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16. Abstract

This report surveys the published literature in geotechnical engineering and related disciplines for applications of probabilistic and statistical methods to soil and rock mechanics problems. Published references up to and including 1982 are summarized to establish the state of the art at that time. A bibliography of approximately 650 items is appended.

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TABLE OF CONTENTS

		PAGE
	DOCUMENTATION PAGE	i
	METRIC CONVERSIONS	ii
	TABLE OF CONTENTS	iii
	LIST OF FIGURES	vi
	LIST OF TABLES	vii
I.	INTRODUCTION	1
II.	PROBABILITY AND STATISTICS PRIMER	1
	2.1 Concepts of Statistical Descriptions	1
	2.1.1 Statistical Description of Data Scatter	1
	2.1.2 Statistical Moments	1
	2.1.3 Distribution Functions	2
	2.2 Distribution Theory	3
	2.2.1 Analytical Forms	3
	2.2.2 Systems of Distribution Functions	5
	2.2.2.1 Pearson Family	5
	2.2.2.2 Other Families	6
	2.3 Statistical Inference	6
	2.3.1 Estimates of Moments	6
	2.3.2 Distribution Parameters	7
	2.3.3 Tests of Distributional Forms	8
	2.3.4 Probability Paper	9
	2.4 Profile Estimation	9
	2.5 Bayesian Inference	12
	2.6 Transformations of Random Variable and Vectors	12
	2.6.1 Variance Propagation	13
	2.6.2 Other Methods of Uncertainty Analysis	14
III.	TAXONOMY OF ANALYTICAL METHODS	16
	3.1 Probabilistic Techniques & Reliability	16
	3.2 Statistical Methods	17
	3.3 Risk Analysis	18
	3.4 Decision Making and Optimization	18

PAGE

IV. APPLICATIONS IN SOIL ENGINEERING 18

4.1 Statistical Analysis 18

4.1.1 Distribution Theory 19
4.1.1.1 Variability of Soil Properties 19
4.1.1.2 Distributional Forms 19

4.1.2 Site Characterization 24

4.1.2.1 Reconnaissance 25
4.1.2.2 Mapping 25
4.1.2.3 Search 26
4.1.2.4 Mechanical Testing 26
4.1.2.5 Frequentist IID Sampling 27
4.1.2.6 Measurement Error 27
4.1.2.7 Bayesian IID Sampling 30
4.1.2.8 Sampling From Random Fields 30

4.1.3 Autocorrelation 31
4.1.4 Quality Control & Construction Monitoring 32

4.2 Reliability Modeling 32

4.2.1 Foundations for Building and Offshore Structures 32

4.2.1.1 Stability 32
4.2.1.2 Settlement 33
4.2.1.3 Deep Foundations 34
4.2.1.4 Finite Element Modeling 34
4.2.1.5 Offshore Structures 35

4.2.2 Slopes and Retaining Structures 35

4.2.3 Factors of Safety 37

4.2.3.1 Evaluation of Design Reliability 37
4.2.3.2 Design Factors of Safety 40

V. APPLICATIONS IN ROCK ENGINEERING 40

5.1 Joint Surveys 40
5.2 Foundation Deformations 43

5.2.1 Empirical Correlations 43
5.2.2 RQD - Modulus Ratio Relations 43
5.2.2.1 Direct Correlations between Descriptors and Rock Mass Deformability 44
5.2.2.2 Rock Mass Classification 44
5.2.2.3 Analytic and Experimental Component Modelling 44

PAGE

5.3 Slope Stability 46

5.3.1 Shear Failure 46

5.3.2 Block Sliding 47

5.3.3 Wedge Instability 48

5.4 Fracture Flow 49

LIST OF FIGURES

Page

Figure 1 -- Coumulative propability plot of cone penetration data taken in copper tailings 23

Figure 2 -- Hypothetical soil property showing effect of trend removal 24

Figure 3 -- Types of statistical sampling 26

LIST OF TABLES

	Page
Table 1 -- Reported Coefficients of Variation for Various Soil Properties	21
Table 2 -- Empirical Autocorrelation Estimate	28
Table 3 -- Typical Reliability Indices for Geotechnical Facilities	45

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I. INTRODUCTION

Every geotechnical engineer recognizes the influence of uncertainty on site characterization, engineering analysis, and project design. Yet, quantified approaches to the analysis of that uncertainty began appearing only as recently as the mid-1960's. Widespread work and dissemination of research results on probabilistic methods in soil and rock engineering are principally a phenomenon of the late 1970's. Today, while broad research interest has been shown in the topic, practical applications remain few. Penetration of the newly developed tools into the practice of geotechnical engineering remains modest.

The purpose of this review and bibliography is to bring together the dispersed literatures pertinent to geotechnical risk and reliability analysis. No pretense is made to exhaustiveness, nor is the document intended as a primer on probabilistic methods. The latter function is served by a companion volume ("Geotechnical risk analysis user's guide"). This report is intended as a reference for the latter volume, and as a point of access to the literature. This review includes materials published through 1982.

II. PROBABILITY AND STATISTICS PRIMER

2.1 Concepts of Statistical Descriptions

Due to limited numbers of samples, testing errors and spatial variability, estimations of engineering profiles for geological materials are always subject to some error. The magnitude of these errors can be specified in a number of ways, using upper and lower bounds for example, but has an important influence on the reliability with which predictions of engineering performance of embankments, foundations or other structures can be made. This section summarizes techniques for describing the amount of error in engineering profiles using simple statistical methods, and serves as a background for later parts of this report.

2.1.1 Statistical Description of Data Scatter

Data as collected in the field or as a result of laboratory testing invariably display some amount of scatter. For typical soils, this scatter can be many tens of percents of the data averages.

2.1.2 Statistical Moments

Given a time series or distance dependent set of data, the simplest way to describe data scatter is with a set of two numbers, one specifying the central magnitude of the measurements, and one specifying the dispersion of measurements about that center. While there are many candidates for these two measures, the most useful and the most common are the so-called first two moments of the data scatter, specifically the average or mean and the variance.

The mean of a set of observations,

$$\underline{x} = x_1, \ldots, x_n \tag{1}$$

is defined as the arithmetical average,

$$m_x = \frac{1}{n} \Sigma x_i \tag{2}$$

From a physical point of view the mean is analogous to the center of gravity of the data along the axis of measurement. The variance of the data is defined as the mean squared variation of the data about m_x, or

$$V(x) = \frac{1}{n-1} \Sigma (x_i - m_x)^2 \tag{3}$$

Again, from the physical point of view, this is analogous to the moment of inertia of the data about m_x. Together, the mean and variance are said to constitute a second-moment description of the data scatter.

For many purposes a convenient measure of dispersion is the square root of the variance, or rms variation, usually called the standard deviation,

$$s_x = \sqrt{V(x)} \tag{4}$$

The standard deviation is measured in the same units as the data themselves, rather than the square of those units, and can be used to describe proportionate dispersion through the coefficient of variation,

$$\Omega_x = s_x/m_x \tag{5}$$

2.1.3 Distribution Functions

While moments of the data scatter describe the location and dispersion of the data, a complete description of the scatter is more easily obtained by the so-called distribution function. Arranging the data in order of increasing magnitude, the fraction of the observations less than some value x is summarized in the cumulative distribution function F(x), sometimes abbreviated cdf. Cdf's (or their complements) are widely used in soil engineering to describe grain size distributions. They may be used to describe the variation of engineering properties (i.e., strength, deformation and flow) data as well.

The derivative of the cdf,

$$f(x) = dF(x)/dx \quad , \tag{6}$$

describes the density of data along the measurement axis, and might be loosely described as a smoothed version of the data histogram. This derivative is typically called the probability density function (pdf), and has the property that its integral between two values of x gives the fraction of observations within the interval (i.e., the probability that a randomly chosen datum would lie in the interval).

Continuing the physical point of view, the moments of the data scatter are related to the pdf in the same way that mechanical moments are related to a continuous solid,

$$m_x = \int x \, f(x) \, dx \tag{7}$$

$$V(x) = \int (x-m_x)^2 \, f(x) \, dx \tag{8}$$

2.2 Distribution Theory

The preceding sections have concerned themselves with the description of observed data. The cdf's or pdf's characterizing those observations are said to be the sample distribution functions. Analytical forms, however, are also available with which to model sample distribution functions, and such models are of convenience both for statistical inference and for engineering modeling.

Certain analytical functions play a central role in statistical theory and data analysis. The more common and useful of these are described in this section.

2.2.1 Analytical Forms

The most common analytical distribution functions are those of the general EXPONENTIAL FORM,

$$f(x) = N \exp(a+bx+cx^2) \, x^d \tag{9}$$

in which a,b,c, and d are constants, and N is a normalizing term to insure that the integral of f(x) over the complete measurement axis is unity. The better known distributions having this form are the normal, lognormal, exponential, and gamma.

The NORMAL DISTRIBUTION, recognized by its characteristic bell-shape, is the most common of all distributionns. It is observed in sample data with such frequency that Galton, in his early work on the distribution of features in human populations, coined its common name, at least for the English literature. In non English literatures the distribution is more commonly called the Gaussian, in honor of Gauss' original proof of the central limit theorem, which says that variables composed of the sum of independent perturbations necessarily tend toward normal distributions as the number of pertubations becomes large. Thus, through the central limit theorem there exists theoretical justification for the widespread use of the normal form, and for the central position occupied by this form in statistical sampling theory.

The LOGNORMAL DISTRIBUTION is that which describes the distribution of a variable, the logarithm of which is normally distributed. Thus, the lognormal is closely related to the normal, and by an extension of the central limit theorem the lognormal distribution describes a variable formed by the product of independent pertubations as the number of perturbations becomes large.

The EXPONENTIAL DISTRIBUTION, sometimes called negative exponential, is a one parameter function and is arguably the simplest of common distributions. While this distribution is often observed in geometric data, as for example the spacings among rock joint traces in outcrop, it is not commonly encountered in strength, deformation and flow data. Theoretical arguments can be made that certain types of data should be exponentially distributed, for example spacings between random events in time or space, but in the general case its use is primarily one of covenience.

The GAMMA DISTRIBUTION is positively skewed, as is the lognormal, and although derived from theoretical arguments pertaining to discrete measurements, its use with continuous measurements is often based on its similarity to the lognormal and its greater convenience.

Only a limited number of NON-EXPONENTIAL FORMS have been proposed for use with geotecnical data. Perhaps the most publicized of these is the beta distribution, which has been advocated by some geotechnical researchers.

The BETA DISTRIBUTION is a four parameter pdf, and is thus very flexible and often appears to model empirical data quite well. A danger in this observation is that, due to the large number of prameters required to specify a beta distribution, the degrees of freedom in fitting empirical data is reduced. Thus, with limited data sets the statistical uncertainty in estimated parameters can be large. With large data sets degrees of freedom is seldom a problem, but many geotechnical data sets are not large. The beta distribution is defined over a segment of the measurement axis rather than the entire axis. Thus, upper and lower bounds on x must be estimated or fixed a priori. This is usually difficult to do, and presents statistical problems.

It also taxes geotechnical intuition for there are often no cogent reasons for specifying bounds that a variable cannot exceed.

2.2.2 Systems of Distribution Functions

Given the large number of analytical forms for distribution functions, attempts have been made to develop systems of distributions to bring order to the taxonomy of functions. The principal attempt in this direction was made by Pearson. Other systems are mentioned for completeness.

2.2.2.1 Pearson Family

Pearson's family of distributions comprises the solutions to the differential equation

$$\frac{f(x)}{dx} = \frac{(x-a)\ f(x)}{b_o + b_1 x + b_2 x^2} \tag{10}$$

in which a, b_o, b_1, and b_2 are constants. Among others, these solutions include the normal, lognormal, gamma, exponential, and beta distributions. They also include other common or useful distributions which are not extensively considered in this report, e.g., Student's.

Members of the Pearson family are usually identified by study of their low order moments, specifically those of order 1 through 4. These moments are calculated from data by extension of the discussion above. The third order moment is the average of the cubes of the observed data. The fourth order moment is the average of the fourth powers of the observed data. These moments may be combined in the two statistics,

$$\beta_1 = \frac{E^2[(x_i-m)^3]}{E^3[(x_i-m)^2]} \tag{11}$$

$$\beta_2 = \frac{E\ [(x_i-m)^4]}{E^2[(x_i-m)^2]} \tag{12}$$

in which E[] is the expectation or average of the quantity within the brackets. β_1 and β_2 can be used to distinguish among members of the Pearson family. The simplest use of the β_1 and β_2 statistics is via the Pearson Diagram with which distributional forms may be identified by inspection.

2.2.2.2 Other Families

Other systems of distributionns have been studied, primarily by representing frequency functions as series expansions, or by considering the transformations of frequency functions to common shapes (e.g., to normal distributions). These are briefly listed here for reference. Discussions of these families can be found in Ord (1972) or in Kendall and Stuart (1977, v.1). The principal series expansion systems of distributions are (1) the Chebyshev-Hermite polynomials, based on polynomial multipliers of the error function integral; (2) Edgeworth's Type A series, and the (3) Gram-Charlier Type A series, each based on series of normal integrals or their derivatives; (4) the Tetrachoric function series, due to K. Pearson; and (5) Charlier's Type B series, based on derivatives of the Poisson pdf.

The principal transformation systems are (1) polynomial transformations to normality, and (2) general (non-polynomial) transformations to normality. In each case a frequency function is catagorized by the nature of the transformation that changes it to a normal.

2.3 Statistical Inference

The discussion above refers to descriptions of data sets. Most geotechnical applications address inferences of properties from limited numbers of data (i.e., finite samples) collected at a site. Since samples may vary, one from the other, inference always introduces some uncertainty. Therefore, estimates drawn from a sample must be reported with error bands, and conclusions (e.g., on distributional shape) must be tested statistically. This section describes techniques used to quantify estimation errors and to test conclusions on distributional forms.

2.3.1 Estimates of Moments

Given a set of data $X = (x_1,\ldots,x_n)$, any mathematical function of X,

$$T(X) = fn(x_1,\ldots,x_n) \tag{13}$$

is said to be a statistic of the data X, or more simply a sample statistic. To estimate properties of the population from which the data came, some sample statistic, or combination of statistics, may be used to form an estimator. Here, an "estimator" is the mathematical function used to make an estimate from data, and an "estimate" is the numerical result of applying the estimator to a set of data.

For any set of data an essentially infinite number of estimators may be defined with which to make estimates of population properties. Thus, criteria must be established for choosing one estimator over another. In traditional statistics, such criteria are based on the so-called sampling distributions of the estimators, that is, on the distribution of estimates made with a given estimator over the possible samples that might be randomly taken from some population. Two important properties of an estimator are its mean and its variance over these possible samples. If the mean equals the parameter to be estimated, then the estimator is said to be unbiased. If the variance is the least of all alternative estimators, the estimator is said to be minimum variance (or minimum-squared-error for biased estimators). The imprecision associated with an estimate is conveniently summarized by the standard deviation of the sampling distribution of the estimator, which is traditionally called the standard error.

Moment estimators use moments of the sample as estimators of the moments of the sampled population from which the sample is taken. For example, the sample mean is used as an estimate of the population mean and the sample variance is used as an estimate of the population variance. The sample mean is an unbiased estimator of the population mean, with sampling variance,

$$V(m_x) = V(x)/n \tag{14}$$

in which n is the number of independent measurements (i.e., sample size). The sample variance, defined as

$$S^2 = \frac{\Sigma\ (x_i - m_x)^2}{n-1} \tag{15}$$

is also an unbiased estimator, with sampling variance approximately equal to

$$V(S^2) = \frac{2\ V^2(x)}{n-1} \tag{16}$$

2.3.2 Distribution Parameters

Parameters defining a distribution may be estimated in ways similar to those for estimating moments. In certain cases, as e.g., the normal distribution, the moments are themselves the parameters defining the distribution. Two types of estimators have been used in this work, moment estimators and maximum likelihood estimators.

Moment estimators of distribution parameters use the functionnal relation between distribution parameters and distribution moments to estimate the parameters, essentially by back calculation. Such estimators are easily calculated, but may be far from efficient, in the sense of having small sampling variances. Maximum likelihood estimators use the conditional probability of having observed the sample X as the criterion of estimation, taking those values of the distribution parameters that maximize this probability as estimates. Such estimates are always efficient, but may be difficult to derive for certain sampling plans.

2.3.3 Tests of Distributional Forms

Since sample data vary from sample to sample, the shape of their frequency distribution (i.e., the sample pdf) also varies. Thus, the sample pdf may appear similar to some common analytical form when in fact the population pdf does not; or conversely, the population pdf may have some common form, but the sample data fail to reflect it. For this reason, the goodness-of-fit of an analytical form to the sample data must be tested statistically.

Chi-squared and Kolmogorov-Smirnov tests are the most common goodness-of-fit tests to assure statistical confidence of distributional forms. The chi-squared test is based on the squared difference between observed cell frequencies in histograms and those predicted from an analytical form. It is a widely used and accepted procedure for establishing levels of confidence for distributional forms (see, e.g., Kendall and Stuart, V.2, 1973). The Kolmogorov-Smirnov test is based on the maximum deviation between observed cumulative frequencies of data and those predicted by an analytical cdf. The K-S test is also widely used and accepted, particularly as a graphical check when using probability grid.

The chi-squared test uses the test statistic

$$\chi^2 = \Sigma \frac{(O-P)^2}{P} \qquad (17)$$

summed over the number of histogram intervals, in which O is the observed number of data and P is the number predicted from the analytical form for the distribution. For data actually sampled from the presumed distributional form the pdf of chi-square can be calculated. Thus, the observed chi-square can be compared with the distribution of values chi-square should have and a conclusion drawn on whether the presumed parent distribution is reasonable.

The K-S test uses the test statistic

$$Dm = \max_{x}\left(\left|F(x) - G(x)\right|\right) \qquad (18)$$

in which F(x) is the analytical cdf and G(x) is the sample cdf. For data actually sampled from a population with cdf F(x), the distributional form of the pdf of Dm can be calculated. This distribution depends on the sample size (i.e., the number of data), but it does not depend on the form of F(x). Thus, the observed Dm can be compared with the distribution of values Dm should have and again a conclusion drawn on whether the presumed parent distribution is reasonable.

2.3.4 Probability Paper

A convenient way to display data and to draw conclusions about distributional forms by inspection is through the use of probability paper. Probability paper is simply a specialized grid on which cumulative distribution functions plot as straight lines. Such paper is widely available for normal and lognormal distributions, and can be straightforwardly generated for most other distributions.

2.4 Profile Estimation

For engineering purposes uncertainty in tailings properties is summarized in a geotechnical design profile. This uncertainty comprises three parts: spatial variability, measurement or model bias, and statistical estimation error. Random measurement error--noise--affects profile uncertainty only to the extent that it increases statistical estimation error and possibly to the extent that it is confused with spatial variability.

As a first approximation data scatter in in situ measurements can be divided into two parts, a spatial or innate part, and a measurement error or noise part. Thus,

$$\text{Data scatter} = \left\{ \begin{array}{c} \text{Spatial variation} \\ + \\ \text{Measurement noise} \end{array} \right. \tag{19}$$

The spatial part is the part that the exploration program intends to characterize. The noise part is spurious. Thus, it is important to distinguish these two types of scatter.

Since spatial variability and measurement error combine to produce data scatter, they can not be directly separated. However, at least three indirect methods can be used. The first is replicate measurement on the same or similar materials, the second is multiple profiling with different instruments, and the third is through the structure of spatial variability. The last is inexpensive and often the most practical method.

The spatial structure of data scatter about a mean trend is summarized by an autocovariance function (or equivalently, a variogram). Adopting the simple, but common model,

$$z(t) = x(t) + e(t) \tag{20}$$

in which z(t) is the measurement at point t in the deposit, x(t) is the actual geotechnical property at t, and e(t) is a corrupting measurement noise, the autocovariance function of z(t) is

$$Cz(r) = E\left[\left(z(t)-m_z\right)\left(z(t+r)-m_z\right)\right] \tag{21}$$

in which m_z is the mean or mean trend of z(t). Assuming stationarity (i.e., statistical homogeneity, Cz(r) simply expresses the covariance of the observations as a function of their spatial separation. Typically, Cz(t) is anisotropic, smaller vertically than horizontally; and sometimes also anisotropic in the horizontal plane.

Similar autocovariance functions can be defined for x(t) and e(t), and these are related to Cz(r) by

$$Cz(r) = Cx(r) + Ce(r) \tag{22}$$

Cx(r) takes on the spatial variance of x at r=0 and decays to zero as r increases. On the other hand, e(t) is presumably independent from one measurement to another; thus, Ce(r) must be a spike of height V(e) at r=0, and zero elsewhere. Thus, extrapolation of the observed Cz(r) back to r=0 allows an estimate of V(e) to be made.

In addition to data scatter, which is associated with variation about the mean trend, two systematic errors affect the estimation of the mean trend itself. First, the measurement procedure may introduce a systematic bias, B,

$$z(t) = B\ x(t) + e(t) \tag{23}$$

$$V(z(t)) = B^2\ V(x(t)) + V(e(t)) \tag{24}$$

Second, the total number of measurements is limited, thus statistical fluctuations introduce estimation errors. Combining, the total variance in an estimated profile becomes

$$V(\hat{x}(t)) = \left(\begin{array}{c} \text{Variance about the mean} \\ + \\ \text{Variance of the mean} \end{array} \right. \qquad (25)$$

in which the caret over x signifies an estimate. Mathematically,

$$V(\hat{x}(t)) = R\ V(x(t)) + V(B)mx^2 + B^2V(m) \qquad (26)$$

in which R is a scale factor, $V(x(t))$ is the spatial variance of the data, $V(B)$ is the variance of the measurement bias (i.e., the uncertainty about the proper value of the calibration coefficient), and $V(m_x)$ is the statistical estimation error on the mean trend. For n widely spaced measurements,

$$V(m) = \frac{V(x(t)) + V(e(t))}{n} \qquad (27)$$

For closely spaced measurements

$$V(m) = \frac{(\underline{1}C_x\underline{1}^t) + V(e(t))}{n} \qquad (28)$$

in which

$$\underline{1} = \{1,\ldots,1\}_n \qquad (29)$$

and C is the covariance matrix of the x(t).

Based on the above development, mean profile and standard deviation envelopes are easily constructed. Note, however, that the standard deviation envelopes, which express the uncertainty in engineering parameters for analysis, must reflect mode of behavior and scale. This dependency is summarized in the factor R. For example, circular shear instability depends on total resistance over a surface of sliding. Thus, spatial variation in part averages out. For very small instabilities R approaches 1, but for very large ones R approaches zero.

2.5 Bayesian Inference

Bayesian techniques are predicated on a degree-of-belief interpretation of "probability", whereas the techniques described above are based on a relative frequency interpretation. This has two important consequences. First, Bayesian techniques allow probabilities to be defined directly on states of nature. For example, using Bayesian techniques one may define the "probability of a fault existing at a given site." Using frequentist techniques this probability is undefined because it has no frequency interpretation. To a frequentist the probability is either 0 or 1, one simply doesn't know which.

The second consequence is that Bayesian techniques require a prior probability to be specified; that is, a probability before subsequent evidence is considered. Within the Bayesian approach data do not speak for themsleves, they only indicate how to update what one believed before seeing them to what one should rationally believe after.

The main vehicle for inference within the degree of belief school of thought is Bayes' Theorem, relating prior probabilities to posterior probabilities by

$$Pr[A|E] \propto Pr[A] \; L[E|A] \qquad (30)$$

in which A = some event, E = data or evidence, and L[E|A] = the likelihood (i.e., conditional probability) of E given A. Pr[A|E] is the conditional probability of A given E, or the probability of A based on havinng seen the evidence in question. Pr[A] is the marginal probability of A before having seen (i.e., irrespective of) E.

As a matter of computational convenience, the mathematical form of the prior distribution Pr[A] is often chosen such that it is closed under multiplication by the likelihood function. Thus, the forms of the prior and posterior are the same, differing only in the values of their parameters.

2.6 Transformations of Random Variable and Vectors

Engineering analyses use the soil property estimates made from measurements by incorporating them in models. These models are based on engineering mechanics and relate soil properties, imposed loads and other aspects of a proposed design fo predicted performance. In traditional design, conservative point estimates of properties, loads, and other conditions are entered into the model, and a point estimate of performance is calculated. For example, in predicting the settlement of a shallow footing on sand the soil property data might be compression moduli, blow counts, or cone resistances, and these would be used as parametric input to one of many settlement formulae. The resulting calculation would lead to a prediction of settlement, perhaps as a function of load. To test the sensitivity of the prediction to uncertainty in soil properties, a number of calculations might be made and settlement plotted as a function of the input parameters. Sensitivity analyses of this type become more difficult when more parameters are uncertain or when the uncertainty is not independent from one parameter to another. Yet, good practice dictates that an attempt be made.

In the risk-based format under discussion, all calculations are based on mean values of parameter estimates. Conservatism is avoided if possible. Uncertainty or error is propagated through the analysis in the form of variances and covariances: Variances and covariances of soil properties, loads, and other parameters are translated to variances and covariances on predictions. The result of the simple calculation of footing settlement would be a best estimate of settlement and an associated variance.

2.6.1 Variance Propagation

Operationally, variance is propagated through an analysis using a first-order approximation. For a geotechnical model relating an input parameter x to a prediction y through the relation,

$$y = g(x) \quad , \tag{31}$$

a Taylor's series expansion truncated to linear terms yields the approximations,

$$\bar{y} \doteq g(\bar{x}) \quad , \tag{32}$$

$$V[y] \doteq \left(\frac{dg}{dx}\right)^2 V[x] \tag{33}$$

In words, the mean or best estimate of the prediction y is the function of the mean or best estimate of the parameter x; the variance of the prediction y is the product of the variance of the parameter x and the square of the derivative of y with respect to x. These simple results are based on a linear approximation to g(x), but for most geotechnical problems they are sufficiently accurate.

If the prediction y depends on a set (i.e., vector) of parameters, the equivalent forms of Eqs. 12 and 13 are,

$$m_y = g(m_{x1},\ldots,m_{xn}) \quad , \tag{34}$$

$$V[y] = \Sigma\ \Sigma\ \frac{dg}{dx_i}\frac{dg}{dx_j}\ C[x_i,x_j] \quad , \tag{35}$$

in which $C[x_i,x_j]$ is the covariance of the two parameters x_i and x_j.

Two special cases deserve note because they are common in practice and lead to simple results. For the case in which y is a linear combination of a set of independent parameters $x_1,\ldots,x_n$, the variance of y is exactly,

$$y = \Sigma_{a_i x_i}: \quad V[y] = \Sigma\ a_i{}^2 V[x_i] \quad . \tag{36}$$

For the case in which y is a power function of a set of independent parameters, the variance of y is approximately,

$$y = \pi_{x_i}\ a_i: \quad Cov^2[y] \doteq \Sigma\ a_i{}^2\ Cov^2\ [x_i] \tag{37}$$

2.6.2 Other Methods of Uncertainty Analysis

The approach to propogating uncertainty through an engineering model used here is based on a first-order propogation of variance. This is a common technique and is called many things in the many disciplines to which it finds application. It is sometimes called "first-order second-moment" (FOSM) analysis, and sometimes simply "error analysis." However, there are several other ways to analyze the effect of input uncertainties on output uncertainties. Among the more often encountered of these other methods in civil engineering practice are adjoint methods, simulation, and response surface techniques.

Adjoint techniques (Hadlock, 1984) evaluate the proportionate effect of a pertubation in input parameter on the resulting purtubation in an output prediction. That is, they lead to an evaluation of the quantity $\{(\Delta y_j/\Delta x_i)\ x_i/y_j\}$, in which y_j is the jth component of the prediction and x_i is the ith input parameter. Adjoint techniques are conveniently applied to large numerical models involving the solution of systems of linear equations. By manipulating the linear algebra of such solutions, adjoint results can be obtained in the course of computations. While adjoint techniques are usually used to obtain sensitivies of a model rather than to perform quantitative uncertainty analysis, the results can be used to numerically obtain derivatives, and thus to provide the means for first-order variance propogation.

Simulation (Harr, 1977) uses many repetitions of deterministic calculations in which values of input parameters are randomly generated from specified probability distributions. The result of simulation is a set of many predictions of each output parameter which are treated as empirical data from which statistical inferences of the means, variances, etc. of output predictions can be made. An advantage of simulation is simplicity. It requires none of the mathematics of variance propogation, adjoint analysis, and related techniques. On the other hand, simulation has three important limitations. It is expensive because the deteministic model must be run many times. For example, at least several hundred trials are typically needed. It requires not only means and variances of input parameters, but entire probability distributions. These may be ambiguous or arbitrary. Finally, the components of uncertainty are lumped together in simulations. Thus, differing effects are hard to unravel. Nevertheless, simulation is an important tool when a model is complicated, involves logical branching, or on other occations when variance propogation and related techniques cannot be used.

Response surface techniques are related both to variance propogation and simulation, finding their most frequent use with models that are numerical, possibly implicit, difficult to analytically propogate variance through, and expensive to run. Response surface techniques are closely related to regression analysis. Multiple runs of the model are made in the vacinity of the mean of the input parameter values and a regression surface of chosen complexity is fit to the output predictions obtained. This regression surface is presumably less complicated than the model function itself, and yet can still be used as an approximation on which variance propogation or other techniques can be used. At the same time, many fewer runs of the model are made than with simulation, and thus cost is reduced. Response surface approaches are often applied to risk analysis problems associated with nuclear power and waste facilities, and to structural reliability problems.

The point estimate method, originally due to Rosenbleuth (1975), uses a limited number of deterministic calculations made at well-chosen sets of input parameter values to approximate the mean and standard deviation of a predicted variable. For example, in the simplest case of Eqn. 26 when both x and y are scalers, three deterministic calculations are made. These use as input, (a) the mean of x, (b) the mean plus one standard deviation of x, and (c) the mean minus one standard deviation of x. The calculated results are used to estimate a mean and standard deviation of y by the relations,

$$m_y \approx \frac{\left| g(m_x+s_x) + g(m_x-s_x) \right|}{2} \quad . \tag{38}$$

$$s_y \approx \frac{| g(m_x+s_x) - g(m_x-s_x) |}{2} \quad . \tag{39}$$

Similar techniques have been proposed for multivariate and correlated input.

The point estimate method gives exact results when g(x) is linear. Thus, in this particular case the point estimate and first-order technique give the same answer. They do not necessarily give the same answer when g(x) is nonlinear. The point estimate method is convenient for many geotechnical uses, although the goodness of its approximation appears not to have been widely studied to date. Nonetheless, its use will probably become more widespread in the future.

III. TAXONOMY OF ANALYTICAL METHODS

Probabilistic and statistical methods in geotechnical engineering are of several distinct forms, having clearly different purposes. As a matter of convenience, they may be divided into four groups by the methods they use and the questions they answer:

- Probabilistic techniques,
- Statistical methods,
- Risk assessment, and
- Economic optimization (decision analysis).

3.1 Probabilistic Techniques & Reliability

Probability theory is an axiomized mathematical theory which can be used to characterize uncertainties about engineering parameters and to describe the relations among such uncertainties. The theory is internally consistent, and once the characteristics of a set of random variables are defined all further results of probabilistic modeling follow necessarily.

Probability theory is used in geotechnical engineering essentially to propogate uncertainties about engineering parameters or variables through geomechanical models to draw conclusions on uncertainties about the predictions of those models. For example, given information about the uncertainty of soil conditions, probability theory could be used to calculate the uncertainty of bearing capacity or settlement predictions made by Terzaghi's superposition formula or 1D compression, respectively. The calculations of uncertainty made using probability theory cannot be right or wrong, per se, they are merely the logically following results of the mechanical model chosen and the characterization of uncertainty in the input parameters.

Geotechnical reliability analysis is the application of probabilistic models to the analysis of geotechnical systems in order to replace conventional safety indices, for example the factor of safety F, with indices based on probabilistic descriptions. The most common probabilistic index is the so-called "probability of failure" p_f. Usually, this is defined as the area under the pdf of predicted performance within the domain defined as "adverse" (i.e., failing). Contrary to appearence, this index is not in fact a prediction of the rate at which facilities perform adversely, but has rather to do with uncertainty in calculations. That is, p_f is the probability that errors in the selection of parameter values for input to the engineering calculations might be so large or in such combination that the analysis should be yielding a prediction of adverse performance although due to the errors it is not. Other common probabilistic indices are based on moments of the distributions of predicted performance. The most important of these are the first-order second-moment reliability index β_{FOSM}, the Hasover-Lind index β_{HL}, and the second-moment reliability index β_{SM}.

In many cases, the purpose of reliability analysis is to predict rates of actual failure, and not simply safety indices. In this review, such cases are considered as risk assessment.

3.2 Statistical Methods

Statistical methods are a set of techniques, sometimes ad hoc, for drawing inferences from observations. These methods use probability theory as a means for describing variability and in some cases uncertainty, but they are not themselves axiomtically based.

The distinction between frequentist and Bayesian statistics is important both philosophically and practically. Frequentist theory defines probability as the frequency of occurrence in series of similar "trials," and thus uses probability to describe variability. Inference within the frequentist school is not based on a unifying foundation, meaning that for the most part conclusions are drawn through a set of ad hoc criteria. Bayesian theory, on the other hand, defines probability as belief or credibility, and thus uses probability to describe mental uncertainty. Inference within the Bayesian school is based entirely on Bayes' Theorem, necessitating a statement of uncertainty prior to the observations, which many find inappropriate.

Statistical methods are used in geotechnical engineering primarily to analyze data on site conditions and environmental loads. To some extent they are applied to validataing model predictions against observed performance. This latter use might be expected to increase as probabilistic analysis become more widespread. The intent of statistical analysis in geotechnical applications is to make efficient use of data and to provide the probabilistic characterizataion of uncertainty necessary for reliability modeling or risk analysis. Increasingly, statistical methods are also being used to plan efficient "scientific" experiments for gathering information or validating models.

3.3 Risk Analysis

Risk analysis in its meaning here is the effort to bring a characterization of all relevant uncertainties together in an analysis to assess the aggregate uncertainty facing a "decision maker." This uncertainty is of many types. One part of a risk analysis is forecasting rates at which real facilities fail. Another is assessing systematic errors which manifest as correlated failures. A proper risk assessment leads to predictions of rates of failure and a quantification of the uncertainty in those predicted rates.

Risk assessment is typically a mixture of statistical analysis, probabilistic modeling, expert opinion, and pragmatism. Its use to date in geotechnical engineering has been limited and often proprietary, for example, in evaluating risks for insurance underwriting. An increasing area of use is in regulatory licensing and evaluation of siting hazards for power plants and other hazardous facilities. It appears likely that risk analysis will also become more widespread in the design of dams and other civil projects.

3.4 Decision Making and Optimization

Optimization of design or project decisions by balancing risk against cost requires not only risk assessment but also an analysis of the costs accruing to failures or other adverse performance. In many cases such failure costs involve only economic attributes, but in others they involve costs which are noncommensurate with monetary attributes: life loss, environmental degradation, social disruption. Decision analysis and optimization attempt to quantify the consequences of facility failures, combine these quantifications with assessments of their associated probabilities, and identify design or project options that are in some sense optimal.

In geotechnical engineering decision analysis approches have been often discussed, but seldom implemented in a serious and comprehensive way. Applications have tended to emphasize either careful assessment of consequences or careful assessment of probabilities, but seldom both. The better of the applications of decision analysis in geotechnical engineering for the most part have delt with regulatory problems such as power plant siting, in which the principal uncertainties and concern do not deal with soil or rock mechanics problems.

IV. APPLICATIONS IN SOIL ENGINEERING

4.1. Statistical Analysis

This section considers empirical work in the geotechnical literature on statistical analysis of soil engineering data.

4.1.1. Distribution Theory

4.1.1.1 Variability of Soil Properties

Soil is a natural material formed by geological processes and thus varies considerably from place to place, even within what are for engineering purposes presumed to be "homogeneous" zones or strata. This variability is not "random" in the colloquial meaning of that term, but spatial. In principle, if one had an extremely large exploration budget and unlimited computing time, the point to point variability of a soil mass could be accomodated in analysis. This is never the case, of course. For practical purposes a soil mass is divided into what are considered to be homogeneous zones, and each zone is assigned an average value or an average trend for the property of interest, and these averages are used in analysis. The variations about such averages are sometimes ignored, sometimes they are accounted for by selecting representative parameter values which are more conservative than averages, and sometimes they are summarized using the mathematics of probability theory. This section considers the third of these options, and in particular summarizes the statistical character of the variation of soil properties as observed empirically.

Most geotechnical engineers are perhaps of the opinion that soil and rock properties are more variable than most other materials encountered in civil construction, but this is not necessarily the case. Table 1 lists a variety of common engineering properties of soils and the associated coefficients of variation of their data scatter as reported in the literature. It must be noted that these data vary considerably in quality, and that the c.o.v.'s reported here are upper bounds on actual soil variability, in that data scatter combines real variability with random measurement error. Without more information than normally presented in the literature--indeed, than normally collected in site investigation--the separation of these sources of variance is not possible. For comparison, peak strength variation in concrete typically has a Cov of 20%; in structural steel, tyically 7 to 10%; and in wood, typically 20% (National Bureau of Standards, 1980).

4.1.1.2 Distributional Forms

As a general rule, the variability of soil engineering properties observed in field or laboratory data tends to regular distributional forms and can be approximated using common density function models. As discussed in Section II, the most typically useful of these are the Normal, logNormal, and Gamma pdf's; and for special applications the Exponential and Beta family.

The use of distributional forms in soil mechanics has engendered considerable debate. Those who oppose the use of probabilistic or statistical techniques for whatever reasons, have found the question of distributional form--although often irrelevant--to be prime for attack. This is principally due to insufficientcy of data with which to demonstrate empirical validity of an assumed pdf. The situation is different in rock mechanics where data, for example from joint surveys, are numerious. In that field arguments center not on which family of pdf's to use for data, but rather the effect of sampling biases and survey plans on differences between sample and sampled population.

In those cases where data are sufficiently numerious to draw confident statistical conclusions on distributional forms, one finds Normal and logNormal forms to be the most prevalent. This comes as little surprise, in that the same conclusion can be drawn for most natural phenomena. As an example, Figure 1 shows cone penetration resistances measured in a copper tailings embankment in the Zambian Copper Belt. Normality of the data scatter is evident by inspection.

LogNormal distributions are often encountered in permeability data, and also in data related to permeability, as for example, consolidation coefficient. These properties depend multiplicatively on other properties of soil or the soil mass, and thus by the Central Limit Theorem logNormal distributions would not be unexpected. In a similar way, grain size distributions often approximate logNormal pdf's, an observation which has been explained by arguments based on sequential crushing of particles leading to multiplicative effects (Aitcheson and Brown, 1957). On occasion, skewed distributions are also observed with strength or deformability data, but such results are often associated with population mixtures. Separating such mixtures in many cases reduces the skew considerably.

The number of workers who have devoted significant attention to empirical distributional forms for soil engineering properties is surprisingly small compared to the number working in analytical models of geotechnical reliability. Most prominent of these workers are Lumb (1966, 1971, 1974), and Schultze (1971, 1972, 1975), other contributions having been made by Singh (1971), Stamatopoulos and Kotzias (1975), Wu and Kraft (1970), Corotis, et al. (1975), and a handful of others. No systematic guide to the selection of distributional forms has resulted from these studies, an effort which should rank high in the list of research priorities.

Since the variability of soil data is spatial, the separation into a deterministic trend and residuals about that trend is arbitrary, yet clearly affects the conclusions drawn about soil property variability. The variation of undrained strength data with depth below is a simple, but useful illustration (Figure 2). The standard deviation of the data taken about the stratum average is about 1.2 units. Removing a linear trend with depth reduces the standard deviation of the residuals to about 0.25 units. Removing a quadratic trend reduces the standard deviation to less than 0.1 units. As higher and higher order trends are fit to the data, more and more of the variability is "explained" deterministically, and therefore less and less is attributable to scatter.

Not surprisingly, detrending data not only changes the magnidute of variability, but may also change its form. Baecher, et al (1983) report a variety of data from mine tailings which show the effect of detrending on

Table 1 -- Reported Coefficients of Variation for Various Soil Properties

Material	Property	COV	Source
Clay	liquid limit	5.9	Lumb[1]
	plastic limit	4 ±	
	clay content	11.4	
	specific gravity	0.5 ±	
	dry density	26.4	
Clay Shale	cohesion (direct shear, DS)	94.8	
	friction coefficient (t), DS	45.6	
Cohesive Till	c -- DS	103.3	
	t -- Ds	17.7	
"undisturbed"	c -- triaxial D	13.5	
	t -- triaxial D	1.6	
	c -- CU	19.9	
	t -- triaxial CU	9.8	
	c -- triaxial UU	18.8	
	t -- triaxial UU	22.3	
compacted	c -- D	24.0	
	t -- D	2.1	
	c -- CD	26.9	
	t -- CD	6.8	
	c -- UU	25.5	
	t -- UU	5.4	
Various Tills	UU	14.8	Morse[1]
		14.7	
		31.0	
		19.8	
		29.0	
Silt	e_o	21.6	
	n	89.4	
Gravelly Sand	e_o	29	
	n	16	
Coarse Sand	n	9.8	
	e_o	16	
Medium Sand	n	10	
	e_o	17.5	
Fine Sand	e_o	13.3	

Table 1 -- Continued

Material	Property	COV	Source
Marine Clay	c	18.4	Singh[2]
London Clay	c	16.2	
Sandy Clay	$\log(C_c)$	34.2	
Silty Sand	t	13.8	
Clay Silt	t	14.8	
	c	31.6	
	c	25.9	
Ottawa Sand (loose)	phi	14	
Ottawa Sand (dense)	phi	12.5	
Clayey Silt	c	51	
(unsoaked)	phi	22	
	s_u	19	
Clayey Silt	c	55	
(soaked)	phi	29	
	S_u	20	
Clayey Silt	c	64	
CH	c -- triaxial UU	15	
	phi -- UU	56	
CL	c -- UU	22	
	phi -- UU	19	
ML	c -- UU	71	
	phi -- UU	12	
CH	c -- DS	63	
	phi -- DS	10.4	
CL	c -- DS	3	
ML	c -- DS	2.5	
Road Subgrade	soil suction	24.2	Miura and
	soil suction	23.2	Fujita[3]
Average over	LL	6.37	Minty, Smith
16 cohesive soils	PL	9.55	and Pratt[3]
Road base coarse	CBR	17.4	Ingles [3]
	density	3.9	
	PI	75.0	
	S_u	36.8	
Plastic Clay	compression ratio	17 to 38	Vanmarke and
	t		Fuleihan[2]
Fine Sands	t	5 to 13	Schultze[2]
Gravel-Sands	t	5	
Coarse Sand	t	8 to 14	

1. First ICASP, Hong Kong.
2. Second ICASP, Aachen.
3. Third ICASP, Sydney.

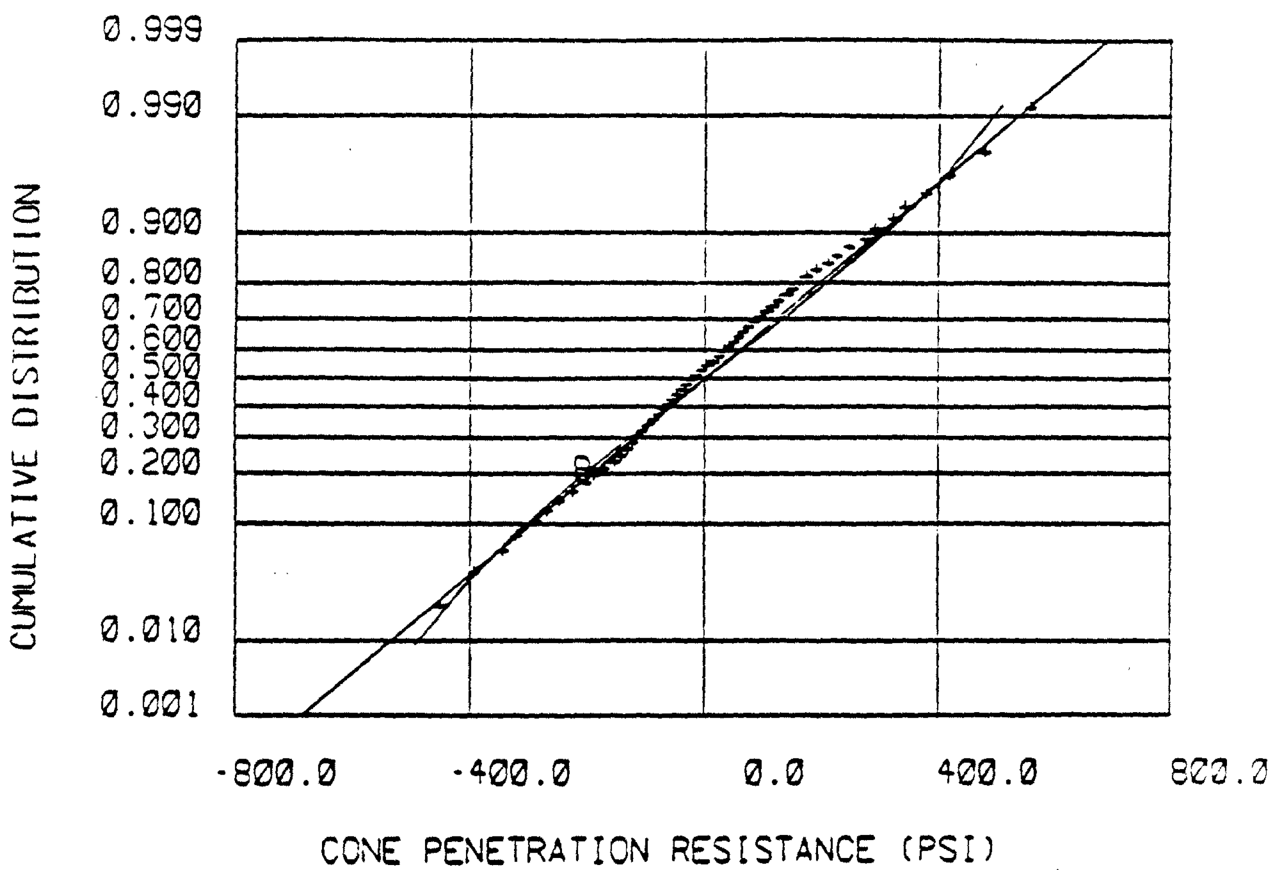

Figure 1 Cumulative probability plot of cone penetration data taken in copper tailings (from Baecher, Marr, Lin, and Consla, 1983).

distributional form as reflected on Pearson diagrams. The nondetrended data have distributions which are skewed and broad shouldered; they have both more skew and more kurtosis than Normal pdf's. Shows the same data with linear trends removed have distributions that are closer to Normal shapes.

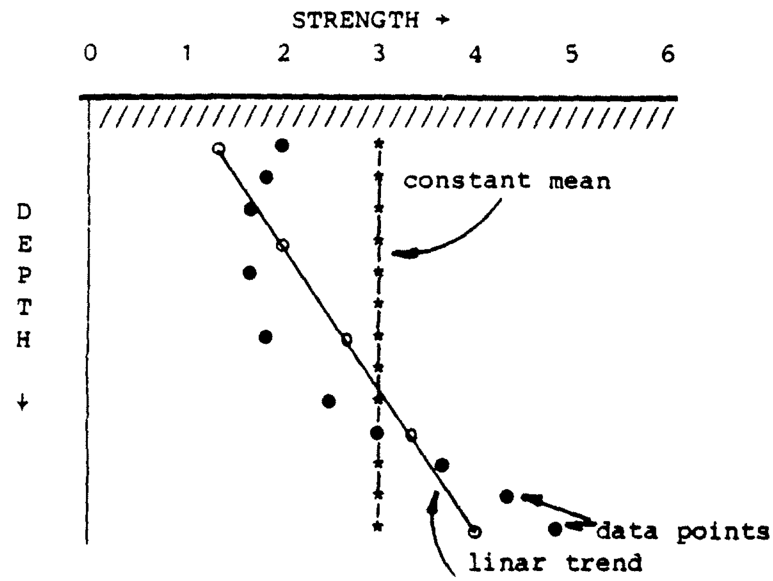

Figure 2 -- Hypothetical soil property showing effect of trend removal.

4.1.2 Site Characterization

Only a handful of attempts have been made to integrate information about soil property variability, geology, subsurface zonation, and hazardous conditions into comprehensive, systematic approaches to site characterization. Dowding (1978) and Baecher (1972) discuss philosophical and practical issues facing such attempts. The work that has been done draws heavily on developments in mineral exploration, in oil & gas reserve estimation, and in rock mechanics (Einstein, et al., 1978, 1979, 1981). This work is discussed in more detail in Sec. 5.

A number of workers have focused attention of specific aspects of the site characterization problem without attempting an integration. In organizing these contributions it is convenient to divide the tasks of site characterization into four main groups:

- Reconnaissance,
- Mapping,
- Search, and
- Mechanical testing.

4.1.2.1 Reconnaissance

Reconnaissance includes those aspects of site characterization that are fundamentally subjective (i.e., inductive). These include the development of hypotheses about site geology, identification of potential hazards, and assessment of how credible the hypotheses and hazards are. Little work has been done on these problems, in part because they are difficult to quantify and in part because they are broad. Nevertheless, many people have identified them as important and justifying attention (Dowding, 1978). The few attempts at understanding the structure of reconnaissance have tended to focus on the quantification of engineering judgement (expert opinion), although in recent years some interest has developed in relation to geological or geotechnical information in regulatory decisions, mostly nuclear siting (Meehan, 1984).

The use of subjective probability for quantifying engineering judgement has been studied by Folayan (1969), Baecher (1972), Hynes (1976), and Nordquist (1976) in geotechnical applications; and by Okrent (1975), TERA Corporation (1979), and Mensing (1983) in seismic hazard assessments. This work confirms the more extensive studies conducted in behavioral decision theory (e.g., Hogarth, 1975) that (1) the quantification of expert opinion as probabilities is possible, (2) experts display systematic biases in their assessments, (3) significant correlations may exist among a group of experts, and (4) assessments are sensitive to techniques of elicitation. Lambe, et al. (1980) and Whitman (1981) have demonstrated the use of subjective probabilities in project specific applications.

4.1.2.2 Mapping

Mapping geological formations or soil deposits is a principal part of site characterizations, and yet one which has benefited little from statistical methodology. The intent of mapping is to divide a site into discrete spatial zones which in some defined sense are internally homogeneous. While such maps typically reflect discrete changes actually existing in nature, as for example, soil or rock type; they may sometimes summarize artificial boundaries of a continuum, as e.g., rock "quality." The present section considers only the former little work on quantitative mapping has appear in the geotechnical literature per se. Among those that have are Lee (1977), Baecher (1972), Nucci (1975), and Wu (personal communication), all essentially building on Switzer's discrete random field model (1967, 1971, 1973).

Switzer's model idealizes the spatial distribution of geological properties as a discrete random field and uses the corelation structure of the field to predict error rates for specified mapping rules, and to form optimal (i.e., minimum expected error) maps from a set of observations. In the simplest case, a dicotomous field is represented by a zero-one assignment to alternate geological state, and this field is characterized by a mean value over space and an autocovariance function. Results are easily graphed as a function of the decay of autocovariance with distance and direction and the accuracy of the procedure is good. Similar work has appeared in the "geostatistics" literature under the name, indicator functions (Matheron, 1965),

A problem with all of the models based on discrete random field theory is that they ignore geological structure. The models are based purely on spatial correlation which is assumed stationary (the same everywhere). Intuitively, one would suspect that a geologist could improve upon such a map using his knowledge of geological theory. However, the few results of scientifically performed experiments leave this question in some doubt (Dahlberg, 1975). Again intuitively, were such an effect to exist, it would probably be most apparent in depostis that were strongly structurally controlled (as, e.g., folded and faulted terrain), and less apparent in deposits that were not (as, e.g., surface soils).

4.1.2.3 Search

Search problems in site characterization deal primarily with finding details of site geology which are suspected to exist and which may pose hazards to the safe performance of a facility. Typical targets for search in geotechnical investigations are solution features, weak or altered lenses, local bedrock weathering, and faults. A number of other features may be of concern at peculiar sites, e.g., abandoned mine openings.

Mathematical search theory as developed in operations research (e.g., Morse, 1971; Stone, 1975 and applied in mineral exploration [e.g., Slichter, 1955; and Brown, 1960) and has been applied to the general issue of geotechnical investigations by Baecher (1972), and to sink holes in particular by Drake (1976), Grant (1973), and Hynes (1976). The use of search theory in geotechnical site investigation is summarized by Baecher (1975).

4.1.2.4 Mechanical Testing

By far, the most statistical work in site charactreization has been done on the question of analyzing mechanical testing data, specifically index and strength property data, and to a much less extent deformation and permeability data. This work can be roughly divided in a two way table, one characteristic being whether data are treated as IID observations from a homogeneous population or as observations of a correlated random field, the other characteristic being whether frequentist or Bayesian inference is used. The

Figure 3 -- Types of Statistical Sampling

	IID	Random Field
Frequentist	X	X
Bayesian	X	X

upper left hand corner of this table is the most populus; the lower right, the least.

Frequentist inference from IID observations intend to estimate the moments or distributional from of the parent populatino. The statistical theory of such techniques is widely available in introductory tests, and therefore not repeated here (see, Benjamin and Cornell, 1970). The two primary approachs to frequentist estimation appearing in the geotechnical literature are the methods of moments and maximum likelihood estimation. The method of moments uses moments of the sample (e.g., sample mean and variance) to estimate parallel moments of the parent population. Maximum likelihood estimation uses the conditional probability of the observations actually recorded as a function of possible parent population parameters as the criterion for estimation, the parameter values maximizing that conditional probabilty being the chosen estimates. In many situations moment and maximum likelihood estimates coincide, but not in all. Where they do not, as e.g., with a Gamma parent, the moment estimator may be much less efficient than the maximum likelihood, in the sense of having a larger variance in repetitive sampling (Baecher and Rackwitz, 1982).

4.1.2.5 Frequentist IID Sampling

Among the earliest work on frequentist IID sampling is that by Wu and Kraft (1967, 1970) resulting in part from Kraft's Ph.D. dissertation (1968). This work was based on sampling from normal populations, leading to point and estimates and confidence limits. About the same time, Lumb began publishing a core series of articles on empirical results of soil sampling (1966, 1967, 1970, 1971, 1974). Other applications are listed in standard deviation 4.1.

4.1.2.6 Measurement Error

An important aspect of statistical inference on mechanical properties is separating the components of variation and error. At a minimum, the scatter observed in data either from the field or laboratory combines two components: Soil variability and random testing error (noise). To the extent that noise is important, the conclusions drawn on data scatter reflect equipment and procedures of measurement and only secondarily the real variability of the soil. The proportion of data scatter variance due to measurement error can be large, as shown by the empirical results in Table 2. It is important to note that most results reported in the literature (e.g., Table 2) do not differentiate soil variability from measurement error.

A second influence of measurement error on data scatter is the alteration of distributional form. Measurement error in most physical measurements tends to have Normal distributions (Benjamin and Cornell, 1969). This reflects its multiple sources and the action of the Central Limit Theorem. Thus, if

TABLE 2 -- Empirical Autocorrelation Estimate

Deposit	Measurement	Autocovariance Distance Horizontal	Autocovariance Distance Vertical	Noise	Reference
North Sea Clay	CPT	20m		0%	Tang, 1979
Silty Bay fill	SPT	35		35	Spikula, 1983
Marine Clay	FV	30	3m	40	SEBJ, 1983
Lacustrine Clay	FV	30	3m	0	"
Cu Tailings					
Chambishi	CPT		1		Baecher et al, 1983
Chibuluma	CPT		2		"
Chino	SPT		<1		"
Chingola	CPT		<1		"
Magna	w/c		0+		"
Magna	FV		1		"
Magna	γd		0+		"
Mindola	CPT		2		"
Morenci	CPT		1		"
Ur Tailings	SPT		3		"
Gypsum Tailings					"
Texas	SPT		0+		"
Texas	γd		1		"
La. 1	SPT		0+		"
Florida 1	SPT		0+		"
Piney Point	Cpt		0+		"
Compacted Clay	w/c		5	0	
Dune Sand	SPT	35		50	Hilldale, 1971
Joints	Dip	5			Einstein et al, 1983
Joints	Density	5			Miller
Cu Tailings	CPT	50	5		Bowles, et al, 1978
Clay Fill	w/c,δd	5		40	NEXUS Assoc., 1985
Alluvial Sand	SPT	20	1		"

observations z are corrupted additively by a Normally distributed noise $e \sim N(o,s)$, as e.g., by

$$z = x + e \tag{40}$$

in which x = the property being measured; then even were x not Normally distriubted, z might approximately be. This trend is exacerbated by the Central Limit Theorem acting on the sum of Eq. 40.

Estimation of random measurement error can be attempted in at least three ways: (1) Replicate measurement on the same specimen, (2) multiple profiling along the same boring, or (3) analysis of the autocovariance function. Replicate measurement is only possible with nondestructive testing, eliminating it from practical application in site characterization. Multiple profiling requires tests of different types that can be performed in the same boring. Analysis of the autocovariance function requires sufficiently many and closely spaced measurements to estimate $C_z(r)$ for small r.

Estimates of noise variances from multiple profiling data can be readily made by calculating covariances among parallel measurements, x_1 and x_2 of the same physical property z. Adopting the common model

$$x_i = B_i z + e_i \tag{41}$$

for the respective types of measurements, in which B_i is a measurement bias and e_i is the noise terms, respective data scatter variances are simply

$$V(x_i) = B_i^2 v(z) + V(\varepsilon_i), \tag{42}$$

and the covariance of the data scatter of two measurements is

$$C(x_i,x_j) = B_i B_j v(z). \tag{43}$$

Thus,

$$V(z) = \frac{C(x_i,x_j)}{B_i B_j}, \tag{44}$$

and

$$V(e_i) = V(x_i) - \frac{B_i}{B_j} C(x_i,x_j). \tag{45}$$

Presuming the relative bias of the measurements is known, the noise variance is readily estimated.

Estimates of noise variance from the autocovariance function are made by adopting Eqn. 40 as a measurement model and algebraically deriving the autocovariance function of the data scatter to be

$$C_x(r) = B^2 C_z(r) + C_e(r) \qquad (46)$$

in which r = separation distance between observations. Since

$$C_e(r) = \begin{cases} V(e) & r=0 \\ 0 & \text{otherwise} \end{cases} \qquad (47)$$

the sample autocovariance function is extrapolated back to the origin to estimate V(e). This techique has been used by Soulie (1983) and Baecher (1982).

4.1.2.7 Bayesian IID Sampling

A number of papers have also been published on applications of Bayesian methods to simple IID sampling of soil properties. Most of these are direct appliations of results summarized, e.g., in such texts as Raiffa and Schlaifer (1961). Among the earliest of these is Tang's (1971) paper to on Normal Bayesian sampling and predictive distributions. Other contributions have been made by Bowles, et al. (1978), Folayan, et al. (1970), Harr (1977), Jowitt and Munro (1975), Kay (1976), Matsuo (1976), among others.

4.1.2.8 Sampling From Random Fields

While a number of workers have presented random field characterizations of soil engineering properties, only a few have delt explicitly with the problem of statistical sampling from correlated fields. The most comprehensive such work has been done by Veneziano in collaboration with coworkers. Veneziano and Faccioli (1975) have presented an analysis of Bayesian sampling from correlated fields with known autocovariance functions. This work treats the optimal design of observation networks to minimize interpolation error, and the variance-covariance structure of statistical interpolation errors among observations. The latter is the Bayesian parallel to BLUE or kriging estimators common in "geostatistics (Spikula, 1983)." Veneziano and Kitanidis (1982) have presented a Bayesian method for optimally contouring the excursions of random fields above or below specified levels based on a network of observations.

Frequentist methods for interpolating random fields among observations are common in the ore reserve literature. A number of deterministic and ad hoc methods are in common use (e.g., squared distance, nearest neighbor, and similar methods), as are the now widely appreciated techniques of "geostatistics." The literature on the latter is large, but summarized by David (1977) and by Journel and Huijbregts (1979). Considerable attention is given in this literature to linear interpolation procedures ("kriging") and in recent years to various non-linear techniques (e.g., "disjunctive kriging"). A recent volume by Henley on nonparametric methods in random field interpolation (1981) promises a new approach for random field estimation. Early parallel work on BLUE techniques for spatial fields, mathematically special cases of kriging, was presented by Matern (1960) and Whittle (1963).

4.1.3 Autocorrelation

Most work in geotechnical statistics and in "geostatistics" assumes the autocovariance function to be known. In fact this is seldom the case, and the statistical problems associated with autocovariance estimation are non-trivial. Two aspects of the problem are important to geotechnical applications, the first is simply the sampling variability or uncertainty of estimates of C(r) (in either the frequentist or Bayesian sense, respectively). The second is propogating uncertainty in the the autocovariance function to (Bayesian) predictive distributions on interpolations.

Much of the relevant work on autocovariance function estimation appears in literatures other than geotechnics, especially those related to statistical time series analysis, signal processing, and econometrics. Some work on estimating variograms appears in the "geostatistics" literature (e.g., Armstrong, 1983). Gelb, et al. (1974) discuss frequentist approaches estimating autocovariance from regularly spaced observations. An important result of this work is that moment estimators of C(r), the type most common in geotechnical applications, are unbiased only asympotically. For short data strings estimation bias can be substantial. Such results are difficult to apply directly to geotechnical data since the latter are almost always taken on non-uniform networks. Veneziano and et al., (198x) have studied the problem of estimating autocovariance from non uniformly spaced data using Kernel estimators. DeGroot (1985) has studied maximum likelihood estimation.

Spikula (1983) has demonstrated that autocovariance distances can also be estimated inversely by minimizing the observed error variance of BLUE interpolations over r_0. This is done by sequentially removing data points from the sample, estimating their value, and calculating the error variance between observed and predicted over all data. This approach is often used in "geostatistical" applications (e.g., Matheron, 1971).

The effect of scale and detrending has been illustrated by Javette (1983), who analyzed vertical autocovariances of water content measurements over a many meter depth of SF Bay Mud in which water contents were measured every foot, and again over a one foot segment in which water contents' were measured every 1/2 inch. In the former case the boring mean was used in calculations, in the latter case the mean over the segment was used. The autocovariance distance in the first case is several feet; in the latter case it is only a few inches. The difference is due to the difference in means, and illustrates that covariance is not a property of the soil, but of the way analysis is undertaken.

4.1.4 Quality Control and Construction Monitoring

Quality control and inspection sampling would appear to be areas of obvious applications of statistical techniques, both because they are routine in embankment on road construction and because statistical methods are integral to quality control in other industries. Yet, only a limit number of applications have been published.

Perhaps the most complete application appearing in the general literature is Kotzias and Stamotopoulis (1975), who use statistical inspection sampling in compaction control for earth dams. They report extensive data on distributional forms for density and water content and discuss various procedures for maintaining compaction control records. However, little guidance is provided for designing sampling plans.

Other discussions of statistical methods in compaction control have been published by Blaut (1975), Fang (1975), Kuhn (1967, 1971, 1972), Kuroda and Kanada (1983), and Leach and Goodram (1976) and NEXUS Associates (1985). Ingles (1971) maintains that more control samples are taken in common highway construction than can be justified economically.

4.2 Reliabilty Modeling

Reliability modeling of foundations, slopes, and other geotechnical problems focuses on quantifying the uncertainty of engineering prediction caused by uncertainties in engineering properties, loads, and other factors in engineering analysis. Most published work on geotechnical reliability deals with foundations and slopes, much more limited work having been done on other problems (e.g., underground openings, retaining structures, construction support).

4.2.1 Foundations for Buildings and Offshore Structures

Interest in the reliability of foundations has come from both traditional geotechnical applications to buildings and earth structures, and from the offshore industry. While the groups working on these problems in part overlap, the problem areas themselves are somewhat distinct, and publications on these two problems appear in separate places.

4.2.1.1 Stability

Reliability analyses of shallow foundations have followed the traditional distinction between stability and settlement predictions. Stability of shallow foundations, notably footings, is typically predicted using Terzaghi's superposition formula

$$q_v = \frac{1}{2}\gamma B N_\gamma + c N_c + q N_q \tag{48}$$

in which γ = soil density, B = footing width, q = sinobaye, and N_γ, N_c, N_q are bearing capacity factors. As in deterministic practice, the effects of load eccentricity and inclination, and foundation shape and size are usually summarized by calibration factors. Soil properties affect the prediction of bearing capacity q_v through γ and c directly, and through N_γ, N_c, and N_q indirectly, as the latter depend on the effective strength parameters c' and ϕ'. Correction factors for load eccentricity (E_γ), inclination (I_γ), shape (S_γ), and size effects (R_γ) may display mild dependence on soil properties, but such dependence is often ignored.

The problem of propagating soil parameter uncertainty through Eqn. 48 has lead to a number of papers on the subject. Among others these include Wu and Kraft (1967), A.-Grivas (1979), Singh (1971), Harr (1977), D'Andrea and Sangrey (1974), Schultze (1972), McAnally (1983), each of whom propogates pdf's on soil properties through Eqn. 48 to obtain a pdf on q_v. FOSM applications have been made by Cornell (1971). Hoeg and Murarka (1971) consider bearing capacity problem as one part of the analyses of gravity retaining walls. Each of these contributions is more an illustration of how uncertainties propagate through the bearing capacity formula than it is a guide to footing design. A more detailed analyses of test and field data for the purpose of developing design quidelines has been presented by Ingra and Baecher (1983). This work used regression analysis to estimate bearing capacity factors and the influence of soil properties on the uncertainty of q_v. Raft and mat foundation stability has been addressed by Schultze and Pottharst (1983).

4.2.1.2 Settlement

Previous studites of probabilistic settlement appearing in the literature can be grouped in three categories: random lumped parameter models, stochastic one dimensional models, and stochastic finite element models. Random lumped parameter models propagate parameter uncertainty through deterministic equations in which soil properties are assumed uniform but uncertain. Stochastic models are distinct from lumped parameter models in that they assume spatial variability in the soil profile. The one dimensional models address variation along vertical lines in the subsurface. Finite element models address variation from one element to the next.

One of the earliest lumped parameter models is that of Wu and Kraft (1967), which propagates uncertainty in standard penetration test data through a Terzaghi & Peck correlation for the load necessary to cause one inch of settlement. This is a full distribution model, assuming the standard penetration test resistance and model uncertainty to be Normally distributed. Ramos (1976) extended this model to include uncertainty in loads, in a second moment formulation.

Hilldale (1971) developed a one dimensional settlement model in which modulus is characterized as a second-order stationary stochastic process. Settlement is calculated by integrating one dimensional deformations induced by a deterministic elastic stress field. Similar models were developed by Resendiz and Herrera (1969) and Diaz (1974). The former model assumes independence of soil properties in adjacent layers, and constant variance within layers. The latter assumes soil properties to be autocorrelated within layers. Both are second modment analyses. Grant, et al. (1974) complemented the approach with an analysis of settlement histories to assess building damage associated with differential settlement.

Uncertainty in one-dimensional settlement on clays has been analyzed by Fuleihan (1975) in conjunction with his analysis of plastic deformation under Atachaflaya dyke system. Of particular interest in this work was the availability of post construction records allowing spatial variability of settlement to be validated against data. More recent, but similar work has been performed by Javette (1983) on a filled bay load overlying San Francisco Bay Mud. Extensive data on soil properties and settlement records are available for the site, allowing a comparison of predicted and observed behavior.

A Bayesian approach to settlement prediction, observation, and updating was adopted by Asoka (1978) in the analysis of settlement performance of an earth embankment. A particularly interesting part of this work is the attempt to identify soil properties inversely from settlement records, and to use this information via Bayes Theorem to update prior estimates. Systems identification techniques have also been used in groundwater modeling by Wilson, et al., (1974), and in rock mechanics by Jurina, et al., 1977).

4.2.1.3 Deep Foundations

Several papers have appeared on the problem of propagating parametric uncertanities through pile capacity formulae. Among others, these include Kovacs et al., 1975; Madhav et al., 1979; Nishida et al., 1971; Rizkellah et al., 1979; Ruiz, 1983; Wagner et al., 1975; and Wakamatsu et al, 1971).

An interesting approach has been taken by Kay (1976, 1977), who recognized both the imprecision of pile capacity formulae and the importance of load tests on design, in developing a Bayesian procedure for combining formulae predictions with load test results in selecting factors of safety. A typical result of Kay's work gives the design factor of safety necessary to attain a given reliability in relation to the number of load tests performed. Kay presumes the spatial variation of pile capacity across a site to be known a priori. Baecher and Rackwitz (1982) relaxed this assumption, but arrive at similar results.

4.2.1.4 Finite Element Modeling

Stochastic finite element techniques for two (or higher) dimensional settlements were suggested by Cornell (1971) in a general discussion of the applicability of second moment techniques to linear systems, and developed by Ditlevsen (1979) for solution by matrix techniques. Application of stochastic finite element methods to rock and soil mechanics has been made by Su, et al. (1969), Cambou (1978), and Baecher and Ingra (1981). Su, et al. and Praseau use Monte Carlo simulations to generate element properties from specified distributions, repeating the calculations to form a sampling distribution. In both studies, element properties were assumed to be independent and identically distributed. This yields low variance output with marked sensitivity to element size, through an "averaging" of random fluctuations over larger and larger "sample sizes," as element dimensions are reduced. Cambou applied second moment approximations to linear solutions of the finite element method which include autocorrelation among properties. The procedure for selecting element properties, and whether the IID assumption was made, was not discussed. Baecher and Ingra (1981) used a similar approach to estimate foundation settlements.

4.2.1.5 Offshore Structures

Hoeg and Murarka (1975) present an analysis of bearing capacity uncertainty as one mode of failure of a gravity retaining wall. The analysis is lumped parameter, without spatial variation in soil properties, but includes horizontal sliding and bearing capacity under an inclined, eccentric load. First-order second-moment analysis of the margin of safety yield surprisingly high probabilities of failure (p_f) at acceptable deterministic factors of safety (FS). The probabilities are calculated under the assumption of Normal distribution of the safety margin. The authors attribute these high probabilities of failure to the sensitivity of bearing capacity factors to friction angle. Kraft and Murff (1975) have suggested a similar procedure for offshore gravity structures.

Hoeg and Tang (1978) consider slip surface stability of an offshore gravity structure, and uncertainties in FS due to uncertainties in loads, strength parameters, geometric variables, and a number of correction factors. Their analysis is for undrained behavior and they conclude that approximately 70% of the uncertainty in FS predictions are due to uncertainty in undrained strength. Another 25% they attribute to uncertainty in loads. However, as the authors note, uncertainties deriving from poorly understood mechanisms such as cyclic loading are not directly included in these calculations.

4.2.2 Slopes and Retaining Structures

More work has appeared on slope stability than on possibly all other areas of geotechnical reliability combined. Early work focused primarily on 2D undrained stability of homogeneous slopes, somewhat later work considering 3D effects. In recent years, attention has turned to effective stress analysis and applications to real projects.

Attempts to determine the risk of slope failure have been made by Wu and Kraft (1970), Yucemen, Tang and Ang (1973), Morla-Catalan and Cornell (1976), Barboteu (1972), Gilbert (1974), and Matsuo and Kuroda (1974). These studies all work within the framework of conventional plane strain analylsis of slope stability. Three dimensional analysis have been performed by Veneziano and others (1977, 1979, 1982), and Bowles, et al., (1978). The latter two are simple extensions of the circular arc method with plane ends. Veneziano's approach is based on work energy in a plasticity model of various geometries.

Matsuo and Kuroda (1974) and Matsuo (1977) among others devoted considerable effort to estimate the variability of soil properties as determined from laboratory and field tests; they concluded that although significant scatter exists in measured strength parameters (c, ,S_u), the scatter in measured soil unit weights is fairly small and can be neglected for all practical purposes. The same researchers suggest that the uncertainty in slope geometry is also small and can be neglected.

Alonso (1976) studied the relative influence of different factors contributing to the probability of failure of the slope using a two-dimensional plane strain mode of failure. In these results then a large difference in the uncertainty attributed to driving and resisting moments; almost all the uncertainty comes from the resisting forces of the slope.

Although Cornell (1971) recognized that soil properties are random spatial variables, several studies choose a random variable representation, (for example Matsuo 1977 and Matsuo and Kuvoda 1974) and use coefficient of variations, and mean values from sets of measured soil properties. The stochastic field representation, however, shows there to be significant difference between the levels of variability of soil properties obtained from field or lab tests and the values for analysis.

Statistical information from a set of soil test data may be of little use without information about soil heterogeneity. The exact shape of the probability density function for "point" soil properties may be even less important. What is often needed is the probability density function of some spatially averaged soil property, which will have narrower probability density functions that the corresponding "point" properties. Assigning to average layer properties the same degree of uncertainty as may have been observed in laboratory test sample often leads to too large a probability of failure. It is worthwhile to note that Wu and Kraft (1970) recognized the difference between "point" properties and "average" properties and proposed reduction of the "point" variance by division with . n where n is the number of the "independent" test results along the failure surface; Tang et al (1976) also proposed a similar approach.

Morla-Catalan and Cornell (1976), used a plane strain idealization of the slope and modelled shear resistance as a two-dimensional random field over the cross section. The cylindrical failure surface was defined by the cylinder axis x_o, y_o, and the cylinder radius r_o. Then the reliability was derived as a nonstationary function of x_o, y_o, r_o, and the probability of slope failure determined by a zero-down-crossing approach.

Alonso (1976) working on plane strain cylindrical failure modes, reduced two-dimensional spatial variability of soil properties to a representative set of random variables through averaging over finite intervals. In this way he was able to use second moment algebra to obtain approximate failure probabilities including spatial variability of cohesion and friction angle, uncertainty in embankment geometry, pore pressure, etc.

Veneziano (1977), and Veneziano and Camacho (1977) proposed a general method to calculate the reliability index of a slope. Their formulation considers three-dimensional failure mode and random shear resistance fields and doesn't pose restrictions on the failure mechanisms or on the random properties fields. The major disadvantage of the above methods is that they study the reliability problem conditionally on a given mode of failure. To overcome this drawback, Veneziano and Antoniano (1979) propose a frequency domain analysis with the only limitation that the shear resistance random field must be homogeneous along the slope. If a random field representation is used, one finds either normal or log-normal fields; Alonso (1976) and Veneziano and Antoniano (1979) demonstrate that slope failure probabilities are rather sensitive to distributional assumptions.

4.2.3 Factors of Safety

Current geotechnical practice employs traditional safety factor concepts for determining the design relaibility of earthqorks and foundations. Although deterministic concepts inherently include a qualitative asessment of uncertainty, the quantitative effects of specific uncertainties remain unknown. Such subjective criteria creates the potential for misleading conclusions. Indeed, Peck (1967) indicates adverse foundation performance is often attributable to the misjudgement of uncertainties in loading or soil conditions.

Limitations of deterministic safety factor concepts are illustrated by Langejan (1965), Meyerhof (1970), Lumb (1970), and Hoeg and Murarka (1975). These works treat probabilistic methods for evaluating the influence of specific uncertainties on design reliability. In this section, concepts for assessing design reliability are considered.

4.2.3.1 Evaluation of Design Reliability

The evaluation of design reliability is usually by a factor of safety. Convention defines factor of safety as the ratio of available resistance R to applied load L. Accordingly, failure occurs for $R/L \leq 1$. To evaluate the probability of this event, probability density functions for resistance $f_R(r)$ and load $f_L(\ell)$ are necessary. If these functions are known, the probability of failure, P_f, is,

$$P_f = P[R \leq L] = \int_0^{\infty} \int_0^{L} f_R(r)\ f_L(\ell)\ dr\ d\ell \tag{49}$$

$$= \int_0^{\infty} F_R(\ell)\ f_L(\ell)\ d\ell \tag{50}$$

where $F_R(\ell)$ is the cumulative distribution function for resistance ($P[R \leq \ell]$). Equation 50 evaluates the probability of resistance being less than load for all values of load.

In practice, the density functions for resistance and load are seldom known. Evaluations of load and resistance result in estimates of means and variances. Consequently, only estimates of the expected value and variance of the factor of safety are possible. The probability of failure may be determined, however, by two approaches: (1) Use the Chebyshev inequality which requires no assumptions on the probability density function for factor of safety, or (2) assume an intuitively satisfying form of the probability density function for factor of safety.

The Chebyshev inequality yields a weak bound on the probabilty of a random variable falling within $\pm\beta$ standard deviations of its mean. Formally, for a random variable X with mean m_X and standard deviation σ_X, the Chebyshev inequality states:

$$P[(m_X - \beta\sigma_X) \leq X \leq (m_X + \beta\sigma_X)] \geq 1 - (1/\beta^2) \tag{51}$$

For a given mean and variance of factor of safety (FS) Equation 51 yields the probability of failure:

$$P_f = P[FS \leq 1] \leq \frac{\sigma_{FS}}{m_{FS} - 1} \tag{52}$$

Although providing the advantages of simplicity and freedom from distribution shape, the Chebyshev bounds are too wide for most engineernig applications. Freeman (1963) shows a slightly more informative inequality by assuming the distribution shape is unimodal and asymptotic. For these two assumptions Equation 52 becomes:

$$P_f = P[\ FS \leq 1] \leq \frac{0.67\ \sigma_{FS}{}^2}{m_{FS} - 1} \tag{53}$$

For coefficients of variation as low as 0.10, the upper bound probability of failure is still 10^{-2}.

Alternatively, a complete probability density function for the factor of safety is often assumed. The availability and convenience of probabilistic information favor the normal or lognormal distribution (Harr,1977; Hoeg and Murarka, 1975; Kraft and Murff, 1975; Ang and Cornell, 1974; and Langejan 1965). In most cases, a definitive selection between distributions is not possible. The sensitivity of probability of failure to assumptions of a probability density function, therefore, is important. As shown by several authors (Ang and Cornell, 1974; Harr, 1977; and Ingra, 1978), the probability of failure is not sensitive to distribution form above 10^{-2}. For small coefficients of variation of factor of safety the agreement extends to 10^{-3}. At probabilities of failure less than 10^{-3} the deviations become significant. The sensitivity of probability of failure to distribution form, however, generally decreases with decreasing uncertainty in the factor of safety.

One additional method for evaluating design reliability is the number of standard deviations, β, the mean factor of safety lies from the failure event:

$$\beta = \frac{m_{FS} - 1}{\sigma_{FS}} \tag{54}$$

In this context, β is frequently called the second-moment reliability index. The interpretation of β is difficult, however, unless specified with some distribution for factor of safety. For example, for an $E[FS] = 3.0$ and $P_f = 10^{-4}$, the normal distribution requires $\beta = 3.72$, whereas the lognormal distribution requires $\beta = 2.2$.

The FOSM reliability index is not invariant with respect to the definition of failure. However, lack of invariance is also a problem with deterministic factors of safety. For example, in calculating the stability of a block of rock lying on an inclined joint surface, the effect of dilatent motion over asperities can be accomodated either by adding an extra friction term to the numerator of the factor of safety (denoted "i" in the rock mechanics literature), or by subtracting an equivalent angle from the slope of the plane in the denominator. Although these are mechanically equivalent formulations, they give numerically different factors of safety. Two factors of safety are possible depending on the definition of failure; similarily, two probabilities of failure are possible using β. An exact probabilistic analysis using correct distributions would give equal probabilities of failure. Exact methods, however, are impractical for design purposes.

4.2.3.2 Design Factors of Safety

The traditional factor of safety is the ratio of available resistance R to applied load L. These values have largely developed from judgement and experience with acceptable performance.

Meyerhof (1976) presents a brief survey of earthwork, retaining wall, and foundation failures. Based on this survey and considerable judgement, Meyerhof presents the comparison between factors of safety and probability of failure shown in Table 3. For geotechnical structures, the probability of failure ranges from 10^{-2} to 10^{-4}. Although the assessment of coefficients of variation for factor of safety are difficult, Meyerhof suggests 0.10 to 0.30 based on limited data indicating similar values for soil strength, applied loads, and engineering models. If the true COV[FS] for foundations is 0.20 or less, the normal distribution reasonably describes the factor of safety. If the true COV[FS] is 0.30, or greater, the lognormal distribution appears appropriate. Considering the sensitivity of p_f to distribution assumptions, the definitive selections of a probability density function is difficult without further knowledge of the level of uncertainty in foundation design.

V. APPLICATIONS IN ROCK ENGINEERING

A considerable amount of work has been done on statistical and probabilistic methods for rock mechanics applications. Much of this work has been applied to mining problems and thus appears in other areas of the literature from most of the soil mechanics work discussed above.

Rock masses unlike soil deposits are commonly modelled as discontinua. The presence of pervasive fracturing (jointing) in natural formations strongly affects engineering behavior. Because these features are too numerous to be deterministically represented in engineering calculations, a statistical approach to sampling the geometric properties of joints and representing them in engineering models developed relatively early in rock mechanics as compared with soil mechanics.

5.1. Joint Surveys

In this section geometric properties of jointing as commonly used in the literature are adopted. While these geometric properties seem to fall naturally into distinct geometric classes, in reality they are only facets of other, more fundamental ways of describing joint geometry. It is, therefore, important, when interpreting the implications of survey results for predicting aggregate rock mass behavior, that these observed geometric properties be viewed as strongly interdependent.

In joint surveys, three geometric properties are commonly of interest: density (e.g, spacing, frequency), size (e.g,. trace length, area), and orientation (e.g., strike and dip of an approximating plane, direction cosines of the pole). The measures adopted here are spacing, trace length, and polar direction cosines. Spacing is measured by the separation of the intersections of adjacent joint traces with a sampling line, either for individual sets of subparallel joints or for all joints. Trace length is typically measured as the linear distance between the end points of the intersection of a joint with an exposed surface. For joints that are strongly nonplanar, other measures are sometimes used. If both ends of a trace are not observable, the length recorded is a censored length. Statistical approaches to joint persistence are recent and not yet well developed.

One of the earlier papers to treat joint trace lengths statistically is Robertson (1970), in which 9000 joint trace length from the DeBeer mine (South Africa) are analyzed. Robertson drew three conclusions from this work: Strike trace length and dip trace length have about the same distribution, possibly implying joints to be circular (disks); Joint trace length data is "censored", in that long traces often extend beyond an outcrop or excavation, and thus are unmeasurable; Trace lengths are Exponentially distributed. Procedural effects draw the third conclusion into question. Data were grouped in 5-foot intervals or histograms. This interval approached, or sometimes exceeded, the sample mean. Potential modes in negatively skewed distributions, such as the lognormal and gamma, would be masked by this grouping.

Steffen, et al. (1975) also discused censoring of joint trace lengths and developed inference equations for known, constant censoring length (i.e., all traces greater than a fixed length are censored). The derivation is for Exponentially distributed trace lengths, although data underlying the assumption of Exponentiality are not reported. The usefullness of this derivation is limited in practice because censoring lengths are actually random variables. Epstein (1954), in a different application, has presented inference equations for randomly censored Exponential variables, which are applicable to joint trace length data (see Part IV).

Censoring has been further discussed by Call, et al. (1976), who also pointed out the bias introduced by truncation (i.e., lower limits on joint traces recorded). Exponential distribution of trace lengths was propsed in this paper but few data are presented.

Barton (1977), McMahon (1974), and Bridges (1976) concluded that trace lengths are logNormally distributed, but base their argument on inspection rather than statistical tests. Bridges concludes that joints are rectangular (oblong?) and that both strike and dip trace lengths are logNormal. Barton concludes that joints are equidimensional with mean area

$$\bar{A} = \frac{4\bar{T}^2}{\pi} \qquad (55)$$

where $\bar{T}$ is the average trace length. However, this study used large histogram intervals (5 foot, with sample means on the order of 5.6 foot), and fixed (i.e., non-random) censoring assumptions. Also, goodness-of-fit tests, as reported, appear to have been performed on the original observations not corrected for censoring.

The conclusions above provide important indirect evidence on joint shape. Kendall and Moran (1963) show that the expected length of random chord of a circle is $\pi r/2$, where r is the circle radius. Santalo (1976) has shown that for any convex shape, the expected random chord length is A/P, where P = perimeter, A = area. Combining these two relations yields Barton's equation. It is not clear from the original report whether (Eqn.55) is an empirical finding. If it is, it supports the contention that joints are roughly circular in their plane.

Cruden (1977) pointed out that the point of censoring of trace lengths is a random variable, and recommended that data be maintained on how many end points are observable. He presents evidence that trace lengths follow a censored Exponential distribution, by fitting distributions to the non-censored part of the sample.

From the literature, three conclusions seem to be supported: Joint trace lengths observed in outcrops and excavations are negatively skewed, and are perhaps either exponentially or logNormally distributed; Joints are approximately equidimensional; Severe sampling biases -- censoring and truncation -- afffect joint surveys and must be accounted for.

Einstein, Baecher, and Veneziano (1979) report a study of joint survey data from seven construction and mining sites of varying geology. These data were principally collected for engineering purposes, including foundation and slope design. Typical results for spacing distributions are shown consistent exponential cumulative density functions (cdf), $F(s) = 1 - \exp\{-\lambda s\}$. While average spacing varies with orientation of the sampling line, exponentiality does not. Trace length distributions do not exhibit the consistent characteristics that spacings do; however, in samples, trace lengths appear lognormal.

For a limited number of cases, data are avialable on trace lengths observed on orthogonal planes. For most of these, length pdf's exhibit little difference between strike and apparent dip directions, when individual joints sets are considered separately. When data are not separated by joint set, the orientation of the sampled face influences the relative proportions of joints from different sets being sample, and thus the trace length distributions.

Much less success has been enjoyed in fitting analytical forms to orientation distributions. In all, 22 data sets were analyzed, and each of the following distributions tested by χ^2 and likelihood ratio methods: Fisher, bivariate Fisher, Bingham, bivariate normal, and uniform. Data were sorted into clusters and maximum likelihood estimates made of distribution parameters. For many data sets no analytical form provided a satisfactory fit based on χ^2. Based on log likelihood ratios, the Bingham and bivariate Fisher appear to provide the better fits.

5.2. Foundation Deformations

Rock mass deformability affects the performance of essentially all structures in and on rock, from underground openings and excavations to foundations. Thus, the prediction of deformability is an important part of rock engineering.

The most direct way of estimating deformability is through field testing. However, for meaningful results, field tests must subject large volumes of rock to significant stress. Therefore, the tests are expensive, time consuming, and must be limited in number. To supplement direct testing and to provide estimates of deformability when field tests are impractical, other procedures have been introduced. Broadly, these derive from empirical correlations, on the one hand, or analytical decompositions, on the other. Many such procedures have been introduced. Empirical correlations attempt to statistically relate deformability to index properties, such as RQD, or to descriptive rock mass classifications. Analytical decompositions attempt to predict deformability by summing deformations over elements of the rock mass, such as intact blocks and joints. Both approaches suffer limitations. Correlations are limited by the character of the case studies from which the baseline data come. Decompositions are limited by an inability to measure and specify parameters of the models. Improvement of these techniques is needed.

5.2.1 Empirical Correlations

Possibly the best known correlation of deformability to indices or descriptions is that of Deere (1967), using RQD. However, others have been proposed, ranging from the refined descriptions of the German-Austrian school (Müller, 1963; Terzaghi, 1946), to intricate quantitative descriptions of Barton (1977) and Bieniawski (1975). Indices or descriptions are correlated:

(1) to deformability by correction factors on material properties that are easily determined (e.g., intact modulus),
(2) directly to a rock mass deformability,
(3) to design features (e.g., structural dimensions).

Obviously, correlations are based on field studies, and are limited by the geologic richness of the calibrating cases.

5.2.2 RQD - Modulus Ratio Relations

Deere's work (1967) in conjunction with co-workers was originally based on field studies at Dworshak Dam. Field plate loading tests were compared with intact modulus and RQD to arrive at an empirical correlation. All of the data are from good quality rock and the lower portion of the curve is therefore poorly defined.

A problem with these correlations is that they are based upon jacking tests of limited load and zone of influence. Further, some RQD's are obtained indirectly by correlation with seismic velocity ratios. Although RQD is often assumed to equal the velocity ratio, this is in fact only an approximation. Finally, only a limited number of tests and sites form the basis for the correlations.

5.2.2.1 Direct Correleoations between Descriptors and Rock Mass Deformability

Quantitative geologic descriptors like fracture spacing and qualitative descriptors of structural features and weathering have been related directly to the deformability observed under one or several structures. Boughton (1968), for example, produced a correlation at dam sites. Although they consider the effect of load magnitude and influence zone appropriately, such correlations are again limited due to their site specificity.

5.2.2.2 Rock Mass Classification

Many classification systems exist which directly relate structural and material properties to design consequences. These systems are particularly common in tunnel design. Terzaghi (1946), Barton (1977), and Bieniawski (1973, 1974, 1976) have all presented classification systems. Surveys to tunnel supports and related geologic conditions are generally used to derive these systems. Since the supports are in most cases overdesigned and since the amount of overdesign is not known, such classifications are not entirely satisfactory. A rock mass classification whose design consequences are not overdesigned is used in the New Austrian Tunneling Method (see e.g. John, 1977), but this classification system is highly qualitative and difficult to learn.

5.2.2.3 Analytic and Experimental Component Modelling

These approaches strive to model the mechanisms underlying rock mass deformation. Most commonly, deformation is found by combining deformation contributions from discontinuities and from intact rock. Discontinuity or joint deformability is usually expressed by stress displacement parameters k_s (shear stiffness). It can also be expressed by constitutive relations or by modeling of the interaction of asperities on joint surfaces.

Goodman (1968) developed a finite element model based upon decomposition. His model includes joint elements and stress-strain relations in the directions normal and shear (parallel) to the joint. Similar models have been developed by other authors.

A simpler but less powerful approach is to model a rock mass as a body with anisotropic deformabilities and to use closed form approaches for the analysis. This requires the assumption of orthagonally oriented discontinuities and of elastic behavior of both intact rock and discontinuities. Kulhawy (1978) used this approach together with a derivation of RQD from fracture spacing.

A comprehensive rock mass model describing general elasto-visco-plastic behavior of intact rock and a more detailed model of discontinuity behavior describing the interaction of discontinuity surfaces have been developed by Roberds and Einstein (1978).

Experimental models analogous to the analytic component models have been developed by Rosengren and Jaeger (1968) and Seeler (1978). Such experimental models can be used to determine relations between discontinuity deformability, intact rock deformability and discontinuity geometry on one hand with "rock mass deformability" on the other hand.

Although component models are quite attractive in their potentially complete description of deformability, they are limited by input parameter uncertainty. Reviews of discontinuity stiffnesses by Kulhawy (1978), Rosso (1970), and others have shown a wide spread of values even for similar rocks. Neither variation nor the distribution of joint geometry parameters are taken

Table 3 -- Typical Reliability Indices for Geotechnical Facilities
(after Meyerhof, 1976)

Facility	Typical Ω	Typical F	Typical β
Earth retaining structures	0.13±	1.3 to 1.5	2.0 to 2.5
Earthworks	0.15±	1.5 to 2.0	2.0 to 3.0
Offshore Foundations	0.20±	1.5 to 2.0	1.5 to 2.5
Onshore Foundations	0.25±	2.0 to 3.0	2.0 to 3.0

into account in the present analytical models. In addition, experimental models are limited by the same restrictions as the empirical descriptions in that only a limited number of different geometries and material properties can be examined.

Dershowitz (1979) developed a component model for rock mass deformation based on stochastic description of orientation, spacing, and stiffness of joints. This model can be made to mimic results such as Deere's, however the model requires several parameter estimates and is thus quite flexible. Later, more advanced 3D modeling efforts using stochastic geometry representations of joints have been made by Dershowitz (1984).

5.3 Slope Stability

5.3.1 Shear Failure

Shear failures are almost universally analyzed by limiting equilibrium of a volume of material bounded by potential failure surfaces. As in soil mechanics, moment equilibrium about a hypothetical point of rotation is the most common approach, being favored for its simplicity, ease of application, and past validation. Moment equilibrium is evaluated by dividing the failing mass into slices and making the assumption that stresses acting at a point on the failure surface are primarily influenced by the weight of rock lying above. Force equilibrium is analyzed on the slices and moments derived. Unless stresses in the slope are analyzed the problem is indeterminate, so simplifying assumptions must be made. This leads to several parallel methods of which the most popular is probably simplified Bishop's.

If stresses in the slope are analyzed, force or moment equilibrium can be evaluated by integrating shear stresses and resistances along potential failure surfaces, as suggested by Canmet (1977). This requires numerical modeling, but allow such factors as lateral stress ratio and non-linear stress-strain relations to affect the prediction.

Difficulties with common techniques of analyzing shear failures -- which stochastic models do nothing to solve or relieve -- are that progressive failure is ignored, stress distriubitons are usually simplified, and three-dimensional effects are not incorporated.

Random lumped parameter analyses of moment equilibrium have been applied both in rock and soil mechanics (Yuceman et al 1973; Wu and Kraft, 1970; Kim, et al., 1978). To the extent that these analyses decompose the slope into large numbers of small zones to which different properties are assigned, they approach stochastic analysis (e.g., Visca and Marek, 1978). Stochastic analysis of cohesive slopes (ϕ=0) have been performed by Morla-Catalan and Cornell (1976), Vanmarcke (1977), Veneziano, et al. (1977) and Matsuo (1976), but have only limited applicability to rock slopes. Most of these stochastic analyses are first-order second-moment approximations.

A difficulty with stochastic analysis of (c',ϕ') slopes using simplified Bishop's method is that the solution is iterative. To calculate force equilibrium on a slice the shear strength mobilizaton at the base of that slice must be known. This is usually assumed as inversely proportional to the overall factor-of-safety; thus the solution is iterative, and common techniques of probabilistic analysis cannot be used. To avoid this difficulty, Alonzo (1976) assumes the mobilization to be an independent random variable, but this seems counter to intuition. Vanmarcke (1980) has used the Ordinary Method of Slices, which resolved forces such that mobilization does not enter the analysis and thus a direct solution is obtainable. Little other work has been done on the problem. Equilibrium calculated from the stress distribution avoids the problem, but is more expensive.

The "iteration problem" is actually not the difficulty it is often made out to be, since the only real issue is how far estimated rock properties are from those that would cause failure, as measured in some unit reflecting uncertainty in those properties -- for example, the standard deviation. The criterion of failure assuming full mobilization and that assuming mobilization inversely proportional to FS converge at FS = 1.0, and their behaviors above or below this point are immaterial. The only thing that is material is the area under the pdf for $F<1.0$. Therefore, stochastic analysis using simplified Bishop's method can be performed by assuming full mobilization.

Deterministic analyses search for the minimum FS over a set of parametrically defined surfaces. This minimum is said to be the FS of the slope. Similarly, stochastic analyses search for the minimum reliabilty (β) surface, and typically take this minimum to be the reliability of the slope. The minimum FS and minimum β surfaces are not necessarily the same. Reliability depends on variances as well as means, thus a low variance (as on a long failure surface) may compensate a lower FS.

In reality, a slope may fail along any of an infinite number of surfaces. Therefore the reliability of the minimum β surface is an upper bound on overall reliability. Morla-Catalan and Cornell (1976) present a techique for addressing systems reliabilty in general shear, but the problem is mathematically difficult. Nevertheless, the β of the minimum reliability surface is a decent approximation to overall reliability, because the FS's of adjacent surfaces are strongly correlated and E[FS] usually rises sharply as surfaces deviate from the minimum.

5.3.2 Block Sliding

Block failures occur, ideally, along a fully or partially persistent discontinuity and are analyzed as simple frictional sliding (Marek & Savely, 1978; Herget, 1978). For full persistence if the angle of the discontinuity is greater than the friction angle plus some dilation term, i, the block is unstable. This simple problem again illustrates the ordinal nature of FS as an index of safety: FS is not invariant with respect to the way dilation is included except at FS = 1.0. Dilation can be added to the friction term giving Tan $(\phi+i)$ or subtracted from the discontinuity angle giving $(\alpha-i)$, but these equally defensible procedures yield different results.

Since there may be many discontinuities in a slopes, with a distribution of orientations, friction angles and dilation, an extreme value distribution is sought yielding

$$P_f = 1 - F_{FS}^{\,n}\,[FS=1.0] \qquad (55)$$

where n is the number of discontinuities (itself a r.v.), $F_{FS}[\cdot]$ is the CDF of FS for a single discontinuity, and P_f is the probability of failure (see, e.g., Call and Kim, 1978).

In practice the above procedure yields an upper bound on P_f because discontinuities may not be persistent. Failure often occurs by en echelon fracturing or sliding to produce a stepped failure surface. Call and Nicholas (1978) have approached this problem by simulating the geometry of jointing and arriving at a distribution of failure paths and volumes. This analysis considers failure to be initiated by a discontinuity at the toe of the slope and randomly generates succeeding discontinuties from an exogenous distribution of length, spacing and orientation. Glynn, Einstein, and Veneziano (1978) have approached the problem by simulating joint geometry in a regular shaped mass and finding minimum en echelon failure paths by dynamic programming. This gives a pdf over "effective persistence" which is used as a pseudo-cohesion term. A difficulty with these analyses is that multiple failure paths are not considered. Multiple paths increase the probability of failure, but the problem is complicated by correlations in resistance across paths. This is caused by spatial correlation in material and joint properties, and by shared segments. A second difficulty is that the analyses are two-dimensional, allowing no direct prediction of the influence of total length of slope.

Since friction angles and (possibly) dilation average over the failure surface, longer surfaces would have smaller coefficients of variation on resisting force than would shorter surfaces. Thus, from the view of random variation the probability of failure for small blocks near the crest -- discounting size effect due to cohesion -- would be higher than for large blocks at the toe.

5.3.3 Wedge Instability

Wedge failures along two or more discontinutieis have received extensive attention. The standard approach is to replace random variation in joint orientation by parallel joints, and to search for the least favorable wedge geometry. The orientation of each set is chosen by inspection from a stereographic projection of joint poles obtained in the joint survey. The factor of safety against slope failure is taken as the minimum of the wedges analyzed by limiting equilibrium. Recently, stress approaches have been developed based on joint stiffness (Glynn, 1979).

The difficulties with this modeling are the same as for block sliding: the criterion for choosing joint orientation is unclear and multiplicity of geometrically congruent sedges is ignored.

Several lumped parameter analyses of wedge failure have been made (Herget, 1978). These are each based on a similar procedure, whether the computations are analytical, numerical, or by simulation. A distrihution of joint poles, or lines of intersection, on some other orientation measure is calculated from the original pole diagram; the region of orientaions satisfying both kinematic and kinetic criteria of failure are identified and the density of the orientation distribution is integrated over this failure region. This gives the probabilty of failure of an individual wedge p_f. The failure probability p_f for a slope containing n independent wedges is then

$$P_f = 1 - [1-p_f]^n \tag{56}$$

Stochastic analysis of wedge failure brings issues into focus that are otherwise not considered. These are extensions of those discussed in the last seciton: the number of wedges in a slope, the mechanics of nonpersistent jointing, and the correlation of wedge resistances. Each is difficult, but important. As suggested by Veneziano (1978) wedges can be indexed by their nodes of intersection on the slope face. However, the pmf of number of nodes is difficult to calculate and does not follow common forms.

The p_f of Eqn. 56 rises rapidly with number of wedges, which does not match experience. The reason is correlaiton. Each "wedge" contains many potential failure surfaces, many of which share segments. Because their resistance depends on common random variables, they are correlated.

This also applies as nesting becomes broader. Further, joints have spatially correlated properties. Thus, the overall correlation many be very strong.

Little work has been done on this problem, given its importance. Even approximation would shed light on predictions of wedge stability and the relation between total length and expected numbers of failures.

5.4 Fracture Flow

The field of fracture flow was pioneered by Snow (1968) in his paper on flow through systems of persistent parallel plates. Recent work of Long et al. (1983), Doe et al. (1983), Smith et al. (1983), and Dershowitz (1983) involves extension of Snow's model, by introduction of statistical joint properties for orientation, persistence, and length, and intensity (joints per area or total joint length per area). These stochastic models are being used to determine questions of scale effects, diffusion, sampling criteria, statistical properties of rock mass permeability, and criteria for equivalent porous media formulations. Dershowitz and Long are currently expanding the stochastic plate model to three dimensions using the Veneziano (1979) and Baecher et al. (1977) models, respectively.

Other approaches to fracture flow include anisotropic equivalent porous media approaches (e.g., Noorishad and Mehran, 1982), stochastic porous media (e.g., Warren and Price, 1961) and further development of Snow's analytical model for persistent joints (Long, et al 1983). A novel approach to stochastic fracture flow modeling is the use of a specially developed printed circuit boards as advocated by Hudson and LaPoint (1982).

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