# DESIGN OF HIGH-THROUGHPUT HUMP YARDS 

By: K. W. GARDINER, D. W. ROSS, M. W. SIDDIQEE, and P. J. WONG

Prepared for:
SOUTHERN PACIFIC COMPANY
65 MARKET STREET
SAN FRANCISCO, CALIFORNIA

## CLIENT PRIVATE

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SRI Project 7685

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This report covers the work done during a six-month study of design of hump yards to achieve increased throughput. The basic functions of hump yard operation were analyzed and those factors that limit throughput were studied in detail. A modified yard design and freight car control policy was developed that will make possible the design of a hump yard with throughputs ranging from 300 ft to 500 ft of car/min.

An evolutionary hump yard design is outlined to permit initial operation at 300 ft of car/min with the capability of increasing this to 400 or 500 ft of car/min by the addition and relocation of the control retarders.
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## I INTRODUCTION

## A. General

The research project reported herein was directed toward exploration of ways to increase the present throughput of railroad hump yards from three or four cars per minute to six and eight cars per minute. The results of this study were to serve as a tool for not only designing new hump yards but also for modernizing existing yards. Since Southern Pacific Railroad is contemplating building a large new hump yard by 1971 in West Colton, California, the research on this project has used the proposed West Colton hump yard as the focal point of the studies. Consequently, the research on this project was scheduled for completion by June 1969, which was the date that preliminary plans and costs must be submitted to the Southern Pacific management for approval. This research was aided by the frequent interchange of ideas and the review of results of the SRI project team by Messrs. H. V. Williamson, B. Gallagher, and B. Flohr of Southern Pacific.

## B. Statement of the Problem

The basic function of a railroad hump yard is to regroup cars from incoming trains to form new outgoing trains. This is accomplished by pushing cars single file over an incline (called a hump) and switching cars to various classification tracks at the bottom of the hump. Two main problems arise in the operation of the yard, namely,
(1) Creating and maintaining sufficient separation between consecutive cars in the switching area to allow for satisfactory switching operation, and
(2) Minimizing impacts between cars when these are regrouped on the classification (bowl) tracks. Impacts between two cars with a velocity difference of $6 \mathrm{ft} / \mathrm{s}$ are considered undesirable because of the possibilities of damage to the cargo.

For purposes of the present study, a minimum headway separation of 50 ft between consecutive cars has been selected. If this separation is not achieved, the switch may not be thrown and a car switched to a wrong classification track. This problem of car separation (resulting in misswitching cars) is compounded by the fact that cars have widely different values of rolling resistance; consequently, the easy-rolling cars tend to overtake the hard-rolling cars. The spacing and velocities of the cars along the route from the crest of the hump to the classification tracks are controlled by retarders, which decelerate cars by pressing steel beams against the sides of the wheels. By properly monitoring the behavior of each car and decelerating the easy-rolling cars, cars are guided through as many as a half-dozen switches to their proper classification tracks.

Once the car has reached the beginning of the classification tracks (called the tangent point), an individual car may be required to roll as far as 3000 ft or as little as 100 ft to the end of the classification track, depending on the number of cars already on the classification track. Because of this great disparity of distances and the rollability differences of cars, the velocities of cars must be adjusted so that severe impacts (above $4 \mathrm{mi} / \mathrm{h}$ ) do not occur as cars couple on the classification tracks.
C. Acknowledgments

We wish to acknowledge the guidance and suggestions received from H. V. Williamson, B. Gallagher, and B. Flohr. Their suggestions derived from extensive experience with the operation and design of hump yards have led to many ideas presented in this report.

## II SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

## A. Summary of Findings

As the desired yard throughput increases, the time separation between cars decreases; consequently, the problems of maintaining sufficient headway between cars of varying rolling characteristics and of holding impact velocities on the classification tracks are aggravated. It was the purpose of this research to pinpoint and alleviate the bases of these impediments to high-throughput hump yard operations.

Relationship between Hump Speed and Speed in the Switching Area-One important consideration in yard operation was discovered to be the relationship of a car's hump velocity to its velocity in the switching area. In particular, for a headway of one car length, (required to avoid misswitching cars), the velocity of a hard roller in the switching area must be at least twice the hump velocity. The implications of this simple result are significant in terms of control of the cars as they roll from the hump to the classification tracks. Specifically, it is shown that retarders located between the crest of the hump and the end of the switching area should be used solely to control separation for switching; coupling speeds on the classification tracks should be controlled by retarders located after the last switch. Presently, typical hump yards use only two retarders (called master and group) to control separation and coupling speeds simultaneously. It is shown that this philosophy can severely restrict the throughput.

Factors Affecting the Grade Design--The design of the grades throughout the yard plays an important part in determining the throughput of the yard. Specifically, as the first switch is moved closer to the hump crest, the initial grade must be steepened in order to produce the desired headway in the vicinity of the first switch. Quantitative values for this relationship are given. Similarly, the grade in the switching area must ensure that even the hardest rolling cars will pass over all the switches with a velocity at least twice the hump velocity.

Placement and Control of Retarders--As a result of this study, it is proposed that three to four retarders be used along the route of the car, rather than the conventional arrangement of master and group retarders. The last retarder is placed at the tangent point of the classification tracks. It is shown that there exists a direct relationship between the hump velocity and the amount of necessary tangent point retarder and that any deviation from this amount of tangent point retarda tion will either limit the hump speed or cause excessive impact velocities on the classification tracks. Schemes for controlling the retarders are proposed. It is shown that the policy of controlling tangent point retarders is especially critical for high-throughput yards.

Design of an Expandable Yard--With an understanding of the problems of hump yards, a design is proposed for a yard which could operate initially at 6 cars $/ \mathrm{min}$ and be modified later for 8 or $10 \mathrm{cars} / \mathrm{min}$. This design is attractive because of the minimization of the initial investment capital required and because the yard can evolve in such a way as to minimize the disruption of the yard during periods of evolution. A more detailed treatment of this concept is given in Part B below.

Need for an Accurate Model of the Rolling Resistance of Cars--The worst-case situation in hump yard operation is when the easiest rolling car follows the hardest rolling car down to the farthest switch before separating. Detailed analysis of this case requires an accurate rolling resistance model and knowledge of the variability in rolling resistance. Because of the importance of an accurate rolling resistance model, several field test results available to Southern Pacific were studied by the SRI team in conjunction with Mr. Barney Gallagher. It was concluded that a rolling resistance model with a static term (Coulomb friction) and a velocity dependent term (viscous friction) was appropriate. It was also concluded that two ranges of rolling resistance should be used, one range in the switching area, and another range in the bowl track area. This accounts for the fact that cars become better lubricated and consequently roll easier during the latter part of their run through the
yard. This model choice was based upon the limited experimental data available to Southern Pacific; more extensive experimental data should be gathered in the future as recommended in Sec. D below.

## B. Yard Designs for Throughputs of 6, 8, and $10 \mathrm{cars} / \mathrm{min}$ and Discussion of Expandable Yard Design

It is shown in later sections of this report that the design value of yard throughput can be directly related to the grades required in all parts of the yard, the size of retarders required between hump crest, and the size of the tangent point retarders. These requirements are summarized in the following paragraphs for throughputs of 6, 8, and 10 cars/min (assuming an average car length of 50 ft ). Subsequent paragraphs describe construction of a yard such that the throughput of the yard is expandable, i.e., the yard can be built with the capability of a higher throughput than initial operating capacity.

Before summarizing the salient points of the West colton yard design for different throughputs, we should define "throughput" so that the conclusions will not be misinterpreted. First, an average car length of 50 ft is assumed; hence, $6 \mathrm{cars} / \mathrm{min}$ is equivalent to about 300 ft of hump feed per minute, 8 cars $/ \mathrm{min}$ is equivalent to about $400 \mathrm{ft} / \mathrm{min}$, and 10 cars $/ \mathrm{min}$ is equivalent to about $500 \mathrm{ft} / \mathrm{min}$. If the average car length increases in the future (as is the trend), the designs discussed below will still permit throughputs of 300 ft of hump feed per minute, $400 \mathrm{ft} / \mathrm{min}$, and $500 \mathrm{ft} / \mathrm{min}$, although the throughput in cars per minute will be slightly less than 6,8 , and 10 cars/min respectively.

Grade Designs--The design principles for the grades in the yard are discussed in Secs. V and VI. The analyses in those sections show that a yard throughput of 6 cars/min requires an (uncompensated) vertical drop from crest to tangent point of about 18 ft ; for a throughput of 8 cars/min it is about 21 ft , and for $10 \mathrm{cars} / \mathrm{min}$ about 24 ft . The bowl track grade is independent of throughput and should be about a 0.06 percent grade, which implies a vertical drop of about 2 ft along the bowl tracks. The guiding principles for the detailed grade profile designs for these vertical drops are discussed in Secs. IV and V.

Retardation in the Front End of the Yard--The estimated total frontend retarder length for throughputs of 6,8 , and 10 cars $/ \mathrm{min}$ is about 700 , 800 , and 980 ft of retarder, respectively. These estimates are based upon the assumption that the retarders extract about 0.0715 ft of velocity head per foot length (a typical figure). These estimates are also based upon the assumption that the yard is not built to be expandable to a higher throughput at some future time. The case of an expandable yard is discussed below.

Tangent Point Retarders--The total length of tangent point retarders for the 80 bowl tracks at West Colton increases with increasing throughput. At a throughput of $6 \mathrm{cars} / \mathrm{min}$, a total of about 2320 ft (29 ft per bowl track) of tangent point retarders is required. This retardation must be increased about 90 percent for a throughput of $8 \mathrm{cars} / \mathrm{min}$. To achieve a throughput of $10 \mathrm{cars} / \mathrm{min}$ after the yard has been modified to operate at 8 cars/min requires an additional 40 percent of tangent point retardation.

In addition to adding retarders for each increase in yard throughput, the tangent point retarders should be sectionalized and the sections should be spread apart. The details are discussed in Sec. VI and VIII. Table III in Sec. VIII summarizes the approximate total retarder lengths required at different stages of the West Colton yard during its transition from a low-throughput yard to a high-throughput yard.

Expandable Yard Design--For a yard with initial low throughput to be increased in future years, it is suggested that the yard grades be selected on the basis of the eventual higher throughput. During the initial low-throughput period, the front-end yard can be made to behave as a shallow-grade yard by placing some extra retarders in the front end so that the speed levels of the cars in the switching area correspond to the speeds of a yard with shallow grades and lower hump velocities. The additional length of these extra retarders in the switching area is quite small: To make a yard, whose grades are designed for 8 cars/min, suitable for initial operation at $6 \mathrm{cars} / \mathrm{min}$, only about 350 ft of additional retarders are required in the switching area. This represents less than 10 percent of the total expense for retarders in the initial operation.

As the throughput increases over the years, the extra retarders in the switching area may be gradually removed (or made inoperative) so that the velocity levels in the switching area increase in relation to increased hump velocity. Additional bowl track retarders may then be added in stages to extract the increased tangent point entrance speeds. Thus the yard can be gradually evolved from a low-throughput to a highthroughput yard without any major structural change.

Table III, in Sec. VIII, summarizes the approximate total retarder lengths and vertical drops required at different stages of the West Colton yard during its evolution from a low-throughput yard to a highthroughput yard.

## C. Conclusions

It has been pointed out that high-throughput yards can be designed and that the basic philosophy is to allow cars to travel through the switching area with velocities of at least twice the hump speed and to use tangent point retarders to bring the cars down to the proper coupling speeds on the classification tracks. Although the extra cost of retarders used in high-throughput yards will be several times greater than in conventional yards, the total capital cost of a high-throughput yard will be only moderately greater than the cost of conventional yards. The important factor is the ratio of the increase in throughput to the increase in total yard cost.

It was found that the factors that limit the throughput include:
(1) Grade profiles that cannot give the hard roller a velocity of twice the hump speed in the switching area.
(2) A control policy in the switching area that slows cars velocities in the switching area below twice the hump speed.
(3) An initial grade that is too shallow with respect to the location of the first switch.
(4) An inadequate design and control of tangent point retarders, which can cause a succession of collisions (cars piling up) at the tangent point.

With a careful understanding of these considerations, it is possible to design an evolving yard with an initial throughput of 6 cars $/ \mathrm{min}$ to eventually have a throughput of 8 or 10 cars $/ \mathrm{min}$.

An examination of the above design considerations indicates that they fall into two categories: grades and retarders. To allow a yard to evolve, it is proposed to design the grades to handle the ultimate desired throughput and to use retarders to modify the effective grade profile for the initial lower throughput. As more throughput is desired, the retarders can be added and repositioned as appropriate to correspond to the higher throughput. This procedure allows the modifications to take place with the minimum disturbance of yard operations since the yard does not have to be regraded and the retarders can be added on one track at a time. Also a more efficient allocation of investment capital results without restricting the future needs of the yard.
D. Recommendations for Further Research
(1) The crucial factor in yard design are the values of rollability for the hardest and easiest rollers. These values are a function of velocity and with the distance a car has rolled from the hump crest. A more thorough quantitative understanding of the rollability model would make possible improved yard designs.
(2) A method of improving present rollability measurements is discussed in Sec. IV-C of this report. The basic idea is to utilize all relevant information concerning a car's rolling behavior through the switching section of the yard to improve the estimate of the car's rollability on the bowl tracks.
(3) The understanding of the key design factors for highthroughput hump yards should be applied to evaluate the potential improvements that can be made in the throughput of existing yards. As was pointed out in this report, a major limitation in throughput is the present placement,
utilization, and control of retarders. By adding tangent point retarders to existing yards and changing the policy of operating the retarders in the switching area, a significant increase in the throughput can be achieved. Since this would not require a new track layout or new grades, hump yards can be modernized with minimum down time.
(4) The assignment of classification tracks and the make-up information of cars going over the hump clearly affect the hump speed. The magnitude and extent to which these considerations affect the hump speed are as yet not completely defined. A detailed study of how the entire classification logic is to be defined for the most efficient overall hump yard operation is recommended for the next phase of the study.

## III ROLLING RESISTANCE OF FREIGHT CARS

## A. Introduction.

In order to design a hump yard properly, it is essential to know the range of values of the rolling resistance associated with a hardrolling car at one extreme and an easy-rolling car at the other extreme. It is this variability that introduces some of the main problems of yard design. For example, easy-rolling cars tend to overtake the hardrolling cars, which introduces a headway-control problem in the switching area. On the bowl tracks, the release velocities of the tangent point retarders must be selected in accordance with the rolling characteristic of each car to avoid undesirable impacts between cars. This requires the measurement of car rolling resistance as well as a suitable control scheme for the tangent point retarders. In this section, the model of rolling resistance that was used for the West Colton design studies is discussed. The experimental data from which this model was developed are also discussed. The latter part of this section discusses the measurements of the car resistance.

## B. Experimental Data and Choice of a Model

In order to design a hump yard quantitatively, a model for car rolling resistance as well as the range of the model parameters is needed. Many different sets of qualitative terminology are used in the literature for rolling resistance, among them "rollability," "car friction," and "car resistance." In this report we have used either "rollability" or "car rolling resistance" to refer to a specific quantitative model. The model that is used was developed after the review of several past experiments. Specifically, the model chosen is composed of two friction terms: a Coulomb friction term and a viscous friction term. Thus, the equation of motion of a freight car rolling on a grade of $\theta$ radians has been assumed to be:

$$
\begin{equation*}
\dot{\mathrm{V}}=\theta \mathrm{g}-\mu \mathrm{g}-\mathrm{KgV} \tag{1}
\end{equation*}
$$

where

$$
\begin{aligned}
& \mathrm{V}=\text { car velocity }(\mathrm{ft} / \mathrm{s}) \\
& \theta=\text { grade in radians } \\
& \dot{\mathrm{V}}=\text { car acceleration }\left(\mathrm{ft} / \mathrm{s}^{2}\right) \\
& \mu=\text { coefficient of Coulomb friction (dimensionless) } \\
& \mathrm{K}=\text { coefficient of viscous friction }\left(\mathrm{s}^{-1}\right) \\
& \mathrm{g}=32.16 \mathrm{ft} / \mathrm{s}^{2}
\end{aligned}
$$

Note: Strictly speaking, the term $\theta \mathrm{g}$ should be $\mathrm{g} \sin \theta$, however for small values of $\theta$ under consideration, $\sin \theta \simeq \theta$.

For our studies the choice of a model for car rolling resistance and the choice of coefficients for this model that corresponded to hard, medium, and easy rollers was made after reviewing the results of two previous series of experiments. The two sets of experiments were:
(1) Experiments conducted in 1956-57 by R. M. Hermes of Stanford Research Institute, and
(2) Experiments conducted in 1960 by representatives of Union Switch and Signal Co. and Cotton Belt Railway.

## 1. Hermes Experiments

In 1956-57, R. M. Hermes of the Stanford Research Institute Control Systems Laboratory conducted tests at the Santa Clara and Roseville yards of Southern Pacific Company to determine the rolling resistance of freight cars. His conclusion was that, for car speeds above approximately $5 \mathrm{ft} / \mathrm{s}$ (about $3-1 / 2 \mathrm{ml} / \mathrm{h}$ ) and car speeds in the normal hump yard range of about 5 to $20 \mathrm{ft} / \mathrm{s}$, the car rolling resistance is essentially viscous friction, i.e. the car rolling resistance is approximately proportional to velocity:

$$
\begin{equation*}
R \cong B V \tag{2}
\end{equation*}
$$

where $V$ is the car velocity and $B$ is a constant of proportionality.
At Roseville, freight cars were observed as they rolled over several calibration sections, each located on a different track. The grades of each section were measured. The sections were about 250 ft long. A total of about 700 runs of about 20 cars (including both
journal-bearing and roller-bearing types) over the calibration sections were made. For each run, the velocity of the car was measured at the entrance to and exit from the calibration section and the travel time between these two points was measured. Car speeds over these sections ranged from about 10 to $20 \mathrm{ft} / \mathrm{s}$ (about 7 to $14 \mathrm{mi} / \mathrm{h}$ ). In this range of car speeds, Hermes found the rolling resistance of cars to be as indicated in Table I. These ranges of car rolling resistance are also shown in Fig. l.

Table I
CAR RESISTANCE ( $1 \mathrm{~b} /$ ton) BASED ON HERMES' TESTS*

|  | at $7 \mathrm{mi} / \mathrm{hr}$ | at $10 \mathrm{mi} / \mathrm{hr}$ | at $14 \mathrm{mi} / \mathrm{hr}$ |
| :--- | :---: | :---: | :---: |
| Average Car | 4.48 | 6.40 | 8.95 |
| Hard Roller | 9.60 | 13.70 | 19.20 |
| Easy Roller | 0.63 | 0.90 | 1.26 |

[^0]
## 2. Pine Bluff Tests

In the Spring of 1960 , tests were conducted at the Pine Bluff, Arkansas yard of the Cotton Belt Railway by the representatives of the Union Switch and Signal Co. and the Cotton Belt Railway. The purpose of the tests was to determine the improvement in coupling speeds that would be obtained through application of a test computer based on the viscous friction model developed by Hermes.

In the tests, cars of all weight categories and both journalbearing and roller-bearing types were released onto two bowl tracks having a fairly uniform 0.08 percent grade with good cross level. On each of the two bowl tracks, the cars were observed as they rolled over a calibration section where the tangent track rolling resistance measurement (in lb/ton) was measured. The velocity of cars at these calibration sections ranged between 8 and $10 \mathrm{mi} / \mathrm{h}$. Additional sections on the


FIGURE 1 RESULTS OF HERMES AND PINE BLUFF TESTS
bowl tracks were observed and stop watch measurements of car velocity were taken on these sections. About 543 cars were involved in the tests. Experimental values of rolling resistance found in the Pine Bluff tests have been tabulated by B. Gallager and are displayed in Fig. 1.
3. Analysis of the Data

After a detailed study of the results of the above noted two independent tests studies, B. Gallagher of Southern Pacific and the SRI team concluded that a suitable model for car resistance would be:

$$
\begin{equation*}
R=A+B V \tag{3}
\end{equation*}
$$

where $A$ and $B$ are constants and $V$ is the car velocity.
In relation to this model, it was noted by B. Gallagher that cars tended to roll more freely as they traversed more and more distance in the yard. The explanation for this behavior seems to be the improvement in lubrication caused by the heat generated by the motion of the car. In view of these observations, it was suggested by B. Gallagher that two sets of the coefficients A and B be used to represent a car's resistance; one set for the crest side of the group retarders and one set for the bowl-track side of the group retarders. Following this suggestion, the limits of car resistance corresponding to hard- and easy-rolling cars in both sections of a yard were found from the Pine Bluff data and are shown in Fig. 2. It can be seen that the Pine Bluff limits of hard and easy rolling cars compare well with the Hermes results in the velocity range of the Hermes experiments. The curves of rolling resistance as a function of car velocity shown in Fig. $\bar{Y}$ were used for designing the grades in the yard and retarder control schemes in the switching area as well as in the bowl tracks.

In terms of Eq. (1) the parameters $\mu \mathrm{g}$ and Kg corresponding to the curves given in Fig. 2 are shown in Table II.


FIGURE 2 FREIGHT CAR RESISTANCE MODEL USED FOR YARD DESIGN

Table II
RANGE OF $\mu \mathrm{g}$ AND Kg USED FOR YARD DESIGN

| Car <br> Characteristic | Crest Side of Group <br> Retarders |  | Bowl Side of Group <br> Retarders |  |
| :---: | :--- | :---: | :---: | :---: |
|  | $\mu \mathrm{g}$ <br> $\left(\mathrm{ft} / \mathrm{s}^{2}\right)$ | Kg <br> $\left(\mathrm{s}^{-1}\right)$ | $\mu \mathrm{g}$ <br> $\left(\mathrm{ft} / \mathrm{s}^{2}\right)$ | Kg <br> $\left(\mathrm{s}^{-1}\right)$ |
| Hard Roller | 0.0610 | 0.0150 | 0.0322 | 0.0113 |
| Easy Roller | 0.00643 | 0.00350 | 0.00643 | 0.00206 |

## C. An Improved Scheme for Estimating Car Rolling Resistance

For satisfactory operation of the yard it is necessary to know the rolling characteristic of each car as it moves down the hump in order that appropriate control be exerted on it by various retarders. As discussed later, the retarders in the switching area could suitably be controlled as a function of entrance velocity of the car so that advance information about car rolling resistance is not necessary for the retarders in the switching area. However, to control the let out velocity at the tangent point retarder properly, it is necessary to know the rolling resistance of the car and also the available unoccupied length on the bowl track. The conventional method of measuring the rolling resistance of cars by an acceleration measurement section placed somewhere near the hump has often proved to be unsatisfactory because of inaccuracies in the measurements and also because the rolling characteristic of a car changes as it moves in the yard. Since it would be economically infeasible to place an acceleration measurement section on each bowl track, a scheme is presented that deals with the possibility of improving one's estimate of a car's rolling resistance at the tangent point by taking the data from the standard acceleration measurement sections and combining it with data on the car's rolling behavior throughout the switching area. The motivation is based on the simple intuitive notion (which can be verified mathematically) that measurement errors are reduced as the number of independent measurements increases. This assumes, of course, that the additional measurements are properly processed. An example of this simple notion is that to estimate the
value of a quantity accurately, one can take many independent measurements and then take the average value of these measurements as the estimate.

A single standard acceleration measurement section can not determine both of the parameters $\mu$ and $K$ given in Eq. (1). What a measurement section can determine is a lumped value of rolling resistance $R$ at the average velocity in the measurement section, i.e.

$$
\begin{equation*}
R=2000\left(\mu+K V_{\text {ave }}\right) 1 \mathrm{~b} / \text { ton } \tag{4}
\end{equation*}
$$

Present hump yards measure rolling resistance at a nominal value of velocity on one or several acceleration measurement section early in the car's run. It is proposed to supplement this estimate of the car's rolling resistance by observing the behavior of the car throughout its run from crest to tangent point.

In the suggested design of high-throughput hump yards, a car may roll through as many as three or four retarders before reaching its final destination. Since the velocity of a car at the exit of each retarder and at the entrance to the next retarder is available, it is possible to process this information to obtain additional information on the car's rolling resistance. Consider the following simple example. Let $\mu \mathrm{g}$ and Kg be the rolling resistance parameters of a car, which are to be determined. Let $R_{0}$ be the value of resistance determined on the acceleration measurement section and $R_{1}, R_{2}$, and $R_{3}$ be the value of the composite resistance determined by using the velocity information at each retarder. Then we can let

$$
\begin{aligned}
& \mu g=a_{0} R_{0}+a_{1} R_{1}+a_{2} R_{2}+a_{3} R_{3} \\
& K g=b_{0} R_{0}+b_{1} R_{1}+b_{2} R_{2}+b_{3} R_{3}
\end{aligned}
$$

where the $a_{i}{ }^{\prime} s$ and $b_{i}$ 's are as yet undetermined coefficients. When the yard is built, controlled tests can be used to determine the coefficient $a_{i}$ and $b_{i}$. In particular, one can set up temporary instrumentation to
measure $\mu \mathrm{g}$ and Kg on each bowl track for a series of test cars. Since $\mu \mathrm{g}, \mathrm{Kg}, \mathrm{R}_{0}, \mathrm{R}_{1}, \mathrm{R}_{2}$, and $\mathrm{R}_{3}$ are known for these test cars, the coefficients $a_{i}$ and $b_{i}$ can be determined using a least-squares fit. Using this procedure, such questions as the accuracy of grades and curved resistance versus tangent resistance are inherently accounted for in the process of fitting a curve to the data.
D. Summary

Our study of experimental data has led us to develop a model for car rolling resistance that combines Coulomb friction and viscous friction. The extremes of resistance corresponding to a hard roller and an easy roller have been established from existing experimental data and are summarized in Fig. I and Table II. Also, it has been proposed that velocity information that is available throughout a car's run be used to obtain an improved estimate of what a car's rolling resistance will be beyond the tangent point. This suggestion is motivated by the fact that a car's rolling resistance changes throughout its run.

## IV MAIN PRINCIPLE OF YARD DESIGN

## A. Introduction

This section discusses the main principle in controlling cars in hump yards to maximize throughput. In the course of this discussion it is shown that, in order to avoid misswitching cars, the minimum velocity in the switching area must be greater than twice the hump speed. Based on this fact, it is shown that the retarders in the switching area should be used solely to control separation of cars, whereas the tangent point retarders are used solely to decelerate cars down to the proper coupling speeds on the classification tracks. Thus, the present practice of using master and group retarders to control both separation and coupling speed is unsatisfactory and can result in restricted throughput.

## B. Minimum Velocity in the Switching Area

In this section, it is shown that, for a desired headway of 50 ft between typical $50-f t$ long cars, the velocity in the switching area must be at least twice the hump speed.

Assume that two identical cars of length $L$, having the same rolling resistance are pushed over the hump with velocity $V_{H}$ and follow each other all the way to the last switch.

Figure 3(a) shows two cars passing over the hump crest with a hump velocity $V_{H}$. The hump throughput in cars/min is:

$$
\begin{equation*}
\text { hump throughput }=\mathrm{L} / \mathrm{V}_{\mathrm{H}} \tag{5}
\end{equation*}
$$

This throughput must remain constant throughout the switching area. If the throughput in this area were less than the hump throughput, this would mean that more cars/min were entering the switching area than were leaving. This will eventually create collision and back-up problems. Likewise, if the throughput in the switching area were greater


FIGURE 3 CARS AT HUMP AND SWITCH LOCATION
than the hump throughput, more cars/min would be leaving the switching area than were entering it, and this is impossible! Therefore:

$$
\begin{align*}
& \text { Switching Area }=\text { Hump Throughput }=\mathrm{L} / \mathrm{V}_{\mathrm{H}}  \tag{6}\\
& \text { Throughput }
\end{align*}
$$

In the switching area, cars are spaced as shown in Fig. 3(b); that is, instead of being bumper to bumper, the cars have gained speed and some headway, h, exists between cars. The size of this headway is dependent upon the speed, $V_{s}$, of cars in the switching area and the throughput. Specifically,

$$
\begin{equation*}
\frac{\mathrm{L}+\mathrm{h}}{\mathrm{~V}_{\mathrm{S}}}=\underset{\text { Throughput }}{\text { Switching Area }} \tag{7}
\end{equation*}
$$

which implies

$$
\begin{equation*}
\mathrm{h}=\text { (Switching Area Throughput) } \mathrm{V}_{\mathrm{S}}-\mathrm{L} \tag{8}
\end{equation*}
$$

Upon combining Eqs. (6) and (8), we see that

$$
\begin{equation*}
h=\left(\frac{L}{V_{H}}\right) V_{S}-L=L \frac{\left(V_{S}-V_{H}\right)}{V_{H}} \tag{9}
\end{equation*}
$$

Now, to prevent misswitching, the headway between cars should be about a car-length, that is:

$$
\mathrm{h} \cong \mathrm{~L}
$$

Therefore, substituting $R=L$ into Eq. (9), we find that $\left(V_{S}-V_{H}\right) / V_{H} \cong$ 1.0, implying $V_{S}=2 \mathrm{~V}_{\mathrm{H}}$. Thus the speed of the cars in the switching area should be at least twice the hump speed, $V_{H}$.

Looking at the problem from the reverse viewpoint, the minimum velocity in the switching area is a fundamental restriction on throughput. In particular, the hump velocity is restricted to be approximately half of the minimum velocity in the switching area.
C. Dichotomy between Front End and Back End of the Yard

Retarders in hump yards have two basic functions:
(1) Control of separation for switching, and
(2) Reduction in velocity-head to avoid collisions in the classification tracks. However, if the grades in the front end have been overdesigned to give the hardest rolling car a velocity greater than $2 \cdot V_{H}$, then the retarders--in addition to controlling separation--will be required to compensate for the excess grade. In summary, as shown in Fig. 4, the front end of the yard must deliver to the back end of the yard cars traveling at $2 \cdot V_{H}$ to avoid misswitches.


FIGURE 4 SEPARATION OF YARD FUNCTIONS

The retarders at the tangent point must be capable of taking these cars traveling at a speed of about $2 \cdot V_{H}$ down to the maximum coupling speed $V_{C}$. The amount of retardation to accomplish this in terms of velocity head is

$$
\begin{equation*}
\text { Retarder Velocity Head }=\frac{\left(2 \cdot \mathrm{~V}_{\mathrm{H}}\right)^{2}-\mathrm{V}_{\mathrm{C}}^{2}}{2 \cdot \mathrm{~g}} \tag{10}
\end{equation*}
$$

where $g=32.16 \mathrm{ft} / \mathrm{s}^{2}$. The only variables in Eq. (10) are the hump speed and the maximum coupling velocity. If a hump speed of $7 \mathrm{ft} / \mathrm{s}$ is desired and the coupling speed of $6 \mathrm{ft} / \mathrm{s}$ is specified, then the amount of retardation power at the tangent point is fixed by Eq. (10). In order to reduce the amount of retarder power required at the tangent point, one must either reduce the desired hump speed $\mathrm{V}_{\mathrm{H}}$ or increase the maximum coupling speed $\mathrm{V}_{\mathrm{C}}$. This means that either the yard throughput must be decreased or higher coupling speeds must be allowed--both undesirable options. It is to be noted from Eq. (10) that if the desired
hump speed $V_{H}$ is less than half of the maximum coupling speed (i.e., $\mathrm{V}_{\mathrm{H}}<I / 2 \mathrm{~V}_{\mathrm{C}}$, then no tangent point retarders are required.

Current placement and operation of the master and group retarders do not separate these functions. The release velocity of present-day group retarders are designed to create separation and also to ensure that coupling speeds in the classification area are small. The requirement that the group retarders control coupling speeds may limit the velocity in the switching area following the group retarder to below $2 \cdot \mathrm{~V}_{\mathrm{H}}$. Consequently, according to the previous discussion, the hump velocity may be severely restricted by this design and control philosophy.

## D. Summary

Two very important factors affecting the throughput of the yard have been pointed out. Firstly, it was shown that the hump speed is restricted by the maximum allowable speed in the switching area. In particular it was pointed out that, for a desired headway of one car length, the speed in the switching area must be at least twice the hump speed, implying that the maximum hump speed cannot exceed one-half of the maximum allowable speed in the switching area. Secondly, it was shown that the conventional practice of using the master and group retarder for controlling both headway and buffing speed is unsatisfactory and is a major source in restricting the throughput of the yard. It is shown that for high throughputs, the retarders in the switching area should be used only for headway control and the buffing speed should be controlled separately by tangent point retarders.

## V DESIGN OF THE FRONT END OF THE YARD

## A. Introduction

In order to avoid misswitches, it is necessary to maintain a certain minimum headway between two consecutive cars traveling in the switching area. The necessary headway is produced by a proper initial steep grade and is then maintained (within acceptable limits) by an appropriate combination of grade profile and retarders, which are placed at suitable locations in the switching area. The required minimum headway depends on the average velocity of the cars during the switching period as well as the minimum time needed for the switch to change over from one position to the other after the lead car has passed through it. For average speeds of 20 to $25 \mathrm{ft} / \mathrm{s}$ in the switching area, a minimum of 35 ft headway between the rear end of the preceding car and the front end of the following car is an absolute necessity for satisfactory switching. However, in order to compensate for the uncertainties in the rolling characteristics of the cars, errors in sensing the speed, and operation of the retarders, etc., a minimum required headway of 50 ft has been selected for the analysis and design of the headway control scheme.

In this section, the principles of grade design, selection of the size and location of retarders and their control policy are discussed in detail. Various aspects of the general principles are elaborated with examples.

## B. Grade Design

The grades from the crest of the hump to the last switch must be designed to give sufficient separation at switch points based on the worst case condition, viz., the hardest roller followed by the easiest roller traveling together to the last switch. In the following, the considerations for designing the initial, intermediate, and final grade profiles in the front end of the hump yard are discussed in detail (see Fig. 5).


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FIGURE 5 GRADES FROM THE CREST TO THE LAST SWITCH

## Initial Front-End Grade

The determining factor in the design of the initial grade is the location of the first switch. For a given hump speed and initial grade, there is an optimum location for the first switch in terms of obtaining maximum separation at the switch: Locating the first switch either closer or further from the hump crest will restrict the hump speed. Also, for a given location of the first switch, any increase in hump speed requires that the initial grade be steepened in order for cars to clear the first switch. Similarly, if the first switch location is moved away from the optimum location--either toward or away from the hump--the initial grade must be steepened.

Figure 6 is a plot of the maximum allowed hump velocity as a function of the location of the first switch on grades of 3 , 4 , and 5 percent. A static rollability model was used in this study, since data from Union Switch and Signal (transmitted via Barney Gallagher) tend to indicate the rolling resistance is a constant during the first

several hundred feet of a car's run. * The plots are based on the hardest rolling car being followed by the easiest rolling car achieving 50-ft. separation at the first switch with no retardation on the hump. With a master retarder on the hump, the initial grade can be more shallow. The following two worst-case rollability spreads were assumed:

Case 1: Hard Roller $=30 \mathrm{lb} /$ ton; easy roller $=0 \mathrm{lb} /$ ton
Case 2: Hard Roller $=28 \mathrm{lb} /$ ton; easy roller $=2 \mathrm{lb} / \mathrm{ton}$
Figure 6 indicates that for a given grade, the maximum hump velocity deteriorates quickly as one moves the first switch from the optimum location toward the hump crest and not as quickly as one moves from the optimum location away from the hump crest. From a practical standpoint, if one cannot place the first switch at the optimum, it is generally desirable to place the first switch further from the hump crest than closer to the hump crest. The distance one holds the initial grade is dictated by such consideration as the maximum allowed velocity and the desired height of the hump.

## 2. Intermediate Front-End Grade

An intermediate grade is chosen to maintain the velocities and separation achieved at the end of initial grade past the first switch and into the long lead. This grade should be designed to keep car velocities below the maximum allowed velocity.

## 3. Final Front-End Grade

As discussed in Sec. III, the minimum velocity allowed in the switching area is twice the hump speed. Consequently, the final grade in the switching area must be designed to give the hardest rolling car a velocity of about twice the hump speed until the last switch is passed.

[^1]
## 4. Curve Compensation

Since the initial track branches out into several bowl tracks (e.g., in case of West Colton, the initial single track at the hump ultimately branches out into about 80 bowl tracks), there will always be tracks that will have some curved portions. A car traveling on a curved track experiences centrifugal frictional forces in addition to rolling friction. Thus, to obtain the effect of, say, a grade of 3 percent on a curved portion, the actual grade must be made slightly more than 3 percent to compensate for the additional frictional forces. An empirical rule in this regard as given by $B$. Gallagher is to add 0.04 ft extra head per degree of central angle, where the central angle is the angle between perpendiculars erected at the extremities of the curved portion as shown in Fig. 7.


FIGURE 7 DEFINITION OF CENTRAL ANGLE

For tracks in which there are some curved portions, additional head using the above noted empirical rule must be added to compensate for the additional frictional forces.

## C. Selection of Sizes and Location of Retarders

From an operational point of view, it would be desirable to place several small retarders at short intervals throughout the switching area to extract continuously the extra kinetic energy gained by easy rollers so that the velocity profile of the controlled easy roller will be almost identical to that of a hard roller. This will result in a uniform constant headway. However, this would not only be uneconomical but
impractical since the presence of various switches at various points in the track as well as the existence of curved portions of the track impose some limitations on the placement of the retarders. A clearance of about 18 ft on each side of a switch is necessary from mechanical and structural considerations. Under certain circumstances it may not be desirable to place the retarders in the curved portions of the track, Thus only certain definite portions of the track are available for placing the retarders. Keeping these constraints and considerations in view, a suitable approach would be to select the size, location and control policy of the retarders so that the resulting controlled velocity profile of the easy rollers intertwines the profile of a hard roller as shown roughly in Fig. 8.


FIGURE 8 PROPOSED CONTROLLED-VELOCITY PROFILE FOR EASY ROLLER

A few trial studies can easily be made by studying the velocity and travelled distance profiles of a hard roller followed by an easy roller, since this represents the worst case as far as closing up the headway is concerned. If the easy roller is allowed to roll freely, it will tend to catch up with the hard roller within a short distance. The first retarder should therefore be placed in the vicinity where
the headway is first indicated to be getting smaller than the desired value. The velocity of the easy roller at this point should be reduced by a certain amount through retarder action. (To establish a suitable value for the reduced speed, a simple control policy is proposed in the next section.) The necessary size and location of the second, third, etc. retarder can similarly be established through further computation of distance and velocity profiles.

## D. A Proposed Control Scheme for the Retarders

After selecting tentatively the location of retarders, the next step is to use a suitable control scheme for the retarders. A simple control law of the form of Eq. (11) was tried and a few test studies gave quite encouraging results.

$$
\begin{equation*}
V_{E}=V_{0}-K_{1}\left(V_{i}-V_{b}\right) \tag{11}
\end{equation*}
$$

where $\quad V_{0}=$ Velocity of the hardest roller at the point it leaves the retarder
$V_{b}=$ Velocity of the hardest roller at the point it enters the retarder
$V_{i}=$ Velocity of any other car at the entrance to retarder
$V_{E}=$ Controlled exit velocity of the other car
$K_{1}=A$ suitable constant to be chosen by trial.
Note that if $V_{i}=V_{b}$, it implies that the incoming car is a hard roller, so

$$
V_{i}-V_{b}=0 \text { and } V_{E}=V_{0}=\text { exit velocity of a hard roller, }
$$

which is what it should be.
It is felt that, with suitable values for $K$ to be found by a few trials, the above control law would be a simple and convenient form of control. The implementation of this law requires the measurement of car speed at the entrance of the retarder. Using this information, the necessary exit velocity of the car can be calculated quickly by the
computer using Eq. (11) and the retarder may be instructed to reduce the velocity to the value found by Eq. (11). The sensing and communication requirements are thus not very elaborate. In this connection it is to be mentioned that easy rollers can be differentiated from hard rollers by several methods. Two common methods are velocity measurements and travel time measurements. Whereas there is no difficulty in modifying the proposed control law so as to utilize time measurements instead of velocity measurements, we feel that control based on velocity measurements is preferable for the following reasons:
(1) Velocity measurement is already necessary for proper operation of retarders to achieve desired exit velocities.
(2) Time separation may vary for reasons other than variations in rollability, e.g., delays in disconnecting the car on the hump. The velocity at the entrance point to a retarder is not dependent on the time reference and is not very sensitive to initial hump velocity because the velocity profile in the yard is essentially governed by the grades and the initial transient dies out within 100 ft from the hump. Therefore, velocity seems to be a better criterion to distinguish a good roller from a bad roller.
(3) The control algorithm based on velocity measurement is quite simple.

## E. An Example

The above-noted general approach is now illustrated with a specific example wherein we consider some specific values for switch location, hump speed, car length, desired headway, etc. These assumed values are used only for convenience and to clarify various ideas. The same general approach is applicable for other similar hump speeds, desired headway, etc.

1. Assumptions
(1) Assume that, for a car rolling freely on a grade, the motion is described by

$$
\begin{equation*}
\dot{\mathrm{V}}=\theta \mathrm{g}-\mu \mathrm{g}-\mathrm{Kg} \mathrm{~V} \tag{12}
\end{equation*}
$$

where $V=$ Speed of the car in $\mathrm{ft} / \mathrm{s}$
$\theta=$ Angle of the grade in radians $\mathrm{g}=32.2 \mathrm{ft} / \mathrm{s}^{2}$
$\mu \mathrm{g}$ and Kg have been assumed to have the following values based on various test results:

$$
\begin{array}{lll}
\text { Hardest Roller: } & \mu \mathrm{g}=0.0611 ; & \mathrm{Kg}=0.015 \\
\text { Average Roller: } & \mu \mathrm{g}=0.0257 ; & \mathrm{Kg}=0.008 \\
\text { Easy Roller: } & \mu \mathrm{g}=0.00643 ; & \mathrm{Kg}=0.0035
\end{array}
$$

(2) Car lengths are assumed to be 50 ft .
(3) Minimum desired headway between centers of the cars is taken as 100 ft .
(4) Assume a humping speed of $7 \mathrm{ft} / \mathrm{s}$.
(5) The first switch is located somewhere between $350-450 \mathrm{ft}$ from the hump. The second, third, etc, switches are located in the area $1000-1500 \mathrm{ft}$ from the hump. It is not easily possible to place any retarders in the section from 1000-1500 ft because of several switches in this section.
2. Design of Grades
a. Initial and Intermediate Grades

Considering the case of an easy roller followed by a hard roller, it was found after a few trials that an initial grade of at least 3 percent for about 200 ft followed by a grade of 2 percent for the next $250-300 \mathrm{ft}$ is necessary to produce a satisfactory headway of

100 ft between the distance 350 ft to 450 ft from the hump where the first switch might be located. This, however, is not a unique grade profile but indicates the minimum initial grade necessary for the given location of the first switch and the given velocity at the hump.

Other combinations--e.g., an initial grade of 5 percent for 200 ft and then a grade of 1.5 percent for the next $200-250 \mathrm{ft-m}$ might produce equally acceptable or even better headway between 350 to 450 ft .

Only after a few trial studies of velocity and distance profiles of a hard and an easy roller can one select a reasonably good profile. In order to compensate for inaccuracies in the car rollability models, it was considered desirable to select an initial grade of slightly more than the minimum theoretical value. It was therefore decided to use a 3.5 percent grade for the first 200 ft and 2 percent for the next 300 ft .
b. Front-End Final Grades

Front-end final grade should be selected so that a hard roller rolling freely and entering this grade with the velocity it attains in the intermediate section is gradually brought down to a speed of about twice the speed at the hump. For the initial and intermediate grade mentioned above, the velocity of a hard roller at the end of the intermediate grade (i.e., at a distance of 500 ft from the hump) was computed to be about $23 \mathrm{ft} / \mathrm{s}$. To reduce this speed to about $14 \mathrm{ft} / \mathrm{s}$ (twice the humping speed) in a rolling distance of 1000 ft (i.e., 500 ft to 1500 ft from the hump), a grade of 0.6 percent is necessary. A shallower grade will reduce the speed to less than $14 \mathrm{ft} / \mathrm{s}$ and a steeper grade will result in an easy roller attaining too high speeds. Thus a suitable value for this part of the yard would be 0.6 percent. Combining the initial and intermediate grade with the final grade a possible grade profile for the yard under the assumed conditions is as shown below in Fig. 9.


FIGURE 9 A SUITABLE GRADE PROFILE FOR THE EXAMPLE
c. Location of Retarders

To establish the needs and location of retarders, the following procedure was followed:
(1) The performance of a hard roller followed by an easy roller, both rolling freely on the above noted grade profile, were computed using a simple computer program that calculates the distance and velocity of the car as a function of time using Eq. (12).
(2) It was found that the headway builds up to 100 ft , when the center of the hard roller (leading car) reaches a distance of 150 ft ; it is maintained at more than 100 ft (highest value 115 ft when leading car at 200 ft from hump) until the lead car reaches a distance of about 500 ft after which it starts decreasing and ultimately becomes zero when the lead car is 700 ft away from the hump. Thus, strictly speaking, no retarder is needed between humping point and the first switch. However, in view of the fact that an actual hard roller may be slightly worse than the assumed model and an actual easy roller can be slightly better than the assumed model (in which case the actual headway might be less than 100 ft ), it was decided to place a retarder between humping point
and the first switch. A suitable location appeared to be at 300 ft away from the hump. Tentatively its length was selected as 40 ft . After the finalization of the exact location of the first switch, some modification in the size and location of the first retarder may be necessary.
(3) Assuming as a first trial that the first retarder is made to release the easy roller with the same exit velocity as that of a hard roller at the exit point, its further performance was computed, assuming a free roll on the track. It was noted that the headway between the leading hard roller and the following easy roller tended to become less than the desired headway, when the lead car is at about 650 ft (from the hump). Thus it was decided to place a second retarder at a distance of 550 ft. First its length was tentatively selected as 40 ft ; after further study, it was changed to 60 ft since a 40-ft retarder did not have the capability to reduce the speed of the easy roller in accordance with the requirement of the control law discussed below.
(4) In view of the fact that no retarder could be placed in the section $1000-1500 \mathrm{ft}$, and rough calculation had indicated that one more retarder will be needed after the second retarder, it was decided to place it as close as possible to the tangent point to obtain maximum effectiveness in the area between 1000-1500 ft. Thus the third retarder was placed at 900 ft with a length of 60 ft as a first trial.

The above-noted considerations indicate the process by which retarder placements were established. We now briefly discuss the control logic employed.

## d. Control Policy for Retarders

Control law of the form of Eq. (11) was used for the three retarders mentioned above. The values of $K$ for the three retarders used were:

| First Retarder | $K=0$ |
| :--- | :--- |
| Second Retarder | $K=1$ |
| Third Retarder | $K=2.5$ |

Figure 10 shows the distance and velocity profiles of a hard roller as well as an easy roller for the example under consideration. It is seen that the headway between a hard roller followed by an easy roller, between an easy roller followed by another easy roller, between an easy roller followed by a hard roller, are all at least 100 ft throughout the distance from 100 to 1500 ft .

With reference to the results of the above example, it is seen that between 800 ft and 1000 ft the headway is just 100 ft . This is not quite desirable since if the hard roller were slightly worse than the assumed model or if the easy roller were slightly better than the assumed model, the actual headway might be quite closer than 100 ft . Some adjustment in the values of $K$ could be made so as to bring the distance profile of the controlled easy roller to come closer to the profile of a hard roller, in which case there will be a few feet of extra headway available to compensate for inaccuracies in the model or in the measurements of speed or the retarder operation.

A few test studies for this specific example and a similar example using the control law in Eq. (12) indicate that, by using a fourth retarder and a slight adjustment in the location of retarders, an error of about $\pm 1 \mathrm{ft} / \mathrm{s}$ in the release velocity of the retarder can be tolerated. This tolerance could be relaxed further if additional retarders are used. Therefore, for any specific case where grade profile, hump speed, switch location, reliability and tolerances in car models, sensing and retarder performance are known, the size, number and location of various retarders can be found out by a few simple test studies


FIGURE 10 VELOCITY AND DISTANCE OF HARD AND EASY ROLLERS CONTROLLED BY $V_{E}=V_{0}-K_{1}\left(V_{1}-V_{b}\right): K_{1}=0$ FOR FIRST DETARDERS USING THE SECOND RETARDER. - $3^{\prime}$ FOR THIRD RETARDER
using the control law of the form shown in Eq. (12), and following the general procedure explained with reference to the example.
F. Summary

Various factors that influence the design of grade profiles and principles of selecting the size, location and control policy of the retarders in the front end of the yard have been discussed. It was shown that there may not exist any unique way of selecting the grades in this area. However for any set of given conditions such as the location of the switches, rollability model of the cars, humping velocity and the desired headway etc. there exist only a few combinations of initial, intermediate and final grade which fulfill all the desired conditions. A suitable grade profile can therefore be selected by a few trial studies. After the selection of the grade profile, the size, location and control policy for the retarders can be finalized by considering the case of a hard roller followed by an easy roller and computing the headway between them. A simple control policy for retarders was proposed in which the retarding force on an easy roller is made directly proportional to the excess speed of these rollers in comparison to the speed of the hard rollers at the corresponding location. An example was presented to elaborate various ideas.

VI GENERAL DESIGN CONSIDERATIONS FOR THE BOWL TRACK AREA

## A. Introduction

Three main design considerations for the bowl track area of the yard are discussed in this section. They are:
(1) Selection of the bowl track grade,
(2) Type and size tangent point retarder to be used, and
(3) Control of the tangent point retarders.

It is shown how the grade of the bowl track is dependent upon the values for the rolling resistance of a hard roller and of an easy roller. The best type of tangent point retarders seem to be the weight-responsive type. It is shown that their size is directly related to the design value of the yard throughput. Finally, it is shown that, for a yard throughput of $8 \mathrm{cars} / \mathrm{min}$, in order to avoid damaging collisions of cars within the tangent point retarder, a particular type of control policy should be used.

The selection of the size, location and the proposed control scheme of the tangent point retarders presented in this section is based on the following desired objectives:
(1) The retarders should be capable of reducing the speed of an easiest roller to about $6 \mathrm{ft} / \mathrm{sec}$ because of the collision constraint.
(2) Consecutive cars destined for the same tracks should not collide in the retarder section with intolerable speed difference.

In the present section, the case of two consecutive cars has been analyzed in detail. The case of three or more consecutive cars destined for the same track requires considerations of some additional factors, e.g., cutting policy and spreading of retarder sections. However, the basic control policy proposed in this section with reference to the
case of two consecutive cars is equally applicable to the cases of three or more consecutive cars. These cases have been considered in Section VII, along with the cutting policies where a modified scheme of spreading the retarder sections to minimize the collision is discussed.

## B. Bowl Track Grade

The grade of the bowl tracks is a compromise value. It should not be so large that an easy roller will accelerate and reach speeds in excess of $6 \mathrm{ft} / \mathrm{s}$ (otherwise an easy roller would sometimes couple with standing cars with an impact velocity greater than $6 \mathrm{ft} / \mathrm{s}$ ). On the other hand, it should not be too shallow, otherwise, a hard roller will stop shortly after passing the tangent point.

As discussed in Sec. III, when a car is rolling on a grade of value $\theta$, the net car acceleration is given by:

$$
\begin{equation*}
\dot{\mathrm{V}}=\theta \mathrm{g}-\mu \mathrm{g}-\mathrm{Kgv} \tag{1}
\end{equation*}
$$

where

$$
\begin{aligned}
& \theta=\text { grade in radians } \\
& \mu=\text { coefficient of Coulomb friction (dimensionless) } \\
& K=\text { coefficient of viscous friction }\left(\mathrm{s}^{-1}\right) \\
& g=32.16 \mathrm{ft} / \mathrm{s}^{2} \\
& V=\text { car speed in } \mathrm{ft} / \mathrm{s}
\end{aligned}
$$

The ranges of $\mu \mathrm{g}$ and Kg are (from Table II):

$$
\begin{align*}
& \mu \mathrm{g}= \begin{cases}0.0322 & \text { for a hard roller } \\
0.00643 & \text { for an easy roller }\end{cases}  \tag{13}\\
& \mathrm{Kg}= \begin{cases}0.0113 & \text { for a hard roller } \\
0.00206 & \text { for an easy roller }\end{cases} \tag{14}
\end{align*}
$$

Using Eqs. (1), (13), and (14), one can show that an easy roller will never exceed $6 \mathrm{ft} / \mathrm{s}$ (as long as it is released from the tangent
point retarder at speeds below $6 \mathrm{ft} / \mathrm{s}$ ), if the grade is less than approximately 0.06 percent. Furthermore, with this grade, a hard roller will roll about 700 ft beyond the tangent point assuming it clears the tangent point with a velocity of about $14 \mathrm{ft} / \mathrm{s}$. This 700 ft of clearance is considered sufficient to prevent blocking of the retarder, and the hard roller will eventually be pushed down the bowl track by other cars.

In summary, a grade of 0.06 percent is satisfactory on the bowl tracks. A steeper grade would require additional retarders placed along the bowl track, and a grade more shallow than 0.06 percent would result in low coupling speeds and also result in too many cars stopping short.

## C. Size of the Tangent Point Retarders

The size (working length) of a tangent point retarder is determined by the maximum energy reduction required for a car at the tangent point and the velocity head of standard lengths of the retarder.

The maximum energy reduction required for a car at the tangent point is a direct function of the yard throughput. As discussed in Sec. IV, all cars must have a velocity of at least twice the hump speed $V_{H}$ in the switching area. Therefore all cars will reach the tangent points with a speed of at least $2 \mathrm{~V}_{\mathrm{H}}$. Since the last control point for cars is at the group retarders, easy rollers will gain kinetic energy from the group retarder to the tangent point. Our studies have shown that an easy roller gains about $3 \mathrm{ft} / \mathrm{s}$ additional velocity in its roll from group retarder to tangent points with an assumed grade of 0.4 percent in this section. Consequently, the tangent point retarders must have a velocity head of approximately:

$$
\begin{equation*}
h=\frac{\left(2 V_{H}+3\right)^{2}-\mathrm{V}_{\text {min }}^{2}}{2 \mathrm{~g}} \tag{15}
\end{equation*}
$$

where

$$
\begin{aligned}
h & =\text { velocity head of tangent point retarder }(f t) \\
V_{H} & =\text { the hump speed }(f t / s)
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{V}_{\min } & =\text { minimum let-out velocity }(\mathrm{ft} / \mathrm{s}) \text { of the tangent point } \\
\mathrm{g} & =32.16 \mathrm{ft} / \mathrm{s}^{2}
\end{aligned}
$$

If the bowl track grade is 0.06 percent it is not necessary to release any car (even an easy roller) from the tangent point retarder at a speed less than $6 \mathrm{ft} / \mathrm{s}$ because this grade will allow an easy roller to coast at a constant $6 \mathrm{ft} / \mathrm{s}$ along the bowl track when the car is released at $6 \mathrm{ft} / \mathrm{s}$, and harder rollers can and should be released at speeds above $6 \mathrm{ft} / \mathrm{s}$, after which they will gradually decelerate as they roll along the bowl track. Therefore, with a 0.06 percent bowl track grade and $V_{\text {min }}$ of $6 \mathrm{ft} / \mathrm{s}$, Eq. (15) becomes:

$$
\mathrm{h}=\frac{\left(2 \mathrm{~V}_{\mathrm{H}}+3\right)^{2}-(6)^{2}}{2 \mathrm{~g}}
$$

Consequently, for a hump speed of $7 \mathrm{ft} / \mathrm{s}$ ( 8 car/min throughput) the velocity head requirement of a tangent point retarder is

$$
h=\frac{(17)^{2}-(6)^{2}}{2(32.16)}=3.94 \mathrm{ft}
$$

The most desirable type of retarder to use at the tangent point from a control point of view is a weight-responsive retarder because the velocity profile of cars in such retarders is independent of car weight and therefore the control of the tangent point retarder is very simple and depends only upon the entrance velocity of the car to the retarder, and the required letout velocity. The actual retarder length to produce the above noted velocity head can easily be calculated knowing the characteristics of the retarders. For example, the Abex weight-responsive retarders have a typical velocity head of 0.0715 ft per ft of working length. Thus, to obtain a head of 3.94 ft , a total working length of about 55 ft of this type of retarder will be needed.

## D. A Proposed Scheme for Sectionalizing and Controlling the Retarders

In the above section the considerations to establish the size of the tangent point retarder were discussed. However to minimize the collisions between two consecutive cars in the retarder section, it is
necessary to sectionalize and control the individual sections appropriately.

The need to sectionalize the retarder arises because the total retarder length i.e., 55 ft is often much greater than the headway between consecutive cars as they travel through the retarder. Therefore, parts of two consecutive cars may frequently happen to be simultaneously in the retarder. If there was only one long section of the retarder, one or the other car could obviously not be controlled correctly. Therefore, to exert the appropriate control on each car, the retarder must be sectionalized into at least two parts, preferably more as will be explained below.

In the following paragraphs a scheme of controlling the retarder and sectionalizing it appropriately is proposed.

In order to minimize the possibilities of collisions within the retarder, the control policy of the retarder should allow each car to clear the retarder in the shortest time possible for its particular input velocity and required letout velocity. In this way, any impacts in the clearance point retarder occur as far into the retarder as possible, thereby minimizing the impact velocity. The control policy that achieves this type of operation is easy to visualize from Fig. 11.


FIGURE 11 PROPOSED CONTROL POLICY OF TANGENT POINT RETARDER

In Fig. 11, Point A corresponds to the maximum entrance speed of a car that could still be slowed down by the retarder to the desired letout velocity. If a car enters the retarder at a speed below this maximum, such as Point B in Fig. 1l, then the car should be allowed to coast (by leaving open the first few sections of the retarder) until it reaches Point $C$ and then be retarded along the curve ACD until the letout velocity is reached at Point D. In this way the car will clear the retarder in the shortest possible time. If the car were slowed immediately upon entering the retarder, as is the present-day practice (i.e., along curve BED), the clearance time would be significantly larger than the minimum clearance time.

In Fig. 11, Path ACD corresponds to the case where the retarder must be closed for the entire time that a car is in the retarder. It is the case in which the maximum possible energy is removed from a car by the retarder, referred to as the "capability curve" of the retarder. E. An Example

Let us consider an example to elaborate the significance of the minimum clearing time policy for the tangent point retarders discussed above.

Consider the case of two consecutive cars. Assume that the first car reaches the tangent point with $15 \mathrm{ft} / \mathrm{s}$ and must be released from the tangent point retarder at $6 \mathrm{ft} / \mathrm{s}$ either because the bowl track is nearly full or because the car is an easy roller. Assume that the second car reaches the tangent point with $17 \mathrm{ft} / \mathrm{s}$ and must also be retarded to $6 \mathrm{ft} / \mathrm{s}$. Assume that the time separation between the two cars is 6 seconds, which is a typical value for a humping speed of $7 \mathrm{ft} / \mathrm{s}$. These values of speeds and time separations have been selected to quantitatitively elaborate the importance of the proposed control policy. The conclusions are, however, equally valid for any other set of values.

Figure 12 shows the velocity and distance traveled as functions of time for both cars on the paths discussed above. If the first car is retarded along profile BED of Fig. 12, we see from Fig. 12 that the
second car collides with it at time $t_{1} \cong 8.2$ seconds, with an impact velocity of about $9.2 \mathrm{ft} / \mathrm{s}$. However, if the first car is retarded along profile $B C D$, the second car does not collide with it until time $t_{2} \cong 10.6$ seconds, and the impact velocity is only about $5.5 \mathrm{ft} / \mathrm{s}$. This illustrates the point that the impact between successive cars within the tangent point retarders is the least when the minimum clearance time control policy is used.

We remark in passing that the other case, in which the first car enters the retarder with a higher speed than the second car, is not critical. In fact, if one reversed the two cars in the above example it can be shown that the impact velocity is less than $1 \mathrm{ft} / \mathrm{s}$.

## F. Implementation of the Proposed Control Policy

The steps necessary to implement the proposed control policy are now briefly described. The capability curve (ACD in Fig. 11) of the tangent point retardex shifts vertically when the letout velocity is changed. Figure 13 shows the capability curve of a typical tangent point retarder of 55 ft length for different letout velocities. From Fig. 13, one could derive an approximate formula for the distance a car should be allowed to roll freely before being retarded, in terms of the required letout velocity and the measured entrance speed. This formula could then be stored in the computer and used to control cars in the tangent point retarder. The 55 ft retarder length could be composed of as many sections as are practically feasible. In case of Abex retarders, sections as small as 11 ft are available. Five of these sections could be placed end to end, giving the required total length of 55 ft . Knowing the distance a car is to be allowed to roll freely, the first one, two, three, etc., sections can be left open. For instance, if only three sections are needed to retard the car, the first two sections can be left open and the last three sections turned on with a closed-loop velocity sensing on perhaps only the last one or two sections. Thus the sensing and communication requirements are not very complex.


FIGURE 13 CAPABILITY CURVES OF THE TANGENT POINT RETARDER

Note in the discussions above that the velocity profile of a car in the tangent point retarder has been considered to be independent of car length. Actually the profile is to some degree dependent upon the car length, and therefore the capability curve of the retarder is to some degree dependent upon the car length. However, analysis shows that the effect is not significant and so the control policy can safely be assumed to be independent of car length.
G. Summary

Our analysis has shown that the bowl track grade should be about 0.06 percent, a grade value that is independent of the throughput for which the yard is designed. The same grade should be used for the bowl tracks of any hump yard.

It has been shown that, for a hump speed of $7 \mathrm{ft} / \mathrm{s}$ (approximately 8 cars $/ \mathrm{min}$ ), the tangent point retarders must have a working length of about 55 ft . These retarders should be built as a string of short sections (Sec. VII discusses this point further).

It has also been shown that it is important to use a particular control policy for the tangent point retarders--the minimum clearance time policy. This policy can be described as follows:
(1) The car speed is measured just prior to entering the tangent point retarder.
(2) Release speed is determined from the fullness of the bowl track and the car's rolling resistance.
(3) The minimum length, $L_{M}$, of the tangent point retarder that is necessary to slow the car to the required release speed is calculated (by computer).
(4) The car is allowed to roll freely into the (open) retarder until it is within a distance of $L_{M}$ from the exit end of the retarder.
(5) Full retardation is then applied until the car clears the retarder.

## VII THE INFLUENCE OF TRAIN MAKE-UP INFORMATION UPON YARD DESIGN AND OPERATION

## A. Introduction

This section explores the implications of typical make-up information upon car cutting policies, tangent point retarder design, and yard operating policies.

Typical consist data (from the Bakersfield yard) were studied to see if groups of two, three or more cars destined for the same bowl track occurred frequently. It was found that indeed they did occur frequently enough to deserve detailed consideration. The question naturally arises whether such groups of cars should be humped as a group or should be cut singly or in some other group arrangement. This question is resolved in Part $C$ where it is concluded that generally it is best to cut the groups into single car cuts.

It is shown in Part D that because groups of cars occur frequently it is desirable to spread the tangent point retarder sections along each bowl track. This is especially important at throughputs of 8 cars/ min or more.

Part E discusses the possibilities of utilizing car make-up information to improve yard operation. Finally, conclusions are presented in Part $F$.

## B. Consist Data from Bakersfield Yard

Several computer printouts indicating the car make-up details of various trains in Bakersfield yard were provided to us by Mr. Williamson. About 20 trains were selected randomly from these printouts and their make-up was studied. It was found that the most frequent group in these trains is the group of two cars. Groups of three cars was next most frequent group. On an average basis, about 20 percent of the cars were in groups of two cars and about 10 percent in groups of three, e.g., in a train of 100 cars, there will typically be ten pairs of two
consecutive cars going to same track and three or four groups of three cars. Groups of more than three cars were quite rare and amounted to less than 1 percent of the total number of cars. Assuming that these observations would be valid for West Colton as well, it was decided to limit the studies to the comparison of single-car cuts with two-car cuts.

Two more factors contributed to the idea of limiting the study to only two-car cuts. First, the length of acceleration measurement section will also have to be increased correspondingly if more than two-car cuts are to be considered; this is not desirable because of loss of active track and costs. Secondly the cutting process becomes more involved if cuts of more than two types are allowed, and is liable to create confusion for the operating personnel.
C. Selection of the Car-Cutting Policy

1. Influence of Different Cutting Policies Upon Headway

There are numerous possible situations where the decision about cutting the cars singly or in groups of two or more might have to be made, e.g.
(1) A roller destined for Track a, followed by two rollers destined for Track b, followed by another roller destined for Track a.
(2) A roller destined for Track a, followed by three rollers destined for Track $b$, followed by another roller destined for Track a.

It is easily recognized that if the factor of rolling resistance variations is combined with variability of train make-up, there will result a multitude of combinations. Fortunately, it does not seem necessary to study all possible combinations: The study of one or two typical combinations suffices to give enough insight into various aspects of the problem. It was therefore decided to study the following two typical cases:
(1) One roller destined for Track a, designated $a_{1}$; followed by two rollers destined for Track $b$, designated $b_{1}$ and $b_{2}$; followed by another roller destined for Track $a$, designated $a_{2}$.
(2) One roller destined for Track a, designated $a_{1}$; followed by four rollers destined for Track $b$, designated $b_{1}, b_{2}$, $b_{3}$ and $b_{4}$; followed by another roller destined for Track a, designated $a_{2}$.

Case (1)--In Case (1), there are two possible cutting combinations:

- Cutting each car singly, i.e., $a_{1}, b_{1}, b_{2}, a_{2}$; or
- Cutting the two cars going to Track $b$ as a group, i.e.,

$$
a_{1},\left(b_{1}+b_{2}\right), a_{2}
$$

In case of the first cutting arrangement, three separate headways must be considered, viz., between $a_{1}$ to $b_{1}, b_{1}$ to $b_{2}$, and $b_{2}$ to $a_{2}$. In case of the second cutting arrangement, only two headways need be studied, viz., $a_{1}$ to $\left(b_{1}+b_{2}\right)$ and $\left(b_{1}+b_{2}\right)$ to $a_{2}$.

In case of first cutting arrangement the headways between $a_{1}$ to $b_{1}$, between $b_{1}$ to $b_{2}$ and between $b_{2}$ to $a_{2}$ are the typical headways between two consecutive cars. Various cases of two consecutive cars were already analyzed in Sec. $V$ and it was shown that for the assumed limits on the rolling resistances the headways remain above or equal to the minimum desired limit. This of course is obvious, since the grades and retarder in the switching area have been designed on the very basis of single-car cuts. The main question is how the headway between $a_{1}$ to $\left(b_{1}+b_{2}\right)$ or $\left(b_{1}+b_{2}\right)$ to $a_{2}$ varies when the two cars going to $b$ are left coupled.

The travelled distance profiles of cars $a_{1},\left(b_{1}+b_{2}\right)$ and $a_{2}$ were computed in a manner similar to those used for Fig. 10, assigning random rolling characteristics to the cars within the limits and using the same control law, and it was found that the headway between rear end of car $a_{1}$ and front end of the combination $\left(b_{1}+b_{2}\right)$ as well as
between back end of $\left(b_{1}+b_{2}\right)$ and front end of $a_{2}$ varies between about 165 ft and 95 ft , i.e., much above the minimum limit of 50 ft .

Case (2)--In Case (2), the following two cutting combinations were studied:

- Single-car cuts, i.e., $a_{1}, b_{1}, b_{2}, b_{3}, b_{4}, a_{2}$
- Cutting the group of four cars going to Track b in two groups, i.e.,

$$
a_{1},\left(b_{1}+b_{2}\right),\left(b_{3}+b_{4}\right), a_{2}
$$

In case of single-car cuts, the headways between $a_{1}$ to $b_{1}$, $b_{1}$ to $b_{2},--b_{4}$ to $a_{2}$ are all covered by the analysis of Fig. 10 and are therefore within admissible limits.

In case of second cutting policy, the headways between $a_{1}$ to $\left(b_{1}+b_{2}\right),\left(b_{1}+b_{2}\right)$ to $\left(b_{3}+b_{4}\right)$, and $\left(b_{3}+b_{4}\right)$ to $a_{2}$ were computed using the same control policy as mentioned above. In each case it was found that the headways remain much above the minimum desired limit, i.e., between 95 to 165 ft .

It is not difficult to see that the results will be similar for other situations. Thus, it can be stated that under the assumed control policy multicar cuts result in larger headways and thereby may be preferable from the point of view of switching. In the following section we discuss the effect of multicar cuts on collisions in the bowl track section.
2. Influence of Cutting Policies
upon Impact Velocities at the Tangent Point Retarders
It was shown in the previous section that cutting a string of cars into groups increases headway between cars in the switching area and is desirable from that point of view. The other question to investigate is the effect of the cutting policy on impact velocities and backup within the tangent point retarder.

For reasons already explained above, it was decided to restrict the analysis to two cutting policies: single-car cuts, and the cutting of groups of two cars.

Our studies indicate that, unless the control logic for the tangent point retarder is dual-mode (one control policy for a single car and a second control policy for a pair of cars), the impact velocities and backup distances are larger for pair-wise cutting than for single-car cutting. The reason for this is that a weight-responsive tangent point retarder actually has more effective retarding power for a pair of cars than it does for a single car. Therefore the deceleration of a pair of cars is more rapid than for a single car and high impact velocities are more likely. To illustrate this point, consider the retardation of a pair of cars each weighing $W$ tons and that of a single car weighing $W$ tons in a tangent point retarder section of length $L$. The velocity vs time plots for these two cases are shown in Fig. 14. (It is assumed that the length of the section is shorter than a car length.)


FIGURE 14 RETARDATION OF A PAIR OF CARS AND A SINGLE CAR

Curve $A B C D E$ is that of a single car. From $A$ to $B$, the first truck of the car is in the retarder and the deceleration is $\mathrm{rg} / 2$ where $r$ is the force-to-weight ratio of the retarder and $g=32.16 \mathrm{ft} / \mathrm{s}^{2}$. At Point B, the first truck leaves the retarder at Point $C$. Then from Point $C$ to $D$ the acceleration is again $\mathrm{rg} / 2$. At Point D the car leaves the retarder.

Curve AFGHIJ is that of a pair of cars. From A to B, the first truck of the leading car is in the retarder and the deceleration is $r g / 4$ (not $r g / 2$ because the weight of the two cars is twice that of a single car, thus the deceleration is half that of a single car). This truck leaves the retarder at F. From G to H, the second truck of the leading car as well as the first truck of the trailing car are in the retarder. If the retarder were a perfect weight-responsive retarder, the force on each of these two trucks would be $r \cdot w$, so that the total force on the pair of cars would be $2 x w$ and therefore the deceleration would be $2 \mathrm{rw} /(2 \mathrm{w} / \mathrm{g})=\mathrm{rg}$. In actuality the retarder is probably not ideal, and the deceleration is about $3 \mathrm{rg} / 4$. At Point $H$ the first truck of the trailing car leaves the retarder, from $I$ to $J$ the deceleration is $\mathrm{rg} / 4$. Finally, the pair of cars leave the retarder at Point $J$.

This example shows two important points: First, the maximum deceleration for a single car is $\mathrm{rg} / 2$, and $3 \mathrm{rg} / 4$ for a pair of cars. Therefore, for a portion of its travel, the pair of cars experience greater deceleration than a single car. As shown in Sec. VI, this generally results in higher impact velocities. The second point to consider is that the final velocity of the pair of cars will be less than that of a single car, since the total work done on the pair of cars is rwL/2 $+3 r w L / 2+r w L / 2=5 r w L / 2$, whereas for the single car it is rwL/ $2+r w L / 2=r w L$. Therefore the ratio of the work done on the pair of cars to the work done on a single car is $5: 2$, yet the ratio of the initial kinetic energy of the pair of cars to that of a single car is $2: 1$. Thus proportionately more work is done by the retarder on a pair of cars than on a single car and so the final velocity of the pair of cars is less than that of a single car. This means that a pair of cars will take a proportionately longer time to clear the retarder.

Because of these two facts, cutting in pairs is likely to result in higher impact velocities in the tangent point retarder than cutting singly. Correspondingly, the backup distance beyond the retarder entrance will be worse.

Unless the retarder control logic has specific provisions for controlling a pair of cars differently than a single car, this problem cannot be avoided.

These conclusions were verified in simulations. A string of ten cars having the same bowl track destination were humped at speeds of $7.0,6.5$, and $6.2 \mathrm{ft} / \mathrm{s}$ and released from the tangent point retarder at $6 \mathrm{ft} / \mathrm{s}$ (the minimum release speed for any car). (A string of ten cars is sufficiently long for us to observe the resulting collisions for any substring of less than ten cars. It is extremely rare for a string of more than ten cars to occur.) Figure 15 shows the impact velocities in the tangent point retarder for cutting these cars singly and in pairs. (The horizontal scale in Fig. 15 is labeled "Car Being Bumped" so that a fair comparison between the two cutting policies can be made. That is, the nth collision of a chain of pair-wise cut cars occurs between the $2 n$th and $(2 n+1)$ th cars in the chain; consequently this collision should be compared with the 2nth collision of a chain of singly-cut cars for a fair comparison of the alternate cutting policies.)

Figure 16 shows the backup distances. As can be seen from these figures, both the impact velocities and backup distances are uniformly worse for cutting the cars in pairs than for cutting the cars singly. Furthermore, it is seen from these plots that if there is a group of four or more cars in the hump feed destined for the same track, the humping speed must be reduced from $7 \mathrm{ft} / \mathrm{s}$ to a suitable lower value since for such a group even a single car cut policy will result in undesirable third, fourth, etc., impacts if hump speed is kept at $7 \mathrm{ft} / \mathrm{s}$. A possible alternative is to spread the retarder sections on a longer length. This alternative is discussed in the next section. However,


FIGURE 15 IMPACT VELOCITIES FOR CUTTING SINGLY AND IN PAIRS


FIGURE 16 BACKUP DISTANCE FOR CUTTING SINGLY AND IN PAIRS
since single-car cuts result in less severe impacts than resulting from pair-wise cutting, irrespective of hump velocity, a general conclusion on cutting policy can be stated as follows:

## 3. Conclusions on Cutting Policy

For the tangent point retarder design and control policy outlined in Sec. VI, the suggested car-cutting policy is:

Cut all cars singly--An exception to this policy is that if a pair of cars exist in the hump feed that are destined for the same bowl track, and these cars are not adjacent to any other cars also destined for this same track then one might as well leave the two cars coupled together, although this is optional.

## D. Desirability of Spreading the Tangent Point Retarder Sections

1. General

As already discussed, when the hump speed is larger than the tangent point retarder release velocity, damaging collisions in the tangent point retarder become a problem. As shown in Sec. VI-C, in the case of only two consecutive cars going to the same bowl track, the impact velocity can be minimized using the appropriate control policy for the tangent point retarders. However, when the hump feed contains groups of three or more consecutive cars destined for the same track, the impact velocities on the third, fourth, etc. impacts become intolerable unless the hump speed is reduced below $7 \mathrm{ft} / \mathrm{s}$. In this section, it is shown that in order to keep the impact velocities of three or more cars below the tolerable limit without reducing the hump speed, an alternative solution is to spread the retarder section on a longer length on the bowl track.

In general, the impact velocities between consecutive cars can be kept small by decelerating the cars very gradually in the tangent point retarder. Of course there are practical limits to how gradual the deceleration can be since a very long tangent point retarder uses
up bowl track capacity. To illustrate the fact that gradual deceleration reduces impact velocities, consider the example depicted in Fig. 17.


FIGURE 17 VELOCITY PROFILES OF SUCCESSIVE CARS

Figure 17 shows the plots of velocity as a function of time for three consecutive cars in the tangent point retarder. Curve ABDF is the velocity of the leading car, Curve $C D F$ is the velocity of the next car, and Curve EF is the velocity of a third car. The velocity curve of the second and third cars are identical to that of the first car (until the cars collide), except that it is delayed in time by $T$ and $2 T$ respectively, where $T$ is the time separation of the cars. At the time that the first two cars collide, the value of the impact velocity is given by the vertical distance between the two velocity curves. For instance, if the cars collide at time $t^{*}$ as shown in Fig. 17, the impact velocity is $\Delta V$. If they had approximately the same weight, the two cars would move along the velocity profile GHDF after the collision. If the third car then collided with the first two, the impact velocity could be as large as $3 \Delta \mathrm{~V} / 2$ (see Fig. 17). In general, if $n$ cars followed each other into the same tangent point retarder, the maximum value of the $(n-1)$ th impact velocity would be $n \Delta V / 2$ (assuming the cars have roughly the same weight). Although this result is based
upon some simplifying assumptions, it illustrates an important point, namely that in order to sustain several consecutive impacts in the tangent point retarder before the maximum impact velocity of $6 \mathrm{ft} / \mathrm{s}$ is reached, the distance $\Delta V$ between velocity profiles of consecutive cars must be considerably smaller than $6 \mathrm{ft} / \mathrm{s}$. Equivalently, the slope of the velocity curves in Fig. 17 must be small, which means that the deceleration must be gradual.

## 2. Some Examples

Let us consider some examples that illustrate the major points made above. The first point to be considered is that the magnitude of impact velocities in the tangent point retarder increase directly with the difference between the peak hump speed and the minimum letout velocity. To demonstrate this fact, consider a consecutive string of 10 cars that are humped at speeds of $6.2,6.5$, and $7.0 \mathrm{ft} / \mathrm{s}$ and all travel to the same bowl track where they are retarded and released from the tangent point retarder at $6 \mathrm{ft} / \mathrm{s}$. The cars are 50 ft long and are cut individually. The tangent point retarder consists of five li-ft sections of weight-responsive retarder laid end to end. Figure 18 shows the impact velocity behavior of the string of cars in the tangent point retarder. We see that for a hump speed of $7.0 \mathrm{ft} / \mathrm{s}$, only two collisions can occur before the impact velocity rises above the maximum allowable impact velocity of $6 \mathrm{ft} / \mathrm{s}$. However, if the hump speed were dropped, for example, to $6.5 \mathrm{ft} / \mathrm{s}$, five collisions could occur before the impact velocities rise above $6 \mathrm{ft} / \mathrm{s}$.

The second major point to be illustrated by example is that the impact velocities can be diminished by making the retardation gradual. To make the retardation gradual, one can space sections of the tangent point retarder. For instance, a yard throughput of 8 cars $/ \mathrm{min}$ requires that the tangent point retarders have a working length of 55 ft . But this 55 ft of retarder can be split into sections, which then can be spaced to make the retardation gradual.

If the retarder sections are placed end to end (flange to flange), then for a portion of a car's travel through the retarder, both


FIGURE 18 IMPACT VELOCITY OF COLLISIONS IN TANGENT POINT RETARDER
trucks will be retarded, thereby doubling the car deceleration. The velocity vs time curve for the car then resembles Curve ABCDE in Fig. 19. By spacing the sections of the retarder properly, it is


FIGURE 19 VELOCITY PROFILE OF A CAR IN TANGENT POINT RETARDER WITH END-TO-END SECTIONS
possible to ensure that only one truck of a car is retarded at a time. In this latter case, the velocity of the car would look like Curve ABE in Fig. 19. This spacing reduces the maximum impact velocity to onehalf the value when the retarder sections are placed end to end.

As discussed previously, for a yard throughput of $8 \mathrm{car} / \mathrm{min}$, the tangent point retarder has a working length of 55 ft . This can be sectioned into five sections, each with a working length of 11 ft . Since most cars are 50 ft long, the spacing shown in Fig. 20 will ensure that for most cars, only one truck will be retarded at a time.


FIGURE 20 SPREADING THE TANGENT POINT RETARDER SECTIONS

To illustrate the effect that this spacing has upon impact velocities consider an example in which 10 cars of length 50 ft are humped at a $7 \mathrm{ft} / \mathrm{s}$ hump speed and then travel to the same bowl track. Consider also that the cars must be slowed to a letout velocity of $6 \mathrm{ft} / \mathrm{s}$ at the tangent point retarder. Figure 21 shows the impact velocity behavior for the string of cars under the assumptions: (a) the retarder sections are placed end to end, and (b) the retarder sections are spaced as shown above.

Figure 21 shows that if the retarder sections are placed end to end and cars are cut singly, it is impossible to hump any more than three consecutive cars at 8 cars/min before the impact velocity constraint is exceeded. Furthermore, when the retarder sections are placed end to end, the first two collisions occur with an impact velocity only slightly less than $6 \mathrm{ft} / \mathrm{s}$. Consequently, if effects due to differences in car length, differences in car speeds at the retarder entrance, and errors in sensing and control are considered, it may well be that the end-to-end design cannot reliably slow even groups of two or three cars without incurring impacts in excess of $6 \mathrm{ft} / \mathrm{s}$. However, if the retarder sections are spaced properly, it is possible to safely hump as many as five consecutive cars at 8 cars/min before the impact velocity constraint is exceeded.

## 3. Concluding Remarks

Consist data for the Bakersfield yard indicates that about 20 percent of cars coming into that yard were (in the hump-feed) in groups of two going to the same bowl track, and that about ten percent of the cars were in groups of three going to the same bowl track, and that a negligible percentage of cars were in groups of four or more cars going to the same track. Assuming that these statistics would be approximately true for West Colton also it is recommended that for a yard throughput of 8 cars/min, the tangent point retarders be sectionalized and spaced as discussed above, to:
(1) Ensure that the yard can operate nearly continuously at a throughput of 8 cars/min, while keeping the


FIGURE 21 IMPACT VELOCITY BEHAVIOR WITH AND WITHOUT SPACING
impact velocities of groups of two and three cars well below the impact velocity constraint.
(2)

Provide a margin of safety for variations in the impact velocities due to differences in car length, differences in car speeds at the retarder entrance, and errors in sensing and control.

We remark at this point, that if, initially, the West Colton yard is operated at a throughput of $6 \mathrm{cars} / \mathrm{min}$, the corresponding hump speed, $5 \mathrm{ft} / \mathrm{s}$, is below the minimum letout velocity of the tangent point retarders ( $6 \mathrm{ft} / \mathrm{s}$ ). Consequently, there is no need to space retarder sections. But if the yard is later modified to operate at a throughput of $8 \mathrm{cars} / \mathrm{min}$, then additional retarder length will be needed; furthermore, the retarder sections should then be spaced as discussed above.

## E. Utilization of Train Make-Up Information <br> to Improve Throughput during Yard Operation

In this section, we discuss briefly the possibilities of utilizing train make-up information to improve the yard operation, e.g., increasing the throughput or assigning the tracks efficiently. Our preliminary studies indicate that under certain favorable car make-up conditions and an appropriate scheme of track designations, the yard can be operated with a hump speed higher than the nominal speed, thereby resulting in a higher throughput.

1. Possibilities of Increasing the Throughput

The design of grades and retarders both in the front end and in the back end has been based on worst case considerations, i.e., in case of switching area, the grades and retarders have been so designed that a minimum headway of 50 ft is maintained till the last swilch is cleared, assuming that two consecutive cars always travel together up to the last switch and there switch over to two different bowl tracks. Similarly, in the case of tangent point retarders, it has been assumed that one car is always followed by another with a nominal time separation of 6 or 7 seconds, i.e., assuming that two consecutive cars are
going to the same bowl track. In actual yard operation it may frequently happen that no consecutive set of cars are going to the adjacent bowl tracks. As an example, consider the following track designations for a ten-car train with reference to the West Colton track arrangements.

| 3 | 15 | 20 | 20 | 15 | 1 | 7 | 17 | 8 | 1 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

Each box represents a car and the number inside it represents its bowl track designation. It can easily be seen that, except for the two cars destined for track 20, no other consecutive cars are going to the same track. Note particularly that

- Car 8 follows Car 1 through only four switches
- Car 17 follows Car 8 through only one switch
- Car 7 follows Car 17 through only two switches
- The second Car 1 follows Car 17 through only four switches, etc. If this table is completed it will be found that in the train of 10 cars under consideration, the maximum number of switches through which two consecutive cars travel together is only four. Therefore for this example the headway of 50 ft need be maintained only up the distance of the fourth switch.

The two cars destined for Track 20 can be left coupled together without creating any switching or collision problem. This observation indicates the possibility of increasing the hump speed, since the nominal hump speed was based on maintaining a headway of 50 ft throughout the switching area, i.e., up to the last switch. It is to be noted that a favorable condition in switching area implies also a favorable condition in bowl tracks, since under the above noted condition or other similar conditions, the time separation between cars arriving on the same bowl tracks will be significantly more than 6 or 7 seconds and the possibilities of collision will be correspondingly reduced. Thus, for each incoming train, the track designation of each car (if it has already been assigned), can be checked with reference to the preceding and following car and the maximum distance through which any two cars travel
together can be established. If this maximum distance is the distance of the last switch, then the train must be humped with nominal hump velocity. If it is significantly less than this distance (e.g., a distance corresponding to only four or three switches), then hump speed can be increased by a certain amount. A table can be prepared in advance indicating the maximum allowable humping velocity corresponding to the maximum distance through which the headway is to be maintained.

## 2. Improved Scheme of Track Designation

The above noted ideas were based on the assumption that car destinations have already been assigned. In actual operation of the yard there may be considerable freedom and flexibility in assigning the car destination itself. Train make-up information can also be utilized to assign favorable track designation, i.e., assigning the short tracks for the maximum number of cars. As an example, suppose an incoming train of 50 cars contains the following cars distributed randomly in the original train.

> 16 cars destined for City a
> 14 cars destined for City b
> 12 cars destined for City c
> 8 cars destined for City d

A favorable track assignment can then be selected on the basis of following considerations.
(1) Try to assign the shortest available track to cars destined for City a, the next shortest to cars destined for City b, etc. Consideration must of course be given to the available empty space on the bowl tracks. The general idea here is to bring the maximum number of cars as fast as possible on the bowl tracks. This helps in improving the yard
operation, since back-up and collision problems are minimized, which in turn can contribute to increased throughput.
(2) Try to assign the tracks such that the distance through which the headway is to be maintained is minimized. This again has the advantage of contributing to increased throughput as already explained above.

The above noted general observations indicate the strong possibility of utilizing the train make-up information to improve the yard operation. It is suggested that further detailed studies be conducted in this respect to develop quantitative relationships between humping speed and track assignment scheme.

## F. Conclusions

It was shown that if a train contains three or more consecutive cars destined for the same track, it is preferable to cut them singly to minimize collisions on the bowl tracks. However, if there are groups of only two cars destined for the same track, preceded and followed by cars destined for other tracks, the two cars can be left coupled and humped together if this is found to be convenient from other yard operating conditions. Otherwise these can be cut singly as well.

It was also shown that in order to achieve nearly continuous operation of the hump at peak throughput of 8 cars/min it is necessary to spread the tangent point retarder sections.

The possibilities of utilizing car make-up information to increase the throughput and to develop an efficient track designation scheme were explored. It was indicated that such a possibility definitely exists and must be studied in further detail.

## VIII EVOLUTION OF A YARD FROM A LOW THROUGHPUT TO A HIGH THROUGHPUT YARD

## A. A Proposed Method

The philosophy of designing the front end and the back end of the yard separately, suggested in Sec. IV, has an additional attractive feature in terms of long term economics of particularly those yards where the initial throughput is low but is expected to increase gradually to a higher value over a period of some years. For such yards, it is suggested that the grades of the yard be selected on the basis of the eventual higher throughput. During the initial low-throughput period, the front-end yard can be made to behave as a shallow grade yard by placing some extra retarders in the front end so that the speed levels of the cars in the switching area correspond to the low hump velocity. The additional length of these extra retarders in the switching area is quite small, as is shown below. The speed levels of the cars at the tangent point will be approximately twice the initial low hump velocity. Therefore, during the initial low-throughput period, only as few tangent point retarders may be used as are necessary corresponding to the low entrance velocity. As the throughput increases over the years, the extra retarders in the switching area may be gradually removed (or rendered inoperative) so that the velocity levels in the switching area increase in relation to increased hump velocity. Additional bowl track retarders may now be added in stages to extract the increased entrance speeds. Thus the yard can be gradually evolved from a low throughput to a high throughput yard without any major structural change. In this connection we have made a few preliminary studies taking West Colton yard as a reference. The results of these studies are indicated in Table III. It is to be noted that the grade in the bowl track is essentially designed on the basis of the easy roller not attaining a speed higher than $6 \mathrm{ft} / \mathrm{s}$ because of impact considerations. Therefore, this grade is designed to be the same irrespective of the throughput level. The exit velocity of a fast roller at the tangent point is brought down to a level of about $6 \mathrm{ft} / \mathrm{s}$ through

Table III
APPROXIMATE TOTAL RETARDER LENGTHS REQUIRED
AT DIFFERENT STAGES OF WEST COLTON YARD

| Desired <br> Throughput | Total Front-End Retarder Length in Feet* |  |  | Back-End Retarders* |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Grade Design 6 cars/min (Total uncompensated grade drop $\approx 18$ ft) | for an Ultimate Thr 8 cars/min (Total uncompensated grade drop $\approx 21 \mathrm{ft}$ ) | ughput of: 10 cars $/ \mathrm{min}$ (Total uncompensated grade drop $\simeq 24 \mathrm{ft}$ ) | Total Length Required (total grade drop $\approx 2 \mathrm{ft}$ ) | Approximate Arrangement (0.0715 ft/ft retarder) |
| $6 \mathrm{cars} / \mathrm{min}$ | 700 | $1050$ | $1410$ | $2320$ | One continucus 29-ft retarder in each bowl track |
| $8 \mathrm{cars} / \mathrm{min}$ |  | $800$ | 1160 | 4400 | 55 ft of retarders in each bowl track over a length of 150 ft |
| $10 \mathrm{cars} / \mathrm{min}$ |  |  | 980 | 6080 | 76 ft of retarders in each bowl track spread over a length of about 420 ft |

Note: The indicated retarder lengths are the estimated sums of all the retarders in the switching area in case of front end and the sum of all the retarders for the 80 tracks in case of back end.
*The estimated lengths are based upon the assumption that the retarders extract about 0.0715 ft of velocity head per ft length. This is a typical figure for weight-responsive retarders.
enough retarder lengths. The required length depends on the entrance velocity. Therefore, for higher throughputs--which means for higher entrance velocities at the tangent point--correspondingly more retarder lengths will be required, as indicated in the table. However, as indicated in the column of approximate arrangement, the increased retarder length in the bowl tracks must also be spread over relatively longer lengths. This may be a limiting factor, since the available bowl track length will be reduced correspondingly.
B. An Example

With reference to Table III, suppose it is desired to have an ultimate throughput of 10 cars/min and that the initial throughput is expected to be only 6 cars/min. The evolution may then proceed as follows:

Stage 1 Desired Throughput: 6 cars/minute
(a) Grades are designed for 10 cars per minute throughput; total drop required in the yard will be about 24 ft .
(b) Retarder length distributed appropriately in front end of yard $=1410 \mathrm{ft}$; the effective grades then correspond to 6 cars/minute throughput.
(c) Retarder length required in bowl tracks = 2320 ft (i.e., about 29 ft per bowl track).

Note that if the front yard grades are designed for 6 cars/min, retarders of length 700 ft in the front end and retarders of 2320 ft in the bowl tracks will still be required. Therefore the additional retarder length needed because of steeper grades is 700 ft , which is only about 24 percent of the total retarder length of the yard.

Stage 2 Throughput to be raised to 8 cars/min
(a) Remove a total of $1410-1160=250 \mathrm{ft}$ of retarders from the front end.
(b) Add a total of $4400-2320=2080 \mathrm{ft}$ of retarders length in the bowl tracks.

Stage 3 Throughput to be raised to 10 cars/min
(a) Remove another $1160-980=180 \mathrm{ft}$ of retarders from front end.
(b) Add another $6080-4400=1680 \mathrm{ft}$ of retarders in the bowl tracks.

In the above noted paragraphs, we have presented the fundamental guidelines for the proposed approach with some approximate calculations. For actual implementation, it will be necessary to conduct computational studies to select the appropriate grades, location and exact size of individual retarders in the switching area, proper location and exact sizes of bowl track retarders, etc.

## C. Summary

It has been shown how the design of both the front and back end of the yard is dependent upon throughput. In addition, a scheme has been proposed for designing a yard on a long term basis. By designing the front and grades slightly steeper and adding only about 20 percent extra retarder lengths, a yard can be evolved from a throughput of about $6 \mathrm{cars} / \mathrm{min}$ to an eventual throughput of $10 \mathrm{cars} / \mathrm{min}$. The scheme offers a flexible and economical approach for evolving a yard from a low throughput to a higher throughput without requiring any major structural modification.

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[^0]:    * It is common practice in railroad engineering literature to use the units of $1 \mathrm{~b} /$ ton for car rolling resistance. It can be shown that, in terms of the parameters given in Eq. (l), the car rolling resistance is equal to $2000(\mu+K V) 1 b / t o n$.

[^1]:    * 

    This results from cancellation of the increase in resistance due to velocity by the initial decrease in resistance due to heating of the journal oil.

