for net section calculations =

27.2 mm (S6.8.3)

Area of the holes = 2 x 24 x 27.2 x 18 = 23 500 mm^2

Considering that the web splice plates are primarily flexural members, only the area of holes in excess of 15% of the gross area needs to be deducted (S6.10.1.4).

Actual percentage of section loss = 23 500 / 6.57E04 = 35.8%

Percentage loss to be considered in net section calculations = 35.8 - 15 = 20.8%

Ratio between net section to gross section = 1.00 -0.208 = 0.792

Considering that the holes are uniformly distributed along the height of the web plate, it is reasonable to assume that reducing both the area and the moment of inertia of the gross section by 20.8% will produce the net section properties.

Net area of both web splice plates:

A_{wspl} = 6.57E04 x 0.792 = 5.203E04 mm^2

Moment of inertia of both web splice plates about their horizontal centroidal axis:

I_{wspl} = 1.824E10 x 0.792 = 1.445E10 mm^4

Moment of inertia of both web splice plates about the neutral axis of different sections:

For noncomposite section (see Figure 13.3.3-a):

I_{nc} = 1.824E10 x 0.792 = 1.445E10 mm^4

$S_{t,nc}$ = 1.998E07 x 0.792 = 1.582E07 mm^3

$S_{b,nc}$ = 1.998E07 x 0.792 = 1.582E07 mm^3

For short-term composite section (see Figure 13.3.3-b):

I_{c-n} = 3.855E10 x 0.792 = 3.053E10 mm^4

$S_{t,c-n}$ = 1.081E08 x 0.792 = 8.562E07 mm^3

$S_{b,c-n}$ = 2.625E07 x 0.792 = 2.079E07 mm^3

Check of Stresses on the Net Section of the Web Splice Plates (S6.13.6.1.4b)

Allowable Flexural Stress at Strength I Limit State = F_y = 250 MPa (S6.13.6.1.4b)

Maximum applied stress in web splice plates = (ratio of web stiffness to section stiffness x M + M_e) /S

where:

M = total factored moment at the splice location

M_e = moment due to the eccentricity of the notional shear

S = the appropriate section modulus corresponding to the sign of the moment

Positive Flexure:

σ_t = -(0.306 x 1.532E10 + 4.875E08) / 8.562E07 = -60.4 MPa < 250 MPa OK

σ_b = (0.306 x 1.532E10 + 4.875E08) / 2.079E07 = 248.9 MPa < 250 MPa OK

Negative Flexure:

σ_t = (0.319 x 1.046E10 + 4.875E08) / 1.582E07 = 241.7 MPa < 250 MPa OK

σ_b = -(0.319 x 1.046E10 + 4.875E08) / 1.582E07 = - 241.7 MPa < 250 MPa OK

Shear (S6.13.5.3)

Nominal shear capacity of two web splice plates:

$V_n = 0.58A_gF_y$

$= (0.58)(6.57E04)(250) = 9.527E06$ N

Shear resistance, $V_r = \phi_v V_n = (1.00)(9.527E06)$
$= 9.527E06$ N

Design shear = 5.385E06 N < V_r = 9.527E06 N OK

Check Fatigue in Web Splice Plates

Gross section properties are required to be used in the stress calculations for the Fatigue Limit State (S6.13.6.1.4a). Fatigue needs to be checked only if the compressive stress due to permanent loads is less than twice the tensile stress due to the fatigue load combination (S6.6.1.2.1). Assuming new construction, the effect of the future wearing surface will be considered only if it reduces the compressive stresses due to permanent dead loads.

Unfactored stresses in splice plates due to permanent loads:

σ_t = -0.319 x (-6.274E08 - 2.29E07 - 2.90E07) / 1.998E07

= 10.85MPa > 0.0

Therefore, fatigue needs to be checked at the top of the web splice plates.

σ_b = 0.319 x (-6.274E08 - 2.29E07) / 1.998E07

= - 10.38 MPa < 0.0

σ_b is compression, therefore, check maximum stresses due to fatigue load combination to determine if fatigue needs to be checked at bottom of web splice plates.

Stress in splice plates due to fatigue load combination:

Positive flexure acting on the short-term composite section:

σ_t = -0.306 x (0.75 x 1.233E09) / 1.081E08

= -2.61 MPa

σ_b = 0.306 x (0.75 x 1.233E09) / 2.625E07

= 10.78 MPa

Negative flexure acting on the noncomposite section:

σ_t = 0.319 x (0.75 x 8.867E08) / 1.998E07

= 10.62 MPa

σ_b = -0.319 x (0.75 x 8.867E08) / 1.998E07

= -10.62 MPa

Permanent load compressive stress at the bottom < twice the maximum tensile stress from the fatigue load combination.

Therefore, fatigue at the bottom of the web splice plates needs to be checked.

Allowable Fatigue Stress Range:

Bolted connections are considered Fatigue Category B (Table S6.6.1.2.3-1). For this category, the allowable fatigue stress range, $F_{r,fat}$, is at least 55 MPa (S6.6.1.2.5).

Fatigue stress range, top of web splice plate
= |(-2.61) - (10.62)|

= 13.23 MPa < 55.0 MPa OK

Fatigue stress range, bottom of web splice plate =
|(10.78) - (-10.62)|

= 21.4 MPa < 55.0 MPa OK

Design of Web Splice Bolts

Assuming:

Y_i = the Y coordinate of a bolt

X_i = the X coordinate of a bolt

Both X and Y are measured relative to the centroidal axes of the bolt group at one side of the splice.

Moment of inertia of the web splice bolts at one side of the splice about the horizontal axis at the center of gravity of the bolt group = ΣY_i^2 = 1.294E07 mm^2.

Moment of inertia of the web splice bolts at one side of the splice about the vertical axis at the center of gravity of the bolt group = ΣX_i^2 = 6.75E04 mm^2.

All coordinates are measured relative to the center of gravity of the bolt group.

Moment of inertia of the bolt group about the neutral axis of the noncomposite section:

$I_{x,nc}$ = 1.294E07 mm^2

$I_{y,nc}$ = 6.75E04 mm^2

$I_{p,nc}$ = 1.294E07 + 6.75E04 = 1.301E07 mm^2

Moment of inertia of the bolt group about the neutral axis of the short-term composite section:

$I_{x,c-n}$ = $I_{x,nc}$ + Number of bolts x (the Y distance between the center of the bolt group to the center of the section)^2

= 1.294E07 + 48 x $(556)^2$ = 2.778E07 mm^2

$I_{y,c-n}$ = 6.75E04 mm^2

$I_{p,c-n}$ = 2.778E07 + 6.75E04 = 2.785E07 mm^2

Maximum Factored Shearing Force Applied to the Most Stressed Bolt

Effect of Moments:

The same loads that were used to design the splice plates are used to design the bolts.

Vertical component of bolt shear due to the portion of moment resisted by the web plus the moment of eccentricity of the notional shear = MX/I_p.

Horizontal component of bolt shear due to the portion of the moment resisted by the web plus the moment of eccentricity of the notional shear = MY/I_p.

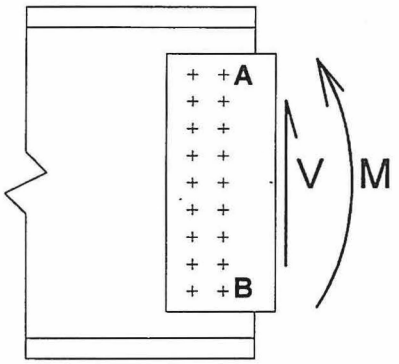
where :

M = total applied factored moment

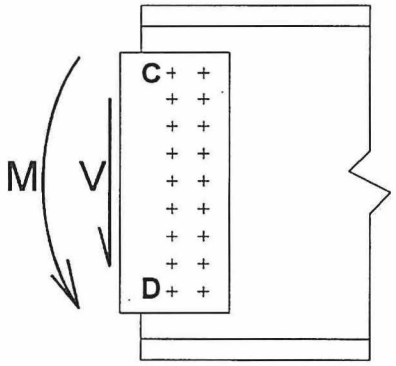
I_p = polar moment of inertia of the appropriate section with respect to the applied moment

X and Y = coordinates of the bolt with respect to the neutral axis of the section

See Figure 13.3-4 for location of the bolts relative to the neutral axis of the section. See Figure 13.3.4 for the location of the most stressed bolts in case of positive and negative flexure.



(a) Case of Positive Flexure



(b) Case of Negative Flexure

Figure 13.3-4 - Location of the Most Stressed Bolts

Case of Positive Flexure:

Vertical force on Bolt A = (0.306 x 1.532E10 + 4.875E08)(37.5)/(2.785E07) = 6970 N

Horizontal force on Bolt A = (0.306 x 1.532E10 + 4.875E08)(306.5)/ (2.785E07) = 56 960 N

Vertical force on Bolt B = 6970 N

Horizontal force on Bolt B = (0.306 x 1.532E10 + 4.875E08)(1418.5)/ (2.785E07) = 263 600 N

Case of Negative Flexure:

Vertical force on Bolt C = (0.319 x 1.046E10 + 4.875E08)(37.5)/ (1.301E07) = 11 020 N

Horizontal force on Bolt C = (0.319 x 1.046E10 + 4.875E08)(862.5)/ (1.301E07) = 253 530 N

Vertical force on Bolt D = 11 020 N

Horizontal force on Bolt D = 253 530 N

Effect of Notional Shear:

Bolt vertical shear due to notional shear = notional shear / number of bolts = 5.385E06/48 = 112 190 N/bolt

Total Shear Force on Bolts

By inspection, the force on bolt B controls the design
Total shear force in bolt B, P_u = [(total horizontal shear)^2 + (total vertical shear)^2]$^{0.5}$ = [(263 600)^2 + (6970 + 112 190)^2]$^{0.5}$ = 289 300 N/bolt

Shear Resistance of Bolts (S6.13.2.7)

Number of slip planes per bolt, N_s = 2

Assume:

D_{bolt} = 24 mm

A_{bolt} = $\pi(24)^2/4$ = 452.4 mm^2

Hole diameter for net area calculations = 24 + 3.2 = 27.2 mm (S6.8.3)

Assume threads are excluded from shear plane.

Bolt Shear Resistance at the Strength Limit State (S6.13.2.7)

P_n = $0.48\ N_s A_{bolt} F_{u,bolt}$

= 0.48(2)(452.4)(830) = 360 472 N

P_r = $\phi_s\ P_n$ = (0.80)(360 472) = 288 380 N

P_u = 289 300 N $\simeq$ P_r = 288 380 N OK

Bolt Bearing Resistance at the Strength Limit State (S6.13.2.9)

By inspection, thickness of the web is less than the sum of thicknesses of the two splice plates. Therefore, bearing on the web controls.

Clear distance between holes = 75.0 - 27.0 = 48.0 mm

Clear distance between holes and edge of splice plate = 50.0 - (27.0/2) = 36.5 mm ---controls

$L_{c,min}$ = 36.5 mm < $2.0D_{bolt}$ = 2.0(24) = 48.0 mm

P_n = $1.2L_{c,min}T_wF_{u,web}$

= 1.2(36.5)(20)(400) = 350 400 N

P_r = $\phi_{bb}P_n$ = (0.80)(350 400) = 280 300 N

P_u = 289 300 N > P_r = 280 300 N

The applied load 3.2% larger than the bearing resistance of the bolt. In situations where the bolts are slightly overstressed, most designers exercise some judgment to determine if the overstress value is within the practical limits considering the ratio between the design loads and the applied loads and considering the approximations in the design procedures. For the purpose of this example, the number of the bolts was not increased.

Bolt Slip Resistance at the Service II Limit State (6.13.22 and S6.13.2.8)

Applied Forces at the Service II Limit State:

Bolt vertical shear due to applied shear force
= shear force / number of bolts

= (1.21E06)/(48) = 25 210 N/bolt

Moment of eccentricity of applied shear = 1.21E06 x 90.5
= 1.095E08 N·mm

Case of Positive Flexure:

By inspection, the force on Bolt B is higher than that on Bolt A.

Vertical Force in Bolt B = (0.306 x 3.141E09 + 1.095E08)(37.5)/(2.785E07) = 1440 N

Horizontal Force in Bolt B = (0.306 x 3.141E09 + 1.095E08)(1418.5)/ (2.785E07) = 54 530 N

Case of Negative Flexure:

Vertical Force in Bolt C = (0.319 x 4.159E09 + 1.095E08)(37.5)/(1.301E07) = 4140 N

Horizontal Force in Bolt C = (0.319 x 4.159E09 + 1.095E08)(862.5)/ (1.301E07) = 95 210 N

Vertical Force in Bolt D = 4140 N

Horizontal Force in Bolt D = 95 210 N

By inspection, the force in Bolts C and D are equal and control the design.

Total shear force in bolt C or D, P_u = [(total horizontal shear)^2 + (total vertical shear)^2]$^{0.5}$ = [(95 210)^2 + (4140 + 25 210)^2]$^{0.5}$ = 99 630 N/bolt

Bolt slip resistance, $P_r = K_h K_s N_s P_t$

where:

Number of shear planes, N_s = 2

Minimum required bolt tension, P_t = 212 000 N (Table S6.13.2.8-1)

K_h = 1.00 for standard hole (Table S6.13.2.8-2)

K_s = 0.33 for surface class A (Table S6.13.2.8-3)

P_r = (1.00)(0.33)(2)(212 000) = 139 900 N (S6.13.2.8)

P_r > Applied force = 99 630 N OK

Design of Flange Splice Plates

Gross and Net Areas:

Due to the equal size of the top and bottom flanges and the use of the same number of gage lines in both of them, the gross and net areas are the same for both flanges.

Flange gross area = 420 x 32 = 13 440 mm^2

Section Modulii to Middle of Top and Bottom Flanges:

For noncomposite section (see Figure 13.3.3-a):

$S_{t,nc}$ = 3.876E10/991.0 = 3.911E07 mm^3

$S_{b,nc}$ = 3.876E10/991.0 = 3.911E07 mm^3

For short-term composite section (see Figure 13.3.3-b):

$S_{t,c\text{-}n}$ = 7.983E10/435.0 = 1.835E08 mm^3

$S_{b,c\text{-}n}$ = 7.983E10/1547.0 = 5.160E07 mm^3

Design Force for Flange Splice Plates

Assume Flange Force = Flange gross area x stress at center of flange based on gross section properties

Positive Flexure:

$P_{t,pos}$ = -13 440 x 1.532E10 / 1.835E08 = -1.122E06 N

$P_{b,pos}$ = 13 440 x 1.532E10 / 5.160E07 = 3.99E06 N

Negative Flexure:

$P_{t,neg}$ = 13 440 x 1.046E10 / 3.911E07 = 3.595E06 N

$P_{b,neg}$ = -13 440 x 1.046E10 / 3.911E07 = -3.595E06 N

Allowable Stresses

Compression: (S6.9)

$F_n = F_y$ = 345.0 MPa

$F_r = \phi_c F_n$ = (0.90)(345.0) = 310.5 MPa

Tension, yield of gross section: (S6.8.2.1)

F_{ny} = F_y = 345.0 MPa

F_{ry} = $\phi_y F_{ny}$ = (0.95)(345.0) = 327.8 MPa

Tension, fracture of net section: (S6.8.2.1)

F_{nu} = F_u = 450.0 MPa

F_{ru} = $\phi_u F_{nu}$ = (0.80)(450.0) = 360.0 MPa

Design of Bottom Flange Splice Plates

Case of Positive Flexure:

$P_{b,pos}$ = 3.99E06 N > 0 Use tension allowables

Total required flange splice plates gross area for yield in tension (S6.8.2.1-1):

A_g = $P_{b,pos}/F_{ry}$

= 3.99E06/327.8 = 12 172 mm^2

Total required flange splice plates net area for fracture in tension (S6.8.2.1-1):

A_{net} = $P_{b,pos}/F_{ru}$

= 3.99E06/360 = 11 083 mm^2

Case of Negative Flexure:

$P_{b,neg}$ = -3.595E06 N < 0 Use compression allowable

Total required flange splice plates gross area for yield in compression:

$A_g = P_{b,pos}/F_r$ = -3.595E06/(310.5) = 11 578 mm^2

Splice plates dimensions are chosen such that the areas of the plates above and below the flange are as close as practicable. This arrangement causes the center of the forces in the splice plates to be almost concentric with the center of the flange and, hence, minimizes the moment due to force eccentricity.

Assume:

420 x 20 mm wide outer splice plate, gross area = 8400 mm^2

175 x 22 mm wide inner plates, gross area of two plates = 7700 mm^2

Four gage lines:

Total provided gross area = 8400 + 7700 = 16 100 mm^2 > 12 172 mm^2 required OK

Net area of splice plates (S6.8.3):

Net section area used in the design is the smaller of the actual net section area or 85% of the gross section area (S6.13.5.2).

Diameter of standard holes for the purpose of calculating net area = diameter of bolt + 3.2 = 27.2 mm (S6.8.3)

Assuming four gage lines, as shown in Figure 13.3.2:

The width lost from the outer splice plate due to the presence of the holes = (4)(27.2) = 108.8 mm^2

Net width of outer flange splice plate = 420 -108.8 = 311.2 mm

Ratio of net to gross area = 311.2 / 420 = 0.74 < 0.85 OK (6.13.5.2)

Net width of each of the inner flange splice plates = 175 - 2 x 27.2 = 120.6 mm

Ratio of net to gross area = 120.6 / 175 = 0.69 < 0.85 OK (6.13.5.2)

Provided net area = 311.2 x 20 + 2 x 120.6 x 22 = 11 530 mm^2 > 11 083 mm^2 required OK

Design of Top Flange Splice Plates

By inspection, top flange has less force than the bottom flange and, hence, requires smaller splice plates area. For the purpose of this example, top flange splice is kept the same as the bottom flange splice.

Check Fatigue in Flange Splice Plates

Top Flange Splice Plates

Maximum stress under permanent loads = (6.274E08 + 2.29E07 + 2.90E07)/(3.911E07)] = 17.36 MPa > 0 Fatigue needs to be checked (S6.6.1.2.1)

Maximum stress from fatigue load combination = 0.75 x 8.867E08 / 3.911E07 = 17.0 MPa

Minimum stress from fatigue load combination = -0.75 x 1.233E09 / 1.835E08 = -5.0 MPa

Applied fatigue stress range = 17.0 -(-5.0) = 22.0 MPa

Allowed stress range for bolted connections, $F_{r,fat}$, is at least 55.0 MPa (S6.6.1.2)

Allowable range > applied range OK

Bottom Flange Splice Plates

All components of permanent load produce compression in the bottom flange. The most critical case is the case of minimum permanent load compressive stress. Therefore, the effect of the future wearing surface is not included.

Compressive stress due to permanent loads = (-6.274E08 - 2.29E07) / 3.911E07 = -16.6 MPa

Maximum stress from fatigue load combination = 0.75 x 1.233E09 / 5.160E07 = 17.9 MPa

|Permanent load compressive strength| < twice the tensile stress due to fatigue load combination, therefore, fatigue needs to be investigated (6.6.1.2).

Minimum stress from fatigue load combination = -0.75 x 8.867E08 / 3.911E07 = -17.0 MPa

Applied fatigue stress range = 17.9 -(-17.0) = 34.9 MPa

Allowable fatigue stress range for bolted splice = 55 MPa

Allowable range > Applied range = 34.9 MPa OK

Bottom Flange Splice Bolts

For 24 mm diameter bolts in a standard hole assuming two shear planes and assuming that threads are excluded from the shear planes, single bolt resistance was calculated earlier as:

Bolt shear resistance at the Strength Limit State (S6.13.2.7):

$P_r = \phi_s P_n = 288\ 380$ N (9)

Bolt Bearing Resistance at the Strength Limit State (S6.13.2.9):

By inspection, thickness of the flange is less than the sum of thicknesses of the two splice plates and the bolt edge distances in the flange is equal to those in the splice plates. Therefore, bearing on the flange controls.

Clear distance between holes = 75.0 - 27.0 = 48.0 mm

Distance from the center of last hole in the flange to the center of the splice = 50 mm

Clear distance from the holes to the edge of the flange = 50.0 - (27.0/2) = 36.5 mm controls

$L_{c,min}$ = 36.5 mm < 2.0 D_{bolt} = 2.0(24) = 48.0 mm

$P_n = 1.2 L_{c,min} T_{flange} F_{u,flange}$

$= 1.2(36.5)(32)(450) = 630\ 700$ N

$P_r = \phi_{bb} P_n = (0.80)(630\ 700) = 504\ 600$ N (10)

From (9) and (10), at the strength limit state, shear resistance of the bolt controls.

Required number of bolts

= Maximum force in flange / bolt resistance

= 3.99E06/288 380 = 13.84 bolts

Assume 14 bolts for both the top and bottom flanges

Bolt Slip Resistance at the Service II Limit State (6.13.22 and S6.13.2.8)

As calculated earlier, bolt resistance $P_r = \phi P_u =$ 139 900 N (S6.13.2.8)

Assume Flange Force = Flange gross area x stress at center of flange based on gross section properties

Positive Flexure:

$P_{t,pos}$ = -13 440 x 3.141E09 / 1.835E08 = -2.301E05 N

$P_{b,pos}$ = 13 440 x 3.141E09 / 5.160E07 = 8.181E05 N

Negative Flexure:

$P_{t,neg}$ = 13 440 x 4.159E09 / 3.911E07 = 1.429E06 N

$P_{b,neg}$ = -13 440 x 4.159E09 / 3.911E07 = -1.429E06 N

By inspection, the maximum forces are equal in both the top and bottom flanges.

Maximum force per bolt = 1.429E06 / 14 = 102 071 N < 139 900 allowed OK

Block Shear Rupture in Bottom Flange (S6.13.4)

Various possible modes of block shear rupture need to be checked. For the purpose of this example, only the three failure modes shown in Figure 13.3.5 were considered for the bottom flange. Since the flange dimensions, splice plates dimensions and bolt pattern are the same for both the top and bottom flange and the top flange forces are less than the bottom flange, there is no need to check the top flange for block shear failure.

For Failure Mode 1 (see Figure 13.3.5a)

The length of the planes of failure in the flange plate and in the splice plates is nearly equal. By inspection, Failure Mode 1 will occur in the flange before the splice plates due to its smaller thickness.

A_{vg} = 2 x (200 + 275) x 32 = 30 400 mm^2

A_{vn} = 30 400 - 2 x (2.5+3.5) x 27.2 x 32 = 19 955 mm^2

$s^2/4g$ = $(75)^2/(4 \times 75) = 18.75$

A_{tg} = 2 x (75 + 18.75) x 32 = 6000 mm^2

A_{tn} = 600 - 27.2 x 32 = 4259 mm^2

A_{tn}/A_{vn} = 4259 / 19 955 = 0.21 < 0.58, therefore:

R_r = ϕ_{bs} (0.58 F_u A_{vn} + F_y A_{tg})

= 0.8 x [(0.58 x 450 x 19 955) + (345 x 6000)] = 5.822E06 N

Maximum applied factored flange force = 3.99E06 N < R_r OK

For Failure Mode 2 (see Figure 13.3.5b)

Similar to Failure Mode 1, Failure Mode 2 will occur in the flange before the splice plates.

A_{vg} = 2 x 275 x 32 = 17 600 mm^2

A_{vn} = 17 600 - 2 x 3.5 x 27.2 x 32 = 11 507 mm^2

A_{tg} = 2 x 125 x 32 = 8000 mm^2

A_{tn} = 8000 - 2 x 0.5 x 27.2 x 32 = 7130 mm^2

A_{tn}/A_{vn} = 7130 / 11507 = 0.62 > 0.58 therefore:

R_r = ϕ_{bs} (0.58 F_y A_{vg} + F_u A_{tn})

= 0.8 x [(0.58 x 345 x 17 600) + (450 x 7130)] = 5.384E06 N

Maximum applied factored flange force = 3.99E06 N < R_r OK

For Failure Mode 3 (see Figure 13.3.5c)

A_{vg} = 2 x 220 x (20 + 22) = 18 480 mm^2

A_{vn} = 18 480 - 2 x 2.5 x 27.2 x (20 + 22) = 12 768 mm^2

A_{tg} = 320 x 20 + 2 x 125 x 22 = 11 900 mm^2

A_{tn} = 11 900 - 3 x 27.2 x 20 - 2 x 1.5 x 27.2 x 22 = 8473 mm^2

A_{tn}/A_{vn} = 8473 / 12 768 = 0.66 > 0.58

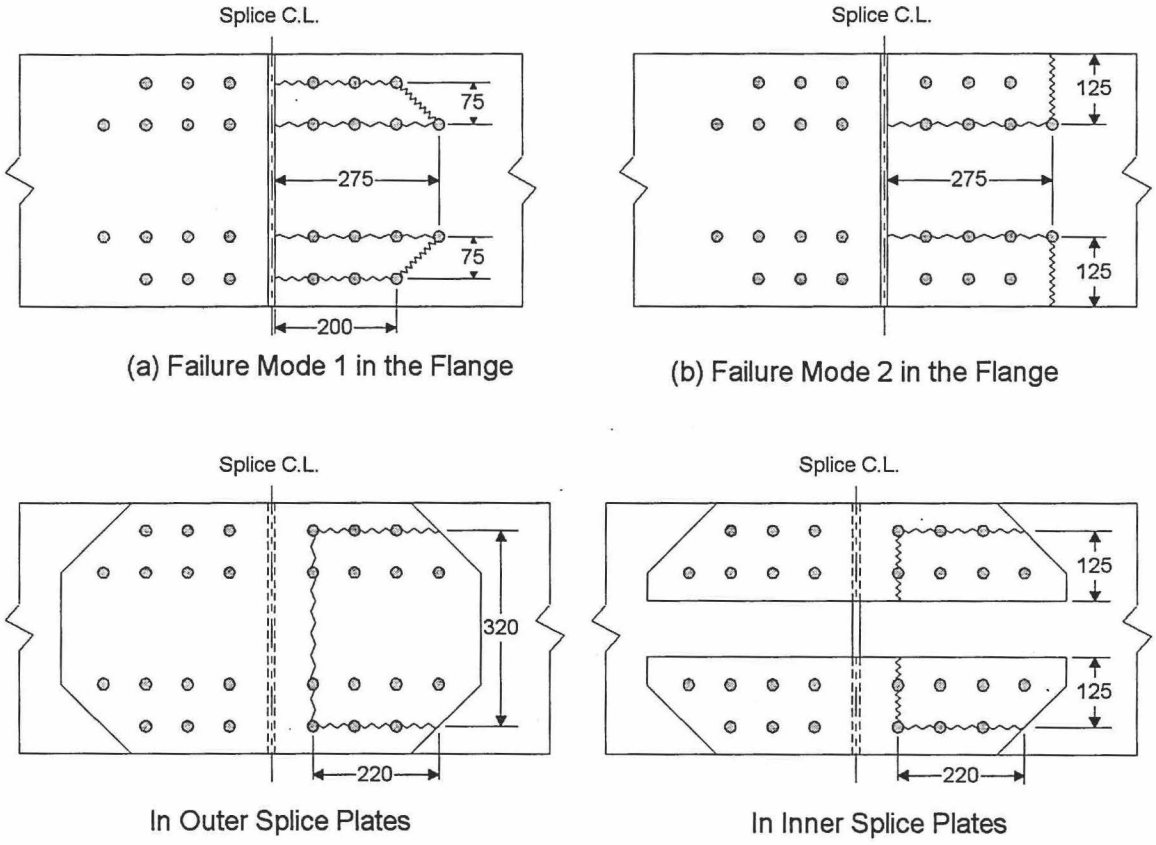
therefore:

R_r = ϕ_{bs} (0.58 F_y A_{vg} + F_u A_{tn})

= 0.8 x [(0.58 x 345 x 18 480) + (450 x 8473)] = 6.009E06 N

Maximum applied factored flange force = 3.99E06 N < R_r OK

Therefore, block shear failure is unlikely.



(a) Failure Mode 1 in the Flange

(b) Failure Mode 2 in the Flange

In Outer Splice Plates

In Inner Splice Plates

(c) Failure Mode 3

Figure 13.3-5 - Three Possible Modes of Block Shear Rupture

LECTURE 14 - INTRODUCTION TO FOUNDATIONS

14.1 OBJECTIVE OF LESSON

The purpose of this lesson is to introduce the design of spread footing and driven pile foundations required by the LRFD Specification. The lesson identifies general considerations which should be part of foundation design, outlines the design procedure, and presents requirements for the geotechnical and structural design of foundations at the service and strength limit states.

14.2 SPREAD FOOTING FOUNDATION DESIGN

14.2.1 General Design Considerations

In addition to the bearing resistance and load-settlement behavior of soil and rock that provide bearing support, the design of spread footing (or shallow) foundations requires consideration of local site constraints which can affect footing performance, including:

- Depth of the bottom of footing below ground surface to ensure adequate resistance to the effects of scour and frost;
- Anchorage of footings bearing on inclined rock surfaces by use of anchors, dowels or keys;
- Variable groundwater levels, including the effect of seepage when footings support walls which do not provide adequate drainage to prevent the build up of water behind the wall;
- Uplift loading; and
- The presence of nearby structures which are sufficiently close to the footing to impose load on the footing or could be influenced by the construction of or loading from the footing.

Of these factors, selection of the depth of embedment and evaluation of the variation of groundwater levels must be considered by the designer for most sites. If undermining by scour is a potential problem, footings must be constructed below the depth of scour. However, because accurate prediction of the depth of scour is difficult, the use of footings

at these sites is permitted only if the footings are below scour depths determined for the check flood for scour per S2.6.4.4.2 and foundations supported on driven piles or drilled shafts should be considered. In regions of the U. S. where ground freezing is possible, foundations should be placed below the maximum depth of frost penetration to prevent damage from frost heave. Figure 14.2.1-1 shows the maximum depth of frost penetration in the continental U. S.

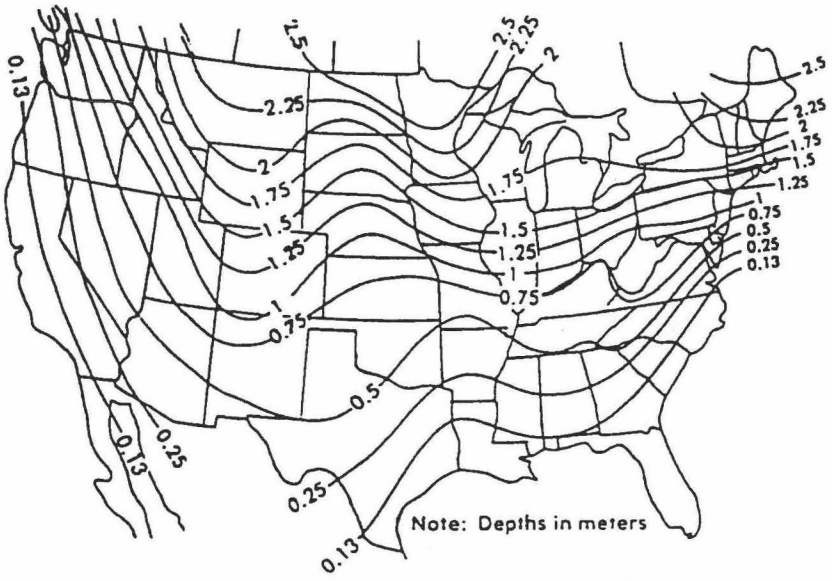


Figure 14.2.1-1 - Maximum Depth of Frost Penetration

Although most subsurface investigations include measurement of the groundwater level at the time of the exploration, the designer must recognize that the level varies with time and may not be at the maximum level at the time of the investigation. The periodic variation can be monitored using piezometers or estimated considering local variations in precipitation and the permeability of soils underlying the foundation. In regions of the U. S. where expansive or collapsible soils are present, variations in moisture content can result in substantial foundation movements if footings are constructed within the zone of influence of these moisture-sensitive soils.

The design of footings requires evaluation of footing movements at the Service Limit State and bearing resistance at the Strength Limit State. Because the design of most footings is controlled by settlement, consideration of settlement herein and in the Specification is presented first followed by evaluation of bearing resistance.

14.2.2 Design Procedure

The design procedure for spread footing foundations involves the following steps:

1. Develop design foundation profile including: soil and rock strata layering; engineering properties of foundation strata; groundwater level; and problem conditions.

2. Estimate loads for strength and service limit states analyses and select resistance factors for conditions analyzed.

3. Identify special conditions which need to be evaluated, such as loss of support due to scour or consolidation settlement of underlying cohesive soil layers.

4. Using presumptive bearing pressures or local experience, select preliminary footing width and length, and select footing depth based on location of suitable bearing strata and requirements to protect against freezing and/or scour of soils.

5. Estimate the settlement of the footing and compare with tolerable settlement. If the settlement is excessive, increase the footing width or revise the bearing level to achieve tolerable settlement.

6. Evaluate the bearing resistance of the foundation materials below the footing considering applicable load combinations.

7. Determine the location of the factored resultant force along the base of the footing and compare with eccentricity criterion. If the resultant load is beyond the acceptable limit, increase the footing width to meet eccentricity criterion.

8. Evaluate sliding resistance along base of footing due to inclined and lateral loads. If sliding resistance is not sufficient, increase footing width or improve footing contact by overexcavation and replacement of underlying materials to achieve acceptable sliding resistance.

9. Perform structural design of footing.

Table 14.2.2-1 summarizes the strength and service limits states that must be evaluated for the design of spread footing foundations.

Table 14.2.2-1 - Strength and Service Limit States for Design of Spread Footing Foundations

Design Consideration	Strength Limit State	Service Limit State
Bearing Resistance	X	
Settlement		X
Sliding Resistance	X	
Overturning (Eccentricity of Base Pressure Resultant)	X	
Structural Capacity	X	

14.2.3 Movement and Bearing Pressure at Service Limit State

14.2.3.1 ANALYSIS OF FOOTING MOVEMENTS

The movement of footings shall be evaluated at the service limit state using the Service I Load Combination from Table S3.4.1-1 in LRFD. Thus, the vertical settlement of footings should be estimated. The potential for lateral displacement of footings should also be considered where the:

- Footing is subjected to inclined or vertical loads;
- Footing is placed on or near an embankment slope;
- Loss of foundation support is possible by scour; and
- Bearing strata (usually rock) are significantly inclined.

Depending on the material providing foundation support, square or nearly square footings stress the foundation material to a depth of about two footing widths (i.e., 2B), and long footings (i.e., L/B > 5) stress the foundation material to a depth of about 4B.

The vertical settlement will be a combination of the elastic, consolidation and secondary compression movements. In general, settlement of footings on cohesionless soils, very

stiff to hard cohesive soils, and rock with tight, unfilled joints will be elastic and will occur as load is applied. For footings on very soft to stiff cohesive soils, the potential for consolidation and secondary compression settlement components should be evaluated in addition to elastic settlement.

Methods for estimating settlement of footings are described in Barker, et al, (1991), as well as various geotechnical engineering textbooks (e.g., Hunt, 1986; Holtz and Kovacs, 1981; and Fang, 1991). The elastic settlements of footings are estimated using elastic theory and a value of elastic modulus based on the results of in-situ or laboratory testing. The consolidation and secondary compression settlement of footings on cohesive soils is estimated using consolidation theory and the results of laboratory consolidation tests.

As an aid in estimating approximate footing dimensions for settlement analysis, Table 14.2.3.1-1 presents presumptive bearing pressures for various types of foundation material. After the approximate footing dimensions are determined, settlement of the footing can be determined as described in S10.6.2.2.3 in LRFD for footings on cohesionless and cohesive soils and rock.

Table 14.2.3.1-1 - Presumptive Bearing Pressures for Spread Footings at the Service Limit State

TYPE OF BEARING MATERIAL	CONSISTENCY IN PLACE	BEARING PRESSURE (MPa)	
		Ordinary Range	Recommended Value of Use
Massive crystalline igneous and metamorphic rock: graphite, diorite, basalt, gneiss, thoroughly cemented conglomerate (sound condition allows minor cracks)	Very hard, sound rock	5.7 to 9.6	7.7
Foliated metamorphic rock: slate, schist (sound condition allows minor cracks)	Hard sound rock	2.9 to 3.8	3.4

TYPE OF BEARING MATERIAL	CONSISTENCY IN PLACE	BEARING PRESSURE (MPa)	
		Ordinary Range	Recommended Value of Use
Sedimentary rock: hard cemented shales, siltstone, sandstone, limestone without cavities	Hard sound rock	1.4 to 2.4	1.9
Weathered or broken bedrock of any kind, except highly argillaceous rock (shale)	Medium hard rock	0.77 to 1.1	0.96
Compaction shale or other highly argillaceous rock in sound condition	Medium hard rock	0.77 to 1.1	0.96
Well-graded mixture of fine- and coarse-grained soil: glacial till, hardpan, boulder clay (GW-GC, GC, SC)	Very dense	0.77 to 1.1	0.96
Gravel, gravel-sand mixture, boulder-gravel mixtures (GW, GP, SW, SP)	Very dense Medium dense to dense Loose	0.57 to 0.96 0.38 to 0.67 0.19 to 0.57	0.67 0.48 0.29
Coarse to medium sand, and with little gravel (SW, SP)	Very dense Medium dense to dense Loose	0.38 to 0.57 0.19 to 0.38 0.096 to 0.29	0.38 0.29 0.14
Fine to medium sand, silty or clayey medium to coarse sand (SW, SM, SC)	Very dense Medium dense to dense Loose	0.29 to 0.48 0.19 to 0.38 0.096 to 0.19	0.29 0.24 0.22
Fine sand, silty or clayey medium to fine sand (SP, SM, SC)	Very dense Medium dense to dense Loose	0.29 to 0.48 0.19 to 0.38 0.096 to 0.19	0.29 0.24 0.22
Homogeneous inorganic clay, sandy or silty clay (CL, CH)	Very stiff to hard Medium stiff to stiff Soft	0.29 to 0.57 0.096 to 0.29 0.048 to 0.096	0.38 0.19 0.048

TYPE OF BEARING MATERIAL	CONSISTENCY IN PLACE	BEARING PRESSURE (MPa)	
		Ordinary Range	Recommended Value of Use
Inorganic silt, sandy or clayey silt, varied silt-clay-fine sand (ML, MH)	Very stiff to hard Medium stiff to stiff Soft	0.19 to 0.38 0.096 to 0.29 0.048 to 0.096	0.29 0.14 0.048

14.2.3.2 MOVEMENT CRITERIA

The tolerable movement of shallow foundations depends on structural criteria, such as the type and size of the supported superstructure, as well as factors, such as the cost and difficulty of implementing repairs in the future, rideability, aesthetics and safety. Based on such factors, limits on foundation movements have been set arbitrarily or based on empirical assumptions (e.g., 25 mm maximum vertical movement) without consideration of actual structural performance. However, due to the effects of creep, relaxation and redistribution of forces in bridge superstructures, bridges can accommodate substantially more settlement than traditionally allowed or anticipated in design. This conclusion is supported by the results of FHWA research (Moulton, et al, 1985) which involved a performance survey of more than 200 bridges supported on shallow foundations. The study showed that angular distortions (i.e., relative settlement of adjacent foundations divided by the span length) of 0.008, or less for simple span structures and 0.004 or less for continuous-span structures, are acceptable. Using these relationships, the maximum tolerable settlement between foundations can be much greater than normally assumed. The final choice for permissible movement should be made by the bridge designer in consultation with a geotechnical engineer.

14.2.4 Bearing and Sliding Resistance at the Strength Limit State

Bearing resistance pertains to the capacity of the foundation materials to support loads distributed from the structure and loads from other sources. If the foundation materials supporting a footing provide adequate bearing resistance, the design will ensure that bearing failure (and associated unacceptable movements) should not occur due to shear distortion in the ground below the footing.

Foundations must be designed to safely support the loads they carry. Accordingly, the foundation must not fail structurally and must provide adequate bearing support for all applicable loading conditions. Structural failure can be avoided by assuring the foundation has sufficient shear and moment capacity to distribute the load into the foundation materials. Evaluation of the structural resistance of foundations is presented in Sections 5, 6 and 8 of the LRFD Specification.

14.2.4.1 RESISTANCE FACTORS

Resistance factors for the geotechnical design of shallow foundations are presented in Table 14.2.4.1-1. A majority of the resistance factors in the table are based on design procedures which are commonly used for allowable stress design (ASD) of shallow foundations. The resistance factors presented in Table 14.2.4.1-1 were developed mostly by calibration with reliability theory where sufficient statistical information was available regarding particular design procedures, tempered with engineering judgment for some cases. Where statistical information was insufficient, resistance factors were chosen by calibration with working stress design so that LRFD and current design would result in a footing with similar dimensions.

14.2.4.2 BEARING RESISTANCE

The LRFD Specification requires that the bearing resistance of a footing subjected to vertical loads be evaluated at the strength limit state using the appropriate strength and/or extreme event load combinations from Table S3.4.1-1 and resistance factors from Table 14.2.4.2-1. If a footing is subjected to horizontal or inclined loading, the design must also provide adequate resistance to sliding (see 14.2.4.4).

Table 14.2.4.1-1 - Resistance Factors for Geotechnical Strength Limit State for Shallow Foundations

METHOD/SOIL/CONDITION		RESISTANCE FACTOR
Bearing Capacity and Passive Pressure	Sand - Semi-empirical procedure using SPT data	0.45
	- Semi-empirical procedure using CPT data	0.55
	- Rational Method -- using ϕ_f estimated from SPT data	0.35
	using ϕ_f estimated from CPT data	0.45
	Clay - Semi-empirical procedure using CPT data	0.50
	- Rational Method using shear resistance measured in lab tests	0.60
	using shear resistance measured in field vane tests	0.60
	using shear resistance estimated from CPT data	0.50
	Rock - Semi-empirical procedure, Carter and Kulhawy (1988)	0.60
	Plate Load Test	0.55
Sliding	Precast concrete placed on sand: using ϕ_f estimated from SPT data	0.90
	using ϕ_f estimated from CPT data	0.90
	Concrete cast-in-place on sand: using ϕ_f estimated from SPT data	0.80
	using ϕ_f estimated from CPT data	0.80

METHOD/SOIL/CONDITION		RESISTANCE FACTOR
	Sliding on clay is controlled by the strength of the clay when the clay shear is less than 0.5 times the normal stress, and is controlled by the normal stress when the clay shear is greater than 0.5 times the normal stress (see Figure 1).	
	Clay (where shear resistance is less than 0.5 times normal pressure)	
	using shear resistance measured in lab tests	0.85
	using shear resistance measured in field tests	0.85
	using shear resistance estimated from CPT data	0.80
	Clay (where the resistance is greater than 0.5 times normal pressure)	0.85
	Passive earth pressure component of sliding resistance	0.50
Overall Stability	Shallow foundations on or near a slope evaluated for overall stability and resistance to a deep-seated failure mode	0.85

The nominal bearing resistance (or ultimate bearing capacity, q_{ult}) of the foundation materials is determined using existing procedures from ASD for estimating q_{ult}. The Specification provides guidance for the determination of q_{ult} based on theoretical estimation procedures using soil properties based on test results and on in-situ test procedures using the SPT, CPT and pressuremeter. The design procedures can include the effects of:

- footing shape
- load inclination
- depth of footing embedment
- soil compressibility

- position of the groundwater table below the footing
- load eccentricity

For footings supported on saturated or nearly saturated cohesive soils, the unfactored value of q_{ult} can be determined as:

$$q_{ult} = cN_{cm} + g\gamma D_f N_{qm} \times 10^{-9} \qquad (14.2.4.2\text{-}1)$$

where:

q_{ult} = ultimate or nominal bearing capacity (MPa)

c = S_u = Undrained shear strength (MPa)

N_{cm}, N_{qm} = modified bearing capacity factors which account for the effects of footing shape, embedment depth, soil compressibility and load inclination (dim)

g = gravitational acceleration constant (9.8066 m/sec^2)

γ = total density of cohesive soil (kg/m^3)

D_f = embedment depth to footing bottom (mm)

The bearing capacity factors N_{cm} and N_{qm} are defined as:

N_{cm} = $5[1 + 0.2(D_f/B)][1 + 0.2(B/L)][1 - 1.3(H/V)]$ for $D_f/B \leq 2.5$, $B/L \leq 1$ and $H/V \leq 0.4$; (14.2.4.2-2)

N_{cm} = $7.5[1 + 0.2(B/L)][1 - 1.3(H/V)]$ for $D_f/B > 2.5$ and $H/V \leq 0.4$ (14.2.4.2-3)

N_{qm} = 1.0 for saturated clay (14.2.4.2-4)

For footings supported on cohesionless soils, the unfactored value of q_{ult} can be determined as:

$$q_{ult} = 0.5g\gamma BC_{w1}N_{\gamma m} \times 10^{-9} + g\gamma C_{w2}D_f N_{qm} \times 10^{-9} \qquad (14.2.4.2\text{-}5)$$

where:

B = width of footing (mm)

L = length of footing

C_{w1}, C_{w2} = coefficients for depth of groundwater level below footing (dim)

$N_{\gamma m}$, N_{qm} = modified bearing capacity factors which account for the effects of footing width and footing embedment (dim)

γ = total density of cohesionless soil (kg/m^3)

The bearing capacity factors $N_{\gamma m}$ and N_{qm} are defined as:

$$N_{\gamma m} = N_\gamma s_\gamma c_\gamma i_\gamma d_\gamma \qquad (14.2.4.2\text{-}6)$$

$$N_{qm} = N_q s_q c_q i_q d_q \qquad (14.2.4.2\text{-}7)$$

where values of each term can be interpolated from Tables 14.2.4.2-1 through 14.2.4.2-8.

Table 14.2.4.2-1 - Bearing Capacity Factors N_γ and N_q for Footings on Cohesionless Soil

(Barker, et al, 1991)

Friction Angle, (ϕ_f) (deg)	N_γ (dim)	N_q (dim)
28	17	15
30	22	18
32	30	23
34	41	29
36	58	38
38	78	49
40	110	64
42	155	85
44	225	115
46	330	180

Table 14.2.4.2-2 - Shape Factor s_q for Footings on Cohesionless Soil

(Barker, et al, 1991)

Friction Angle, (ϕ_f) (deg)	s_q (dim)			
	L/B = 1	L/B = 2	L/B = 5	L/B = 10
28	1.53	1.27	1.11	1.05
30	1.58	1.29	1.11	1.06
32	1.62	1.31	1.12	1.06
34	1.67	1.34	1.13	1.07
36	1.73	1.36	1.14	1.07
38	1.78	1.39	1.16	1.08
40	1.84	1.42	1.17	1.08
42	1.90	1.45	1.18	1.09
44	1.96	1.48	1.19	1.10
46	2.03	1.52	1.21	1.10

Table 14.2.4.2-3 - Shape Factor s_γ for Footings on Cohesionless Soil

(Barker, et al, 1991)

L/B	s_γ (dim)
1	0.60
2	0.80
5	0.92
10	0.96

Table 14.2.4.2-4 - Soil Compressibility Factors c_γ and c_q for Square Footings on Cohesionless Soil

(Barker, et al, 1991)

Relative Density (%)	Friction Angle, (ϕ_f) (deg)	$c_\gamma = c_q$ (dim)			
		q = 0.024 MPa	q = 0.048 MPa	q = 0.096 MPa	q = 0.192 MPa
20	30	1.00	1.00	0.92	0.89
30	32	1.00	1.00	0.85	0.77
40	35	1.00	0.97	0.82	0.75
50	37	1.00	0.96	0.81	0.73
60	40	1.00	0.86	0.72	0.65
70	42	0.96	0.80	0.66	0.60
80	45	0.79	0.66	0.54	0.48
100	50	0.52	0.42	0.35	0.31

Note: q equals the initial vertical effective stress at the footing depth, i.e., vertical stress at bottom of footing prior to excavation, corrected for water pressure.

Table 14.2.4.2-5 - Soil Compressibility Factors c_γ and c_q for Strip Footings on Cohesionless Soil

(Barker, et al, 1991)

Relative Density (%)	Friction Angle, (ϕ_f) (deg)	$c_\gamma = c_q$ (dim)			
		q = 0.024 MPa	q = 0.048 MPa	q = 0.096 MPa	q = 0.192 MPa
20	30	0.85	0.75	0.65	0.60
30	32	0.80	0.68	0.58	0.53
40	35	0.76	0.64	0.54	0.49
50	37	0.73	0.61	0.52	0.47
60	40	0.62	0.52	0.43	0.39
70	42	0.56	0.47	0.39	0.35
80	45	0.44	0.36	0.30	0.27
100	50	0.25	0.21	0.17	0.15

Note: q equals the initial vertical effective stress at the footing depth.

Table 14.2.4.2-6 - Load Inclination Factors i_γ and i_q for Load Inclined in Direction of Footing Width

(Barker, et al, 1991)

H/V	i_γ (dim)			i_q (dim)		
	Strip	L/B = 2	Square	Strip	L/B = 2	Square
0.10	0.73	0.76	0.77	0.81	0.84	0.85
0.15	0.61	0.65	0.67	0.72	0.76	0.78
0.20	0.51	0.55	0.57	0.64	0.69	0.72
0.25	0.42	0.46	0.49	0.56	0.62	0.65
0.30	0.34	0.39	0.41	0.49	0.55	0.59
0.35	0.27	0.32	0.34	0.42	0.49	0.52
0.40	0.22	0.26	0.28	0.36	0.43	0.46
0.45	0.17	0.20	0.22	0.30	0.37	0.41
0.50	0.13	0.16	0.18	0.25	0.31	0.35
0.55	0.09	0.12	0.14	0.20	0.26	0.30
0.60	0.06	0.09	0.10	0.16	0.22	0.25
0.65	0.04	0.06	0.07	0.12	0.17	0.21
0.70	0.03	0.04	0.05	0.09	0.13	0.16

Note: H and V are unfactored loads

Table 14.2.4.2-7 - Load Inclination Factors i_γ and i_q for Load Inclined in Direction of Footing Length

(Barker, et al, 1991)

H/V	i_γ (dim)			i_q (dim)		
	Strip	L/B = 2	Square	Strip	L/B = 2	Square
0.10	0.81	0.78	0.77	0.90	0.87	0.85
0.15	0.72	0.68	0.67	0.85	0.81	0.78
0.20	0.64	0.59	0.57	0.80	0.74	0.72
0.25	0.56	0.51	0.49	0.75	0.68	0.65
0.30	0.49	0.44	0.41	0.70	0.62	0.59
0.35	0.42	0.37	0.34	0.65	0.56	0.52
0.40	0.36	0.30	0.28	0.60	0.51	0.46
0.45	0.30	0.25	0.22	0.55	0.45	0.41
0.50	0.25	0.20	0.18	0.50	0.40	0.35
0.55	0.20	0.16	0.14	0.45	0.34	0.30
0.60	0.16	0.12	0.10	0.40	0.29	0.25
0.65	0.12	0.09	0.07	0.35	0.25	0.21
0.70	0.09	0.06	0.05	0.30	0.20	0.16

Note: H and V are unfactored loads.

Table 14.2.4.2-8 - Depth Factor d_q for Cohesionless Soils

(Barker, et al, 1991)

Friction Angle, (ϕ_f)	D_f/B (dim)	d_q (dim)
32	1	1.20
	2	1.30
	4	1.35
	8	1.40
37	1	1.20
	2	1.25
	4	1.30
	8	1.35
42	1	1.15
	2	1.20
	4	1.25
	8	1.30

Note: Values of d_q are applicable if the soils above the footing bottom are as competent as the soils below the footing. If the soils are weaker, use $d_q = 1.0$.

The nominal bearing resistance, R_n, of the foundation materials is determined by multiplying q_{ult} by the footing area. The adequacy of the footing at the strength limit state is then computed by multiplying R_n by the appropriate resistance factor, ϕ, and determining whether the factored resistance equals or exceeds the factored loads using the design equation for LRFD. If the factored resistance is less than the factored loads, the footing size must be increased until design equation for LRFD is satisfied.

14.2.4.3 LOAD ECCENTRICITY

If the location of load application is eccentric to the centroid of the footing, compliance of the footing design with tolerable movement criteria and adequate bearing resistance must be based on a reduced effective footing area (i.e., B′ x L′) within the dimensions of the footing so that the load is acting at the centroid of the reduced area. For the purposes of geotechnical design, the bearing pressure can be assumed to be uniformly applied to the effective footing area. As shown in Figure 14.2.4.3-1, the reduced dimensions for an eccentrically loaded footing can be taken as:

$$B' = B - 2e_B \qquad (14.2.4.3\text{-}1)$$

$$L' = L - 2e_L \qquad (14.2.4.3\text{-}2)$$

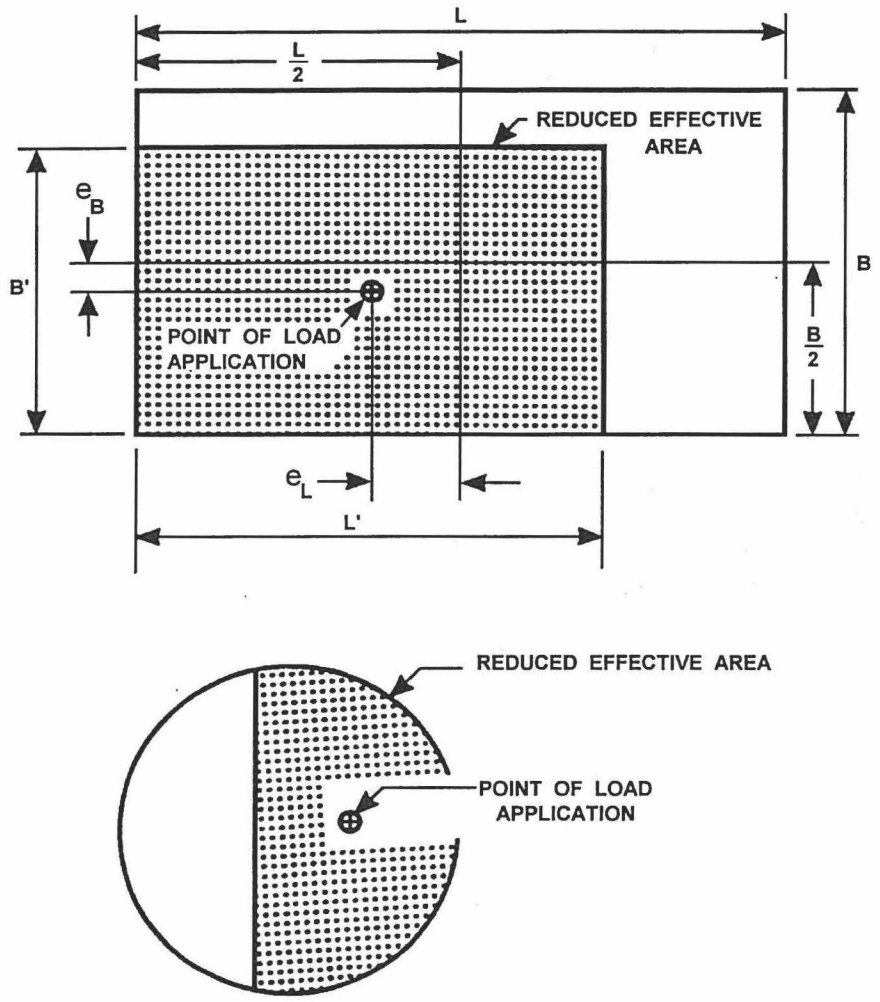


Figure 14.2.4.3-1 - Reduced Footing Dimensions

where e_B and e_L are the eccentricities parallel to the B and L dimensions, respectively. On this basis, footings subjected to eccentric loads must be designed to ensure that:

- The factored bearing resistance is not less than the factored loads; and

- Except for the extreme event limit states, the eccentricity of footings based on factored loads is less than B/4 or L/4 for footings on soil, and less than 3B/8 or 3L/8 for footings on rock. More liberal eccentricities may be allowed for the extreme event limit state.

For structural design, the bearing pressure at the base of the footing is usually assumed to vary linearly across the bottom of the footing. This assumption results in a slightly

conservative triangular or trapezoidal contact pressure distribution.

14.2.4.4 SLIDING RESISTANCE

If footings support inclined loads or are founded on an inclined base, failure by sliding along the footing base must be evaluated. As described in S10.6.3.3, the factored resistance against failure by sliding, Q_R, is determined as:

$$Q_R = (\phi_r Q_r + \phi_{ep} Q_{ep}) \qquad (14.2.4.4\text{-}1)$$

where:

ϕ_r = resistance factor for shear between the footing and foundation material from Table S10.5.4-1 (dim)

Q_r = nominal shear resistance between footing and foundation material (N)

ϕ_{ep} = resistance factor for passive resistance from Table 14.2.4.1-1 (dim)

Q_{ep} = nominal passive resistance of foundation material available throughout the design life of the footing (N)

For most cases, the passive resistance component of sliding resistance is ignored or reduced because it is difficult to assure that loss of ground (e.g., temporary excavation) or loss of contact by shrinkage will not occur in the future.

For footings supported on cohesionless soil, Q_r can be determined as:

$$Q_r = V \tan\delta \qquad (14.2.4.4\text{-}2)$$

where:

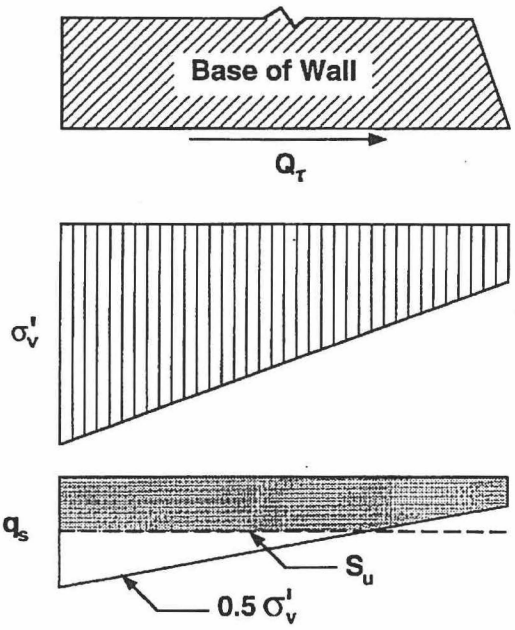
V = total vertical force (N)

$\tan\delta$ = $\tan\phi_f$ for concrete cast directly against soil, or $0.8\tan\phi_f$ for a precast footing

ϕ_f = soil angle of friction (deg)

For footings supported on cohesive soil, Q_r can be determined as the lesser of:

- the cohesion of the foundation soil, or
- one-half the normal stress on the interface between the footing and soil as shown in Figure 14.2.4.4-1 below.



q_s = unit shear resistance, equal to S_u or $0.5\ \sigma'_v$, whichever is less

Q_T = area under q_s diagram (shaded area)

S_u = undrained shear strength (MPa)

σ'_v = vertical effective stress (MPa)

Figure 14.2.4.4-1 - Procedure for Estimating Sliding Resistance of Footings on Clay

It should be noted that the latter case is applicable only when the footing is supported on a thin (e.g., 150 mm) thickness of compacted granular soil prior to footing construction.

Evaluation of footing failure by sliding is necessary, for example, for the design of a retaining wall. For this case, the magnitudes of earth load which tend to stabilize and destabilize the footing can be determined as described in Article S3.11.5 of the Specification and Section 14.2.2 herein.

14.3 DRIVEN PILE FOUNDATIONS DESIGN

14.3.1 General Design Considerations

In addition to determining the design pile section to meet the service and strength limit states criteria for axial and lateral loading, the design of driven pile foundations required consideration of factors which can affect pile performance including:

- pile spacing and the effects of group action (S10.7.15, S10.7.3.8 and S10.7.3.10)
- the possibility of scour and its effect on capacity and movement for piles installed near or into flowing water channels (S10.7.1.3)
- negative or downdrag loads resulting from piles driven through soft compressible soil which are settling due to applied surcharges (S10.7.1.4)
- the effect of variable groundwater levels and buoyancy (S10.7.1.7)
- protection against deterioration due to corrosion of steel piles, sulfate, chloride and acid attack on concrete piles, and decay of timber piles due to wet/dry cycles and insects (S10.7.1.8)
- uplift loading of single piles and pile groups (S10.7.1.9 and S10.7.3.7)
- battering of piles to improve the resistance of a pile group to lateral loading (S10.7.1.6 and S10.7.3.9)

14.3.2 Design Procedure

The design procedure for driven pile foundations involves the following steps:

1. Develop design foundation profile including soil and rock strata layering, engineering properties of foundation strata, groundwater level and problem conditions.
2. Estimate loads for strength and service limit states analyses and select resistance factors for conditions analyzed.

3. Identify special conditions which need to be evaluated, such as loss of lateral support due to scour or negative loading on end bearing piles due to settling ground.

4. Select pile type(s) and lengths suitable for site conditions, local availability and cost.

5. Estimate axial (compression) pile capacity considering both soil/rock and structural capacity, and investigate drivability.

6. Determine the number, location and spacing of piles in a group required to support factored load combinations.

7. Estimate pile group efficiency and adjust number and spacing of piles to provide adequate capacity.

8. For bearing pile group, check for possible punching of pile group into weak stratum underlying bearing level.

9. Estimate the settlement of the pile group and compare with tolerable settlement. If the settlement is excessive, revise the pile design by increasing the length and/or number of piles.

10. If pile group is subjected to uplift loading, check uplift capacity.

11. Evaluate the structural capacity of pile group to lateral loading.

12. Estimate the lateral displacement of the pile group and compare with tolerable displacement. If the displacement is excessive, revise the pile design by increasing the length and/or number of piles.

13. Perform structural design of pile in accordance with Sections 5, 6 or 8 of LRFD depending on pile type.

14. Determine whether special construction control is required, such as dynamic monitoring or static load testing.

Table 14.3.2-1 summarizes the strength and service limits states that must be evaluated for the design of driven pile foundations.

Table 14.3.2-1 - Strength and Service Limit States for Design of Driven Pile Foundations

Design Consideration	Strength Limit State	Service Limit State
Bearing Resistance of Single Pile/Group	X	
Structural Capacity of Single Pile	X	
Punching	X	
Settlement of Pile Group		X
Tensile Capacity of Uplift-Loaded Piles	X	
Structural Capacity of Laterally-Loaded Piles	X	
Lateral Displacement of Pile or Pile Group		X

14.3.3 Movement at the Service Limit State

14.3.3.1 ANALYSIS OF PILE DISPLACEMENTS

The displacement of driven piles in cohesionless soil shall be evaluated for all applicable load combinations using the Service I Load Combination from Table S3.4.1-1. For piles in cohesive soil, the requirement to consider transient loads may be omitted. The axial displacement of piles should be estimated. The lateral displacement of piles should also be evaluated where the:

- pile is subjected to inclined or vertical loads,
- pile is placed on or near an embankment slope, and
- loss of lateral foundation support by scour is possible.

The Specification provides guidance for estimating the settlement of pile groups in cohesive and cohesionless soils. The procedure for estimating the settlement of a pile group in cohesive soil uses an equivalent footing for the pile group as shown in Figure 14.3.3.1-1. When the equivalent footing

procedure is used, the methods presented in S10.6.2 for estimating the immediate and consolidation settlements of footings on cohesive soil should be followed.

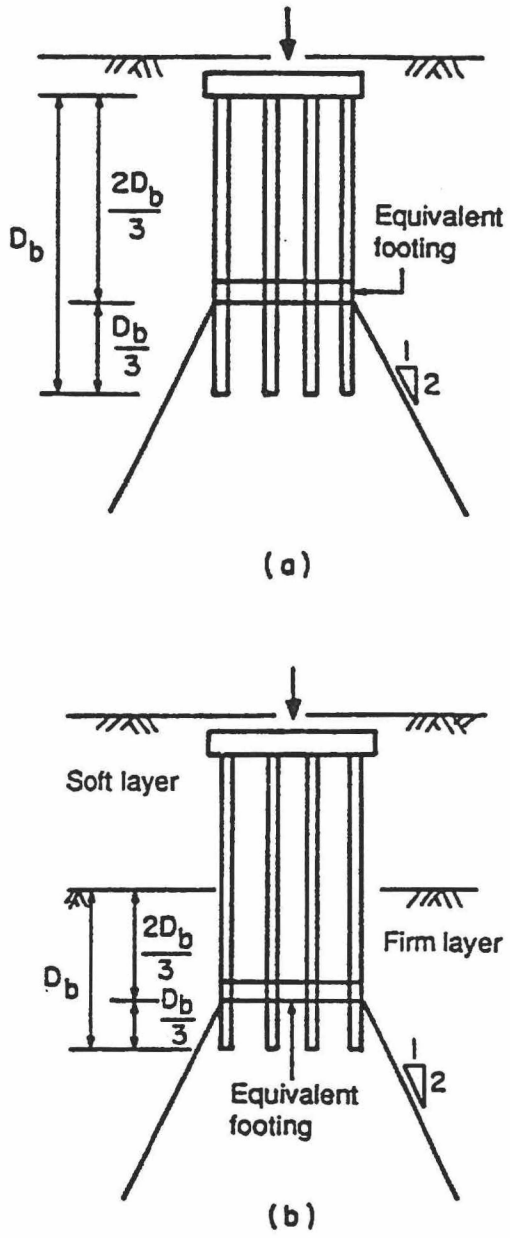


Figure 14.3.3.1-1 - Location of Equivalent Footing

For a pile group in cohesionless soil, semi-empirical procedures are presented in LRFD which are based on the results of SPT and CPT in-situ testing. If SPT results are available, the settlement of a pile group bearing in sand can be estimated using:

$$\rho = \frac{360 q I \sqrt{X}}{N_{corr}} \qquad (14.3.3.1\text{-}1)$$

where:

$$I = 1 - 0.125 \frac{D'}{X} \leq 0.5 \qquad (14.3.3.1\text{-}2)$$

$$N_{corr} = \left[0.77 \log_{10}\left(\frac{1.92}{\sigma'_v}\right)\right] N \qquad (14.3.3.1\text{-}3)$$

where:

ρ = settlement of pile group (mm)

q = net foundation pressure applied at a depth of $2D_b/3$ as shown in Figure 14.3.3.1-1 (MPa)

X = width or smallest dimension of pile group (mm)

I = influence factor of the effective group embedment (dim)

D′ = effective depth taken as $2D_b/3$ (mm)

D_b = depth of pile group embedment (mm)

N_{corr} = average corrected SPT blow count within a depth of X below the group (blows/300 mm)

σ'_v = effective vertical stress (MPa)

If CPT results are available, the settlement of a pile group bearing in sand can be estimated using:

$$\rho = \frac{qXI}{2q_c} \qquad (14.3.3.1\text{-}4)$$

where:

q_c = average static cone resistance within a depth of X below the group (MPa)

Reference is made (SC10.7.2.4) to a procedure for estimating the lateral displacement of pile groups (e.g., Barker,

et al, 1991). This method is based on various simplifying assumptions regarding uniformity of the soil profile and fixity of the pile in the cap. Therefore, the procedure can be used for preliminary evaluation and when the problem does not substantially violate the assumptions of the procedure. During the past decade, computer programs have become available for the lateral load-deflection analysis of driven piles and drilled shafts based on the p-y curve method of analysis. For individual vertical piles, these programs include COM624 (Reese, 1984) for individual flexible vertical piles in level ground, LTBASE (Borden and Gabr, 1987) for individual flexible vertical piles in level and sloping ground, COM624P (Version 2) (Wang and Reese, 1993) for individual flexible and rigid vertical piles in level and sloping ground, and GROUP1 (Reese, et al, 1988) for a group of flexible vertical and battered piles in level ground can be used.

14.3.3.2 TOLERABLE MOVEMENT CRITERIA

The tolerable axial and lateral movement of driven piles should be established using the structural criteria described for shallow foundations. In some cases, the tolerable lateral movement is fixed at a small displacement (e.g., $\leq$12 mm based on observed acceptable performance and engineering judgment) to ensure acceptable structure performance.

14.3.4 Resistance at the Strength Limit State

The resistance of driven pile foundations at the strength limit state should include consideration of:

- bearing resistance,
- uplift resistance,
- punching of a pile group through a strong layer into a weaker underlying layer,
- structural resistance of the piles under vertical and lateral loads, and
- anticipated damage from pile driving, if any.

Large displacements are typically required to cause passive failure of the soil in response to lateral loading of piles. Therefore, lateral resistance of the supporting soil need not be evaluated as long as tolerable movement criteria are met.

14.3.4.1 RESISTANCE FACTORS

Resistance factors for the geotechnical design of driven pile foundations are presented in Table 14.3.4.2-1. A majority of the resistance factors in the table are based on design procedures which are commonly used for allowable stress design (ASD) of axially-loaded piles (i.e., α, β and λ methods). Development of the resistance factors was based primarily on the use of reliability theory where a statistically sufficient source of information was available. Where sufficient information was not available, selection of resistance factors was based on calibration with working stress designs and engineering judgment.

14.3.4.2 AXIAL LOADING

The factored bearing resistance of piles subjected to axial loading, ϕ_R, can be determined as:

$$Q_r = \phi Q_n = \phi_{qp} Q_p + \phi_{qs} Q_s \qquad (14.3.4.2\text{-}1)$$

for which:

$$Q_p = q_p A_p \qquad (14.3.4.2\text{-}2)$$

$$Q_s = q_s A_s \qquad (14.3.4.2\text{-}3)$$

where:

ϕ_{qp}, ϕ_{qs} = resistance factors for tip resistance and side resistance in Table 14.2.4.1-1 (dim)

Q_p, Q_s = pile tip and side resistance (N)

q_p, q_s = unit tip and side resistance (N)

A_p, A_s = area of the pile tip and side surface (mm^2)

A further reduction in P_n for piles should be considered when pile driving difficulty is expected. In the past, a reduction multiplier factor of about 0.875 was considered when moderate driving difficulty was expected, and a factor of about 0.75 was considered when difficult driving was expected. For further details see Davisson (1983).

Table 14.3.4.1-1 - Resistance Factors for Geotechnical Strength Limit State in Axially Loaded Piles

(Table S10.5.4-2 in LRFD)

METHOD/SOIL/CONDITION		RESISTANCE FACTOR
Ultimate Bearing Resistance of Single Piles	Skin Friction: Clay	
	α-method (Tomlinson, 1987)	0.70
	β-method (Esrig & Kirby, 1979)	0.50
	λ-method (Vijayvergiya & Focht, 1972)	0.50
	End Bearing: Clay and Rock	
	Clay (Skempton, 1951)	0.70
	Rock (Canadian Geotech. Society, 1985)	0.50
	Skin Friction and End Bearing:Sand	
	SPT-method	0.45
	CPT-method	0.55
	Skin Friction and End Bearing: All Soils	
	Load Test	0.80
	Pile Driving Analyzer	0.70
Block Failure	Clay	0.65
Uplift Resistance of Single Piles	α-method	0.60
	β-method	0.40
	λ-method	0.45
	SPT-method	0.35
	CPT-method	0.45
	Load Test	0.80
Group Uplift Resistance	Sand	0.55
	Clay	0.55

Using Equation 14.3.4.2-1, LRFD provides procedures for estimating the axial capacity of driven piles in soil using semi-empirical methods and in-situ testing, and for piles bearing on or in rock. When using Equation 14.3.4.2-1, it should be recognized that the pile capacity is comprised of components

resulting from side resistance and tip resistance. Thus, for point-bearing piles, the tip resistance is the predominant source of pile capacity, and for friction piles, side resistance is the predominate source of pile capacity. For combination point-bearing and friction piles, pile capacity can be estimated by adding the relative contributions from tip and side resistance. It should be noted, however, that because the maximum values of side and tip resistance are mobilized at different displacements, care should be taken in using the numerical sum of the individual resistances for resistances less than Q_N. Additionally, because the values of ϕ_{qp} and ϕ_{qs} depend on the method used to estimate the pile bearing resistance, the values of ϕ_{qp} and ϕ_{qs} will usually differ.

The semi-empirical methods presented in LRFD include the α-method which is applicable for estimating q_s for displacement and non-displacement piles in cohesive soils, the β-method which is applicable for estimating q_s for displacement piles in normally to lightly overconsolidated cohesive soils, and the λ-method which is applicable for estimating q_s for displacement piles in cohesive soils. The in-situ test methods presented in LRFD include procedures for estimating q_s and q_p in cohesionless soils based on SPT and CPT test results, and q_p in rock based on information from coring to define the spacing and width of fractures and joints, and laboratory testing to determine the compressive strength of intact rock core.

As an example of procedures presented in LRFD to estimate the factored bearing resistance, Q_R, of a axially-loaded driven pile foundation, the procedure presented for a pile in cohesive soil is presented herein. Using the α-method, the nominal unit shaft resistance, q_s, can be determined as:

$$q_s = \alpha S_u \qquad (14.3.4.2\text{-}4)$$

where:

S_u = mean undrained shear strength along the length of pile (MPa)

α = adhesion factor applied to S_u (dim)

The value of α can be determined using Figure 14.3.4.2-1 which shows how α varies, depending on the installation condition, pile length in cohesive soil and S_u.

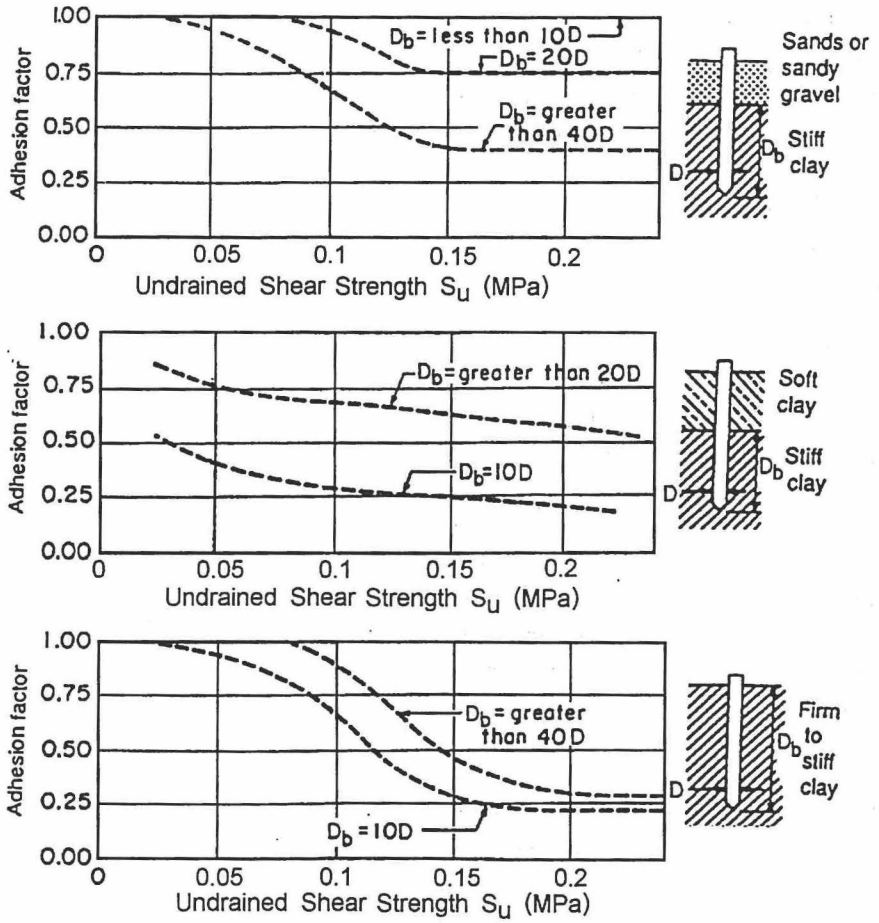


Figure 14.3.4.2-1 - Design Curves for Adhesion Factors for Piles Driven into Clay Soils

(Figure 10.7.3.3.2a-1 from LRFD)

The nominal unit tip resistance, q_p, can be determined as:

$$q_p = 9S_u \qquad (14.3.4.2\text{-}5)$$

where:

S_u = undrained shear strength of clay within 2 to 4 diameters of the pile tip (MPa)

The specification also provides guidance for determining the factored pile capacity based on the results of static pile load testing and dynamic monitoring of piles during pile driving operations.

The Specification also provides guidance for estimating pile capacity based on the results of static pile load testing and dynamic monitoring of piles during pile driving operations.

14.3.4.3 LATERAL LOADING

Most pile foundations supporting highway structures are subjected to lateral loading due to earth and water pressures, wind, earthquake and live loads. For bridges, the axial load is usually greater than the lateral load, while for structures such as noise walls and overhead sign structures, the lateral loads usually exceed the axial loads.

Pile foundations must be designed to resist lateral loads without structural failure of the pile (i.e., without reaching the strength limit state), and without deflecting excessively (i.e., without exceeding the tolerable lateral movement at the service limit state.) The analysis of a laterally-loaded pile or pile group is a complex soil-structure interaction problem. If the foundation materials are homogeneous (e.g., uniform cohesionless or cohesive soil deposit) and the pile head is fixed, simplified methods of analysis (e.g., Barker, et al, 1991) can be used. However, because the foundation loading conditions are usually more complex and the fixity of the pile is intermediate between the fixed- and free-head case, the analysis of laterally loaded piles and pile groups is usually accomplished using computer programs based on the p-y curve method of lateral load-deflection analysis. These programs include COM624 (Reese, 1984) for individual vertical piles in level ground, LTBASE (Borden and Gabr, 1987) for individual vertical piles in level and sloping ground, and GROUP1 (Reese, et al, 1988) for a group of vertical and battered piles in level ground can be used. Each of these methods of analysis incorporate p-y curves which model the nonlinear lateral load-deflection behavior for each soil type along the length of the pile. For pile groups, the effect of closely-spaced piles (i.e., where the center-to-center (CTC) spacing between piles is less than about 6 pile diameters) can be considered in the analyses by modifying the p-y relationships using efficiency relationships.

Design of laterally-loaded piles using the above methods can be performed using the following procedure:

1. For the pile section selected based on the service and strength limit state design for axial loading, determine the maximum lateral groundline deflection and the maximum factored moment for an individual pile at the strength limit state using an appropriate method of analysis and factored axial and lateral loads.

2. If the lateral groundline deflection exceeds a tolerable deformation or the maximum factored moment exceeds the factored moment capacity of the pile obtained from Sections 5, 6 or 8, select a new pile section and repeat Step 1.

3. If neither the lateral groundline deflection nor the factored moment criteria in Step 2 is exceeded, compute the maximum lateral groundline lateral deflection for the pile at the service limit state using an appropriate method of analysis and unfactored axial and lateral loads.

4. If the lateral groundline deflection exceeds the tolerable movement criteria, select a new trial section and repeat Step 1.

5. If the lateral groundline deflection does not exceed the tolerable movement criteria, the pile section is acceptable for the design loads.

For these analyses, lateral load-deflection relationships used to determine deflections should be not be factored, because these relationships are full-range and model non-linear behavior up to ultimate strength of the soil.

14.3.4.4 BATTER PILES

If the lateral loads acting on a foundation are large, batter piles are often used because they are more effective than vertical piles in transmitting lateral load into soil. Batter piles are more efficient in this regard because they transmit lateral loads predominately as an axial force into the soil, and because piles are substantially stiffer in axial loading than in lateral loading. In some instances, however, battered piles should not be used. These applications include:

- Installations where negative loading or downdrag forces can develop and cause bending, and

- applications in areas of high seismicity (i.e., Seismic Zones 3 and 4) where high lateral forces from earthquake loading can result in local overstressing of piles near the cap/batter pile contact, and punching of piles through the cap.

14.3.4.5 GROUP BEHAVIOR

The design of pile groups is similar to the design of an individual pile, such that the nominal resistance of the pile group, Q_r is:

$$Q_r = \phi Q_n = \phi_g Q_g \qquad (14.3.4.5\text{-}1)$$

where:

Q_g = nominal resistance of the group (N)

ϕ_g = group resistance factor from Table 14.2.4.1-1

In cohesive soils, the axial load resistance of a pile group may be less than the sum of the resistance of the individual piles due to overlapping of zones of shear deformation in the soil surrounding the piles. In stiff cohesive soils, there is no loss in resistance due to group effects. However, in soft cohesive soils where the pile cap is not in contact with the underlying soil, the resistance of a pile group will be less than the cumulative resistance of an equal number of individual piles if the CTC spacing between piles is less than six pile diameters. For closer pile spacings, the resistance of each pile is multiplied by an efficiency factor, η, such that:

η = 0.65 for CTC spacing of 2.5 diameters

η = 1.0 for CTC spacing of 6.0 diameters.

For intermediate spacings, the value of η can be determined by linear interpolation.

The value of Q_g for a group of piles in cohesive soil should be determined as the lesser of:

- the sum of the modified individual resistances of each pile in the group, or
- the resistance of an equivalent pier consisting of the piles and the block of soil bounded within the piles.

For the equivalent pier method:

- the peak shear strength of the soil should be used to determine the side resistance,
- the total base area of the pier should be used to determine the end bearing resistance, and

- any resistance provided by the cap should be ignored.

The resistance factor for block failure should be used to determine the factored resistance of the pile group determined on the equivalent pier basis, regardless of whether the pile cap is or is not in contact with the ground. If the resistance of an equivalent number of individual piles is adjusted by η, the resistance factor for a single pile in Table S10.5.4-2 should be used to determine the factored resistance of the pile group. The commentary to S10.7.3.10.2 provides guidance regarding determination of Q_g using the equivalent pier method.

For a pile group in cohesionless soil, the value of η is 1.0, regardless of the spacing of piles or contact between the pile cap and the ground. No reduction in capacity is used for a pile group in sand due to the increase in soil density that usually occurs when piles are driven in sand. For this case, the resistance factor for axial capacity of a single pile in Table 10.5.4-2 should be used to determine the factored resistance of the pile group.

If a pile group is embedded in a stiff soil deposit overlying a weaker deposit, the potential exists for a punching failure of the pile group through the stiff soil into the soft soil. In addition, the potential for settlements in the soft layer must be evaluated. If the distance between the pile tips of the group and the top of the weaker layer is greater than 10 times the width of the group, Q_g equals the value based on the soil properties into which the group is embedded. If the distance is less than ten times the width of the group, Q_g can be determined as:

$$Q_g = (Q_g)_0 + \frac{[(Q_g)_1 - (Q_g)_0]H}{10B} < (Q_g)_1 \qquad (14.3.4.5\text{-}2)$$

where:

$(Q_g)_0$ = nominal group capacity for the weaker layer (N)

$(Q_g)_1$ = nominal group capacity for the stronger layer (N)

B = pile group width (mm)

H = distance between pile tips and weaker layer (mm)

14.3.4.6 STRUCTURAL DESIGN

In addition to meeting the requirements for the structural design of piles in accordance with Sections 5, 6 and 8 of LRFD, the potential for buckling should be evaluated when piles extend through air or water before achieving lateral fixity at some depth into the ground. The provisions for stability of compression members in Sections 5, 6 and 8 of LRFD require the determination of the depth to fixity to estimate the equivalent unsupported length of the pile. For these cases, the depth to fixity, D_f in mm, can be determined as:

- for piles in cohesive soil:

$$D_f = 1.4\left[\frac{E_p I_p}{E_s}\right]^{0.25} \qquad (14.3.4.6\text{-}1)$$

- for piles in cohesionless soil:

$$D_f = 1.8\left[\frac{E_p I_p}{n_h}\right]^{0.20} \qquad (14.3.4.6\text{-}2)$$

where:

E_p = modulus of elasticity of the pile section (MPa)

I_p = moment of inertia of the pile section (mm^4)

E_s = soil modulus for cohesive soils = 67Su (MPa)

S_u = undrained shear strength of cohesive soil (MPa)

n_h = rate of increase of soil modulus with depth for cohesionless soil (MPa/mm) (Table S10.7.4.2-1)

If the spacing between piles is less than eight pile diameters, the modulus of cohesive soil should be reduced in the previous relationship. If the spacing between piles is three pile diameters, the soil modulus should be reduced by 25%, and the reduction for spacings between eight and three piles diameters can be determined by interpolation.

In Equations 1 and 2, the loading condition has been assumed to be axial load only and the piles are assumed to be fixed at their ends. Since the equations give depth to fixity from the ground line, the Engineer must determine the boundary conditions at the top of the pile to determine the total unbraced length of the pile. If other loading or pile tip conditions exist, see Davisson and Robinson (1965).

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LECTURE 15 - INTRODUCTION TO WALLS

15.1 OBJECTIVE OF LESSON

The purpose of this lesson is to introduce the design of conventional walls and abutments, anchored walls and mechanically-stabilized earth walls required by the LRFD Specification. The lesson identifies general considerations which should be part of wall design, outlines the design procedure, and presents requirements for the geotechnical and structural design of walls at the service and strength limit states.

15.2 CONVENTIONAL RETAINING WALL AND ABUTMENT DESIGN

15.2.1 General Design Considerations

In addition to determining the design of the wall section to meet the service and strength limit states criteria for applied loads, the design of conventional walls and abutments requires consideration of factors which can affect wall and abutment performance, including:

- quality and drainage of backfill materials (S11.6.1.1)
- integral abutments and the effects of constraining wall movements (S11.6.1.2)
- load effects in abutments due to the weight of earth overlying the rear face or base of the abutment (S11.6.1.3)
- requirements for placement of construction and expansion joints (S11.6.1.5)
- structural and geometric design of wingwalls and cantilevered walls (S11.6.1.4)
- scour of foundations subjected to flowing water and subsurface erosion or piping due to elevated water levels in poorly-drained backfill materials (S11.6.3.5)
- seismic loading (S11.6.5 and Appendix S11.11)

15.2.2 Design Procedure

The design procedure for conventional walls and abutments involves the following steps:

1. Select wall type and preliminary wall proportions considering grade separation requirements, site constraints and foundation conditions.

2. Develop design foundation profile including soil and rock strata layering, engineering properties of foundation strata, groundwater level and problem conditions.

3. Estimate applied loads and earth pressure for strength and service limit states analyses and select resistance factors for conditions analyzed.

4. Check stability criteria along the base of the wall at the strength limit state for: (a) location of factored resultant force; (b) factored bearing resistance, and (c) factored sliding resistance; and revise wall proportions as necessary to meet stability criteria.

5. Design deep foundation for wall if acceptable factored bearing and sliding resistance cannot be developed using a wall base of reasonable width.

6. Check overall stability against deep-seated foundation failure.

7. Check vertical and horizontal wall displacements with tolerable movement criteria at service limit strength.

8. Design structural elements according to the provisions of Sections 5, 6 and/or 8 of LRFD.

9. Compare cost of wall design with other wall systems.

Table 15.2.2-1 summarizes the strength and service limits state that must be evaluated for the design of conventional retaining walls and abutments.

Walls should also be investigated for the extreme event limit state using a resistance factor of 1.0 unless specified otherwise.

Table 15.2.2-1 - Strength and Service Limit States for Design of Conventional Retaining Walls and Abutments

Design Consideration	Strength Limit State	Service Limit State
Location of Base Resultant Force	X	
Bearing Resistance	X	
Sliding Resistance	X	
Overall Stability	X	
Settlement and Horizontal Movement		X
Structural Capacity of Wall Elements	X	

15.2.3 Movement at the Service Limit State

15.2.3.1 ANALYSIS OF WALL DISPLACEMENTS

The Specification requires that the displacement of conventional walls and abutments be evaluated at the service limit state for all applicable load combinations. Usually this will be the Service I Load Combination from Table S3.4.1-1 in LRFD. Vertical and lateral wall movements can be estimated using the procedures described in Section 10 of LRFD for foundations and considering, as applicable, differential movements along the base of walls supported on footings due to variations in contact pressure. Lateral movements of walls on shallow foundations can be estimated assuming the wall displaces as a rigid body due to the effects of earth loads and differential settlements along the base of the wall.

15.2.3.2 TOLERABLE MOVEMENT CRITERIA

The tolerable movement criteria for abutments should follow the guidance provided in Section 14.2.3.2 for spread footing foundations. For other retaining walls, tolerable movement criteria should be developed with consideration of the function and type of wall, anticipated service life and consequences of unacceptable movements (e.g., affect of wall movements on adjacent facilities).

15.2.4 Resistance at the Strength Limit State

Abutments and conventional retaining walls must be proportioned to ensure adequate resistance at the strength limit state based on consideration of:

- eccentricity of the base resultant force due to lateral and inclined loading
- bearing resistance
- sliding resistance to lateral and inclined loading
- overall stability against deep-seated failure of foundation materials below the structure
- structural resistance of wall components

These failure modes are illustrated in Figure 15.2.4-1.

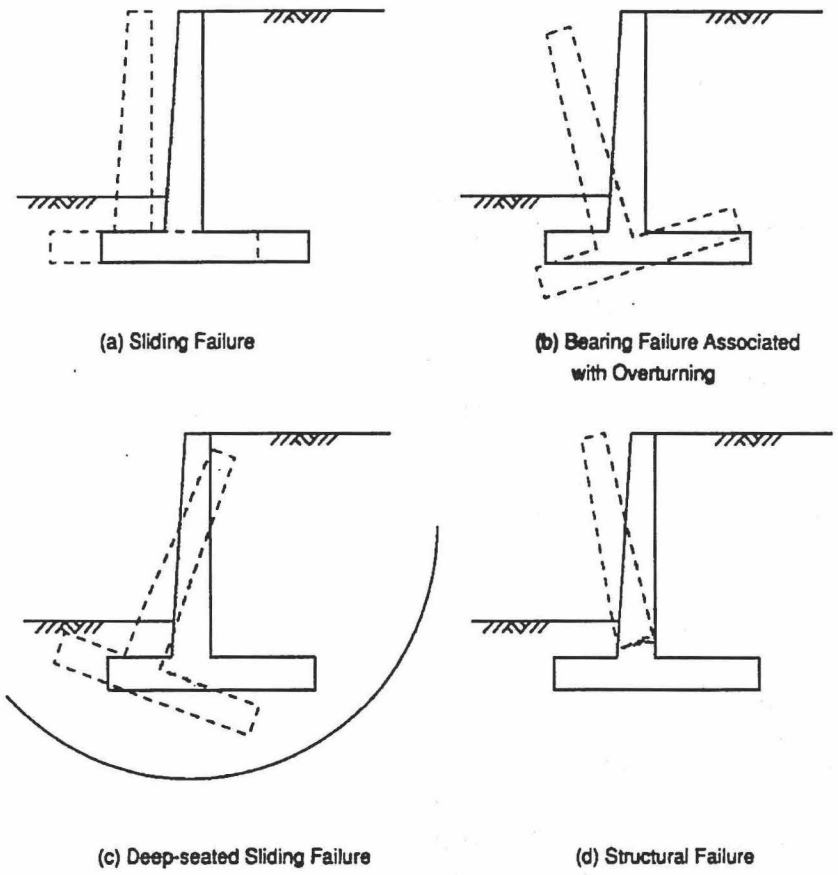


Figure 15.2.4-1 - Failure Modes for Retaining Walls

15.2.4.1 RESISTANCE FACTORS

Resistance factors for the design of conventional walls and abutments supported on shallow foundations are the same as those used for the design of spread footings presented in Table 14.2.4.1-1. If the stability criteria for wall design cannot be achieved using a spread footing for foundation support of the wall, deep foundation design should follow the provisions of S10.7 and S10.8 for driven pile and drilled shaft foundations, respectively. The location of the resultant force along the wall base was developed by comparison with current allowable stress design procedures for walls supported on soil and rock. The provisions of Sections 5, 6 and/or 8 of LRFD should be followed in evaluating the structural resistance of wall components.

15.2.4.2 LOAD FACTORS

Article S3.4.1 of LRFD specifies that certain permanent loads, including earth loads, be factored using maximum and minimum load factors γ_p as required in Table 15.2.4.2-1.

Table 15.2.4.2-1 - Load Factors for Permanent Structure and Earth Loads for Walls

Type of Load	Load Factor Maximum	Load Factor Minimum
DC: Components and Attachments	1.25	0.90
EH: Horizontal Earth Pressure		
• Active Pressure	1.50	0.90
• At-Rest Pressure	1.35	0.90
EV: Vertical Earth Pressure		
• Retaining Structure	1.35	1.00
ES: Earth Surcharge	1.50	0.75

Criteria in S3.4.1 require the following:

- Load factors shall be selected to produce the total extreme factored force effect, and for each combination, both positive and negative extremes shall be investigated.

- For permanent force effects, the load factor which produces the more critical combination shall be selected from Table 15.2.4.2-1.

- For load combinations where one force effect decreases the effect of another force (e.g., top/sidewall design of box culvert), the minimum value shall be applied to the load that reduces the force effect.

- If a permanent load increases the stability or load carrying capacity of a structure component (e.g., load from soil backfill on heel of wall), the minimum value for that permanent load shall also be investigated.

Applying these criteria for the evaluation of the sliding resistance of walls:

- The vertical earth load on the rear of a cantilevered retaining wall would be multiplied by γ_{pmin} (1.00) and the weight of the structure would be multiplied by γ_{pmin} (0.90) because these forces result in an increase in the contact stress (and shear strength) at the base of the wall and foundation.

- The horizontal earth load on a cantilevered retaining wall would be multiplied by γ_{pmax} (1.50) for an active earth pressure distribution because the force results in a more critical sliding force at the base of the wall.

Similarly, the values of γ_{pmax} for structure weight (1.25), vertical earth load (1.35) and horizontal active earth pressure (1.50) would represent the critical load combination for an evaluation of foundation bearing resistance.

15.2.4.3 OVERALL STABILITY

The overall stability of an abutment or retaining wall, retained slope and foundation materials should be evaluated for all walls using limit equilibrium methods of slope stability analysis. This analysis applies to deep failure of the soil mass, e.g., slope stability, not the stability of the wall itself which is covered in Article 15.2.4.4. These analyses should be conducted using a load factor of 1.35 for all earth, water and externally-applied loads used in the analysis, and a resistance factor of 0.85 (see Table S10.5.4-1 in LRFD) should be applied to the shear strength of soil and rock materials. Because resistance to slope instability can be affected by geologic features (e.g., bedding planes and fracturing), the anisotropic strength behavior of soils and layered rock and rate of loading effects, it is important that the strength properties and method of analysis correctly represent these conditions in the analysis. Although the potential for overall slope failure should be considered for all walls and abutments, slope stability is of

particular concern for walls which are supported on weak foundation materials and/or are backfilled with poor draining soils that result in an elevated water level behind the wall.

15.2.4.4 LOCATION OF RESULTANT FORCE

Allowable stress methods for the design of conventional walls and abutments require evaluation of overturning as a potential failure mode. For this condition, walls on soil are designed for a minimum factor of safety (FS) of 2.00, and walls on rock are designed for a minimum FS of 1.75. At the strength limit state, LRFD evaluates overturning based on the location of the bearing pressure resultant as follows:

- For foundations on soil, the resultant of the factored reaction force shall be within the middle one-half of the base of the wall footing.

- For foundations on rock, the resultant of the factored reaction force shall be within the middle three-fourths of the base of the wall footing.

The location of the resultant force within the specified locations considers the use of a plastic bearing pressure distribution at the strength limit state.

The effect of earthquake is investigated using the extreme event limit state of Table S3.4.1-1 with resistance factors ϕ = 1.0 and an accepted methodology.

For foundations on soil, the location of the resultant of the reaction forces must be within the middle one-third of the base for γ_{EQ} = 0.0 and within the middle one-half of the base for γ_{EQ} = 1.0.

For foundations on rock, the location of the resultant of the reaction forces must be within the middle one-half of the base for γ_{EQ} = 0.0 and within the middle two-thirds of the base for γ_{EQ} = 1.0.

For values of γ_{EQ} between 0.0 and 1.0, the restrictions of the location of the resultant are obtained by linear interpolation of the values given in this Article.

15.2.4.5 BEARING RESISTANCE

The bearing resistance of foundations supporting walls and abutments is determined following the provisions of S10 for the design of foundations. Care must be taken to perform these analyses for all appropriate load combinations.

15.2.4.6 SLIDING RESISTANCE

The sliding resistance of foundations supporting walls and abutments is determined following the provisions of S10 for the design of foundations. Care must be taken to perform these analyses for all appropriate load combinations.

15.2.4.7 CONTINUATION OF RETAINING WALL DESIGN EXAMPLE

The loads and load combinations required for each limit state to be investigated were calculated in Section 4.3.9. This section continues with foundation design aspects.

Step 4: Calculate the Settlement of the Retaining Wall on its Cohesive Foundation

Assume embankment construction has been performed earlier and that consolidation settlement from the embankment loading beneath the wall has already occurred.

Divide the 6 m of cohesive soil below the wall foundation into four layers with the following thicknesses:

$H_1 = 1$ m
$H_2 = 1$ m
$H_3 = 2$ m
$H_4 = 2$ m

Depth of footing below the existing ground surface is 1 m.

The depth to the center of each layer from the existing ground surface is

$d_1 = 1.5$ m
$d_2 = 2.5$ m
$d_3 = 4$ m
$d_4 = 6$ m

(A) Calculate the existing overburden pressure at the center of each layer - Using effective unit density:

$P_{O1} = (d_1)(760)g/(10^6) = 0.0112$ MPa
$P_{O2} = (d_2)(760)g/(10^6) = 0.0186$ MPa
$P_{O3} = (d_3)(760)g/(10^6) = 0.0298$ MPa
$P_{O4} = (d_4)(760)g/(10^6) = 0.0447$ MPa

(B) Calculate Increase in vertical pressure resulting from loading of the wall

Investigate vertical movement at the service limit state SA10.6.2.1.

Because the underlying material is cohesive, the determination of settlement is based on the permanent loads. Using Service I loading from Table 4.3.9-5, the total vertical load for calculation of settlement is 281 379 -34 167 = 247 212 N/m.

Considering an evenly distributed loading over the width of the wall foundation (3 m), the increase in pressure at the base of the footing is $P_v = (247\ 212)/(3x10^6) = 0.0824$ MPa.

Table 15.2.4.7-1 - Calculation of Overburden Pressures

Layer i	$(d_i-1)/B$	P_{vi}/P_v	P_{vi} MPa	P_{Oi} MPa	H m	H_1 log A $A = 1 + P_{vi}/P_{Oi}$
1	0.17	0.95	0.0783	0.0112	1	0.9025
2	0.50	0.80	0.0659	0.0186	1	0.6574
3	1.00	0.55	0.0453	0.0298	2	0.7026
4	1.67	0.34	0.0280	0.0447	2	0.5124
TOTAL						2.7749

where:

B = 3 m

d_i-1 is the depth to the center of layer i below the footing

P_{vi}/P_v is obtained using the Boussinesq stress contours at the center of a continuous foundation.

(C) Estimate the settlement using equations in Das, 1984

$S = \Sigma [H_i\{\log(P_{Oi} + P_{vi})/P_{Oi}\}]C_{cr}/(1 + e_o)$

with $C_{cr} = 0.012$ and $e_o = 0.63$

$S = 20$ mm

Step 5: Check Eccentricity

Table 15.2.4.7-2 - Summary for Eccentricity Check

Group/Item Units	V_{TOT} N/m	H_{TOT} N/m	M_v N-m/m	M_h N-m/m	X_o m	e_B m	e_{max} m
Strength I-a	299,235	145,766	536,649	313,206	0.75	0.75	0.75
Strength I-b	385,759	145,766	685,656	313,206	0.97	0.53	0.75
Strength IV	345,390	102,416	587,747	204,833	1.11	0.39	0.75
Service I	281,379	93,049	494,068	198,483	1.05	0.45	0.75

where:

X_o = location of the resultant = $(M_v\text{-}M_h)/V_{TOT}$

e_B = eccentricity = $0.5B\text{-}X_o$

Using the provisions of either Article S10.6.3.1.5 or S11.6.3.3, $e_{max} = B/4$

For all cases, e_B is less than or equal to e_{max}, therefore, the design is adequate with regards to eccentricity.

The adequacy for bearing capacity is developed based on a rectangular distribution of soil pressure as indicated in S10.6.3.1.5, a figure which is repeated below.

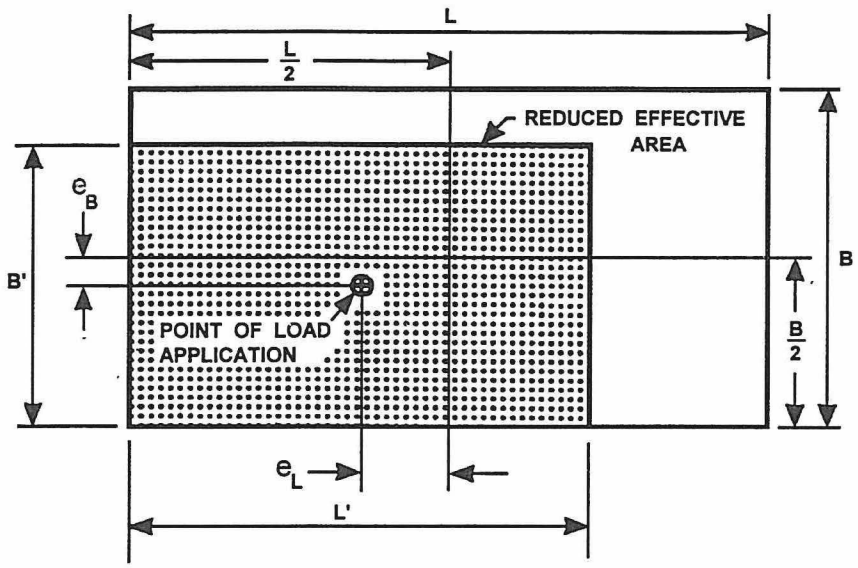


Figure 15.2.4.7-1 - Reduced Footing Dimensions

Step 6: Check Bearing Capacity

(A) Estimate the Bearing Pressures

Table 15.2.4.7-3 - Summary of Maximum Bearing Pressures

Group/Item Units	V_{TOT} N/m	X_o m	e_B m	q_{max} MPa	q_{min} MPa	$q_{uniform}$ MPa
Strength I-a	299,235	0.75	0.75	0.2672	0	0.2004
Strength I-b	385,759	0.97	0.53	0.2664	0	0.1998
Strength IV	345,390	1.11	0.39	0.2052	0.0250	0.1556
Service I	281,379	1.05	0.45	0.1781	0.0095	0.1339

For a rectangular distribution use B = B′ and L = L′:

L′ = L

q = $V_{TOT}/(L' B')$

L = wall length

B′ = B - 2 e_B

For a triangular base pressure distribution:

q_{max} = pressure at toe

= $2V_{TOT}/(3X_o)$

$q_{uniform}$ = average pressure for an assumed rectangular distribution

= $V_{TOT}/(2X_o)$

For a trapezoidal base pressure distribution using full-width, B = 3 m:

q_{max} = pressure at toe

= $(V_{TOT}/B) + [(6)(V_{TOT})(e_B)/B^2]$

q_{min} = pressure at heel

= $(V_{TOT}/B) - [(6)(V_{TOT})(e_B)/B^2]$

$q_{uniform}$ = average pressure for an assumed rectangular distribution

= $0.5\ V_{TOT}/X_o$

(B) Estimate the Bearing Capacity

The factored resistance at the strength limit state is defined by the following:

$q_r = \phi\ q_{ult}$

where:

ϕ = resistance factor = 0.60

From Table S10.5.4-1 for clay using the rational method:

q_{ult} = nominal bearing resistance

From S10.6.3.1.2b for the case with saturated clays

$q_{ult} = cN_{cm} + g\ \gamma_2\ D_f\ N_{qm}\ (10^{-9})$

where:

c = 0.15 MPa

N_{cm} and N_{qm} are modified bearing capacity factor which are functions of shape, embedment and load inclination and are defined as follows:

From Equation 14.2.4.2-2 using B = B' and Strength 1-A

$N_{cm} = 5(1 + 0.2D_f/B)(1 + 0.2B/L)(1\text{-}1.3H/V)$

$N_{cm} = 3.324$

where H and V are the unfactored horizontal and vertical loads, respectively, and L = 50 m (assumed)

From Equation 14.2.4.2-7

N_{qm} = 1.0 for cohesive soil

The shape factor (s_q), the soil compressibility factor (c_q) and the embedment depth factor (d_q) have all been assumed to be 1.0.

$q_{ult} = 0.15\ (3.324) + 9.81\ (1760)\ (1000)\ (1.0)\ (10^{-9})$

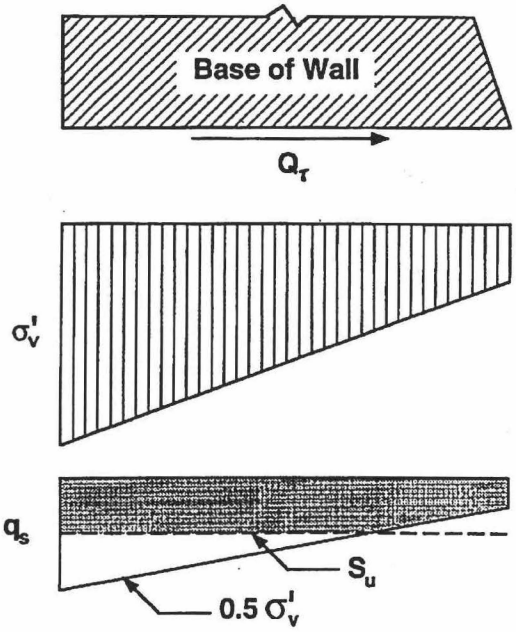
$q_{ult} = 0.515$ MPa

$q_r = 0.309$ MPa

The estimated bearing capacity of the soil exceeds the maximum and uniform bearing pressures of the wall foundation for the strength and service load groups analyzed.

Step 7: Check Sliding

Sliding of walls on clay is checked using Figure S10.5.4-1 which is repeated below as Figure 15.2.4.7-2.



q_s = unit shear resistance, equal to S_u or $0.5\,\sigma'_v$, whichever is less

Q_r = area under q_s diagram (shaded area)

S_u = undrained shear strength (MPa)

σ'_v = vertical effective stress (MPa)

Figure 15.2.4.7-2 - Procedure for Estimating Sliding Resistance for Walls on Clay

From Equation S10.6.3.3-1, the factored resistance against failure by sliding is taken as:

$$Q_R\ (N) = \phi_r\, Q_r + \phi_{ep}\, Q_{ep}$$

with Q_r and Q_{ep} in N

Using S10.6.3.3:

Q_r = the lesser of the following

- cohesion of the clay

- one-half the normal stress on the interface between the footing and the soil

where the cohesion q_s = 0.1500 MPa

Table 15.2.4.7-4 - Summary of Sliding Resistance

Group/Item Units	$q_{max}/2$ MPa	$3(X_o)$ m	Q_r N/m	$\phi_r Q_r$ N/m	H_{TOT} N/m
Strength I-a	0.1336	2.24	149,617	127,175	145,766
Strength I-b	0.1332	2.90	192,880	163,948	145,766
Strength IV	0.1026	3.33	170,657	145,058	102,416
Service I	0.0891	3.15	140,331	119,281	93,049

Since $q_{max}/2$ is less than q_s in all cases, $q_{max}/2$ is used in the calculation of Qt:

$Q_r = (0.5)(q_{max}/2)(3X_o)$

$\phi_r = 0.85$

Since the sliding resistance ($\phi_r Q_r$), calculated above, is less than the horizontal loading for group Strength I-a, the effect of the passive resistance must be considered to provide adequate resistance against sliding, S11.6.3.6.

Assuming that the 0.5 m of soil above the top of the footing may some day be removed, a depth of 0.5 m (thickness of the wall footing) will be used for calculation of the passive resistance.

Using Figure 5.3.9-3, presented in Step 1, since $\phi = 0$, $K_p = 1$

By Equation 5.22 - Das (1984), the total passive pressure per meter of wall length is

$Q_{ep} = P_p = 0.5\ \gamma_s\ g\ H^2\ K_p + 2\ cH\sqrt{K_p} \times 10^6$

which reduces to

$Q_{ep} = P_p = (0.5)(1760\ g)(0.5^2) + (2)(0.15)(0.5) \times 10^6$

$Q_{ep} = P_p = 152{,}158$ N/m

Table S10.5.4-1 gives $\phi_{ep} = 0.50$ for passive earth pressure component of sliding resistance

$\phi_{ep} Q_{ep}$ = 76,079 N/m

Q_R = 127,175 + 76,079 = 203,254 N/m

From Table 15.2.4.7-4, the worst case for sliding is for Loading Group Strength I-a. Since Q_R is greater than H_{TOT} = 145,766, adequate protection against sliding is provided.

15.2.4.8 STRUCTURAL DESIGN

The design of conventional walls and abutments must meet the requirements of Section S5 for the design of concrete structures. Barker, et al, (1991) provides the following guidance with regard the structural design of wall components:

- Wall Stem
  - The wall stem is designed as a cantilever supported above the base.
  - The wall stem of a counterfort (or buttress) can be designed as a horizontal fixed or continuous beam supported by counterforts or buttresses. The wall stem of these structures can also be designed as a plate supported in three sides by the base slab and counterforts (or buttresses), and free at the top.
- Base Slab
  - The base slab of a cantilevered wall is designed as a cantilever fixed at the wall.
  - The base slab of a counterfort or buttress wall is designed as a fixed or continuous beam spanning between the counterforts or buttresses.
  - The maximum bending moment for base slab design is taken at the face of the wall stem for the toe and the back of the wall stem for the heel.
  - The maximum design shear forces are taken at a distance equal to effective depth of the slab from the face of the wall stem for the toe section and at the back of the wall stem for the heel section.

- Counterforts and Buttresses

  - Counterforts are designed as T-beams, whereas buttresses are designed as rectangular beams. In counterfort and buttress walls, tension reinforcement should be provided as should a combination of horizontal and vertical bars or stirrups at the junction of the counterfort or buttress with the wall stem.

15.3 ANCHORED RETAINING WALL DESIGN

15.3.1 General Design Considerations

As shown in Figure 15.3.1-1, anchored walls are comprised of anchors constructed using prestressing strand or steel bars, discrete vertical wall elements (i.e., usually rolled steel section) spaced approximately 2 to 3 m along the length of the wall and facing (i.e., timber, precast concrete lagging or shotcrete).

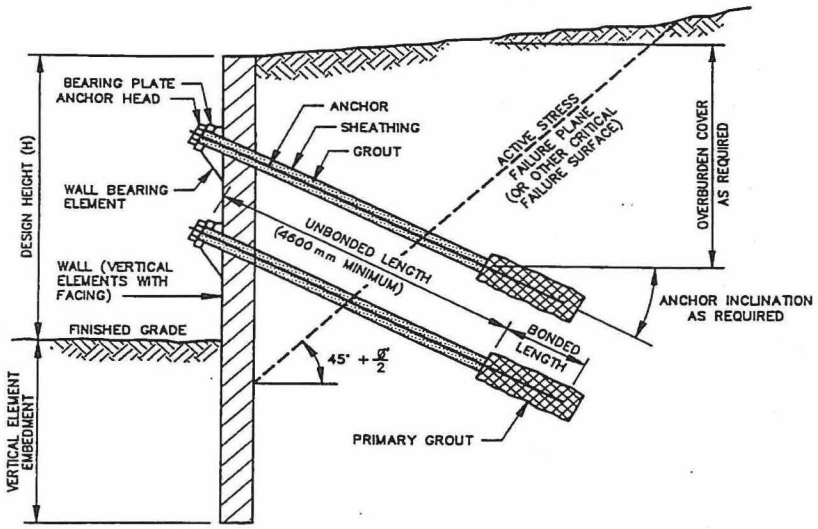


Figure 15.3.1-1 - Anchored Wall Nomenclature

Anchors are installed into drilled holes and extend through the wall face to a bearing plate supported on the vertical wall element or a wale between adjacent vertical wall elements. Because most anchored walls are constructed from the top down, wall construction proceeds by:

- Installing discrete vertical wall elements by driving or concreting into a predrilled hole

- Excavating to a level of between 2 to 4 m to install the upper level of anchors, and installing facing support, as required

- Drilling the anchor hole at an angle of about 10 to 30 degrees to the horizontal, and installing and grouting the anchor in the bond length

- Stressing and testing each anchor, locking off the anchor at the design load, and grouting the unbonded length if the anchor has grease and sheathing corrosion protection

- Repeating the excavation, anchor hole drilling, installation, grouting and testing, and facing installation for each anchor level to the base of the wall

- Constructing a wall facing in front of the wall for permanent construction.

The geometry of wall construction, shown in Figure 15.3.1-1, is typical for most anchored walls. The length of anchor is designed to provide an adequate bond length to resist loads on the wall and to extend a reasonable distance beyond the critical failure surface. Although the inclination of anchors is usually limited to maximum of about 30° to the horizontal to minimize axial loads on the vertical wall elements, steeper inclinations can be used to avoid interference with buried structures or to reach more suitable materials which provide a higher bond capacity. The vertical spacing of anchors can be optimized to provide a balanced design between the number of anchors and bending of the vertical wall element.

In addition to determining the design wall section to meet the service and strength limit states criteria for applied loads, the design of anchored walls requires consideration of factors which can affect wall and abutment performance, including:

- seismic loading (S11.8.6 and Appendix S11.1);

- corrosion protection (S11.8.7);

- anchor stressing and testing during construction (S11.8.8.1)

- drainage control to mitigate build up of water pressure behind the wall (S11.8.9)

15.3.2 Design Procedure

The design procedure for anchored walls involves the following steps:

1. Identify site constraints (e.g., inadequate right-of-way for anchor installation) that could affect anchor wall construction.

2. Develop design profile including: soil and rock type and layering, engineering properties of retained and foundation strata, groundwater level, and problem conditions (e.g., corrosion).

3. Estimate applied loads and earth pressure for strength and service limit states analyses and select resistance factors for conditions analyzed.

4. Determine number and location of anchors to resist pullout from factored earth and applied loads.

5. Select spacing of vertical wall elements and design elements to resist factored earth and applied loads between elements and the vertical component of the factored anchor loads according to the provisions of Sections 5 or 6 of LRFD, depending on the type of wall element.

6. Evaluate the resistance of the embedment zone against end bearing and passive earth failure.

7. Select the type and design the facing between vertical wall elements to resist the factored loads on the facing considering the effects of soil arching between vertical wall elements according to the provisions of Sections 5 or 8 of LRFD, depending on the type of facing selected.

8. Check vertical and horizontal wall displacements with tolerable movement criteria at service limit strength.

9. Check overall stability against deep-seated foundation failure.

10. Determine whether special construction control is required, such as preconstruction anchor stressing and testing or wall performance monitoring during construction.

11. Compare cost of wall design with other wall systems.

Table 15.3.2-1 summarizes the strength and service limits states that must be evaluated for the design of anchored walls.

Walls should also be investigated for the extreme event limit state using a resistance factor of 1.0 unless specified otherwise.

Table 15.3.2-1 - Strength and Service Limit States for Design of Anchored Walls

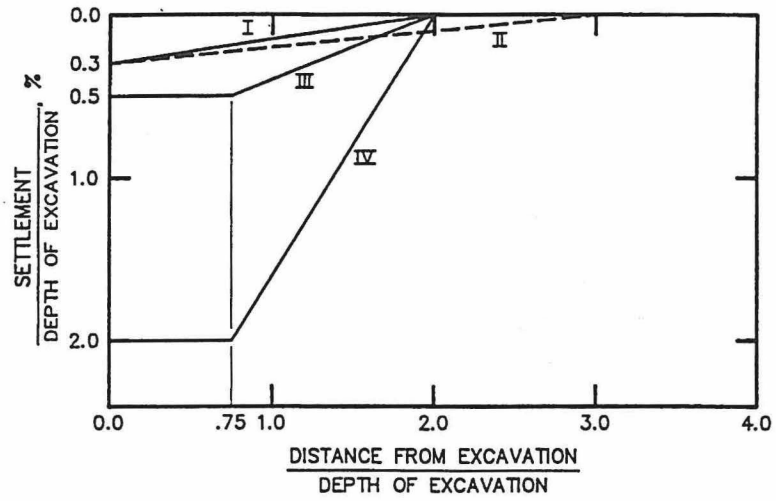
Design Consideration	Strength Limit State	Service Limit State
Anchor Pullout	X	
Structural Resistance of Anchor	X	
Structural Resistance of Vertical Wall Element	X	
Passive and Bearing Resistance of Embedment	X	
Structural Resistance of Facing Elements	X	
Overall Stability	X	
Settlement and Horizontal Movement		X

15.3.3 Movement at the Service Limit State

15.3.3.1 ANALYSIS OF WALL DISPLACEMENTS

The displacement of anchored walls shall be evaluated at the service limit state for all applicable load combinations using the Service I Load Combination from Table S3.4.1-1 in LRFD. Because the vertical and lateral displacement of anchored walls is a complex soil-structure interaction problem, deformation analyses can be performed using modified forms of beam on elastic foundation theory or finite element analyses. For many projects, however, these types of analyses are usually not warranted unless deformation-sensitive structures are in close proximity to the wall. As an alternative, for walls built using well proven anchored wall construction technology, Figure 15.3.3.1-1 can be used as a guide to estimate settlement of the ground surface behind anchored walls.

Although not shown in the figure, the maximum lateral movement for walls supporting sand and stiff to hard clay soils usually does not exceed about 0.3% of the depth of excavation. It should be noted that the relationships in the figure do not include the effects of settlements caused by other construction activities (e.g., dewatering, ground heave at the base of the wall, or poor construction quality during anchor, or vertical wall element or facing installation).



CURVE I = sand

CURVE II = stiff to very hard clay

CURVE III = soft to medium clay - with high resistance to base heave

CURVE IV = soft to medium clay - with marginal resistance to base heave

Figure 15.3.3.1-1 - Settlement Profiles Behind Anchored Walls (Modified after Clough and O'Rourke 1990)

15.3.3.2 TOLERABLE MOVEMENT CRITERIA

The tolerable movement of the wall should be developed based on consideration of the effects of possible wall movements on other structures and facilities near the wall.

15.3.4 Resistance at the Strength Limit State

Anchored walls must be proportioned to ensure adequate resistance at the strength limit state based on consideration of:

- Pullout and structural failure of anchor due to imposed loads
- Passive and bearing resistance of soil or rock into which vertical wall element is embedded
- Structural capacity of vertical wall elements and facing
- Overall stability against deep-seated failure of foundation materials below the structure

15.3.4.1 RESISTANCE FACTORS

Resistance factors for the design of anchored walls are presented in Table 15.3.4.1-1. The resistance factors were developed based on engineering judgment and calibration with current allowable stress design procedures for anchored walls. The provisions of Sections 5, 6 and/or 8 of LRFD should be followed in evaluating the structural resistance of wall components.

15.3.4.2 ANCHOR PULLOUT

Prestressed anchors used for anchored wall construction must be designed to resist pullout of the bonded length of anchor in soil or rock. The minimum bonded length, L, is determined as:

$$L \geq \frac{Q_u}{\phi Q_a} \qquad (15.3.4.2\text{-}1)$$

where:

L = bond length (mm)

Q_u = factored anchor load from contributory area (N) based on a soil pressure diagram consistent with S3.11

Q_a = unit resistance between grout and soil or rock in anchor bond zone (N/mm)

ϕ = resistance factor from Table S11.5.6-1

Tables 15.3.4.2-1 and 15.3.4.2-2 provide conservative values of Q_a for anchors in soil and rock, respectively. These values are intended for preliminary design or evaluation of the feasibility of straight shaft anchors installed in small diameter holes which are grouted using low pressures. Pressure-grouted anchors usually achieve higher capacities. Because anchor

capacity in soil and rock can be strongly influenced by the method of anchor hole advancement, hole diameter, anchor type, type of grout and grouting pressure, selection of the anchor type and determination of Q_a for final design should be made by the specialty geotechnical contractor selected for wall construction.

Several methods are available for determining the factored anchor load, Q_u. In the proportional method, the top anchor row is assumed to support the tributary area of pressure between the top of the wall and the mid-point between the upper two anchor levels; and the bottom anchor row is assumed to support the pressure between the base of the wall and the mid-point between the two lowest anchor rows. Optionally, the embedded portion of the vertical wall element can be assumed to support the pressure between the base of the exposed wall and the mid-point between the base and the lowest anchor row. For either option and intermediate anchor rows are assumed to support the pressures from the midway points between vertically adjacent anchor rows.

Table 15.3.4.1-1 - Resistance Factors for Anchored Walls

WALL-TYPE AND CONDITION		RESISTANCE FACTOR
Bearing resistance of vertical elements		S10.5 applies
Overturning	Passive resistance of vertical elements	
	• in soil	0.60
	• in rock	0.60
	Anchor pullout resistance	
	• Sand	
	correlation with SPT resistance-corrected for overburden pressure	0.65
	pullout load tests	0.70
	• Clay	
	correlation with unconfined compressive strength	0.65
	using shear strength from lab tests	0.65
	using shear strength from field tests	0.65
	pullout load tests	0.70
	• Rock	
	correlation with rock-type only	0.55
	using minimum shear resistance measured in lab tests - soft rock only	0.60
	laboratory rock-grout bond tests	0.75
	pullout load tests	0.80
Tensile resistance of anchor	Permanent	
	• Yielding of the gross section	0.90
	• Fracture of the net section	0.75
	Temporary	
	• Yielding of the gross section	1.00
	• Fracture of the net section	0.85
Flexural capacity of vertical elements	• Permanent	0.90
	• Temporary	1.00

Table 15.3.4.2-1 - Ultimate Unit Resistance of Anchors in Soil

Soil Type	Compactness or SPT Resistance (Blows per 300 mm)		Anchor Resistance Q_a (N/mm)
Sand and Gravel	Loose	4-10	145
	Medium	10-30	220
	Dense	30-50	290
Sand	Loose	4-10	100
	Medium	10-30	145
	Dense	30-50	190
Sand and Silt	Loose	4-10	75
	Medium	10-30	100
	Dense	30-50	130
Soil Type	Unconfined Compressive Strength (MPa)		Anchor Resistance Q_a (N/mm)
Silt-Clay Mixtures	Stiff	0.10-0.24	30
	Hard	0.24-0.38	60

Table 15.3.4.2-2 - Ultimate Unit Resistance of Anchors in Rock

Rock Type	Ultimate Unit Anchor Resistance Q_a (N/mm)
Granite or Basalt	730
Dolomitic Limestone	585
Soft Limestone/Sandstone	440
Slate and Hard Shale	365
Soft Shales	145

15.3.4.3 PASSIVE AND BEARING RESISTANCE

The resistance of vertical wall elements to passive and bearing failure should be evaluated using the procedures in Section S10.7 in LRFD for pile foundations.

15.3.4.4 STRUCTURAL RESISTANCE OF VERTICAL WALL ELEMENTS

Discrete vertical wall elements must be designed to resist all applicable earth and water pressure, surcharge, anchor and seismic loadings, and the vertical component of the anchor loads and other vertical loads within the tributary area between adjacent vertical wall elements. In designing these elements, fixed horizontal support can be assumed at each anchor level and at the bottom of the wall if the elements are sufficiently embedded below the base of the wall.

Unless beam on elastic foundation, finite element or other methods of soil-structure interaction analysis are used, the maximum bending moments in the vertical wall elements may be determined as follows if the proportional method is used to compute anchor forces and wall element stresses.

- The wall section above the top row of anchors is designed as a cantilever and M_{max} is determined as:

  $$M_{max} = 0.5pLx^2 \qquad (15.3.4.4\text{-}1)$$

  where:

  M_{max} = factored maximum flexural moment (N-mm)

  p = average factored lateral pressure carried by vertical wall element (MPa)

  L = spacing between vertical wall elements (mm)

  x = vertical spacing of anchors (mm)

- Wall sections between rows are designed as simply-supported beams and M_{max} may be determined as:

  $$M_{max} = 0.125pLx^2 \qquad (15.3.4.4\text{-}2)$$

This moment is considered to be the positive design moment between anchors and the negative design moment at the anchors.

- If the wall is not embedded or the lateral resistance of the embedded length is neglected, the wall section below the lowest anchor level may be designed as a cantilever and M_{max} may be determined using Equation 15.3.4.4-1.

- If the embedded portion of the vertical wall element is assumed to provide support and embedment is not required to resist unbalanced forces due to base instability, calculate the maximum bending moment in the section of wall below the lowest row of anchors may be determined as:

$$M_{max} = 0.333pLx^2 \qquad (15.3.4.4\text{-}3)$$

- Wall sections of vertical wall elements spanning three or more equally spaced anchor levels may be designed as continuous beams and M_{max} may be determined as:

$$M_{max} = 0.1pLx^2 \qquad (15.3.4.4\text{-}4)$$

If the variation in lateral pressure with depth is large, moment diagrams should be constructed to provide improved accuracy.

15.3.4.5 FACING ELEMENTS

The maximum spacing between vertical wall elements should be determined based on the relative stiffness of the vertical elements and the type and condition of soil to be supported. The horizontal spacing between vertical elements typically varies between 2 to 3 m. If timber facing is specified, the timber shall be stress-grade pressure treated in conformance with Section 8 of LRFD.

Facing may be designed assuming simple support between elements, with or without soil arching, or assuming the facing behaves as a continuous support across several elements. Based on these assumptions, the value of the maximum factored flexural moment on a unit width or height of facing may be determined as follows:

- For simple spans without soil arching

$$M_{max} = 0.125pL^2 \qquad (15.3.4.5\text{-}1)$$

where:

M_{max} = factored maximum flexural moment on a unit width or height of facing (N-mm)

p = average factored lateral pressure acting on vertical wall element (MPa)

L = spacing between vertical wall elements or other facing supports (mm)

- Simple span (soil arching)

$$M_{max} = 0.083pL^2 \qquad (15.3.4.5\text{-}2)$$

- Continuous

$$M_{max} = 0.10pL^2 \qquad (15.3.4.5\text{-}3)$$

If the variation in lateral pressure with depth is large, moment diagrams should be constructed to provide improved accuracy.

Equation 15.3.4.5-1 is applicable for simply supported facing behind which the soil will not arch between vertical supports (e.g., in soft cohesive soils or for rigid concrete facing placed tightly against the in-place soil). Equation 15.3.4.5-2 is applicable for simply supported facing behind which the soil will arch between vertical supports (e.g., in granular or stiff cohesive soils with flexible facing or rigid facing behind which there is sufficient space to permit the in-place soil to arch). Equation 15.3.4.5-3 is applicable for facing which is continuous over several vertical supports (e.g., reinforced shotcrete).

15.3.4.6 OVERALL STABILITY

The overall stability of anchored wall should be evaluated using the procedures described for conventional retaining walls and abutments. In performance of the slope stability analyses, consideration should be given to using a method of analysis (e.g., STABL5; Carpenter, 1986) that includes the effects of anchor forces in assessing the resistance of a slope to instability.

15.4 MECHANICALLY-STABILIZED EARTH RETAINING WALLS

15.4.1 General Design Considerations

Mechanically-stabilized earth (MSE) walls are comprised of a reinforced soil mass and a discrete modular facing which is vertical or near vertical.

The reinforced soil mass consists of select granular backfill meeting the requirements of Division II. The tensile reinforcements may be proprietary and may employ either metallic (i.e., strip- or grid-type) or polymeric (i.e., sheet-, strip-, or grid-type).

MSE walls may be used where conventional gravity, cantilever, or counterforted concrete retaining walls are considered, and they are particularly well suited where substantial total and differential settlements are anticipated. MSE walls should not be used:

- where utilities other than highway drainage would have to be constructed within the reinforced fill zone,
- with galvanized metallic reinforcements exposed to surface or groundwater contaminated by acid mine drainage or other industrial pollutants as indicated by low pH and high chlorides and sulfates, and
- where floodplain erosion or scour may undermine the reinforced fill zone, or any supporting foundation.

MSE walls may be considered for use under the following special conditions:

- When two intersecting walls form an enclosed angle of 70° or less, the affected portion of the wall shall be designed as an internally tied bin structure with at-rest earth pressure coefficients.
- Where metallic reinforcements are used in areas of anticipated stray currents within 60 m of the structure, a corrosion expert should evaluate the potential need for corrosion control requirements.

The size of the reinforced soil mass is based on consideration of:

- requirements for stability and geotechnical strength as described for conventional walls,

- requirements for structural resistance within the reinforced soil mass and the panel units, and for the development of reinforcement extending beyond assumed failure zones, and

- traditional requirements for reinforcement length of not less than 70% of the wall height, or 2.4 m.

In addition to determining the design wall section to meet the service and strength limit states criteria for applied loads, the design of MSE walls requires consideration of factors which can affect wall performance. These factors include:

- Seismic loading (S11.9.6)

- Use in supporting a stub abutment (S11.9.7);

- Drainage control to mitigate build up of water pressure behind wall (S11.9.9)

- Scour protection and erosion control (S11.9.10)

15.4.2 Design Procedure

The design procedure for MSE walls involves the following steps:

1. Develop design foundation and wall backfill profiles, including foundation soil and rock strata layering; random backfill and reinforced backfill compositions; engineering properties of the foundation strata and backfill material(s); and groundwater level.

2. Define additional pertinent problem conditions, including wall height; ground surface geometry (i.e., slope at base and top of wall); traffic or live surcharge loads; reinforcement type and properties; and design life.

3. Estimate loads for strength and service limit states analyses and select resistance factors for conditions analyzed.

4. Identify special conditions which need to be evaluated, such as potential loss of support at base of wall due to removal of soil, scour or erosion.

5. Select an appropriate reinforcement length based on wall height.

6. Evaluate the safety against soil failure assuming the reinforced soil mass to be a rigid body (external stability). Evaluate the reinforced soil mass and foundation soil and rock strata at all applicable strength limit states for the following failure modes: overturning (eccentricity of resultant of reaction forces); sliding; and bearing (magnitude of resultant of reaction forces).

7. Evaluate the overall stability of the reinforced soil mass against deep soil failure using an acceptable limiting equilibrium method.

8. Evaluate the internal stability of the wall elements at all applicable strength limit states. Elements to be considered include the reinforcing strips or grids (to be evaluated for pullout failure and rupture failure giving consideration to loss due to corrosion), wall face elements, and connections between reinforcement and wall face elements.

9. In areas of known seismicity, investigate the effects of earthquakes for each of the external and internal failure modes at the Extreme Event I Limit State. The liberalized eccentricities of Article 15.2.4.4 apply.

10. Evaluate settlement of the reinforced soil mass at the applicable service limit states.

11. If unacceptable safety is determined against external or internal failure, alter the appropriate design parameters such as: reinforcement type, length and spacing or backfill type (i.e., backfill with greater shear resistance). If bearing pressures or settlement magnitudes are unacceptable, improvements to the foundation material should be considered.

12. Incorporate drainage measures into the design to prevent saturation of the reinforced backfill and to intercept any surface flows containing aggressive elements.

Table 15.4.2-1 summarizes the strength and service limit states that must be evaluated for the design of MSE walls.

Walls should also be investigated for extreme event limit states using a resistance factor of 1.0 unless specified otherwise.

Table 15.4.2-1 - Strength and Service Limit States for Design of MSE Walls

Design Consideration	Strength Limit State	Service Limit State
Sliding	X	
Bearing Resistance	X	
Overturning	X	
Overall Stability	X	
Rupture of Reinforcing Elements	X	
Pullout of Reinforcing Elements	X	
Resistance of Wall Face Elements	X	
Resistance of Wall Reinforcing Element Connections	X	
Settlement		X

15.4.3 Movement at the Service Limit State

15.4.3.1 ANALYSIS OF WALL DISPLACEMENTS

The displacement of MSE walls and MSE abutments must be evaluated at the service limit state for all applicable load combinations using Load Combination Service I from Table S3.4.1-1 in LRFD. Vertical wall movements can be estimated using the procedures described in Section S10 of LRFD for foundations and considering, as applicable, differential movements along the base of walls supported on footings due to variations of contact pressure.

15.4.3.2 TOLERABLE MOVEMENT CRITERIA

The tolerable settlement of MSE walls is limited by the longitudinal deformability of the facing and the ultimate purpose of the structure. Limiting tolerable differential settlement for systems with panels less than 2.8 m^2 in area and a maximum joint width of 19 mm are presented in Table 15.4.3.2-1. These limits are intended to prevent open spaces in the joints. Where foundation conditions indicate large differential settlements are

possible over a short horizontal distance, a vertical full-height slip joint must be used.

Table 15.4.3.2-1 - Relationship Between Joint Width and Limiting Distortion of MSE Wall Facing

Joint Width (mm)	Limiting Vertical Distortion
19	1/100
13	1/200
6	1/300

The tolerable movement criteria for MSE abutments should follow the guidance provided in Section 16.2.3.2 for spread footing foundations.

15.4.4 Resistance at the Strength Limit State

Mechanically-stabilized earth walls must be proportioned to ensure adequate resistance at the strength limit state based on consideration of:

- Overturning resistance of the reinforced soil zone
- Sliding of the reinforced soil zone
- Resistance of soil or rock upon which the reinforced soil zone bears
- Overall stability against deep-seated failure of foundation materials below the structure
- Structural capacity of wall facing and connections
- Pullout and rupture resistance of reinforcing elements

15.4.4.1 RESISTANCE FACTORS

Resistance factors for the design of MSE walls are presented in Table 15.4.4.1-1. The resistance factors were developed based on engineering judgment and calibration with current allowable stress design procedures for MSE walls. The provisions of Sections 5, 6 and/or 8 of LRFD should be followed in evaluating the structural resistance of wall components.

When applying resistance factors for polymeric reinforcements, it should be noted that the resistance factors for tensile strength incorporate factors for the effects of creep, aging, environmental losses and construction damage. The resistance factors for the strength limit state in the table include a multiplier of 0.33 for the effects of construction damage and 0.5 for the effects of post-construction environmental and aging strength losses. If product-specific data is available that indicates that the effects of construction damage and/or environmental deterioration and aging are less severe, higher multipliers may be used to increase the resistance factor.

15.4.4.2 SAFETY AGAINST SOIL FAILURE

Similar to requirements in 15.2.3 for conventional retaining walls and abutments, MSE walls shall be evaluated at the strength limit state for bearing and sliding resistance, the location of the resultant force at the base of the wall and overall stability.

Table 15.4.4.1-1 - Resistance Factors for Mechanically-Stabilized Earth Walls

Mechanically-Stabilized Earth Walls		
Bearing resistance		Section S10.5 applies
Sliding		Section S10.5 applies
Tensile resistance of metallic reinforcement	Strip reinforcements • Yielding of gross section less sacrificial area • Fracture of net section less sacrificial area Grid reinforcements • Yield of gross section less sacrificial area • Fracture of net section less sacrificial area Connectors • Yielding of gross section less sacrificial area • Fracture of net section less sacrificial area	 0.85 0.70 0.75 0.60 0.75 0.60
Strength limit state tensile resistance of polymeric reinforcement	From laboratory creep tests of 10,000 hours minimum duration From wide-width tensile test-ASTM D4595 • Polyethylene • Polypropylene • Polyester • Polyamine • High Density Polyethylene	0.27 @ ULT / @ 5% Strain 0.05 / 0.08 0.05 / 0.08 0.11 / 0.16 0.09 / 0.14 0.09 / 0.14
Service limit state tensile resistance of polymeric reinforcement	From laboratory creep tests of 10,000 hours minimum duration From limit state strength of "4b"	0.41 0.66
Ultimate pullout resistance in soil		0.90

For the evaluation of sliding resistance, the coefficient of friction at the base of the reinforced soil zone should be evaluated using the angle of friction, the foundation material, or a maximum value of 30° in the absence of specific strength data for the foundation materials.

For the evaluation of bearing resistance, an equivalent footing shall be assumed having a width equal to the length of the reinforcing elements and a length equal to the length of the wall. Bearing pressures shall be computed using a uniform base pressure distribution over an effective width of footing in accordance with S10.6.3.1 and S10.6.3.2 in LRFD. The effect of eccentricity and load inclination is included in the evaluation of bearing resistance using an effective width of ($B' = B - 2e_B$) instead of the actual width.

15.4.4.3 INTERNAL STABILITY OF REINFORCEMENTS

MSE walls shall be evaluated for internal failure by slip or rupture of the reinforcements. For these evaluations, the factored horizontal force acting on the reinforcement at any level, P_i, shall be:

$$P_i = \sigma_H h_i \qquad (15.4.4.3\text{-}1)$$

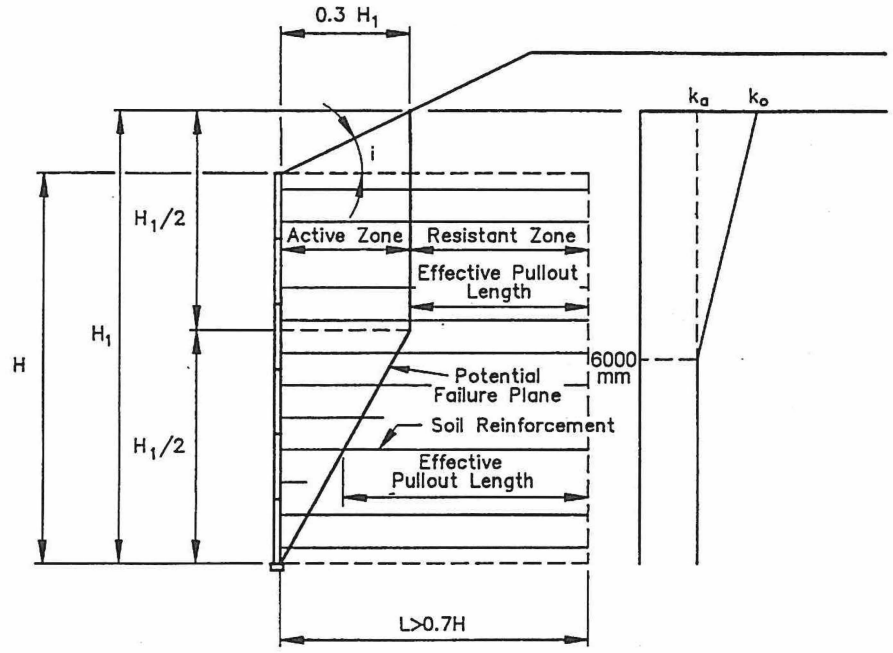
where:

h_i = height of reinforced soil zone contributing horizontal load to the reinforcement at Level i, determined as the vertical distance from the mid-point between Layer i and the next overlying layer to the mid-point between Layer i and the next underlying layer (mm)

σ_H = factored horizontal stress at Layer i, determined as described below, for inextensible and extensible reinforcements (MPa)

Inextensible Reinforcements

The internal stability of structures constructed with metallic strip or grid reinforcements shall be analyzed by considering that the in situ reinforced zone can be divided in two zones, the active and resistant zones. The failure surface is assumed to be bilinear as shown in Figure 15.4.4.3-1.



$$H_1 = H + \frac{\tan i \times .3H}{(1-.3 \tan i)}$$

Figure 15.4.4.3-1 - Determination of Failure Plane and Earth Pressure Coefficients for MSE Wall with Inextensible Reinforcements

The factored horizontal stress, σ_H, at each reinforcement level is:

$$\sigma_H = \gamma_p \sigma_V k \qquad (15.4.4.3\text{-}2)$$

where:

γ_P = load factor for earth pressure in Table S3.4.1-2 of LRFD

k = horizontal pressure coefficient (dim)

σ_V = pressure due to the resultant vertical forces at the reinforcement level determined using a uniform pressure distribution over the effective width ($B - 2e_B$) from S10.6.3.1.5 in LRFD (MPa)

In determining σ_V at each level of reinforcement, evaluation of equilibrium shall consider only the forces acting at that level.

Structures shall be designed using $k = k_o$ at H_1 above the top of the leveling pad and decreasing linearly to $k = k_a$ at 6000 mm as shown in Figure 15.4.4.3-1. Below a depth of 6000 mm depth, $k = k_a$. The earth pressure coefficients k_a and k_o shall remain the same, regardless of the external loading conditions. The values of k_a and k_o shall be taken from S3.11.5.7 in LRFD.

The maximum friction angle used to determine the horizontal force within the reinforced soil mass shall be 34°, unless the specific project select backfill is tested for frictional strength by triaxial or direct shear testing. Live loads shall be positioned for extreme force effect within the zone subjected to live loads. For this loading, the provisions of S3.11.6 in LRFD must be followed.

Extensible Reinforcements

The internal stability for MSE structures constructed with polymeric reinforcements must be analyzed using a tie-back wedge method approach. This approach assumes that the full shear strength of the reinforced fill is mobilized and active lateral earth pressures, k_a, are developed. Because it is assumed that the full shear strength of the reinforced fill is mobilized, the soil mass bounded by the failure plane moves as a rigid body resulting in the development of active lateral earth pressures. The assumed failure plane is defined by the Rankine active earth pressure zone defined by a straight line passing through the wall toe and oriented at an angle of $45° + \phi_f'/2$ from the horizontal, for both the horizontal and sloping backfill conditions. The value of k_a is determined following S3.11.5.7 in LRFD.

The tensile force at each level of reinforcement is a function of the vertical stress induced by gravity, uniform normal surcharge multiplied by k_a and active thrust. Reinforcement tensions induced by vertical or horizontal line loads, or by point loads should be added by superposition to the tensile forces induced by the reinforced wall fill soil and the retained backfill. The method of computation shall assume an unyielding rigid wall rotating about its toe.

The value of k_a in the reinforced soil mass is assumed to be independent of all external loads, except sloping fills. If testing of the site-specific select backfill is not available, the maximum friction angle used to compute the horizontal stress within the reinforced soil mass shall not exceed 34°. Where site-specific tests are performed, the soil strength shall be evaluated at residual stress levels.

15.4.4.4 PULLOUT OF REINFORCING ELEMENTS

The pullout resistance of reinforcements shall be evaluated at each level using only the effective pullout length which extends beyond the theoretical failure surface. The length of reinforcements shall meet the following criteria:

- The minimum length of the resistant zone shall be 900 mm
- The minimum total length of reinforcement shall be 2400 mm
- The reinforcement length shall be equal at all levels

For ribbed or smooth steel reinforcing strips, the nominal pullout capacity, $P_N = P_{fs}$ in newtons, shall be determined as:

$$P_{fs} = gf^*\gamma_s Z A_s \times 10^{-9} \qquad (15.4.4.4\text{-}1)$$

where:

g = gravitational constant (9.8066 m^2/sec)

f^* = apparent coefficient of friction at each reinforcement level (dim)

γ_s = unfactored soil density (kg/m^3)

Z = depth below effective top of wall or to reinforcement (mm)

A_s = total top and bottom surface area of reinforcement along effective pullout length, less sacrificial thickness (mm^2)

In the absence of pullout data for ribbed reinforcement strips in backfill materials conforming to Division II for MSE backfills, the value of f^* shall not exceed 2.0 at ground level and may be assumed to decrease linearly to a value of $\tan\phi_f$ at a depth of 6000 mm, where ϕ_f is the friction angle of the backfill in the reinforced zone. If the uniformity coefficient (D_{60}/D_{10}) of backfill is less than 4, but otherwise meets Division II requirements for MSE backfills, f^* should not exceed 1.2.

For smooth steel reinforcing strips, f^* shall be constant at all depths and determined as:

$$f^* = \tan\psi \leq 0.4 \qquad (15.4.4.4\text{-}2)$$

where:

ψ = soil-reinforcement angle of friction (deg)

For steel grid reinforcing systems with transverse bar spacings of 150 mm or greater, the generalized relationship for the nominal pullout capacity, $P_N = P_{fg}$ in newtons, shall be taken as:

$$P_{fg} = gN_p\gamma_s ZnA_b \times 10^{-9} \qquad (15.4.4.4\text{-}3)$$

where:

g = gravitational constant (9.8066 m/sec^2)

N_p = pullout based on site-specific pullout tests, or as defined by Figure 15.4.4.4-1 (dim)

γ_s = unfactored soil density (kg/m^3)

Z = depth below effective top of wall or to reinforcement (mm)

n = number of transverse bearing members behind failure plane (dim)

A_b = surface area of transverse reinforcement in bearing, less sacrificial thickness of cross bars (mm^2)

Figure 1 indicates that $\phi_f \geq 34°$. This is because the data for Figure 1 is derived from data based on ϕ_f of 34° or more.

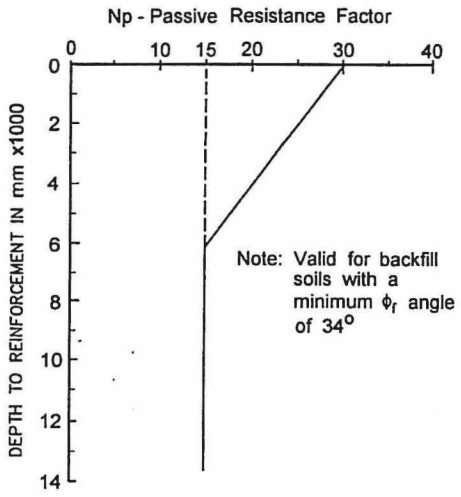


Figure 15.4.4.4-1 - Pullout Factors for Inextensible Mesh and Grid Reinforcement

For grid reinforcements with transverse spacing less than 150 mm, the nominal pullout capacity shall be calculated using the following expression:

$$P_{fg} = 2gwl\gamma_s Z f_d \tan\phi_f \times 10^{-9} \qquad (15.4.4.4\text{-}4)$$

where:

g = gravitational constant (9.8066 m/sec^2)

w = width of mat (mm)

l = length of mat beyond failure plane (mm)

γ_s = unfactored soil density (kg/m^3)

Z = depth below effective top of wall or to reinforcement (mm)

f_d = coefficient of resistance to direct sliding of reinforcement (dim)

ϕ_f = internal angle of friction of reinforced soil zone (dim)

The value of f_d may be assumed to vary from 0.45 for continuous sheets to 0.80 for bar mats with transverse spacing of 150 mm. Values of f_d must be determined experimentally for each grid geometry.

For polymeric reinforcements, Equation 15.4.4.4-4 is applicable where f_d is developed for a range of normal stresses in accordance with GRI-GG-5 (GRI, 1994). Experimental values of f_d may be limited by the limit state tensile load, T_1.

15.4.4.5 DESIGN LIFE

The long-term durability of steel and polymeric reinforcements must be considered in the design of MSE walls to ensure suitable performance throughout the design life of the structure.

The structural design of galvanized steel soil reinforcements and connections shall be made on the basis of a thickness, E_c, defined as:

$$E_c = E_n - E_s \qquad (15.4.4.5\text{-}1)$$

where:

E_c = thickness of metal reinforcement at the end of service life (mm)

E_n = nominal thickness of steel reinforcement at construction (mm)

E_s = sacrificial metal thickness expected to be lost by uniform corrosion during the service life of the structure (mm)

For structural design, sacrificial thicknesses shall be computed for each exposed surface as follows:

- Loss of galvanizing = 0.015 mm/year for the first 2 years

  = 0.004 mm/year for subsequent years

- Loss of carbon steel = 0.012 mm/year after zinc depletion

Other corrosion-resistant coatings, if specified, shall be electrostatically applied, resin-bonded epoxy type, with a minimum application thickness of 0.40 mm.

In determining the requirements for corrosion protection, the following should be considered:

- Galvanizing is preferred to use of epoxy-bonded coatings because the long-term performance of epoxy-bonded coatings buried in soil is unknown.

- If used, epoxy coatings should have a minimum thickness of 0.38 mm to 0.46 mm when placed in granular fills with sharp angular fragments.

- The sacrificial thickness should be provided in addition to the epoxy coating.

- Alloys, such as aluminum and stainless steel, should not be used for reinforcements.

The durability of polymeric reinforcements is influenced by time, temperature, mechanical damage, stress levels, microbiological attack and changes in the molecular structure by radiation or chemical exposure. The long-term stress-strain-time behavior of these materials shall be determined from the results of controlled laboratory creep tests conducted for a minimum duration of 10,000 hours for a range of load levels on samples of the finished product in accordance with ASTM D5262 (1994). These samples must be tested in the direction in which the load will be applied, and the results extrapolated to the required design life using procedures outlined in ASTM D2837 (1994). From this testing, the reinforcement tensile strength shall be the lesser of:

- T_l - The highest load level at which the log time-creep-strain rate continues to decrease with time within the required lifetime without either brittle or ductile failure

- T_5 - The tension level at which total strain is not expected to exceed 5% within the design life of the structure

The effects of aging, chemical and microbiological attack, environmental stress cracking, stress relaxation, hydrolysis and variations in the manufacturing process, as well as the effects of construction damage, shall be evaluated and extrapolated to the required design life.

In determining the durability requirements for polymeric reinforcements, the following should be considered:

- In the absence of 10,000-hour creep test data, creep reduction factors may be estimated from ASTM D4595 (1994), Mitchell and Villet (1987) and Christopher and Holtz (1985).

- Short-term EPA 9090 test data is not sufficient to provide data regarding long-term strength reduction of polymeric reinforcements. For polyoleofin products, oxidation strength losses can be determined using oven aging tests (Wisse, 1990).

- Environmental and aging losses for polymeric materials vary widely depending on composition of the polymer and manufacturing process. Based on hydrolysis losses alone, up to 60% of the maximum strength of polymers can be lost during a 75-year service life under poor moisture and temperature conditions. Polypropylenes are only slightly affected by hydrolysis. Hydrolysis strength losses of polyester products can be determined following (McMahon, 1959). Aging losses have not been evaluated.

- Limited testing of the effects of construction damage on polymeric reinforcements suggest minimum reduction factors to tensile strength of 20% for sand backfills, and as much as 70% if the backfills are gravels or crushed rock.

15.4.4.6 STRUCTURAL DESIGN OF FACE PANEL

Design of the concrete face panel shall be in accordance with the provisions of Section 5 of LRFD.

15.4.5 Example Problem - Mechanically Stabilized Earth (MSE) Wall

Retaining Sandy Soils

An MSE wall is proposed as follows:

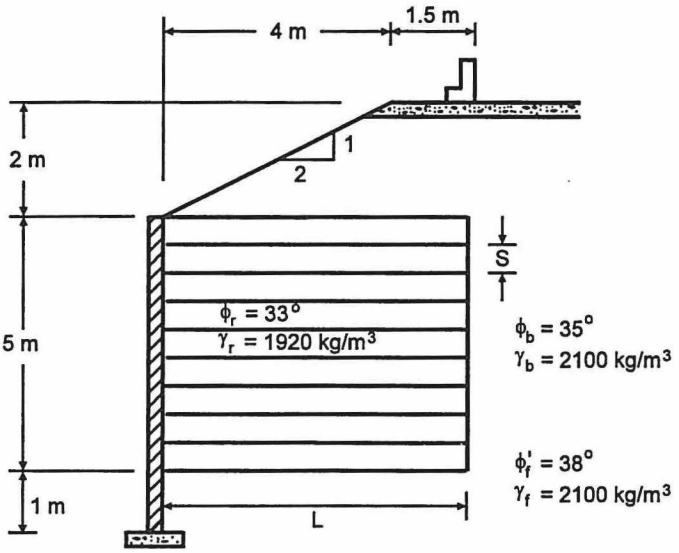


Figure 15.4.5-1 - Problem Geometry

During the subsurface exploration, it was determined that the foundation soils consist predominantly of sand to a depth of 10 m beneath the base of the proposed MSE wall. Bedrock underlies the sand. After correcting for depth, the average N-value for the sand is 20. The angle of internal friction is based on the SPT data.

The seasonal high groundwater table has been identified at a depth of 2 m below the bottom of the MSE wall.

Apply a live load surcharge above the 2:1 slope.

Assume the leveling pad to be constructed of precast concrete.

Inextensible ribbed strip reinforcement is 50 mm wide by 4 mm thick and has a center-to-center spacing of 700 mm. The steel reinforcing is galvanized.

Consider a design life of 100 years.

SOLUTION

Step 1: Calculate the Unfactored Loads

(A) Determine Length of Soil Reinforcement

The minimum soil reinforcement length should be taken as the greater of either 70% of the wall height as measured from the leveling pad (4.2 m) or 2400 mm (S11.9.5.1.4). For this example, try L = 5400 mm.

H = 6.0 m

L = 5.4 m

(B) Determine Vertical Earth Pressure (EV)

Total Unit Weight of reinforced backfill:

1920 kg/m^3

Weight of reinforced backfill:

P_{EV1} = (6)(L)(1920)g
= 610 260 N/m of length

Total Unit Weight of fill above reinforced backfill:

2100 kg/m^3

Weight of fill above reinforced backfill
P_{EV2} = [(1/2)(4)(2) + (2)(L-4)](2100)g
= 140 087 N/m of length

(C) Determine Live Load Surcharge (LS)

The live load surcharge is applied where vehicular load is expected to act on the backfill within a distance equal to the wall height behind the wall (S3.11.6.2).

Since the live load surcharge is not applied above the reinforcing strips, only the horizontal forces will be considered.

From Table S3.11.6.2-1, an equivalent height of soil for the design vehicular loading (h_{eq}) is estimated as:

h_{eq} = 760 mm

Use soil density of the backfill = 2100 kg/m^3

Determine active earth pressure coefficient, k_a:

$$k_a = \cos I \frac{\cos I - \sqrt{\cos^2 I - \cos^2 \phi_f}}{\cos I + \sqrt{\cos^2 I - \cos^2 \phi_f}} \qquad \text{(S3.11.5.7-3)}$$

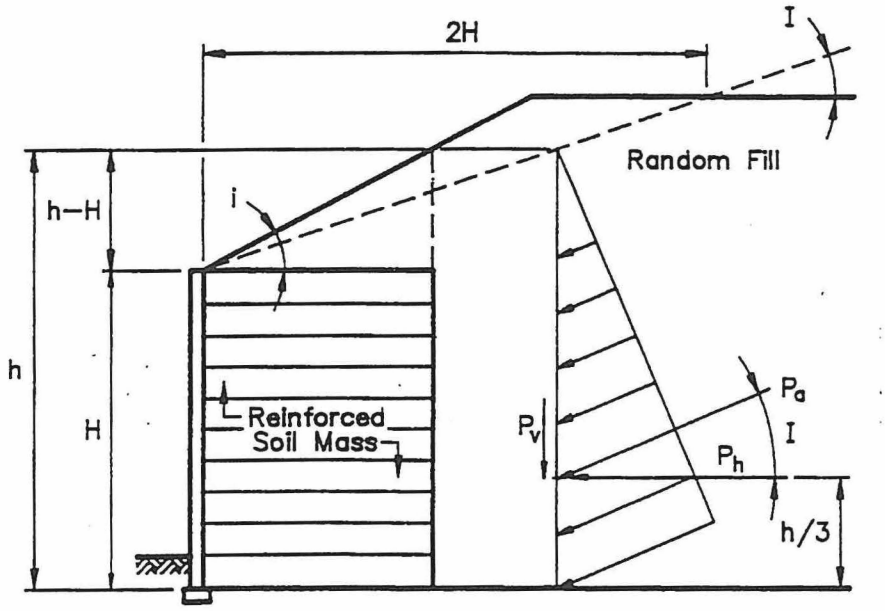


Figure 15.4.5-2 - Earth Pressure Distribution for MSE Wall with Broken Back Backfill Surface

I = arctan[2/(2H)] = 9°

cos (I) = 0.99

sin (I) = 0.16

cos ϕ_b = 0.82

K_a = 0.28

h = H + 2 = 8.0 m

Using Equation S3.11.6.2-1:

$\Delta p = (k)\ (\gamma)\ (g)\ (h_{eq})\ 10^{-9}$

$$= 0.28\left(2100\ \frac{Kg}{m^3}\right)\left(9.806\ \frac{m}{Sec^2}\right)(760\ mm)\ x10^{-9}\ \frac{m^3}{mm^3}$$

Δp = 0.0044 MPa

The live load horizontal earth pressure acting over the entire wall given by a rectangular distribution is:

$P_{LS} = (\Delta p)(h)$

= 0.0044 x 8000 x 1000 = 35 143 N/m of length

(D) Determine Horizontal Earth Pressure (EH)

The basic earth pressure should be assumed to be linearly proportional to the depth of earth and, using Equation S3.11.5.7-1, taken as:

$P_a = 0.5\ (\gamma)g(h^2)K_a$

and

$\phi = 35°$

Use soil density of the backfill =

2100 kg/m^3

$h = H+2 = 8.0$ m

$K_a = 0.28$

The horizontal earth pressure acting over the entire wall, given by a triangular pressure distribution, is:

$P_a = (0.5)(2100)g(h^2)(0.28) = 184\ 965$ N/m of length

P_a horiz. $= P_a$ COS (I) $= 182\ 448$ N/m of length

P_a vert. $= P_a$ SIN (I) $= 30\ 408$ N/m of length

(E) Summary of Unfactored Loads

Table 15.4.5-1 - Vertical Loads/Moments

Item	V_{unf} N/m	Arm about 0 m	Moment about 0 N-m/m
P_{EV1}	610 260	2.7	1 647 703
P_{EV2}	140 087	3.5	490 853
P_a vert.	30 408	5.4	164 203
Total	780 755		

Table 15.4.5-2 - Horizontal Loads/Moments

Item	H_{unf} N/m	Arm about 0 m	Moment about 0 N-m/m
$P_{EH} = P_a$ horiz.	182 448	2.7	486 529
P_{LS}	35 143	4.0	140 573
Total	217 592		

Step 2: Determine the Appropriate Load Factors Using Tables S3.4.1-1 and S3.4.1-2

Table 15.4.5-3 - Load Factors and Load Combinations

GROUP	EV 1	EV 2	EH (Active)	LS
Strength I-a	1	1	1.5	1.75
Strength I-b	1.35	1.35	1.5	1.75
Strength IV	1.35	1.35	1.5	0
Service I	1	1	1	1

Shrinkage and temperature loading is not required for MSE walls (S11.9.2).

For the loading cases identified, additional load combination groups are either redundant or have loadings which are not applicable.

Step 3: Calculate the Factored Loads and Moments

Table 15.4.5-4 - Vertical Factored Loads

GROUP	$P_{EV\,1}$ N/m	$P_{EV\,2}$ N/m	P_{EV} Vertical N/m	TOTAL Vertical N/m
Unfactored	610 260	140 087	30 408	780 755
Strength I-a	610 260	140 087	45 612	795 959
Strength I-b	823 852	189 117	45 612	1 058 581
Strength IV	823 852	189 117	45 612	1 058 581
Service I	610 260	140 087	30 408	780 755

Table 15.4.5-5 - Horizontal Factored Loads

GROUP	P_{EH} Horizontal N/m	P_{LS} N/m	TOTAL Horizontal N/m
Unfactored	182 448	35 143	217 592
Strength I-a	273 672	61 501	335 173
Strength I-b	273 672	61 501	335 173
Strength IV	273 672	0	273 672
Service I	182 448	35 143	217 592

Table 15.4.5-6 - Factored Moments (M_v)

GROUP	$P_{EV\,1}$ N-m/m	$P_{EV\,2}$ N-m/m	P_{EV} Vertical N-m/m	TOTAL Vertical N-m/m
Unfactored	1 647 703	490 853	164 203	2 302 760
Strength I-a	1 647 703	490 853	246 305	2 384 862
Strength I-b	2 224 399	662 652	246 305	3 133 356
Strength IV	2 224 399	662 652	246 305	3 133 356
Service I	1 647 703	490 853	164 203	2 302 760

Table 15.4.5-7 - Factored Moments (Mh)

GROUP	P_{EH} Horizontal N-m/m	PLS N-m/m	TOTAL Horizontal N-m/m
Unfactored	486 529	140 573	627 102
Strength I-a	729 793	246 003	975 796
Strength I-b	729 793	246 003	975 796
Strength IV	729 793	0	729 793
Service I	486 529	140 573	627 102

Step 4: Calculate the Settlement of the MSE Wall (S10.6.2.1)

Assume embankment construction behind the MSE wall has been performed previously and that any settlement from the embankment has already occurred.

The average settlement based on D'Appolonia, et al, (1968):

$\rho = q\ B\mu_o\ \mu_1/M$

where:

q = $V_{TOT}/L = 780\ 755/[(B)(10^{-6})] = 0.1446$ MPa

B = 5.4 m

M = 350 tsf = 33.5 MPa

M can be correlated with average N-value (20) from SPT data, D'Appolonia, et al, (1968)

for D/B = 0.19 $\mu_o = 0.9$

for H/B = 1.9 and L/B > 10 $\mu_1 = 0.7$

$\rho = 0.015$ m = 15 mm

Step 5: Check Overturning (Eccentricity)

Table 15.4.5-8 - Summary for Eccentricity Check

Group/Item	V_{TOT} N/m	M_v N-m/m	M_h N-m/m	X_o m	e m	$q_{uniform}$ MPa
Strength I-a	795 959	2 384 862	975 796	1.77	0.93	0.2248
Strength I-b	1 058 581	3 133 356	975 796	2.04	0.66	0.2597
Strength IV	1 058 581	3 133 356	729 793	2.27	0.43	0.2331
Service I	780 755	2 302 760	627 102	2.15	0.55	0.1819

where:

X_o = location of the resultant = $(M_v-M_h)/V_{TOT}$

e = eccentricity = $B/2-X_o$

S11.6.3.3 - Require that the location of the resultant shall be in the middle half of the base.

e_{max} = $B/4 = 1.35$ m

For all cases, e is less than e_{max}, therefore, the design is adequate with regards to Overturning.

Since e is less than B/6 = 0.9 m, for groups Strength I-b through Strength IV, the base pressure will be trapezoidal. Since e is greater than B/6 for group Strength I-a, the base pressure distribution will be triangular. A uniform distribution will be used for both cases.

The uniform pressure is taken as the average pressure of an assumed rectangular distribution.

$q = V_{TOT}/(B-2e) = V_{TOT}/2X_o$

Step 6: Check Sliding

The factored resistance against failure by sliding from S11.9.4.1 is taken as:

$Q_R = \phi_t Q_t + \phi_{ep} Q_{ep}$

Because of the potential for loss of soil in front of the wall, assume

$\phi_{ep} Q_{ep} = 0$

The resistance factor for sliding resistance from Table S10.5.4-1 is:

$\phi_t = 0.9$

Nominal shear resistance is taken as:

$Q_t = V \tan \delta$

$\tan \delta = 0.8 \tan \phi_f = 0.8 \tan 38° = 0.63$

Using the smallest factored total vertical force from Table 15.4.5-4, Strength I-a:

$V = 795\ 959$ N/m

$Q_R = 451\ 309$ N/m

Compare Q_R to the corresponding horizontal load for Strength I-a from Table 15.4.5-5 which is 335 173 N/m. Therefore, there is adequate protection against sliding.

Step 7: Check Bearing Resistance (S11.9.4.2)

For calculation of bearing resistance, per running meter of MSE wall, use Equation S10.6.3.1.1-1 and width (B) equal to the length of reinforcement (L) = 5.4 m.

$q_R = \phi\ q_{ult}$

ϕ = 0.35 from Table S10.5.4-1

Using Equation S10.6.3.1.2C-1:

$$q_{ult} = (0.5)\ g\ (\gamma)(B)(C_{w1})(N_{\gamma m})(1E\text{-}6) + g\ (\gamma)(C_{w2})(D_f)(N_{qm})(1E\text{-}6)$$

where:

$\gamma = 2100$ kg/m^3

Using Table S10.6.3.1.2c-1 and interpolating for C_{w1} for a depth of 3m below the ground surface:

$D_f = 1.0$

$1.5B + D_f = 9.1$

$C_{w1} = 0.62$

C_{w2} = 1.0

Using Equation 14.2.4.2-6:

$N_{\gamma m} = N_\gamma s_\gamma c_\gamma i_\gamma d_\gamma$

From Table 14.2.4.2-1:

$N_\gamma = 78$

From Table 14.2.4.2-3 and assuming L/B = 10:

$s_\gamma = 0.96$

Using Table 14.2.4.2-5 for strip footings and assuming q = (1)(2100) g = 0.021 MPa:

$c_\gamma = 0.70$

For strip footings from Table 14.2.4.2-6 with H/V = 0.28 (using unfactored values for H/V):

$i_\gamma = 0.37$

From Barker, et al, (1991), Page 22, $d_\gamma = 1.0$:

$N_{\gamma m} = 78\ (0.96)\ (0.7)\ (0.37)\ (1.0) = 19.4$

Using Equation 14.2.4.2-7:

$N_{qm} = N_q s_q c_q i_q d_q$

From Table 14.2.4.2-1:

$N_q = 49$

From Table 14.2.4.2-2 and assuming L/B = 10:

$s_q = 1.08$

Using Table 14.2.4.2-5 - for strip footings and assuming q = (1)(2100) g = 0.021 MPa:

$c_q = 0.70$

For H/V = 0.28 and Table 14.2.4.2-6 for strip footings and using unfactored values for H/V:

$i_q = 0.52$

From Table 4.7 Barker, et al, (1991)

$d_q = 1.0$

$N_{qm} = 49\ (1.08)\ (0.70)\ (0.52)\ (1.0) = 19.3$

$q_{ult} = 1.07$ MPa

$q_R = 0.37$ MPa

A uniform base pressure distribution over an effective footing width shall be used for comparison to the bearing resistance - See Table 15.4.5-8.

Comparing q_R to $q_{uniform}$ from Table 15.4.5-8. Adequate bearing capacity is available.

Step 8: Overall Stability

Verify overall stability against a deep-seated soil failure using a limit equilibrium method of analyses.

Step 9: Internal Stability

(A) Calculate the Factored Horizontal Force Acting on the Reinforcement

Consider the slip or the rupture of the reinforcement strips

By Equation S11.9.5.2.1-1, the factored horizontal force acting on any single layer of reinforcement shall be:

$P_i = \sigma_H h_i$

where:

h_i = height of reinforced soil zone at Level i

σ_H = factored horizontal stress at Layer i = $\gamma_p \sigma_v k$

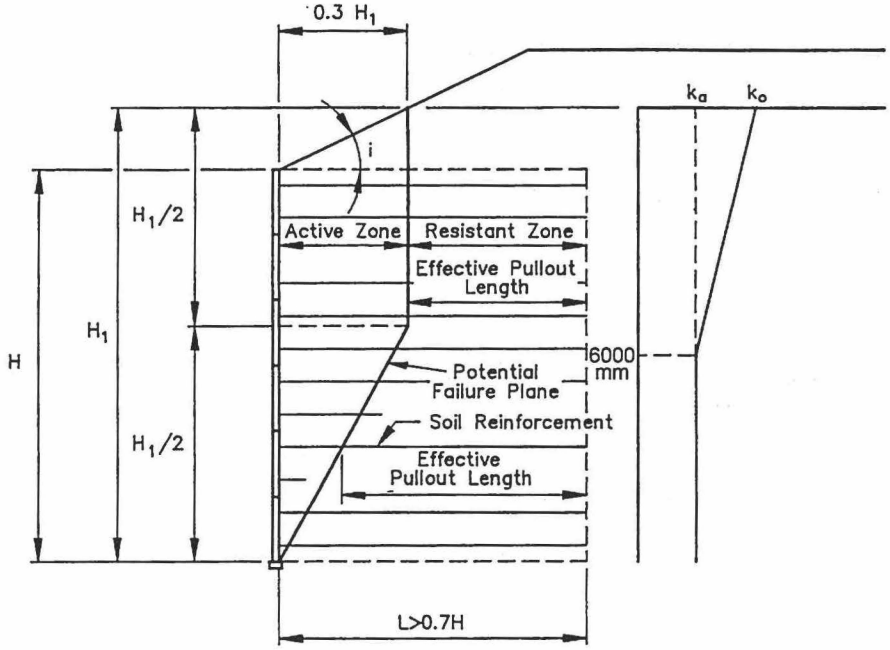
γ_p = the load factor for vertical earth pressure or earth surcharge applied to the unfactored σ_v. The load factor is typically applied to V_{TOT} used to calculate σ_v and the moments used to calculate X_o (and e).

σ_v = pressure due to resultant of factored vertical forces at Level i, based on overall stability of the wall section above Level i. The value of σ_v is determined using a uniform pressure distribution

over an effective width of L-2e = $V_{TOT}/(B-2e)$ = $V_{TOT}/2X_o$, for effective width see S10.6.3.1.5.

k = lateral earth pressure coefficient which varies from k_o to k_a

Use geometry shown in general Figure S11.9.5.2.2-1, reproduced below, and problem-specific Figure 15.4.5-3



$$H_1 = H + \frac{\tan i \times .3H}{(1-.3 \tan i)}$$

Figure S11.9.5.2.2-1 - Determination of Failure Plane and Earth Pressure Coefficients MSE Wall with Inextensible Reinforcements

Using Equation S3.11.5.7-5:

$k_o = 1 - \sin(35) = 0.43$

This value applied at 1.1 m above the top of the wall.

Using Equation S3.11.5.7-4:

$k_a = \tan^2(45-35/2) = 0.27$

This value applies at 4.9 m below the top of the wall and lower.

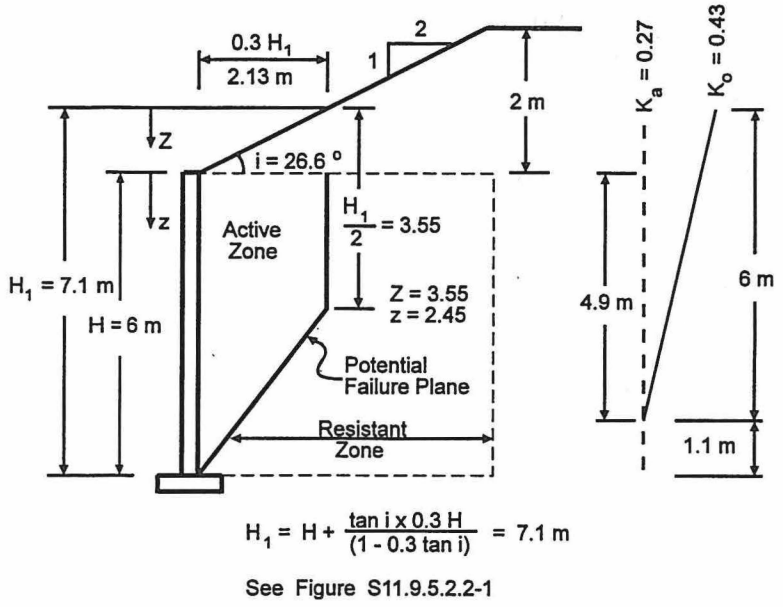


Figure 15.4.5-3 - Failure Plane

Place strips at a vertical spacing of 750 mm beginning at 375 mm below the top of the wall.

Live load surcharge is not included in the calculation of the internal stability.

Based on Table 15.4.5-8, the load case which produces the smallest eccentricity and the corresponding maximum uniform pressure when live load surcharge is neglected is Strength IV (EV = 1.35 and EH = 1.5).

Table 15.4.5-9 - Calculation of Vertical Load at the Level of Reinforcement per M of Wall

Layer	z mm	V_{TOT} N/m	M_v N-m/m	M_h N-m/m	e m	σ_v MPa
1	375	244 044	820 229	19 081	0.583	0.0576
2	1125	349 538	1 111 847	43 467	0.357	0.0746
3	1875	455 717	1 407 164	82 875	0.206	0.0914
4	2625	562 581	1 706 182	140 910	0.082	0.1075
5	3375	670 131	2 008 900	221 178	0.032	0.1256
6	4125	778 366	2 315 319	327 284	0.146	0.1524
7	4875	887 286	2 625 437	462 833	0.263	0.1820
8	5625	996 891	2 939 256	631 431	0.385	0.2153

- V_{TOT} is calculated based on the weight of the soil overlying the strip in question and includes the load factor of 1.35 and 1.50 as appropriate.

- M_h is calculated based on the horizontal earth pressures with a load factor of 1.5.

- $e = B/2X_o = B/2 - (M_v-M_h)/V_{TOT}$, note: Eccentricity is taken as a positive value on either side of the mid-point of the strip.

- $\sigma_v = V_{TOT}/(B-2e)$

Sample Calculations for Layers 1 and 2

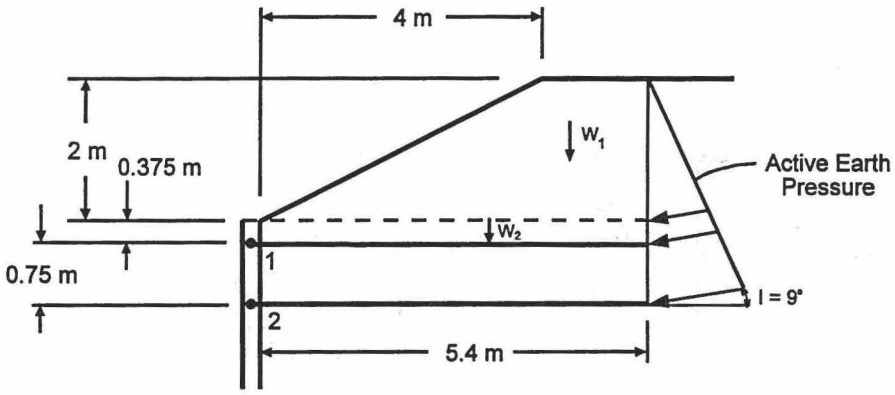
$V_{TOT} = V_1 + V_2 + V_3$

V_1 = weight of soil overlying strip in sloped portion above wall (factored)

V_2 = weight of soil overlying strip below top of wall (factored)

V_3 = vertical component of factored active earth pressure

Layer 1



$V_1 = \gamma_p W_1$

$= 1.35\ (2100\ kg/m^3)\ (9.81\ m/s^2)\ [1/2\ (4000\ mm)\ (2000\ m) + (5400\ mm\text{-}4000\ m)(2000\ mm)]\ (10^{-6})$

$= 111\ 245\ N/m + 77\ 872\ N/m = 189\ 117\ N/m$

$V_2 = \gamma_p W_2$

$= 1.35\ (1920\ kg/m^3)\ (9.81\ m/s^2)\ (375\ mm)\ (5400\ mm)\ (10^{-6})$

$= 51\ 491\ N/m$

$V_3 = \gamma_p{}^* P_a \sin i$

$= 1.35 \sin (9°)\ [0.5(2100\ kg/m^3)\ (9.81\ m/s^2)\ (2375\ mm)^2\ (0.28)]\ (10^{-6})$

$= 3436\ N/m$

*Note: $\gamma_p = 1.5$ for earth surcharge (ES)

At Layer 1, $V_{TOT} = 189\ 117\ N/m + 51\ 491\ N/m + 3436\ N/m$

$= 244\ 044\ N/m$

Moment About Front Face of Wall at Connection to Strip

$M_{v1} = (111\ 245\ N/m)\ (4\ m)\ (2/3) + (77\ 872\ N/m)\ (4\ m + 1/2\ (5.4\ m\text{-}4\ m)) + (51\ 491\ N/m)\ (5.4\ m/2) + (3436\ N/m)\ (5.4\ m)$

$M_{v1} = 296\ 653 + 365\ 998 + 139\ 026 + 513 + 5994$

$= 808\ 184\ N{\cdot}mm/m$

$M_{h1} = 1.5 \cos (9°)\ (2100\ kg/m^3)\ (9.81\ m/s^2)\ (0.28)\ [(0.5(2.375)^2\ (2.375)/3]$

$= 19\ 081\ N{\cdot}m/m$

Uniform Pressure on Effective Base Width - (For determination of pullout force and connection design force)

$e = ABS\ [5.4\ m/2 - (820\ 231 - 19\ 081)/244\ 044) = 0.583\ m$

$\sigma_v = \frac{244\ 044\ N/m}{(5.4\ m - 2(0.583))}(10^{-6})$

$= 0.0576\ MPa$

Layer 2

V_1 = 189 117 N/m - Same as Layer 1

V_2 = 1.35 (1920 kg/m^3) (9.81 m/s^2) (1125 mm) (5400 mm) (10^{-6})

= 154 472 N/m

V_3 = 1.35 sin (9°) (2100 kg/m^3) (9.81 m/s^2) (0.28) [0.5 (3125 mm)^2] (10^{-6})

= 5949 N/m

V_{TOT} = 349 537 N/m

M_{v2} = (111 245 N/m) (4m) (2/3) + (77 872 N/m) (4 m + 1/2 (5.4 m-4 m)) + (154 472 N/m) (5.4 m/2) + (5949 N/m) (5.4 m)

= 1 111 844 N/m

M_{h2} = 1.5 cos (9°) (2100 kg/m^3) (9.81 m/s^2) (0.28) [0.5 (3.125)^2 (3.125)/3]

= 43 467 N·m/m

e = ABS (5.4 m/2 - (1 111 844-43 467)/349 537]

= 0.357 m

$$\sigma_v = \frac{349\ 537}{(5.4-2(0.357))}(10^{-6})$$

= 0.0746 MPa

Table 15.4.5-10 - Summary of Horizontal Load per M of Wall

Layer	z (mm)	K	σ_v MPa	σ_H MPa	h_i mm	P_i N/mm
1	375	0.39	0.0576	0.0225	750	16.89
2	1125	0.37	0.0746	0.0276	750	20.73
3	1875	0.35	0.0914	0.0320	750	24.03
4	2625	0.33	0.1075	0.0355	750	26.65
5	3375	0.31	0.1256	0.0390	750	29.26
6	4125	0.29	0.1524	0.0443	750	33.22
7	4875	0.27	0.1820	0.0493	750	36.95
8*	5625	0.27	0.2153	0.0581	750	43.60

*Foundation soil assumed to partially support this layer

(B) Calculate the Corrosion Losses

Original dimensions of the reinforcing in mm is:

width = 50
thickness = 4

The thickness of the galvanizing is given as 0.086 mm/side.

For structural design, sacrificial thicknesses computed for each exposed surface as follows from S11.9.8.

Loss of galvanizing = 0.015 mm/year for first 2 years
0.004 mm/year for subsequent years

Loss of carbon steel = 0.012 mm/year after zinc depletion

Thickness after first two years (per side):

0.086-2(0.015) = 0.056 mm

Service life for remaining zinc:

0.056/0.004 = 14 years

Total service life for zinc = 16 years

Loss of steel:

Remaining Design life = 100 - 16 = 84 years

Loss of steel over design life (per side)

= (84) 0.012 = 1.01 mm

Cross-sectional area after 100 years:

width = 47.98 mm

thickness = 1.98 mm

(C) Check for Breakage of the Strip

With a c/c spacing of 0.70 m, there will be 1.43 strip per meter of width

Using Layer 7, the maximum loaded strip without support from the foundation soil, P_i = 0.0474 x 1000 mm x 750 mm = 35 550 N/m

Tension Load Per Strip = 24 885 N

Maximum Tensile Stress = Tension Load/x-section area

$$= \frac{24\ 866}{(47.98)(1.98)}$$

= 262 MPa

Allowable Tensile Stress in the Reinforcement = ϕF_u

where:

ϕ is the tensile resistance of metallic strip reinforcement for yielding of gross section less sacrificial area

ϕ = 0.90 Table 11.5.6-1

F_u is the minimum tensile strength

F_u = 400 MPa

Factored Tensile Stress = 360 MPa > 262 MPa

Since the allowable tensile stress is greater than the maximum stress per strip of reinforcement, adequate protection against breakage is provided.

(D) Design of Panel Connection

By S11.9.5.1.2, the horizontal force used to design the connection to the panels may be taken as not less than 85% of the maximum calculated force as determined above, except for the lower one-half of the structure where it shall be 100% of the maximum force calculated above.

Horizontal forces, as given in Table 15.4.5-11, should be used for design of connections.

Table 15.4.5-11 - Summary of Maximum Horizontal Forces for Connection of Strip to Facing

Layer	P_i N/mm	Force/Strip N	Connection Design Force/Strip N
1	16.89	11 821	10 048
2	20.73	14 513	12 336
3	24.03	16 820	14 297
4	26.65	18 655	15 856
5	29.26	20 485	20 485
6	33.22	23 252	23 252
7	36.95	25 865	25 865
8*	43.60	30 520	30 520

*Connection at Layer 8 shall be designed for force in Layer 7 (see note Table 15.4.5-9)

Sample Calculation for Layer 1:

- P_1 = 16.89 N/mm
- Force/Strip = (16.89 N/mm) (700 mm)
  = 11 821 N
- Connection Design Force = (0.85) (11 821 N)
  = 10 048 N

(E) Calculate the Pullout Resistance of the Reinforcement using Equation S11.9.5.3-1

$$P_{fs} = g\, f^*\, y_s\, Z\, A_s\, 10^{-9}$$

where:

f^* = apparent coefficient of friction at each reinforcement level

at the ground level, $f^* = 2.0$

at 6 m, $f^* = \tan \phi_f = 0.65$

ϕ_f = the friction angle of the backfill within the reinforced volume = 33°

γ_s = soil density = 1920 kg/m^3

Z = depth below effective top of wall (H_1 on Figure 15.4.5-3)

A_s = total top and bottom surface area of reinforcement along the effective pullout length beyond the failure plane specified in Figure 15.4.5-3 less any sacrificial thickness.

From Table S11.5.6-1:

Ultimate Pullout Resistant Factor = 0.85

Table 15.4.5-12 - Pullout Capacity

Layer	Z (mm)	f^*	Pullout Length (mm)	A_s (mm^2)	P_{fs} N	ϕP_{fs} N
1	1475	1.72	3270	313 815	14 991	12,742
2	2225	1.58	3270	313 815	20 737	17,626
3	2975	1.43	3270	313 815	25 218	21,435
4	3725	1.29	3375	323 892	29 347	24,945
5	4475	1.15	3825	367 078	35 542	30,211
6	5225	1.01	4275	410 263	40 621	34,528
7	5975	0.86	4725	453 449	44 061	37,452
8	6725	0.72	5175	496 634	45 340	38,539

Z is measured down from a height H_1 above the bottom of the wall as opposed to z which is measured down from the top of the wall (see Figure 15.4.5-3).

From S11.9.5.3:

- Pullout length must be at least 900 mm
- Minimum Total length shall be 2400 mm

Since the pullout capacity provided by the resisting zone of Figure 15.4.5-3 (ϕ P_{fs} of Table 15.4.5-12) is greater than the horizontal force applied to each strip (Table 15.4.5-11), adequate protection against pullout is provided.

Sample Calculation for Layer 1:

Z = 7100 mm - 6000 mm + 375 mm = 1475 mm (see Figure 15.4.5-3)

f^* = 2.0 - (2.0 - 0.65) 1475/7100 = 1.72

f^* varies from 2.0 @ Z = 0 to tan 33° = 0.65 @ Z = 7100 mm

Pullout Length = 5400 mm - 0.3 H_1 = 3270 mm (Length of Resistant Zone, see Figure 15.4.5-3)

A_s = 2 * (3270 mm) * (47.98 mm) = 313 789 mm^2

P_{fs} = $gf^* \gamma_s Z A_s (10^{-9})$

= (9.81) (1.72) (1920) (1475) (313 789) (10^{-9})

= 14 994 N

ϕ P_{fs} = 0.85 (14 994) = 12 745 N

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WORK PERIOD #2 - CONCRETE BOX CULVERTS

Example 1 - Box Culverts at Grade

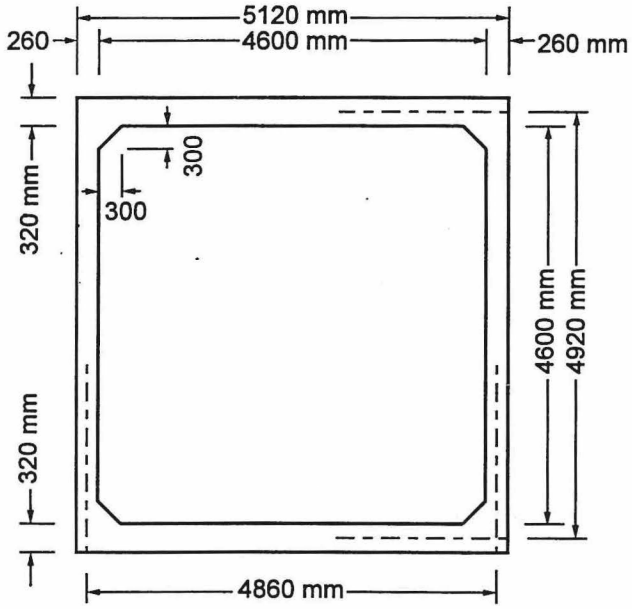


Figure 1 - Dimensions

Assumptions:

Concrete compressive strength: Top slab 28 MPa
Other Slabs 24 MPa

Steel yield strength = 400 MPa

Concrete density = 2400 kg/m^3

Thickness of future wearing surface = 50 mm

Spacing between expansion joints in the culvert = 20 000 mm

Number of traffic lanes = 4 lanes

Reinforcement bars:

#15: diameter = 16 mm

cross-section area = 200 mm^2

#20: diameter = 19.5 mm

cross-section area = 300 mm^2

The steel at the compression side was ignored in the design of all sections.

Concrete cover: (S5.12.3)

Top slab:	Top:	60 mm, including 10 mm integral wearing surface
	Bottom:	25 mm
Bottom slab:	Top:	60 mm, including 10 mm sacrificial layer for wear caused by water flow
	Bottom:	75 mm, including 25 mm to be ignored in strength calculations to account for the uneven bottom surface.
Walls:		50 mm both sides

Note that most jurisdictions increase the thickness of the top cover of bottom slab to account for possible wear caused by the water flow inside the culvert. For the same reason, some jurisdictions also increase the cover at the inside face of the walls. These provisions are not explicitly included in the Specification. However, the designers are allowed to modify the concrete cover thickness specified in Article S5.12.3 based on the site-specific conditions and the policies of the culvert owner. Therefore, the set of concrete cover thicknesses shown above were adopted for this example, but the designer may chose to use different values.

When a check is based on an uncracked section, the section area and moment of inertia were calculated based on the concrete gross area, ignoring the effect of the reinforcing steel on the section properties. This was done for simplicity. The designer may chose to take advantage of the transformed section area and transformed moment of inertia.

Assume that the reinforcement will be arranged to provide ductile failure.

η_D = 0.95 for strength limit state (S1.3.3)

η_D = 1.0 for other limit states

By nature, plate structures, such as culvert walls and top and bottom slabs allow the redistribution of forces in case of yielding of some reinforcement bars. Therefore, catastrophic failure is not likely and multiple load paths may be assumed resulting in:

η_R = 0.95 for strength limit state (S1.3.4)

η_R = 1.0 for other limit states

Assume that the structure is of operational importance

$\eta_i \geq 1.05$ for strength and extreme event limit state (S1.3.5)

Use $\eta_i = 1.05$ for strength and extreme event limit state

η_i = 1.0 for other limit states

For strength limit state $\eta = \eta_D \eta_R \eta_i \geq 0.95$ (S1.3.2.1)

η = 0.95 x 0.95 x 1.05 = 0.948 < 0.95 N.G

Use η = 0.95

For extreme event limit state $\eta = \eta_D \eta_R \eta_i$ = 1.0 x 1.05
= 1.05

For other limit states η = 1.0

An accurate analysis of a box culvert would normally consider the soil-structure interaction of the culvert and the surrounding soil and would analyze them as one system. This type of analysis would require accurate determination of soil parameters. Considering the relatively small size of box culvert projects, accurate determination of the soil parameters may be relatively expensive. In addition, a tight control on the fill material and compaction of the fill would be required. Therefore, simpler analysis methods that do not consider accurate determination of the soil-structure interaction are usually employed in box culvert design.

Vertical loads are transmitted to the bottom slab through the walls. The loads transmitted to the bottom slab can be modeled as line loads acting near the edge of the slab along the center of the walls. This, combined with the relatively thin, flexible bottom slab makes the soil pressure under a box culvert not uniform, even under a symmetric uniform load on the top slab. Since the actual distribution of soil pressure is hard to determine, most designers consider the soil reaction to be uniformly distributed under the bottom slab for all symmetric and unsymmetric cases of loading. Some other designers prefer to consider the reaction to vary linearly across the width of the culvert. In both cases, a unit width strip is designed disregarding the interaction between the adjacent strips. Concentrated loads are divided by a certain distribution width to determine the share of each design strip. In this example, the soil pressure was assumed to be uniformly distributed under all cases of loading. The value of the soil pressure was calculated to provide upward force equal to the vertical forces on the culvert. To make it easier to follow the example, the load cases are identified as Load Case 1 through Load Case 14.

Loads

All loads are calculated for a strip 1.0 mm wide.

1. **Dead Loads:**

1a. Dead load of components only "D_c" (Load Case I)

Dead weight of top slab = 320 mm x 1.0 mm x 2400 x 10^{-9}) kg/mm^3 x 9.81m/sec^2 = 7.53 x 10^{-3} N/mm

Net upward reaction on bottom slab:

Due to weight of walls and haunches =
(2 x 4600 x 260 + 4 x 300 x 300/2) x (1/5120) x 2400 x 10^{-9} x 9.81 = 11.83 x 10^{-3} N/mm

Due to weight of top slab = 7.53 x 10^{-3} N/mm

Total net upward reaction on bottom slab =
11.83 x 10^{-3} + 7.53 x 10^{-3} = 19.36 x 10^{-3} N/mm

Note that the reaction is assumed to be uniformly distributed. Hence, the effect of the self-weight of the bottom slab is cancelled by the reaction caused by this weight and, therefore, is not considered in calculating the net upward soil reaction.

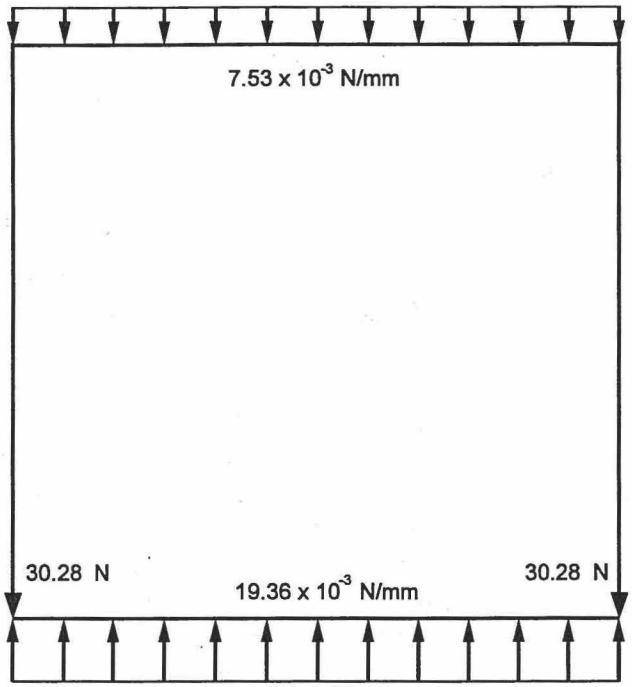


Figure 2 - Weight of the Components, D_c

Load factor for dead load of components: Table S3.4.1-2

1b. Dead load of future wearing surface (D_w) (Load Case 2):

Assume 50 mm thick future wearing surface with a density of 2250 kg/m^3.

Load on top slab = upward reaction on bottom slab

D_w = 50 mm x 1.0 mm x (2250 x 10^{-9}) kg/mm^3 x 9.81 = 1.1 x 10^{-3} N/mm

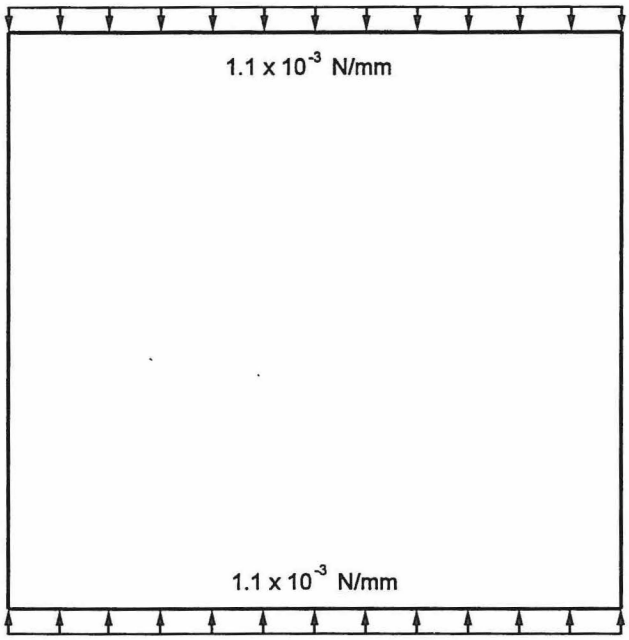


Figure 3 - Weight of Future Wearing Surface, D_w

Load factor for D_w: Maximum = 1.50 (Table 3.4.1-2)

Ignore the effect of future wearing surface if it produces a force effect that does not add to the force effect being maximized.

2. Live Loads:

Live load effects are calculated using the equivalent strip method. The effect of the design lane load is combined with the effect of either the design truck or the design tandem, whichever produces larger force effect. (S3.6.1.3.3)

Equivalent Strip Width

Deck span between centerlines of walls = 4600 + 260
= 4860 mm

For spans larger than 4600 mm, the equivalent strip width equations of Article S4.6.2.3 are applicable (S4.6.2.1.3). Multiple presence factors need not be applied when using these equations (S3.6.1.1.2). Notice that for culverts with smaller vents, i.e., distance between centerlines of adjacent walls less than 4600 mm, strip width would be calculated using Table S4.6.2.1.3-1 and the multiple presence factors would be applied.

$E = 250 + 0.42\sqrt{L_1 W_1}$ (S4.6.2.3)

L_1 = 4860

W_1 = the smaller of actual width (20 000 mm) or

9000 mm
= 9000 mm

E = 3027 mm

Multiple lanes loaded:

$$E = 2100 + 0.12\sqrt{L_1 W_1} < \frac{W}{N_L} \qquad \text{(S4.6.2.3)}$$

W_1 = the smaller of actual width (20 000 mm) or 18 000 mm = 18 000 mm

N_L = number of traffic lanes on the segment of the culvert between two expansion joints = 4 lanes

W/N_L = 20 000/4 = 5000 mm

$$E = 2100 + 0.12\sqrt{4860 \times 18\,000} = 3222\ mm < \frac{W}{N_L} \quad \text{OK}$$

Width of the equivalent strip to be used for design is the smaller of the two values of E calculated above, i.e., E = 3027 mm. The effect of the design truck or tandem is to be distributed over this width for top slab, bottom slab and side walls (S12.11.2.2).

Dynamic load allowance for buried structures:

$$IM = 40\ (1 - 4.1 \times 10^{-4}\ D_E) \geq 0\% \qquad \text{(S3.6.2.2)}$$

where:

D_E = thickness of earth fill above the structure
= 0.0 for culverts on grade

IM = 40%

The dynamic load allowance is applied to the design truck and the design tandem, but is not applied to the design uniform lane load.

For 1.0 mm wide strip:

Load from the heavy axles of the design truck, including dynamic load allowance = (weight of axle including IM)/E = 145 000 x 1.4/3027 = 67.1 N

Load from the axles of the design tandem, including dynamic load allowance = 110 000 x 1.4/3027 = 50.9 N

Load from design lane load = 9.3/E = 9.3/3027 =

3.1×10^{-3} N/mm

Notice that for culverts with smaller dimensions, the width of the equivalent strip would be smaller than the actual width of the design lane load (3000 mm). In these cases, it will not be logical to distribute the load over the smaller equivalent strip width. A more logical approach is to distribute the design lane load over 3000 mm, despite the smaller width of the equivalent strip. However, the effects of the design truck and design tandem will be distributed over a width equal to the calculated equivalent strip width in all cases.

For Strength I load combination:

Load factor for live load = 1.75 (S3.4.1)

Notice that in the analysis of this example, the loads from the design truck and the design tandem were not distributed in the direction parallel to traffic. However, the truck and tandem wheel loads can be distributed over a length equal to the length of the tire contact area. (S3.6.1.2.5)

The design truck and the design tandem loads are positioned to produce the maximum moment at the section under consideration. The design lane load is applied to the areas of the span that will contribute to the extreme force effect being considered. The lane load is not interrupted to provide space for the design truck or tandem (SC3.6.1.3.1).

For the purpose of this example, only the cases of loading, shown in Figure 4, were considered in the analysis. The maximum moments and shears obtained using these cases of loading are expected to be close to the maximum effects that would be obtained using a more detailed analysis. This simple approach was adopted to allow easy duplication of the analysis. In the design of an actual structure, the user will need more detailed analysis to determine the maximum force effect at each section.

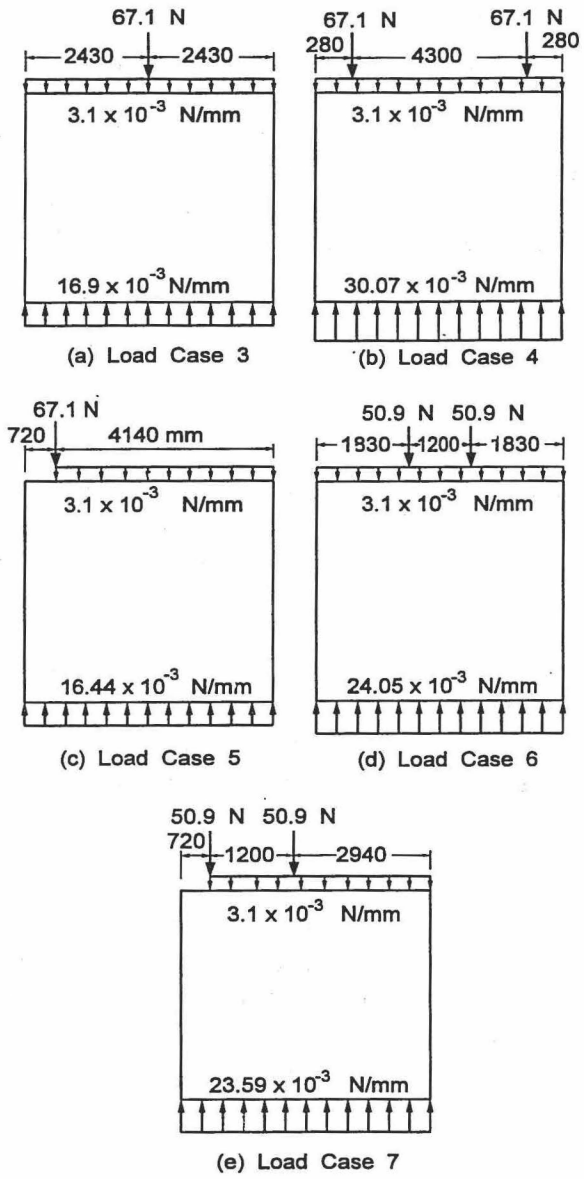


Figure 4 - Live Loads, Cases of Loading

3. Lateral Earth Pressure (EH) (Load Case 8)

For rigid frames, the movement of the walls is relatively small and the at-rest soil pressure is the adequate pressure to be used in that case.

Assume:

Soil Density, γ_s = 1600 kg/m^3 (S3.5.1)

Friction angle of drained soil, ϕ_f = 35°

Z = depth from ground surface

K_o = $1 - \sin\phi = 0.426$ (S3.11.5.1)

$P = K_o \gamma_s g Z (10^{-9})$

$= 0.426 \times 1600 \times 9.81 \times Z \times (10^{-9})$

$= 6.6865 \times 10^{-6} Z$ MPa

Load factors:

Maximum = 1.35 (S3.4.1)
Minimum for culverts = 0.5 (S3.11.7)

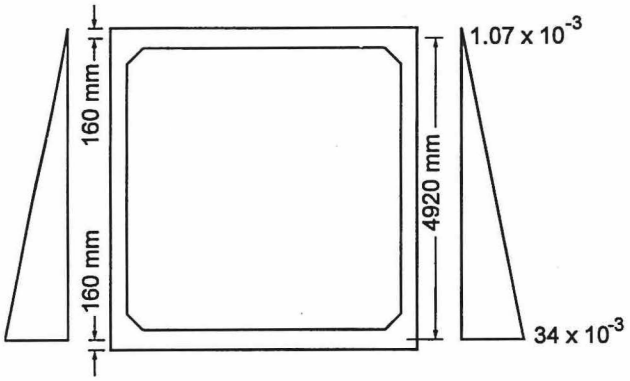


Figure 5 - Horizontal Earth Pressure, EH

4. Live Load Surcharge (LS) (Load Cases 9, 10 and 11)

From Table S3.11.6.2.1, live load surcharge equivalent fill depth, h_{eq}, is 1200 and 760 mm for wall heights of 3000 and 6000 mm, respectively. Culvert height from bottom of bottom slab to top of fill = 320 + 4600 + 320 = 5240 mm

By interpolation, $h_{eq} = 1200 - (1200-760)\left(\frac{5240-3000}{6000-3000}\right)$

= 871 mm

$D_p = k_o \gamma_s g h_{eq} (10^{-9})$

$= 0.426 \times 1600 \times 9.81 \times 871 \times 10^{-9}$

$= 5.824 \times 10^{-3}$ MPa

This load is applied as a uniform horizontal load acting on the outside walls of the culvert. In this example, it was assumed that the load may be acting on either side of the culvert (Load Cases 9 and 10) or acting on both sides simultaneously (Load Case 11).

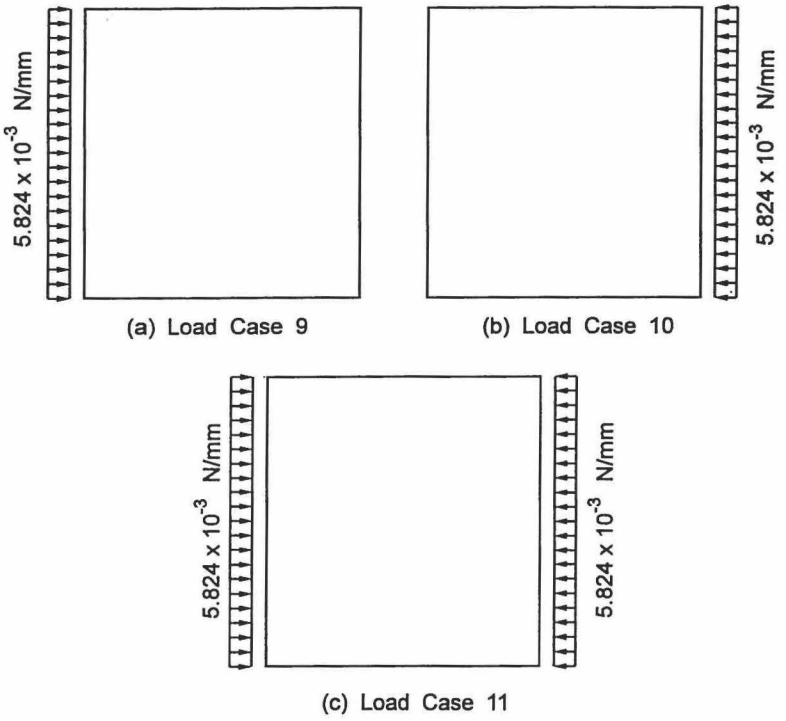


Figure 6 - Live Load Surcharge

Check for Fatigue in Reinforcement Bars

No fatigue problems were reported for culvert structures. However, since the top slabs of culverts on grade may be treated as deck slabs with main reinforcement parallel to traffic, fatigue in reinforcement bars needs to be checked. The fatigue in the wall reinforcement near the top may also be checked. For other elements of the culvert and for the top slabs of culverts under fill, the fatigue resistance may be affected by the presence of the soil. There is no research data to support that assumption, and a need for more research to study fatigue in box culverts.

For the purpose of this example, the three load cases shown in Figure 7 (Case 12 through 14), were considered in the determination of the forces resulting from the fatigue truck. The given loads include 15% dynamic load allowance (S3.6.2.1). The configuration of the fatigue truck is given in Article S3.6.1.4. The 9000 mm spacing between the heavy axles of the truck does not allow the two axles to be positioned on the culvert at the same time. Therefore, the maximum load on the bottom slab, Load Case 13, is produced by positioning the front axle together with the middle axle on the culvert. In the following calculations, fatigue is checked only in the reinforcement of the top slab and the wall near the top corner.

The multiple presence factor does not need to be considered when checking fatigue. Therefore, the force effect, calculated using the single-lane strip width equations of Article S4.6.2.3, need to be divided by 1.2 to remove the multiple lane distribution factor that is included in the equation (S3.6.1.1.2). Loads for 1.0 mm wide strip:

From the rear axles = $\frac{145\ 000\ x\ 1.15}{1.2\ x\ 3027}$ = 45.9 N/mm

From the front axle = $\frac{35\ 000\ x\ 1.15}{1.2\ x\ 3027}$ = 11.1 N/mm

Location of design sections for negative moment:

at the end of the haunch (S12.11.4.2)

Location of design sections for shear:

at a distance d from the end of the haunch (S5.13.3.6.1)

For the purpose of this example, the design section for positive moment was taken at mid-point of each member. All elements of the culvert were modeled as prismatic elements, i.e., the effect of the stiffness of the haunch on the distribution of the moments was ignored.

In determining the maximum force effect at a design section, force effects produced by permanent loads that exist all the time after building the culvert, i.e., weight of the structure and soil above, if exist, and lateral earth pressure due to the weight of the soil, are factored using the maximum load factor if they add to the force effect being maximized, otherwise, minimum load factor is applied. Other loads, such as future wearing surface, live loads and live load surcharge, are factored using maximum load factors if they add to the force effect being maximized, otherwise, they are ignored in the analysis. For live load cases (Cases 3 through 7), only the case that produces maximum force effect at the section being designed is considered in the analysis. The same applies to live load surcharge cases (Cases 9 through 11).

Stress range for fatigue stresses is calculated using the maximum positive and maximum negative moments produced by Load Cases 12 through 14. If the three load cases produce moments that has the same sign, the stress range is determined as the stress produced by the load case that produces the maximum moment. The minimum stress, due to the fatigue load combination, is taken equal to zero.

Table 1 gives the loads at the design sections produced by Load Cases 1 through 11. These loads are to be used in designing the sections for both strength and service limit states. Table 2 gives the section moments and concurrent axial forces produced by Load Cases 12 through 14 at the design sections for moment in top slab and near the top of the side wall. These loads are to be used for checking the reinforcement for fatigue.

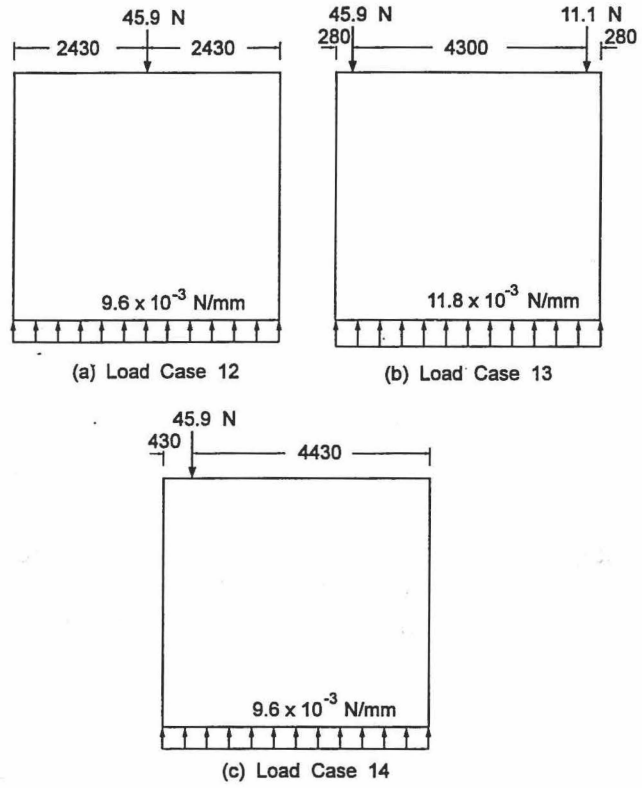


Figure 7 - Load Cases for Fatigue

Design Sections

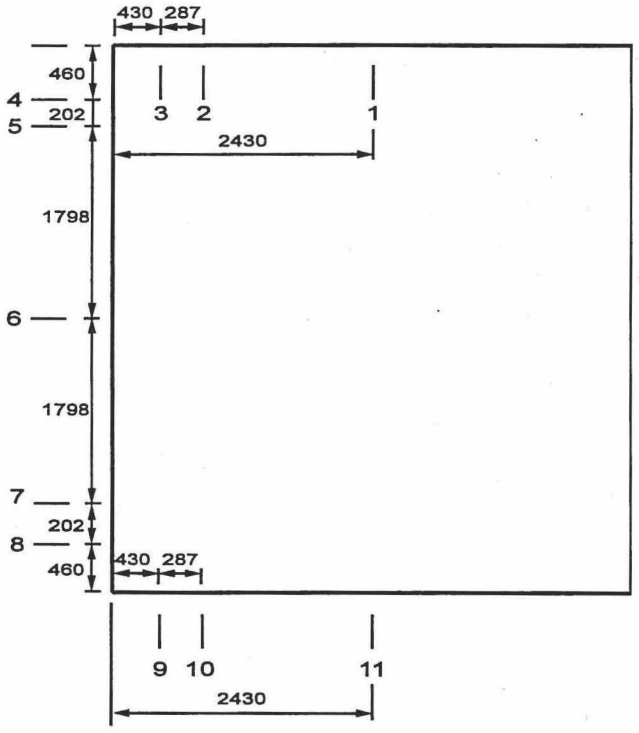


Figure 8 - Location of the Sections Considered in the Design

printed on September 28, 1995

Table 1 - Section Forces

SECTION		LOAD CASE										
		DEAD WT.	FWS	LIVE LOAD					EARTH PR.	LL SURCHARGE		
		CASE 1	CASE 2	CASE 3	CASE 4	CASE 5	CASE 6	CASE 7	CASE 8	CASE 9	CASE 10	CASE 11
1	MOMENT	20207	2498	72632	24583	24534	79635	57592	-20528	-3824	-3854	-7679
	AXIAL	3	0	-2	5	1	-2	-1	-29	-7	-7	-14
	SHEAR	0	0	34	0	9	38	27	0	5	-5	0
2	MOMENT	9158	883	10613	20035	35945	10017	38076	-20528	5071	-12751	-7679
	AXIAL	3	0	-2	5	1	-2	-1	-29	-7	-7	-14
	SHEAR	13	2	38	5	63	43	80	0	-5	5	0
3	MOMENT	5147	298	-667	18383	17837	-2536	15061	-20528	6562	-14241	-7679
	AXIAL	3	0	-2	5	1	-2	-1	-29	-7	-7	-14
	SHEAR	16	2	40	6	63	44	80	0	-5	5	0
4	MOMENT	-3360	-750	-17266	-5457	-9643	-21073	-19168	-7609	11471	-13177	-1705
	AXIAL	-21	-3	-41	-75	-63	-46	-80	0	5	-5	0
	SHEAR	3	0	-2	5	1	-2	-1	-28	-5	-7	-12
5	MOMENT	-3946	-750	-16924	-6379	-9797	-20745	-19057	-2180	12257	-11728	528
	AXIAL	-22	-3	-41	-75	-63	-46	-80	0	5	-5	0
	SHEAR	3	0	-2	5	1	-2	1	-26	-3	-7	-10
6	MOMENT	-9165	-750	-13876	-14587	-11167	-17816	-18069	29888	8780	1163	9942
	AXIAL	-33	-3	-41	-75	-63	-46	-80	0	5	-5	0
	SHEAR	3	0	-2	5	1	-2	1	-6	7	-7	0

SECTION		LOAD CASE										
		DEAD WT.	FWS	LIVE LOAD					EARTH PR.	LL SURCHARGE		
		CASE 1	CASE 2	CASE 3	CASE 4	CASE 5	CASE 6	CASE 7	CASE 8	CASE 9	CASE 10	CASE 11
7	MOMENT	-14384	-750	-10827	-22794	-12535	-14888	-17080	5360	-13525	14054	528
	AXIAL	-44	-3	-41	-75	-63	-46	-80	0	5	-5	0
	SHEAR	3	0	-2	5	1	-2	1	36	18	-7	10
8	MOMENT	-14970	-750	-10485	-23716	-12690	-14559	-16970	-2636	-17207	15502	-1705
	AXIAL	-46	-3	-41	-75	-63	-46	-80	0	5	-5	0
	SHEAR	3	0	-2	5	1	-2	1	47	20	-7	12
9	MOMENT	2134	298	6391	3424	2798	8948	5968	-25624	-22476	14797	-7679
	AXIAL	-3	0	2	-5	-1	2	1	-58	7	-21	-14
	SHEAR	-39	-2	-37	-66	-36	-52	-51	0	-9	9	0
10	MOMENT	10766	788	13927	17114	10228	19590	16607	-25624	-20270	12590	-7679
	AXIAL	-3	0	2	-5	-1	2	1	-58	7	-21	-14
	SHEAR	-34	-2	-30	-54	-29	-42	-42	0	-9	9	0
11	MOMENT	40854	2497	40191	64824	36516	56356	54156	-25624	-3855	-3842	-7679
	AXIAL	-3	0	2	-5	-1	2	1	-58	7	-21	-14
	SHEAR	0	0	0	0	0	0	-1	0	-9	9	0

- All forces are calculated for 1.0 mm wide strip
- All forces are in Newtons and moments are in N·mm
- A dynamic load allowance of 40% is included in the live load effects

Table 2 - Effect of Fatigue Truck

Section		Load Case		
		Fatigue Truck		
		Case 12	Case 13	Case 14
1	Moment	44859	7471	8074
	Axial	-1	2	1
3	Moment	-1041	11255	15770
	Axial	-1	2	1
4	Moment	-10372	-1683	-2879
	Axial	-23	-44	-42

• A dynamic load allowance of 15% is included

It is obvious that the axial forces are small relative to the moments and their effect on the design is minimal. For hand calculations, ignoring the effect of the axial forces at the sections where the force eccentricity is relatively large compared to the thickness of the section will result in a minor change in the results, but the analysis will be much easier to perform. However, the user of this manual may face cases where larger axial forces exist, such as the case of culverts under deep fill. Therefore, the effect of the axial forces was included in most calculations to generalize the method of analysis to all other cases of box culverts.

Positive Moment in Top Slab, Section 1

Maximum positive moment at Section 1:

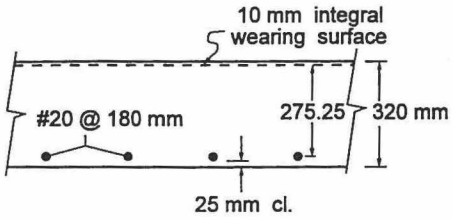
Design factored moment, M_u
$= \eta$ (1.25 DL + 1.5 FWS + 1.75 LL + 0.5 EH)

= 0.95 x [1.25 x 20 207 + 1.5 x 2498 + 1.75 x 79 635 + 0.5(-20 528)] = 150 200 N·mm/mm

Concurrent axial force = 0.95 [(1.25 x 3 + 0 + 1.75(-2) + 0.5(-29)] = -14 N/mm

Axial force is negligible compared to moment. Therefore, axial force is neglected in the determination of the reinforcement at this section. Ignoring the effect of the axial forces at the sections where eccentricity of the applied force is relatively large compared to the thickness of the section will

result in a minor change in the results, but the analysis will be much easier to perform.



Assume #20 bars:

$$d = 320 - 25 - \frac{19.5}{2} - 10 = 275.25 \ mm$$

Notice that the top 10 mm of the slab was ignored in calculating the effective depth d.

$$R_u = \frac{M_u}{\phi b d^2} = \frac{150\ 200}{0.9\,x1\,x(275.25)^2} = 2.203$$

$$\rho = 0.85\frac{f'_c}{f_y}\left[1 - \sqrt{1 - \frac{2R_u}{0.85 f'_c}}\right] = 0.00578$$

Required area of steel = ρd = 0.00584 x 275.25 = 1.59 mm^2/mm

Maximum allowed spacing of #20 bars = 300/1.59 = 188 mm

Use #20 @ 180 mm

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{300\,x400}{0.85\,x28\,x180} = 28.01 \ mm$$

for f'$_c$ = 28 MPa, β_1 = 0.85 (S5.7.2.2)

$$C = \frac{a}{\beta_1} = \frac{28.01}{0.85} = 32.95 \ mm$$

$$\frac{c}{d} = \frac{32.95}{275.25} = 0.12 < 0.42 \quad \text{OK} \qquad \text{(S5.7.3.3.1)}$$

Minimum reinforcement requirements:

$$\rho_{min} = 0.03 \frac{f_c'}{f_y} \qquad \text{(S5.7.3.3.2)}$$

$$\rho_{min} = 0.03 \, x \frac{28}{400} = 0.0021$$

$$\rho_{min} < \rho \; provided = \frac{300}{180 \, x 275.25} = 0.006 \qquad \text{OK}$$

Check for crack control at Section 1

Service load moment = 20 207 + 2498 + 79 635 - 20 528
= 81 810 N·mm/mm

Concurrent axial force = 3 + 0 - 2 - 29 = -28 N/mm

Modulus of rupture, $f_r = 0.63\sqrt{28}$
= 3.33 MPa (S5.4.2.6)

Tensile stress in the concrete assuming uncracked section with 10 mm of wear took place at the top surface and ignoring the effect of the reinforcement on the moment of inertia of the section =

$$\frac{-28}{310} + \frac{81\ 810}{(310)^2/6} = 5.02 \text{ MPa} > 0.8\ f_r = 2.67 \text{ MPa}$$

Therefore, check of cracking under service loads is required. (S5.7.3.4)

Maximum allowable service load stress in the reinforcement

$$f_{sa} = \frac{Z}{(d_c A)^{1/3}} \leq 0.6 f_y = 240 \; MPa \qquad \text{(S5.7.3.4-1)}$$

Clear concrete cover = 25 mm < 50 mm OK

d_c = 25 + 19.5/2 = 34.75 mm

A = 2 d_c x bar spacing

For deck slabs, assuming severe exposure conditions, Z = 23 000 N/mm. For buried structures, Z = 17 500 N/mm (S5.7.3.4-1)

In general, top slabs for culverts on grade may be treated as deck slabs. However, for this example, Z for the top slab was taken equal to 17 500 N/mm. This was decided to make the design consistent for all elements of the culvert.

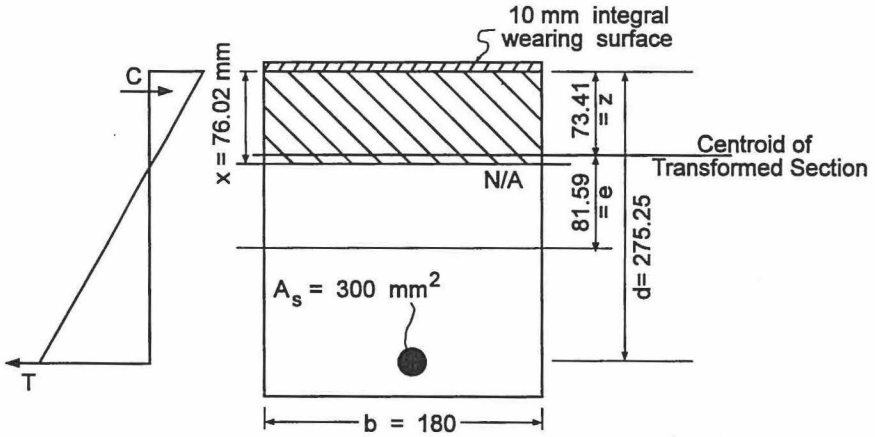
$$f_{sa} = \frac{17500}{(34.75\,x2\,x34.75\,x180)^{1/3}} = 231\ MPa$$

$f_{sa} < 0.6\ f_y = 240$ MPa OK

For a rectangular section under the combined effect of axial force and moment, the location of the neutral axis under service load conditions can be determined using a closed form solution or, more common, using an iterative solution. In the iterative solution used herein, an arbitrary location of the neutral axis is assumed. Transformed section area and moment of inertia, A_{Trans} and I_{Trans}, respectively, are calculated based on the assumed location of the neutral axis. The stresses and total tensile and compression forces in the steel and concrete, respectively, are then determined. The equilibrium of the forces acting on the section is checked. If a state of equilibrium exists, the assumed position of the neutral axis is right. Otherwise, a new location is assumed and the process is repeated until the forces are in equilibrium.

Modular ratio for $f'_c = 28$ MPa $= 8$ (S5.4.2.4 and S5.4.3.2)

Ignore the top 10 mm (integral wearing surface). Ignore the steel at the compression side of the section. Assume the neutral axis is at a distance x = 76.02 mm from the compression face of the effective section.



$$I_{Trans} = \frac{b[Z^3 + (X-Z)^3]}{3} + A_s n (d-Z)^2$$

$$I_{Trans} = \frac{180[(73.41)^3 + (76.02-73.41)^3]}{3} + 300\,x\,8(275.25-73.41)^2$$
$$= 119\,x10^6\ mm^4$$

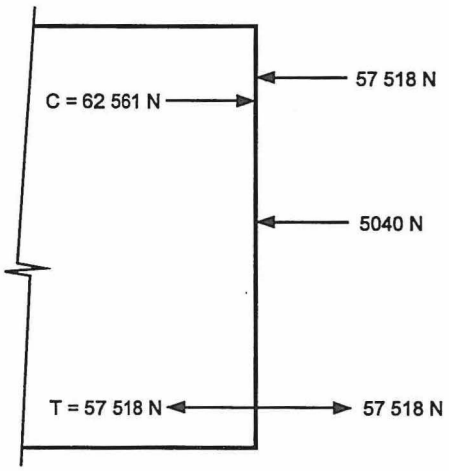
$$A_{Trans} = bx + A_s n = 180\,x\,76.02 + 300\,x\,8 = 16\,084\ mm^2$$

Tensile stress in steel:

$$f_s = n\left[\frac{P}{A_{Trans}} + \frac{(M+P_e)(d-Z)}{I_{Trans}}\right]$$
$$= 8x\left[\frac{-28x180}{16\,084} + \frac{(81\,810x180 - 28x180x81.59)\,x(275.25-73.41)}{119x9.144}\right]$$
$$= 191.7\ MPa$$

Maximum compression stress in concrete:

$$f_c = \frac{P}{A_{Trans}} + \frac{(M+P_e)Z}{I_{Trans}} = \frac{-28x180}{16\,084} - \frac{(81\,810x180 - 28x180x81.59)\,x73.4}{119x10^6}$$
$$= -9.144\ MPa$$



Total tensile force in steel =
$A_s f_s = 300 \times 191.73 = 57\,518$ N

Total compression force in concrete =
$0.5\ bx\ f_c = 0.5 \times 180 \times 76.02 \times (-9.144) = -62\,561$ N

Total external axial force = $28 \times 180 = 5040$ N

$62\,561 \simeq 57\,518 + 5040 = 62\,558$ N

Forces are in equilibrium

The assumed location of the neutral axis is right.

f_s = 191.7 MPa < f_{sa} = 231 MPa OK

In the analysis of other sections, the right location of the neutral axis (distance x) and the stress in the reinforcement will be given without detailed calculations. The user of this manual can confirm that the given location of the neutral axis satisfy the force equilibrium by conducting a complete analysis as shown above.

Distribution reinforcement at Section 1:

Distribution reinforcement is required for culverts with less than 600 mm of fill (S12.11.2).

Required distribution reinforcement as a percent of main reinforcement = $1750/\sqrt{S} \leq 50\%$

S for slabs monolithic with walls = distance face-to-face of supports (S9.7.2.3).

Take S equal to the clear distance between the ends of haunches = 4000 mm

% distribution reinforcement = $1750/\sqrt{4000}$ = 27.7%.

Main reinforcement: #20 @ 180 mm = 1.67 mm^2/mm
Required distribution reinforcement = 27.7/100 x 1.67 = 0.46 mm^2/mm

Use #10 bars: bar area = 100 mm^2

Maximum allowed spacing = 100/0.46 = 216 mm

Use #10 @ 210 mm

Check negative moment at Section 1:

Design factored negative moment, M_u =
η [0.9 DL + 1.35 EH + 1.75 LS]

0.95 x [0.9 x 20 207 + 1.35 x (-20 580) + 1.75 x (-7679)] = -21 883 N·mm/mm

Concurrent axial force =
0.95 [0.9 x 3 + 1.35 x (-29) + 1.75(-14)] = -58 N/mm

Assuming #15 reinforcement bars:

d = t - top cover - 1/2 bar diameter = 320 - 60 - 16/2 = 252 mm

Based on the applied loads, the required reinforcement was found to be less than the minimum allowed reinforcement, ρ_{min} = 0.0021.

Provide ρ_{min}

Required minimum reinforcement = #15 @ 370 mm

The actual spacing to be provided will be determined after the negative moment reinforcement at the top corner of the box is determined. Usually some of corner top reinforcement is extended to provide reinforcement for negative moment near the center.

Check for crack control under service load:

M = 20207 - 20580 - 7679 = -8052 N·mm/mm

Concurrent axial force = 3-29-14 = -40 N/mm

Assuming 10 mm of wear took place at the top surface of the culvert.

Concrete tensile stress under service loads assuming uncracked section = $\frac{-40}{310} + \frac{8052}{(310)^2/6}$

= 0.37 MPa < 0.8 f_r = 2.67 MPa

Therefore, checking for crack control under service load is not required. (S5.7.3.4)

Check for Fatigue in Reinforcement Bars at Section 1

From Table 2, the force effects of the fatigue truck at Section 1 are:

Maximum positive moment = 44 859 N·mm/mm

Concurrent axial force = -1 N/mm

Maximum negative moment = 0

Fatigue needs to be checked if the stresses due to permanent loads, including prestressing, are tension, and if the

stresses due to permanent loads are compression, but less than twice the tensile stresses produced by the fatigue load combination (S5.5.3.1). Section properties for fatigue check will be based on cracked section if the stresses due to unfactored permanent loads and 1.5 the fatigue truck are tension and exceed $0.25\sqrt{f'_c}$ (S5.5.3.1).

From Tables 1 and 2

Maximum moment due to unfactored permanent loads = 20 207 + 2498 - 20 580 = 2125 N·mm/mm

Concurrent axial force = 3 +0 -29 = -26 N·mm/mm

Tensile stresses exist in positive moment reinforcement under permanent load. Therefore, check of fatigue in positive moment reinforcement is required. (S5.5.3.1)

Maximum moment due to unfactored permanent loads and 1.5 fatigue truck = 2125 + 1.5 x 44 859 = 69 414 N·mm/mm

Concurrent axial force = -26 -1.5 x 1 = -28 N/mm

Maximum tensile stress in the concrete assuming uncracked section $= \frac{-28}{310} + \frac{69\ 414}{(310)^2/6}$

$= 4.24$ MPa $> 0.25\sqrt{f'_c} = 1.32$ MPa

Therefore, check for fatigue should be based on cracked section. (S5.5.3.1)

Maximum fatigue factored moment (incl. DL) = 2125 + 0.75 x 44 859 = 35 769 N·mm/mm

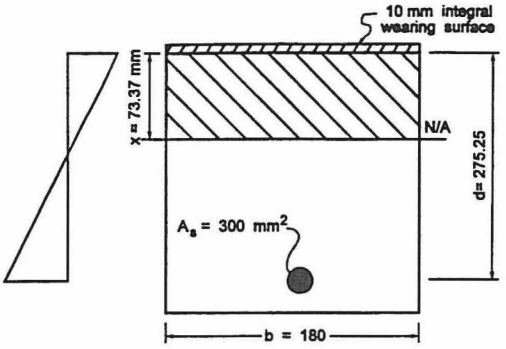
Concurrent axial load = -26 -0.75 x (1.0) = -26.75 N/mm

Minimum fatigue factored moment (including DL) = 2125 + 0 = 2125 N·mm/mm

Concurrent axial load = -26 N/mm

The value of the axial load is relatively small, therefore, it can be ignored without significant effect.

At this level of loading, the stresses can be assumed to be proportional to the strains.



X = 73.37 mm

I_{Trans} = 121.51×10^6 mm^2

$f_{s\,(max)}$ = 85.58 MPa

$f_{s\,(min)}$ = 5.08 MPa

stress range = 80.5 MPa

Allowable stress range, $f_f = 145 - 0.33\, f_{min} + 55\ (r/h)$ (S5.5.3.2)

f_{min} = minimum stress under fatigue load combination (including dead load)

= 5.08 MPa

r/h = can be taken as 0.3

f_f = $145 - 0.33 \times 5.08 + 55 \times 0.3 =$ 159.8 MPa

f_f > applied stress range OK

Moments due to the unfactored permanent load at Section 1 produce compressive stresses at the top surface of the concrete section. Fatigue truck moments at Section 1 always have positive sign and, hence, produce compressive stresses at the top surface of the section. Therefore, no need to check fatigue in the top reinforcement at Section 1.

Design of Reinforcement at the Top Corner

a. **Determination of required reinforcement at Section 3** (negative moment in top slab, f'_c = 28 MPa)

Maximum negative factored moment, M_u = η [0.9 DL + 1.75 LL + 1.35 EH + 1.75 LS]

0.95 x [0.9 x 5147 + 1.75 x (-2536) + 1.35 x (-20528) + 1.75 (-1424)] = -49 818 N·mm/mm

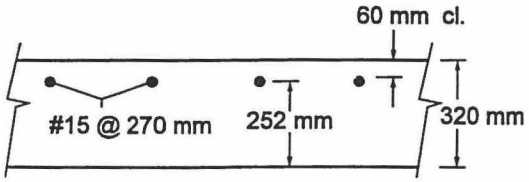
Concurrent axial force = 0.95 x [0.9 x 3 + 1.75 x (-2) + 1.35 x (-29) + 1.75 x (-14)] = -58 N/mm

Assuming #15 bars

$$d = 320 - 60 - \frac{16}{2} = 252 \ mm$$

By calculating the required area of reinforcement

Maximum allowed spacing of #15 bars = 270 mm



$$\rho = \frac{200}{270 \, x 252} = 0.0029 > \rho_{min} = 0.0021 \qquad \text{OK}$$

$$a = \frac{200 \, x 400}{270 \, x 0.85 \, x 28} = 12.45 \ mm$$

$$c = \frac{a}{0.85} = 14.65 \ mm$$

$$\frac{c}{d} = \frac{14.65}{252} = 0.06 < 0.42 \qquad \text{OK}$$

Minimum required reinforcement = #15 @ 270 mm (1)

b. **Determination of required reinforcement at Section 4** (negative moment near the top of wall)

f'_c = 24 MPa

f_r = $0.63\sqrt{f'_c} = 3.09 \ MPa$

ρ_{min} = $0.03 \frac{f'_c}{f_y} = 0.0018$

Maximum negative factored moment, M_u =
η [1.25 DL + 1.5 FWS + 1.75 LL + 1.35 EH + 1.75 LS]

= 0.95 x [1.25 x (-3360) + 1.5 x (-750) + 1.75 x (-21 073) + 1.35 x (-7609) + 1.75 x (-13 177)] = -71 760 N·mm/mm

Concurrent axial force = 0.95 [1.25 x (-21) + 1.5 x (-3) + 1.75 x (-46) +0 + 1.75 x (-5)] = -114 N/mm

Due to the relatively larger axial force in the walls, the effect of the axial force at the strength limit state will be considered.

Resistance factor for sections subjected mainly to axial compression = 0.75 (S5.5.4.2.1)

Resistance factor for sections subjected mainly to moment = 0.9 (S5.5.4.2.1)

For sections subjected to moment and axial compression simultaneously, the resistance factor ϕ decreases linearly from 0.9 to 0.75 as the axial load resistance ϕ P_n increases from 0 to 0.1 f'_c A_g. (S5.5.4.2.1)

0.1 f'_c A_g = 0.1 x 24 x 260 = 624 N

At Section 4:

P_u = 114 N = ϕ P_n < 0.1 f'_c A_g

$$\phi = 0.9 - \frac{P_u}{0.1\, f'_c A_g} x (0.9-0.75)$$

$$\phi = 0.9 - \frac{114}{624} x (0.9-0.75) = 0.872$$

Assuming #15 @ 210 mm

$$d = 260 - 50 - \frac{16}{2} = 202\ mm$$

$$T = \frac{200 x 400}{210} = 381\ N/mm$$

$$C = T + \frac{P_u}{\phi} = 381 + \frac{114}{0.872} = 514\ N/mm$$

$$a = \frac{C}{0.85\, f_c'} = \frac{514}{0.85\, x24} = 25.2\ mm$$

$$M_n = T\left(d - \frac{a}{2}\right) + \frac{P_u}{\phi}\left(\frac{d}{2} - \frac{a}{2}\right)$$

$$M_n = 381\left(202 - \frac{25.2}{2}\right) + \frac{114}{0.872}\left(\frac{202}{2} - \frac{25.2}{2}\right)$$
$$= 83\ 720\ N{\cdot}mm/mm$$

$$\phi M_n = 0.872\, x 83\ 720 = 73\ 000\ N{\cdot}mm/mm$$
$$\phi M_n > M_u = 71\ 760\ N{\cdot}mm/mm \qquad OK$$

$$\rho_{Provided} = \frac{200}{210\, x202} = 0.0047 > \rho_{min} = 0.0018 \qquad \text{OK}$$

$$c = \frac{a}{0.85} = \frac{25.2}{0.85} = 29.65\ mm$$

$$\frac{c}{d} = \frac{29.65}{202} = 0.15 < 0.42 \qquad \text{OK}$$

Minimum required reinforcement at Section 4 = #15 @ 210 mm. (2)

Maximum allowed spacing of #15 bars to satisfy strength requirements for both the top slab and the walls near the top corner = the smaller of (1) and (2) = 210 mm

Check for Crack Control Under Service Load near the Top Corner

a. At Section 3

Service load moment = DL + LL + EH + LS

= 5147 - 2536 - 20 528 - 14 241

= -32 158 N·mm/mm

Concurrent axial force = 16 + 44 + 0 + 5 = 65 N/mm

Assuming 10 mm of wear took place at the top surface, tensile stress in the concrete assuming uncracked section =

$$\frac{65}{310} + \frac{32\ 158}{(310)^2/6} = 2.22\ MPa < 0.8 f_r = 2.67\ MPa$$

No need to check for crack control under service load. (S5.7.3.4)

b. At Section 4

Maximum service load moment = DL + FWS + LL + EH + LS

= -45 970 N·mm/mm

Concurrent axial load = -75 N/mm

Tensile stress in the concrete assuming uncracked section =

$$\frac{-75}{260} + \frac{45\ 970}{(260)^2/6} = 3.79\ MPa > 0.8 f_{cr} = 2.67\ MPa$$

Crack control check is required (S5.7.3.4)

Clear concrete cover = 50 mm

d_c = 50 mm cover + 8 mm bar diameter = 58 mm

For buried structures Z = 17 500 N/mm (S5.7.3.4)

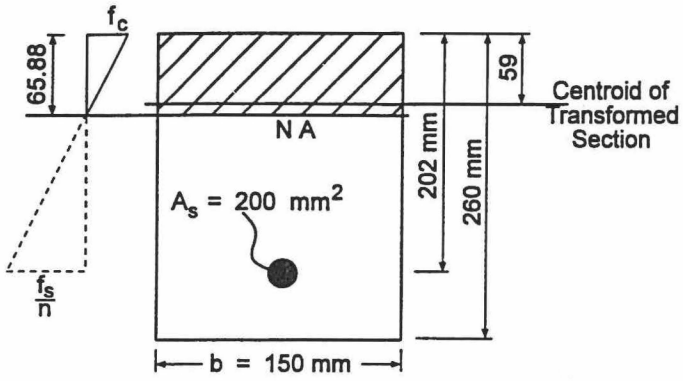
$$f_{sa} = \frac{Z}{(d_c A)^{1/3}} = \frac{17\ 500}{(58 \times 58 \times 210 \times 2)^{1/3}} = 156.0\ MPa$$

For concrete with f'_c = 24 MPa, the modular ratio n = 9

x = 57.56 MPa

f_s = 232.8 MPa > f_{sa}

Reinforcement spacing needs to be reduced



Reduce reinforcement spacing to 150 mm

x = 65.88 mm

f_{sa} = 174.5 MPa

f_s = 171.2 MPa < f_{sa} OK

Use #15 @ 150 mm

Check of Fatigue in Reinforcement Bars

a. At Section 3

Maximum moment due to unfactored permanent load
= 5147 +298 -20 528 = -15 083 N·mm/mm

Concurrent axial force = 3 +0 -29 = -26 N/mm

Maximum negative moment produced by the fatigue truck
= -1041 N·mm

Concurrent axial force = -1.0 N/mm

Maximum positive moment produced by the fatigue truck (from Table 2)

= 15 770 N·mm/mm

Concurrent axial force = 1 N/mm

The unfactored permanent loads produce tension at the top surface of the slab, therefore, check of fatigue in the negative moment reinforcement at this section is required.

Maximum negative moment due to permanent load and 1.5 fatigue truck = -15 083 -1.5 x 1041 = -16 644 N·mm/mm

Concurrent axial force = -26 -1.5 x 1.0 = -27.5 N·mm/mm

Maximum tensile stress in the concrete under permanent load and 1.5 the fatigue truck =

$$\frac{-27.5}{310} + \frac{16\ 644}{(310)^2/6} = 0.95\ MPa \quad = 1.15\ \text{MPa} < 0.25\sqrt{f'_c} = 1.32$$

Therefore, the section properties for fatigue check do not need to be based on the cracked section (S5.5.3.2).

Where the section properties are based on the uncracked section, the steel stresses will be equal to the stress in the adjacent concrete multiplied by the modular ratio. For simplicity, use the concrete gross section properties.

Maximum tensile stress in the steel due to 0.75 fatigue truck

$$= 0.75n\left(\frac{P}{A} + \frac{Mc}{I}\right)$$

$$= .75\,x8\left[\frac{-1\,x150}{310\,x150} + \frac{(1041\,x150)\,x(310/2 - 58)}{((310)^3\,x150)/12}\right]$$

= 0.22 MPa

Minimum stress in the steel due to 0.75 fatigue truck =

$$0.75\,x8\,x\left[\frac{1.0\,x150}{310\,x150} - \frac{(15\ 770\,x150)\,x(310/2 - 58)}{((310)^3\,x150)/12}\right] = -3.68\ MPa$$

Similarly, minimum stress in the steel due to 0.75 fatigue truck, plus dead load, f_{min} = 0.05 MPa

Stress range = 0.22 + 3.68 = 3.90 MPa

Allowable stress range = 145 - 0.33 f_{min} + 55 (r/h)

= 145 - 0.33 x (0.05) + 55 x 0.3

= 161.5 MPa > applied stress range = 3.90 MPa OK

In general, when the tensile stress from the permanent loads and 1.5 the fatigue truck is less than $0.25\sqrt{f'_c}$, the applied stress range will always be below the allowable stress range and, hence, there is no need to check fatigue in the reinforcement.

b. At Section 4

Moment due to unfactored permanent loads
= -3360 -750 -7609 = -11 719 N·mm/mm

Concurrent axial force = -20 -3 + 0 = -24 N/mm

Maximum negative moment produced by the fatigue truck
= -10 372 N·mm/mm

Concurrent axial force = -23 N/mm

No positive moment was produced by the fatigue truck.

Tensile stresses exist in the negative moment reinforcement under permanent loads. Therefore, check of fatigue in these bars is required (S5.5.3.1).

Maximum moment due to unfactored permanent load and 1.5 fatigue truck = -11 719 - 1.5 x 10 372

= 27 277 N·mm/mm

Concurrent axial force = -24 -1.5 x 23 = -58.5 N

Tensile stress in the concrete assuming uncracked section

$$= \frac{-58.5}{260} + \frac{27\ 277}{(260)^2/6} = 2.42\ MPa > 0.25\sqrt{f'_c} = 1.22\ MPa$$

Check for fatigue is required to be based on cracked section. (S5.5.3.1)

Maximum fatigue factored moment (including dead load)
= -11 719 -0.75 x (10 372) = -19 498 N·mm

Concurrent axial force = -24 -0.75 x (23) = -41 N/mm

X = 68.9 mm

I_{Trans} = 47.1 x 10^6 mm^4

f_s = 63.29 MPa

Similarly, minimum stress in the reinforcement, produced by the dead load alone, can be calculated as f_{min} = 38.30 MPa

Applied stress range = 63.29 - 38.30

= 24.99 MPa

f_f = $145 - 0.33 f_{min} + 55$ (r/h)

= 145 - 0.33 x 38.3 + 55 x 0.3 = 148.9 MPa > applied stress range OK

The neutral axis of the cracked section for fatigue load analysis was determined assuming the section is subjected only to the factored moment and axial force produced by the fatigue truck. Due to the small value of the axial force, the designer may choose to ignore the effect of the axial force, and the location of the neutral axis becomes a property of the section that is not dependant on the applied forces. On the other hand, when determining the location of the neutral axis, the designer may chose to consider the force effects of the permanent loads, in addition to the force effect of the fatigue truck. In all cases, the fatigue stress range should be determined using 0.75 the fatigue truck effects.

In addition to the above analysis, the reinforcement required to resist the positive moment at Sections 3 and 4 need to be checked for strength, service and fatigue limit states. In most cases, the reinforcement extending from the positive moment regions of the top slab and the wall will be sufficient to satisfy all requirements.

Positive Moment at Mid-Span of the Walls, Section 6

$M_u = \eta$ [0.9 DL + 1.35 EH + 1.75 LS]

M_u = 0.95 [0.9 x (-9165) + 1.35 x 29 888 + 1.75 x 9942]

M_u = 47 023 N·mm/mm

Concurrent axial force = 0.95 (0.9 x -33 + 0 +0) = -28 N/mm

Assuming #15 bars

Required reinforcement: #15 @ 310 mm

$$\rho = \frac{200}{310 x 202} = 0.0032 > \rho_{min} = 0.0018 \quad \text{OK}$$

a = 12.7 mm

c = a/0.85 = 14.9 mm

$\frac{c}{d} < 0.42$ OK

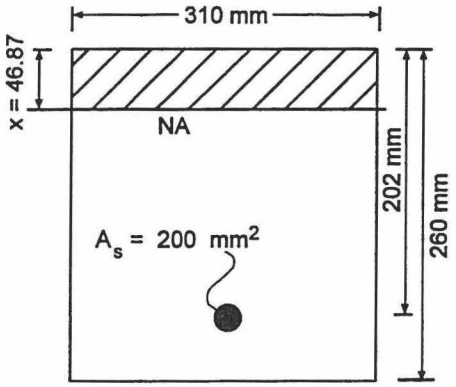
Service load moment = -9165 + 29 888 + 9942 = 30 665 N·mm/mm

Concurrent axial force = -33 N/mm

Tensile stress in the concrete assuming uncracked section =

$$\frac{-33}{260} + \frac{30\ 655}{(260)^2/6} = 2.59\ MPa > 0.8\,f_r = 2.47\ MPa$$

Therefore, check for cracking under service load is required.



x = 46.87 mm

f_s = 233.1 MPa

d_c = 58 mm

$$f_{sa} = \frac{17\ 500}{(58\,x2\,x58\,x310)^{1/3}} = 137.0\ MPa < f_s \quad N.G.$$

Reduce reinforcement spacing to 200 mm

x = 56.4 mm

f_s = 155 MPa

f_{sa} = 158.5 MPa > f_s OK

Check for negative moment at Section 6:

M_u = η[1.25 DL + 1.5 FWS + 1.75 LL + 0.5 EH]

= 0.95 x [1.25 x (-9165) + 1.5 x (-750) + 1.75 x (-18 069) + 0.5 x 29 888] =
- 27 800 N·mm/mm

Concurrent axial force = 0.95 x [1.25 x (-33) + 1.5 x (-3) + 1.75 x (-80) + 0] = -176 N/mm

Required reinforcement to satisfy strength requirements = #15 @ 1200 mm

$$\rho = \frac{200}{1200\,x202} = 0.0083 < \rho_{min} = 0.0018 \quad N.G.$$

Provide $\rho_{min} = 0.0018$

Minimum required reinforcement = #15 @ 550 mm

The actual reinforcement will be determined after the negative moment reinforcement at both the top and bottom corners are determined.

Negative moment reinforcement at the mid-point of the wall is usually provided by extending some of the negative moment reinforcement at the top and bottom corners to the center portions of the wall. The spacing of this reinforcement should satisfy the maximum spacing requirement of Article S5.10.3.2.

Determination of Required Reinforcement at the Bottom Corner

a. **At Section 8**
(negative moment at bottom of wall)

M_u = η x [1.25 DL + 1.5 FWS + 1.75 LL + 1.35 EH + 1.75 LS]

= 0.95 x [1.25 x (-14 970) + 1.5 x (-750) + 1.75 x (-23 716) + 1.35 x (-2636) + 1.75 x (-17 207)] = -90 260 N·mm/mm

Concrete axial force = 0.95 [1.25 x (-46) + 1.5 x (-3) + 1.75 x (-75) + 1.35 x (0) + 1.75 x 5] = -175 N/mm

$$\phi = 0.9 - \frac{P_u\,x\,(0.9-0.75)}{0.1\,Ag\,f'_c} = 0.9 - \frac{175\,x\,(0.9-0.75)}{0.1\,x260\,x24}$$
$$= 0.858$$

Minimum allowed reinforcement to satisfy strength requirement = #15 @ 160 mm (3)

$$\rho_{provided} = \frac{200}{160 \times 202} = 0.0062 > \rho_{min} = 0.0018 \qquad \text{OK}$$

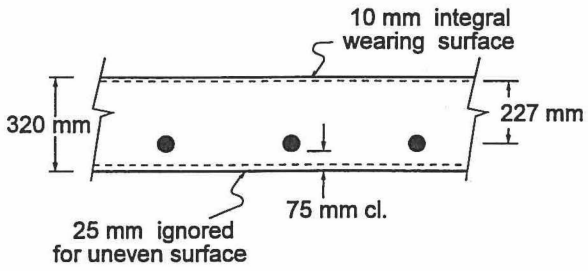
a = 24.5 mm

c = 28.8 mm

$\frac{c}{d} < 0.42$ OK

b. At Section 9
(negative moment at end of bottom slab)

Assume #15 bars



d = t - bottom cover - 1/2 bar diameter - 10 mm for wearing at top surface

= 320 - 75 - 8 - 10 = 227 mm

M_u = η x [0.9 DL + 1.35 EH + 1.75 LS]

= 0.95 [0.9 x 2134 + 1.35 x (-25624) + 1.75 x (-22476)] = 68 400 N·mm/mm

Concurrent axial force = 0.95 [0.9 x (-3) + 1.35 (-58) + 1.75 x (7)] = -65 N/mm

Ignore the bottom 25 mm in all calculations. This is done to account for the uneven bottom surface and for the low quality of the concrete directly in contact with the soil.

$t_{effective}$ = 320-10-25 = 285 mm

$0.1\ f'_c\ A_g$ = 0.1 x 24 x 285

= 684 N

$$\phi = 0.9 - \frac{P_u x (0.9-0.75)}{0.1 f_c' A_g}$$

$$= 0.886$$

Minimum allowed reinforcement to satisfy strength requirements = #15 @ 240 mm (4)

From (3) and (4), minimum allowed reinforcement at the bottom corner to satisfy strength requirements for both the wall and bottom slab is #15 @ 160 mm.

Check cracking under service load

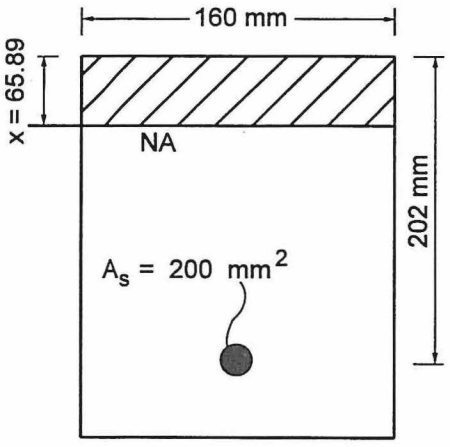
a. At Section 8

Service load moment = -14 970 -750 -23 716 -2636 -17 207 = -59 279 N·mm/mm

Concurrent axial force = -46 -3 -75 + 5 = -119 N/mm

Tensile stress in the concrete under service load assuming uncracked section = $\frac{-119}{260} + \frac{59\ 279}{(260)^2/6} = 4.8\ MPa > 0.8 f_r = 2.47$

Check for cracking under service load is required.



x = 65.89 mm

f_s = 228 MPa

f_{sa} = 171 MPa < f_s N.G.

Reduce bar spacing to 130 mm

$x = 71.4$ mm

$f_s = 189$ MPa

$f_{sa} = 183.0$ MPa $\simeq f_s$ OK

Required reinforcement for crack control at Section 8 is #15 @ 130 mm

b. At Section 9

Maximum service load negative moment
= 2134 - 25 624 - 22 476

= -45 966 N·mm/mm

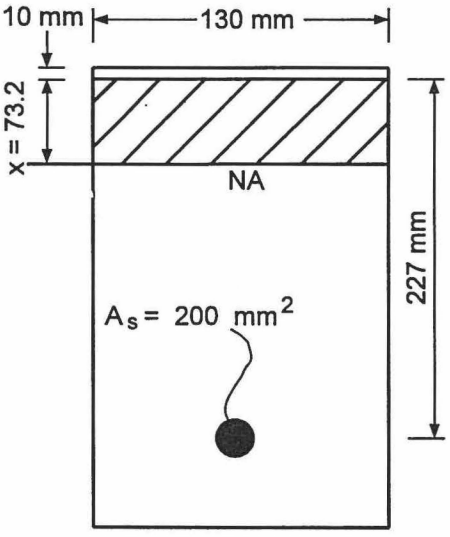
Concurrent axial force = -3 -58 + 7 = -54 N/mm

Tensile stress in concrete assuming uncracked section and ignoring 10 mm at the top and 25 mm at the bottom, as explained earlier =

$$\frac{-54}{285} + \frac{45966}{(285)^2/6} = 3.2\ MPa > 0.8 f_{cr} = 2.47\ MPa$$

Check for cracking is required.

Assuming reinforcement #15 @ 130 mm



$x = 73.2$ mm

f_s = 136 MPa

f_{sa} = 183 MPa OK

Therefore, #15 @ 130 mm satisfy both strength and cracking requirements for both the bottom slab and the side walls near the bottom corner.

In addition to the above analysis, positive moment reinforcement at both sections near the bottom corner need to be designed. The reinforcement extending from the positive moment regions of the wall and the bottom slab is usually sufficient to resist positive moments near the corner.

Check Positive Moment at Mid-Span of Bottom Slab, Section 11

Maximum factored positive moment,

M_u = η x [1.25 DL + 1.5 FWS + 1.75 LL + 0.5 EH]

= 0.95 x [1.25 x 40 854 + 1.5 x 2497 + 1.75 x 64 824 + 0.5 x (-25 624)]

= 148 000 N·mm/mm

Concurrent force = 0.95 x [1.25 x (-3) + 0 + 1.75 x (-5) + 0.5 x (-58)] = -40 N/mm

Due to the small value of the applied axial force relative to the applied moment, axial force can be ignored in the determination of the required reinforcement.

Ignore the bottom 25 mm for uneven bottom surface

Assume #20 bar

d = 320 - 60 mm top cover - 25 mm at the bottom - 1/2 bar diameter = 225.25 mm

Required reinforcement: #20 @ 140 mm

Check cracking under service load

Service load positive moment = 40 854 + 2497 + 64 824 - 25 624 = 82 551 N·mm/mm

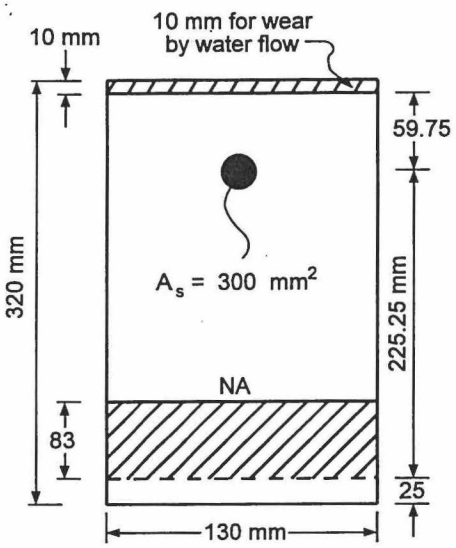
Concurrent axial force = -3 -5 -58 = -66 N/mm

Tensile stress in the concrete assuming uncracked section and ignoring the top 10 mm and bottom 25 mm =

$$\frac{-66}{285} + \frac{82551}{(285)^2/6} = 5.87\ MPa > 0.8\ f_r$$

Check for cracking under service load is required

Required reinforcement spacing = 130 mm



x = 83 mm

f_s = 172.9 MPa

f_{sa} = 179.4 MPa > f_s OK

Negative moment reinforcement at this section is still to be designed. The reinforcement bars extending from the ends of the slab are usually enough to satisfy the negative moment requirements at the center provided that minimum reinforcement requirements are satisfied.

In an actual design, the designer will need to design more sections, usually sections at the tenth points of each element. By knowing the required reinforcement at each section, some of the reinforcement bars can be terminated if they are not required to extend along the full-length of the member. The designer will also need to check the development length requirements for all terminated bars. According to the requirements of Article S5.5.11.2, the maximum bar spacing requirements (S5.10.3.2) need also be satisfied at all sections.

Design for Shear

Shear in Top Slab, Section 2

Maximum factored shear = η [1.25 DL + 1.5 FWS + 1.75 LL + 1.75 LS]

= 0.95 [1.25 x 13 + 1.5 x 2 + 1.75 x 80 + 1.75 x 5] = 160 N/mm

For culverts under less than 600 mm of fill, apply the provisions of Article S5.8.3.3. (S5.14.5.3)

Nominal shear strength of reinforced concrete elements without shear reinforcement:

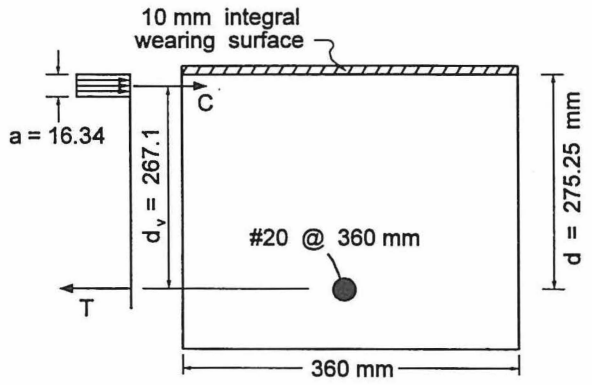
$V_n = 0.083 \beta \sqrt{f'_c}\, b_v d_v$ (S5.8.3.3)

β = 2 for reinforced concrete elements

b_v = 1.0 for 1.0 mm wide strip

d_v = larger of: d - a/2, 0.72 h or 0.9d (S5.8.2.7)

Assuming half the bottom reinforcement is terminated before this section, the reinforcement at the design section for shear is #20 @ 360 mm.



$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{300 x 400}{0.85 x 24 x 360} = 16.34 \ mm$$

$$d - \frac{a}{2} = 275.25 - \frac{16.34}{2} = 267.1 \ mm \quad (5)$$

0.9d = 0.9 x 275.25 = 247.7 mm (6)

$0.72h$ = $0.72 \times 310 = 223.2$ mm (7)

d_v = largest of (5), (6) and (7) = 267.1 mm

V_n = $0.083 \times 2 \times \sqrt{28} \times 267.1 = 234.6$ N

ϕ = 0.9 for shear (S5.5.4.2)

ϕV_n = $0.9 \times 234.6 = 211.1$ N mm $> V_u$ OK

Note: If the concrete did not provide enough shear capacity, either increase the thickness or provide shear reinforcement.

Shear in Walls

Shear needs to be checked at the design section for shear near both the top and bottom of the walls (Sections 5 and 7 of Figure 8, respectively).

Most of the shear forces in culvert walls are due to the applied horizontal earth pressure. Culverts on grade have a smaller horizontal earth pressure acting on their walls and, hence, the shear capacity of the concrete section is usually sufficient to resist the applied force. For the purpose of this example, shear in walls is not checked. For the analysis to be complete, the designer needs to conduct this check.

Shear in Bottom Slab, Section 10

Maximum factored shear,

V_u = η[1.25 DL + 1.5 FWS + 1.75 LL + 1.75 LS]

= 0.95 x [1.25 x (-34) + 1.5 x (-2) +1.75 + (-54) + 1.75 x (-9)]

= -148 N

Assume half the top reinforcement near mid-span of the bottom slab is extended to this section,

A_s = #20 @ 260 mm

d = 225.25 mm

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{300 \times 400}{0.85 \times 24 \times 260} = 22.6 \text{ mm}$$

d_v = largest of

d - a/2 = 214.0 mm

0.9d = 202.73 mm

or 0.72H = 162.2 mm

d_v = 214 mm

$\phi V_c = \phi \times 0.083 \times \beta\sqrt{f'_c}\, b_v\, d_v$

$= 0.9 \times 0.083 \times 2\sqrt{24} \times 1 \times 214$

= 156.6 N

$\phi V_c > V_u$ OK

Design of Longitudinal Reinforcement

Shrinkage and temperature reinforcement are required along the inside face of walls and slabs of cast-in-place box culverts (S12.11.4.3.1). For boxes on grade, the top surface of top slab also needs to be protected. The reinforcement perpendicular to the direction of the traffic at the bottom of the top slab for culverts under less than 600 mm of fill need to be designed as distribution reinforcement (S5.14.4.1).

The shrinkage and temperature reinforcement, A_s, should satisfy: $A_s \geq 0.75\, A_g/f_y$. This reinforcement needs to be distributed equally between the two surfaces of the section (S5.10.8.1). For solid walls, the required reinforcement need not to exceed $0.0015\, A_g$ (S5.10.8.1).

For f_y = 400 MPa, required reinforcement ratio

$$\frac{0.75}{400} = 0.0018 > 0.0015$$

Use reinforcement ratio = 0.0015

Shrinkage and Temperature Reinforcement in Top Slab

Total required reinforcement = $0.0015\, A_g$

$= 0.0015 \times 320 = 0.48\ mm^2/mm$

Required reinforcement at each surface
$= 0.5 \times 0.48 = 0.24\ mm^2/mm$

a. At top surface

Use #10 @ 300 mm

Provided reinforcement = 100/300 = 0.33 mm^2/mm

> 0.24 mm^2/mm required OK

b. At bottom surface

Provided distribution reinforcement = #10 @ 210 mm

A_s provided = 100/210 ≐ 0.48 mm^2/mm > 0.24 mm^2/mm required OK

Similarly, #10 @ 300 mm will satisfy the shrinkage and temperature reinforcement requirements along the inner face of the walls and top surface of bottom slab.

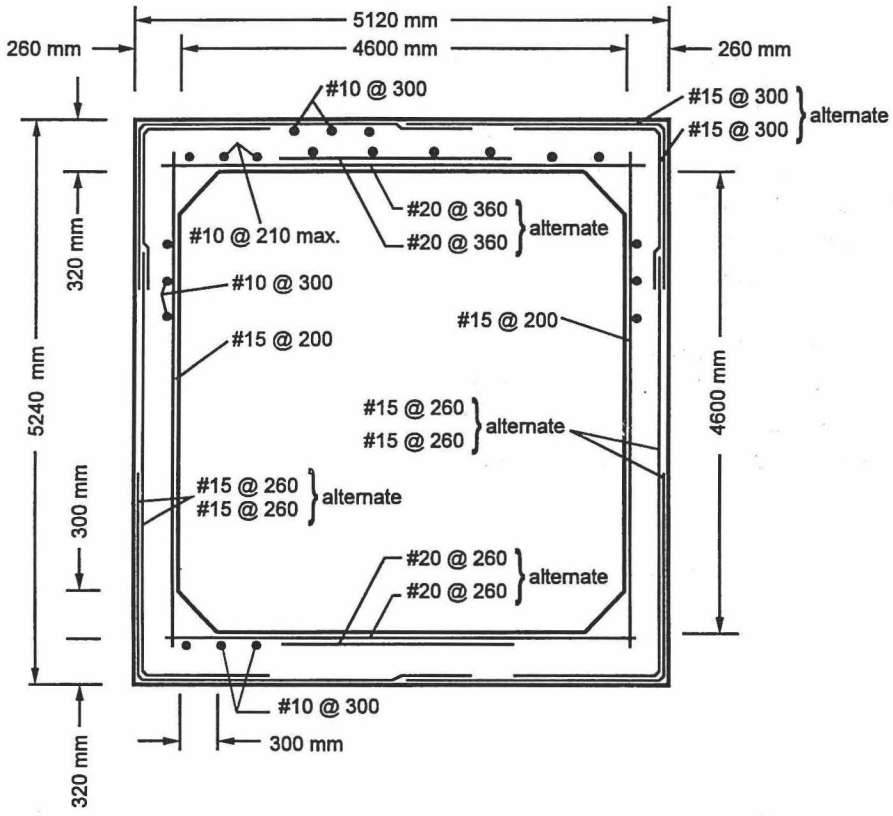


Figure 9 - Culvert Reinforcement

Example 2: Loads on Box Culverts Below Grade

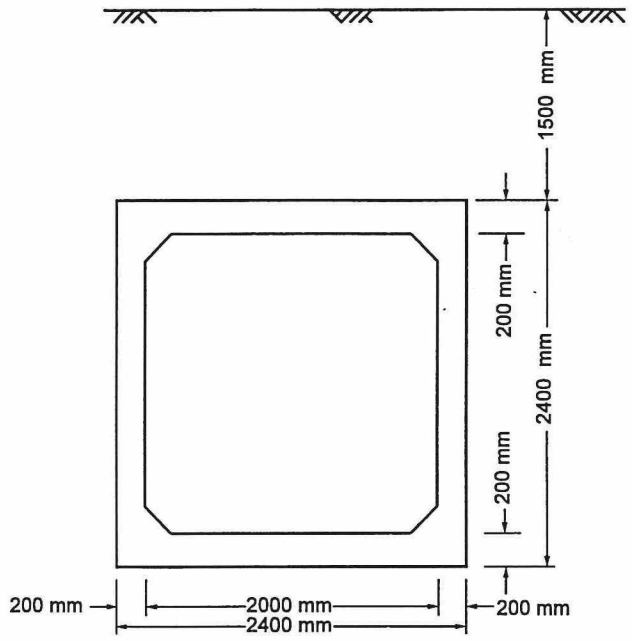


Figure 1 - Box Culvert Geometry

Fill depth above top slab = 1500 mm

1. Permanent Loads

Loads from self-weight of the culvert and weight of future wearing surface can be calculated using the procedures utilized for culverts on grade (see Example 1). The calculations of the vertical load of soil on the top slab of culvert are presented below. The effect of the self-weight, future wearing surface and weight of earth fill should be analyzed as three separate cases of loading because each of the three loads is multiplied by a different load factor (see Table S3.4.1-2). Also, to maximize the force effect at a section, the designer may need to ignore the effect of the future wearing surface, if its effect has the opposite sign to the force effect being maximized.

Vertical load of soil on top slab of culvert:

Assume that the size of excavation is relatively larger than the width of culvert, i.e., embankment installation conditions (S12.11.2.1).

Total vertical soil load on 1.0 mm wide strip of top slab,

$W_E = gf_e \gamma_s B_c H \times 10^{-9}$ (S12.10.2.1)

g = acceleration of gravity = 9.81 m/sec^2

γ_s = 1600 kg/m^3

B_c = outside dimension of culvert = 2400 mm

H = depth of backfill = 1500 mm

f_e = soil-structure interaction factor for embankment installation = $1 + 0.2\ H/B_c = 1.125$ (S12.10.2.1.1)

W_E = $9.81 \times 1.125 \times 1600 \times 2400 \times 1500 \times 10^{-9}$ = 63.6 N/mm

Intensity of uniform load on top slab of design strip =

$$\frac{W_E}{B_c} = 0.0265\ N/mm/mm$$

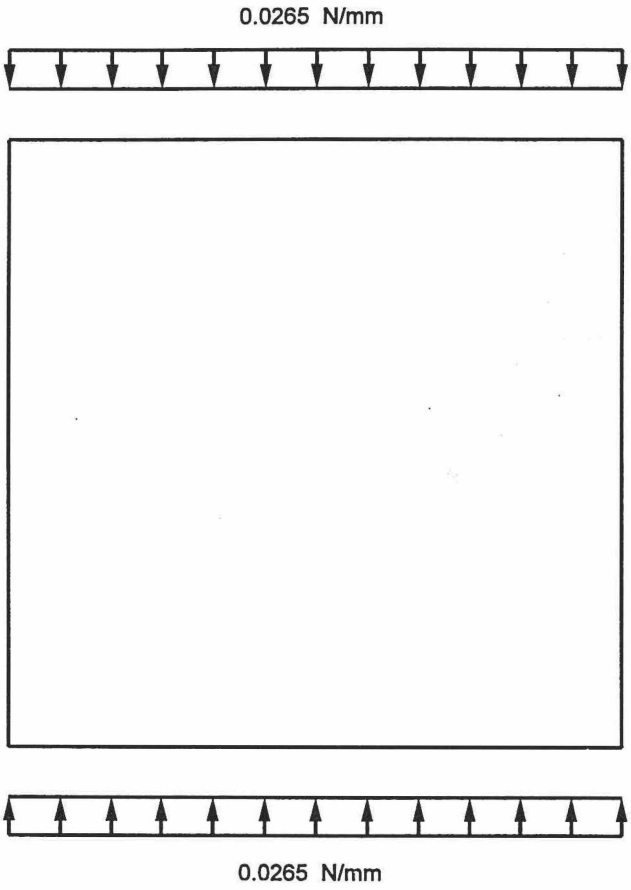


Figure 2 - Dead Load of Soil above Culvert

Load factors:

	Maximum	Minimum
weight of structures	1.25	0.9
weight of future wearing surface	1.5	0.65
weight of soil	1.3	0.9

Basic Lateral Earth Pressure, EH, and Earth Surcharge, ES:

Assume same coefficient of at-rest soil pressure as calculated for Example 1, $k_o = 0.426$

$EH = k_o \gamma_s g z (10^{-9})$

$= 0.426 \times 1600 \times 9.81 \times 2200 \times 10^{-9} = 14.7 \times 10^{-3}$ MPa

$ES = k_o \gamma_s g H_1 (10^{-9})$

$= 0.426 \times 1600 \times 9.81 \times 1600 \times 10^{-9} = 10.7 \times 10^{-3}$ MPa

Maximum load factor for EH and ES is 1.35 and 1.5, respectively (Table S3.4.1-2). Minimum load factor is 0.9 for both loads. If the effect of the horizontal earth pressure reduces the force effect being maximized, a load factor of 0.5 should be used (S3.11.7).

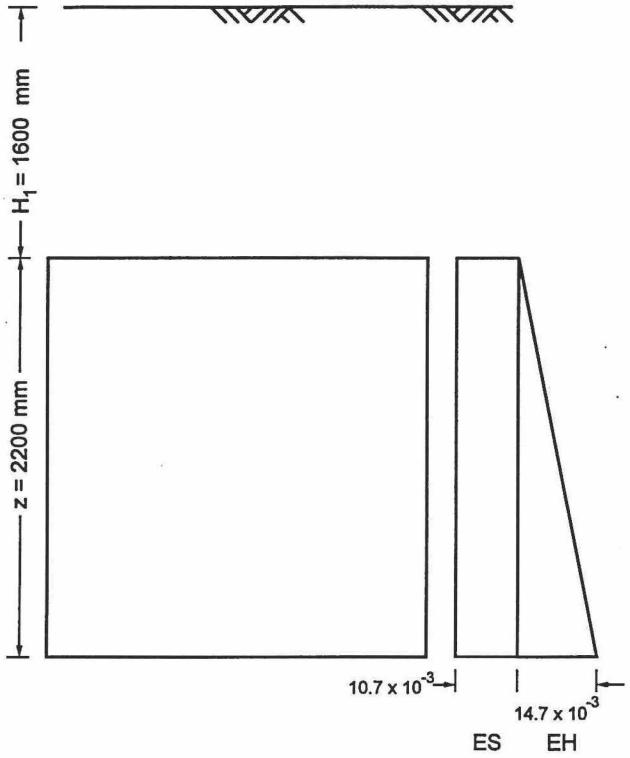


Figure 3 - Earth Pressure and Earth Surcharge

Live Load Surcharge

Height from bottom of bottom slab to top of fill =
200 + 2000 + 200 + 1500 = 3900 mm

From Table S3.11.6.2-1

h_{eq} = 1200 mm For total height of fill = 3000 mm

h_{eq} = 760 mm For total height of fill = 6000 mm

By interpolation for total height of fill = 3900 mm

h_{eq} = 1068 mm

$\Delta p = k_o \gamma_s g h_{eq} \times 10^{-9}$

$= 0.426 \times 1600 \times 9.81 \times 1068 \times 10^{-9} = 7.14 \times 10^{-3}$ MPa

Live load surcharge is uniformly distributed along the full-height of exterior walls. (S3.11.6.2)

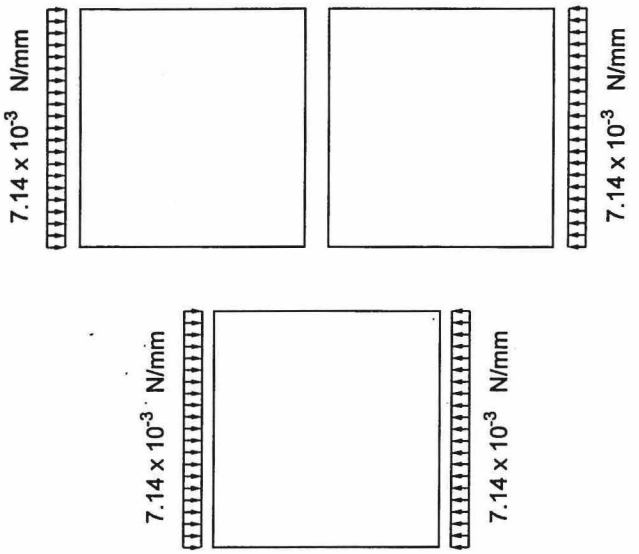


Figure 4 - Live Load Surcharge

Load factor for live load surcharge at the Strength I load combination is 1.75.

Live Loads

Wheel loads are assumed to be uniformly distributed over a rectangular area with sides equal to the tire contact area, assuming 40% dynamic load allowance, at the surface. The sides of the rectangle are increased by 1.15 times the depth of select granular fill or the depth of fill in all other cases. (S3.6.1.2.6)

Assume select granular fill

Tire contact area: (S3.6.1.2.5)

width	=	510 mm
length, ℓ	=	2.28 γ (1 + IM/100) P
γ	=	load factor = 1.75
IM	=	40% for determining tire contact area for culverts (SC3.6.1.2.6)
P	=	72.5 kN for design truck 55.0 kN for design tandem
ℓ	=	2.28 x 1.75 x (1 + 40/100)P

= 405 mm for design truck

= 307 mm for design tandem

Dynamic load allowance for culverts:

IM = $40 (1-4.1 \times 10^{-4} D_E) \geq 0$ (S3.6.2.2)

D_E = depth of earth above structure = 1500 mm

IM = $40 (1 - 4.1 \times 10^{-4} \times 1500) = 15.4\%$

Distribution of Design Truck Wheel Loads

Distribution width per wheel = width of tire contact area + 1.15 depth of select granular fill = 510 + 1.15 x 1500 = 2235 mm

Distribution length per wheel = length of tire contact area + 1.15 depth of select granular fill = 405 + 1.15 x 1500 = 2130 mm

The distribution width for a wheel is larger than the distance between the centers of the two wheels in the same axle of the truck (1800 mm). Therefore, the distribution areas overlap (see Figure 4a) and the total load from the two wheels is assumed to be distributed uniformly over the area within the outer boundaries of the overlapped areas. (S3.6.1.2.6)

Distribution width of the axle = 1800 + 2235 = 4035 mm

If two trucks are positioned side-by-side, the distribution width for their axles also overlap. The total load from the two axles is then distributed over the area within the boundaries of the two axles (see Figure 4b).

Assume the distance between the adjacent wheels from the two trucks = 1200 mm

Distribution width for both axles = 4035 + 3000 = 7035 mm

Multiple presence factor, MPF = 1.2 for single truck
= 1.0 for two trucks

Load intensity on the culvert

$$= \frac{Total\ Load}{Distribution\ Area} x (1 + IM) x MPF$$

For single truck =

$$\frac{145000}{4035\,x2130}\,x(1+0.154)\,x1.2 = 23.4\,x10^{-3}\ MPa \qquad (1)$$

For two trucks = $\frac{2\,x145000}{7035\,x2130}\,x(1+0.154)\,x1.0 = 22.3\,x10^{-3}\ MPa$

(2)

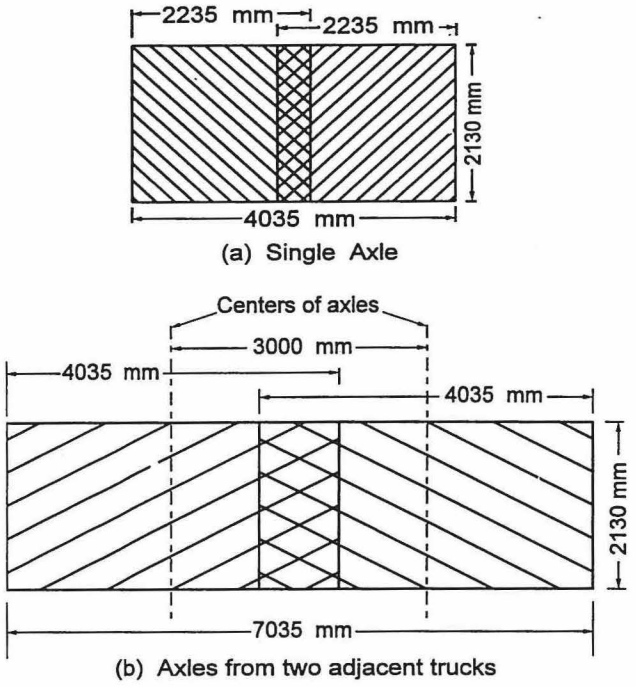


Figure 5 - Distribution Area for Truck Axles

The distribution width for the lane load may be assumed constant and equal the width of the lane load at the roadway level (3000 mm). The effect of the lane load on culverts is relatively small and, for culvert under fill, the load intensity on the culvert top is not uniform. The load intensity near the center of the lane load is usually higher. Assuming a constant distribution width for the lane loads will make the analysis easier without any significant effect on the results.

Intensity of lane load:

$$\text{Single lane loaded} = \frac{9.3}{3000}\,x1.2 = 3.7\,x10^{-3}\ MPa \qquad (3)$$

$$\text{Two lanes loaded} = \frac{9.3}{3000}\,x1.0 = 3.1\,x10^{-3}\ MPa \qquad (4)$$

From (1), (2), (3) and (4), case of a single lane loaded is more critical. The load from the design truck may be positioned to produce maximum effect at the section being designed. The lane load should be applied on the regions of the top slab, such as to maximize the load effect. The loads on the top slab of 1.0 mm wide design strip are shown in Figure 6.

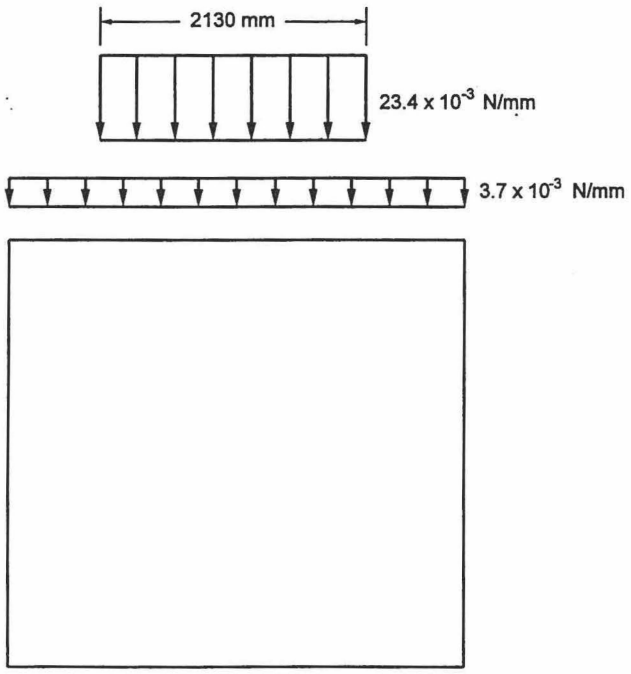


Figure 6 - Design Loads from the Design Truck and Design Lane Loads (One Lane Loaded)

Distribution of Design Tandem Wheel Loads

Distribution width per wheel =
510 + 1.15 x 1500 = 2235 mm

Distribution length per wheel =
307 + 1.15 x 1500 = 2032 mm

Case of single lane loaded:

The area from the four wheels in the design tandem overlap. Therefore, the total load is distributed over the total area within the boundaries of the four wheel distribution areas.

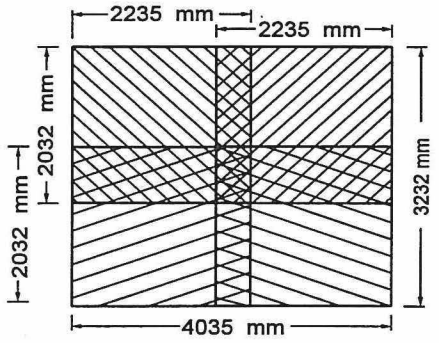
Total distribution width = Distance between centers of wheels in the same axle + width of wheel distribution area =
1800 + 2235 = 4035 mm

Total distribution length = Distance between centers of wheels in the two axles of the design tandem + length of wheel distribution area = 1200 + 2032 = 3232 mm

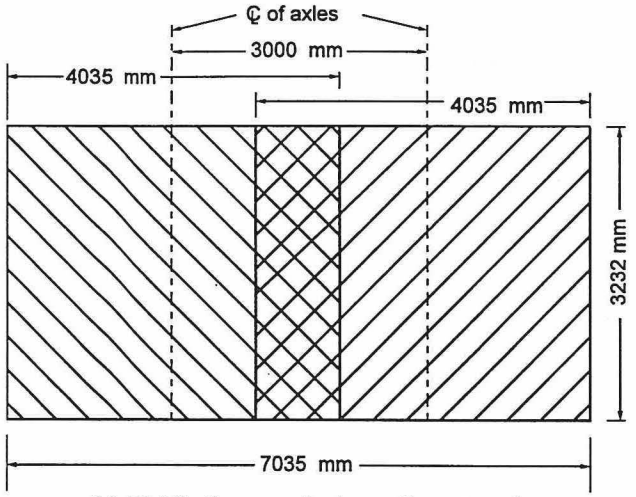
Case of two lanes loads:

Total distribution length = 3000 + 4035 = 7035 mm

Total distribution length = 3232 mm



(a) Distribution area for one tandem



(b) Distribution area for two adjacent tandems

Figure 7 - Distribution Area for Tandem Axles

Load intensity on top of culvert:

Single lane loads =

$$\frac{2x110\ 000}{4035\,x3232}\,x(1+0.154)\,x1.2 = 23.4\,x10^{-3}\ MPa$$

Two lanes loaded =

$$\frac{2 x 2 x 110\ 000}{7035 x 3232} x (1 + 0.154) x 1.0 = 22.3 x 10^{-3}\ MPa$$

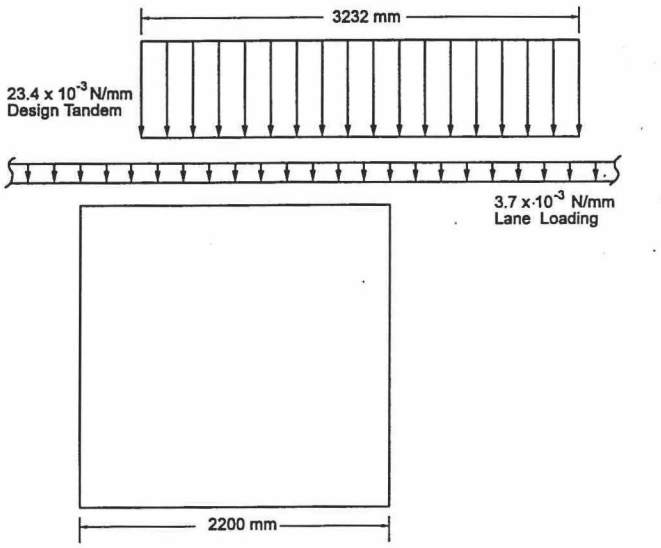


Figure 8 - Design Loads from the Design Tandem and Design Lane Loads (One Lane Loaded)

The effect of the lane load is the same as calculated for the case of the design trucks.

Therefore, the case of single lane loaded controls. The loads on the top slab of a 1.0 mm wide design strip are shown in Figure 8. As shown, the distribution length of the tandem wheel is larger than the total width of the culvert. This load can be moved to produce maximum load effect, but it cannot be broken such that the length of the load is less than 3232 mm. This is of greater significance in multi-vent culverts where some of the load will be applied to regions that produces force effects with the opposite sign to the force effect being maximized. Lane load can be broken into patches of load as required to maximize the force effect being maximized.

After the loads have been determined, the analysis and design of the culvert may be conducted similar to those presented for Example 1.

LECTURE 16 - OVERVIEW OF RAILINGS, JOINTS AND BEARINGS

16.1 OBJECTIVE OF THE LESSON

The objective of this lesson is to summarize the design provisions for railing systems, deck joints and bridge bearings. Design examples for both concrete and post-type railings are included to familiarize the student with the analytical design procedures required for design for vehicular collision.

16.2 OVERVIEW OF RAILING SYSTEMS

16.2.1 Traffic Railing

16.2.1.1 RAILING SYSTEMS REQUIREMENTS

Traffic railings are used on bridges constructed for the exclusive use of highways. The primary purpose of traffic railings shall be to contain and redirect vehicles using the structure. All new vehicle traffic barrier systems, traffic railings and combination railings must be shown to be structurally and geometrically crashworthy.

Consideration should be given to:

- satisfying applicable warrants
- protection of the occupants of a vehicle in collision with the railing,
- protection of other vehicles near the collision,
- protection of persons and property on roadways and other areas underneath the structure,
- possible future rail upgrading,
- railing cost effectiveness, and
- appearance and freedom of view from passing vehicles.

In high-speed rural areas, the approach end of a parapet or a railing is required to either have a crashworthy configuration or be shielded by a crashworthy traffic barrier. In addition, an approach railing system is required at the beginning of all bridge railings in high-speed rural areas. The bridge approach railing system should include a transition from the

guardrail system to the rigid bridge railing system which is capable of providing lateral resistance to an errant vehicle. The approach guardrail system shall have a crashworthy end terminal at its nosing.

Railings, their connection to the deck and the deck overhang, are to be designed to satisfy the requirements of:

- the strength limit state under all applicable load combinations, other than collision, per se, and; the requirements of the extreme event limit state under vehicular collision conditions.

16.2.1.2 PERFORMANCE LEVEL SELECTION CRITERIA

Three performance levels are specified by the Specification. It is intended that these levels correspond to three of the six severity levels indicated in the update of NCHRP Report 350. The choice of a performance level is a function of the bridge alignment, traffic volume and speed. The three performance levels and the recommended site conditions for the use of each of them are listed below.

- PL-1 - Performance Level One - Used for short, low level structures on rural highway systems, secondary expressways, and areas where a small number of heavy vehicles are expected and speeds are either posted or reduced.

- PL-2 - Performance Level Two - Used for high-speed main line structures on freeways, expressways, highways, and areas with a mixture of heavy vehicles and tolerable maximum speeds.

- PL-3 - Performance Level Three - Used for freeways with variable cross slopes, reduced radius of curvature, higher volume of mixed heavy vehicles and maximum tolerable speeds. Site-specific justification is required for use of this performance level.

A higher performance level may be specified for unusual conditions. A lesser performance level may be specified by the Owner if justified by specific location warrants. In fact, the purpose of requiring "site-specific justification" for PL-3 was to encourage consideration of other performance levels. Regardless of this, some jurisdictions are simply upgrading existing concrete railings to PL-3, or showing by crash test that existing standard railings are acceptable.

16.2.1.3 RAILING DESIGN

Due to the difficulty of analyzing the behavior of railing systems during vehicular collision, full-scale crash-testing is required to validate all new railing systems. The crash-test criteria depends on the required performance level. Table 16.2.1.3-1 lists the test requirements for each performance level.

where:

B = Width of Vehicle

G = Height of center of gravity from deck

θ = Collision angle

Table 16.2.1.3-1 - Bridge Railing Performance Levels and Crash-Test Criteria

PERFORMANCE LEVEL	TEST VEHICLE DESCRIPTIONS AND IMPACT ANGLES			
	Small Automobiles	Pickup Truck	Medium Single-Unit Truck	Van-Type Tractor-Trailers
	W = 8000 N	W = 24 000 N	W = 80 000 N	W = 220 000 N
	B = 1700 mm	B = 2000 mm	B = 2300 mm	B = 2450 mm
	G = 500 mm	G = 690 mm	G = 1240 mm	G = 1630 mm
	θ = 20°	θ = 20°	θ = 15°	θ = 15°
	TEST SPEEDS (km/hr)			
PL-1	70	70	N/A	N/A
PL-2	100	100	80	N/A
PL-3	100	100	N/A	80

Once a system is crash-tested it can be used without further analysis and/or testing, provided that the proposed installation does not have features which are not present in the tested configuration that might detract from the performance of the tested railing system.

When a minor detail is changed on, or an improvement is made to a railing system that has already been tested and approved, engineering judgment and analysis should be used when determining the need for additional crash- testing.

Due to the high cost of crash-testing, analytical procedures for the design of both concrete and metal parapets were presented in an Appendix. These procedures are intended to be used for preliminary design of test specimens. In case of slight modification of a crash-tested parapet, the analytical procedures can be used to estimate the change in parapet strength. In such a case, the modified parapets can be used without further crash-testing. The design forces to be used in the design of parapets and the distribution length of the force in each direction are given in Table 16.2.1.3-2. The given values were developed through crash-testing of numerous parapet systems. These values represent an update to the values given in the 1989 AASHTO Guide Specification for Bridge Railing.

The distribution length for collision forces varies from one performance level to another based on the test vehicle. For example, L_t, the contact length between the vehicle and the parapet, is equal to the diameter of a single wheel for PL-1 and PL-2 and the length of a tandem for PL-3.

Table 16.2.1.3-2 - Design Forces for Traffic Railings

Design Forces and Designations	Railing Performance Levels		
	PL-1	PL-2	PL-3
F_t Transverse (N)	120 000	240 000	516 000
F_L Longitudinal (N)	40 000	80 000	173 000
F_v Vertical (N) Down	20 000	80 000	224 000
L_t and L_L (mm)	1220	1070	2440
L_v (mm)	5500	5500	12 200
H_e (min) (mm)	510	810	1020
Minimum H Height of Rail (mm)	510	810	1020

The location and distribution of collision forces is shown in Figure 16.2.1.3-1. The following notations apply:

R1, R2: The resistance of the rail elements

R': Total rail resistance

Y': height of total rail resistance above riding surface

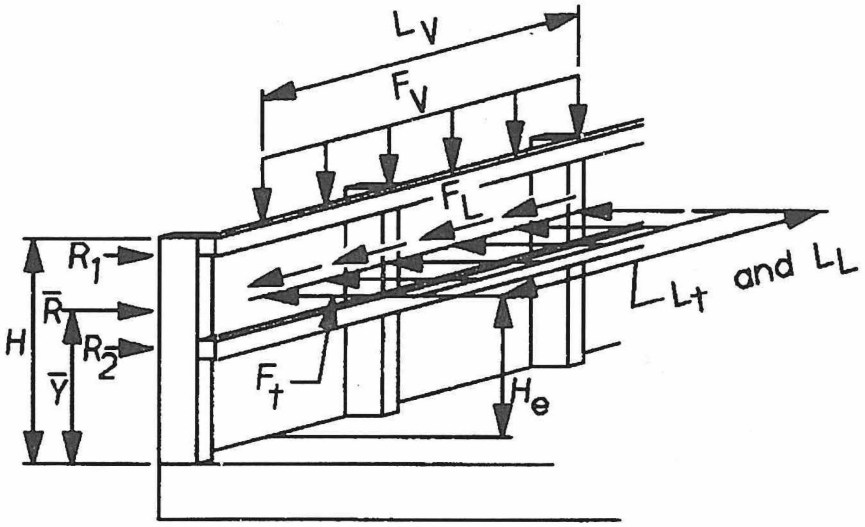


Figure 16.2.1.3-1 - Metal Bridge Railing Design Forces, Vertical Location and Horizontal Distribution Length

The following geometric requirements need to be satisfied for the railing geometry to be considered satisfactory.

- Geometric Requirements:

  $\Sigma A/H \geq 0.25$

  See Figure 16.2.1.3-2 for the definition of values for A and H for different types of railing systems.

- Setback requirements:

  Railing geometry should give a combination above or within the shaded area in Figure 16.2.1.3-3.

- Setback/clear distance requirements:

  Railing should give a combination of geometric variable which are within or below the shaded area in Figure 16.2.1.3-4. The definition of the setback, S, for each

type of railing system is illustrated in the Figures 16.2.1.3-2 and 16.2.1.3-4.

The following notation applies.

ΣA = Σ heights of railing elements

C = clear distance between rails

H = height of rail system

The goal of these requirements is to minimize the possibility of a contact between the vehicle and the posts of the post-type railing system during a collision.

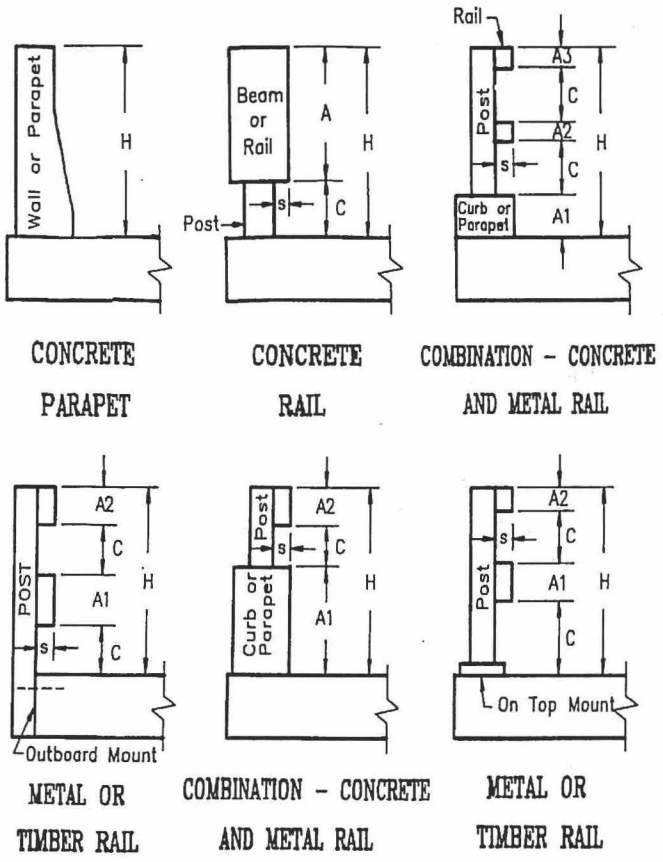


Figure 16.2.1.3-2 - Typical Traffic Railings

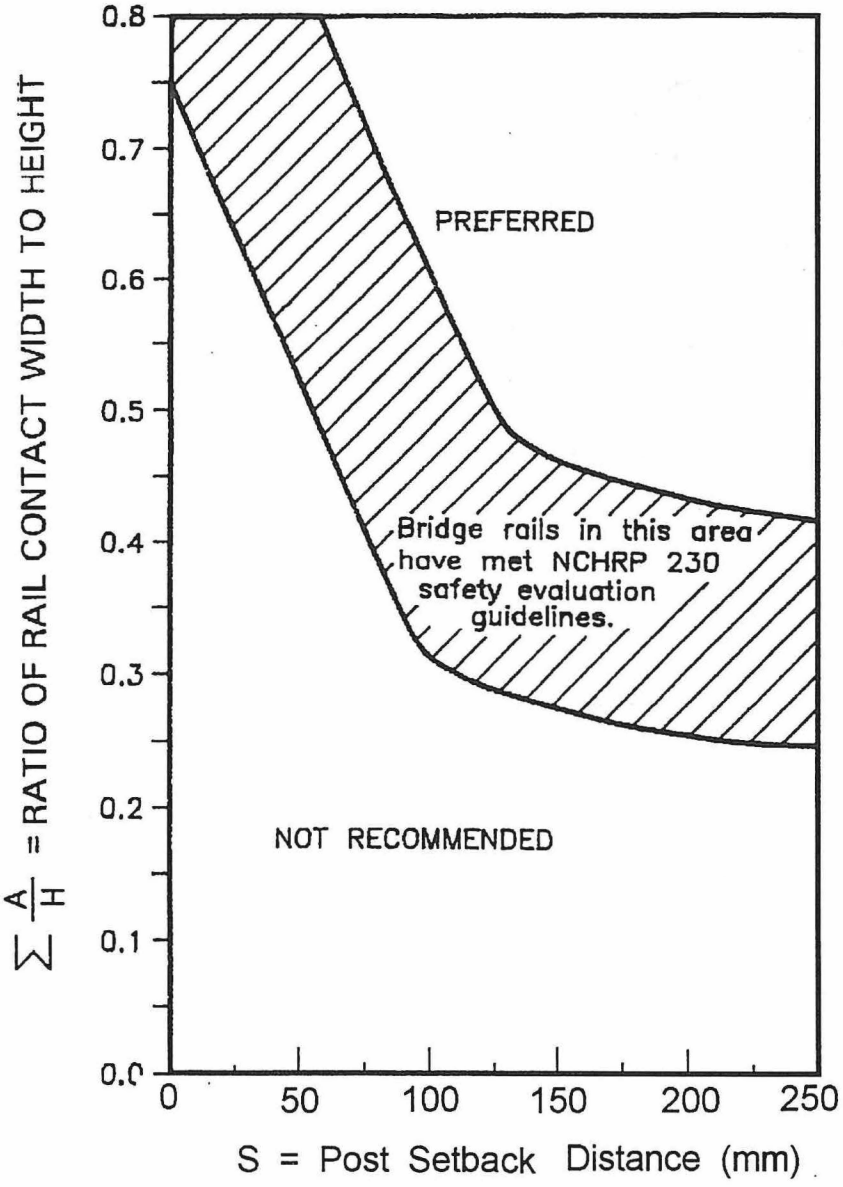


Figure 16.2.1.3-3 - Post Setback Criteria

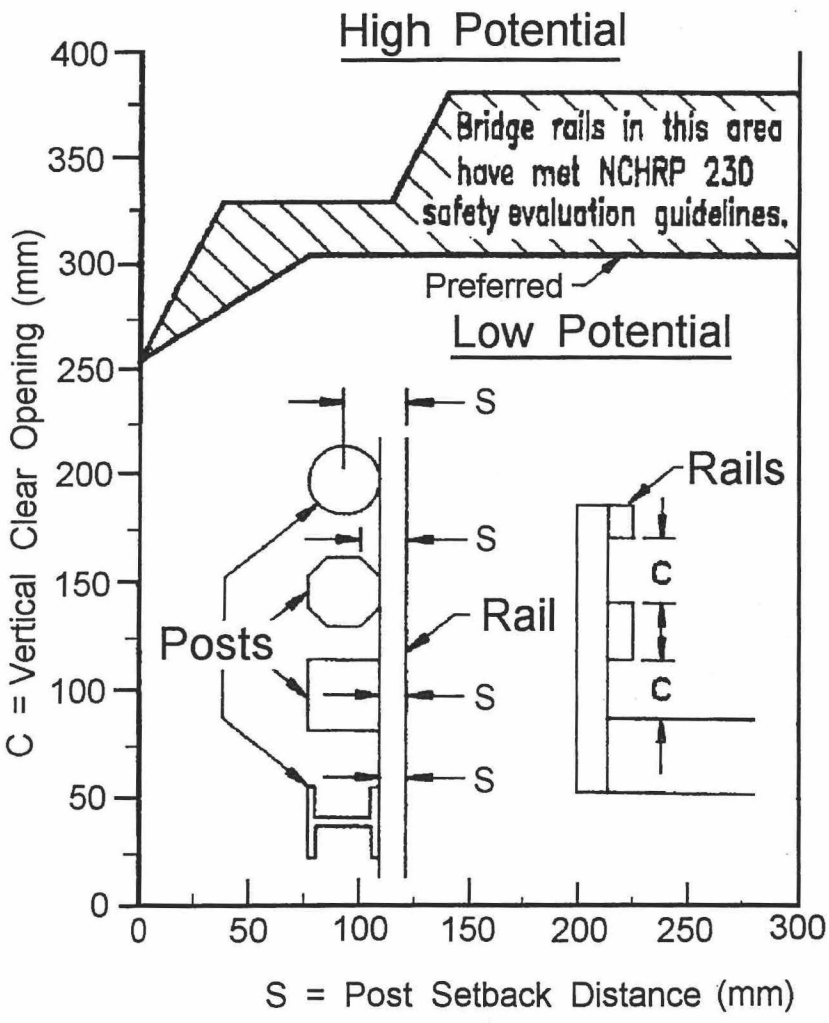


Figure 16.2.1.3-4 - Potential for wheel, bumper or hood impact with post

The theoretical design procedures for concrete railings is based on the yield line theory. Figures 16.2.1.3-5 and 16.2.1.3-6 show the assumed failure mechanism for collision within a parapet segment and near an expansion joint, respectively. Based on these failure mechanisms, the load capacity of the parapet decreases significantly near expansion joint locations. Hence, the number of the expansion joints in the parapet should be minimized and they should be located at expansion joints in the deck. This will also enhance performance of the deck overhang and decrease the deflections of the superstructure.

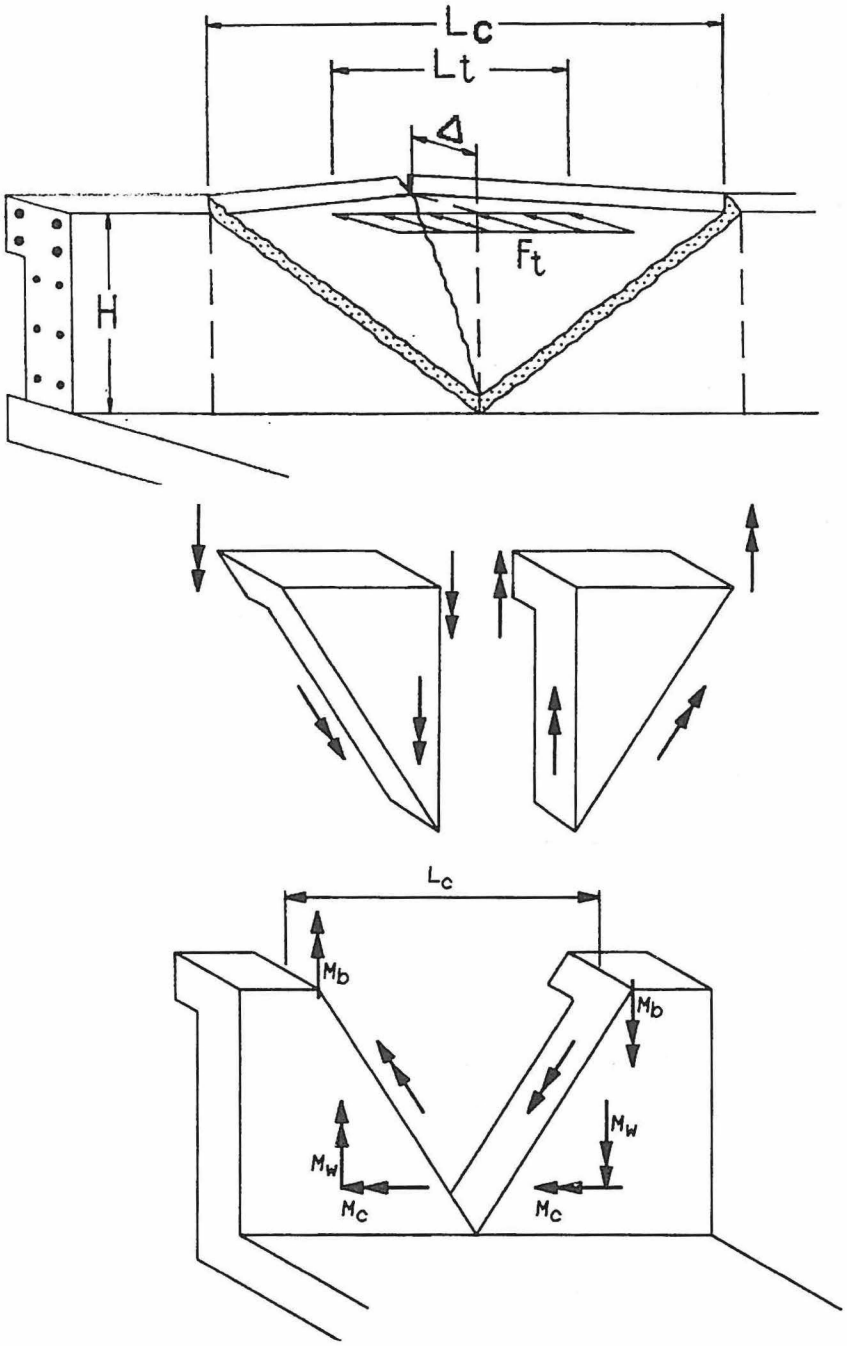


Figure 16.2.1.3-5 - Yield Line Analysis of Concrete Parapet Walls for Impact within Wall Segment

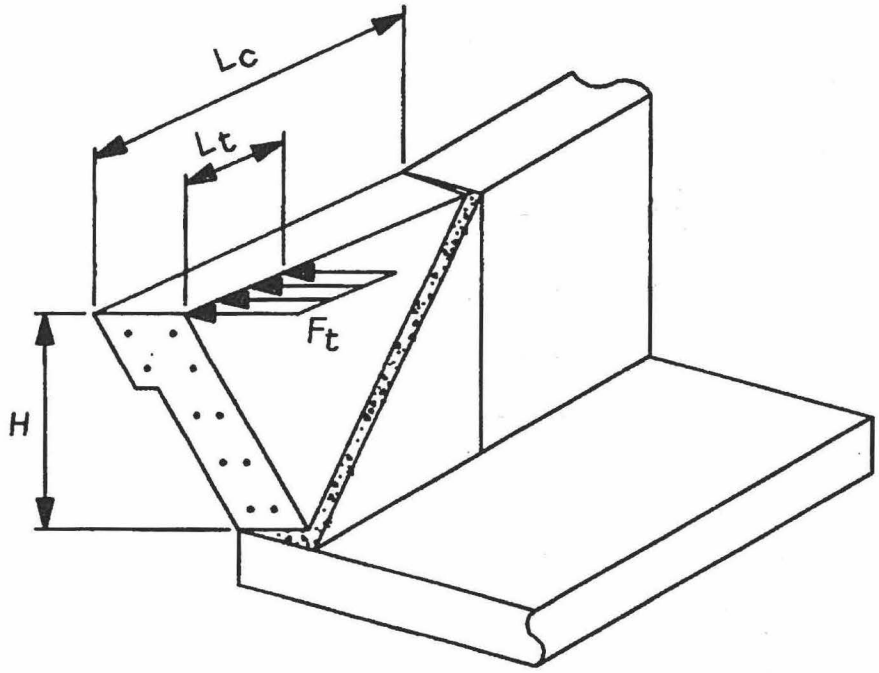


Figure 16.2.1.3-6 - Yield Line Analysis of Concrete Parapet Walls for Impact Near End of Wall Segment

The first step to determine the load capacity of a concrete parapet is to determine the moment of resistance of the cross-section. M_c, shown in Figure 16.2.1.3-5, is the moment of resistance about a horizontal axes parallel to the longitudinal axes of the concrete parapet. M_c is calculated per unit length of the parapet and, in case of parapets with sloped face, is taken as the average along the height of the parapet. The total moment of resistance about the vertical axes of the parapet cross-section is designated ($M_b + M_wH$). M_w is the moment of resistance of a unit-height strip of the parapet about the vertical axis. If the parapet has a reinforced top, the flexural resistance of the top zone in excess of M_w is designated M_b. Figure 16.2.1.3-7 shows the resisting moments for a parapet with vertical faces.

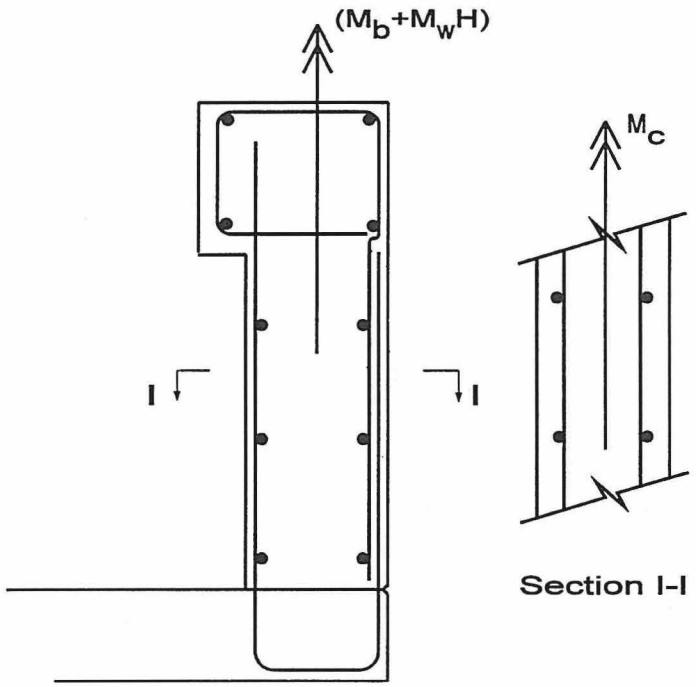
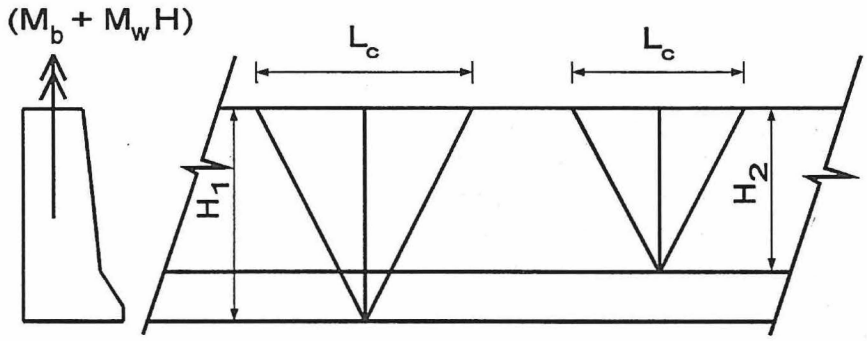


Figure 16.2.1.3-7 - Definition of M_c and $M_b + M_wH$

For a typical concrete parapet with sloped face, two failure mechanisms that are different in height may develop, depending on the dimensions and reinforcement of the parapet. Figure 16.2.1.3-8 shows the two mechanisms and the height of parapet, H, failure length, L_c, corresponding to each of the two mechanisms.



Failure Mechanism 1 Failure Mechanism 2

Figure 16.2.1.3-8 - Possible Failure Mechanism for Typical Concrete Parapets

The length of failure mechanisms can be determined as follows with the appropriate substitution of H_1 or H_2 for H:

- For collision within a segment:

$$L_c = \frac{L_t}{2} + \sqrt{\left(\frac{L_t}{2}\right)^2 + \frac{8H(M_b + M_w H)}{M_c}} \qquad (16.2.1.3\text{-}1)$$

- For collision near an expansion joint:

$$L_c = \frac{L_t}{2} + \sqrt{\left(\frac{L_t}{2}\right)^2 + H\left(\frac{M_b + M_w H}{M_c}\right)} \qquad (16.2.1.3\text{-}2)$$

The load capacity of the parapet, R_w, can then be calculated as:

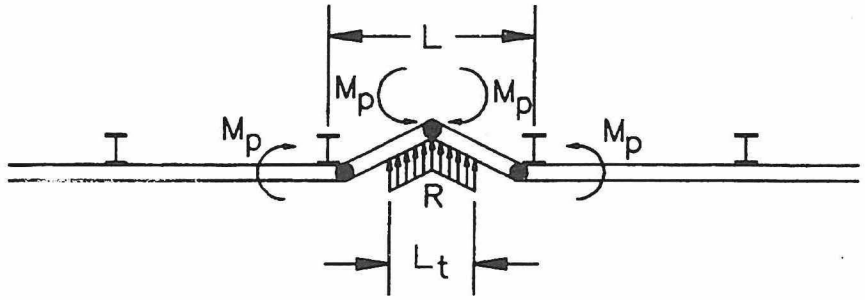
- For collision within a segment:

$$F_w = \left(\frac{1}{L_c - \frac{L_t}{2}}\right)\left[8(M_b + M_w H) + \frac{M_c L_c^2}{H}\right] \qquad (16.2.1.3\text{-}3)$$

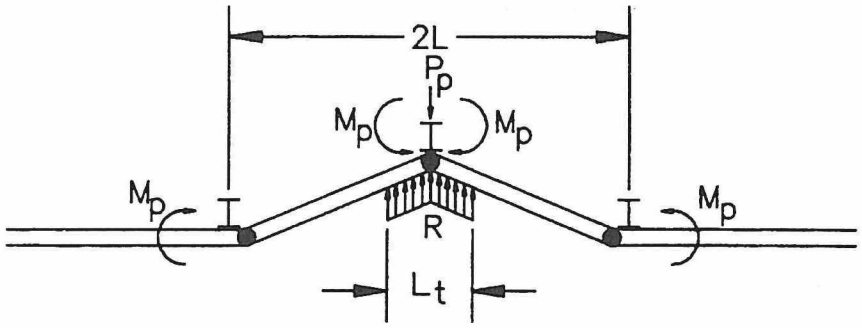
- For collision near an expansion joint:

$$R_w = \left(\frac{1}{L_c - \frac{L_t}{2}}\right)\left[(M_b + M_w H) + \frac{M_c L_c^2}{H}\right] \qquad (16.2.1.3\text{-}4)$$

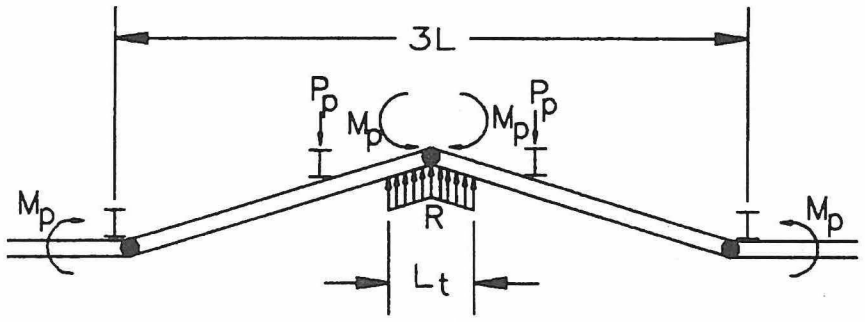
The analytical design procedure for post-type railing system is also based on an assumed failure mechanisms. Post-and-rail railing systems fail through yielding of the different elements. The load capacity can be calculated using the plastic capacity of each element and by assuming a failure mode that engage a certain number of rail spans. The failure is controlled by the mode that has the least load capacity. Figure 16.2.1.3-9 shows some of the possible failure modes involving one, two or three spans of the rail. The load capacity equations are based on the assumption that the deck overhang and post anchorage to the deck will not fail before the post.



Single–Span Failure Mode



Two–Span Failure Mode



Three–Span Failure Mode

Figure 16.2.1.3-9 - Possible Failure Modes for Post-and-Beam Railings

The load capacity of the post- and rail-type railing systems, R, can be calculated as follows:

- For failure modes involving an odd number of railing spans:

$$R = \frac{16M_p + (N-1)(N+1)P_pL}{2N - L_t} \qquad (16.2.1.3\text{-}5)$$

- For failure modes involving an even number of railing spans:

$$R = \frac{16M_p + N^2P_pL}{2N} \qquad (16.2.1.3\text{-}6)$$

where:

N = number of railing spans involved in the failure mechanism

M_p = plastic moment of horizontal rails

L = post spacing

P_p = ultimate resistance of single post due to a load at $\bar{Y}$ from top of deck

Notice that M_p should be based on consideration of the interaction of the post moments from both the transverse and longitudinal collision loads. This will also be reflected on the calculated value of P_p.

The number of posts engaged in the failure mechanism should be enough to make the load on the posts at the two ends of the failure mechanism equal or less than their yield load. For any failure mechanism, the posts at the ends of the failure mode will fail if the following condition is not satisfied:

Post yield load capacity $> \frac{1}{2}[R-(N-1)P_p]$

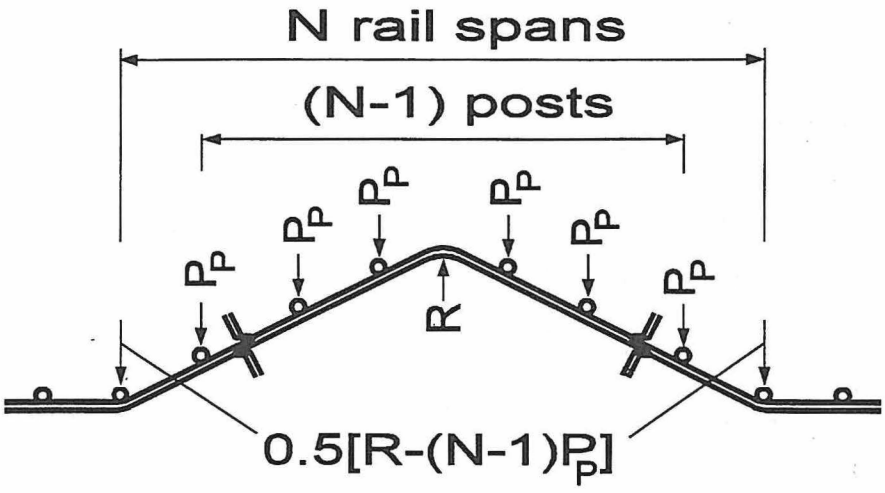


Figure 16.2.1.3-10 - Load on Posts at End of Failure Mechanism

In addition to the strength requirements of the railing system, the rail should have adequate height to prevent the rollover of the vehicles. The effective height of the railing system, $\bar{Y}$,should exceed the required minimum effective height, H_e, that is determined based on the characteristics of the rail and the type of vehicle using the following equation:

$$H_e = G - \frac{WB}{2F_t} \qquad (16.2.1.3\text{-}7)$$

where:

G = height of vehicle center of gravity above bridge deck as specified in Table 16.2.1.3-1 (mm)

W = weight of vehicle corresponding to the required performance level, as specified in Table 16.2.1.3-1 (N)

B = out-to-out wheel spacing on an axle as specified in Table 16.2.1.3-1 (mm)

F_t = transverse force corresponding to the required performance level as specified in Table 16.2.1.3-2 (N)

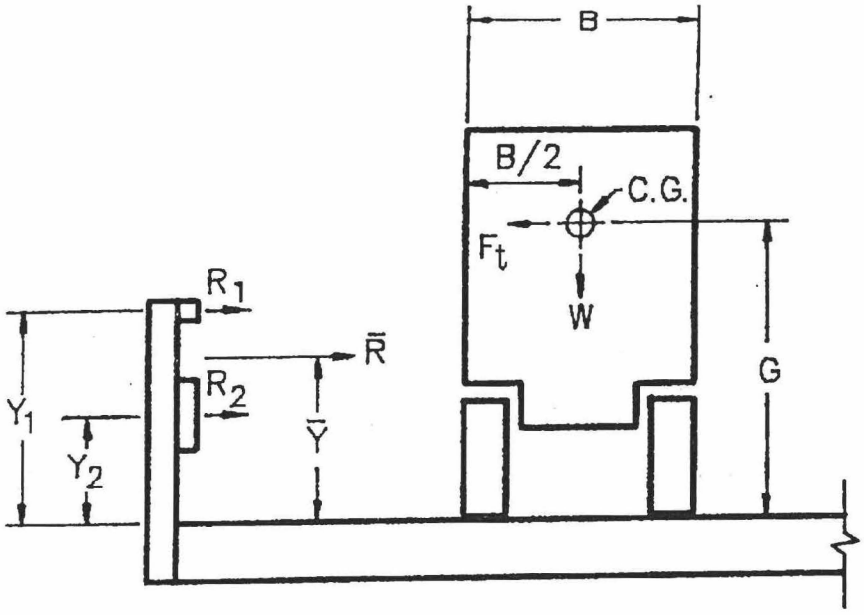


Figure 16.2.1.3-11 - Traffic Railing Forces

The following conditions need to be satisfied for a parapet to be considered satisfactory:

$R' \geq F_t$

$\bar{Y} \geq H_e$

The first condition ensures that the parapet can resist the applied collision force while the second condition ensures that vehicle rollover will not occur.

The required minimum height of concrete parapets having vertical face is 685 mm and the minimum for sloped face (shown in Figure 16.2.1.3-12) is 810 mm for Performance Level 2 and 1060 mm for performance Level 3. These heights do not precisely match those obtained from Equation 16.2.1.3-7 and were chosen based on crash-test results and on past experience with concrete parapets. Sloped-face test specimens that included an encroachment of 50 mm, leaving a 25 mm vertical lip at the bottom, were successfully crash-tested. Therefore, the 75 mm vertical lip does not need to be increased to account for future wearing surface.

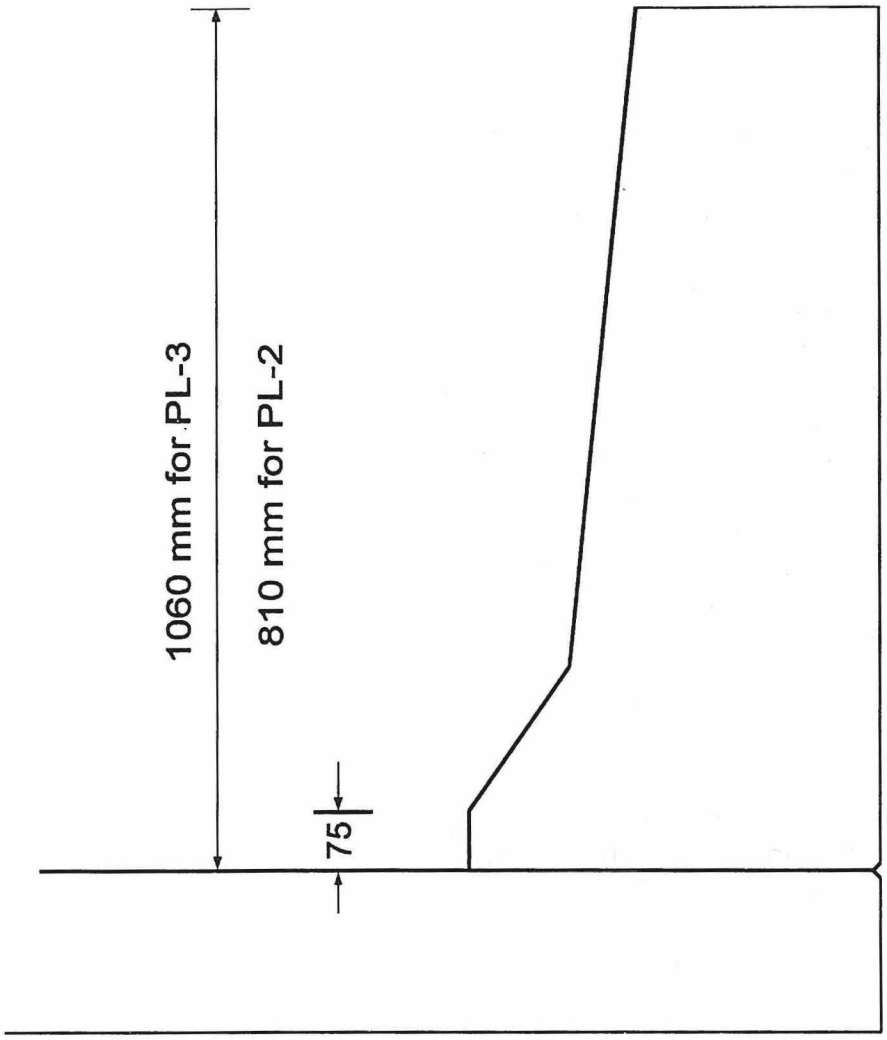


Figure 16.2.1.3-12 - Required Height for Typical Concrete Railing

16.2.2 Pedestrian Railing

Pedestrian railings are required along the edges of pedestrian bridges and along the outer edges of sidewalks when the highway traffic is separated from the pedestrian traffic by a traffic railing. The height and opening dimensions of pedestrian railings are required to be chosen to safeguard the pedestrians using the bridge and the public passing underneath. The minimum acceptable height of pedestrian railings is 1060 mm measured from the top of the walkway. It is common to use post-type railing as a pedestrian railing. In this case, the maximum allowed clear opening between rail elements is 150 mm, in the lower 685 mm of the railing, and the maximum spacing in the upper portion is the smaller of 380 mm and the required maximum spacing determined from Figure 16.2.1.3-3.

In addition, the setback requirements of Figure 16.2.1.3-4 need to be satisfied.

The elements of a post-type railing need to be designed to resist a load w = 0.73 N/mm, both transversely and vertically, acting simultaneously on each longitudinal element. In addition, a concentrated load of 890 N, which may act simultaneously with the above loads at any point and in any direction, will be applied at the top of the rail.

Chain-link railing systems are usually used if there is a possibility of pedestrians or vehicles passing under the bridge. In this case, the railing system is required to protect not only the pedestrians on the bridge, but also the public underneath by being able to retain the debris that may exist on the bridge. Therefore, the chain-link mesh should be capable of retaining an average beverage bottle. Such system need to be designed to resist a pressure of 7.2×10^{-4} MPa acting normal to the entire surface of the rail.

16.2.3 Bicycle Railings

Bicycle railings are required on bridges specifically designed to carry bicycle traffic and on bridges where specific protection of bicyclists is deemed necessary. The height of a bicycle railing is required to be not less than 1370 mm, measured from the top of the riding surface. This height is divided into a top and a bottom zones, each is 685 mm high. The rail spacing in each zone is required to satisfy the respective provisions for pedestrian railings. The need for rub-rails attached to a rail or fence to prevent snagging are controversial among many bicyclists. Where provided, rub-rails should be deep enough to protect a wide range of bicycle handlebar heights. If the rail height exceeds 1370 mm above the riding surface, design loads are to be determined by the designer. The design loads for the lower 1370 mm of the bicycle railing shall not be less than those specified for pedestrian railings.

16.2.4 Combination Railings

On high-speed urban expressways where a pedestrian walkway is provided, the walkway area is required to be separated from the adjacent roadway by a combination railing. On low-speed bridges where the walkway is not separated from the roadway, a combination railing is required along the edges of the bridge. Figure 16.2.4-1 shows an example for both types of combination railings. The combination railing must conform to the geometric and load requirements of either the

pedestrian or bicycle railings, whichever is applicable. The traffic railing portion of the combination railing is required to be designed for the applicable requirements of traffic railings. When designing for vehicular collision, pedestrians and bicycle design loads need not be applied simultaneously with the vehicular collision loads. Combination railings along the edges of the bridge, i.e., railings with barrier curbs, are required to be crashworthy with or without the sidewalk and need to be crash-tested at a speed of 70 km/hr (S13.7.1.1).

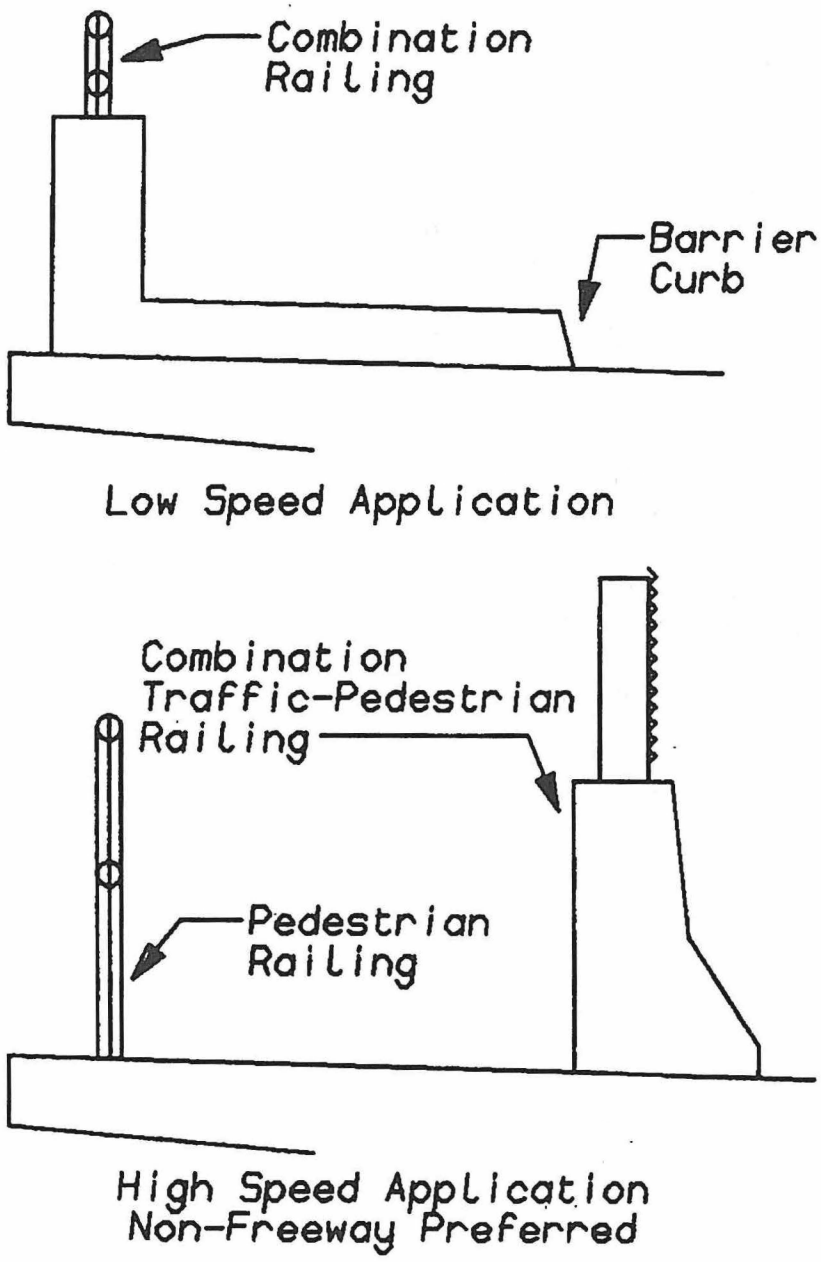


Figure 16.2.4-1 - Combination Traffic/Pedestrian Walkway

16.2.5 Curbs and Sidewalks

A sidewalk curb located on the highway traffic side of a bridge railing is considered as an integral part of the railing and needs to be included in the crash-test specimens. When curb and gutter sections with sidewalks are used on roadway approaches, the curb height for raised sidewalks on the bridge should be no more than 200 mm. If a barrier curb is required, the curb height should not be less than 150 mm. If the height of the curb on the bridge differs from that off the bridge, it should be uniformly transitioned over a distance greater than or equal to 20 times the change in height.

16.2.6 Deck Overhang Requirements

The proper design of the deck overhangs is as important as the design of the railing itself. The deck is designed to be stronger than the railing system, such that the failure will be contained in the railing and not to extend in the deck. This makes repairs of damage due to vehicular collision easier and less expensive.

The design forces in the slab due to collision differ according to type of railing system. Three design cases need to be considered, regardless the type of rail:

- Case 1: transverse and longitudinal collision forces at the extreme event limit state
- Case 2: vertical collision force at the extreme event limit state
- Case 3: conventional design (dead load and live load) at the strength limit state

In case of continuous concrete parapet, Case 2 produces relatively small force effects and, hence, it can be ignored.

Concrete Parapets

In case of vehicular collision, force is transmitted to the deck overhang as a tensile force (see Figure 16.2.6-1). In case of a concrete parapet, the tensile force per unit width of the deck, T, is calculated as:

$$T = \frac{R_w}{L_c + 2H} \qquad (16.2.6\text{-}1)$$

For a deck overhang supporting a continuous concrete parapet to be satisfactory at the face of the parapet, it should have a moment of resistance, M_s, in the presence of T, at least equal to the sum of the moment of resistance of the parapet at its base and the dead load moments, i.e.,

$$M_s \geq M_{DL} + M_c. \qquad (16.2.6\text{-}2)$$

where:

M_s = Moment resistance of the overhang in the presence of the "T"

M_c = Moment resistance of the parapet at its base

M_{DL} = Dead load moment in the overhang at a section at the inside face of the parapet

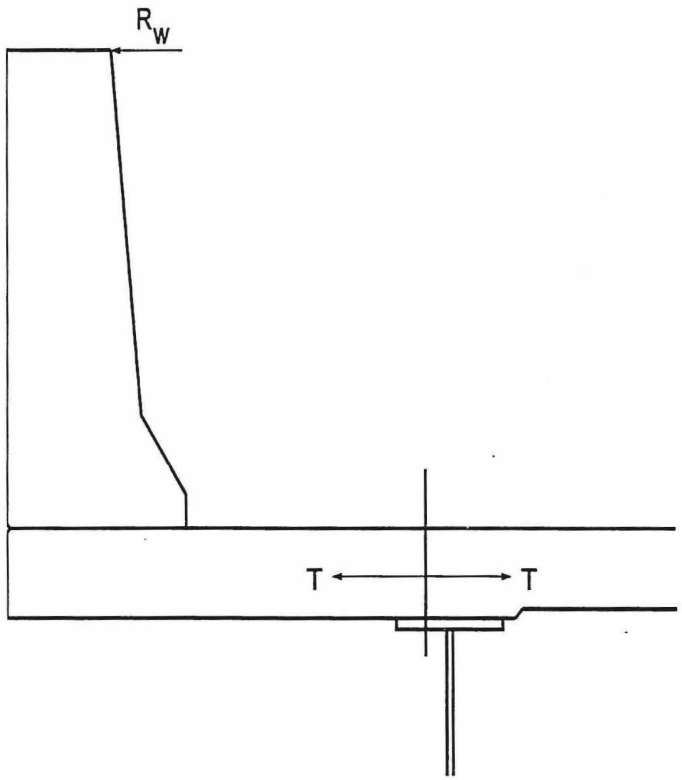


Figure 16.2.6-1 - Tension Force in the Deck Overhang Produced by Collision Forces

Even though the colliding vehicle is on the bridge, it was observed during crash-testing that the wheels near the parapet were not in contact with the deck at the moment of collision. Therefore, the required strength of the overhang does not contain a term to account for the live load moments in Case 1.

At other sections in the overhang, at some distance from the parapet, the value of T and M_c per unit length is decreased by spreading them at an angle, say 30°, as shown in Figure 16.2.6-2, from a length L_c + 2H at the base of the parapet for T, and L_c at the base of the parapet for M_c. In full-scale load tests of actual bridges with continuous concrete parapets, it was observed that the parapets distribute the loads over considerable length of the bridge. The 30° angle is an arbitrary value which is thought to give a conservative value.

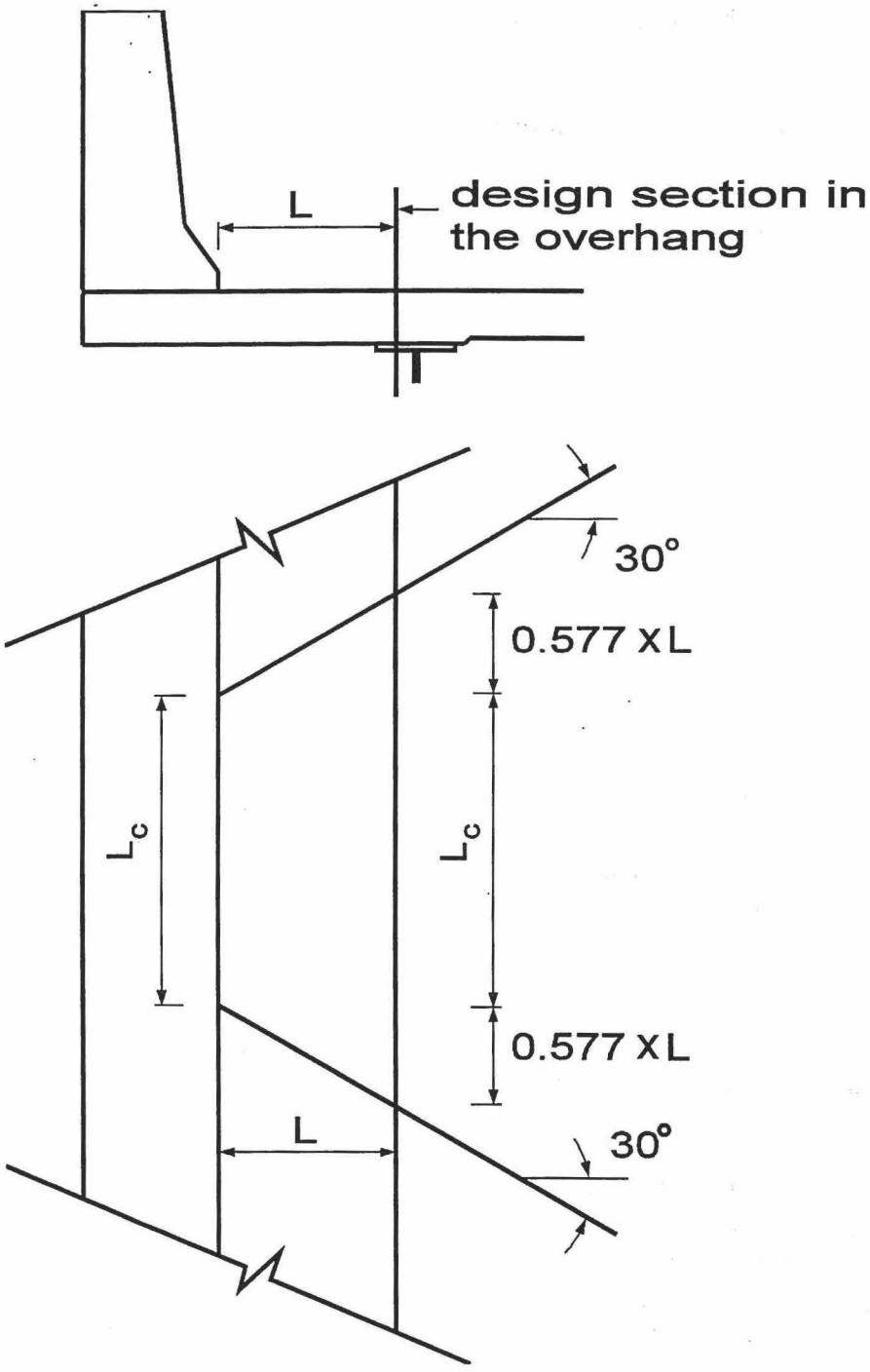


Figure 16.2.6-2 - Distribution of M_c in the Overhang

Post-Type Railings

In case of post-type railing, the design forces for the deck overhang are calculated for each design case as follows:

Case 1: Transverse and longitudinal collision forces

The overhang moment and tensile force can be calculated as:

$$M_d = \frac{M_{post}}{W_b + D} \qquad (16.2.6\text{-}3)$$

$$T = \frac{P_p}{W_b + D} \qquad (16.2.6\text{-}4)$$

where:

M_{post} = yield moment of the post

M_d and T are both force per unit width.

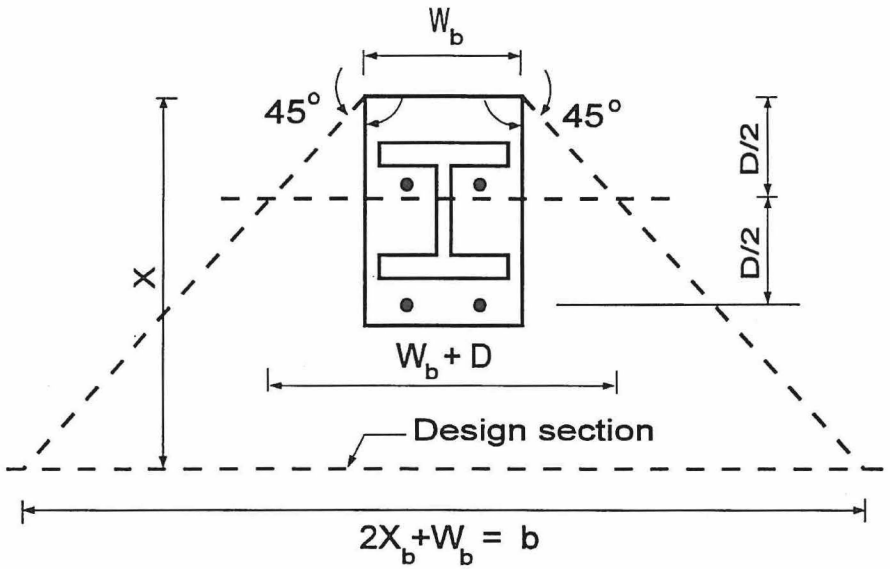


Figure 16.2.6-3 - Distribution of Moments in the Deck Overhangs Supporting Post-and-Rail Railings

The factored shear load may be taken as:

$$V_u = A_f F_y \qquad (16.2.6\text{-}5)$$

The factored resistance of deck overhangs to punching shear may be taken as:

$$V_r = \phi V_n \qquad (16.2.6\text{-}6)$$

$$V_n = V_c\left[W_b + h + 2\left(E + \frac{B}{2} + \frac{h}{2}\right)\right]h \qquad (16.2.6\text{-}7)$$

$$V_c = \left(0.166 + \frac{0.332}{\beta_c}\right)\sqrt{f'_c} \le \phi\, 0.332\sqrt{f'_c} \qquad (16.2.6\text{-}8)$$

$$\frac{B}{2} + \frac{h}{2} \le B \qquad (16.2.6\text{-}9)$$

for which:

$\beta_c = W_b/D$

where:

h = depth of slab (mm)

W_b = width of base plate (mm)

A_f = area of post compression flange (mm^2)

F_y = yield strength of post compression flange (MPa)

b = length of deck resisting post strength or shear load = h + W_b

B = distance between centroids of tensile and compressive stress resultants in post (mm)

D = depth of base plate (mm)

E = edge distance from line of support to centroid of compressive stress resultant in post (mm)

f'_c = 28-day compressive strength of concrete (MPa)

ϕ = resistance factor = 1.0

The assumed distribution of forces for punching shear shall be as shown in Figure 16.2.6-4.

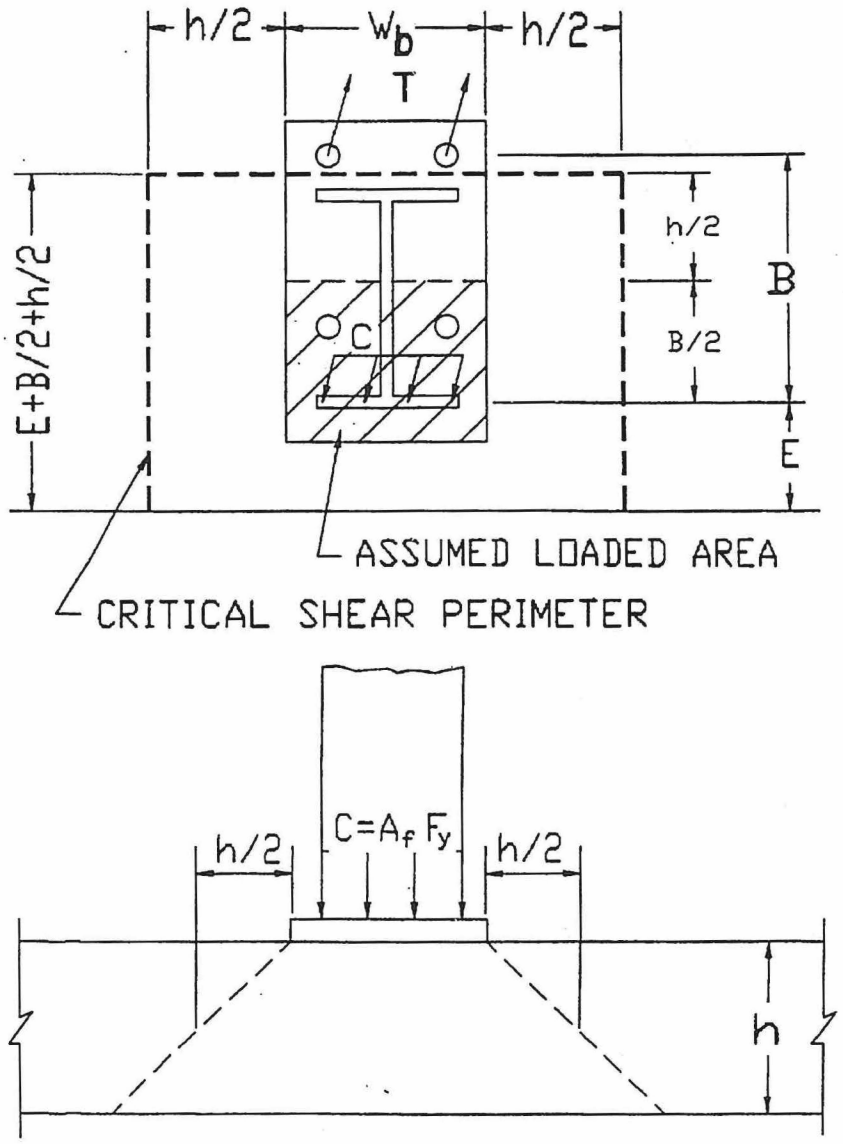


Figure 16.2.6-4 - Punching Shear Failure Mode

Case 2: Vertical collision force

The punching shear force and overhang moment can be calculated as:

$$P_v = \frac{F_v L}{L_v} \qquad (16.2.6\text{-}10)$$

$$M_d = \frac{P_v X}{b}, b \le L \qquad (16.2.6\text{-}11)$$

where:

L = post spacing

P_u and M_d are both force per unit length.

Case 3: Conventional design for dead load and live load

16.2.7 Design Examples

TRAFFIC PARAPET DESIGN EXAMPLES

1 - Concrete parapets

Materials:

F_y = 400 MPa

F'c = 24 MPa

Reinforcement bars:

#10: A = 100 mm^2, d = 11.3 mm
#15: A = 200 mm^2, d = 16.0 mm
#20: A = 300 mm^2, d = 19.5 mm

Performance Level - 3

Minimum required transverse resistance = 516 000 N
(Table 16.2.1.3-2)

L_t = 2440 mm (Table 16.2.1.3-2)

Example 1

CONSTANT THICKNESS PARAPET

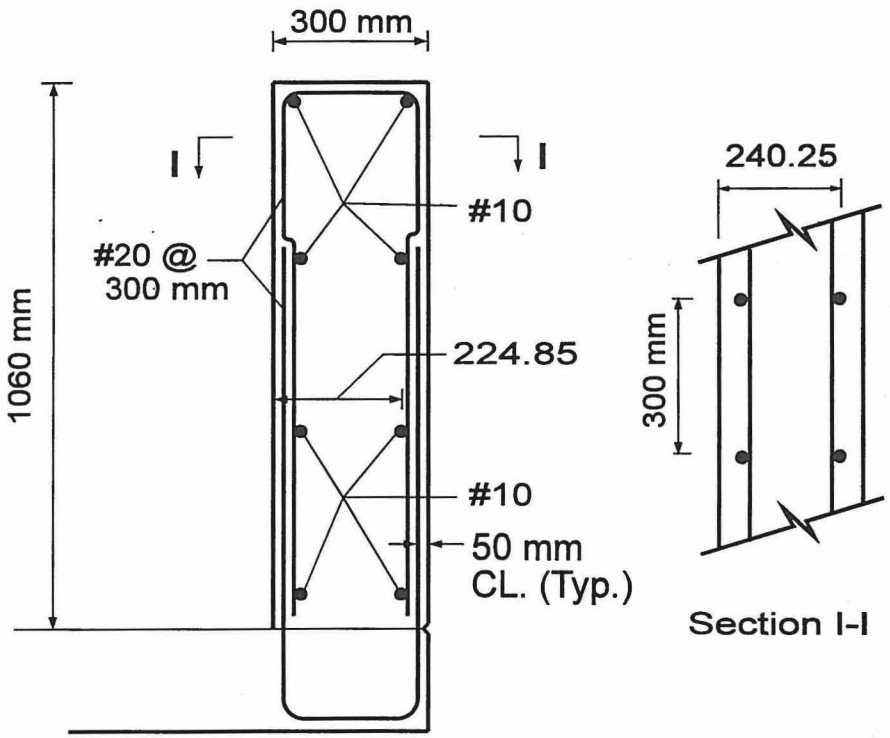


Figure 16.2.7-1 - Reinforcement of Constant Thickness Concrete Parapet

Calculate M_c

d = 240.25 mm

A_s = 300 mm^2

ϕ = 1.0 for extreme event limit state (S1.3.2.1)

a = $\frac{A_s F_y}{0.85 f'_c b} = \frac{300 * 400}{0.85(24)(300)} = 19.6\ mm$

$M_c = \frac{\phi A_s F_y}{b}\left(d - \frac{a}{2}\right)$

M_c = 92 000 N·mm/mm

M_b = 0 - No top beam

Calculate M_wH

$d = 300-50-19.5-\frac{11.3}{2} = 224.85\ mm$

$A_s = 4\ \#10 = 400\ mm^2$

$a = 7.4\ mm$

$M_wH = \phi F_y A_s\left[d-\frac{a}{2}\right] = 1.0 * 400 * 400\left[224.85-\frac{7.4}{2}\right]$
$= 35\,384\,000\ N{\cdot}mm$

COLLISION WITHIN A SEGMENT

$L_t = 2440\ mm$ for PL-3

$L_c = \frac{L_t}{2}+\sqrt{\left(\frac{L_t}{2}\right)^2+\frac{8H(M_b+M_wH)}{M_c}} = 3399\ mm$

$R_w = \frac{8(M_b+M_wH)}{L_c-\frac{L_t}{2}}+\frac{M_c(L_c)^2}{H\left(L_c-\frac{L_t}{2}\right)}$

$= 130\,000 + 460\,000 =$
$590\,000\ N > 516\,000\ N$

COLLISION NEAR AN EXPANSION JOINT

$L_c = \frac{L_t}{2}+\sqrt{\left(\frac{L_t}{2}\right)^2+\frac{H(M_b+M_wH)}{M_c}} = 2597\ mm$

$R_w = \frac{(M_b+M_wH)}{L_c-\frac{L_t}{2}}+\frac{M_c(L_c)^2}{H(L_c-L_t)}$

$= 25\,700 + 425\,000 =$
$450\,700\ N < 516\,000N\ NG$

ALTERNATIVES TO INCREASE THE LOAD CAPACITY NEAR EXPANSION JOINTS

Use sleeves and dowels to transfer a portion of the force across the joint without restraining the movement of the expansion joint, or increase the parapet reinforcement near the expansion joint. If the vertical reinforcement is increased, it may lead to heavier deck reinforcement due to the need to have the deck resistance greater than M_c from the railing.

Concrete Parapets

Example 2

ALTERNATIVE PARAPET SECTION

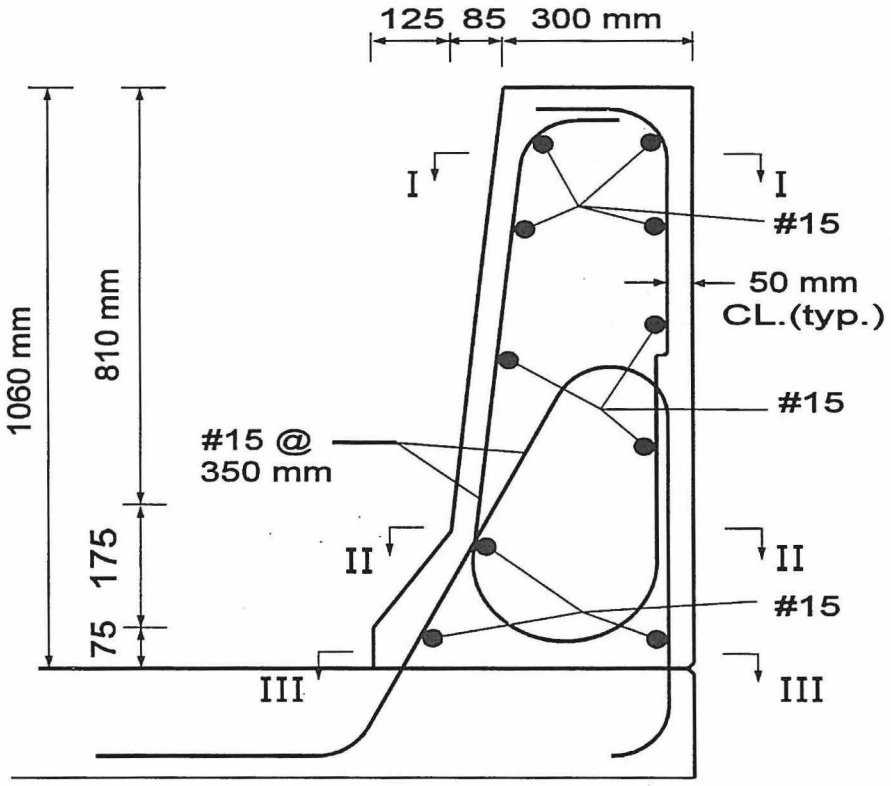


Figure 16.2.7-2 - Reinforcement of a Typical Concrete Parapet

The moment capacity, M_c, at Sections I, II and III can be calculated as for the constant thickness parapet.

At Section I:

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{200 \times 400}{0.85 \times 24 \times 350} = 11.2 \ mm$$

$$d = 300 - 50 - \frac{16}{2} = 242 \ mm$$

$\phi = 1.0$ for extreme event limit state (S1.3.2.1)

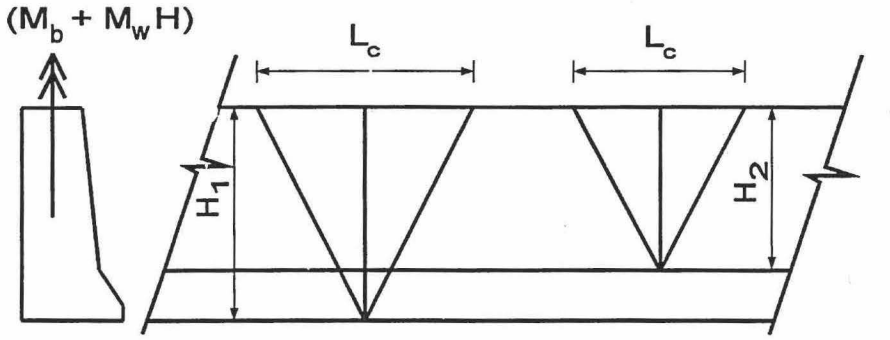
$$M_c\text{-}I = \frac{\phi A_s f_y}{b}\left(d - \frac{a}{2}\right) = 54\ 000\ N{\cdot}mm/mm$$

Similarly, and by considering the thickness of the concrete at each section,

M_c-II = 73 500 N·mm/mm

M_c-III = 102 000 N·mm/mm

FAILURE MECHANISMS



Failure Mechanism 1 Failure Mechanism 2

Figure 16.2.7-3 - Possible Failure Mechanisms

Calculate M_c Average

Assuming Failure Mechanism 1, M_c Average =

$$\left[\frac{54\,000+73\,500}{2} * 810 + \frac{73\,500+102\,000}{2} x250\right] / 1060 = 69\,400\ N{\cdot}mm/mm$$

Assuming Failure Mechanism 2, M_c Average =

$$\frac{[54\,000 + 73\,500]}{2} = 63\,750\ N{\cdot}mm/mm$$

M_b = 0, No top beam

Calculate M_wH

Divide the section into three portions and calculate the moment capacity of each portion about the vertical axis.

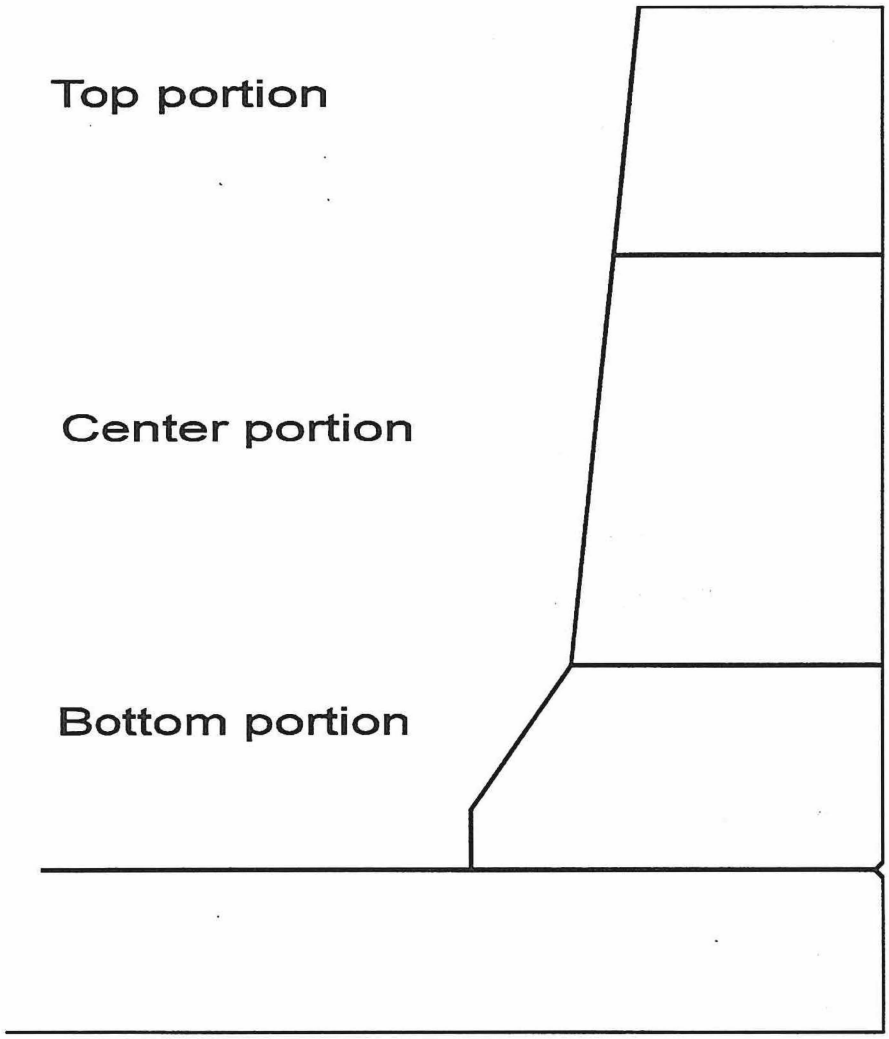


Figure 16.2.7-4 - Dividing the Parapet for Calculations of M_b + M_wH

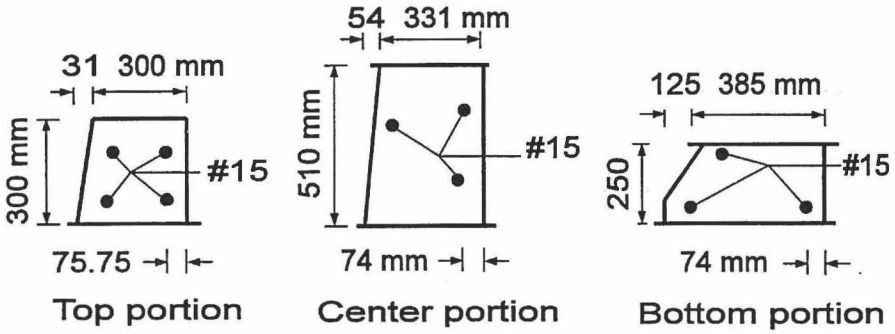


Figure 16.2.7-5 - Analysis of $M_b + M_wH$

Top portion

A_s = 2 #15 for both positive and negative moment

d_{ave} = $\left(300 + \frac{31}{2}\right) - 75.75 = 239.75\ mm$

a = $\frac{A_s f_y}{0.85 f'_c b} = \frac{400 \times 400}{0.85 \times 24 \times 300} = 26.14\ mm$

$M_{positive} = M_{negative}$ = $\phi A_s f_y \left(d - \frac{a}{2}\right) = 36\ 270\ 000\ N{\cdot}mm$

Center Portion

$A_{spositive}$ = 1 #15 = 200 mm^2

$d_{positive}$ = $\left(331 + \frac{54}{2}\right) - 74 = 284\ mm$

$a_{positive}$ = $\frac{200 \times 400}{0.85 \times 24 \times 510} = 7.69\ mm$

$M_{positive}$ = 22 410 000 N·mm

$A_{snegative}$ = 2 #15 = 400 mm^2

$d_{negative}$ = 284 mm

$a_{negative}$ = $\frac{400 \, x \, 400}{0.85 \, x \, 24 \, x \, 510} = 15.38 \ mm$

$M_{negative}$ = 41 210 000 N·mm

M_{ave} = $\frac{(M_{positive} + M_{negative})}{2} = 33\,310\,000 \ N{\cdot}mm$

Bottom Portion

$A_{spositive}$ = 400 mm^2

$d_{positive}$ = 373.5 mm

$a_{positive}$ = 31.37 mm

$M_{positive}$ = 57 250 000 N·mm

$A_{snegative}$ = 200 mm^2

$d_{negative}$ = 436 mm

$a_{negative}$ = 15.67 mm

$M_{negative}$ = 34 253 000 N·mm

M_{ave} = 45 751 500 N·mm

In calculating M_wH of the parapet, the moment resistance of the center and bottom portions are determined as follows:

The mechanism equations given in the Specification and duplicated herein are based on equal positive and negative hinge live moments. For collision within a segment, use the average of the positive and negative values. This is acceptable because the yield line mechanism for this case will have some positive moment hinges and some negative moment hinges.

For collision near expansion joint, use the moment resistance that corresponds to a moment causing tension along the inside face of parapet, i.e., $M_{positive}$. This is required because the only yield line to form is caused by a moment causing tension along the inside face.

COLLISION WITHIN A SEGMENT

M_wH:

Failure Mechanism 1:

M_wH = 36 270 000 + 33 310 000 + 45 751 000

= 115 331 000 N·mm

Failure Mechanism 2:

M_wH = 36 270 000 + 33 310 000

= 69 580 000 N·mm

Load Capacity:

Performance Level-3 (PL-3)

L_t = 2440 mm

Failure Mechanism 1:

H = 1060 mm

$$L_c = \frac{L_t}{2} + \sqrt{\left(\frac{L_t}{2}\right)^2 + \frac{8H(M_b + M_w H)}{M_c}} = 5167\ mm$$

$$R_w = \frac{8(M_b + M_w H)}{L_c - \frac{L_t}{2}} + \frac{M_c (L_c)^2}{H\left(L_c - \frac{L_t}{2}\right)} = 676\,600\ N > 516\,000\ N$$

Failure Mechanism 2:

H = 810 mm

L_c = 4142 mm

R_w2 = 654 000 N > 516 000 N

Parapet Load Capacity = smaller of R_w1 and R_w2 = 654 000 N

COLLISION NEAR EXPANSION JOINTS

M_c: No change

M_wH:

Failure Mechanism 1:

M_wH = 36 270 000 + 22 410 000 + 57 250 000

= 115 930 000 N·mm

Failure Mechanism 2:

M_wH = 36 270 000 + 22 410 000

= 58 680 000 N·mm

Load Capacity:

R_wI = 396 100 N < 516 000 N

R_wII = 427 310 N < 516 000 N

Parapet capacity needs to be increased near expansion joints or lower performance level may be accepted in the expansion joint region.

Example 3

RAIL AND POST PARAPET

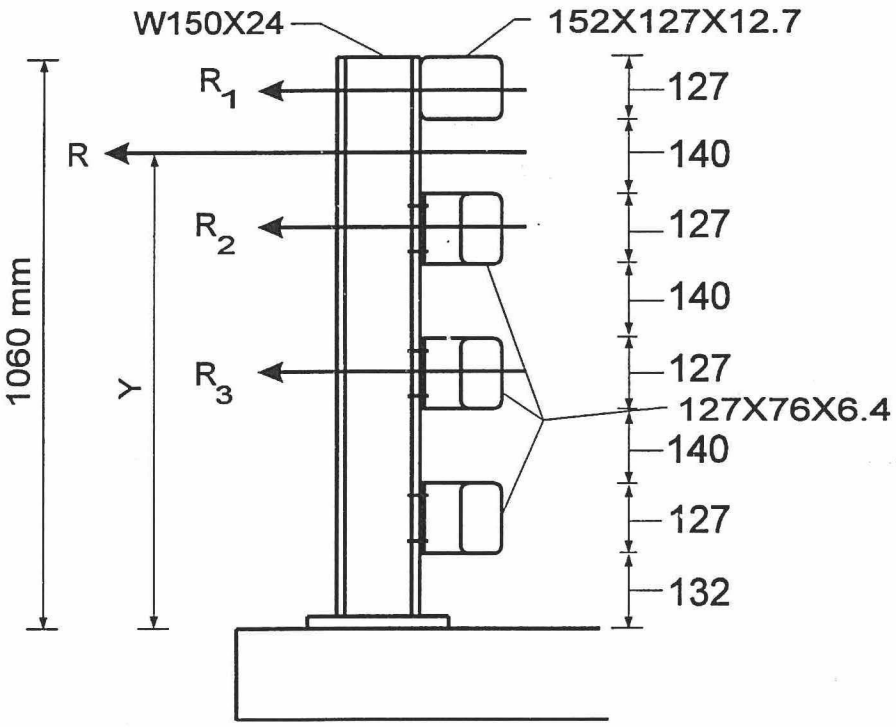


Figure 16.2.7-6 - Geometry of Post-and-Rail Railing System

Given:

F_y = 250 MPa, f'_c = 28 MPa

Post Spacing = 1800 mm

Plastic Moment Capacity, M_p

Top Rail:

Plastic Modulus, Z = 296 000 mm^3

M_p Top Rail = $f_y Z$ = 250 x 296 000
= 74 000 000 N·mm

Other Rails:

Plastic Modulus, Z = 65 100 mm^3

M_p for each rail = 250 x 65 100
= 16 275 000 N·mm

Post:

Plastic Modulus Z = 192 000 mm^3

M_p Post = 250 x 192 000
= 48 000 000 N·mm

In calculating M_p for this example, no consideration was given to the effect of the longitudinal collision force on the plastic moment of the post.

For PL-2, L_t = 1070 mm (Table 16.2.1.3-2)

Post setback = 152 mm Figures 16.2.7-6 and 16.2.1.3-4

Maximum clear distance = 140 mm

$\Sigma A = 4 \times 127 = 508$ mm

$\Sigma A/H = 508/1060 = 0.48 > 0.25$

From Figure 16.2.3-3 and 16.2.3-4, the chosen geometry of the parapet is acceptable.

Bottom rail probably will not yield. Therefore, this rail was ignored in the load resistance calculations of the railing system.

$$\overline{Y} = \frac{\Sigma R_i Y_i}{\Sigma R_i}$$

where:

R_i = load resistance of the i_{th} rail

Y_i = height of the i_{th} rail above the top surface of the deck

ΣR_i = total resistance of the rails = $\overline{R}$

$\overline{Y}$ = the height of $\overline{R}$ above the top surface of the deck

For rails made from the same material, the load resistance of the rails relative to each other is the same as the ratio between their moment capacity. Therefore, the plastic

moment capacity of the rails can be used to determine $\overline{Y}$ as follows:

$$\overline{Y} = [74 \times 10^6 \times 996.5 + 16.275 \times 10^6 \times (729.5 + 462.5)]/[74 \times 10^6 + 2 \times 16.275 \times 10^6]$$

$$= 874.2 \text{ mm}$$

Required minimum effective height of rail, $H_e = G - \frac{WB}{2F_t}$

(16.2.1.3-1)

For PL-2

G = 1240 mm (Table 16.2.1.3-1)

W = 80 000 N (Table 16.2.1.3-1)

B = 2300 mm (Table 16.2.1.3-1)

F_t = 240 000 N (Table 16.2.1.3-2)

H_e = 857 mm < $\overline{Y}$ = 874.2 mm

RAIL LOAD CAPACITY

M_p = ΣM_p for top three rails

= 106 550 000 N·mm

P_p = Post plastic moment/ $\overline{Y}$

= 48 000 000/874.2 = 54 900 N

L_t = 1070 mm (Table 16.2.1.3-2)

See Figures 16.2.1.3-9 and 16.2.1.3-10 for various failure mechanisms.

For failure modes involving an odd number of railing spans, N

$$R = \frac{16M_p + (N-1)(N+1)P_pL}{2NL - L_t}$$

N = 1 R = 674 000 N

N = 3 R = 256 000 N

N = 5 R = 241 600 N

N = 7 R = 267 000 N

For failure modes involving an even number of spans, N

$$R = \frac{16M_p + N^2 P_p L}{2NL - L_t}$$

N = 2 R = 343 000 N

N = 4 R = 247 000 N

N = 6 R = 256 000 N

N = 8 R = 290 000 N

The load capacity of the railing system is determined by the mechanism that has the least load capacity. Other mechanisms will have no chance to form because the system would fail before the load reaches the required level to form these mechanisms. Therefore, the load capacity of the railing system = 241 000 N > 240 000 N required for PL-2 (Table 16.2.1.3-2).

Notice that for a given post cross-section and spacing, the number of rail spans involved in the failure mechanism increases as the plastic moment capacity of the rails, M_p, increases. For example, if M_p was doubled, i.e., M_p = 213 000 N·mm, while the post size and spacing were kept the same, the failure mechanism that would have the least load capacity would involve seven spans and would have a load capacity of 338 000 N.

Check load on posts at end of failure mode:

# of failed posts = # of rail spans engaged in the failure mode (N)-1 = 4

Load on failed posts = 4 * P_p = 219 600 N

Load on the post at each end of the failure mode =

$$\left(\frac{R - 219\,600}{2}\right) = 10\,500\ N < P_p = 54\,900\ N$$

Check top rail for vertical force

F_v = 80 000 N (Table 16.2.1.3-2)

L_v = 5500 mm (Table 16.2.1.3-2)

$$\text{Vertical Load} = \frac{80\ 000}{5500} = 14.55\ N/mm$$

$$\text{Moment} = \frac{WL^2}{10} = \frac{14.55x(1800)^2}{10} = 4\ 714\ 200\ N{\cdot}mm$$

Plastic moment of top rail under vertical force = 65 250 000 N·mm > 4 714 200 N·mm

- Design the connection between the top rail and the post to resist the vertical load and the moment from the load eccentricity.

- Check the connection between the rails and the post for the effect of longitudinal collision forces.

- Design the post-to-base weld, base plate and anchor bolts to resist the plastic moment of the post.

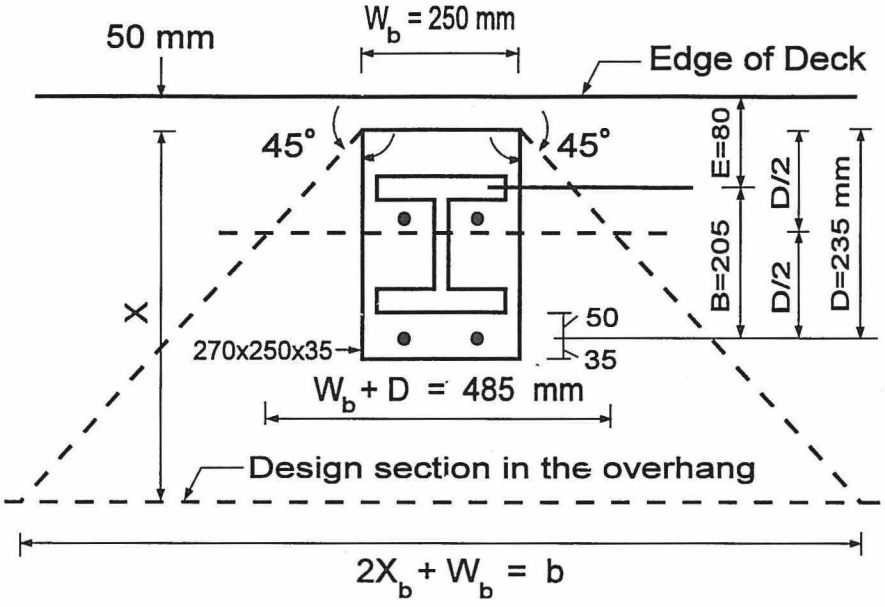


Figure 16.2.7-7 - Distribution of Forces in the Deck Overhang

Anchors and base plate are designed to have higher resistance than the post. Post resistance is based on plastic moment, i.e., highest possible resistance. Base plate and anchor bolts resistance is based on yield strength, i.e., base plate and anchor bolts will not have significant deformations before the post collapse.

Force in anchor bolts $M_{post}/B \simeq 48\ 000\ 000/205$
$= 234\ 146$ N

Assumed characteristics of anchor bolts:

f_y = 400 MPa

ϕ = 0.75

A_{net} = 0.75 A_g

d_{bolt} = 27 mm

Resistance of two bolts =

$$2 x 0.75 x 0.75 x \frac{\pi (27)^2}{4} x 400$$

= 257 650 N

> 234 146 N

Base Plate

Assume design section is at outside edge of flange (see Figure 16.2.7-7)

f_y = 250 MPa

Assume t = 35 mm

$$M = \frac{234\,146 * 50}{250} = 46\,829\ N{\cdot}mm/mm$$

$$\sigma = \frac{(46\ 829)\ (6)}{(35)^2} = 229\ MPa < f_y = 250\ MPa$$

DESIGN FORCES IN DECK SLAB

Design Case 1:

Transverse collision force

$$M_d = \frac{M_{post}}{W_b + D} = \frac{48\ 000\ 000}{250 + 235} = 98\ 969\ N{\cdot}mm/mm$$

$$T = \frac{P_p}{W_b + D}$$

$$= \frac{54\ 900}{250 + 235} = 113\ N/mm$$

To have a more economical deck design, it is desirable to use smaller posts to reduce M_d & T. However, this will result in the failure of larger number of rail spans when a vehicle collides with the railing system.

Design the deck overhang to resist M_d & T simultaneously using conventional reinforced concrete design procedures.

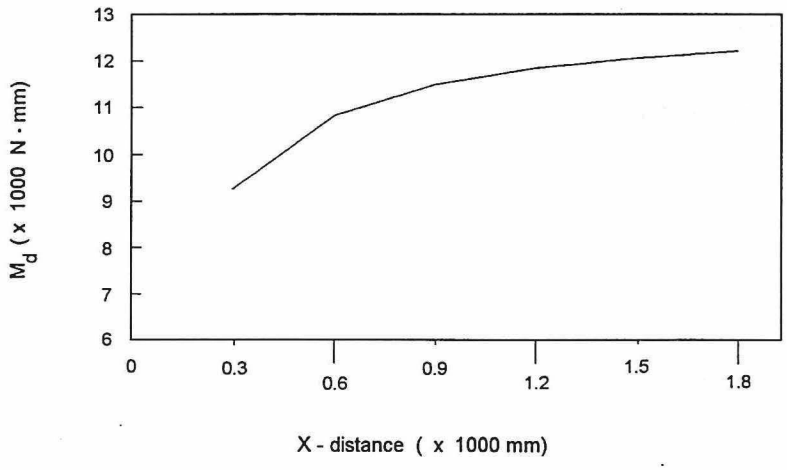
Design Case 2:

Calculate vertical collision force

$$P_v = \frac{F_v L}{L_v} = \frac{80\ 000\ x\ 1800}{5500} = 26\ 180\ N$$

$$M_d = \frac{P_v X}{b} = \frac{P_v X}{2X + W_b}$$

For a given performance level, M_d depends on the overhang length and post spacing. The variation of M_d with X in the given case is shown below.



Calculate punching shear (see Figure 16.2.6-4)

$V_u = A_f f_y = 1050.6 * 250 = 262\ 650$ N

Resistance:

Assume h = 200 mm
f'_c = 28 MPa
E = 80.15 mm
B = 205 mm

$$\frac{B}{2} + \frac{h}{2} \le B \quad (16.2.6\text{-}9)$$

$$\beta_c = \frac{W_b}{D} = \frac{250}{235} = 1.063$$

Concrete punching shear strength

Resistance factor $\phi = 1.0$

$$V_c = \left(0.166 + \frac{0.332}{\beta_c}\right)\sqrt{f'_c} \le \phi\, 0.332\sqrt{f'_c} \quad (16.2.6\text{-}8)$$

$$\left(0.166 + \frac{0.332}{\beta_c}\right)\sqrt{f'_c} = \left(0.166 + \frac{0.332}{1.063}\right)\sqrt{28} = 2.53 \ MPa$$

$$\phi\, 0.332\sqrt{f'_c} = (1.0)\,(0.332)\sqrt{28} = 1.76 \ MPa$$

V_c = smaller of 2.53 and 1.76 = 1.76 MPa

Punching shear resistance

$V_r = \phi\, V_n = V_n$

$$V_n = V_c\left[W_b + h + 2\left(E + \frac{B}{2} + \frac{h}{2}\right)\right]h \ , \text{ assume h = 200 mm}$$

= 357 400 N > V_u

16.3 OVERVIEW OF BRIDGE JOINTS

16.3.1 General

Based on past experience, deck joints have been the source of numerous problems. Therefore, the number of movable deck joints in a structure should be minimized. Preference should be given to continuous deck systems and superstructures. Integral bridges, bridges without movable deck joints, should be considered where the length of the superstructure and flexibility of the substructures are such that secondary stresses due to restrained movement are controlled within tolerable limits.

Where used, deck joints components are arranged to accommodate the translation and rotation of the structure at joint locations. The type of joints and surface gaps should be chosen to accommodate the movement of motorcycles, bicycles and pedestrians, as required, and should neither significantly impair the riding characteristics of the roadway, nor cause damage to vehicles. The joints should be detailed to prevent damage to the structure from water, deicing chemicals and roadway debris without significantly impairing the riding characteristics of the roadway, or causing damage to vehicles. Special details are required for joints and joint anchors for grid and timber decks, and orthotropic deck superstructures.

Longitudinal deck joints should be used on a limited basis. They should be provided only where necessary to modify the effects of differential lateral and/or vertical movement between the superstructure and substructure. Where practical, elastomeric bearings or combination bearings with the capacity for lateral movement should be used instead of longitudinal joints to accommodate differential lateral movement.

Factored force effects and factored range of movements are to be considered in the design of deck joints and their supports. Superstructure movements include those due to placement of bridge decks, volumetric changes, such as shrinkage, temperature, moisture and creep, passage of vehicular and pedestrian traffic, pressure of wind, and the action of earthquakes. Substructure movements include differential settlement of piers and abutments, tilting, flexure, and horizontal translation of wall-type abutments responding to the placement of backfill, and shifting of stub abutments due to the consolidation of embankments and in-situ soils.

Section 3 of the Specification requires that load factors of 1.0 and 1.2 be used when considering the effect of uniform temperature change on the movements at the service and strength limit states, respectively. Using the 1.2 factor is intended to ensure that the gap will not be closed under the most adverse conditions anticipated. The load factor to be used when considering the effect of temperature gradient, γ_{TG}, is to be determined on a project-specific basis. In snow regions, force effects that may be imposed on the joints by snagging snowplow blades need to be considered in the design of joint armor, armor connections and anchors. Resistance factors and modifiers shall be taken as specified in Sections 1, 5, 6, 7 and 8, as appropriate for the joint and deck materials.

The following factors are required to be considered in determining force effects and movements:

- properties of materials in the structure, including coefficient of thermal expansion, modulus of elasticity and Poisson's ratio,
- effects of temperature, creep and shrinkage,
- sizes of structural components,
- construction tolerances,
- method and sequence of construction,
- resistance of the joints to movements,
- substructure movements due to embankment construction,
- foundation movements associated with the consolidation and stabilization of subsoils,
- structural restraints, for which the length of superstructure affecting the movement at one of its joints shall be the length from the joint being considered to the structure's neutral point.
- skew and curvature.
- static and dynamic structural responses and their interaction, and
- approach pavement growth. Rigid approach pavements composed of cobblestone, brick, or jointed concrete will experience growth or substantial longitudinal pressure due to restrained growth.

For a curved superstructure that is laterally unrestrained by guided bearings, the direction of longitudinal movement at a bearing joint may be assumed to be parallel to the chord of the deck centerline taken from the joint to the neutral point of the structure.

16.3.2 Selection

The need of deck joints is dependant on several factors which include temperature range, possible range of foundation settlement, rigidity of the substructure, and type, material, and length of the superstructure.

Where a floorbeam design which can tolerate differential longitudinal movements resulting from relative temperature and live load response of the deck and independent supporting members, such as girders and trusses, is not practical, relief joints in the deck slab, movable joints in the stringers, and movable bearings between the stringers and floorbeams should be used. Movable joints may also be provided at abutments of single-span structures exposed to appreciable differential settlement. Intermediate deck joints should be considered for multiple-span bridges where differential settlement would result in significant overstresses.

The location and type of deck joints should be chosen to ensure proper drainage and minimize the possibility of discharging drainage on or near bridge bearing or superstructure and substructure components. The following general rules will help achieve that goal:

- Open deck joints should be avoided over roadways, railroads, sidewalks, other public areas and at the low point of sag vertical curves.

- Deck joints should be positioned with respect to abutment backwalls and wingwalls to prevent the discharge of deck drainage that accumulates in the joint recesses onto bridge seats.

- Open deck joints should be located only where drainage can be directed to bypass the bearings and be discharged directly below the joint.

- Closed or waterproof deck joints should be considered where joints are located directly above structural members and bearings that would be adversely affected by debris accumulation. Where deicing chemicals are used on bridge decks, sealed or waterproofed joints should be provided.

- For straight bridges, the longitudinal elements of deck joints, such as finger plates, curb and barrier plates, and modular joint seal support beams should be placed parallel to the longitudinal axis of the deck. For curved

and skewed structures, allowance for deck end movements consistent with that provided by the bearings need to be considered.

16.3.3 Design Requirements

Deck joints are required to accommodate all ranges of possible movements including those induced during construction. These movements include probable abutment and pier movements resulting from embankment consolidation and the effect of prestress-induced shortening on the width of seals and the size of bearings. Proper construction staging may minimize these effects.

The maximum and minimum joint gap openings at the factored extreme movement are specified in Article S14.5.3.2. Specifying a minimum allowable gap is intended to provide enough room to accommodate construction tolerances without closing the gap. A wide joint gap may cause undesirable riding characteristics and, therefore, a maximum gap width is specified. If bicycle traffic is anticipated, the use of special covering floor plates in shoulder areas should be considered.

Joints in concrete decks may be armored with steel shapes, weldments or castings. Such armor is required to be recessed below roadway surfaces and be protected from snowplows.

Additional precautions to prevent damage by snowplows should be considered where the skew of the joints coincides with the skew of the plow blades, typically 30° to 35°.

To ensure joint straightness and fit of components, the use of shapes, bars and, plates 12.5 mm or thicker, is required. To ensure appropriate fit and function, the Specification requires that following requirements be included in the contract documents:

- joint components be fully assembled in the shop for inspection and approval,
- joints and seals be shipped to the job site fully assembled, and
- assembled joints in lengths up to 18 000 mm be furnished without intermediate field splices.

During installation, deck joints need to be adjusted to account for installation temperature. In the absence of more

accurate information, the installation temperature is considered as the mean shade air temperature under the structure for the 48 hours prior to joint installation in concrete structures and for the 24 hours prior to joint installation for structures where the main members are made of steel. Connections of joint supports to primary members should allow horizontal, vertical and rotational adjustments. To facilitate joint adjustment and to take full advantage of the shop preassembly of the joints, construction procedures and practices should be developed to allow joint adjustment for installation temperatures without altering the orientation of joint parts established during shop assembly.

Except for short bridges where installation temperature variations would have only a negligible effect on joint width, plans for each bearing joint should include required joint installation widths for probable installation temperatures. For concrete structures, use of a concrete thermometer and measurement of temperature in expansion joints between superstructure units may be considered.

16.3.4 Joint Types

Special requirements and limitations of each of the following deck joint types are presented in Article S14.5.6.

- Open joints
- Closed Joints
- Waterproofed Joints
- Joint Seals
- Poured Seals
- Compression and Cellular Seals
- Sheet and Strip Seals
- Plank Seals
- Modular Seals

16.4 OVERVIEW OF BEARINGS

16.4.1 Load and Movement Capabilities

The bearing chosen for a particular application is required to have appropriate load and movement capabilities. Table 16.4.1-1 and Figure 16.4.1-1 may be used as a guide. Information in Table 16.4.1-1 is based on general judgment and observation, and there will obviously be some exceptions. Bearings which are listed as suitable for limited application may work if the load and rotation requirements are not excessive.

The following notations are applicable to Table 16.4.1-1:

S	=	Suitable
U	=	Unsuitable
L	=	Suitable for limited applications
R	=	May be suitable, but requires special considerations or additional elements such as sliders or guideways
Long.	=	Longitudinal axis
Trans.	=	Transverse axis
Vert.	=	Vertical axis

Table 16.4.1-1 - Bearing Suitability

Type of Bearing	Movement		Rotation About Bridge Axis Indicated			Resistance to Loads		
	Long.	Trans.	Long.	Trans.	Vert.	Long.	Trans.	Vert.
Plain Elastomeric Pad	L	L	S	S	L	L	L	L
Fiberglass Reinforced Pad	S	S	S	S	L	L	L	L
Cotton Duck Reinforced Pad	U	U	U	U	U	L	L	S
Steel-reinforced Elastomeric Bearing	S	S	S	S	L	L	L	S
Plane Sliding Bearing	S	S	U	U	S	R	R	S
Curved Sliding Spherical Bearing	R	R	S	S	S	R	R	S
Curved Sliding Cylindrical Bearing	R	R	U	S	U	R	R	S
Disc Bearing	R	R	S	S	L	S	R	S
Double Cylindrical Bearing	R	R	S	S	U	R	R	S
Pot Bearing	R	R	S	S	L	S	S	S
Rocker Bearing	S	U	U	S	U	U	R	S
Knuckle Pinned Bearing	U	U	U	S	U	S	R	S
Single Roller Bearing	S	U	U	S	U	U	R	S
Multiple Roller Bearing	S	U	U	U	U	U	U	S

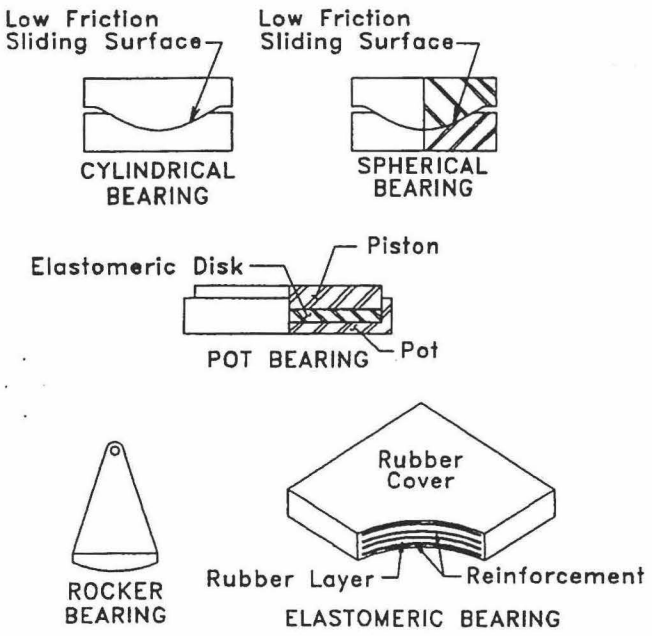


Figure 16.4.1-1 - Common Bearing Types

16.4.2 Forces in the Structure Caused by Restraint of Movement

Restraint of movement results in a corresponding force or moment in the structure. Therefore, horizontal forces due to rolling friction or shear deformation of a flexible element in the bearing, and moments induced in the bridge by restraint of movement at the bearings are required to be considered in the design. These force effects should be calculated taking into account the stiffness of the bridge and the bearing. In some cases, the bearing stiffness depends on time and temperature, as well as on the magnitude of the movement.

In addition to horizontal forces, both the substructure and superstructure are required to be designed for the factored moment, M_u, transferred by the bearings. M_u is the moment due to the tangential force caused by friction resistance at the curved surface of curved sliding bearings. This force acts about the center of the curved surface and is transmitted by the bearing. The moment imposed on individual components of the bridge structure may be different from M_u, depending on the location of the axis of rotation

16.4.3 Overview of Special Design Provisions for Bearings

16.4.3.1 METAL ROCKER AND ROLLER BEARINGS

Due to the fact that cylindrical bearings contain no deformable parts and are susceptible to damage if the superstructure rotates about an axis other than the axis of the line of bearings, the rotation axis of the bearing is required to be aligned with the axis about which the largest rotations of the supported member occur. In addition, provisions should be made to ensure that the bearing alignment does not change during the life of the bridge. In general, these bearings are unsuitable for bridges in which the axis of rotation may vary significantly under different situations, such as bridges with a large skew. They are also unsuitable for use in seismic regions because the transverse shear caused by earthquake loading can cause substantial overturning moment (SC14.7.1.1). Multiple roller bearings need to be connected by gearing to ensure that individual rollers remain parallel to each other and at their original spacing.

The allowable contact stresses in roller and rocker bearings are a function of the bearing diameter. Provisions for geometric requirements for the bearings and for the allowable contact stresses are presented in the Specification.

16.4.3.2 PTFE SLIDING SURFACES

PTFE, polytetrafluorethylene, may be used in sliding surfaces of bridge bearings to accommodate translation or rotation. PTFE may be provided in sheets or in mats woven from fibers. Unfilled sheets shall be made from PTFE resin alone, while filled sheets are made from PTFE resin uniformly blended with glass fibers, carbon fibers or other chemically-inert filler. Adding reinforcing fibers reduces creep, i.e., cold flow and wear. The filler content is required not to exceed 15% for glass fibers and 25% for carbon fibers. Sheet PTFE may contain dimples to act as reservoirs for silicone grease lubricant, effective to -35° C. The reservoirs should cover more than 20%, but less than 30% of the contact surface. These requirements are intended to provide enough lubrication while maintaining ample contact area. Dimples should not be placed to intersect the edge of the contact area so that the grease does not escape.

The friction coefficient depends on many factors, such as sliding speed, contact pressure, lubrication, temperature and properties such as the finish of the mating surface. The material properties which influence the friction coefficient are

not well understood, but the crystalline structure of the PTFE is known to be important, and it is strongly affected by the quality control exercised during the manufacturing process.

To ensure uniform bearing and to allow for wear, a minimum thickness of the PTFE layer is specified. During the first few cycles of movement, small amounts of PTFE transfer to the mating surface and contribute to the very low friction achieved subsequently. This initial wear is acceptable and desirable. However, PTFE continues to wear with time and movement and is exacerbated by deteriorated or rough surfaces. Continuing wear is undesirable because it usually causes higher friction and reduces the thickness of the remaining PTFE. Unlubricated, flat PTFE wears more severely than the lubricated material. The evidence on the rate of wear is tentative. High travel speeds, such as those associated with traffic movements, appear to be more damaging than the slow ones due to thermal movements. However, they may be avoided by placing the sliding surface on an elastomeric bearing which will absorb small longitudinal movements. No allowance for wear is made in the Specification due to the limited research available to quantify or estimate the wear as a function of time and travel. However, wear may ultimately cause the need for replacement of the PTFE, so it is wise to allow for future replacement in the original design.

Recessing is the most effective way of preventing creep in unfilled PTFE. The Specification requires that sheet PTFE which is not confined be bonded to a metal surface or to an elastomeric layer. It allows sheet PTFE confined in a recess in a rigid metal backing plate for one-half its thickness to be either bonded or unbonded. When the PTFE is bonded to the top cover layer of an elastomeric bearing, this layer should be relatively thick and hard to avoid rippling of the PTFE.

The PTFE is to be used in conjunction with a mating surface. Stainless steel is the most commonly used mating surface and is the material required by the Specification. Anodized aluminum has been sometimes used in spherical and cylindrical bearings produced in other countries and may be considered if acceptable documentation of experience is provided. The finish of the mating surface is extremely important, since it affects the coefficient of friction. Friction testing is required for the PTFE and its mating surface because of the many variables involved.

The service limit state coefficient of friction of the PTFE sliding surface is given in Table S14.7.2.5-1. However, it should be acknowledged that the value of the coefficient of

friction of PTFE sliding surfaces is affected by many factors. The friction factor decreases with lubrication and increasing contact stress, but increases with sliding velocity, increasing roughness of the mating surface and decreasing temperature. Dynamic friction is smaller than static friction, and the dynamic coefficient of friction is larger for the first cycle of movement than it is for later cycles. Where friction is required to resist loads, the design coefficient of friction under dynamic loading may be taken as not more than 10% of the specified values.

The friction factors used in the 1992 AASHTO Specifications are suitable for use with dimpled, lubricated PTFE. They are too small for the flat, dry PTFE commonly used in the United States. The values presented in the LRFD Specification were modified to recognize this fact. However, coefficients of friction, somewhat smaller than those given in the Specification, are possible with care and quality control. Testing is the only reliable method for certifying the coefficient of friction and bearing behavior and, based on test results, lesser values may be used.

Contact stresses between the PTFE and the mating surface are determined at the strength limit state and are assumed to vary linearly across the contact surface. The average contact stress is to be determined by dividing the load by the projection of the contact area onto a plane perpendicular to the direction of the load. The contact stress at the edge is to be determined based on the factored load and the extreme factored moment transferred by the bearing. The contact pressure must be limited to the allowables given in Table S14.7.2.5-1. These limits are intended to prevent excessive creep or plastic flow of the PTFE, which causes the PTFE disc to expand laterally under compressive stress and may, therefore, contribute to separation or bond failure.

16.4.3.3 BEARINGS WITH CURVED SLIDING SURFACES

Bearings with curved sliding surfaces consist of two metal parts with matching curved surfaces and a low friction sliding interface. The curved surfaces may be either cylindrical or spherical and the two surfaces must have equal nominal radii.

The geometry of a spherical bearing controls its ability to resist lateral loads, its moment rotation behavior and its frictional characteristics. The geometry is relatively easy to define, but it has some consequences which are not widely appreciated. The stress may vary over the contact surface of spherical or cylindrical bearings due to the fact that the

surfaces cannot be machined as accurately as a flat smooth surface. It is important that the radius of the convex and concave surfaces be within appropriate limits. If these limits are exceeded, the bronze may crack due to hard bearing contact or there may be excessive wear and damage due to creep or cold flow of the PTFE. The stress limits used in the Specification are based on average contact stress levels.

16.4.3.4 POT BEARINGS

The basic form of pot bearings provides for rotational movements. The rotational elements of the pot bearing consist of at least a pot, a piston, an elastomeric disc and sealing rings. Pot bearings may be provided with a PTFE slider to provide for both rotation and horizontal movement. They may also be provided by a guide to limit the movements to one direction.

Softer elastomers permit rotation more readily and are preferred. Therefore, the elastomeric disc is made from a compound based on virgin natural rubber or virgin neoprene conforming to the requirements of Section 18.2 of Division II. The nominal hardness shall lie between 50 and 60 on the Shore A scale. In addition, the elastomeric disk needs to be thick enough to prevent the seal from escaping and the bearing from locking up, even under the most adverse conditions. The two factors that affect the required thickness of the disk are the design angle and the internal diameter of the pot. The minimum allowable thickness of the elastomeric disk is six times the compression of the edge of the disk due to the rotation of the piston. In calculating the design angle, the designer should account for both the anticipated movements due to loads and those due to fabrication and installation tolerances, including the rotation imposed on the bearing due to out-of-level of other bridge components, such as undersides of prefabricated girders, and permissible misalignments during construction. Vertical deflection caused by compressive load should also be taken into account, because it will reduce the available clearance.

The allowable average stress on the elastomeric disc is largely limited by the ability of the seal to prevent escape of the elastomer. The Specification requires that the average stress on the elastomer at the service limit state not to exceed 24 MPa. This level of stress has been used as a practical upper limit for some years and most bearings have generally performed satisfactorily, but a few seal failures have occurred.

A seal is required to be used between the pot and the piston. Performance requirements under both service and

strength limit states are included in the Specification. At the service limit state, seals must be adequate to prevent escape of elastomer under compressive load and simultaneously applied cyclic rotations. At the strength limit state, seals must also be adequate to prevent escape of elastomer under compressive load and simultaneously applied static rotation.

The pot and piston are required to be made from carbon steel or from stainless steel. Corrosion resistant steels, such as ASTM A709M, Grade 345W, are not recommended for applications where they may come into contact with saltwater or be permanently damp, unless their whole surface is completely corrosion protected. The yield strength and hardness of the piston is required not to exceed that of the pot. This provision is meant to avoid wear or damage on the inside surface of the pot and the consequent risk of seal failure.

Under the least favorable combination of factored displacements and rotations, the dimensions of the elements of a pot bearing are required to satisfy the following requirements:

- The pot is deep enough to permit the seal and piston rim to remain in full contact with the vertical face of the pot wall.

- Contact or binding between metal components do not prevent further displacement or rotation.

16.4.3.5 STEEL REINFORCED ELASTOMERIC BEARINGS

Steel reinforced elastomeric bearings have greater strength and superior performance compared to other elastomeric bearings. Reinforced bearings consist of alternate layers of steel reinforcement and elastomer bonded together. In addition to any internal reinforcement, bearings may have external steel plates bonded to either, or both, the upper or lower elastomer layers.

The strength and stiffness of the bearing in resisting compressive load are controlled by the thickest layer. Therefore, the Specification requires that all internal layers of elastomer be of the same thickness with the top and bottom cover layers no thicker than 70% of the internal layers. Tapered elastomer layers cause larger shear strains and bearings made with them may fail prematurely due to delamination or rupture of the reinforcement. Therefore, tapered elastomer layers are not allowed and any taper should be achieved by tapering the reinforcement layers.

The shear modulus, G, is the most important material property for design, and it is, therefore, the preferred means of specifying the elastomer. The shear modulus of the elastomer at 23° C is to be used as the basis for design. Hardness has been widely used in the past because the test for it is quick and simple. However, the results obtained from the hardness are variable and correlate only loosely with shear modulus. If the material is specified by hardness, the shear modulus to be considered in the design is taken as the least favorable value from the range of values corresponding to that hardness. Specifying the material by hardness thus imposes a slight penalty in design.

The Specification requires that shear modulus of the elastomer be between 0.55 and 1.2 MPa and a nominal hardness between 50 and 60 on the Shore A scale. Materials with a nominal hardness greater than 60 are prohibited because they generally have a smaller elongation at break, greater stiffness and greater creep than their softer counterparts. This inferior performance is generally attributed to the larger amounts of filler present. Their fatigue behavior does not differ in a clearly discernible way from that of softer materials.

Creep varies from one compound to another and is generally more prevalent in harder elastomers, but is seldom a problem if high-quality materials are used. This is particularly true because the deflection limits are based on serviceability and are likely to be controlled by live load, rather than total load. The creep values given in Table 16.4.3.5-1 are representative of neoprene and are conservative for natural rubber.

Table 16.4.3.5-1 - Shear Modulus, G (MPa)

	Hardness (Shore A)		
	50	60	70
Shear Modulus @ 23° C	0.66-0.90	0.90-1.38	1.38-2.07
Creep deflection @ 25 years divided by instantaneous deflection	1.72	2.41	3.10

Shear modulus increases as the elastomer cools, but the extent of stiffening depends on the elastomer compound, time and temperature. It is, therefore, important to specify a material with low temperature properties which are appropriate for the bridge site. For the purposes of bearing design, all bridge sites are classified as being in temperature Zones A, B, C, D or E for which design data are given in Table 16.4.3.5-2. In the absence of more precise information, Figure 16.4.3.5-1 may be used as a guide in selecting the zone required for a given region. The low temperature classification is intended to limit the force on the bridge substructure to 1.5 times the service limit state design force under extreme environmental conditions.

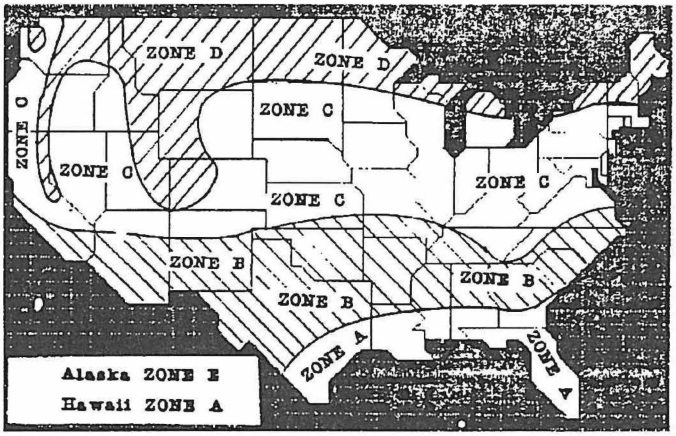


Figure 16.4.3.5-1 - Temperature Zones

Table 16.4.3.5-2 - Low Temperature Zones and Minimum Grades of Elastomer

LOW TEMPERATURE ZONE	A	B	C	D	E
50-Year Low Temperature (°C)	-18	-30	-35	-43	< - 43
Maximum number of consecutive days when the temperature does not rise above 0° C	3	7	14	N/A	N/A
Minimum low temperature elastomer grade	0	2	3	4	5
Minimum low temperature elastomer grade when special force provisions are incorporated	0	0	2	3	5

Steel reinforced bearings are designed to resist relatively high stresses. Their integrity depends on good quality control during manufacturing, which can only be assured by rigorous testing. Bearings designed to meet the Code provisions are required to be tested in accordance with the requirements for steel reinforced elastomeric bearings as specified in Article 18.2.7 of Division II of the Specification.

The shape factor of a layer of an elastomeric bearing is defined as the plan area of the layer divided by the area of perimeter free to bulge. The compressive resistance of the reinforced elastomeric bearings is a function of the applied shear stress and the shape factor. Holes are strongly discouraged in steel-reinforced bearings. However, if holes are used, their effect should be accounted for when calculating the shape factor because they reduce the loaded area and increase the area free to bulge.

The relationship between the shear stress and the applied compressive load depends directly on shape factor, with higher shape factors leading to higher capacities. If movements are accommodated by shear deformations of the elastomer, they cause shear stresses in the elastomer. These add to the

shear stresses caused by compressive load, so a lower load limit is allowed in case of combined shear and compressive loading.

Increases in the load to simulate the effects of impact are not required. This is because the impact stresses are likely to be only a small proportion of the total load, and also because the compressive stress limits are based on fatigue damage, the limits of which are not clearly defined.

Laminated elastomeric bearings have a nonlinear load deflection curve in compression. In the absence of information specific to the particular elastomer to be used, Figure 16.4.3.5-2 may be used as a guide.

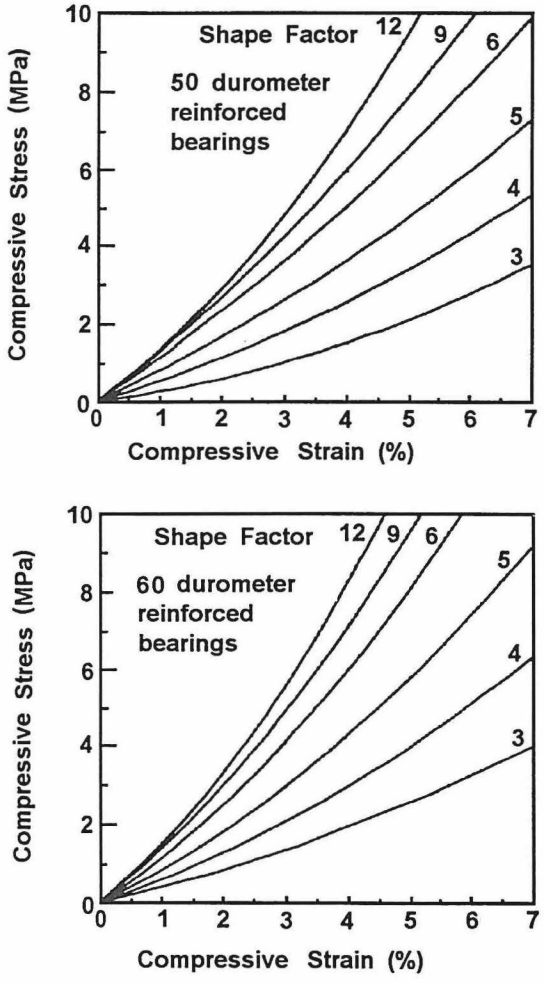


Figure 16.4.3.5-2 - Stress-Strain Curves

Deflections of elastomeric bearings due to total load and to live load alone must be considered separately. Limiting instantaneous deflections is important to ensure that deck

joints and seals are not damaged. A maximum relative deflection across a joint of 3.2 mm is suggested by the Specification. However, joints and seals that are sensitive to relative deflections may require tighter limits.

Long-term deflections should be considered where joints and seals between sections of the bridge rest on bearings of different design, and when estimating redistribution of forces in continuous bridges caused by settlement. Provided high-quality materials are used, the effects of creep of the bearings are unlikely to cause problems.

The maximum shear deformation of the bearing, Δ_s, is required to be taken as the extreme relative displacement between the superstructure and the substructure at the bearing location, determined for the service limit state. The magnitude of the displacements is a function of the length and geometry of the superstructure, type of materials, temperature range and distribution, substructure stiffness, construction procedures and the applied loads. This modification is required due to the fact that pier deflections sometimes accommodate a significant portion of the bridge movement, and this may reduce the movement which must be accommodated by the bearing. Construction methods may increase the bearing movement because of poor installation tolerances or poor timing of the bearing installation. If the bridge girders are lifted to allow the bearings to realign after some of the girder shortening has occurred, that may be accounted for in design. If a low friction sliding surface is installed, it will allow the movement of the superstructure relative to the substructure. In such a case, Δ_s need not be taken larger than the deformation corresponding to first slip. In any case, maximum shear deformation of the bearing must not exceed half the total elastomer thickness in order to avoid rollover at the edges and delamination due to fatigue.

Fatigue tests which formed the basis for the maximum shear deformation provision were conducted to 20 000 cycles, which represents one expansion/contraction cycle per day for approximately 55 years. The provisions will, therefore, be unconservative if the shear deformation is caused by high-cycle loading due to braking forces or vibration. The maximum shear deformation due to these high-cycle loadings should be restricted to no more than +/- 0.10 the total elastomer thickness, unless better information is available. At this strain amplitude, the experiments showed that the bearings have an essentially infinite fatigue life.

Limits on the stress in the elastomer, at the service limit state, under the effect of combined compression and rotation are presented in Article S14.7.5.3.5. These limits address two conditions. No point in the bearing is allowed to undergo net uplift, and excessive compressive stress on an edge is prevented. Uplift is required to be prevented because strain reversal in the elastomer significantly decreases its fatigue life. The interaction between compressive and rotation capacity in a bearing is illustrated in Figure 16.4.3.5-3, which is analogous to the interaction diagram for a reinforced concrete column. The following definitions apply to Figure 16.4.3.5-3:

n = number of interior layers of elastomer

h_{ri} = height of the i_{th} elastomer layer (mm)

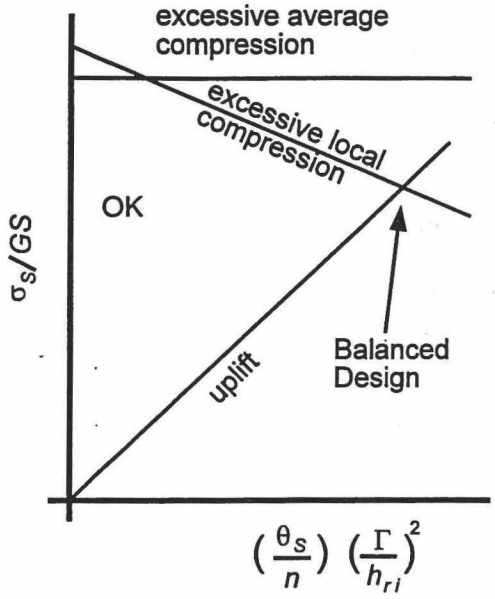
σ_s = stress in elastomer (MPa)

Γ = length of pad if rotation is about its transverse axis, or width of pad if rotation is about its longitudinal axis (mm)

Θ_s = rotation about any axis of the pad (RAD)

Since a high shape factor is best for resisting compression, but a low one accommodates rotation most readily, the best choice represents a compromise between the two. The "balanced design" point in Figure 16.4.3.5-3, where uplift and compressive stress are simultaneously critical, will in many cases provide the most economical solution for a given plan geometry. Table 16.4.3.5-1 gives coordinates of the balance point for different bearing shapes.

Design rotations should be taken as the maximum sum of the effects of initial lack-of-parallelism and subsequent girder end rotation due to imposed loads and movements. A rectangular bearing should normally be oriented so that its long side is parallel to the axis about which the largest rotation occurs. The critical location in the bearing for both compression and rotation is then at the mid-point of the long side. If rotation occurs about both axes, uplift and excessive compression should be investigated in both directions.



← Not Found in TSPs ?

Figure 16.4.3.5-3 Elastomeric Bearing - Interaction Between Compressive Stress and Rotation Angle

Table 16.4.3.5.2 - Steel Reinforced Elastomeric Bearings - Balanced Design

Bearing Type	$\frac{\sigma_s}{GS}$	$\left(\frac{\theta_s}{n}\right)\left(\frac{\Gamma}{h_{ri}}\right)^2$
Fixed Rectangular	1.636	1.636
Movable Rectangular	1.364	1.364
Fixed Circular	2.00	2.667
Movable Circular	1.667	2.222

Bearings are also required to be investigated for instability at the service limit state load combinations. Equations to determine the limiting average stress for both rectangular and circular bearings are presented in Article S14.7.5.3.7. The average compressive stress given by these equations is limited to half the predicted buckling stress calculated using theoretical equations and modified to account for changes in geometry during compression. They were calibrated against experimental results. This provision will

permit taller bearings and reduced shear forces compared to those permitted under previous specifications.

The reinforcement should sustain the tensile stresses induced by compression of the bearing. With the present load limitations, the minimum steel plate thickness practical for fabrication will usually provide adequate strength. Equations to calculate the minimum required reinforcement thickness are presented in Article S14.7.5.3.7. Holes in the reinforcement cause stress concentrations and, therefore, holes should be discouraged. If holes exist in the reinforcement, the minimum thickness is required to be increased by a factor equal to twice the gross width divided by the net width. This increase in steel thickness accounts for both the material removed and the stress concentrations around the hole.

16.4.3.6 ELASTOMERIC PADS

Elastomeric pads have characteristics which are different from those of steel reinforced elastomeric bearings. The Code provisions for elastomeric pads apply to the design of:

- plain elastomeric pads, PEP,
- pads reinforced with discrete layers of fiberglass, FGP, and
- pads reinforced with closely spaced layers of cotton duck, CDP

Plain elastomeric pads are weaker and more flexible because they are restrained from bulging by friction alone. Slip inevitably occurs, especially under dynamic loads, causing larger compressive deflections and higher internal strains in the elastomer.

In pads reinforced with layers of fiberglass, the reinforcement inhibits the deformations found in plain pads. However, elastomers bond less well to fiberglass and the fiberglass is weaker than steel, so the fiberglass pad is unable to carry the same loads as a steel reinforced pads. Fiberglass pads have the advantage that they can be cut to size from a large sheet of vulcanized material.

Pads reinforced with closely spaced layers of cotton duck typically display high compressive stiffness and strength, obtained by the use of very thin elastomeric layers. However, the thin layers also give rise to very high rotational stiffness which could easily lead to edge loading and a higher shear

stiffness than that to be found in bearings with thicker layers. This high shear stiffness leads to larger forces in the bridge, unless it is offset by the use of the PTFE slider on top of the elastomeric pad.

The elastomer needs to satisfy the same material requirements required for reinforced elastomeric pads, except that a nominal hardness between 50 and 70 on the Shore A scale is allowed. The Specification requires the shear force on the structure induced by deformation of the elastomer be based on a G value not less than that of the elastomer at 23° C. Effects of relaxation need not be considered. In general, the three types of pad, PEP, FGP and CDP behave differently, so information relevant to the particular type of pads should be used for design.

16.4.3.7 BRONZE OR COPPER ALLOY SLIDING SURFACES

Bronze or copper alloy sliding surfaces have a long history of application in the United States with relatively satisfactory performance of the different materials. However, there is virtually no research to substantiate the properties and characteristics of these bearings. Successful past experience is the best guide currently available.

Bronze or copper alloy may be used for:

- flat sliding surfaces to accommodate translational movements,
- curved sliding surfaces to accommodate translation and limited rotation, and
- pins or cylinders for shaft bushings of rocker bearings or other bearings with large rotations.

The magnitude of the horizontal forces transmitted to the substructure is a function of the vertical load and the coefficient of friction. The best available experimental evidence suggests that lubricated bronze can achieve a coefficient of friction on the order of 0.07 during its early life while the lubricant projects above the bronze surface. The coefficient of friction is likely to increase to approximately 0.10 after the surface lubrication wears away and the bronze starts to wear down into the recessed lubricant. Copper alloy or plain bronze would cause considerably higher friction. In the absence of better information, conservative coefficients of friction of 0.1 for self-lubricating bronze components and 0.4 for other types are recommended for design.

16.4.3.8 DISC BEARINGS

A disc bearing functions by deformation of a Polyether Urethane disc, which should be stiff enough to resist vertical loads without excessive deformation and yet flexible enough to accommodate the imposed rotations without lift-off or excessive stress on other components such as PTFE. Elements of a disc bearing are proportioned such that hard contact between metal components, which prevents further displacement or rotation, will not occur under the least favorable combination of design displacements and rotations at the strength limit state. Furthermore, the urethane disc should be positively located to prevent its slipping out of place.

The elastomeric disc is made from a compound based on Polyether Urethane, using only virgin materials. The appropriate material properties must be selected as an integral part of the design process, because the softest urethanes may require a limiting ring to prevent excessive compressive deflection, whereas the hardest ones may be too stiff causing an excessive resisting moment. Also, harder elastomers generally have higher ratios of creep to elastic deformation.

16.4.3.9 GUIDES AND RESTRAINTS

Guides may be used to prevent movement of a bearing in one direction. If the horizontal force becomes too large to be carried reliably and economically on a guided bearing, a separate guide system may be used. Restraints may be used to permit only limited movement in one or more directions. Guides and restraints shall have a low-friction material at their sliding contact surfaces.

16.4.3.10 OTHER BEARING SYSTEMS

The Specification allows the use of bearing systems, other than those discussed above, if approved by the bridge owner. In such case, the bearings should be carefully tested to determine whether they are adequate to resist the forces and deformations imposed on them at the service, strength and extreme event limit states without material distress and without inducing deformations detrimental to their proper functioning. The tests are required to be designed to demonstrate any potential weakness in the system under individual compressive, shear or rotational loading, or combinations thereof. Testing under sustained and cyclic loading is also required. Two problems are often faced in planning test programs, first, the availability of bearings test equipment, and, second, the high cost of testing that may be prohibitive.

LECTURE 17A - TIMBER DECKS

17A.1 OBJECTIVE OF THE LESSON

The objective of this lesson is to introduce the provisions for the design of stress-laminated timber decks and steel girder splices to the student through design examples. In addition, an example for the calculations of thermal stresses in a two-span continuous concrete box girder is also included.

17A.2 STRESS-LAMINATED DECK EXAMPLE

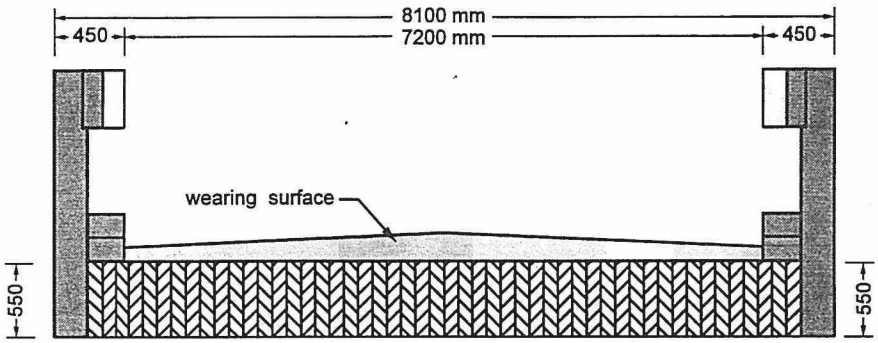


Figure 17A.2-1 - Bridge Cross-Section

Assumptions

Simple Span Bridge

Span length = 12 000 mm

Total width = 8100 mm

Clear width = 7200 mm

Number of traffic lanes = 2

Thickness of bituminous wearing surface = 75 mm

Thickness of future wearing surface = 40 mm

Density of wearing surface = 2250 kg/m^3

Density of hard wood = 960 kg/m^3

Material: Douglas Fir-Larch, 75 mm thick laminations

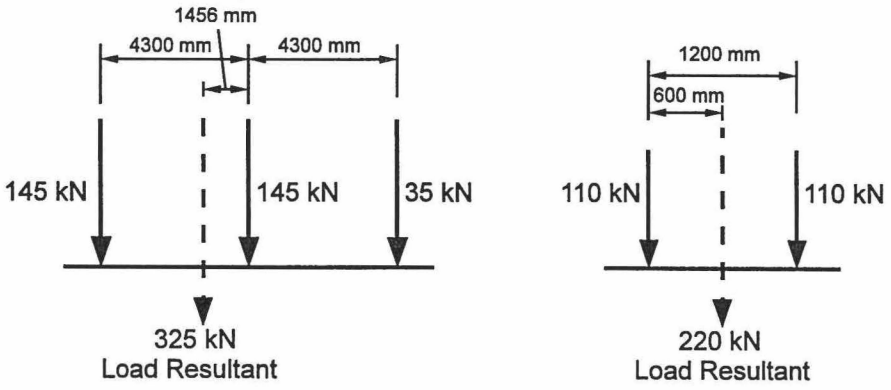
Determination of minimum deck thickness:

The Specification does not give a limit for the depth/span ratio for timber structures. However, the behavior of stress-laminated decks is close to that of simple span concrete slab bridges for which the Specification gives an optional minimum thickness of 1.2 (S + 3000/30). For a span length of 12 000 mm, the required thickness would be 600 mm. For the purpose of this example, a deck thickness of 550 mm was assumed as the actual depth of the laminates. Notice that the actual depth may be different from the nominal depth based on the type of wood cutting procedures.

Design Moments

For a simple span, maximum moments due to the weight of the structure, future wearing surface and uniform lane load occur at mid-span. Maximum moments produced by a series of concentrated loads, such as the case of the design truck and the design tandem, occur at a distance from mid-span equal to half the distance between the resultant of the concentrated loads and the load nearest to the resultant. The sections of maximum moments from the design truck and design tandem are located at 727.0 and 300.0 mm from mid-span, respectively.

In general, as the span gets longer, the difference between the maximum moments and the mid-span moment gets smaller. For preliminary analysis, considering only the mid-span section should give sufficiently accurate design. Therefore, only mid-span moments were considered in this example.



(a) design truck (b) design tandem

Figure 17A.2-2 - Location of live load resultant

All dead and live load moments are calculated for a 1.0 mm wide design strip.

Moments Due to Weight of Deck

Intensity of the load = (960 x 10^{-9} Kg/mm^3) x 550 mm x 9.81 m/sec^2 = 5.18 x 10^{-3} N/mm

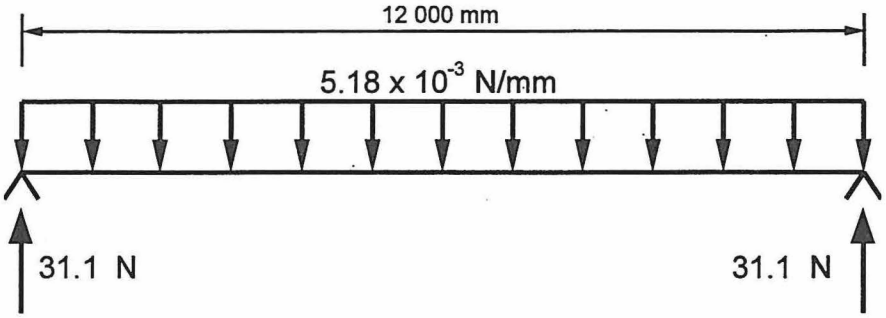


Figure 17A.2-3 - Design Dead Load

Mid-Span Moment = 5.18 x 10^{-3} x (12 000)^2/8 = 93 240 N·mm

Moment Due to Wearing Surface

Combined thickness of initial and future wearing surfaces = 75 + 40 = 115 mm

Intensity of uniform load = (2250 x 10^{-9}) x 115 x 9.81 = 2.54 x 10^{-3} N/mm

Mid-Span Moment = 45 720 N·mm

Live Load Moments

Width of Equivalent Strip

For decks where the primary load path is parallel to traffic, Article S4.6.2.3 applies (S4.6.2.1.3). Considering the following notations:

E = width of equivalent strip per lane (mm)

L_1 = the lesser of actual span and 18 000 mm

W_1 = lesser of actual width of the bridge or 9000 mm for single-lane loading, or 18 000 mm for multi-lane loading

W = actual width of the bridge

N_L = number of design lanes

Case of single-lane loaded:

$$E = 250 + 0.42\sqrt{L_1 W_1}$$

$$E = 250 + 0.42\sqrt{12\,000 \times 8100} = 4391\ mm$$

Case of multi-lane loading:

$$W/NL = \frac{8100}{2} = 4050\ mm$$

$$E = 2100 + 0.12\sqrt{L_1 W_1} \leq \frac{W}{N_L}$$

$$E = 2100 + 0.12\sqrt{12\,000 \times 8100} = 3283 mm < \frac{W}{N_L}$$

The total moments produced by the live loads on one traffic lane are divided by the equivalent strip width to determine the live load moment per unit width. To obtain maximum moments, the smaller of the two values of E is considered in the analysis.

Therefore, use E = 3283 mm

Notice that the multiple presence factors are included in the expressions used to determine the equivalent strip width. Therefore, no multiple presence factor is applied to the force effects (S3.6.1.1.2).

The equivalent strip width in this example, 3283 mm, is wider than the actual width of the design lane. In this case, the calculated strip width may be used to determine the force effects of the design truck, design tandem and design lane load per unit width. For shorter spans, the equivalent strip width per lane may be smaller than the actual width of the design lane load. In this case, the calculated equivalent strip width should be used to determine the design truck and design tandem force effects per unit width and the actual width of the

design lane loads used to determine the force effect of the uniform load per unit width.

Dynamic load allowance for wood components
= 0.5 x 0.33 = 0.165 (S3.6.2.3)

The dynamic load allowance is applied only to the design truck and design tandem.

a. Moments due to design lane loading

Design lane load per traffic lane = 9.3 N/mm (S3.6.2.3)

Intensity of load for a 1.0 mm wide design strip = 9.3/3283 = 2.83×10^{-3} N/mm

Mid-Span Moment = 50 940 N·mm

b. Moments due to the design truck

Dynamic load allowance = 0.165

At mid-span (see Figure 4), for 1.0 mm wide design strip

M = 1.165 x (201 920 x 6000 - 145 000 x 4300)/3283 = 208 664 N·mm

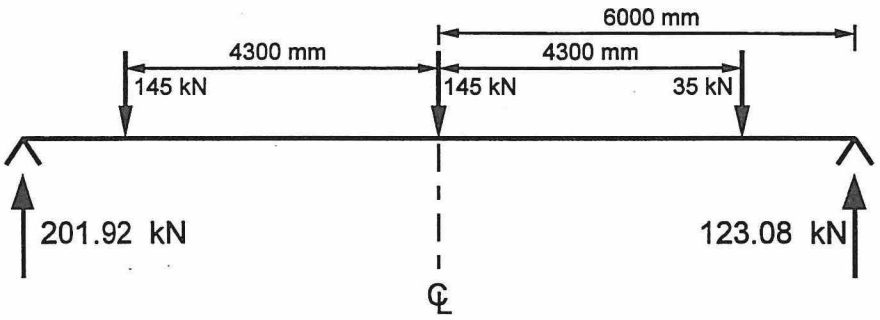


Figure 17A.2-4 - Position of the Design Truck for Moment Calculation

c. Moment due to the design tandem

Dynamic load allowance = 0.165

At mid-span (see Figure 5), for 1.0 mm design strip

M = 1.165 x 99 000 x 6000/3283 = 210 786 N·mm

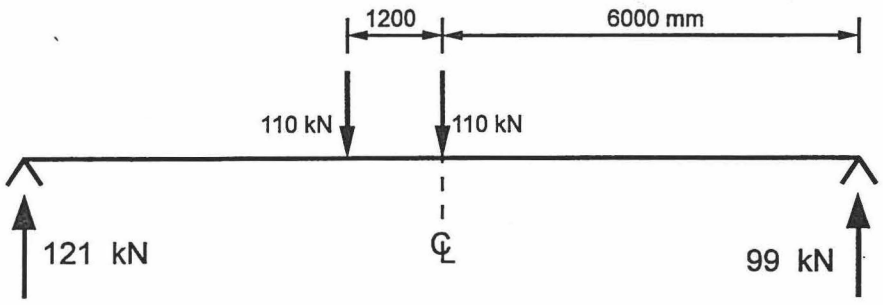


Figure 17A.2-5 - Position of the Design Tandem for Moment Calculations

Design Factored Moment

Strength I Load Combination

Load factors (Table S3.4.1.1):

Weight of superstructure: 1.25

Weight of wearing surface: 1.5

Live load: 1.75

The moment from the design tandem at mid-span is larger than the moment from the design truck. Therefore, live load moments are calculated for lane load + tandem load.

Factored moment = 1.25 x 93 240 + 1.5 x 45 720 + 1.75 x (50 940 + 210 786)
= 643 150 N·mm

Ductility, Redundancy and Operational Importance Factors:

- By nature, timber decks develop significant deformations prior to collapse. Therefore, the structure is considered a ductile structure.

  η_D = 0.95 for ductile structures at the strength limit state

  η_D = 1.0 for all other limit states (S1.3.3)

- The load path in stress-laminated decks is not a continuous path due to the presence of butt joints of the laminates. If any laminate breaks under load, the load will be redistributed through a different path as it does at the locations of butt joints. Therefore, the failure of

some laminates does not necessarily mean that the deck system will collapse and, hence, redundant load paths may be considered to exist. For redundant load paths:

η_R = 0.95 for redundant load paths at the strength limit state

η_R = 1.0 for all other limit states (S1.3.4)

- Assume that the bridge is deemed not of operational importance. Therefore, $\eta_i \geq 0.95$ (S1.3.5). For the purpose of this example, assume $\eta_i = 0.95$.

$\eta = \eta_D \eta_R \eta_i > 0.95$ (S1.3.2.1)

$\eta = 0.95 \times 0.95 \times 0.95 = 0.857$

Minimum allowed $\eta = 0.95$ (S1.3.2.1)

Therefore, use $\eta = 0.95$

Design factored moment, $M_u = 643\ 150 \times 0.95$

$M_u = 611\ 200$ N·mm

Moment Resistance

Assume No. 2 visually graded Douglas Fir-Larch with 75 mm thick laminations.

Base resistance for bending, $F_{bo} = 17$ MPa (S8.4.1.1.4)

Base modulus of elasticity, $E_o = 10\ 000$ MPa (S8.4.1.1.4)

Size factor, $C_F = 1.0$ (S8.4.4.2)

Moisture content factor, $C_M = 1.0$ (S8.4.4.3)

Deck factor, $C_D = 1.5$ (S5.4.4.4)

Nominal resistance, $F_b = F_{bo} C_F C_M C_D$ (S8.4.4.1)
$= 17 \times 1.0 \times 1.0 \times 1.5 = 25.5$ MPa

Nominal modulus of elasticity $E = E_o C_M$ (S8.4.4.1)

$= 10\ 000$ MPa

For a 1.00 mm wide strip of a 550 thick deck

$M_n = F_b S$

$$S = \frac{bt^2}{6} = \frac{1.0\,x\,(550)^2}{6} = 50\ 417\ mm^3$$

$M_n = 25.5 \times 50\ 417 = 1\ 286\ 000$ N·mm

Resistance factor for moment $\phi = 0.85$ (S8.5.2.2)

$M_R = \phi\, M_n = 1\ 093\ 000$ N·mm

$M_R > M_u = 611\ 000$ N·mm OK

Design for Shear

Shear need not be checked in stress-laminated deck. (S9.9.3.2)

Deflections

Maximum deflections from different loads do not occur at the same section. For example, maximum deflection from the truck load occurs at a section near the center of the span, while maximum deflection from dead loads and lane loads occur at mid-span. For simplicity, all deflections are calculated at mid-span.

For the purpose of this example, the effect of the curbs and barriers on the stiffness is ignored. The additional deflections due to the weight of prestressing bars and anchor plates are also ignored.

Camber

Dead Load Deflection:

Deflection due to self-weight of the deck, wearing surface and future wearing surface:

$$\Delta_{DL} = \frac{5}{384}\frac{WL^4}{EI}$$

where:

W = weight of (deck + wearing surface, including future wearing surface)

= $(5.18 + 2.54) \times 10^{-3} = 7.72 \times 10^{-3}$ N/mm

L = 12 000 mm

E = modulus of elasticity = 10 000 MPa

$$I = \frac{bt^3}{12} = \frac{1.0\,x\,550^3}{12} = 13\ 864\ 583\ mm^4$$

Δ_{DL} = 15.0 mm

Camber the deck a distance = $3\Delta_{DL}$ = 45 mm (S8.12.3)

Optional Criteria for Live Load Deflections

Assume both design lanes loaded and the stiffness of the full-width of the bridge (8100 mm) is active in resisting deflection. (S2.5.2.6.2)

Deflection due to uniform lane load:

Load intensity from two lanes loaded = 2 x 9.3 = 18.6 N/mm

$$I = \frac{bt^3}{12} = \frac{8100\,x\,550^3}{12} = 1.123\,x\,10^{11}\ mm^4$$

$$\Delta_{lane\ load} = \frac{5\,WL^4}{384\,EI} = 4.5\ mm$$

Deflection due to design truck:

The dynamic load allowance is to be considered when calculating the deflection due to truck load. For the position of load shown in Figure 17A.2-6,

Δ_{truck} = 16.4 mm including 16.5% dynamic load allowance

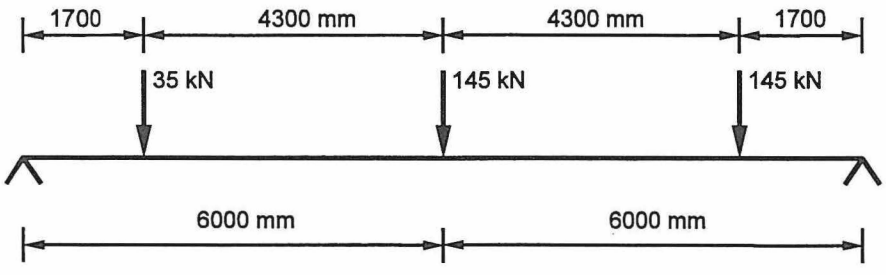


Figure 17A.2-6 - Position of Design Truck for Deflection Calculations

Notice that the above position of the load was chosen for simplicity. Shifting the load may result in slightly larger deflections at mid-span and may be considered in the design of actual structure.

Design live load deflection: (S2.5.2.6.2)

Maximum design live load deflection is:

- Deflection due to design truck alone = 16.4 mm (1)
- Deflection due to uniform lane loading + 25% of the design truck = 4.5 + 16.4/4
  = 8.6 mm (2)

Maximum design LL deflection = larger of (1) and (2) = 16.4 mm

Limit on LL deflection in the optional deflection criteria = 1/425 (S2.5.2.6.2)

Maximum deflection/span = 16.4/12 000 = 1/732 < 1/425 OK

Design of prestressing system: (S9.9.5.6.3)

Assume prestressing system similar to that shown in Figure 17A.2-7.

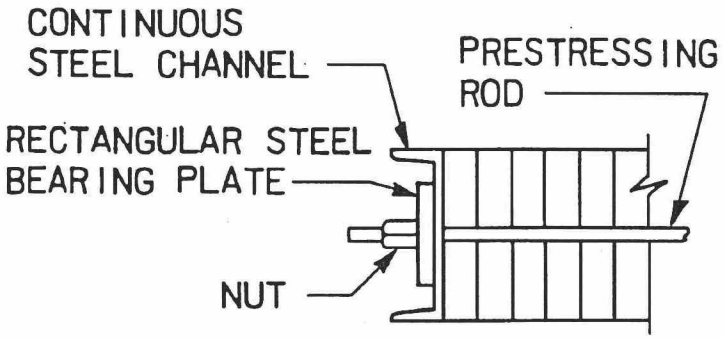


Figure 17A.2-7 - Channel Bulkhead Anchorage Configuration

Requirements:

$$R_{sw} = \frac{A_s}{sh} \leq 0.0016$$

P_{pt} = 0.7 hs

$P_{Bu} = \phi F A_B \geq P_{pt}$

where:

R_{sw} = the steel-wood ratio

s = spacing of the prestressing elements (mm)

h = depth of deck (mm)

A_s = area of steel bar or strand (mm^2)

P_{pt} = the prestressing force per prestressing element (N)

A_B = the effective bearing area

P_{BU} = factored compressive resistance of the wood under bulkhead (N)

ϕ = resistance factor for compression perpendicular to grain = 0.9 (S8.5.2.2)

F = 2.93 for Douglas Fir-Larch (Table S9.9.5.6.3-1)

Spacing of prestressing element is required to be greater or equal to 2.5 times the deck thickness or 15 times the hole diameter. (S9.9.5.4)

Assume #35 prestressing bars with tensile strength, f_{pu}, of 1035 MPa were used.

Bar diameter = 35.7 mm

Assume hole diameter, d = 50 mm

Bar cross-sectional area, A_s = 1000 mm^2

Assume bar spacing, S = 1400 mm

$$\frac{s}{h} = \frac{1400}{550} = 2.55 > 2.5 \qquad \text{(S9.9.5.4)}$$

$$\frac{s}{d} = \frac{1400}{50} = 28 > 15 \qquad \text{(S9.9.5.4)}$$

$$\frac{d}{h} = \frac{50}{550} = 0.091 < 0.2 \qquad \text{(S9.9.5.4)}$$

Allowable stress at transfer, $f_{pt} = 0.7\ f_{pu} = 724.5$ MPa

Allowable force per prestressing element, P_{pt}
$= 724.5 \times 1000 = 724\ 500$ N

Required force per prestressing bar = 0.7 hs =
$0.7 \times 550 \times 1400 = 539\ 000$ N < allowable force. Therefore, prestressing bar size is sufficient.

Assume continuous channel bulkhead (Figure 17A.2-7) depth of the channel should be as close as possible to the depth of the deck.

Assume MC460 x 86 (equivalent to MC18 x 58)

depth = 457 mm

Factored compressive resistance of wood under bulkhead, P_{Bu}
$= 0.9 \times 2.93 \times 1400 \times 457 = 1.69 \times 10^6$ N > P_{pt} OK

The channel needs to be checked as a continuous beam of Spans S = 1400 mm under the effect of bearing pressure:

Load factor for post-tensioning jacking force, $\gamma = 1.2$
(S3.4.3)

Bearing load on channel =

$$\frac{\gamma P_{pt}}{S} = \frac{1.2 \times 539\,000}{1400} = 462\ N/mm$$

For continuous spans 1400 mm each:

$$M = M_{pos} \simeq M_{neg} \simeq \frac{w\ell^2}{10} = \frac{462(1400)^2}{10} = 90.55 * 10^6\ N \cdot mm$$

Assume $f_y = 350$ MPa

$S = 87.1 \times 10^3\ mm^3$

$M_R = f_y S = 350 \times 87.1 \times 10^3 = 30.49 \times 10^6$ N·mm

$M_R < M$ NG

A stronger bulkhead is needed.

Since the largest available channel was tried and was found not sufficient, a special built-up section may be required.

LECTURE 17B - SHIP COLLISION

17B.1 OVERVIEW OF VESSEL COLLISION PROVISIONS

17B.1.1 Background Information on the Development of Vessel Collision Guidelines

Earlier editions of the AASHTO Standard Specifications for Highway Bridge Design do not contain guidelines for the design of bridges with respect to vessel collision. It was only after a marked increase in the frequency and severity of vessel collisions with bridges that studies of the vessel collision problem have been initiated. Catastrophic bridge failures due to vessel collision, such as those of the Causeway Bridge over Lake Pontchartrain, Louisiana (1964, 1974); Sidney Lanier Bridge, Georgia (1974); Tasman Bridge, Australia (1975), and Almo Bridge, Sweden (1980), have claimed several lives each. Overall, an average of one vessel bridge collision has occurred worldwide every year. The 1980 collapse of the Sunshine Skyway Bridge over Tampa Bay, Florida, which caused 35 fatalities was the impetus for several in-depth studies. Important steps in the development of modern ship collision design principles and specification include:

- In 1983, a "Committee on Ship/Barge Collision", appointed by the Marine Board of the National Research Council in Washington, D. C., published the findings of its study on the risk and consequences of ship collisions with bridges spanning navigable coastal waters in the U.S.

- In June 1983, a colloquium on "Ship Collision with Bridges and Offshore Structures" was held in Copenhagen under the auspices of the International Association for Bridge and Structural Engineering (IABSE) to bring together latest developments on the subject.

- In 1984, "Criteria for the Design of Bridge Piers with Respect to Vessel Collision in Louisiana Waterways" was developed for the Louisiana Department of Transportation and Development and the Federal Highway Administration by Modjeski and Masters, Inc., Consulting Engineers.

- In 1988, a pooled-fund research project was sponsored by 11 states and the Federal Highway Administration to develop vessel collision design provisions applicable to

all of the U. S. The final report of this project, for which Greiner Engineering was the principal investigator, was adopted by AASHTO as a Guide Specification in February 1991.

- In 1993, "Ship Collision with Bridges - The Interaction between Vessel Traffic and Bridge Structures" structural engineering document was published by the International Association for Bridge and Structural Engineering (IABSE). This document includes the latest research findings to-date.

Research work in the area of vessel collision with bridges continues. Several aspects, such as the magnitude of the collision loads to be used in design, are not yet well established. As further research results become available, appropriate code updates could be expected.

17B.1.2 Background Information on the Main Factors Affecting the Vessel Collision Problem

The main factors affecting the risk and the consequences of vessel collisions are related to the vessel, waterway and bridge characteristics.

17B.1.2.1 VESSEL CHARACTERISTICS

General knowledge on the operation of vessels and their characteristics is essential for safe bridge design. The types of vessels using the U. S. waterways include ships and barges.

The consequences of a ship or barge collision with a bridge are affected by factors, such as the vessel size, type, loading condition, speed and direction.

17B.1.2.1.1 Ships

Ships use deep draft waterways. Their size may be determined based on the deadweight tonnage (DWT) (1 tonne = 1000 kg). The DWT is the mass of cargo that the vessel can carry when fully loaded. There are three main classes of ships: bulk carriers, product carriers/tankers and freighter/containers. Appendix A, Typical Ship Characteristics, contains information on ship profiles, dimensions and sizes. The ship characteristics are listed as a function of the class of ship and its deadweight tonnage. They include typical ship lengths, beams and bow depths. Ship drafts, displacement tonnages, mast and deck house clearance heights are given for both fully-loaded and ballasted conditions. These dimensions

are typical values, and due to the large variety of existing vessels, they should be regarded as averages.

The steering of ships in coastal waterways is a difficult process. It involves constant communications between the ship master, the helmsman and the engine room. There is a time delay before a ship starts responding to an order to change speed or course, and the response of the ship itself is quite slow. Therefore, the ship master has to be familiar with the waterway and be aware of obstructions, navigation and weather conditions in advance. Very often local pilots are used to navigate the ships through a given portion of a coastal waterway. When the navigation conditions are difficult, tug boats are used to assist ships in making turns. Ships need speed to be able to steer. Therefore, the speed of ships going downstream can be quite high, at least 8 km/hr over the stream velocity. Fully-loaded ships are more maneuverable, and in deep water they are directionally stable and can make turns with a radius equal to 1 to 2 times the length of the ship. However, as the underkeel clearance decreases to less than half the ship's draft, many ships tend to become directionally unstable, which means that they require constant steering to keep them traveling in a straight line. In the coastal waterways of the U. S., the underkeel clearance of many laden ships may be far less than this limit, in some cases as small as 5% of the ship's draft. Ships riding in ballast with shallow draft are less maneuverable than loaded ships, and, in addition, they can be greatly affected by winds and currents.

17B.1.2.1.2 Barges

Barges use both the deep draft and the shallow draft waterways. The majority of the existing bridges cross shallow draft waterways where the vessel fleet comprises barges (with tugs or tows) only. The size of barges is usually determined based on the cargo carrying capacity in U. S. tons (1 U. S. ton = 910 kg). The types of inland barges include open and covered hoppers, deck barges and tank barges. They are rectangular in shape and their dimensions are quite standard so they can travel in tows. The number of barges per tow can vary from 1 to over 20 and their configuration is affected by the conditions of the waterway. In most cases, the tows are pushed by a tug. Appendix B, Typical Barge Characteristics, contains information on barge dimensions and capacity, as well as barge tow configurations.

It is very difficult to control and steer barge tows, especially in waterways with high stream velocities and cross currents. Taking a turn in a fast waterway with high current is

a dangerous undertaking. Sometimes bridge piers and fenders are used to line-up the tow before the turn. The licensing requirements for tug boat operators are much less stringent than those for pilots or ship masters. Bridges located in a high velocity waterway near a bend in the channel will probably be hit by barges several times during their lifetime. In general, there is a high likelihood that any bridge element that can be reached by a barge will be hit during the life of the bridge.

17B.1.2.2 WATERWAY CHARACTERISTICS

The determination as to whether a waterway is navigable is usually made by the U. S. Coast Guard.

The characteristics of the waterway in the vicinity of the bridge site also have a great influence on the risk of vessel collision. The width and depth of the navigation channel, the stream velocity, the channel alignment, its cross-section geometry, the water elevation and the hydraulic conditions at the bridge site are all factors that must be taken into account. They will be further discussed in the next sections.

17B.1.2.3 BRIDGE CHARACTERISTICS

The bridge characteristics to be considered in design for vessel collision are related to the bridge layout, geometry and strength. Ideally, all bridge elements should be out of the reach of vessels navigating the waterway. The bridge piers should be located outside the waterway, and the superstructure should be high enough to clear all vessels. However, economic and engineering constraints limit the span of bridges and their vertical clearance. In general, bridge piers can be designed for or protected from vessel collision. However, it is usually not economically feasible to design bridge superstructures for vessel collision.

17B.1.3 Initial Planning

It is very important to consider the vessel collision aspect as early as possible in the planning process for a new bridge, since it can have a significant effect on the total cost of the bridge. Decisions related to the bridge location, its type and its layout should take into account the waterway geometry, the navigation channel layout and the vessel traffic.

The location of a bridge structure over a waterway is usually predetermined based on other considerations. However, whenever possible, the following guidelines should be followed:

- Bridges should be located away from turns in the channel. The distance to the bridge should be such that vessels can "shape up" before passing the bridge, usually at least eight times the length of the vessel. An even larger distance is preferable when high currents and winds are likely at the site. Local vessel pilots may be consulted on this issue.

- The bridge should be designed to cross the navigation channel at right angles and should be symmetrical with respect to the channel.

- An adequate distance should exist from locations with congested navigation, berthing maneuvers or other navigation problems.

- Locations where bridge piers can be placed in shallow water, so that they cannot be reached by vessels out of control before grounding, should be preferred.

The layout of the bridge should maximize the horizontal and vertical clearances for navigation. Piers should be placed away from the reach of vessels. The bridge should not be a hazard to navigation. Bridge protection alternatives should also be considered during the initial planning phase. Finding the optimum bridge layout for different degrees of protection is an iterative process which is further discussed in the next sections.

17B.1.4 General Provisions

17B.1.4.1 OBJECTIVE OF SPECIFICATIONS

The objective of the vessel collision specifications is to minimize the risk of catastrophic failure due to a collision of a ship or barge with a bridge component.

This objective may be achieved by:

- placing the bridge components away from vessel reach

- designing the bridge components exposed to collision to resist collision forces, and/or

- providing adequate protection, either independent of the bridge or integral with the bridge.

The magnitude of the collision loads a bridge may be subjected to is uncertain, and there is a large variation in their

estimations. Since designing for the largest load caused by collision with the largest vessel that may travel the waterway is economically undesirable, a certain amount of risk is considered as acceptable so that lower load levels can be specified.

17B.1.4.2 FLOW CHART FOR THE DESIGN OF BRIDGE COMPONENTS FOR VESSEL COLLISION

The steps in the design of bridge components for ship collision are summarized in the flow chart in Figure 17B.1.4.2-1.

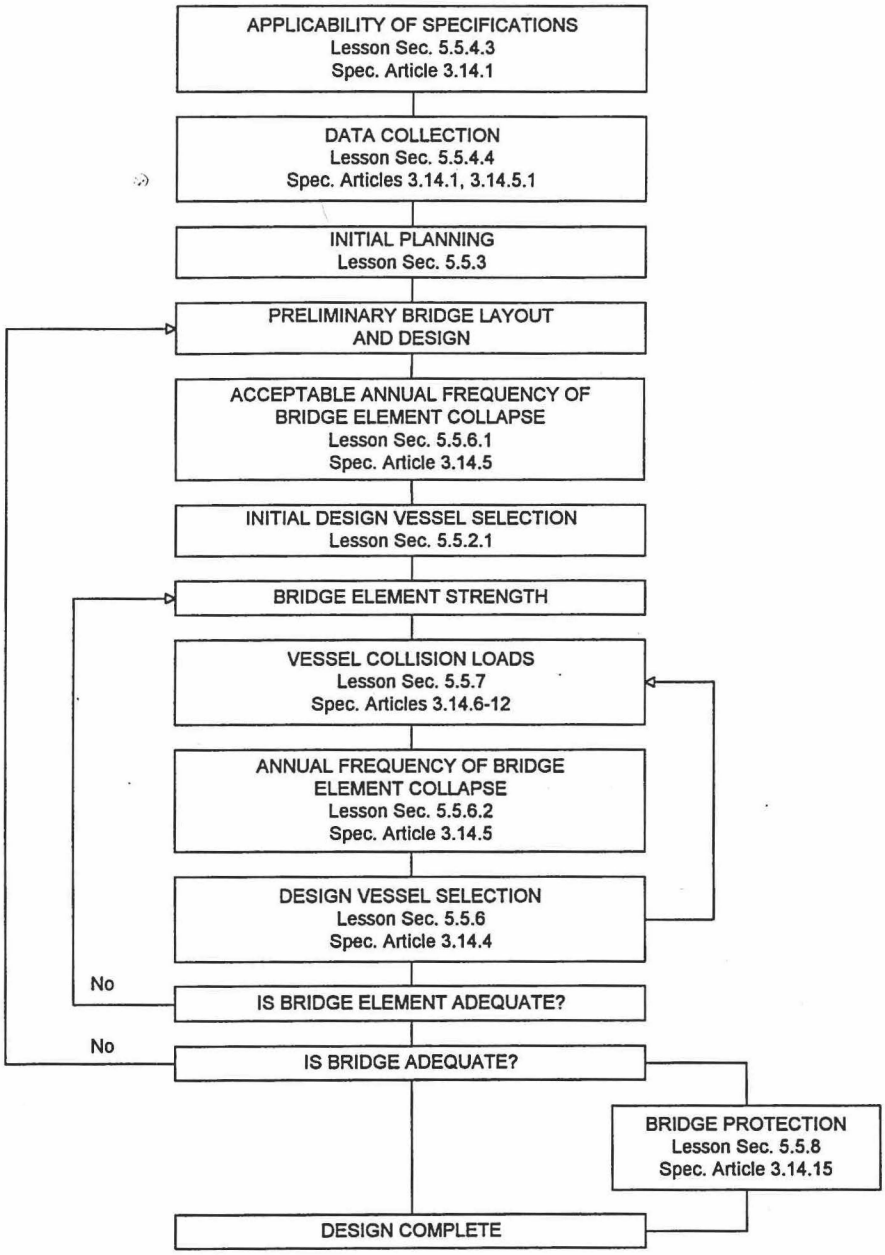


Figure 17B.1.4.2-1 - Flow Chart for Design for Vessel Collision

17B.1.4.3 APPLICABILITY OF SPECIFICATIONS

The vessel collision specifications apply to bridge components in navigable waterways with a water depth of over 0.6 m. The vessels considered include merchant ships larger than 1000 DWT and typical inland barges. The premise of the specifications is that if a bridge component can be reached by a vessel it should be designed for vessel collision.

17B.1.4.4 DATA COLLECTION

The data collection process is an important and, sometimes the most time consuming, part of designing a bridge for vessel collision. The data needed for this purpose include:

- Types of vessels and size distributions
- Typical vessel speeds and loading conditions
- Geometry of the waterway and navigation channel
- Stream velocity, hydraulic and environmental conditions

In order to determine the vessel size distribution at the bridge site, detailed information is needed on the present and projected vessel traffic.

Some of the sources for collecting data are listed below:

- U. S. Army Corps of Engineers, District Offices,
- Port Authorities and Industries along the Waterway,
- Local Pilot Associations and Merchant Marine Organizations,
- U. S. Army Corps of Engineers, "Products and Services Available to the Public", Water Resources Support Center, Navigation Data Center, Fort Belvoir, Virginia, NDC Report - 89-N-1, August 1989,
- U. S. Army Corps of Engineers, "Waterborne Commerce of the United States (WCUS), Parts 1 through 5", Water Resources Support Center (WRSC), Fort Belvoir, Virginia,
- U. S. Army Corps of Engineers, "Lock Performance Monitoring (LPM) Reports", Water Resources Support Center (WRSC), Fort Belvoir, Virginia,

- Shipping registers, such as American Bureau of Shipping Register, New York, and Lloyd's Register of Shipping, London, and

- National Oceanic and Atmospheric Administration (NOAA), "Tide Tables; Tidal Current Tables; Tidal Current Charts; U.S. Coast Pilots; Distance Tables and Nautical Charts", National Ocean Services, Rockville, Maryland.

Additionally, the U. S. Coast Guard should be contacted regarding site specific accident history and known navigation problems or aberrancy. The Coast Guard can sometimes also provide an indication of cargo classification, which may relate to fender flexibility or spark prevention.

17B.1.5 Minimum Impact Requirements

The minimum impact load for substructure design should be that of a single empty hopper barge (10.7 m x 60 m), with a displacement of 180 metric tons drifting at a velocity equal to the yearly mean current at the bridge site.

The minimum impact load for the design of a superstructure that is not high enough to clear ships in a deep draft waterway may be taken as the impact load specified for ship mast collision in Article S3.14.10.3.

17B.1.6 Design Vessel Selection

The selection of a typical vessel for bridge design is based on an "acceptable" risk level and a collision risk model. An iterative procedure is specified, in which a trial design vessel is selected for each bridge component exposed to collision and a resulting annual frequency of bridge component collapse is computed as explained in Section 17B.1.6.2. The value obtained is compared to the "acceptable" annual frequency of bridge component collapse, and, if different, a new design vessel associated with a new pier strength requirement is selected and the process is repeated.

The design vessel selection procedure may be summarized as follows (see also the design procedure flow chart):

Step 1 - Make an initial design vessel selection (see Section 17B.1.2.1)

Step 2 - Determine preliminary bridge element strength

Step 3 - Compute vessel collision loads for each of the vessel size categories selected (see Section 17B.1.7)

Step 4 - Compute annual frequency of bridge element collapse for each vessel size category (see Section 17B.1.6.2)

Step 5 - Compare the sum of the annual frequency of bridge element collapse for all vessel size categories to the acceptable annual frequency of bridge element collapse (see Section 17B.1.6.1)

Step 6 - If the annual frequency of bridge element collapse is not satisfactory, revise the bridge element strength, determine the vessel size that is associated with a collision load equal to the new bridge element strength and go back to Step 3.

17B.1.6.1 ACCEPTABLE ANNUAL FREQUENCY OF BRIDGE ELEMENT COLLAPSE

For critical bridges, the maximum annual frequency of whole bridge collapse was set at 0.0001. Critical bridges are defined as those bridges that are expected to continue to function after an impact, because of social/survival or security/defense requirements. For regular bridges, the maximum annual frequency of bridge collapse was set at 0.001.

The acceptable annual frequency of collapse for each bridge element exposed to vessel collision is computed as follows:

- For waterways whose width is less than 6.0 times the length overall of the design vessel, LOA, the acceptable annual acceptable frequency of collapse for each pier and superstructure component is determined by distributing the annual acceptable frequency of bridge collapse among all the exposed piers and superstructure components. The simplest such distribution would be to divide the annual acceptable frequency of collapse by the number of exposed components within 6.0 times LOA. A more rational process would be to assign low frequencies to more critical components and distribute the remainder to less critical components.

- For wide waterways, whose width is greater than 6.0 times LOA, the acceptable risk is distributed only over those bridge elements that are located within a 3.0

times LOA band on each side of the centerline paths for inbound and outbound vessel transit.

The acceptable levels of risk for bridge collision design were established subjectively by comparing them to risks associated with natural disasters and other engineering projects. The risk of vessel collision with a bridge refers to both its probability of occurrence and the severity of its consequences.

17B.1.6.2 ANNUAL FREQUENCIES OF BRIDGE ELEMENT COLLAPSE

The annual frequency of collapse of a bridge element, AF, is computed from:

$$AF = (N)(PA)(PG)(PC) \qquad (17B.1.6.2\text{-}1)$$

bridge strength/impact load

bridge/waterway geometry

navigation conditions

vessel traffic

where:

N = annual number of vessels classified by type, size and loading condition which can strike a bridge element

PA = probability of vessel aberrancy

PG = geometric probability of a collision between an aberrant vessel and a bridge pier or span

PC = probability of bridge collapse due to a collision with an aberrant vessel

17B.1.6.2.1 General Remarks

It is mainly the geometric probability term, PG, that the designer will vary to achieve the acceptable annual frequency of collapse. The designer also controls the probability of collapse, PC, which varies with the bridge strength to impact load ratio. The impact load, in turn, varies with the design vessel.

Once the layout and the structural characteristics of the bridge have been selected, the selection of design vessels is done through several iterations. If adjustments in the bridge characteristics are made, the iteration process is repeated.

In order to reduce the number of iterations, the initial trial design vessel for the main piers may be selected based on a vessel size that has over 50 passages per year for critical bridges and over 200 passages per year for regular bridges. It should be noted that the water depth at the location of the piers limits the vessels that can reach these piers.

17B.1.6.2.2 Vessel Traffic Distribution, N

The number of vessels, N, passing under the bridge based on size, type and loading condition and available water depth has to be developed for each pier and span component to be evaluated. All vessels of a given type and loading condition have to be divided into discrete groupings of vessel size by DWT. It is recommended that the DWT intervals used not exceed 20 000 DWT for vessels smaller than 100 000 DWT and 50 000 DWT for ships larger than 100 000 DWT.

Once the vessels have been grouped and their frequency distribution has been established, information on typical vessel characteristics may be obtained from Appendices A and B, as a function of the vessel size in DWT, or from site specific vessel surveys.

17B.1.6.2.3 Probability of Aberrancy, PA

The probability of vessel aberrancy reflects the likelihood that a vessel would be out of control in the vicinity of a bridge. This may occur as a result of pilot error, mechanical failure, or adverse environmental conditions. The probability of aberrancy is mainly related to the navigation conditions at the bridge site. The designer does not have much control over this, except to locate the bridge away from navigation hazards, as explained in Section 17B.1.4. Vessel traffic regulations, vessel traffic management systems and aids to navigation can improve the navigation conditions and reduce the probability of aberrancy.

The probability of vessel aberrancy may be evaluated based on site specific information that includes historical data on vessel collision, rammings and groundings in the waterway, vessel traffic, navigation conditions and bridge/waterway geometry.

As an alternative, the Specification recommends the following formulation:

$$PA = (BR)\ (R_B)\ (R_C)\ (R_{XC})\ (R_D) \qquad (17B.1.6.2.3\text{-}1)$$

where:

PA = probability of aberrancy

BR = aberrancy base rate

R_B = correction factor for bridge location

R_C = correction factor for currents acting parallel to vessel transit path

R_{XC} = correction factor for cross currents acting perpendicular to vessel transit path

R_D = correction factor for vessel traffic density

The recommended base rates, BR, are 0.6×10^{-4} for ships and 1.2×10^{-4} for barges.

The correction factor for bridge location, R_B, is related to existence of a turn or a bend in the channel as shown in Figure 17B.1.6.2.3-1. It is computed as follows:

- Straight Region: For a bridge located in a straight region:
  $$R_B = 1.0 \qquad (17B.1.6.2.3\text{-}2)$$

- Transition Region: For a bridge located in a transition region, R_B can be computed by:
  $$R_B = \left[1 + \frac{\theta}{90°}\right] \qquad (17B.1.6.2.3\text{-}3)$$

  where:

  θ = angle of the turn (degrees)

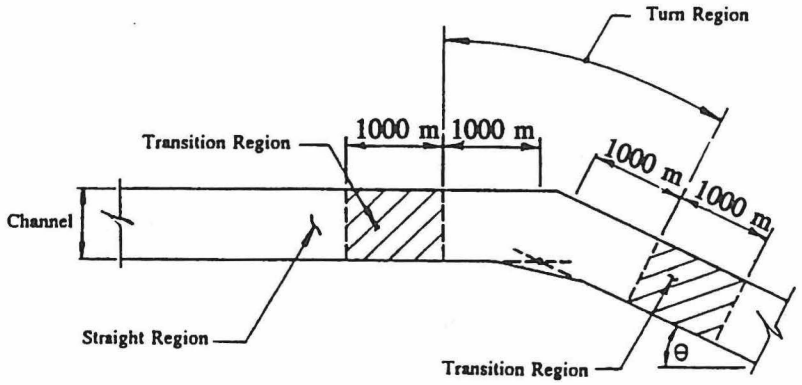
- Turn/Bend Region: For a bridge located in a turn or bend region, R_B can be computed by:
  $$R_B = \left[1 + \frac{\theta}{45°}\right] \qquad (17B.1.6.2.3\text{-}4)$$

The correction factor for currents acting parallel to vessel transit path, R_C, can be computed from:

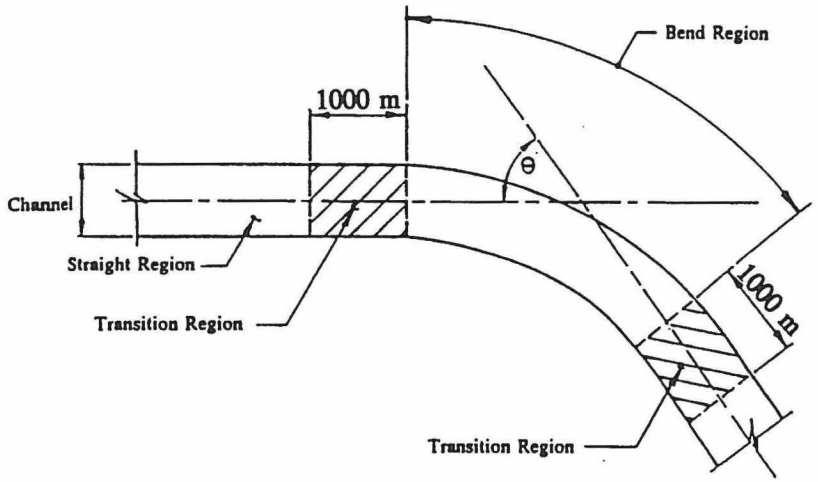
$$R_C = \left[1 + \frac{V_c}{19}\right] \qquad \text{(17B.1.6.2.3-5)}$$

where:

V_C = current component parallel to vessel path (km/hr)



a. Turn in Channel.



b. Bend in Channel.

Figure 17B.1.6.2.3-1 - Waterway Regions for Bridge Location

The correction factor for cross currents acting perpendicular to vessel transit path, R_{XC}, can be computed from:

$$R_{XC} = (1.0 + 0.54\ V_{XC}) \qquad \text{(17B.1.6.2.3-6)}$$

where:

V_{XC} = current component perpendicular to vessel path (km/hr)

The correction factor for vessel traffic density, R_D, in the waterway can be estimated as follows:

- Low Density, R_D = 1.0: vessels rarely meet, pass, or overtake each other in the immediate vicinity of the bridge.

- Average Density, R_D = 1.3: vessels occasionally meet, pass, or overtake each other in the immediate vicinity of the bridge.

- High Density, R_D = 1.6: vessels routinely meet, pass, or overtake each other in the immediate vicinity of the bridge.

The probability of aberrancy has been determined for various bridge projects worldwide, as shown in Table 17B.1.6.2.3-1.

Table 17B.1.6.2.3-1 - Summary of Probability of Aberrancy, PA, Values

LOCALITY	TYPE OF DATA	PROBABILITY OF VESSEL ABERRANCY ($x10^{-4}$)
Dover Straits - Collisions	Statistics	5 to 7
Dover Straits - Groundings	Statistics	1.4 to 1.6
Japanese Straits - Groundings	Statistics	0.7 to 6.7
Japanese Straits - Collisions	Statistics	1.3
Worldwide	Statistics	0.5
Tasman Bridge, Australia	Estimate	0.6 to 1.0
Great Belt Bridge, Denmark	Estimate	0.4
Sunshine Skyway Bridge, Florida	Statistics Statistics	1.3 (Ships) 2.0 (Barges)
Annacis Island Bridge, Canada	Estimate	3.6
Francis Scott Key Bridge & William Preston Lane Bridges, Maryland	Statistics	1.0 (Ships) 2.0 (Barges)
Dames Point Bridge, Florida	Statistics	1.3 (Ships) 4.1 (Barges)
Laviolette Bridge, Canada	Statistics	0.5
Centennial Bridge, Canada	Statistics	5.0
Louisiana Waterways	Statistics	0.8 to 1.9 (Ships) 1.5 to 3.0 (Barges)
Gibraltar Straits - Strandings, Morocco	Statistics	2.2
Gibraltar Straits - Strandings, Morocco	Statistics	1.2

17B.1.6.2.4 Geometric Probability, PG

The geometric probability is the probability that a vessel that has lost control while approaching a bridge will hit a given bridge element. It is mainly a function of the geometry of the bridge and the waterway. There are several other factors that can affect the likelihood that an aberrant vessel will strike a bridge element. They include the original vessel location, course, rudder position and velocity at the time of failure, the

type, size, dimensions, draft and maneuvering characteristics of the vessel, and the hydraulic and environmental conditions at the bridge site.

A normal probability density function about the centerline of the vessel transit path is recommended for estimating the likelihood of an aberrant vessel being within a certain impact zone along the bridge axis, as shown in Figure 17B.1.6.2.4-1. Using a normal distribution accounts for the fact that aberrant vessels are more likely to pass under the bridge closer to the navigation channel than further away from it. The standard deviation of the distribution equals the length of the design vessel, LOA, and, bridge elements beyond three times the standard deviation from the centerline of vessel transit path need not be included in the analysis (other than the minimum impact requirements, see Section 17B.1.5). For simplicity, the vessel length of the initial trial design vessel (see Section 17B.1.6.2.1) may be used for LOA. The probability that an aberrant vessel is located within a certain zone is the area under the normal probability density function within that zone, which can be computed from a normal probability function table, such as the one included in Lecture 1.

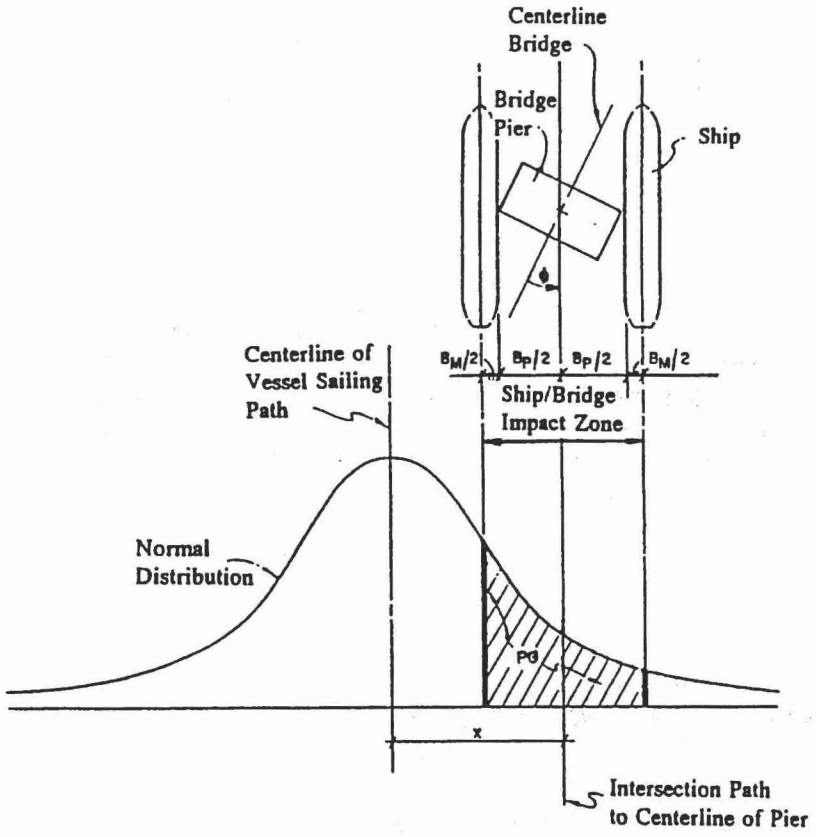


Figure 17B.1.6.2.4-1 - Geometric Probability of Pier Collision

17B.1.6.2.5 Probability of Collapse, PC

The probability of collapse, PC, has been introduced to account for the large uncertainties in the vessel collision impact loads, P, and the ultimate strength of a bridge element, H. A complimentary probability of collapse distribution as a function of the ratio H/P is assumed that results in the following relationships:

- For $0.0 \leq H/P < 0.1$, PC is computed as:

$$PC = 0.1 + 9\left[0.1 - \frac{H}{P}\right] \qquad \text{(17B.1.6.2.5-1)}$$

- For $0.1 \leq H/P < 1.0$, PC is computed as:

$$PC = \frac{\left[1 - \frac{H}{P}\right]}{9} \qquad \text{(17B.1.6.2.5-2)}$$

- For $H/P \geq 1.0$, PC is computed as:

$$PC = 0.0 \qquad \text{(17B.1.6.2.5-3)}$$

where:

H = ultimate bridge element strength (N)

P = vessel impact force (N)

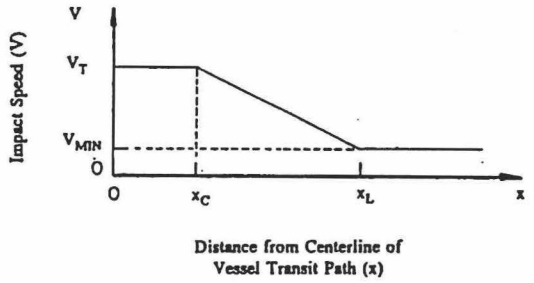
Examination of these relationships reveals that the ratio H/P should be kept at a level larger than 0.1. If this ratio is smaller than 0.1, the probability of collapse has a very sharp increase towards 1.0. If the H/P ratio is larger than 0.1, the probability of collapse decreases from 0.1 to 0.0 as the H/P ratio approaches 1.0.

17B.1.7 Vessel Collision Loads

The evaluation of the effects of the collision of a design vessel with a bridge involves computations of design vessel velocity, vessel collision energy and forces, and vessel depth of damage.

17B.1.7.1 DESIGN VESSEL VELOCITY

The selection of a design vessel velocity for a given bridge element is described in Figure 17B.1.7.1-1 below.



V = design impact speed (mps)
V_T = typical vessel transit speed in the channel (mps)
V_{MIN} = minimum design impact speed (mps)
x = distance to bridge element from centerline of vessel transit path (m)
x_C = distance to edge of channel from centerline of vessel transit path (m)
x_L = distance equal to 3xLOA from centerline of vessel transit path (m)

Figure 17B.1.7.1-1 - Design Impact Speed Distribution

The typical vessel speed in the channel, V_T, represents the typical speeds of the vessels close in size to the design vessels. In high velocity waterways, the upstream and the downstream traffic can have significantly different velocities and should be separated.

As we move away from the navigation channel, the design vessel velocity decreases to a minimum value, V_{MIN}, that is equal to the yearly mean current at the location of the bridge.

17B.1.7.2 VESSEL COLLISION ENERGY

The vessel collision energy may be evaluated using the formulation given in Article 3.14.7 of the Specification. Since the specified vessel collision loads are not dependent on the computation of the vessel collision energy, the vessel energy evaluation is not covered in detail. The designer is referred to in Article 3.14.7 if an energy approach is needed.

17B.1.7.3 SHIP COLLISION FORCE ON PIER

The estimation of the load on a bridge pier during a ship collision is a very complex problem. The actual force is time-dependent, and, among other factors, it depends on the size and construction of the vessel, its velocity, the degree of water ballast in the forepeak of the bow, the geometry of the collision and the geometry and strength characteristics of the pier. There is a very large scatter among the collision force values

recommended in various vessel collision guidelines or used in various bridge projects.

The following formula is recommended in Article 3.14.8 of the Specification for estimating the head-on ship collision impact force, P_S, on a rigid pier:

$$P_S = 1.2 \times 10^5 \, V\sqrt{DWT} \qquad (17B.1.7.3\text{-}1)$$

where:

P_S = equivalent static vessel impact force (N)

DWT = dead weight tonnage of vessel (metric ton)

V = vessel impact velocity (m/s)

This formulation was primarily developed from research conducted by Woisin in West Germany during 1967 to 1976 on physical ship models to generate data for protecting the reactors of nuclear power ships from collisions with other ships. The scatter in the results of these tests is of the order of $\pm$50%. The formula recommended uses a 70% fractile of an assumed triangular distribution with zero values at 0% and 100% and a maximum value at the 50% level.

Research in this area continues and updates in the computation of loads can be expected.

17B.1.7.4 SHIP BOW DAMAGE LENGTH

An estimate of the ship bow damage depth may be obtained using the formula given in Article 3.14.9 of the Specification. Since the estimation of the ship bow damage length is not directly related to the bridge design process, this topic is not covered here. If such an estimate is needed, the designer is referred to Article 3.14.9 of the Specification.

17B.1.7.5 SHIP COLLISION FORCE ON SUPERSTRUCTURE

Formulas for computing design ship collision loads on a bridge superstructure are given in Article 3.14.10 of the Specification. The loads are expressed as a function of the design ship impact force, P_S, as follows:

- Ship Bow Impact Force, P_{BH} in N:

$$P_{BH} = (R_{BH})\,(P_S) \qquad (17B.1.7.5\text{-}1)$$

where:

R_{BH} = reduction coefficient equal to the ratio of exposed superstructure depth to the total bow depth

- Ship Deck House Impact Force, P_{DH} in N:

$$P_{DH} = (R_{DH})\ (P_S) \qquad (17B.1.7.5\text{-}2)$$

where:

R_{DH} = reduction coefficient equal to:

0.10 for ship larger than 100 000 DWT and,

$$0.2 - \left(\frac{DWT}{100\,000}\right)\ (0.10)$$ for ships under 100 000 DWT (17B.1.7.5-3)

- Ship Mast Impact Force, P_{MT} in N:

$$P_{MT} = 0.10\ P_{DH} \qquad (17B.1.7.5\text{-}4)$$

where P_{DH} is the ship deck house impact force

The magnitude of the impact loads computed for ship bow, and even deck house, collision is quite high relative to the strength of most superstructure designs. Also, there is great uncertainty associated with predicting ship collision loads on superstructures because of the limited data available and the ship/superstructure load interaction effects. It is, therefore, suggested that superstructures, and also weak or slender parts of the substructure, be located out of the reach of a ship's hull or bow.

17B.1.7.6 BARGE COLLISION FORCE ON PIER

The barge collision loads recommended for the design of piers are shown in the Figure 17B.1.7.6-1 below as a function of the tow length and the impact speed.

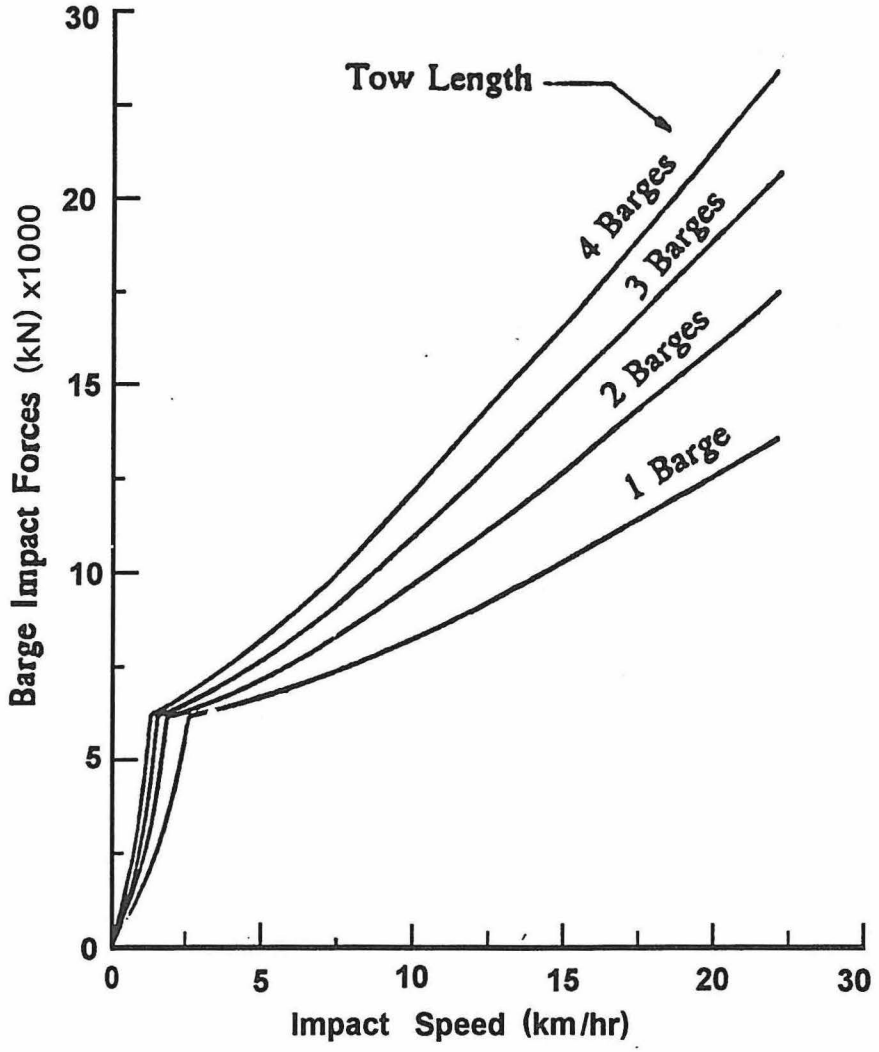


Figure 17B.1.7.6-1 - Typical Hopper Barge Impact Forces

The loads in this Figure 17B.1.7.6-1 were computed using a standard hopper barge. The characteristics of a standard hopper barge may be found in Appendix B - Typical Barge Characteristics.

The impact force recommended for design barges larger than the standard hopper barge is determined by increasing the standard barge impact force by the ratio of the width of larger barge to the width of the standard hopper barge.

Articles 3.14.11 and 3.14.12 of the Specification contain formulas for computing the barge collision loads based on the barge energy and the depth of damage (indentation). However, in view of the approximations involved, and the limited research data on barge collision loads available, an "accurate" estimate of the magnitude of the barge collision loads is not needed, and, using a graphical format for

specifying these loads is adequate. If an estimate of the barge depth of damage is needed, the designer is referred to Article 3.14.12 of the Specification.

17B.1.7.7 APPLICATION OF IMPACT FORCES

The application of the impact forces on the substructure is as follows:

- either 100% of the design impact force in a direction parallel to the navigation channel, or
- 50% of the design impact force in a direction normal to the channel.

The criteria for the evaluation of the substructure are as follows:

- for overall stability, the design impact force is applied as a concentrated force at the mean high water level, as shown in Figure 17B.1.7.7-1.
- for local collision forces, the design impact force is applied as a vertical line load equally distributed along the ship's bow depth for ships, and along the head block depth for barges, as shown in Figures 17B.1.7.7-2 and 17B.1.7.7-3, respectively.

For superstructure design, the impact forces are applied transverse to the superstructure component in a direction parallel to the navigation channel.

Any bridge component exposed to physical contact by any portion of the design vessel's hull or bow has to be designed to resist the applied loads. The bow overhang, rake or flair distance of ships and barges has to be taken into account as shown in Figures 17B.1.7.7-4 and 17B.1.7.7-5.

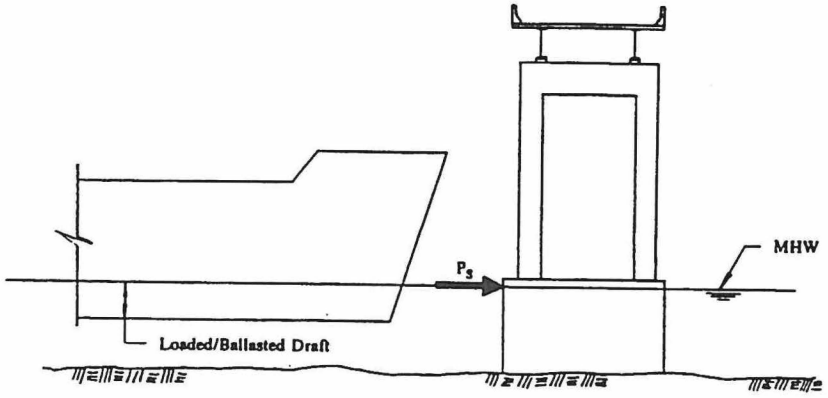


Figure 17B.1.7.7-1 - Ship Impact Concentrated Force on Pier (for Foundation Design and Overall Stability)

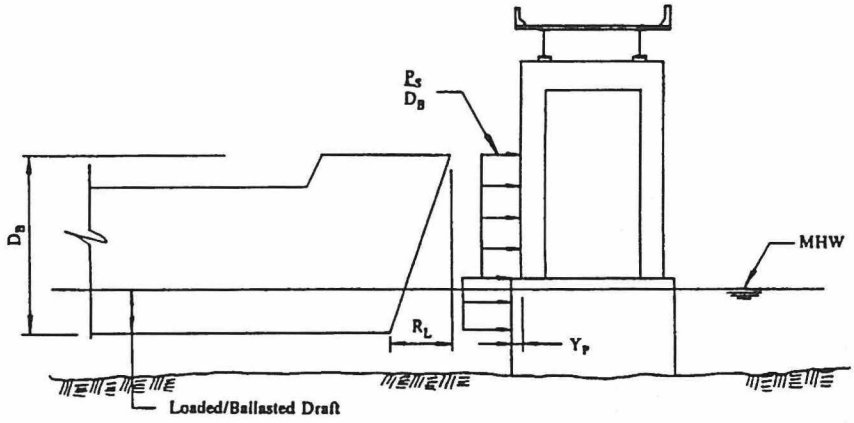


Figure 17B.1.7.7-2 - Ship Impact Line Load for Local Collision Force on Pier (for Structure Check and Design)

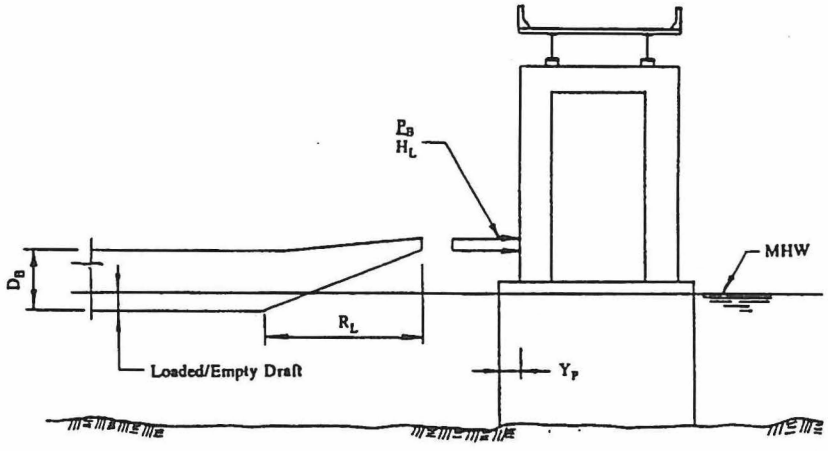


Figure 17B.1.7.7-3 - Barge Impact Line Load for Local Collision Force on Pier (for Structure Check and Design)

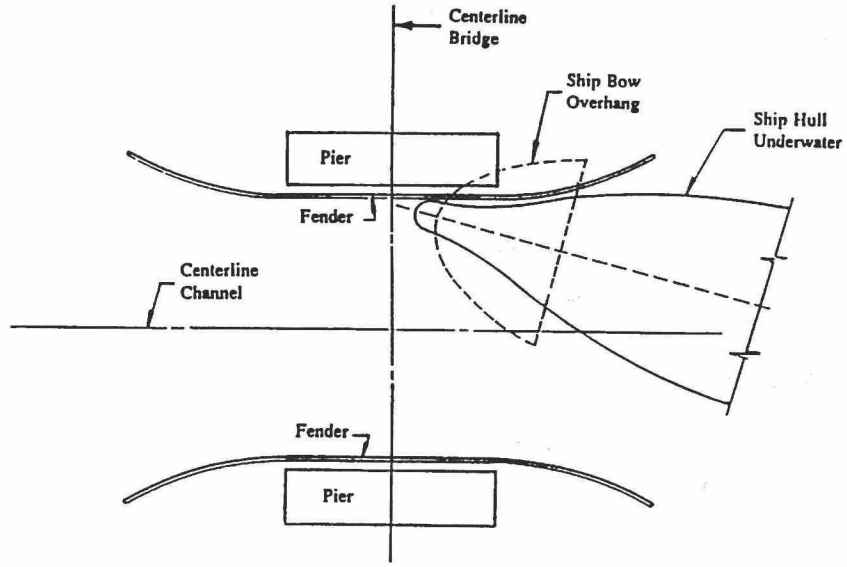


Figure 17B.1.7.7-4 - Plan of Ship Bow Overhang Impacting Pier Behind Fender

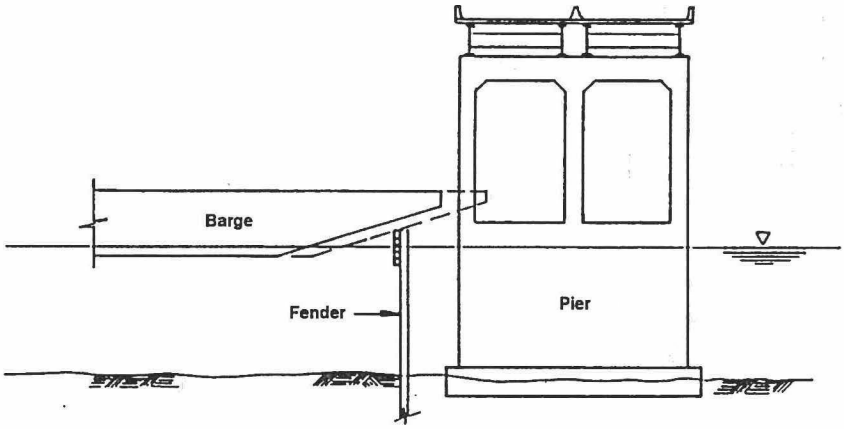


Figure 17B.1.7.7-5 - Elevation of Barge Overhang Impacting Pier Behind Fender

17B.1.8 Bridge Protection

Article 3.14.15 of the Specification contains an overview of bridge protection and prevention measures, and the designer is referred to it. Additional information and guidance may be found in [2, 3 and 4].

17B.2 EXAMPLE BRIDGE DESCRIPTION

Consider the bridge shown in Figure 1. For the existing bridge, the following two items will be evaluated:

1. What is the annual frequency of collapse, AF, of one of the main piers of the suspension bridge; and

2. If AF is unacceptable, what level of impact force should be used to develop pier protection alternatives.

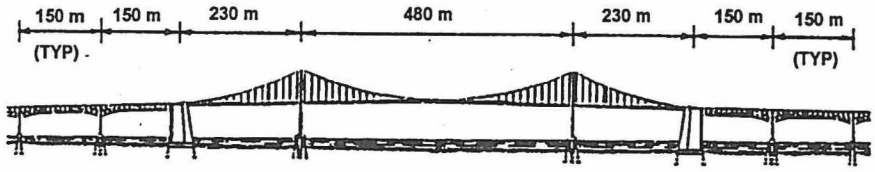


Figure 17B.2-1 - Bridge Profile for Method II Example

WATERWAY CHARACTERISTICS

The following waterway characteristics shall be assumed for the sample analysis:

- navigable channel is 240 m wide
- two-way traffic for merchant vessels exist, therefore, the centerline of vessel transit is located 60 m on each side of the channel centerline
- water depth at the main piers is greater than 20 m
- annual mean water current is approximately 0.6 km/hr (i.e., V_{MIN} = 0.17 mps)

BRIDGE CHARACTERISTICS

The following main pier characteristics shall be assumed for the sample analysis:

- pier width is 18 m
- pier resistance to impact equals 90 000 000 N
- bridge importance classification = Critical

VESSEL FREQUENCY

The bridge is to be evaluated for a 50-year period of time. The number of vessels using the waterway is increasing annually. The forecasted vessel traffic data for the merchant fleet 50 years in the future is shown in Table 1. Using the 50-year forecast for the number of vessel transits will result in a conservative (i.e., over estimation) of the current risk. (To be consistent with other design lives in LRFD Specification, this could be 75 years.)

Table 17B.2-1 - Ship Frequency Data (Yr. 2040)

Size(metric tons)	Type*	No. of Passages N (per year)
10 000	F/C	3000
20 000	F/C	2000
40 000	B/T	100
60 000	B/T	100
80 000	B/T	300
100 000	B/T	100
150 000	B/T	60
TOTAL		5660

* F/C = Freighter/Container Vessel
B/T = Bulk Carrier/Tanker Vessel

VESSEL CHARACTERISTICS

The vessel characteristics shown in Table 2 were estimated for each ship classification using the data in Section S3.5.2.

Table 17B.2-2 - Ship Characteristics

Size (metric tons)	Type	Length LOA (m)	Beam B_M (m)	Draft, D_L (m)
10 000	F/C	144	20	8.2
20 000	F/C	196	27.5	10.5
40 000	B/T	208	30	11.4
60 000	B/T	235	33	12.3
80 000	B/T	259	36.5	13.2
100 000	B/T	275	42	16.1
150 000	B/T	314	44.5	18.0

The LOA dimension required for the impact speed and geometric probability distribution is equal to a vessel selected using the 50th ship or 5% criterion. For this example, the maximum of 50 ships would control. Counting back 50 ships from the vessel distribution in Table 2 results in a 150 000 metric ton vessel with LOA = 314 m. This LOA shall be considered a constant and applicable to all vessel classifications in accordance with Section S3.7.

DESIGN IMPACT SPEED

Discussion with the local pilot's association who are responsible for navigating all foreign flagged ships into the harbor indicates that the typical transit speed, V_T, for ships in the portion of the waterway where the bridge is located is 24 km/hr (6.7 mps). From Figure 2, the design impact speed, V, for the main pier was computed as,

$$V = 6.7 - \left[(240-120) \left[\frac{6.7-0.17}{942-120} \right] \right] = 5.75 \ mps$$

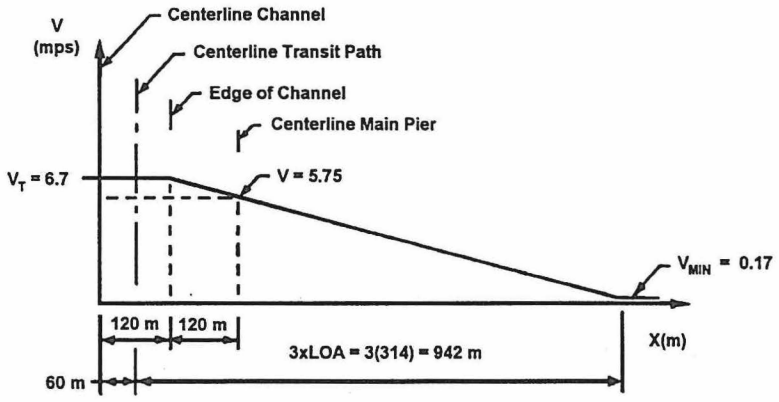


Figure 17B.2-2 - Main Pier Design Impact Speed

PROBABILITY OF ABERRANCY

The probability of vessel aberrancy, PA, is estimated below:

BR = 0.6×10^{-4}, for ships

R_B = 1.0, since bridge is in a Straight Region

R_C = 1 + (0.6/19) = 1.032, for V_C = 0.6 km/hr

R_{XC} = 1.0 (no cross-currents at site)

R_D = 1.6, for High Density vessel traffic

therefore,

$PA = BR(R_B)(R_C)(R_{XC})(R_D)$
$= (0.6x10^{-4})(1.0)(1.032)(1.0)(1.6)$
$\cong 1.0x10^{-4}$

GEOMETRIC PROBABILITY

The geometric probability of ship collision, PG, is computed using the normal distribution shown in Figure 3. The standard deviation of the normal distribution is σ = LOA = 314 m. For the 150 000 metric ton vessel with B_M equal to 44.5 m, the value of PG is computed from normal distribution tables as

At $x_1 = (0.47)\sigma$; $1\text{-}F(x_1) = 0.3192$

At $x_2 = (0.67)\sigma$; $1\text{-}F(x_2) = 0.2514$

therefore,

$PG = [1\text{-}F(x_1)]\text{-}[1\text{ -}F(x_2)]$
$= (0.3192\text{-}0.2514) = 0.0678$

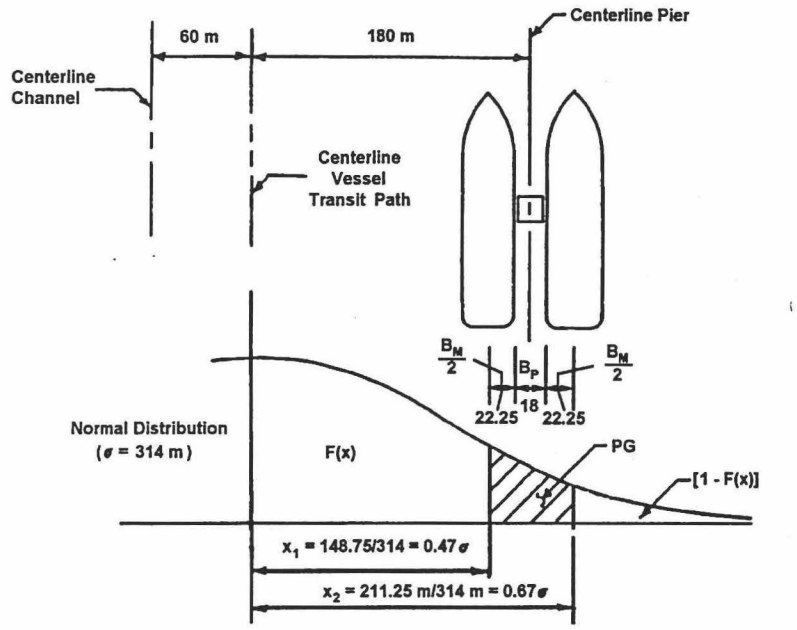


Figure 17B.2-3 - Geometric Probability of Vessel Collision with the Main Pier

A summary of PG for each vessel classification is shown in Table 5.

SHIP COLLISION FORCE

For a 80 000 metric ton Bulk Carrier/Tanker colliding at the design speed, V = 5.75 mps, the impact force becomes

$$P_s = 1.2 \times 10^5 (M)^{1/2}(V)$$
$$= 1.2 \times 10^5 (80\ 000)^{1/2}(5.75)$$
$$= 195\ 161\ 472\ N$$

A summary of vessel impact forces for each vessel classification is shown in Table 3.

Table 17B.2-3 - Ship Collision Forces

Size (metric tons)	(Type)	P_s (N)
10 000	F/C	69 000 000
20 000	F/C	97 580 736
40 000	B/T	138 000 000
60 000	B/T	169 014 792
80 000	B/T	195 161 472
100 000	B/T	218 197 159
150 000	B/T	267 235 851

PROBABILITY OF COLLAPSE

The ultimate lateral resistance capacity of the main pier, H_p, is equal to 90 000 000 N. For the 20 000 metric ton Freighter/Container vessel, PC is computed as:

$$H_p/P_s = 90\ 000\ 000/97\ 580\ 736 = 0.922$$

$$PC = 0.111(1-H/P)$$
$$= 0.111(1-0.922) = 0.0086$$

A summary of PC values for each vessel classification is shown in Table 4.

Table 17B.2-4 - Probability of Collapse

Size (metric ton)	(H_p/P_s)	PC
10 000	1.304	0.0000
20 000	0.922	0.0086
40 000	0.652	0.0386
60 000	0.532	0.0519
80 000	0.461	0.0599
100 000	0.412	0.0653
150 000	0.337	0.0737

ANNUAL FREQUENCY OF COLLAPSE

The annual frequency of main pier collapse, AF, is computed using

$$AF = (N)(PA)(PG)(PC)$$

The results of the computation for each ship classification and the pier total are summarized in Table 5. In this table, the ship sizes have been arranged in decreasing order from the largest (150 000 metric tons) to the smallest (10 000 metric tons) in order to sum the cumulative annual frequencies, ΣAF. This ordering allows a relatively quick determination of the impact of vessel size on the ΣAF and for comparison against the acceptable annual frequency, AF_p.

Table 17B.2-5 - Annual Frequency of Main Pier Collapse ($H_p = 90 \times 10^6$ N)

Ship (metric ton)	N	PA ($\times 10^{-4}$)	PG	PC	AF ($\times 10^{-6}$)	ΣAF ($\times 10^{-6}$)
150 000	60	1.0	0.0678	0.0737	30	30
100 000	100	1.0	0.0676	0.0653	44	74
80 000	300	1.0	0.0657	0.0599	118	192
60 000	100	1.0	0.0624	0.0519	32	224
40 000	100	1.0	0.0631	0.0386	24	248
20 000	2000	1.0	0.0604	0.0086	104	352
10 000	3000	1.0	0.0476	0.0000	----	----

From Table 5, the ΣAF for all ship classifications equals 0.000 352. The main pier return period of collapse, RP, becomes

$RP = (1/AF) = (1/0.000\ 352) = 2841$ years

RISK ACCEPTANCE CRITERIA

For a Critical Bridge, the total bridge acceptable annual frequency of collapse, AF_C, equals 0.0001. This total acceptance, AF_C, must be distributed among the exposed bridge elements within a distance 3xLOA on each side of the inbound and outbound vessel transit paths. For the suspension bridge in Figure 1, the distance from the centerline of channel to the edge of the analysis area is

$$x = 60 + (3 \times LOA)$$
$$= 60 + 3(314) = 1002 \text{ m}$$

Within this distance are five piers on each side of the channel centerline for a total of ten piers. The distribution of AF_C among these ten piers is determined by the designer. One method would be to equally spread the acceptable risk among all piers (i.e., $AF_C/10$), however, this is not desirable since it fails to take into account the importance and higher cost associated with the main pier of the suspension bridge. A better method would be to apportion the risk to each pier based on its percentage value of the replacement cost of the structure in the central analysis area. For the sample problem, the two main piers and the superstructure they support represents 50% of the replacement cost of the bridge in the central analysis area. Each main pier will, therefore, be apportioned 25% of the total acceptable annual frequency, so that

$$AF_P = (0.25)(AF_C) = (0.25)(0.0001) = 0.000\ 025$$

$$RP_P = (1/AF_P) = 40\ 000 \text{ years}$$

Comparing the allowable value of $AF_p = 0.000\ 025$ with the actual value of $\Sigma AF = 0.000\ 352$ in Table 5 indicates that under these future year traffic conditions, the main pier will be relatively vulnerable to catastrophic vessel collision. Also, from a comparison with Table 5, it can be seen that even the 150 000 metric ton vessels with $AF = 0.000\ 030$ exceed the acceptance criteria. The vulnerability is a result of the relatively weak resistance (H_p = 90 000 000 N) of the main pier compared to the magnitude and frequency of the vessel impact forces.

REVISED ANNUAL FREQUENCY

What would the pier strength have to be in order to meet the acceptance criteria? Or restated in another way, what should the impact force be to develop a pier protection system to protect the main pier? To answer this question requires a trial and error process in which values of H_p are assumed, new AF's are computed, and a comparison with AF_p is made.

For the first trial, assume H_p = 200 000 000 N. This value of H_p is approximately equal to the impact force, P_s, of a 90 000 metric ton ship. The revised annual frequency of collapse is shown in Table 6.

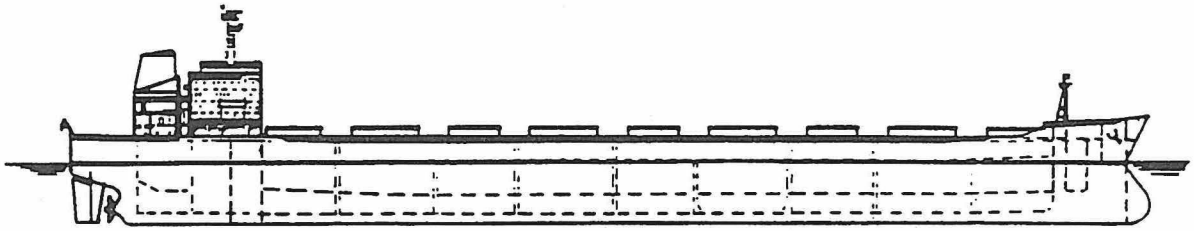
The new ΣAF = 0.000 017 from Table 6 is less than the AF_p = 0.000 025 acceptance criteria, therefore, a 90 000 metric ton design vessel would be required for the design of a pier protection system for the main pier.

Table 17B.2-6 - Annual Frequency of Main Pier Collapse ($H_p = 200 \times 10^6$ N)

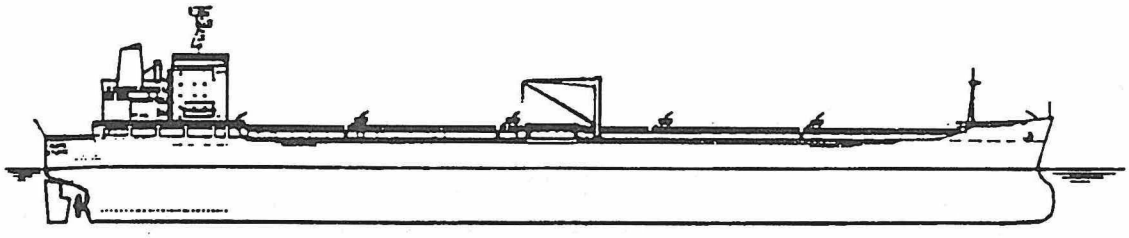
Ship (metric ton)	N	PA ($\times 10^{-4}$)	PG	PC	AF ($\times 10^{-6}$)	ΣAF ($\times 10^{-6}$)
150 000	60	1.0	0.0678	0.0279	11	11
100 000	100	1.0	0.0676	0.0093	6	17
80 000	300	1.0	0.0657	0.0000	----	----
60 000	100	1.0	0.0624	0.0000	----	----
40 000	100	1.0	0.0631	0.0000	----	----
20 000	2000	1.0	0.0604	0.0000	----	----
10 000	3000	1.0	0.0476	0.0000	----	----

APPENDIX A

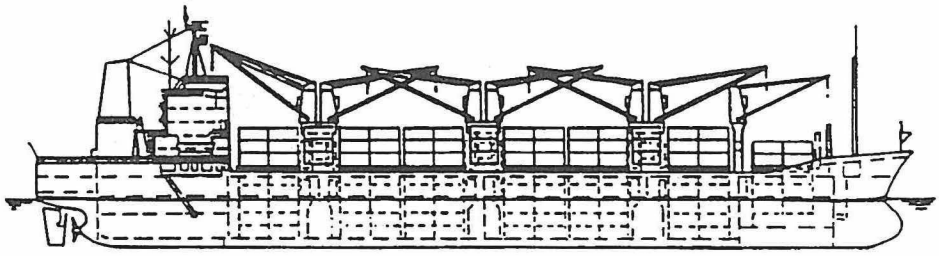
Typical Ship Characteristics



a. 81 400 DWT Bulk Carrier (length = 260 m).



b. 67 000 DWT Product Carrier/Tanker (length = 240 m).



c. 22 000 DWT Multi-Purpose Freighter/Container Ship (length = 200 m).

Figure A17B-1 - Typical Ship Profiles

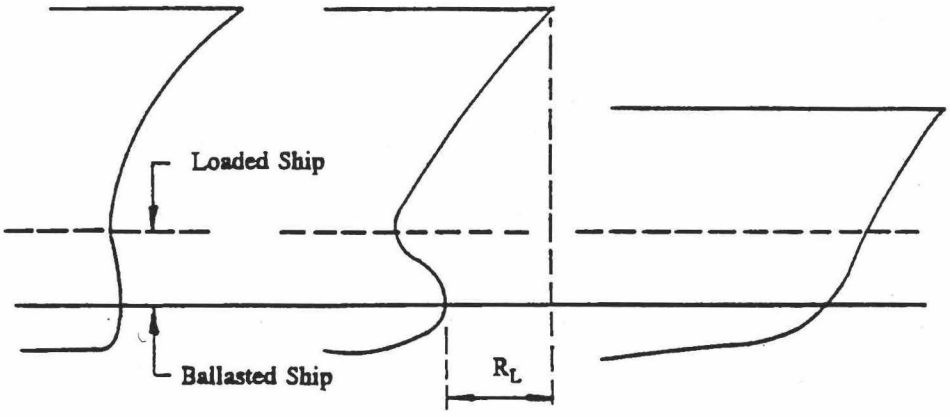
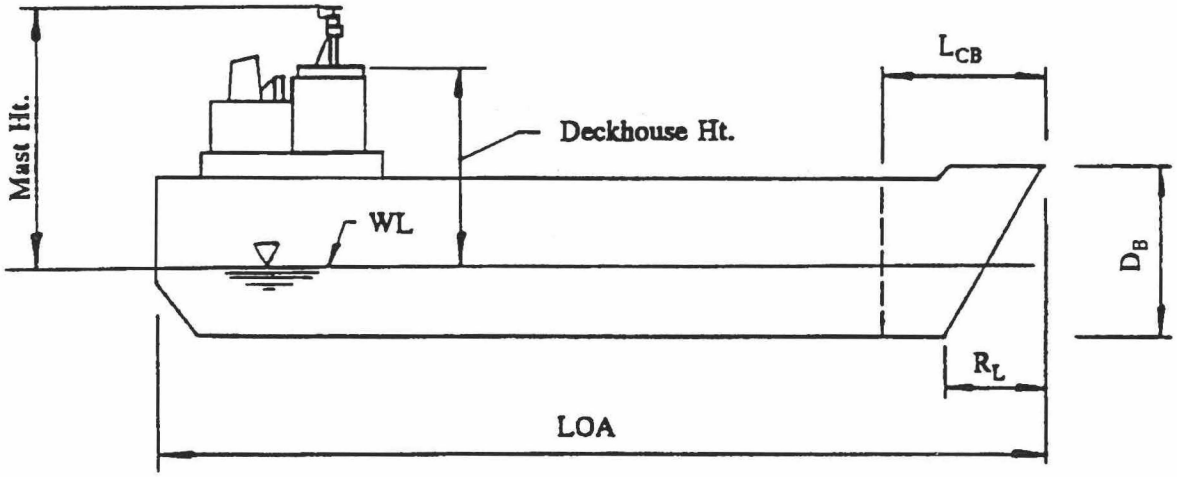


Figure A17B-2 - Common Ship Bow Shapes



D_B = bow depth
R_L = rake length of bow ≅ 0.25 D_B
L_{CB} = length to collision bulkhead ≅ 0.1(LOA)
LOA = length overall of ship
WL = waterline

Figure A17B-3 - Typical Ship Bow and Vertical Clearance Dimensions

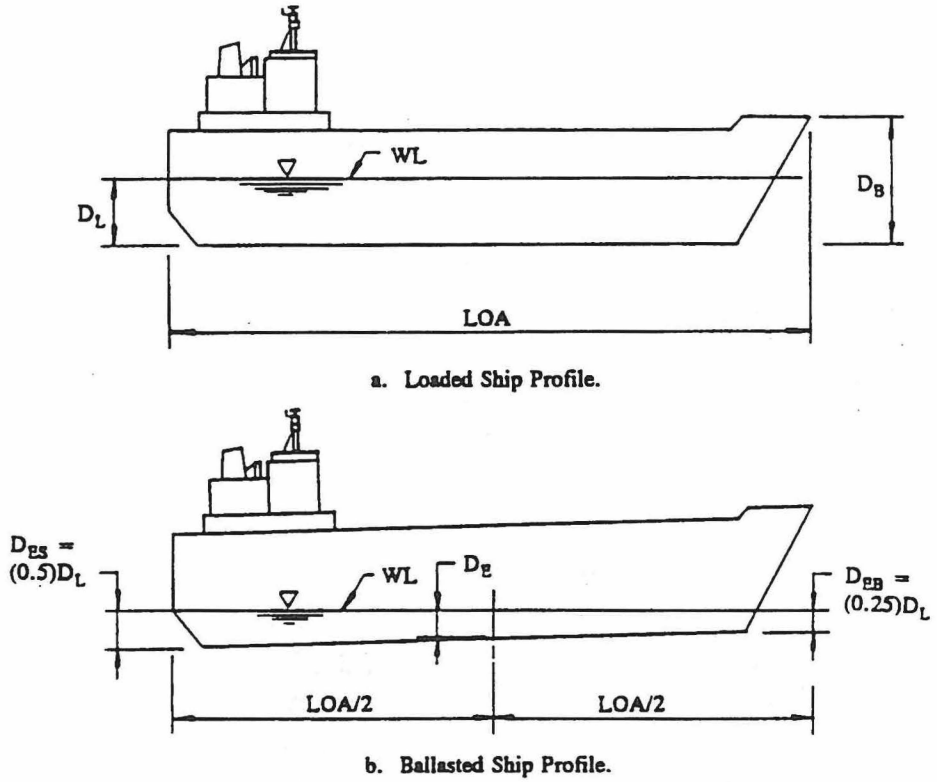


Figure A17B-4 - Typical Ship Characteristics

Ships DWT (tonnes)	Length LOA (m)	Beam B_m (m)	Bow Depth, D_B (m)	Fully Loaded		Ballasted		
				Draft D_L (m)	Displacement, W_L (tonnes)	Draft D_{EB} (m)	Draft D_{ES} (m)	Displacement, W_E (tonnes)
1000	60	8.9	8.3	4.3	1500	1.07	2.16	600
3000	88	12.7	11.6	6.8	4200	1.71	3.41	1600
5000	104	14.9	13.8	6.5	6800	1.62	3.26	2600
10 000	140	18.7	17.6	8.1	13 100	2.04	4.05	4900
15 000	157	21.5	19.6	9.0	19 300	2.26	4.51	7200
20 000	170	23.7	20.8	9.6	25 500	2.41	4.82	9600
25 000	176	25.1	21.6	9.8	31 500	2.47	4.91	11 800
30 000	192	27.3	22.6	10.6	37 500	2.65	5.30	14 100
40 000	208	30.2	23.7	11.4	49 400	2.87	5.70	18 500
50 000	222	32.6	24.4	11.9	61 100	2.99	5.94	22 900
60 000	235	33.3	25.5	12.3	72 800	3.08	6.16	27 300
80 000	259	36.6	26.3	13.2	95 800	3.30	6.61	35 900
100 000	275	42.0	28.3	16.1	118 600	4.02	8.05	44 500
150 000	313	44.5	30.4	18.0	174 700	4.51	9.02	65 500

Figure A17B-5 - Typical Bulk Carrier Ship Characteristics

Ships DWT (tonnes)	Length LOA (m)	Beam B_m (m)	Bow Depth, D_B (m)	Fully Loaded		Ballasted		
				Draft D_L (m)	Displace-ment, W_L (tonnes)	Draft D_{EB} (m)	Draft D_{ES} (m)	Displace-ment, W_E (tonnes)
1000	57	9.4	7.6	4.2	1400	1.07	2.10	500
3000	85	12.8	10.8	5.9	4100	1.49	2.96	1500
5000	102	14.7	12.7	6.9	6700	1.74	3.44	2500
10 000	139	19.0	16.3	8.1	13 000	2.04	4.05	4900
15 000	157	21.7	18.3	9.0	19 300	2.26	4.51	7200
20 000	171	23.8	19.8	9.8	25 400	2.47	4.91	9500
25 000	176	25.5	20.9	10.1	31 500	2.53	5.06	11 800
30 000	194	27.2	21.9	10.6	37 500	2.65	5.30	14 100
40 000	211	29.9	23.1	11.7	49 500	2.93	5.85	18 600
50 000	226	32.1	23.9	12.5	61 400	3.14	6.25	23 000
60 000	236	34.0	24.9	12.8	73 200	3.20	6.40	27 500
80 000	260	37.3	25.5	13.9	96 500	3.47	6.95	36 200
100 000	270	39.0	25.9	14.6	119 700	3.66	7.32	44 900
120 000	279	42.3	26.9	15.5	142 600	3.87	7.77	53 500
150 000	291	44.2	27.6	17.9	176 800	4.48	8.96	66 300

Figure A17B-6 - Typical Product Carrier/Tanker Ship Characteristics

Ships DWT (tonnes)	Length LOA (m)	Beam B_m (m)	Bow Depth, D_B (m)	Fully Loaded		Ballasted		
				Draft D_L (m)	Displace-ment, W_L (tonnes)	Draft D_{EB} (m)	Draft D_{ES} (m)	Displace-ment, W_E (tonnes)
1000	58	9.5	7.0	4.2	1400	1.05	2.10	500
3000	86	13.2	11.9	5.9	4200	1.48	2.96	1600
5000	103	15.4	13.7	6.8	7000	1.71	3.41	2600
7000	129	17.6	16.1	7.5	9700	1.88	3.75	3600
10 000	144	19.4	17.7	8.2	13 800	2.06	4.11	5200
12 000	152	20.1	18.5	8.8	16 600	2.21	4.42	6000
16 000	188	25.7	23.2	9.4	24 800	2.35	4.70	9300
20 000	196	27.6	24.5	10.5	31 600	2.62	5.24	11 850
24 000	212	30.0	25.0	10.5	36 700	2.62	5.24	13 800
27 000	219	31.2	26.2	11.2	42 200	2.81	5.61	15 800
33 000	263	32.2	26.4	11.5	51 600	2.88	5.76	19 400
49 700	290	32.3	28.9	11.0	77 000	2.76	5.52	28 900
54 500	275	39.4	29.4	12.5	84 500	3.12	6.25	31 700

Figure A17B-7 - Typical Freighter/Container Ship Characteristics

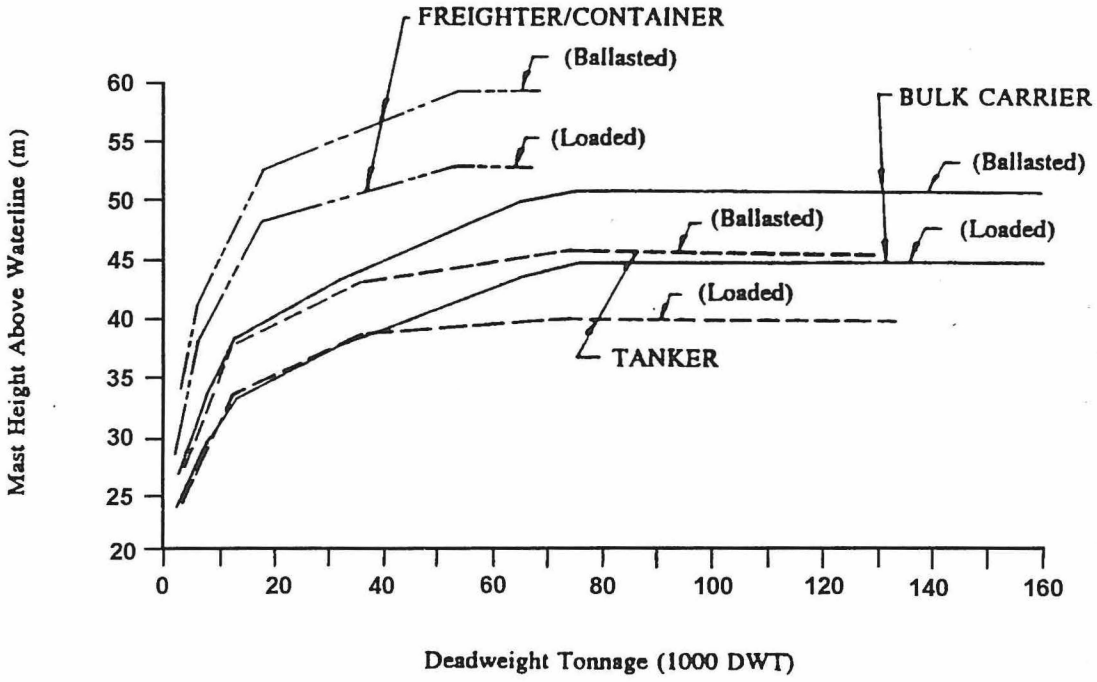


Figure A17B-8 - Typical Ship Mast Clearance Heights

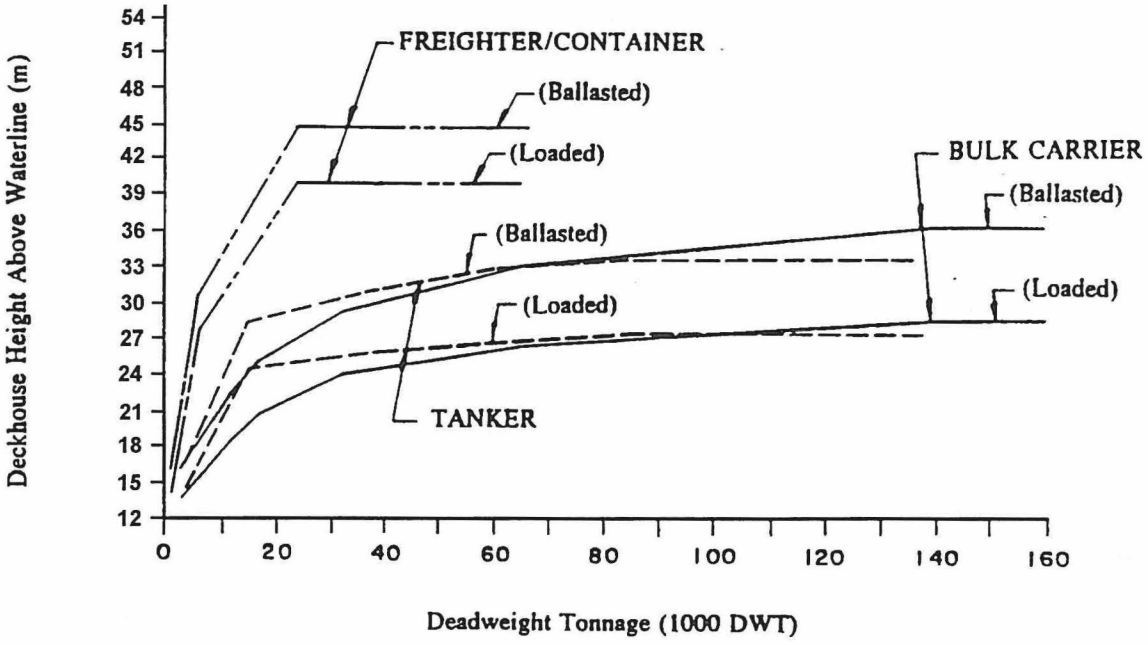
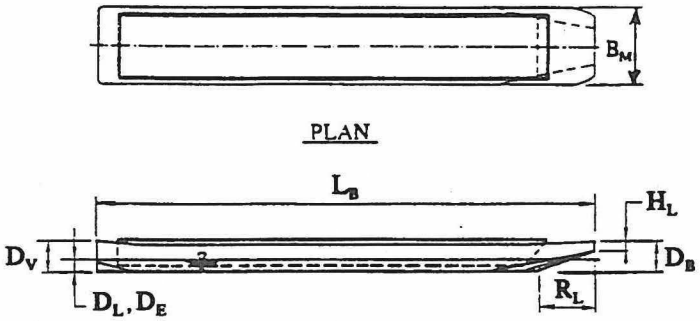


Figure A17B-9 - Typical Ship Deck House Clearance Heights

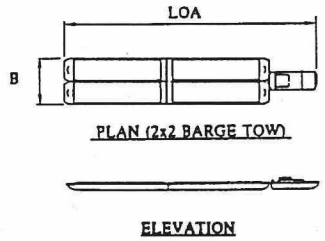
APPENDIX B

Typical Barge Characteristics



	Jumbo Hopper	Oversize Tank	Special Deck
L_B = length (mm)	60 000	88 000	76 000
B_M = width (mm)	10 700	16 200	22 000
D_V = depth of vessel (mm)	3600	3600	5200
D_E = empty (light) draft (mm)	520	520	760
D_L = loaded draft (mm)	2650	2650	5500
D_B = depth of bow (mm)	4000	4000	5500
R_L = bow rake length	6000	7600	9100
H_L = head log height (mm)	600-900	600-900	900-1500
C_c = cargo capacity (metric tons)	1550	3360	4550
W_E = empty displacement (metric tons)	180	550	1180
W_L = loaded displacement (metric tons)	1730	3900	5700

Figure B17B-1 - Typical Barge Characteristics



Barge Type	Average Size of Barge in Tow	Average Number of Barges Along Length of Tow
Jumbo Hopper	10 700 x 60 000	3
Oversize Tank	16 100 x 88 400	2
Special Deck	21 900 x 76 200	1

Figure B17B-2 - Typical Barge Tow Configuration

Barge Type	Size	% of Barges In System 1975	Length (mm)	Width (mm)	Maximum Loaded Draft (mm)	Capacity (metric tons)
Open Hopper	Small	6	36 600	9100	2100	570
Open Hopper	Standard	14	53 300	7900	2700	960
Open Hopper	Jumbo	27	59 400	10 700	2700	1550
Open Hopper	Oversize	1	74 700	10 700	3000	2180
Covr'd Hopper	Jumbo	22	60 000	10 700	2700	1550
Deck Barge	Small	10	30 500/45 700	7900/9800	1800	320/550
Deck Barge	Jumbo	2	60 000	10 700	2700	1550
Deck Barge	Oversize	2	61 000	15 200	2700	1870
Tank Barge	Small	3	41 100	12 [illegible]0	2700	1180
Tank Barge	Jumbo	4	60 000	10 700	2700	1550
Tank Barge	Oversize	9	56 400/88 400	16 200	2700	2300/3400

Figure B17B-3 - Typical Characteristics of Barges on the Inland Waterways System

Towboat Type	kilowatts	% of Towboats 1975	Typical Length (mm)	Typical Width (mm)	Typical Draft (mm)
Harbor Boat	< 450	29	15 200	4900	1400
Line Haul	450-900	40	23 800	7000	2100
Line Haul	900-1800	14	36 600	9100	2700
Line Haul	1800-3200	10	44 500	10 700	2700
Line Haul	3200-6200	7	48 800	13 700	2700
Line Haul	> 6200	1	56 400	16 800	2700

Figure B17B-4 - Typical Characteristics of Tow Boats on the Inland Waterways System

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