Report No. FH-11-7473-1

A STOCHASTIC MODEL FOR PREDICTION OF ACCUMULATIVE DAMAGE IN HIGHWAY PAVEMENTS

Department of Civil Engineering
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139

January 1971

FINAL REPORT

Prepared for
FEDERAL HIGHWAY ADMINISTRATION
Office of Research
Washington, D. C. 20591

TECHNICAL REPORT STANDARD TITLE PAGE

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
FH-11-7473-1		

4. Title and Subtitle	5. Report Date
A Stochastic Model for Prediction of Accumulative Damage in Highway Pavements	January 1971
	6. Performing Organization Code
7. Author(s)	8. Performing Organization Report No.
F. Moavenzadeh, J. R. Soussou, H.K. Findakly	R71-6
9. Performing Organization Name and Address	10. Work Unit No.
Department of Civil Engineering Massachusetts Institute of Technology Cambridge, Massachusetts 02139	11. Contract or Grant No. FH-11-7473
12. Sponsoring Agency Name and Address	13. Type of Report and Period Covered
Federal Highway Administration Office of Research Washington, D. C. 20591	Final
	14. Sponsoring Agency Code

15. Supplementary Notes

16. Abstract A review of the state of the art of pavement damage is presented with special emphasis upon load associated factors affecting the initiation, propagation, and accumulation of damage.

The pavement is viewed as a structure sensitive engineering system. Such systems are defined as those in which the distribution of damage or the failure of a component results in a decrease in the level of performance rather than the abrupt incidence of total failure. Failure for such systems is thus the limiting extent of the damage which has been accumulated as a consequence of structural deterioration over a range of stress, strain, time, and temperature conditions in an operational environment.

A framework for a rational method of analysis of pavement structures is proposed. This method utilizes a three-layer viscoelastic model, a cumulative damage model, and a simulation technique. In this framework the input variables of environment, layer material properties and geometry are described in a stochastic manner utilizing the Monte Carlo simulation technique.

17. Key Words	18. Distribution Statement
Pavement, Structural Integrity, Linear Viscoelasticity, Accumulative Damage, Stochasticity of Response.	Availability is unlimited. Document may be released to the National Technical Information Service, Springfield, Virginia, 22151, for sale to the public

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price
Unclassified	Unclassified	152	

Form DOT F 1700.7 (8-69)

TABLE OF CONTENTS

I. Introduction 1

I.1 Performance of Engineering Structures 1

I.2 The Structural Integrity of the Pavement 2

II. Review of Literature 5

II.1 Material Characterization 6

II.1.1 Functional Relations 6

II.1.2 Mathematical Representation 9

II.2 Stress Analysis 13

II.3 Distress and Failure of Engineering Materials 16

II.3.1 Mechanisms of Damage 17

II.3.2 Failure Criteria 23

II.3.3 Parameters of Damage in the Repeated Load Mode 27

II.3.4 Summary 36

III. Damage and Distress in Highway Pavements 39

III.1 Introduction 39

III.2 Models for Pavement Damage 47

III.3 Stochastic Nature of Damage 51

III.4 Materials and Environmental Variabilities 52

III.5 Monte Carlo Simulation 54

III.6 Summary 56

IV. Pavement Cumulative Damage Model 58

IV.1 Formulation of Damage Law 58

IV.1.1 Primary Response 59

IV.1.2 Limiting Response 61

IV.2 Input Requirements 69

IV.2.1 Materials Characterization 69

IV.2.2 History of the Environment 71

IV.2.3 History of Load Applications 73

IV.3 Applications to Highway Pavements 73

IV.3.1 Primary Response 73

IV.3.2 Limiting Response 77

IV.3.3 Systems Simulation 78

IV.4 Summary 82

V. Results and Discussion 83

V.1 Introduction 83

V.2 The Simplified Model 84

V.2.1 Description of the Model 85

V.2.2 Results 88

V.3 Repeated Load Model 93

V.3.1 Basic Assumptions 98

V.3.2 Three-Layer Linear Viscoelastic Model 100

V.3.3 Limiting Response 101

V.3.4 Results 106

VI. Summary 122

List of References 125

Appendix I Computer Program

1. Formulation of the Primary Response
2. Formulation for Damage F(t)
3. Input Description
4. Solution for Primary Response Model
5. Output

LIST OF FIGURES

Figure 1.	Determination of the First Three Multiple Integrals Using Multiple Steps Inputs	11
Figure 2.	Determination of the First Three Multiple Integrals Using Sinusoidal Inputs	12
Figure 3.	Two-Dimensional Simulation of the Performance of a Pavement Structure	41
Figure 4.	Failure Envelopes	68
Figure 5.	Variation of the Relaxation Function with Temperature at a Fixed Time	87
Figure 6.	Creep Functions of the Simulated Pavement System	89
Figure 7.	Relative Frequency Distribution of Uniformly Distributed Temperatures as Generated by an IBM/System 360 Random Number Generator	90
Figure 8.	Relative Frequency Distribution of Vertical Strain at $t/\tau = 0.0$ for Uniformly Distributed Random Inputs	91
Figure 9.	Relative Frequency Distribution of Vertical Strain at $t/\tau = 10.0$ for Uniformly Distributed Random Inputs	92
Figure 10.	Relative Frequency Distribution of Normally Distributed Temperatures as Generated by an IBM/System 360 Random Number Generator	94
Figure 11.	Relative Frequency Distribution of Vertical Strain at $t/\tau = 0.0$ for Normally Distributed Random Inputs	95
Figure 12.	Relative Frequency Distribution of Vertical Strain at $t/\tau = 10.0$ for Normally Distributed Random Inputs	96
Figure 13.	Summary of Simulation of Vertical Strains for Uniformly Distributed Random Inputs	97
Figure 14.	Ratio of Tensile Strength for Damaged Materials (Tensile Test) vs. Undamaged Materials	104

Figure 15. Recovery Function K_{REL} for a Fatigued Dense Bituminous Mix 105

Figure 16. Temperature Variations 107

Figure 17. Comparison of Different Sequences of Temperatures 108

Figure 18. Example of Damage Function Obtained for 15 Simulation Runs 110

Figure 19. Relative Frequency Distribution of Residual Strain Factor After 1-Month Period 112

Figure 20. Relative Frequency Distribution of Residual Strain Factor After 6-Month Period 113

Figure 21. Relative Frequency Distribution of Residual Strain Factor After 12-Month Period 114

Figure 22. Relative Frequency Distribution of Damage After 1 Month 115

Figure 23. Relative Frequency Distribution of Damage After 6 Months 116

Figure 24. Relative Frequency Distribution of Damage After 12 Months 117

Figure 25. Frequency Distribution of Residual Strains at Three Observation Times 118

Figure 26. Frequency Distribution of Damage Function at Three Observation Times 119

Figure 27. Frequency Distribution of the Number of Load Applications (100 Samples) 121

I. INTRODUCTION

An understanding of what constitutes failure in pavement structures and what factors contribute to damage initiation, propagation, and accumulation is important to the development of a rational method of design. This study reviews the state of the art in the area of pavement damage with a view to identifying the pertinent damage mechanisms, and presents a damage formulation for highway pavements.

I.1 PERFORMANCE OF ENGINEERING STRUCTURES

The performance of a pavement structure in a given traffic and climatic environment may be defined as its ability to provide an acceptable level of serviceability, with a specified degree of reliability for an assumed level of maintainability (1). The impairment or loss in the ability to provide the necessary services in a given locale may then be considered as the 'failure' of the pavement. When viewed in this context, 'failure' becomes a loss in performance; it is the extent to which the pavement has failed to render itself serviceable, and it results from an accumulation of damage over a given time period.

The failure age or the life of the pavement is then, the time during which serviceability deteriorates to an unacceptable level as determined by the users.

The question of what is the unacceptable level of serviceability at which the pavement must be considered failed is a highly subjective matter. It depends on the user's evaluation of the performance, and it involves many intangible and not easily quantifiable factors such as cost, comfort, convenience and safety. The problem is further compounded by the fact that there does not exist a usually accepted and comprehensive function describing the pavement serviceability in terms of damage parameters. Presently one can only assume that pavement failure is a many sided problem, and it is the result of a series of interacting complex processes none of which is clearly understood.

Therefore, in order to present an integrated comprehensive picture of failure, each of its components has to be studied in detail. From such studies, methods may be developed for analyzing the effect that each component has on the behavior of the structure. Finally, all of those methods may be grouped together in a meaningful way so that for any given environment, the performance history of a pavement can be predicted. This study investigates 'failure' from the "structural integrity" viewpoint with special emphasis on load-associated factors affecting the structural integrity. It must therefore, be emphasized that this is only one aspect of the 'failure' problem.

I.2 THE STRUCTURAL INTEGRITY OF THE PAVEMENT

The structural integrity of a pavement may be defined as its ability to resist destruction and functional impairment in a particular traffic and climatic environment.

Under the combined destructive action of these elements several distress mechanisms develop within the structure and propagate either independently of each other or through interacting complex processes to produce eventually any or all of these broad groups of distresses: disintegration, distortion, and fracture (rupture).

In order to develop a thorough understanding of these mechanisms one has to trace the path from the time of initiation, through propagation to that of global manifestation.

Although it is realized that field behavior results from a series of interacting complex processes, and that all distress mechanisms must be analyzed in order to present not only a realistic but also a totally comprehensive picture of damage behavior, only the distress occuring as a consequence of mechanical loading has been investigated. The motivation for doing this stems from the fact that an extensive amount of work has been done experimentally on the modes of damage associated with mechanical loading. This makes it possible, to a certain extent, to adopt certain assumptions as to the manner of damage propagation in this mode of loading.

This study is presented in five broad sections. The discussion begins with a close study of the conditions governing the initiation, propagation and attainment of critical size of the defect area in engineering materials under arbitrary loading histories. It provides the background work necessary for the development of the cumulative damage model.

Having established the conditions governing the physical failure of engineering materials in general, the pavement system is investigated with a view to identifying the variables which affect its performance and subsequently bring about ultimate distress in a given environment. On the basis of the conclusions arrived at, a framework for cumulative damage is developed, and examined through computer simulation techniques.

II. REVIEW OF LITERATURE

An engineering structure can be modeled as a transfer function for which the input functions are determined by the environment where it operates and the output functions by some damage parameters. The determination of the transfer function of the system may be made either by a statistical analysis of the input and output functions of a large number of actual systems, or by using models derived from the principles of mechanics. The statistical approach was preferred in some cases (AASHO) but it is not satisfactory because it does not lend itself to the prediction of the behavior when new materials or new environments are used.

In all the mechanistic models for the transfer function, the output functions, which are also called limiting responses are related to the stress and strain fields within the system, so that the geometry of the structure is not an explicit variable. Stresses, strains and their derivatives are called the primary response of the structure and are used by all the mechanistic models.

The development of a mechanistic model for the transfer function of a highway pavement, therefore, requires:

a. Determination of Constitutive Equations of the Materials Making up the Structure, i.e. Characterization of the Materials.

b. Development of the Formulations Required for the Stress Analysis of the Structure. These Formulations will, of course, Depend on the form of the Constitutive Equations.

c. Definition of the damage parameters and development of the model which relates them to the primary response.

II.1 MATERIAL CHARACTERIZATION

II.1.1 Functional Relations

Studies of pavement materials show that linear relations are often merely a crude approximation of reality. In particular, granular materials were found to exhibit distinct nonlinear stress-strain relations (e.g. Biarez (2)). Similarly clays and other cohesive materials which are often used as subgrade materials change properties with age, load level, and duration of the load, so that linear elastic characterization is not always sufficient. Finally, both portland cement concrete and asphaltic mixtures exhibit aging and some other time-dependent behaviors.

A large amount of research has been devoted to the study of time-dependent behavior of asphaltic mixes. Most of the results show that for small enough strains the material can be characterized by the linear viscoelastic theory. Deviations from linear viscoelastic behavior are apparent for the high temperatures, long time response and large stresses or strains. Attempts were made to establish the limits of linear viscoelastic behavior (3). Under repetitive triaxial loadings, Terrel (4) observed marked nonlinear time dependent behavior. Under these conditions, uniaxial characterization of materials is not always satisfactory and a more general approach to the characterization of nonlinear time dependent materials is required.

In this context, characterization of a material means the determination of the Constitutive Equation, i.e. the properties of the material which relate the stress and strain tensors at each point in the material.

Some physical and mathematical requirements bound the arbitrariness of the form of a Constitutive Equation. The principle of causality limits the number of variables to be considered; the principle of determinism restricts the dependency of the constitutive equation to the past history of the particles of the studied body: and the principle of material frame indifference (5) states that the constitutive equation is form invariant to rigid motions of frame of reference. These are the basic principles. In the case of the so-called simple materials, the constitutive equation can generally be expressed as:

$$\underset{\sim}{T}(t) = \underset{\tau=\infty}{\overset{\tau=t}{\underset{\sim}{G}}} [\underset{\sim}{E}(\tau)] \qquad (1)$$

where τ is the independent variable which measures the history time $-\infty<\tau\leq t$, $\underset{\sim}{E}(\tau)$ is the history of the Lagrangian strain tensor and $\underset{\sim}{G}$ is the response stress functional of the material. $\underset{\sim}{T}(t)$ is the stress tensor. Similarly

$$\underset{\sim}{E}(t) = \underset{\tau=-\infty}{\overset{\tau=t}{\underset{\sim}{H}}} [\underset{\sim}{T}(\tau)] \qquad (2)$$

or

$$\underset{\tau=-\infty}{\overset{\tau=t}{\underset{\sim}{F}}} [\underset{\sim}{E}(\tau), \underset{\sim}{T}(\tau)]=0$$

are other possible forms of the constitutive equation. Through considerations of invariance, these functionals $\underset{\sim}{G}$, $\underset{\sim}{H}$ or $\underset{\sim}{F}$ can be restricted to certain forms involving specific combinations of the invariants of the tensors $\underset{\sim}{E}$ and $\underset{\sim}{T}$ (6). For definitiveness we will consider the Rivlin functional $\underset{\sim}{G}$ in what follows.

In the most general case the functional relation $\underset{\sim}{G}$ has 81 components. However, considerations of isotropy reduce these independent components to two, while sometimes five independent functionals are necessary such as for cross-isotropic axisymmetric systems.

In the review which follows we will limit ourselves to the uniaxial case where a single functional is to be determined. The various representations of nonlinear functionals will be reviewed. However, at the present time the experimental procedures for such determinations seem prohibitive, and the model which yield the primary responses would not be capable of handling the added complexities. Thus emphasis will be put on the development of a method of representation which yield the best first degree approximation of the Constitutive Equation in conditions simulating as much as possible the type of stress paths encountered in the field. Such necessity has been early recognized in studies of soil mechanics (7) and nonlinear elastic materials (8).

II.1.2 Mathematical Representations

Integral representations of constitutive equations are flexible to use and a variety of ways is suggested in order to express them. Leaderman (9) suggested that the nonlinear functional be represented by

$$\sigma(t) = \int_{-\infty}^{t} E(t-\tau)\frac{\partial g[e(\tau)]}{\partial \tau}\, d\tau \tag{3}$$

where $g[e(\tau)]$ is now a function of the history of the strain e and E (t) is the relaxation function. Similarly, one can also propose

$$\sigma(t) = \int_{-\infty}^{t} E[t-\tau,\ e(\tau)]\frac{\partial e(\tau)}{\partial \tau}\, d\tau \tag{4}$$

as a nonlinear constitutive equation, where the relaxation function is a function of $t-\tau, e(\tau)$. Note that the response of the material may be nonlinear even if $e(\tau) \cong \varepsilon(\tau)$, i.e., the strains are infinitesimal.

Many forms of nonlinear representations have been proposed, however their extension to the three-dimensional case often revealed their defectiveness because they did not satisfy the principle of material frame-indifference, i.e., the constitutive equation is not invariant when the frame of reference is given an arbitrary solid motion.

Among the properly invariant stress constitutive equations, the most widely used one is that of Green and Rivlin (10). It is an expansion into an infinite series of multiple integrals similar to the Taylor series expansion of a function. This expansion may be written as

$$y(t) = \int_{-\infty}^{t} h_1(\tau)x(t-\tau)d\tau + \iint_{-\infty}^{t} h_2(\tau_1,\tau_2)x(t-\tau_1)x(t-\tau_2)d\tau_1 d\tau_2$$

$$+\ldots+\int_{-\infty}^{t}\ldots\int_{-\infty}^{t} h_n(\tau_1,\ldots,\tau_n)x(t-\tau_1)\ldots x(t-\tau_n)d\tau_1\ldots d\tau_n$$

$$+\ldots \tag{5}$$

where x(t) and y(t), the input and output functions respectively may be a stress and a strain or vice-versa. $h_n(\tau_1, \ldots, \tau_n)$ are the kernel functions to be determined. A linear system will be represented by a single kernel $h_1(\tau)$.

The procedures used for the determination of these kernel functions may be divided into stress controlled and strain controlled experiments. Each group may be arbitrarily subdivided in turn into:

1. Transient Input (see Figure 1)
2. Sinusoidal Input (see Figure 2)
3. Random Input

The procedure used in the latter case is described in Reference (11). Since it uses random loadings, it can better simulate the field condition and it yields the best linear approximation for a nonlinear material. This is often desirable so as to simplify the primary model.

Transient and sinusoidal inputs are now commonly used for the characterization of linear viscoelastic materials. The characterization of nonlinear viscoelastic material is still impractical and very time consuming.

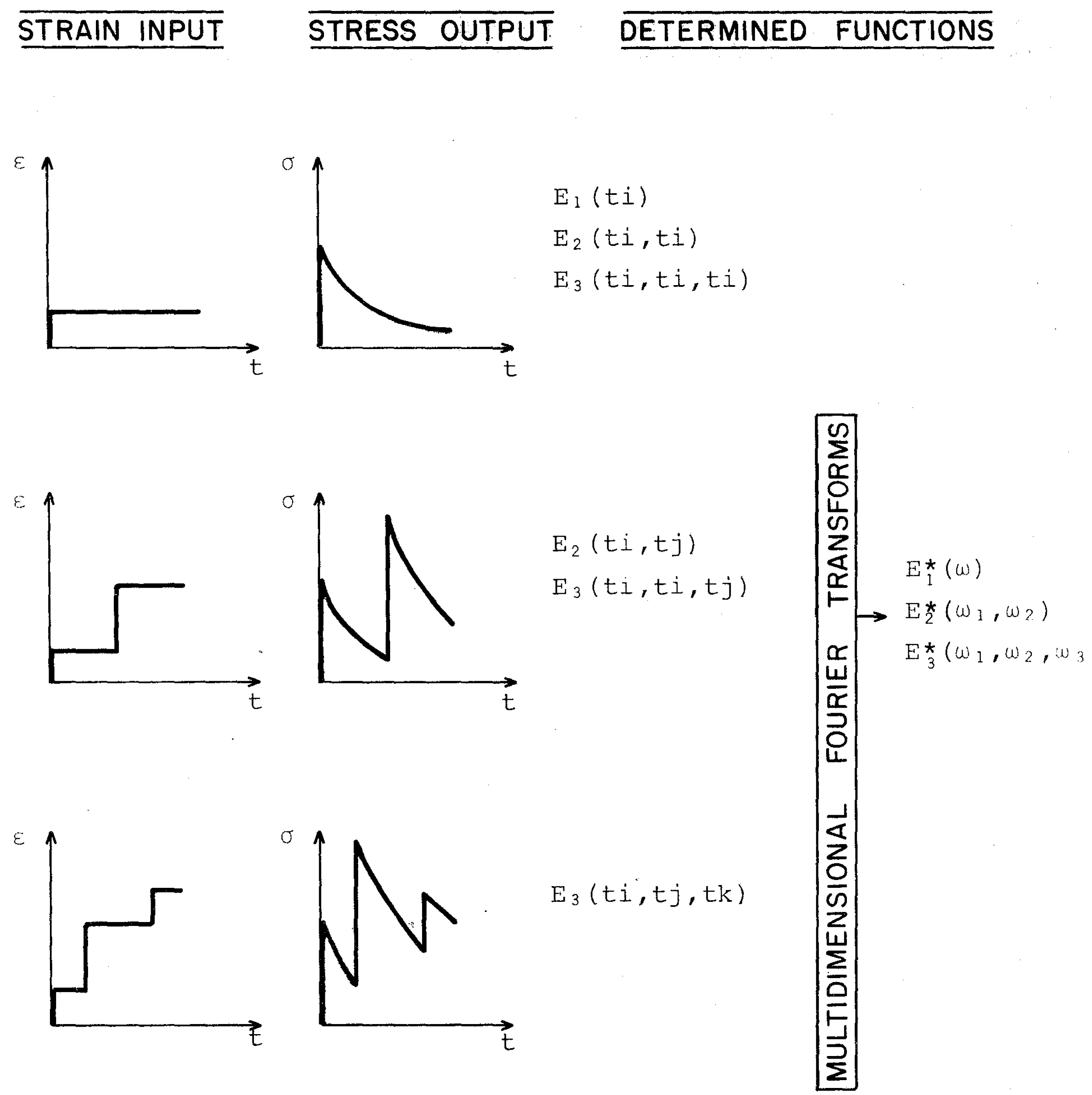


FIGURE 1: DETERMINATION OF THE FIRST THREE MULTIPLE INTEGRALS USING MULTIPLE STEPS INPUTS

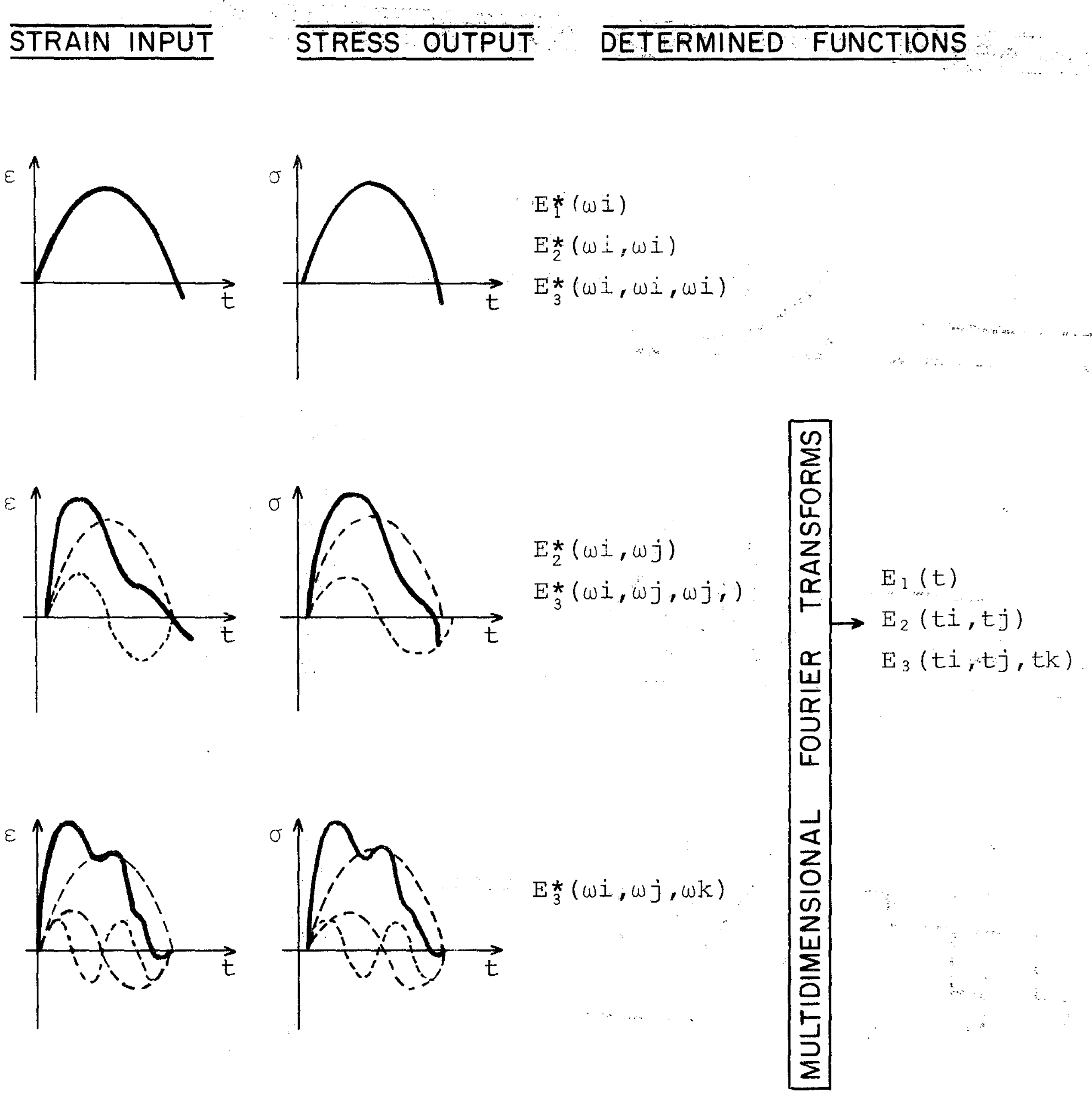


FIGURE 2: DETERMINATION OF THE FIRST THREE MULTIPLE INTEGRALS USING SINUSOIDAL INPUTS

The new method of determining nonlinear constitutive equations using random loading histories seems promising for various reasons:

a. It yields the properties in the form of an orthogonal expansion. This property guarantees convergence, makes the calculation easier and minimizes the error, because the error in the representation of a function by a finite orthogonal set of functions is the minimum square error.

b. Since a truncated representation of a nonlinear system depends on the type of input function which is applied, it is preferable that the input functions simulate as much as possible the expected type of input to which the system will be submitted later. Thus random testing may often be superior since it would approximate best the conditions to which the system may be submitted.

c. Extension of this approach to three-dimensional characterization seems also easier than the extension of the classical methods using multiple-step and sinusoidal inputs. This is due to the fact that the number of tests to perform does not increase dramatically but remains equal to one. The length of the computations will be, however, the limiting factor in such cases.

d. This method may also be applied for field testing by considering the whole structure as a single nonlinear system.

II.2 STRESS ANALYSIS

Once the geometry of a structure is defined and the Constitutive Equation of its materials assumed, mathematical models can be derived for the prediction of the primary responses. In the case of pavement systems, two types of constitutive equations have been assumed, one based on the linear elastic theory and the other on the viscoelastic theory.

The simplest model, the homogeneous half-space, is one in which no change in material properties occurs with depth or with horizontal extent. Boussinesq (12) solved the problem of a point load on a homogeneous half-space in 1885. His work was subsequently modified by various authors (13, 14), until Ahlvin and Ulery (15) presented a comprehensive table for the stresses and displacement at any arbitrary point in a half-space under a uniformly distributed circular load, for different values of Poisson's ratio.

The homogeneous half-space has, since, been used extensively by several investigators as that representing a layered pavement structure. The major shortcomings of such an assumption is its lack of capability to account for the presence of different types of materials in the strata which exist in a real pavement structure.

In order to account for these variables in the mathematical model, semi infinite bodies composed of distinctly different materials in layered form are used and the distribution of the stresses and displacement boundary conditions are analyzed.

Westergaard (16) developed the initial solutions for an elastic plate resting on an elastic subgrade which could undergo only vertical displacements or provide vertical reactions (a Winkler foundation). Since then, several modified forms of his method have appeared in the literature.

The first solution for both a two-layer and a three-layer system using the elastic theory was given by Burmister (17). He formulated the problem for N-layered elastic systems by assuming the existence of a stress function involving Bessel and exponential functions from which he was able to develop and present the general equations and solutions for both two and three-layered systems.

The use of a layered elastic system as a mathematical model for a pavement structure has been quite extensive, and several of the presently available design methods for flexible pavements are indeed based upon such analyses (18). This model is based on the assumptions that each layer is composed of materials which are isotropic, homogeneous, weightless and linear elastic. Also the boundary conditions at the interfaces assume a continuity of stresses or strains.

The linear elastic assumption is often unrealistic. Indeed, time dependent stress-strain relations are commonly observed in paving materials (19, 20). Thus the pavement structure is expected to be rate sensitive, and its behavior should depend upon its entire loading history.

The theory of linear viscoelasticity was found to be suitable for both the characterization of the paving materials and the development of mathematical models for stress analysis in pavement structures (21,22,23).

The viscoelastic models for the analysis of stress and displacement in a pavement structure differ from those of layered elastic systems only in the material characterization used for each layer. The geometry, boundary conditions and loading functions are exactly similar in the two models. Such similarities between the models have resulted in the development of a technique, known as the correspondence principle, whereby the solutions to the elastic problems can be used to obtain the viscoelastic solutions of the same problem.

The technique of solution was presented in a previous study (24) for the cases of stationary load, repeated load, and moving load. The viscoelastic method of analysis permits the computation of the primary responses of a three layer system under various types of loading histories. It can be extended to account for the stochasticity of the loading and environment histories. This method of analysis is restricted to materials having linear properties. The case of nonlinear properties is handled only by linearization of these properties. An important use of this method is in the study of the effects of changes in the different parameters characterizing the layered system.

II.3 DISTRESS AND FAILURE OF ENGINEERING MATERIALS

The intensive interest that has developed over the past several years concerning the accumulation of damage in engineering materials and structures has its roots in the following questions:

a. The problem of the life prediction of an engineering material or structure under an arbitrary load history in a given environment,

b. the amount and distribution of damage in the material or structure under the arbitrary loading spectrum mentioned above, and

c. the manner and rate of accumulation of damage.

This section presents a concept of damage by examining the processes of fracture and flow in solid materials. It describes what the damage is, how it manifests itself and which parameters can be employed to describe it.

Several observations are made about the distribution and propagation of damage within a material that is under an arbitrary loading history. Some well known theories and criteria which have been postulated for the failure of engineering materials are discussed. The damage of materials in a repeated loading environment is closely examined.

II.3.1 Mechanisms of Damage

Damage may be defined as a structure-sensitive property of all solid materials; structure sensitivity is imparted to it through the influence of defects in the form of microscopic and macroscopic cracks, dislocations and voids which may have been artificially or naturally introduced into the material, thereby rendering it inhomogeneous. Characteristically, structure sensitive phenomena involve processes which grow gradually and accelerate rapidly once an internal irregularity or defect size exceeds a certain limit. Damage may therefore, be said to occur in a similar fashion.

The progression of damage in an engineering material or engineering structure may occur under the application of uniaxial or multiaxial stationary or repeated loads. The damage progression has been categorized by two different conditions: ductile and brittle. The ductile condition is operative if a material has undergone considerable plastic deformation or flow before rupture. The brittle condition, on the other hand, occurs if localized stress and energy concentrations cause a separation of atomic bonds before the occurrence of any appreciable plastic flow. Note here that no mention is made of a ductile or a brittle material per se. According to van Karman (25), this implies that failure is not in itself a single physical phenomenon, but rather a condition brought about by several different processes that may lead to the disintegration of a body by the action of mechanical forces. Damage may therefore progress within a material under the different mechanisms of fracture and flow depending on the environmental stress, strain and temperature conditions. For instance, low carbon steel exhibits fibrous and shear types of fracture at room temperature, below -80°C brittle fracture occurs and integranular creep fracture is dominant in slow straining at 600°C and above (26). A material may, therefore, have several characteristic strength values, when several fracture mechanisms operate at different critical levels of the stress or strain components.

Though the mechanisms of damage initiation and propagation in the two failure modes are different, they have three major points in common:

a. A particular combination of stress or strain concentration is required to create a defect nucleus,

b. a different combination of stress or strain quantities is then required for the propagation of the defect and,

c. a critical combination of stress and strain concentrations is required for the transition from relatively slow to fast propagation to catastrophic failure.

The distribution and progression of damage in solid material is in itself a random process which is both spatial and temporal. Hirata (27) working on glass panes, and Joffe (28) on pyrex-glass filaments concluded that the distribution of internal cracks must be spatial by demonstrating considerable variability in the breaking strength values of these materials. Yokobori (29) in his investigation of the creep fracture of copper under a uniaxial load, demonstrated a considerable scatter in the values of time to fracture of a group of specimens taken from the same stock. Further evidence of these random processes is to be found in the works of Yokobori on fatigue fracture (29), creep fracture and ductile fracture (30) (31), brittle fracture and yielding in steel (32).

The above concepts of damage progression to failure suggest that an engineering material or structure can fail under a given system of external loadings when either of the following two criteria is satisfied:

The distribution of internal flaws is such that;

1. Excessive deformation is attained (usually for ductile behavior), or
2. a fracture threshold is reached under an arbitrary loading history (usually for brittle behavior).

From the foregoing discussion, it should be evident that the fracture of an engineering material is a statistical process brought about by the interaction of several complex mechanisms.

Accordingly, over the years several reasons have been advanced as explanations for the observed behavior of 'damage' in an engineering material, and based on such explanations several theories have emerged. Researchers have approached the problem both deterministically and statistically from the molecular and phenomenological levels. On the molecular basis, the differences between the fracture mechanisms involved are emphasized, since at this level, the material is essentially discontinuous.

On the macro level, the criteria for fracture are basically similar, and utilize the concepts of continuum mechanics. The fracture laws are generally based on either local or global energy, stress or strain concentrations within the material.

Theories like the Eyring rate process (33) developed for viscous materials, Knauss theory (34) for viscoelastic materials and Weibull's theory (35) for brittle materials have attempted to explain on the basis of a statistical model some of the phenomena observed when materials like metals, textiles, concrete and others fracture under applied stress. The basic assumption is that an assembly of unit damage processes grows in a probabilistic way to yield an observed macroscopic effect with temperature fluctuations and activation energy distribution playing a significant role.

Weibull (35) studied the manner that probabilistic postulation about the size of the specimen would affect the fracture strength of a material that fails in a brittle manner. He assumed that the probability of failure $P(\sigma_c)$ of a unit volume as a function of applied stress σ_c is given by

$$P(\sigma_c) = 1-\exp[-(\sigma_c/\sigma_o)^m]$$

where σ_o and m are constants dependent on material characteristics.

σ_o relates to some inherent ultimate strength
m relates to material inhomogeneity

Using this approach he showed that the strength of a specimen of volume V is proportional to $\sigma_o V^{-1/m}$.

Though this result has met with some success, Frenkel and Kontorova (36) claim that Weibull's approach is devoid of physical reasoning because of his assumption for $P(\sigma_c)$. They assumed that the specimen had flaws distributed in it in a Gaussian manner and obtained;

$$P(\sigma_c) = \frac{1}{\sqrt{2\pi}} \exp[-(\sigma_c - \bar{\mu})^2/2\nu^2 \qquad (7)$$

where;

$\bar{\mu}$ = mean strength of specimen of volume, V.
ν = strength variance of specimen of volume, V.

From the above equation for $P(\sigma_x)$ they determined that the strengths of specimens with volume V (with many flaws) is given to a first approximation by

$$\text{Strength} = \bar{\mu} - (2\nu)^{1/2} \sqrt{\log nV - 2V\Pi} \qquad (8)$$

where

$\bar{n}$ = number of flaws/cubic centimeter

Such statistical theories have a significant advantage over deterministic concepts, because they account for the role of chance in the behavior of materials.

II.3.2 Failure Criteria

From the phenomenological point of view failure criteria can be developed and applied to the material which is assumed to be continuous. These failure criteria may be suggested by the failure mechanism of the materials involved or they may be derived by trial and error.

A general failure criteria would predict the failure due to any type of loading history. As seen above such a unified criterion is not known for any material. Instead different criteria are developed to account for various classes of loading histories. This approach is used in number of engineering practices. For example a maximum shear criterion is used for parts of a structure subjected to a given stress field, while a maximum tensile stress criterion is used for other parts of the same structure. Following a rational mechanics approach it seems that a general criterion should involve combinations of stress or strain tensors invariants. Novozhilov (37) has given physical interpretations to these invariants.

Different failure criteria may be operating simultaneously. However for a given class of loading histories it is possible to determine a general envelope involving stress or strain invariants.

In practice some of the used failure criteria involve stress invariants such as the distortional theory

(38) or the octahedral shear stress theory (38) while others, such as the maximum shear stress theory (38) do not involve these invariants. Note that although these three criteria work well for metals and can be justified on a atomic scale because of the mode of crystal slip in a polycrystal, their application to the failure of other materials (sand, clay...) is questionable. For these granular materials frictional resistance is proportional to the hydrostatic pressure. Coulomb failure criteria is found to be more suitable for these materials. Coulomb postulated that plastic deformation will start on a slip plane through the material when the normal stress on the plane produces a frictional component which when coupled with the molecular cohesive strength of the material results in the shear resistance of the plane. The outcome of this was the Mohr-Coulomb theory (39), which was met with reasonable success in soil mechanics. Although, the criterion neglects the influence of the intermediate principal stress on failure, Bishop (40) and others have found it to be a satisfactory first approximation for three-dimensional situations as well. Some of the deterministic failure criteria try to account for the existence of defects or cracks in the material. This is namely the case with the Griffith theory (41) which states that the reason for the difference between the observed and calculated strengths is the presence of internal flaws. This theory predicts the strength of material for a given maxi-

mum crack size when the surface energy and the elastic constants of the materials are known.

The Griffith theory has been used quite successfully to predict the occurence of brittle fracture in many materials among which are glass, cast iron, rocks, asphalts and polymers. This theory applies best for a purely brittle material.

Another related approach is Irwin's theory (42) which define a stress intensity factor K related to the crack size, the geometry of the structure and the loading condition. This factor is limited for design purposes by the critical stress intensity factor for which a crack starts to propagate. This K factor is defined for three different modes of crack propagation called: K_I for the opening mode, K_{II} for the edge sliding mode and K_{III} for the tearing mode. Irwin's theory is closely related to Griffith's theory and is successfully applied to the design of brittle materials. This approach was used in the design of flexible pavements (43). A generalization of this concept to time dependent materials (93) is the most promising method of analysis of crack propagation in time dependent materials. This method of analysis is still limited by the difficulty of determination of the stress intensity factors in three dimensional problems. Moreover the method of analysis is still limited to the case of a constant Poisson's ratio, and the computations are still too lengthy to be practical. The method of analysis does not account for crack healings. For time-dependent materials a further complexity in the usual methods of analysis is brought about by the time variable. The above mentioned enveloped become dependent on the time parameter. Most of the failure envelopes determined for time dependent materials correspond to uniaxial loadings. These envelopes are characterized by an idetermination in their values, i.e., statistical variations are the rule rather than the exception. Also failure envelopes corresponding to long periods of loading (creep type) generally show more variations than failure envelopes corresponding to short times of loading.

Examples of such envelopes are a log stress at failure vs. log strain at failure envelope suggested by T. Smith (44) for constant rate of loading histories, and stress-time envelopes described in Bartenev and Zuyev (45) obtained from creep tests.

Thus in the most general case a failure envelope is a limit involving combinations of the strain or stress tensor invariants, and possibly the time:

$$F[I_1(\tau), I_2(\tau) \Big|_{\tau=-\infty}^{\tau=t}, I_3(\tau), t, \text{temperature, etc...}] < K$$

Many simplifications are necessary in order to give this relationship a tractable form. In particular the important simplification of neglecting the influence of the past histories of I_1, I_2, I_3 may be achieved as will be seen later.

II.3.3 Parameters of Damage in the Repeated Loading Mode

In many materials, the initiation, progression and ultimate manifestation of distress in the form of fracturing under a repeated load occurs under the action of two separate processes: crack initiation and crack growth which are governed by different criteria. In metals, this behavior has been attributed to localized slip and plastic deformation (46), and to the cyclic motion of dislocations. In polymers and asphaltic mixtures, the cracks initiate from air holes, inhomogeneities and probably molecular chain orientations and molecular density distributions (47).

The mechanism of crack propagation has been explained by many researchers from a consideration of the energy balance at the crack tip which deforms as cycling progresses. The propagation is slow when a considerable amount of plastic deformation occurs at the crack tip which as a result of this becomes "blunted". It is fast when the released portion of the stored energy exceeds the energy demand for creating new surfaces.

Erikson and Work (48) discovered that the history of load application had a significant influence on the progression of damage. On the application of a high prestress followed by a low stress the degree of damage created was greater than when the application was vice versa. The authors explained this occurrence by suggesting that on the first few cycles of load application, a certain number and distribution of crack sites form depending on the stress level, and the application of subsequent loads merely causes propagation from these sites.

In order to handle the problem of life prediction for any material in a given repeated load environment, several molecular and phenomenological theories have appeared in the literature.

a. <u>Molecular Theories</u>: In some of the molecular theories developed the statistical mechanics principles and kinetic reaction rates concept have been utilized. Coleman (49) and Machlin (50) employed the Eyring rate process theory to study respectively the fatigue characteristics of nylon fibers and metals.

Coleman's theory implies that for every material a constant strain level exists at which fracture will occur, but a variety of experimental results shows that this is not the case. Moreover, it does not account for progressive internal damage as only failure conditions are represented. Mott (51) and Orowan (26) have presented fatigue theories for metals which take into account the fact that plastic deformation and strain-hardening occurs during fatigue. Mott's theory attributes the formation of microcracks to the occurrence of dislocation within the material. Orowan's assumes the presence of a plastic zone within which a crack forms and propagates. Both these theories have given good agreement with experimental results at times. The discrepancy observed is mainly due to the fact that damage is a stochastic phenomenon while the theories are deterministic in nature. To increase their accuracy a statistical approach is needed.

b. Phenomenological Theories: Despite the fact that several molecular mechanisms have been shown to be operative during fatigue growth in a material, one suspects that the process itself may not be that fundamental in nature. Therefore, instead of searching for molecular theories, a possible coherent picture can be found from the continuum mechanics approach (with certain reservations).

This has been the motivation behind several phenomenological theories of cumulative damage - the Miner theory (52), Corten and Dolan's (53), and Valluri's (54) to name a few. The underlying concept in these theories can be illustrated by the work of Newmark (55).

In this approach, it is assumed that when a material is in a given load and climatic environment the degree or percentage of internal damage F_i is at any time commensurate with the appropriate number of load repetitions - N_i. (i.e., for $0 \leq F_i \leq 1$, $0 \leq N_i \leq N_f$) With this assumption, a damage curve exists for every constant stress or strain repeated mode of loading. Since damage in effect implies that a loss in original capacity can result from either the creation and growth of plastic zones or the initiation and propagation of cracks, the strain developed in the material under load, the crack length, and the rate of crack growth can all be used as damage determinants.

The above reasoning is currently utilized in constant amplitude stress or strain fatigue tests.

Generally speaking, the combined effects of damage and recovery processes resulting from microstructural changes imply that the damage curve should have different forms for different stress levels and loading histories. Some damage theories, such as those of Miner and Williams, assume a unique degree of damage caused by a stress cycle-ratio $(\frac{n}{N})$ applied at any time. Williams' theory (56) makes a similar assumption but with respect to time-ratios $(\frac{t}{T_f})$, where t = elapsed time from start of experiment, and T_f is time to failure. In both theories a linear summation of the ratios results, at failure, with the following expressions:

$$\text{i.e.} \quad \sum_{i=1}^{n} \left(\frac{n_i}{N_i}\right) = 1 \text{ (Miner) and}$$
$$\sum_{i=1}^{n} \left(\frac{t_i}{T_{fi}}\right) = 1 \text{ (Williams)}$$

with;

n_i = number of cycles applied at stress level-S_i
N_i = number of cycles to failure at stress level-S_i
t_i = elapsed time of application of strain rate level-R_i
T_{f_i} = elapsed time to failure at strain rate level-R_i

Both theories have a major shortcoming in that prior history and sequence of events cannot be accounted for. Despite this shortcoming, Miner's theory has been successful when applied to rate-insensitive materials. Williams' theory had similar success when used for rate-sensitive materials. In Corten and Dolan's theory (53) the damaging effect under a stress cycle is considered dependent on the state of damage at any instant, and the expression for damage is

$$F_i = mrN^a \tag{9}$$

where;

N = number of cycles

r = coefficient of damage propagation which is a function of stress level

a = damage rate at a given stress level which increases with number of cycles

m = number of damage nuclei

The Corton and Dolan approach is a rational attempt to modify Miner's theory. The determination of the significant parameters 'm' and 'r', however, requires the performance of a considerable number of experiments. In the simplest case, where the loading history can be characterized by two sinusoidal stress conditions, the parameters 'r' and 'm' may take on many different values depending upon the absolute magnitudes of the two sinusoids and their relative relationships. In addition, rate effects cannot be adequately accounted for. Consequently in terms of usefulness over a wide class of materials and circumstance, the Miner theory is preferable.

Several researchers (57,58,59) have related the rate of crack growth to the localized energy and elastic stress conditions existing at the crack tip. The expression obtained when this fracture mechanics approach is used can be given in general form, as

$$\frac{dc}{dN} = Ac^{k}\sigma^{\ell} \tag{10}$$

where;

A, k, ℓ are constants and

c = crack length,
σ = stress at tip of crack,
N = number of load cycles.

The constants k, ℓ, are dependent on the properties of the material tested, and on the boundary conditions of the problem in question (57,58,59). Liu (60) noted that when $\ell=2.0$ and $k=1.0$, the plastic zone-size ahead of the crack tip is small in comparison with the crack length and specimen thickness. Paris and Erdogan (58) found that the use of values 2.0 and 4.0 for k and ℓ respectively yielded good agreement with experimental results. Paris (61) by considering the energy dissipated per cycle of load application as being proportional to the plastic zone-size ahead of the crack tip found

$$\frac{dc}{dN} \sim \frac{dW}{dN}$$

where

$$\frac{dW}{dN} = A_1(\Delta K)^4 = A_1 c^2 \sigma^4 \tag{11}$$

W = energy dissipated
A_1 = constant

c = crack length

σ = stress at crack tip

K = stress intensity factor = $\sqrt{\sigma^2 c \Pi}$

It is evident that these analyses have attempted to take rate effects into account in an indirect manner. When conditions of fracture are brittle in nature, then these analyses are accurate. This concept was used to study the crack propagation in asphaltic beams (43). However, in the presence of tearing action, the properties of time dependent material change with time and thereby the coefficients of the above formulas are no more constant. Indeed in reference 43 they are shown to be very sensitive to temperature changes. Since temperature changes are qualitatively equivalent to time variations, one can deduce that they are also dependent on the rate (or frequency) of application of the loads.

A limitation of this approach is that similarly to Miner's law it implies the concept of "cycles". Therefore it cannot account for cracks due to static fatigue. It cannot also account for the effects of mean loads, nor does it account for the possibility of crack healing. Despite these shortcomings, analyses of this type are attractive in the sense that the fatigue process has been linked to microphenomena on a phenomenological basis. This approach works also reasonably well if the considered loading histories are restricted to specific classes of histories.

In order to take rate effects, order effects and prior history effects into account Dong (62) postulated a cumulative damage theory to predict the life of a material under any arbitrary loading history. His assumptions were:

1. The material is undergoing an arbitrary loading history,
2. life has full value at zero history and zero value at failure,
3. temperature conditions are isothermal, and
4. damage and recovery processes can be accounted for.

The mathematical expression obtained is

$$\ell(t) = f[\gamma_{ij}(\tau)]_{\tau=-\infty}^{\tau=t}$$

where;

$\ell(t)$ = life remaining in the material at time t after damage has accumulated during $-\infty \leq \tau \leq t$,

τ = generic time,

t = present time,

γ_{ij} = any set of variables that can be used to describe loading history,

$f[\ ]$ = damage functional.

This theory can account for the effect of prior history and sequence of events in damage behavior because the damage functional represents an infinite series expansion of hereditary integrals of the linear and non-linear type.

$$\ell(t) = L + \int_{-\infty}^{t} \beta_{ij}(t,\tau)\gamma_{ij}(\tau)d\tau$$
$$+ \int_{-\infty}^{t}\int_{-\infty}^{t} \beta_{ijkl}(t,\tau_1,\tau_2)\gamma_{ij}(\tau_1)\gamma_{kl}(\tau_2)d\tau_1 d\tau_2$$
$$+ - - - + - - - \qquad (12)$$

where;

L = life at zero history,
t = present time,
β_{ij} = linear damage kernels,
β_{ijkl} = nonlinear damage kernels, and

the integral expression represents cumulative damage which is zero for $t = 0$ and equal to $-L$ at failure, when $t = t_f$.

The damage behavior of any rate-dependent or rate-independent material can be predicted under any arbitrary loading history using this approach. In the fatigue loading mode however, it can be shown that when a special form is chosen for a linear kernel β_1, the Miner and Williams theories are recoverable (69). This shows that the inability of Miner and Williams' theories to demonstrate the influence of prior history and sequence of events on failure is due to the restrictive form of their damage kernels.

The above discussion on the damage created within a material in the repeated loading mode lends much credence to the general proposition that its manner of accumulation is a consequence of the fact that engineering materials and structures are inhomogeneous. Under load, various regions of stress concentration exist within the material, and because of its inhomogeneous nature, a distribution of strengths is created such that some regions are weaker than others. When the strength of a weak region is exceeded it is quite possible that a crack may initiate and cause a redistribution of stresses with attendant crack formation in other regions. As the load is repeatedly applied they propagate and grow to a size which eventually renders the material or structure unserviceable. When this event occurs, fatigue damage is completed.

II.3.4 Summary

In light of the several important observations that have been made in regard to cumulative damage in the repeated loading mode, there are a few substantial points to remember in the course of developing a cumulative damage theory for any material or structure:

a. Damage is a function of the inherent inhomogeneity of materials and structures; its initiation, progression, and attainment of a critical magnitude are therefore stochastic processes.

b. For a given temperature, damage sites are nucleated under unique stress or strain conditions within the material. They propagate under stress or strain states different from initial conditions until a critical state is reached.

c. The state of damage at any time is a function of the material property and load history. i.e., damage is not unique, it is a function of stress level and microstructural changes within the material.

d. Assumptions have to be made regarding the manner in which damage propagates, and regarding the parameters utilized to delineate its progression. The damage surface is essentially exponential in most materials, but the characteristics of the surface have to be determined from the stress and microstructural conditions existing in the material under load. For instance, if stress, strain, time and temperature conditions within the material are such that brittle fracture is warranted, then the rate of damage accumulation is one of fast growth to failure. If a tearing kind of fracture is warranted, gradual accumulation of damage is experienced.

The next question that arises in view of the basic premise of this investigation is the possibility of developing a cumulative theory of damage for pavement structures composed of different engineering materials, utilizing the basic concepts of damage progression presented above. Such a theory may be able to bridge the rather wide gap between states of loading in the field and the relatively simple experiments on a mathematical model of the structure. The ability of the structure to adjust itself to these loadings should yield in symbolic terms, with some degree of reliability, the relationships between the external loadings and the physical constants which measure the competence of the system. To do this, it will be necessary to know how a pavement fails in practice. The information thus collected must be interpreted in the light of the failure mechanisms governing the performance of the materials comprising the pavement. When this is done, an adequate failure theory will begin to emerge. To this end the performance of a pavement structure in a repeated loading environment is examined in the next section within the context of internal damage development.

III. DAMAGE AND DISTRESS IN HIGHWAY PAVEMENTS

III.1 INTRODUCTION

An engineering system in which damage or the failure of a component results in a decrease in the level of performance rather than the abrupt incidence of total failure may be called a "Structure-Sensitive System". For these systems, internal damage develops within an operational environment, over a given time period, and the failure is the ultimate condition which results from a loss in performance. Failure is, thus, the extent of the damage which has been accumulated as a consequence of structural deterioration over a range of stress, strain, time and temperature conditions in an operational environment.

The performance level of a structure-sensitive engineering system in an operational environment may be defined as the degree to which the stated functions of the system are executed within the environment. This level at any point in time is dependent on the history of the magnitude and the distribution of the applied load, the quality of the construction and the materials used, their spatial distribution, and the extent to which proper maintenance practices are executed over the entire life of the facility. The reliability of the system consequently depends upon the above parameters which are in turn dependent on the variabilities of nature. In other words, the performance level of the system at any instant has a probability of occurrence and a frequency distribution of values associated with it.

Figure 3 illustrates that the performance of the system diminishes in some way until an unacceptable level is attained. This behavior occurs as a result of the combined action of the load and the weather in a given environment. In this environment, pockets of local distress develop within the system (e.g., stress induced inhomogeneity, corrosion), propagate in a manner which depends upon the composition and spatial distribution of the structural materials and bring about a loss in structural integrity with time.

The base level VP in Figure 3 represents an unacceptable level of performance, as determined by the users and engineers of the facility. The time within which the performance drops to this level characterizes the time at which the extent of damage in the structure becomes intolerable. At any instant of time t_i, P_i represents a level of performance, and associated with it is the degree of damage D_i developed within the period $t = o$ to $t = t_i$. At time zero, the performance of the system is presumably a hundred percent of its initial value since it has not yet been put into use. At the time when the internal damage becomes intolerable the performance of the structure is zero percent of its initial value. The integrity level of the system at any time is, therefore, one minus the amount of damage accumulated within that time

$$P_i(t_i) = 1 - D(t_i) \quad (1)$$

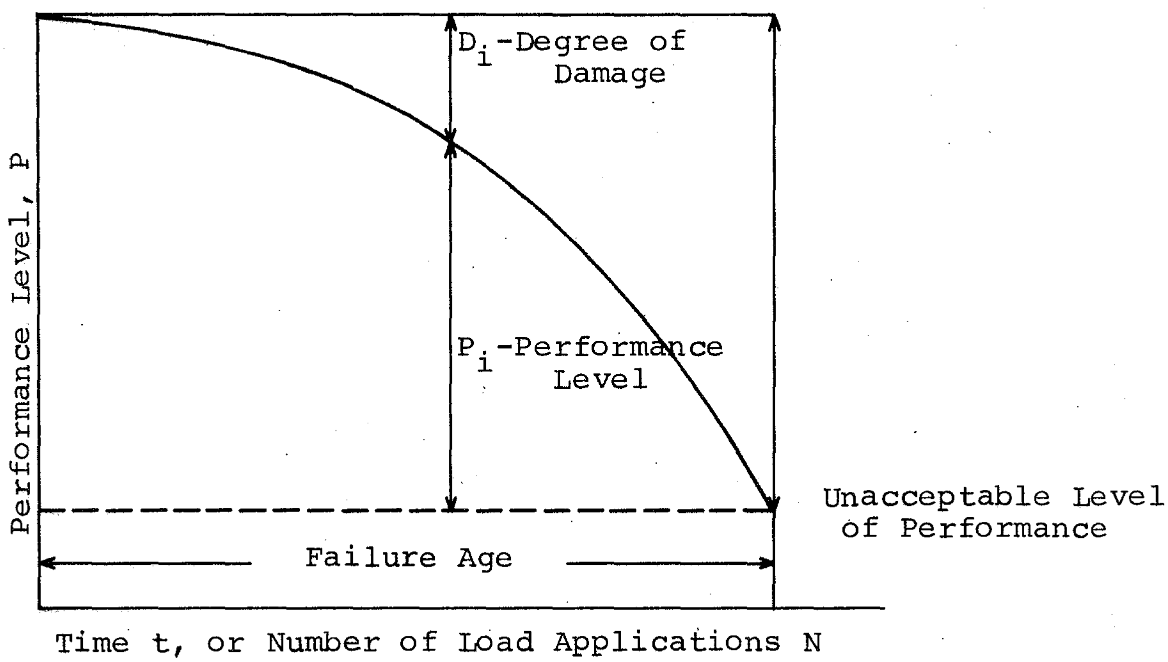


FIGURE 3: TWO DIMENSIONAL SIMULATION OF THE PERFORMANCE OF A PAVEMENT STRUCTURE

In view of the previous discussion on damage it is obvious that the quantities P_i and D_i are probabilistic in nature. Thus, depending on the temporal and spatial distributions of damage within the structure the real instantaneous performance level will be on, below, or above the drawn curve. This means that each point on the curve has a probability of occurrence and a frequency distribution of values associated with it and this fact must always be acknowledged.

The preceding discussion was conducted in the two-dimensional domain, the real picture is, however, more complex. The observed response of the structure depends on rate effects (19) (63), the position and magnitude of the applied load (20), climate (71), materials type, previous traffic history, temperature (20), and constructional variables (65). It can therefore be linear or non-linear depending upon the manner in which these variables combine. If the system behavior can be characterized as linearly or non-linearly elastic, plastic or viscoelastic, the response will have similar characteristics. Then at any point t_i in time a 'performance surface' which is a function of these variables exists such that its inverse is a 'damage surface'.

Within this context, at any instant, and at some point on the surface, a prediction of the percentage of total life already used and that remaining within the structure can be made. Therefore, the damage surface as well as the performance surface is n-dimensional with construction, maintenance, load and environmental parameters playing a significant role in its determination. The development of such a surface is not immediately possible. This, however, does not mean that the problem is intractable, because the possibility of reducing 'n' may exist. In fact such a technique is used in the development of yield surfaces for metals (66).

All the significant factors that have a role to play in internal damage progression can be generally accounted for, providing that they are translated, through the properties of the layer materials and the response behavior of the pavement structure for a given quality of construction and maintenance operations, into stress and strain quantities. In other words, the magnitude and type of the stress and strain concentration (tensile or shear) within the pavement structure is a function of not only the characteristics of the applied load but also of the spatial distribution of layer material properties and local defects. A knowledge of the material properties yields information on the kind of structural response to expect. From such information postulations can be made about the manner of internal damage progression. This technique takes into consideration the two most significant structural properties - material properties and response behavior - which reflect the influence of all the others. It can therefore be used to classify pavements into three broad groups - the frictional group, the flexural group and the frictional-flexural group - so that the stress-strain parameters of damage progression in each group can be identified.

The frictional type pavement is composed of granular materials in which load transfer occurs at interparticle contact points by purely frictional action. The deformation that takes place under load is purely of the shear or flow type, and for each application of the load a permanent deformation results. Such pavement structures, generally require a thin type of wearing course which can deflect conveniently with the rest of the structure under repeated loading. In order to protect the underlying materials, the wearing course should possess good ductile properties as opposed to brittle properties since toughness in this case is more important than tensile strength. However, when the deformation becomes excessive, cracks may appear in the surface due to the randomly distributed cumulative shear action in the subgrade. Therefore, in a frictional-type pavement, damage can be considered to develop as a result of shear action. Consequently the damage parameter must somehow be associated with shear stresses and shear strains.

In a flexural type pavement, the materials in the layers are capable of resisting the applied load through the action of tensile stresses which develop as a result of the flexing action. This implies that bending is the only mode of deformation and upon the repeated application of load, repeated flexing results. In such a pavement, fatigue action is important, and though the overall shear support of the components is adequate, cracks develop very early due to the accumulation of tensile strains. These propagate slowly or rapidly in a random manner depending on the properties of the layer materials and the rate of the repeated flexing action. The fatigue properties of the materials in the layers are therefore a prime concern during the design stage of such facilities. Damage in such pavements is propagated in the fatigue loading mode under the action of tensile stresses and tensile strains.

The third type of pavement possesses both frictional and flexural materials. Its structural integrity under repeated load is impaired by the destructive tensile and shear action that is manifested within the layer components. It is conceivable that if one action tensile or shear should dominate in creating damage within the structure, the failure would occur in that mode. On the other hand, it is also possible that both actions may play a significant rôle during the life of the facility depending upon the environmental conditions. The damage parameter is, therefore, associated with both tensile and shear stresses and strains.

The above classification of pavements accounts for almost all types of pavements currently in existence (67). It also makes possible the tractability of the damage progression within such structures. One can generally say that the damage 'build up' occurs in three different modes. When the behavior of the pavement structure is completely frictional, damage initiates and progresses by plastic or shear flow until the appearance of surface cracks terminates or aggravates the situation. When flexural behavior is pertinent the damage, initiation and progression occur by the development and growth of internal cracks, under the action of tensile stresses and strains. However, in the frictional-flexural type of pavement, the damage initiates and progresses by shear flow and/or by crack growth. Consequently a pavement structure may show signs of distress either from the independent action of excessive deformations, the isolated action of 'fatigue', or from both failure mechanisms working together. This indicates that in order to analyze the response behavior of a pavement structure and predict the failure behavior, a number of models which would account for such behavior in a given traffic and climatic environment should be developed.

At the present time, three such models seem to be appropriate:

a) A model is needed for the representation of the linear and non-linear behavior of paving materials,

b) the pavement system must be modeled in terms of the geometrics of the applied load and the structure so that the utilization of the former model within such a framework will aid in the prediction of the developed stresses and strains in a given environment and,

c) a model that must be capable of handling linear and non-linear damage behavior.

Finally, in order to achieve realistic predictions, it is essential to combine these models in a probabilistic manner, since the progression of damage as has been demonstrated is stochastic in nature.

Since the objectives of this study are to provide a better understanding of mechanisms of damage and distress in pavement structures, the remainder of the discussion is devoted to the item c and the stochastic nature of the problem.

III.2 MODELS FOR PAVEMENT DAMAGE

The pavements discussed in this section belong to the frictional-flexural group and are therefore representative of many current pavement sections.

In this type of structure 'fatigue' damage may occur in the surface layer when it behaves in a flexural manner.

The occurence of fatigue in pavements has been observed or noted for a considerably long period of time. Porter (68) in 1942 observed that pavements do in fact undergo fatigue. In 1953, Nijboer and Van der Poel (69) related fatigue cracks to the bending stresses caused by moving wheel loads. Hveem (70) also correlated the performance of flexible pavements with deflections under various repeated axle loads. The AASHO & WASHO tests (71) confirmed these observations by relating the cracking and initial failure of pavements to repeated loading of the type discussed by Seed et al (72).

The field observations of this kind of behavior led to the laboratory investigation. Many researchers have conducted laboratory experiments so as to determine the fatigue properties of paving materials and to investigate the possibility of extrapolating laboratory results to existing field conditions. To this end, Hennes & Chen (73) conducted tests on asphalt beams resting on steel springs and subjected to sinusoidal deformation with a variety of constant amplitude magnitudes. They discovered that as the frequency of application is increased, the creep-rupture compliance of the material decreases. On the conduction of similar tests by Hveen (70), on beams cut from actual pavements the same results were obtained.

Monismith (74) in his tests on asphalt beams supported on flexible diaphragms mounted on springs under constant stress amplitudes discovered that increases in the stiffness of the material resulted in corresponding increases in fatigue life. Sall and Pell (75) conducted similar tests from which the tensile strain to failure (ε_T) versus the number of cycles to failure, or fatigue life (N_f) relation, was found to be $N_f = 1.44 \times 10^{-16} (1/\varepsilon_T)^6$. They further found that this expression does not vary with temperature, rate of loading and type of asphalt. These results are not surprising since one should expect such factors to affect the developed stresses and not the strains through the stiffness of the material. For the mixtures tested, no endurance limit was observed up to 10^8 application, as is to be expected, since the mode of failure is one of crack initiation and propagation to failure at each stress level. The general conclusion arrived at by several authors from such tests indicate that the fatigue life of an asphaltic paving material is a function of several variables - the tensile strain level to which the specimen is subjected, the amount of asphalt, the age of the mixture, temperature, the stiffness of the mixture, its density and void ratio.

Another important factor in such tests is the mode of loading. In controlled stress tests for example, fatigue life increases not only as the stiffness of the sample increases, but also as the temperature decreases. However, in strain-controlled tests, the fatigue life decreases as stiffness increases; for this test, at low temperature no change is observed in fatigue life and as temperature increases the fatigue life increases as well (76),(77),(78). Controlled stress and strain behavior can be explained from a consideration of either the time-temperature superposition principle or the amount of energy stored in the sample when such tests are performed. In controlled stress tests the minimum energy stored per load repetition can be achieved by minimizing deflection and causing a resultant increase in fatigue life. In controlled strain tests the reverse is true. This implies that for a specimen of a given initial stiffness and initial strain, failure under a controlled stress mode of loading will occur sooner. Therefore, when extrapolating laboratory results to field conditions such considerations have a significant role to play. Monismith (79) through the use of a mode factor suggested that for surface layers less than 2" thick, the controlled strain mode of loading results, while for those layers 6" thick or greater, the controlled stress mode of loading is applicable. For thickness between these, an intermediate mode of loading is appropriate.

Tests have also been performed on granular and other paving materials so as to determine the significant characteristics of their behavior under repeated loading (72) (80). Although the approach is empirical it, however, points up the important fact that the response of granular and treated materials in pavement sections depends upon the characteristics of the applied loading, the material and the existing confining stress.

In an attempt to apply the experimental results to predict the occurrence of damage in a real pavement Deacon and Monismith (81), suggested a modification of the Miner's theory of linear summation of cycle ratios. They point out that such an approach has the desirable features of procedural simplicity, and a wide range of applicability to different types of compound loading. Their analysis, however, is rather difficult to interpret, also, the sequence of events and prior history cannot be accounted for in such an approach.

Majidzadeh et al. (82), in their recent work have shown that crack initiation and propagation in asphaltic mixtures can be predicted using the fracture mechanics approach. They have found that the critical stress intensity factor which is a function of material's elastic properties and the driving force at the tip of the crack is an inherent property of the asphaltic mixtures at low temperatures. Since the crack geometry affects the value of the stress intensity factor, the authors suggest an analytical technique for its calculation for some practical cases.

III.3 STOCHASTIC NATURE OF DAMAGE

Factors contributing to the initiation, propagation and accumulation of pavement damage can be divided into three categories; a) material properties and pavement geometry, b) loading variables, and c) climatic conditions. A substantial variability is associated with the measurement or specification of each of these factors and as a result the pavement response is stochastic. To account for these variabilities, the damage model should have the capability of yielding statistical estimates of the pavement response. In otherwords the model should be able to estimate the probability that damage will occur in a certain mode. To achieve this use can be made of simulation techniques. The following discussion describes the possibility of using one such technique - Monte Carlo Method - to predict the stochastic behavior of damage accumulation.

III.4 MATERIALS AND ENVIRONMENTAL VARIABILITIES

The behavior of a material in a given environment can be represented by a set of responses R_i. The material itself is defined by a set of relevant properties Y_k, and the environment could be specified by a set of conditions X_j.

In deterministic approach, it is usually assumed that there exists a functional relation between each response term and the associated material properties and environmental condition. Material properties will also vary systematically with environment. So;

$$R_i = g_i[Y_1, \ldots, Y_k, \ldots, Y_L, X_1, \ldots, X_{ji}, \ldots, X_n] \quad (2)$$

$$Y_k = \nu_k[X_1, \ldots, X_j, \ldots, X_n] \quad (3)$$

However, both material properties and environmental conditions are subject to considerable variability in a random manner over fairly wide ranges. When environmental conditions are correlated, that is, when there is an interaction between these parameters, such as the interaction of moisture and temperature and the effect of one on the other, their joint frequency distribution $f(X_1, X_2, \ldots X_n)$ will yield the density function as the necessary data to be input for the environmental conditions. If they are not correlated, their independent frequency distributions could sufficiently define the environment. The vectors X_j are therefore treated as random variables with probability density functions f_{x_j} and associated cumulative distributions F_{x_j}.

Material properties are inherently variable and the terms Y_k are also considered to be random variables with density functions f_{Y_k} and cumulative distirbutions F_{Y_k}.

Since the material properties are dependent on the environmental conditions, statistical correlation is implied by equation (3). Complete information of inherently correlated material properties can be given by the joint density function $fY_1, Y_2, \ldots, Y_L$ rather the density functions f_{Y_k}.

Variability in material properties and environmental conditions implies variability in the material behavior or response. Variability in material behavior is represented by a set of density functions f_{R_i} or alternatively by the cumulative distribution F_{R_i}.

To evaluate f_{R_i}, data should be available about the density functions f_{x_j}, f_{Y_k}. Even if these density functions are somehow evaluated, considerable difficulty can arise in determining f_{R_i} by analytical methods. Such difficulties can be encountered if f_{x_j}, f_{Y_k} are not normal and the equations giving the functional relations between the three basic vectors are not linear. In these cases a numerical solution can be obtained by Monte Carlo method.

III.5 MONTE CARLO SIMULATION

The Monte Carlo technique is a simulation method for the evaluation of the cumulative distribution F_{R_i} in an algorithmic form suitable for computer programming (83). The method is probabilistic in its approach and considers a conditional probability distribution of the form.

$$F_{Y_k|x_j} (Y_k \leq y_k | X_j = x_{j\ell}), \; j = 1,2,\ldots p \qquad (4)$$

where $x_{j\ell}$ is any set of values of X_j from populations with cumulative distributions F_{x_j}. Similarly the inherent interdependence of material properties can be taken into account in the same algorithm, i.e.,

$$F_{Y_k|x_j} (Y_k | X_j) \cdot F_{x_j} (X_j) = F_{Y_k} (Y_k) \qquad (5)$$

The algorithm involves an interation process from which a number n of sample is drawn for the values of R_i. From the sample of n simulations, histograms, means, variances and percentage points can be obtained. If the number n of the simulation is very large, the histogram can accurately represent the continuous distribution of the parent population.

This simulation method is a simple numerical method which gives statistical answers to specific problems which are not amenable to analytical procedures due to their inherent complexity and interacting factors. This method is approximate in nature, but quite good accuracy can be achieved if the number of simulations is sufficiently large. The sufficiency conditions here depend on the statistical data available or required.

In this case the decision as to how many samples to be drawn out should be preceded by something like sensitivity studies.

Several techniques have been developed based on such studies, these are basically variance reducing techniques to increase information in the "Interesting Regions" of the distribution F_{R_i}, and hence to decrease the information in the <u>non-interesting</u> regions or ranges.

The other factors that have an influence on the cumulative frequency distribution are the probability density function of the parameters involved, i.e., the environmental variables and the material properties, their interactions and correlations. In case of interacting parameters, a joint density function can be used instead of the single density functions.

The probability density functions of the different parameters that are being simulated can generally be obtained by some statistical tests. Sampling from the real, statistically measured distribution is preferable to obtaining samples from assumed distribution, because the more realistic these density functions are, the better are the results of simulation. However, in the absence of the statistical data for density functions of the parameters under consideration, special care should be given in assuming such density functions. This could be done by looking into the literature for statistical representation of the same or similar parameter.

III.6 SUMMARY

The results of the review of literature on pavement damage due to the mechanical loading indicate that; in order to develop a general damage function for pavements, the pavement system may be considered as a black box which possesses some unknown properties and is subjected to some variable inputs (load and environment).

The properties of the black box are dependent not only on the distribution of the inputs but also on the materials and geometrical configuration of the structure. The outputs are the responses of interest such as the deflection, the extent of cracking, etc...

In such structures, damage accumulates, and either a modified form of Dong's general theory, or a modified form of Miner's linear law is needed to account for accumulation of damage, Any damage concept developed for highway pavement should be adaptable to account for the stochastic nature of the problem. A direct probabilistic method should be used if possible, otherwise a method utilizing simulation techniques should be developed.

IV. PAVEMENT CUMULATIVE DAMAGE MODEL

This section presents the framework for a comprehensive damage model which is being developed at M.I.T. The model, although in the developing stage, is capable of accounting for the randomness of the material properties and environmental factors, as well as accumulation of damage in the form of deformation and cracking. First the formulation of the problem is presented, and then the input requirements are described. Finally the application to highway pavements is presented.

IV.1 FORMULATION OF DAMAGE LAW

To develop a general damage law one must relate the input variables (loads, temperature, time, etc.) to output variables (deflections, cracks, etc.). The sought relationships is generally difficult to obtain, and its application to each specific case requires a great deal of modifications. To simplify the complexity of the required general relation, the damage formalization can be handled in two steps. The first step yields the stress and strain fields within the system. Stress, strains and the displacements are called the primary responses and are then used as inputs to the second step in the model which yields damage parameter or limiting responses. This separation of the damage formulation into two independent parts assumes that the limiting responses are dependent solely on the primary responses, a fact that seems to be supported in the literature.

IV.1.1 Primary Response

The geometrical model that is used in this study is a semi-infinite half space consisting of three distinct layers. It is assumed that each layer has distinct material properties which can be characterized as linear elastic or linear viscoelastic. The load is considered to be uniform, normal to the surface, and acting over a circular area.

An assumption of incompressibility is made so that one constitutive relationship is sufficient to define the viscoelastic equation of state for each layer. This constitutive equation is assumed in terms of a viscoelastic equivalent to the elastic compliance. That is, for the i^{th} layer:

$$\frac{1}{E}: \text{(equiv.)} = [D_i(o)(\ ) - \int_o^t (\ ) \frac{\partial D_i(t-\tau)}{\partial \tau} d\tau]$$

where;

$D_i(t)$ = the creep compliance of the i^{th} layer

In order to obtain the viscoelastic solution for the stresses and displacements for other loading conditions, the correspondence principle is utilized. This principle states that if the elastic constants in the elastic solutions to a given boundary value problem are replaced by operator forms of the stress-strain relations, then the viscoelastic solution will be obtained.

The above principle was used by Ashton and Moavenzadeh (84) to obtain the viscoelastic solution for the stresses and displacements induced in a three layer viscoelastic system subjected to a stationary load. It was then subsequently modified by using the principles of the response of initially relaxed linear systems to imposed excitations to obtain the solutions for the moving and repeated load history conditions.

This method of analysis has been selected largely because the viscoelastic behavior of many paving materials can be approximated by stress-strain relations of the linear and non-aging type, such as hereditary integrals. The steps involved in its application to a boundary value problem can be concisely stated as follows:

1. The elastic solution for the surface deflection of the system due to a stationary applied load is first obtained (84).
2. The "correspondence principle" is applied to the above solution in the form of hereditary integrals for the stress-strain relations, to obtain the viscoelastic solutions (84).
3. The expressions for the surface deflection due to other types of loading are then developed through the use of Duhamel's superposition integral for linear systems (85).

Elliott and Moavenzadeh (24) have analyzed three loading conditions: a stationary load, a repeated load, and the case where the load is moving at a constant velocity. In all these cases the environmental conditions were assumed to remain constant. In this study the results of their study are used and extended to account for varying environmental conditions.

IV.1.2 Limiting Response

The structural damage of a pavement structure is divided into two parts, not necessarily independent of each other: excessive deformation and cracking. The excessive deformation can be directly predicted by any primary model of a multilayer system which has accumulative capabilities (86), providing it is modified to account for the stochastic nature of the environmental factors. Cracking is assumed to arise mainly from fatigue behavior and its accumulation thus can be measured in different manners. For example one can relate the crack nucleation and growth to specific combinations of the primary responses such as the stress intensity factor K (43) and then evaluate the probabilities of having a given distribution of cracks. Another method is to use a more phenomenological approach and represent the accumulation of cracking by a damage functional F(t) which depends on the past histories of the stress and strain tensors. With some assumptions of continuity this functional can be expanded into a series of multiple integrals (62):

$$F(t) = \int_o^t K_1(t,s)V(s)ds + \iint_o^t K_2(t,s_1,s_2)V(s_1)V(s_2)$$

$$ds_1 ds_2 + \ldots + \int \ldots \int_o^t K_n(t,s_1,\ldots,s_n)V(s_1)\ldots V(s_n)ds_1\ldots ds_n$$

The measure of damage $F(t)$ is not however, uniquely defined. The damage may be measured by the density of cracking, or by the value of dynamic modulus of the layer materials at a given frequency since this modulus decreases as the density of cracking increases (e.g. Ref. 87). The creep compliance of the material can be used as a measure of crack propagation as in Ref. 43 and 94, or the number of remaining cycles before complete failure under a given mode of loading can be used for this purpose as suggested in Ref. 88. Any of these measures can be used, and it is convenient to normalize them so that the damage functional equals 0 when the material is intact and increases to 1 at failure. In the above equation $V(s)$ is a function involving stress and/or strain invariants, s is an arbitrary parameter which may have a meaning of time or cycles, etc. This representation of the damage functional is general and accounts for accumulation of damage as well as recovery processes such as healing, and of aging effects.

The review of literature showed that for various asphaltic and bituminous mixtures failure envelopes were related to a strain measure. In the general case of triaxial loading conditions the strain measure should be expressed as a combination of invariants. In the absence of results of triaxial tests, we will use the derivative of the major principal strain as a strain measure in the damage functional. Thus:

$$F(t) = \int_o^t K_1(t,s)\ \varepsilon(s)ds + \iint_o^t K_2(t,s_1,s_2)\varepsilon(s_1)\varepsilon(s_2)ds_1 ds_2 + ..$$

Where ( ) represents a differentiation with respect to the argument. When s has a meaning of a time, this expansion is similar to the representation of the time response of a nonlinear viscoelastic material, while when s has a meaning of a cycle it may be related to the dynamic representation of a nonlinear viscoelastic material and may be determined as a transfer function of a system subjected to a cyclic loading. Dong (62) has showed that in the latter case, a discretization of these integrals results in Miner's law (52).

In order to simplify this expansion an assumption is made that three different damage processes may be recognized: a damage process depending on the number and amplitude of cycles, a healing process (or a recovery process) depending on the elapsed time since the damage was created, and an aging process whereas the materials properties are changing with time. The damage functional may now be written as:

$$F(t) = \int_o^t K_1(s,t-s)\varepsilon(s)ds +$$

$$\iint_o^t K_2(s_1,t-s_1,s_2,t-s_2)\varepsilon(s_1)\varepsilon(s_2)ds_1 ds_2 + \ldots$$

This equation implies that the kernels are functions of the running time s (cumulative and aging processes) and of the lapse of time t-s (recovery process).

In a first approach to the problem the second and higher order kernels will be neglected in the damage expression. We will further assume that the sought first order kernel may be factorized:

$$K_1(s,t-s) = K_{CUM}(s)\ K_{REC}(t-s,s)$$

i.e., the cumulative and recovery process are independent. The aging process is included in both K_{CUM} and K_{REC} through the dependency on the time s.

In order to determine these kernels it is necessary to choose a measure for the damage and normalize it as mentioned above. Let N be the number of cycles to failure (i.e. inadmissible density of cracking) under a given type of random load during relatively short time (no aging or recovery takes place). A damaged material will undergo only N' cycles under the same conditions before failing. The amount of damage is represented by $\frac{N-N'}{N}$. N and N' can be measured on control specimens. Note that in this case F(t) is not a measure of the amount of cracking but is a function of it.

a. Cumulative Kernel

For a small period of time over which there is neither aging nor damage recovery we have for the increment of damage $\Delta F(\tau)$:

$$\Delta F(\tau) = \int_0^S K_{CUM}(S-s)\ \frac{\partial \varepsilon}{\partial s}\ ds$$

Dong (35) proved that if s = number of cycles of a given strain amplitude, the above integral is identical to Miner's representation, i.e.

$$\Delta F(\tau) = \sum_{i=1}^{m} \frac{dn_i(\tau)}{N_i(\tau)}$$

where $dn_i(\tau)$ is the number of cycles of amplitude $\Delta\varepsilon_i$ which are applied at time τ, and $N_i(\tau)$ is the number of cycles $\Delta\varepsilon_i$ which would cause failure. Hence the general expression for the damage becomes:

$$F(t) = \int_o^t \Delta F(\tau)\ K_{REC}(t-\tau,\tau)\,d\tau$$

$$F(t) = \int_o^t K_{REC}(t-\tau,\tau)\left(\sum_{i=1}^{m} \frac{dn_i(\tau)}{N_i(\tau)}\right)d\tau$$

The aging process is accounted for in the dependency of K_{REC} and N on the time τ.

For a uniaxial case we will consider a random stress history to be composed of a mean value and a cyclic component. In the triaxial case these may be replaced by a mean value of the hydrostatic stress and a cyclic component of the octahedral stress. If sinusoidal stresses with constant amplitude are applied to various specimens of a material, a fatigue envelope (S-N curve) is usually obtained in the form of a stress or strain amplitude vs. number of cycles to failure. Miner's law may be applied to such diagrams to predict the results under varying amplitudes. Since this envelope is found to be generally independent of temperature and rate of loading when it is given in the form of strain amplitude vs. number of cycles to failure we will concentrate on the use of such diagrams. These diagrams and Miner's law, however do not account readily for the order in which successive loads are applied and the effects of varying amplitudes. The order in which the loads are applied will be accounted for implicitly since strain amplitudes are obtained as the primary responses of the three layer viscoelastic model, and thus they are functions of the sequence of loading. To take into consideration the effects due to varying amplitudes and non-zero mean, an approach - suggested by Freudenthal and Heller (89) and based on the statistical character of fatigue, can be used. The basis of this statistical evaluation of the results is the three-parameter distribution function of the smallest values (89). This function derives from the assumption of a distribution function of the strength (or strain at failure) of cohesive bonds in the material and a distribution function of the induced stresses (or strains) when a macroscopic stress (or strain) S is applied to the structure. The probability function P(S) of the strength (or strain at failure) of the material is obtained as a convolution of the two previous distribution functions. The relation between the mode of loading P(S) and N determines the trend of a theoretical S-N diagram. Based on the determination of a single S-N diagram obtained for a given mode of loading (e.g., sinusoidal with constant amplitudes), one can predict the probable S-N diagrams for other modes of loadings. Freudenthal and Heller (89) related the effect of the change in the load spectrum to the changes in the S-N diagram.

This results in a modification of Miner's law to include an interaction factor $\omega_i > 1$ to account for interaction effects of various stress amplitudes. Thus Miner's law becomes:

$$\sum_{i=1}^{m} n_i/\omega_i N_i = 1$$

ω_i depends on the load spectrum and results in a fictitious envelope shown in Figure 4. The cumulative process can therefore be given by an expression such as:

$$\sum_{i=1}^{m} \frac{dn_i}{\omega_i N_i [\Delta\varepsilon(\tau),\tau]}$$

where τ indicates that the number of cycles to failure may vary because of aging and that the envelope is to be determined for different values of τ. The increment of damage is also a function of the average strain amplitude applied during the increment of time τ.

The determination of $N_i(\Delta\varepsilon)$ can also be derived from the knowledge of the mechanisms of cracking. For example it can be determined as in Reference (43) through the use of the concept of stress intensity factors.

b. Recovery Kernel

The recovery kernel $K_{REC}(t-\tau,t)$ is a function of the time $(t-\tau)$ elapsed since the application of the damage increment and of the age of the material. From Reference (88)

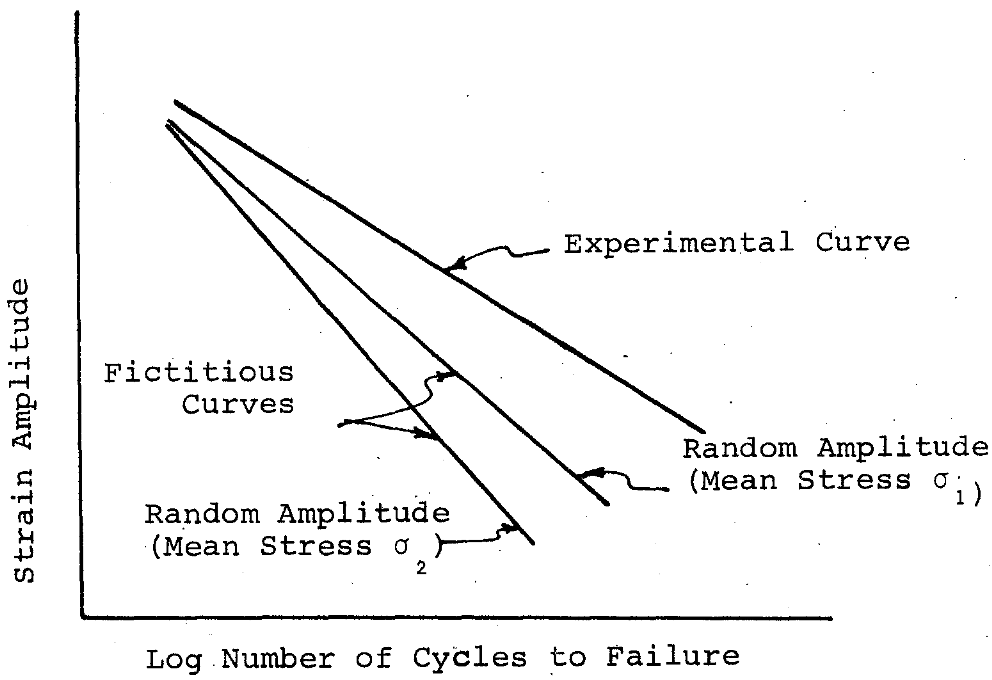


FIGURE 4 - FAILURE ENVELOPES

it is apparent that healing requires the presence of a minimum compressive stress. Thus we will assume that the argument $(t-\tau)$ can be replaced by $(t^*-\tau)$ where

$$t^* = \int_{\tau}^{t} H[\sigma(s) - \sigma_{min}]ds$$

H [ ] is the Heaviside step function which is equal to 1 when its argument is positive and zero elsewhere. σ_{min} is the minimum compressive stress which triggers healing. Thus $(t^*-\tau)$ is the accumulated time during which a minimum compressive stress is present.

To determine $K_{REC}(T)$ two identical specimens (or sets of specimens) should be given the same amount of damage F. F is determined by testing one of the two specimens (control specimen) and measuring the amount of damage which should still be applied to fail the specimen. The second specimen is left to rest for a time T, and then failed to determine the amount of recovery $K_{REC}(T)$.

c. Aging

Aging is accounted for, through changes in the characteristics of the constitutive equation, as well as in the cumulative and recovery kernels.

IV.2 INPUT REQUIREMENTS

IV.2.1 Materials Characterization

For the determination of the damage function F it is important to determine the stress and/or strain invariants or as in the simplified formulation above to determine the major principal strains. In order to do this, it is necessary to determine the properties of the materials in the layers. These properties are generally dependent upon the manner in which they are prepared and constructed, their thickness and confining stress, the rate of loading, and the history of the environmental variables. Since all of these factors are statistical quantities, one must expect the properties to be also statistically distributed within the layers.

The materials properties assumed to be pertinent here are the compliances or creep functions, the Poisson's ratio and height of the layer-materials. Poisson's ratio and the height of each layer are considered as deterministic quantities. While the height of the layers can change with different structural sections, Poisson's ratio is set equal to one half, for all sections.

The properties of the material for each layer will be represented by a creep compliance function for a viscoelastic layer and a creep compliance for an elastic layer.

For a viscoelastic layer the following representation will be assumed:

$$D_j(t) = D_{E_j} + \sum_{i=1}^{n} G_i^j e^{-t\delta_i} \quad j = 1,2,3$$

where;

j = the layer numbers

$D_j(t)$ = the value of creep function at time t

D_{E_j} = the zero time value of the creep function i.e., $\sum_{i=1}^{n} G_i^j = 0$

G_i^j = the coefficient in Dirichlet series $\sum_{i=1}^{n} G_i^j e^{-t\delta_i}$

δ_i = the exponent in exponential term corresponding to coefficient G_i^j

$\delta_n = 0, \ldots D(\infty) = D_{E_j} + G_n^j$

To include the statistical characteristics of the properties in the analysis, a method similar to that described in Reference (11) should be used. In this method a random loading is used as an input in the tests designed to determine the creep compliances. The resulting functions are least square approximations of the social properties.

IV.2.2 History of the Environment

The main measurable quantities which reflect the influence of the environment are temperature and humidity. The temperature within the system will be assumed to be uniformly distributed. Later stages of the study may introduce the spatial distribution of temperature as a function of the atmospheric temperatures and wind condition. Data are available on such distributions and analytical means of computation exist (e.g., see Reference (90)), but the present models for viscoelastic layered systems do not have this capability. The same observations as made for temperature can be made for the moisture or humidity distribution within the system. The history of environment will be generated in a random manner so as to approximate the climate in a given area.

Many viscoelastic materials were shown to be "thermo-rheologically simple", i.e., to fulfill the time-temperature superposition principle. Similar principles of superposition were found to be true for other types of environmental changes (91). Thus if $\phi(t)$ is the value of the environment at time t, and ϕ_o is the reference value of the environment, viscoelastic properties at $\phi(t)$ may be derived from visco-elastic properties at ϕ_o through different operations of scaling. A general expression is:

$$D[t, \phi(t)] = \alpha[\phi(t)] + \beta[\phi(t)]\ D[\gamma[\phi(t)]\ t, \phi_o]$$

$\phi(t)$ is a function of the values of the temperature or moisture content etc..., α is a vertical shifting of the creep compliance, β is a vertical scaling of the transient response, and γ corresponds to a horizontal shifting for a semi-log plot (change in time scale). These techniques of scaling apply well to a wide variety of viscoelastic materials, and they allow for an easy introduction of the environment effects in viscoelastic analyses.

This representation is used to describe the effect of changes of properties with temperature and moisture. Moreover, since the repeated load program can be directly related to the results of the stationary load program, these scaling techniques will be used to represent the response of the stationary load program for different values of the environment variables.

IV.2.3 History of Load Applications

The traffic load intensity on a pavement system is statistically distributed in time, space and magnitude. In this investigation a single wheel load applied over a circular area is considered to be appropriate on the assumption that the equivalent single wheel load concept is valid. The rate of load application and the magnitude of the load can be varied. The loading function is assumed to be a Heaviside characterized by a load duration and a period which is the time between two consecutive load applications. At this stage of the study the load magnitude as well as its' duration are assumed constant. The period of the load is constant during a time period (day, week, month etc...) and is a function of the number of load applications during that period.

IV.3 APPLICATION TO HIGHWAY PAVEMENTS

IV.3.1 Primary Response

Since a major part of the problem of determining the damage functionals is to predict stresses, strains and deflections under a random load and environment histories, it was necessary to modify the repeated load program (86) to give it this capability. The response $P_s(t)$ of a linear viscoelastic system to any varying load $P(t)$ may be written as:

$$P_s(t) = \int_{-\infty}^{t} SR(t-\tau) \frac{\partial}{\partial \tau} P(\tau) d\tau$$

where SR(t) is the response to a step load (stationary load program). If we want to introduce the effect of the history of the environment $\phi(t)$ where $\phi(t)$ may be the history of temperature, moisture and other environmental variables, the response is given by (91):

$$P_s(t) = \int_{-\infty}^{t} SR[t-\tau, \underset{s=\tau}{\overset{t}{\phi}} (s)] \frac{\partial}{\partial \tau} P(\tau) d\tau$$

and

$$SR[t-\tau, \underset{s=\tau}{\overset{t}{\phi}} (s)] = \alpha[\phi(t)] + \beta[\phi(t)] \times$$

$$SR[\int_{\tau}^{t} \gamma[\phi(\theta)]d\theta, \phi_o] - \int_{\tau}^{t} SR[\int_{\tau}^{t} \gamma[\phi(\theta)]d\theta, \phi_o] d\beta[\phi(s)]$$

where α, β and γ are scaling factors which were described above and which are functions of the environment $\phi(t)$. ϕ_o is the reference value for the environment. The determination of α, β and γ is made by curve-fitting techniques (11).

When SR(t) is given in the form of a series of exponentials:

$$SR(t) = \sum_{i=1}^{N} G_i e^{-t\delta_i}$$

for a step load under a constant value $\phi(t)$, the response becomes:

$$SR[t,\phi] = \alpha[\phi] + \beta[\phi] \{ \sum_{i=1}^{N} G_i e^{-t\delta_i [\phi]} \}$$

while in the more general case of a variable environment history:

$$SR[t-\tau, \underset{s=\tau}{\overset{t}{\phi}} (s)] = \alpha[\phi(t)] + \beta[\phi(t)] \times \{\sum_{i=1}^{N} G_i e^{-\delta_i t^*}\} - \int_{\tau}^{t} \{\sum_{i=1}^{N} G_i e^{-\delta_i s^*}\} \, d\beta[\phi(s)]$$

The notation x* is defined by:

$$x^* = \int_{\tau}^{x^*} \gamma[\phi(\theta)]d\theta$$

At the present time only the temperature is considered in the variable $\phi(t)$ and it is assumed that the temperature remains constant for a time period and changes as a step function at the end of every time period. Thus:

$$SR[t-\tau, \underset{s=\tau}{\overset{t}{\phi}} (s)] = \alpha[\phi(t)] + \beta[\phi(t)] \sum_{i=1}^{N} G_i e^{-\delta_i t^*}\} - \sum_{m=\ell}^{j} \sum_{i=1}^{N} [G_i e^{-\delta_i s^*_m}] \times [\frac{T(t_m) - T(t_{m-1})}{T_o}]$$

This formulation can be used to compute the primary responses due to arbitrary histories of loads and environment variables. In the present work we have adopted the concept of equivalent single wheel loads where the magnitude of the applied load is maintained constant. In the more general case the magnitude of the applied load will also present a statistical distribution. In the present analysis, however the magnitude of the load, as well as the duration of its application are considered to be constant. The load is described by:

$$P(\tau) = \sin^2\omega\tau[H(0) - H(\text{duration}) + H(1 \times \text{period}) - H(1 \times \text{period} + \text{duration}) + H(2 \times \text{period})$$

$-H(2 \text{ x period} + \text{duration}) + \ldots]$

where H is the Heaviside step function. Hence:

$$\frac{\partial P(\tau)}{\partial \tau} = \begin{cases} \omega \sin \omega\tau & \text{while a load is applied} \\ 0 & \text{otherwise} \end{cases}$$

The unit step response is described by:

$$SR[t-\tau, \overset{t}{\underset{s=\tau}{\phi}}(s)] = \alpha[\phi(t)] + \beta[\phi(t)]\{\sum_{i=1}^{N} G_i e^{-\delta_i t^*}\}$$

$$\sum_{m=\ell}^{j} \sum_{i=1}^{N} [G_i e^{-\delta_i s^*_m}] \times [\frac{T(t_m) - T(t_{m-1})}{T_o}]$$

Based on results from Reference (92) we will use as typical values in the computer program:

$$\alpha[\phi(t)] = 0$$

$$\beta[\phi(t)] = T(t)/T_o$$

$$\gamma[\phi(t)] = 1/a_T(t) = 10^{[10,000(\frac{1}{T(t)} - \frac{1}{T_o})]}$$

where $T(t)$ and T_o are respectively the present temperature and the reference temperature, in °K; $a_T(t)$ is the present value of the shift factor. In this case we can write:

$$SR[t-\tau, \overset{t}{\underset{s=\tau}{\phi}}(s)] = (T_j/T_o)\{\sum_{i=1}^{N} G_i e^{-\delta_i t_j^*}\}$$

$$- \sum_{m=\ell}^{j} \sum_{i=1}^{N} [G_i e^{-\delta_i s^*_m}] \times [\frac{T(t_m) - T(t_{m-1})}{T_o}]$$

where t_j^* and s_m^* reduces to

$$t_j^* = \int_0^{t_j} \gamma(\tau)\,d\tau$$

and

$$s_m^* = \int_0^{s_m} \gamma(\tau)\,d\tau$$

Finally the formulation of the response under a repeated load and a varying temperature is given in Appendix I.

IV.3.2 Limiting Responses

The limiting responses which will be considered for a pavement are: rutting, slope variance, and cracking. The first two are directly predictable from the primary response while the latter requires the development of a damage model. Although cracking may result from a single load application, it is more often created by repeated loading, i.e., by fatigue. The amount of damage created by fatigue is represented by the function F(t) defined previously.

The evaluation of F(t) requires the knowledge of the kernels involved. The form of these kernels was suggested in the review of the literature.

The cumulative kernel is given by the fatigue envelope relating strain amplitudes and number of cycles to failure. This fatigue envelope is defined by a relationship of the form $N = K(\frac{1}{\Delta\varepsilon})^n$ (Reference 90), where N = cycles to failure at a particular strain level, K is of the order of 10^{-6} to 10^{-10} for various asphalt concrete mixtures and n varies between 2.8 and 5. $\Delta\varepsilon$ will be defined as the average difference between two consecutive peaks and valleys in the strain function. The strain level is the average strain level for the whole period K and n can be given in terms of mean strain level (as is done commonly for metals). The recovery kernel is more difficult to obtain because of the scarcity of data. It is possible to use some experimental results such as in Reference (88) which reports the rate of healing and recovery of some asphaltic mixes.

Still less data is available for evaluating the effects of aging. This process is important to include because it accounts for some of the non-load associated failures.

IV.3.3 Systems Simulation

Flow Chart (1) summarizes the steps involved in modeling the pavement system and simulating load and environmental histories. This section will describe the different steps involved in the formulation of this model. A more detailed description of these steps is presented in the next section.

The first stage in the model consists in dividing the time into periods during which the environment variable (i.e., temperature) is assumed to be constant. These time periods may be days, weeks, months etc... depending on the assumptions made for the analysis. The duration of the load and its magnitude are assumed constant. The number of load applications for each time period is generated randomly so that it has given statistical characteristics, e.g., uniform distribution over a given range. Similarly the value of the temperature for each time period is also a random variable which has a specific frequency distribution such as a normal distribution over a given range of temperatures.

The characterization of the materials properties yields environmental scaling factors $\alpha_i, \beta_i, \gamma_i$ for the i^{th} layer. Then the responses of the three layer viscoelastic model to static loads at different values of the temperature T are curve-fitted to obtain the unit step response of the system at a reference temperature SR(t, T ref.), as well as scaling factors $\alpha(T)$, $\beta(T)$ and $\gamma(T)$ for the overall system.

In the present analysis since there were no particular assumptions on the values of these coefficients for each layer, the values of $\alpha(T)$, $\beta(T)$ and $\gamma(T)$ were assumed to be those found for a particular sand-asphalt mixture (90). These values were assumed to be the same for both the deflection and strain unit step response of the layered system. Note that the variability of the materials properties is not included, but this can be done by associating frequency distributions for each of α, β, γ and the coefficients describing $SR(t, T_{ref.})$.

The simulation program proceeds then to compute the total residual deflection at the end of the j^{th} time period as well as the mean circumferential strain and average circumferential strain amplitudes during the j^{th} time period. The strains are computed at the bottom of the top layer, and the strain amplitudes are computed as half the difference between successive peaks and valleys of the resulting strain.

These results are then readily related to the three principal measures of damage: Rutting, Slope Variance and Cracking.

Rutting is measured by the amount of residual deflection. A large number of simulation runs yields a probability of occurrence of a maximum deflection before a given number of time periods. The variability of the materials properties can be easily accounted for in such results, however for the present time deterministic values were chosen for the materials properties in order to keep at a minimum the required number of simulation runs.

Slope variance is measured by differential deflections. These differential deflections occur mainly because of variabilities in the system's properties. Therefore it is important to evaluate the correlations between the properties of the system at two points separated by a distance d (e.g., d = 2 ft.). The knowledge of these correlations coefficient allows one to determine the probabilities of having a given amount of differential settlement before the j^{th} time period. This determination is done through a series of simulation runs representing each of the two points. Each of the points is assumed to be a a three-layered half-space system, i.e., the differential settlement is not accounted for, directly in the mathematical model. This part of the damage was not evaluated because the computation process appears lengthy and suggests to further investigate first the derivation of closed form probabilistic solutions. If such solutions can be obtained they would complete or replace advantageously the simulation procedures. At a later stage of this study the effects of differential settlement on the loading history can be represented by a feedback loop which would multiply the loads magnitude.by a dynamic coefficient related to the amount of differential settlement. This feedback loop is represented by a dashed line in Flow Chart 1. Finally Cracking is measured by the function F(t) as essentially a fatigue phenomenon. At the end of each time period the increment of damage $\Delta F(\tau)$ is evaluated using a Miner's type of law. The number of cycles applied during that time period is known, and $N(\tau)$ is obtained from the value of the average amplitude of the strain $\Delta\varepsilon_t$ and its mean value ε_t. The total value of F(t) is obtained by convolution of $\Delta F(\tau)$ with the recovery kernel $K_{REC}(t-\tau)$. The effect of aging is included by changing at different stages of the simulation the functions describing $N(\tau)$ and $K_{REC}(t-\tau)$. Hence a series of simulation runs yields the probabilities of obtaining F = 1 before a given time t_j. At a later stage of the study a feedback loop from the value of F(t) to the history of the environment can be added. Such a loop would account for facts such as the changes of moisture content of the subgrade due to moisture infiltration through newly formed cracks.

IV.4 SUMMARY

This section presented the framework for a Pavement Cumulative Damage Model. This model uses the Primary Responses as an intermediary step in the process of computing the limiting responses. The primary responses are obtained for linear elastic and viscoelastic layered systems under varying loads and environmental conditions. These primary responses are used to predict three components of the damage: rutting, slope variance and fatigue cracking. The damage functional assumes three independent mechanisms: a cumulative fatigue process, a healing or recovery process and an aging process. The model can include some of the nonlinearities of the damage functional in feedback loops which account for interactions between the output variables (cracks, deflections etc...) and the input variables (loads, environmental variables...). Simulation techniques are applied to this model in order to account for the stochasticity of the input variables.

V. RESULTS AND DISCUSSION

V.1 INTRODUCTION

The development of the framework for the study of damage in pavement structures, presented in the preceding chapter, was based on three main assumptions:

a. The theories behind the model should be general enough to apply to different types of materials under various conditions. For instance Dong's expansion which was used allow for such generality.

b. These general theories should be specialized into forms easy to manipulate. This is necessary so that the model does not become workable only on paper. These simplified forms (such as Miner's law) are then used as first order approximations to the more accurate representations.

c. The model should be modular so that it is possible to progressively improve it by modifying or changing parts of it. This is part of the reason why the model is divided into primary and limiting response parts which are further subdivided into subcomponents as shown in Flow Chart 1. Sensitivity analyses determine the parts of the model which can be further simplified and the parts which are to be developed or more examined.

The model at the present time uses simulation techniques in order to account for the stochasticity of its input variables. At the present time these techniques appear to be the most promising in view of the complexity of the system and because of the constraint that the model be modular. For specific cases closed form solutions can and should be developed because of the savings in computing time.

This chapter will present first a simplified model which was used to demonstrate the application of the simulation techniques. And then it presents the present model and describes its use. Finally a description is given of the expected refinements and further developments.

V.2 THE SIMPLIFIED MODEL

In this analysis the static load program was used to provide the primary response. The environmental conditions were introduced as random variables. However the environmental conditions were held constant for a given run. The repeated load response can be derived from these results but only under constant environmental conditions. In other words this simplified model generates an arbitrary set of environmental conditions and keeps them constant during the applications of the successive loads. Under these conditions one could readily obtain probabilistic closed form solutions. This simplified model was used however as a step in the development of the more complete model.

V.2.1 Description of the Model

The properties of the viscoelastic layers (creep compliances) are represented as usual by the Dirichlet series

$$D_j(t) = \sum_{i=1}^{n} G_i^j e^{-t\delta_i} \ldots$$

for the three layers j = 1,2,3. These coefficients were defined previously in Section III.2.1. The environmental variables were assumed to be temperature and moisture. These variables were considered to influence the coefficients of the Dirichlet series and not the exponents. This is not very realistic for viscoelastic material but was done so to keep the model very simple. Two statistical distributions were assumed for the temperatures in this analysis: a uniform distribution in the range (T_1, T_2) where T_1 and T_2 are the end points of an assumed working range of temperatures and a normal (Gaussian) distribution characterized by its mean and its variance. The creep compliance of the material is assumed to vary with temperature in the following manner:

$$D_j(t) = \sum_{i=1}^{n} G_i^j \, e^{-\alpha_i^j (T + C)} \quad j = 1,2,3 \ldots$$

$$\alpha_1 = 0$$

where G_i^j and α_i^j are constants for a given layer. These constants should be actual fact be random variables of a certain distribution. $\sum_{i=1}^{n} G_i^j$ represents the value of the compliance at $T = -C$ or at $T = 0$ when $C = 0$. At a given temperature T in the range (T_1, T_2), the value of the compliance $D_j(t)$ can vary between $D_j^L(t)$ and $D_j^U(t)$ where the subscirpts of L and U refer to the lower and upper bound values of the function under consideration. The position of D_j for a given time will depend on the moisture content of the layer. Since no direct relationship can be developed to determine the coupled effect of temperature and moisture as of now, further work on this point is needed.

Two curves are therefore arbitrarily drawn for the representation of the functional relation that has been assumed to exist between material property and temperature, Figure 5. The lower bound curve is for the worst condition of moisture and the upper bound curve is for the best condition of moisture. The effect of moisture content is therefore implicitly included in the analysis.

The distributions of material property are also are also assumed to be rectangular and normal; any other type of distributions can easily be handled. The distribution of temperatures is of the following form:

$$T = T_1 + d_1 (T_2 - T_1) \ldots$$

where T_1 and T_2 are defined as previously and d_1 is a random number uniformly distributed in the range $0 < d_1 < 1$, or a number normally distributed over $[-\infty, +\infty]$.

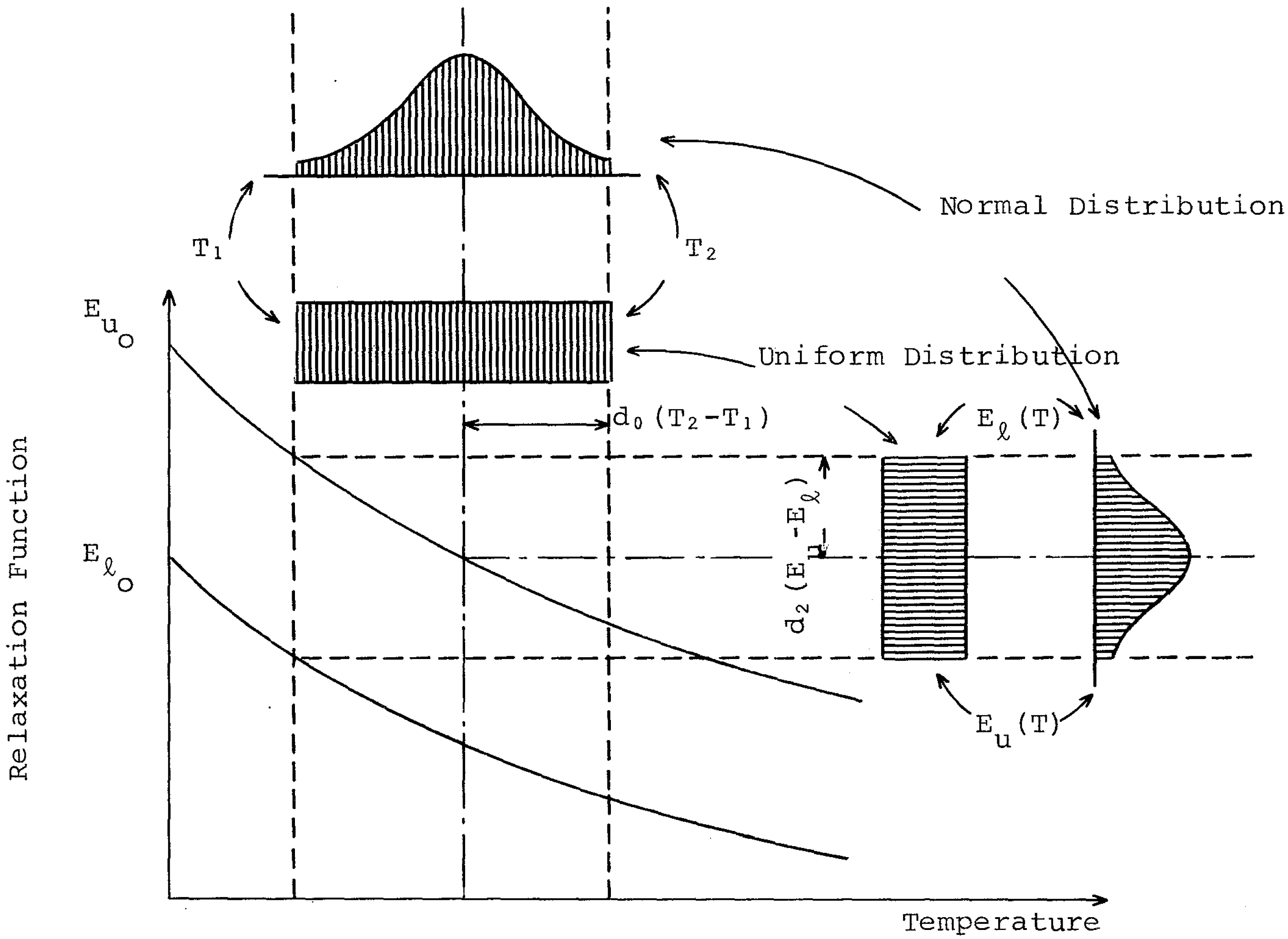


FIGURE 5: VARIATION OF THE RELAXATION FUNCTION WITH TEMPERATURE AT A FIXED TIME

For the creep compliance, the following equation results for the uniform distribution:

$$D_j = D_j^L + d_2 \; (D_j^U - D_j^L) \; \ldots$$

where D_j, D_j^L, D_j^U are defined as previously and, d_2 is similar to d_1. Figure 6 shows the variability of the creep compliances of the three layers of the system.

V.2.2 Results

Since the properties of the structure are random variables as described above, the response variables will also be stochastic quantities. The model yields the frequency distributions of the input variables. Two cases were considered; a uniform distribution (rectangular) and a normal distribution. In both cases the response parameters may be in terms of displacements, stresses and strains developed at any point in the structure under stationary, repeated or moving load (but constant environmental conditions). To demonstrate the capability of the model we confine our presentation to only the frequency distributions of the strains at different instants of time.

Figure 7 shows the simulation of a uniform distribution of temperatures as obtained by a random number generator. 100 samples were used and it is apparent that much larger sample sizes may be needed to obtain a better representation. Figures 8 and 9 show the distributions

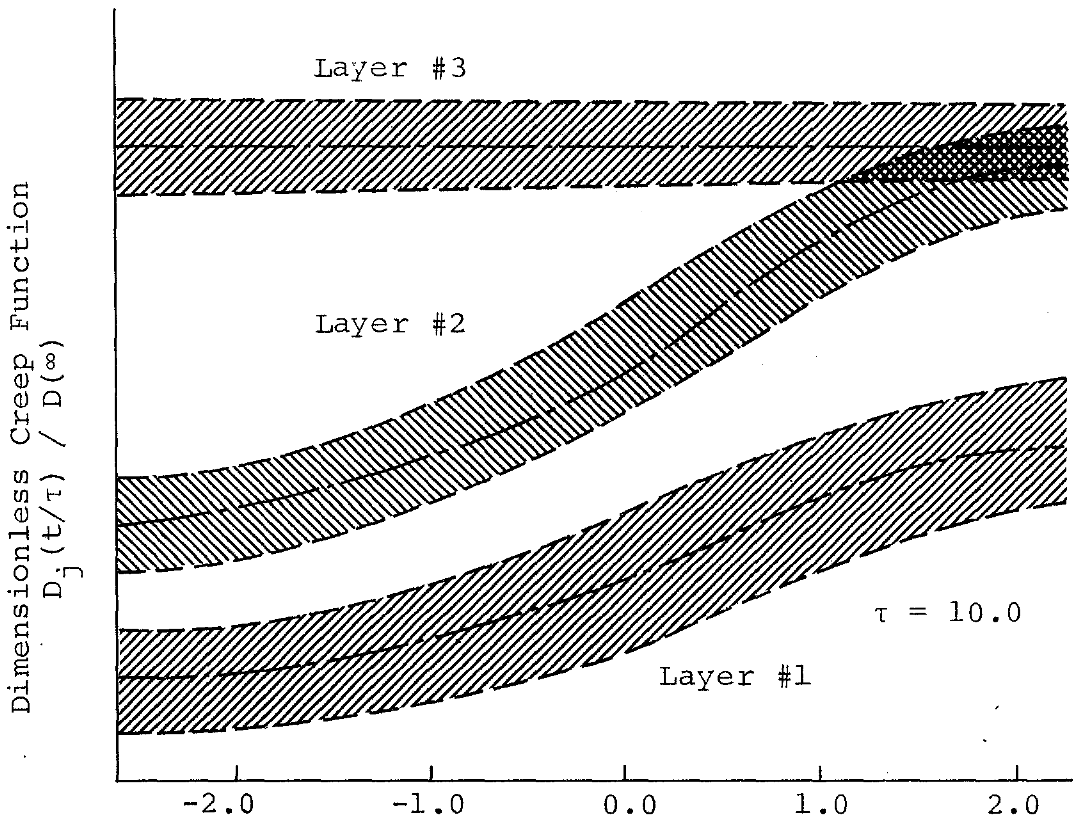


FIGURE 6: CREEP FUNCTIONS OF THE SIMULATED PAVEMENT SYSTEM

$$D_j(t) = \sum_i G_i e^{-t\delta_i}$$

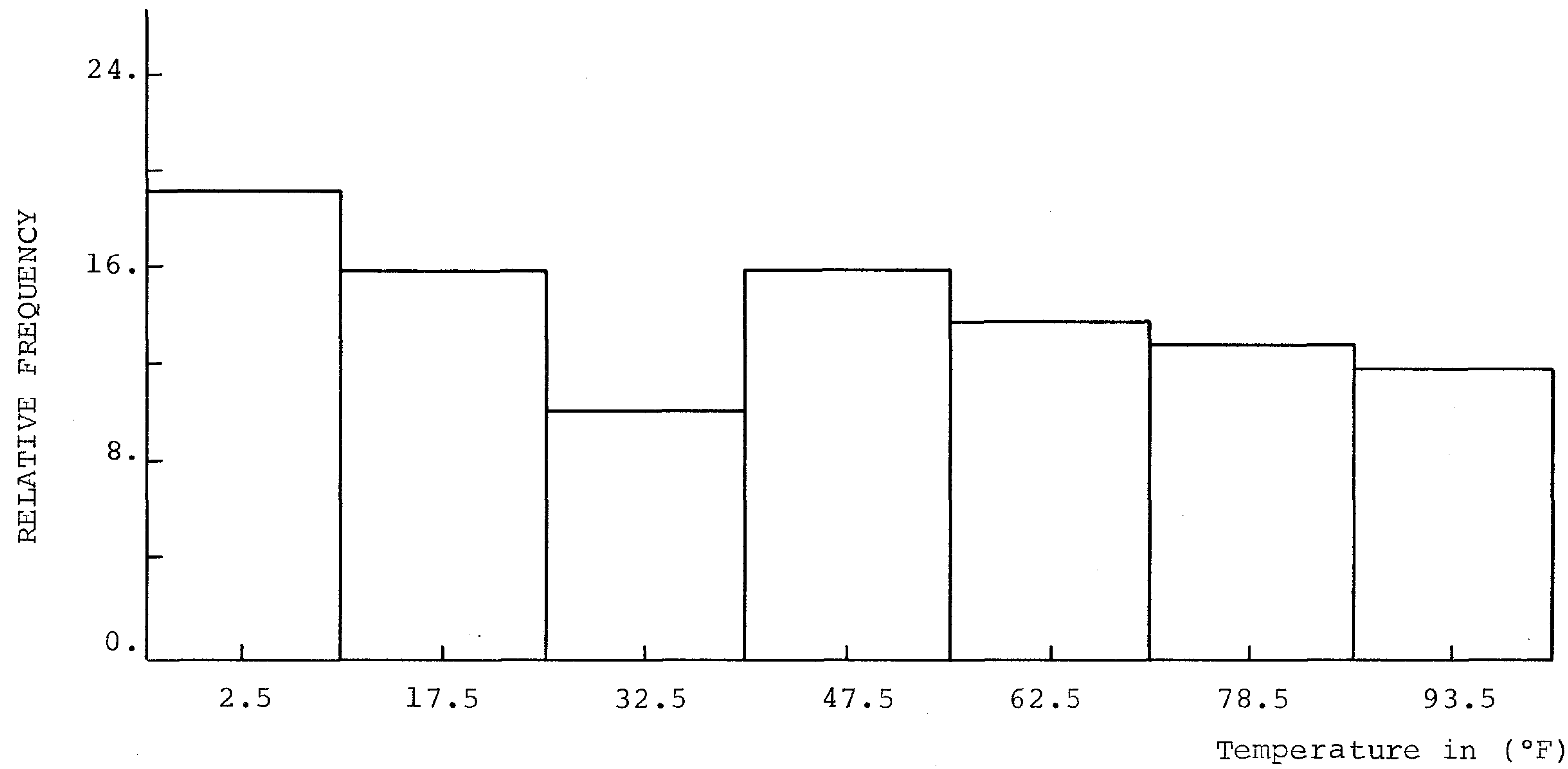


FIGURE 7: RELATIVE FREQUENCY DISTRIBUTION OF UNIFORMLY DISTRIBUTED TEMPERATURES AS GENERATED BY AN IBM / SYSTEM 360 RANDOM NUMBER GENERATOR

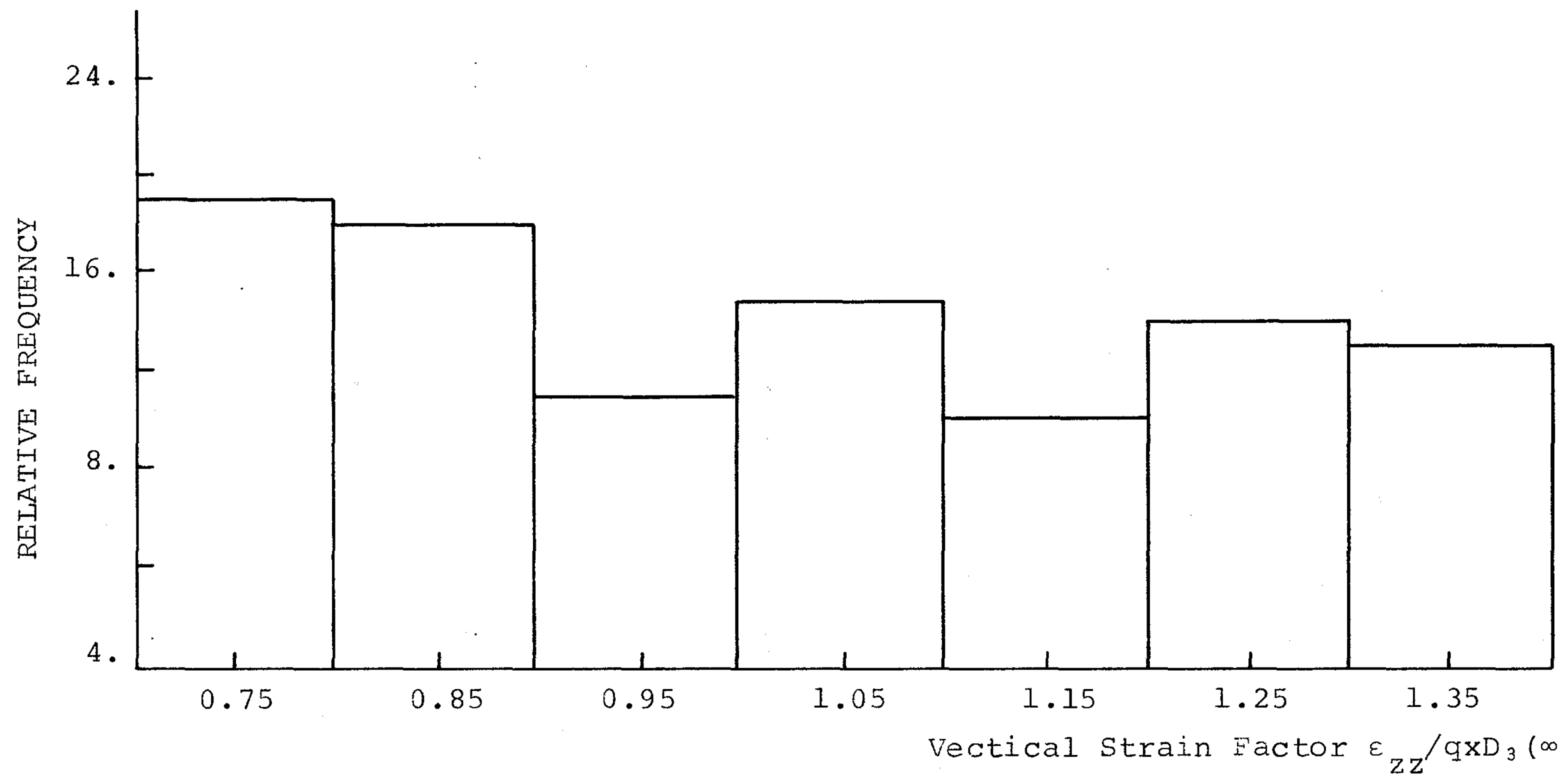


FIGURE 8: RELATIVE FREQUENCY DISTRIBUTION OF VERTICAL STRAIN AT $t/\tau = 0.0$ FOR UNIFORMLY DISTRIBUTED RANDOM INPUTS

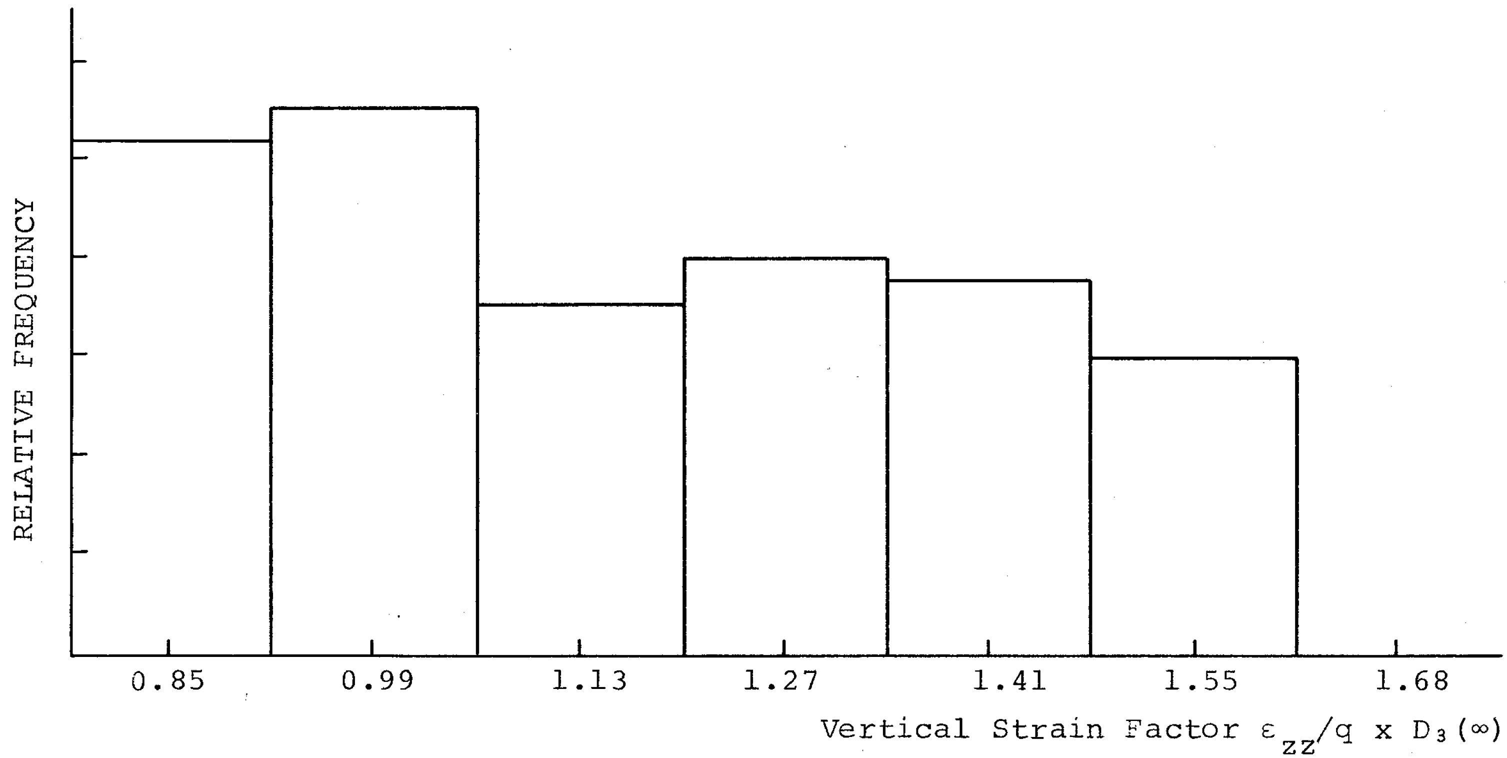


FIGURE 9: FREQUENCY DISTRIBUTION OF VERTICAL STRAIN AT $t/\tau = 10.0$ FOR UNIFORMLY DISTRIBUTED RANDOM INPUTS

of deflections at times 0.0, and 10. respectively. It is apparent that these responses also show a tendency towards a uniform distribution which shifts to larger values of the deflection, as the time of the observation increases.

Figure 10 shows the simulation of normally distributed temperatures. Here also it is apparent that much larger sample sizes are needed in order to obtain more accurate representations. Figures 11 and 12 which show the distributions of the deflections at times 0 and 10 suggest that the responses of this model show the same statistical characteristics as the inputs that produced them. The shape of these distributions was checked by the usual "goodness-of-fit" methods such as the X^2 test (95).

This example illustrates how the stochastic nature of the input variables, such as the temperature and physical properties of the materials, affects the results. The responses are represented by frequency distributions so that the designer can account for the uncertainty associated with the occurrence of each possible response. The results of Figures 11 and 12 may also be summarized graphically such as in Figure 13. This figure shows the statistical properties of the resulting responses.

V.3 REPEATED LOAD MODEL

The simplified model was used only for demonstration purpose of simulation techniques. In order to account for the effects of changing environmental conditions it was

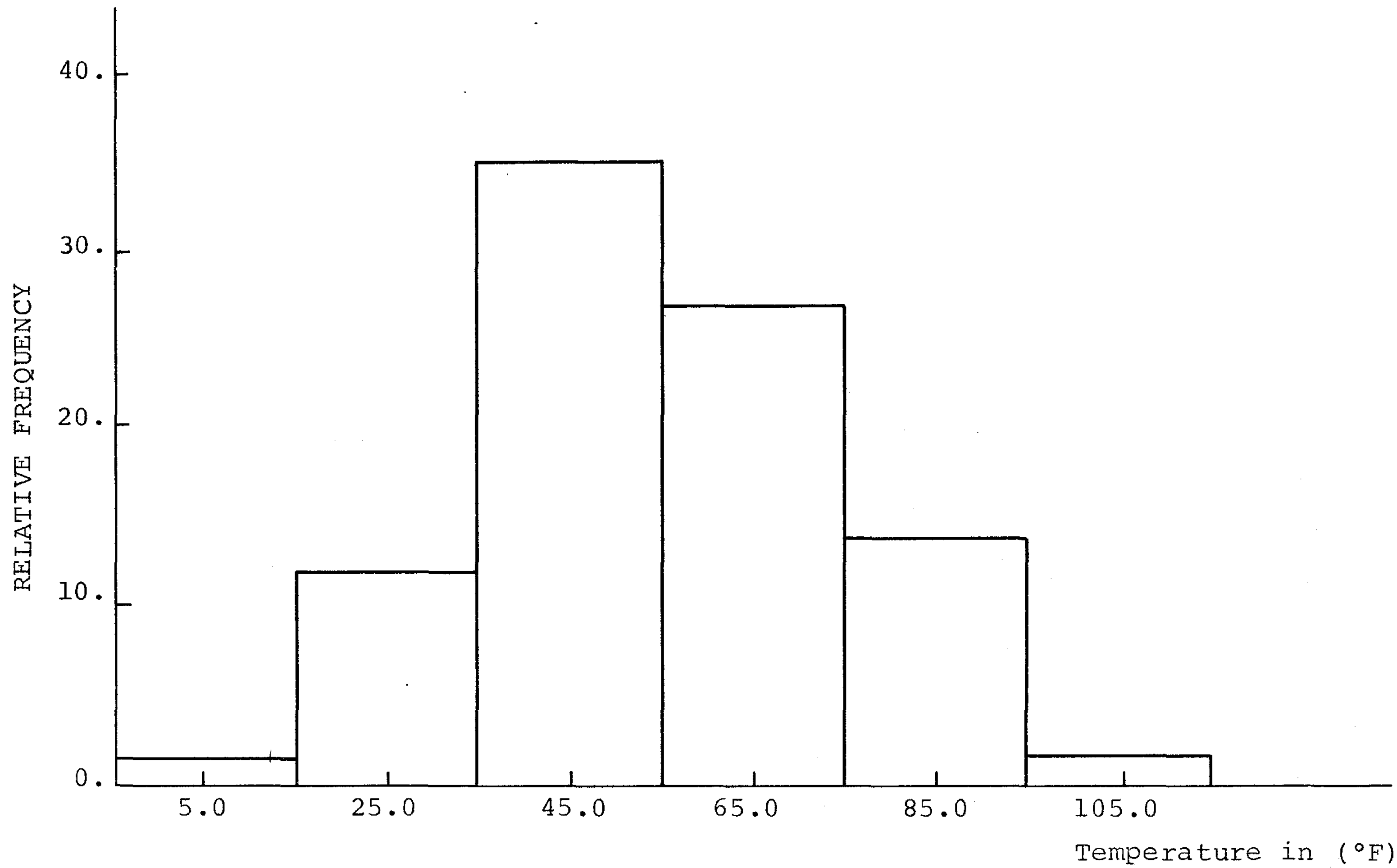


FIGURE 10: RELATIVE FREQUENCY DISTRIBUTION OF NORMALLY DISTRIBUTED TEMPERATURES AS GENERATED BY AN IBM / SYSTEM 360 RANDOM NUMBER GENERATOR

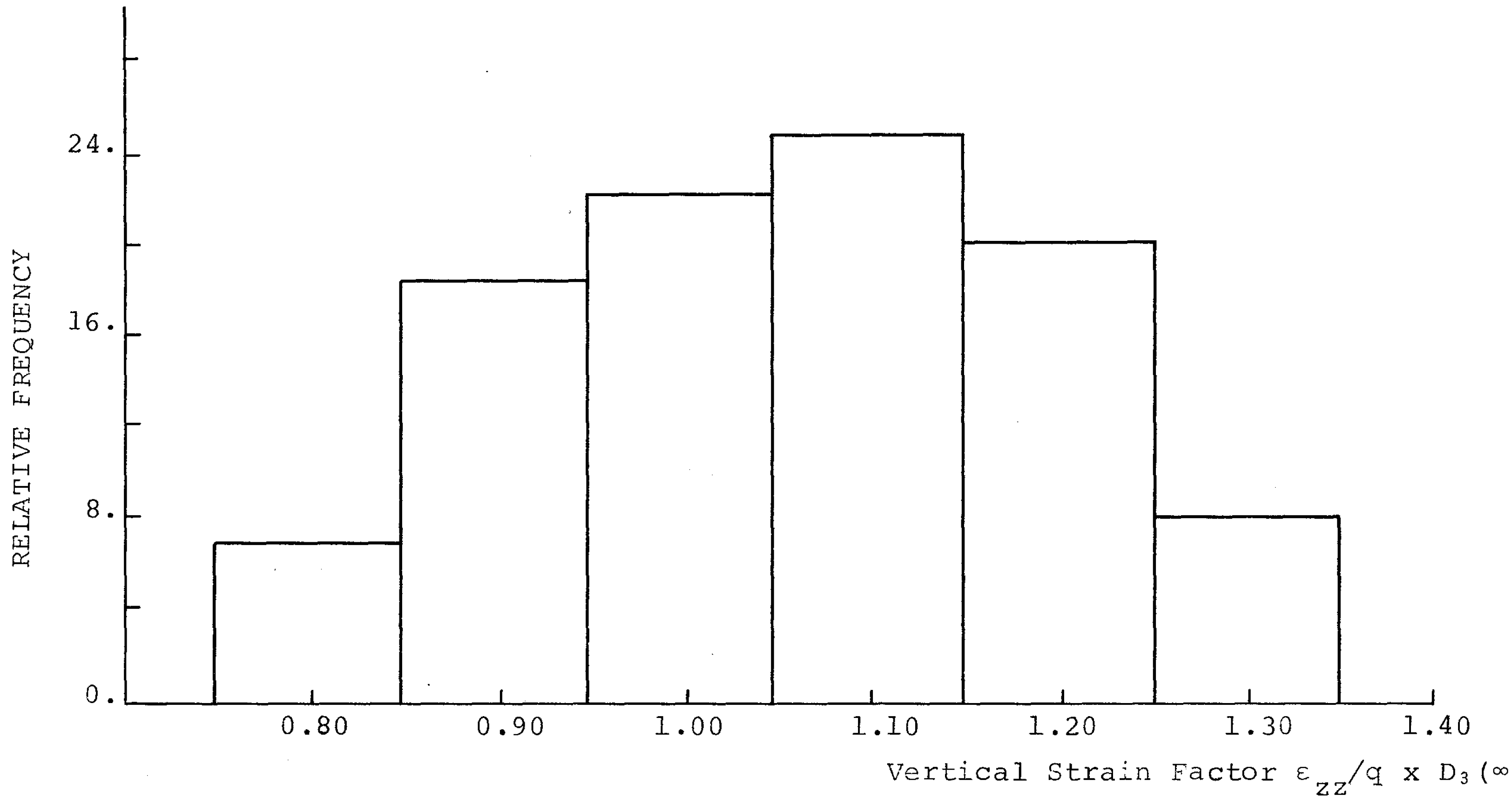


FIGURE 11: RELATIVE FREQUENCY DISTRIBUTION OF VERTICAL STRAIN AT $t/\tau = 0.0$ FOR NORMALLY DISTRIBUTED RANDOM INPUTS

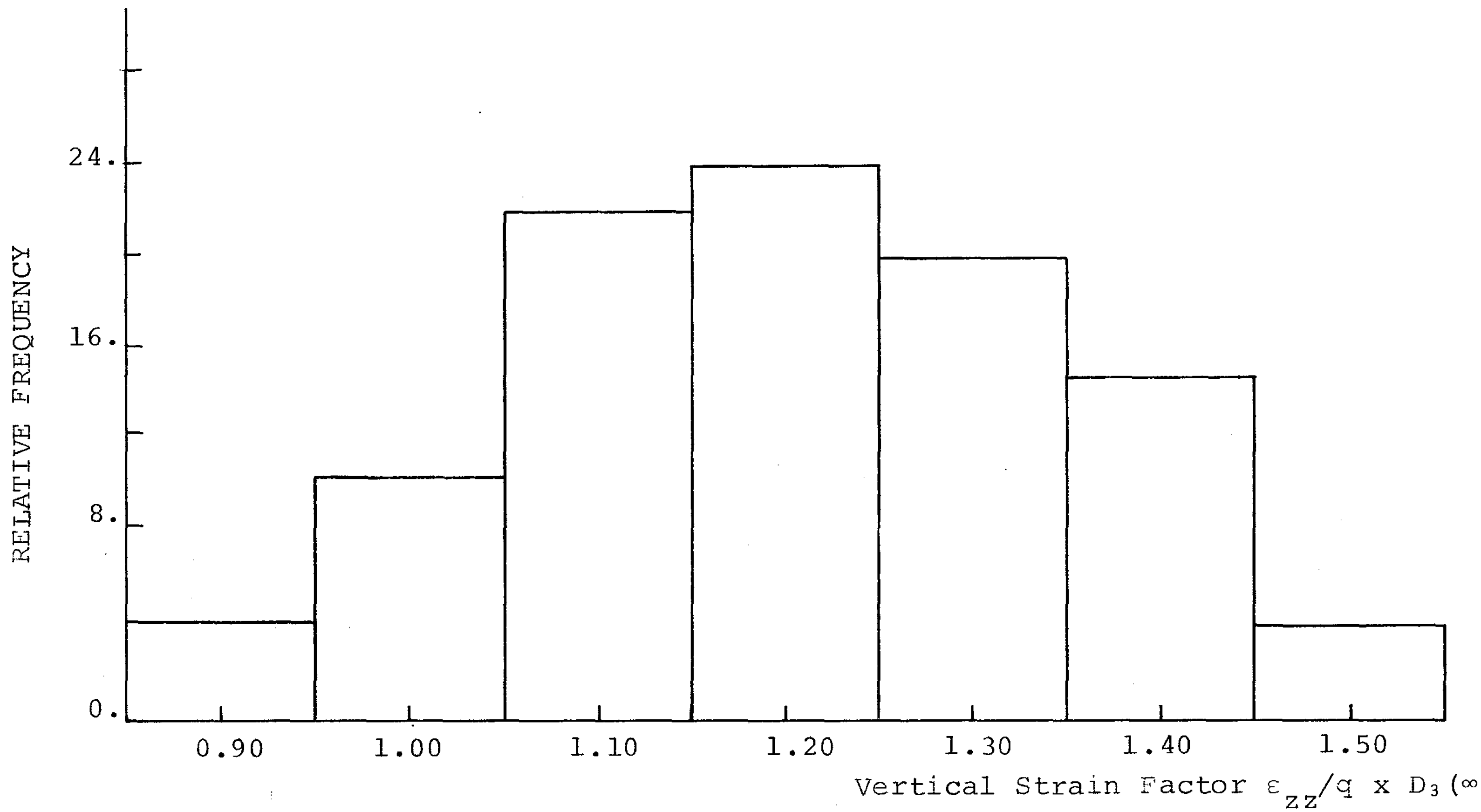


FIGURE 12: RELATIVE FREQUENCY DISTRIBUTION OF VERTICAL STRAIN AT $t/\tau = 10$. FOR NORMALLY DISTRIBUTED RANDOM INPUTS

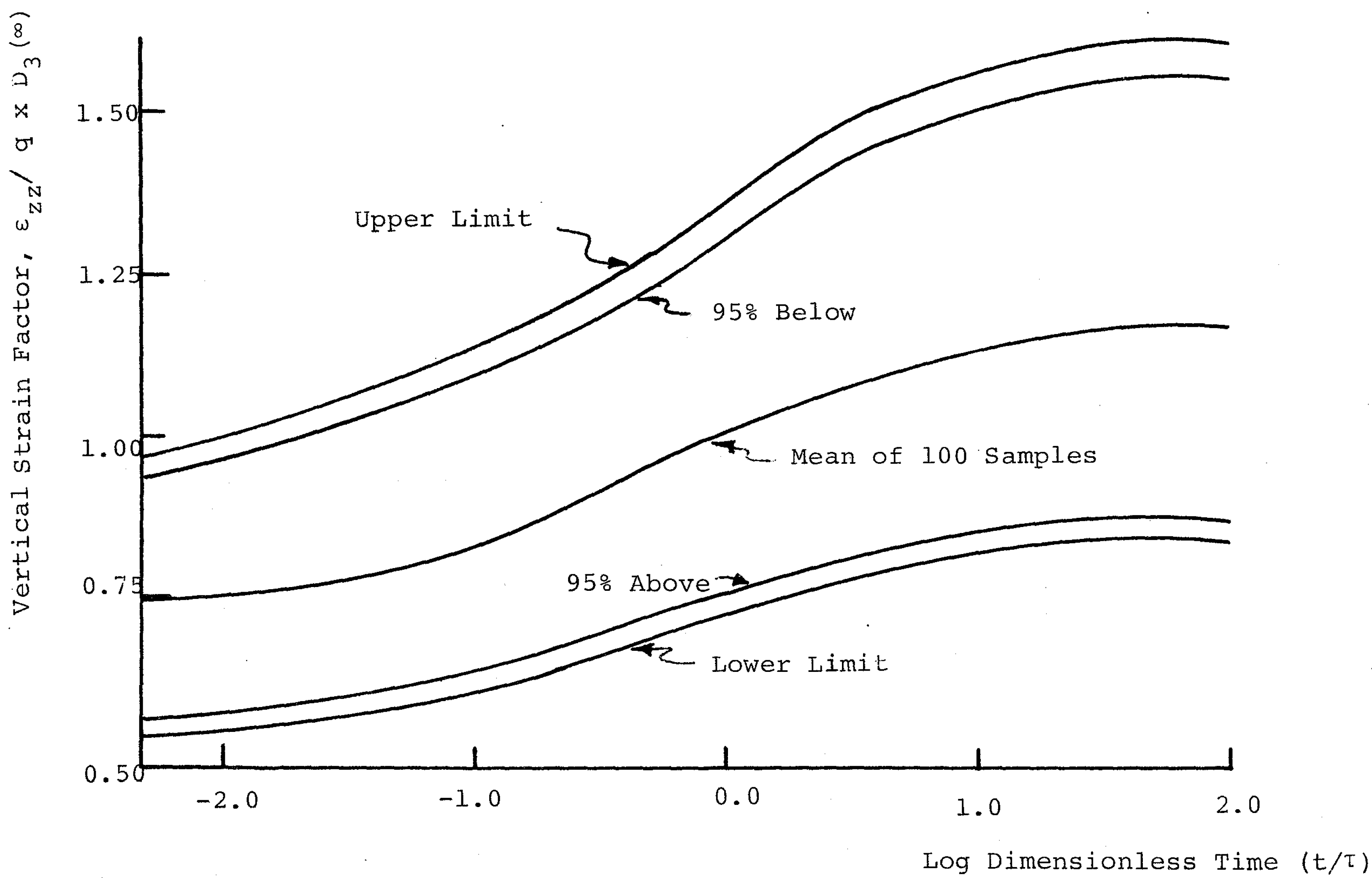


FIGURE 13. SUMMARY OF SIMULATION OF VERTICAL STRAINS FOR UNIFORMLY DISTRIBUTED RANDOM INPUTS.

necessary to structure the model described in Flow Chart 1. The model yields the stresses, strains and displacements (primary responses) at the critical points of the pavement. These are in turn used to predict three major appearances of damage, namely slope variance, rutting and cracking. Slope variance and rutting are easily derived from the primary responses. Cracking is treated as a fatigue phenomenon combined with recovery and aging. We will follow Flow Chart 1 step by step in order to detail it.

V.3.1 Basic Assumptions

a. History of Load

The history of loads which will ultimately be considered is a load with variable amplitude (weight of vehicle) and a variable duration (velocity of vehicle) being applied at variable periods (intervals of time between successive load applications). However in the present analysis the amplitude is constant (equivalent single wheel; this assumption can be checked at later stages of the analysis). The duration of the load is constant while the period of application is a function of the number of applied loads in the basic unit of time (such as one day, one month or one year). In the example which is treated the basic unit of time is a month period.

The number of applied loads is different for every basic unit of time. It is considered to be normally distributed. For the month period the mean of the distribution was 15,000 load applications with a variance of 3,000. A single load application is represented by a Haversine function.

b. History of Environment

The history of environment accounts for the changes of temperature and moisture. The environment is designated by the variable ϕ which is a function of time, temperature and moisture. Data on the history of the environment can be collected as shown in Reference 90. The changes in the environment were limited in the present study to the changes of temperature. The temperature was assumed to be normally distributed with a mean of 20° C and a variance of 7° C. It is maintained constant during the basic unit of time. At further stages of this study the mean and variance of the temperature would be functions of the seasonal changes. The effects of the moisture would also be included by giving it in terms of a single variable ϕ which combines the effects of temperature and moisture.

c. Materials Properties

The materials properties are determined using the scaling techniques described in Section II.2.2. The required α, β and γ coefficients are determined by curve fitting methods and least square optimization. In the present case we used the data of Reference 92 as representative values:

$$\alpha[\phi(T)] = 0$$

$$\beta[\phi(T)] = T(t)/T_o$$

$$\gamma[\phi(T)] = \frac{1}{a_T(t)} = 10^{[10000(\frac{1}{T(t)} - \frac{1}{T_{ref}})]} .$$

These data were obtained from the characterization of a sand-asphalt mixture and are only used for demonstration purposes.

Note that in the simplified model described above the temperature effects are accounted for by changes similar to those produced by β. γ-type coefficients were not used then because of the complexities they introduced in the primary response model. For most viscoelastic materials the main effect of environmental changes is a time-scale modification in the primary responses, so that the γ-type coefficients are the most important of the three.

V.3.2 Three-Layer Linear Viscoelastic Model

This model was modified to account for the effects of the stochasticity of the environment history. The changes were described in Section IV.3.1 and further details can be found in Appendix I. In its present form the repeated load program used the output of the static load program. Therefore a main point of simplification is stressed here: instead of giving the properties of the materials as functions of time and environment, the response function of the static load program denoted SR(t) is given as the function of time and environment. Therefore the coefficients α, β and γ are used in conjunction with this response SR(t). Note that SR(t) is given in the form of Dirichlet series similar to those used to represent the creep complicances of the materials.

Thus at this point we can obtain the deflection at the surface as well as stresses and strains at the points judged to be critical. The model subdivides at this point into three parts. These three parts are treated as being independent. At later stages of this study the interaction of these three parts may be accounted for by introducing feedbacks in the model (see, e.g., the dotted lines of Flow Chart I).

V.3.3 Limiting Response

a. Rutting

Rutting is assumed to be produced by excessive deformation. Therefore the simulation model can directly yield the proabilities of having a certain amount of rutting at a given time in the history of the structure given certain characteristics of the random temperature history. This section will be broadened up by including in it the characteristics of the spatial distribution of properties. In its present form the program will predict the probabilities of deflections at a given point.

b. Slope Variance

This section is not implemented yet and requires the determination of the variation in properties between two points separated by a distance d. These differences in properties follow some random distribution which is to be described. The model can be used then to predict the probabilities of having some differential settlements.

c. Cracking

This particular manifestation of damage is assumed to be mainly associated with fatigue. The damage function is designated by F(t) and its computation follows the method outlined in Section IV.1.2. Note that F(t) is a linear function of the number of applied cycles. This means that when F(t) = 0.5, half the life of the pavement is used up (it may recover some of it). It does not mean, however, that the density of cracking is half of what it will be at failure. The relationship (nonlinear) between these two values is still to be determined.

Typical values for the failure envelope were taken from the literature (90). These envelopes were obtained from bending tests under constant stress levels. More accurate failure envelopes should be obtained by trying to simulate typical histories of the triaxial state of stresses which develop at the critical joints in the pavements. When the amplitudes of the loads will also be considered to be random, the method of Reference

89 will be used to relate the failure envelopes to the frequency distribution of the amplitudes.

In the example which was studied, the failure envelope was given by

$$N = k \left(\frac{1}{\Delta\varepsilon} \right)^n$$

where $k = 5,\ 10^{-14}$ and $n = 4.5$. $\Delta\varepsilon$ is the strain amplitude and is measured as half the difference between two consecutive peaks and valleys in the strain function. For temperatures above 22° C, K was increased by a function of temperature to account for the fact that at higher temperatures very large strains occur but they do not contribute to cracking.

The recovery kernel is obtained by curve fitting with series of exponentials experimental results such as Figures 14 and 15 which are taken from Reference 88. These figures show that for broken and fatigued bituminous mixes a complete recovery is obtainable after three months at 10° C. This recovery function is a function of temperature also. In the example which was treated, the temperature dependence of this function was simplified to this form

$$K_{REC}(t) = 0.2\left(1 + \sum_{i=1}^{5} e^{-\alpha_i t^*}\right)$$

t^* = total amount of time during which no load is applied AND the temperature is above 22° C.

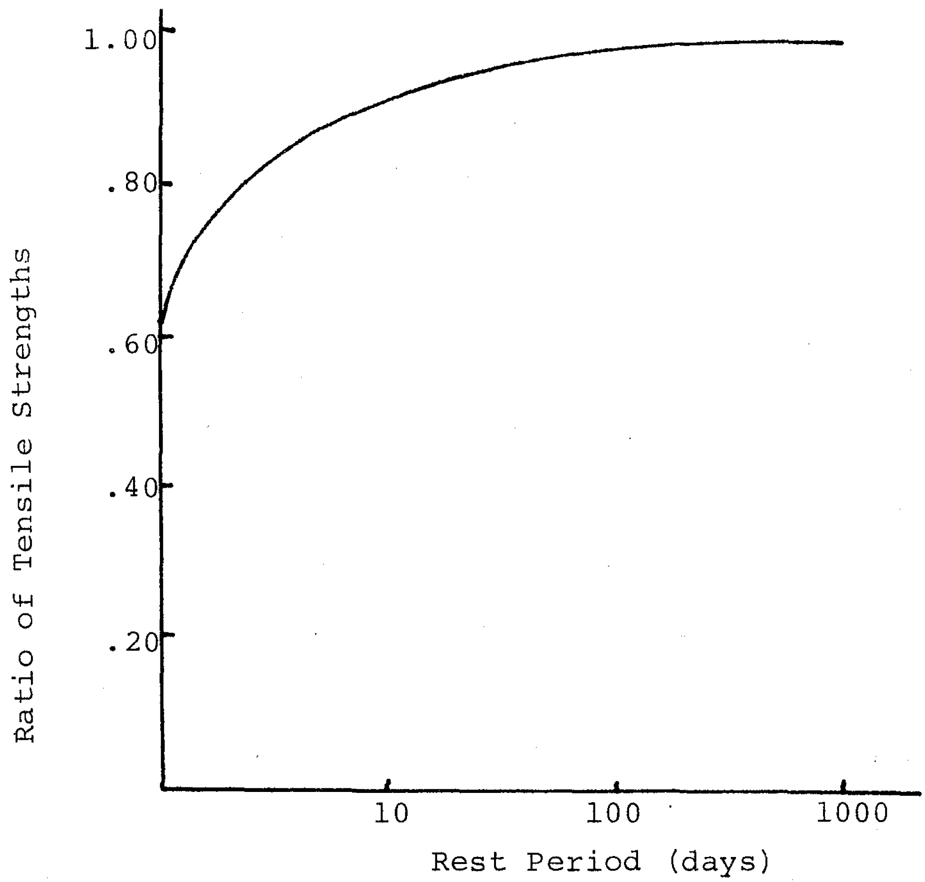


FIGURE 14. RATIO OF TENSILE STRENGTH FOR DAMAGED MATERIAL (TENSILE TEST) VS. UNDAMAGED MATERIALS (REF. 88)

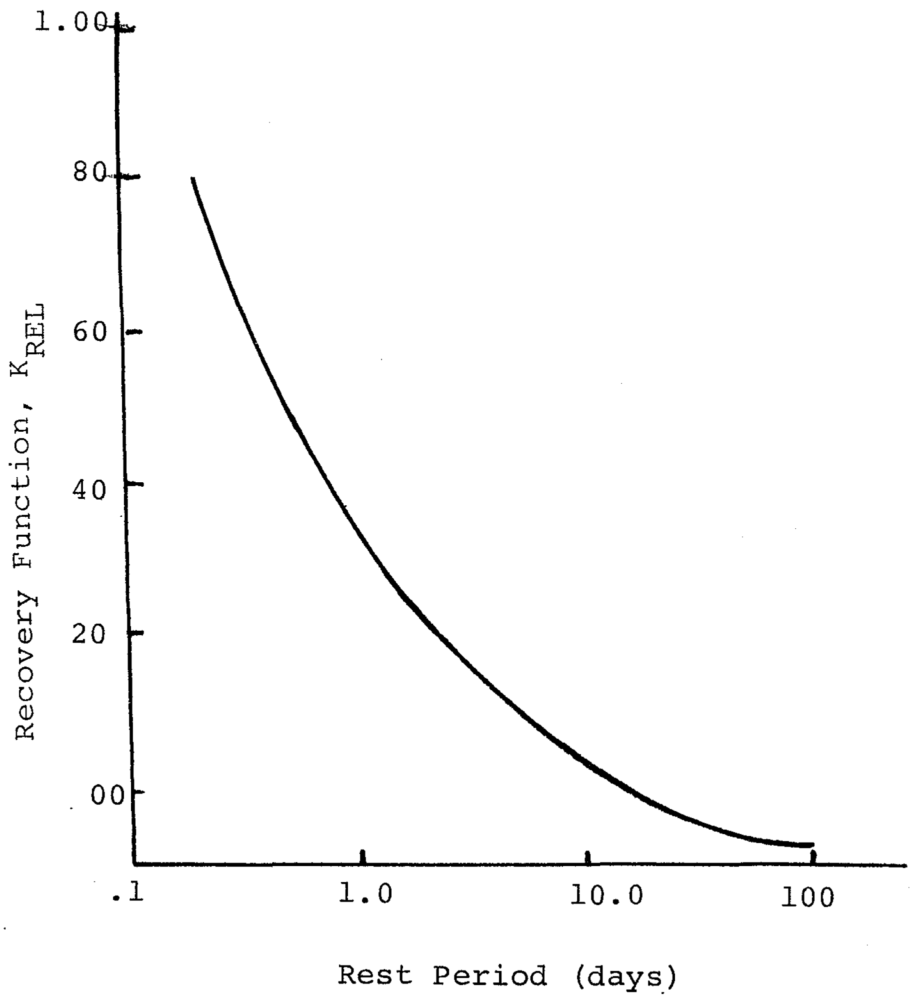


FIGURE 15. RECOVERY FUNCTION K_{REL} FOR A FATIGUED DENSE BITUMINOUS MIX (REF. 88)

α_i were chosen so that the recovery is completed in a period of three months.

Finally aging is accounted for by modifying the above failure envelope and recovery kernel as time goes on (for instance after every two years). Very little data, however, is available to describe these effects. Therefore, these effects are not included at the present time.

V.3.4 Results

First, a series of 12 different temperatures were generated by the random number generator. These temperatures were ordered by increasing order (a), then by decreasing order (b), and finally by a successively increasing and decreasing order (c). Figure 16 shows the three considered series of temperatures. These series were used as inputs to the analysis and the number of loads applications was assumed constant and equal to 15,000 loads. The resulting residual strains are shown in Figure 17. These residual strains at the first interface were used in lieu of the residual deflections at the surface because they present the same type of behavior and permit the qualitative study of the trends of this behavior. Figure 17 shows that the residual strains after a period of twelve months were almost equal for the series (b) and (c) because the temperatures corresponding to the last five months of these series

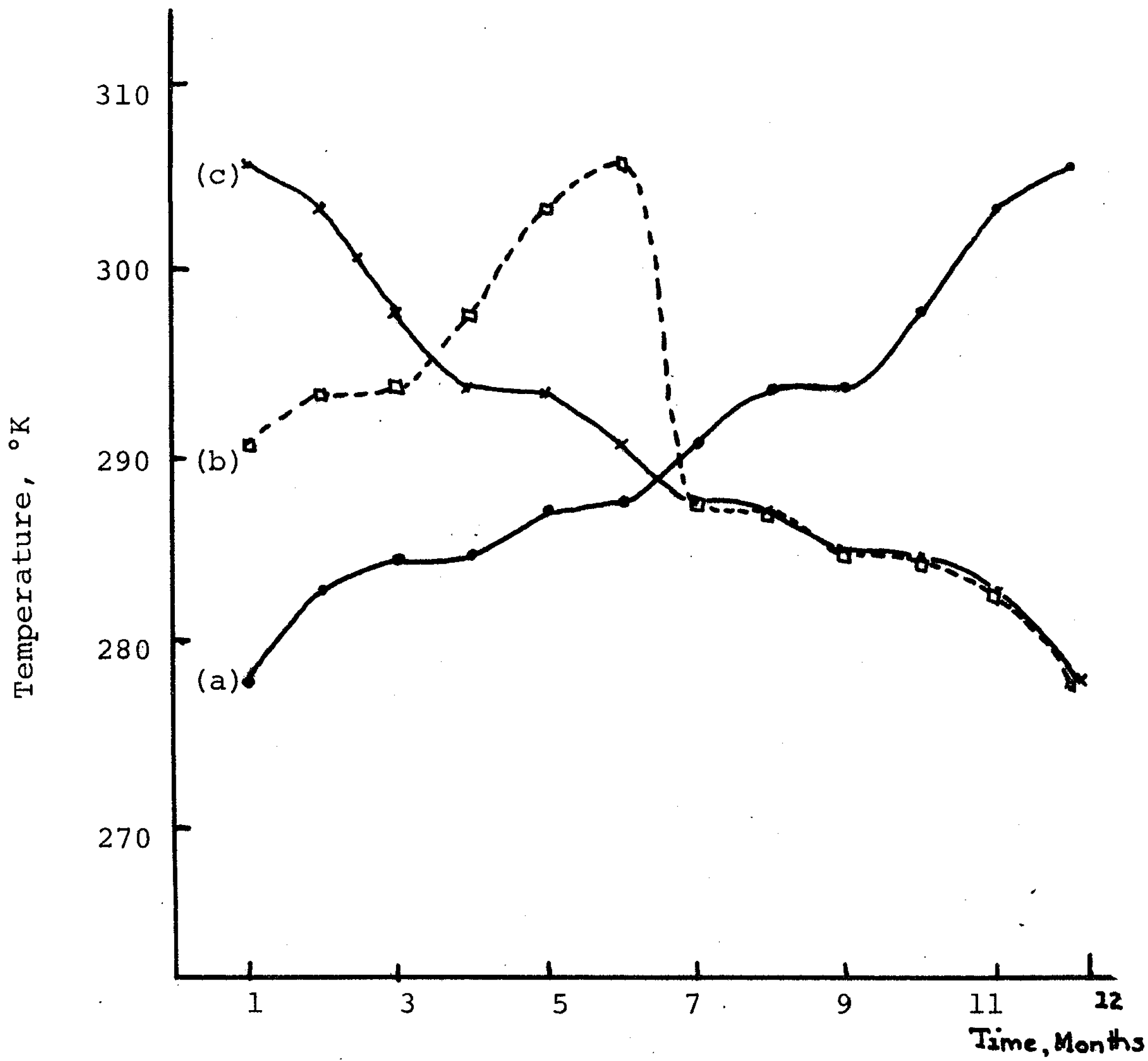


FIGURE 16. TEMPERATURE VARIATIONS

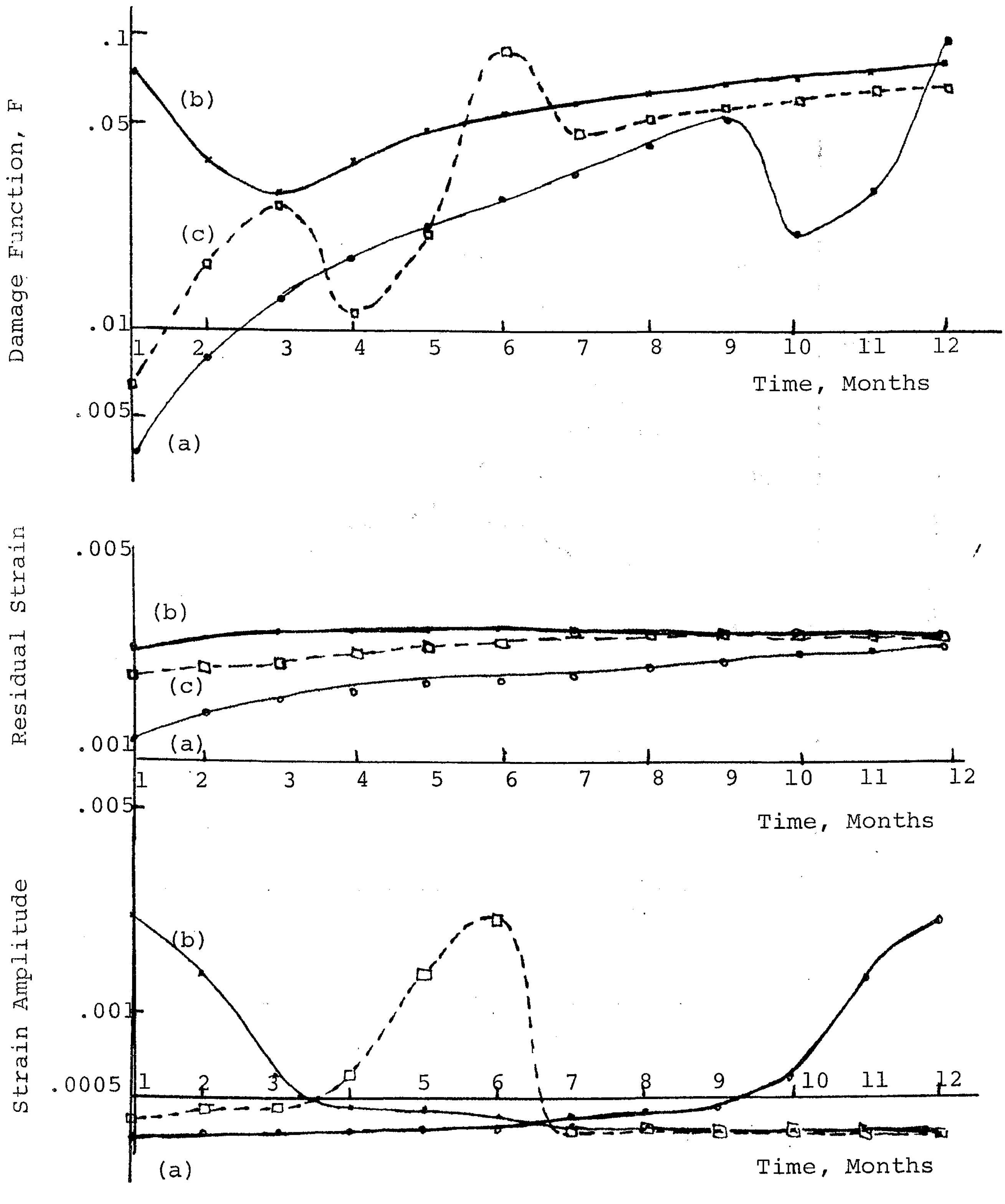


FIGURE 17. COMPARISON OF DIFFERENT SEQUENCES OF TEMPERATURES

were identical. The effect of the difference of temperature history in the first months was negligible. The residual strains due to the increasing order series (a) were a little different. On the whole, for the assumed materials properties, the residual strains were not very sensitive to changes in the sequence of temperatures.

Figure 17 shows also the strain amplitudes corresponding to each basic unit of time (month). These amplitudes were essentially functions of the present temperatures. Hence their variations are directly related to the temperature changes.

The same Figure 17 shows also the damage function, F(t), for the three sequences of temperatures (a), (b), and (c). The irregularities in the shape of F(t) are due to the strong nonlinearities arbitrarily introduced by the data used in the formulations. It can be seen, however, that the model accounts for the differences in sequence of the temperatures, and that these sequences are more important for the determination of the damage function F(t) than they are for the determination of the residual strains. It is not possible to derive more conclusions from the computed behavior because the data are not real.

A simulation was then conducted with the same model. Temperatures and number of loads (assumed constants for a month period) were generated as normal variables on the computer. Figure 18 shows the computed

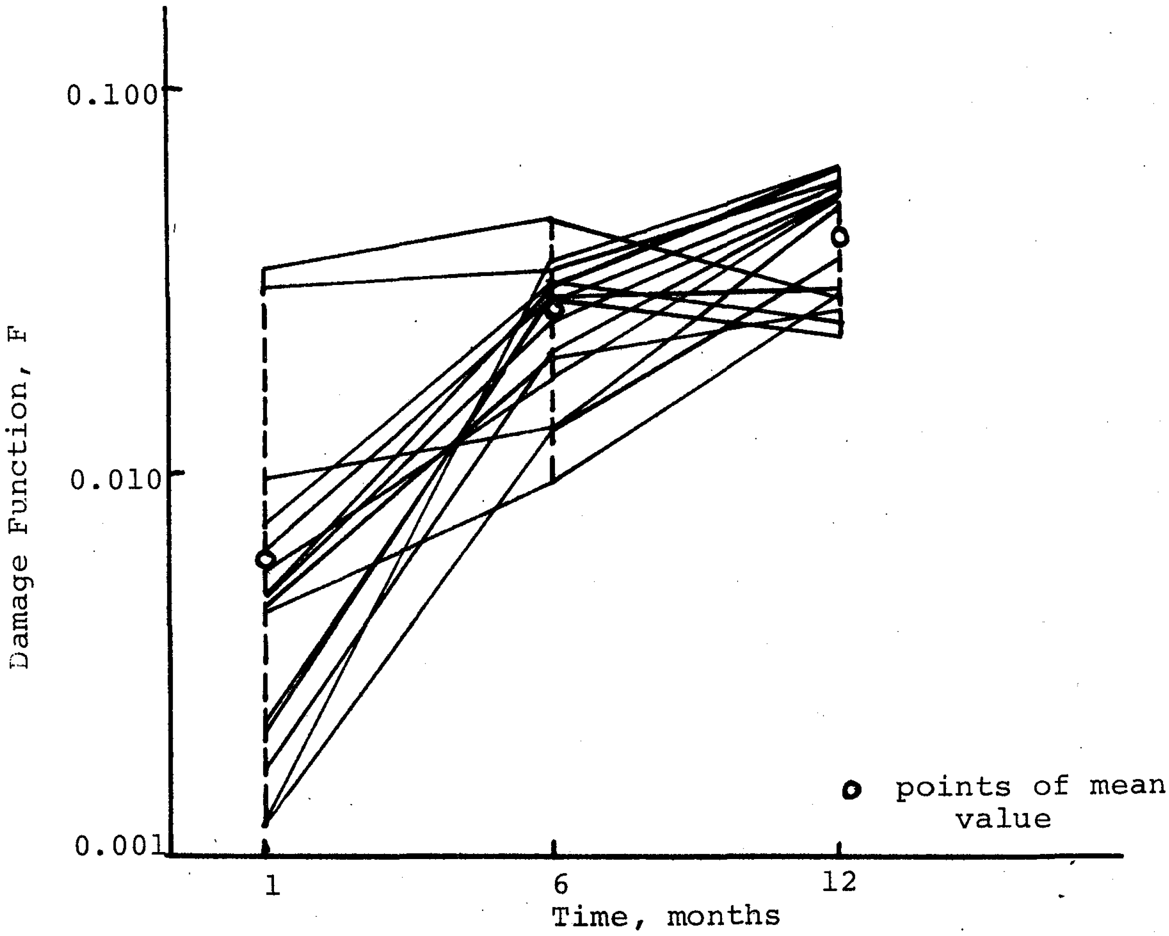


FIGURE 18. EXAMPLE OF DAMAGE FUNCTION OBTAINED FOR 15 SIMULATION RUNS

damage functions after successively 1, 6, and 12 months corresponding to 15 cases. From such figures one can guess the trends in the responses, i.e., the averages, variances, extremes, etc. To measure these trends, however, it is necessary to determine the distribution functions of the residual strains and the damage at different observation times. For a 100 samples the resulting distributions of the residual strains are shown in Figures 19 to 21. Figure 19 shows the results after a month period, while Figures 20 and 21 present the results after 6 and 12 months. The distributions of the damage after 1, 6, and 12 months are shown successively in Figures 22, 23, and 24. It is noticed that the distributions of the residual strains have more tendency towards normal distributions than the distributions of damage. The nonlinearities introduced in the damage function affect their distribution functions, which appear to be skewed.

Figure 25 is a plot of the three distribution functions of the residual strains after 1, 6, and 12 months plotted at the same scale. One can readily observe the shift in the distribution functions as time increases.

Figure 26 is a plot of the distribution functions of the log of the damage after 1, 6, and 12 months. From this plot one can also observe how the distributions are shifted as time increases. This figure also suggests that the distributions of the damage functions appear to be better represented by a log normal distribution

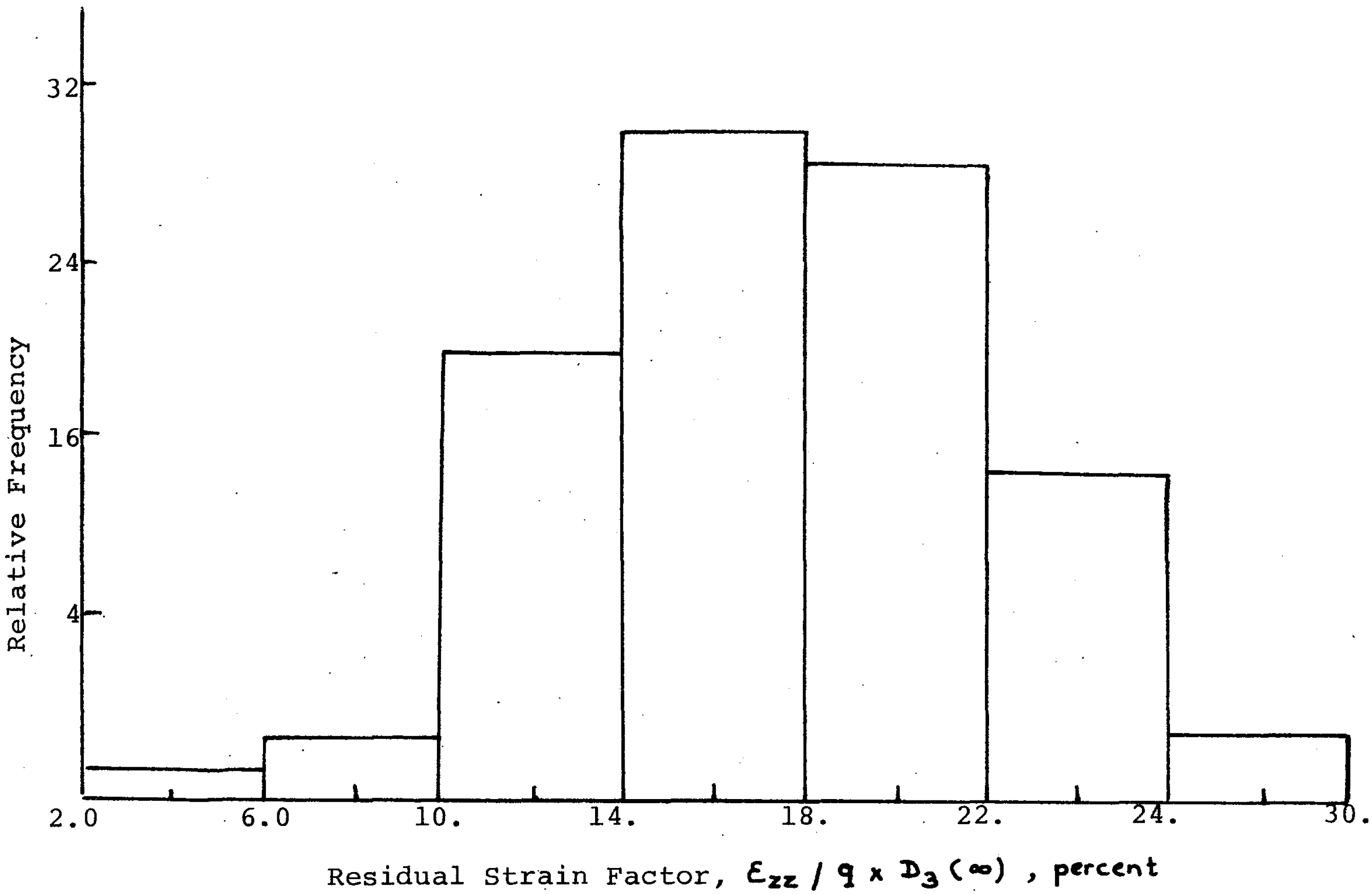


FIGURE 19 - RESIDUAL STRAIN FACTOR AFTER 1 MONTH PERIOD

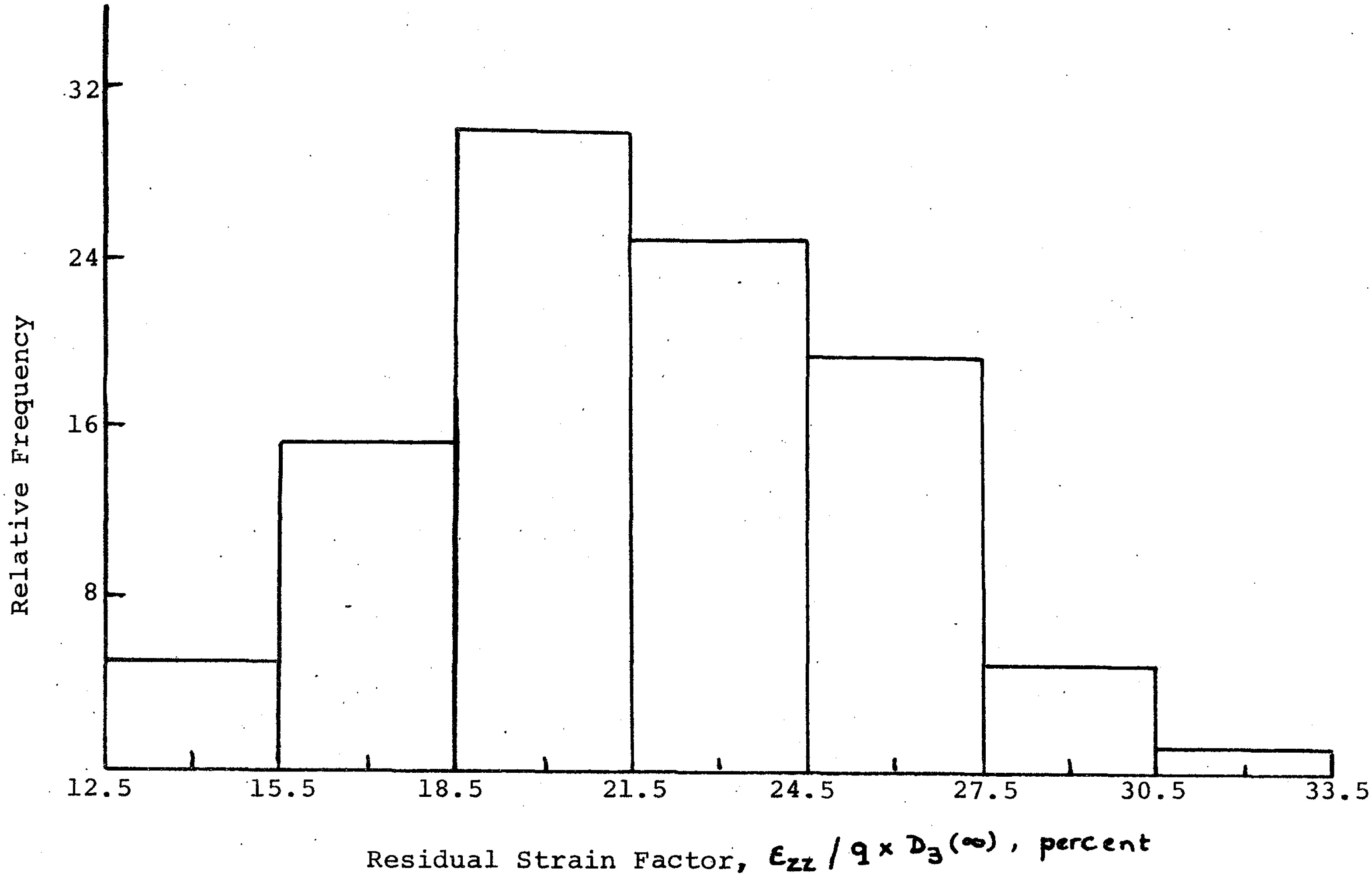


FIGURE 20 - RESIDUAL STRAIN FACTOR AFTER 6-MONTH PERIOD

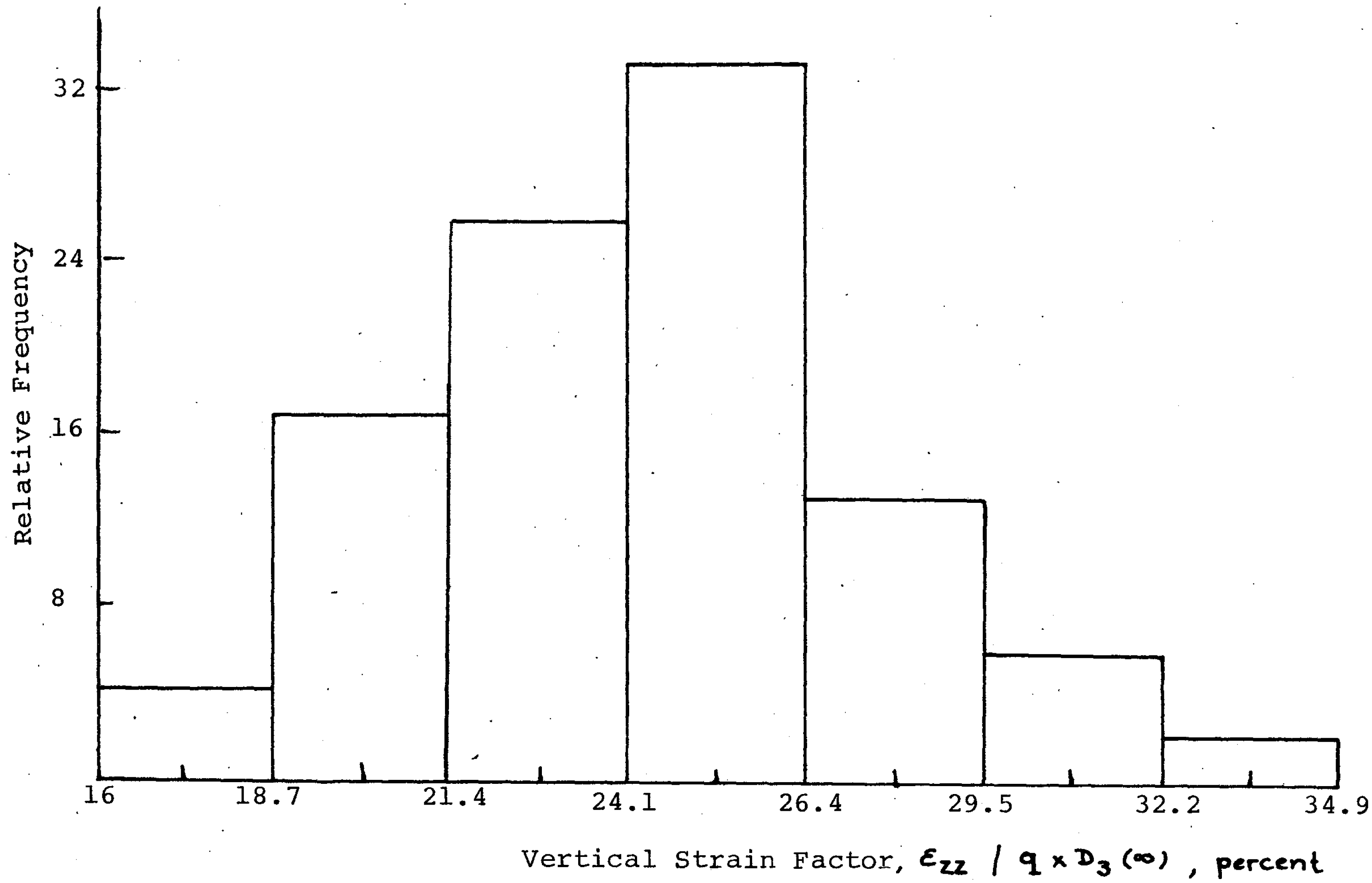


FIGURE 21 - RELATIVE FREQUENCY DISTRIBUTION OF RESIDUAL **STRAIN FACTOR**, AFTER 12 Months PERIOD.

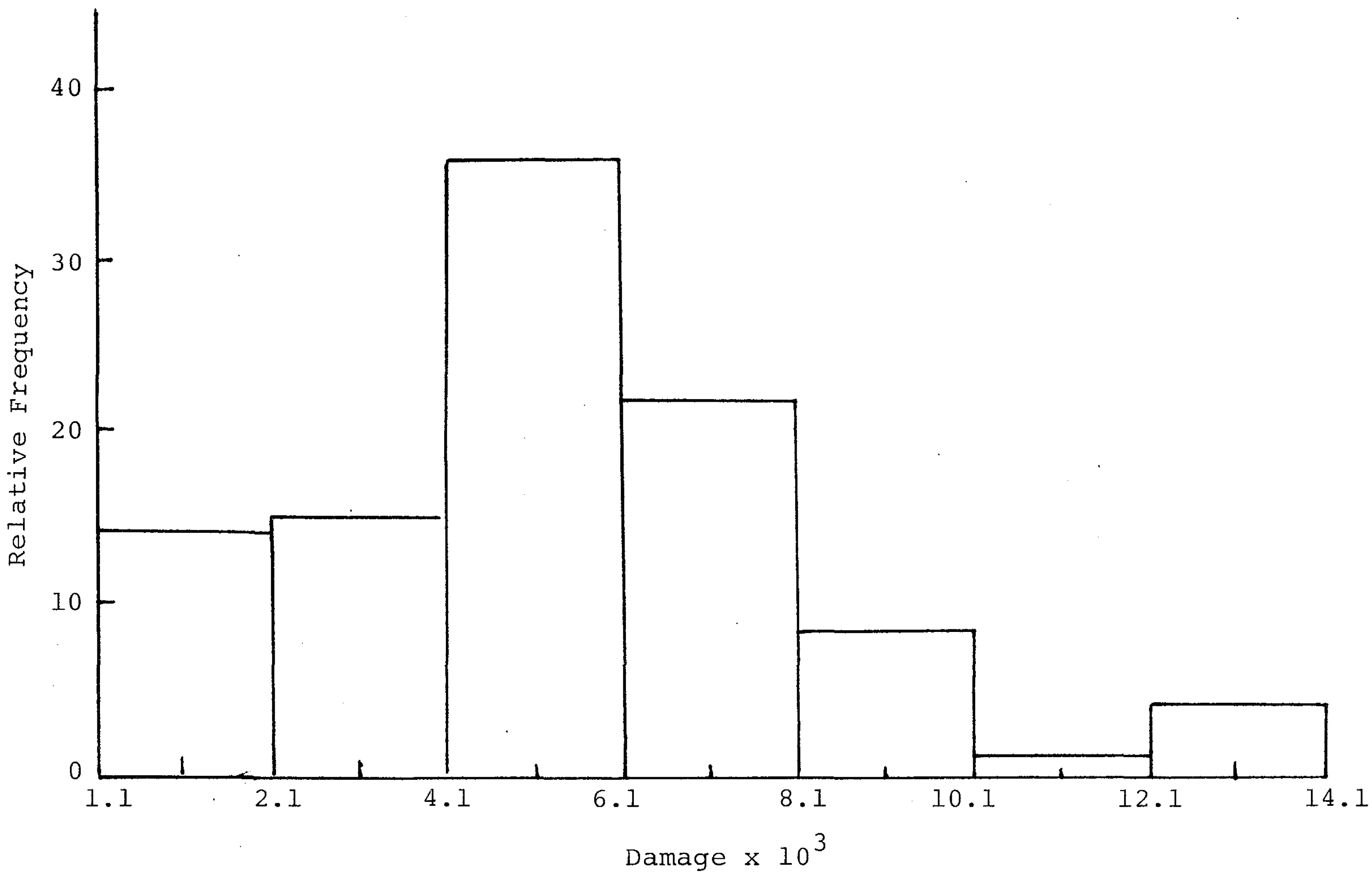


FIGURE 22 - FREQUENCY DISTRIBUTION OF DAMAGE AFTER 1 MONTH

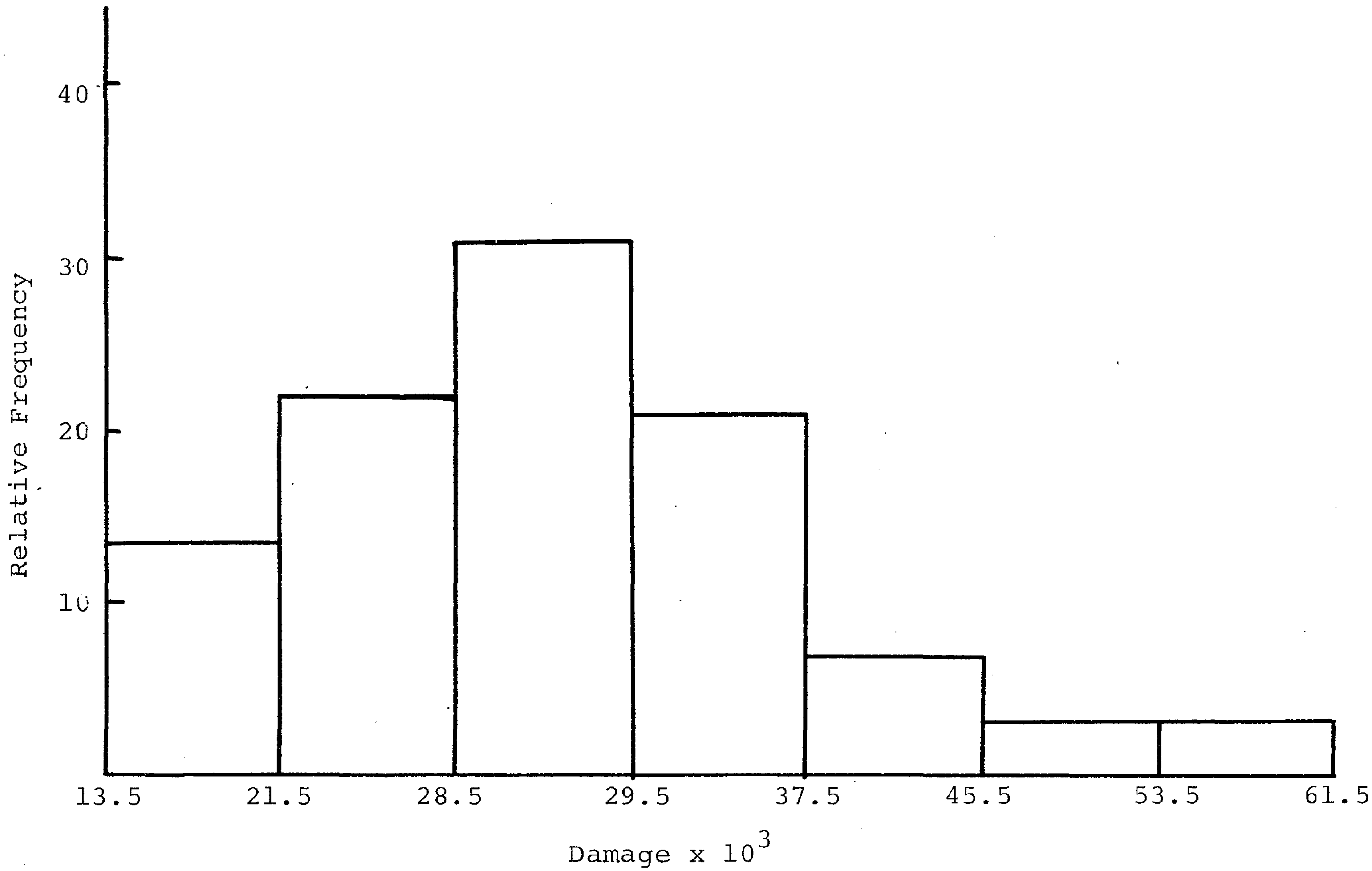


FIGURE 23 - RELATIVE FREQUENCY DISTRIBUTION OF DAMAGE AFTER 6 MONTHS

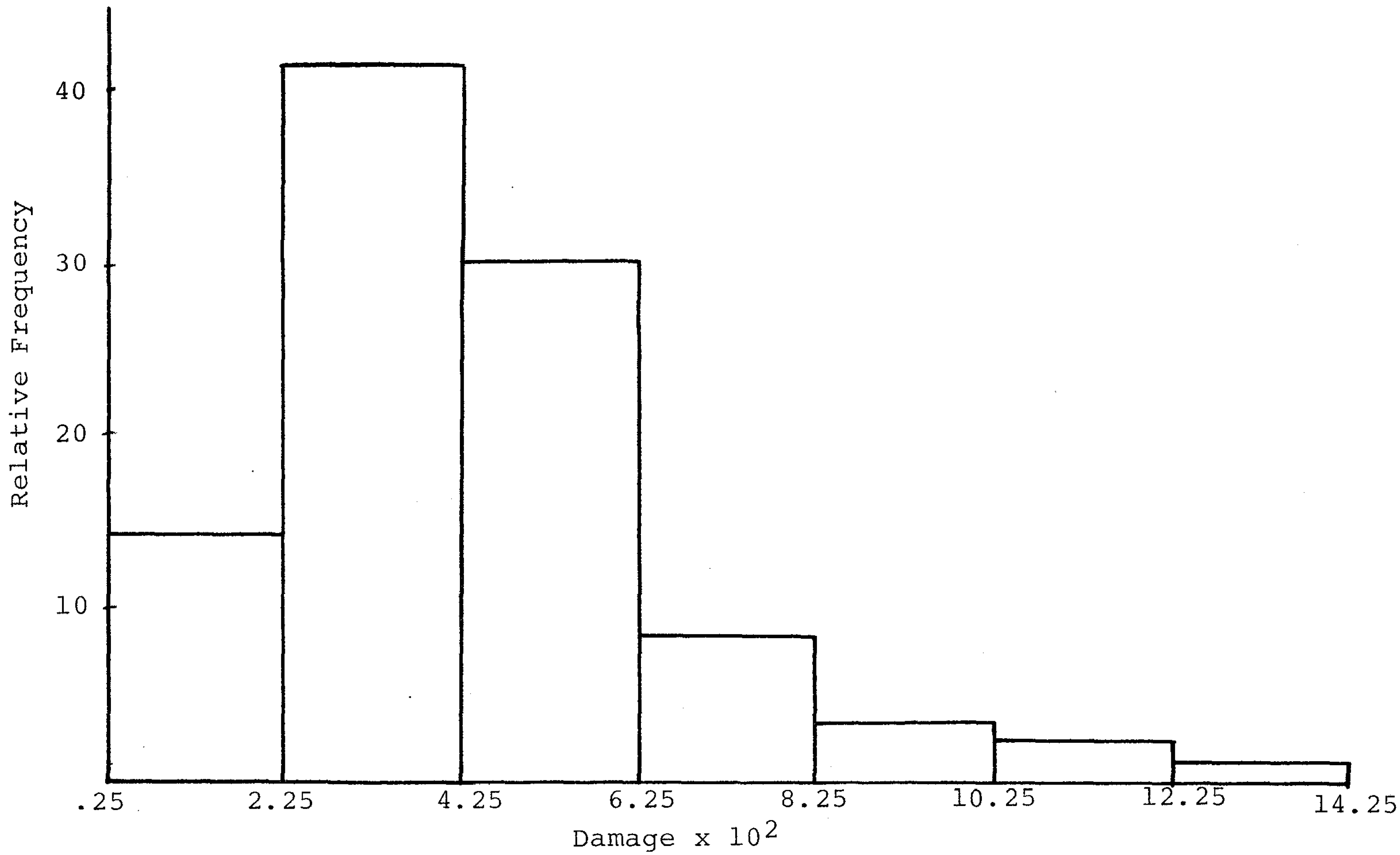


FIGURE 24. RELATIVE FREQUENCY DISTRIBUTION OF DAMAGE AFTER 12 MONTHS

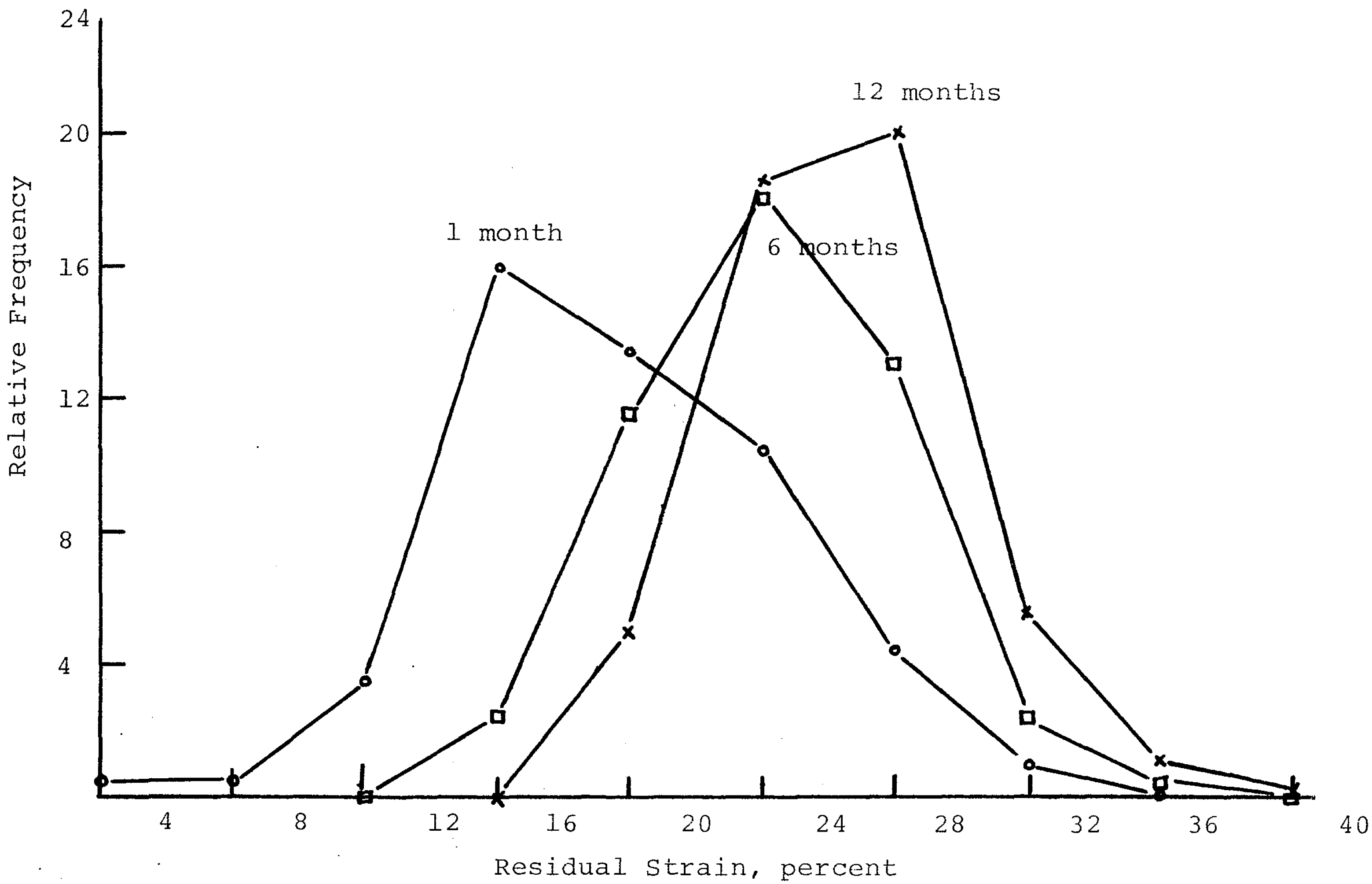


FIGURE 25 - FREQUENCY DISTRIBUTION OF RESIDUAL STRAINS AT THREE OBSERVATION TIMES.

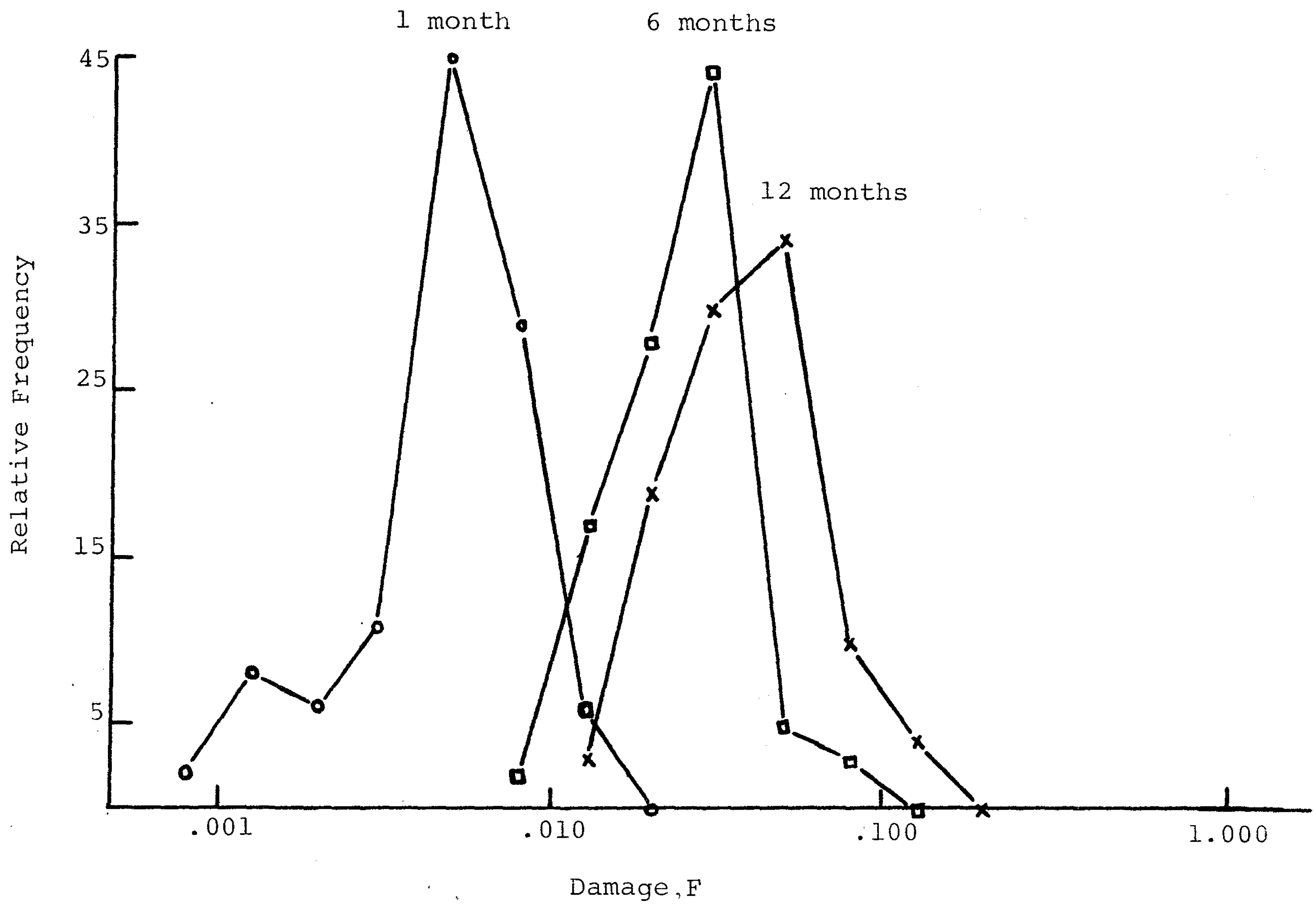


FIGURE 26 - FREQUENCY DISTRIBUTION OF DAMAGE FUNCTION
AT THREE OBSERVATION TIMES. (log scale)

(compared with Figures 22, 23, and 24).

In Figure 27 the histogram of the generated number of load applications was shown to check that the input values were effectively normal variates.

These results show how the simulation techniques can be used along with the developed model. This method of analysis allows the designer to evaluate the probability of occurence of some amount of damage at a given age of a structure. The model can easily account for seasonal changes, other environmental variables, and aging effects. The data used in the different parts of the model came from different sources, so it does not necessarily represent a coherent set of realistic values. Therefore, one can only use the results to evaluate the capabilities of the model. It is not possible, however, to draw specific conclusions as for the real behavior of a highway system. That is the shape of the various distribution that is only characteristic of the particular systems which were considered. Hence, there is a need to obtain the required experimental data corresponding to a realistic system in order to conduct sensitivity analyses on the model and determine the parts which need to be modified.

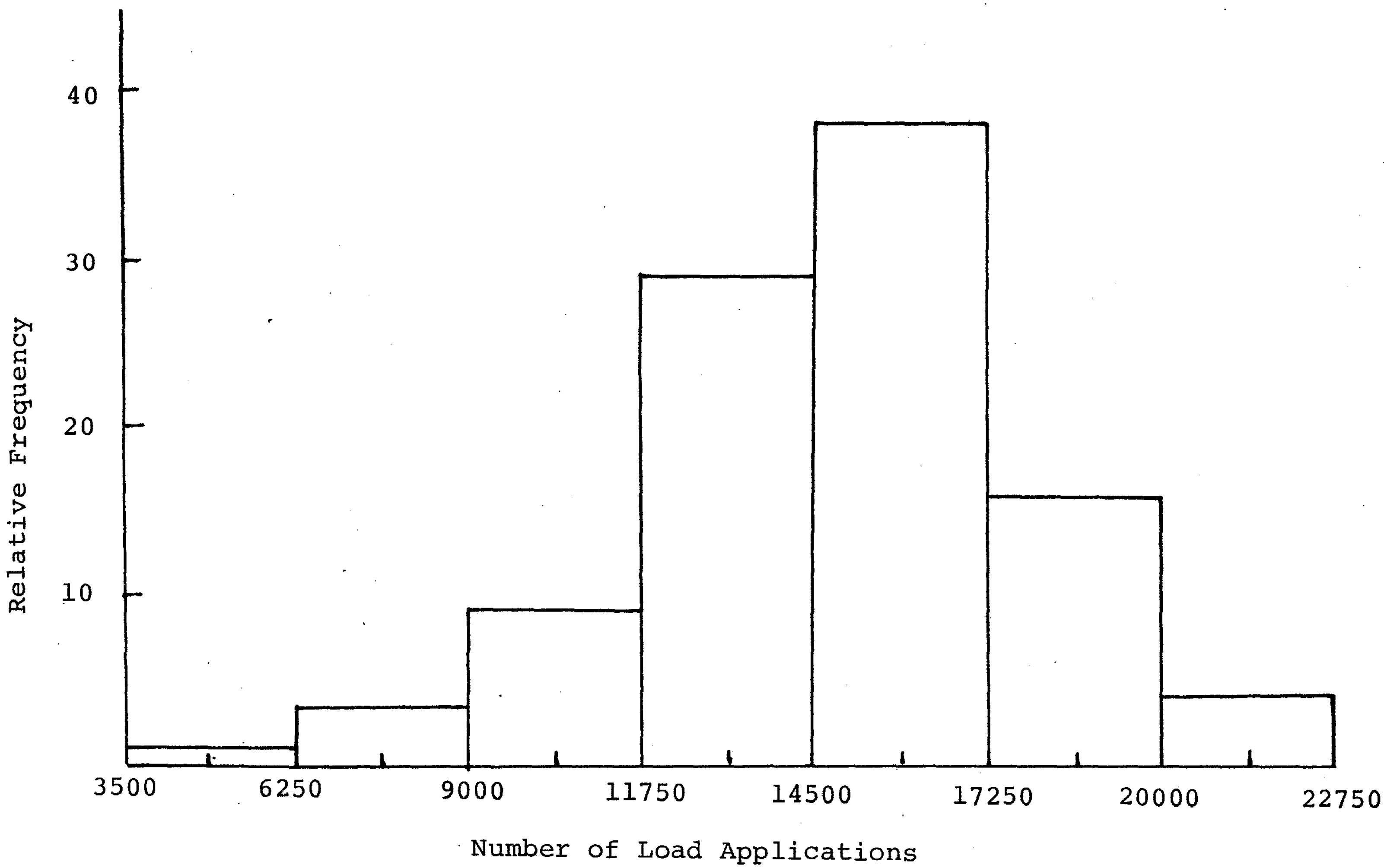


FIGURE 27. FREQUENCY DISTRIBUTION OF THE NUMBER OF LOAD ~~RELATIVE~~ APPLICATIONS
(100 Samples)

VI. SUMMARY

The objective of this study is to identify the modes of damage, their initiation, propagation, and accumulation in the flexible pavement structures. The simulated framework is limited to only load associated damage and its influence on the structural integrity of the pavement. The study is performed by first reviewing the concept of damage in engineering material, and thus providing the necessary background work for the development of a damage concept in structure sensitive engineering systems. Then, the pavement system and its modes of damage are reviewed with special emphasis upon the mode of damage associated with the repeated loading.

The results substantiate the following conclusions:

1. Pavement failure is a many sided problem, and it is the result of a series of interacting complex processes, none of which is completely understood. The question of what constitute the failure is highly subjective and it depends upon the user's evaluation of the facility.
2. The damage in pavement structures is accumulative, and depends upon the external excitation--loading and environmental variables--and the physical factors which measure the competence of the system.

3. The input variables and the capabilities of the pavement to resist the initiation and growth of damage can at best be represented in a stochastic manner.
4. Development of any comprehensive model for analysis of damage in pavement structure should take into account: a) the subjective nature of definition of failure, b) cumulative nature of damage, and c) variabilities present in materials properties, environmental factors, and load.
5. Simulations procedures have been used successfully to predict the behavior, failure and variability of solid propellants (96). They are powerful tools for the analysis of damage in pavement structures and should be used when closed form probabilistic solutions are not readily obtainable.

The framework of a comprehensive model for analysis of damage in highway pavement is also presented. This framework consists of a three-layer viscoelastic model, and a cummulative damage concept used in conjunction with a simulation technique. Parts of the model, such as the determination of differential settlements, are still to be implemented. As more experimental data becomes available, the model can be completed. For instance it can easily account for aging effects if the proper data is available.

Realistic data values are also necessary in order to test the model. Sensitivity analyses will then determine the elements of the model to be modified or expanded.

This model demonstrates how the designer can account for the stochasticity of the input parameters. It also shows some of the advantages and drawbacks of the simulation techniques, for instance the flexibility in the model and the length in computing time. The results are presented in the form of probability distribution functions, rather than in the classical deterministic form.

LIST OF REFERENCES

1. Moavenzadeh, F., and Lemer, A.C., "An Integrated Approach to Analysis and Design of Pavement Structure", M.I.T., Civil Engineering Report, R68-58, July 1968.

2. Biarez, J. "Contribution a'l'Etude des Propriétés Mecaniques des Sols et des Materiaux Pulvérulents" D.Sc. Thesis, University of Grenoble, 1962.

3. Monismith, C.L., Alexander, R.L. and Secor, K.E., "Rheological Behaviour of Asphalt Concrete" Proc. Association of Asphalt Paving Technologists, 1966, Vol. 35.

4. Terrel, R.L. "Factors influencing the resilient characteristics of asphalt treated aggregates" Ph.D. Dissertation, University of California, Berkeley, Aug. 1967.

5. Noll, W. "A Mathematical Theory of the Mechanical Behavior of Continuous Media" Arch. Rational Mech. Anal. 2, 197-226, 1958.

6. Rivlin, R.S. "Nonlinear Viscoelastic Solids", SIAM Review, 7, 323-340, 1965.

7. Lambe, T.W. and Whitman, R.V. "An Introduction to Soil Mechanics", McGraw-Hill, 1970.

8. Dehlen, G.L. "The Effect of Non-Linear Material Response on the Behaviour of Pavements Subjected to Traffic Loads", Ph.D. Dissertation, University of California, Berkeley 1969.

9. Leaderman, M. "Large Longitudinal Retarded Elastic Deformation of Rubberlike Netwoek Polymers" Trans. Soc. Rheol. 6, p. 361, 1962.

10. Green, A.E. and Rivlin, R.S. "The Mechanics of Non-linear Materials with Memory" Arch. Rational Mech. Anal., 1, 1-34, 1957.

11. Moavenzadeh, F., and Elliott, J.F., "Moving Loads on a Viscoelastic Layered System," Research Report R68-37, Department of Civil Engineering, Materials Research Laboratory, School of Engineering, Massachusetts Institute of Technology, Cambridge, Mass. June, 1968.

12. Boussinesq, J., Application des Potentials, Paris, 1885.

13. Terazawa, K., Journal of the College of Sciences, Imperial University, Tokyo, December 1916.

14. Love, A.E.H., "The Stress Produced on a Semi-Infinite Body by Pressure on part of the Boundary," Philosophical Transactions of the Royal Society, Series A., Vol. 228.

15. Ahlvin, R.G. and Ulery, H.H. "Tabulated Values for Determining the Complete Pattern of Stresses, Strains and Deflections Beneath a Uniform Circular Load on a Homogeneous Half Space," Highway Research Board Bulletin, No. 342, 1962.

16. Westergaard, H.M., "Stresses in Concrete Pavements Computed by Theoretical Analysis," Public Roads, Vol. 7, No. 2, April, 1926.

17. Burmister, D.M., "The General Theory of Stresses and Displacements in Layered Soil System, I, II, III," Journal of Applied Physics, Vol. 16, No. 2, pp. 89-96; No. 3, pp. 126-127; No. 5, pp. 296-302; 1945.

18. Burmister, D.M., "The Theory of Stresses and Displacements in Layered Systems and Application to the Design of Airport Runways," Highway Research Board Proceedings, 1954.

19. Davis, M.M., McLeod, N.W. and Bliss, E.J. "Symposium on Pavement Design and Evaluation-Report and Discussion of Preliminary Results," Proceedings, Canadian Good Roads Association, 1960.

20. Yoder, E.J. "Flexible Pavement Deflections - Methods of Analysis and Interpretation" Purdue University Engineering Reprints CE19A, July 1963.

21. Achenbach, J.D. and Sun, C., "Dynamic Response of a Beam on a Viscoelastic Subgrade," Proceedings of the American Society of Civil Engineers, Journal of Engineering Mechanics, Vol. 91, No. EM5, October 1965.

22. Kraft, D.C. "Analysis of a Two-Layer Viscoelastic System," Proceedings of the American Society of Civil Engineers, Journal of the Engineering Mechanics Division, Vol. 91, No. EM6, Part I, December 1965.

23. Pister, K.S., "Viscoelastic Plates on Viscoelastic Foundations" Journal of the American Society of Civil Engineers pp. 43-54. February 1961.

24. Elliott, J.F. and Moavenzadeh, F. "Moving Load on Viscoelastic Layered Systems Phase II" M.I.T. Department of Civil Engineering, R69-64, 1969.

25. von Karman, T., Mitt. Forch. Ver. Deut. Ing., pt. 118, 37-68 (1912).

26. Orowan, E., "Fracture and Strength of Solids, The Physical Society, Report on Progress in Physics, Vol. XII, P. 185, 1949.

27. Hirata, M., Scientific Paper, Institute-Phys. Chem. Research, 16 (1931) 187.

28. Joffe, A., International Conference on Physics II. The Solid State of Matter, Phys. Soc. London (1934) 72.

29. Yokobori, T., Journal of the Physics Society, Japan, 7 (1951) 44.

30. ______________, Journal of the Physics Society, Japan, 6 (1951) 81.

31. ______________, Journal of the Physics Society, Japan, 7 (1952) 48.

32. ______________, Journal of the Physics Society, Japan, 8 (1953) 265.

33. Gladstone, S., Laidler, K.J., and Eyring, H., The Theory of Rate Processes, McGraw-Hill, New York, 1941.

34. Knauss, W.G., "The Time Dependent Fracture of Visco-elastic Materials", Proceedings of the First International Conference on Fracture, Sendai, Japan, 1965.

35. Weibull, W., Ingen. Vetensk Akad. (Stockholm) Proc. No. 151 (1939) a; (1939)b. Ibid., Proc. No. 153.

36. Frenkel, J., and Kontorova, T.A., "A Statistical Theory of the Brittle Strength of Real Crystals", Journal of Physics, U.S.S.R., 7, 108 (1943).

37. Novozhilov, V.V. "The Physical Meaning of the Stress Invariants of the Theory of Plasticity", Applied Math. and Mech. (Acad. of Science U.S.S.R.), Vol. 16, 1952.

38. Nadai, A., Theory of Flow and Fracture of Solids, Vol. 1, Second Edition, McGraw-Hill Book Company, 1950.

39. Ford, H., Advanced Mechanics of Materials, Wiley, 1963.

40. Bishop, A.W., "The Strength of Soils as an Engineering Material." Geotechnique, Vol. 16. No. 2, June 1966, pp. 99-128.

41. Griffith, A.A., Phil. Trans. Roy. Soc.A, 221, 163 (1920); First International Congress of Applied Mechanics (Delft) 5, (1924)

42. Irwin, G.R. "Fracture Mechanics" Proc. of the First Symposium on Naval Structural Mechanics, Stanford University, August 1958.

43. Majidzadeh, K. et al. "Analysis of Fatigue and Fracture of Bituminous Paving Mixtures" The Ohio State University, Report RF 2845, May 1970.

44. Smith, T.L. "Ultimate Tensile Properties of Elastomers Comparison of Failure Envelopes of Unfilled Vulcanizates", Journal of Applied Physics, Vol. 35, No. 1, Januray 1964.

45. Bartenev, G.M. and Zuyev, Y.S. "Strength and Failure of Viscoelastic Materials", Pergamon Press, 1968.

46. McEvily, A.J. and Boettner, R.C. "On Fatigue Crack Propagation in F.C.C. Metals", Acta Metallurgica, Vol. 11, p. 725, 1963.

47. McEvily, A.J., Boettner, R.C. and Johnston, T.L. "On the Formation and Growth of Fracture Cracks in Polymers," Fatigue an Inetrdisciplinary Approach, Proceedings of the 10th Sagamore Army Mat. Res. Conf., Syracuse University Press, 1964.

48. Erikson, William H. and Work, Clyde E., "A Study of An Accumulation of Fatigue Damage in Steel," Proceedings, ASTM, Vol. 61, 1961.

49. Coleman, B.D. "Application of the Theory of Absolute Reaction Rates to the Creep Failure of Polymeric Filaments" Journal of Polymer Science, Vol. XX, 1956.

50. Machlin, E.S. "Dislocation Theory of the Fatigue of Metals," National Advisory Committee for Aeronautics, Technical Notes, 1948.

51. Mott, N.F. Symposium on Dislocations in Crystals, International Conference Theoretical Physics, held at Nikko, Japan, September 1953, Abstract 56.

52. Miner, M.A., "Cumulative Damage in Fatigue," Journal of Applied Mechanics, Vol. 4, Trans. ASME Vol. 12, 1945 pp. A159-A164.

53. Corten, H.T. and Dolan, T.J., "Cumulative Fatigue Damage," Proceedings, International Conference on the Fatigue of Metals. Am. Soc. Mech. Engrs., 1956.

54. Valluri, S.R., "A Unified Engineering Theory of High Stress Level Fatigue," Institute of the Aeronautical Sciences Paper Number 61-149-1843, June 1961.

55. Newmark, N.M., Fatigue and Fracture of Metals, Wiley and Sons, 1952.

56. Williams, M.L., "Initiation and Growth of Viscoelastic Fracture," Proceedings of the First International Conference on Fracture, Vol. 2. Sendai, Japan, 1965.

57. Frost, N.E., "The Growth of Fatigue Cracks," First International Conference on Fracture, Sendai, Japan, Vol. 3, 1965.

58. Paris, P. and Erdogan, F.J. "A Critical Analysis of Crack Propagation Laws" Journal of Basic Engineering, Transaction of ASME, Series D, Vol. 85, 1963.

59. Weiss, V., "Analysis of Crack Propagation in Strain Cycling Fatigue," Fatigue an Interdisciplinary Approach, Proceedings of the 10th Sagamore Army Mat. Res. Conf., Syracuse University, 1964.

60. Liu, Discussion on article by P.C. Paris, "The Fracture Mechanics Approach to Fatigue," Fatigue as an Interdisciplinary Approach, Proceedings 10th Sagamore Army Mat. Res. Conf. Syracuse University Press, 1964.

61. Paris, P.C. "The Fracture Mechanics Approach to Fatigue," Fatigue, An Interdisciplinary Approach, Proceedings of the 10th Sagamore Army Materials Research Conference, Syracuse University Press, 1964.

62. Dong, R.G. "A Functional Cumulative Damage Theory and its Relation to two well-known theories," Lawrence Radiation Lab., University of California, UCRL, January 1967.

63. Quinn, Bayard E., and Thompson, David R., "Effect of Pavement Condition Upon Dynamic Vehicle Reactions" HRB, 1962.

64. Monismith, C.L., Kasianchuk, D.A., and Epps, J.A., "Asphalt Mixture Behavior in Repeated Flexure: A Study of an In-Service Pavement near Morro Bay, California," Report No. TE67-4, Department of Civil Engineering, Institute of Transportation and Traffic Engineering, University of California, Berkeley.

65. Committee on Structural Design of Roadways, "Problems of Designing Roadway Structures," Transportation Engineering Journal, Proceedings ASCE, May 1969.

66. Fung, Y.C., Foundations of Solid Mechanics, Prentice-Hall, Inc.

67. Clegg, B. and Yoder Eldon J., "Structural Analysis and Classification of Pavements," Fourth Australia-New Zealand Conference on Soil Mechanics and Foundation Engineering.

68. Porter, O.J., "Foundations for Flexible Pavements," Proceedings, Highway Research Board, Vol. 22, 1942.

69. Nijboer, L.W. and van der Poel, C., "A Study of Vibration Phenomena in Asphaltic Road Constructions," Proceedings, Association of Asphalt Paving Technology, Vol. 22, 1953, pp. 197-231.

70. Hveem, R.N., "Pavement Deflections and Fatigue Failures," Highway Research Board, Bulletin 114, 1955, pp. 43-73.

71. The WASHO Road Test, HRB, Special Report 22.

72. Speed, H.B., Chan, C.K. and Lee, C.E., "Resilience Characteristics of Subgrade Soils and Their Relation to Fatigue Failures in Asphalt Pavements," International Conference on the Structural Design of Asphalt Pavements, Ann Arbor, Michigan, 1962.

73. Hennes, R.G. and Chen, H.H., "Dynamic Design of Bituminous Pavements - the Trend in Engineering, University of Washington, 1950.

74. Monismith, C.L., "Flexibility Characteristics of Asphalt Paving Mixtures," Proceedings AAPT, Vol. 27 1958, pp. 74-106.

75. Saal, R.N.J. and Pell, P.S., "Fatigue of Bituminous Road Mixes," Kolloid Zeitschrift. Bd. 171, Heft 1, 1960, pp. 61-71.

76. Monismith, C.L. "Significance of Pavement Deflection" Proceedings AAPT, Vol. 31, 1962, pp. 231-253.

77. "Three Year Evaluation of Shell Avenue Test Road," 44th Annual Meeting HRB (1965).

78. Deacon, J.A., Fatigue of Asphalt Concrete, Graduate Report Institute of Transportation and Traffic Engineering, University of California, Berkeley, 1965.

79. Monismith, C.L., "Asphalt Mixture Behavior in Repeated Flexure," Report No. T 65-9 Department of Civil Engineering, Institute of Transportation and Traffic Engineering, University of California, Berkeley.

80. Larew, H.G., and Leonards, G.A., "A Strength Interior for Repeated loads," Proceedings HRB, Vol. 41 (1962) pp. 529-556.

81. Deacon, J.A., and Monismith, C.L. "Laboratory Flexural Fatigue Testing of Asphalt Concrete With Emphasis on Compound Loading Tests," HRR No. 158, HRB, Washington, D.C.

82. Majidzadeh, K., Kaufmann, F.M., Ramsamaaj, D.V., "Fatigue Design for Pavement Systems," A Paper Presented to the Annual Meeting of the American Society of Civil Engineers, Boston, July 1970.

83. Kabaila, A.P., and Warner, R.F., "Monte Carlo Simulation of Variables Material Response" Proceedings, International Conference in Structure, Solid Mechanics, and Engineering Design in Civil Engineering Materials, Southampton, England, 1969.

84. Ashton, J.E. and Moavenzadeh, F., "The Analysis of Stresses and Displacements in a three-layered Viscoelastic System," International Conference on the Structural Design of Asphalt Pavements, 1967.

85. Gross, R.V. "Modern Operational Mathematics in Engineering,"McGraw-Hill, New York, 1944.

86. Moavenzadeh, F., and Elliott, J.F., "Moving Loads on a Viscoelastic Layered System," Research Report R68-37, Department of Civil Engineering, Materials Research Laboratory, School of Engineering, Massachusetts Institute of Technology, Cambridge, Mass. June, 1968.

87. Valyer, P.J. "Research on Mechanical Phenomena in Roads and Asphalt Mixes", Revue Générale des Routes et des Aerodromes, No. 437, 37 (1968).

88. Bazin, P. and Saunier, J.B., "Deformability, Fatigue and Healing Properties of Asphalt Mixes," Second Int. Conf. Struc. Des. of Asphalt Pavements, Ann Arbor, Michigan, 1967.

89. Freudenthal, A.M., and Heller, R.Z., "On Stress Interaction in Fatigue and a Cumulative Damage Rule," J. Aero Space Science, Vol. 26, No. 7, July 1959.

90. Kasianchuk, D.A., "Fatigue Considerations in the Design of Asphalt Concrete Pavements," Ph.D. dissertation, Graduate Division, Univ. of Cal., Berkeley, 1969.

91. Stouffer, D.C., "On Linear Viscoelastic Materials with Aging or Environment Dependent Properties," Ph.D. dissertation, Univ. of Michigan, 1968.

92. Moavenzadeh, F., Soussou, J.E., "Linear Viscoelastic Characterization of Sand-Asphalt Mixtures," M.I.T. Civil Engineering Department Report No. R67-32, August 1968.

93. Knauss, W.G. and Dietmann, H. "Crack Propagation Under Variable Load Histories in Linearly Visco-elastic Solids" Int. J. Engng. Sci. Vol. 8, pp. 643-656, 1970.

94. Glucklich, J. "Static and Fatigue Fracture of Portland Cement Mortar in Flexure" First International Conference on Fracture, Sendai, Japan, Vol. 3, 1965, pp. 1343.

95. Findakly, H. "Stochastic Approach to the Analysis of Highway Pavements" S.M. thesis, Civil Engng, M.I.T., 1971.

96. Majerus, J.N., Briar, H.P. and Wiegand, J.H., "Behavior and Variability of Solid Propellants and Criteria for Failure and for Rejection" Journal of Spacecraft and Rockets, Vol. 2, No. 6, November 1965.

APPENDIX I

COMPUTER PROGRAM

1. Formulation of the Primary Response

The response function $P_s(t)$ of a linear viscoelastic system to a repeated load in a variable environment may be written as a convolution integral of the following form:

$$P_s(t) = \int_{-\infty}^{t} SR[t - \tau,\ \phi^{t}_{\tau=-\infty}(\tau)]\ \frac{\partial P(\tau)}{\partial \tau}\ d\tau$$

where $SR(t - \tau, \phi)$ is the response of the system to a unit step load in a constant environment ϕ. If this step response is

$$SR(t - \tau) = \sum_{i=1}^{n} G_i\ e^{-(t-\tau)\delta_i}$$

it was shown in Reference 91 that

$$SR(t-\tau,\ \phi^{t}_{\tau=-\infty}(\tau)) = \alpha[\phi(t)] + \beta[\phi(t)][\sum_{i=1}^{n} G_i\ e^{-t^*\delta_i}] - \int_{\tau}^{t} \{\sum_{i=1}^{n} G_i\ e^{-s^*\delta_i}\}\ d\beta[\phi(s)]$$

where

$$x^* = \int_{\tau}^{x} \gamma[\phi(\theta)]\ d\theta.$$

For the particular values

$$\alpha[\phi(t)] = 0,$$

$$\beta[\phi(t)] = T(t)/T_o,$$

$$\gamma[\phi(t)] = \frac{1}{a_T} = 10^{[10{,}000\ (\frac{1}{T(t)} - \frac{1}{T_o})]}$$

where T_o is the reference temperature which has been taken to be equal to 298° K, this expression becomes:

$$SR[t-\tau, \underset{\tau=-\infty}{\phi^t(\tau)}] = \frac{T(t)}{T_o}\ [\sum_{i=1}^{n} G_i\ e^{-\delta_i t^*}] - \int_\tau^t \sum_{i=1}^{n} G_i\ e^{-\delta_i s^*}\ \frac{dT(s)}{T_o}\ .$$

If the load is applied as a haversine function, $\sin^2\omega\tau$, for a duration d and if the time elapsed between two applications of the load is p_K, then we have for a basic time period:

$$P_s(t) = \int_0^t \{\ \frac{T(t)}{T_o}\ [\sum_{i=1}^{n} G_i\ e^{-\delta_i t^*}] - \int_\tau^t \sum_{i=1}^{n} G_i\ e^{-\delta_i s^*}\ \frac{dT(s)}{T_o}\}\ \frac{\partial P(\tau)}{\partial \tau}\ d\tau.$$

Let the temperature change as a step function at the end of each basic time period then:

$$P_s(t_J) = \int_0^{t_J} \{\frac{T(t)}{T_o} [\sum_{i=1}^{n} G_i e^{-\delta_i t^*}] - \sum_{M=K}^{J-1} \sum_{i=1}^{n} G_i e^{-\delta_i s_m^*} (\frac{T_{M+1}-T_M}{T_o})\}$$

$$x \frac{\partial P(\tau)}{\partial \tau} d\tau$$

where τ was symbolized by the index K while t was replaced by t_J or more concisely J. By replacing

$$\int_0^{t_J} \quad \text{by} \quad \sum_{K=2}^{J}$$

and changing the order of summation, we obtain:

$$P_s(t_J) = \sum_{K=2}^{J} \sum_{i\neq 1}^{n} \int_{t_{K-1}}^{t_K}$$

$$\{ \frac{T(t)}{T_o} G_i e^{-\delta_i t^*} - \sum_{M=K}^{J-1} G_i e^{-\delta_i s^*} (\frac{T_{M+1}-T_M}{T_o})\}$$

$$x \frac{\partial P(\tau)}{\partial \tau} d\tau$$

$$P_s(t_J) = \sum_{K=2}^{J} \sum_{i=1}^{n} \frac{G_i}{T_o} \int_{t_{K-1}}^{t_K} \{T(t) e^{-\delta_i t^*} - \sum_{M=K}^{J-1} e^{-\delta_i s^*} (T_{M+1}-T_M)\}$$

$$\frac{\partial P(\tau)}{\partial \tau} d\tau.$$

The variables t^* and s^* can be written in a discretized form:

$$t^* = \gamma_K(t_K-\tau) + \sum_{KK=K+1}^{J} \gamma_{KK}(t_{KK}-t_{KK-1})$$

$$s^* = \gamma_K(t_K-\tau) + \sum_{MM=K+1}^{M} \gamma_{MM}(t_{MM}-t_{MM-1}).$$

Therefore further factorization can be obtained:

$$P_s(t_J) = \sum_{K=2}^{J} \sum_{i=1}^{n} (A_i - \sum_{M=K}^{J-1} B_i)$$

$$\{ \int_{t_{K-1}}^{t_K} e^{-\delta_i \gamma_K (t_K-\tau)} \frac{\partial P(\tau)}{\partial \tau} d\tau \}$$

where

$$A_i = \frac{T(t)}{T_o} G_i \; e^{-\delta_i \sum_{KK=K+1}^{J} \gamma_{KK}(t_{KK}-t_{KK-1})}$$

$$B_i = \frac{T_{M+1}-T_M}{T_o} G_i \; e^{-\delta_i \sum_{MM=K+1}^{M} \gamma_{MM}(t_{MM}-t_{MM-1})}$$

and the integral is evaluated below in a closed form. Since

$$\frac{\partial P(\tau)}{\partial \tau} = \begin{cases} \omega \sin 2\omega\tau & \text{when the load is on} \\ 0 & \text{otherwise} \end{cases}$$

$$\int_{t_{K-1}}^{t_K} e^{-\delta_i K(t_K-\tau)} \frac{\partial P(\tau)}{\partial \tau} d\tau =$$

$$\sum_{L=1}^{LL} [e^{-\delta_i \gamma_K (t_K-\tau)} \frac{\gamma_K \delta_i \sin 2\omega\tau - 2\omega \cos 2\omega\tau}{\gamma_K^2 \delta_i^2 + 4\omega^2}]_{(L-1)P_K}^{(L-1)P_K+d}$$

that is, it is the sum of LL terms corresponding to LL applications of the load. Each term is evaluated between the beginning of its application $(L-1)p_K$ and the end of its application $(L-1)p_K+d$, when p_K is the period between successive loads and d is their duration. This expression can be calculated as:

$$\int_{t_{K-1}}^{t_K} e^{-\delta_i \gamma_K (t_K-\tau)} \frac{\partial P(\tau)}{\partial \tau} d\tau = C_i \exp(\delta_i \gamma_K p_K) \exp(-\delta_i \gamma_K d) \times$$

$$[1 - \exp(-\gamma_K p_K \delta_i)] + \frac{1 + \exp(0.5\delta_i \gamma_K d)}{2[1 + (\frac{\gamma_K \delta_i}{2})^2]}$$

where

$$C_i = \frac{1 - \exp(\gamma_K \delta_i d)}{2[1 + (\frac{\gamma_K \delta_i}{2})^2][\exp(\delta_i p_K \gamma_K) - 1]}$$

$$\int_{t_{K-1}}^{t_K} e^{-\delta_i \gamma_K (t_K-\tau)} \frac{\partial P(\tau)}{\partial \tau} d\tau = \frac{1}{2[1 + (\delta_i \gamma_K / 2\omega)^2]}$$

$$\{ \frac{\exp(-\delta_i \gamma_K d)-1}{1 - \exp(-\delta_i \gamma_K p_K)} [1-\exp(-\gamma_K p_K \delta_i)] + [1 + \exp(0.5\delta_i \gamma_K d)]\}$$

for the peak values and

$$\int_{t_{K-1}}^{t_K} e^{-\delta_i \gamma_K (t_K-\tau)} \frac{\partial P(\tau)}{\partial \tau} d\tau = \frac{[1-\exp(\delta_i \gamma_K d)][1-\exp(-\gamma_K p_K \delta_i)]}{2[1 + (\frac{\gamma_K \delta_i}{2})^2][\exp(\gamma_K \delta_i p_K)-1]}$$

for the residual response. The chosen numerical values for G_i and δ_i were:

$G_1 = - 0.0087$ $\delta_1 = 0.05$

$G_2 = - 0.0030$ $\delta_2 = 0.005$

$G_3 = - 0.0032$ $\delta_3 = 0.0005$

$G_4 = - 0.0034$ $\delta_4 = 0.00005$

$G_5 = - 0.0039$ $\delta_5 = 0.000005$

$G_6 = - 0.0010$ $\delta_6 = 0.0000005$

$G_7 = + 0.0232$ $\delta_7 = 0.0$

2. Formulation for Damage F(t)

For the Kth time period, the increment of damage is:

$$\Delta f_K = \frac{n_K}{N_K(\Delta\varepsilon, \varepsilon)}$$

n_K is the given number of repetitions of the load

N_K is the function of the average amplitude $\Delta\varepsilon$

$$N_K = K \left(\frac{1}{\Delta\varepsilon} \right)^m$$

where K and m can be made functions of the mean strain ε, when data is available to set such relationships. K was taken equal to 5.10^{-14} and m = 4.5. When the temperature T was above 295° K, K was taken equal to 5.10^{-14} $[1 + 100(T - 295)]$. The total damage after J time periods is:

$$F_J = \sum_{K=2}^{J} \Delta F_K \times K_{REC}(t^*_{JK})$$

where t^*_{JK} equals the time interval between t_K and t_J during which the temperature T is greater than 295° K.

And $K_{REC}(t)$ is given in exponential form:

$$K_{REC}(t) = \sum_{i=1}^{N} R_i \; e^{-\alpha_i t}.$$

The assumption was that only 80% of the damage is recoverable, and that the recovery takes place in four decades of time. Thus the used data were:

$$N = 5$$
$$R_i = 0.2 \qquad i = 1, 2, \ldots 5$$
$$\alpha_1 = 0.0$$
$$\alpha_2 = 0.01$$
$$\alpha_3 = 0.001$$
$$\alpha_4 = 0.0001$$
$$\alpha_5 = 0.00001$$

3. Input Description

The input data to the program are the following:

DURA → Duration of the load on the system: Assumed to be constant all over.

G( ), DELTA( ) → Coefficients of the exponential series representation of the static response to to a unit load = $\Sigma_i G_i e^{-t\delta_i}$. Those coefficients are obtained by running the stationary load program and obtaining the required response at the point of interest, then curve-fit them using the least squares fitting method (the curve fitting program is available), to a series of the form shown above. Then the G_i's and δ_i's are obtained at decades apart from each point.

N → Number of terms in the exponential series i.e., $\sum_{i=1}^{N} G_i e^{-(t-\tau)\delta_i}$.

TMEAN, TVAR → The mean temperature and the variance of a certain statistical distribution of the prevailing temperature in the pavement system--assumed to be normal. The values of the temperature are given in °F.

NMEAN, NVAR → The mean value and the variance for the number of load applications in the basic time period. The distribution also is assumed to be normal.

TREF → A reference temperature in °K. Taken to be 298° K.

4. Solution for the Primary Response Model

The program is based on the following assumption:

1) The number of load applications is constant for a one basic time unit (month).
2) The temperature over the whole basic time unit is assumed to be constant.

3) The duration of the load on the system is constant for the whole interval, i.e., assuming the speed of the vehicles practically constant over the system.

4) The only term in the environment to vary is the temperature.

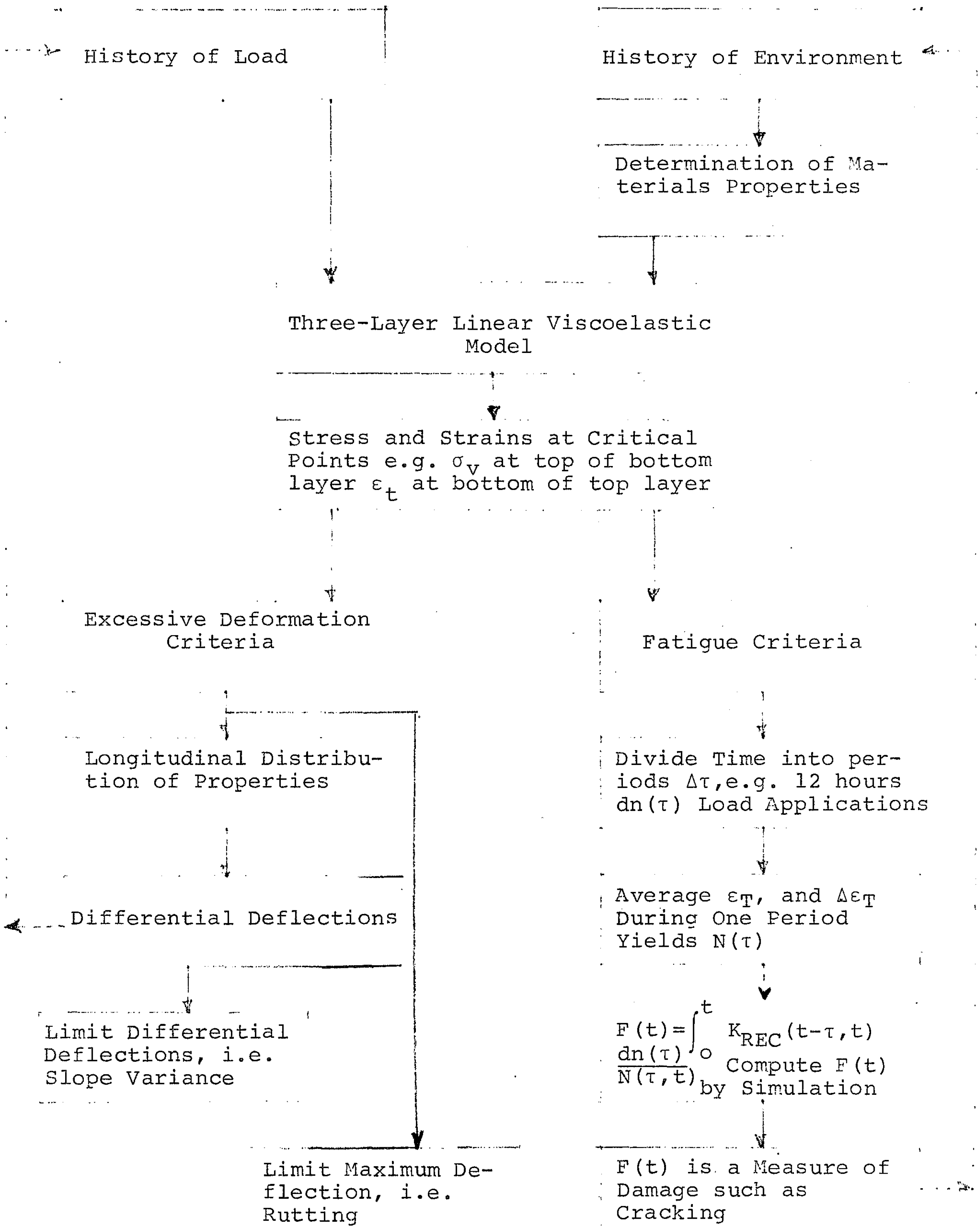
Thus the program generates a value for the number of load applications and a value for temperature randomly from the assumed probability density function and computes the response for the first month, subject to a repeated load of the form: $\sin^2 \omega\tau$. The response to the load is evaluated only when the load is applied on the system.

The response of the first month is stored, and a new set of temperatures and number of load cycles is generated. This process is repeated and the value of the response is printed whenever required, until the whole desired interval is covered.

5. Output

The output of the program is a set of values for stresses, strains, or deflections at some desired intervals of time. These intervals of time may be one day intervals, one week intervals, one month intervals, or one year intervals, or as desired.

FLOW CHART #1


History of Load
History of Environment
Determination of Materials Properties
Three-Layer Linear Viscoelastic Model
Stress and Strains at Critical Points e.g. σ_v at top of bottom layer ε_t at bottom of top layer
Excessive Deformation Criteria
Fatigue Criteria
Longitudinal Distribution of Properties
Divide Time into periods $\Delta\tau$, e.g. 12 hours $dn(\tau)$ Load Applications
Differential Deflections
Average ε_T, and $\Delta\varepsilon_T$ During One Period Yields $N(\tau)$
Limit Differential Deflections, i.e. Slope Variance
$F(t)=\int_0^t K_{REC}(t-\tau,t)\frac{dn(\tau)}{N(\tau,t)}$ Compute F(t) by Simulation
Limit Maximum Deflection, i.e. Rutting
F(t) is a Measure of Damage such as Cracking

```
C.....THIS IS A SIMULATION PROGRAM FOR THE PERFORMANCE OF A THREE-LAYER
C.....HALF-SPACE VISCOELASTIC PAVEMENT SYSTEM UNDER VARIABLE OPERATIONAL
C.....ENVIRONMENT AND REPEATED LOADING ACTION.  THE PERIOD OF LOADING
C.....IS ASSUMED TO CHANGE AT CERTAIN TIME INTERVALS.
C.....THE MAIN INPUT TO THIS PROGRAM ARE GEE(I) AND DELTA(I), WHICH
C.....REPRESENT THE RESPONSE OF THE SYSTEM TO STATIC LOAD OF A UNIT
C.....MAGNITUDE UNDER CONSTANT CONDITIONS OF ENVIRONMENT.  THIS RESPONSE
C.....IS THEN FITTED USING THE LEAST SQUARE FITTING METHOD TO AN
C.....EXPONENTIAL SERIES.  GEE( ) AND DELTA( ) ARE THE COEFFICIENTS OF
C.....THIS SERIES, WHICH IN THIS CASE IS THE DIRICHLET SERIES.
      IMPLICIT REAL*8(A-H,O-$)
      REAL*4 OUT,RLR,SNGL,FLOAT
      COMMON TM(100),NLR(100),PERIOD(100),GEE(20),DELTA(20),DURA,TREF,
     1FSOLU,FSOLL,PSOLU,PSOLL,DF(100)
      DIMENSION LIJ(50),A(10)
C     READ IN DATA
      READ(5,5001) TREF,TVAR,TMEAN,DURA,IX
 5001 FORMAT(4F10.4,I10)
      READ (5,5002) NMEAN,NVAR,JK
 5002 FORMAT(3I10)
      READ(5,5002) NX,N,NK
C     NX IS ZERO FOR STRAINS, OTHERWISE IT CAN HAVE ANY INTEGER VALUE.
      READ(5,5003) (GEE(I),I=1,N)
 5003 FORMAT(5E15.8)
      READ(5,5003) (DELTA(I),I=1,N)
      DO 100 I=1,N
  100 DELTA(I)=DELTA(I)/1000.
      READ(5,5003)(A(I),I=1,NK)
      WRITE(6,205)(A(I),I=1,NK)
  205 FORMAT(4X,'A = ',F6.2)
      READ(5,5003) CK
      WRITE(6,200)(DELTA(I),I=1,N)
  200 FORMAT(4X,'DELTA = ',E15.8)
      WRITE(6,201)(GEE(I),I=1,N)
  201 FORMAT(4X,'GEE = ',E15.8)
      WRITE(6,202) CK,TREF
```

```
  202 FORMAT(4X,'CK = ',E15.8,' REFERENCE TEMPERATURE = ',F8.3)
      OMEGA=3.1415926535/DURA
      WRITE(6,1147) DURA
 1147 FORMAT(6X,'DURA = ',F5.3)
      READ(5,10)(LIJ(J),J=1,JK)
C     LIJ IS THE NUMBER OF TIME PERIODS OVER WHICH THE SOLUTION IS REQ-
C     UIRED IN EACH CASE.
   10 FORMAT(15I5)
C     LIJ IS THE NUMBER OF PERIODS OVER WHICH THE PROGRAM WILL OPERATE.
C     SET ARRAYS TO ZERO
      CALL ERASE(TM,100,NLR,100,PERIOD,100,DF,100)
      JJJ=LIJ(JK)+1
      READ(5,5002)LS
C     THE LOOP THROUGH 41 ALLOWS FOR THE SIMULATION OF 'LS' NUMBER OF
C     SAMPLES FOR THE RESPONSE TERMS.
      DO 41 LL=1,LS
  112 DO 120 K=2,JJJ
      CALL GAUSS(IX,FLOAT(NVAR),FLOAT(NMEAN),RLR)
      NLR(K)=RLR+0.50
      PERIOD(K)=30000./NLR(K)
      IRATIO=PERIOD(K)/DURA+0.50
      NLR(K)=30000./PERIOD(K)+0.5
C     GENERATE RANDOM DISTRIBUTION OF TEMPERATURES
      CALL GAUSS(IX,SNGL(TVAR),SNGL(TMEAN),OUT)
      TM(K)=OUT
      TM(1)=TM(2)
      TM(K)=(TM(K)-32.0)*5.0/9.0+273.0
      IF(TM(K).LT.267.) TM(K)=267.
      IF(TM(K).GT.318.) TM(K)=318.
  120 CONTINUE
  111 WRITE(6,5004)(TM(K),K=2,JJJ)
 5004 FORMAT(4X,'TEMPERATURES GENERATED ARE ',10F10.2)
      DO 40 J=1,JK
      NLL=0
      FSOLL=0.0
      FSOLU=0.0
```

```
      PSOLL=0.0
      PSOLU=0.0
      DUMMY=0.0
      II=LIJ(J)+1
      DO 18 K=2,II
      WRITE(6,5005) NLR(K)
 5005 FORMAT(1H0,3X,'NUMBER OF LOAD APPLICATIONS = ',I5)
      WRITE(6,5006) PERIOD(K)
 5006 FORMAT(4X,' PERIOD= ',F10.6)
      NLL=NLL+NLR(K)
      PSOL1=0.0
      PSOL2=0.0
      CALL RPETED(OMEGA,K,II,N,NX,PSOL1,PSOL2)
      FSOLU=FSOLU+PSOLU
      FSOLL=FSOLL+PSOLL
      III=II-1
      WRITE(6,5007) FSOLU,FSOLL,III,NLL
      IF(NX.NE.0) GO TO 18
      DF(K)=(PSOL1-PSOL2)/2.
      WRITE(6,5) DF(K),K
      KKK=K-1
      DUMMY=DAMAGE(A,CK,NK,K)
      WRITE(6,5008) DUMMY,KKK
   18 CONTINUE
 5007 FORMAT(4X,'CUMULATIVE DYNAMIC RESPONSE ',E15.8,' CUMULATIVE RESIDU
     1AL RESPONSE ',E15.8,' AFTER ',I4,' DAYS AND ',I6,' LOADS ')
    5 FORMAT(4X,'STRAIN AMPLITUDE = ',E15.8,' AT DAY ',I4)
      GO TO 40
      IF(NX.NE.0) GO TO 40
C     IF THE STRAINS ARE CALCULATED THE DAMAGE IS COMPUTED.
      DUMMY=DAMAGE(A,CK,NK,II)
      WRITE(6,5008) DUMMY,III
 5008 FORMAT(4X,'DAMAGE = ',F10.5,' AFTER ',I4,' MONTHS')
 40   CONTINUE
   41 CONTINUE
      CALL EXIT
      END
```

```
      SUBROUTINE RPETED(OMEGA,K,II,N,NX,PSOL1,PSOL2)
      IMPLICIT REAL*8(A-H,O-$)
      COMMON TM(100),NLR(100),PERIOD(100),GEE(20),DELTA(20),DURA,TREF,
     1FSOLU,FSOLL,PSOLU,PSOLL,DF(100)
      DIMENSION GX(100),GG(20),AA(20),A1(20)
      CALL ERASE(GX,100,AA,20,GG,20,A1,20)
      KK=K+1
      NN=NLR(K)
      III=II-1
      B=0.0
      BD=0.
      XX=0.0
      E=0.0
      D=0.0
      PSOL1=0.0
      PSOL2=0.0
      B1=0.0
      E1=0.0
      D1=0.0
      BD1=0.0
      T2=NN*PERIOD(K)
      T1=T2-PERIOD(K)+DURA/2.
      DO 17 I=1,N
      SUM=SUMM(KK,II,I,T2)
      IF(SUM.GT.40.) GO TO 201
      AI=TM(II)*DEXP(-SUM)/TREF
      GO TO 203
  201 AI=0.0
  203 BI=0.
      IF(AI.EQ.0.0) GO TO 204
      IF(III.LT.K)GO TO 204
      DO 20 M=K,III
      MM=K+1
      GR=SUMM(MM,M,I,T2)
      IF(GR.GT.40.) GO TO 501
      GX(M)=(TM(M+1)-TM(M))*DEXP(-GR)/TREF
```

```
      GO TO 20
501   GX(M)=0.0
  20  BI=BI+GX(M)
 204  GG(I)=(AI-BI)*GEE(I)
      W=GAMA(K)*DELTA(I)
      WP=W*PERIOD(K)
      DEN=1./(1.+(W/(2.*OMEGA))**2)*0.5
      IF(T2*W.GT.50.) GO TO 15
      TX=1.-DEXP(-T2*W)
      GO TO 9920
15    TX=1.
9920  IF(WP.GT.50.) GO TO 117
      EXT=DEXP(WP)
      IF(W.LT.1.E-06) EXT=0.5
      EXD=DEXP(W*DURA)
      AA(I)=TX*(1.-EXD)*DEN/(EXT-1.)*GG(I)
      A1(I)=TX*DEN*(1.-EXD)/(EXT-1.0)*GEE(I)
      IF(W.LT.1.E-06) EXT=1.
      EXD2=DEXP(W*DURA*0.5)
      D=AA(I)*EXT/EXD2
      D1=A1(I)*EXT/EXD2
      B=(1.+EXD2)*DEN*GG(I)
      B1=(1.+EXD2)*DEN*GEE(I)
      BD=B+D+BD
      BD1=B1+D1+BD1
      GO TO 210
117   BD=BD+DEN*GG(I)
      BD1=BD1+DEN*GEE(I)
      S=W*(PERIOD(K)-DURA)
      IF(S.GT.60.) GO TO 225
      AA(I)=-TX*DEXP(-S)*DEN*GG(I)
      A1(I)=-TX*DEXP(-S)*DEN*GEE(I)
      GO TO 210
 225  AA(I)=0.0
      A1(I)=0.0
210   E=AA(I)+E
```

```
      E1=E1+A1(I)
      XX=XX+GG(I)
   17 CONTINUE
      PSOLL=E
      PSOLU=BD-XX
      KKK=K-1
      IF(NX.NE.0) GO TO 222
C     IF THE STRAINS ARE CALCULATED, THE SSTRAIN AMPLITUDE FOR EACH
C     TIME PERIOD CONSIDERED NEED TO BE COMPUTED TO COMPUTE THE DAMAGE.
      PSOL1=BD1*TM(K)/TREF
      PSOL2=E1*TM(K)/TREF
      WRITE(6,5007) PSOL1,PSOL2,KKK,NLR(K)
 5007 FORMAT(4X,'DYNAMIC RESPONSE = ',E15.8,' RESIDUAL RESPONSE = ',
     1E15.8,' AFTER ',I5,' DAYS FOR ',I6,' LOAD APPLICATIONS')
  222 CONTINUE
      RETURN
      END
```

```
      REAL FUNCTION DAMAGE*8(A,CK,NK,II)
      IMPLICIT REAL*8(A-H,O-$)
      COMMON TM(100),NLR(100),PERIOD(100),GEE(20),DELTA(20),DURA,TREF,
     1FSOLU,FSOLL,PSOLU,PSOLL,DF(100)
      DIMENSION DEL(10),A(10)
      CALL ERASE(DEL,NK)
C     INITIALIZE DELTA'S
      DEL(1)=0.0
      DEL(2)=0.01
      DO 110 I=3,NK
 110  DEL(I)=DEL(I-1)/10.0
      DAMAGE=0.0
      DO 125 J=2,II
      TX=0.0
      DO 15 I=J,II
      IF(TM(I).LT.295.) GO TO 15
C     THIS IS AN ARBITRARILY ASSIGNED VALUE OF TEMPERATURE(20 DEGREES)
C     BELOW WHICH NO HEELING OR RECOVERY OCCURS.
      IF(J.EQ.II) GO TO 15
      TX=TX+NLR(I)*(PERIOD(I)-DURA)
   15 CONTINUE
      REC=0.0
      DO 13 K=1,NK
      IF(TX*DEL(K).GT.60.) GO TO 13
      REC=REC+DEXP(-TX*DEL(K))*A(K)
   13 CONTINUE
      DK=CK
      IF(TM(J).GT.295.) DK=((TM(J)-295.)*10.+1.)*CK
      WRITE(6,10) DK,DF(J),J
   10 FORMAT(4X,'CK = ',E15.8,'  DF = ',E15.8,' J = ',I4)
      XI=DK*(1./DF(J))**4.5
      DELF=DFLOAT(NLR(J))/XI
      WRITE(6,12) XI,DELF,REC,NLR(J),J
   12 FORMAT(4X,'XI = ',E15.8,' DELF = ',E15.8,' REC = ',F7.5,' NLR = ',
     1I7,' J = ',I3)
      DAMAGE=DAMAGE+DELF*REC
```

```
  125 CONTINUE
      RETURN
      END
```

```
      REAL FUNCTION SUMM*8 (IK,IJ,I,T2)
      IMPLICIT REAL*8(A-H,O-$)
      COMMON TM(100),NLR(100),PERIOD(100),GEE(20),DELTA(20),DURA,TREF,
     1FSOLU,FSOLL,PSOLU,PSOLL,DF(100)
      SUMM=0.0
      IF(IJ-IK) 20,21,21
   21 CONTINUE
      DO 10 JI=IK,IJ
   10 SUMM=SUMM+GAMA(JI)*T2*DELTA(I)
   20 CONTINUE
      RETURN
      END
```

```
      REAL FUNCTION GAMA*8 (KLM)
      IMPLICIT REAL*8(A-H,O-$)
      COMMON TM(100),NLR(100),PERIOD(100),GEE(20),DELTA(20),DURA,TREF,
     1FSOLU,FSOLL,PSOLU,PSOLL,DF(100)
      X=10000./TREF-10000./TM(KLM)
      GAMA=10.**X
      RETURN
      END
```