U.S. Department
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**Federal Highway
Administration**

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Materials Control and Acceptance - Quality Assurance

National Highway Institute

MATERIAL CONTROL AND ACCEPTANCE QUALITY ASSURANCE

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TABLE OF CONTENTS

CHAPTER 1: INTRODUCTION

INSTRUCTIONAL OBJECTIVES

1. Introduce the students and instructors.
2. Present basic quality-related definitions.
3. Present types of specifications and how they work.
4. Introduce the major topics for the course.

DESIRED STUDENT ACHIEVEMENTS

1. Develop interaction among students and instructors.
2. Become conversant with basic quality terminology.
3. Understand concepts of various types of specifications.
4. Understand the objective of the course.

CHAPTER 1: INTRODUCTION

1.

In the fall of 1992, the **National Policy on the Quality of Highways** was set forth " . . . to make a continuing commitment for quality products, information and services . . . ".

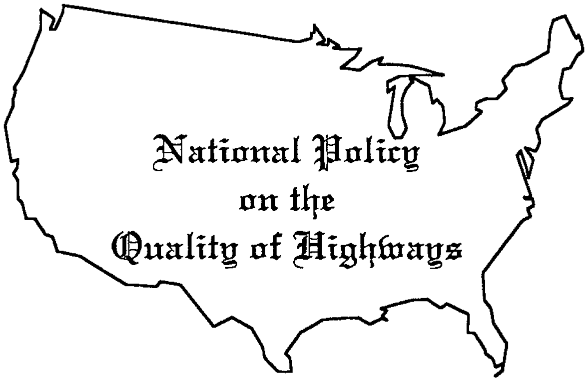
2.

Among the principles in this policy, the following relate directly to Quality Management:

- Proper design, construction specifications related to performance, adherence to specifications, use of quality materials, use of qualified personnel;
- Adequate assurances of quality achievement . . . by owner agencies;
- Incentives that reward achievements . . . in providing a demonstrated level of value-added quality;
- Cooperative development of quality management systems and specifications between Federal, State, and local agencies, academia, and industry.

3.

As part of this National Policy and through the cooperative efforts of FHWA, AASHTO, and transportation industry representatives, a **National Quality Initiative (NQI)** has been developed to promote the improvement of our highways



Principles

- Quality Materials
- Assurances of Quality
- Incentives for Quality
- Quality Management Systems

National Quality Initiative

NQI

NOTES

4.

There are signs of a strong emphasis on quality wherever one looks today. The United States has established and for a number of years has awarded the **Malcolm Baldrige National Quality Award**. This is the highest quality award that a U.S. company can win. Awards are made annually to recognize U.S. companies that excel in quality management and quality achievement. Up to two awards may be given in each of three eligibility categories: manufacturing companies, service companies, and small businesses. In 1992, five awards were made. This is the most that have been awarded in one year since the award began in 1988.

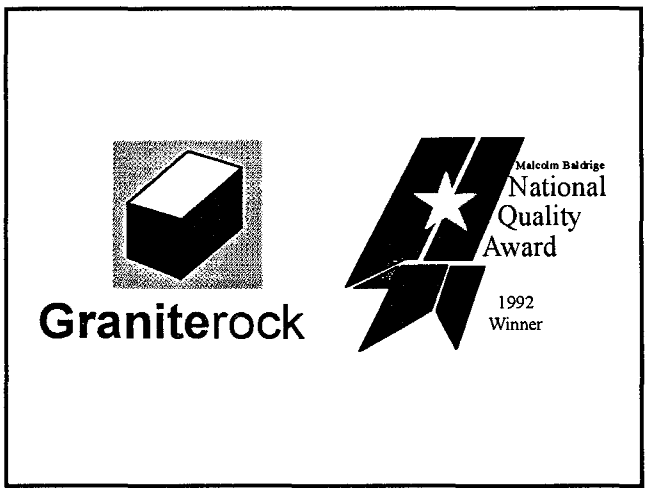
5.

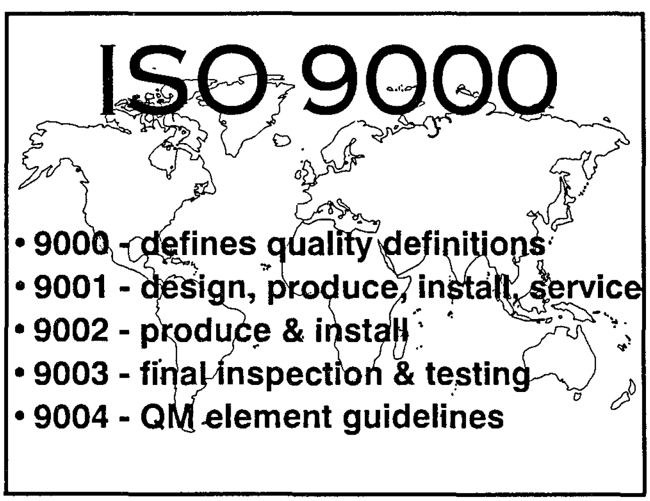
Also in 1992, for the first time a contractor was the recipient of the award. Granite Rock Company, headquartered in Watsonville, CA, won in the small business category. Granite Rock produces rock, sand, and gravel aggregates, ready-mix concrete; asphalt concrete; road treatments and recycled road-base materials. This is obviously a major accomplishment, and is an excellent example of what can be achieved in the highway construction industry.

6.

Another quality-related topic that has received much press lately is **ISO 9000**. ISO stands for the International Organization for Standardization which is headquartered in Geneva, Switzerland. ISO develops and promotes common standards worldwide. ISO has 90 member countries, including the United States. ISO 9000 is a series of five international standards for Quality Management and Quality Assurance. The series is generic and is not specific to any one industry. The standards refer to quality system elements that are to be implemented, not to the means to implement them. ISO 9000 and ISO 9004 are advisory in nature. ISO 9001-9003 constitute a three-level series of external quality assurance standards for use in contractual situations. ISO 9000 provides a road map for the use of the other standards in the series and defines key quality terms. ISO 9001 specifies a model for use when a contract between two parties requires the demonstration of a supplier's capability to design, produce, install, and service a product. ISO 9002 specifies a model for quality assurance in production and installation. ISO 9003 is a model for quality assurance in final inspection and testing. ISO 9004 provides quality management and quality system element guidelines for use by any producer in developing and implementing a quality system and in determining the extent to which each quality system element is applicable.


Malcolm Baldrige
National
Quality
Award


Graniterock
Malcolm Baldrige
National
Quality
Award
1992
Winner


ISO 9000
• 9000 - defines quality definitions
• 9001 - design, produce, install, service
• 9002 - produce & install
• 9003 - final inspection & testing
• 9004 - QM element guidelines

NOTES

7.

ISO 9000 registration consists of certification by a properly accredited third party that a company's quality system, as documented and implemented, satisfies the requirements of the appropriate ISO 9000 standard. The ISO 9000 series is very popular in Europe and is gaining momentum world-wide. Many U.S. companies are in the process of becoming ISO 9000 certified because it is widely believed that it will be necessary to have such certification to be able to do work in Europe or for European clients in the future.

8.

This course is one effort to support the NQI. This course deals with **Quality Management (QM)**, and specifically with its relationship to **Quality Assurance (QA)**. As occurs in any evolutionary process, and certainly in specification development, new terminology develops, and confusion with the new terms is likely. Before we can begin, therefore, it is necessary to present definitions of terms that will be used in this course.

9.

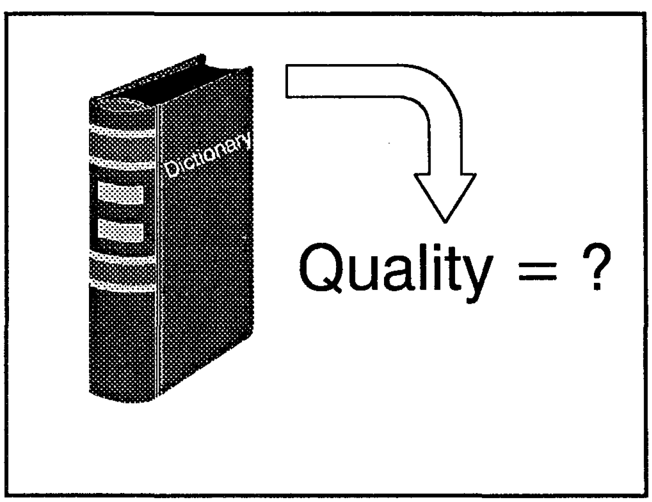
A good place to begin a discussion of any aspects of quality, is with the **definition of quality**. This may seem like an easy task, but in reality the term quality is likely to have different connotations to different people. The American Heritage College Dictionary defines quality as "degree or grade of excellence." Is this a good definition for quality as it applies to highway material? How would you define quality?

National Quality Initiative

NQI

Quality Management

Quality Assurance



NOTES

10.

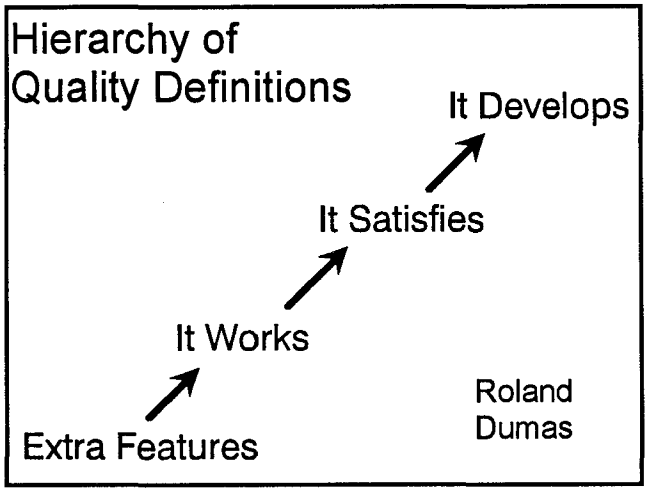
There are many possible definitions of quality. Roland Dumas (1) has defined a **hierarchy of quality definitions**. Studies by Dumas have shown that there is a correlation between the maturity of an organization's approach to managing quality and the definition of quality used. The first level, *extra features*, was a common definition in the automobile industry in the 1950's and 1960's. The definition fails when the customer realizes that the basic product is not reliable, thus rendering the extra features irrelevant. Definitions such as "conformance to requirements," "do it right the first time," and "meet the specifications," are typical of the second level, *it works*. These definitions are effective for accounting purposes in determining the cost of quality, but they do not necessarily reflect the customer's actual needs. As Dumas says, "After all, there are products that meet the requirement but turn out to be nonfunctional or even dangerous in the long run." Deming has stated that "Quality has no meaning without reference to the customer." This thought helps lead to the third level, *it satisfies*. There are many organizations currently moving to this level. Their quality measures focus on the customer rather than on their own internal processes. At the final level, *it develops*, the organization strives to develop its customers through education and exposure to ways to better utilize its products and services to receive greater value. This is not product-use training, but rather a service that enhances the value of the product or service to the customer.

11.

Thanks, in part, to the Japanese and to a vast multitude of books on the market, the term **Total Quality Management**, or **TQM**, is probably not new to anyone in this course. TQM is a new management philosophy which encompasses much more than just quality. It is a new way of managing an organization and deals with all aspects of the organization's operations. One definition of TQM that has been applied to the engineering and construction industry is **a customer-focused, strategic and systematic approach to continuous performance improvement**. TQM might be better thought of as TMC, for total management change, since it represents a new way of looking at the world as it applies to managing an organization.

12.

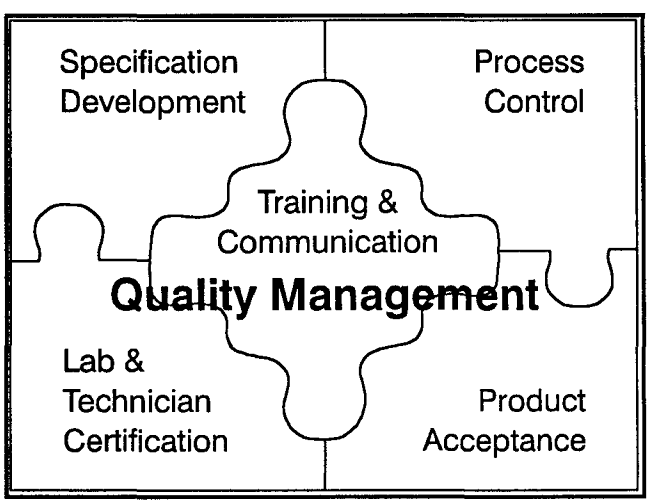
In this course, we will use the term **Quality Management (QM)** to represent all aspects of producing and accepting a quality product. As such, it includes topics as diverse as specification development and implementation, process control, product acceptance, training, communications, and laboratory and technician certification.



NOTES

Total Quality Management

a customer-focused, strategic and systematic approach to continuous performance improvement



13.

Quality Assurance (QA) is one component of QM. QA is made up of many parts and can be accomplished many ways. A popular definition of QA is **those activities which concern making sure the quality of a product is what it should be**. This definition has two parts. "Making sure the quality of a product is" is the first part and deals with the decisions necessary to determine conformity to the specifications. This aspect of QA is discussed in this course. The second part -- "what it should be" -- deals with the basic engineering properties of the material or construction process. A broad interpretation of this definition of QA would appear to be synonymous with the definition of QM. In practice, QA has evolved to the point that QM and QA are often used interchangeably.

14.

Whatever form QA takes, to be effective it must include the functions of **process control**, **acceptance**, and **independent assurance**. QM includes QA as well as all aspects necessary to implement and carry out a QA process.

15.

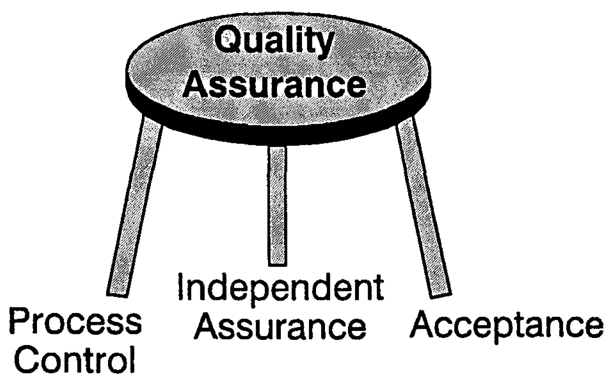
The **process control** function, often referred to as Quality Control (QC), has become recognized as an important part of QA. Quality cannot be tested or inspected into a product after-the-fact. It must be present in the product from step one. And, in virtually every instance, process control is best performed by the contractor or producer; that party to the contract who has control over the product at its initial stage.

NOTES

Quality Assurance

Making Sure the Quality of a Product Is

What it Should Be



Independent Laboratory

Supplier

Process Control

Contractor

Highway Agency?

16.

The **acceptance** function can be varied and may consist of acceptance testing, monitor testing, or process control monitoring. However, one essential ingredient is inspection. The need for an inspection activity is as important in a QA specification as in other types. The acceptance function is generally performed by the owner, i.e., the state highway agency, although some states have experimented with basing acceptance decisions on tests conducted by the contractor.

17.

Independent assurance (IA) is a management tool that requires a third party, not directly responsible for process control or acceptance, to provide an independent assessment of the product.

18.

The most recognized form for an IA program is that which the FHWA requires of state highway agencies for FHWA-funded construction projects.

NOTES

Acceptance Function

- Acceptance testing
- Monitor testing
- Process control monitoring
- INSPECTION

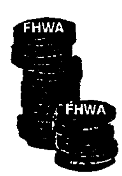
- Performed by the owner

Independent Assurance

Independent, third party assessment of the product.

Independent Assurance

FHWA requires IA



on FHWA-funded projects.

19.

The purpose of the FHWA IA program is to make independent checks on the reliability of the results obtained in acceptance sampling and testing and not for directly determining the quality and acceptability of the materials and workmanship. The tests are to be performed by State personnel not normally responsible for process control or acceptance.

20.

For the IA program to be effective, inspectors who witness test procedures or obtain IA samples should make a strong effort to maintain good rapport with inspectors responsible for acceptance. A reasonable philosophy might be the following.

> It is not the purpose of IA to find things being performed incorrectly, but rather to verify that procedures are proper. However, for the IA program to be effective, all variations from specified procedures and significant differences between acceptance tests and IA tests must be reported.

IA inspectors must be knowledgeable on all sampling and testing procedures so they can recommend correction to procedures before leaving the project.

21.

Prior to discussing the many ways of conducting materials control and acceptance procedures, we first make a brief comparison among the many specification types and discuss the evolution that has taken place in specifications over the last 30 years.

NOTES

Independent Assurance

- to be performed by personnel NOT directly responsible for process control or acceptance sampling and testing
- provides an independent check on the reliability of the results obtained for acceptance

It is not the purpose of IA to find things being performed incorrectly, but rather to verify that procedures are proper.

However, for the IA program to be effective, all variations from specified procedures & significant differences between acceptance & IA tests must be reported.

Evolution of Specifications

1950's ↗ 1960's ↗ 1970's ↗ 1980's ↗ 1990's

Today?

22.

The **method specification**, which was widely used in the 1940's and 1950's, puts maximum **control** and **responsibility** in the hands of the specifying agency. The contractor is required to follow step-by-step procedures using specified materials and equipment. In the extreme case, this specification can be considered an equipment and labor rental specification. As an example, in asphalt concrete compaction, the specifying agency may tell the contractor what equipment to use, when to roll, and how many passes to make with each roller.

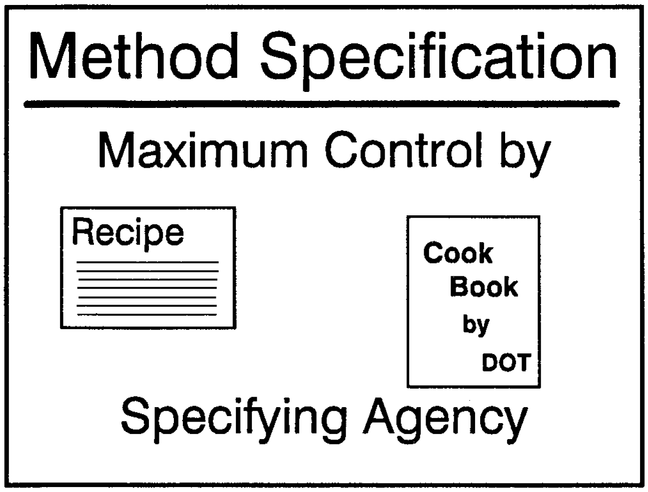
In the early use of method specifications, little or no testing was done. The specifying agency based acceptance primarily on inspection. This type of specification is still used, but usually is accompanied by a greater degree of testing than in the 1950's.

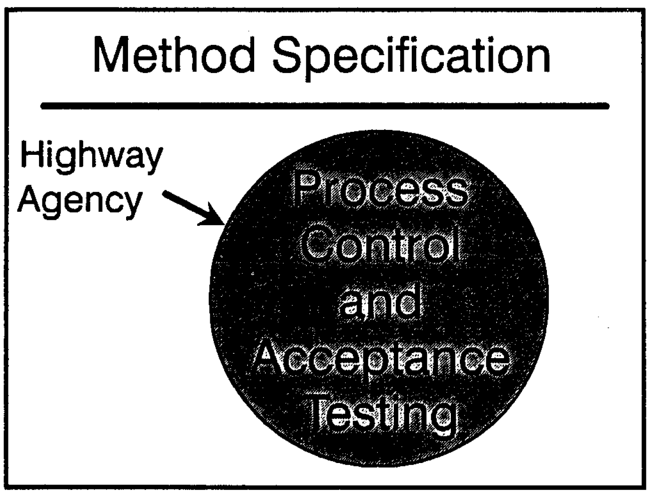
23.

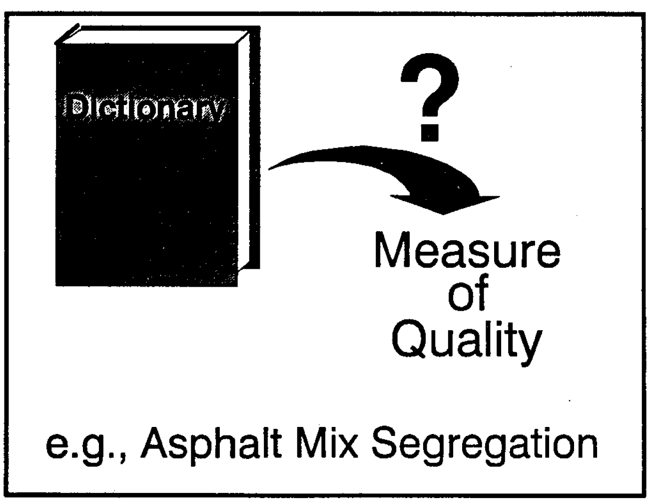
With this type of specification, little process control and acceptance testing are performed. Rarely are they separate functions. The agency inspector in the above example verifies that the rollers meet the specification in terms of size and weight and then must count the number of passes of each roller and monitor the compaction temperature. The contractor exercises minimal control and has little responsibility. Independent assurance is not normally performed under this type of specification. However, it should be performed by having an independent party verify the roller properties, number of passes, and temperatures.

24.

There are areas where method specifications can be used advantageously. These are areas where a measure of quality is particularly difficult to define. Asphalt mix segregation is one such case. Segregation is an undesirable feature, but the degree of segregation that is allowable is difficult to specify. Thus, method specifications can be used to specify what a contractor must do to prevent segregation.


Method Specification
Maximum Control by
Recipe
Cook
Book
by
DOT
Specifying Agency


Method Specification
Highway
Agency
Process
Control
and
Acceptance
Testing


Dictionary
?
Measure
of
Quality
e.g., Asphalt Mix Segregation

NOTES

25.

The disadvantages of method specifications are:

- Contractors may not be allowed to use the most economical or "innovative" procedures to produce the product sought.
- They are inspector labor intensive.
- If the quality is measured and found to be less than desirable, the contractor has no legal responsibility to improve it.
- The quality attained is difficult to relate to the performance of the finished product.

26.

In 1958, the construction of the AASHO Road Test provided the impetus toward **end-result specifications**. This specification type started the move toward testing, as opposed to inspection, as the main measure of specification compliance. The development of the acceptance limits used on the AASHO Road Test came from a panel of engineers who used their expertise to determine limits that they thought could be met by the contractor and which would lead to the desired performance of the product.

27.

The testing program undertaken at the AASHO Road Test proved that some of these subjectively derived limits could not be met consistently. Although this project took place over 30 years ago, its impact on specification development has been monumental, and continues today. One example of the lack of consistency in developing the limits which resulted in the inability to meet the specifications is shown in the results of the Marshall test properties.

One important conclusion from the AASHO Road Test was **(2)** "that with many more well-trained inspectors than could economically be used in normal construction, with high-speed testing techniques, with a large-scale materials laboratory on the site, with the ability to control in detail the contractor's construction procedures, with a highly competent and cooperative contractor who was well paid for everything he was required to do, and with the eyes of the highway fraternity on the back of our necks, we were still unable to meet the specifications for many of the construction items within a country mile."

NOTES

Method Specifications
Disadvantages

1. May not allow innovation.
2. Inspector intensive.
3. Contractor - **no legal responsibility.**
4. Quality $\xleftrightarrow{?}$ Performance.

End-Result Specification

Specifying agency sets limits for the Contractor

JMF ± tolerance $\bar{X} \geq$ Limit

AASHO Road Test

Marshall Test Results

Characteristic	% in tolerances
Stability	92
Flow	98
Voids, total mix	91
Voids filled	18

28.

Specifications that have limits based solely on subjective judgment are often difficult to meet due to a lack of definition of the capabilities of the production process and the desired product. An estimate of the desired target value and the variability that can be tolerated are necessary to establish realistic specification limits. The acceptance plan must be based on a scientific and engineering analysis to establish practical target values and to identify inherent variabilities. Only when this is done can the specification be considered truly attainable and defensible.

29.

The disadvantages of specifications using limits based on subjective judgment are:

- The limits are often too tight and the target value not well defined.
- Acceptance decisions are often based on the results of a single test or on only the sample mean. This is very risky from the standpoint of the specifying agency accepting undesirable product. It also discourages accumulating testing results to obtain an estimate of the population.
- The responsibilities for process control and acceptance testing are often not clearly defined.

30.

One approach which can often lead to problems is to combine a method specification with end-result testing. This is potentially a very controversial specification. If the contractor is directed as to what equipment to use and how to use it, and an end-result requirement is also imposed on the product, arguments frequently result. Combining stipulated methods with required end results generally renders the specification legally indefensible.

NOTES

Specifiction Limits
based on subjective judgement

 6.00" + 0.00"

6" pavement?
process capability?
variability?

Disadvantages - Spec Limits
based on Subjective Judgement

1. Target ? ± tolerances

2. often n = 1

3. process control responsibility?

Method Specification

End-result Testing

= DANGER

31.

The next step in the evolution of specifications was a more rational form of end-result specification in which, among other requirements, the acceptance limits are derived using mathematical concepts proven valid in writing defense specifications. These mathematical concepts are proven and have been used in some states for nearly 30 years. This was the introduction of the **QA specification**.

32.

The **quality assurance (QA) specification** is a more rational form of end-result specification. The development of a QA specification requires four principal steps.

- Determine the properties of the desired product while considering the cost to produce the product.
- Define the product in precise and measurable terms.
- Place process control responsibilities on the producer.
- Develop an acceptance plan and carry out the specified testing.

33.

QA specifications separate process control from acceptance testing and clearly state in the contract which party is responsible for each. A QA specification recognizes variability and accommodates it in the acceptance procedures. The acceptance procedures are established to balance the risks to the contractor and to the specifying agency. The specification may include equipment requirements that do not compromise other requirements in the specification.

Quality Assurance Specification

Derived from:

Mil Standard 414

Sampling Procedures and
Tables for Inspection
by
Variables for Percent Defective

U.S. Department of Defense, 1957

NOTES

QA Specifications

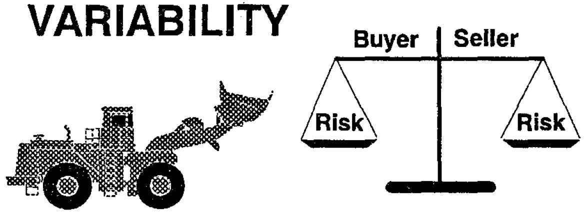
1. Determine desired properties.
2. Define product.
3. Process control by producer.
4. Well-defined acceptance plan.

QA Specifications

Process Control

Acceptance Testing

VARIABILITY



34.

QA specifications are often blamed for the initiation of price adjustment schedules for material that does not meet specifications. However, this blame is unfounded because all specifications should address what consequences result from product that does not meet the specified requirements. Many QA specifications choose to use a price adjustment schedule to define and accept marginal product that has some value instead of requiring complete removal or plant shut-down. Price adjustment schedules are an optional part of QA specifications. Their development and use should be based on a thorough engineering and economic analysis.

35.

Ideally, the specification should be related to performance. The establishment of this ideal relationship has led to a special type of end-result specification known as **performance-related specifications**. This type of specification has two additional requirements to those previously mentioned for QA specifications. The first is that the specification requirements are related to the performance of the product.

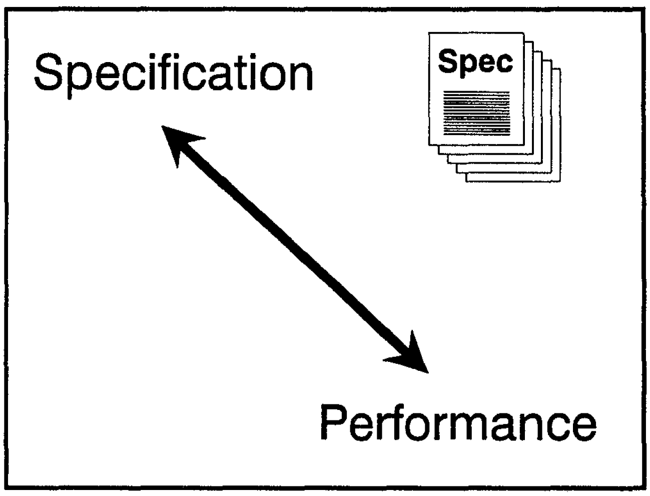
36.

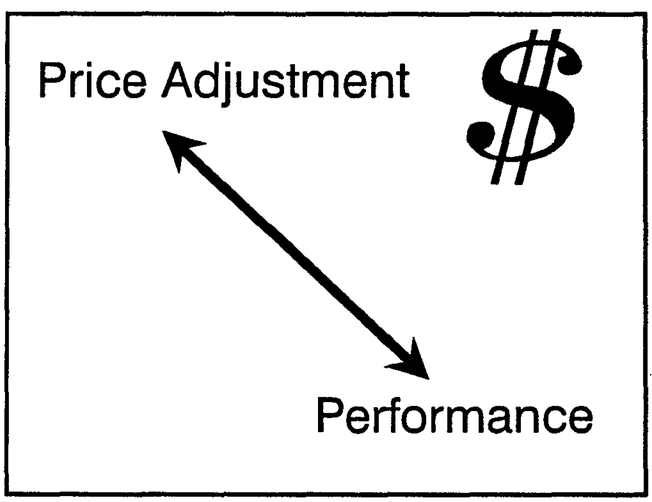
The second requirement is that the price adjustment system is related to the performance or life of the product as reflected by the specification requirements. There are relatively few such specifications currently in existence.

QA Specifications

$\neq$

Price Adjustments





NOTES

37.

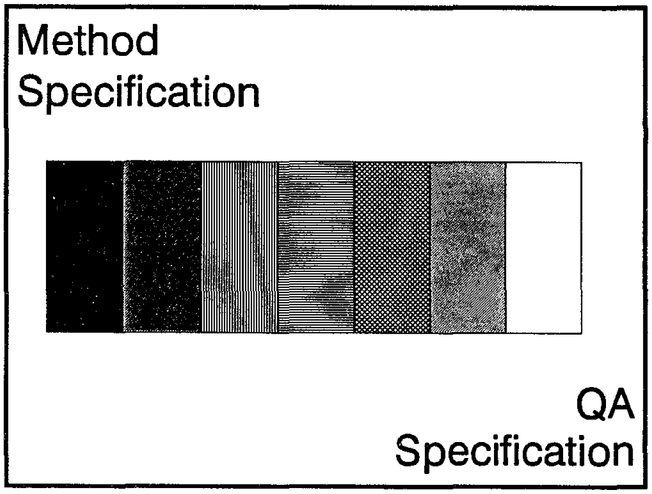
At present, a spectrum of specifications exists. On one end of the scale are the method specifications, with only inspection. Next are method specifications with indirect end-result testing. Next are end-result specifications using intuitively derived acceptance limits and sometimes containing equipment requirements. On the other end of the scale are QA specifications requiring contractor process control, using statistically derived limits for acceptance, and specifying the degree of independent assurance.

38.

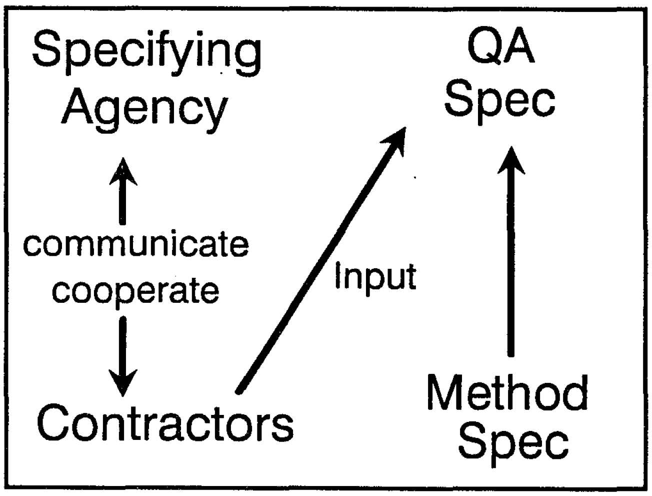
For the implementation of QA specifications, experience has taught that it is essential that industry representatives are brought into QA discussions at an early stage. In this manner, contractors and material suppliers can provide necessary input to the specification development process. This very logical step can reduce potential conflicts and misunderstandings that may hamper specification implementation.

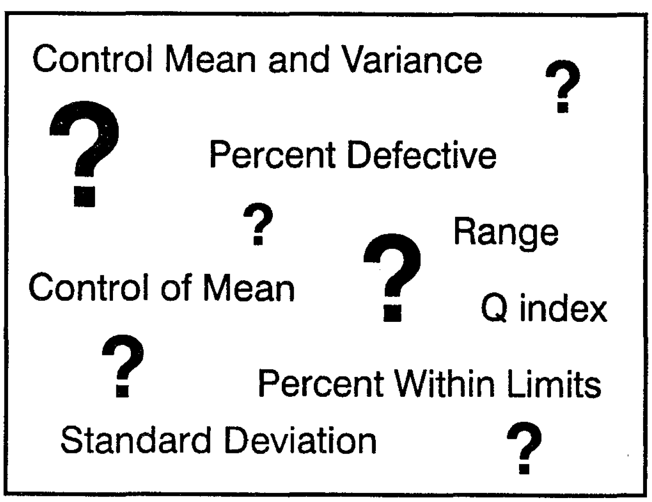
39.

Although there are almost as many forms of QA specifications as there are agencies using them, they all can be categorized and analyzed. However, this takes some basic level of statistical knowledge.


Method
Specification
QA
Specification

NOTES


Specifying
Agency
communicate
cooperate
Contractors
Input
QA
Spec
Method
Spec


Control Mean and Variance
?
?
Percent Defective
?
Range
Control of Mean
?
Q index
?
Percent Within Limits
Standard Deviation
?

40.

Statistics is a tool that can aid in decision making. The level of statistical knowledge necessary for using a QA specification is very basic, and certainly no more difficult than calculating most test results. The analysis of a QA specification requires a more thorough knowledge of statistical concepts, and the development of QA specifications requires a good working understanding of statistical applications.

41.

Statistics is the science that deals with the treatment and analysis of numerical data. This course will hopefully provide the ability to understand the analysis of a QA specification. While a thorough understanding of statistical theory is not necessary to use and understand a QA specification, it is necessary to properly develop and determine the risks associated with the specification. To develop a QA specification, the specification writer must know the product desired, how to define it, and determine the risk of accepting nonconforming material and rejecting conforming material. This takes technical training and experience as well as statistical training. There is an old cliché that says "A little knowledge is a dangerous thing." Analogously, a poorly written QA specification may do more harm than good.

42.

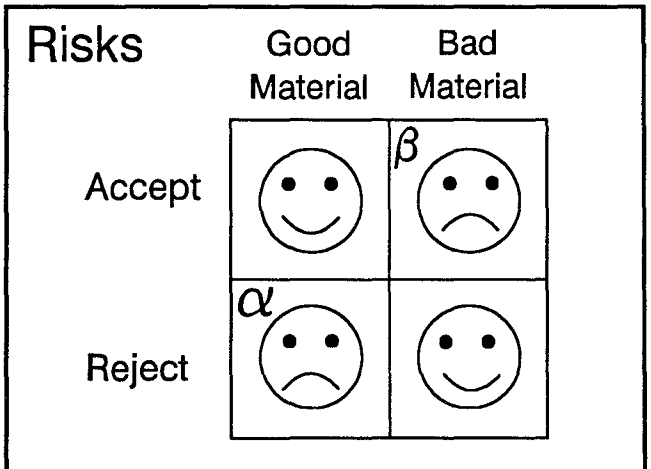
In all specification types, a decision must be made regarding compliance with the specifications. This involves risks, i.e., the chance of making the wrong decision. There are two types of risk. The seller's risk, α, is the risk of rejecting "good" material. The buyer's risk, β, is the risk of accepting "bad" material. For a well written specification, the risks should ideally be balanced between the buyer and the seller.

NOTES

a tool

Statistics

science of treatment and analysis of numerical data



43.

As we will see as the course progresses, one of the greatest advantages of a QA specification is that, if properly developed, the risks can be quantified. If the risks can be quantified, they can be balanced.

44.

This course covers a broad range of topics leading to an understanding of how statistically-based QA specifications are developed. Sampling theory will first be related to real life sampling. A basic understanding of data analysis is important in understanding materials testing. The concept of population and the relationship between samples and population will be presented. The usefulness of the concept of the normal curve will be illustrated. One of the most important concepts that will be taught is that of variability. This is the one concept that is most misunderstood in specification development and enforcement. The importance of process control and acceptance as separate functions in the QA process will be discussed.

45.

Toward the end of the week, the concepts of risk and how they can be evaluated using operating characteristics (OC) curves will be presented. Specification development and design will be thoroughly covered, and the topic of implementing QA specifications will be discussed.

NOTES

Quantifying Risks -

A primary value of QA Specifications

Topics Covered

- Sampling Theory
- Data Analysis
- Population
- Sample/Population
- Normal Curve
- Variability
- Process Control
- Acceptance

Topics Covered

- Risks
- Operating Characteristics Curves
- Specification Development
- Specification Design
- Implementation

46.

The key points of this chapter are summarized in the slide below.

Key Points

1. Current emphasis on quality.
2. Specification types & evolution.
3. Components of a QA system.
4. Statistics is an important tool.
5. Course topics.

NOTES

References

1. Matthews, M. F. and Burati, J. L., "Quality Management Organizations and Techniques," *Source Document 51*, The Construction Industry Institute, Austin, TX, 1989.

2. Carey, W. N. and Shook, J. F., "The Need for Change in Construction Control Procedures," *Proceedings of the National Conference on Statistical Quality Control Methodology in Highway and Airfield Construction*, Charlottesville, VA, 1966.

3. Oswald, T. H. and Burati, J. L., "Guidelines for Implementing Total Quality Management in the Engineering and Construction Industry," *Source Document 74*, The Construction Industry Institute, Austin, TX, 1992.

CHAPTER 2: SAMPLING THEORY

INSTRUCTIONAL OBJECTIVES

1. Introduce the phases of statistical analysis.
2. Discuss applications of sampling.
3. Show relationship between sample and population.
4. Introduce concepts of random sampling.
5. Conduct Sampling Experiment #1.

DESIRED STUDENT ACHIEVEMENTS

1. Understand how a sample relates to the population.
2. Understand the myth of a single representative sample.
3. Learn how to perform random and stratified random sampling.

1.

This session is the first in a series designed to introduce the **basic phases of statistical analysis** along with the basic concepts and tools necessary to evaluate materials and construction processes.

The objective of statistical analysis, as it relates to construction materials, is to derive an understanding of these materials by utilizing **data** obtained from a small portion of the total quantity of materials produced.

2.

The procedures used to achieve this objective can be explained by dividing statistical analysis into four phases:

Collection of Data. The planned process of obtaining a relatively small number of measurements (sample data) from a fairly large quantity of material (lot or population). Proper sampling procedures are essential for the collection of valid, meaningful data.

Organization of Data. Assembling of data into systematic groups or classifications from which logical conclusions can be drawn.

Analysis of Data. Numerical determination of statistical measures which describe the important characteristics of the data.

Interpretation of Data. Using the basic sample results to infer broader statements about the total quantity of material (lot or population). An understanding of basic statistical and probability concepts is necessary to ensure proper interpretation of the sample results and to understand how and why data are often misinterpreted.

3.

This chapter considers the first phase: **collection of data**. Specifically, how **sampling theory** is applied to collect data for organization, analysis, and interpretation is considered. Data are of little value if they are collected in a haphazard, unplanned manner.

NOTES

Why Collect

?

Data

Phases of
Statistical Analysis

1. Collection of Data
2. Organization of Data
3. Analysis of Data
4. Interpretation of Data

Phases of
Statistical Analysis

1. Collection of Data
2. Organization of Data
3. Analysis of Data
4. Interpretation of Data

4.

Typically, data collection involves **sampling** to determine the characteristics (slump, asphalt content, etc.) of a larger quantity of material, commonly known as a **lot** or **population**. The **lot** is usually too large to test or evaluate in its entirety (complete enumeration), so we must evaluate a portion of the lot (sample) in order to make decisions about the total lot.

5.

The word **sample** can be used to refer to a **sample** of aggregate or asphalt concrete that is collected for testing to evaluate the material under consideration. **Sample** also has another meaning when used in a statistical application. To distinguish between these uses, in this module, the term **specimen** is used to refer to the material that is obtained for testing.

6.

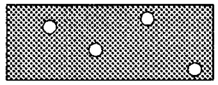
Some definitions are in order if we are to understand the process by which **data** are obtained by selecting a **sample** from a **lot** or **population**.

Population or Lot. The totality of measurements or counts that is obtainable from all of the objects which possess some common specified characteristic. A common definition of a lot is an isolated quantity of material produced essentially by the same process.

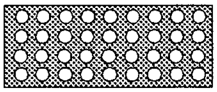
Sample. A set of measurements or counts that constitute a part or all of the population.

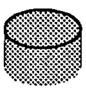
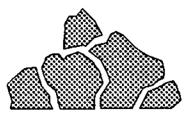
Data. Factual information, such as measurements or statistics, used as a basis for reasoning, discussion, or decision making.

NOTES

Sampling 

vs.

 Complete Enumeration

 ← Sample → 
(Specimen)

$X_1 X_2 X_3 X_4$ ← Sample
$n = 4$

Sampling vs.
Complete Enumeration

Definitions:

Population
Sample
Data

7.

The large cube-shaped object can be thought of as a **lot** consisting of the smaller cube-shaped blocks which represent potential **specimens**. Assume that the cube represents a lot of hot mix and that we wish to know the asphalt content of the hot mix in the lot. It should be kept in mind that it is always the properties of the lot that we wish to identify. Obviously, to determine the "best" estimate of asphalt content, every bit of material in the lot should be tested (**complete enumeration**).

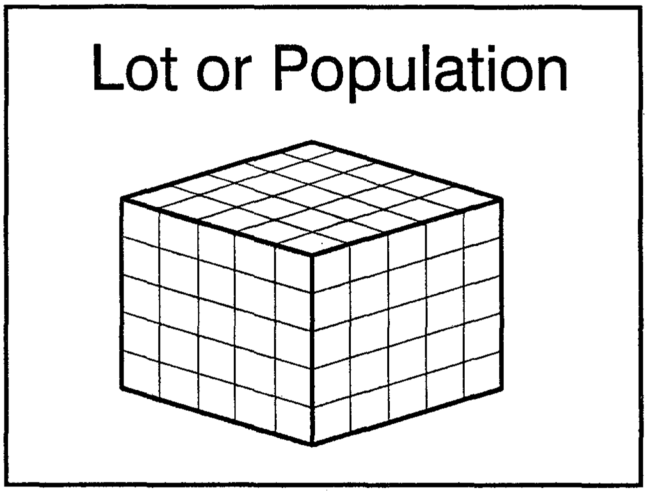
8.

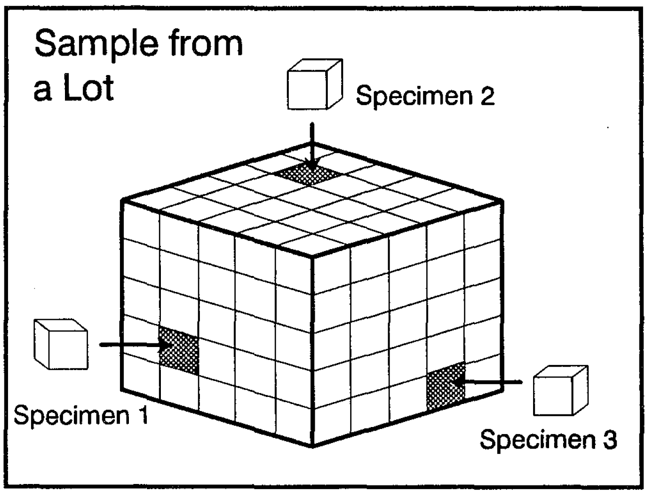
Complete enumeration is never feasible. For destructive testing, the entire lot would be destroyed. For nondestructive testing, there is neither time nor labor available for complete enumeration. Therefore, **sampling** is the only practical solution. A **sample** can be selected from the lot of material, and the **data** from the sample can be used to estimate the asphalt content of the hot mix in the lot in order to make a **decision** regarding its acceptability.

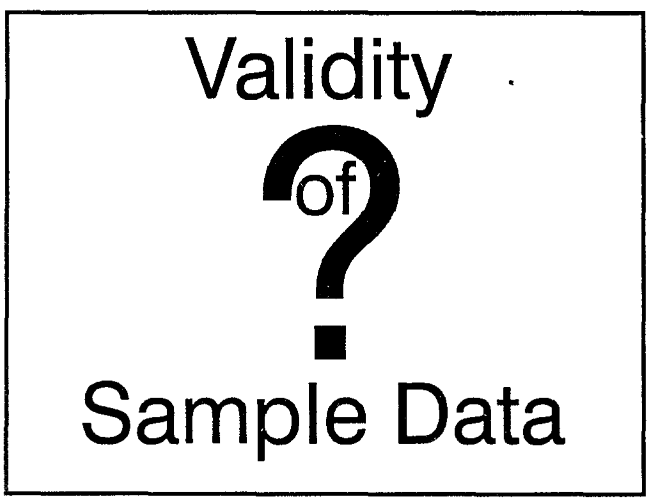
9.

The relationship between the properties of the **sample** and the properties of the **population** is an important aspect of statistical theory and practice since "good" estimates of the properties of a population require valid samples. Two concepts that are of particular importance for ensuring sample validity are **random sampling** and **controlled conditions**.

NOTES


Lot or Population


Sample from
a Lot
Specimen 2
Specimen 1
Specimen 3


Validity
of
?
Sample Data

10.

Obtaining valid samples is not automatic. The following are two possible procedures for obtaining samples:

Random Sampling. A sampling procedure whereby any individual measurement in the population is as likely to be included as any other.

Biased Sampling. A sampling procedure whereby certain individual measurements have a greater chance of being included than others.

From a statistical standpoint, **random sampling** is an absolute necessity. Biased sampling occurs when the inspector uses "judgment" regarding where or when to take the sample. The use of random sampling is necessary when obtaining samples to use for determining specification compliance. This should not be confused with inspection used to identify materials or construction obviously not in compliance.

11.

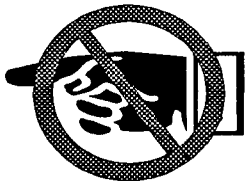
If the collected **data** are to be useful for generalization to the **lot** or **population**, it is essential that they are obtained under **controlled conditions**. This includes the requirement that the set of data to be treated as a single group should represent **homogeneous data** (i.e., measurements or counts made under the same testing situation and that represent construction material produced under essentially the same conditions).

12.

Data can either be **continuous** or **discrete**. **Continuous** data result from a **measurement** process and are usually the result of reading a scale (e.g., a ruler or pressure gauge). Data of this type are referred to as continuous variable data since all values along a continuous scale within a particular range are possible.

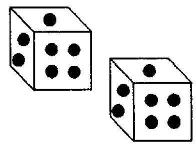
Discrete data result from a counting process or from a "yes or no" decision process. This type of data could result from counting the number of dowel bars placed, or the number of reinforcing bars in a bundle. By definition, then, **discrete** data are not observed on a continuous scale. This distinction is important when attempting to organize and present the data that have been obtained.

Random vs. Biased Sampling



Biased

Random

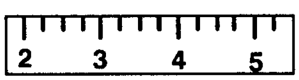


Controlled Conditions

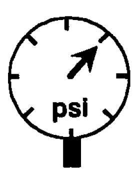
For example,

measurements made under the same testing situation and which represent material produced under essentially the same conditions.

Continuous Data

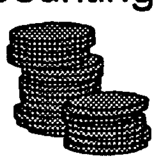


Measurement Process



Counting Process

1-2-3



Discrete Data

NOTES

13.

When a record is made of a measured characteristic, such as compressive strength in p.s.i., the quality is said to be expressed by a **variable** format. Since most construction materials must meet certain requirements, it is common to express specifications in a **variable** format by giving the acceptable upper and lower limits for a measured value.

There are instances, however, where it is only necessary to keep a record of the number of articles conforming, or failing to conform, to the specified requirements. This approach results in the collection of **attribute data**. This might be the case when it is only necessary to compare a particular characteristic to a given standard. It either meets the standard or it does not.

Two typical applications of attribute data are (1) screening tests which can be "go" or "no go", and (2) acceptance of individual items such as lengths of drainage pipe.

However, the use of variable data is more efficient than attribute data. It would therefore be a mistake to try to convert a variable data specification into an attribute data specification.

14.

As mentioned earlier, it is always the properties of the **lot** or **population** that are needed. Sampling is the only effective means for **estimating** the acceptability of a lot or population. However, how many specimens (1, 2, 3, etc.) should comprise the sample to arrive at a sound estimate? Getting samples which represent the population is the objective of any sampling plan.

15.

Illustrative Example: Sampling

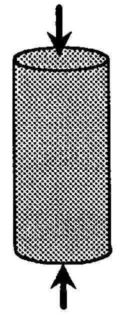
The figure at right illustrates the results of 30 concrete air content tests that we will assume represent a total enumeration of all the possible air content results for a **lot** of structural concrete. If the specification limits for air content for this mix are 5% $\pm$ 1%, then any air content result between 4% and 6% is within the specification requirements. A review of the figure indicates that 12 of the tests (40%) are outside specification limits. Conversely, 60% are within specification limits. Obviously, a sample of size one (often referred to as a single sample) cannot show this type of proportioning.

NOTES

Variable Data

Quantitative

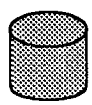
Strength = 3400 psi



Does strength meet spec requirement? Yes or No?

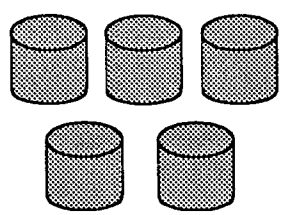
Qualitative

Attribute Data



Single Sampling

Multiple Sampling



2.1	4.1	3.4	3.7	7.3
5.0	5.2	4.0	2.0	5.3
4.8	5.9	5.4	4.7	5.1
6.0	7.1	1.9	5.2	6.8
4.9	6.0	4.2	6.2	5.8
3.0	5.0	3.9	7.7	5.8

Lot or Population
4 < spec < 6

16.

The results of a sample of five air content tests taken from the **lot** are shown at right. If a decision were to be based on only the first test result (2.1%), then the lot would be rejected. If, however, we take a second test and the result is 5.2%, which is within the specification limits, some indecision is introduced since one sample meets the requirement while the other does not. Increasing the number of tests in the sample presents a clearer picture regarding how the air content is **varying** within the lot. This **variation** can not be shown on the basis of a sample of size one, but is indicated as the number of tests (specimens) increases (i.e., **multiple sampling**). We will consider the question of **how many** specimens per sample should be taken in subsequent chapters.

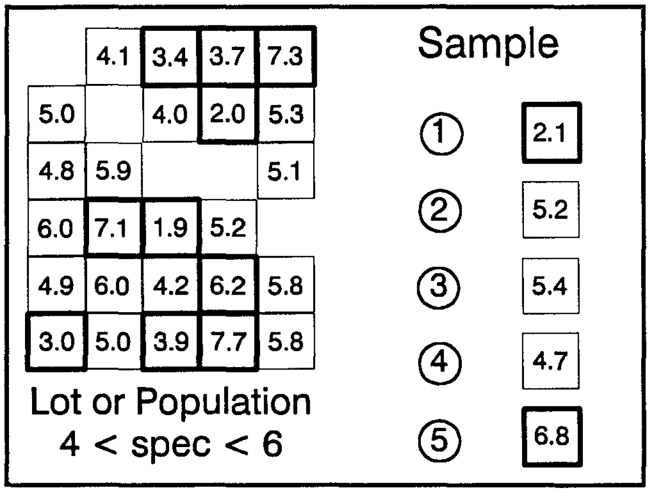
17.

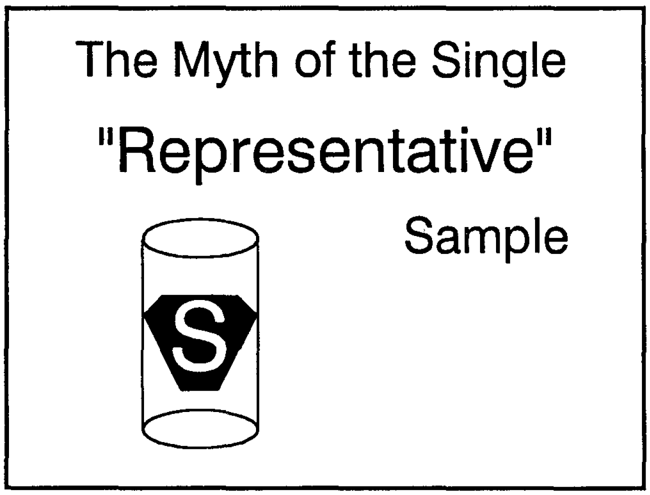
The idea persists that a test on a **single sample** shows the "true" quality of the material, and that if any test result is not within some limit, there is something wrong with the material, construction, sampling or testing. Thus, terms such as investigational, check, and referee samples are in common use to either confirm or document these "failures." Nature dislikes identities; variation is the rule. Therefore, any acceptance or process control sampling must account for variability of materials or construction. **Multiple sampling** accomplishes this objective.

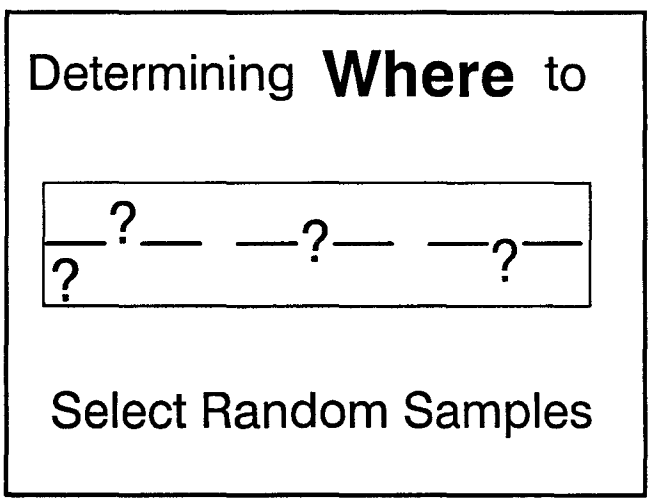
18.

The selection of the sampling locations within the **lot** must be entirely **random**. "Random" does not mean "haphazard"; it means that the sample is selected without bias. In practice it may be difficult to train technicians who have been accustomed to inspection to select samples without regard to quality. Their tendency is to make sure that defective materials are represented in the sample, or possibly that only acceptable materials are included in the sample, thus unconsciously biasing the sample.

NOTES


Sample
4.1 3.4 3.7 7.3
5.0 4.0 2.0 5.3
4.8 5.9 5.1
6.0 7.1 1.9 5.2
4.9 6.0 4.2 6.2 5.8
3.0 5.0 3.9 7.7 5.8
Lot or Population
4 < spec < 6
1 2.1
2 5.2
3 5.4
4 4.7
5 6.8


The Myth of the Single
"Representative"
Sample
S


Determining Where to
? ? ? ?
Select Random Samples

19.

The most common method for determining when or where to obtain samples is through the use of a **random number table**.

A random number table is a collection of random digits. Random number tables come in many forms -- some are short, some long, some grouped by pairs of digits, some with as many as 5 digits per group. When using a random number table, the key is **bias must be avoided**.

A brief example of a random number table is presented at the right. The random numbers are presented in pairs of digits and, for the methods that we will consider, can be thought of as two-place decimal fractions. For example, the random number 57 in the table would be 0.57.

Sampling locations can be determined on the basis of time, tonnage, volume, distance, area, etc. The only other necessary information is the size of the lot to be sampled and the number of specimens to comprise the sample.

When selecting a group of random numbers, one can enter the table at any point (but never at the same point twice) and select the required amount of numbers. The numbers can be selected by columns or rows, by going left or right, up or down, selecting alternate numbers or any other pattern desired.

In addition to using tables, random numbers can be generated by computer and some pocket calculators.

Random Number Table

57	50	75	54	10	74	60	01	27	43	34	07	27	68	50
40	50	91	86	58	29	21	78	01	43	45	57	18	24	06
89	04	52	53	41	28	37	00	49	97	02	05	16	56	92
72	04	68	85	80	73	08	87	32	97	05	32	54	70	48
10	58	38	62	63	72	14	09	70	41	03	52	96	47	78
14	90	56	86	07	22	10	94	05	58	60	97	09	34	33
39	80	82	77	32	50	72	56	82	48	29	40	52	42	01
06	28	89	80	83	13	74	67	00	78	18	47	54	06	10
86	50	75	84	01	36	76	66	79	51	90	36	47	64	93
87	51	76	49	69	91	82	60	89	28	93	78	56	13	68
17	46	85	09	50	58	04	77	69	74	73	03	95	71	86
17	72	70	80	15	45	31	82	83	74	21	11	57	82	53
77	40	27	72	14	43	23	60	02	01	45	52	16	42	37
66	25	22	91	48	36	93	68	72	03	76	62	11	39	90
14	22	56	85	14	46	42	75	67	88	96	29	77	88	22
68	47	92	76	86	46	16	28	35	54	94	75	08	99	23
26	94	03	68	58	70	29	73	41	35	53	14	03	33	40
85	15	74	79	54	32	97	92	65	75	57	60	04	08	81
11	10	00	20	40	12	86	07	46	97	96	64	48	94	39
16	50	53	44	84	40	21	95	25	63	43	65	17	70	82
26	45	74	77	74	51	92	43	37	29	65	39	45	95	93
95	27	07	99	53	59	36	78	38	48	82	39	61	01	18
67	89	75	43	87	54	62	24	44	31	91	19	04	25	92
97	34	40	87	21	16	86	84	87	67	02	07	11	20	59
73	20	88	98	37	68	93	59	14	16	26	25	22	96	63
75	24	63	38	24	45	86	25	10	25	61	96	27	93	35
64	05	18	81	59	96	11	96	38	96	54	69	28	23	91
26	89	80	93	54	33	35	12	54	62	77	97	45	00	24
45	42	72	68	42	83	60	94	97	00	13	02	12	48	92
01	39	09	22	86	77	28	14	40	77	93	91	08	36	47
87	37	92	52	41	05	56	70	70	07	86	74	31	71	57
20	11	74	52	04	15	95	66	00	00	18	74	39	24	23
01	75	87	53	79	40	41	92	15	85	66	67	43	68	06
19	47	60	72	46	43	66	79	45	43	59	04	79	00	33
36	16	81	08	51	34	88	88	15	53	01	54	03	54	56
76	53	01	35	86	34	67	35	48	76	80	95	90	90	17
64	89	47	42	96	24	80	52	40	37	20	63	61	04	02
19	64	50	93	03	23	20	90	25	60	15	95	33	47	64
09	37	67	07	15	38	31	13	11	65	88	67	67	43	97
80	15	73	61	47	64	03	23	66	53	98	95	11	68	77

20.

Random sampling ensures that each portion of a **lot** has the same chance of being selected for the sample. **Stratified random sampling** additionally involves the selection of two or more defined parts of a given **lot**. **Stratified sampling** is used to ensure that the specimens for the sample are obtained from throughout the lot, and are not concentrated in one portion or section of the lot.

21.

The next two figures illustrate the basic principle of **stratified random sampling**. The large rectangle represents a **lot**, perhaps one day's paving from which cores are to be obtained. Using a random number table, it is possible (but not necessarily likely) that all of the cores could be selected within the first half of the lot.

22.

To avoid this possibility, the **lot** can be stratified into a number of **sublots** equal to the sample size to be selected from the lot. One core is then randomly selected from within each **sublot**. This ensures that each portion of the lot has the same chance of being selected while, at the same time, ensuring that the sample is spread out over the entire lot.

An example will help to illustrate the use of random number tables and stratified random sampling.

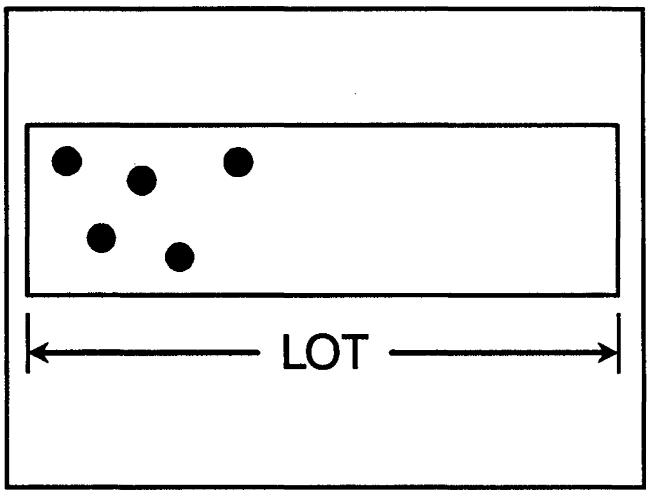
NOTES

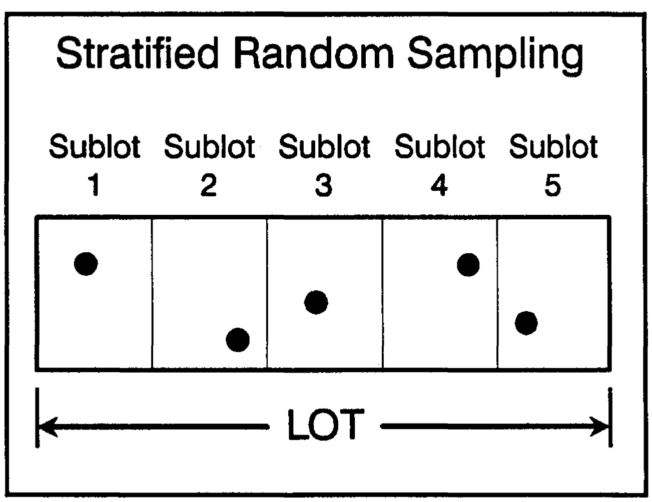
Random Sampling

Stratified

Random

Sampling





23.

Suppose one is to sample a bituminous mixture from the roadway to obtain cores for density determination. The specifications state that the **lot size** shall be 5,000 linear feet of pavement, and that the **sample** consists of 5 cores per **lot**. If we assume that the pavement width is 12 feet and the lot begins at station 100+00, then we can use a random number table to select the sampling locations.

24.

Sublot Size. The lot begins at station 100+00 and ends at station 150+00 (i.e., 5,000 feet in length). Five equal sublots are required, so the sublot length is 5,000/5 = 1,000 feet. The sublot locations are represented on the figure at right.

25.

Now that the sublot boundaries have been identified, the location of the core within each sublot must be determined. To accomplish this, the location must be randomized in the longitudinal as well as the transverse direction.

NOTES

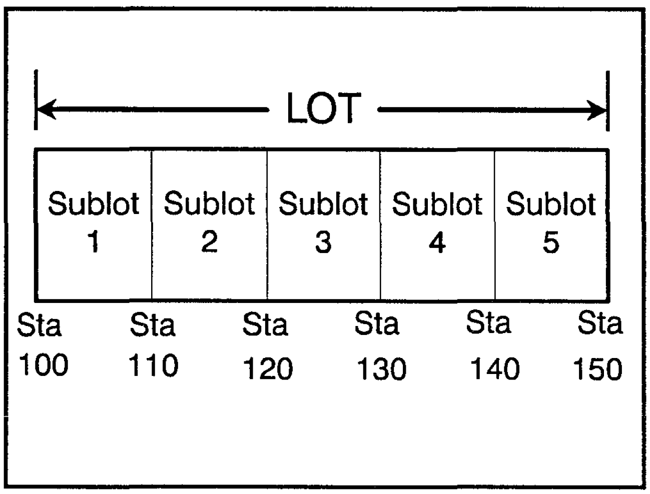
EXAMPLE:

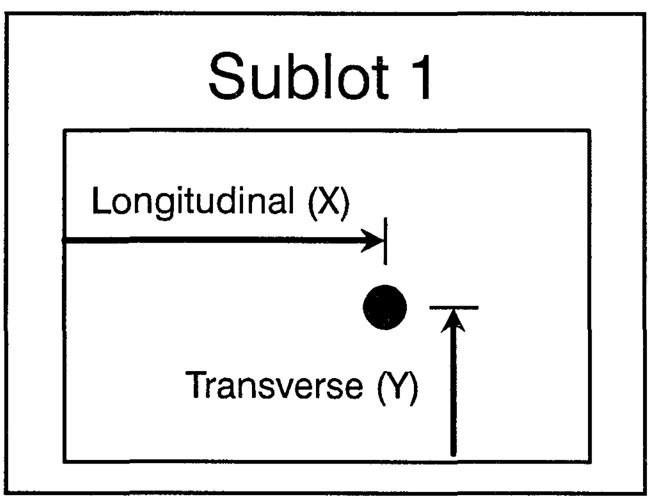
LOT Size: 5,000 LF

Pavement Width: 12 FT

5 Cores per LOT

LOT begins at Sta 100+00





26.

The **random number table** from ***slide 19*** can be used to determine both the transverse and longitudinal locations for the cores. Two sets (columns, rows, etc.) of random numbers are selected, one for the transverse position, the other for the longitudinal position.

A set of 5 random numbers for the longitudinal (X) position and 5 random numbers for the transverse (Y) position of the sample may be chosen by using the second block of numbers from the table.

27.

These X and Y **random numbers** are multiplied by the **sublot** length and paving width, respectively, as shown in the example below:

Sublot #1 (start at station 100+00)

Coordinate X = 0.74 x 1,000 = 740 ft.
Coordinate Y = 0.29 x 12 = 3.5 ft.

Sublot #2 (start at station 110+00)

Coordinate X = 0.60 x 1,000 = 600 ft.
Coordinate Y = 0.21 x 12 = 2.5 ft.

28.

The longitudinal distance (X) is added to the beginning station of the **sublot** and the companion transverse distance (Y) is measured from a selected edge of pavement (assume the right edge for this example).

Sample 1

Sta 100+00 + 740 ft. = Sta 107+40
@ 3.5 ft. from right edge.

Sample 2

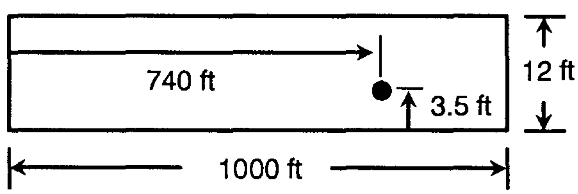
Sta 110+00 + 600 ft. = 116+00
@ 2.5 ft. from right edge.

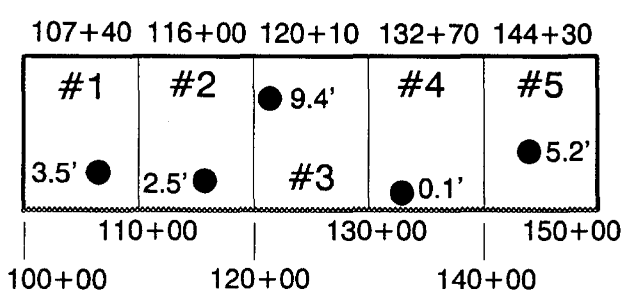
NOTES

74	60	01	27	43	X
29	21	78	01	43	Y
28	37	00	49	97	
73	08	87	32	97	
72	14	09	70	41	

Sublot 1

Coordinate X = .74 x 1000 = 740 ft.
Coordinate Y = .29 x 12 = 3.5 ft.





29.

Sampling Experiment #1

On a typical construction project, **sample data** are **collected** and **organized** for an important material characteristic such as portland cement concrete compressive strength. The **analysis** of the **sample data** is often intended to provide an estimate of the properties of the entire **lot** of concrete.

In a construction situation, there is usually very little information available regarding the true **population** from which the **sample** is taken. At this point, we are going to reverse the typical situation by performing a sampling experiment in which the **population** is defined (known) at the beginning of the "collection of data" phase. The **population** consists of the 200 concrete compressive strengths shown at right.

Concrete Compressive Strength Data

3882	3990	4022	4115	4117	4120	4147
4172	4182	4193	4208	4209	4214	4229
4229	4230	4250	4290	4301	4309	4315
4319	4319	4336	4355	4393	4409	4412
4416	4416	4450	4500	4509	4514	4519
4519	4520	4541	4541	4542	4555	4563
4572	4572	4573	4573	4590	4590	4592
4598	4602	4613	4614	4619	4625	4636
4640	4645	4645	4659	4661	4663	4665
4671	4673	4675	4684	4690	4691	4691
4693	4705	4713	4714	4717	4718	4718
4719	4722	4741	4741	4750	4755	4760
4760	4771	4772	4773	4773	4775	4776
4779	4784	4791	4791	4798	4809	4810
4811	4814	4814	4820	4833	4841	4841
4846	4851	4853	4855	4868	4871	4871
4873	4876	4881	4881	4881	4883	4890
4890	4899	4900	4900	4901	4903	4905
4917	4934	4945	4951	4952	4952	4952
4953	4960	4960	4971	4973	4981	4981
4983	4991	4995	4998	5004	5007	5018
5030	5033	5038	5060	5061	5093	5095
5098	5103	5103	5104	5104	5106	5109
5113	5116	5120	5123	5146	5146	5190
5209	5214	5214	5215	5218	5225	5275
5280	5283	5288	5304	5305	5313	5319
5336	5371	5409	5412	5470	5513	5614
5630	5663	5709	5717	5735	5748	5756
5833	5890	5905	5960			

30.

Procedure for Sampling Experiment #1

Each participant will select a random sample from the lot by using the procedure just completed in the example in ***slides 23-28***. The random number table in ***slide 19*** should be used. The lot is illustrated in the figure at right. In the event that the random location should coincide with a line on the grid, either longitudinally or transversely or both, the next higher location in the same direction should be selected.

Each participant will be polled for his or her selected value and the entire class will record the values in the table below.

Results of Sampling Experiment #1

1	________	11	________	21	________	31	________
2	________	12	________	22	________	32	________
3	________	13	________	23	________	33	________
4	________	14	________	24	________	34	________
5	________	15	________	25	________	35	________
6	________	16	________	26	________	36	________
7	________	17	________	27	________	37	________
8	________	18	________	28	________	38	________
9	________	19	________	29	________	39	________
10	________	20	________	30	________	40	________

Station	24–21	21–18	18–15	15–12	12–9	9–6	6–3	3–0
25+00–24+00	4691	4625	4719	4713	4659	4855	4934	4775
24+00–23+00	4230	4973	5033	5146	4520	5093	4846	5113
23+00–22+00	4290	4903	4995	4900	5120	4811	4229	5190
22+00–21+00	4717	5018	5288	4661	4983	5123	4590	4319
21+00–20+00	4573	4952	4809	4572	4871	5630	5412	4905
20+00–19+00	5756	4952	4673	4814	4881	5218	3990	4555
19+00–18+00	4773	4810	4899	5275	5214	4416	4718	4853
18+00–17+00	4172	4951	4881	5095	4741	5004	4541	4755
17+00–16+00	4791	4901	4890	4590	4115	5060	5098	4820
16+00–15+00	4772	4971	5513	4714	4500	4981	4573	4022
15+00–14+00	4636	4722	5409	4675	5104	5038	4412	5106
14+00–13+00	4690	5109	4833	4563	4301	4981	4409	5225
13+00–12+00	5313	5103	4851	4871	4336	4750	5280	4182
12+00–11+00	5007	5283	4705	4841	5905	4779	5319	4953
11+00–10+00	5116	4645	4117	4881	4542	5104	4514	5061
10+00–9+00	4693	4900	5215	4776	5146	4945	4890	3882
9+00–8+00	5709	4798	5304	4613	4509	4841	5614	4960
8+00–7+00	4120	4519	4760	5735	5103	4952	4771	4917
7+00–6+00	4663	5305	4784	4416	4718	5371	4315	4665
6+00–5+00	4883	4991	5209	4640	4814	4791	5470	5030
5+00–4+00	4876	4309	4602	5336	4645	4760	4393	4773
4+00–3+00	4592	4208	4572	4868	5960	4960	4319	4998
3+00–2+00	4741	5748	4450	5890	4619	4684	4519	4193
2+00–1+00	4691	4614	5717	4671	4214	4229	4873	4355
1+00–0+00	4250	5663	4598	4209	4541	5833	4147	5214

31.

Key points for this chapter are presented in the slide below.

Key Points

1. Sampling vs. total enumeration.
2. Sample vs. population.
3. Random vs. biased sampling.
4. Variable vs. attribute data.
5. Multiple sampling.
6. Random number tables.

References

1. Sigma Partnership, *Practical Applications of Statistical Quality Control in Highway Construction for Engineers and Technicians*, Federal Highway Administration, 1979.

2. Willenbrock, J. H., *Statistical Quality Control of Highway Construction, Vol. I*, Federal Highway Administration, 1976.

CHAPTER 3: ORGANIZATION OF DATA

INSTRUCTIONAL OBJECTIVES

1. Show the importance of organizing data.

2. Demonstrate graphical forms of data analysis.

DESIRED STUDENT ACHIEVEMENTS

1, Understand importance of organizing data.

2. Understand forms of data organization.

3. Learn how to plot frequency histograms and polygons.

CHAPTER 3: ORGANIZATION OF DATA

1.

In Sampling Experiment #1 we obtained data by sampling from a population. These data were in the form of concrete compressive strength test results. We now consider what to do with the data we collect. This brings us to the second phase of statistical analysis, organization of data.

2.

A large group of measurements and test results cannot provide any useful information until they are organized in preparation for analysis. Until the data are organized into a form that is intelligible and understandable, they are just a collection of numbers. The human mind cannot easily comprehend a large series of separate facts or numbers.

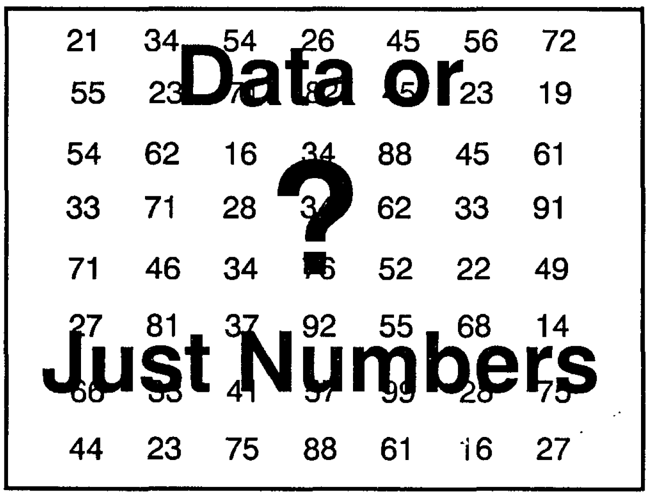
3.

For data to be meaningfully evaluated they must first be organized into a format that will allow an understanding of the trends that are present among the data. This is necessary if further analysis is to be conducted and inferences drawn from the sample data. Data can be organized in two formats: **tabular** and **graphical**.

NOTES

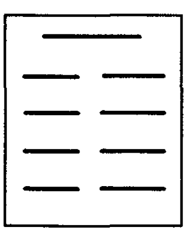
Phases of Statistical Analysis

1. Collection of Data
2. **Organization of Data**
3. Analysis of Data
4. Interpretation of Data



Organization of DATA

Tabular



Graphical

4.

This table presents a set of 30 air content test results for portland cement concrete. The results are presented as a set of individual test values. The test results represent "raw" data that have not been grouped or organized in any fashion to allow a better understanding of the meaning of the results.

5.

The first step that could be taken in an effort to make these numbers more intelligible is to order the data from the smallest to the largest. This provides more information than was provided by the ungrouped raw data. For example, the largest and smallest values and the maximum spread or range of the data are now obvious. However, we are still faced with 30 separate facts to consider. Experience has shown that the mind grasps a pictorial presentation of data faster than any other form. Consequently, several methods of grouping data for presentation have been developed based upon charts, graphs, and curves.

6.

The first presentation method that we will consider is the **frequency table**. As its name implies, this is a tabular method that is a necessary first step for the graphical presentation methods. A frequency table for a set of observations is an arrangement that shows the frequency of occurrence of the values of the variable in ordered classes.

Each group of observations is called a **class**. The **frequency** for any class is the number of observations with measurements falling within that class, while the **relative frequency** for any class is the frequency for that class divided by the total number of observations (data points). The **class limits** are the values that determine the upper and lower limits of a class. The interval between two class limits is called the **class interval**.

NOTES

Air Content for Concrete

Ungrouped Data

2.1	4.1	3.4	3.7	7.3
5.0	5.2	4.0	2.0	5.3
4.8	5.9	5.4	4.7	5.1
6.0	7.1	1.9	5.2	6.8
4.9	6.0	4.2	6.2	5.8
3.0	5.0	3.9	7.7	6.3

Air Content for Concrete

Ordered Data

1.9	2.0	2.1	3.0	3.4
3.7	3.9	4.0	4.1	4.2
4.7	4.8	4.9	5.0	5.0
5.1	5.2	5.2	5.3	5.4
5.8	5.9	6.0	6.0	6.2
6.3	6.8	7.1	7.3	7.7

Frequency Table

Data are GROUPED to show frequency of occurrence in ordered "classes"

7.

Constructing a frequency table is often a "trial and error" process. The following guidelines illustrate the process:

1. Determine the **range** of data.
2. Decide on the number of **classes**.
3. Determine the **class interval**.
4. Establish the **class limits**.
5. Ensure that the classes are **mutually exclusive**.

8.

The process of developing a frequency table can be illustrated with the concrete air voids data from ***slide 5***. (Repeated here on the right hand page)

- Determine the **range** of the data: largest value minus smallest value = 7.7 - 1.9 = 5.8
- Decide on the number of **classes** into which to divide the data. A rule of thumb is 5 to 15 classes depending upon the form of the data. As a first try, select 8.
- Determine the **class interval** by dividing the range of the data by the number of classes to be used. Generally, the class intervals should be the same width.

  (range / number of classes) = (5.8/8) = 0.725

  To keep the intervals even, round off to 1.0 (i.e., 1.0%). An interval of 0.75% would be another possible choice.
- Establish the **class limits** so that the lower limit of the lowest class is a number lower than the lowest data value.

  The lowest data point is 1.9%, so the first lower class limit must be set below this point. Since we rounded up from 0.725% to 1.0% for the class intervals, we have some flexibility and can set the lower class limit at 1.0% so as to have class limits fall on integer values, i.e., 1.0 to 2.0, 2.0 to 3.0, etc. The **class midpoint** is the value half-way between the upper and lower class limits.
- Ensure that all classes are **mutually exclusive**. The class limits are defined so that a value can be placed in only one class.

(continued on page 8)

Guidelines for Frequency Table Development

1. Determine RANGE of data
2. Decide on number of CLASSES
3. Determine CLASS INTERVAL
4. Establish CLASS LIMITS
5. Ensure classes are MUTUALLY EXCLUSIVE

NOTES

Air Content for Concrete

Ordered Data

1.9	2.0	2.1	3.0	3.4
3.7	3.9	4.0	4.1	4.2
4.7	4.8	4.9	5.0	5.0
5.1	5.2	5.2	5.3	5.4
5.8	5.9	6.0	6.0	6.2
6.3	6.8	7.1	7.3	7.7

(continued from page 6)

If, for example, the first class limits are assigned as 1.0 to 2.0 and the second class limits as 2.0 to 3.0, then a problem can arise if a data value is 2.0. Should it be assigned to the first class or the second class? One way to avoid this problem is to establish the class limits half-way between two possible observations. This implies that the class limits will have one more significant figure than the data values. For example, the class limits could be set at 0.95 to 1.95, 1.95 to 2.95, etc.

The same objective can be accomplished by establishing a decision rule that any values falling on a class limit are assigned to the higher class. This is the approach that is used in the example frequency table.

- Place each data value in its proper class to form a tallied frequency column.
- Count the number of tallied frequencies in each class to form a frequency column.
- Divide the frequency in each class by the total number of data values to form a relative frequency column.

Comments: In the frequency table and the graphical approaches that are discussed next, certain information about **individual** data points is lost. For example, from the frequency table it is no longer possible to know the values of the highest and lowest data points. However, the advantages gained by grouping the data far outweigh this loss of individual detail. For example, the frequency table provides us with an appreciation for the **center** (value around which the data are centered) and **spread** of the data.

Frequency Table for Air Content

Class Limits	Class Midpoint	Tally	Frequency	Relative Frequency
1.0 - 2.0	1.5	/	1	0.033
2.0 - 3.0	2.5	//	2	0.067
3.0 - 4.0	3.5	////	4	0.133
4.0 - 5.0	4.5	//////	6	0.2
5.0 - 6.0	5.5	/////////	9	0.3
6.0 - 7.0	6.5	/////	5	0.167
7.0 - 8.0	7.5	///	3	0.1
8.0 - 9.0	8.5		0	0

9.

Remembering that the mind grasps pictorial representations of data faster than other forms, we will now consider the graphical forms of data presentation. The **frequency histogram** is an extension of the frequency table. Individual rectangles whose heights are proportional to the frequencies in each class are erected on the horizontal axis. The base of each rectangle is set equal to the class intervals. If the class intervals are equal in width, the area of an individual rectangle represents the number of observations within the class, while the total area under the figure represents the total number of data values.

10.

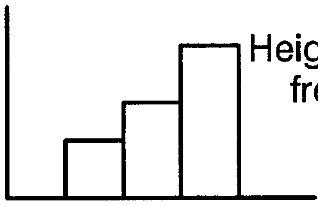
The **frequency table** shown in ***slide 8*** results in the **frequency histogram** shown here. The histogram is a graphical representation of the "tally" and "frequency" columns in the frequency table. A frequency table is usually developed as the first step in preparing all of the graphical presentation formats.

11.

If we divide the frequency in each class by the total number of observations in all classes, then we have the relative frequency for each class. These relative frequencies can then be plotted to form **a relative frequency histogram**. For example, the relative frequency in the first class (1.0 to 2.0) is 1/30 or 0.033. The sum of the relative frequencies in all classes must add up to 1.0 (100%).

NOTES

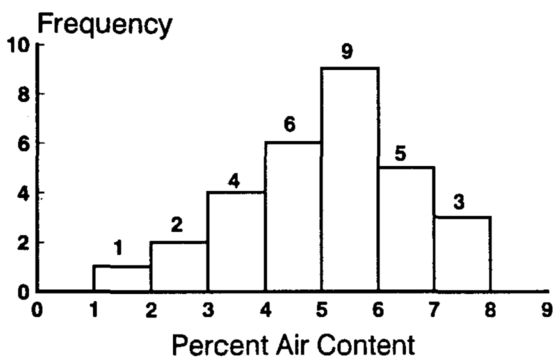
Graphical Forms
Frequency Histogram



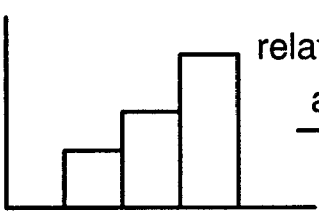
Height = frequency in class

base = class interval

Frequency Histogram



Graphical Forms
Relative Frequency Histogram



relative frequency = $\frac{\text{actual frequency}}{\text{total number}}$

base = class interval

12.

The **relative frequency histogram** for the concrete air content example shows that 3/10 of the test results were between 5% and 6%. The concept of relative frequency is extremely important because it provides us with a relatively simple definition of the term **probability** (i.e., probability is the relative frequency of a particular occurrence). In this case, we can say that the probability is 30% that a single test result will be between 5% and 6%. It is also common for specifications to require an estimate of the percentage of the material that is within certain limits. The relative frequency histogram provides this type of information for a data set.

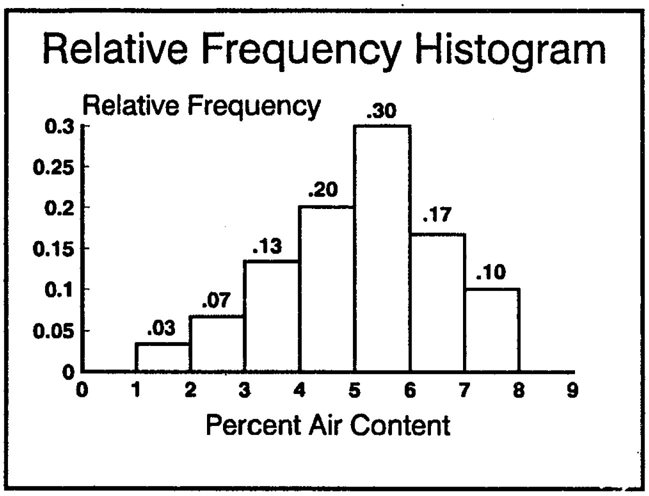
13.

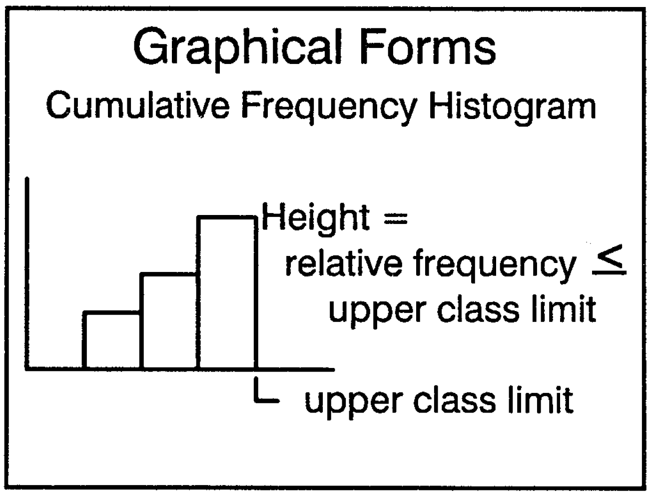
A **cumulative relative frequency histogram** presents the percentage of the data that is **less than or equal to** a certain value. The histogram is constructed from the frequency table or the relative frequency histogram by summing the relative frequencies for each class. The last class must have a cumulative frequency of 1.0 (100%).

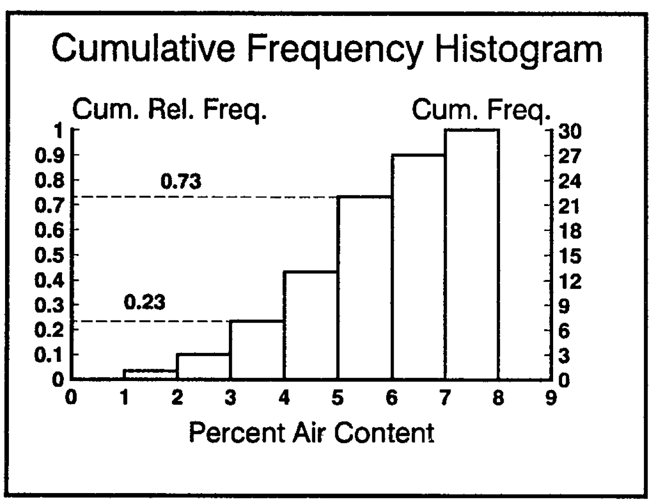
14.

The cumulative frequency histogram for the concrete air content example can be interpreted to indicate that 23% of the data values are less than 4.0% (i.e., less than the lower class limit for the class 4.0 to 5.0) and that 73% of the data values are less than 6.0%. The shape of the histogram is the same for a cumulative frequency and cumulative relative frequency basis.

NOTES


Relative Frequency Histogram
Relative Frequency
0.3
0.25
0.2
0.15
0.1
0.05
0
.03
.07
.13
.20
.30
.17
.10
0 1 2 3 4 5 6 7 8 9
Percent Air Content


Graphical Forms
Cumulative Frequency Histogram
Height =
relative frequency ≤
upper class limit
upper class limit


Cumulative Frequency Histogram
Cum. Rel. Freq.
Cum. Freq.
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.73
0.23
30
27
24
21
18
15
12
9
6
3
0
0 1 2 3 4 5 6 7 8 9
Percent Air Content

15.

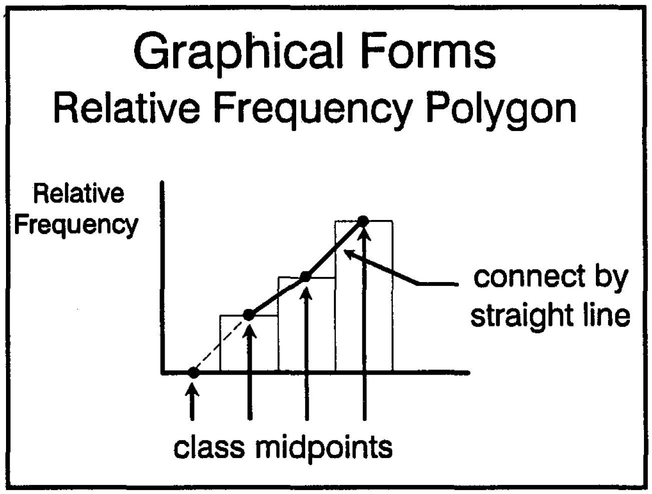
If an ordinate (value on the vertical axis) equal to the relative frequency in each class is erected at the class midpoint (i.e., the center of each class interval) and the ends of these ordinates are joined by straight lines, a **relative frequency polygon** is formed. This assumes that the class midpoint can be used to represent the class.

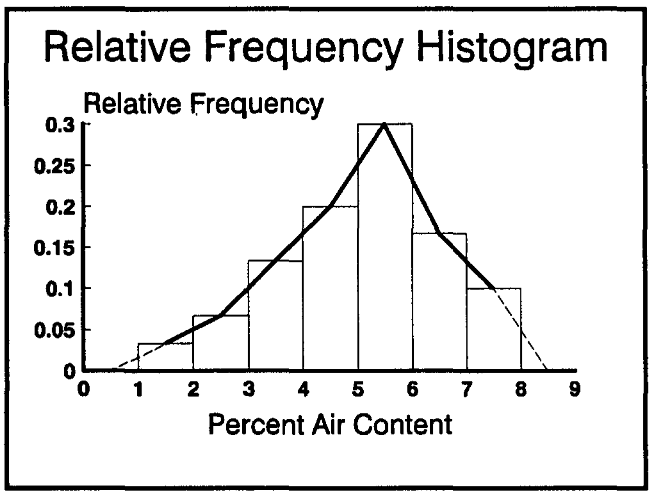
16.

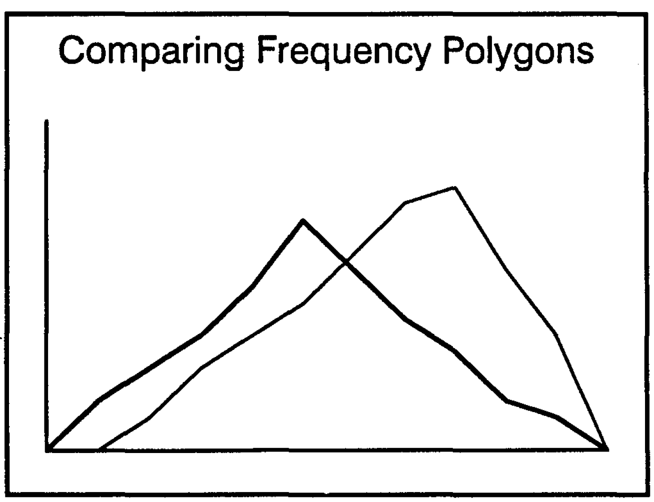
In this plot, the **relative frequency polygon** has been superimposed on the relative frequency histogram for the concrete air content example. As a matter of convention, the polygon has been extended to meet the horizontal axis at the midpoints of the classes below and above the limits of the histogram. These extensions of the polygon are sometimes shown as dotted lines to indicate that they are extensions beyond the range of the data.

17.

In many cases, the relative frequency polygon presents a more effective picture of the variation in the distribution of the data even though the histogram shows the frequency of each class most clearly. Two histograms can not be effectively superimposed upon the same graph for the purpose of comparison. On the contrary, two frequency polygons (particularly two relative frequency polygons if the sizes of the two data sets are different) can be easily compared.


Graphical Forms
Relative Frequency Polygon
Relative Frequency
connect by straight line
class midpoints


Relative Frequency Histogram
Relative Frequency
0.3
0.25
0.2
0.15
0.1
0.05
0
0 1 2 3 4 5 6 7 8 9
Percent Air Content


Comparing Frequency Polygons

NOTES

18.

A **cumulative relative frequency polygon** is a straight line graph having the upper class limits represented on the horizontal axis and the cumulative relative frequencies on the vertical axis. The cumulative relative frequency corresponding to each upper class limit is marked by a point and consecutive points are connected by straight lines.

19.

In this plot, the **cumulative relative frequency polygon** has been superimposed on the cumulative relative frequency histogram for the concrete air content example. The graph indicates that 23% of the data values are less than or equal to 4.0%, while 73% are less than or equal to 6.0%.

20.

Illustrative Example

A listing of 200 concrete test results is presented below. Determine an appropriate class interval and class limits if 11 classes are selected for the frequency table and histogram.

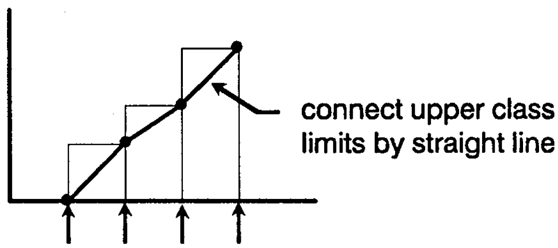
Concrete Compressive Strength Data

3882	3990	4022	4115	4117	4120	4147	4172	4182	4193	4208	4209	4214	4229
4229	4230	4250	4290	4301	4309	4315	4319	4319	4336	4355	4393	4409	4412
4416	4416	4450	4500	4509	4514	4519	4519	4520	4541	4541	4542	4555	4563
4572	4572	4573	4573	4590	4590	4592	4598	4602	4613	4614	4619	4625	4636
4640	4645	4645	4659	4661	4663	4665	4671	4673	4675	4684	4690	4691	4691
4693	4705	4713	4714	4717	4718	4718	4719	4722	4741	4741	4750	4755	4760
4760	4771	4772	4773	4773	4775	4776	4779	4784	4791	4791	4798	4809	4810
4811	4814	4814	4820	4833	4841	4841	4846	4851	4853	4855	4868	4871	4871
4873	4876	4881	4881	4881	4883	4890	4890	4899	4900	4900	4901	4903	4905
4917	4934	4945	4951	4952	4952	4952	4953	4960	4960	4971	4973	4981	4981
4983	4991	4995	4998	5004	5007	5018	5030	5033	5038	5060	5061	5093	5095
5098	5103	5103	5104	5104	5106	5109	5113	5116	5120	5123	5146	5146	5190
5209	5214	5214	5215	5218	5225	5275	5280	5283	5288	5304	5305	5313	5319
5336	5371	5409	5412	5470	5513	5614	5630	5663	5709	5717	5735	5748	5756
5833	5890	5905	5960										

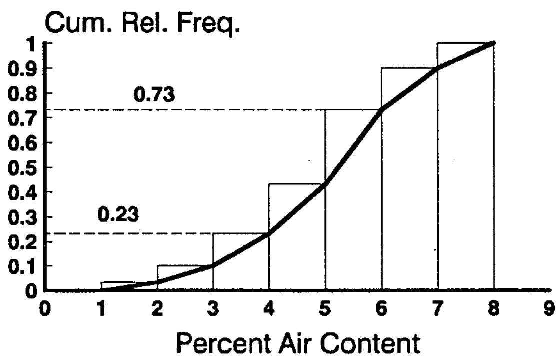
NOTES

Graphical Forms

Cumulative Relative Frequency Polygon



Cumulative Frequency Histogram



Concrete Compressive Strength Data

3882 3990 4022 ----

---- ---- ---- ---- ----
---- ---- ---- ---- ----
---- ---- ---- ---- ----
---- ---- ---- ---- ----
---- ---- ---- ---- ----
---- ---- ---- ---- ----
---- ---- ---- ---- ----
---- ---- ---- ---- ----
---- ---- ---- ---- ----

5833 5890 5905 5960

21.

The frequency polygon is superimposed on the frequency histogram for the 200 concrete compressive strength test results in this plot. A total of 11 classes were selected for this plot. What can be determined from the plot regarding the center and spread of the data?

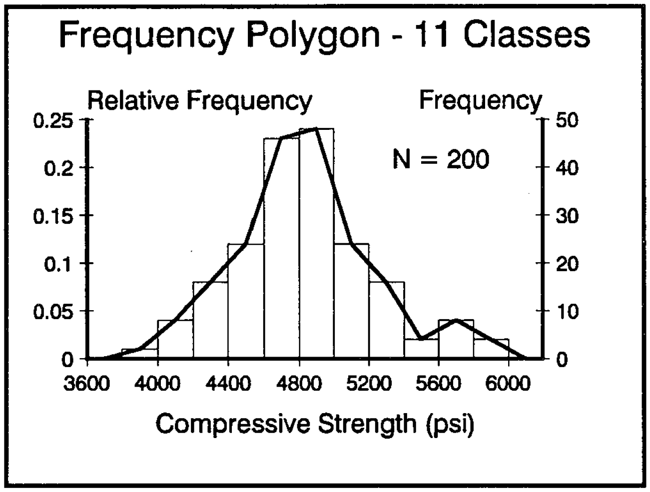
22.

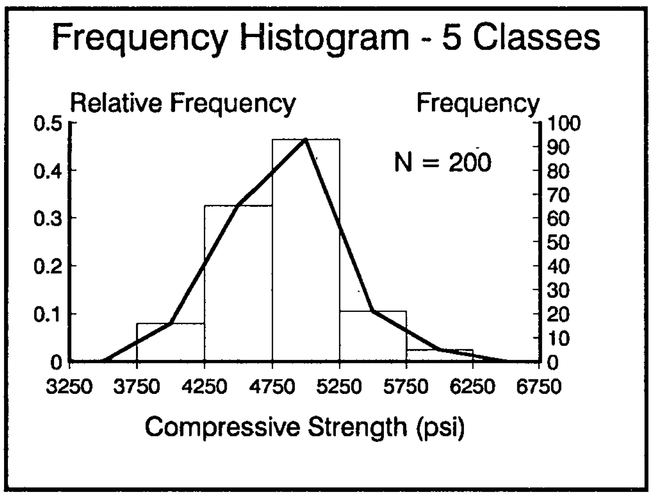
The frequency histogram based on 5 classes for the 200 concrete compressive strength test results is shown in this plot. The range of the data is still clearly depicted, but not as much detail regarding the shape of the distribution is evident when compared with the previous histogram based on 11 classes.

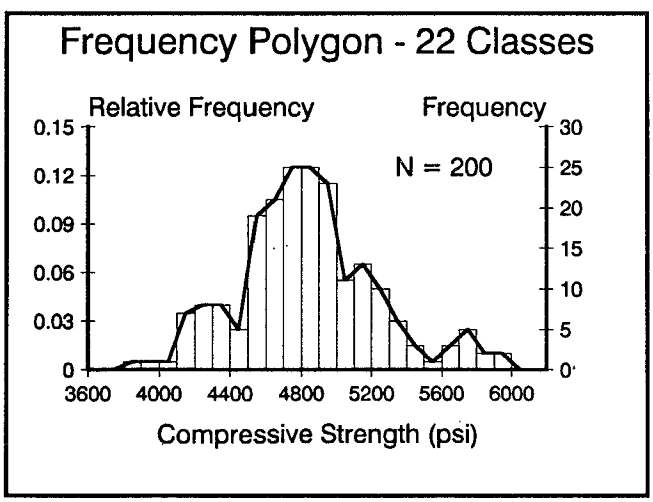
23.

The frequency histogram for the 200 concrete compressive strength test results is shown for 22 classes in this plot. In this case, the overall shape of the data and general trends are not as obvious.

NOTES


Frequency Polygon - 11 Classes
Relative Frequency
Frequency
N = 200
0.25
0.2
0.15
0.1
0.05
0
50
40
30
20
10
0
3600 4000 4400 4800 5200 5600 6000
Compressive Strength (psi)


Frequency Histogram - 5 Classes
Relative Frequency
Frequency
N = 200
0.5
0.4
0.3
0.2
0.1
0
100
90
80
70
60
50
40
30
20
10
0
3250 3750 4250 4750 5250 5750 6250 6750
Compressive Strength (psi)


Frequency Polygon - 22 Classes
Relative Frequency
Frequency
N = 200
0.15
0.12
0.09
0.06
0.03
0
30
25
20
15
10
5
0
3600 4000 4400 4800 5200 5600 6000
Compressive Strength (psi)

24.

The frequency histogram for the 200 concrete compressive strength test results is shown for 44 classes in this plot. While this is an extreme case, it shows the effect of too many classes. There are many localized peaks, and it is more difficult to identify overall trends in the shape of the data. The number of classes should be selected to provide the appropriate level of detail for the data that are available. Too few classes will not provide sufficient information, while too many classes may highlight localized irregularities in the data that are not significant to the overall trend of the data.

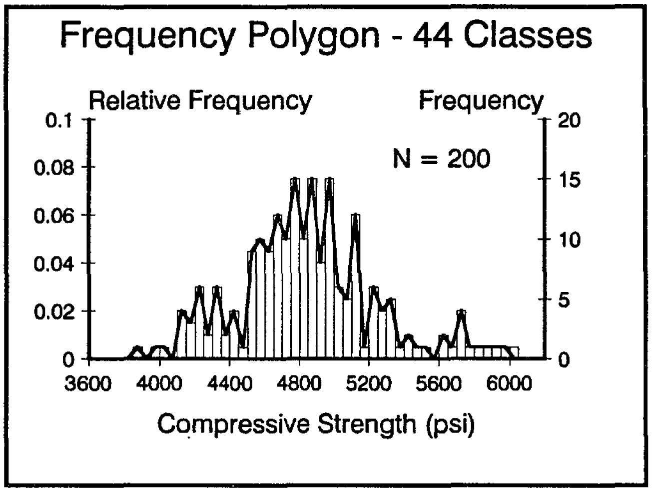
25.

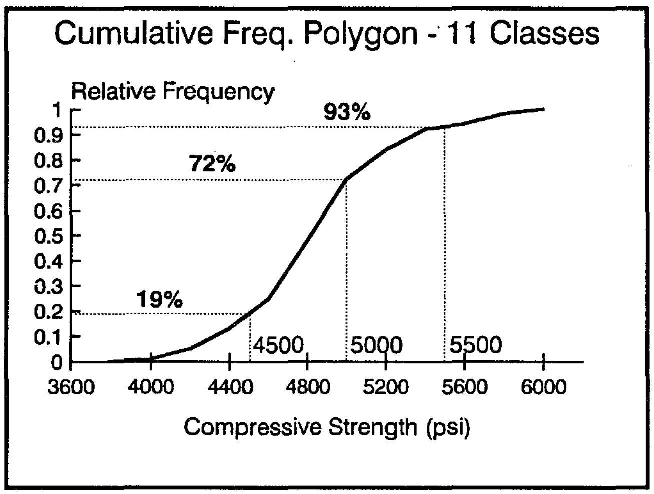
In this plot, the cumulative frequency polygon is superimposed on the cumulative frequency histogram for the 200 concrete compressive strength test results using 11 classes. From the cumulative frequency polygon, it can be determined that 19% of the concrete strengths were less than or equal to 4500 p.s.i., 72% were less than or equal to 5000 p.s.i., and 93% were less than or equal to 5500 p.s.i.

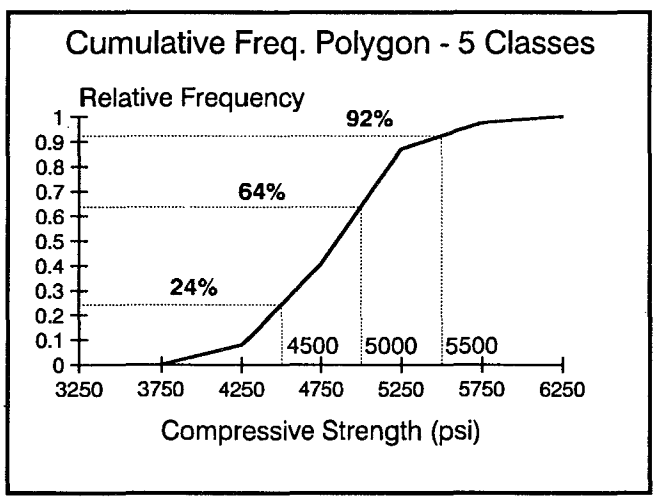
26.

The cumulative frequency polygon based on 5 classes is shown in this plot. The values determined for percent of the test results less than or equal to 4500 p.s.i., 5000 p.s.i. and 5500 p.s.i. differ slightly from those determined for the plot based on 11 classes. The results are therefore dependent upon the selection of the number of classes.

NOTES


Frequency Polygon - 44 Classes
Relative Frequency
Frequency
N = 200
0.1
0.08
0.06
0.04
0.02
0
20
15
10
5
0
3600 4000 4400 4800 5200 5600 6000
Compressive Strength (psi)

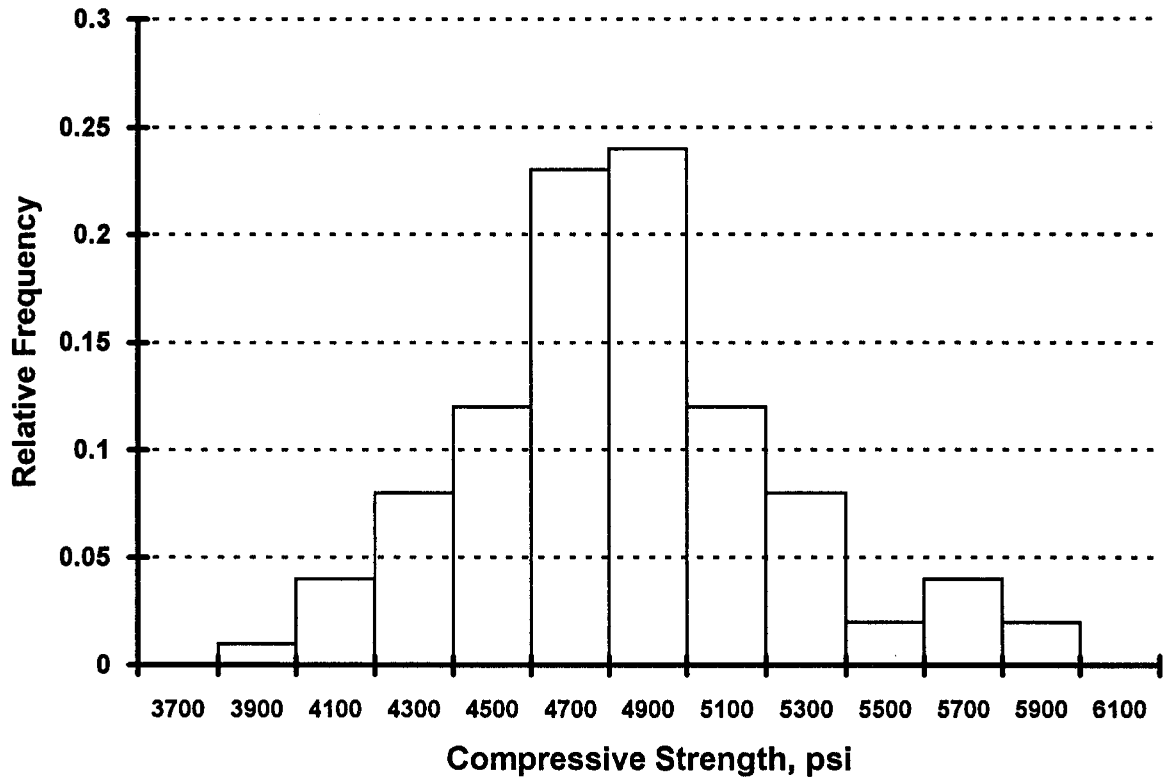

Cumulative Freq. Polygon - 11 Classes
Relative Frequency
93%
72%
19%
4500
5000
5500
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
3600 4000 4400 4800 5200 5600 6000
Compressive Strength (psi)


Cumulative Freq. Polygon - 5 Classes
Relative Frequency
92%
64%
24%
4500
5000
5500
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
3250 3750 4250 4750 5250 5750 6250
Compressive Strength (psi)

27.

Example

In this exercise, we will organize the data from Sampling Experiment #1. Complete the frequency table on the next page for the data from Sampling Experiment #1. Then plot the relative frequency histogram superimposed on the histogram for the population.



Frequency Table for Sampling Experiment #1

Class	Limits	Midpoint	Frequency	Rel. Freq.
1	3799.5-3999.5	3900	________	________
2	3999.5-4199.5	4100	________	________
3	4199.5-4399.5	4300	________	________
4	4399.5-4599.5	4500	________	________
5	4599.5-4799.5	4700	________	________
6	4799.5-4999.5	4900	________	________
7	4999.5-5199.5	5100	________	________
8	5199.5-5399.5	5300	________	________
9	5399.5-5599.5	5500	________	________
10	5599.5-5799.5	5700	________	________
11	5799.5-5999.5	5900	________	________

28.

Workshop: Organization of Data

1. Prepare a frequency table (using 7 classes) for the data in the table at right.

2. Construct a relative frequency histogram and polygon for the data.

3. Construct a cumulative relative frequency histogram and polygon for the data.

4. If the specifications require that 90% of the material exceed a density of 110 p.c.f., do you believe that the material sampled meets the specifications? Why or why not?

29.

The key points of this chapter are summarized in the slide at the right.

References

1. Sigma Partnership, *Practical Applications of Statistical Quality Control in Highway Construction for Engineers and Technicians*, Federal Highway Administration, 1979.

2. Willenbrock, J. H., *Statistical Quality Control of Highway Construction, Vol. I*, Federal Highway Administration, 1976.

NOTES

Proctor Density Test Results
(pounds per cubic foot, pcf)

107.5	109.4	110.5	107.0	112.0
110.1	95.5	101.3	104.3	99.7
101.5	98.0	106.0	102.5	103.3
101.3	93.5	107.0	114.0	110.8
124.0	105.0	111.4	103.5	94.0
113.2	90.8	97.9	106.7	100.1

Key Points

1. Organization of data.
2. Tabular vs. graphical methods.
3. Frequency & relative frequency.
4. Frequency tables & histograms.
5. Cumulative frequency.

CHAPTER 4: ANALYSIS OF DATA

INSTRUCTIONAL OBJECTIVES

1. Introduce concepts of center and spread.

2. Discuss measures of center.

3. Discuss measures of spread.

4. Conduct Sampling Experiment #2.

DESIRED STUDENT ACHIEVEMENTS

1. Learn how to calculate mean, median, population standard deviation, sample standard deviation, and coefficient of variation.

2. Understand meaning of degrees of freedom.

3. Gain an understanding of the relationship between samples of size one and samples of size greater than one.

CHAPTER 4: ANALYSIS OF DATA

1.

This is the third phase of statistical analysis (i.e., the quantitative **analysis of data**).

2.

This chapter covers the two values necessary for **estimating** the **population**. It is reiterated that the properties of the population (called population parameters) are what are needed, but only the properties of the sample (called sample statistics) are available. Thus the sample statistics are used to estimate the properties of the population. The two measures for which estimates are needed are the **center** and **spread**.

3.

There are several measures of center. The most common, sometimes called the expected value, is the arithmetic mean. The mean, or average, is determined as shown in the slide at right. The mean of a population is usually denoted as μ, while a sample mean is often denoted as an **X** with a bar over it and called **X-bar.** Many modern pocket calculators have a mean function.

NOTES

Phases of Statistical Analysis

1. Collection of Data
2. Organization of Data
3. **Analysis of Data**
4. Interpretation of Data

→ Center ←

← Spread →

Arithmetic Mean

→ Center ←

$$\mu = \frac{X_1 + X_2 + \cdots + X_N}{N} = \frac{\sum X_i}{N}$$

$$\bar{X} = \frac{X_1 + X_2 + \cdots + X_n}{n} = \frac{\sum X_i}{n}$$

4.

Another measure of the center of a set of data is the **median**. The median is the value for which half the data are smaller and half the data are larger. One advantage of the median is that it is not affected by extreme values (sometimes called outliers) as the mean may be.

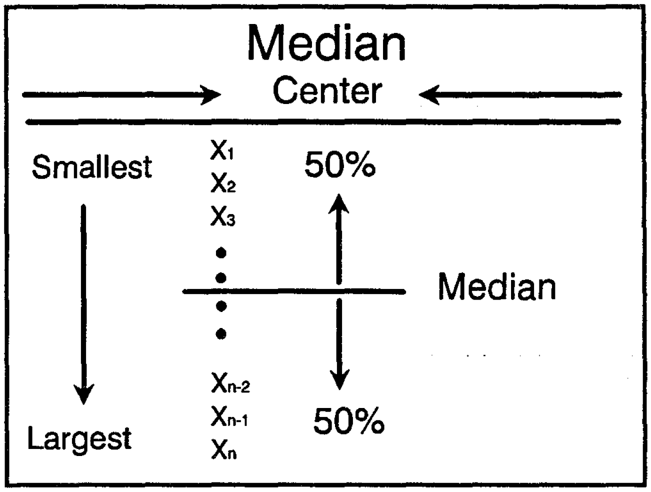
5.

For example, for the two sets of data, the extreme value in data set B has a large impact on the mean, but has no effect on the median.

6.

The second parameter necessary to define the population is a measure of spread or variability. One common and simple measure is the **range, R**. The range is the difference between the largest and smallest values. However, because of its simplistic nature, the range does not provide the best estimate of the variability. Since it uses only two values, it does not take into account the other values that may be available for estimating the variability. It is therefore not an efficient use of the data.

NOTES



Data Set A

1, 2, 3, 4, 5, 6, 7

Mean = Median =

Data Set B

1, 2, 3, 4, 5, 6, 84

Mean = Median =

Range, R

R = Max. X - Min. X

(Largest - Smallest)

← Spread →

7.

A better measure of variability is the standard deviation, **s** or **σ**. It is common practice to use **s** when dealing with a **sample** standard deviation and **σ** when dealing with a **population** standard deviation. This distinction is discussed in more detail in a later chapter.

8.

The basic measure of variability, from which the standard deviation is derived, is the **variance** which is the mean squared deviation of the individual values from the mean.

However, this measure of variability produces the basic units of measure in units squared, which is often difficult to use. Thus the standard deviation, the square root of the variance, allows the use of the basic unit of measure. Note that if the deviations from the average were not squared, but only summed, the minus values would offset the positive values, providing a less than optimum measure of spread.

9.

The **standard deviation** is the square root of the variance and is in the basic measurement units of the data. Many hand calculators contain a standard deviation function.

NOTES

Standard Deviation

S or σ

← **Spread** →

← **Variability** →

Variance

$$\sigma^2 = \frac{\sum (X_i - \mu)^2}{N}$$

$$S^2 = \frac{\sum (X_i - \bar{X})^2}{n - 1}$$

Standard Deviation

$$\sigma = \sqrt{\frac{\sum (X_i - \mu)^2}{N}}$$

$$S = \sqrt{\frac{\sum (X_i - \bar{X})^2}{n - 1}}$$

10.

Notice that the denominator used to calculate the sample variance, s^2, and the sample standard deviation, s, is n-1, while the denominator used for the population variance, σ^2, and population standard deviation, σ, is N. The denominator in this equation is known as the **degrees of freedom** and is the number of independent observations available for determining the variability. If n values are available, and the mean is calculated from these values, then there are only n-1 independent data points available for calculating the standard deviation. In other words, one of the independent data points is lost for calculating sample variance or sample standard deviation since an estimated value, $\overline{X}$, is used in the calculation.

A simple analogy for remembering the relationship between n and n-1 is that with a single value, n, an estimate of the average (although a poor one) can be obtained. However, for an estimate of variability, at least two values are needed; thus n-1 in this case is 2-1 = 1.

11.

One additional measure of variability is the coefficient of variation, **V** or **COV**, defined as the standard deviation as a percentage of the mean. Since the coefficient of variation is dimensionless, it can be used to provide a comparison of variability among different measurements. As difficult as quality is to measure, there is some feeling that a higher degree of consistency (lower coefficient of variation) is an indicator of better quality, other factors being the same.

12.

Example. Calculate the average, sum of squares, standard deviation and state the degrees of freedom for the following asphalt content sample results:

Lot 1: 6.2, 5.9, 6.0, 5.9.

Lot 2: 4.0, 4.7, 4.8, 4.5.

NOTES

Degrees of Freedom

N for σ

n - 1 for S

Coefficient of Variation, %

$$V = \frac{s}{\overline{X}} \times 100$$

Lot 1

X_i	$\overline{X}$	$(X_i - \overline{X})$	$(X_i - \overline{X})^2$
6.2			
5.9			
6			
5.9			

Lot 2

X_i	$\overline{X}$	$(X_i - \overline{X})$	$(X_i - \overline{X})^2$
4			
4.7			
4.8			
4.5			

13.

Sampling Experiment #2

When Sampling Experiment #1 was performed (Chapter 2, ***slide 29***), 40 individual compressive strength test results were collected. These results were organized into a frequency histogram in Chapter 3. The spread of the **sample** test results was nearly as great as the spread of the entire **lot**. This implies that a single test result may be quite far from the true average of the lot. Individual high tests and low tests are both a part of the same lot and have the same probability of occurrence, but both may be far from the true average. In Sampling Experiment #2, samples of size 4 will be obtained from the same lot used in Sampling Experiment #1; and each sample of size 4 will be averaged to evaluate the lot.

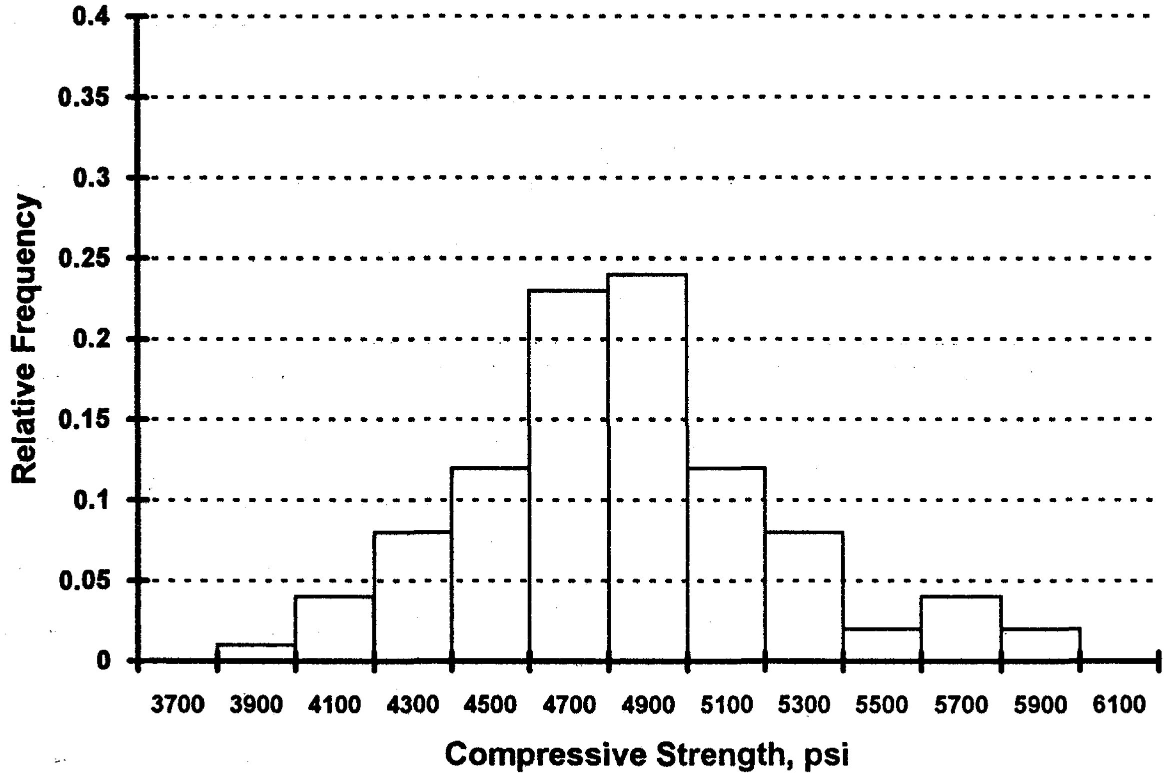
In Sampling Experiment #1, it was observed that an **individual** test result did not necessarily provide a good approximation of the center of the lot. Even the average of the 40 individual test results did not determine the **exact** value of the average of the lot. A logical question that could be asked, then, is **would the average of a number of sample means be a better estimate of the true mean of the lot?** Sampling Experiment #2 will illustrate the behavior of a distribution of these sample mean values (or sample means) as opposed to the distribution of individual values. The same lot of 200 concrete compressive strength test results that was used in Sampling Experiment #1 will be used in Sampling Experiment #2.

Each participant will randomly select 4 compressive strength results from the **lot** shown at right using the same procedure that was used for Sampling Experiment #1. The random number table from ***slide 19*** in Chapter 2 will be used to identify the sample locations. Each participant will then determine the mean of the 4 strengths. The mean values will be recorded in the table below. The frequency table and relative frequency histogram will then be completed for the sample means. This histogram represents the **distribution of sample means**.

24	21	18	15	12	9	6	3	0
								25+00
4691	4625	4719	4713	4659	4855	4934	4775	
								24+00
4230	4973	5033	5146	4520	5093	4846	5113	
								23+00
4290	4903	4995	4900	5120	4811	4229	5190	
								22+00
4717	5018	5288	4661	4983	5123	4590	4319	
								21+00
4573	4952	4809	4572	4871	5630	5412	4905	
								20+00
5756	4952	4673	4814	4881	5218	3990	4555	
								19+00
4773	4810	4899	5275	5214	4416	4718	4853	
								18+00
4172	4951	4881	5095	4741	5004	4541	4755	
								17+00
4791	4901	4890	4590	4115	5060	5098	4820	
								16+00
4772	4971	5513	4714	4500	4981	4573	4022	
								15+00
4636	4722	5409	4675	5104	5038	4412	5106	
								14+00
4690	5109	4833	4563	4301	4981	4409	5225	
								13+00
5313	5103	4851	4871	4336	4750	5280	4182	
								12+00
5007	5283	4705	4841	5905	4779	5319	4953	
								11+00
5116	4645	4117	4881	4542	5104	4514	5061	
								10+00
4693	4900	5215	4776	5146	4945	4890	3882	
								9+00
5709	4798	5304	4613	4509	4841	5614	4960	
								8+00
4120	4519	4760	5735	5103	4952	4771	4917	
								7+00
4663	5305	4784	4416	4718	5371	4315	4665	
								6+00
4883	4991	5209	4640	4814	4791	5470	5030	
								5+00
4876	4309	4602	5336	4645	4760	4393	4773	
								4+00
4592	4208	4572	4868	5960	4960	4319	4998	
								3+00
4741	5748	4450	5890	4619	4684	4519	4193	
								2+00
4691	4614	5717	4671	4214	4229	4873	4355	
								1+00
4250	5663	4598	4209	4541	5833	4147	5214	
								0+00
24	21	18	15	12	9	6	3	0

Results of Sampling Experiment #2

1	______	11	______	21	______	31	______
2	______	12	______	22	______	32	______
3	______	13	______	23	______	33	______
4	______	14	______	24	______	34	______
5	______	15	______	25	______	35	______
6	______	16	______	26	______	36	______
7	______	17	______	27	______	37	______
8	______	18	______	28	______	38	______
9	______	19	______	29	______	39	______
10	______	20	______	30	______	40	______



Frequency Table for Sampling Experiment #2

Class	Limits	Midpoint	Frequency	Rel. Freq.
1	3799.5-3999.5	3900	______	______
2	3999.5-4199.5	4100	______	______
3	4199.5-4399.5	4300	______	______
4	4399.5-4599.5	4500	______	______
5	4599.5-4799.5	4700	______	______
6	4799.5-4999.5	4900	______	______
7	4999.5-5199.5	5100	______	______
8	5199.5-5399.5	5300	______	______
9	5399.5-5599.5	5500	______	______
10	5599.5-5799.5	5700	______	______
11	5799.5-5999.5	5900	______	______

14.

Workshop Problems:

1. Calculate the average, sum of squares, and standard deviation, for each group of aggregate gradation results:

Group 1: 49, 47, 52, 50, 51, 49, 52, 50

Group 2: 49, 47, 52, 58, 51, 49, 52, 50

2. Calculate the average, sum of squares, and standard deviation for this set of PCC pavement thickness measurements:

10.5, 10.3, 10.5, 10.5, 10.4, 10.4, 10.7, 10.5, 10.3, 10.7

15.

Key points for this chapter are presented in the slide at right.

References:

1. Mann, Lawrence, *Applied Engineering Statistics for Practicing Engineers,* Barnes & Noble, Inc., 1970.

Group 1

X_i	$\overline{X}$	$(X_i - \overline{X})$	$(X_i - \overline{X})^2$
49			
47			
52			
50			
51			
49			
52			
50			

Group 2

X_i	$\overline{X}$	$(X_i - \overline{X})$	$(X_i - \overline{X})^2$
49			
47			
52			
58			
51			
49			
52			
50			

Problem 2

X_i	$\overline{X}$	$(X_i - \overline{X})$	$(X_i - \overline{X})^2$
10.5			
10.3			
10.5			
10.5			
10.4			
10.4			
10.7			
10.5			
10.3			
10.7			

NOTES

Key Points

1. Center.
2. Spread.
3. Degrees of freedom.
4. Coefficient of variation.

CHAPTER 5: THE NORMAL DISTRIBUTION

INSTRUCTIONAL OBJECTIVES

1. Introduce the concept of probability.

2. Introduce the concept of probability density function.

3. Introduce the concept of the normal distribution.

4. Demonstrate how to calculate areas under the normal curve.

DESIRED STUDENT ACHIEVEMENTS

1. Gain a basic understanding of probability.

2. Learn how to relate histograms to probability density functions.

3. Be able to calculate areas under the normal curve.

CHAPTER 5: THE NORMAL DISTRIBUTION

1.

Up to this point we have covered the first three of the phases of statistical analysis. These have concentrated on **collecting, organizing** and **analyzing** sample data. It is even more important to understand how the information provided by the sample data is used to generalize about the **population** or **lot** from which the sample data were obtained. This is the fourth phase of statistical analysis: **interpretation of data**.

2.

The sample data must be interpreted to make **statistical inferences** about the **lot** (or **population)** of material that was sampled. For instance, we might be interested in drawing inferences about the acceptability of **a lot** of portland cement concrete based on the compressive strength of a **sample** of five cores.

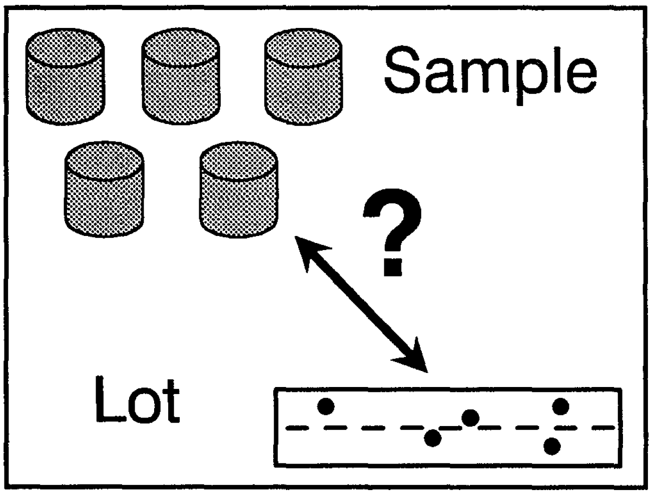
3.

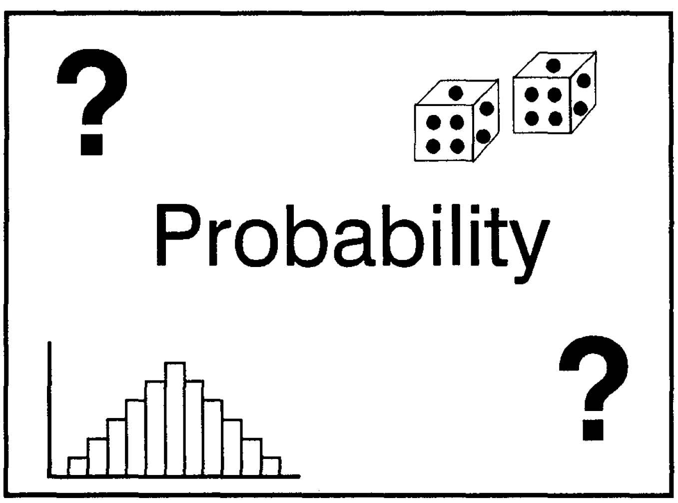
To be able to make inferences about **lots** or **populations** based on sample data, we must introduce and understand some basic points about **probability**. We will do this by investigating how histograms and relative frequency histograms can be used to calculate probabilities of certain outcomes.

NOTES

Phases of Statistical Analysis

1. Collection of Data
2. Organization of Data
3. Analysis of Data
4. **Interpretation of Data**





4.

Example: discrete data

A paving contractor is using 5 trucks (A, B, C, D, E) to deliver asphalt concrete to a highway project. On any given day the number of loads delivered by each truck will vary. The histogram at right illustrates the distribution of 50 truckloads that were delivered to the project on a certain day.

5.

The relative frequency histogram for the distribution of trucks is shown at right. We can interpret the term "relative frequency" in two ways:

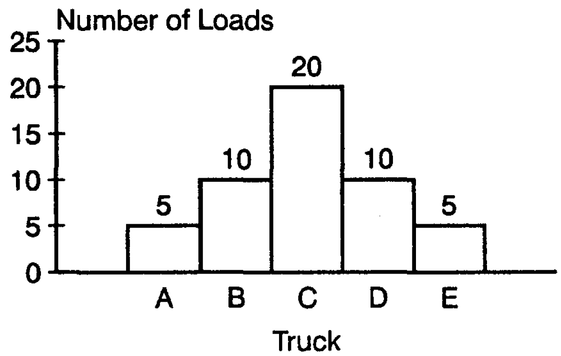
1. 40% (20/50) of the truckloads were delivered by truck C.

2. If a truck were randomly selected for sampling on the day in question, there is a 40% chance (0.40 or 40% probability) that truck C would have been selected.

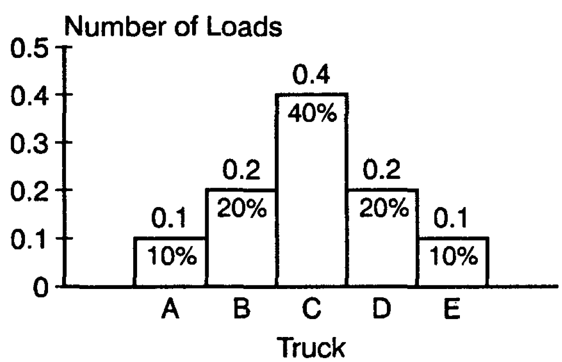
6.

Probability can be defined as a measure of chance. A specific truck's relative frequency is therefore a measure of the probability that it will be the one that is selected if a random selection process is used. The sum of all of the relative frequencies is 1.00 (or 100%). In other words, the sum of the probabilities of all of the possible outcomes (e.g., the 5 different trucks) is 100% (or 1.00).

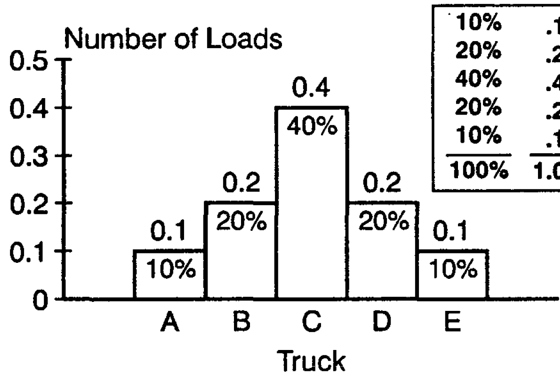
Example: Discrete Data



Example: Discrete Data



Example: Discrete Data



10%	.1
20%	.2
40%	.4
20%	.2
10%	.1
100%	1.00

NOTES

7.

For the **discrete data** case, the probability can be obtained from the relative frequency histogram in two ways:

1. The height of each rectangle represents the probability.

2. The ratio of the area of each rectangle to the total area of all the rectangles indicates the probability. If the total area of the histogram is set equal to 100 units (or 1.0), then the area of each rectangle directly indicates the probability.

8.

Example: continuous data

The interpretation of the relative frequency histogram is different when we are dealing with measurements of construction materials since measurements are usually **continuous data**. We will illustrate this with the concrete compressive strength test results that were used in Sampling Experiments #1 and #2. The relative frequency histogram for this population is shown at right.

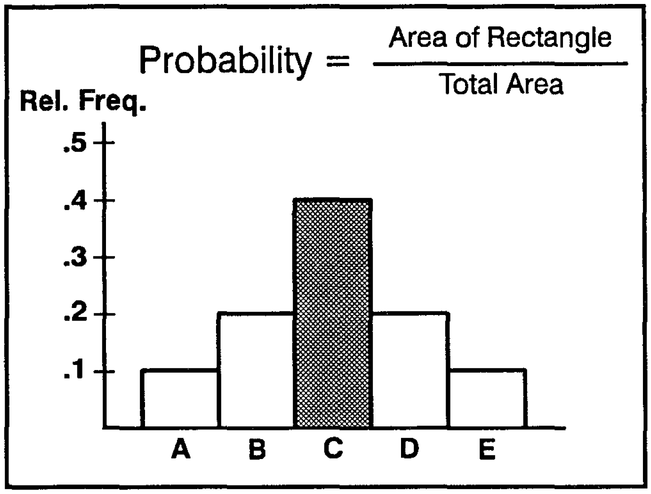
9.

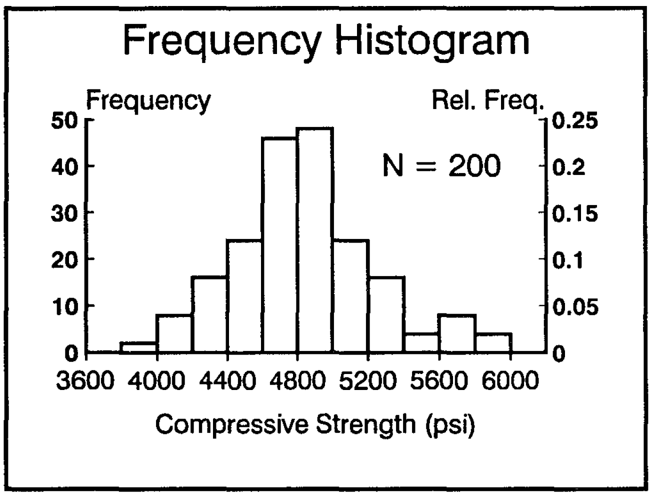
If we set the total area under the histogram equal to 100 units (or 1.0), what does the 24% area (and height) of the shaded rectangle mean? There are again two possible interpretations of relative frequency for the continuous data case:

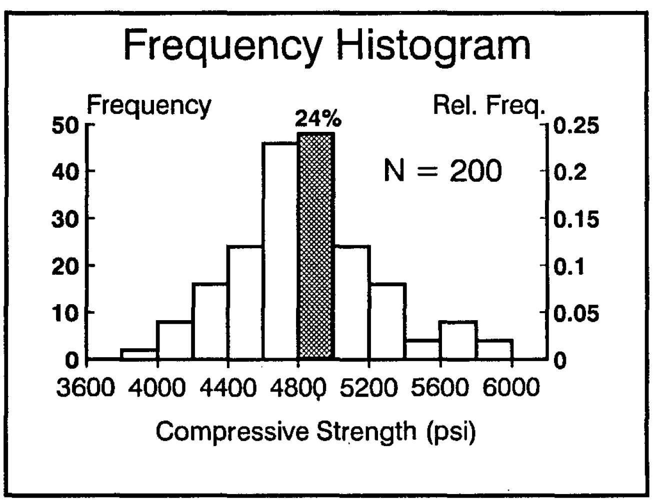
1. 24% of the 200 test results are between 4800 psi and 5000 psi.

2. If a test result were randomly selected from the population (or lot), there is a 24% chance (24% or 0.24 probability) that the test result will have a value **between** 4800 psi and 5000 psi.

NOTES


Probability = Area of Rectangle / Total Area
Rel. Freq.
.5
.4
.3
.2
.1
A
B
C
D
E


Frequency Histogram
Frequency
Rel. Freq.
N = 200
50
40
30
20
10
0
0.25
0.2
0.15
0.1
0.05
0
3600 4000 4400 4800 5200 5600 6000
Compressive Strength (psi)


Frequency Histogram
Frequency
24%
Rel. Freq.
N = 200
50
40
30
20
10
0
0.25
0.2
0.15
0.1
0.05
0
3600 4000 4400 4800 5200 5600 6000
Compressive Strength (psi)

10.

In the **discrete** data example, the areas of the rectangles represent the probability that each of the possible outcomes will occur (where outcome is type of truck).

In the **continuous** data example, the areas of the rectangles represent the probability that an outcome will be ***between*** two possible values (where outcome is a test result).

11.

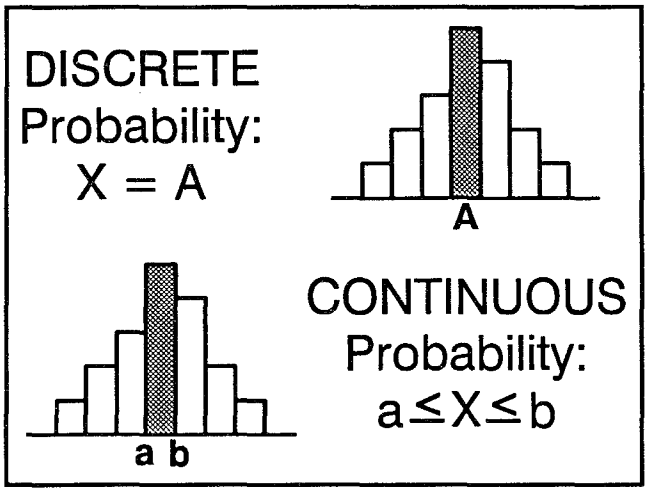
The reason for the different interpretations is due to the infinitely large number of possible outcomes in the **continuous** data case. The probability that a continuous random variable will have a single specific value is, therefore, very, very small. It is actually one possible outcome from an infinite number of outcomes (i.e., $1 \div \infty$, or zero).

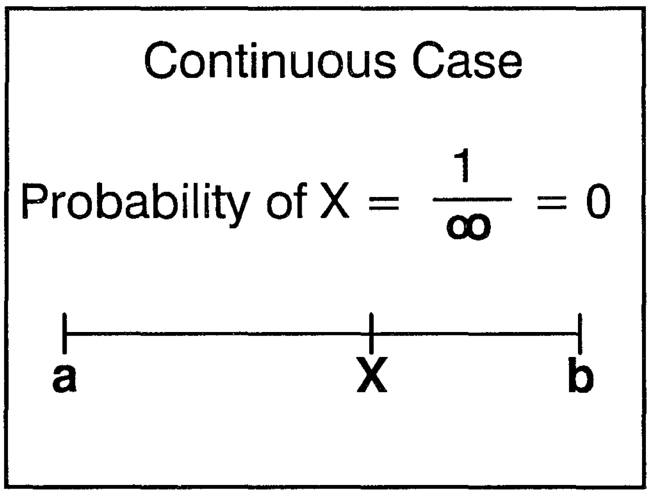
Most people would probably refuse a bet which asked them to wager that the next compressive strength test result would be **exactly** 4322.65 psi. However, they may be willing to consider the wager if they were asked to bet that the next test result would be between 4800 psi and 5000 psi.

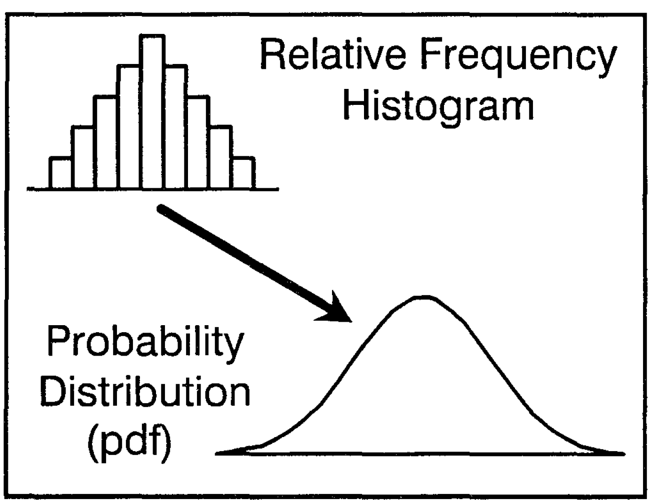
12.

We have now shown that the areas of the rectangles of a relative frequency histogram can be used to determine the probability that an outcome will be between two values. There are limitations to using histograms to determine probabilities. They are limited to the results of the sample from which they were developed, and probabilities can only be determined for the class intervals that were used for developing the histogram. If we are to make inferences about the total lot or population from which the sample was taken, then we must develop a method for making more general probability statements. We will approximate the relative frequency histogram with a smooth curve that we will call a **probability distribution**. Statisticians call such a curve a **probability density function**, or **pdf**.

NOTES


DISCRETE
Probability:
X = A
A
CONTINUOUS
Probability:
a≤X≤b
a b


Continuous Case
Probability of X = 1/∞ = 0
a
X
b


Relative Frequency
Histogram
Probability
Distribution
(pdf)

13.

If we increase the number of observations (n) used in developing the histogram and correspondingly reduce the width of the class interval (w) used in plotting the histogram, as $n \rightarrow \infty$ and $w \rightarrow 0$, then the histogram approaches a smooth curve.

14.

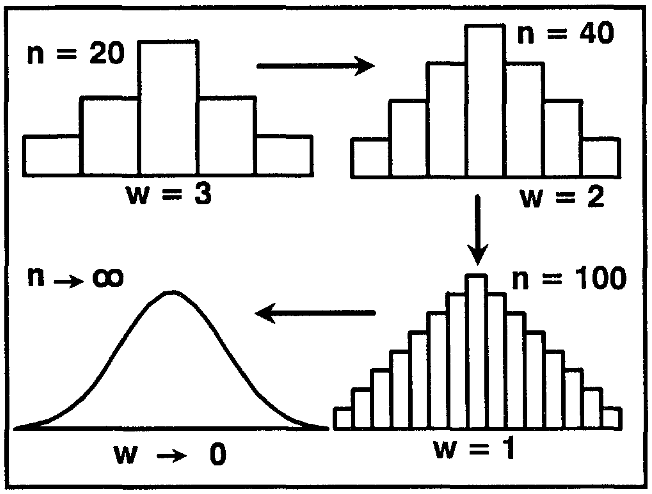
The smooth curve is drawn so that

1. the area under the curve between two values corresponds to the area under the histogram between the same two values, **and**

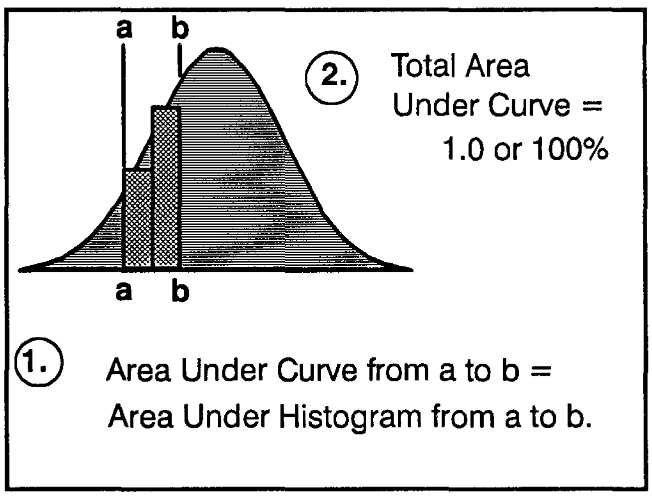
2. the total area under the curve is equal to 1.0 (i.e., 100%).

15.

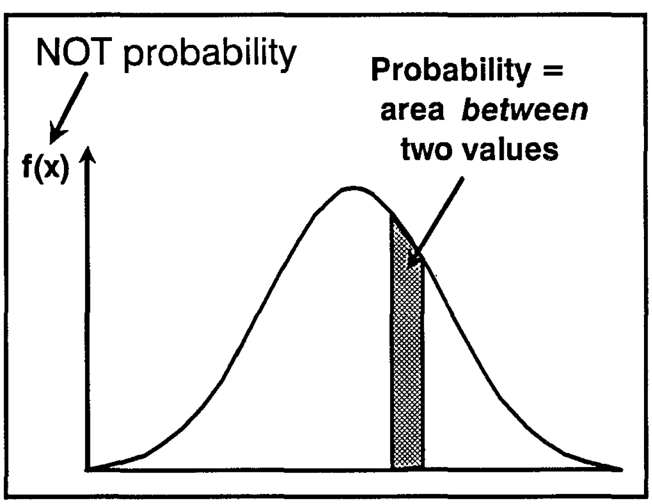
It is extremely important to note that although the axis of the relative frequency histogram can be expressed in terms of relative frequency, **the vertical axis of the probability distribution is *NOT* probability**, it is simply f(x). Probability can only be expressed as the area under the curve **between** two values.



n = 20
n = 40
w = 3
w = 2
n → ∞
n = 100
w → 0
w = 1



a
b
2.
Total Area Under Curve = 1.0 or 100%
a
b
1.
Area Under Curve from a to b = Area Under Histogram from a to b.



NOT probability
f(x)
Probability = area *between* two values

NOTES

16.

Although the shapes of smooth curve probability functions can vary greatly, experience has shown that particular types of data tend to exhibit certain particular shapes. For illustrative purposes, some of the more common shapes are discussed and related to concrete compressive strength.

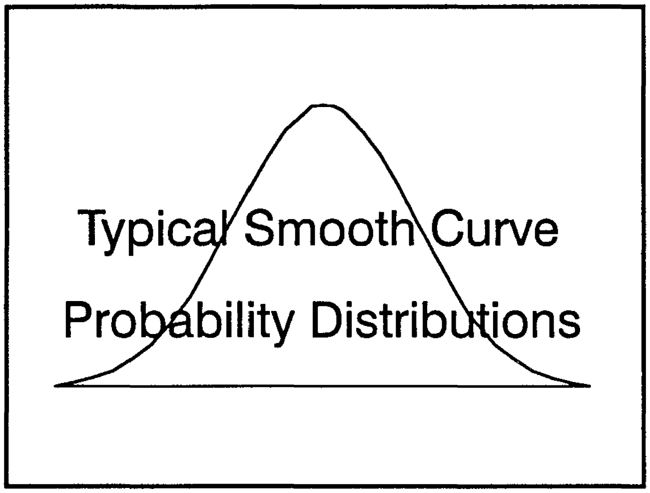
17.

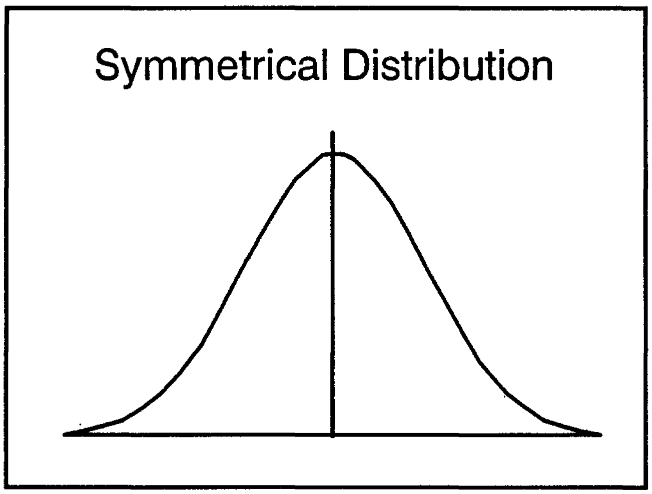
Symmetrical Probability Distribution

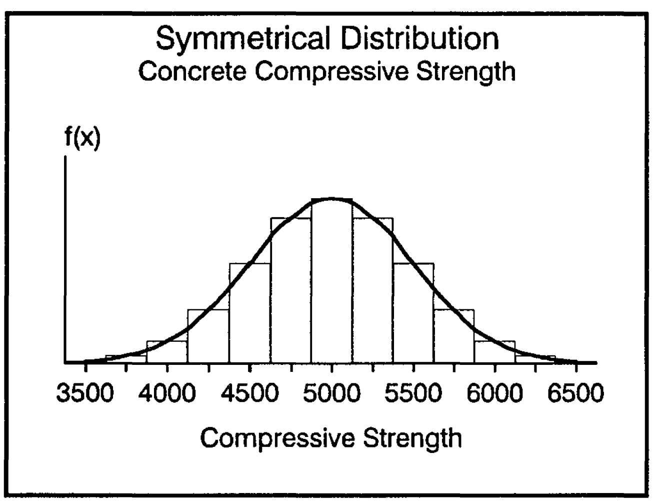
A symmetrical distribution has a shape that is a mirror image on opposite sides of a line dividing the middle of the distribution. The most famous and important symmetrical distribution is the **normal** or **Gaussian** probability distribution. The normal distribution is a unimodal (a single peak) distribution which has been found to adequately describe many populations that occur in nature, research, and industry (including highway materials).

18.

The slide at right indicates a normal distribution superimposed over the relative frequency histogram for a population of concrete compressive strengths. The normal probability distribution is completely described if the **mean** and **standard deviation** are known. The normal distribution is discussed in great detail in this chapter because it has been found to apply to many highway material and construction situations.


Typical Smooth Curve
Probability Distributions


Symmetrical Distribution


Symmetrical Distribution
Concrete Compressive Strength
f(x)
3500
4000
4500
5000
5500
6000
6500
Compressive Strength

NOTES

19.

Skewed Probability Distribution

A probability distribution that is not symmetrical, although it may be unimodal, is called a **skewed** distribution. **Positive skewness** indicates that the distribution has a longer tail extending toward the higher values on the right side. **Negative skewness** indicates the distribution has a longer tail extending towards the lower values on the left side.

20.

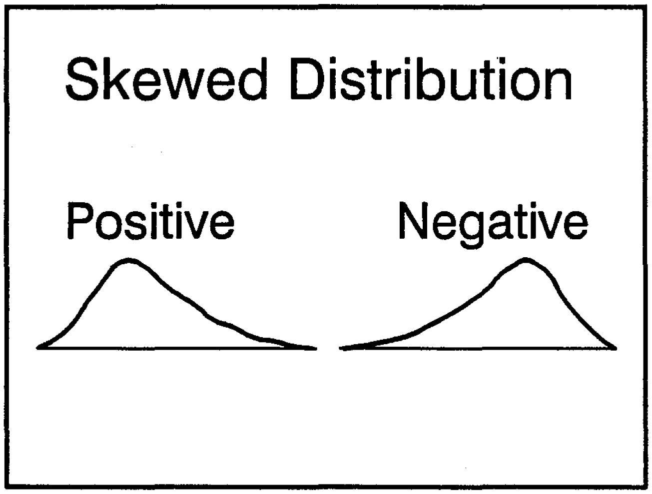
For concrete compressive strengths, skewed distributions could result if there is some factor in the concrete production process that causes either comparatively low strengths or high strengths. The figure at right illustrates a situation where poor sample preparation, storage and/or testing procedures lead to the occurrence of relatively low strength test results.

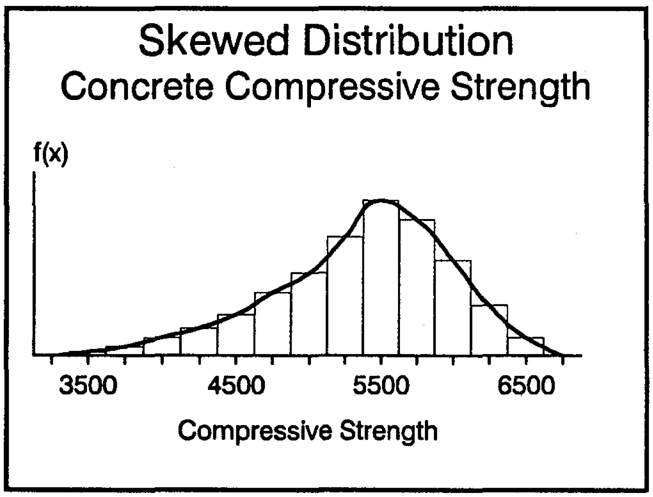
21.

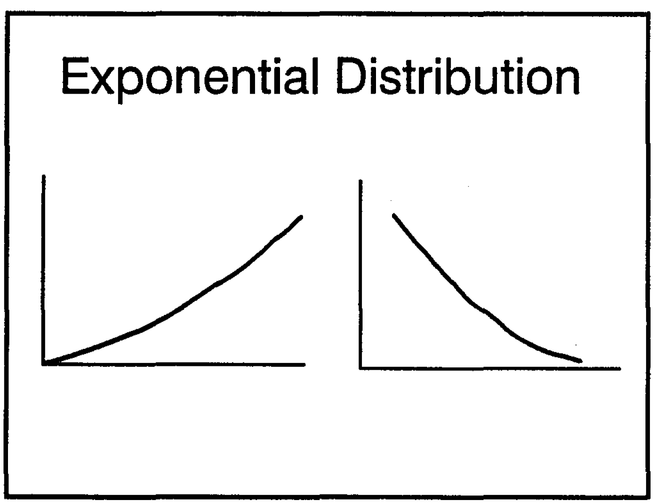
Exponential Probability Distribution

Exponential, distributions exhibit their maximum frequency at the final or initial parts of the distribution, with declining frequencies as the values move away from the maximum location. Since data do not naturally occur in the form of such a distribution, we should become suspicious if such distributions are encountered on a project.

NOTES


Skewed Distribution
Positive
Negative


Skewed Distribution
Concrete Compressive Strength
f(x)
3500
4500
5500
6500
Compressive Strength


Exponential Distribution

22.

An exponential distribution could possibly result from a practice that concrete testing labs sometimes follow -- releasing the compression load on the specimen once it has reached the minimum required compressive strength even though the cylinder has not yet failed. This practice eliminates the data that are higher than some minimum strength. It is therefore not possible to determine the true shape of the distribution or the true range of variability of the compressive strengths.

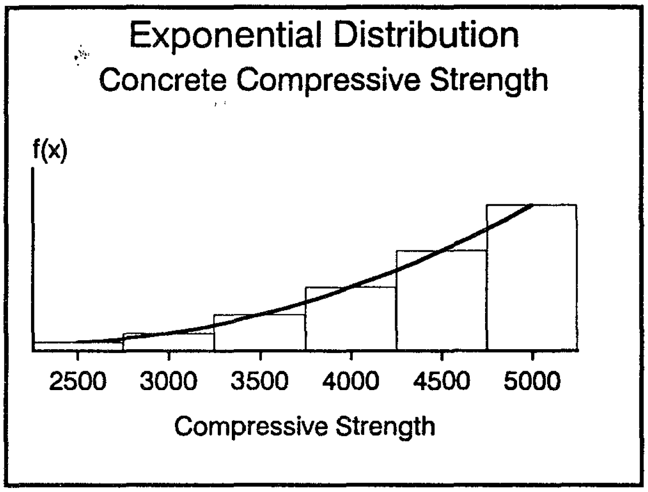
23.

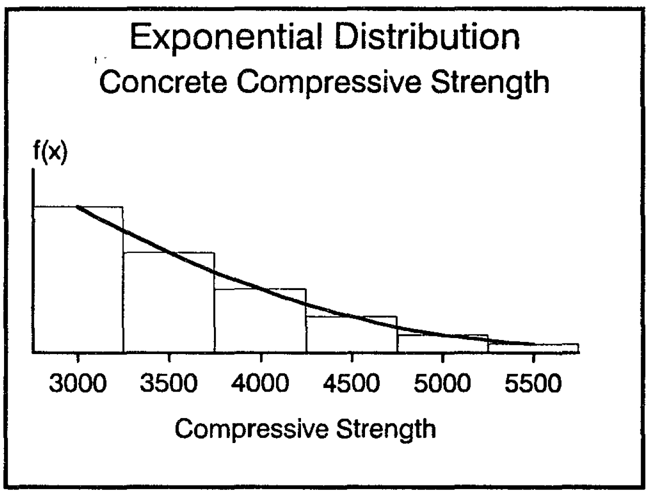
The distribution at right might occur if the compressive strength results are being reported to deceive the highway agency by reporting only those results that are higher than the minimum strength requirement, while modifying or failing to report strengths below the specified minimum.

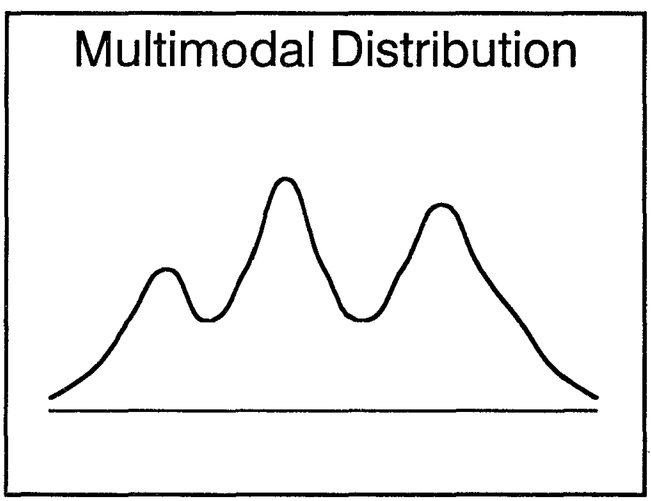
24.

Multimodal Probability Distributions

Multimodal probability distributions exhibit more than one peak, or **mode**. A distribution with **two peaks**, for instance, would be called **Bimodal**.


Exponential Distribution
Concrete Compressive Strength
f(x)
2500 3000 3500 4000 4500 5000
Compressive Strength


Exponential Distribution
Concrete Compressive Strength
f(x)
3000 3500 4000 4500 5000 5500
Compressive Strength


Multimodal Distribution

NOTES

25.

One of the primary reasons for concern if a probability distribution appears bimodal is that **sampling was probably not performed on a single controlled process**. That is, data from two separate populations were mistakenly combined. The figure at right illustrates how data from two different populations might combine to form a single bimodal distribution.

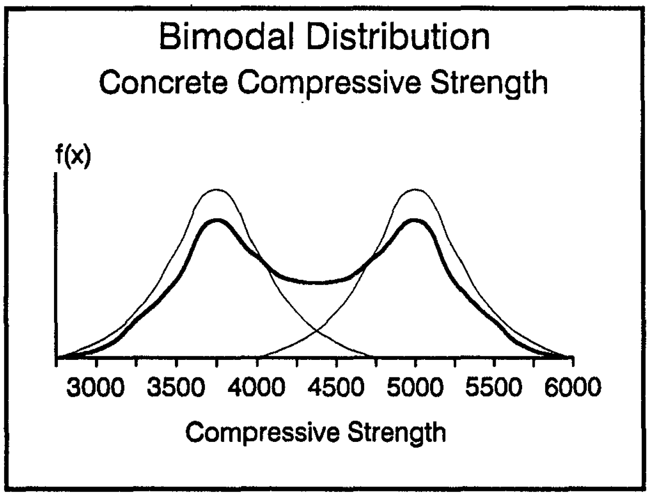
26.

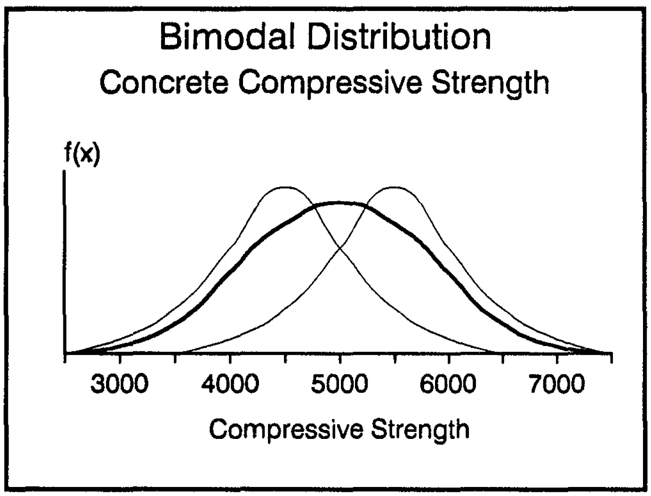
A bimodal distribution serves as a warning that the data were likely from two different populations. Unfortunately, however, data taken from two different processes will not always plot to form a bimodal distribution. The combined distribution may still be unimodal, but with a broader base than either of the two individual populations. The incorrect conclusion will then be that the data are from a single population with a high degree of variability.

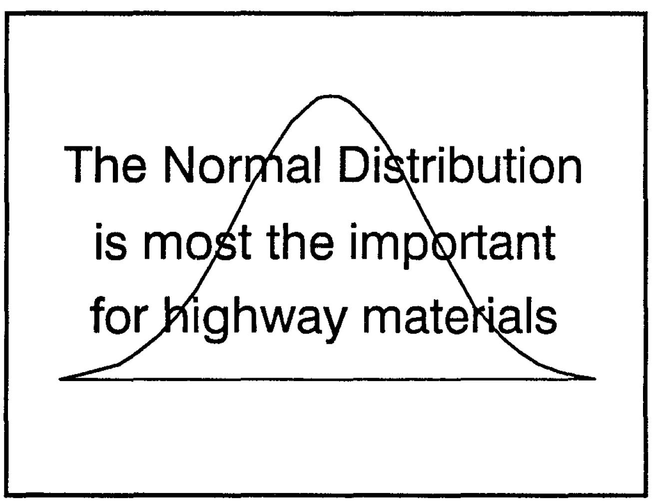
27.

The Normal Probability Distribution

From extensive research, it has been concluded that numerous measurements that occur in highway construction, and in nature in general, distribute themselves about some average value with the majority of the measurements grouped near the mean and with progressively fewer results recorded as one proceeds away from the mean. The **normal distribution**, therefore, is the most important probability distribution for the case of highway materials and construction. Besides being extremely useful in the analysis of acquired data, the **normal distribution** is a key factor with regard to the inferences about the population that are made from the sample data.


Bimodal Distribution
Concrete Compressive Strength
f(x)
3000 3500 4000 4500 5000 5500 6000
Compressive Strength


Bimodal Distribution
Concrete Compressive Strength
f(x)
3000 4000 5000 6000 7000
Compressive Strength


The Normal Distribution
is most the important
for highway materials

NOTES

28.

Properties of the Normal Distribution

We will begin our study of the normal distribution by looking at some of the properties or features of this important distribution. Some of these have already been presented, but are reiterated here along with some additional properties. A concrete compressive strength example is used to illustrate the features of the normal distribution. This population, which is shown at right, has a mean of 5,000 psi and a standard deviation of 500 psi. From the plot it can be seen that **the distribution is *unimodal*,** that **most of the compressive strength test results are clustered near the middle (the mean) of the distribution,** and that **the normal distribution is symmetric about the vertical axis through the mean.**

29.

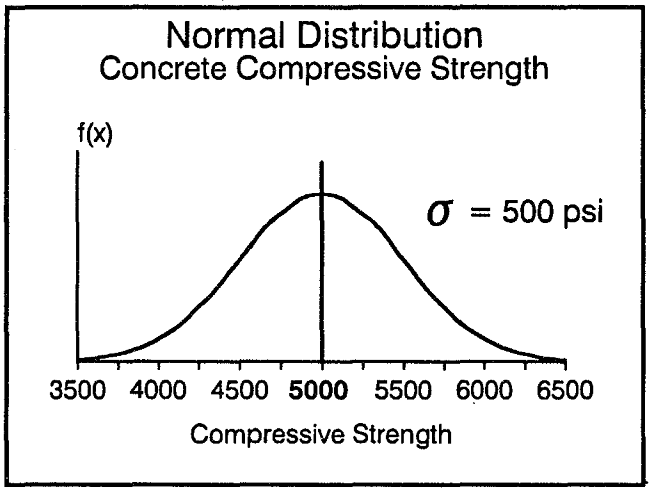
Like all other probability distributions, **the total area under the normal distribution is equal to 100% (or 1.0).**

30.

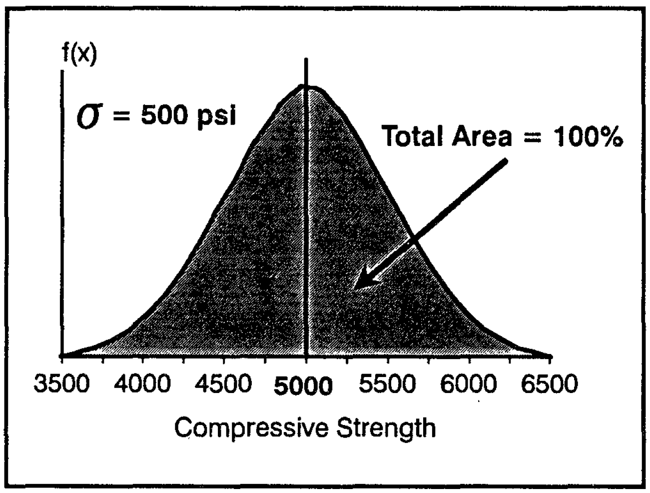
Also like other probability distributions, **the area shown at right under the normal distribution can be interpreted in two ways**:

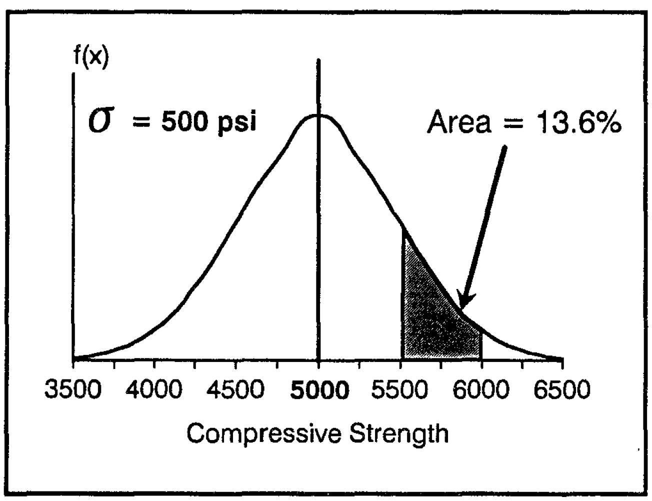
1. 13.6% of the test results in the population occur between 5,500 psi and 6,000 psi, **and**

2. if a randomly selected test result were obtained from a population (lot) with mean concrete compressive strength of 5,000 psi and standard deviation of 500 psi, there is a 13.6% probability (i.e., chance) that the strength will be between 5,500 psi and 6,000 psi.


Normal Distribution
Concrete Compressive Strength
f(x)
σ = 500 psi
3500 4000 4500 5000 5500 6000 6500
Compressive Strength

NOTES


f(x)
σ = 500 psi
Total Area = 100%
3500 4000 4500 5000 5500 6000 6500
Compressive Strength


f(x)
σ = 500 psi
Area = 13.6%
3500 4000 4500 5000 5500 6000 6500
Compressive Strength

31.

A particular normal distribution is uniquely and completely described if the mean and standard deviation associated with the distribution are known. A different normal distribution is created whenever the value of *either* the *mean* or *standard deviation* is changed. There are, therefore, an infinite number of possible normal distributions, each with a different combination of mean and standard deviation. The figure at right shows two distributions with the **same standard deviation**, but with **different means**.

32.

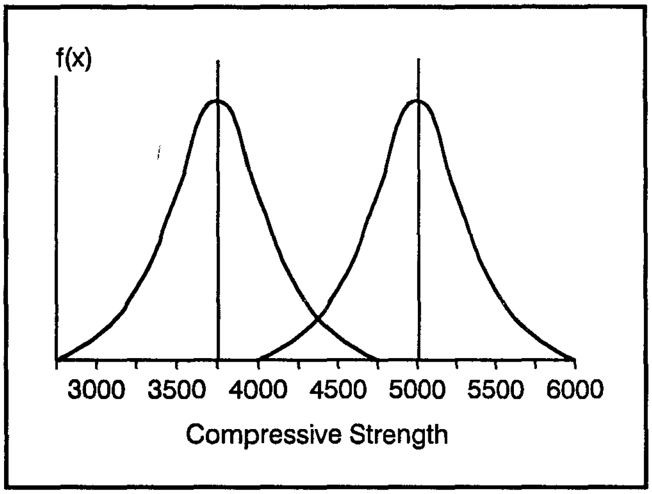
The figure at right shows several normal distributions with the **same mean**, but each with a **different standard deviation**.

33.

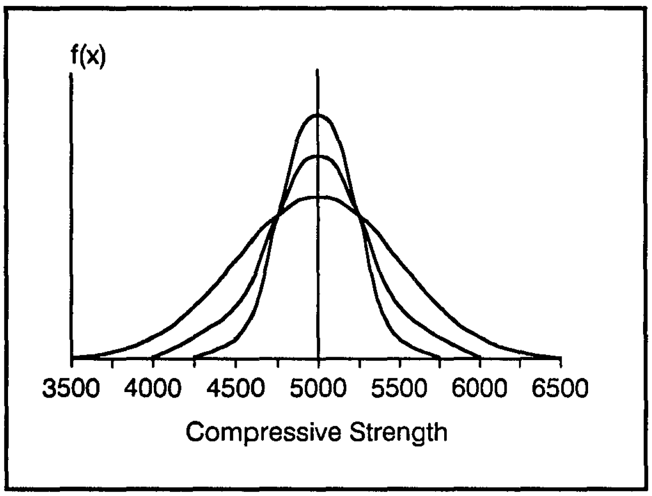
The normal distribution has a complicated equation that can be used to represent it in terms of its mean and standard deviation.

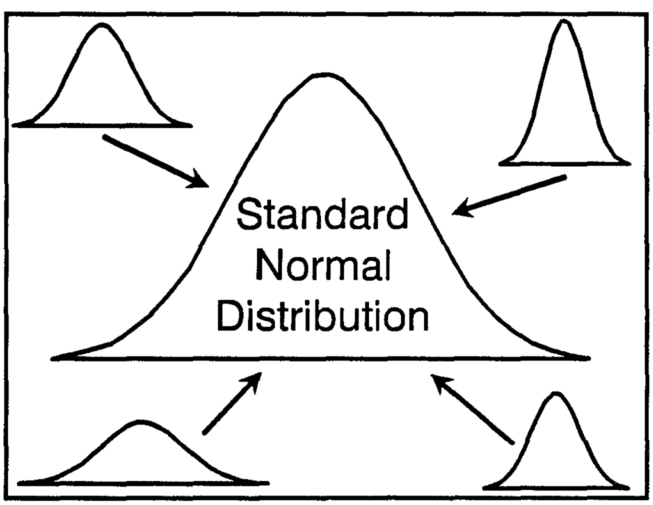
$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{X-\mu}{\sigma})^2}$$

One way to calculate the area under the curve, therefore, would be to integrate the equation between the limits in which we are interested. Fortunately, we do not need to do this because **all normal distributions can be transformed into a *Standard Normal Distribution* for which areas are readily available in the form of a reference table.**


f(x)
3000 3500 4000 4500 5000 5500 6000
Compressive Strength

NOTES


f(x)
3500 4000 4500 5000 5500 6000 6500
Compressive Strength

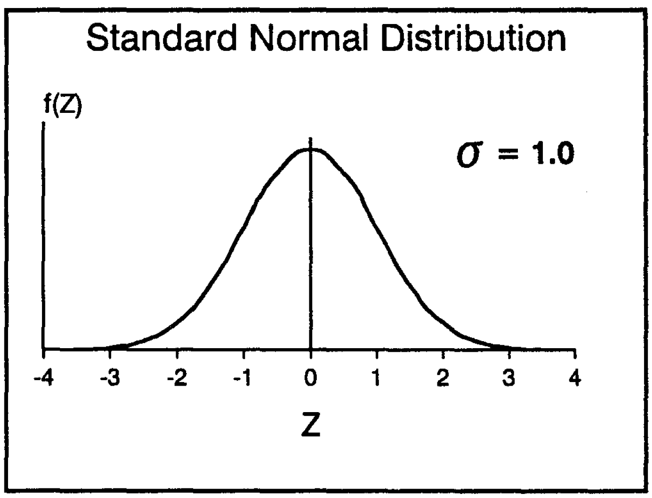

Standard
Normal
Distribution

34.

The equation for the **standard normal distribution** is

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{\frac{-z^2}{2}}$$

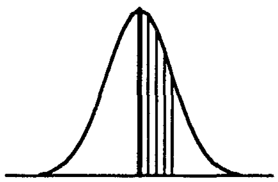
The standard normal distribution is a normal distribution with **mean** = 0, and **standard deviation** = 1. The horizontal axis for the standard normal distribution is usually designated as the **Z axis**. The values on the Z axis are equal to the number of standard deviations above (positive Z values), or below (negative Z values) the mean of 0.



35.

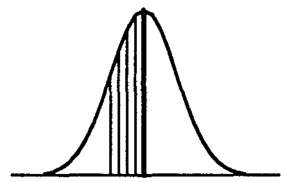
A tabulation of the areas under the Standard Normal Distribution is presented in the table on the facing page. The use of this table and the significance of the results that can be obtained by using the table are illustrated in the example problems that follow.

Areas Under the Standard Normal Distribution



Z

or



-Z

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3183
.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998

36.

Example 1

What is the area under the Standard Normal Distribution that is less than or equal to Z = 0?

Note:

When solving problems using the standard normal distribution **it is always helpful to draw a simple free-hand sketch to clarify the problem statement**. It is *always* worth the little bit of time that is required to draw a sketch of any of these problems, regardless of how straightforward or simple they appear.

37.

Example 2

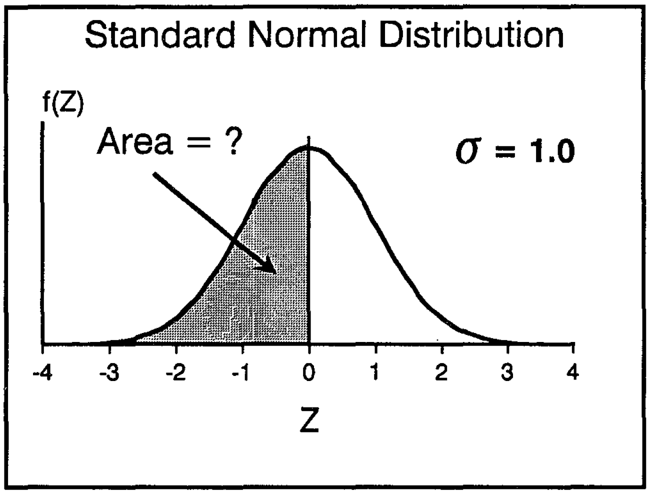
What is the area under the standard normal distribution between Z = 0 and Z = +0.33?

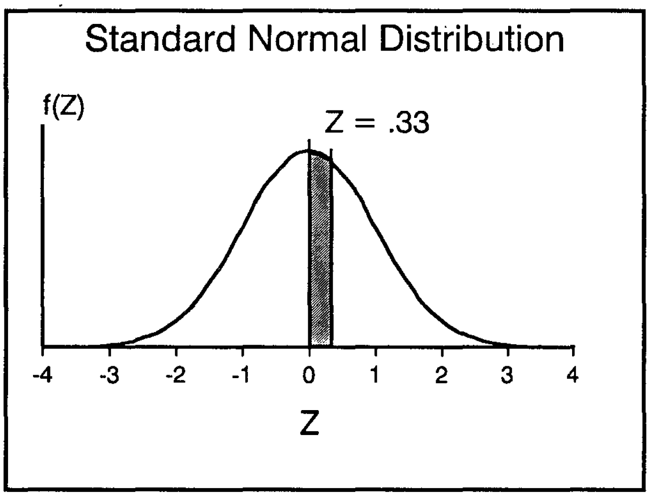
38.

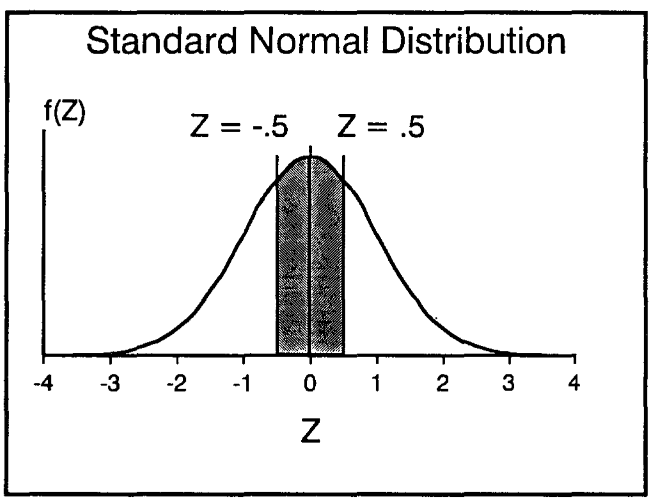
Example 3

If one Z value were randomly obtained from the standard normal distribution, what are the chances (i.e., what is the probability) that it will be between Z = -0.5 and Z = +0.5?

NOTES


Standard Normal Distribution
f(Z)
Area = ?
σ = 1.0
-4 -3 -2 -1 0 1 2 3 4
Z


Standard Normal Distribution
f(Z)
Z = .33
-4 -3 -2 -1 0 1 2 3 4
Z


Standard Normal Distribution
f(Z)
Z = -.5
Z = .5
-4 -3 -2 -1 0 1 2 3 4
Z

39.

What is the area under the standard normal distribution above Z = +3.0?

40.

Workshop: Standard Normal Distribution

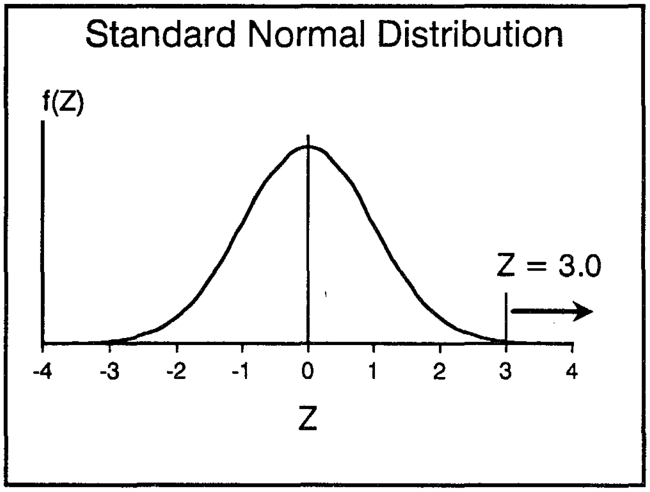
1. What percentage of the area under the standard normal distribution is between Z = -1.0 and Z = +1.0?

2. What percentage of the area under the standard normal distribution is between Z = -2.0 and Z = +2.0?

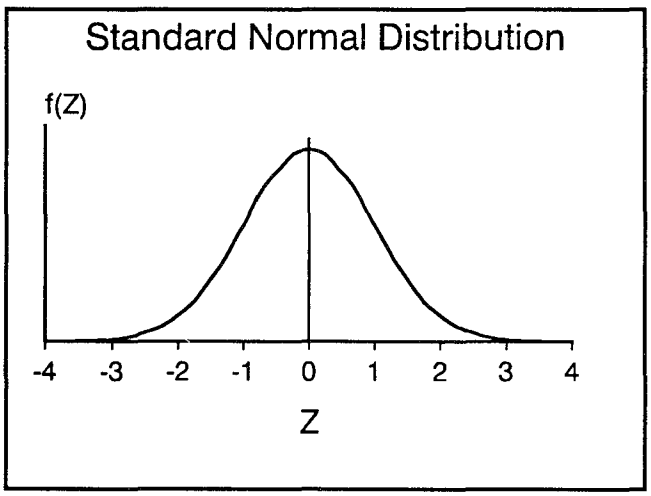
3. What percentage of the area under the standard normal distribution is between Z = -3.0 and Z = +3.0?

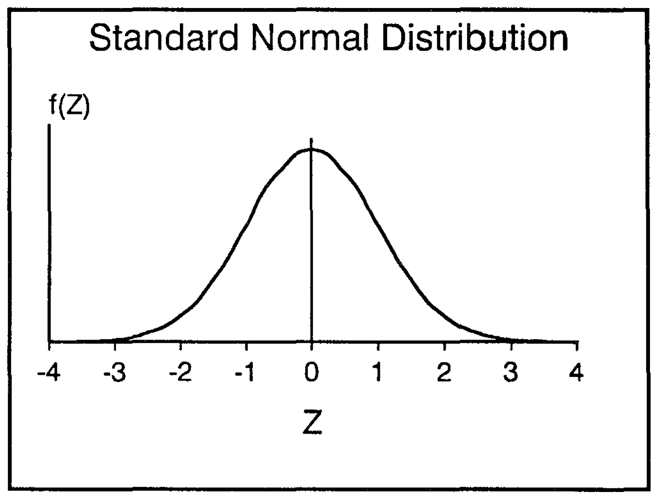
4. If a single Z value is randomly obtained from a standard normal distribution, what is the probability that it will be greater than 1.75?

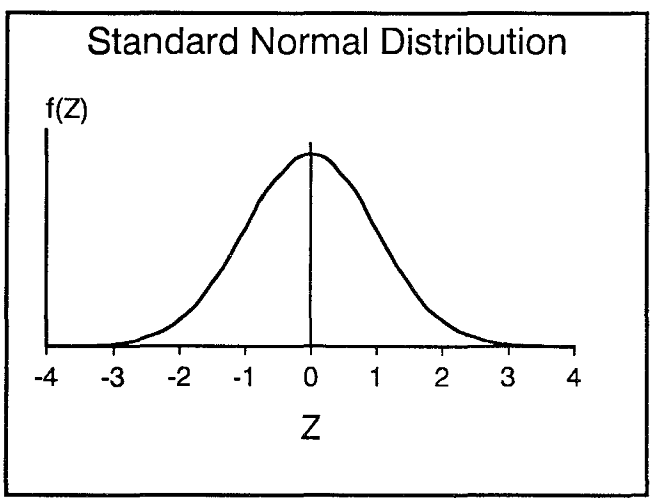
5. If a single Z value is randomly obtained from a standard normal distribution, what is the probability that it will be between Z = -1.0 and Z = -0.5?

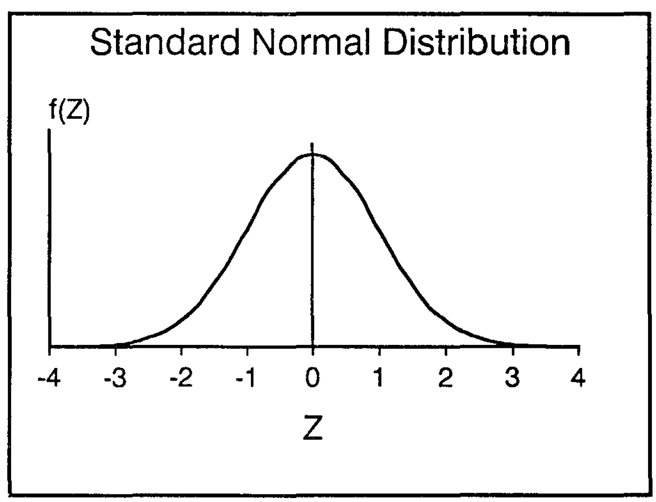

Standard Normal Distribution
f(Z)
Z = 3.0
-4 -3 -2 -1 0 1 2 3 4
Z

NOTES


Standard Normal Distribution
f(Z)
-4 -3 -2 -1 0 1 2 3 4
Z


Standard Normal Distribution
f(Z)
-4 -3 -2 -1 0 1 2 3 4
Z


Standard Normal Distribution
f(Z)
-4 -3 -2 -1 0 1 2 3 4
Z


Standard Normal Distribution
f(Z)
-4 -3 -2 -1 0 1 2 3 4
Z

41.

How can our new knowledge of the standard normal distribution be related to the highway materials and construction situation? It was noted previously that there is a family of normal distributions that can all be transformed to the standard normal distribution. This relationship allows the standard normal distribution to be applied to practical construction situations.

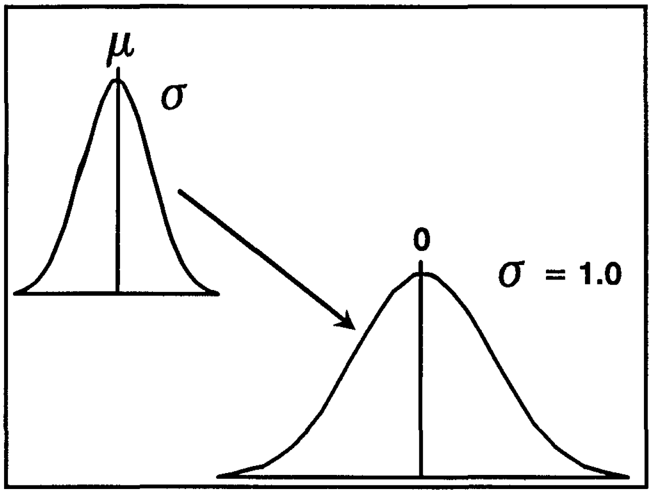
42.

Any normal distribution can be transformed into a standard normal distribution by remembering that **the Z axis is in units of *standard deviation* from the mean**. Therefore, a value from a normal distribution with any given mean and standard deviation can be transformed into the standard normal distribution by the equation shown.

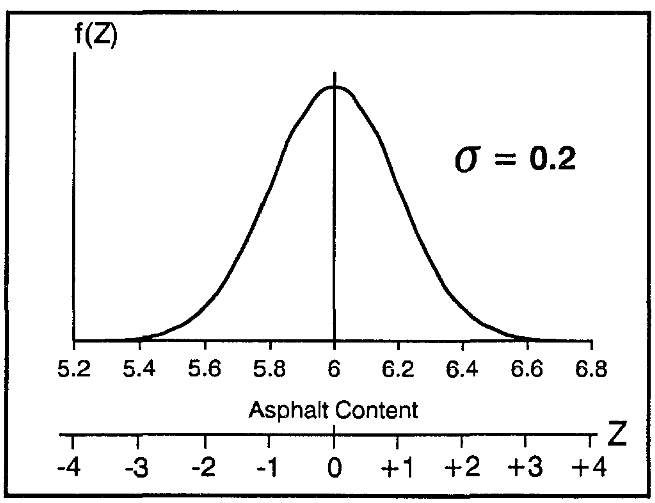
43.

The transformation equation converts the raw value to a value that is measured in standard deviation units above (+) or below (-) the mean. This is illustrated by the two horizontal scales shown on the figure at right. The normal distribution in question represents the asphalt contents for a lot of asphalt concrete. Any value of asphalt content can also be represented as being a given number of standard deviations above or below the mean (shown on the Z axis). Once the Z value is known, the standard normal distribution can be used to determine areas and hence probabilities.

NOTES



$$Z = \frac{X - \mu}{\sigma}$$



44.

Example 1

If the compressive strength for a concrete batching process is assumed to follow a normal probability distribution with a mean of 4,000 psi and a standard deviation of 500 psi, what percentage of the distribution is between 3,000 psi and 4,000 psi?

45.

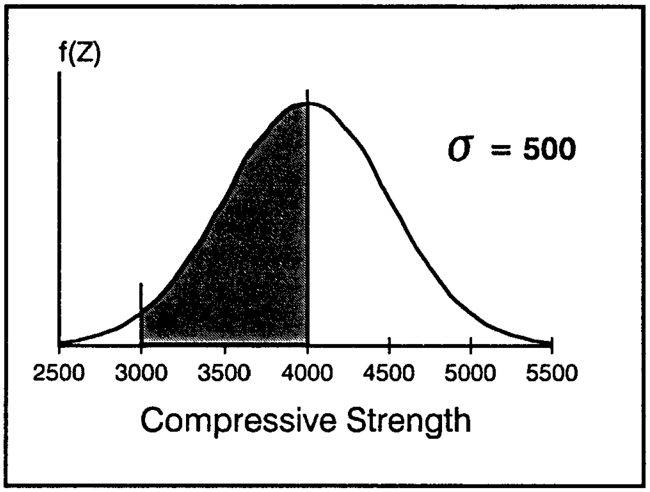
Example 2

For the distribution from Example 1, what is the probability that a single compressive strength test will give a value below the design strength of 3,000 psi?

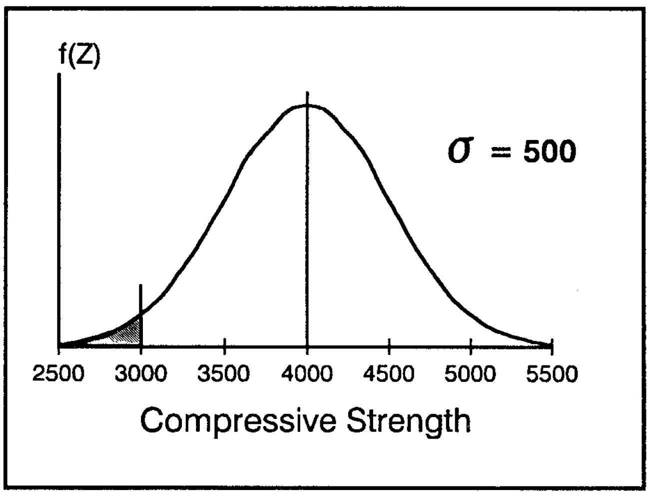
46.

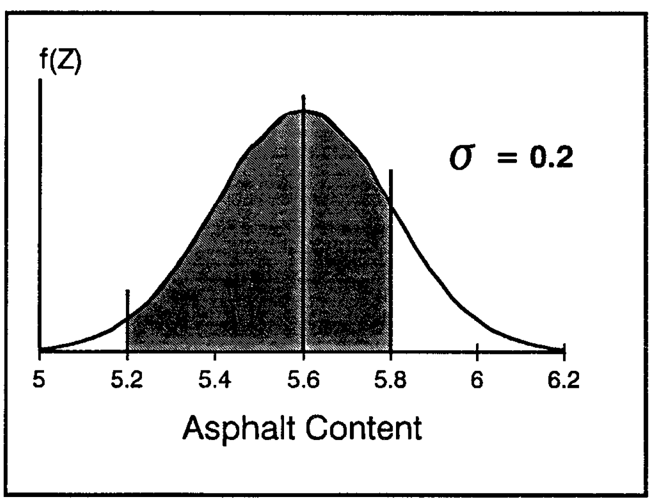
Example 3

A contractor is producing asphalt concrete with an asphalt content that is assumed to be normally distributed with mean = 5.6% and standard deviation = 0.2%. If the Job Mix Formula is 5.5%, with an allowable tolerance of ± 0.3%, what percentage of the asphalt concrete will be within the allowable tolerance limits?


f(Z)
σ = 500
2500 3000 3500 4000 4500 5000 5500
Compressive Strength

NOTES


f(Z)
σ = 500
2500 3000 3500 4000 4500 5000 5500
Compressive Strength


f(Z)
σ = 0.2
5 5.2 5.4 5.6 5.8 6 6.2
Asphalt Content

47.

Distribution of Sample Means

In Sampling Experiment #1 (Chapter 2 and 3), it was found that the spread of samples of size n=1 was nearly as great as that for the population. However, in Sampling Experiment #2 (Chapter 4), it was found that the spread of the means of samples of size n=4 was considerably less than that of the population. This would tend to imply that although the mean of a single sample of size n=4 would not necessarily be equal to the population mean, it would have a better chance of being close to the population mean than would a single sample result of size n=1.

48.

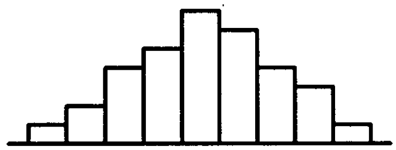
It has been proven mathematically that the mean and standard deviation of the distribution of sample means are related to the mean and standard deviation of the population. The mean of the distribution of samples means will be equal to the mean of the population, while the standard deviation of the sample means will be equal to the population standard deviation divided by the square root of the sample size. Sample size here is the number of values used in calculating the sample mean, and not the number of sample means.

49.

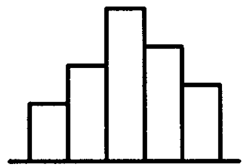
The distribution of sample means is one of the theoretical bases for the use of multiple sampling (i.e., use of sample sizes of $n > 1$). The shape of the distribution of sample means can be derived from the **central limit theorem**. The central limit theorem states that regardless of the shape of the population distribution, the means of samples of size $n > 1$ drawn from the population, will approach a normal distribution as $n \to \infty$. This is true regardless of the shape of the population from which the samples are drawn.

NOTES

Sampling Experiment #1



Sampling Experiment #2



	Population	Distribution of Sample Means
Mean	μ	μ
Standard Deviation	σ	$\frac{\sigma}{\sqrt{n}}$

Central Limit Theorem

Means of samples of size $n > 1$ approach a normal distribution as

$$n \longrightarrow \infty$$

50.

To illustrate the central limit theorem, 1000 samples of size n=5 were drawn from probability distributions that were rectangular and triangular shaped. The sample means were then calculated for each of the 1000 samples The histograms for the 1000 sample means were then plotted. It can be seen from these histograms that even though the parent populations are decidedly non-normal, the distributions of sample means are approximately normal. This is true despite the fact that the sample sizes were only n =5.

51.

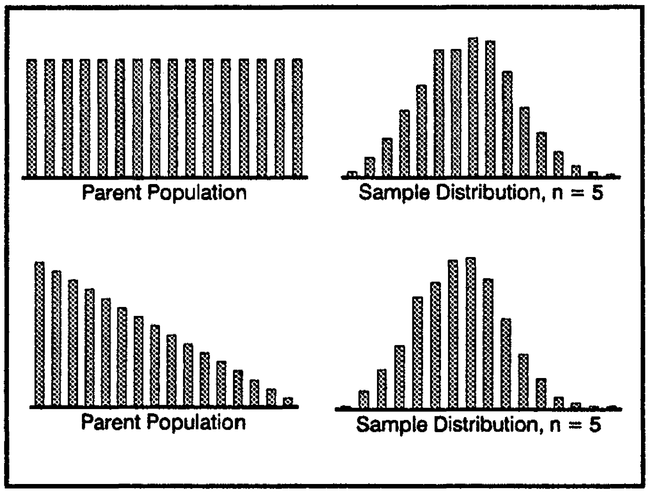
From the relationships stated above, as the sample size increases, the spread of the distribution of sample means decreases. This is one justification for using multiple samples (i.e., $n > 1$). Also, because of the central limit theorem, multiple sampling allows us to approximate the distribution of sample means by the normal distribution without knowing the shape of the population distribution.

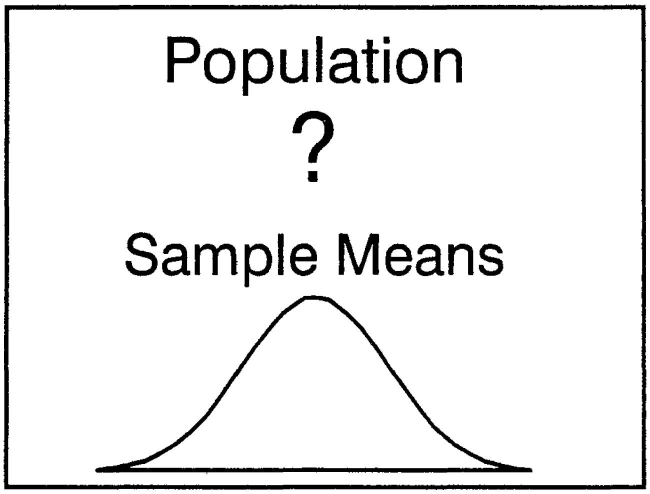
52.

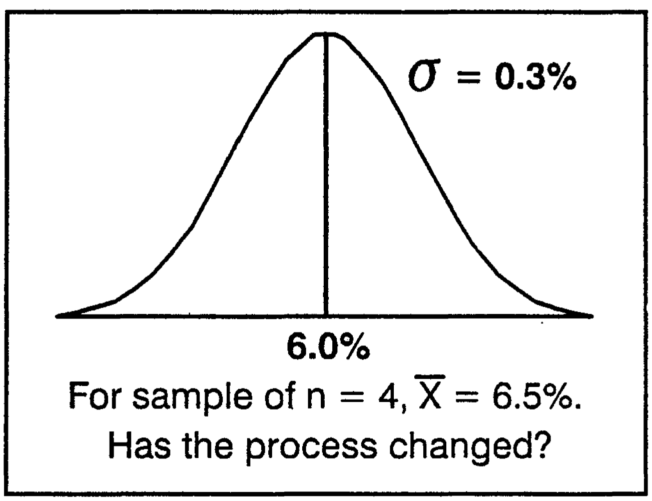
Illustrative Examples: Distribution of Sample Means

Assume that an asphalt plant is supplying mix that has an asphalt content with a mean of 6.0% and a standard deviation of 0.3%; and that it is assumed that the population is normally distributed. After a plant breakdown, a sample of size $n = 4$ is obtained to determine if the process has changed. The sample mean is 6.5%. Based on this sample mean, how likely is it that the process has not changed?

NOTES


Parent Population
Sample Distribution, n = 5
Parent Population
Sample Distribution, n = 5


Population
?
Sample Means


σ = 0.3%
6.0%
For sample of n = 4, X̄ = 6.5%.
Has the process changed?

53.

Solution:

The distribution of sample means of samples of size n = 4 will follow a normal distribution with mean equal to the population mean and standard deviation equal to the population standard deviation divided by the square root of 4. Three standard deviation limits for the distribution of sample means would be 5.55 to 6.45. Based on the sample mean of 6.5%, is it likely that the process has changed or remains the same?

54.

Workshop: Distribution of Sample Means

The following are simplified problems that indicate how the limits of a specification for sample means might be established. The actual development of statistically-based specification is more complicated than the examples presented below. The design and development of specifications is discussed in later chapters.

1. The population parameters for asphalt content are known to be normally distributed with mean = 6.0% and standard deviation = 0.3%. Based on this population, develop upper and lower specification limits within which all asphalt content test results of sample size n = 1 must fall.

2. Develop upper and lower specification limits within which the mean asphalt content of samples of size n = 4 must fall.

3. Develop upper and lower specification limits within which the mean asphalt content of samples of size n = 9 must fall.

55.

The following steps are involved in transforming construction material data to allow a standard normal curve to be used to make probability statements about the data:

1. **Collect** the data using random sampling.

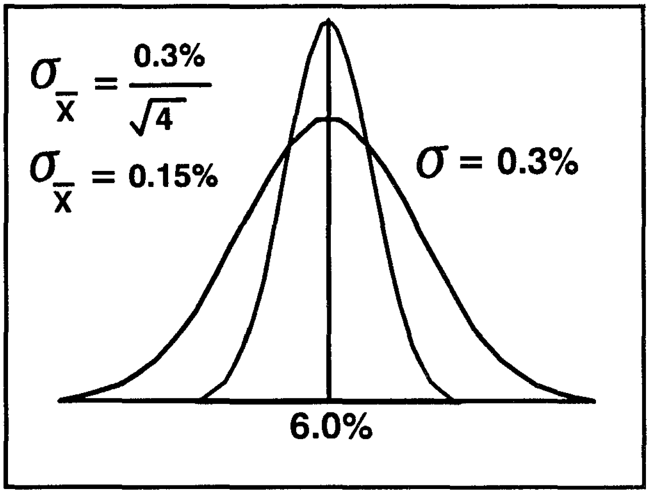
2. **Organize** the data into a relative frequency histogram.

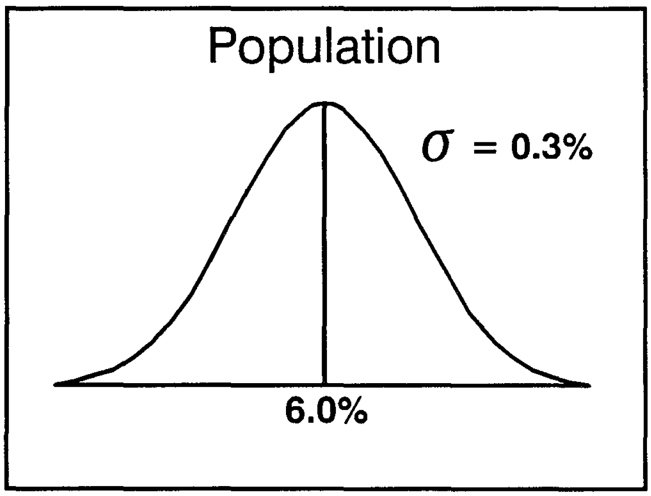
3. If reasonably symmetrical and bell-shaped, approximate the histogram with a normal distribution.

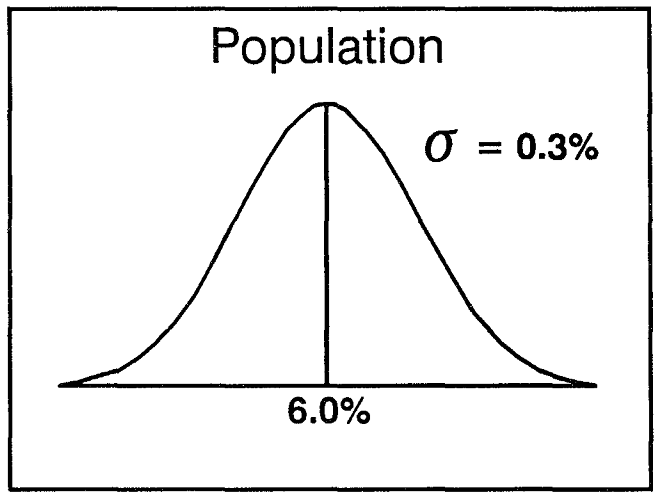
4. **Analyze** the data to determine the mean and standard deviation.

5. Transform the normal distribution into the standard normal distribution to **Interpret** the data.

NOTES







Collect	→	random sampling
Organize	→	relative frequency histogram
Approximate	→	normal distribution
Analyze	→	$\bar{X}$ and s
Interpret	→	standard normal distribution

56.

Trying to decide whether or not a histogram of the data appears to be normally distributed is obviously subjective. There are statistical techniques called goodness-of-fit tests designed to determine how well a theoretical probability distribution (such as normal) fits a set of data. The most common of these are the **chi-square (χ^2)** and **Kolmogorov-Smirnov (K-S)** tests. These tests are described in numerous statistics books, and are not discussed here.

57.

One other graphical method for testing normality is the use of **normal probability paper**. This approach is discussed in detail in References 3 and 4.

58.

Key points for this chapter are shown in the slide at right.

References

1. Johnson, R., *Elementary Statistics*, Duxbury Press, North Scituate, Massachusettes, 1976.

2. Miller, I. and Freund, J. E., *Probability and Statistics for Engineers, 2nd Edition*, Prentice Hall, 1977.

3. Willenbrock, J. H., *Statistical Quality Control of Highway Construction Vol. 1*, Federal Highway Administration, Washington, DC, 1976.

4. Dixon, W. J. and Massey, F. J., *Introduction to Statistical Analysis, 3rd Edition*, McGraw-Hill, 1969.

Goodness-of-Fit Tests

- Chi-square test

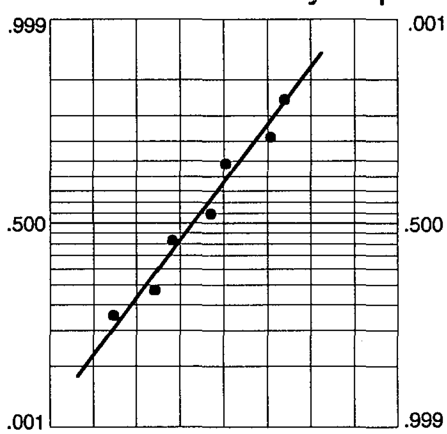
  χ^2-test

- Kolmogorov-Smirnov test

  K-S test

NOTES

Normal Probability Paper



Key Points

1. Relative frequency & probability.
2. Probability distributions.
3. The normal distribution.
4. The standard normal curve.
5. The Z-statistic.
6. Areas under the normal curve.
7. Distribution of sample means.

Workshop Problems:

1. The job mix formula for the aggregate for a particular bituminous mix specifies a tolerance of 52 to 62 percent passing for one of the sieves. The records of the plant indicate that the average percent passing based on many gradation tests was 55 percent and that the standard deviation was 4 percent. What percent of the plant's output probably will meet the specification on a future project?

2. Based on a large number of density tests on a compacted embankment section, it is determined that the average of the measured values of the relative density was 98% and the standard deviation was 4%. The specifications require that at least 85% of the measured results shall exceed 92% relative density. Does the embankment section meet the specification requirements?

3. The average thickness of the first 2 miles of a concrete pavement, based on a large number of cores, was 9.80 inches and the standard deviation was 0.21 inches. If the same paving process is used for the remaining five miles of the project:

   a. How many of the 25 cores that will be taken on the remaining five miles would you expect to indicate a thickness greater than or equal to 10 inches?

   b. If a single, randomly selected core is checked, what is the probability that it will be less than or equal to 9.50 inches?

4. The job mix formula for an asphalt concrete mix calls for an asphalt content of 5.5%. The highway agency tolerance from this value is ± 0.5%. The contractor has compiled enough historical data on this mix to know that his mean is 5.6% and standard deviation is 0.25%. If the contractor gets paid at his full unit bid price for all material that is within the tolerance limits, how many of the 25,000 tons of material on the project will probably be paid at the full unit price?

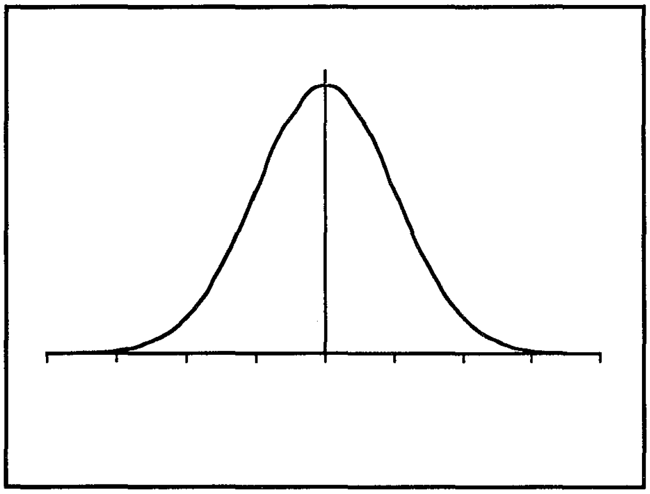
5. A contractor is building a project that includes 5 miles of bituminous pavement. Assume that he will follow the same rolling pattern throughout the project. After considerable testing on the first two miles of pavement, the contractor claims that he is achieving an in-place density of 127 pcf with a standard deviation of 1.50 pcf.

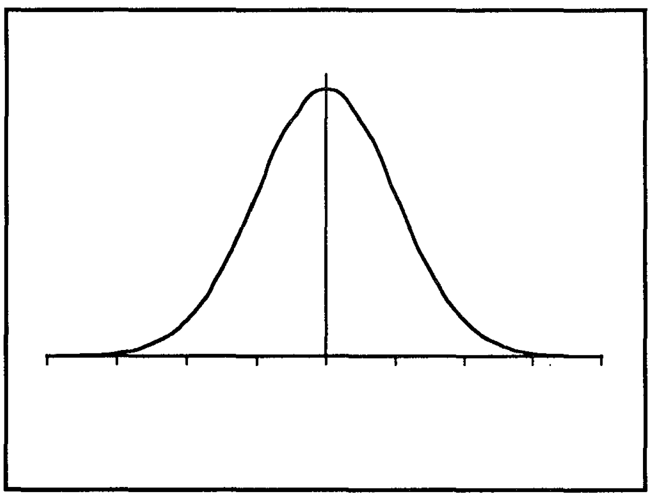
   At the beginning of the third mile a highway agency inspector requests that a control strip be identified. A total of 30 nuclear density readings are obtained from the control strip at random locations.

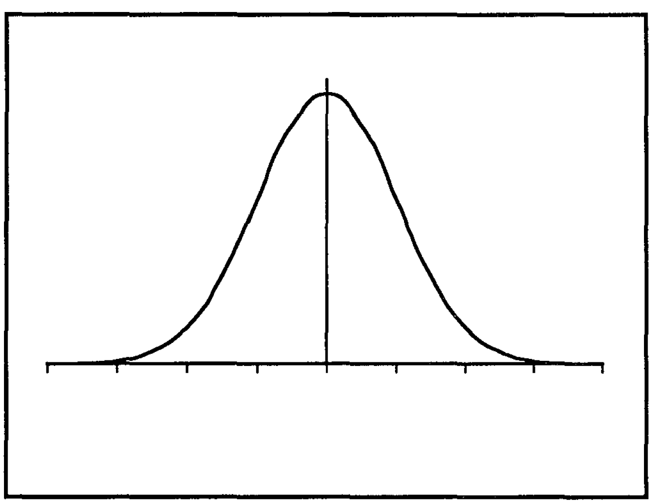
   a. What is the probability that the mean of these 30 test results will be between 126.7 pcf and 127.5 pcf?

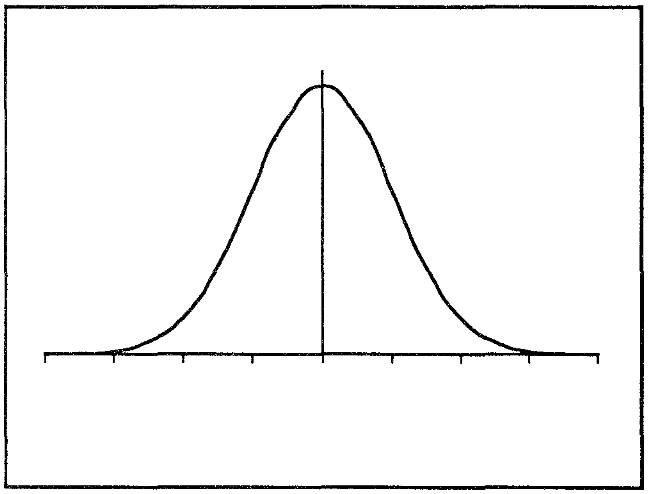
   b. If the mean of the 30 test results were equal to 126.0, how likely is it that the contractor's claims regarding the process mean and standard deviation are correct?

NOTES









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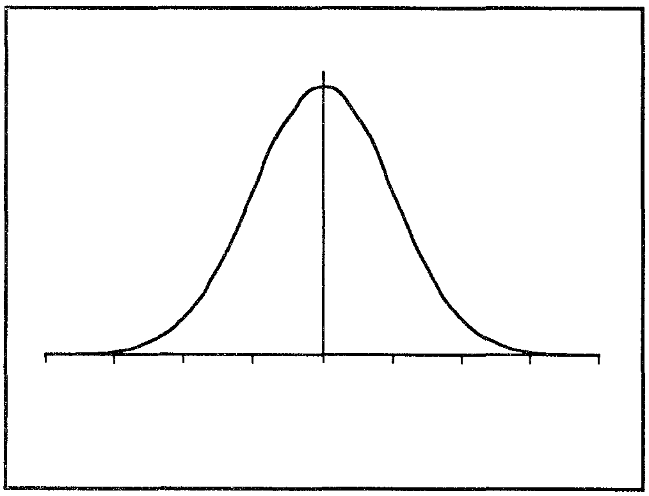
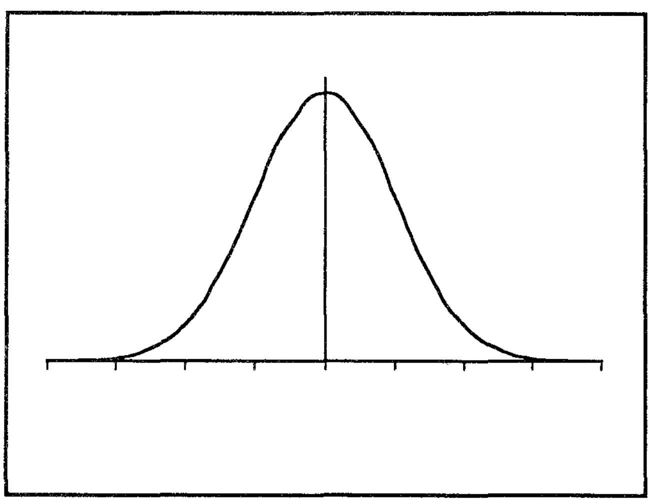




CHAPTER 6: SOURCES OF VARIABILITY

INSTRUCTIONAL OBJECTIVES

1. Introduce the concepts of precision, accuracy, and bias.
2. Show how sources of variability are related.
3. Show typical variabilities of highway products.
4. Discuss precision and bias statements.

DESIRED STUDENT ACHIEVEMENTS

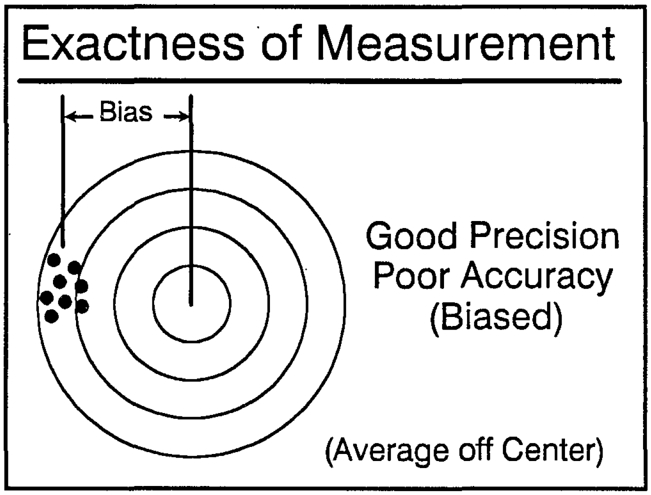
1. Learn what precision, accuracy, and bias mean.
2. Learn how to calculate sources of variability.
3. Understand how to use precision and bias statements.

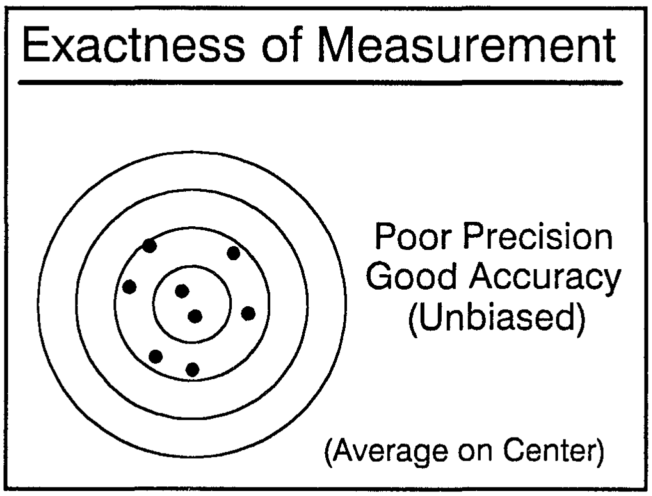
CHAPTER 6: SOURCES OF VARIABILITY

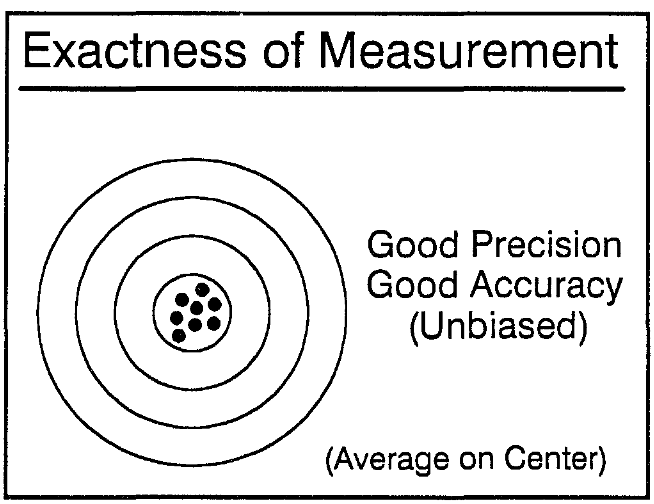
1.

The concepts of precision and accuracy are fundamental to the understanding of variability. **Precision** refers to the variability of repeat measurements under carefully controlled conditions. **Accuracy** is the conformity of results to the true value -- absence of bias. **Bias** is a tendency of an estimate to deviate in one direction from a true value. The analogy of a target is a practical way of understanding the relationship between precision and accuracy. A practical difficulty in measuring accuracy is that the **true** value must be determined. Ideally, the true value should be determined by a method of high precision and with as little bias as possible.

NOTES


Exactness of Measurement
Bias
Good Precision
Poor Accuracy
(Biased)
(Average off Center)


Exactness of Measurement
Poor Precision
Good Accuracy
(Unbiased)
(Average on Center)


Exactness of Measurement
Good Precision
Good Accuracy
(Unbiased)
(Average on Center)

2.

It is essential to understand that all highway products are inherently variable. Once this is understood, it is important to understand that there are various sources of variability and that variability should be considered an important parameter in the quality of the product.

The source, or component, of variability is a straightforward mathematical relationship based on the Pythagorean theorem. The previously discussed variability, **σ**, is termed **overall variability**.

As mentioned earlier, **s** is used to designate the standard deviation of a sample and uses the degrees of freedom **n-1** in the denominator, under the radical. However, for the standard deviation of a population, **σ** is used to indicate that the variability is known from a great deal of data. In the equations at right, **n** is used to denote sample size, while **N** represents the total number in the population.

It is obvious that, as the sample size increases, **s** approaches **σ** because of the smaller percentage difference between **n-1** and **n**. As a rule of thumb, when n is 30 or larger, s is approximately equal to σ and n can be used to calculate standard deviation.

3.

Variability can come from many sources usually called errors -- sampling error, testing error, etc. These terms mean sampling variability and testing variability, not mistakes. The sources of variability are related by the Pythagorean theorem, given in the formula at right, where

σ_o^2 = overall variance

σ_m^2 = material variance

σ_t^2 = testing variance.

Remember that the variance is the standard deviation squared.

4.

This figure describes the relationship.

NOTES

Sample Standard Deviation:

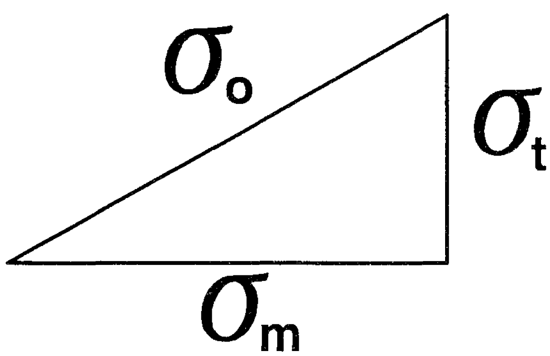
$$S = \sqrt{\frac{\sum (X_i - \bar{X})^2}{n - 1}}$$

Population Standard Deviation:

$$\sigma = \sqrt{\frac{\sum (X_i - \mu)^2}{N}}$$

$$\sigma_o^2 = \sigma_m^2 + \sigma_t^2$$

overall material testing



5.

Furthermore, one leg of the triangle can be considered the hypotenuse of a second triangle so as to include more sources of variability.

6.

These added sources of variability are related by the equation at right, where

σ_t^2 = testing variance

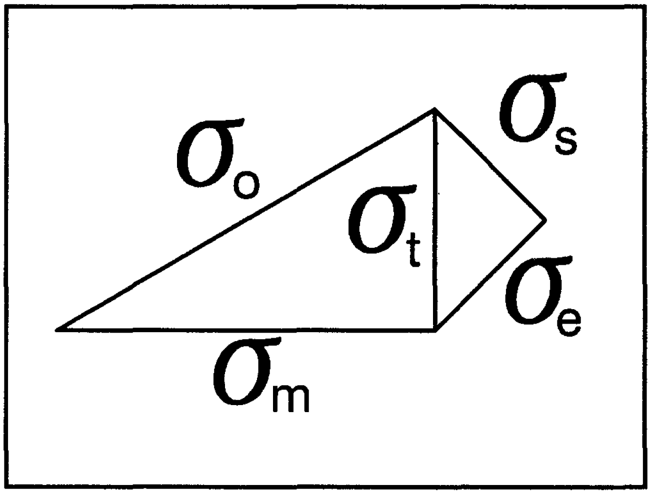
σ_e^2 = test method variance

σ_s^2 = sampling variance.

7.

The relationship that relates the overall variability to each source is given by the equation at right.

NOTES



$$\underbrace{\sigma_t^2}_{\text{testing}} = \underbrace{\sigma_e^2}_{\text{test method}} + \underbrace{\sigma_s^2}_{\text{sampling}}$$

$$\sigma_o^2 = \sigma_m^2 + \sigma_e^2 + \sigma_s^2$$

8.

As an example, the West Virginia Highway Department conducted a concrete study several years ago to determine the sources of variability in their concrete testing problem. They found this relationship.

9.

Example Problem

If, in the above example, σ_s had equaled 300 and the other variabilities had remained the same, what would σ_o be?

$\sigma_o^2 =$

$\sigma_o =$

10.

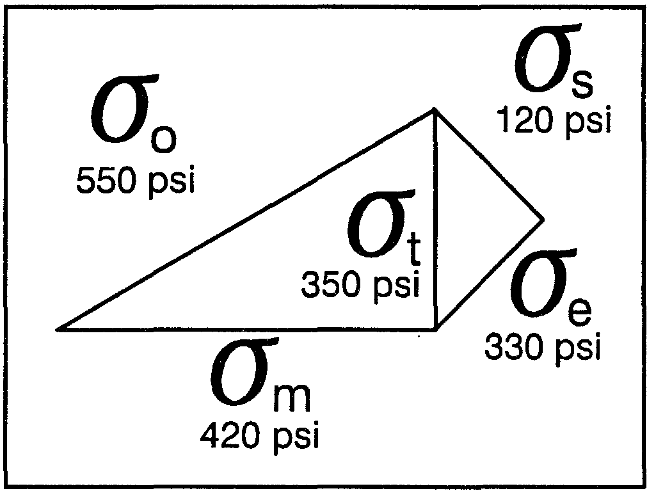
Example Problem

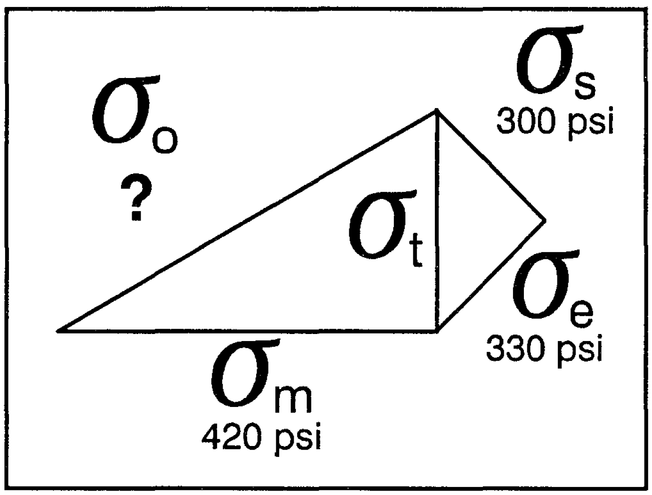
At an asphalt plant, the property being measured is asphalt content. Calculate σ_o if σ_m = 0.15%, σ_e = 0.05%, and σ_s = 0.10%.

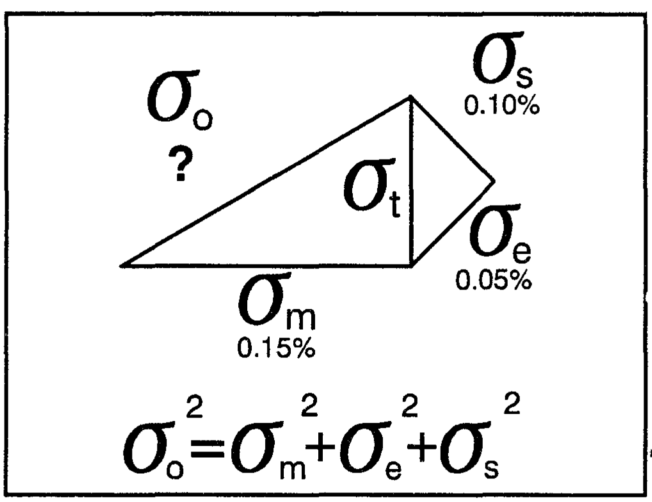
$\sigma_o^2 =$

$\sigma_o =$

NOTES


σ_o
550 psi
σ_s
120 psi
σ_t
350 psi
σ_e
330 psi
σ_m
420 psi


σ_o
?
σ_s
300 psi
σ_t
σ_e
330 psi
σ_m
420 psi


σ_o
?
σ_s
0.10%
σ_t
σ_e
0.05%
σ_m
0.15%
$\sigma_o^2=\sigma_m^2+\sigma_e^2+\sigma_s^2$

11.

Concrete Variability

At least three properties are considered important for controlling the quality of **portland cement concrete**: strength of hardened concrete, and air content and slump of fresh concrete. The strength can be determined by testing cylinders or cores. Typical standard deviations are shown in the table (**1**).

To those unaccustomed to statistical analyses, these values may appear high. Experience has shown that a strict degree of control is necessary to achieve a variability of this magnitude. It is important to realize that the production of concrete is an inherently variable process. Recognition of variability and obtaining quantitative values are essential to meaningful analyses of concrete test data regardless of whether or not specifications are written in terms of statistical parameters. Use of these measures will be helpful in stating quantitatively what is meant by **reasonable compliance**.

12.

Asphalt Concrete Variability

Many studies have been conducted on the variability of asphalt concrete (**2,3**). The variabilities of primary interest are those for mix gradation and asphalt content, and for air voids in the compacted mat. A typical range of variabilities for gradation taken from extraction tests is shown in the table at right.

13.

Typical ranges of asphalt contents from extraction tests and air voids from cores taken from field projects are shown at right.

NOTES

Typical σ Values - PCC

Property	Std Dev
Compressive Strength (Avg. - 2 cylinders)	525 psi
Compressive Strength (Single cores)	725 psi
Slump	0.5 inch
Air Content (Pressure method)	0.6 percent

Typical σ Values - Asph. Grad.

Sieve	Range of Std Dev
3/4"	1.5 - 4.5
3/8"	2.5 - 5.0
No. 4	2.5 - 5.0
No. 8	2.5 - 4.0
No. 30	2.0 - 3.5
No. 50	1.0 - 2.0
No. 200	0.6 - 1.0

Typical σ Values - Asph. Conc.

Property	Range of Std Dev
Asphalt Content	0.15% - 0.30%
Air Voids (cores)	1.3% - 1.5%

14.

Research (4) on the variability of void properties of Marshall-compacted specimens shows these standard deviations to be typical.

15.

Pavement Thickness Variability

Finished pavements, like other materials, are inherently variable (6). Typical thickness variabilities are shown in this table.

16.

Example Problem

Calculate 1, 2, and 3 standard deviation limits for an asphalt concrete with an average of 6.1% asphalt and a standard deviation of 0.25%.

Typical σ Values - Asph. Conc.

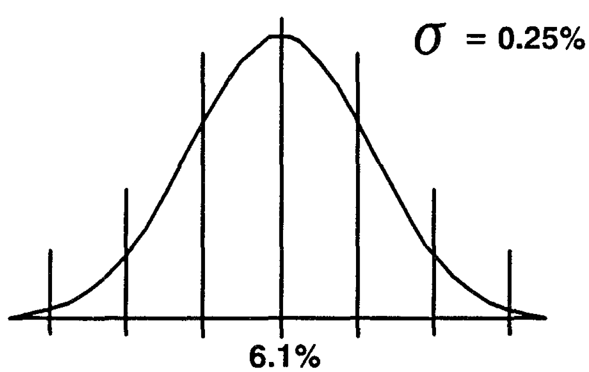
Property	Typical Std Dev
Marshall Voids	
Voids Total Mix	0.9
Voids Mineral Agg.	0.9
Voids Filled Asphalt	4.0

NOTES

Typical Variabilities Pavement Thickness - NJ

Design	$\bar{X}$	σ	Pvmt Type
1.5"	1.6"	0.30	AC surf/binder
2.0"	2.1"	0.35	AC surf/binder
3.0"	3.25"	0.45	AC surf/binder
4.0"	4.25"	0.60	AC base
6.0"	6.25"	0.75	AC base

Example Problem:



17.

Many standardized test methods such as those in ASTM contain precision and bias statements. It is important to realize how to use such statements.

18.

For a precision statement, a one-sigma limit (**1S**) is used to denote the estimate of the standard deviation of the population of the test. This limit is usually established for a single operator and also for multi-laboratory precision.

Another expression found in precision statements is the acceptable range of results, called the "difference two-sigma limit" (**D2S**). The index indicates a maximum acceptable difference between two results obtained on test portions of the same material. It is the difference between two individual test results that would be equaled or exceeded in the long run in only 1 case in 20. $\text{D2S} = 2 \times \sqrt{2} \times (1\text{S})$.

19.

As stated previously, bias is a systematic error between a test value and the true value. Although **bias statements** are highly desirable, many test methods have no true value. Stated differently, the test values can only be established through the test method. Where bias is found, the method may be adjusted to eliminate the bias.

NOTES

Precision and Bias Statements

Precision Statement
Theoretical Max. Specific Gravity

	(1S)	(D2S)
Single Oper.	0.0040	0.011
Multi-lab	0.0064	0.019

Bias Statement

1. No bias - test value is defined by test method.
2. Method yields results 0.12 units higher than reference value.

20.

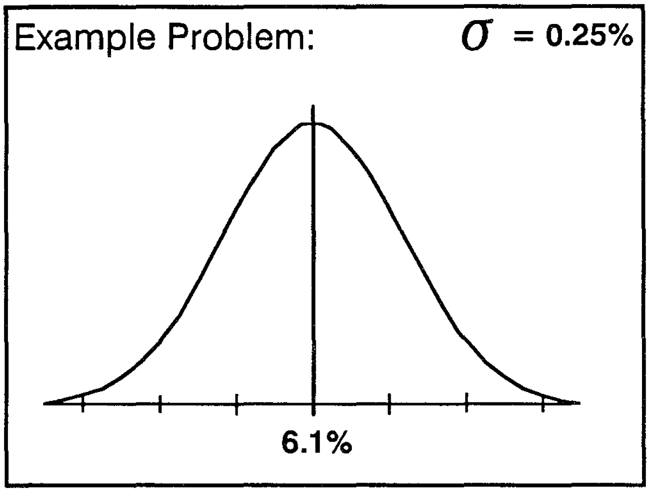
Workshop Problems

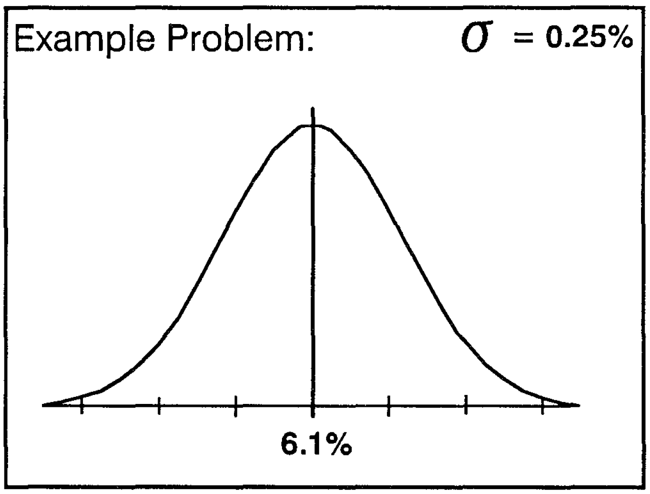
1. For a population with a mean of 6.1% and standard deviation of 0.25%, how much of the population would be between 5.7 and 6.3 percent asphalt content?

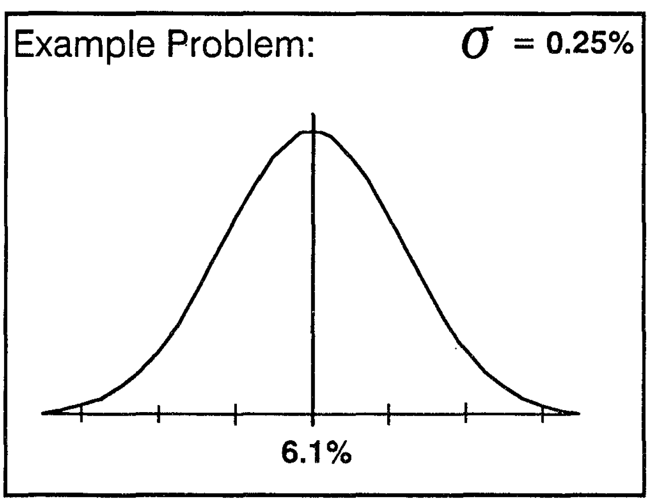
2. How much between 5.8 and 6.4 percent for the same population?

3. What would be the limits, if you desired 90% of the population to meet the specification?

NOTES


Example Problem:
σ = 0.25%
6.1%


Example Problem:
σ = 0.25%
6.1%


Example Problem:
σ = 0.25%
6.1%

21.

Key points for this chapter are presented in the slide below.

Key Points

1. Bias and accuracy.
2. Precision.
3. Sources of variability.
4. Typical variabilities.

NOTES

References

1. Newton, H. H., Jr., "Variability of Portland Cement Concrete," *Proceedings of National Conference on Statistical Quality Control*, 1966.

2. Yoder, E. J. and Witczak, M. W., *Principles of Pavement Design, Second Edition*, John Wiley and Sons, 1975.

3. Hughes, C. S., "Virginia's Experience with a Quality Assurance and Acceptance Specification for Asphaltic Concrete." *Transportation Research Record No. 385*, 1972.

4. Hughes, C. S., "Field Management of Asphalt Concrete Mixes." *Virginia Transportation Research Council, Report VTRC 89-R6*, 1988.

5. Weber, W. G., Grey, R. L., and Cady, P. D.,. "Evaluating Procedures for Determining Concrete Pavement Thickness and Reinforcement Location, Phase II, Intermediate Report." *NCHRP Project 10-8, Penn. DOT*, 1972.

6. Afferton, Kenneth C., "New Jersey's Thickness Specification for Bituminous Pavement." *TRB Circular No. 172*, 1975.

CHAPTER 7: PROCESS CONTROL

INSTRUCTIONAL OBJECTIVES

1. Discuss the importance of process control.

2. Show how to determine control chart limits.

3. Show how to interpret control charts.

DESIRED STUDENT ACHIEVEMENTS

1. Understand the meaning of process control plans.

2. Learn how to calculate control chart limits.

3. Learn how to plot and interpret control charts.

CHAPTER 7: PROCESS CONTROL

1.

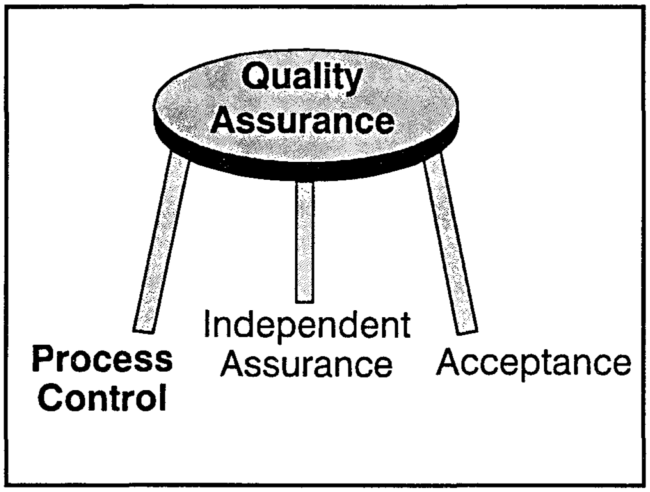
In Chapter 1 we introduced the three functions of QA: **process control**, **acceptance** and **independent assurance**. In this session we concentrate on **process control**.

2.

The specifications must consider which party is to be responsible for process control. In the past, the highway agency has often assumed this responsibility. This can lead to legal problems if the material does not meet the specification requirements since the contractor generally meets its legal obligations if it follows the process directions given by the highway agency. The party that does the work is generally in the best position to control its own process. For this reason, most QA specifications call for the contractor to be responsible for process control.

3.

Some specifications require the contractor to submit a **process control plan** to the highway agency for review or approval prior to the beginning of work. The plan outlines the details of how many tests are to be run, the frequency of the testing, and criteria for when action will be taken to put a process back in control if it goes out. Other factors that might be addressed in a process control plan include amount and frequency of plant inspections, verification of calibrations, and type and amount of documentation that will be maintained.



Independent Laboratory

Supplier

Process Control

Contractor

Highway Agency?

Process Control Plan

- type & frequency of tests
- plant inspections
- calibrations
- documentation
- criteria for action

NOTES

4.

The process control procedure that we will concentrate on in this chapter is the use of **control charts**, particularly **statistical control charts**. Control charts provide a means of verifying that a process is in control. It is important to stress that control charts **do not** get or keep a process under control. You must still control the process. Control charts simply provide a visual warning mechanism to identify when the contractor or material supplier should look for possible problems with the process.

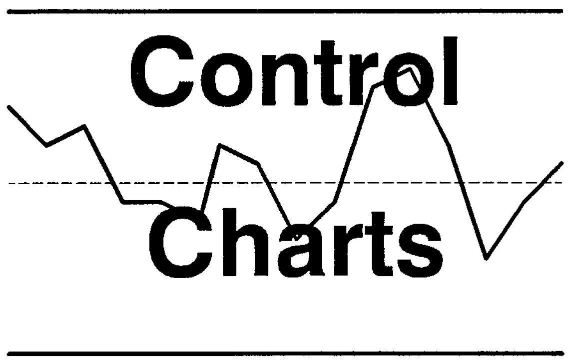
5.

As we should recognize by now, variation of construction materials is inevitable and unavoidable. The purpose of control charts, then, is not to eliminate variability, but to distinguish between the inherent or **chance causes** of variability and a system of **assignable causes**. Chance causes (sometimes known as common causes) are a part of every process, and can be reduced but generally not eliminated. Assignable causes (sometimes known as special causes) are factors that we can eliminate, thereby reducing variability.

6.

Duplicate measurements will not always be identical. Every process has some inherent variability that cannot be eliminated, but can possibly be reduced by changing the process. Due to this variability, individual results can not be accurately predicted. However, groups from a constant process or system tend to be predictable.

NOTES



Variation

- Chance Causes
- Assignable Causes

Chance Causes

1. *Everything* varies
2. *Individuals* are <u>un</u>predictable
3. *Groups* from a <u>constant</u> system tend to be predictable

7.

For example, it is not possible to predict how long an individual will live. However, insurance companies have actuarial tables that predict with relatively high accuracy what percentage of the population will live to various ages. This, indeed, is the basis for establishing insurance premiums. We can do the same thing with a construction materials process, provided the process is in control.

8.

Chance causes are something that a contractor or material supplier must learn to live with. They cannot be eliminated, but it may be possible to reduce their effects. The second cause of variation, assignable causes, can create major problems. However, **assignable causes *can*** be eliminated **IF** we can identify them. Examples of assignable causes might be when the gradation for an aggregate blend goes out of specification due to a hole in one of the sieves or because the cold feed conveyor setting is incorrectly adjusted.

9.

Control charts have been used for years in the manufacturing industry and have been successfully employed in construction materials applications. Some of the benefits that have been attributed to control charts are presented in the slide at right.

Example: Chance Causes

1. people live to different ages
2. no one knows how long he or she will live
3. insurance companies can predict the percentage of people who will live to certain ages

Assignable Causes

- can be eliminated
- IF we can identify them

Benefits of Control Charts

- early detection of trouble
- decrease variability
- establish process capability
- reduce price adjustment costs
- decrease inspection frequency
- basis for altering spec limits
- permanent record of quality
- provide a basis for acceptance
- instill quality awareness

NOTES

10.

There are many different ways to chart data (e.g., test results). One of the simplest is known as a run chart or trend chart. This chart is a plot of the data in time sequence. The intent is to identify trends in the data. In a construction materials situation, the run chart can be plotted with the specification limits indicated. In this way, it is easy to identify test results that are outside the specification requirements.

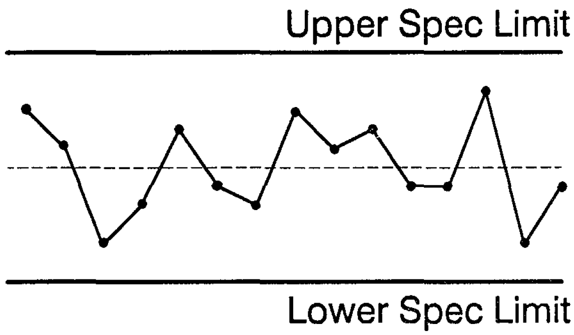
11.

These simple run charts are usually based on a sample size of n = 1, and therefore fail to consider variability within the sample. **These charts do not provide the capability to distinguish *chance causes* from *assignable causes*.** To do this, we must use **statistical control charts**.

12.

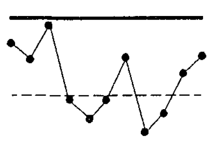
Many different types of statistical control charts are available. We will concentrate on two that are most useful for construction materials. These are the **control chart for means** ($\overline{X}$, called X-bar Chart) and the **control chart for ranges** (R Chart).

Run Chart

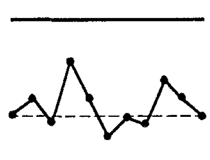


Statistical Control Charts

Control Chart for Means



Control Chart for Ranges



NOTES

13.

The key element in the use of statistical control charts is the proper designation of the **control limits** that are set for a given process. These control limits are not necessarily the same as the tolerance or specification limits. The control limits are established by the process capabilities (i.e., by the variability of the process). If the control limits are not within the allowable specification tolerance limits, then it will be necessary to modify the process to reduce variability to bring the control limits within the specification limits.

14.

Statistical control charts for means rely on the fact that, for a normal distribution, essentially all of the values fall within ± 3 standard deviations from the mean. **The normal distribution can be used because the distribution of sample means is normally distributed.**

15.

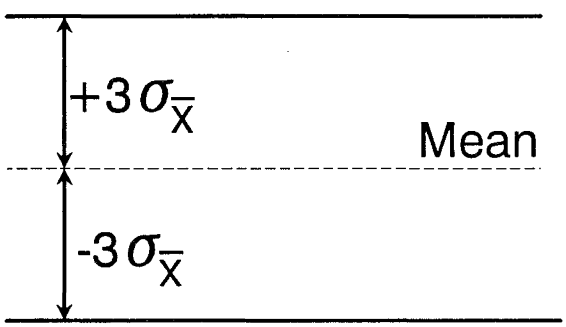
The **range chart** considers the variability of the material and prevents extremely large positive and negative results from canceling out and not being detectable on the control chart for means. The range, which is the easiest measure of spread to use in the field, is usually used in place of the standard deviation.

NOTES

Control

Limits

Control Chart for Means

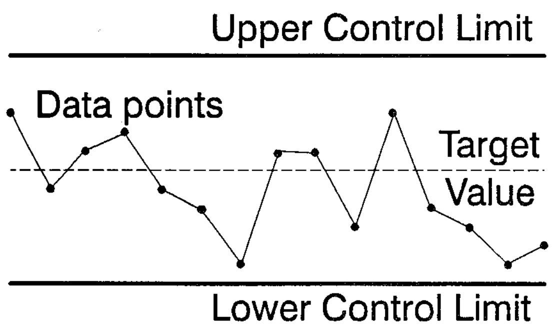


Range Chart

considers variability
of the sample

NOTES

Statistical Control Chart



Population Parameters

1. Known (or specified)

2. Estimated
   - past data
   - early production (20+ values)

Estimating Population Parameters

X_i	$\bar{X}$	R
7 5 9 6 8	7.0	4
4 9 7 3 6	5.8	6
2 7 5 8 7	5.8	6
6 6 9 4 7	6.4	5
3 9 7 4 5	5.6	6
7 8 5 7 6	6.6	3
	37.2	**30**

$\bar{\bar{X}} = 37.2/6 = 6.2$ $\bar{R} = 30/6 = 5$

19.

A **statistical control chart** can be viewed as a normal distribution curve on its side. For a normal curve, only about 0.27% (1 out of 370) of the measurements should fall outside ± 3 standard deviations from the mean. Therefore, **control limits** (indicating that an investigation for an assignable cause should be conducted) are set at $+3\sigma_{\bar{X}}$ and $-3\sigma_{\bar{X}}$.

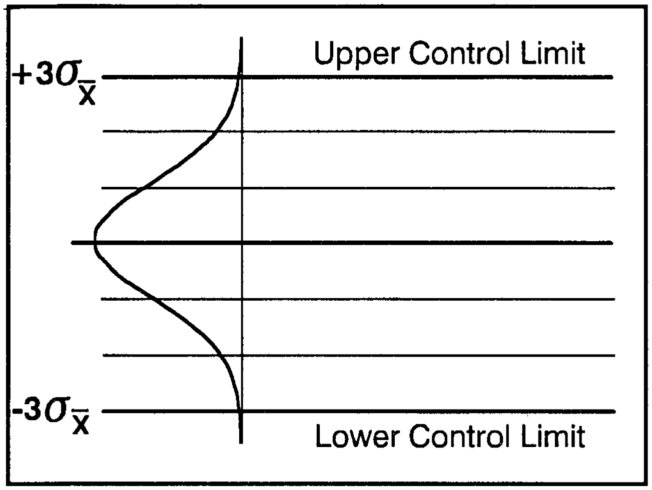
20.

Statistical control charts are used to determine **when** an assignable cause is acting to change the process. For example, action is taken (defined as attempting to identify the assignable cause of the change in the mean or variability) when one point falls outside the control limits. Another cause for action is the occurrence of 8 points in a row on either side of the target line.

21.

Several formulas are used to determine the control limits for statistical control charts for averages and ranges. These equations are different whether the population parameters (mean and standard deviation) are known or estimated. Since the population parameters are rarely, if ever, known, we will consider only the case where the mean and standard deviation are estimated.

NOTES



Look for Assignable Cause if:

- one point is outside of control limits
- eight consecutive points are on one side of the target value

Control Chart Equations

1. Mean & Standard Deviation Known

2. Mean & Standard Deviation Estimated

22.

When the mean and standard deviation are not known, they are estimated (slide at right) by the **grand mean** and the **average range**. For the $\overline{X}$ chart, the grand mean becomes the target value and the control limits are calculated as shown.

23.

The factor A_2 depends upon the sample size, and is given in the table at right. A_2 incorporates the following items into a single factor to establish the control limits:

- a factor to estimate the standard deviation, σ, from the range, R
- division by $\sqrt{n}$ to account for $\sigma_{\overline{X}} = (\sigma / \sqrt{n})$
- multiplication by 3 to account for 3 standard deviations from the mean.

Thus, $\overline{\overline{X}} \pm (A_2 \times \overline{R})$ is an estimate for $\overline{\overline{X}} \pm 3(\sigma / \sqrt{n})$.

24.

For the R chart, the target value is the average range and the control limits are calculated as shown. The control limits for R charts are defined as $\overline{R} \pm 3\sigma_R$. However, no simple formula gives either the expected average range, $\overline{R}$, or the standard deviation of the range, σ_R. These can however be estimated using the factors D3 and D4. Using these factors, the 3-sigma limits can be calculated from an observed $\overline{R}$ as

$$\overline{R} + 3\sigma_R = D_4\overline{R}$$

$$\overline{R} - 3\sigma_R = D_3\overline{R}$$

NOTES

$\overline{X}$ Chart

Target Value = $\overline{\overline{X}}$

$$UCL = \overline{\overline{X}} + (A_2 x \overline{R})$$

$$LCL = \overline{\overline{X}} - (A_2 x \overline{R})$$

Factors for Control Charts

n	A_2
2	1.88
3	1.02
4	0.73
5	0.58
6	0.48
7	0.42

R Chart

Target Value = $\overline{R}$

$$UCL = D_4 x \overline{R}$$

$$LCL = D_3 x \overline{R}$$

25.

The factors $\mathbf{D_3}$ and $\mathbf{D_4}$ depend on sample size, and are given in the table at right.

26.

Statistical control charts are **always** based on the average and range (or σ) of a subgroup of size $n > 1$. There are several reasons for this. One of the main reasons is that the distribution of sample means tends to be normally distributed. Therefore, even if the underlying population from which the samples are taken is not normal, the distribution of sample means will be approximately normal. This allows the use of 3σ control limits to identify when the process is out of control. Secondly, the use of $n > 1$ is necessary to allow the calculation of ranges for the R charts.

27.

In determining subgroups and subgroup size, the values comprising the subgroup should be as homogeneous as possible (i.e., from the same population). A common approach is to take all subgroup samples from the same lot such as one day's production.

NOTES

Factors for Control Charts

n	A_2	D_3	D_4
2	1.88	0	3.27
3	1.02	0	2.58
4	0.73	0	2.28
5	0.58	0	2.12
6	0.48	0	2.00
7	0.42	0.08	1.92

Statistical Control Charts

n > 1

ALWAYS

Subgroups

Values *within* subgroups *must* be logically related

28.

Example: Control Charts when Mean and Standard Deviation are Unknown

The data on the next page can be used to illustrate the calculation for a control chart when the population parameters are unknown and are estimated from the early production process. The table contains the gradation results for percent passing the #4 sieve for 40 production days (4 tests per day). We can use the average and range of the first 20 subgroups to estimate the mean and standard deviation of the population. When this is done

$$\overline{\overline{X}} = \frac{18.4 + 18.0 + \cdots + 18.4 + 16.4}{20} = \frac{365.9}{20} = 18.3$$

$$\overline{R} = \frac{2.1 + 4.1 + \cdots + 4.7 + 3.9}{20} = \frac{83.6}{20} = 4.2$$

Having found these values, the upper and lower control limits can be calculated from the formulas in ***slides 22 and 24*** and the table in ***slide 25***:

$\overline{X}$ Chart

$$UCL = \overline{\overline{X}} + (A_2 \times \overline{R}) = 18.3 + (0.73 \times 4.2) = 21.4$$

$$LCL = \overline{\overline{X}} - (A_2 \times \overline{R}) = 18.3 - (0.73 \times 4.2) = 15.2$$

$$\text{Target Value} = \overline{\overline{X}} = 18.3$$

R Chart

$$UCL = D_4 \times \overline{R} = 2.28 \times 4.2 = 9.6$$

$$LCL = D_3 \times \overline{R} = 0.0 \times 4.2 = 0.0$$

$$\text{Target Value} = \overline{R} = 4.2$$

Once the target value and control limits are established, the control charts can be constructed using the data in the table on the next page.

Data for Demonstration Example

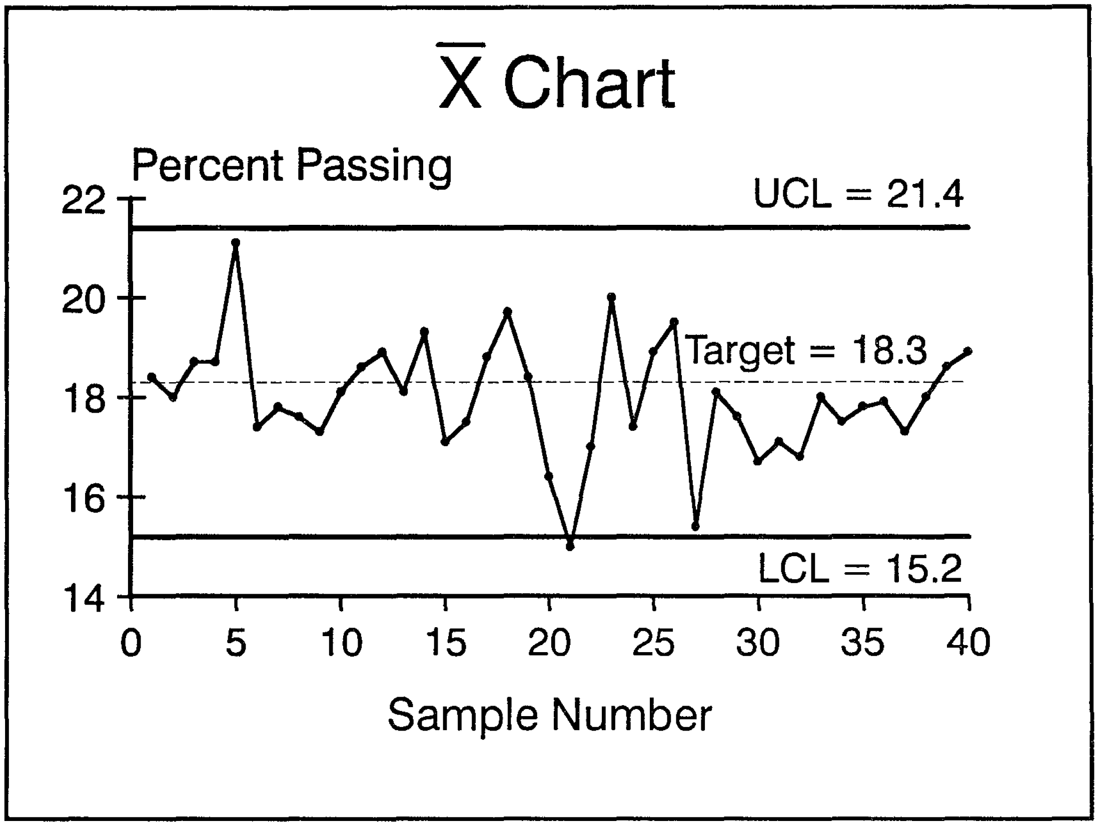
No.	X_1	X_2	X_3	X_4	$\overline{X}$	R
1	18.9	18.2	19.3	17.2	18.4	2.1
2	18.2	16.3	17.2	20.4	18.0	4.1
3	18.5	19.5	17.8	19.1	18.7	1.7
4	19.7	17.6	18.3	19.2	18.7	2.1
5	23.5	22.5	14.9	23.6	21.1	8.7
6	16.6	16.9	17.4	18.8	17.4	2.2
7	19.0	17.9	15.8	18.4	17.8	3.2
8	14.5	17.7	18.0	20.1	17.6	5.6
9	18.5	16.3	17.3	17.2	17.3	2.2
10	15.2	20.4	20.4	16.4	18.1	5.2
11	19.5	16.4	20.7	17.7	18.6	4.3
12	17.4	17.9	17.7	22.4	18.9	5.0
13	15.6	18.1	18.7	19.8	18.1	4.2
14	22.2	17.1	16.7	21.2	19.3	5.5
15	20.1	12.8	18.5	17.0	17.1	7.3
16	19.6	18.0	17.4	14.8	17.5	4.8
17	19.5	19.9	20.1	15.7	18.8	4.4
18	19.9	20.7	19.8	18.3	19.7	2.4
19	20.9	19.9	16.5	16.2	18.4	4.7
20	14.2	18.1	17.1	16.2	16.4	3.9
21	16.7	13.6	11.4	18.4	15.0	7.0
22	18.7	17.3	16.1	15.8	17.0	2.9
23	22.7	18.3	23.8	15.3	20.0	8.5
24	17.3	18.2	16.8	17.2	17.4	1.4
25	20.8	16.7	16.0	22.1	18.9	6.1
26	17.5	21.3	19.1	20.2	19.5	3.8
27	13.6	16.8	19.2	12.1	15.4	7.1
28	19.5	18.3	16.5	18.1	18.1	3.0
29	17.7	18.5	17.4	16.9	17.6	1.6
30	16.4	17.5	15.2	17.8	16.7	2.6
31	15.3	14.5	17.3	21.2	17.1	6.7
32	13.7	18.4	16.1	19.1	16.8	5.4
33	13.4	20.3	18.8	19.5	18.0	6.9
34	14.6	21.9	18.5	14.9	17.5	7.3
35	16.0	20.4	14.7	20.0	17.8	5.7
36	17.2	18.5	15.8	20.0	17.9	4.2
37	18.8	15.0	20.2	15.2	17.3	5.2
38	21.4	17.7	13.1	19.6	18.0	8.3
39	16.5	18.8	20.0	19.2	18.6	3.5
40	19.4	18.6	15.4	22.0	18.9	6.6

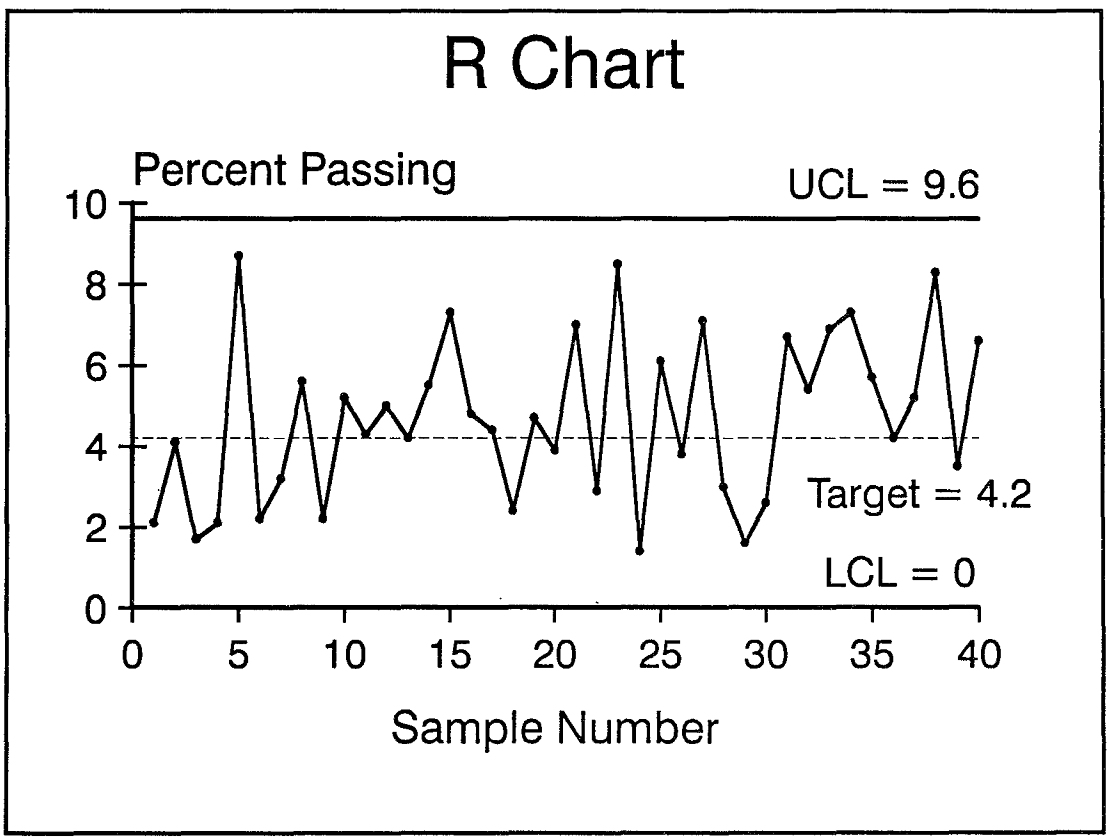
29.

The $\overline{X}$ chart is shown at right. Are there any indications of the process being out of control?

30.

The R chart is shown at right. Are there any indications of the process being out of control?


X̄ Chart
Percent Passing
UCL = 21.4
Target = 18.3
LCL = 15.2
22
20
18
16
14
0
5
10
15
20
25
30
35
40
Sample Number


R Chart
Percent Passing
UCL = 9.6
Target = 4.2
LCL = 0
10
8
6
4
2
0
0
5
10
15
20
25
30
35
40
Sample Number

31.

Now that we can calculate and plot average and range charts, we must address the question of how to interpret them to identify assignable causes.

32.

What do we mean by **lack of control**? A process can be out of control (or lack control) in one of three ways:

- the process mean changes while the process standard deviation remains constant
- the process standard deviation changes while the process mean remains constant
- both the process mean and standard deviation change.

33.

As noted by Grant (1), with the process standard deviation remaining constant, the mean can change, or shift, in several ways.

1. **Sustained sudden shift in mean.**

   This could be indicative of a situation where the aggregate supplier for an asphalt concrete mix is changed during the project.

Interpreting $\bar{X}$ & R Charts

NOTES

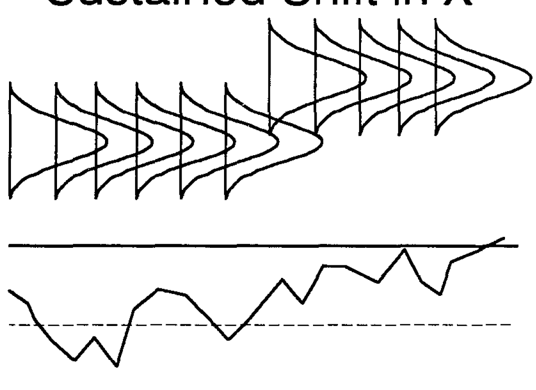
Lack of Control

Change in $\bar{X}$, R constant

Change in R, $\bar{X}$ constant

Change in both $\bar{X}$ & R

Sustained Shift in $\bar{X}$



34.

2. Trend in mean.

This could be indicative of a progressive change brought on by machine wear.

35.

3. Irregular shift in mean.

This could be indicative of the operator making continuous, but unnecessary adjustments to the process settings.

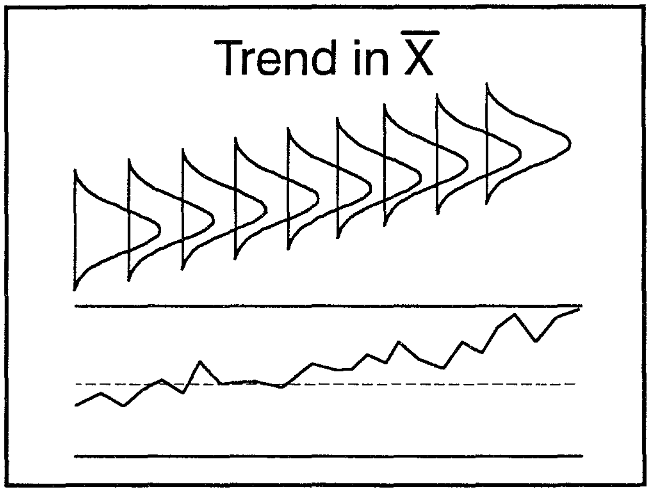
36.

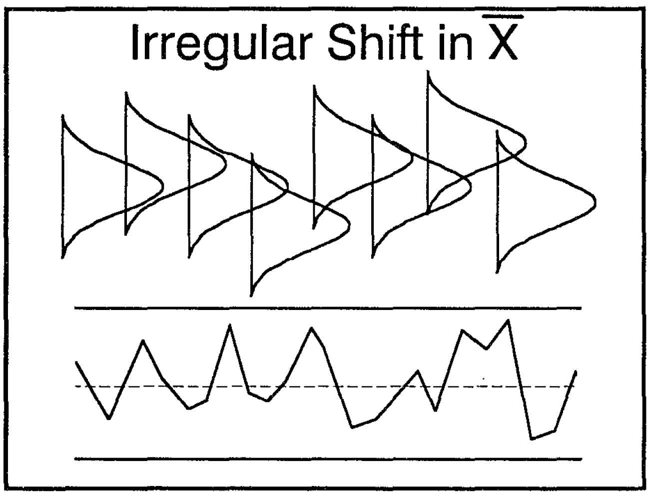
Also, standard deviation (and hence the range) can change in several ways while the mean remains constant. This would generally be related to an increase in R.

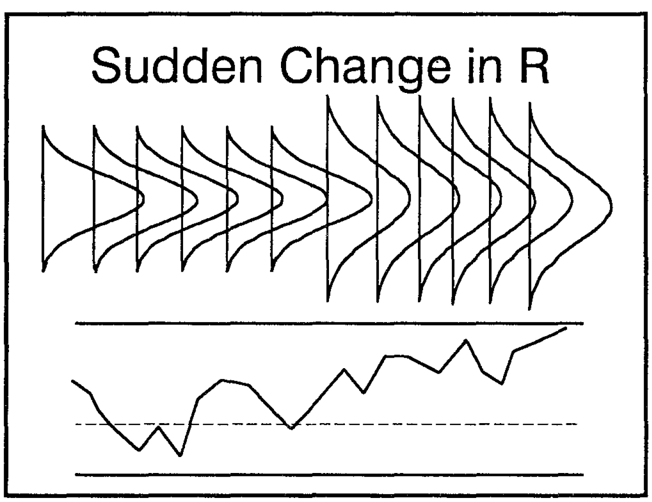
1. A sudden change in range.

This could be indicative of a situation where the aggregate supplier for an asphalt concrete mix is changed during the project.

NOTES


Trend in $\overline{X}$


Irregular Shift in $\overline{X}$


Sudden Change in R

37.

2. A gradual change in Range.

This could be indicative of a progressive change such as machine wear.

38.

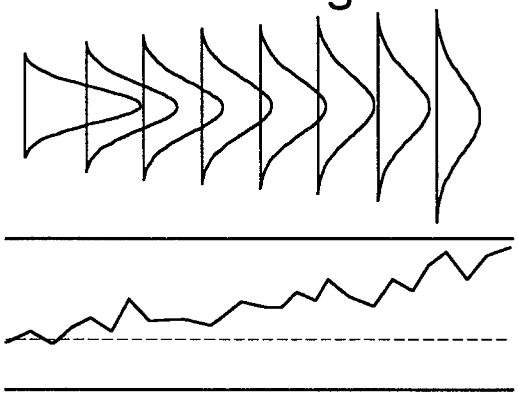
A process could also exhibit irregular shifts in **both** mean and range. This would be a process that was really in trouble!

39.

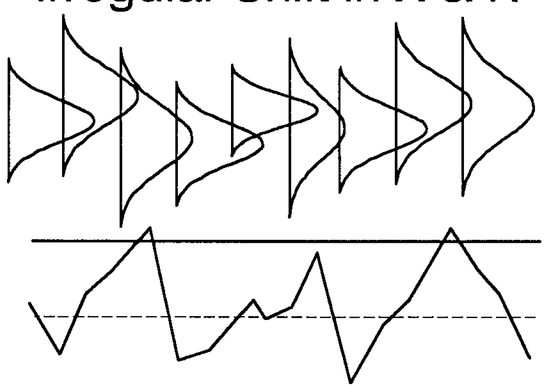
There are numerous rules for interpreting Control Charts. Two that are recommended are shown at right. These were presented earlier in this chapter. A number of other criteria are presented and discussed in references (1) and (2).

NOTES

Gradual Change in R



Irregular Shift in $\bar{X}$ & R



Lack of Control?

One point outside UCL or LCL

Eight points on one side of target value

40.

An intimate knowledge of the process being controlled is vital to the successful use of control charts. A control chart tells **when** to look for possible trouble, but it cannot, by itself, tell **where** to look or **what** assignable cause will be found.

41.

Key points for this chapter are presented in the slide at right.

Workshop Problem

Control Charts for Mean and Standard Deviation Unknown

Data are presented in the table two pages forward for percent passing the 3/8" sieve. The data have been grouped into subgroups of size n = 2. The average and range for each subgroup have been computed. Calculate the control limits, based on the first 15 subgroups, for X-bar and range charts. Using the graph paper following the table, plot the first 30 subgroups on X-bar and range charts. Is the process in control?

References

1. Grant, E. I. and Leavenworth, R. S., *Statistical Quality Control, 5th Edition*, McGraw-Hill Book Company, 1988.
2. *The Memory Jogger*™, GOAL/QPC, Methuen, MA.

Control Charts

Tell WHEN to look, but NOT WHERE to look nor WHAT the assignable cause is

NOTES

Key Points

1. Process control plan.
2. Chance vs assignable causes.
3. Control charts for $\overline{X}$ and R.
4. Control limits.
5. Plotting control charts.
6. Interpreting control charts.

Data for the Workshop Problem

No.	X_1	X_2	$\overline{X}$	R
1	34.6	47.1	40.85	12.5
2	28.0	30.9	29.45	2.9
3	29.4	35.1	32.25	5.7
4	35.8	49.2	42.50	13.4
5	45.9	26.4	36.15	19.5
6	31.4	32.1	31.75	0.7
7	36.7	49.3	43.00	12.6
8	36.3	33.4	34.85	2.9
9	26.3	51.8	39.05	25.5
10	29.1	27.2	28.15	1.9
11	38.5	29.3	33.90	9.2
12	31.5	37.2	34.35	5.7
13	24.1	38.0	31.05	13.9
14	43.2	39.8	41.05	3.4
15	35.0	29.1	32.05	5.9
16	42.8	35.7	39.25	7.1
17	42.0	50.4	46.20	8.4
18	30.2	53.5	41.85	23.3
19	21.6	47.5	34.55	25.9
20	25.1	28.7	26.90	3.6
21	30.4	24.6	27.50	5.8
22	33.4	21.2	27.30	12.2
23	33.4	34.2	33.80	0.8
24	26.5	38.0	32.25	11.5
25	35.0	49.0	42.00	14.0
26	31.3	43.5	37.40	12.2
27	35.2	39.4	37.30	4.2
28	45.1	41.9	43.50	3.2
29	34.1	37.3	35.70	3.2
30	47.6	33.8	40.70	13.8

CHAPTER 8: ACCEPTANCE PLANS AND RISKS

INSTRUCTIONAL OBJECTIVES

1. Introduce and discuss elements of acceptance plans.
2. Define elements needed in acceptance plans.
3. Illustrate buyer's and seller's risks.
4. Illustrate the use of Operating Characteristics (OC) curves.

DESIRED STUDENT ACHIEVEMENTS

1. Understand the elements of acceptance plans.
2. Understand how probability relates to risks.
3. Understand the importance of OC curves.

1.

The functions involved in acceptance of a product include in-process inspection, monitoring of the contractor's or supplier's process control, and acceptance testing. Acceptance testing is very important. It is performed to provide the specifying agency with information that relates the product to the specification limit(s). There are many factors and elements to consider in specification design. One of the principal elements is the **acceptance plan.**

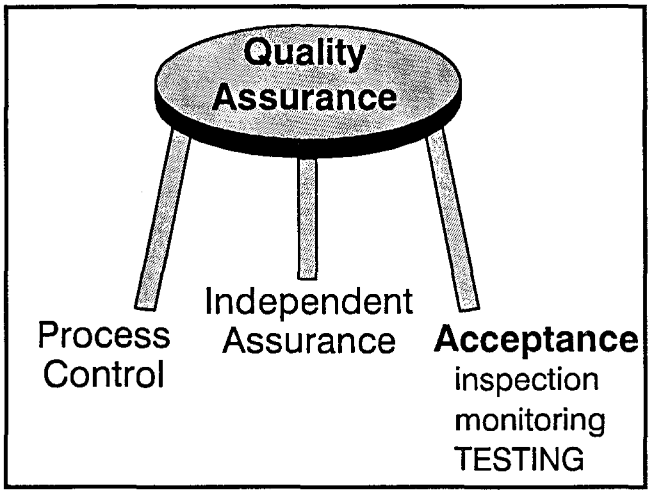
2.

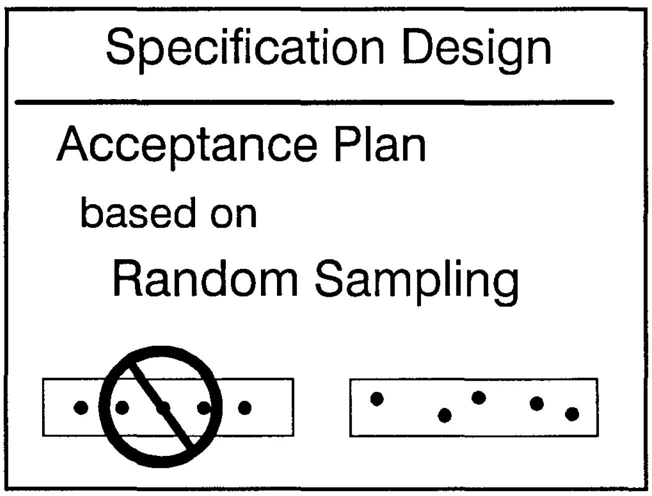
Since it is impractical to test 100% of the product, an appropriate sampling methodology must be developed. If the **sampling** is not done in some **random** fashion, it is likely that bias will enter into the sampling program. The factors discussed below are combined to form an acceptance plan.

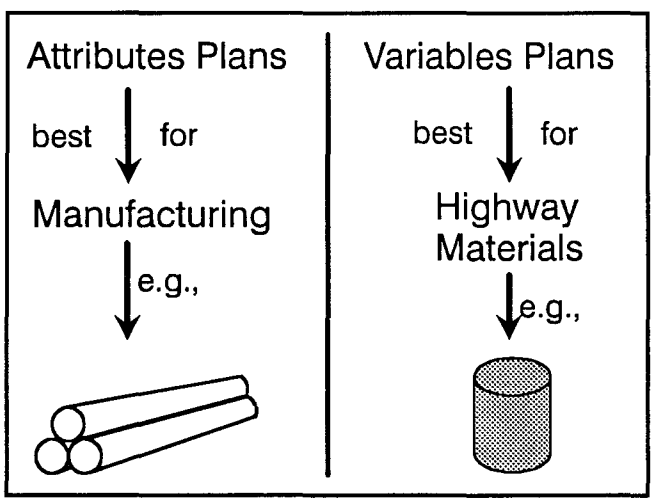
3.

Acceptance plans can be written for either **attributes** or **variables**. In an attributes plan, the sample either **passes or fails** and the lot is either accepted or rejected. In a variables plan, the test characteristics are measured and the values of the measurements are important pieces of information. Attributes plans are most appropriate in manufacturing processes. A specification to accept asphalt coated corrugated pipe might use an attributes plan, for instance. This chapter concentrates on **variables acceptance plans** which are more common to **highway materials** specifications.

NOTES


Quality
Assurance
Process
Control
Independent
Assurance
Acceptance
inspection
monitoring
TESTING


Specification Design
Acceptance Plan
based on
Random Sampling


Attributes Plans
best for
Manufacturing
e.g.,
Variables Plans
best for
Highway
Materials
e.g.,

4.

Before an effective acceptance plan can be written, a determination must be made of the typical variability associated with the process or material. Prior knowledge of the product is essential. Factors such as typical inherent process variability and the level of variability that can be tolerated must be identified. Where can this information be obtained?

5.

Historical records are a potential source of variability data, and are a source that state highway agencies tend to use. However, this can be a dangerous source due to potential unknown bias and lack of random sampling procedures that may have been used when the data were obtained. Most highway construction and materials processes have had studies, such as those presented in Chapter 6, conducted to provide variability data. The results of these studies should be verified as applicable to the particular state before they are used in developing specifications.

6.

A better source of variability data are special studies conducted to determine the needed information. These studies should use random sampling procedures and unbiased reporting of test results. Such a study may be necessary when initially developing the statistically-based acceptance plan, and in cases where new processes are introduced for which no measures of variability have been previously established.

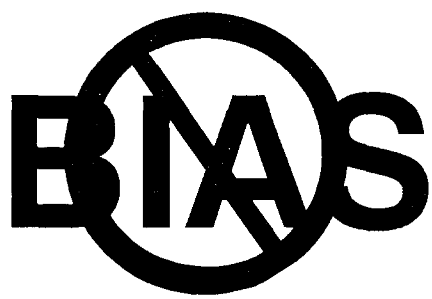
NOTES

Specification Development

Know the product!

What is σ ?

Historical Data



MUST NOT be Biased

Variability Information?

Special Research Studies

- Random sampling
- Unbiased reporting
- New processes

7.

The acceptance plan should consider at least the following factors:

- Method of Test and Point of Sampling
- Lot size
- Sample size
- Acceptance limits
- Risks
- Operating Characteristics (OC) Curve

Analyzing risks and developing an OC curve are ways to describe the effectiveness of a specification.

8.

The **method of test** to judge compliance and the **point of sampling** must be stated in the specification. The method of test must be stated because different methods have different within-test variabilities. This impacts the overall variability and thus the specification limits. While there are often many choices for the point of sampling, **a single point must be specified**. Again, the variability is often influenced by the point of sampling. Both of these elements should be the same as those used when establishing the acceptance limits of the specification.

9.

A **lot** is the amount of product that is to be judged acceptable or unacceptable on the basis of a sample comprised of a stated number of test results. Since the number of specimens in the sample usually remains constant for a lot of a particular product, the determination of the most appropriate lot size is basically an economic decision. If the lot is very large (e.g., an entire project), the cost of rejecting the product or adjusting the payment can have severe negative consequences on the contractor. On the other hand, if the lot is very small (e.g., a load of material), the cost of testing may be more than the benefits provided. Generally, a lot is defined in terms of time, production or area.

NOTES

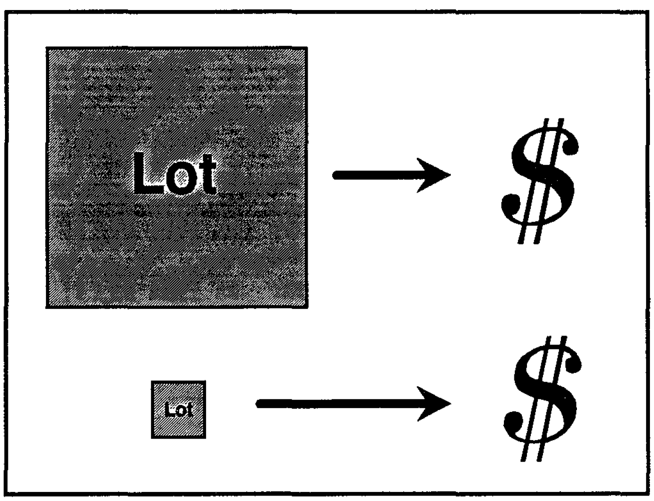
Acceptance Plans

Test method/point of acceptance
Lot size
Sample size
Tolerances
Risks
OC curves

Test Method $\longrightarrow \sigma_e$

Point of Sampling $\longrightarrow \sigma_s$

$$\sigma_o = ?$$



10.

The number of specimens comprising the sample taken to judge the compliance of a lot is often termed the **sample size**. This is not to be confused with the amount of material (size of sample) sampled for testing. The proper sample size is associated with the risk that the specification writer uses in the specification development. In most QA specifications, the sample size ranges from 3 to 5.

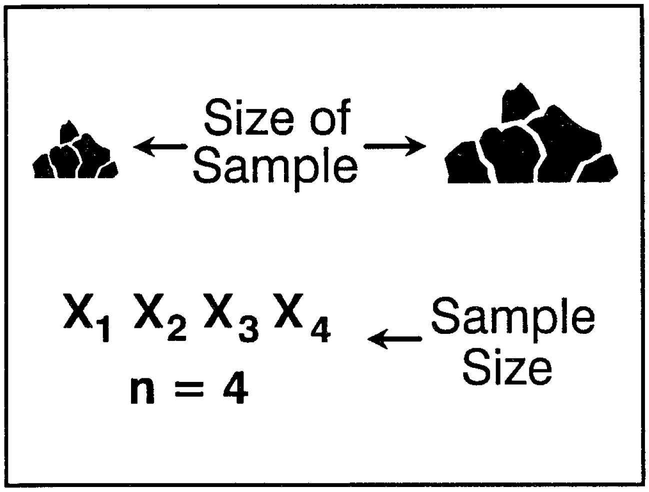
11.

Acceptance limits, which are an important part of an acceptance plan, can be established in several ways. Acceptance plans based on using the mean of the sample are discussed in this chapter to illustrate risk determination. Acceptance plans using both the mean and standard deviation of the sample are discussed in the next chapter.

12.

Establishing limits for the mean requires defining acceptable and unacceptable material. These are both engineering decisions. The first decision defines acceptable material and should address the material that will provide satisfactory service at an affordable cost when used for the intended purpose. What constitutes acceptable material is often determined based on what has performed well in the past. The level at which the material is just considered acceptable is known as the **acceptable quality level (AQL)**. Statistics has been a valuable tool in defining the characteristics (mean and standard deviation) of acceptable material. Caution should be exercised if a lower variability is chosen for the specification than has been shown to be readily achievable. Arbitrarily "tightening the specs" can increase the cost of the material above that which may be cost effective.

NOTES



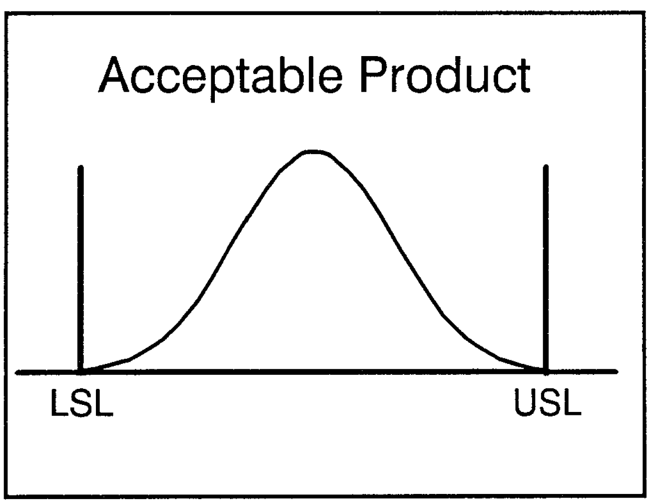
Acceptance Limits

Based on:

1. sample mean

   $LSL < \bar{X} < USL$

2. sample mean & standard deviation



13.

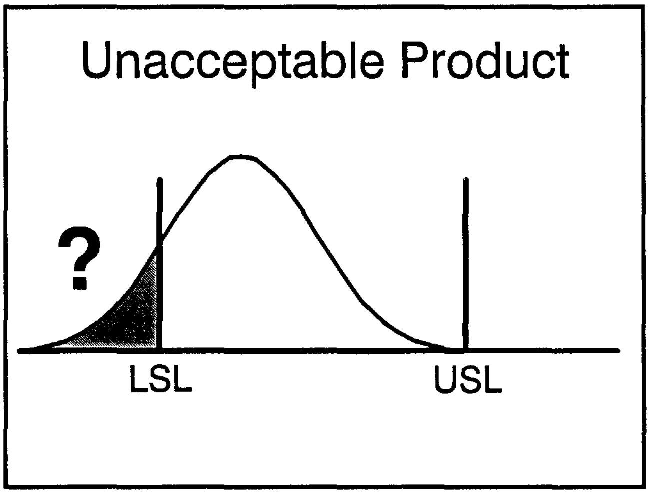
Once acceptable material has been defined, a sometimes more difficult decision must be made regarding what constitutes unacceptable material. Unacceptable material is that which is unlikely to perform as planned. It should have a low probability of being accepted or will be accepted only under the conditions of a reduced payment schedule. The level at which the material is considered unacceptable and requires removal and replacement is known as the **rejectable quality level (RQL)**.

14.

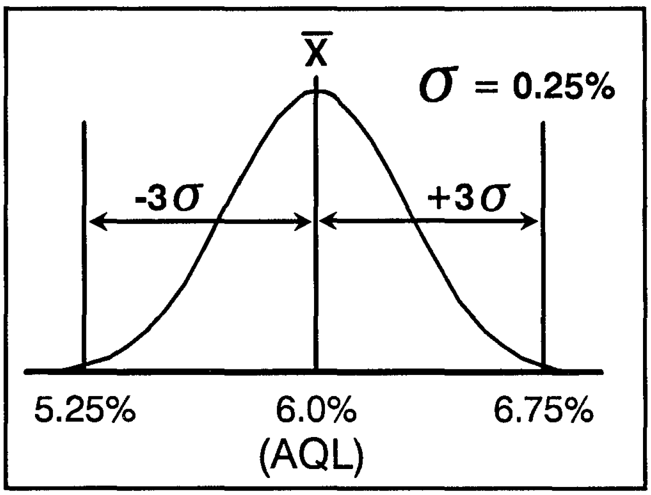
An example of the above definitions and their use may help explain how acceptance limits can be established. While it is preferable to use an acceptance approach which considers both mean and standard deviation, control of only the mean is simpler to understand and is used here as a simplified example. It has been determined that for asphalt content, acceptable material has a standard deviation of about 0.25% (Chapter 6) when the mean is close to the target (job mix) value. If the job mix formula has established the target as 6.0% asphalt content, acceptable material can be defined as shown at right. In this example, the AQL is a lot (population) with a mean of 6.0% and a standard deviation of 0.25%

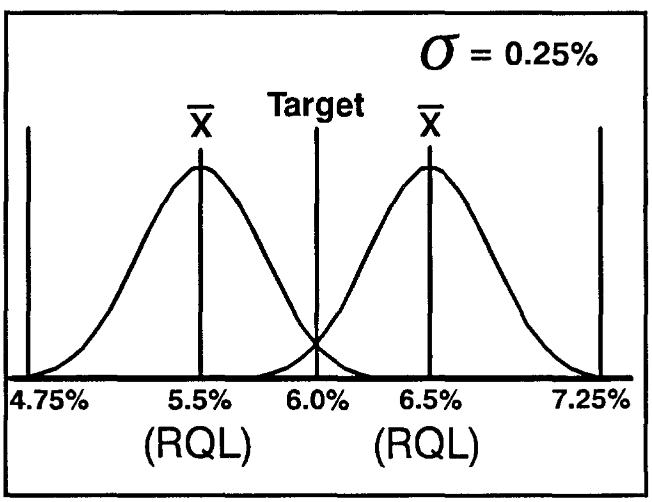
15.

Additionally, unacceptable material might be defined as that for which the mean differs from the target value by 0.5% or more, as long as the standard deviation does not exceed 0.25%. (Other definitions of unacceptable material would be equally valid.) Unacceptable material is then defined as shown at right. The RQL is a lot (population) with a standard deviation of 0.25% and a mean as low as 5.5% or as high as 6.5%.


Unacceptable Product
?
LSL
USL

NOTES


X̄
σ = 0.25%
-3σ
+3σ
5.25%
6.0%
(AQL)
6.75%


σ = 0.25%
X̄
Target
X̄
4.75%
5.5%
(RQL)
6.0%
6.5%
(RQL)
7.25%

16.

One additional and important concept that must be understood in developing acceptance limits is that of probability and how it relates to **risks**. This concept is the primary basis for understanding why QA specifications are so much more powerful than other specifications for deciding whether or not a product meets a specification.

17.

From which population does the indicated sample result come? Remember, the purpose of a sample is to estimate the population. Since we never know from what population a sample actually comes, using the sample estimates involves risks: risks of making the **wrong** decision.

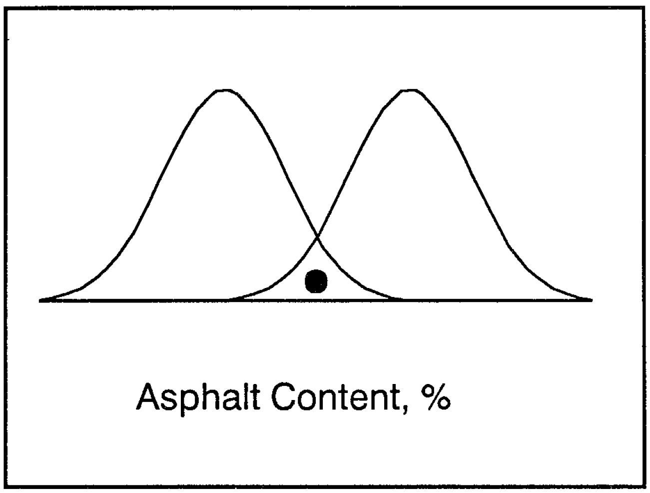
18.

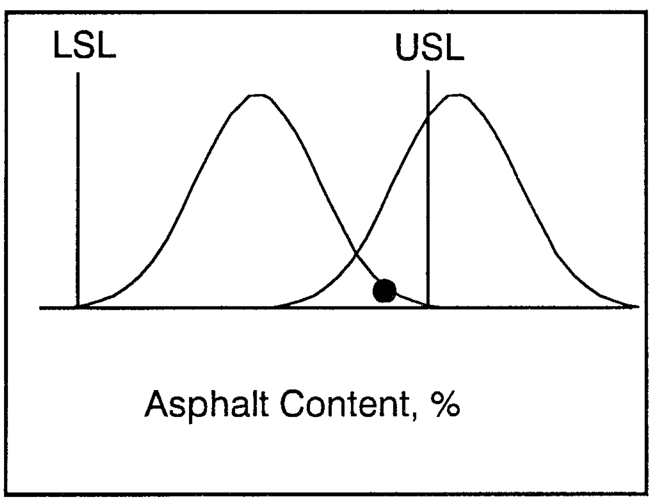
This sample mean result is within the specification limits, but does it represent "good" material? It could come from either of the distributions shown, or from many others.

NOTES

Quantifying Risks -

A primary value of QA Specifications





19.

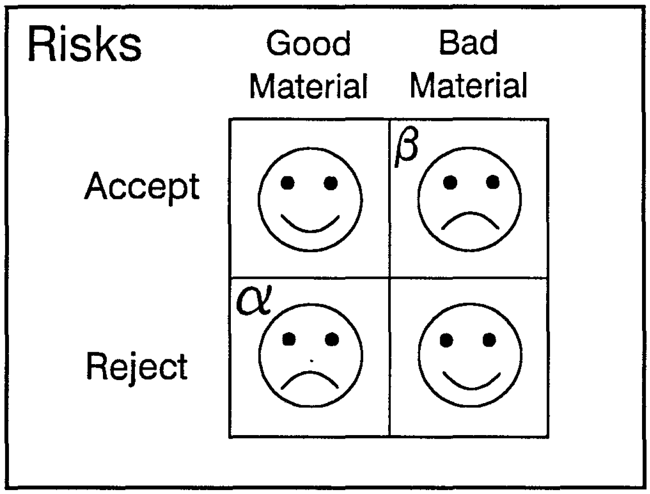
There are two types of risk; seller's (or producer's) risk (α) and buyer's (or agency) risk (β). Seller's risk (α) is the probability of rejecting a lot when the lot is acceptable. Buyer's risk (β) is the probability of accepting a lot when the lot is unacceptable.

20.

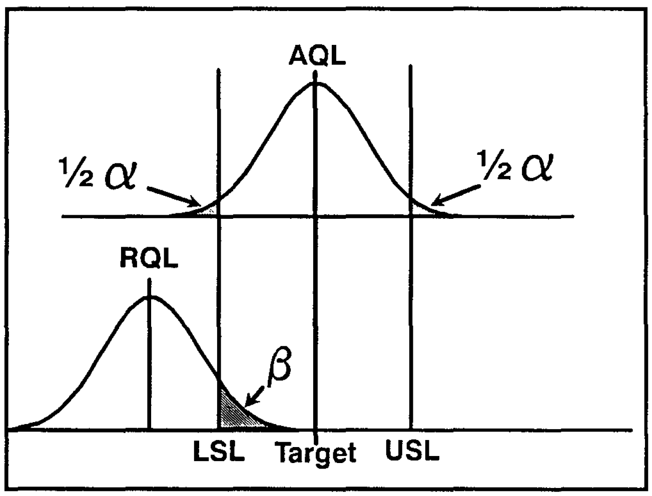
The two types of risks are illustrated at right. The normal distribution on the top represents a lot that is acceptable (the mean is at the AQL), while the normal distribution on the bottom represents a lot that is unacceptable (the mean is at the RQL). If acceptance decisions are based on the results of a single test, then the α risk, the risk that an AQL lot will be rejected, is the probability that a single test result from the AQL lot will be outside of the specification limits. This probability is represented by the sum of the two shaded areas in the top distribution. The β risk, the risk that an RQL lot will be accepted, is the probability that a single test result from the RQL lot will be within the specification limits. This probability is represented by the shaded region in the bottom distribution.

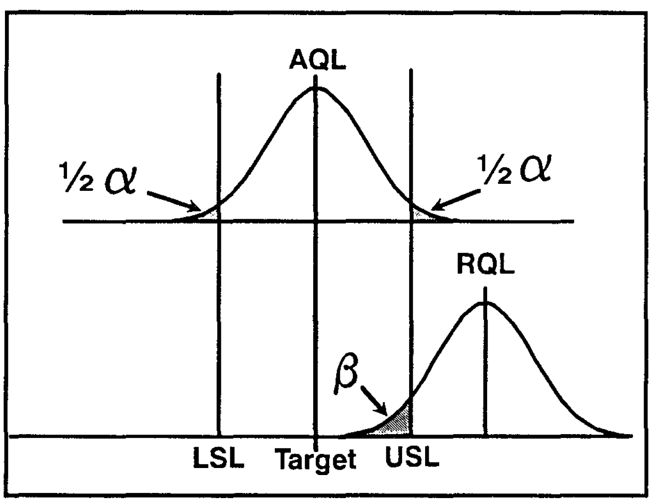
21.

The RQL lot could also be on the high side of the AQL, or target value for the population mean. Since the normal distribution is symmetrical, the case in the slide at right is identical to the one in the previous paragraph where the RQL was on the low side of the AQL.


Risks
Good Material
Bad Material
Accept
Reject
β
α

NOTES


AQL
½ α
½ α
RQL
β
LSL
Target
USL


AQL
½ α
½ α
RQL
β
LSL
Target
USL

22.

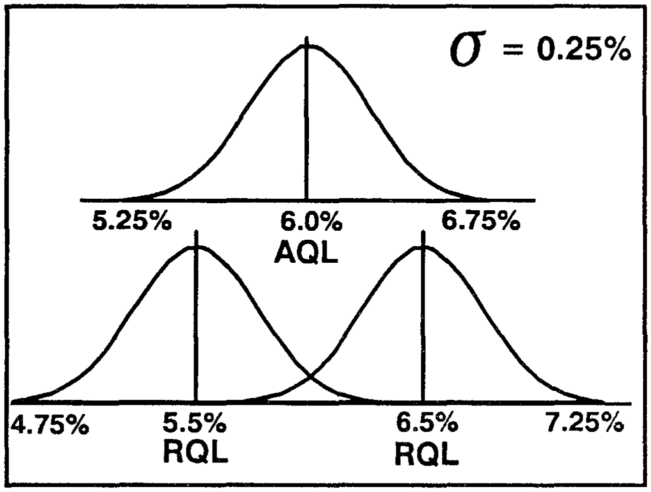
To demonstrate the concept of α and β risks, consider the earlier definitions of an AQL lot (mean = 6.0; standard deviation = 0.25) and an RQL lot (mean $\leq$ 5.5 or mean $\geq$ 6.5; standard deviation = 0.25). If possible, we would like to always accept a lot that is acceptable, but we do not want to accept a lot defined as unacceptable. Where should the specification limits be set, and what sample size should be used to satisfy these conditions?

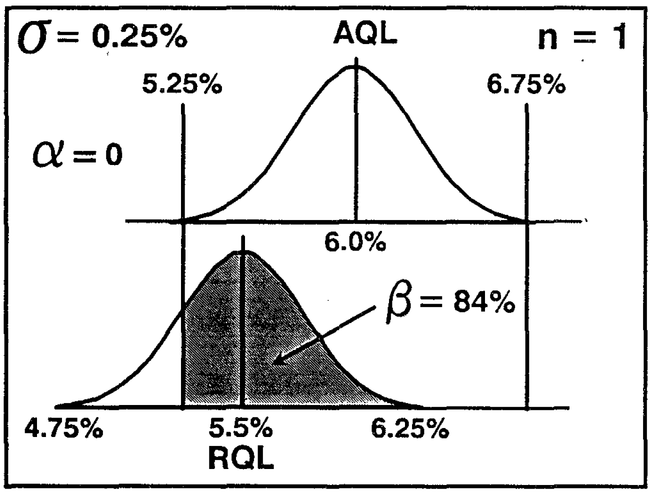
23.

Consider first the situation where acceptance is based on a single test, i.e., sample size n = 1. To ensure essentially a 100% chance of accepting our definition of an AQL lot, the limits would be set at $\pm 3\sigma$ limits, or 5.25% and 6.75%. Thus, α is essentially 0%; one of our objectives. However, β, (the probability of accepting an RQL lot -- with mean $\leq$ 5.5% or mean $\geq$ 6.5%) would be 84% (see table of areas under normal curve). It is obvious that there is only a 16 chance that an RQL lot would correctly be rejected (i.e., 84% probability of incorrectly being accepted). The slide at right shows the probability of accepting a lot with an average of 5.5%. The same probability exists if the average is 6.5%. As mentioned earlier, it is important to try to balance α and β. In this specification an α = 0% and a β = 84% is an imbalance that is not acceptable.

24.

A relationship exists among α, β, and n. For a constant n, if α goes up, β goes down and vice versa.





for
n = constant

as $\alpha\uparrow$, $\beta\downarrow$

or

as $\alpha\downarrow$, $\beta\uparrow$

NOTES

25.

Trying to bring α and β into a better balance without changing n shows how β can be reduced by allowing α to increase. Suppose the limit of ±3σ is tightened to ±2σ limits. What does this do to α and β? With ±2σ limits, α has been increased to about 5% and β has been reduced to 50%. This still is likely to be considered too high a risk of accepting an unacceptable lot. Tightening the limits further to increase α and decrease β, is an option, but not a very realistic one. This essentially does what we do not want to do; it rejects more acceptable lots. Tightening limits beyond those that are considered practical is likely not to be cost effective.

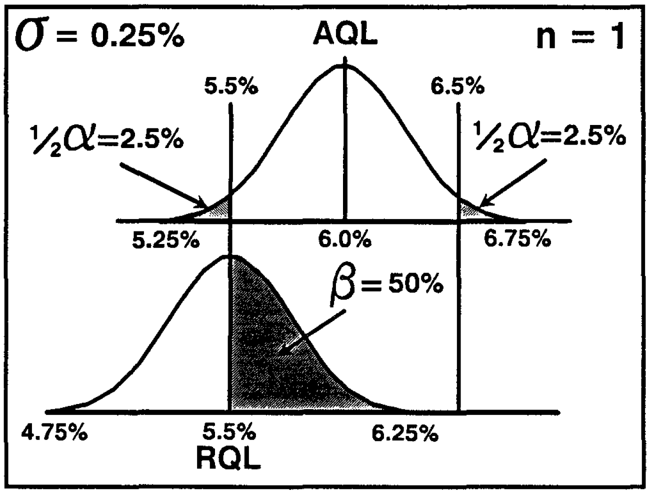
26.

If we want to decrease **both** α and β, we must increase n.

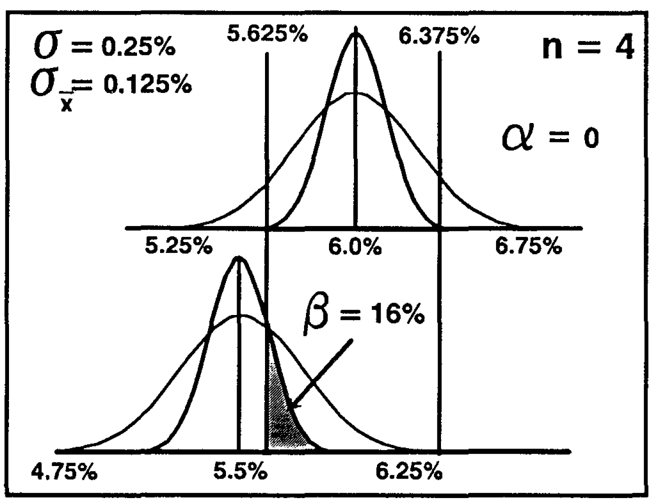
27.

Assume we select n = 4, so that acceptance decisions are based on an average of 4 test results. We know $\sigma_{\overline{X}} = (\sigma / \sqrt{n})$, or for our example, $\sigma_{\overline{X}} = (0.25 / \sqrt{4}) = 0.125\%$. Thus, if we want an α = 0% and we base acceptance on n = 4, the limits become ±0.375% (i.e., $3 \times \sigma_{\overline{X}}$) as shown in the slide at right. By increasing the sample size to n = 4 and setting α = 0%, β is reduced to 16%.

NOTES



for n = constant as $\alpha\uparrow$, $\beta\downarrow$ or as $\alpha\downarrow$, $\beta\uparrow$	for $\alpha\downarrow$ & $\beta\downarrow$ n MUST increase



28.

The buyer's risk can be reduced further using the same concept used earlier. That is, by increasing α to 5% by using $\pm 2\sigma_{\bar{X}}$ limits and by using n = 4, the limits now become 5.75% and 6.25% as shown at right. Under these conditions, β is only 2.5%, very likely an acceptable level for risk of accepting an unacceptable lot.

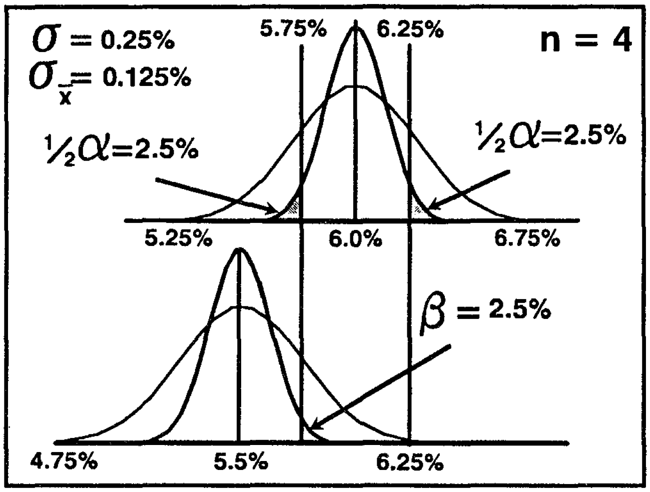
29.

The changes in β as α and n change are shown in the table at right.

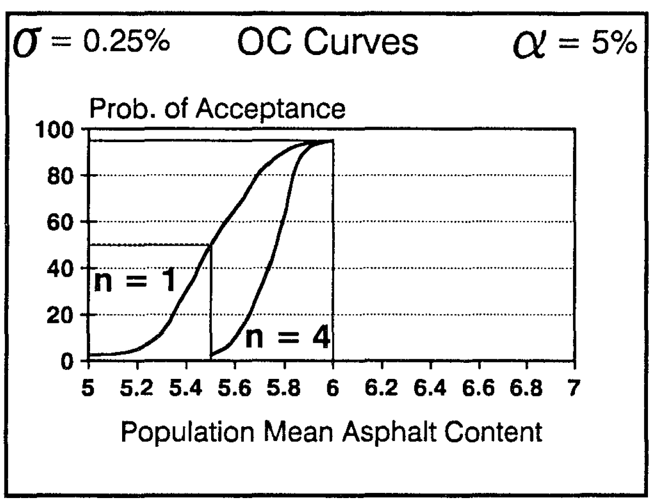
30.

The example α and β calculations above are for lots at the AQL and RQL levels. Similar values for the probability of accepting lots with other population means can also be calculated. The relationship between the lot mean and the probability that the lot will be accepted can be illustrated by the use of **operating characteristics (OC) curves**. For a given standard deviation and α value, and various sample sizes, a set of curves can be drawn to characterize the probability of accepting lots with various population means. On the OC curve, β is indicated as the probability of accepting a lot with a mean at the RQL. While α is 100% minus the probability of accepting a lot with a mean at the AQL. In the figure at right, α was chosen as 5% (0.05) so that when a lot has a mean of 6.0% (i.e., the AQL), the probability of acceptance is 95%. When a lot has a mean of 5.5% (i.e., the RQL), β = 50% for n = 1 and 2.5% for n = 4. Obviously, OC curves can be extremely useful to verify that the risks are balanced when developing a specification.

NOTES



n	Seller's Risk α	Buyer's Risk β
1	0%	84%
1	5%	50%
4	0%	16%
4	5%	2.5%



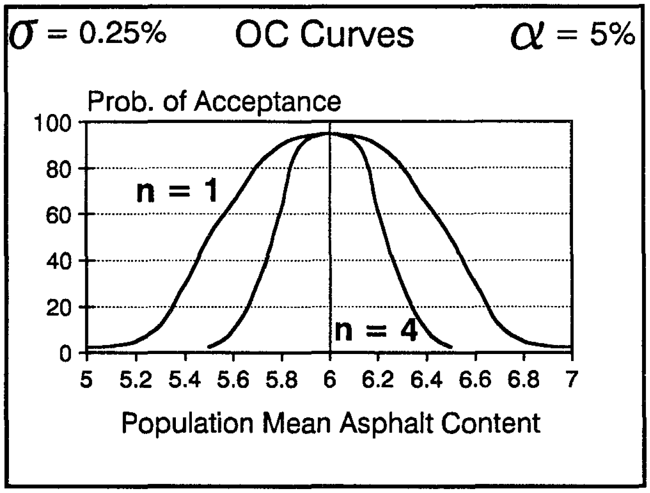
31.

The calculations for this example were done for population mean asphalt contents on the low side of the target value of 6.0%. Similar calculations could be made for population means on the high side of the target. The complete OC curves for α = 5% (0.05) and n=1 and n=4 are shown at right.

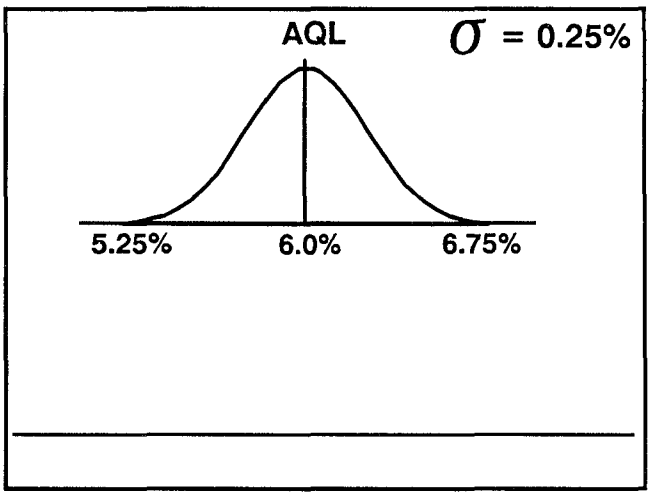
Workshop

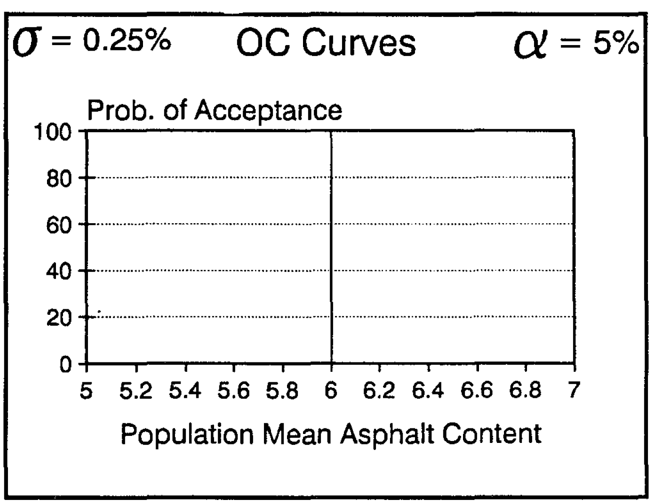
32.

For the OC curve in the previous example, each group of students will develop a point on the curve for a different sample size and population mean. The groups' results will be summarized and plotted to verify the shape of the OC curve.


σ = 0.25%
OC Curves
α = 5%
Prob. of Acceptance
100
80
60
40
20
0
n = 1
n = 4
5 5.2 5.4 5.6 5.8 6 6.2 6.4 6.6 6.8 7
Population Mean Asphalt Content

NOTES


AQL
σ = 0.25%
5.25%
6.0%
6.75%


σ = 0.25%
OC Curves
α = 5%
Prob. of Acceptance
100
80
60
40
20
0
5 5.2 5.4 5.6 5.8 6 6.2 6.4 6.6 6.8 7
Population Mean Asphalt Content

33.

Although the above approach can be used for establishing limits for accepting the sample mean of a lot, it has a serious disadvantage. The standard deviation is assumed to be known and constant. This is often termed a standard deviation known specification. If these acceptance limits are used, and the contractor has a standard deviation other than 0.25%, the risks are no longer those for which the specification was designed. One solution to this problem is to establish a tolerance on the variability and develop a specification to accept the standard deviation. However, another approach which uses an estimate of the percentage of the lot that is within the specification limits to determine acceptability is generally accepted as the best procedure for considering both mean and standard deviation. This method is presented in the next chapter.

34.

Guidelines for selecting α and β risks are presented (2) in AASHTO Standard R-9.

35.

Key points for this chapter are presented in the slide at right

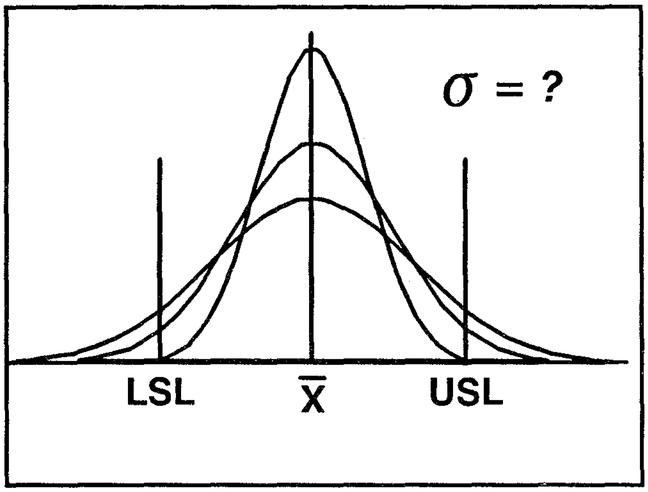
References

1. Smith, Jr. N. L., *Materials Certification and Material Certification Effectiveness, NCHRP Synthesis 102*, Washington, DC, 1983.

2. AASHTO Materials, Part I Specification; *R-9 Acceptance Sampling Plans for Highway Construction*, 1984.

3. Weed, R. M., Statistical *Specification Development, 2nd Edition, FHWA/NJ-88/017*, Federal Highway Administration, Washington, DC, 1989.

NOTES



Suggested Risks - AASHTO

Criticality	α	β
Critical	0.050	0.005
Major	0.010	0.050
Minor	0.005	0.100
Contractual	0.001	0.200

Key Points

1. Acceptance plans.
2. Probability and risks.
3. OC Curves.

CHAPTER 9: PWL ACCEPTANCE PLANS

INSTRUCTIONAL OBJECTIVES

1. Discuss disadvantages of acceptance based on only the sample mean.

2. Discuss acceptance based on percent within limits (PWL).

3. Introduce the term Quality Index.

4. Illustrate how to estimate PWL for a lot.

DESIRED STUDENT ACHIEVEMENTS

1. Understand the disadvantages of acceptance based on only control of the mean.

2. Understand the concept of acceptance based on percent within limits (PWL).

3. Understand how to estimate PWL using the Quality Index.

CHAPTER 9: PWL ACCEPTANCE PLANS

1.

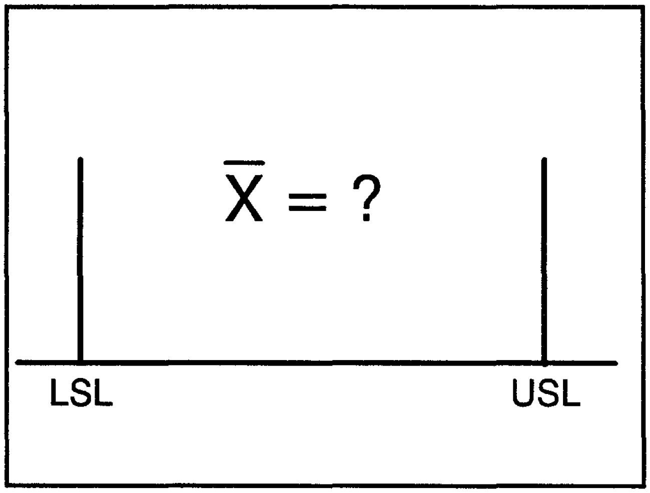
In the last chapter the concept of developing acceptance limits and using test results to determine the acceptability of a lot was presented. The acceptance approach presented was based on considering only the sample mean in the acceptance decision. There are several disadvantages to this simplified approach. These are discussed in this chapter, and a better approach which incorporates both the sample mean and standard deviation is presented.

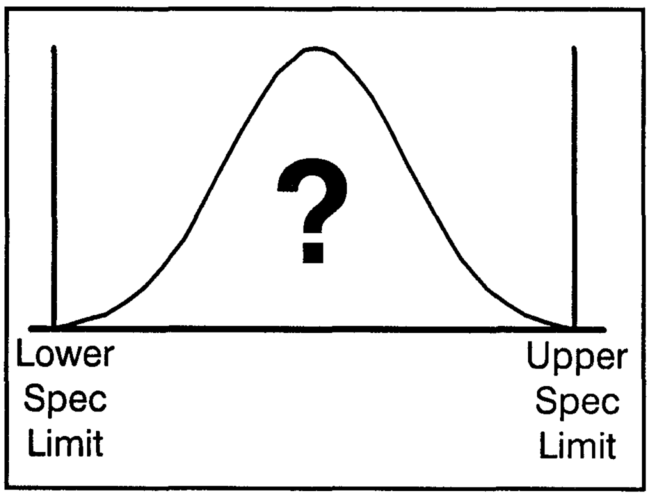
2.

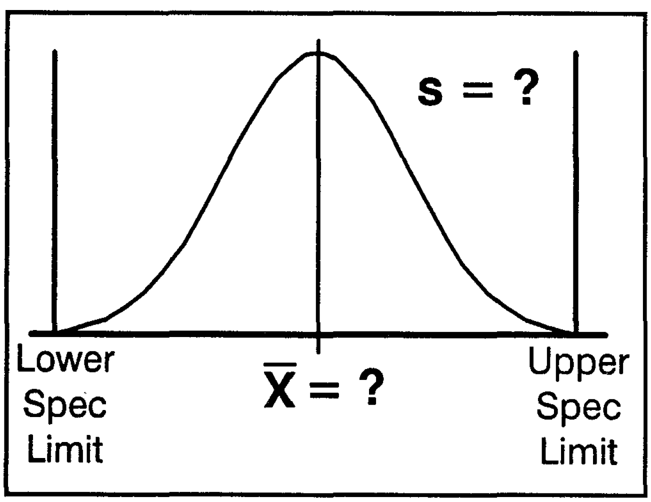
The specifying agency must determine the relationship between the lot (population) and the specification limit(s). Information regarding this relationship can be obtained from the acceptance testing process.

3.

Two sample statistics -- the **mean** and the **standard deviation** -- are needed to estimate the lot (population).


$\bar{X}$ = ?
LSL
USL


?
Lower Spec Limit
Upper Spec Limit


s = ?
Lower Spec Limit
$\bar{X}$ = ?
Upper Spec Limit

NOTES

4.

Comparing individual sample values to the specification limit without accumulating the results can lead to erroneous conclusions about the population. For example, suppose the lower specification limit for asphalt content is 5.8%.

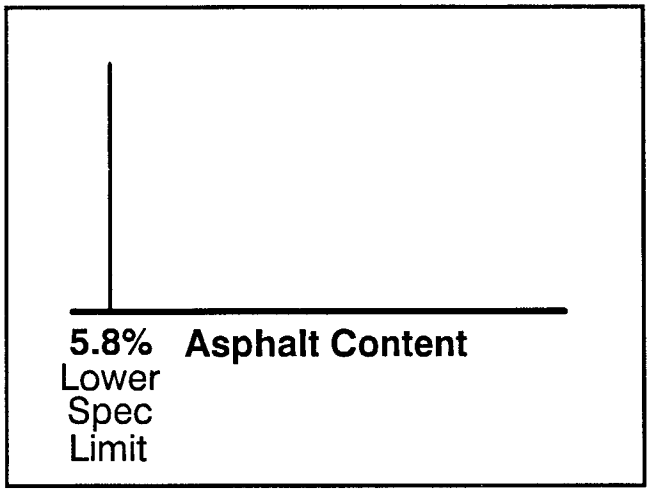
5.

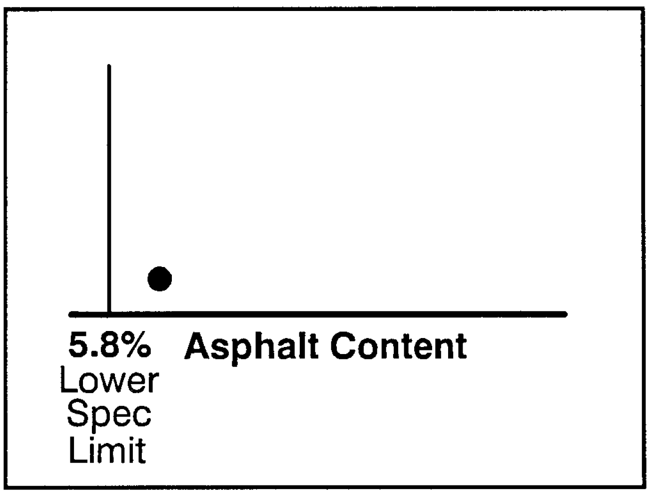
The first asphalt content test result is 5.9%. Since this result meets the specification requirement, the test passes and the result is filed away and forgotten.

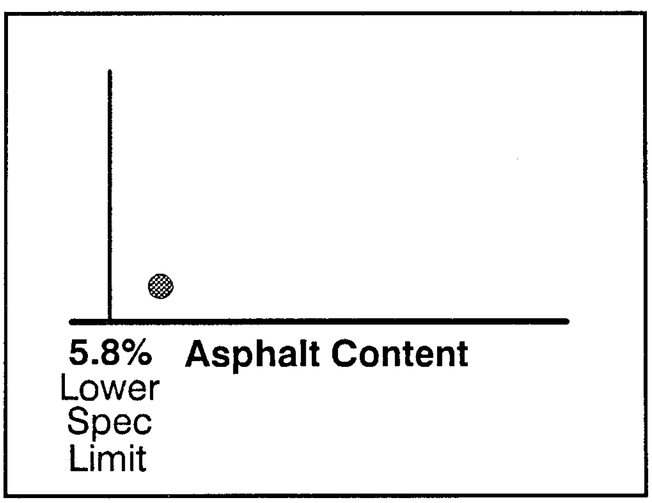
6.

The second asphalt content test result is also 5.9%. This also meets the specification limit so the test passes and the results are once again filed and forgotten.

NOTES


5.8%
Asphalt Content
Lower
Spec
Limit


5.8%
Asphalt Content
Lower
Spec
Limit


5.8%
Asphalt Content
Lower
Spec
Limit

7.

The third test result is also 5.9%. Admittedly, this would be a very unusual condition. However, the point to be made is that once a test passes, it is often recorded and forgotten. If this happens, a variable specification effectively becomes an attribute specification and the ability to estimate the population from the sample is greatly reduced.

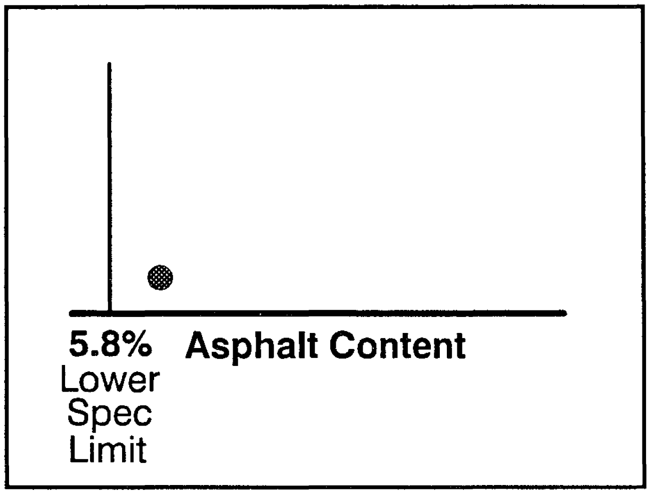
8.

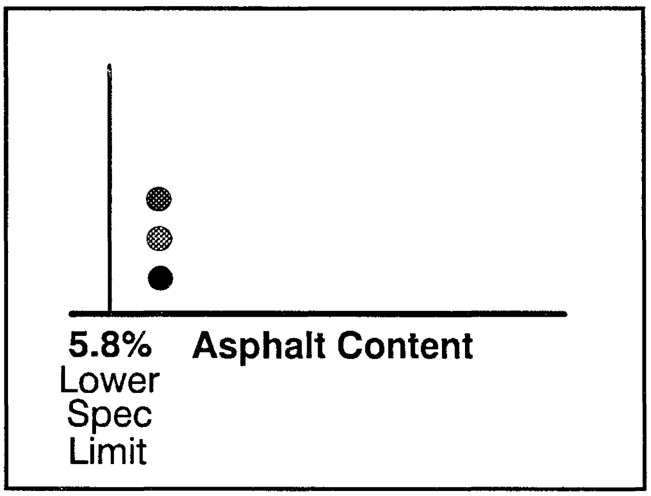
Unless these three results are considered collectively, it is likely that the conclusion concerning the population would be that it is within the specifications.

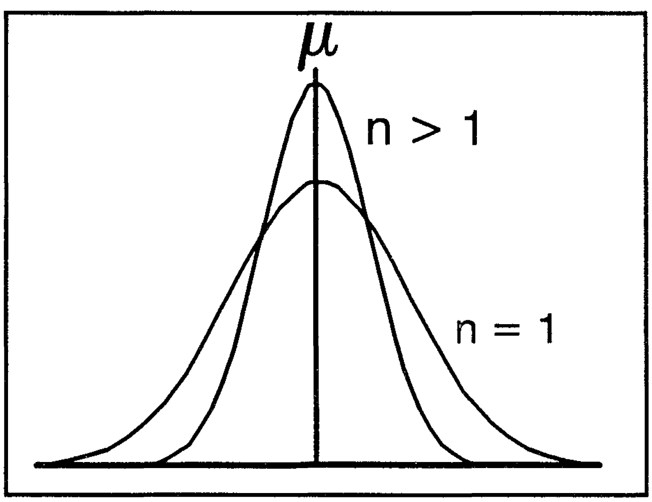
9.

The most accurate estimate of the lot (population) mean is the sample mean. It has already been shown that means of samples of size $n > 1$ vary less than individuals (i.e., samples of size $n = 1$). Chapter 8 discusses the influence of sample size on the sample means and the establishment of specification limits.

NOTES


5.8%
Asphalt Content
Lower
Spec
Limit


5.8%
Asphalt Content
Lower
Spec
Limit


μ
n > 1
n = 1

10.

Chapter 4 discusses the lack of efficiency of using the range to estimate the standard deviation. Therefore, the sample standard deviation is the best estimate of the population standard deviation.

11.

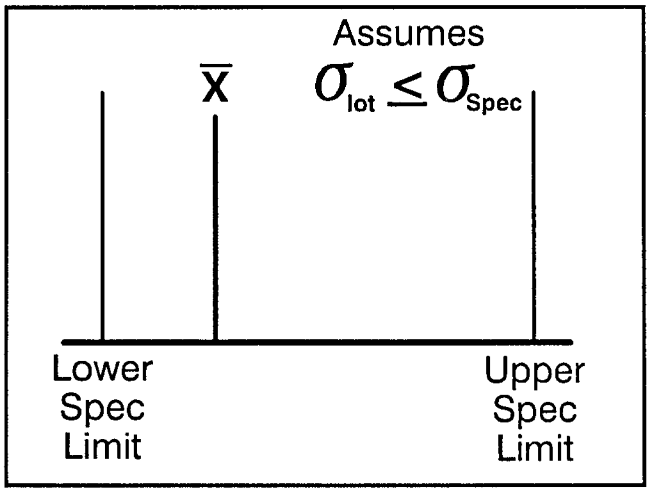
As in the example presented in Chapter 8, some specifications use only the sample mean to estimate the lot mean and assume that the lot standard deviation is equal to or less than the standard deviation used to establish the specification limits. These specifications are called **standard deviation (or variability) known specifications**. It is rare in highway projects that the standard deviation is really known. Standard deviation known specifications have been used primarily because they are simple to understand and to employ.

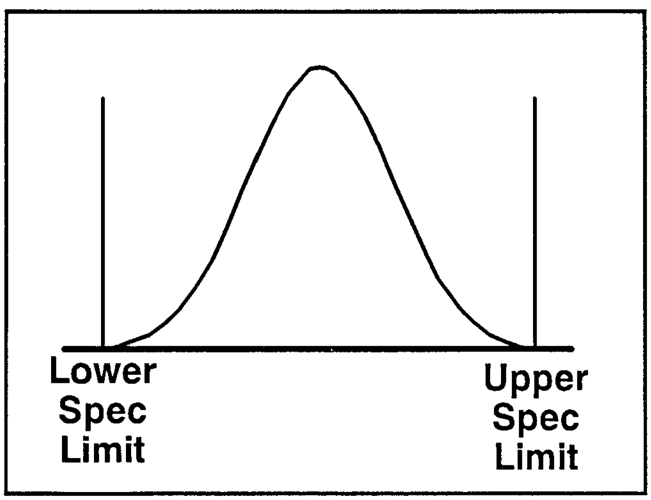
12.

If a contractor is operating with a standard deviation equal to that used in the development of the specification limits, controlling the mean works in the way the specification was designed.

NOTES

$$s \cong \sigma$$





13.

However, there is no incentive for the contractor to reduce the process standard deviation below that used in developing the specification. For example, if the process standard deviation were less than that used to establish the specification limits, the process mean could be closer to the limit without violating the specification requirements. This fact is not acknowledged if control of the mean is based solely on limits established from an assumed "known" population standard deviation.

In fact, the incentive is for the contractor to balance a low result with a high result (or vice versa) to bring the average within the specification limits. This effectively increases the standard deviation and produces a bimodal distribution.

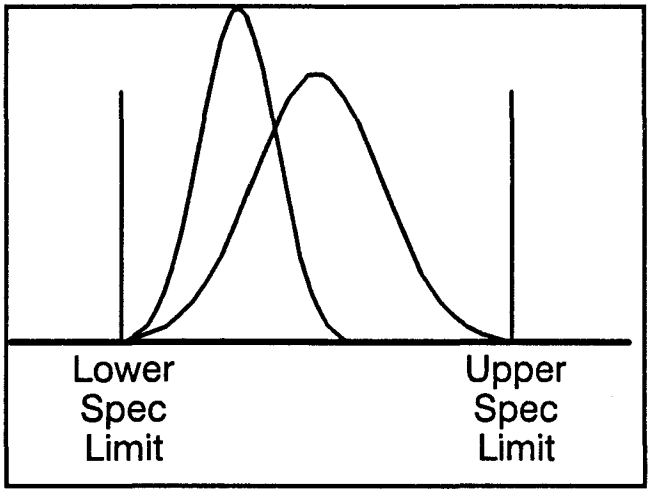
14.

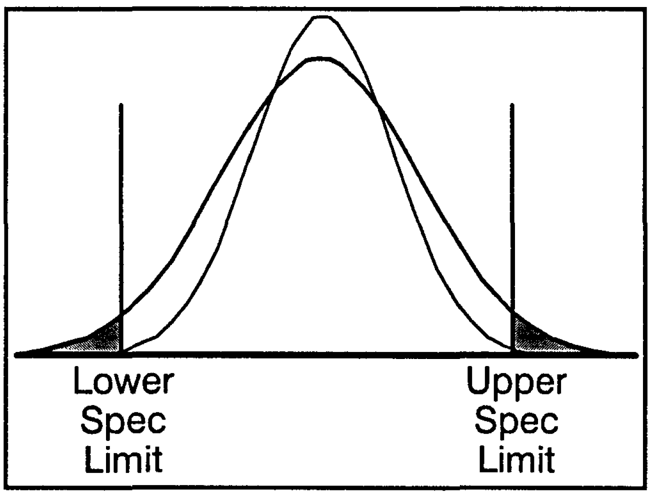
On the other hand, if the process standard deviation is larger than that used in establishing the specification limits, since only the sample mean is determined, the specifying agency will very likely not know that the process is more variable than intended. The only way the larger variability will show up is in results outside the specification limits, and these will probably be attributed to having the wrong population mean rather than to increased variability.

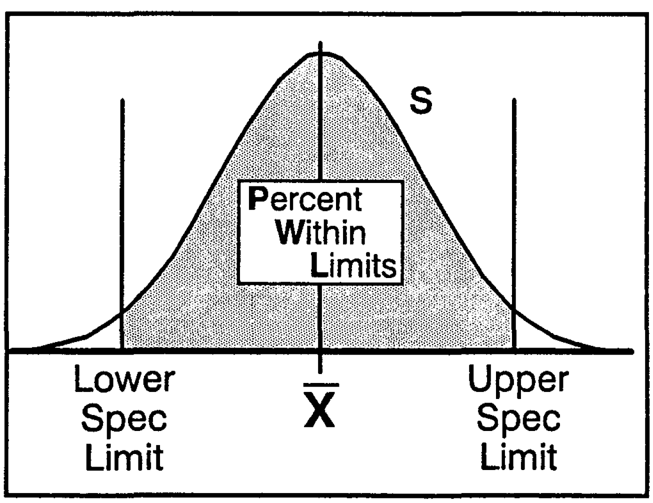
15.

A preferred method for establishing acceptance testing requirements uses the sample mean and the sample standard deviation to estimate the percentage of the population (lot) that is within the specification limits. This is called the percent within limits (PWL) method, and is similar in concept to the area under the normal curve.

NOTES


Lower
Spec
Limit
Upper
Spec
Limit


Lower
Spec
Limit
Upper
Spec
Limit


S
Percent
Within
Limits
Lower
Spec
Limit
$\overline{X}$
Upper
Spec
Limit

16.

Conceptually, the PWL method is based on the normal distribution. The area under the normal curve can be calculated (as shown in Chapter 5) to determine the percentage of the distribution that is within certain limits. Similarly, the percentage of the lot that is within the specification limits can be estimated. Instead of using the Z-value and the standard normal curve, a similar statistic, the quality index (Q), is used to estimate PWL.

17.

Because the sample size for each lot is likely to be small, and because the normal distribution is only appropriate for sample sizes of $n > 30$, another distribution similar to the normal must be used. Thus, Q is used instead of Z. *The table at the end of this chapter relates the values of Q and PWL for various sample sizes **n**.*

18.

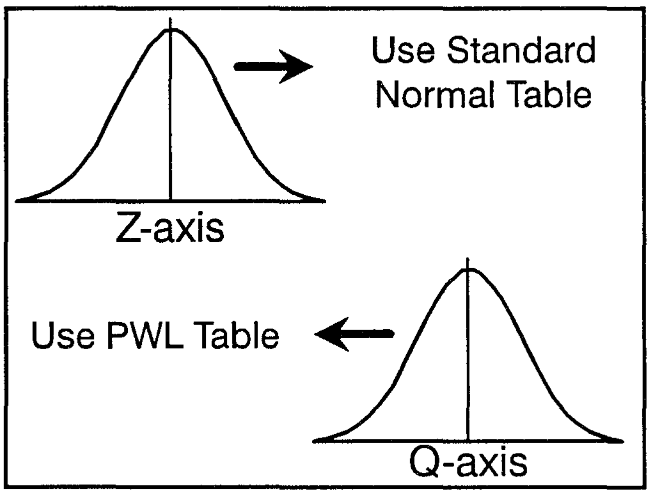
The PWL method, which is known as a **standard deviation (or variability) unknown specification**, has one particular advantage over the standard deviation known specification. As the standard deviation of the lot decreases, the lot mean can approach the specification limit and the lot can still be acceptable. This is often an incentive for a contractor to produce a more uniform product. Another way to look at a PWL specification is that the contractor's process variability (standard deviation) determines the required location of the process mean.

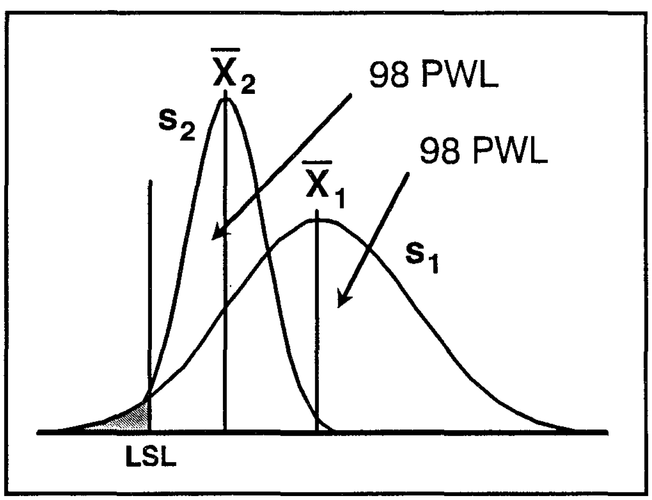
Quality Index (Q)

$$Q_L = \frac{\overline{X} - LSL}{s}$$

or

$$Q_U = \frac{USL - \overline{X}}{s}$$





NOTES

19.

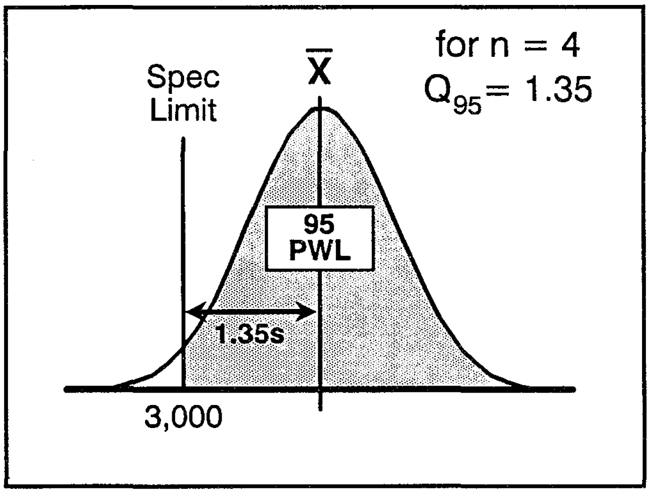
An example of a portland cement concrete specification can be used to explain this concept. The minimum specification limit for strength is 3000 psi. The sample size must be greater than 1 since the sample standard deviation is necessary to estimate PWL. For this specification, n = 4. Furthermore, the specification requires that at least 95% of the lot exceed the minimum strength (i.e., PWL ≥ 95). The PWL table at the end of the chapter shows that the minimum Q value is 1.35 for 95 PWL and a sample size of n = 4. Whenever the mean is 1.35s above the specification limit, the lot is accepted.

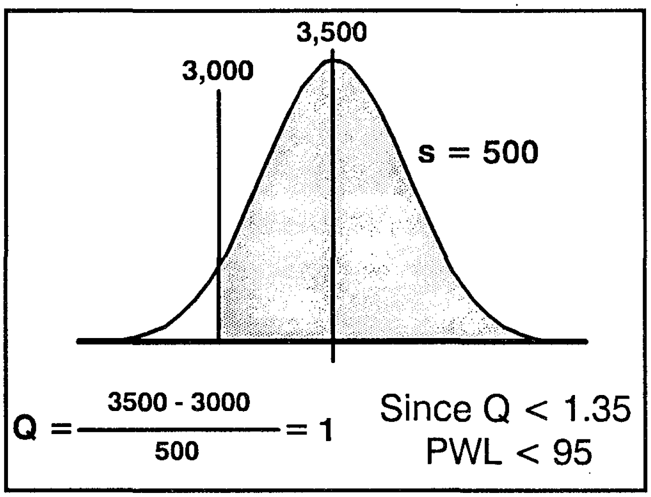
20.

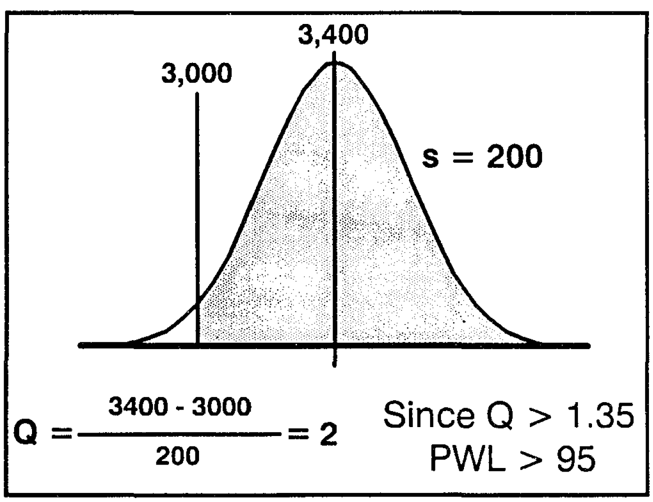
Lot 1 has a sample mean of 3,500 psi and a sample standard deviation of 500 psi. How does this lot relate to the specification limit? Does it meet the specification requirement?

21.

Lot 2 has a sample mean of 3,400 psi and a sample standard deviation of 200 psi. How does this lot relate to the specification limit? Does it meet the specification requirement?


for n = 4
Q_{95} = 1.35
Spec Limit
$\overline{X}$
95 PWL
1.35s
3,000


3,500
3,000
s = 500
$Q = \frac{3500 - 3000}{500} = 1$
Since Q < 1.35
PWL < 95


3,400
3,000
s = 200
$Q = \frac{3400 - 3000}{200} = 2$
Since Q > 1.35
PWL > 95

NOTES

22.

The question of retests often arises in acceptance testing. It is important to recognize that retesting essentially increases the sample size. It was shown in Chapter 8 that in specifications which control the mean, when the sample size increases, the specification limits must be adjusted. With PWL specifications, this increased sample size is recognized by using the larger value of n when entering the table to determine the estimated PWL value. As n increases, the PWL estimate based on Q approaches the area calculated from the Z value and the standard normal distribution.

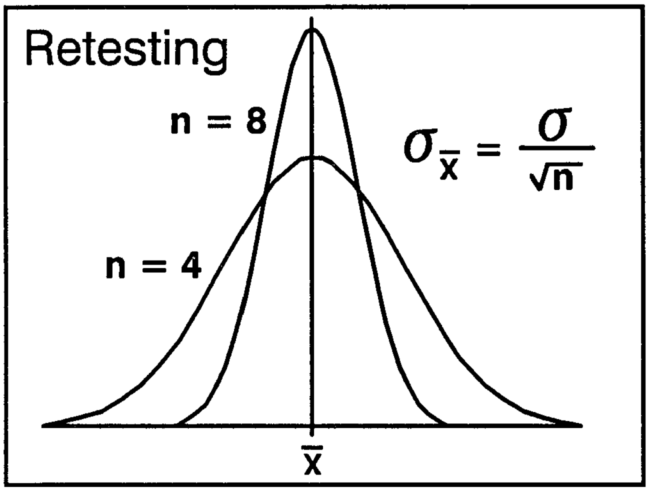
23.

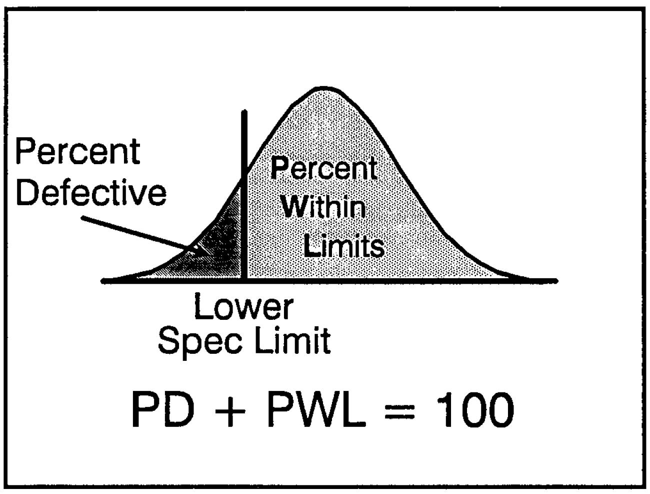
The PWL approach is based on Military Standard 414 which was published in 1957. That standard is based on the use of **percent defective (PD)** rather than PWL. Conceptually, they both estimate percentages of the population related to the specification limits. More state highway agencies have chosen to base acceptance on PWL than PD.

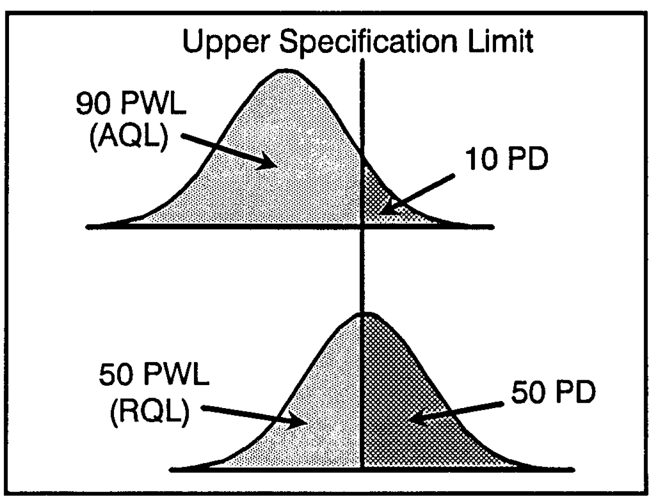
24.

Just as acceptable and unacceptable lots were defined in the acceptance plans based on controlling the mean, **acceptable quality level (AQL)** and **rejectable quality level (RQL)** must be defined for PWL or PD acceptance plans. The AQL is the level of PWL at or above which the product is considered acceptable. The RQL is the level of PWL at or below which the product is considered unacceptable.

NOTES


Retesting
n = 8
n = 4
$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$
$\bar{x}$


Percent
Defective
Percent
Within
Limits
Lower
Spec Limit
PD + PWL = 100


Upper Specification Limit
90 PWL
(AQL)
10 PD
50 PWL
(RQL)
50 PD

25.

The **quality index** (Q) is determined by the equations relating mean, standard deviation, and the specification limit(s). It is important to keep in mind the conceptual relationship between Z, for the standard normal distribution, and Q, for small sample sizes. (Q is a measure of the number of sample standard deviations that the sample mean is away from the specification limit.)

26.

An example (1) will help to explain how an acceptance plan based on PWL works. An acceptance plan for the percent passing the #200 sieve for an aggregate base course is desired. Experience indicates that bases with 7% or less -#200 material perform well and if the -#200 material exceeds 10%, they perform poorly. A typical standard deviation for this material has been found to be about 1.1%. It is believed that the base will perform well as long as 90% or more (i.e., most) of the material has a -#200 value of 7% or less. Thus, the AQL is 90 PWL. This is a relatively conservative definition because, even if the standard deviation were considerably larger than the typical value, there is little chance that any of the material in the normal distribution representing AQL quality would reach the known critical value of 10% passing the #200 sieve.

27.

We must now determine the PWL value to be defined as the RQL. If we place the extreme upper tail of a normal distribution with a standard deviation of 1.1% at the known critical value of 10%, then the mean of that distribution will be at 10% - (3 x 1.1%) = 6.7%. The table of the standard normal curve (Chapter 5) can be used to determine that this corresponds to approximately 60% of the population below the known satisfactory value of 7%. PWL (or approximately 40% above the satisfactory value). On those few occasions where the standard deviation was larger than the typical value of 1.1%, a relatively small portion of the distribution would extend above the critical value of 10%. As the amount of material below 7.0% -#200 material decreases below 60%, however, progressively more will exceed the critical value of 10% and performance problems might be expected to develop. Thus, the RQL is chosen as 60 PWL.

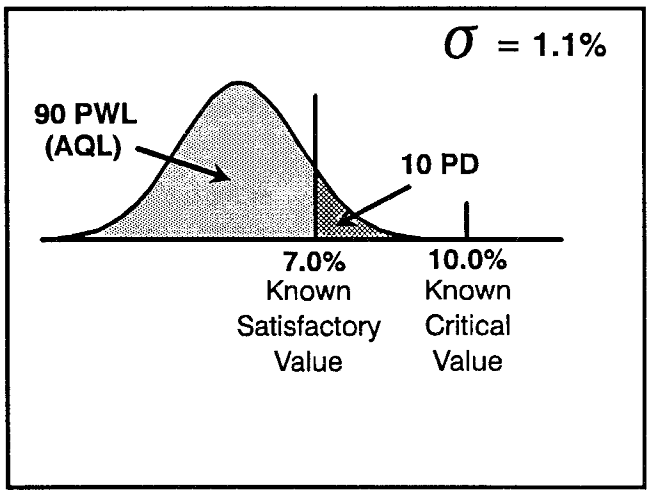
NOTES

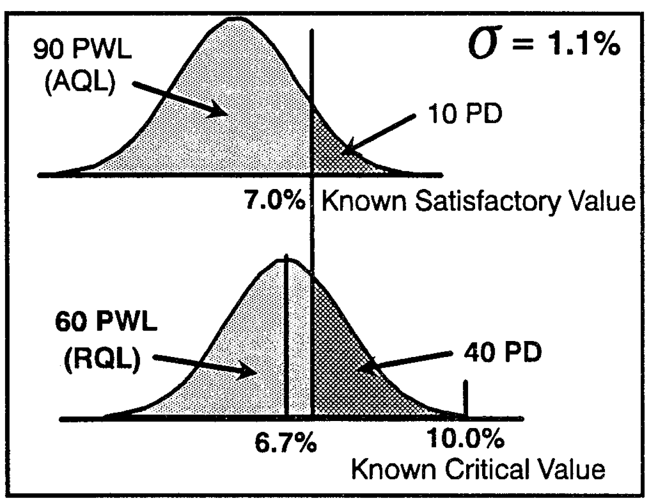
Quality Index (Q)

$$Q_L = \frac{\overline{X} - LSL}{s}$$

or

$$Q_U = \frac{USL - \overline{X}}{s}$$





28.

The specifying agency chooses 7.0% as the upper specification limit and decides to use a sample size of n = 5 for acceptance.

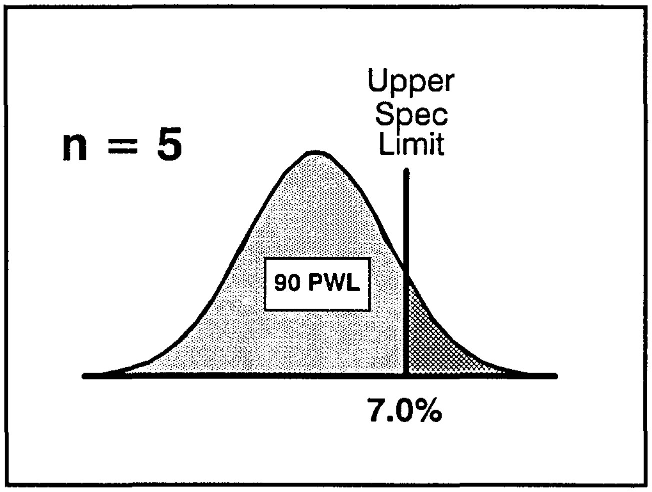
29.

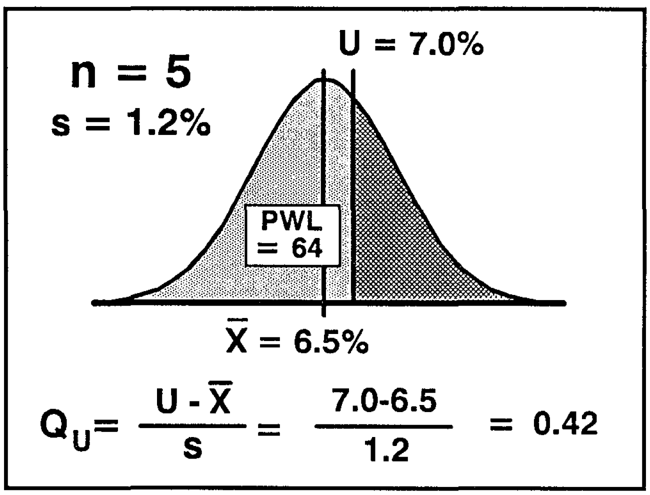
The first lot has a sample mean = 6.5% and standard deviation = 1.2%. Does this lot meet the PWL requirement of 90%? The Q value for this lot is 0.42. From the PWL table at the end of the chapter, it is found that a Q value of 0.42 provides only a little more then 64 PWL for a sample size of 5. Thus, this lot does not meet the specification requirement of 90 PWL.

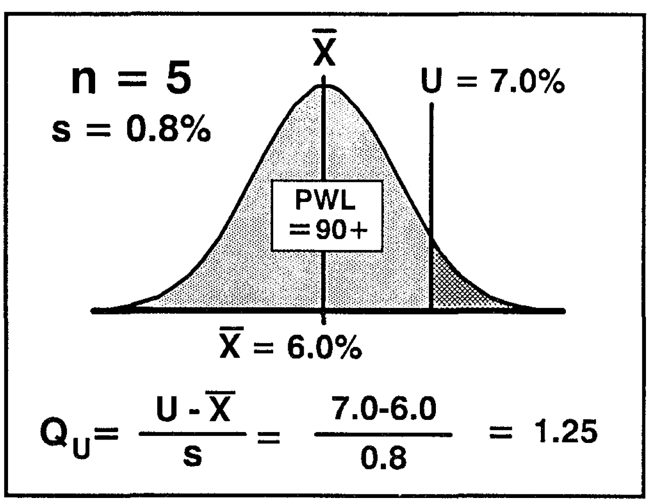
30.

The second lot has a sample mean = 6.0% and a standard deviation = 0.8%. Does this lot meet the PWL requirement of 90%? The Q value for this lot is 1.25. From the PWL table it is found that a Q value of 1.25 provides between 90 and 91 PWL. Thus, this lot meets the requirement.

NOTES


n = 5
Upper Spec Limit
90 PWL
7.0%


n = 5
s = 1.2%
U = 7.0%
PWL = 64
$\bar{X}$ = 6.5%
$Q_U = \frac{U - \bar{X}}{s} = \frac{7.0-6.5}{1.2} = 0.42$


n = 5
s = 0.8%
$\bar{X}$
U = 7.0%
PWL =90+
$\bar{X}$ = 6.0%
$Q_U = \frac{U - \bar{X}}{s} = \frac{7.0-6.0}{0.8} = 1.25$

31.

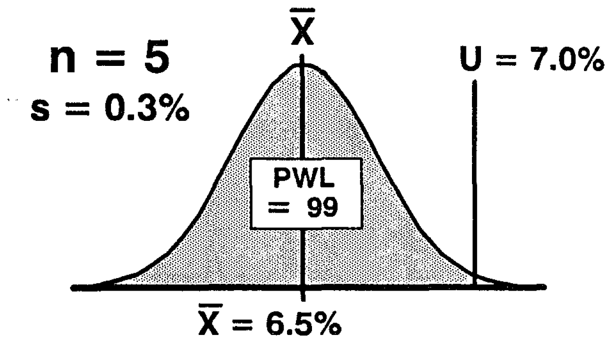
The third lot has a sample mean = 6.5% and a standard deviation of 0.3%. Does this lot meet the PWL requirement of 90%? The Q value for the lot is 1.67. From the PWL table it is found that a Q value of 1.67 provides nearly 99 PWL. Thus, this lot meets the requirement.

32.

Example Problems

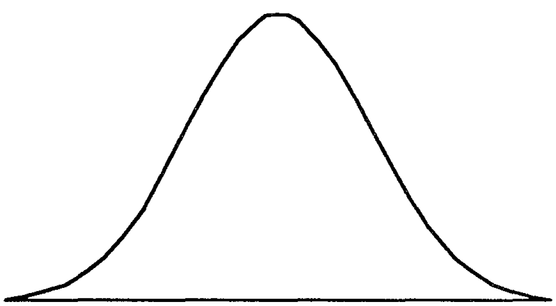
Using the data for the 3 lots that follow, determine whether or not each of the lots meets the specification for -#200 material.

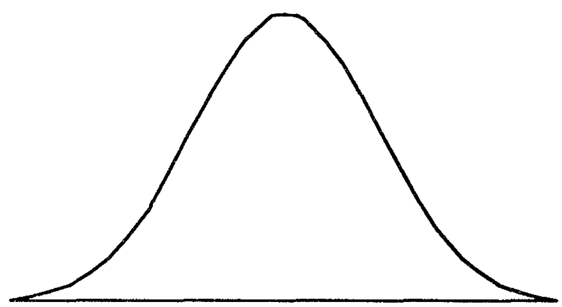
Lot #1	6.5	5.9	7.0	5.1	5.7
Lot #2	8.0	6.6	6.2	5.2	6.4
Lot #3	5.7	6.7	6.8	7.8	6.9

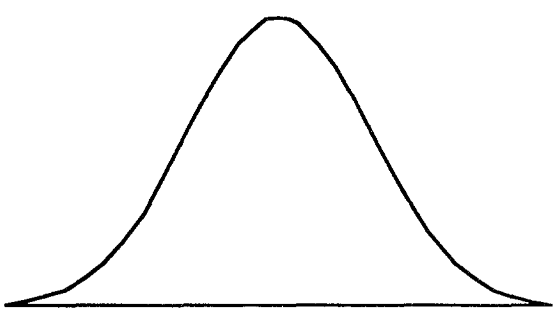


$$Q_U = \frac{U - \bar{X}}{s} = \frac{7.0 - 6.5}{0.3} = 1.67$$

NOTES







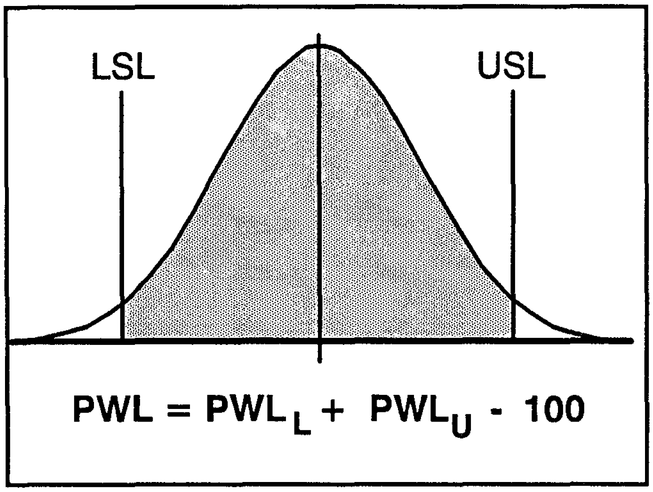
33.

Suppose we have a specification, such as asphalt content, in which there is both an upper and lower limit (called a two-sided specification). It is now possible for the lot to have material that is outside of both specification limits. How can we handle the PWL estimate using Q values?

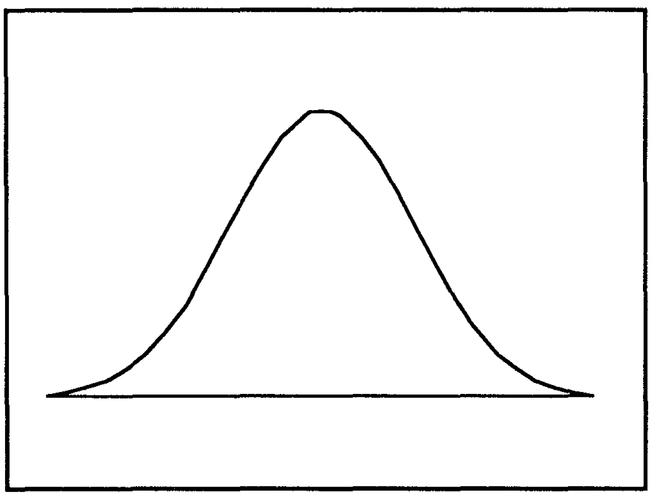
34.

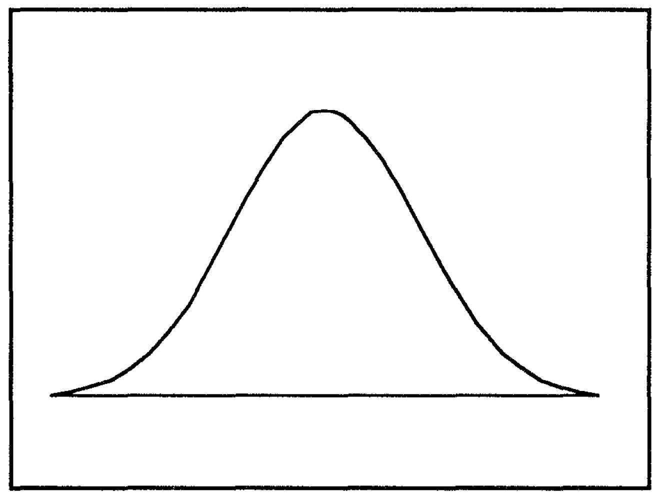
Suppose we have an asphalt content specification that requires 90 PWL with specification limits off 5.6% to 6.4%. Do the following lots meet the specification requirements?

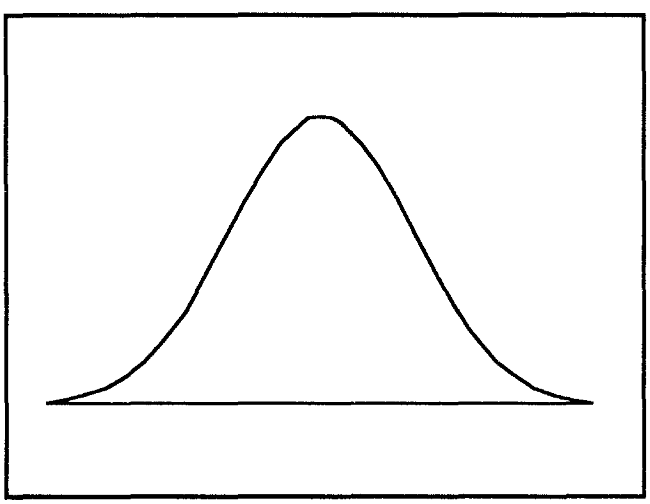
Lot #A	$\overline{X} = 5.9, \; s = 0.4$
Lot #B	$\overline{X} = 5.8, \; s = 0.3$
Lot #C	$\overline{X} = 6.0, \; s = 0.3$



NOTES







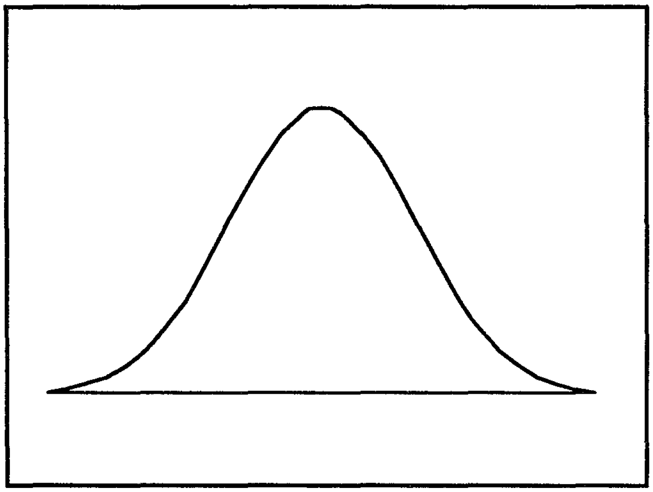
35.

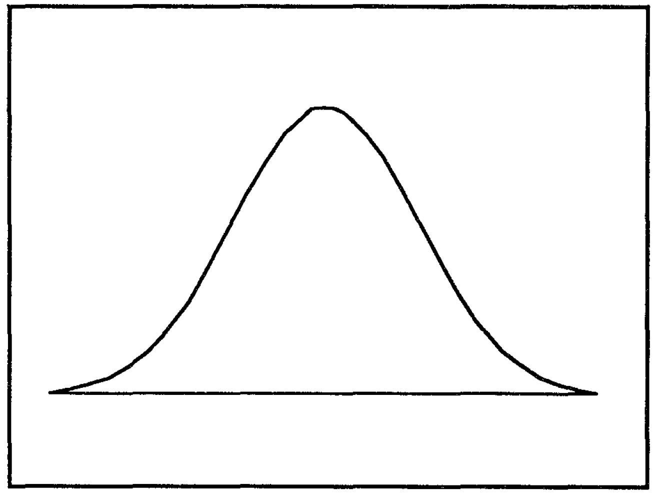
Up to now, the calculated Q value has been positive in all of our examples. What happens if Q is zero? or negative? Suppose we have a specification with a sample size of n = 4 and a lower specification limit of 100. Do each of the following lots meet the requirement?

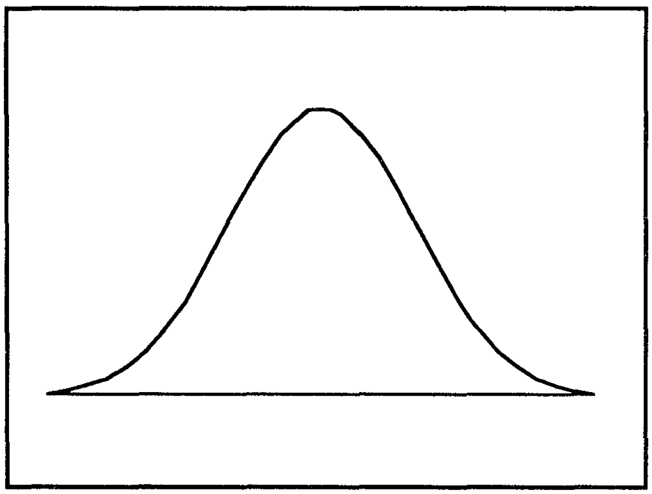
Lot #I	$\overline{X} = 105$, $s = 5$
Lot #II	$\overline{X} = 100$, $s = 5$
Lot #III	$\overline{X} = 98$, $s = 5$

36.

One further requirement in specification design is the decision on how to address material that does not meet the specifications. Material correction or removal, and plant shut downs are two methods that have been typically used. Such methods are costly and do not provide positive incentives to keep the process in control.







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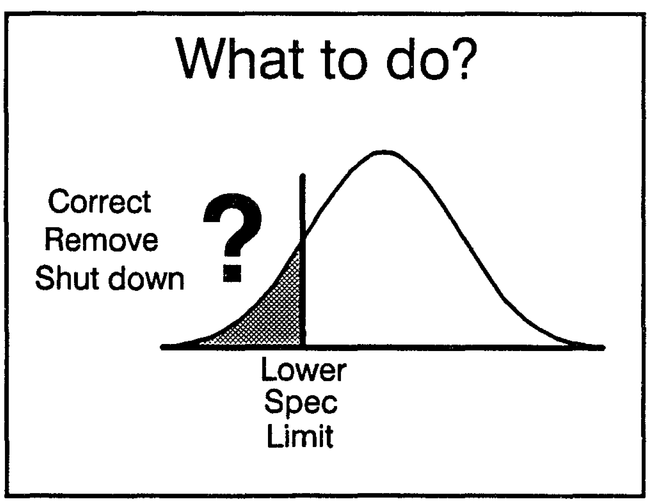


37.

In addition to these methods, alternative approaches using price adjustment schedules have been developed. Both negative and positive price adjustments have been used. The state-of-the-art price adjustment philosophy is that the price should be adjusted commensurate with the estimated performance of the product. If the performance is estimated to be adversely affected by 10%, the price adjustment should stipulate that the product be paid for at 90% of the bid price. Likewise, if the estimated performance is greater than that specified a positive price adjustment is possible.

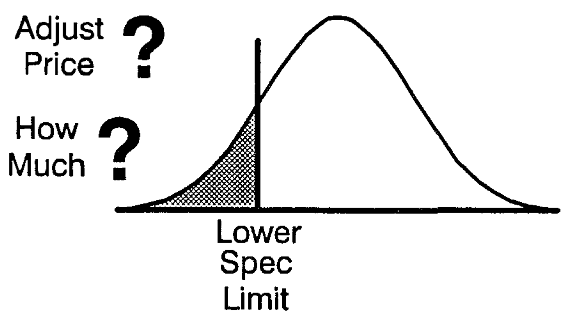
38.

In a survey of state specification practices taken by AASHTO in 1992, of 46 states replying, 36 (78%) use specified price adjustment clauses, including 16 (35%) that use positive pay adjustments (bonuses).

39.

Many different forms of price adjustment system have been developed. Stepped pay factors have been used. However, slight changes in the product characteristic can result in large changes in the pay factor. These changes will average out statistically in the long run. However, it can have impacts on payment levels on individual projects that may not be fair to either the contractor or the state agency.

What to do?



NOTES

AASHTO Survey

78% of state highway agencies use specified price adjustment clauses

35% of state highway agencies use positive pay factors

Stepped Pay Factors

Load Ratio	Pay Factor
>1.50	1.12
1.30-1.49	1.08
1.10-1.29	1.04
0.90-1.09	1.00
0.70-0.89	0.94
0.50-0.69	0.88
<0.50	0.60

40.

A better approach is to use a continuous payment schedule. These are often based on an equation or series of equations. An example of a continuous version of the discrete payment schedule from the previous paragraph is shown at right. A continuous payment schedule avoids the problem of relatively small changes in the measured property leading to relatively large differences in payment level.

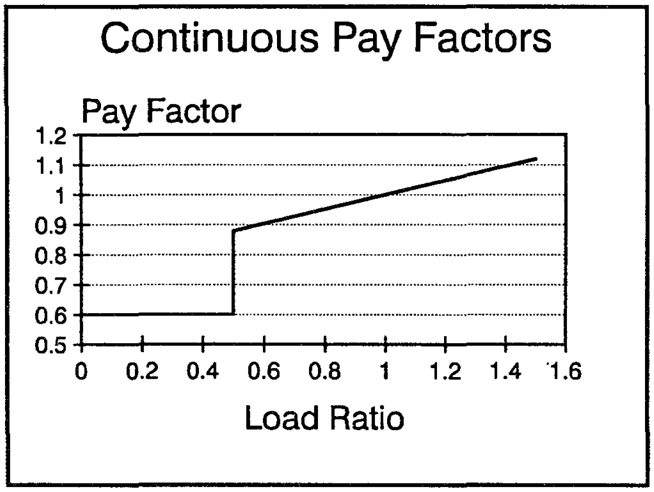
41.

Another option for payment schedules is to use the PWL approach and relate the pay factor to the estimated PWL. In this approach, the sample mean and standard deviation would be used to calculate a Q value. The Q value would then be used to determine the estimated PWL value. The PWL value would then be used in an equation to determine the pay factor for the lot.

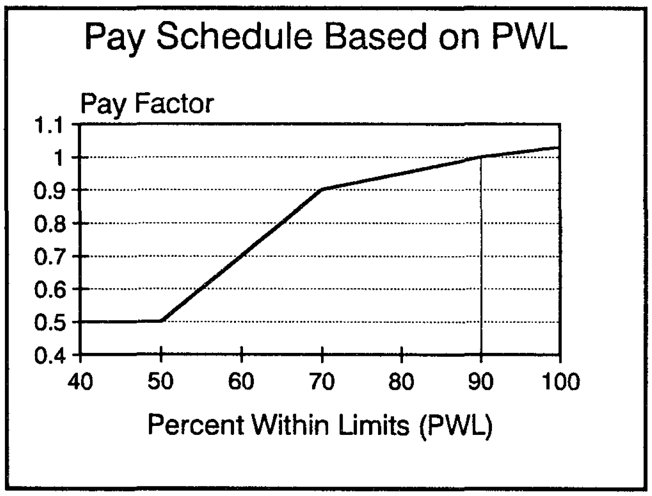
42.

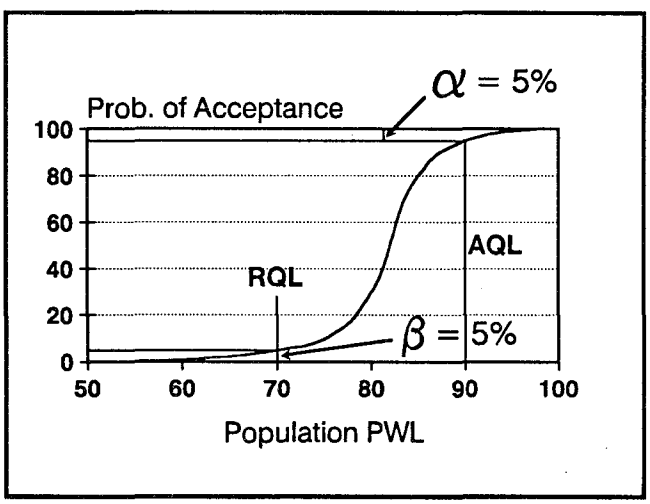
In Chapter 8, OC curves were developed for the example specification based on only the sample mean. OC curves can also be developed for PWL acceptance plans. In the curve shown, if 90 PWL is the AQL, then α = 5% (100% - the 95% probability of acceptance at the AQL). If 70 PWL is the RQL, then β = 5% (the probability of acceptance at the RQL).

The development of these curves is more complicated than the simple example presented previously. However, tables and computer programs are available that allow the development of OC curves for various cases. An enhanced version of a program for developing PWL and PD OC curves is currently under development for the FHWA by Richard Weed of the New Jersey Department of Transportation.


Continuous Pay Factors
Pay Factor
1.2
1.1
1
0.9
0.8
0.7
0.6
0.5
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
Load Ratio

NOTES


Pay Schedule Based on PWL
Pay Factor
1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
40
50
60
70
80
90
100
Percent Within Limits (PWL)


Prob. of Acceptance
α = 5%
100
80
60
40
20
0
AQL
RQL
β = 5%
50
60
70
80
90
100
Population PWL

43.

When price adjustment schedules are used, OC curves based on probability of acceptance are not appropriate since the decision is no longer simply "accept or reject." In the case of payment schedules, the appropriate OC curve plots population PWL on the horizontal axis and **expected payment** on the vertical axis. Expected payment is the average payment in the long run that would be received for a lot with a population PWL as indicated. If the contractor produces at the AQL, he should receive full payment. The α risk therefore would be 100 (or full payment) minus the expected payment at the AQL, or about 2% payment for the curve shown. Similarly, the β risk would be the difference between the expected payment at the RQL and what the state would like to pay for RQL material. For the curve shown, if the state desires to pay 40% for RQL material (defined here as 50 PWL), then the β risk is approximately 10% payment.

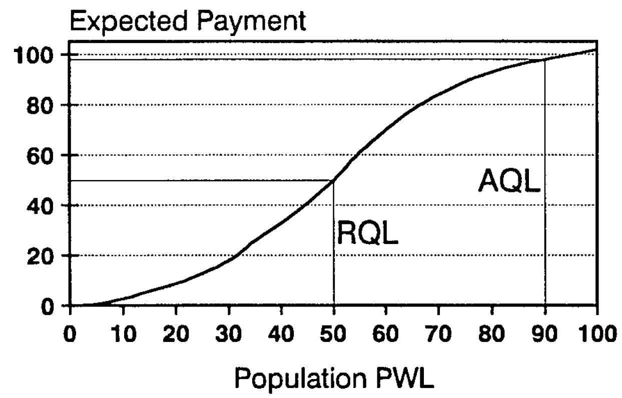
44.

The final item to consider in specification design is whether or not retesting procedures are to be allowed and, if they are, how are they to be handled. At times when a lot has a negative price adjustment applied, the contractor may dispute the results that led to the price adjustment. The specifying agency may agree that the results look suspicious. What should be done? Should the results be thrown out? Not unless it is known that the sampling or testing was in error. Should another set of tests be taken? This is an acceptable practice if all of the test results, the original and the retests, are used and if the fact that the lot is being represented by twice as many test results is considered.

45.

Other decisions resulting from retests concern who pays for the extra sampling and testing. One approach is that if the additional test results indicate the lot is acceptable, the specifying agency pays for the testing, and if the test results indicate the lot should have a negative price adjustment, the contractor pays for the additional tests as well as the price adjustment.

NOTES



Retests Allowed?

Sample Size n = 4

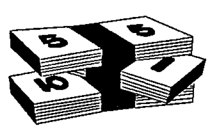
↓

Retest

↓

Sample Size n = 8

Retest - Who Pays?



Final Decision	
Accept	Spec. Agency
Reject	Contractor

46.

Key points for this chapter are presented in the slide below.

Key Points

1. Disadvantages of acceptance by sample mean.
2. Acceptance based on PWL.
3. Use of quality index to estimate PWL.

NOTES

References

1. Weed, R. M., "Revision of a Flawed Acceptance Standard," *Transportation Research Record 1056*, Washington, DC, 1986, pp. 21-35.

2. Weed, R. M., Statistical *Specification Development, 2nd Edition, FHWA/NJ-88/017*, Federal Highway Administration, Washington, DC, 1989.

3. Afferton, K. C., Freidenrich, J., and Weed, R. M., "Managing Quality: Time for a National Policy," *Transportation Research Record 1340*, Washington, DC, 1992, pp. 3-39.

Quality Index Values for Estimating Percent Within Limits

PWL	n = 3	n = 4	n = 5	n = 7	n = 10	n = 15
99	1.16	1.47	1.68	1.89	2.04	2.14
98	1.15	1.44	1.61	1.77	1.86	1.93
97	1.15	1.41	1.55	1.67	1.74	1.80
96	1.15	1.38	1.49	1.59	1.64	1.69
95	1.14	1.35	1.45	1.52	1.56	1.59
94	1.13	1.32	1.40	1.46	1.49	1.51
93	1.12	1.29	1.36	1.40	1.43	1.44
92	1.11	1.26	1.31	1.35	1.37	1.38
91	1.10	1.23	1.27	1.30	1.32	1.32
90	1.09	1.20	1.23	1.25	1.26	1.27
89	1.08	1.17	1.20	1.21	1.21	1.22
88	1.07	1.14	1.16	1.17	1.17	1.17
87	1.06	1.11	1.12	1.12	1.13	1.13
86	1.05	1.08	1.08	1.08	1.08	1.08
85	1.03	1.05	1.05	1.05	1.04	1.04
84	1.02	1.02	1.02	1.01	1.00	1.00
83	1.00	0.99	0.98	0.97	0.96	0.96
82	0.98	0.96	0.95	0.94	0.93	0.92
81	0.96	0.93	0.92	0.90	0.89	0.89
80	0.94	0.90	0.88	0.87	0.85	0.85
79	0.92	0.87	0.85	0.83	0.82	0.82
78	0.89	0.84	0.82	0.80	0.79	0.78
77	0.87	0.81	0.79	0.77	0.76	0.75
76	0.84	0.78	0.76	0.74	0.72	0.72
75	0.82	0.75	0.73	0.71	0.69	0.69
74	0.79	0.72	0.70	0.67	0.66	0.66
73	0.77	0.69	0.67	0.64	0.63	0.62
72	0.74	0.66	0.64	0.61	0.60	0.59
71	0.71	0.63	0.60	0.58	0.57	0.56
70	0.68	0.60	0.58	0.55	0.54	0.54
69	0.65	0.57	0.55	0.53	0.51	0.51
68	0.62	0.54	0.52	0.50	0.48	0.48
67	0.59	0.51	0.49	0.47	0.46	0.45
66	0.56	0.48	0.46	0.44	0.43	0.42
65	0.53	0.45	0.43	0.41	0.40	0.40
64	0.49	0.42	0.40	0.38	0.37	0.37
63	0.46	0.39	0.37	0.35	0.35	0.34
62	0.43	0.36	0.34	0.33	0.32	0.31
61	0.39	0.33	0.31	0.30	0.30	0.29
60	0.36	0.30	0.28	0.25	0.25	0.25

CHAPTER 10: IMPLEMENTATION

INSTRUCTIONAL OBJECTIVES

1. Indicate necessary steps for successful implementation of a QA specification.

2. Discuss various parts of a QM program.

DESIRED STUDENT ACHIEVEMENTS

1. Understand the implementation process for QA specifications.

1.

The use of QA specifications, which were initially called "statistically-based" specifications, began about 30 years ago. Several states began the implementation process then and were able to carry it through successfully. Other states either tried unsuccessfully or met opposition and did not pursue the development of QA specifications. Still others did not see the advantages of QA specifications. One of the greatest deterrents to the initiation and implementation of QA specifications has been contractor opposition. Many contractors have viewed the use of QA specifications as a means of reducing pay with little or no benefits to them.

2.

The successful implementation of QA specifications has demonstrated the need for cooperation among the specifying agency, the private sector, and the FHWA. This cooperation is essential.

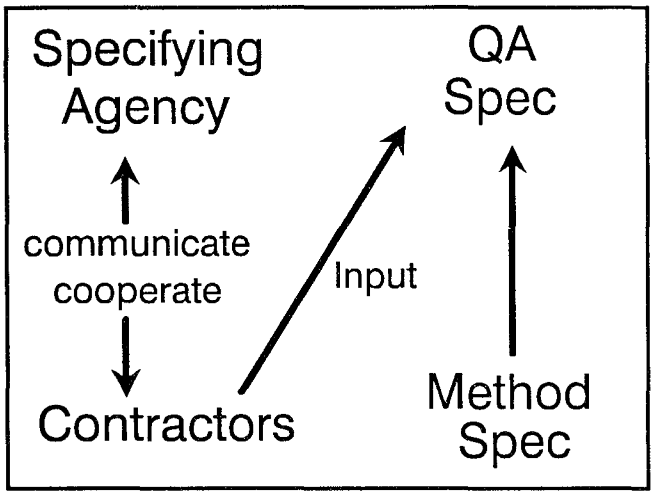
3.

Experience has shown that several steps can be taken to make the implementation process easier.

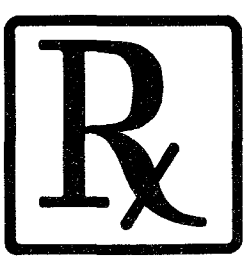
NOTES

QA Implementation
"Statistically-Based" Specifications
State Highway Agencies

Yes	Maybe	No
25	15	10
Why?	How?	When?



Implementation



4.

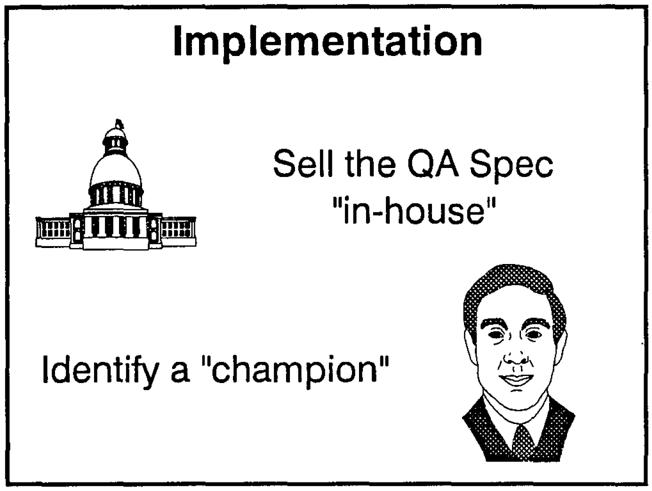
The first step is for the specifying agency to determine that it **wants** to develop and use a QA specification. More than one state has not been able to start the implementation process because it could not sell its own people on the advantages of a QA specification. "We're locking the fox up in the chicken coop" has been heard more than once. The program needs a **champion**, someone convinced that the implementation of a QA program is beneficial and who will keep the implementation process going.

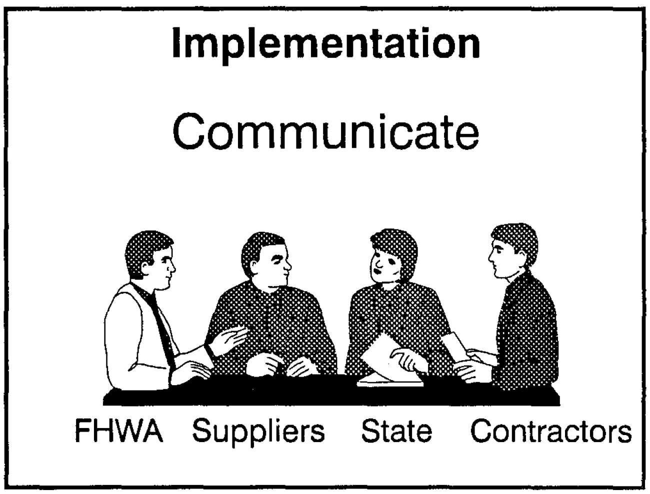
5.

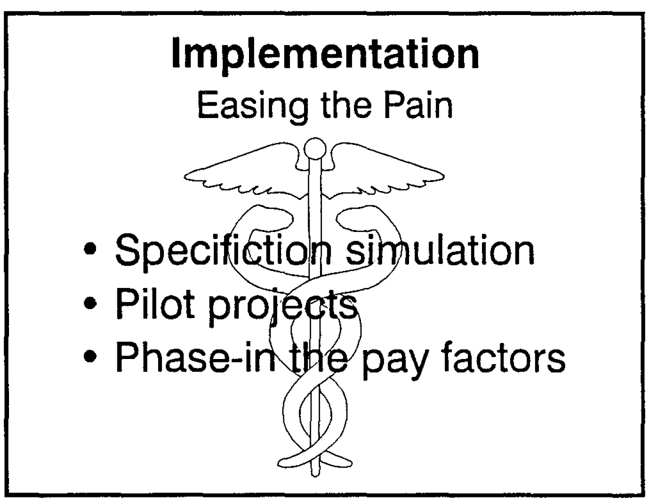
The next step is to open the communication lines among the specifying agency, the contractors' associations, and the FHWA. This should be done as the first draft of the specification is developed. We all fear the unknown. We imagine disaster accompanies change. By putting the first draft on the table for everyone to see and comment on, the concerns of all involved can be addressed. Some opposition can be expected. However, if the specification has been well thought out, the engineering decisions behind the specification development can be defended.

6.

There are several ways of easing into the specification, after receiving input. One is specification **simulation**. Using typical data to show how the specification works in terms of acceptance and the payment schedule. Another way is to let **pilot projects** containing the QA specification. This allows the contractors to develop bidding strategies, to see how the specification works in a limited manner, and allows the specifying agency to examine the outcome and fine tune the specification if desired. Still another implementation procedure is to **phase-in the pay factors**. A schedule of 25%, 50%, and 100% over three years is one strategy.


Implementation
Sell the QA Spec "in-house"
Identify a "champion"


Implementation
Communicate
FHWA
Suppliers
State
Contractors


Implementation
Easing the Pain
• Specifiction simulation
• Pilot projects
• Phase-in the pay factors

NOTES

7.

There are other parts to a Quality Management program that should be considered during implementation. One is **joint training**. There are two aspects to this item -- training and joint training. It is important that the level of training for implementation be considered. Technicians, inspectors, plant operators, and construction foremen only need to know how the specification operates in relationship to their jobs, not how the specification was developed. But it is important that the training be given jointly to state and contractor personnel. This allows both parties to understand the concerns of the other and it helps improve communications and cooperation between the two parties when both have been through the same training side-by-side.

8.

Similar to joint training is **technician certification**. Both state and contractor technicians should have the same basic skills. They should know how to sample and test properly. This has proven to be most effective in a common classroom and laboratory effort. When they have gone through a cenification program together, they are more comfortable discussing their mutual concerns in a project setting. Technician certification is not a one-time effort. As technology advances, specifications evolve, etc., re-certification and certification of new technicians become ongoing commitments.

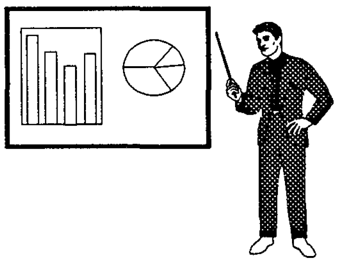
9.

Another important part of implementing a QM program is assuring that all laboratories doing testing have been certified to meet some minimum standard. Sooner or later a difference in test results will occur. This can be minimized by requiring a minimum level of equipment through **laboratory certification**.

Quality Management Program

Additional Considerations

- Joint Training



Quality Management Program

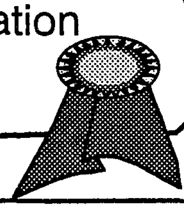
Additional Considerations

- Joint Training
- Technician Certification

Quality Management Program

Additional Considerations

- Joint Training
- Technician Certification
- Laboratory Certification



NOTES

10.

Key points for this chapter are presented in the slide below.

Key Points

1. Multi-party communication and cooperation.
2. Sell in-house first.
3. Ease in slowly.
4. Joint training effort.
5. Laboratory and technician certification.

NOTES

CHAPTER 11: SUMMARY

INSTRUCTIONAL OBJECTIVES

1. Summarize the major elements covered in the course.

DESIRED STUDENT ACHIEVEMENTS

1. Understand the major elements of a successful QM system.

CHAPTER 11: SUMMARY

1.

Quality Assurance (QA) is part of a broad, overall Quality Management (QM) system. QA itself is a system. For this system to work properly, the three functions of **process control**, **acceptance**, and **independent assurance** must be in place and working.

2.

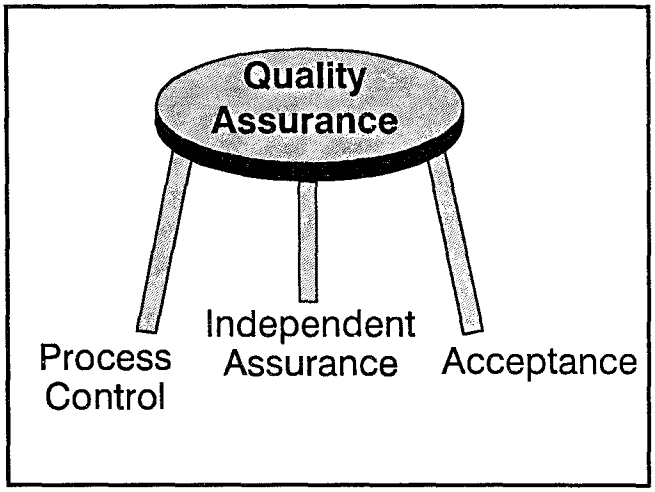
There are many different specification types. Each state highway agency must use the one best suited to its organization. However, risks are present in any specification, and to determine the effectiveness of the specification, the risks should be analyzed.

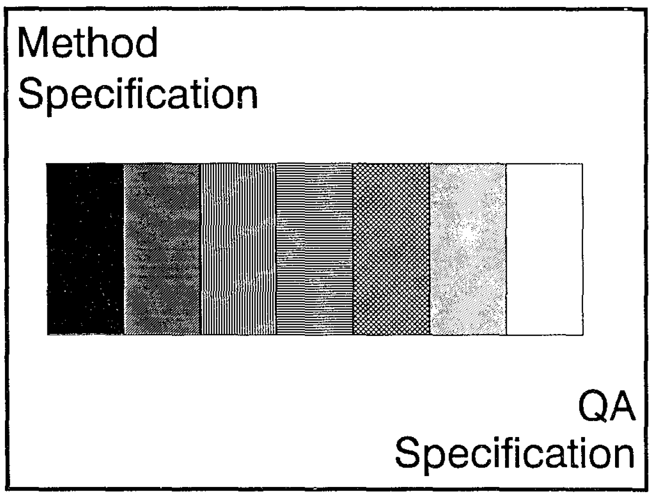
3.

The advantages of a QA specification are:

- Responsibility for process control and acceptance are separated and clearly defined.
- Risks are quantified.
- Specifications can be performance-related.
- Pay factors can be related to performance.

NOTES





Advantages of QA Specifications

- Process control & acceptance are separated & clearly defined.
- Risks are quantified.
- Specifications can be performance-related.
- Pay factors can be related to peformance.

4.

Inspection is essential in all types of specifications.

5.

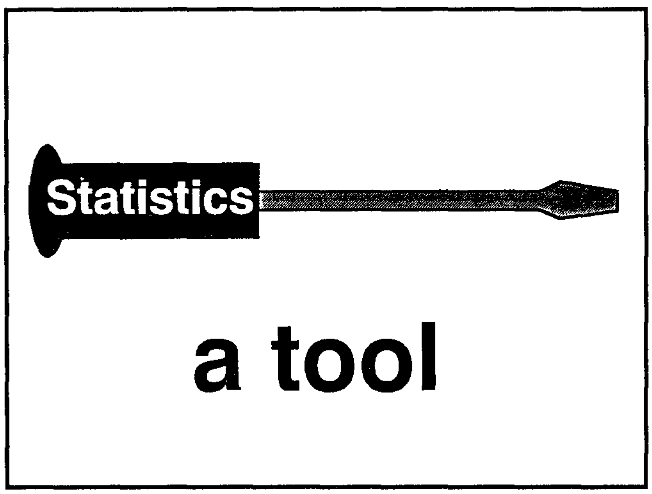
Statistics can be a valuable tool in analyzing specifications and estimating the population of the materials and construction products received.

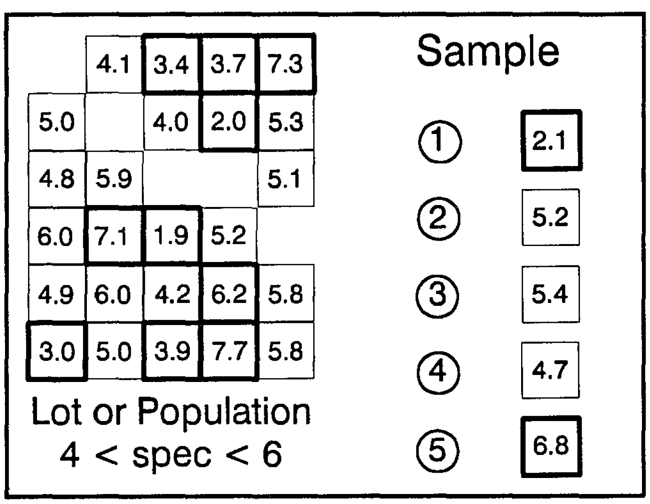
6.

Understanding the relationship between a sample and the population from which it was taken is important for both process control and acceptance.

NOTES

Inspection is essential in all types of specifications.





7.

For data to be analyzed properly, they must be obtained under controlled conditions from a homogeneous process.

8.

Random or stratified random sampling is an absolute necessity.

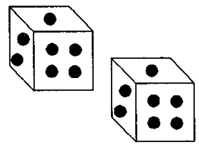
9.

What a single, "representative" sample does not provide is as important as what it may provide. What is does not provide is any estimate of the variability of the material or process. What **it may** provide is an indication of the center of the material or process.

NOTES

Data Collection

Random Sampling



Controled Conditions

Random Sampling

Stratified

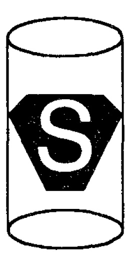
Random

Sampling

The Myth of the Single

"Representative"

Sample



10.

When test measurements are recorded and forgotten, a variable specification effectively becomes an attribute specification and the ability to estimate the population from the sample is greatly reduced. Test results should not be viewed as simply within or outside specification limits, but should be viewed collectively with regard to what they can indicate about the center and variability of the process they represent.

11.

Variability is to be expected. In fact, the lack of variability should be questioned.

12.

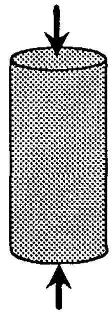
The normal distribution applies to most highway material and processes produced under controlled conditions. The use of this distribution makes the analyses of these products much simpler.

NOTES

Variable Data

Quantitative

Strength = 3400 psi



Does strength meet spec requirement? Yes or No?

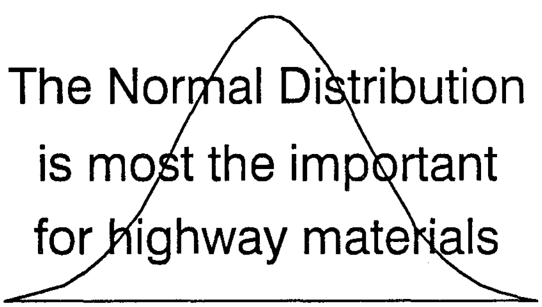
Qualitative

Attribute Data

Asphalt Concrete Density
Test Results

? ? ? ? ? ?

139.1
139.0
139.0
139.1



13.

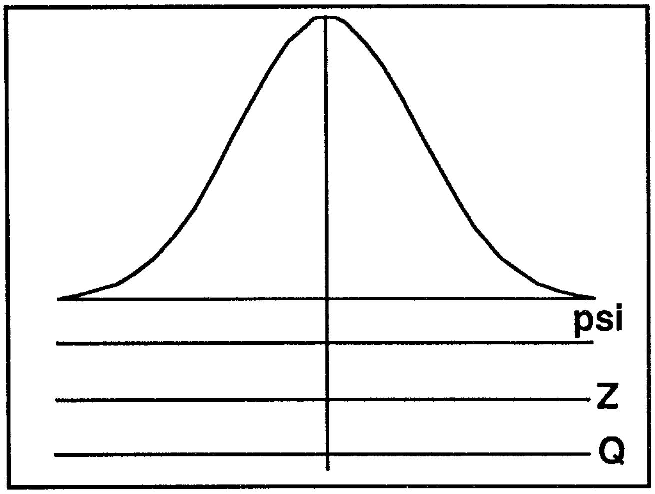
Describing the population in engineering terms (e.g., p.s.i.), the standard normal distribution (using the Z-axis), and in PWL terms (using the Q-axis) only requires a change of scale.

14.

The central limit theorem is an important concept to remember, particularly when there is a question as to whether the process is normally distributed.

15.

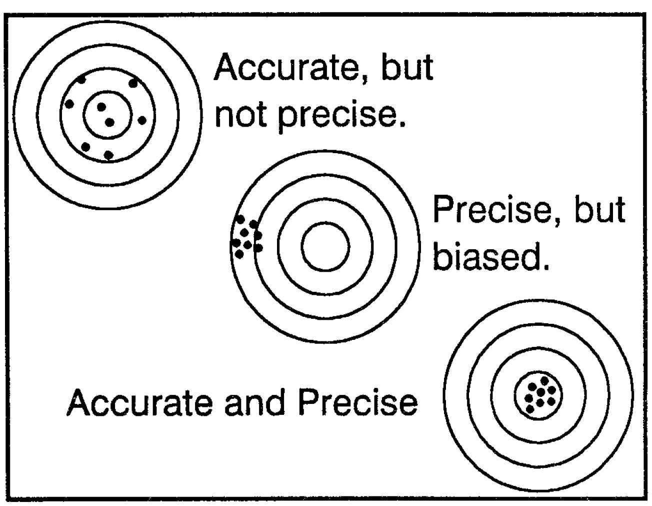
Precision, accuracy, and bias do not mean the same things and each term should be understood and used properly.



Central Limit Theorem

Means of samples of size $n > 1$ approach a normal distribution as

$$n \longrightarrow \infty$$



NOTES

16.

Process control is a very important part of the QA system. When done throughout production, as it should be, it provides a good first step in producing quality highway products.

17.

Control charts distinguish between the chance causes of variability, which are always present in a process and can be reduced but not eliminated, and the assignable causes which can be eliminated if identified.

18.

Statistical control charts were developed more than 60 years ago, and have been used for many years in the manufacturing industry and have been successfully employed in construction materials applications. Some of the benefits that have been attributed to control charts are presented in the slide at right.

NOTES

Independent Laboratory	Supplier
Process Control	
Contractor	Highway Agency?

Variation

- Chance Causes
- Assignable Causes

Benefits of Control Charts

- early detection of trouble
- decrease variability
- establish process capability
- reduce price adjustment costs
- decrease inspection frequency
- basis for altering spec limits
- permanent record of quality
- provide a basis for acceptance
- instill quality awareness

19.

Since QA is a system, the relationship among process control, acceptance, and independent assurance differs among specifying agencies depending on their experience with the QA system.

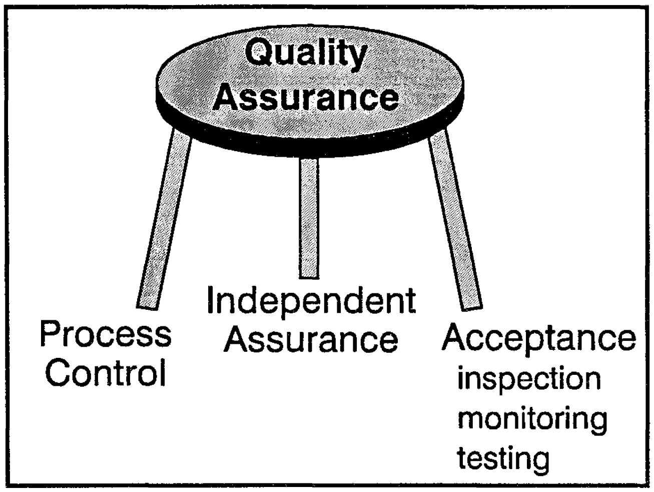
20.

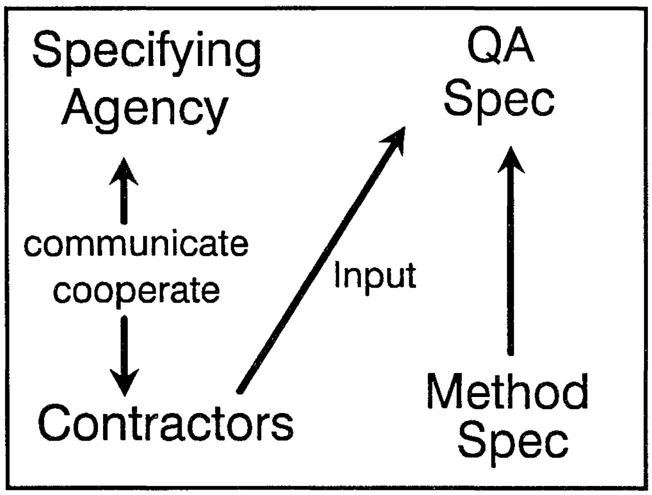
Communication and cooperation between the public and private sectors are important ingredients in specification development and implementation, particularly those using pay factors.

21.

There are many sources from which variability data can be obtained, but it is important that the data were obtained in a random and unbiased manner.

NOTES





Variability Information?

Special Research Studies

- Random sampling
- Unbiased reporting
- New processes
- Valid σ values

22.

There are two types of risk; seller's risk (α) and buyer's risk (β). Seller's risk (α) is the probability of rejecting a lot when the lot is acceptable. Buyer's risk (β) is the probability of accepting a lot when the lot is unacceptable.

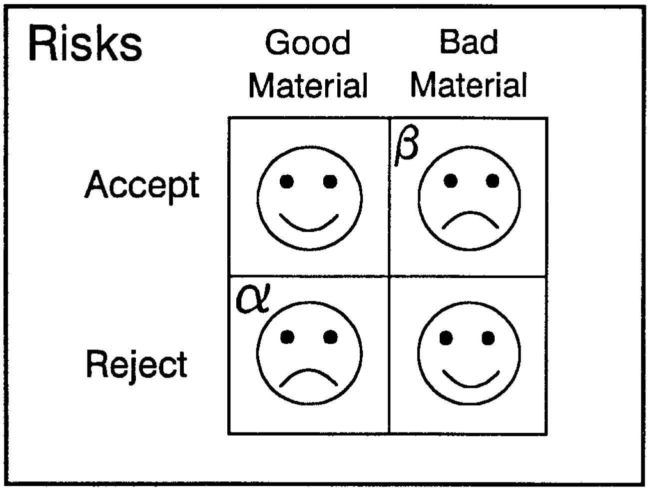
23.

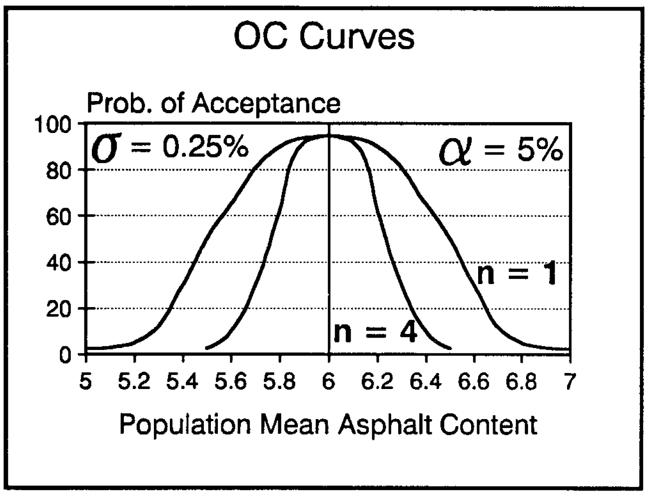
Operating characteristics curves illustrate the risks associated with various levels of quality. These levels of quality could, for example, be indicated by population mean or population PWL. OC curves for a specification are extremely important because they indicate how the risks are related to each other and to various sample sizes and populations. **OC curves should be developed for all specifications before they are implemented, and should be updated to reflect any changes in the acceptance limits or procedures.**

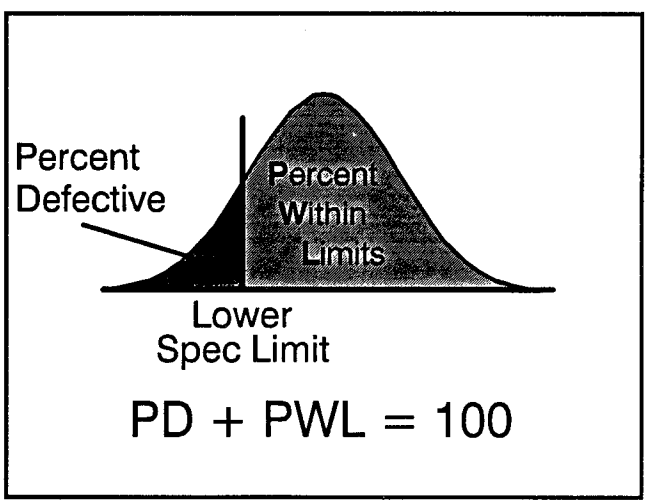
24.

In a PWL specification the terms percent defective and percent within limits are complementary.

NOTES


Risks
Good Material
Bad Material
Accept
Reject
β
α


OC Curves
Prob. of Acceptance
σ = 0.25%
α = 5%
n = 1
n = 4
100
80
60
40
20
0
5
5.2
5.4
5.6
5.8
6
6.2
6.4
6.6
6.8
7
Population Mean Asphalt Content


Percent Defective
Percent Within Limits
Lower Spec Limit
PD + PWL = 100

25.

Positive price incentives are becoming more popular as specifying agencies determine which properties are most closely related to performance.

26.

Key points for this chapter are presented in the slide at right.

NOTES

AASHTO Survey

78% of state highway agencies use specified price adjustment clauses

35% of state highway agencies use positive pay factors

Key Points

1. QA is a system.
2. Acceptance testing and inspection.
3. Process control is important.
4. Statistics is a valuable tool.
5. PWL specifications.
6. Probability and risks.

APPENDIX I: SYMBOLS AND NOTATION

α	alpha risk, seller's risk, producer's risk, contractor's risk
β	beta risk, buyer's risk, consumer's risk, agency's risk
χ^2	chi-square test statistic
μ	population mean
σ	population standard deviation
$\sigma_{\overline{X}}$	standard deviation of the distribution of sample means
σ^2	population variance
σ^2_o	overall variance
σ^2_m	material variance
σ^2_t	testing variance
σ^2_e	test method variance
σ^2_s	sampling variance
∞	infinity
AASHO	American Association of State Highway Officials
AASHTO	American Association of State Highway & Transportation Officials
AQL	Acceptable Quality Level
A_2	factor used to determine control limits for $\overline{X}$ charts
COV	coefficient of variation
D2S	difference two-sigma limit
D_3	factor used to determine the lower control limit for range charts
D_4	factor used to determine the upper control limit for range charts
FHWA	Federal Highway Administration
IA	independent assurance
ISO	International Organization for Standardization
K-S	Kolmogorov-Smirnov goodness-of-fit- test
LCL	lower control limit
LSL	lower specification limit
n	number of values in a sample; sample size
N	number of values in a population
NQI	National Quality Initiative
OC	operating characteristics
PD	percent defective
PWL	percent within limits
Q	quality index
QA	quality assurance
QC	quality control
QM	quality management

APPENDIX I: SYMBOLS AND NOTATION

R	range
$\overline{R}$	average range
s	sample standard deviation
TQM	total quality management
UCL	upper control limit
USL	upper specification limit
V	coefficient of variation
X_i	individual data value
$\overline{X}$	sample mean
$\overline{\overline{X}}$	grand mean
Z	number of standard deviations a value is above or below the mean
1S	one-sigma limit

APPENDIX II: GLOSSARY OF TERMS

Acceptable Quality Level (AQL) - The quality level (e.g., percent within limits value) at which the material is just considered acceptable.

Acceptance Plan - An agreed upon process for evaluating the acceptability of a lot of material. It should consider point of sampling, method of test, sample size, acceptance limits, risks, and operating characteristics.

Accuracy - Refers to the absence of bias in a measurement. The degree of conformity of the measurement to the true value of the quality characteristic being measured.

Adjusted Payment - An increased or reduced payment in the contract price based on using the sample results to measure the conformity of the lot to the specification requirements.

Arithmetic Mean ($\overline{X}$ or μ) - A measure of the center of a set of data (i.e., the average value). For a sample set of observations the symbol used is $\overline{X}$ which is defined as the sum of all of the observations divided by the number of observations. For a population the symbol used is μ which defines the true value of the center of the population.

Assignable Cause - A relatively large factor, usually due to error or process change, which contributes to variation and whose effects are of such importance that the expenditure of time and money for its identification is justified.

Attribute Data - Data which are from a counting rather than a measurement process. Examples of attribute data include screening tests which are conducted on a go or no-go basis.

Average - See Arithmetic Mean

Average Range ($\overline{R}$) - The sum of a the individual sample ranges divided by the number of ranges.

Bias - An error, constant in direction, common to each of a set of values, which cannot be eliminated by any process of averaging.

Biased Sample - A sample obtained by a biased sampling process, i.e., a sampling process for which each portion of the population does not have an equal chance of being included in the sample. Non-random, or judgment sampling, may be subject to bias.

Bimodal Distribution - A set of data or a population which exhibits two modes when plotted as a frequency distribution.

Buyer's Risk (β) - The probability of accepting unsuitable material or construction as a result of using a particular acceptance plan. It is the risk the highway agency takes of accepting material that does not comply with the specification requirements.

Center - The central value about which a set of measurements tends to cluster. It may be thought of as the single value which can be used to represent all of the values in a set of observations.

Central Limit Theorem - A theorem which states that given any population with mean, μ, and variance, σ^2, as the sample size, n, increases without limit, the distribution of sample means approaches a normal distribution with mean, μ, and variance, σ^2/n.

Central Tendency - See Center

Chance Cause - The natural, inherent, variation which occurs in any process.

Class - A group of observations which all satisfy a given set of conditions.

Class Frequency - The number of observations falling into a particular class.

Class Interval - The difference in value between the upper and lower limits for a given class.

Class Limits - The upper and lower values which define a given class.

Class Mid-point - The center value of a particular class.

Coefficient of Variation - The ratio of the standard deviation and the arithmetic mean. It gives a measure of spread relative to the mean. It is generally expressed as a percentage.

Continuous Data - Values obtained from a measurement process for which all values, within a given range, are possible.

Control Charts - Graphical plots of process control which detect when assignable causes are acting on a process and when a systematic variation from the expected results is occurring in a continuous production line process.

Cumulative Frequency Histogram or Polygon - A frequency histogram or polygon which is constructed by adding up the total number of occurrences of values less than or equal to a designated value.

Data - measurements collected for a planned purpose and suitable for the inference of conclusions.

Degrees of Freedom - The number of free choices. For example, given that $X_1 + X_2 + X_3 = 10$, any two of X_1, X_2, or X_3, may be assigned at will (two free choices = two degrees of freedom) but once these two have been determined, the value of the third variable is fixed.

APPENDIX II: GLOSSARY OF TERMS

Discrete Variable - A variable for which the possible values are observed on a discrete or integer scale.

Dispersion - See Spread

Frequency - The number of items or observations that occur within a given interval.

Frequency Histogram - A type of bar chart which displays in terms of area the relative number of measurements for different classes. The width of the bar represents the class interval, while the height represents the number of measurements in the interval.

Frequency Polygon - A broken line graph constructed by drawing line segments which join the mid-points at the top of each column or bar in the frequency histogram.

Frequency Table - A tabular presentation of statistical data, generally showing the number of classes, class limits, class midpoint, tallied frequency, relative frequency, and cumulative relative frequency.

Gaussian Distribution - See Normal Distribution.

Grand Mean ($\bar{\bar{X}}$) - The sum of the arithmetic means of sets or groups of data divided by the total number of sets or groups.

Histogram - See Frequency Histogram

Judgment Sampling - Sampling based solely on the judgment of the sampler. The sampler decides when and where a sample should be taken.

Lot - An isolated quantity of material from a single source. A measured amount of construction assumed to be produced by the same process.

Lower Class Limit - Value which determines the lower limit for a particular class when constructing a frequency table, frequency histogram, or frequency polygon.

Lower Control Limit (LCL) - A process control criterion associated with the control chart technique. It is the limiting value above which the contractor or producer must hold his process if it is to remain in control.

Lower Specification Limit (LSL) - The minimum limiting value used for determining acceptable material within the specification requirements.

Mean - See Arithmetic Mean.

Median - For a set of numbers, the median is the value for which half of the numbers are larger and half are smaller.

Military Standards - A handbook of tables based on the concepts of lot-by-lot sampling inspection by either attributes or variables. These standards, often referred to as Mil Standards, are published by the Federal Government.

Mode - The value of a variable which is possessed by the greatest number of the members of the population.

Multimodal Data - Data, which when plotted as a frequency distribution, exhibit more than one mode.

Negative Skewness - A frequency distribution that has the side to the left of the central maximum value longer than the side to the right.

Normal Distribution - A curve, having a bell shape, that is determined by values of μ and σ, and is often used to describe the distribution of individual measurements (such as construction materials properties).

Operating Characteristics (OC) Curves - A graphical presentation of a sampling plan that shows the relationship between the quality of a lot (population) and the probability of its acceptance, or, where price adjustments are used, its expected payment.

Outlier - An extreme individual measurement or extreme sample mean.

Parameter - A constant or coefficient that describes some characteristic of a population.

Percent Within Limits (PWL) - The estimated percentage of a lot of material that is within the specification limits.

Population - Any set of individuals (or measurements) having some common observable characteristic. The set may be finite or infinite. In many cases a population will be finite but so large that it must be treated as though it were infinite.

Positive Skewness - A frequency distribution that has the side to the right of the central maximum value longer that the side to the left.

Precision - Refers to the variability of a method of measurement when used to make repeated measurements under carefully controlled conditions.

Probability - The relative frequency of occurrence of various events over the long run.

Probability Sampling - See Random Sampling.

Reproducibility - The range within which repeated measurements are made by the same operator on the same apparatus. Essentially, the precision of a test.

Sample - A small part of a lot (or population) that is taken to make inferences about the entire lot. A sample can be made up of one or more increments, or test portions.

Sampling Plan - See Acceptance Plan.

Seller's Risk (α) - The probability of having acceptable material or construction rejected as a result of using a particular acceptance plan. It is the risk the contractor or producer takes of having acceptable material rejected.

Skewness - A condition where the tail of a frequency distribution to one side of the central maximum value is longer than that on the other side.

Specifications - A statement containing a description of requirements or enumeration of particulars, such as terms of a contract or details required of materials and/or construction.

Specification Limits - Limits established, preferably by statistical analysis, for determining acceptable construction material within the specification requirements.

Standard Deviation (σ or s) - A term used in statistics to indicate the spread of a set of data or a population. It is the square root of the average difference between the individual measurements and their mean. The symbol σ is used to represent the standard deviation of a population, while the term s is used for the standard deviation of a sample. The equations for standard deviation are given below:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n}(X_i - \overline{X})^2}{n}} \qquad s = \sqrt{\frac{\sum_{i=1}^{n}(X_i - \overline{X})^2}{n-1}}$$

Standard Error of the Mean ($\sigma_{\overline{X}}$) - The standard deviation of the distribution of sample means (i.e., the standard deviation of a number of sample $\overline{X}$'s).

Standard Normal Distribution - A normal distribution with a mean of zero and a standard deviation of one. Measured values of a normal distribution are usually transformed to the Z-statistic (i.e., the standard normal form where Z is the number of standard deviations a particular value is above or below the mean) to facilitate computing areas under the normal distribution curve.

Statistic - An expression or numerical value that describes some characteristic of the distribution of measurements of a sample.

Statistics - The science that deals with the treatment and analysis of numerical data. Also, a collection of numerical data.

Stratified Sampling - Selecting each of two or more parts independently from a corresponding part. Stratified sampling is inherent in any acceptance sampling based on lots.

Sublots - Equal divisions or portions of a lot.

Symmetry - Correspondence in size and shape of parts about a given axis. For example, the bell-shaped curve of a truly normal distribution is said to be symmetrical about the mean (center) value.

Systematic Error - Errors arising from causes that act consistently under given circumstances, such as a rule calibrated at one temperature and used and read at another.

Systematic Sampling - Selection of successive observations at uniform intervals in a sequence of times, areas, lengths, etc.

Unimodal Data - Data which, when plotted as a frequency distribution, exhibit only one mode or maximum.

Upper Class Limit - Value which determines the upper limit for a particular class when constructing a frequency table, frequency histogram, or frequency polygon.

Upper Control Limit (UCL) - A process control criterion associated with the control chart technique. It is the limiting value below which the contractor or producer must hold his process if it is to remain in control.

Upper Specification Limit (USL) - The maximum limiting value used for determining acceptable material within the specification requirements.

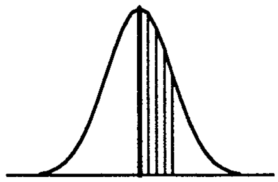
Variable - A measurement that can have a series of different values.

Variable Sampling - Sampling in which the characteristic of interest is measured rather than the type of qualitative classification used in attribute sampling.

Variance (σ^2) - A statistical measure of spread or dispersion. It is the square of the standard deviation, or, more correctly, the standard deviation is the square root of the variance.

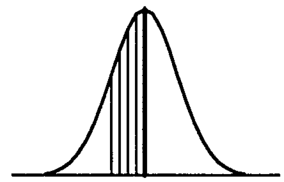
Variation - Differences in measured values of a characteristic within a stable pattern due to chance, or outside the normal pattern due to assignable cause.

Areas Under the Standard Normal Distribution



Z

or



-Z

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3183
.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998

Quality Index Values for Estimating Percent Within Limits

PWL	n = 3	n = 4	n = 5	n = 7	n = 10	n = 15
99	1.16	1.47	1.68	1.89	2.04	2.14
98	1.15	1.44	1.61	1.77	1.86	1.93
97	1.15	1.41	1.55	1.67	1.74	1.80
96	1.15	1.38	1.49	1.59	1.64	1.69
95	1.14	1.35	1.45	1.52	1.56	1.59
94	1.13	1.32	1.40	1.46	1.49	1.51
93	1.12	1.29	1.36	1.40	1.43	1.44
92	1.11	1.26	1.31	1.35	1.37	1.38
91	1.10	1.23	1.27	1.30	1.32	1.32
90	1.09	1.20	1.23	1.25	1.26	1.27
89	1.08	1.17	1.20	1.21	1.21	1.22
88	1.07	1.14	1.16	1.17	1.17	1.17
87	1.06	1.11	1.12	1.12	1.13	1.13
86	1.05	1.08	1.08	1.08	1.08	1.08
85	1.03	1.05	1.05	1.05	1.04	1.04
84	1.02	1.02	1.02	1.01	1.00	1.00
83	1.00	0.99	0.98	0.97	0.96	0.96
82	0.98	0.96	0.95	0.94	0.93	0.92
81	0.96	0.93	0.92	0.90	0.89	0.89
80	0.94	0.90	0.88	0.87	0.85	0.85
79	0.92	0.87	0.85	0.83	0.82	0.82
78	0.89	0.84	0.82	0.80	0.79	0.78
77	0.87	0.81	0.79	0.77	0.76	0.75
76	0.84	0.78	0.76	0.74	0.72	0.72
75	0.82	0.75	0.73	0.71	0.69	0.69
74	0.79	0.72	0.70	0.67	0.66	0.66
73	0.77	0.69	0.67	0.64	0.63	0.62
72	0.74	0.66	0.64	0.61	0.60	0.59
71	0.71	0.63	0.60	0.58	0.57	0.56
70	0.68	0.60	0.58	0.55	0.54	0.54
69	0.65	0.57	0.55	0.53	0.51	0.51
68	0.62	0.54	0.52	0.50	0.48	0.48
67	0.59	0.51	0.49	0.47	0.46	0.45
66	0.56	0.48	0.46	0.44	0.43	0.42
65	0.53	0.45	0.43	0.41	0.40	0.40
64	0.49	0.42	0.40	0.38	0.37	0.37
63	0.46	0.39	0.37	0.35	0.35	0.34
62	0.43	0.36	0.34	0.33	0.32	0.31
61	0.39	0.33	0.31	0.30	0.30	0.29
60	0.36	0.30	0.28	0.25	0.25	0.25